

Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "4 Trig functions\4.1 Sine"

Test results for the 538 problems in "4.1.0 (a sin)^m (b trig)^n.m"

- Problem 35: Result unnecessarily involves higher level functions.

$$\int (c \sin[a + b x])^{1/3} dx$$

Optimal (type 4, 517 leaves, 1 step):

$$-\frac{1}{b} \sqrt{\frac{3}{2} (3 - i \sqrt{3})} c^{1/3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{1 - \frac{(c \sin[a + b x])^{2/3}}{c^{2/3}}}}{\sqrt{3 + i \sqrt{3}}}\right], \frac{3 i - \sqrt{3}}{3 i + \sqrt{3}}\right]$$

$$\text{Sec}[a + b x] \sqrt{1 - \frac{(c \sin[a + b x])^{2/3}}{c^{2/3}}} \sqrt{\frac{i + \sqrt{3}}{3 i + \sqrt{3}} + \frac{2 (c \sin[a + b x])^{2/3}}{(3 - i \sqrt{3}) c^{2/3}}} \sqrt{\frac{i - \sqrt{3}}{3 i - \sqrt{3}} + \frac{2 (c \sin[a + b x])^{2/3}}{(3 + i \sqrt{3}) c^{2/3}}} +$$

$$\frac{1}{2 \sqrt{2} b} 3 (1 - i \sqrt{3}) \sqrt{3 - i \sqrt{3}} c^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{1 - \frac{(c \sin[a + b x])^{2/3}}{c^{2/3}}}}{\sqrt{3 - i \sqrt{3}}}\right], \frac{3 i + \sqrt{3}}{3 i - \sqrt{3}}\right] \text{Sec}[a + b x]$$

$$\sqrt{1 - \frac{(c \sin[a + b x])^{2/3}}{c^{2/3}}} \sqrt{\frac{i + \sqrt{3}}{3 i + \sqrt{3}} + \frac{2 (c \sin[a + b x])^{2/3}}{(3 - i \sqrt{3}) c^{2/3}}} \sqrt{\frac{i - \sqrt{3}}{3 i - \sqrt{3}} + \frac{2 (c \sin[a + b x])^{2/3}}{(3 + i \sqrt{3}) c^{2/3}}}$$

Result (type 5, 59 leaves):

$$\frac{\text{Cos}[a + b x] \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, \text{Cos}[a + b x]^2\right] \text{Sin}[a + b x] (c \sin[a + b x])^{1/3}}{b (\text{Sin}[a + b x]^2)^{2/3}}$$

■ **Problem 36: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(c \sin[a + b x])^{1/3}} dx$$

Optimal (type 4, 252 leaves, 1 step):

$$-\frac{1}{\sqrt{2} b c^{1/3}} 3 \sqrt{3 - i \sqrt{3}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2} \sqrt{1 - \frac{(c \sin[a + b x])^{2/3}}{c^{2/3}}}}{\sqrt{3 - i \sqrt{3}}}\right], \frac{3 i + \sqrt{3}}{3 i - \sqrt{3}}\right]$$

$$\operatorname{Sec}[a + b x] \sqrt{1 - \frac{(c \sin[a + b x])^{2/3}}{c^{2/3}}} \sqrt{\frac{i + \sqrt{3}}{3 i + \sqrt{3}} + \frac{2 (c \sin[a + b x])^{2/3}}{(3 - i \sqrt{3}) c^{2/3}}} \sqrt{\frac{i - \sqrt{3}}{3 i - \sqrt{3}} + \frac{2 (c \sin[a + b x])^{2/3}}{(3 + i \sqrt{3}) c^{2/3}}}$$

Result (type 5, 59 leaves):

$$-\frac{\operatorname{Cos}[a + b x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \operatorname{Cos}[a + b x]^2\right] \operatorname{Sin}[a + b x]}{b (c \sin[a + b x])^{1/3} (\operatorname{Sin}[a + b x]^2)^{1/3}}$$

■ **Problem 37: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(c \sin[a + b x])^{2/3}} dx$$

Optimal (type 4, 271 leaves, 1 step):

$$\left(3^{3/4} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{c^{2/3} - (1 - \sqrt{3}) (c \sin[a + b x])^{2/3}}{c^{2/3} - (1 + \sqrt{3}) (c \sin[a + b x])^{2/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \operatorname{Sec}[a + b x] (c \sin[a + b x])^{1/3} \right.$$

$$\left. \frac{(c^{2/3} - (c \sin[a + b x])^{2/3}) \sqrt{\frac{c^{4/3} \left(1 + \frac{(c \sin[a + b x])^{2/3}}{c^{2/3}} + \frac{(c \sin[a + b x])^{4/3}}{c^{4/3}}\right)}{(c^{2/3} - (1 + \sqrt{3}) (c \sin[a + b x])^{2/3})^2}}}{2 b c^{5/3} \sqrt{\frac{(c \sin[a + b x])^{2/3} (c^{2/3} - (c \sin[a + b x])^{2/3})}{(c^{2/3} - (1 + \sqrt{3}) (c \sin[a + b x])^{2/3})^2}}}\right)$$

Result (type 5, 59 leaves):

$$-\frac{\operatorname{Cos}[a + b x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, \operatorname{Cos}[a + b x]^2\right] \operatorname{Sin}[a + b x]}{b (c \sin[a + b x])^{2/3} (\operatorname{Sin}[a + b x]^2)^{1/6}}$$

■ **Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[a + b x] \operatorname{Sin}[a + b x] dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\sin[a + b x]^2}{2 b}$$

Result (type 3, 37 leaves) :

$$\frac{1}{2} \left(-\frac{\cos[2 a] \cos[2 b x]}{2 b} + \frac{\sin[2 a] \sin[2 b x]}{2 b} \right)$$

■ **Problem 62: Result more than twice size of optimal antiderivative.**

$$\int \sin[a + b x] \tan[a + b x] dx$$

Optimal (type 3, 23 leaves, 3 steps) :

$$\frac{\text{ArcTanh}[\sin[a + b x]]}{b} - \frac{\sin[a + b x]}{b}$$

Result (type 3, 67 leaves) :

$$-\frac{\text{Log}\left[\cos\left[\frac{1}{2}(a + b x)\right] - \sin\left[\frac{1}{2}(a + b x)\right]\right]}{b} + \frac{\text{Log}\left[\cos\left[\frac{1}{2}(a + b x)\right] + \sin\left[\frac{1}{2}(a + b x)\right]\right]}{b} - \frac{\sin[a + b x]}{b}$$

■ **Problem 63: Result more than twice size of optimal antiderivative.**

$$\int \sec[a + b x] \tan[a + b x]^2 dx$$

Optimal (type 3, 34 leaves, 2 steps) :

$$-\frac{\text{ArcTanh}[\sin[a + b x]]}{2 b} + \frac{\sec[a + b x] \tan[a + b x]}{2 b}$$

Result (type 3, 69 leaves) :

$$\frac{1}{2 b} \left(\text{Log}\left[\cos\left[\frac{1}{2}(a + b x)\right] - \sin\left[\frac{1}{2}(a + b x)\right]\right] - \text{Log}\left[\cos\left[\frac{1}{2}(a + b x)\right] + \sin\left[\frac{1}{2}(a + b x)\right]\right] + \sec[a + b x] \tan[a + b x] \right)$$

■ **Problem 87: Result more than twice size of optimal antiderivative.**

$$\int \sec[a + b x]^4 \tan[a + b x]^4 dx$$

Optimal (type 3, 31 leaves, 3 steps) :

$$\frac{\tan[a + b x]^5}{5 b} + \frac{\tan[a + b x]^7}{7 b}$$

Result (type 3, 77 leaves) :

$$\frac{2 \tan[a + b x]}{35 b} + \frac{\sec[a + b x]^2 \tan[a + b x]}{35 b} - \frac{8 \sec[a + b x]^4 \tan[a + b x]}{35 b} + \frac{\sec[a + b x]^6 \tan[a + b x]}{7 b}$$

■ **Problem 88: Result more than twice size of optimal antiderivative.**

$$\int \sec[a + bx]^6 \tan[a + bx]^4 dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\tan[a + bx]^5}{5b} + \frac{2 \tan[a + bx]^7}{7b} + \frac{\tan[a + bx]^9}{9b}$$

Result (type 3, 98 leaves):

$$\frac{8 \tan[a + bx]}{315b} + \frac{4 \sec[a + bx]^2 \tan[a + bx]}{315b} + \frac{\sec[a + bx]^4 \tan[a + bx]}{105b} - \frac{10 \sec[a + bx]^6 \tan[a + bx]}{63b} + \frac{\sec[a + bx]^8 \tan[a + bx]}{9b}$$

■ **Problem 93: Result more than twice size of optimal antiderivative.**

$$\int \sin[a + bx]^3 \tan[a + bx] dx$$

Optimal (type 3, 38 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\sin[a + bx]]}{b} - \frac{\sin[a + bx]}{b} - \frac{\sin[a + bx]^3}{3b}$$

Result (type 3, 84 leaves):

$$-\frac{\text{Log}\left[\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]\right]}{b} + \frac{\text{Log}\left[\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]\right]}{b} - \frac{5 \sin[a + bx]}{4b} + \frac{\sin[3(a + bx)]}{12b}$$

■ **Problem 94: Result more than twice size of optimal antiderivative.**

$$\int \sin[a + bx] \tan[a + bx]^3 dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$-\frac{3 \text{ArcTanh}[\sin[a + bx]]}{2b} + \frac{3 \sin[a + bx]}{2b} + \frac{\sin[a + bx] \tan[a + bx]^2}{2b}$$

Result (type 3, 116 leaves):

$$\frac{1}{4b} \left(6 \text{Log}\left[\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]\right] - 6 \text{Log}\left[\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]\right] + \frac{1}{\left(\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]\right)^2} - \frac{1}{\left(\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]\right)^2} + 4 \sin[a + bx] \right)$$

■ **Problem 126: Result more than twice size of optimal antiderivative.**

$$\int \csc[a + bx] \sec[a + bx] dx$$

Optimal (type 3, 11 leaves, 2 steps) :

$$\frac{\text{Log}[\text{Tan}[a + b x]]}{b}$$

Result (type 3, 31 leaves) :

$$2 \left(-\frac{\text{Log}[\text{Cos}[a + b x]]}{2 b} + \frac{\text{Log}[\text{Sin}[a + b x]]}{2 b} \right)$$

■ **Problem 140: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[a + b x]^2 \text{Sec}[a + b x] dx$$

Optimal (type 3, 23 leaves, 3 steps) :

$$\frac{\text{ArcTanh}[\text{Sin}[a + b x]]}{b} - \frac{\text{Csc}[a + b x]}{b}$$

Result (type 3, 90 leaves) :

$$-\frac{\text{Cot}\left[\frac{1}{2}(a + b x)\right]}{2 b} - \frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] - \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right]}{b} + \frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] + \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right]}{b} - \frac{\text{Tan}\left[\frac{1}{2}(a + b x)\right]}{2 b}$$

■ **Problem 142: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[a + b x]^2 \text{Sec}[a + b x]^3 dx$$

Optimal (type 3, 49 leaves, 4 steps) :

$$\frac{3 \text{ArcTanh}[\text{Sin}[a + b x]]}{2 b} - \frac{3 \text{Csc}[a + b x]}{2 b} + \frac{\text{Csc}[a + b x] \text{Sec}[a + b x]^2}{2 b}$$

Result (type 3, 132 leaves) :

$$-\frac{1}{4 b} \left(2 \text{Cot}\left[\frac{1}{2}(a + b x)\right] + 6 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] - \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right] - 6 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] + \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right] - \frac{1}{\left(\text{Cos}\left[\frac{1}{2}(a + b x)\right] - \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right)^2} + \frac{1}{\left(\text{Cos}\left[\frac{1}{2}(a + b x)\right] + \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right)^2} + 2 \text{Tan}\left[\frac{1}{2}(a + b x)\right] \right)$$

■ **Problem 144: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[a + b x]^2 \text{Sec}[a + b x]^5 dx$$

Optimal (type 3, 70 leaves, 5 steps) :

$$\frac{15 \text{ArcTanh}[\text{Sin}[a + b x]]}{8 b} - \frac{15 \text{Csc}[a + b x]}{8 b} + \frac{5 \text{Csc}[a + b x] \text{Sec}[a + b x]^2}{8 b} + \frac{\text{Csc}[a + b x] \text{Sec}[a + b x]^4}{4 b}$$

Result (type 3, 219 leaves) :

$$\begin{aligned}
 & - \frac{\text{Cot}\left[\frac{1}{2}(a+bx)\right]}{2b} - \frac{15 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(a+bx)\right] - \text{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{8b} + \frac{15 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(a+bx)\right] + \text{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{8b} + \\
 & \frac{1}{16b \left(\text{Cos}\left[\frac{1}{2}(a+bx)\right] - \text{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^4} + \frac{7}{16b \left(\text{Cos}\left[\frac{1}{2}(a+bx)\right] - \text{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^2} - \\
 & \frac{1}{16b \left(\text{Cos}\left[\frac{1}{2}(a+bx)\right] + \text{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^4} - \frac{7}{16b \left(\text{Cos}\left[\frac{1}{2}(a+bx)\right] + \text{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^2} - \frac{\text{Tan}\left[\frac{1}{2}(a+bx)\right]}{2b}
 \end{aligned}$$

■ **Problem 150: Result more than twice size of optimal antiderivative.**

$$\int \text{Cot}[a+bx]^2 \text{Csc}[a+bx] \, dx$$

Optimal (type 3, 34 leaves, 2 steps) :

$$\frac{\text{ArcTanh}[\text{Cos}[a+bx]]}{2b} - \frac{\text{Cot}[a+bx] \text{Csc}[a+bx]}{2b}$$

Result (type 3, 75 leaves) :

$$\begin{aligned}
 & - \frac{\text{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{8b} + \frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} - \frac{\text{Log}\left[\text{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} + \frac{\text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{8b}
 \end{aligned}$$

■ **Problem 153: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[a+bx]^3 \text{Sec}[a+bx]^2 \, dx$$

Optimal (type 3, 49 leaves, 4 steps) :

$$\begin{aligned}
 & - \frac{3 \text{ArcTanh}[\text{Cos}[a+bx]]}{2b} + \frac{3 \text{Sec}[a+bx]}{2b} - \frac{\text{Csc}[a+bx]^2 \text{Sec}[a+bx]}{2b}
 \end{aligned}$$

Result (type 3, 143 leaves) :

$$\begin{aligned}
 & \left(\text{Csc}[a+bx]^4 \left(2 - 6 \text{Cos}[2(a+bx)] + 2 \text{Cos}[3(a+bx)] + 3 \text{Cos}[3(a+bx)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(a+bx)\right]\right] - 3 \text{Cos}[3(a+bx)] \text{Log}\left[\text{Sin}\left[\frac{1}{2}(a+bx)\right]\right] \right) + \right. \\
 & \left. \text{Cos}[a+bx] \left(-2 - 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(a+bx)\right]\right] + 3 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(a+bx)\right]\right] \right) \right) / \left(2b \left(\text{Csc}\left[\frac{1}{2}(a+bx)\right]^2 - \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right) \right)
 \end{aligned}$$

■ **Problem 155: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[a+bx]^3 \text{Sec}[a+bx]^4 \, dx$$

Optimal (type 3, 66 leaves, 5 steps) :

$$-\frac{5 \operatorname{ArcTanh}[\cos[a+bx]]}{2b} + \frac{5 \operatorname{Sec}[a+bx]}{2b} + \frac{5 \operatorname{Sec}[a+bx]^3}{6b} - \frac{\operatorname{Csc}[a+bx]^2 \operatorname{Sec}[a+bx]^3}{2b}$$

Result (type 3, 205 leaves):

$$\frac{1}{3b \left(\operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2 - \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right)^3} \\ 2 \operatorname{Csc}[a+bx]^8 \left(22 - 40 \cos[2(a+bx)] + 13 \cos[3(a+bx)] - 30 \cos[4(a+bx)] + 13 \cos[5(a+bx)] + 15 \cos[3(a+bx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(a+bx)\right]\right] + \right. \\ \left. 15 \cos[5(a+bx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(a+bx)\right]\right] - 15 \cos[3(a+bx)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(a+bx)\right]\right] - \right. \\ \left. 15 \cos[5(a+bx)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(a+bx)\right]\right] + \cos[a+bx] \left(-26 - 30 \operatorname{Log}\left[\cos\left[\frac{1}{2}(a+bx)\right]\right] + 30 \operatorname{Log}\left[\sin\left[\frac{1}{2}(a+bx)\right]\right] \right) \right)$$

■ **Problem 159: Result more than twice size of optimal antiderivative.**

$$\int \cos[a+bx]^3 \cot[a+bx]^4 dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{3 \operatorname{Csc}[a+bx]}{b} - \frac{\operatorname{Csc}[a+bx]^3}{3b} + \frac{3 \operatorname{Sin}[a+bx]}{b} - \frac{\operatorname{Sin}[a+bx]^3}{3b}$$

Result (type 3, 121 leaves):

$$\frac{17 \cot\left[\frac{1}{2}(a+bx)\right]}{12b} - \frac{\cot\left[\frac{1}{2}(a+bx)\right] \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{24b} + \\ \frac{11 \operatorname{Sin}[a+bx]}{4b} + \frac{\operatorname{Sin}[3(a+bx)]}{12b} + \frac{17 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{12b} - \frac{\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{24b}$$

■ **Problem 161: Result more than twice size of optimal antiderivative.**

$$\int \cos[a+bx] \cot[a+bx]^4 dx$$

Optimal (type 3, 37 leaves, 3 steps):

$$\frac{2 \operatorname{Csc}[a+bx]}{b} - \frac{\operatorname{Csc}[a+bx]^3}{3b} + \frac{\operatorname{Sin}[a+bx]}{b}$$

Result (type 3, 103 leaves):

$$\frac{11 \cot\left[\frac{1}{2}(a+bx)\right]}{12b} - \frac{\cot\left[\frac{1}{2}(a+bx)\right] \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{24b} + \frac{\operatorname{Sin}[a+bx]}{b} + \frac{11 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{12b} - \frac{\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{24b}$$

■ **Problem 163: Result more than twice size of optimal antiderivative.**

$$\int \cot [a + b x]^3 \operatorname{Csc} [a + b x] dx$$

Optimal (type 3, 26 leaves, 2 steps) :

$$\frac{\operatorname{Csc} [a + b x]}{b} - \frac{\operatorname{Csc} [a + b x]^3}{3 b}$$

Result (type 3, 93 leaves) :

$$\frac{5 \cot \left[\frac{1}{2} (a + b x) \right]}{12 b} - \frac{\cot \left[\frac{1}{2} (a + b x) \right] \operatorname{Csc} \left[\frac{1}{2} (a + b x) \right]^2}{24 b} + \frac{5 \tan \left[\frac{1}{2} (a + b x) \right]}{12 b} - \frac{\sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right]}{24 b}$$

■ **Problem 166: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc} [a + b x]^4 \operatorname{Sec} [a + b x] dx$$

Optimal (type 3, 38 leaves, 4 steps) :

$$\frac{\operatorname{ArcTanh} [\operatorname{Sin} [a + b x]]}{b} - \frac{\operatorname{Csc} [a + b x]}{b} - \frac{\operatorname{Csc} [a + b x]^3}{3 b}$$

Result (type 3, 148 leaves) :

$$-\frac{7 \cot \left[\frac{1}{2} (a + b x) \right]}{12 b} - \frac{\cot \left[\frac{1}{2} (a + b x) \right] \operatorname{Csc} \left[\frac{1}{2} (a + b x) \right]^2}{24 b} - \frac{\operatorname{Log} \left[\cos \left[\frac{1}{2} (a + b x) \right] - \sin \left[\frac{1}{2} (a + b x) \right] \right]}{b} +$$

$$\frac{\operatorname{Log} \left[\cos \left[\frac{1}{2} (a + b x) \right] + \sin \left[\frac{1}{2} (a + b x) \right] \right]}{b} - \frac{7 \tan \left[\frac{1}{2} (a + b x) \right]}{12 b} - \frac{\sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right]}{24 b}$$

■ **Problem 168: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc} [a + b x]^4 \operatorname{Sec} [a + b x]^3 dx$$

Optimal (type 3, 66 leaves, 5 steps) :

$$\frac{5 \operatorname{ArcTanh} [\operatorname{Sin} [a + b x]]}{2 b} - \frac{5 \operatorname{Csc} [a + b x]}{2 b} - \frac{5 \operatorname{Csc} [a + b x]^3}{6 b} + \frac{\operatorname{Csc} [a + b x]^3 \operatorname{Sec} [a + b x]^2}{2 b}$$

Result (type 3, 215 leaves) :

$$\begin{aligned}
& - \frac{13 \cot\left[\frac{1}{2}(a+bx)\right] - \cot\left[\frac{1}{2}(a+bx)\right] \csc\left[\frac{1}{2}(a+bx)\right]^2}{12b} - \frac{5 \log\left[\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right]}{24b} + \frac{5 \log\left[\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right]}{2b} \\
& + \frac{1}{4b \left(\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right)^2} - \frac{1}{4b \left(\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right)^2} \\
& - \frac{13 \tan\left[\frac{1}{2}(a+bx)\right]}{12b} - \frac{\sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]}{24b}
\end{aligned}$$

■ **Problem 170: Result more than twice size of optimal antiderivative.**

$$\int \csc[a+bx]^4 \sec[a+bx]^5 dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$\frac{35 \operatorname{ArcTanh}[\sin[a+bx]]}{8b} - \frac{35 \csc[a+bx]}{8b} - \frac{35 \csc[a+bx]^3}{24b} + \frac{7 \csc[a+bx]^3 \sec[a+bx]^2}{8b} + \frac{\csc[a+bx]^3 \sec[a+bx]^4}{4b}$$

Result (type 3, 277 leaves):

$$\begin{aligned}
& - \frac{19 \cot\left[\frac{1}{2}(a+bx)\right] - \cot\left[\frac{1}{2}(a+bx)\right] \csc\left[\frac{1}{2}(a+bx)\right]^2}{12b} - \frac{35 \log\left[\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right]}{8b} + \\
& \frac{35 \log\left[\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right]}{8b} + \frac{1}{16b \left(\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right)^4} + \frac{11}{16b \left(\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right)^2} \\
& - \frac{1}{16b \left(\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right)^4} - \frac{11}{16b \left(\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right)^2} - \frac{19 \tan\left[\frac{1}{2}(a+bx)\right]}{12b} - \frac{\sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]}{24b}
\end{aligned}$$

■ **Problem 176: Result more than twice size of optimal antiderivative.**

$$\int \cot[a+bx]^4 \csc[a+bx] dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$\frac{3 \operatorname{ArcTanh}[\cos[a+bx]]}{8b} + \frac{3 \cot[a+bx] \csc[a+bx]}{8b} - \frac{\cot[a+bx]^3 \csc[a+bx]}{4b}$$

Result (type 3, 113 leaves):

$$\frac{5 \csc\left[\frac{1}{2}(a+bx)\right]^2}{32b} - \frac{\csc\left[\frac{1}{2}(a+bx)\right]^4}{64b} - \frac{3 \log\left[\cos\left[\frac{1}{2}(a+bx)\right]\right]}{8b} + \frac{3 \log\left[\sin\left[\frac{1}{2}(a+bx)\right]\right]}{8b} - \frac{5 \sec\left[\frac{1}{2}(a+bx)\right]^2}{32b} + \frac{\sec\left[\frac{1}{2}(a+bx)\right]^4}{64b}$$

■ **Problem 178: Result more than twice size of optimal antiderivative.**

$$\int \cot [a + b x]^2 \csc [a + b x]^3 dx$$

Optimal (type 3, 55 leaves, 3 steps) :

$$\frac{\text{ArcTanh}[\cos [a + b x]]}{8 b} + \frac{\cot [a + b x] \csc [a + b x]}{8 b} - \frac{\cot [a + b x] \csc [a + b x]^3}{4 b}$$

Result (type 3, 113 leaves) :

$$\frac{\csc \left[\frac{1}{2} (a + b x) \right]^2}{32 b} - \frac{\csc \left[\frac{1}{2} (a + b x) \right]^4}{64 b} + \frac{\text{Log} \left[\cos \left[\frac{1}{2} (a + b x) \right] \right]}{8 b} - \frac{\text{Log} \left[\sin \left[\frac{1}{2} (a + b x) \right] \right]}{8 b} - \frac{\sec \left[\frac{1}{2} (a + b x) \right]^2}{32 b} + \frac{\sec \left[\frac{1}{2} (a + b x) \right]^4}{64 b}$$

■ **Problem 181: Result more than twice size of optimal antiderivative.**

$$\int \csc [a + b x]^5 \sec [a + b x]^2 dx$$

Optimal (type 3, 70 leaves, 5 steps) :

$$-\frac{15 \text{ArcTanh}[\cos [a + b x]]}{8 b} + \frac{15 \sec [a + b x]}{8 b} - \frac{5 \csc [a + b x]^2 \sec [a + b x]}{8 b} - \frac{\csc [a + b x]^4 \sec [a + b x]}{4 b}$$

Result (type 3, 190 leaves) :

$$-\frac{7 \csc \left[\frac{1}{2} (a + b x) \right]^2}{32 b} - \frac{\csc \left[\frac{1}{2} (a + b x) \right]^4}{64 b} - \frac{15 \text{Log} \left[\cos \left[\frac{1}{2} (a + b x) \right] \right]}{8 b} + \frac{15 \text{Log} \left[\sin \left[\frac{1}{2} (a + b x) \right] \right]}{8 b} +$$

$$\frac{7 \sec \left[\frac{1}{2} (a + b x) \right]^2}{32 b} + \frac{\sec \left[\frac{1}{2} (a + b x) \right]^4}{64 b} + \frac{\sin \left[\frac{1}{2} (a + b x) \right]}{b \left(\cos \left[\frac{1}{2} (a + b x) \right] - \sin \left[\frac{1}{2} (a + b x) \right] \right)} - \frac{\sin \left[\frac{1}{2} (a + b x) \right]}{b \left(\cos \left[\frac{1}{2} (a + b x) \right] + \sin \left[\frac{1}{2} (a + b x) \right] \right)}$$

■ **Problem 183: Result more than twice size of optimal antiderivative.**

$$\int \csc [a + b x]^5 \sec [a + b x]^4 dx$$

Optimal (type 3, 89 leaves, 6 steps) :

$$-\frac{35 \text{ArcTanh}[\cos [a + b x]]}{8 b} + \frac{35 \sec [a + b x]}{8 b} + \frac{35 \sec [a + b x]^3}{24 b} - \frac{7 \csc [a + b x]^2 \sec [a + b x]^3}{8 b} - \frac{\csc [a + b x]^4 \sec [a + b x]^3}{4 b}$$

Result (type 3, 268 leaves) :

$$\begin{aligned}
& - \frac{1}{24 b \left(\operatorname{Csc}\left[\frac{1}{2}(a+b x)\right]^2 - \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \right)^3} \\
& \operatorname{Csc}[a+b x]^{10} \left(-204 + 658 \operatorname{Cos}[2(a+b x)] - 228 \operatorname{Cos}[3(a+b x)] + 140 \operatorname{Cos}[4(a+b x)] - 76 \operatorname{Cos}[5(a+b x)] - 210 \operatorname{Cos}[6(a+b x)] + \right. \\
& \quad \left. 76 \operatorname{Cos}[7(a+b x)] - 315 \operatorname{Cos}[3(a+b x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right] - 105 \operatorname{Cos}[5(a+b x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right] + \right. \\
& \quad \left. 105 \operatorname{Cos}[7(a+b x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right] + 3 \operatorname{Cos}[a+b x] \left(76 + 105 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right] - 105 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right] \right) + \right. \\
& \quad \left. 315 \operatorname{Cos}[3(a+b x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right] + 105 \operatorname{Cos}[5(a+b x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right] - 105 \operatorname{Cos}[7(a+b x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right] \right)
\end{aligned}$$

■ **Problem 243: Result unnecessarily involves higher level functions.**

$$\int (d \operatorname{Cos}[a+b x])^{9/2} \operatorname{Csc}[a+b x]^3 dx$$

Optimal (type 3, 113 leaves, 7 steps):

$$-\frac{7 d^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{d \operatorname{Cos}[a+b x]}}{\sqrt{d}}\right]}{4 b} + \frac{7 d^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d \operatorname{Cos}[a+b x]}}{\sqrt{d}}\right]}{4 b} - \frac{7 d^3 (d \operatorname{Cos}[a+b x])^{3/2}}{6 b} - \frac{d (d \operatorname{Cos}[a+b x])^{7/2} \operatorname{Csc}[a+b x]^2}{2 b}$$

Result (type 5, 78 leaves):

$$\frac{1}{6 b \sqrt{d \operatorname{Cos}[a+b x]}} d^5 \left((-5 + 2 \operatorname{Cos}[2(a+b x)]) \operatorname{Cot}[a+b x]^2 + 21 (-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Csc}[a+b x]^2\right] \right)$$

■ **Problem 245: Result unnecessarily involves higher level functions.**

$$\int (d \operatorname{Cos}[a+b x])^{5/2} \operatorname{Csc}[a+b x]^3 dx$$

Optimal (type 3, 91 leaves, 6 steps):

$$-\frac{3 d^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{d \operatorname{Cos}[a+b x]}}{\sqrt{d}}\right]}{4 b} + \frac{3 d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d \operatorname{Cos}[a+b x]}}{\sqrt{d}}\right]}{4 b} - \frac{d (d \operatorname{Cos}[a+b x])^{3/2} \operatorname{Csc}[a+b x]^2}{2 b}$$

Result (type 5, 65 leaves):

$$-\frac{d^3 (\operatorname{Cot}[a+b x]^2 - 3 (-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Csc}[a+b x]^2\right])}{2 b \sqrt{d \operatorname{Cos}[a+b x]}}$$

■ **Problem 246: Result unnecessarily involves higher level functions.**

$$\int (d \operatorname{Cos}[a+b x])^{3/2} \operatorname{Csc}[a+b x]^3 dx$$

Optimal (type 3, 91 leaves, 6 steps):

$$\frac{d^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{d \cos[a+bx]}}{\sqrt{d}}\right]}{4b} + \frac{d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d \cos[a+bx]}}{\sqrt{d}}\right]}{4b} - \frac{d \sqrt{d \cos[a+bx]} \operatorname{Csc}[a+bx]^2}{2b}$$

Result (type 5, 76 leaves):

$$\frac{1}{6b} (d \cos[a+bx])^{3/2} (-\cot[a+bx]^2)^{3/4} \left(3 (-\cot[a+bx]^2)^{1/4} + \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Csc}[a+bx]^2\right] \right) \operatorname{Sec}[a+bx]^3$$

■ **Problem 247: Result unnecessarily involves higher level functions.**

$$\int \sqrt{d \cos[a+bx]} \operatorname{Csc}[a+bx]^3 dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d \cos[a+bx]}}{\sqrt{d}}\right]}{4b} - \frac{\sqrt{d} \operatorname{ArcTanh}\left[\frac{\sqrt{d \cos[a+bx]}}{\sqrt{d}}\right]}{4b} - \frac{(d \cos[a+bx])^{3/2} \operatorname{Csc}[a+bx]^2}{2bd}$$

Result (type 5, 62 leaves):

$$\frac{d (\cot[a+bx]^2 + (-\cot[a+bx]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Csc}[a+bx]^2\right])}{2b \sqrt{d \cos[a+bx]}}$$

■ **Problem 248: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Csc}[a+bx]^3}{\sqrt{d \cos[a+bx]}} dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$-\frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{d \cos[a+bx]}}{\sqrt{d}}\right]}{4b \sqrt{d}} - \frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{d \cos[a+bx]}}{\sqrt{d}}\right]}{4b \sqrt{d}} - \frac{\sqrt{d \cos[a+bx]} \operatorname{Csc}[a+bx]^2}{2bd}$$

Result (type 5, 69 leaves):

$$\frac{d (-\cot[a+bx]^2)^{3/4} \left((-\cot[a+bx]^2)^{1/4} - \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Csc}[a+bx]^2\right] \right)}{2b (d \cos[a+bx])^{3/2}}$$

■ **Problem 249: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Csc}[a+bx]^3}{(d \cos[a+bx])^{3/2}} dx$$

Optimal (type 3, 115 leaves, 7 steps):

$$\frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{d \cos[a+bx]}}{\sqrt{d}}\right]}{4bd^{3/2}} - \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{d \cos[a+bx]}}{\sqrt{d}}\right]}{4bd^{3/2}} + \frac{5}{2bd \sqrt{d \cos[a+bx]}} - \frac{\operatorname{Csc}[a+bx]^2}{2bd \sqrt{d \cos[a+bx]}}$$

Result (type 5, 91 leaves) :

$$\frac{-(-\cot [a+b x]^2)^{3/4}(-4+\cot [a+b x]^2)+5 \cot [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc [a+b x]^2\right]}{2 b d \sqrt{d \cos [a+b x]}(-\cot [a+b x]^2)^{3/4}}$$

■ **Problem 250: Result unnecessarily involves higher level functions.**

$$\int \frac{\csc [a+b x]^3}{(d \cos [a+b x])^{5/2}} dx$$

Optimal (type 3, 115 leaves, 7 steps) :

$$-\frac{7 \operatorname{ArcTan}\left[\frac{\sqrt{d \cos [a+b x]}}{\sqrt{d}}\right]}{4 b d^{5/2}}-\frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{d \cos [a+b x]}}{\sqrt{d}}\right]}{4 b d^{5/2}}+\frac{7}{6 b d(d \cos [a+b x])^{3/2}}-\frac{\csc [a+b x]^2}{2 b d(d \cos [a+b x])^{3/2}}$$

Result (type 5, 92 leaves) :

$$\frac{(-\cot [a+b x]^2)^{1/4}(4-3 \cot [a+b x]^2)+7 \cot [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \csc [a+b x]^2\right]}{6 b d(d \cos [a+b x])^{3/2}(-\cot [a+b x]^2)^{1/4}}$$

■ **Problem 251: Result unnecessarily involves higher level functions.**

$$\int \frac{\csc [a+b x]^3}{(d \cos [a+b x])^{7/2}} dx$$

Optimal (type 3, 137 leaves, 8 steps) :

$$\frac{9 \operatorname{ArcTan}\left[\frac{\sqrt{d \cos [a+b x]}}{\sqrt{d}}\right]}{4 b d^{7/2}}-\frac{9 \operatorname{ArcTanh}\left[\frac{\sqrt{d \cos [a+b x]}}{\sqrt{d}}\right]}{4 b d^{7/2}}+\frac{9}{10 b d(d \cos [a+b x])^{5/2}}+\frac{9}{2 b d^3 \sqrt{d \cos [a+b x]}}-\frac{\csc [a+b x]^2}{2 b d(d \cos [a+b x])^{5/2}}$$

Result (type 5, 102 leaves) :

$$\left(45 \cot [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc [a+b x]^2\right]+(-\cot [a+b x]^2)^{3/4}(40-5 \cot [a+b x]^2+4 \sec [a+b x]^2)\right) / \left(10 b d^3 \sqrt{d \cos [a+b x]}(-\cot [a+b x]^2)^{3/4}\right)$$

■ **Problem 257: Result unnecessarily involves higher level functions.**

$$\int (d \cos [a+b x])^{9/2} \sqrt{c \sin [a+b x]} dx$$

Optimal (type 4, 132 leaves, 4 steps) :

$$\frac{7 d^3 (d \cos[a + b x])^{3/2} (c \sin[a + b x])^{3/2}}{30 b c} + \frac{d (d \cos[a + b x])^{7/2} (c \sin[a + b x])^{3/2}}{5 b c} + \frac{7 d^4 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{20 b \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 109 leaves):

$$\frac{1}{120 b (\sin[a + b x]^2)^{3/4}} d^4 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} \left(-14 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] \sin[2(a + b x)] + (\sin[a + b x]^2)^{3/4} (20 \sin[2(a + b x)] + 3 \sin[4(a + b x)]) \right)$$

■ **Problem 258: Result unnecessarily involves higher level functions.**

$$\int (d \cos[a + b x])^{5/2} \sqrt{c \sin[a + b x]} dx$$

Optimal (type 4, 95 leaves, 3 steps):

$$\frac{d (d \cos[a + b x])^{3/2} (c \sin[a + b x])^{3/2}}{3 b c} + \frac{d^2 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{2 b \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 87 leaves):

$$\frac{1}{6 b (\sin[a + b x]^2)^{3/4}} d^2 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + (\sin[a + b x]^2)^{3/4} \right) \sin[2(a + b x)]$$

■ **Problem 259: Result unnecessarily involves higher level functions.**

$$\int \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} dx$$

Optimal (type 4, 53 leaves, 2 steps):

$$\frac{\sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{b \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 69 leaves):

$$\frac{\sqrt{d \cos[a + b x]} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] \sqrt{c \sin[a + b x]} \sin[2(a + b x)]}{3 b (\sin[a + b x]^2)^{3/4}}$$

■ **Problem 260: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c \sin[a + b x]}}{(d \cos[a + b x])^{3/2}} dx$$

Optimal (type 4, 93 leaves, 3 steps):

$$\frac{2 (c \sin[a + b x])^{3/2}}{b c d \sqrt{d \cos[a + b x]}} - \frac{2 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{b d^2 \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 92 leaves):

$$\frac{2 (c \sin[a + b x])^{3/2} \left(2 \cos[a + b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + 3 (\sin[a + b x]^2)^{3/4} \right)}{3 b c d \sqrt{d \cos[a + b x]} (\sin[a + b x]^2)^{3/4}}$$

■ **Problem 261: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c \sin[a + b x]}}{(d \cos[a + b x])^{7/2}} dx$$

Optimal (type 4, 134 leaves, 4 steps):

$$\frac{2 (c \sin[a + b x])^{3/2}}{5 b c d (d \cos[a + b x])^{5/2}} + \frac{4 (c \sin[a + b x])^{3/2}}{5 b c d^3 \sqrt{d \cos[a + b x]}} - \frac{4 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{5 b d^4 \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 110 leaves):

$$\left(2 \sqrt{c \sin[a + b x]} \left(4 \cos[a + b x]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] \sin[a + b x] + 3 (\sin[a + b x]^2)^{3/4} (\sin[2(a + b x)] + \tan[a + b x]) \right) \right) / (15 b d^2 (d \cos[a + b x])^{3/2} (\sin[a + b x]^2)^{3/4})$$

■ **Problem 262: Result unnecessarily involves higher level functions.**

$$\int (d \cos[a + b x])^{3/2} \sqrt{c \sin[a + b x]} dx$$

Optimal (type 3, 320 leaves, 11 steps):

$$\frac{\sqrt{c} d^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a + b x]}}{\sqrt{c} \sqrt{d \cos[a + b x]}}\right]}{4 \sqrt{2} b} + \frac{\sqrt{c} d^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a + b x]}}{\sqrt{c} \sqrt{d \cos[a + b x]}}\right]}{4 \sqrt{2} b} + \frac{\sqrt{c} d^{3/2} \operatorname{Log}\left[\sqrt{c} - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a + b x]}}{\sqrt{d \cos[a + b x]}} + \sqrt{c} \tan[a + b x]\right]}{8 \sqrt{2} b} - \frac{\sqrt{c} d^{3/2} \operatorname{Log}\left[\sqrt{c} + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a + b x]}}{\sqrt{d \cos[a + b x]}} + \sqrt{c} \tan[a + b x]\right]}{8 \sqrt{2} b} + \frac{d \sqrt{d \cos[a + b x]} (c \sin[a + b x])^{3/2}}{2 b c}$$

Result (type 5, 82 leaves):

$$\frac{1}{2 b (\sin[a + b x]^2)^{3/4}} (d \cos[a + b x])^{3/2} \sqrt{c \sin[a + b x]} \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a + b x]^2\right] + (\sin[a + b x]^2)^{3/4} \right) \tan[a + b x]$$

■ **Problem 263: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c \sin[a + bx]}}{\sqrt{d \cos[a + bx]}} dx$$

Optimal (type 3, 280 leaves, 10 steps):

$$\begin{aligned} & - \frac{\sqrt{c} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{c} \sqrt{d \cos[a+bx]}}\right]}{\sqrt{2} b \sqrt{d}} + \frac{\sqrt{c} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{c} \sqrt{d \cos[a+bx]}}\right]}{\sqrt{2} b \sqrt{d}} + \\ & \frac{\sqrt{c} \operatorname{Log}\left[\sqrt{c} - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{d \cos[a+bx]}} + \sqrt{c} \operatorname{Tan}[a + bx]\right]}{2 \sqrt{2} b \sqrt{d}} - \frac{\sqrt{c} \operatorname{Log}\left[\sqrt{c} + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{d \cos[a+bx]}} + \sqrt{c} \operatorname{Tan}[a + bx]\right]}{2 \sqrt{2} b \sqrt{d}} \end{aligned}$$

Result (type 5, 67 leaves):

$$- \frac{\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a + bx]^2\right] \sqrt{c \sin[a + bx]} \sin[2(a + bx)]}{b \sqrt{d \cos[a + bx]} (\sin[a + bx]^2)^{3/4}}$$

■ **Problem 267: Result unnecessarily involves higher level functions.**

$$\int (d \cos[a + bx])^{3/2} (c \sin[a + bx])^{3/2} dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$\frac{c d \sqrt{d \cos[a + bx]} \sqrt{c \sin[a + bx]}}{6 b} - \frac{c (d \cos[a + bx])^{5/2} \sqrt{c \sin[a + bx]}}{3 b d} + \frac{c^2 d^2 \operatorname{EllipticF}\left[a - \frac{\pi}{4} + bx, 2\right] \sqrt{\sin[2a + 2bx]}}{12 b \sqrt{d \cos[a + bx]} \sqrt{c \sin[a + bx]}}$$

Result (type 5, 85 leaves):

$$- \frac{1}{6 b (\sin[a + bx]^2)^{1/4}} c d \sqrt{d \cos[a + bx]} \sqrt{c \sin[a + bx]} \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \cos[a + bx]^2\right] + \cos[2(a + bx)] (\sin[a + bx]^2)^{1/4} \right)$$

■ **Problem 268: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \sin[a + bx])^{3/2}}{\sqrt{d \cos[a + bx]}} dx$$

Optimal (type 4, 93 leaves, 3 steps):

$$- \frac{c \sqrt{d \cos[a + bx]} \sqrt{c \sin[a + bx]}}{b d} + \frac{c^2 \operatorname{EllipticF}\left[a - \frac{\pi}{4} + bx, 2\right] \sqrt{\sin[2a + 2bx]}}{2 b \sqrt{d \cos[a + bx]} \sqrt{c \sin[a + bx]}}$$

Result (type 5, 67 leaves):

$$\frac{\text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \text{Cos}[a + b x]^2\right] (c \text{Sin}[a + b x])^{3/2} \text{Sin}[2(a + b x)]}{b \sqrt{d \text{Cos}[a + b x]} (\text{Sin}[a + b x]^2)^{5/4}}$$

■ **Problem 269: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \text{Sin}[a + b x])^{3/2}}{(d \text{Cos}[a + b x])^{5/2}} dx$$

Optimal (type 4, 98 leaves, 3 steps):

$$\frac{2 c \sqrt{c \text{Sin}[a + b x]}}{3 b d (d \text{Cos}[a + b x])^{3/2}} - \frac{c^2 \text{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\text{Sin}[2 a + 2 b x]}}{3 b d^2 \sqrt{d \text{Cos}[a + b x]} \sqrt{c \text{Sin}[a + b x]}}$$

Result (type 5, 93 leaves):

$$\left(2 (c \text{Sin}[a + b x])^{3/2} \left(2 \text{Cot}[a + b x]^2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \text{Cos}[a + b x]^2\right] + (\text{Sin}[a + b x]^2)^{1/4} \right) \text{Tan}[a + b x] \right) / \left(3 b d^2 \sqrt{d \text{Cos}[a + b x]} (\text{Sin}[a + b x]^2)^{1/4} \right)$$

■ **Problem 270: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \text{Sin}[a + b x])^{3/2}}{(d \text{Cos}[a + b x])^{9/2}} dx$$

Optimal (type 4, 133 leaves, 4 steps):

$$\frac{2 c \sqrt{c \text{Sin}[a + b x]}}{7 b d (d \text{Cos}[a + b x])^{7/2}} - \frac{2 c \sqrt{c \text{Sin}[a + b x]}}{21 b d^3 (d \text{Cos}[a + b x])^{3/2}} - \frac{2 c^2 \text{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\text{Sin}[2 a + 2 b x]}}{21 b d^4 \sqrt{d \text{Cos}[a + b x]} \sqrt{c \text{Sin}[a + b x]}}$$

Result (type 5, 103 leaves):

$$\frac{1}{21 b d^5 (\text{Sin}[a + b x]^2)^{1/4}} + \frac{2 c \sqrt{d \text{Cos}[a + b x]} \sqrt{c \text{Sin}[a + b x]}}{\left(4 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \text{Cos}[a + b x]^2\right] + (-2 - \text{Sec}[a + b x]^2 + 3 \text{Sec}[a + b x]^4) (\text{Sin}[a + b x]^2)^{1/4} \right)}$$

■ **Problem 271: Result unnecessarily involves higher level functions.**

$$\int \sqrt{d \text{Cos}[a + b x]} (c \text{Sin}[a + b x])^{3/2} dx$$

Optimal (type 3, 320 leaves, 11 steps):

$$\frac{c^{3/2} \sqrt{d} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{c} \sqrt{d} \cos[a+bx]}{\sqrt{d} \sqrt{c} \sin[a+bx]}\right]}{4 \sqrt{2} b} - \frac{c^{3/2} \sqrt{d} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{c} \sqrt{d} \cos[a+bx]}{\sqrt{d} \sqrt{c} \sin[a+bx]}\right]}{4 \sqrt{2} b} - \frac{c^{3/2} \sqrt{d} \operatorname{Log}\left[\sqrt{d} + \sqrt{d} \cot[a+bx] - \frac{\sqrt{2} \sqrt{c} \sqrt{d} \cos[a+bx]}{\sqrt{c} \sin[a+bx]}\right]}{8 \sqrt{2} b} +$$

$$\frac{c^{3/2} \sqrt{d} \operatorname{Log}\left[\sqrt{d} + \sqrt{d} \cot[a+bx] + \frac{\sqrt{2} \sqrt{c} \sqrt{d} \cos[a+bx]}{\sqrt{c} \sin[a+bx]}\right]}{8 \sqrt{2} b} - \frac{c (d \cos[a+bx])^{3/2} \sqrt{c} \sin[a+bx]}{2 b d}$$

Result (type 5, 80 leaves):

$$-\frac{1}{6 b d (\sin[a+bx]^2)^{1/4}} c (d \cos[a+bx])^{3/2} \sqrt{c} \sin[a+bx] \left(\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+bx]^2\right] + 3 (\sin[a+bx]^2)^{1/4} \right)$$

■ **Problem 272: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \sin[a+bx])^{3/2}}{(d \cos[a+bx])^{3/2}} dx$$

Optimal (type 3, 313 leaves, 11 steps):

$$-\frac{c^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{c} \sqrt{d} \cos[a+bx]}{\sqrt{d} \sqrt{c} \sin[a+bx]}\right]}{\sqrt{2} b d^{3/2}} + \frac{c^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{c} \sqrt{d} \cos[a+bx]}{\sqrt{d} \sqrt{c} \sin[a+bx]}\right]}{\sqrt{2} b d^{3/2}} +$$

$$\frac{c^{3/2} \operatorname{Log}\left[\sqrt{d} + \sqrt{d} \cot[a+bx] - \frac{\sqrt{2} \sqrt{c} \sqrt{d} \cos[a+bx]}{\sqrt{c} \sin[a+bx]}\right]}{2 \sqrt{2} b d^{3/2}} - \frac{c^{3/2} \operatorname{Log}\left[\sqrt{d} + \sqrt{d} \cot[a+bx] + \frac{\sqrt{2} \sqrt{c} \sqrt{d} \cos[a+bx]}{\sqrt{c} \sin[a+bx]}\right]}{2 \sqrt{2} b d^{3/2}} + \frac{2 c \sqrt{c} \sin[a+bx]}{b d \sqrt{d} \cos[a+bx]}$$

Result (type 5, 89 leaves):

$$\frac{2 c \sqrt{c} \sin[a+bx] (\cos[a+bx]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+bx]^2\right] + 3 (\sin[a+bx]^2)^{1/4})}{3 b d \sqrt{d} \cos[a+bx] (\sin[a+bx]^2)^{1/4}}$$

■ **Problem 276: Result unnecessarily involves higher level functions.**

$$\int (d \cos[a+bx])^{9/2} (c \sin[a+bx])^{5/2} dx$$

Optimal (type 4, 166 leaves, 5 steps):

$$\frac{c d^3 (d \cos[a+bx])^{3/2} (c \sin[a+bx])^{3/2}}{20 b} + \frac{3 c d (d \cos[a+bx])^{7/2} (c \sin[a+bx])^{3/2}}{70 b} -$$

$$\frac{c (d \cos[a+bx])^{11/2} (c \sin[a+bx])^{3/2}}{7 b d} + \frac{3 c^2 d^4 \sqrt{d} \cos[a+bx] \operatorname{EllipticE}\left[a - \frac{\pi}{4} + bx, 2\right] \sqrt{c} \sin[a+bx]}{40 b \sqrt{\sin[2a+2bx]}}$$

Result (type 5, 122 leaves):

$$-\frac{1}{1120 b (\sin[a + b x]^2)^{3/4}} c^2 d^4 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} \left(28 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] \sin[2(a + b x)] + (\sin[a + b x]^2)^{3/4} (-15 \sin[2(a + b x)] + 14 \sin[4(a + b x)] + 5 \sin[6(a + b x)]) \right)$$

■ **Problem 277: Result unnecessarily involves higher level functions.**

$$\int (d \cos[a + b x])^{5/2} (c \sin[a + b x])^{5/2} dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$\frac{c d (\cos[a + b x])^{3/2} (c \sin[a + b x])^{3/2}}{10 b} - \frac{c (\cos[a + b x])^{7/2} (c \sin[a + b x])^{3/2}}{5 b d} + \frac{3 c^2 d^2 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{20 b \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 99 leaves):

$$-\frac{1}{40 b (\sin[a + b x]^2)^{3/4}} c^2 d^2 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} \left(2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] \sin[2(a + b x)] + (\sin[a + b x]^2)^{3/4} \sin[4(a + b x)] \right)$$

■ **Problem 278: Result unnecessarily involves higher level functions.**

$$\int \sqrt{d \cos[a + b x]} (c \sin[a + b x])^{5/2} dx$$

Optimal (type 4, 95 leaves, 3 steps):

$$-\frac{c (\cos[a + b x])^{3/2} (c \sin[a + b x])^{3/2}}{3 b d} + \frac{c^2 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{2 b \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 85 leaves):

$$-\frac{1}{6 b (\sin[a + b x]^2)^{3/4}} c^2 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + (\sin[a + b x]^2)^{3/4} \right) \sin[2(a + b x)]$$

■ **Problem 279: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \sin[a + b x])^{5/2}}{(d \cos[a + b x])^{3/2}} dx$$

Optimal (type 4, 94 leaves, 3 steps):

$$\frac{2 c (c \sin[a + b x])^{3/2}}{b d \sqrt{d \cos[a + b x]}} - \frac{3 c^2 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{b d^2 \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 85 leaves):

$$\frac{2 c (c \sin[a + b x])^{3/2} \left(\cos[a + b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + (\sin[a + b x]^2)^{3/4} \right)}{b d \sqrt{d \cos[a + b x]} (\sin[a + b x]^2)^{3/4}}$$

■ **Problem 280: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \sin[a + b x])^{5/2}}{(d \cos[a + b x])^{7/2}} dx$$

Optimal (type 4, 133 leaves, 4 steps):

$$\frac{2 c (c \sin[a + b x])^{3/2}}{5 b d (d \cos[a + b x])^{5/2}} - \frac{6 c (c \sin[a + b x])^{3/2}}{5 b d^3 \sqrt{d \cos[a + b x]}} + \frac{6 c^2 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{5 b d^4 \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 111 leaves):

$$-\left(2 c^3 \left(2 \cos[a + b x]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + (-1 + 3 \cos[a + b x]^2) (\sin[a + b x]^2)^{3/4} \right) \tan[a + b x]^2 \right) / \left(5 b d^3 \sqrt{d \cos[a + b x]} \sqrt{c \sin[a + b x]} (\sin[a + b x]^2)^{3/4} \right)$$

■ **Problem 281: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \sin[a + b x])^{5/2}}{(d \cos[a + b x])^{11/2}} dx$$

Optimal (type 4, 168 leaves, 5 steps):

$$\frac{2 c (c \sin[a + b x])^{3/2}}{9 b d (d \cos[a + b x])^{9/2}} - \frac{2 c (c \sin[a + b x])^{3/2}}{15 b d^3 (d \cos[a + b x])^{5/2}} - \frac{4 c (c \sin[a + b x])^{3/2}}{15 b d^5 \sqrt{d \cos[a + b x]}} + \frac{4 c^2 \sqrt{d \cos[a + b x]} \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \sin[a + b x]}}{15 b d^6 \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 119 leaves):

$$-\frac{1}{45 b d^6 (\sin[a + b x]^2)^{3/4}} 2 c \sqrt{d \cos[a + b x]} \sec[a + b x]^5 (c \sin[a + b x])^{3/2} \left(4 \cos[a + b x]^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a + b x]^2\right] + (-5 + 3 \cos[a + b x]^2 + 6 \cos[a + b x]^4) (\sin[a + b x]^2)^{3/4} \right)$$

■ **Problem 282: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \sin[a + b x])^{5/2}}{\sqrt{d \cos[a + b x]}} dx$$

Optimal (type 3, 320 leaves, 11 steps):

$$\frac{3 c^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{c} \sqrt{d} \cos[a+bx]}\right]}{4 \sqrt{2} b \sqrt{d}} + \frac{3 c^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{c} \sqrt{d} \cos[a+bx]}\right]}{4 \sqrt{2} b \sqrt{d}} + \frac{3 c^{5/2} \operatorname{Log}\left[\sqrt{c} - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{d} \cos[a+bx]} + \sqrt{c} \tan[a+bx]\right]}{8 \sqrt{2} b \sqrt{d}} - \frac{3 c^{5/2} \operatorname{Log}\left[\sqrt{c} + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{d} \cos[a+bx]} + \sqrt{c} \tan[a+bx]\right]}{8 \sqrt{2} b \sqrt{d}} - \frac{c \sqrt{d} \cos[a+bx] (c \sin[a+bx])^{3/2}}{2 b d}$$

Result (type 5, 82 leaves):

$$\frac{\cot[a+bx] (c \sin[a+bx])^{5/2} \left(3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+bx]^2\right] + (\sin[a+bx]^2)^{3/4}\right)}{2 b \sqrt{d} \cos[a+bx] (\sin[a+bx]^2)^{3/4}}$$

■ **Problem 283: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \sin[a+bx])^{5/2}}{(d \cos[a+bx])^{5/2}} dx$$

Optimal (type 3, 315 leaves, 11 steps):

$$\frac{c^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{c} \sqrt{d} \cos[a+bx]}\right]}{\sqrt{2} b d^{5/2}} - \frac{c^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{c} \sqrt{d} \cos[a+bx]}\right]}{\sqrt{2} b d^{5/2}} - \frac{c^{5/2} \operatorname{Log}\left[\sqrt{c} - \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{d} \cos[a+bx]} + \sqrt{c} \tan[a+bx]\right]}{2 \sqrt{2} b d^{5/2}} + \frac{c^{5/2} \operatorname{Log}\left[\sqrt{c} + \frac{\sqrt{2} \sqrt{d} \sqrt{c \sin[a+bx]}}{\sqrt{d} \cos[a+bx]} + \sqrt{c} \tan[a+bx]\right]}{2 \sqrt{2} b d^{5/2}} + \frac{2 c (c \sin[a+bx])^{3/2}}{3 b d (d \cos[a+bx])^{3/2}}$$

Result (type 5, 88 leaves):

$$\frac{2 c (c \sin[a+bx])^{3/2} \left(3 \cos[a+bx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+bx]^2\right] + (\sin[a+bx]^2)^{3/4}\right)}{3 b d (d \cos[a+bx])^{3/2} (\sin[a+bx]^2)^{3/4}}$$

■ **Problem 287: Result unnecessarily involves higher level functions.**

$$\int \frac{\sin[a+bx]^{7/2}}{\cos[a+bx]^{7/2}} dx$$

Optimal (type 3, 226 leaves, 12 steps):

$$\frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{\sqrt{2} b} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{\sqrt{2} b} - \frac{\operatorname{Log}\left[1 + \cot[a+bx] - \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{2 \sqrt{2} b} + \frac{\operatorname{Log}\left[1 + \cot[a+bx] + \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{2 \sqrt{2} b} - \frac{2 \sqrt{\sin[a+bx]}}{b \sqrt{\cos[a+bx]}} + \frac{2 \sin[a+bx]^{5/2}}{5 b \cos[a+bx]^{5/2}}$$

Result (type 5, 94 leaves) :

$$-\left(2\sqrt{\sin[a+bx]}\left(5\cos[a+bx]^4\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+bx]^2\right]+3(2+3\cos[2(a+bx)])\left(\sin[a+bx]^2\right)^{1/4}\right)\right)/\left(15b\cos[a+bx]^{5/2}\left(\sin[a+bx]^2\right)^{1/4}\right)$$

■ **Problem 289: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\sin[x]}}{\sqrt{\cos[x]}} dx$$

Optimal (type 3, 122 leaves, 10 steps) :

$$-\frac{\operatorname{ArcTan}\left[1-\frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}}+\frac{\operatorname{ArcTan}\left[1+\frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}}+\frac{\operatorname{Log}\left[1-\frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}+\tan[x]\right]}{2\sqrt{2}}-\frac{\operatorname{Log}\left[1+\frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}+\tan[x]\right]}{2\sqrt{2}}$$

Result (type 5, 36 leaves) :

$$-\frac{2\sqrt{\cos[x]}\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[x]^2\right]\sin[x]^{3/2}}{\left(\sin[x]^2\right)^{3/4}}$$

■ **Problem 290: Result unnecessarily involves higher level functions.**

$$\int \frac{\sin[x]^{5/2}}{\sqrt{\cos[x]}} dx$$

Optimal (type 3, 143 leaves, 11 steps) :

$$-\frac{3\operatorname{ArcTan}\left[1-\frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{4\sqrt{2}}+\frac{3\operatorname{ArcTan}\left[1+\frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{4\sqrt{2}}+\frac{3\operatorname{Log}\left[1-\frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}+\tan[x]\right]}{8\sqrt{2}}-\frac{3\operatorname{Log}\left[1+\frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}+\tan[x]\right]}{8\sqrt{2}}-\frac{1}{2}\sqrt{\cos[x]}\sin[x]^{3/2}$$

Result (type 5, 49 leaves) :

$$-\frac{\sqrt{\cos[x]}\sin[x]^{3/2}\left(3\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[x]^2\right]+\left(\sin[x]^2\right)^{3/4}\right)}{2\left(\sin[x]^2\right)^{3/4}}$$

■ **Problem 291: Result unnecessarily involves higher level functions.**

$$\int \frac{(d\cos[a+bx])^{7/2}}{\sqrt{c}\sin[a+bx]} dx$$

Optimal (type 4, 132 leaves, 4 steps) :

$$\frac{5d^3\sqrt{d\cos[a+bx]}\sqrt{c}\sin[a+bx]}{6bc}+\frac{d(d\cos[a+bx])^{5/2}\sqrt{c}\sin[a+bx]}{3bc}+\frac{5d^4\operatorname{EllipticF}\left[a-\frac{\pi}{4}+bx, 2\right]\sqrt{\sin[2a+2bx]}}{12b\sqrt{d\cos[a+bx]}\sqrt{c}\sin[a+bx]}$$

Result (type 5, 140 leaves) :

$$-\frac{1}{30 b c (\sin [a+b x])^{1/4}}$$

$$d^3 \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]} \left(-30 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos [a+b x]^2\right] + 25 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \cos [a+b x]^2\right] + \right.$$

$$\left. 6 \cos [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{5}{4}, \frac{9}{4}, \cos [a+b x]^2\right] - 5 \cos [2(a+b x)] (\sin [a+b x]^2)^{1/4} \right)$$

■ **Problem 292: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \cos [a+b x])^{3/2}}{\sqrt{c \sin [a+b x]}} dx$$

Optimal (type 4, 92 leaves, 3 steps) :

$$\frac{d \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]}}{b c} + \frac{d^2 \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\sin [2 a + 2 b x]}}{2 b \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]}}$$

Result (type 5, 69 leaves) :

$$-\frac{(d \cos [a+b x])^{3/2} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{5}{4}, \frac{9}{4}, \cos [a+b x]^2\right] \sin [2(a+b x)]}{5 b \sqrt{c \sin [a+b x]} (\sin [a+b x]^2)^{1/4}}$$

■ **Problem 293: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]}} dx$$

Optimal (type 4, 53 leaves, 2 steps) :

$$\frac{\operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{\sin [2 a + 2 b x]}}{b \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]}}$$

Result (type 5, 67 leaves) :

$$-\frac{\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \cos [a+b x]^2\right] \sin [2(a+b x)]}{b \sqrt{d \cos [a+b x]} \sqrt{c \sin [a+b x]} (\sin [a+b x]^2)^{1/4}}$$

■ **Problem 294: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d \cos [a+b x])^{5/2} \sqrt{c \sin [a+b x]}} dx$$

Optimal (type 4, 97 leaves, 3 steps) :

$$\frac{2\sqrt{c\sin[a+bx]}}{3bcd(d\cos[a+bx])^{3/2}} + \frac{2\operatorname{EllipticF}\left[a - \frac{\pi}{4} + bx, 2\right]\sqrt{\sin[2a+2bx]}}{3bd^2\sqrt{d\cos[a+bx]}\sqrt{c\sin[a+bx]}}$$

Result (type 5, 104 leaves):

$$\left(2\left(-4\cos[a+bx]^2\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+bx]^2\right] + (2 + \cos[2(a+bx)])\left(\sin[a+bx]^2\right)^{1/4}\right)\tan[a+bx]\right) / \left(3bd^2\sqrt{d\cos[a+bx]}\sqrt{c\sin[a+bx]}\left(\sin[a+bx]^2\right)^{1/4}\right)$$

■ **Problem 295: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d\cos[a+bx])^{9/2}\sqrt{c\sin[a+bx]}} dx$$

Optimal (type 4, 134 leaves, 4 steps):

$$\frac{2\sqrt{c\sin[a+bx]}}{7bcd(d\cos[a+bx])^{7/2}} + \frac{4\sqrt{c\sin[a+bx]}}{7bcd^3(d\cos[a+bx])^{3/2}} + \frac{4\operatorname{EllipticF}\left[a - \frac{\pi}{4} + bx, 2\right]\sqrt{\sin[2a+2bx]}}{7bd^4\sqrt{d\cos[a+bx]}\sqrt{c\sin[a+bx]}}$$

Result (type 5, 103 leaves):

$$\frac{1}{7bcd^5\left(\sin[a+bx]^2\right)^{1/4}} \left(2\sqrt{d\cos[a+bx]}\sqrt{c\sin[a+bx]}\left(-8\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+bx]^2\right] + (4 + 2\sec[a+bx]^2 + \sec[a+bx]^4)\left(\sin[a+bx]^2\right)^{1/4}\right)\right)$$

■ **Problem 296: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{d\cos[a+bx]}}{\sqrt{c\sin[a+bx]}} dx$$

Optimal (type 3, 280 leaves, 10 steps):

$$\frac{\sqrt{d}\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{c}\sqrt{d\cos[a+bx]}}{\sqrt{d}\sqrt{c\sin[a+bx]}}\right]}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{c}\sqrt{d\cos[a+bx]}}{\sqrt{d}\sqrt{c\sin[a+bx]}}\right]}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\operatorname{Log}\left[\sqrt{d} + \sqrt{d}\cot[a+bx] - \frac{\sqrt{2}\sqrt{c}\sqrt{d\cos[a+bx]}}{\sqrt{c\sin[a+bx]}}\right]}{2\sqrt{2}b\sqrt{c}} + \frac{\sqrt{d}\operatorname{Log}\left[\sqrt{d} + \sqrt{d}\cot[a+bx] + \frac{\sqrt{2}\sqrt{c}\sqrt{d\cos[a+bx]}}{\sqrt{c\sin[a+bx]}}\right]}{2\sqrt{2}b\sqrt{c}}$$

Result (type 5, 69 leaves):

$$\frac{\sqrt{d\cos[a+bx]}\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+bx]^2\right]\sin[2(a+bx)]}{3b\sqrt{c\sin[a+bx]}\left(\sin[a+bx]^2\right)^{1/4}}$$

■ **Problem 300: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}} dx$$

Optimal (type 3, 174 leaves, 10 steps):

$$\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{\sqrt{2} b} - \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{\sqrt{2} b} - \frac{\text{Log}\left[1 + \text{Cot}[a+bx] - \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{2\sqrt{2} b} + \frac{\text{Log}\left[1 + \text{Cot}[a+bx] + \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{2\sqrt{2} b}$$

Result (type 5, 57 leaves):

$$\frac{2 \cos[a+bx]^{3/2} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+bx]^2\right] \sqrt{\sin[a+bx]}}{3 b (\sin[a+bx]^2)^{1/4}}$$

■ **Problem 301: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[a+bx]^{3/2}}{\sin[a+bx]^{3/2}} dx$$

Optimal (type 3, 199 leaves, 11 steps):

$$\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[a+bx]}}{\sqrt{\cos[a+bx]}}\right]}{\sqrt{2} b} - \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[a+bx]}}{\sqrt{\cos[a+bx]}}\right]}{\sqrt{2} b} - \frac{\text{Log}\left[1 - \frac{\sqrt{2} \sqrt{\sin[a+bx]}}{\sqrt{\cos[a+bx]}} + \text{Tan}[a+bx]\right]}{2\sqrt{2} b} + \frac{\text{Log}\left[1 + \frac{\sqrt{2} \sqrt{\sin[a+bx]}}{\sqrt{\cos[a+bx]}} + \text{Tan}[a+bx]\right]}{2\sqrt{2} b} - \frac{2\sqrt{\cos[a+bx]}}{b\sqrt{\sin[a+bx]}}$$

Result (type 5, 78 leaves):

$$\frac{2\sqrt{\cos[a+bx]} \left(-\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+bx]^2\right] \sin[a+bx]^2 + (\sin[a+bx]^2)^{3/4}\right)}{b\sqrt{\sin[a+bx]} (\sin[a+bx]^2)^{3/4}}$$

■ **Problem 302: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[a+bx]^{5/2}}{\sin[a+bx]^{5/2}} dx$$

Optimal (type 3, 201 leaves, 11 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{\sqrt{2} b} + \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{\sqrt{2} b} + \\
& \frac{\text{Log}\left[1 + \text{Cot}[a+bx] - \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{2\sqrt{2} b} - \frac{\text{Log}\left[1 + \text{Cot}[a+bx] + \frac{\sqrt{2} \sqrt{\cos[a+bx]}}{\sqrt{\sin[a+bx]}}\right]}{2\sqrt{2} b} - \frac{2 \cos[a+bx]^{3/2}}{3 b \sin[a+bx]^{3/2}}
\end{aligned}$$

Result (type 5, 80 leaves):

$$\frac{2 \cos[a+bx]^{3/2} \left(-\text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[a+bx]^2\right] \sin[a+bx]^2 + (\sin[a+bx]^2)^{1/4}\right)}{3 b \sin[a+bx]^{3/2} (\sin[a+bx]^2)^{1/4}}$$

■ **Problem 303: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[a+bx]^{7/2}}{\sin[a+bx]^{7/2}} dx$$

Optimal (type 3, 226 leaves, 12 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[a+bx]}}{\sqrt{\cos[a+bx]}}\right]}{\sqrt{2} b} + \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[a+bx]}}{\sqrt{\cos[a+bx]}}\right]}{\sqrt{2} b} + \\
& \frac{\text{Log}\left[1 - \frac{\sqrt{2} \sqrt{\sin[a+bx]}}{\sqrt{\cos[a+bx]}} + \text{Tan}[a+bx]\right]}{2\sqrt{2} b} - \frac{\text{Log}\left[1 + \frac{\sqrt{2} \sqrt{\sin[a+bx]}}{\sqrt{\cos[a+bx]}} + \text{Tan}[a+bx]\right]}{2\sqrt{2} b} - \frac{2 \cos[a+bx]^{5/2}}{5 b \sin[a+bx]^{5/2}} + \frac{2 \sqrt{\cos[a+bx]}}{b \sqrt{\sin[a+bx]}}
\end{aligned}$$

Result (type 5, 93 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{\cos[a+bx]} \left(5 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[a+bx]^2\right] \sin[a+bx]^4 + (\sin[a+bx]^2)^{3/4} (1 - 6 \sin[a+bx]^2)\right)\right) / \\
& (5 b \sin[a+bx]^{5/2} (\sin[a+bx]^2)^{3/4})
\end{aligned}$$

■ **Problem 324: Result unnecessarily involves higher level functions.**

$$\int \frac{\sin[a+bx]^{1/3}}{\cos[a+bx]^{1/3}} dx$$

Optimal (type 3, 128 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\sqrt{3} \text{ArcTan}\left[\frac{1 - 2 \sin[a+bx]^{2/3}}{\cos[a+bx]^{2/3}}\right]}{2 b} - \frac{\text{Log}\left[1 + \frac{\sin[a+bx]^{2/3}}{\cos[a+bx]^{2/3}}\right]}{2 b} + \frac{\text{Log}\left[1 - \frac{\sin[a+bx]^{2/3}}{\cos[a+bx]^{2/3}} + \frac{\sin[a+bx]^{4/3}}{\cos[a+bx]^{4/3}}\right]}{4 b}
\end{aligned}$$

Result (type 5, 57 leaves):

$$\frac{3 \operatorname{Cos}[a + b x]^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \operatorname{Cos}[a + b x]^2\right] \operatorname{Sin}[a + b x]^{4/3}}{2 b \left(\operatorname{Sin}[a + b x]^2\right)^{2/3}}$$

■ **Problem 325: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sin}[a + b x]^{2/3}}{\operatorname{Cos}[a + b x]^{2/3}} dx$$

Optimal (type 3, 224 leaves, 11 steps):

$$\begin{aligned} & -\frac{\operatorname{ArcTan}\left[\sqrt{3} - \frac{2 \operatorname{Sin}[a + b x]^{1/3}}{\operatorname{Cos}[a + b x]^{1/3}}\right]}{2 b} + \frac{\operatorname{ArcTan}\left[\sqrt{3} + \frac{2 \operatorname{Sin}[a + b x]^{1/3}}{\operatorname{Cos}[a + b x]^{1/3}}\right]}{2 b} + \frac{\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[a + b x]^{1/3}}{\operatorname{Cos}[a + b x]^{1/3}}\right]}{b} + \\ & \frac{\sqrt{3} \operatorname{Log}\left[1 - \frac{\sqrt{3} \operatorname{Sin}[a + b x]^{1/3}}{\operatorname{Cos}[a + b x]^{1/3}} + \frac{\operatorname{Sin}[a + b x]^{2/3}}{\operatorname{Cos}[a + b x]^{2/3}}\right]}{4 b} - \frac{\sqrt{3} \operatorname{Log}\left[1 + \frac{\sqrt{3} \operatorname{Sin}[a + b x]^{1/3}}{\operatorname{Cos}[a + b x]^{1/3}} + \frac{\operatorname{Sin}[a + b x]^{2/3}}{\operatorname{Cos}[a + b x]^{2/3}}\right]}{4 b} \end{aligned}$$

Result (type 5, 55 leaves):

$$\frac{3 \operatorname{Cos}[a + b x]^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \operatorname{Cos}[a + b x]^2\right] \operatorname{Sin}[a + b x]^{5/3}}{b \left(\operatorname{Sin}[a + b x]^2\right)^{5/6}}$$

■ **Problem 326: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sin}[a + b x]^{4/3}}{\operatorname{Cos}[a + b x]^{4/3}} dx$$

Optimal (type 3, 249 leaves, 12 steps):

$$\begin{aligned} & -\frac{\operatorname{ArcTan}\left[\sqrt{3} - \frac{2 \operatorname{Cos}[a + b x]^{1/3}}{\operatorname{Sin}[a + b x]^{1/3}}\right]}{2 b} + \frac{\operatorname{ArcTan}\left[\sqrt{3} + \frac{2 \operatorname{Cos}[a + b x]^{1/3}}{\operatorname{Sin}[a + b x]^{1/3}}\right]}{2 b} + \frac{\operatorname{ArcTan}\left[\frac{\operatorname{Cos}[a + b x]^{1/3}}{\operatorname{Sin}[a + b x]^{1/3}}\right]}{b} + \\ & \frac{\sqrt{3} \operatorname{Log}\left[1 + \frac{\operatorname{Cos}[a + b x]^{2/3}}{\operatorname{Sin}[a + b x]^{2/3}} - \frac{\sqrt{3} \operatorname{Cos}[a + b x]^{1/3}}{\operatorname{Sin}[a + b x]^{1/3}}\right]}{4 b} - \frac{\sqrt{3} \operatorname{Log}\left[1 + \frac{\operatorname{Cos}[a + b x]^{2/3}}{\operatorname{Sin}[a + b x]^{2/3}} + \frac{\sqrt{3} \operatorname{Cos}[a + b x]^{1/3}}{\operatorname{Sin}[a + b x]^{1/3}}\right]}{4 b} + \frac{3 \operatorname{Sin}[a + b x]^{1/3}}{b \operatorname{Cos}[a + b x]^{1/3}} \end{aligned}$$

Result (type 5, 83 leaves):

$$\frac{3 \operatorname{Sin}[a + b x]^{1/3}}{b \operatorname{Cos}[a + b x]^{1/3}} + \frac{3 \operatorname{Cos}[a + b x]^{5/3} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \operatorname{Cos}[a + b x]^2\right] \operatorname{Sin}[a + b x]^{1/3}}{5 b \left(\operatorname{Sin}[a + b x]^2\right)^{1/6}}$$

■ **Problem 327: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sin}[a + b x]^{5/3}}{\operatorname{Cos}[a + b x]^{5/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2 \cos [a+b x]^{2 / 3}}{\sin [a+b x]^{2 / 3}}}{\sqrt{3}}\right]}{2 b}+\frac{\operatorname{Log}\left[1+\frac{\cos [a+b x]^{4 / 3}}{\sin [a+b x]^{4 / 3}}-\frac{\cos [a+b x]^{2 / 3}}{\sin [a+b x]^{2 / 3}}\right]}{4 b}-\frac{\operatorname{Log}\left[1+\frac{\cos [a+b x]^{2 / 3}}{\sin [a+b x]^{2 / 3}}\right]}{2 b}+\frac{3 \sin [a+b x]^{2 / 3}}{2 b \cos [a+b x]^{2 / 3}}$$

Result (type 5, 81 leaves):

$$\frac{3 \sin [a+b x]^{2 / 3}\left(\cos [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \cos [a+b x]^2\right]+2\left(\sin [a+b x]^2\right)^{1 / 3}\right)}{4 b \cos [a+b x]^{2 / 3}\left(\sin [a+b x]^2\right)^{1 / 3}}$$

■ **Problem 328: Result unnecessarily involves higher level functions.**

$$\int \frac{\sin [a+b x]^{7 / 3}}{\cos [a+b x]^{7 / 3}} d x$$

Optimal (type 3, 155 leaves, 9 steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2 \sin [a+b x]^{2 / 3}}{\cos [a+b x]^{2 / 3}}}{\sqrt{3}}\right]}{2 b}+\frac{\operatorname{Log}\left[1+\frac{\sin [a+b x]^{2 / 3}}{\cos [a+b x]^{2 / 3}}\right]}{2 b}-\frac{\operatorname{Log}\left[1-\frac{\sin [a+b x]^{2 / 3}}{\cos [a+b x]^{2 / 3}}+\frac{\sin [a+b x]^{4 / 3}}{\cos [a+b x]^{4 / 3}}\right]}{4 b}+\frac{3 \sin [a+b x]^{4 / 3}}{4 b \cos [a+b x]^{4 / 3}}$$

Result (type 5, 80 leaves):

$$\frac{3 \sin [a+b x]^{4 / 3}\left(2 \cos [a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \cos [a+b x]^2\right]+\left(\sin [a+b x]^2\right)^{2 / 3}\right)}{4 b \cos [a+b x]^{4 / 3}\left(\sin [a+b x]^2\right)^{2 / 3}}$$

■ **Problem 329: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos [a+b x]^{1 / 3}}{\sin [a+b x]^{1 / 3}} d x$$

Optimal (type 3, 128 leaves, 8 steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2 \cos [a+b x]^{2 / 3}}{\sin [a+b x]^{2 / 3}}}{\sqrt{3}}\right]}{2 b}-\frac{\operatorname{Log}\left[1+\frac{\cos [a+b x]^{4 / 3}}{\sin [a+b x]^{4 / 3}}-\frac{\cos [a+b x]^{2 / 3}}{\sin [a+b x]^{2 / 3}}\right]}{4 b}+\frac{\operatorname{Log}\left[1+\frac{\cos [a+b x]^{2 / 3}}{\sin [a+b x]^{2 / 3}}\right]}{2 b}$$

Result (type 5, 57 leaves):

$$\frac{3 \cos [a+b x]^{4 / 3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \cos [a+b x]^2\right] \sin [a+b x]^{2 / 3}}{4 b\left(\sin [a+b x]^2\right)^{1 / 3}}$$

■ **Problem 330: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos [a+b x]^{2 / 3}}{\sin [a+b x]^{2 / 3}} d x$$

Optimal (type 3, 225 leaves, 11 steps):

$$\frac{\text{ArcTan}\left[\sqrt{3} - \frac{2 \cos[a+bx]^{1/3}}{\sin[a+bx]^{1/3}}\right]}{2b} - \frac{\text{ArcTan}\left[\sqrt{3} + \frac{2 \cos[a+bx]^{1/3}}{\sin[a+bx]^{1/3}}\right]}{2b} - \frac{\text{ArcTan}\left[\frac{\cos[a+bx]^{1/3}}{\sin[a+bx]^{1/3}}\right]}{b} -$$

$$\frac{\sqrt{3} \text{Log}\left[1 + \frac{\cos[a+bx]^{2/3}}{\sin[a+bx]^{2/3}} - \frac{\sqrt{3} \cos[a+bx]^{1/3}}{\sin[a+bx]^{1/3}}\right]}{4b} + \frac{\sqrt{3} \text{Log}\left[1 + \frac{\cos[a+bx]^{2/3}}{\sin[a+bx]^{2/3}} + \frac{\sqrt{3} \cos[a+bx]^{1/3}}{\sin[a+bx]^{1/3}}\right]}{4b}$$

Result (type 5, 57 leaves):

$$-\frac{3 \cos[a+bx]^{5/3} \text{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \cos[a+bx]^2\right] \sin[a+bx]^{1/3}}{5b (\sin[a+bx]^2)^{1/6}}$$

■ **Problem 331: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[a+bx]^{4/3}}{\sin[a+bx]^{4/3}} dx$$

Optimal (type 3, 250 leaves, 12 steps):

$$\frac{\text{ArcTan}\left[\sqrt{3} - \frac{2 \sin[a+bx]^{1/3}}{\cos[a+bx]^{1/3}}\right]}{2b} - \frac{\text{ArcTan}\left[\sqrt{3} + \frac{2 \sin[a+bx]^{1/3}}{\cos[a+bx]^{1/3}}\right]}{2b} - \frac{\text{ArcTan}\left[\frac{\sin[a+bx]^{1/3}}{\cos[a+bx]^{1/3}}\right]}{b} -$$

$$\frac{\sqrt{3} \text{Log}\left[1 - \frac{\sqrt{3} \sin[a+bx]^{1/3}}{\cos[a+bx]^{1/3}} + \frac{\sin[a+bx]^{2/3}}{\cos[a+bx]^{2/3}}\right]}{4b} + \frac{\sqrt{3} \text{Log}\left[1 + \frac{\sqrt{3} \sin[a+bx]^{1/3}}{\cos[a+bx]^{1/3}} + \frac{\sin[a+bx]^{2/3}}{\cos[a+bx]^{2/3}}\right]}{4b} - \frac{3 \cos[a+bx]^{1/3}}{b \sin[a+bx]^{1/3}}$$

Result (type 5, 78 leaves):

$$-\frac{3 \cos[a+bx]^{1/3} \left(-\text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \cos[a+bx]^2\right] \sin[a+bx]^2 + (\sin[a+bx]^2)^{5/6}\right)}{b \sin[a+bx]^{1/3} (\sin[a+bx]^2)^{5/6}}$$

■ **Problem 332: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[a+bx]^{5/3}}{\sin[a+bx]^{5/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$\frac{\sqrt{3} \text{ArcTan}\left[\frac{1 - \frac{2 \sin[a+bx]^{2/3}}{\cos[a+bx]^{2/3}}}{\sqrt{3}}\right]}{2b} + \frac{\text{Log}\left[1 + \frac{\sin[a+bx]^{2/3}}{\cos[a+bx]^{2/3}}\right]}{2b} - \frac{\text{Log}\left[1 - \frac{\sin[a+bx]^{2/3}}{\cos[a+bx]^{2/3}} + \frac{\sin[a+bx]^{4/3}}{\cos[a+bx]^{4/3}}\right]}{4b} - \frac{3 \cos[a+bx]^{2/3}}{2b \sin[a+bx]^{2/3}}$$

Result (type 5, 80 leaves):

$$-\frac{3 \cos[a+bx]^{2/3} \left(-\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \cos[a+bx]^2\right] \sin[a+bx]^2 + (\sin[a+bx]^2)^{2/3}\right)}{2b \sin[a+bx]^{2/3} (\sin[a+bx]^2)^{2/3}}$$

■ **Problem 333: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[a + bx]^{7/3}}{\sin[a + bx]^{7/3}} dx$$

Optimal (type 3, 155 leaves, 9 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \cos[a+bx]^{2/3}}{\sin[a+bx]^{2/3}}}{\sqrt{3}}\right]}{2b} + \frac{\operatorname{Log}\left[1 + \frac{\cos[a+bx]^{4/3}}{\sin[a+bx]^{4/3}} - \frac{\cos[a+bx]^{2/3}}{\sin[a+bx]^{2/3}}\right]}{4b} - \frac{\operatorname{Log}\left[1 + \frac{\cos[a+bx]^{2/3}}{\sin[a+bx]^{2/3}}\right]}{2b} - \frac{3 \cos[a + bx]^{4/3}}{4b \sin[a + bx]^{4/3}}$$

Result (type 5, 80 leaves):

$$-\frac{3 \cos[a + bx]^{4/3} \left(-\operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \cos[a + bx]^2\right] \sin[a + bx]^2 + (\sin[a + bx]^2)^{1/3}\right)}{4b \sin[a + bx]^{4/3} (\sin[a + bx]^2)^{1/3}}$$

■ **Problem 366: Result more than twice size of optimal antiderivative.**

$$\int (d \cos[a + bx])^n (c \sin[a + bx])^{5/2} dx$$

Optimal (type 5, 76 leaves, 1 step):

$$-\frac{c (d \cos[a + bx])^{1+n} \operatorname{Hypergeometric2F1}\left[-\frac{3}{4}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[a + bx]^2\right] (c \sin[a + bx])^{3/2}}{bd (1+n) (\sin[a + bx]^2)^{3/4}}$$

Result (type 5, 158 leaves):

$$\left((d \cos[a + bx])^n \operatorname{Cot}[a + bx] \right. \\ \left. - (3+n) \operatorname{Hypergeometric2F1}\left[-\frac{3}{4}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[a + bx]^2\right] - (3+n) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[a + bx]^2\right] + \right. \\ \left. (1+n) \cos[a + bx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3+n}{2}, \frac{5+n}{2}, \cos[a + bx]^2\right] \right) (c \sin[a + bx])^{5/2} \Big/ (2b (1+n) (3+n) (\sin[a + bx]^2)^{3/4})$$

■ **Problem 450: Result unnecessarily involves higher level functions.**

$$\int \sqrt{b \operatorname{Sec}[e + fx]} (a \sin[e + fx])^{9/2} dx$$

Optimal (type 3, 449 leaves, 13 steps):

$$\begin{aligned}
& - \frac{21 a^{9/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{a} \sqrt{b \cos[e+fx]}}\right] \sqrt{b \cos[e+fx]} \sqrt{b \sec[e+fx]}}{32 \sqrt{2} \sqrt{b} f} + \\
& \frac{21 a^{9/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{a} \sqrt{b \cos[e+fx]}}\right] \sqrt{b \cos[e+fx]} \sqrt{b \sec[e+fx]}}{32 \sqrt{2} \sqrt{b} f} + \\
& \frac{21 a^{9/2} \sqrt{b \cos[e+fx]} \operatorname{Log}\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{b \cos[e+fx]}} + \sqrt{a} \tan[e+fx]\right] \sqrt{b \sec[e+fx]}}{64 \sqrt{2} \sqrt{b} f} - \\
& \frac{21 a^{9/2} \sqrt{b \cos[e+fx]} \operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{b \cos[e+fx]}} + \sqrt{a} \tan[e+fx]\right] \sqrt{b \sec[e+fx]}}{64 \sqrt{2} \sqrt{b} f} - \frac{7 a^3 b (a \sin[e+fx])^{3/2}}{16 f \sqrt{b \sec[e+fx]}} - \frac{a b (a \sin[e+fx])^{7/2}}{4 f \sqrt{b \sec[e+fx]}}
\end{aligned}$$

Result (type 5, 109 leaves):

$$\begin{aligned}
& - \frac{1}{32 f (\sin[e+fx])^{3/4}} a^4 \sqrt{b \sec[e+fx]} \sqrt{a \sin[e+fx]} \\
& \left(21 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+fx]^2\right] \sin[2(e+fx)] + (\sin[e+fx])^{3/4} (9 \sin[2(e+fx)] - \sin[4(e+fx)]) \right)
\end{aligned}$$

■ **Problem 451: Result unnecessarily involves higher level functions.**

$$\int \sqrt{b \sec[e+fx]} (a \sin[e+fx])^{5/2} dx$$

Optimal (type 3, 414 leaves, 12 steps):

$$\begin{aligned}
& - \frac{3 a^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{a} \sqrt{b \cos[e+fx]}}\right] \sqrt{b \cos[e+fx]} \sqrt{b \sec[e+fx]}}{4 \sqrt{2} \sqrt{b} f} + \frac{3 a^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{a} \sqrt{b \cos[e+fx]}}\right] \sqrt{b \cos[e+fx]} \sqrt{b \sec[e+fx]}}{4 \sqrt{2} \sqrt{b} f} + \\
& \frac{3 a^{5/2} \sqrt{b \cos[e+fx]} \operatorname{Log}\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{b \cos[e+fx]}} + \sqrt{a} \tan[e+fx]\right] \sqrt{b \sec[e+fx]}}{8 \sqrt{2} \sqrt{b} f} - \\
& \frac{3 a^{5/2} \sqrt{b \cos[e+fx]} \operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{b \cos[e+fx]}} + \sqrt{a} \tan[e+fx]\right] \sqrt{b \sec[e+fx]}}{8 \sqrt{2} \sqrt{b} f} - \frac{a b (a \sin[e+fx])^{3/2}}{2 f \sqrt{b \sec[e+fx]}}
\end{aligned}$$

Result (type 5, 87 leaves):

$$- \frac{1}{4 f (\sin[e+fx])^{3/4}} a^2 \sqrt{b \sec[e+fx]} \sqrt{a \sin[e+fx]} \left(3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+fx]^2\right] + (\sin[e+fx])^{3/4} \right) \sin[2(e+fx)]$$

■ **Problem 452: Result unnecessarily involves higher level functions.**

$$\int \sqrt{b \operatorname{Sec}[e + f x]} \sqrt{a \operatorname{Sin}[e + f x]} dx$$

Optimal (type 3, 376 leaves, 11 steps):

$$\begin{aligned} & - \frac{\sqrt{a} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \operatorname{Sin}[e + f x]}}{\sqrt{a} \sqrt{b \operatorname{Cos}[e + f x]}}\right] \sqrt{b \operatorname{Cos}[e + f x]} \sqrt{b \operatorname{Sec}[e + f x]} + \sqrt{a} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \operatorname{Sin}[e + f x]}}{\sqrt{a} \sqrt{b \operatorname{Cos}[e + f x]}}\right] \sqrt{b \operatorname{Cos}[e + f x]} \sqrt{b \operatorname{Sec}[e + f x]}}{\sqrt{2} \sqrt{b} f} + \\ & \frac{\sqrt{a} \sqrt{b \operatorname{Cos}[e + f x]} \operatorname{Log}\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \operatorname{Sin}[e + f x]}}{\sqrt{b \operatorname{Cos}[e + f x]}} + \sqrt{a} \operatorname{Tan}[e + f x]\right] \sqrt{b \operatorname{Sec}[e + f x]}}{2 \sqrt{2} \sqrt{b} f} - \\ & \frac{\sqrt{a} \sqrt{b \operatorname{Cos}[e + f x]} \operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \operatorname{Sin}[e + f x]}}{\sqrt{b \operatorname{Cos}[e + f x]}} + \sqrt{a} \operatorname{Tan}[e + f x]\right] \sqrt{b \operatorname{Sec}[e + f x]}}{2 \sqrt{2} \sqrt{b} f} \end{aligned}$$

Result (type 5, 67 leaves):

$$- \frac{\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Cos}[e + f x]^2\right] \sqrt{b \operatorname{Sec}[e + f x]} \sqrt{a \operatorname{Sin}[e + f x]} \operatorname{Sin}[2(e + f x)]}{f (\operatorname{Sin}[e + f x]^2)^{3/4}}$$

■ **Problem 456: Result unnecessarily involves higher level functions.**

$$\int \sqrt{b \operatorname{Sec}[e + f x]} (a \operatorname{Sin}[e + f x])^{7/2} dx$$

Optimal (type 4, 128 leaves, 5 steps):

$$- \frac{5 a^3 b \sqrt{a \operatorname{Sin}[e + f x]}}{6 f \sqrt{b \operatorname{Sec}[e + f x]}} - \frac{a b (a \operatorname{Sin}[e + f x])^{5/2}}{3 f \sqrt{b \operatorname{Sec}[e + f x]}} + \frac{5 a^4 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \operatorname{Sec}[e + f x]} \sqrt{\operatorname{Sin}[2 e + 2 f x]}}{12 f \sqrt{a \operatorname{Sin}[e + f x]}}$$

Result (type 5, 90 leaves):

$$\begin{aligned} & \frac{1}{12 f \sqrt{b \operatorname{Sec}[e + f x]}} \\ & a^3 b \sqrt{a \operatorname{Sin}[e + f x]} \left(2 (-6 + \operatorname{Cos}[2(e + f x)]) + 5 \operatorname{Csc}[e + f x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Sec}[e + f x]^2\right] (-\operatorname{Tan}[e + f x]^2)^{3/4} \right) \end{aligned}$$

■ **Problem 457: Result unnecessarily involves higher level functions.**

$$\int \sqrt{b \operatorname{Sec}[e + f x]} (a \operatorname{Sin}[e + f x])^{3/2} dx$$

Optimal (type 4, 91 leaves, 4 steps):

$$-\frac{a b \sqrt{a \sin[e+f x]}}{f \sqrt{b \sec[e+f x]}} + \frac{a^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e+f x]} \sqrt{\sin[2 e + 2 f x]}}{2 f \sqrt{a \sin[e+f x]}}$$

Result (type 5, 83 leaves):

$$\frac{1}{2 f \sqrt{b \sec[e+f x]}} b \operatorname{Csc}[e+f x]^3 (a \sin[e+f x])^{3/2} \left(-1 + \cos[2(e+f x)] + \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e+f x]^2\right] (-\tan[e+f x]^2)^{3/4} \right)$$

■ **Problem 458: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{b \sec[e+f x]}}{\sqrt{a \sin[e+f x]}} dx$$

Optimal (type 4, 53 leaves, 3 steps):

$$\frac{\operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e+f x]} \sqrt{\sin[2 e + 2 f x]}}{f \sqrt{a \sin[e+f x]}}$$

Result (type 5, 66 leaves):

$$\frac{\operatorname{Cot}[e+f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e+f x]^2\right] \sqrt{b \sec[e+f x]} (-\tan[e+f x]^2)^{3/4}}{f \sqrt{a \sin[e+f x]}}$$

■ **Problem 459: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{b \sec[e+f x]}}{(a \sin[e+f x])^{5/2}} dx$$

Optimal (type 4, 95 leaves, 4 steps):

$$-\frac{2 b}{3 a f \sqrt{b \sec[e+f x]} (a \sin[e+f x])^{3/2}} + \frac{2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e+f x]} \sqrt{\sin[2 e + 2 f x]}}{3 a^2 f \sqrt{a \sin[e+f x]}}$$

Result (type 5, 75 leaves):

$$\frac{2 \operatorname{Cot}[e+f x] \sqrt{b \sec[e+f x]} (-1 + \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e+f x]^2\right] (-\tan[e+f x]^2)^{3/4})}{3 a^2 f \sqrt{a \sin[e+f x]}}$$

■ **Problem 460: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{b \sec[e+f x]}}{(a \sin[e+f x])^{9/2}} dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$-\frac{2b}{7af\sqrt{b\sec[e+fx]}(a\sin[e+fx])^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec[e+fx]}(a\sin[e+fx])^{3/2}} + \frac{4\operatorname{EllipticF}\left[e - \frac{\pi}{4} + fx, 2\right]\sqrt{b\sec[e+fx]}\sqrt{\sin[2e+2fx]}}{7a^4f\sqrt{a\sin[e+fx]}}$$

Result (type 5, 111 leaves):

$$-\left(2\cos[2(e+fx)](b\sec[e+fx])^{3/2} \left((-2 + \cos[2(e+fx)])\operatorname{Csc}[e+fx]^2 + 2\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e+fx]^2\right](-\tan[e+fx]^2)^{3/4} \right) \right) / (7a^3bf(-2 + \sec[e+fx]^2)(a\sin[e+fx])^{3/2})$$

■ **Problem 461: Result unnecessarily involves higher level functions.**

$$\int \frac{\sin[e+fx]^{9/2}}{\sqrt{b\sec[e+fx]}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{7b\sin[e+fx]^{3/2}}{30f(b\sec[e+fx])^{3/2}} - \frac{b\sin[e+fx]^{7/2}}{5f(b\sec[e+fx])^{3/2}} + \frac{7\operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right]\sqrt{\sin[e+fx]}}{20f\sqrt{b\sec[e+fx]}\sqrt{\sin[2e+2fx]}}$$

Result (type 5, 99 leaves):

$$\frac{1}{480bf\sqrt{\sin[e+fx]}}\sqrt{b\sec[e+fx]} \left(4(25 - 14\cos[2(e+fx)] + 3\cos[4(e+fx)])\sin[e+fx]^2 - 84\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \sec[e+fx]^2\right](-\tan[e+fx]^2)^{1/4} \right)$$

■ **Problem 462: Result unnecessarily involves higher level functions.**

$$\int \frac{\sin[e+fx]^{5/2}}{\sqrt{b\sec[e+fx]}} dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{b\sin[e+fx]^{3/2}}{3f(b\sec[e+fx])^{3/2}} + \frac{\operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right]\sqrt{\sin[e+fx]}}{2f\sqrt{b\sec[e+fx]}\sqrt{\sin[2e+2fx]}}$$

Result (type 5, 86 leaves):

$$\frac{1}{24bf\sqrt{\sin[e+fx]}}\sqrt{b\sec[e+fx]} \left(5 - 6\cos[2(e+fx)] + \cos[4(e+fx)] - 6\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \sec[e+fx]^2\right](-\tan[e+fx]^2)^{1/4} \right)$$

■ **Problem 463: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\sin[e + f x]}}{\sqrt{b \sec[e + f x]}} dx$$

Optimal (type 4, 51 leaves, 3 steps):

$$\frac{\text{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[e + f x]}}{f \sqrt{b \sec[e + f x]} \sqrt{\sin[2e + 2f x]}}$$

Result (type 5, 75 leaves):

$$-\frac{1}{2 b f \sqrt{\sin[e + f x]}} \sqrt{b \sec[e + f x]} \left(-1 + \cos[2(e + f x)] + \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \sec[e + f x]^2\right] (-\tan[e + f x]^2)^{1/4} \right)$$

■ **Problem 464: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{b \sec[e + f x]} \sin[e + f x]^{3/2}} dx$$

Optimal (type 4, 81 leaves, 4 steps):

$$-\frac{2 b}{f (b \sec[e + f x])^{3/2} \sqrt{\sin[e + f x]}} - \frac{2 \text{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[e + f x]}}{f \sqrt{b \sec[e + f x]} \sqrt{\sin[2e + 2f x]}}$$

Result (type 5, 64 leaves):

$$\frac{\sqrt{b \sec[e + f x]} \left(-2 + \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \sec[e + f x]^2\right] (-\tan[e + f x]^2)^{1/4} \right)}{b f \sqrt{\sin[e + f x]}}$$

■ **Problem 465: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{b \sec[e + f x]} \sin[e + f x]^{7/2}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{2 b}{5 f (b \sec[e + f x])^{3/2} \sin[e + f x]^{5/2}} - \frac{4 b}{5 f (b \sec[e + f x])^{3/2} \sqrt{\sin[e + f x]}} - \frac{4 \text{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[e + f x]}}{5 f \sqrt{b \sec[e + f x]} \sqrt{\sin[2e + 2f x]}}$$

Result (type 5, 84 leaves):

$$\frac{1}{5 b f \sin[e + f x]^{5/2}} \sqrt{b \sec[e + f x]} \left(-3 + \cos[2(e + f x)] + 2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \sec[e + f x]^2\right] \sin[e + f x]^2 (-\tan[e + f x]^2)^{1/4} \right)$$

■ **Problem 466: Result unnecessarily involves higher level functions.**

$$\int \frac{\sin[e + f x]^{3/2}}{\sqrt{b \sec[e + f x]}} dx$$

Optimal (type 3, 363 leaves, 12 steps):

$$\frac{\sqrt{b} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b \cos[e + f x]}}{\sqrt{b} \sqrt{\sin[e + f x]}}\right]}{4 \sqrt{2} f \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}} - \frac{\sqrt{b} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b \cos[e + f x]}}{\sqrt{b} \sqrt{\sin[e + f x]}}\right]}{4 \sqrt{2} f \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}} -$$

$$\frac{\sqrt{b} \operatorname{Log}\left[\sqrt{b} + \sqrt{b} \cot[e + f x] - \frac{\sqrt{2} \sqrt{b \cos[e + f x]}}{\sqrt{\sin[e + f x]}}\right]}{8 \sqrt{2} f \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}} + \frac{\sqrt{b} \operatorname{Log}\left[\sqrt{b} + \sqrt{b} \cot[e + f x] + \frac{\sqrt{2} \sqrt{b \cos[e + f x]}}{\sqrt{\sin[e + f x]}}\right]}{8 \sqrt{2} f \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}} - \frac{b \sqrt{\sin[e + f x]}}{2 f (b \sec[e + f x])^{3/2}}$$

Result (type 5, 75 leaves):

$$-\frac{b \sqrt{\sin[e + f x]} \left(\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[e + f x]^2\right] + 3 (\sin[e + f x]^2)^{1/4}\right)}{6 f (b \sec[e + f x])^{3/2} (\sin[e + f x]^2)^{1/4}}$$

■ **Problem 467: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{b \sec[e + f x]} \sqrt{\sin[e + f x]}} dx$$

Optimal (type 3, 328 leaves, 11 steps):

$$\frac{\sqrt{b} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b \cos[e + f x]}}{\sqrt{b} \sqrt{\sin[e + f x]}}\right]}{\sqrt{2} f \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}} - \frac{\sqrt{b} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b \cos[e + f x]}}{\sqrt{b} \sqrt{\sin[e + f x]}}\right]}{\sqrt{2} f \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}} -$$

$$\frac{\sqrt{b} \operatorname{Log}\left[\sqrt{b} + \sqrt{b} \cot[e + f x] - \frac{\sqrt{2} \sqrt{b \cos[e + f x]}}{\sqrt{\sin[e + f x]}}\right]}{2 \sqrt{2} f \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}} + \frac{\sqrt{b} \operatorname{Log}\left[\sqrt{b} + \sqrt{b} \cot[e + f x] + \frac{\sqrt{2} \sqrt{b \cos[e + f x]}}{\sqrt{\sin[e + f x]}}\right]}{2 \sqrt{2} f \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}}$$

Result (type 5, 60 leaves):

$$-\frac{2 b \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[e + f x]^2\right] \sqrt{\sin[e + f x]}}{3 f (b \sec[e + f x])^{3/2} (\sin[e + f x]^2)^{1/4}}$$

■ **Problem 472: Result unnecessarily involves higher level functions.**

$$\int \frac{(a \sin[e + f x])^{9/2}}{(b \sec[e + f x])^{3/2}} dx$$

Optimal (type 3, 490 leaves, 14 steps):

$$\begin{aligned}
& - \frac{7 a^{9/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{a} \sqrt{b \cos[e+fx]}}\right] \sqrt{b \cos[e+fx]} \sqrt{b \sec[e+fx]}}{128 \sqrt{2} b^{5/2} f} + \frac{7 a^{9/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{a} \sqrt{b \cos[e+fx]}}\right] \sqrt{b \cos[e+fx]} \sqrt{b \sec[e+fx]}}{128 \sqrt{2} b^{5/2} f} + \\
& \frac{7 a^{9/2} \sqrt{b \cos[e+fx]} \operatorname{Log}\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{b \cos[e+fx]}} + \sqrt{a} \tan[e+fx]\right] \sqrt{b \sec[e+fx]}}{256 \sqrt{2} b^{5/2} f} - \\
& \frac{7 a^{9/2} \sqrt{b \cos[e+fx]} \operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{b \cos[e+fx]}} + \sqrt{a} \tan[e+fx]\right] \sqrt{b \sec[e+fx]}}{256 \sqrt{2} b^{5/2} f} - \\
& \frac{7 a^3 (a \sin[e+fx])^{3/2}}{192 b f \sqrt{b \sec[e+fx]}} - \frac{a (a \sin[e+fx])^{7/2}}{48 b f \sqrt{b \sec[e+fx]}} + \frac{(a \sin[e+fx])^{11/2}}{6 a b f \sqrt{b \sec[e+fx]}}
\end{aligned}$$

Result (type 5, 125 leaves):

$$\begin{aligned}
& \left(a^4 \sec[e+fx]^2 \sqrt{a \sin[e+fx]} \left(-21 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+fx]^2\right] \sin[2(e+fx)] + \right. \right. \\
& \left. \left. (\sin[e+fx]^2)^{3/4} (\sin[2(e+fx)] - 7 \sin[4(e+fx)] + 2 \sin[6(e+fx)]) \right) \right) / (384 f (b \sec[e+fx])^{3/2} (\sin[e+fx]^2)^{3/4})
\end{aligned}$$

■ **Problem 473: Result unnecessarily involves higher level functions.**

$$\int \frac{(a \sin[e+fx])^{5/2}}{(b \sec[e+fx])^{3/2}} dx$$

Optimal (type 3, 453 leaves, 13 steps):

$$\begin{aligned}
& - \frac{3 a^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{a} \sqrt{b \cos[e+fx]}}\right] \sqrt{b \cos[e+fx]} \sqrt{b \sec[e+fx]}}{32 \sqrt{2} b^{5/2} f} + \frac{3 a^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{a} \sqrt{b \cos[e+fx]}}\right] \sqrt{b \cos[e+fx]} \sqrt{b \sec[e+fx]}}{32 \sqrt{2} b^{5/2} f} + \\
& \frac{3 a^{5/2} \sqrt{b \cos[e+fx]} \operatorname{Log}\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{b \cos[e+fx]}} + \sqrt{a} \tan[e+fx]\right] \sqrt{b \sec[e+fx]}}{64 \sqrt{2} b^{5/2} f} - \\
& \frac{3 a^{5/2} \sqrt{b \cos[e+fx]} \operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e+fx]}}{\sqrt{b \cos[e+fx]}} + \sqrt{a} \tan[e+fx]\right] \sqrt{b \sec[e+fx]}}{64 \sqrt{2} b^{5/2} f} - \frac{a (a \sin[e+fx])^{3/2}}{16 b f \sqrt{b \sec[e+fx]}} + \frac{(a \sin[e+fx])^{7/2}}{4 a b f \sqrt{b \sec[e+fx]}}
\end{aligned}$$

Result (type 5, 93 leaves):

$$\begin{aligned}
& - \left(a (a \sin[e+fx])^{3/2} \left(3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+fx]^2\right] + (-1 + 2 \cos[2(e+fx)]) (\sin[e+fx]^2)^{3/4} \right) \right) / \\
& (16 b f \sqrt{b \sec[e+fx]} (\sin[e+fx]^2)^{3/4})
\end{aligned}$$

■ **Problem 474: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a \sin[e + f x]}}{(b \sec[e + f x])^{3/2}} dx$$

Optimal (type 3, 418 leaves, 12 steps):

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e + f x]}}{\sqrt{a} \sqrt{b \cos[e + f x]}}\right] \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}}{4 \sqrt{2} b^{5/2} f} + \frac{\sqrt{a} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e + f x]}}{\sqrt{a} \sqrt{b \cos[e + f x]}}\right] \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}}{4 \sqrt{2} b^{5/2} f} +$$

$$\frac{\sqrt{a} \sqrt{b \cos[e + f x]} \operatorname{Log}\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e + f x]}}{\sqrt{b \cos[e + f x]}} + \sqrt{a} \operatorname{Tan}[e + f x]\right] \sqrt{b \sec[e + f x]}}{8 \sqrt{2} b^{5/2} f} -$$

$$\frac{\sqrt{a} \sqrt{b \cos[e + f x]} \operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e + f x]}}{\sqrt{b \cos[e + f x]}} + \sqrt{a} \operatorname{Tan}[e + f x]\right] \sqrt{b \sec[e + f x]}}{8 \sqrt{2} b^{5/2} f} + \frac{(a \sin[e + f x])^{3/2}}{2 a b f \sqrt{b \sec[e + f x]}}$$

Result (type 5, 82 leaves):

$$\frac{(a \sin[e + f x])^{3/2} \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e + f x]^2\right] + (\sin[e + f x]^2)^{3/4}\right)}{2 a b f \sqrt{b \sec[e + f x]} (\sin[e + f x]^2)^{3/4}}$$

■ **Problem 475: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(b \sec[e + f x])^{3/2} (a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 411 leaves, 12 steps):

$$\frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e + f x]}}{\sqrt{a} \sqrt{b \cos[e + f x]}}\right] \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}}{\sqrt{2} a^{3/2} b^{5/2} f} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e + f x]}}{\sqrt{a} \sqrt{b \cos[e + f x]}}\right] \sqrt{b \cos[e + f x]} \sqrt{b \sec[e + f x]}}{\sqrt{2} a^{3/2} b^{5/2} f} -$$

$$\frac{\sqrt{b \cos[e + f x]} \operatorname{Log}\left[\sqrt{a} - \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e + f x]}}{\sqrt{b \cos[e + f x]}} + \sqrt{a} \operatorname{Tan}[e + f x]\right] \sqrt{b \sec[e + f x]}}{2 \sqrt{2} a^{3/2} b^{5/2} f} +$$

$$\frac{\sqrt{b \cos[e + f x]} \operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{2} \sqrt{b} \sqrt{a \sin[e + f x]}}{\sqrt{b \cos[e + f x]}} + \sqrt{a} \operatorname{Tan}[e + f x]\right] \sqrt{b \sec[e + f x]}}{2 \sqrt{2} a^{3/2} b^{5/2} f} - \frac{2}{a b f \sqrt{b \sec[e + f x]} \sqrt{a \sin[e + f x]}}$$

Result (type 5, 89 leaves):

$$\frac{2 \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e + f x]^2\right] \sin[e + f x]^2 - (\sin[e + f x]^2)^{3/4}\right)}{a b f \sqrt{b \sec[e + f x]} \sqrt{a \sin[e + f x]} (\sin[e + f x]^2)^{3/4}}$$

■ **Problem 477: Result unnecessarily involves higher level functions.**

$$\int \frac{(a \sin[e + f x])^{7/2}}{(b \sec[e + f x])^{3/2}} dx$$

Optimal (type 4, 172 leaves, 6 steps):

$$-\frac{a^3 \sqrt{a \sin[e + f x]}}{12 b f \sqrt{b \sec[e + f x]}} - \frac{a (a \sin[e + f x])^{5/2}}{30 b f \sqrt{b \sec[e + f x]}} + \frac{(a \sin[e + f x])^{9/2}}{5 a b f \sqrt{b \sec[e + f x]}} + \frac{a^4 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e + f x]} \sqrt{\sin[2 e + 2 f x]}}{24 b^2 f \sqrt{a \sin[e + f x]}}$$

Result (type 5, 103 leaves):

$$-\left(a^5 \left(-4 + 17 \cos[2(e + f x)] - 16 \cos[4(e + f x)] + 3 \cos[6(e + f x)] - 20 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e + f x]^2\right] (-\tan[e + f x]^2)^{3/4} \right) \right) / (480 b f \sqrt{b \sec[e + f x]} (a \sin[e + f x])^{3/2})$$

■ **Problem 478: Result unnecessarily involves higher level functions.**

$$\int \frac{(a \sin[e + f x])^{3/2}}{(b \sec[e + f x])^{3/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$-\frac{a \sqrt{a \sin[e + f x]}}{6 b f \sqrt{b \sec[e + f x]}} + \frac{(a \sin[e + f x])^{5/2}}{3 a b f \sqrt{b \sec[e + f x]}} + \frac{a^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e + f x]} \sqrt{\sin[2 e + 2 f x]}}{12 b^2 f \sqrt{a \sin[e + f x]}}$$

Result (type 5, 87 leaves):

$$\frac{1}{12 b f \sqrt{b \sec[e + f x]}} a \sqrt{a \sin[e + f x]} \left(-2 \cos[2(e + f x)] + \csc[e + f x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e + f x]^2\right] (-\tan[e + f x]^2)^{3/4} \right)$$

■ **Problem 479: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(b \sec[e + f x])^{3/2} \sqrt{a \sin[e + f x]}} dx$$

Optimal (type 4, 94 leaves, 4 steps):

$$\frac{\sqrt{a \sin[e + f x]}}{a b f \sqrt{b \sec[e + f x]}} + \frac{\operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \sec[e + f x]} \sqrt{\sin[2 e + 2 f x]}}{2 b^2 f \sqrt{a \sin[e + f x]}}$$

Result (type 5, 84 leaves):

$$-\frac{1}{2 b^2 f \sqrt{a \sin[e + f x]}} \cot[e + f x] \sqrt{b \sec[e + f x]} \left(-1 + \cos[2(e + f x)] - \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \sec[e + f x]^2\right] (-\tan[e + f x]^2)^{3/4} \right)$$

■ **Problem 480: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(b \operatorname{Sec}[e + f x])^{3/2} (a \operatorname{Sin}[e + f x])^{5/2}} dx$$

Optimal (type 4, 100 leaves, 4 steps):

$$-\frac{2}{3 a b f \sqrt{b \operatorname{Sec}[e + f x]} (a \operatorname{Sin}[e + f x])^{3/2}} - \frac{\operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \operatorname{Sec}[e + f x]} \sqrt{\operatorname{Sin}[2 e + 2 f x]}}{3 a^2 b^2 f \sqrt{a \operatorname{Sin}[e + f x]}}$$

Result (type 5, 78 leaves):

$$-\frac{\operatorname{Cot}[e + f x] \sqrt{b \operatorname{Sec}[e + f x]} \left(2 + \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Sec}[e + f x]^2\right] (-\operatorname{Tan}[e + f x]^2)^{3/4}\right)}{3 a^2 b^2 f \sqrt{a \operatorname{Sin}[e + f x]}}$$

■ **Problem 481: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(b \operatorname{Sec}[e + f x])^{3/2} (a \operatorname{Sin}[e + f x])^{9/2}} dx$$

Optimal (type 4, 137 leaves, 5 steps):

$$-\frac{2}{7 a b f \sqrt{b \operatorname{Sec}[e + f x]} (a \operatorname{Sin}[e + f x])^{7/2}} + \frac{2}{21 a^3 b f \sqrt{b \operatorname{Sec}[e + f x]} (a \operatorname{Sin}[e + f x])^{3/2}} - \frac{2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \operatorname{Sec}[e + f x]} \sqrt{\operatorname{Sin}[2 e + 2 f x]}}{21 a^4 b^2 f \sqrt{a \operatorname{Sin}[e + f x]}}$$

Result (type 5, 119 leaves):

$$\left(\operatorname{Cos}[2(e + f x)] \operatorname{Csc}[e + f x]^4 \sqrt{a \operatorname{Sin}[e + f x]} \left((5 + \operatorname{Cos}[2(e + f x)]) \operatorname{Sec}[e + f x]^2 - 2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Sec}[e + f x]^2\right] (-\operatorname{Tan}[e + f x]^2)^{7/4} \right) \right) / \left(21 a^5 b f \sqrt{b \operatorname{Sec}[e + f x]} (-2 + \operatorname{Sec}[e + f x]^2) \right)$$

■ **Problem 482: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(b \operatorname{Sec}[e + f x])^{3/2} (a \operatorname{Sin}[e + f x])^{13/2}} dx$$

Optimal (type 4, 174 leaves, 6 steps):

$$-\frac{2}{11 a b f \sqrt{b \operatorname{Sec}[e+f x]} (a \operatorname{Sin}[e+f x])^{11/2}} + \frac{2}{77 a^3 b f \sqrt{b \operatorname{Sec}[e+f x]} (a \operatorname{Sin}[e+f x])^{7/2}} +$$

$$\frac{4}{77 a^5 b f \sqrt{b \operatorname{Sec}[e+f x]} (a \operatorname{Sin}[e+f x])^{3/2}} - \frac{4 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{b \operatorname{Sec}[e+f x]} \sqrt{\operatorname{Sin}[2 e + 2 f x]}}{77 a^6 b^2 f \sqrt{a \operatorname{Sin}[e+f x]}}$$

Result (type 5, 131 leaves):

$$\left(2 \operatorname{Cot}[2(e+f x)] \operatorname{Csc}[2(e+f x)] \sqrt{a \operatorname{Sin}[e+f x]} \right.$$

$$\left. \left((23 + 6 \operatorname{Cos}[2(e+f x)] - \operatorname{Cos}[4(e+f x)]) \operatorname{Csc}[e+f x]^4 + 8 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Sec}[e+f x]^2\right] (-\operatorname{Tan}[e+f x]^2)^{3/4} \right) \right) /$$

$$\left(77 a^7 b f \sqrt{b \operatorname{Sec}[e+f x]} (-2 + \operatorname{Sec}[e+f x]^2) \right)$$

■ **Problem 488: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+f x]^n \operatorname{Sin}[e+f x]^m dx$$

Optimal (type 5, 86 leaves, 2 steps):

$$\frac{\operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[e+f x]^2\right] \operatorname{Sec}[e+f x]^{-1+n} \operatorname{Sin}[e+f x]^{-1+m} (\operatorname{Sin}[e+f x]^2)^{\frac{1-m}{2}}}{f(1-n)}$$

Result (type 6, 2938 leaves):

$$\left(2(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right.$$

$$\left. \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \right)^{-1+n} \operatorname{Sec}[e+f x]^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x] \right)^n \operatorname{Sin}[e+f x]^{2m} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] \right) /$$

$$\left(f(1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] - \right.$$

$$2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] - \right.$$

$$\left. n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right)$$

$$\left(\left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \right)^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x] \right)^n \right. \right.$$

$$\left. \operatorname{Sin}[e+f x]^m \right) / \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] - \right.$$

$$2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] - \right.$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m \tan\left[\frac{1}{2}(e+fx)\right] \\
& \left(-2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad (3+m) \left(-\frac{1}{3+m} (1+m) (1+m-n) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, n, 2+m-n, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \quad \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3+m} (1+m) n \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1+n, 1+m-n, 1+ \right. \right. \\
& \quad \left. \left. \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \quad \left((1+m-n) \left(-\frac{1}{5+m} (3+m) (2+m-n) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, n, 3+m-n, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+m} (3+m) n \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1+n, 2+m-n, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - n \left(-\frac{1}{5+m} (3+m) (1+m-n) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1+n, 2+m-n, \right. \right. \\
& \quad \left. \left. 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+m} (3+m) (1+n) \operatorname{AppellF1}\left[\right. \right. \\
& \quad \left. \left. 1+\frac{3+m}{2}, 2+n, 1+m-n, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) \Bigg) / \\
& \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - n \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right) + \right. \\
& \left(2 (3+m) n \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \right. \\
& \quad \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{-1+n} \sin[e+fx]^m \tan\left[\frac{1}{2}(e+fx)\right] \\
& \quad \left. \left(-\cos\left[\frac{1}{2}(e+fx)\right] \sec[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \tan[e+fx] \right) \right) \Bigg) \Bigg) / \\
& \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right.
\end{aligned}$$

$$2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \\ \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \Bigg) \Bigg)$$

■ **Problem 489: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sec[e+fx]^n (a \sin[e+fx])^m dx$$

Optimal (type 5, 89 leaves, 2 steps):

$$-\frac{1}{f(1-n)} a \operatorname{Hypergeometric2F1} \left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[e+fx]^2 \right] \sec[e+fx]^{-1+n} (a \sin[e+fx])^{-1+m} (\sin[e+fx]^2)^{\frac{1-m}{2}}$$

Result (type 6, 2946 leaves):

$$\left(2(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \\ \left. \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \sec[e+fx]^n \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m (a \sin[e+fx])^m \tan \left[\frac{1}{2} (e+fx) \right] \right) / \\ \left(f(1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\ \left. \left. 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \right. \\ \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \\ \left(\left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^n \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \right)^n \right. \right. \\ \left. \left. \sin[e+fx]^m \right) / \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \right. \\ \left. \left. 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \right. \\ \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) + \\ \left(2m(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \cos[e+fx] \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \right. \\ \left. \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \right)^n \sin[e+fx]^{-1+m} \tan \left[\frac{1}{2} (e+fx) \right] \right) / \\ \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right.$$

$$\begin{aligned}
& 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \\
& \quad \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \Bigg) + \\
& \left(2 (3+m) (-1+n) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) / \\
& \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) + \\
& \left(2 (3+m) \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m \tan \left[\frac{1}{2} (e+fx) \right] \left(-1 / (3+m) (1+m) (1+m-n) \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad \left. \left. 1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + 1 / (3+m) (1+m) n \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) \right) / \\
& \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) - \\
& \left(2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m \tan \left[\frac{1}{2} (e+fx) \right] \right. \\
& \quad \left. \left(-2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \right. \\
& \quad \left. (3+m) \left(-\frac{1}{3+m} (1+m) (1+m-n) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3+m}(1+m)n \text{AppellF1}\left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - 2 \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left((1+m-n) \left(-\frac{1}{5+m}(3+m)(2+m-n) \text{AppellF1}\left[1 + \frac{3+m}{2}, n, 3+m-n, 1 + \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+m}(3+m)n \text{AppellF1}\left[1 + \frac{3+m}{2}, 1+n, 2+m-n, 1 + \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - n \left(-\frac{1}{5+m}(3+m)(1+m-n) \text{AppellF1}\left[1 + \frac{3+m}{2}, 1+n, 2+m-n, \right. \right. \\
& \left. \left. 1 + \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+m}(3+m)(1+n) \text{AppellF1}\left[\right. \right. \\
& \left. \left. 1 + \frac{3+m}{2}, 2+n, 1+m-n, 1 + \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left((1+m-n) \text{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - n \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) + \\
& \left(2(3+m)n \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \right. \\
& \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \right)^{-1+n} \text{Sin}[e+fx]^m \text{Tan}\left[\frac{1}{2}(e+fx)\right] \\
& \left. \left(-\text{Cos}\left[\frac{1}{2}(e+fx)\right] \text{Sec}[e+fx] \text{Sin}\left[\frac{1}{2}(e+fx)\right] + \text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \text{Tan}[e+fx] \right) \right) \Bigg) \Bigg) / \\
& \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left((1+m-n) \text{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \right. \\
& \left. \left. \left. n \text{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 490: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (b \text{Sec}[e+fx])^n \text{Sin}[e+fx]^m dx$$

Optimal (type 5, 89 leaves, 2 steps):

$$-\frac{1}{f(1-n)} b \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[e+fx]^2\right] (b \operatorname{Sec}[e+fx])^{-1+n} \operatorname{Sin}[e+fx]^{-1+m} (\operatorname{Sin}[e+fx]^2)^{\frac{1-m}{2}}$$

Result (type 6, 2940 leaves):

$$\begin{aligned} & \left(2(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\ & \quad \left. \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} (b \operatorname{Sec}[e+fx])^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^n \operatorname{Sin}[e+fx]^{2m} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\ & \left(f(1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \right. \\ & \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right. \\ & \quad \left. \left(\left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^n \right. \right. \\ & \quad \left. \left. \operatorname{Sin}[e+fx]^m \right) \right) / \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \right. \\ & \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\ & \left(2m(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cos}[e+fx] \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \right. \\ & \quad \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^n \operatorname{Sin}[e+fx]^{-1+m} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\ & \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \right. \\ & \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\ & \left(2(3+m)(-1+n) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \right. \\ & \quad \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^n \operatorname{Sin}[e+fx]^m \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) / \end{aligned}$$

$$\begin{aligned}
& \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) + \\
& \left(2 (3+m) \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec [e+fx] \right)^n \sin [e+fx]^m \tan \left[\frac{1}{2} (e+fx) \right] \left(-1 / (3+m) (1+m) (1+m-n) \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad \left. \left. \left. 1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + 1 / (3+m) (1+m) n \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) / \\
& \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) - \\
& \left(2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec [e+fx] \right)^n \sin [e+fx]^m \tan \left[\frac{1}{2} (e+fx) \right] \right) \\
& \left(-2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \right. \right. \\
& \quad \left. \left. (3+m) \left(-\frac{1}{3+m} (1+m) (1+m-n) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \right. \right. \\
& \quad \left. \left. \left. \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \frac{1}{3+m} (1+m) n \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) - 2 \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \\
& \left((1+m-n) \left(-\frac{1}{5+m} (3+m) (2+m-n) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, n, 3+m-n, 1 + \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \frac{1}{5+m} (3+m) n \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1+n, 2+m-n, 1 + \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - n \left(-\frac{1}{5+m} (3+m)(1+m-n) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1+n, 2+m-n, \right. \right. \\
& \left. \left. 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+m} (3+m)(1+n) \operatorname{AppellF1}\left[\right. \right. \\
& \left. \left. 1+\frac{3+m}{2}, 2+n, 1+m-n, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big/ \\
& \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - n \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left(2(3+m)n \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \right. \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{-1+n} \sin[e+fx]^m \tan\left[\frac{1}{2}(e+fx)\right] \\
& \left. \left. \left. \left(-\cos\left[\frac{1}{2}(e+fx)\right] \sec[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \tan[e+fx] \right) \right) \right) \Big/ \\
& \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left((1+m-n) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \right. \right. \\
& \left. \left. \left. n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \Big/
\end{aligned}$$

■ **Problem 491: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (b \sec[e+fx])^n (a \sin[e+fx])^m dx$$

Optimal (type 5, 92 leaves, 2 steps):

$$-\frac{1}{f(1-n)} a b \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[e+fx]^2\right] (b \sec[e+fx])^{-1+n} (a \sin[e+fx])^{-1+m} (\sin[e+fx]^2)^{\frac{1-m}{2}}$$

Result (type 6, 2948 leaves):

$$\begin{aligned}
& \left(2(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \\
& \left. \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} (b \sec[e+fx])^n \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m (a \sin[e+fx])^m \tan\left[\frac{1}{2}(e+fx)\right] \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left. 1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + 1 / (3+m)(1+m)n \\
& \text{AppellF1}\left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \Bigg/ \\
& \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \right. \\
& \quad 2 \left((1+m-n) \text{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \\
& \quad \quad \left. \left. n \text{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(2(3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^n \sin[e+fx]^m \tan\left[\frac{1}{2}(e+fx)\right] \right) \\
& \left(-2 \left((1+m-n) \text{AppellF1}\left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. n \text{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad (3+m) \left(-\frac{1}{3+m} (1+m)(1+m-n) \text{AppellF1}\left[1 + \frac{1+m}{2}, n, 2+m-n, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \\
& \quad \quad \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3+m} (1+m)n \text{AppellF1}\left[1 + \frac{1+m}{2}, 1+n, 1+m-n, 1 + \right. \right. \\
& \quad \quad \left. \left. \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \quad \left((1+m-n) \left(-\frac{1}{5+m} (3+m)(2+m-n) \text{AppellF1}\left[1 + \frac{3+m}{2}, n, 3+m-n, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \right. \\
& \quad \quad \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+m} (3+m)n \text{AppellF1}\left[1 + \frac{3+m}{2}, 1+n, 2+m-n, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \quad \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - n \left(-\frac{1}{5+m} (3+m)(1+m-n) \text{AppellF1}\left[1 + \frac{3+m}{2}, 1+n, 2+m-n, \right. \right. \\
& \quad \quad \left. \left. 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+m} (3+m)(1+n) \text{AppellF1}\left[\right. \right. \\
& \quad \quad \left. \left. 1 + \frac{3+m}{2}, 2+n, 1+m-n, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg/ \\
& \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - n \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \Big)^2 + \\
& \left(2 (3+m) n \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \right. \\
& \quad \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \right)^{-1+n} \sin[e+fx]^m \tan \left[\frac{1}{2} (e+fx) \right] \right. \\
& \quad \left. \left(-\cos \left[\frac{1}{2} (e+fx) \right] \sec[e+fx] \sin \left[\frac{1}{2} (e+fx) \right] + \cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \tan[e+fx] \right) \right) \Big) / \\
& \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m-n, \frac{3+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+m-n) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m-n, \frac{5+m}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \Big) \Big)
\end{aligned}$$

- **Problem 495: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \csc[e+fx] (b \sec[e+fx])^n dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$-\frac{\operatorname{Hypergeometric2F1} \left[1, \frac{1+n}{2}, \frac{3+n}{2}, \sec[e+fx]^2 \right] (b \sec[e+fx])^{1+n}}{b f (1+n)}$$

Result (type 6, 2658 leaves):

$$\begin{aligned}
& \left((-2+n) \operatorname{AppellF1} \left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2, \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \\
& \quad \left. \cot \left[\frac{1}{2} (e+fx) \right]^2 \csc[e+fx] \sec[e+fx]^{-1+n} (b \sec[e+fx])^n \right) / \\
& \left(f (-1+n) \left(2 (-2+n) \operatorname{AppellF1} \left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2, \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2 \right] \cos \left[\frac{1}{2} (e+fx) \right]^2 + \right. \right. \\
& \quad \left(n \operatorname{AppellF1} \left[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2, \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \\
& \quad \left. \left. 2 \operatorname{AppellF1} \left[2-n, -n, 2, 3-n, \frac{1}{2} \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2, \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \cos[e+fx] \right) \\
& \left(- \left(\left((-2+n) \operatorname{AppellF1} \left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2, \cos[e+fx] \sec \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]^{-1+n}\right) / \\
& \left((-1+n) \left(2(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right. \\
& \quad \left. \left(n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \cos[e+fx] \right) \right) + \\
& \left((-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \quad \left. \cot\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]^n \sin[e+fx] \right) / \\
& \left(2(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \quad \left(n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \\
& \quad \left. 2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \cos[e+fx] \right) + \\
& \left((-2+n) \cot\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]^{-1+n} \left(-\frac{1}{2-n} (1-n) n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \left(-\frac{1}{2} \sec\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right. \\
& \quad \left. \frac{1}{2-n} (1-n) \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \quad \left. \left(-\sec\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \\
& \left((-1+n) \left(2(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right. \\
& \quad \left. \left(n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \cos[e+fx] \right) \right) - \\
& \left((-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]^{-1+n} \right. \\
& \quad \left. (-2(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \sin\left[\frac{1}{2}(e+fx)\right] - \left(n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \\
& \quad \left. 2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sin[e+fx] + \\
& 2(-2+n) \cos\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{1}{2-n} (1-n) n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(-\frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \frac{1}{2} \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& \quad \left. \frac{1}{2-n} (1-n) \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \quad \left. \left(-\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& \cos[e+fx] \left(n \left(\frac{1}{3-n} (1-n) (2-n) \operatorname{AppellF1}\left[3-n, 2-n, 1, 4-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left(-\frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \frac{1}{2} \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + \frac{1}{3-n} (2-n) \operatorname{AppellF1}\left[3-n, 1-n, \right. \\
& \quad \left. 2, 4-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(-\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \cos[e+fx] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) - 2 \left(-\frac{1}{3-n} (2-n) n \operatorname{AppellF1}\left[3-n, 1-n, 2, 4-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(-\frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \frac{1}{2} \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& \quad \left. \frac{1}{3-n} 2(2-n) \operatorname{AppellF1}\left[3-n, -n, 3, 4-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \quad \left. \left(-\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \Big/ \\
& \left((-1+n) \left(2(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right. \\
& \quad \left. \left(n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2}\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2, \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \cos[e+fx] \right) \right) \Big)
\end{aligned}$$

■ **Problem 496: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \csc[e+fx]^3 (b \operatorname{Sec}[e+fx])^n dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}\left[2, \frac{3+n}{2}, \frac{5+n}{2}, \text{Sec}[e+f x]^2\right] (b \text{Sec}[e+f x])^{3+n}}{b^3 f (3+n)}$$

Result (type 6, 5198 leaves):

$$\begin{aligned} & \left(\text{Csc}[e+f x]^3 (b \text{Sec}[e+f x])^n \left(\frac{1 + \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \right)^n \right. \\ & \left(-\text{AppellF1}\left[1, n, -n, 2, \text{Cot}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Cot}\left[\frac{1}{2}(e+f x)\right]^2\right] / \left(n \left(\text{AppellF1}\left[2, n, 1-n, 3, \text{Cot}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Cot}\left[\frac{1}{2}(e+f x)\right]^2\right] \right)^2 + \right. \right. \\ & \quad \left. \left. \text{AppellF1}\left[2, 1+n, -n, 3, \text{Cot}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Cot}\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \right) + \\ & \quad \left. 2 \text{AppellF1}\left[1, n, -n, 2, \text{Cot}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Cot}\left[\frac{1}{2}(e+f x)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right) + \\ & \left(\text{AppellF1}\left[1, n, -n, 2, \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right) / \\ & \left(2 \text{AppellF1}\left[1, n, -n, 2, \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + n \left(\text{AppellF1}\left[2, n, 1-n, 3, \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right)^2 + \right. \\ & \quad \left. \text{AppellF1}\left[2, 1+n, -n, 3, \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right) + \\ & \left(2(-2+n) \text{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right), 1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right] \text{Cot}\left[\frac{1}{2}(e+f x)\right]^2 \left(-1 + \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right) \right) / \\ & \left((-1+n) \left(-2(-2+n) \text{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right), 1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right] + \right. \right. \\ & \quad \left. \left(n \text{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right), 1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right] - \right. \right. \\ & \quad \left. \left. 2 \text{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right), 1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right] \right) \left(-1 + \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right) \right) \right) / \\ & \left(4 f \left(\frac{1}{4} n \left(\frac{1 + \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \right)^{-1+n} \left(\frac{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \text{Tan}\left[\frac{1}{2}(e+f x)\right]}{1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2} + \frac{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \text{Tan}\left[\frac{1}{2}(e+f x)\right] \left(1 + \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right)}{\left(1 - \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right)^2} \right) \right. \right. \\ & \quad \left. \left(-\text{AppellF1}\left[1, n, -n, 2, \text{Cot}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Cot}\left[\frac{1}{2}(e+f x)\right]^2\right] / \right. \right. \\ & \quad \left. \left(n \left(\text{AppellF1}\left[2, n, 1-n, 3, \text{Cot}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Cot}\left[\frac{1}{2}(e+f x)\right]^2\right] + \text{AppellF1}\left[2, 1+n, -n, 3, \text{Cot}\left[\frac{1}{2}(e+f x)\right]^2, \right. \right. \right. \right. \\ & \quad \left. \left. \left. -\text{Cot}\left[\frac{1}{2}(e+f x)\right]^2 \right) \right) + 2 \text{AppellF1}\left[1, n, -n, 2, \text{Cot}\left[\frac{1}{2}(e+f x)\right]^2, -\text{Cot}\left[\frac{1}{2}(e+f x)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right) + \right. \end{aligned}$$

$$\begin{aligned}
& \left(\text{AppellF1}\left[1, n, -n, 2, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) / \\
& \left(2 \text{AppellF1}\left[1, n, -n, 2, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \left(\text{AppellF1}\left[2, n, 1-n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[2, 1+n, -n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(2(-2+n) \text{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), 1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right]^2 \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) / \\
& \left((-1+n) \left(-2(-2+n) \text{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), 1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left(n \text{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), 1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \text{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2}\left(1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right), 1-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) \right) + \\
& \frac{1}{4} \left(\frac{1+\tan\left[\frac{1}{2}(e+fx)\right]^2}{1-\tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^n \left(-\left(-\frac{1}{2} n \text{AppellF1}\left[2, n, 1-n, 3, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 - \right. \right. \\
& \quad \left. \frac{1}{2} n \text{AppellF1}\left[2, 1+n, -n, 3, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 \right) / \\
& \left(n \left(\text{AppellF1}\left[2, n, 1-n, 3, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[2, 1+n, -n, 3, \cot\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \right) + 2 \text{AppellF1}\left[1, n, -n, 2, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(\text{AppellF1}\left[1, n, -n, 2, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) / \\
& \left(2 \text{AppellF1}\left[1, n, -n, 2, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \left(\text{AppellF1}\left[2, n, 1-n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[2, 1+n, -n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(\tan\left[\frac{1}{2}(e+fx)\right]^2 \left(\frac{1}{2} n \text{AppellF1}\left[2, n, 1-n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \frac{1}{2} n \text{AppellF1}\left[2, 1+n, -n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\
& \left(2 \text{AppellF1}\left[1, n, -n, 2, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \left(\text{AppellF1}\left[2, n, 1-n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 + \text{AppellF1}\left[2, 1+n, -n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 + \\
& \left(\text{AppellF1}\left[1, n, -n, 2, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \left(n \left(\frac{2}{3}(1-n) \text{AppellF1}\left[3, n, 2-n, 4, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 - \right. \right. \right. \\
& \quad \left. \frac{4}{3}n \text{AppellF1}\left[3, 1+n, 1-n, 4, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 - \right. \\
& \quad \left. \frac{2}{3}(1+n) \text{AppellF1}\left[3, 2+n, -n, 4, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& 2 \text{AppellF1}\left[1, n, -n, 2, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \\
& 2 \left(-\frac{1}{2}n \text{AppellF1}\left[2, n, 1-n, 3, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 - \frac{1}{2}n \right. \\
& \quad \left. \text{AppellF1}\left[2, 1+n, -n, 3, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \cot\left[\frac{1}{2}(e+fx)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big/ \\
& \left(n \left(\text{AppellF1}\left[2, n, 1-n, 3, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[2, 1+n, -n, 3, \cot\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \right) + 2 \text{AppellF1}\left[1, n, -n, 2, \cot\left[\frac{1}{2}(e+fx)\right]^2, -\cot\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left(\text{AppellF1}\left[1, n, -n, 2, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(n \left(\text{AppellF1}\left[2, n, 1-n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[2, 1+n, -n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& 2 \left(\frac{1}{2}n \text{AppellF1}\left[2, n, 1-n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{1}{2}n \text{AppellF1}\left[2, 1+n, -n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + n \tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left(-\frac{2}{3}(1-n) \text{AppellF1}\left[3, n, 2-n, 4, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{4}{3}n \text{AppellF1}\left[3, 1+n, 1-n, 4, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{2}{3}(1+n) \text{AppellF1}\left[3, 2+n, -n, 4, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \Big/ \\
& \left(2 \text{AppellF1}\left[1, n, -n, 2, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \left(\text{AppellF1}\left[2, n, 1-n, 3, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 - \\
& \left(-\frac{1}{3}(7-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-7+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2\right) - \\
& \left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^3 \right. \\
& \left. \left(2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. 3 \left(-\frac{1}{3}(4-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \left. \left. \frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) + 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left. \left((-4+n) \left(-\frac{3}{5}(5-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 6-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \right. \\
& \left. \left. \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) + \right. \\
& \left. n \left(-\frac{3}{5}(4-n) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \frac{3}{5}(1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 4-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) \right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2\right) + \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right. \\
& \left. \left(2 \left((-5+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 6-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \left((-6+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 7-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 6-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right)^2 + \\
& \left(\operatorname{AppellF1} \left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \left(2 \left((-7+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \\
& \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] + 3 \left(-\frac{1}{3} (7-n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \\
& \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] + \frac{1}{3} n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \right. \\
& \quad \left. -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] \right) + 2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \\
& \left((-7+n) \left(-\frac{3}{5} (8-n) \operatorname{AppellF1} \left[\frac{5}{2}, n, 9-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] + \right. \right. \\
& \quad \left. \frac{3}{5} n \operatorname{AppellF1} \left[\frac{5}{2}, 1+n, 8-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] \right) + \\
& \quad n \left(-\frac{3}{5} (7-n) \operatorname{AppellF1} \left[\frac{5}{2}, 1+n, 8-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] + \right. \\
& \quad \left. \frac{3}{5} (1+n) \operatorname{AppellF1} \left[\frac{5}{2}, 2+n, 7-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] \right) \right) \right) \Bigg) \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 7-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \left((-7+n) \operatorname{AppellF1} \left[\frac{3}{2}, n, 8-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] + n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 7-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 498: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (b \operatorname{Sec}[e+f x])^n \operatorname{Sin}[e+f x]^4 dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$\frac{b \operatorname{Hypergeometric2F1} \left[-\frac{3}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[e+f x]^2 \right] (b \operatorname{Sec}[e+f x])^{-1+n} \operatorname{Sin}[e+f x]}{f (1-n) \sqrt{\operatorname{Sin}[e+f x]^2}}$$

Result (type 6, 6231 leaves):

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big)^2 + \\
& \left(2 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)\right. \\
& \left.2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. 3 \left(-\frac{1}{3}(4-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left. \left((-4+n) \left(-\frac{3}{5}(5-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 6-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \right. \\
& \left. \left. \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + \right. \\
& \left. n \left(-\frac{3}{5}(4-n) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \frac{3}{5}(1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 4-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big) \Big) \Big) \Big) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big)^2 - \\
& \left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(2 \left((-5+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \right. \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right. \\
& \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + 3 \left(-\frac{1}{3}(5-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left. \left((-5+n) \left(-\frac{3}{5}(6-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 7-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned} & \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 6-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \\ & n \left(-\frac{3}{5} (5-n) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 6-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\ & \left. \frac{3}{5} (1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\ & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 5-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-5+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 6-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\ & \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 5-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg) \end{aligned}$$

■ **Problem 499: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (b \operatorname{Sec}[e+fx])^n \operatorname{Sin}[e+fx]^2 dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$\frac{b \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[e+fx]^2\right] (b \operatorname{Sec}[e+fx])^{-1+n} \operatorname{Sin}[e+fx]}{f(1-n) \sqrt{\operatorname{Sin}[e+fx]^2}}$$

Result (type 6, 4143 leaves):

$$\begin{aligned} & \left(24 \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-3+n} (b \operatorname{Sec}[e+fx])^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^n \operatorname{Sin}[e+fx]^2 \right. \\ & \left. \tan\left[\frac{1}{2}(e+fx)\right] \left(\left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 2-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) / \right. \right. \\ & \left. \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 2-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-2+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \right. \\ & \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) - \right. \\ & \left. \operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \left. \left. 2 \left((-3+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \right. \\ & \left. \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\ & \left(f \left(12 \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-2+n} \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^n \left(\left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 2-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right. \right. \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. \left(\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + 3 \left(-\frac{1}{3}(3-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \Big) + 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left((-3+n) \left(-\frac{3}{5}(4-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 5-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 4-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + \right. \\
& \left. n \left(-\frac{3}{5}(3-n) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 4-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \frac{3}{5}(1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 3-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big) \Big) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-3+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& 24 n \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-3+n} \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{-1+n} \tan\left[\frac{1}{2}(e+fx)\right] \\
& \left(\left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 2-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) / \right. \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 2-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-2+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 3-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-3+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 4-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 3-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big) \\
& \left(-\cos\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \tan[e+fx] \right) \Big) \Big) \Big)
\end{aligned}$$

Problem 501: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Csc}[e + f x]^2 (b \text{Sec}[e + f x])^n dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$\frac{b \text{Csc}[e + f x] \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \text{Cos}[e + f x]^2\right] (b \text{Sec}[e + f x])^{-1+n} \sqrt{\text{Sin}[e + f x]^2}}{f (1-n)}$$

Result (type 6, 3228 leaves):

$$\begin{aligned} & \left(\text{Cot}\left[\frac{1}{2}(e + f x)\right] \text{Csc}[e + f x]^2 \text{Sec}[e + f x]^n (b \text{Sec}[e + f x])^n \right. \\ & \left(-\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] / \left(\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \right. \\ & \quad 2n \left(\text{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \\ & \quad \left. \left. \text{AppellF1}\left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) + \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) / \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 2n \left(\text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \\ & \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \left. \right) / \\ & \left(2f \left(-\frac{1}{4} \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x]^n \left(-\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] / \right. \right. \right. \\ & \quad \left(\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 2n \left(\text{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \\ & \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \text{AppellF1}\left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) + \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) / \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 2n \left(\text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \\ & \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \left. \right) + \\ & \left. \frac{1}{2} n \text{Cot}\left[\frac{1}{2}(e + f x)\right] \text{Sec}[e + f x]^{1+n} \text{Sin}[e + f x] \left(-\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] / \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + 2n \left(\text{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \text{AppellF1}\left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + 2n \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left(-\frac{1}{3}(1-n) \text{AppellF1}\left[\frac{3}{2}, n, 2-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \frac{2}{3}n \text{AppellF1}\left[\frac{3}{2}, 1+n, 1-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \quad \left. \frac{1}{3}(1+n) \text{AppellF1}\left[\frac{3}{2}, 2+n, -n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Bigg) \Bigg) / \\
& \left(\text{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2n \left(\text{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 - \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(2n \left(\text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \text{Tan}\left[\frac{1}{2}(e+fx)\right] + 3 \left(\frac{1}{3}n \text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \left. \frac{1}{3}n \text{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + \right. \\
& \quad \left. 2n \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{3}{5}(1-n) \text{AppellF1}\left[\frac{5}{2}, n, 2-n, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \left. \frac{6}{5}n \text{AppellF1}\left[\frac{5}{2}, 1+n, 1-n, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5}(1+n) \text{AppellF1}\left[\frac{5}{2}, 2+n, -n, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) \Bigg) / \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2n \left(\text{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 502: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e+fx]^4 (b \text{Sec}[e+fx])^n dx$$

Optimal (type 5, 73 leaves, 2 steps):

$$\frac{b \operatorname{Csc}[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[e + f x]^2\right] (b \operatorname{Sec}[e + f x])^{-1+n} \sqrt{\operatorname{Sin}[e + f x]^2}}{f (1-n)}$$

Result (type 6, 6799 leaves):

$$\left(\operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^3 \operatorname{Csc}[e + f x]^4 (b \operatorname{Sec}[e + f x])^n \left(\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \right)^n \right. \\ \left(-\operatorname{AppellF1}\left[-\frac{3}{2}, n, -n, -\frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] / \left(\operatorname{AppellF1}\left[-\frac{3}{2}, n, -n, -\frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - \right. \right. \\ \left. \left. 2n \left(\operatorname{AppellF1}\left[-\frac{1}{2}, n, 1-n, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) + \right. \right. \\ \left. \left. \operatorname{AppellF1}\left[-\frac{1}{2}, 1+n, -n, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) - \\ \left(9 \operatorname{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) / \\ \left(\operatorname{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 2n \left(\operatorname{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \\ \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \operatorname{AppellF1}\left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) + \\ \left(27 \operatorname{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^4 \right) / \\ \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 2n \left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \\ \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) + \\ \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, n, -n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^6 \right) / \\ \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, n, -n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 2n \left(\operatorname{AppellF1}\left[\frac{5}{2}, n, 1-n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \\ \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, -n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \left. \right) / \\ \left(24 f \left(-\frac{1}{16} \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2 \left(\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \right)^n \left(-\operatorname{AppellF1}\left[-\frac{3}{2}, n, -n, -\frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) / \right. \right.$$

$$\begin{aligned}
& \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& 2 n \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{3}{5}(1-n) \operatorname{AppellF1}\left[\frac{5}{2}, n, 2-n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{1}{2}(e+fx)\right) + \frac{6}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 1-n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \quad \left. \frac{3}{5}(1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, -n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) \Bigg) \Bigg) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 n \left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 - \\
& \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, n, -n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^6 \left(2 n \left(\operatorname{AppellF1}\left[\frac{5}{2}, n, 1-n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, -n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + 5 \left(\frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, n, 1-n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, -n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) + \right. \\
& \quad \left. 2 n \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{5}{7}(1-n) \operatorname{AppellF1}\left[\frac{7}{2}, n, 2-n, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \left. \frac{1}{2}(e+fx)\right) + \frac{10}{7} n \operatorname{AppellF1}\left[\frac{7}{2}, 1+n, 1-n, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{5}{7}(1+n) \operatorname{AppellF1}\left[\frac{7}{2}, 2+n, -n, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) \Bigg) \Bigg) / \\
& \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, n, -n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 n \left(\operatorname{AppellF1}\left[\frac{5}{2}, n, 1-n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, -n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \Bigg) \Bigg)
\end{aligned}$$

Test results for the 348 problems in "4.1.10 (c+d x)^m (a+b sin)^n.m"

- Problem 28: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \operatorname{Csc}[a + b x]^2 dx$$

Optimal (type 4, 113 leaves, 6 steps):

$$-\frac{i(c+dx)^3}{b} - \frac{(c+dx)^3 \operatorname{Cot}[a+bx]}{b} + \frac{3d(c+dx)^2 \operatorname{Log}[1 - e^{2i(a+bx)}]}{b^2} - \frac{3id^2(c+dx) \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{b^3} + \frac{3d^3 \operatorname{PolyLog}[3, e^{2i(a+bx)}]}{2b^4}$$

Result (type 4, 384 leaves):

$$\begin{aligned} & -\frac{1}{4b^4} d^3 e^{-ia} \operatorname{Csc}[a] \\ & \frac{(2b^2 x^2 (2b e^{2ia} x + 3i(-1 + e^{2ia}) \operatorname{Log}[1 - e^{2i(a+bx)}]) + 6b(-1 + e^{2ia}) x \operatorname{PolyLog}[2, e^{2i(a+bx)}] + 3i(-1 + e^{2ia}) \operatorname{PolyLog}[3, e^{2i(a+bx)}]) + 3c^2 d \operatorname{Csc}[a] (-bx \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[bx] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[bx]]) \operatorname{Sin}[a])}{b^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\ & \frac{\operatorname{Csc}[a] \operatorname{Csc}[a+bx] (c^3 \operatorname{Sin}[bx] + 3c^2 dx \operatorname{Sin}[bx] + 3cd^2 x^2 \operatorname{Sin}[bx] + d^3 x^3 \operatorname{Sin}[bx])}{b} - \\ & \left(3cd^2 \operatorname{Csc}[a] \operatorname{Sec}[a] \left(b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + 1 \right) / \left(\sqrt{1 + \operatorname{Tan}[a]^2} \right) (i bx (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2ibx}] - \right. \\ & \quad \left. 2(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}] + \pi \operatorname{Log}[\operatorname{Cos}[bx]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + \right. \\ & \quad \left. i \operatorname{PolyLog}[2, e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \right) \operatorname{Tan}[a] \right) / \left(b^3 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) \end{aligned}$$

■ **Problem 29: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 \operatorname{Csc}[a + b x]^2 dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$-\frac{i(c+dx)^2}{b} - \frac{(c+dx)^2 \operatorname{Cot}[a+bx]}{b} + \frac{2d(c+dx) \operatorname{Log}[1 - e^{2i(a+bx)}]}{b^2} - \frac{id^2 \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{b^3}$$

Result (type 4, 245 leaves):

$$\begin{aligned} & \frac{2cd \operatorname{Csc}[a] (-bx \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[bx] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[bx]]) \operatorname{Sin}[a]}{b^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \frac{\operatorname{Csc}[a] \operatorname{Csc}[a+bx] (c^2 \operatorname{Sin}[bx] + 2cdx \operatorname{Sin}[bx] + d^2 x^2 \operatorname{Sin}[bx])}{b} - \\ & \left(d^2 \operatorname{Csc}[a] \operatorname{Sec}[a] \left(b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + 1 \right) / \left(\sqrt{1 + \operatorname{Tan}[a]^2} \right) (i bx (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2ibx}] - \right. \\ & \quad \left. 2(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}] + \pi \operatorname{Log}[\operatorname{Cos}[bx]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + \right. \\ & \quad \left. i \operatorname{PolyLog}[2, e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \right) \operatorname{Tan}[a] \right) / \left(b^3 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) \end{aligned}$$

■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 \operatorname{Csc}[a + b x]^3 dx$$

Optimal (type 4, 180 leaves, 9 steps) :

$$\begin{aligned} & - \frac{(c+dx)^2 \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} - \frac{d^2 \operatorname{ArcTanh}[\operatorname{Cos}[a+bx]]}{b^3} - \frac{d(c+dx) \operatorname{Csc}[a+bx]}{b^2} - \frac{(c+dx)^2 \operatorname{Cot}[a+bx] \operatorname{Csc}[a+bx]}{2b} + \\ & \frac{id(c+dx) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{b^2} - \frac{id(c+dx) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{b^2} - \frac{d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right]}{b^3} + \frac{d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right]}{b^3} \end{aligned}$$

Result (type 4, 471 leaves) :

$$\begin{aligned} & - \frac{d(c+dx) \operatorname{Csc}[a]}{b^2} + \frac{(-c^2 - 2cdx - d^2x^2) \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} + \\ & \frac{1}{2b^3} \left(b^2 c^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 2d^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 2b^2 cdx \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + b^2 d^2 x^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - b^2 c^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - \right. \\ & \quad \left. 2d^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 2b^2 cdx \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - b^2 d^2 x^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + 2ibd(c+dx) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] - \right. \\ & \quad \left. 2ibd(c+dx) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right] - 2d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] + 2d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] \right) + \\ & \frac{(c^2 + 2cdx + d^2x^2) \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} + \frac{\operatorname{Sec}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right] (-cd \operatorname{Sin}\left[\frac{bx}{2}\right] - d^2x \operatorname{Sin}\left[\frac{bx}{2}\right])}{2b^2} + \\ & \frac{\operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right] (cd \operatorname{Sin}\left[\frac{bx}{2}\right] + d^2x \operatorname{Sin}\left[\frac{bx}{2}\right])}{2b^2} \end{aligned}$$

■ **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int (c+dx) \operatorname{Csc}[a+bx]^3 dx$$

Optimal (type 4, 109 leaves, 6 steps) :

$$\begin{aligned} & - \frac{(c+dx) \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} - \frac{d \operatorname{Csc}[a+bx]}{2b^2} - \frac{(c+dx) \operatorname{Cot}[a+bx] \operatorname{Csc}[a+bx]}{2b} + \frac{id \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{2b^2} - \frac{id \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{2b^2} \end{aligned}$$

Result (type 4, 292 leaves) :

$$\begin{aligned} & - \frac{dx \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} - \frac{c \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{8b} - \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} + \frac{c \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} + \frac{1}{2b^2} \\ & d \left((a+bx) \left(\operatorname{Log}\left[1 - e^{i(a+bx)}\right] - \operatorname{Log}\left[1 + e^{i(a+bx)}\right] \right) - a \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] + i \left(\operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] - \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right] \right) \right) + \\ & \frac{dx \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} + \frac{c \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{8b} + \frac{d \operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right] \operatorname{Sin}\left[\frac{bx}{2}\right]}{4b^2} - \frac{d \operatorname{Sec}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right] \operatorname{Sin}\left[\frac{bx}{2}\right]}{4b^2} \end{aligned}$$

■ **Problem 52: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[a+bx]^2}{(c+dx)^{9/2}} dx$$

Optimal (type 4, 247 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{16 b^2}{105 d^3 (c + d x)^{3/2}} - \frac{128 b^{7/2} \sqrt{\pi} \cos\left[2 a - \frac{2 b c}{d}\right] \operatorname{FresnelC}\left[\frac{2 \sqrt{b} \sqrt{c+d x}}{\sqrt{d} \sqrt{\pi}}\right]}{105 d^{9/2}} + \frac{128 b^{7/2} \sqrt{\pi} \operatorname{FresnelS}\left[\frac{2 \sqrt{b} \sqrt{c+d x}}{\sqrt{d} \sqrt{\pi}}\right] \sin\left[2 a - \frac{2 b c}{d}\right]}{105 d^{9/2}} \\
 & - \frac{8 b \cos[a + b x] \sin[a + b x]}{35 d^2 (c + d x)^{5/2}} + \frac{128 b^3 \cos[a + b x] \sin[a + b x]}{105 d^4 \sqrt{c + d x}} - \frac{2 \sin[a + b x]^2}{7 d (c + d x)^{7/2}} + \frac{32 b^2 \sin[a + b x]^2}{105 d^3 (c + d x)^{3/2}}
 \end{aligned}$$

Result (type 4, 988 leaves):

■ **Problem 59: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin[a + bx]^3}{(c + dx)^{7/2}} dx$$

Optimal (type 4, 356 leaves, 19 steps):

$$\begin{aligned} & - \frac{2 b^{5/2} \sqrt{2 \pi} \operatorname{Cos}\left[a - \frac{bc}{d}\right] \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right]}{5 d^{7/2}} + \frac{6 b^{5/2} \sqrt{6 \pi} \operatorname{Cos}\left[3 a - \frac{3bc}{d}\right] \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right]}{5 d^{7/2}} \\ & \frac{6 b^{5/2} \sqrt{6 \pi} \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right] \operatorname{Sin}\left[3 a - \frac{3bc}{d}\right]}{5 d^{7/2}} + \frac{2 b^{5/2} \sqrt{2 \pi} \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right] \operatorname{Sin}\left[a - \frac{bc}{d}\right]}{5 d^{7/2}} \\ & \frac{16 b^2 \operatorname{Sin}[a + bx]}{5 d^3 \sqrt{c + dx}} - \frac{4 b \operatorname{Cos}[a + bx] \operatorname{Sin}[a + bx]^2}{5 d^2 (c + dx)^{3/2}} - \frac{2 \operatorname{Sin}[a + bx]^3}{5 d (c + dx)^{5/2}} + \frac{24 b^2 \operatorname{Sin}[a + bx]^3}{5 d^3 \sqrt{c + dx}} \end{aligned}$$

Result (type 4, 1429 leaves):

$$\begin{aligned} & \frac{3}{4} \left(\operatorname{Cos}[a] \left(\frac{1}{5 d} 2 \left(\frac{b}{d}\right)^{5/2} \operatorname{Sin}\left[\frac{bc}{d}\right] \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} - \frac{2}{3} \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} + \sqrt{2 \pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} \right) - \frac{1}{5 d} \right. \right. \\ & \left. \left. 2 \left(\frac{b}{d}\right)^{5/2} \operatorname{Cos}\left[\frac{bc}{d}\right] \left(\frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} + \frac{2}{3} \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} - 2 \left(-\sqrt{2 \pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} \right) \right) \right) \right) + \right. \\ & \left. \operatorname{Sin}[a] \left(-\frac{1}{5 d} 2 \left(\frac{b}{d}\right)^{5/2} \operatorname{Cos}\left[\frac{bc}{d}\right] \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} - \frac{2}{3} \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} + \sqrt{2 \pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} \right) - \right. \right. \right. \\ & \left. \left. \frac{1}{5 d} 2 \left(\frac{b}{d}\right)^{5/2} \operatorname{Sin}\left[\frac{bc}{d}\right] \left(\frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} + \frac{2}{3} \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} - 2 \left(-\sqrt{2 \pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} \right) \right) \right) \right) \right) + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} \left(-\text{Cos}[3a] \left(\frac{1}{5d} 18\sqrt{3} \left(\frac{b}{d}\right)^{5/2} \text{Sin}\left[\frac{3bc}{d}\right] \left(\frac{\text{Cos}\left[\frac{3b(c+dx)}{d}\right]}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} - \right. \right. \right. \\
& \left. \left. \left. \frac{2}{3} \left(2 \left(\frac{\text{Cos}\left[\frac{3b(c+dx)}{d}\right]}{\sqrt{3} \sqrt{\frac{b}{d} \sqrt{c+dx}}} + \sqrt{2\pi} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + \frac{\text{Sin}\left[\frac{3b(c+dx)}{d}\right]}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} \right) - \frac{1}{5d} 18\sqrt{3} \left(\frac{b}{d}\right)^{5/2} \text{Cos}\left[\frac{3bc}{d}\right] \right. \right. \\
& \left. \left. \left(\frac{\text{Sin}\left[\frac{3b(c+dx)}{d}\right]}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} + \frac{2}{3} \left(\frac{\text{Cos}\left[\frac{3b(c+dx)}{d}\right]}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} - 2 \left(-\sqrt{2\pi} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \frac{\text{Sin}\left[\frac{3b(c+dx)}{d}\right]}{\sqrt{3} \sqrt{\frac{b}{d} \sqrt{c+dx}}} \right) \right) \right) \right) - \right. \\
& \left. \text{Sin}[3a] \left(-\frac{1}{5d} 18\sqrt{3} \left(\frac{b}{d}\right)^{5/2} \text{Cos}\left[\frac{3bc}{d}\right] \left(\frac{\text{Cos}\left[\frac{3b(c+dx)}{d}\right]}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} - \right. \right. \right. \\
& \left. \left. \left. \frac{2}{3} \left(2 \left(\frac{\text{Cos}\left[\frac{3b(c+dx)}{d}\right]}{\sqrt{3} \sqrt{\frac{b}{d} \sqrt{c+dx}}} + \sqrt{2\pi} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + \frac{\text{Sin}\left[\frac{3b(c+dx)}{d}\right]}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} \right) - \frac{1}{5d} 18\sqrt{3} \left(\frac{b}{d}\right)^{5/2} \text{Sin}\left[\frac{3bc}{d}\right] \right. \right. \\
& \left. \left. \left(\frac{\text{Sin}\left[\frac{3b(c+dx)}{d}\right]}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} + \frac{2}{3} \left(\frac{\text{Cos}\left[\frac{3b(c+dx)}{d}\right]}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} - 2 \left(-\sqrt{2\pi} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \frac{\text{Sin}\left[\frac{3b(c+dx)}{d}\right]}{\sqrt{3} \sqrt{\frac{b}{d} \sqrt{c+dx}}} \right) \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 68: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left(\frac{x^2}{\text{Sin}[e+fx]^{3/2}} + x^2 \sqrt{\text{Sin}[e+fx]} \right) dx$$

Optimal (type 4, 62 leaves, 3 steps):

$$-\frac{16 \text{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + fx\right), 2\right]}{f^3} - \frac{2x^2 \text{Cos}[e+fx]}{f \sqrt{\text{Sin}[e+fx]}} + \frac{8x \sqrt{\text{Sin}[e+fx]}}{f^2}$$

Result (type 5, 185 leaves) :

$$\left(8 e^{-i f x} \sqrt{2 - 2 e^{2 i (e+f x)}} \left(3 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 i (e+f x)} \right] + e^{2 i f x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, e^{2 i (e+f x)} \right] \right) \operatorname{Sec}[e] \right) /$$

$$\left(3 \sqrt{-i e^{-i (e+f x)} (-1 + e^{2 i (e+f x)})} f^3 \right) - \frac{\operatorname{Sec}[e] \left((8 + f^2 x^2) \operatorname{Cos}[f x] + (-8 + f^2 x^2) \operatorname{Cos}[2 e + f x] - 8 f x \operatorname{Cos}[e] \operatorname{Sin}[e + f x] \right)}{f^3 \sqrt{\operatorname{Sin}[e + f x]}}$$

■ **Problem 109: Result more than twice size of optimal antiderivative.**

$$\int \frac{c + d x}{a + a \operatorname{Sin}[e + f x]} dx$$

Optimal (type 3, 60 leaves, 3 steps) :

$$- \frac{(c + d x) \operatorname{Cot} \left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right]}{a f} + \frac{2 d \operatorname{Log} \left[\operatorname{Sin} \left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right] \right]}{a f^2}$$

Result (type 3, 148 leaves) :

$$\left(-d f x \operatorname{Cos} \left[e + \frac{f x}{2} \right] + 2 d \operatorname{Cos} \left[\frac{f x}{2} \right] \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right] + 2 c f \operatorname{Sin} \left[\frac{f x}{2} \right] + d f x \operatorname{Sin} \left[\frac{f x}{2} \right] + \right.$$

$$\left. 2 d \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right] \operatorname{Sin} \left[e + \frac{f x}{2} \right] \right) / \left(a f^2 \left(\operatorname{Cos} \left[\frac{e}{2} \right] + \operatorname{Sin} \left[\frac{e}{2} \right] \right) \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \right)$$

■ **Problem 112: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^3}{(a + a \operatorname{Sin}[e + f x])^2} dx$$

Optimal (type 4, 309 leaves, 10 steps) :

$$- \frac{i (c + d x)^3}{3 a^2 f} - \frac{2 d^2 (c + d x) \operatorname{Cot} \left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right]}{a^2 f^3} - \frac{(c + d x)^3 \operatorname{Cot} \left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right]}{3 a^2 f} -$$

$$\frac{d (c + d x)^2 \operatorname{Csc} \left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right]^2}{2 a^2 f^2} - \frac{(c + d x)^3 \operatorname{Cot} \left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right] \operatorname{Csc} \left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right]^2}{6 a^2 f} + \frac{2 d (c + d x)^2 \operatorname{Log} \left[1 - i e^{i (e+f x)} \right]}{a^2 f^2} +$$

$$\frac{4 d^3 \operatorname{Log} \left[\operatorname{Sin} \left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2} \right] \right]}{a^2 f^4} - \frac{4 i d^2 (c + d x) \operatorname{PolyLog} \left[2, i e^{i (e+f x)} \right]}{a^2 f^3} + \frac{4 d^3 \operatorname{PolyLog} \left[3, i e^{i (e+f x)} \right]}{a^2 f^4}$$

Result (type 4, 867 leaves) :

$$\begin{aligned}
& - \frac{1}{a^2 f^3 (\cos[e] + i (1 + \sin[e]))} 2 d (\cos[e] + i \sin[e]) \left(2 i d^2 x + i c^2 f^2 x + c d f^2 x^2 \cos[e] + \right. \\
& \quad \frac{1}{3} d^2 f^2 x^3 (\cos[e] - i \sin[e]) - i c d f^2 x^2 \sin[e] + (2 d^2 + c^2 f^2) x (\cos[e] - i \sin[e]) (1 - i \cos[e] + \sin[e]) + \frac{1}{f} \\
& \quad d^2 (2 f x + 2 \operatorname{ArcTan}[\cos[e + f x] + i \sin[e + f x]] + i \log[1 + \cos[2(e + f x)] + i \sin[2(e + f x)]) (i \cos[e] + \sin[e]) \\
& \quad (\cos[e] + i (1 + \sin[e])) + \frac{1}{2} c^2 f (2 f x + 2 \operatorname{ArcTan}[\cos[e + f x] + i \sin[e + f x]] + i \log[1 + \cos[2(e + f x)] + i \sin[2(e + f x)]]) \\
& \quad (i \cos[e] + \sin[e]) (\cos[e] + i (1 + \sin[e])) + \\
& \quad c d (f x (f x + 2 i \log[1 - i \cos[e + f x] + \sin[e + f x]]) + 2 \operatorname{PolyLog}[2, i \cos[e + f x] - \sin[e + f x]]) (i \cos[e] + \sin[e]) \\
& \quad (\cos[e] + i (1 + \sin[e])) + \frac{1}{3 f} d^2 (f^2 x^2 (f x + 3 i \log[1 - i \cos[e + f x] + \sin[e + f x]]) + 6 f x \operatorname{PolyLog}[2, i \cos[e + f x] - \sin[e + f x]]) + \\
& \quad \left. 6 i \operatorname{PolyLog}[3, i \cos[e + f x] - \sin[e + f x]]) (i \cos[e] + \sin[e]) (\cos[e] + i (1 + \sin[e])) \right) + \\
& \frac{1}{3 a^2 f^3 (\cos[\frac{e}{2}] + \sin[\frac{e}{2}]) (\cos[\frac{e}{2} + \frac{f x}{2}] + \sin[\frac{e}{2} + \frac{f x}{2}])^3} \left(-3 c^2 d f \cos\left[\frac{f x}{2}\right] - 6 c d^2 f x \cos\left[\frac{f x}{2}\right] - 3 d^3 f x^2 \cos\left[\frac{f x}{2}\right] + \right. \\
& \quad 6 c d^2 \cos\left[e + \frac{f x}{2}\right] + 6 d^3 x \cos\left[e + \frac{f x}{2}\right] - 6 c d^2 \cos\left[e + \frac{3 f x}{2}\right] - c^3 f^2 \cos\left[e + \frac{3 f x}{2}\right] - \\
& \quad 6 d^3 x \cos\left[e + \frac{3 f x}{2}\right] - 3 c^2 d f^2 x \cos\left[e + \frac{3 f x}{2}\right] - 3 c d^2 f^2 x^2 \cos\left[e + \frac{3 f x}{2}\right] - d^3 f^2 x^3 \cos\left[e + \frac{3 f x}{2}\right] + \\
& \quad 12 c d^2 \sin\left[\frac{f x}{2}\right] + 3 c^3 f^2 \sin\left[\frac{f x}{2}\right] + 12 d^3 x \sin\left[\frac{f x}{2}\right] + 9 c^2 d f^2 x \sin\left[\frac{f x}{2}\right] + 9 c d^2 f^2 x^2 \sin\left[\frac{f x}{2}\right] + \\
& \quad \left. 3 d^3 f^2 x^3 \sin\left[\frac{f x}{2}\right] - 3 c^2 d f \sin\left[e + \frac{f x}{2}\right] - 6 c d^2 f x \sin\left[e + \frac{f x}{2}\right] - 3 d^3 f x^2 \sin\left[e + \frac{f x}{2}\right] \right)
\end{aligned}$$

■ **Problem 119: Result more than twice size of optimal antiderivative.**

$$\int \frac{c + d x}{a - a \sin[e + f x]} dx$$

Optimal (type 3, 59 leaves, 3 steps):

$$\frac{2 d \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right]\right]}{a f^2} + \frac{(c + d x) \operatorname{Tan}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right]}{a f}$$

Result (type 3, 155 leaves):

$$\begin{aligned}
& \left(d f x \cos\left[e + \frac{f x}{2}\right] + 2 d \cos\left[\frac{f x}{2}\right] \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + 2 c f \sin\left[\frac{f x}{2}\right] + d f x \sin\left[\frac{f x}{2}\right] - \right. \\
& \quad \left. 2 d \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] \sin\left[e + \frac{f x}{2}\right] \right) / \left(a f^2 \left(\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \right)
\end{aligned}$$

■ **Problem 132: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^{3/2}}{x^2} dx$$

Optimal (type 4, 263 leaves, 9 steps):

$$\begin{aligned} & -\frac{3}{4} a f \operatorname{CosIntegral}\left[\frac{f x}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \operatorname{Sin}\left[\frac{1}{4}(2 e - \pi)\right] \sqrt{a + a \sin[e + f x]} + \\ & \frac{3}{4} a f \operatorname{CosIntegral}\left[\frac{3 f x}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \operatorname{Sin}\left[\frac{1}{4}(6 e + \pi)\right] \sqrt{a + a \sin[e + f x]} - \\ & \frac{2 a \operatorname{Sin}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right]^2 \sqrt{a + a \sin[e + f x]}}{x} - \frac{3}{4} a f \operatorname{Csc}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \operatorname{Sin}\left[\frac{1}{4}(2 e + \pi)\right] \sqrt{a + a \sin[e + f x]} \operatorname{SinIntegral}\left[\frac{f x}{2}\right] + \\ & \frac{3}{4} a f \operatorname{Cos}\left[\frac{1}{4}(6 e + \pi)\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \sqrt{a + a \sin[e + f x]} \operatorname{SinIntegral}\left[\frac{3 f x}{2}\right] \end{aligned}$$

Result (type 4, 226 leaves):

$$\begin{aligned} & \frac{1}{4 \sqrt{2}} \frac{i \left(-i a e^{-i(e+f x)} \left(i + e^{i(e+f x)}\right)^2\right)^{3/2}}{\left(i + e^{i(e+f x)}\right)^3 x} \\ & \left(2 - 6 i e^{i(e+f x)} - 6 e^{2 i(e+f x)} + 2 i e^{3 i(e+f x)} + 3 e^{i e + \frac{3 i f x}{2}} f x \operatorname{ExpIntegralEi}\left[-\frac{1}{2} i f x\right] + 3 i e^{2 i e + \frac{3 i f x}{2}} f x \operatorname{ExpIntegralEi}\left[\frac{i f x}{2}\right] + \right. \\ & \left. 3 i e^{\frac{3 i f x}{2}} f x \operatorname{ExpIntegralEi}\left[-\frac{3}{2} i f x\right] + 3 e^{\frac{3}{2} i(2 e+f x)} f x \operatorname{ExpIntegralEi}\left[\frac{3 i f x}{2}\right]\right) \end{aligned}$$

■ **Problem 133: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^{3/2}}{x^3} dx$$

Optimal (type 4, 332 leaves, 13 steps):

$$\begin{aligned} & -\frac{9}{16} a f^2 \operatorname{Cos}\left[\frac{3}{4}(2 e - \pi)\right] \operatorname{CosIntegral}\left[\frac{3 f x}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \sqrt{a + a \sin[e + f x]} - \\ & \frac{3}{16} a f^2 \operatorname{CosIntegral}\left[\frac{f x}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \operatorname{Sin}\left[\frac{1}{4}(2 e + \pi)\right] \sqrt{a + a \sin[e + f x]} - \frac{3 a f \operatorname{Cos}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \operatorname{Sin}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \sqrt{a + a \sin[e + f x]}}{2 x} - \\ & \frac{a \operatorname{Sin}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right]^2 \sqrt{a + a \sin[e + f x]}}{x^2} - \frac{3}{16} a f^2 \operatorname{Cos}\left[\frac{1}{4}(2 e + \pi)\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \sqrt{a + a \sin[e + f x]} \operatorname{SinIntegral}\left[\frac{f x}{2}\right] + \\ & \frac{9}{16} a f^2 \operatorname{Csc}\left[\frac{e}{2} + \frac{\pi}{4} + \frac{f x}{2}\right] \operatorname{Sin}\left[\frac{3}{4}(2 e - \pi)\right] \sqrt{a + a \sin[e + f x]} \operatorname{SinIntegral}\left[\frac{3 f x}{2}\right] \end{aligned}$$

Result (type 4, 295 leaves):

$$\begin{aligned}
& - \frac{1}{16 \sqrt{2} \left(i + e^{i(e+fx)} \right)^3 x^2} \\
& i \left(-i a e^{-i(e+fx)} \left(i + e^{i(e+fx)} \right)^2 \right)^{3/2} \left(-4 + 12 i e^{i(e+fx)} + 12 e^{2i(e+fx)} - 4 i e^{3i(e+fx)} + 6 i f x + 6 e^{i(e+fx)} f x + 6 i e^{2i(e+fx)} f x + \right. \\
& 6 e^{3i(e+fx)} f x + 3 i e^{i e + \frac{3ifx}{2}} f^2 x^2 \text{ExpIntegralEi} \left[-\frac{1}{2} i f x \right] + 3 e^{2i e + \frac{3ifx}{2}} f^2 x^2 \text{ExpIntegralEi} \left[\frac{i f x}{2} \right] - \\
& \left. 9 e^{\frac{3ifx}{2}} f^2 x^2 \text{ExpIntegralEi} \left[-\frac{3}{2} i f x \right] - 9 i e^{\frac{3}{2} i (2e+fx)} f^2 x^2 \text{ExpIntegralEi} \left[\frac{3 i f x}{2} \right] \right)
\end{aligned}$$

■ **Problem 163: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^3}{a+b \sin[e+fx]} dx$$

Optimal (type 4, 495 leaves, 12 steps):

$$\begin{aligned}
& - \frac{i(c+dx)^3 \text{Log} \left[1 - \frac{i b e^{i(e+fx)}}{a - \sqrt{a^2 - b^2}} \right]}{\sqrt{a^2 - b^2} f} + \frac{i(c+dx)^3 \text{Log} \left[1 - \frac{i b e^{i(e+fx)}}{a + \sqrt{a^2 - b^2}} \right]}{\sqrt{a^2 - b^2} f} - \frac{3 d (c+dx)^2 \text{PolyLog} \left[2, \frac{i b e^{i(e+fx)}}{a - \sqrt{a^2 - b^2}} \right]}{\sqrt{a^2 - b^2} f^2} + \frac{3 d (c+dx)^2 \text{PolyLog} \left[2, \frac{i b e^{i(e+fx)}}{a + \sqrt{a^2 - b^2}} \right]}{\sqrt{a^2 - b^2} f^2} \\
& - \frac{6 i d^2 (c+dx) \text{PolyLog} \left[3, \frac{i b e^{i(e+fx)}}{a - \sqrt{a^2 - b^2}} \right]}{\sqrt{a^2 - b^2} f^3} + \frac{6 i d^2 (c+dx) \text{PolyLog} \left[3, \frac{i b e^{i(e+fx)}}{a + \sqrt{a^2 - b^2}} \right]}{\sqrt{a^2 - b^2} f^3} + \frac{6 d^3 \text{PolyLog} \left[4, \frac{i b e^{i(e+fx)}}{a - \sqrt{a^2 - b^2}} \right]}{\sqrt{a^2 - b^2} f^4} - \frac{6 d^3 \text{PolyLog} \left[4, \frac{i b e^{i(e+fx)}}{a + \sqrt{a^2 - b^2}} \right]}{\sqrt{a^2 - b^2} f^4}
\end{aligned}$$

Result (type 4, 1486 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{a^2 - b^2} f^4 \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e])} \\
& i \left(3 i \sqrt{a^2 - b^2} c^2 d f^3 x \text{Log} \left[1 + \frac{b (\cos[2e + fx] + i \sin[2e + fx])}{i a \cos[e] + \sqrt{(-a^2 + b^2)} (\cos[e] + i \sin[e])^2 - a \sin[e]} \right] (\cos[e] + i \sin[e]) + \right. \\
& 3 i \sqrt{a^2 - b^2} c d^2 f^3 x^2 \text{Log} \left[1 + \frac{b (\cos[2e + fx] + i \sin[2e + fx])}{i a \cos[e] + \sqrt{(-a^2 + b^2)} (\cos[e] + i \sin[e])^2 - a \sin[e]} \right] (\cos[e] + i \sin[e]) + \\
& i \sqrt{a^2 - b^2} d^3 f^3 x^3 \text{Log} \left[1 + \frac{b (\cos[2e + fx] + i \sin[2e + fx])}{i a \cos[e] + \sqrt{(-a^2 + b^2)} (\cos[e] + i \sin[e])^2 - a \sin[e]} \right] (\cos[e] + i \sin[e]) + \\
& \left. 3 \sqrt{a^2 - b^2} d f^2 (c+dx)^2 \text{PolyLog} \left[2, -\frac{b (\cos[2e + fx] + i \sin[2e + fx])}{i a \cos[e] + \sqrt{(-a^2 + b^2)} (\cos[e] + i \sin[e])^2 - a \sin[e]} \right] (\cos[e] + i \sin[e]) - \right.
\end{aligned}$$

$$\begin{aligned}
& 3 \sqrt{a^2 - b^2} d f^2 (c + d x)^2 \text{PolyLog}\left[2, \frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{-i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 + a \sin[e]}\right] (\cos[e] + i \sin[e]) + \\
& 6 i \sqrt{a^2 - b^2} c d^2 f \text{PolyLog}\left[3, -\frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 - a \sin[e]}\right] (\cos[e] + i \sin[e]) + \\
& 6 i \sqrt{a^2 - b^2} d^3 f x \text{PolyLog}\left[3, -\frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 - a \sin[e]}\right] (\cos[e] + i \sin[e]) - \\
& 6 \sqrt{a^2 - b^2} d^3 \text{PolyLog}\left[4, -\frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 - a \sin[e]}\right] (\cos[e] + i \sin[e]) + \\
& 6 \sqrt{a^2 - b^2} d^3 \text{PolyLog}\left[4, \frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{-i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 + a \sin[e]}\right] (\cos[e] + i \sin[e]) + \\
& 3 \sqrt{a^2 - b^2} c^2 d f^3 x \text{Log}\left[1 - \frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{-i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 + a \sin[e]}\right] (-i \cos[e] + \sin[e]) + \\
& 3 \sqrt{a^2 - b^2} c d^2 f^3 x^2 \text{Log}\left[1 - \frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{-i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 + a \sin[e]}\right] (-i \cos[e] + \sin[e]) + \\
& \sqrt{a^2 - b^2} d^3 f^3 x^3 \text{Log}\left[1 - \frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{-i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 + a \sin[e]}\right] (-i \cos[e] + \sin[e]) + \\
& 6 \sqrt{a^2 - b^2} c d^2 f \text{PolyLog}\left[3, \frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{-i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 + a \sin[e]}\right] (-i \cos[e] + \sin[e]) + \\
& 6 \sqrt{a^2 - b^2} d^3 f x \text{PolyLog}\left[3, \frac{b (\cos[2 e + f x] + i \sin[2 e + f x])}{-i a \cos[e] + \sqrt{-a^2 + b^2} (\cos[e] + i \sin[e])^2 + a \sin[e]}\right] (-i \cos[e] + \sin[e]) - \\
& 2 i c^3 f^3 \text{ArcTan}\left[\frac{b \cos[e + f x] + i (a + b \sin[e + f x])}{\sqrt{a^2 - b^2}}\right] \sqrt{(-a^2 + b^2) (\cos[2 e] + i \sin[2 e])}
\end{aligned}$$

■ **Problem 168: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^3}{(a + b \sin[e + f x])^2} dx$$

Optimal (type 4, 925 leaves, 22 steps):

$$\begin{aligned}
& \frac{i (c+dx)^3}{(a^2-b^2) f} - \frac{3 d (c+dx)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^2} - \frac{i a (c+dx)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f} - \frac{3 d (c+dx)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^2} + \frac{i a (c+dx)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f} + \\
& \frac{6 i d^2 (c+dx) \operatorname{PolyLog}\left[2, \frac{i b e^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^3} - \frac{3 a d (c+dx)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2} + \frac{6 i d^2 (c+dx) \operatorname{PolyLog}\left[2, \frac{i b e^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^3} + \\
& \frac{3 a d (c+dx)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2} - \frac{6 d^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^4} - \frac{6 i a d^2 (c+dx) \operatorname{PolyLog}\left[3, \frac{i b e^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3} - \frac{6 d^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^4} + \\
& \frac{6 i a d^2 (c+dx) \operatorname{PolyLog}\left[3, \frac{i b e^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3} + \frac{6 a d^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(e+fx)}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^4} - \frac{6 a d^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(e+fx)}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^4} + \frac{b (c+dx)^3 \operatorname{Cos}[e+fx]}{(a^2-b^2) f (a+b \operatorname{Sin}[e+fx])}
\end{aligned}$$

Result (type 4, 7006 leaves):

$$\begin{aligned}
& \frac{1}{(a^2-b^2) f^2} 3 a c^2 d \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \right. \\
& \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(-e + \frac{\pi}{2} - fx\right) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{-a^2+b^2}}\right] - 2 \left(-e + \operatorname{ArcCos}\left[-\frac{a}{b}\right]\right) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) + \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2} i \left(-e + \frac{\pi}{2} - fx\right)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[e+fx]}}\right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + \right. \\
& \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2} i \left(-e + \frac{\pi}{2} - fx\right)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[e+fx]}}\right] - \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{\left(a - i \sqrt{-a^2+b^2}\right) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)}\right] +
\end{aligned}$$

$$\begin{aligned}
& \left(-\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right])}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right])}\right] \right) \right) + \\
& \frac{1}{(a^2-b^2)f^3} 6acd^2 \operatorname{Cot}[e] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2(-e+\frac{\pi}{2}-fx) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \right. \\
& \left. \left. 2(-e+\operatorname{ArcCos}\left[-\frac{a}{b}\right]) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]}{\sqrt{-a^2+b^2}}\right] + \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2}i(-e+\frac{\pi}{2}-fx)}}{\sqrt{2}\sqrt{b}\sqrt{a+b \operatorname{Sin}[e+fx]}}\right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + \right. \\
& \left. 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i(-e+\frac{\pi}{2}-fx)}}{\sqrt{2}\sqrt{b}\sqrt{a+b \operatorname{Sin}[e+fx]}}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right])}\right] +
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{ArcCos}\left[-\frac{a}{b}\right] + 2i \text{ArcTan}\left[\frac{(-a+b) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \text{Log}\left[1 - \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right])}\right] + \\
& i \left(\text{PolyLog}\left[2, \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right])}\right] - \right. \\
& \left. \text{PolyLog}\left[2, \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right])}\right] \right) \right) + \\
& \left(3acd^2 e^{ie} \left(f^2 x^2 \text{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} - \sqrt{(-a^2+b^2)} e^{2ie}}\right] - f^2 x^2 \text{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2+b^2)} e^{2ie}}\right] - \right. \right. \\
& 2ifx \text{PolyLog}\left[2, \frac{i b e^{i(2e+fx)}}{a e^{ie} + i \sqrt{(-a^2+b^2)} e^{2ie}}\right] + \\
& 2ifx \text{PolyLog}\left[2, -\frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2+b^2)} e^{2ie}}\right] + 2 \text{PolyLog}\left[3, \frac{i b e^{i(2e+fx)}}{a e^{ie} + i \sqrt{(-a^2+b^2)} e^{2ie}}\right] - \\
& \left. \left. 2 \text{PolyLog}\left[3, -\frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2+b^2)} e^{2ie}}\right] \right) \right) / \left((a^2-b^2) \sqrt{(-a^2+b^2)} e^{2ie} f^3 \right) + \\
& \left(3ad^3 e^{ie} \text{Cot}[e] \left(f^2 x^2 \text{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} - \sqrt{(-a^2+b^2)} e^{2ie}}\right] - f^2 x^2 \text{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2+b^2)} e^{2ie}}\right] - \right. \right. \\
& 2ifx \text{PolyLog}\left[2, \frac{i b e^{i(2e+fx)}}{a e^{ie} + i \sqrt{(-a^2+b^2)} e^{2ie}}\right] + 2ifx \text{PolyLog}\left[2, -\frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2+b^2)} e^{2ie}}\right] + \\
& \left. \left. 2 \text{PolyLog}\left[3, \frac{i b e^{i(2e+fx)}}{a e^{ie} + i \sqrt{(-a^2+b^2)} e^{2ie}}\right] - 2 \text{PolyLog}\left[3, -\frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2+b^2)} e^{2ie}}\right] \right) \right) / \\
& \left((a^2-b^2) \sqrt{(-a^2+b^2)} e^{2ie} f^4 \right) + \frac{1}{2(a^2-b^2) \sqrt{(-a^2+b^2)} e^{2ie} f^4}
\end{aligned}$$

d³

e^{-ie}

Csc[e]

$$\begin{aligned}
& \left(2 e^{2ie} \sqrt{(-a^2 + b^2) e^{2ie}} f^3 x^3 - 3 a e^{ie} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} - \sqrt{(-a^2 + b^2) e^{2ie}}}\right] - \right. \\
& 3 a e^{3ie} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} - \sqrt{(-a^2 + b^2) e^{2ie}}}\right] - 3 i \sqrt{(-a^2 + b^2) e^{2ie}} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} - \sqrt{(-a^2 + b^2) e^{2ie}}}\right] + \\
& 3 i e^{2ie} \sqrt{(-a^2 + b^2) e^{2ie}} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} - \sqrt{(-a^2 + b^2) e^{2ie}}}\right] + \\
& 3 a e^{ie} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2 + b^2) e^{2ie}}}\right] + 3 a e^{3ie} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2 + b^2) e^{2ie}}}\right] - \\
& 3 i \sqrt{(-a^2 + b^2) e^{2ie}} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2 + b^2) e^{2ie}}}\right] + \\
& 3 i e^{2ie} \sqrt{(-a^2 + b^2) e^{2ie}} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2 + b^2) e^{2ie}}}\right] + \\
& 6 \left(\sqrt{(-a^2 + b^2) e^{2ie}} (-1 + e^{2ie}) + i a e^{ie} (1 + e^{2ie}) \right) f x \operatorname{PolyLog}\left[2, \frac{i b e^{i(2e+fx)}}{a e^{ie} + i \sqrt{(-a^2 + b^2) e^{2ie}}}\right] + \\
& 6 \left(\sqrt{(-a^2 + b^2) e^{2ie}} (-1 + e^{2ie}) - i a e^{ie} (1 + e^{2ie}) \right) f x \operatorname{PolyLog}\left[2, -\frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2 + b^2) e^{2ie}}}\right] - \\
& 6 a e^{ie} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2e+fx)}}{a e^{ie} + i \sqrt{(-a^2 + b^2) e^{2ie}}}\right] - 6 a e^{3ie} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2e+fx)}}{a e^{ie} + i \sqrt{(-a^2 + b^2) e^{2ie}}}\right] - \\
& 6 i \sqrt{(-a^2 + b^2) e^{2ie}} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2e+fx)}}{a e^{ie} + i \sqrt{(-a^2 + b^2) e^{2ie}}}\right] + \\
& 6 i e^{2ie} \sqrt{(-a^2 + b^2) e^{2ie}} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2e+fx)}}{a e^{ie} + i \sqrt{(-a^2 + b^2) e^{2ie}}}\right] + 6 a e^{ie} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2 + b^2) e^{2ie}}}\right] + \\
& 6 a e^{3ie} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2 + b^2) e^{2ie}}}\right] - 6 i \sqrt{(-a^2 + b^2) e^{2ie}} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2e+fx)}}{i a e^{ie} + \sqrt{(-a^2 + b^2) e^{2ie}}}\right] +
\end{aligned}$$

$$\begin{aligned}
& \left. 6 i e^{2 i e} \sqrt{(-a^2 + b^2) e^{2 i e}} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2e+fx)}}{i a e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right]\right) + \\
& \frac{1}{(a^2 - b^2) \sqrt{(-a^2 + b^2) e^{2 i e}} f^4} a d^3 e^{i e} \left(f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2e+fx)}}{i a e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] \right) - \\
& 3 i f^2 x^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(2e+fx)}}{a e^{i e} + i \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + 3 i f^2 x^2 \operatorname{PolyLog}\left[2, -\frac{b e^{i(2e+fx)}}{i a e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + \\
& 6 f x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2e+fx)}}{a e^{i e} + i \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - 6 f x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2e+fx)}}{i a e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + \\
& \left. 6 i \operatorname{PolyLog}\left[4, \frac{i b e^{i(2e+fx)}}{a e^{i e} + i \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - 6 i \operatorname{PolyLog}\left[4, -\frac{b e^{i(2e+fx)}}{i a e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right]\right) + \\
& \frac{2 i a c^3 \operatorname{ArcTan}\left[\frac{i b \cos[e] - i(-a+b \sin[e]) \tan\left[\frac{fx}{2}\right]}{\sqrt{-a^2 + b^2 \cos[e]^2 + b^2 \sin[e]^2}}\right]}{(a^2 - b^2) f \sqrt{-a^2 + b^2 \cos[e]^2 + b^2 \sin[e]^2}} + \frac{6 i a c^2 d \operatorname{ArcTan}\left[\frac{i b \cos[e] - i(-a+b \sin[e]) \tan\left[\frac{fx}{2}\right]}{\sqrt{-a^2 + b^2 \cos[e]^2 + b^2 \sin[e]^2}}\right] \operatorname{Cot}[e]}{(a^2 - b^2) f^2 \sqrt{-a^2 + b^2 \cos[e]^2 + b^2 \sin[e]^2}} + \\
& \frac{1}{(a^2 - b^2) f} \\
& 6 \\
& b \\
& c \\
& d^2 \\
& \operatorname{Csc}[e] \\
& \left(-\frac{x^2 \cos[e]}{2 b} + \frac{1}{b f} \right. \\
& \left. x \left(f x \cos[e] - \frac{2 a \operatorname{ArcTan}\left[\frac{\sec\left[\frac{fx}{2}\right] (\cos[e] - i \sin[e]) (b \cos\left[e + \frac{fx}{2}\right] + a \sin\left[\frac{fx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2}}\right]}{\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2}}\right] \cos[e] (\cos[e] - i \sin[e]) - \operatorname{Log}[a + b \sin[e + f x]] \sin[e] \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b f} \left(-\frac{1}{f} a \operatorname{Cos}[e] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(e - \operatorname{ArcCos}\left[-\frac{a}{b}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2e-\pi+2f x)\right]}{\sqrt{-a^2+b^2}}\right] \right) + \right. \right. \\
& \quad \left. \left. (-2e+\pi-2f x) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2e+\pi+2f x)\right]}{\sqrt{-a^2+b^2}}\right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2e-\pi+2f x)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \operatorname{Log}\left[\right. \right. \\
& \quad \left. \left. \frac{(a+b) \left(-a+b-i\sqrt{-a^2+b^2} \right) \left(1+i \operatorname{Cot}\left[\frac{1}{4}(2e+\pi+2f x)\right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2e+\pi+2f x)\right] \right)} \right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2e-\pi+2f x)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[\frac{(a+b) \left(i a-i b+\sqrt{-a^2+b^2} \right) \left(i+\operatorname{Cot}\left[\frac{1}{4}(2e+\pi+2f x)\right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2e+\pi+2f x)\right] \right)} \right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \left(-\operatorname{ArcTanh}\left[\right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2e-\pi+2f x)\right]}{\sqrt{-a^2+b^2}} \right] + \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2e+\pi+2f x)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4}i(-2e+\pi-2f x)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[e+f x]}} \right] + \right. \\
& \quad \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2e-\pi+2f x)\right]}{\sqrt{-a^2+b^2}}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2e+\pi+2f x)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right. \\
& \quad \left. \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4}i(2e-\pi+2f x)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[e+f x]}} \right] + i \left(\operatorname{PolyLog}\left[2, \frac{(a-i\sqrt{-a^2+b^2}) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2e-\pi+2f x)\right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2e+\pi+2f x)\right] \right)} \right] - \right. \\
& \quad \left. \left. \operatorname{PolyLog}\left[2, \frac{(a+i\sqrt{-a^2+b^2}) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2e-\pi+2f x)\right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2e+\pi+2f x)\right] \right)} \right] \right) \right) \right) \right) + \\
& \quad \frac{2 a x \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{f x}{2}\right] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) \left(b \operatorname{Cos}\left[e+\frac{f x}{2}\right]+a \operatorname{Sin}\left[\frac{f x}{2}\right] \right)}{\sqrt{a^2-b^2} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}}\right]}{\sqrt{a^2-b^2} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} \right] \operatorname{Cos}[e] (\operatorname{Cos}[e]-i \operatorname{Sin}[e])}{\sqrt{a^2-b^2} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} + \frac{(e+f x) \operatorname{Log}[a+b \operatorname{Sin}[e+f x]] \operatorname{Sin}[e]}{f} -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{f} \left(\frac{(e+fx) \operatorname{Log}[a+b \operatorname{Sin}[e+fx]]}{b} - 1/b \left(-\frac{1}{2} i \left(-e + \frac{\pi}{2} - fx \right)^2 + 4 i \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[\frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]}{\sqrt{a^2-b^2}} \right] \right) \right. \\
& \left. \left(-e + \frac{\pi}{2} - fx + 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{\left(a - \sqrt{a^2-b^2} \right) e^{i \left(-e + \frac{\pi}{2} - fx \right)}}{b} \right] + \right. \\
& \left. \left(-e + \frac{\pi}{2} - fx - 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{\left(a + \sqrt{a^2-b^2} \right) e^{i \left(-e + \frac{\pi}{2} - fx \right)}}{b} \right] - \left(-e + \frac{\pi}{2} - fx \right) \operatorname{Log}[a+b \operatorname{Sin}[e+fx]] - \right. \\
& \left. i \left(\operatorname{PolyLog} \left[2, -\frac{\left(a - \sqrt{a^2-b^2} \right) e^{i \left(-e + \frac{\pi}{2} - fx \right)}}{b} \right] + \operatorname{PolyLog} \left[2, -\frac{\left(a + \sqrt{a^2-b^2} \right) e^{i \left(-e + \frac{\pi}{2} - fx \right)}}{b} \right] \right) \operatorname{Sin}[e] \right) \\
& \left(3 b c^2 d \operatorname{Csc}[e] \left(-b f x \operatorname{Cos}[e] + b \operatorname{Log}[a+b \operatorname{Cos}[fx]] \operatorname{Sin}[e] + b \operatorname{Cos}[e] \operatorname{Sin}[fx] \right) \operatorname{Sin}[e] + \frac{2 i a b \operatorname{ArcTan} \left[\frac{i b \operatorname{Cos}[e] - i \left(-a+b \operatorname{Sin}[e] \right) \operatorname{Tan} \left[\frac{fx}{2} \right]}{\sqrt{-a^2+b^2 \operatorname{Cos}[e]^2+b^2 \operatorname{Sin}[e]^2}} \right] \operatorname{Cos}[e]}{\sqrt{-a^2+b^2 \operatorname{Cos}[e]^2+b^2 \operatorname{Sin}[e]^2}} \right) \right) / \\
& \left((a^2-b^2) f^2 \right. \\
& \left. (b^2 \operatorname{Cos}[e]^2 + b^2 \operatorname{Sin}[e]^2) \right) + \\
& \left(\operatorname{Csc} \left[\frac{e}{2} \right] \operatorname{Sec} \left[\frac{e}{2} \right] \left(-a c^3 \operatorname{Cos}[e] - 3 a c^2 d x \operatorname{Cos}[e] - 3 a c d^2 x^2 \operatorname{Cos}[e] - a d^3 x^3 \operatorname{Cos}[e] - b c^3 \operatorname{Sin}[fx] - \right. \right. \\
& \left. \left. 3 b c^2 d x \operatorname{Sin}[fx] - 3 b c d^2 x^2 \operatorname{Sin}[fx] - b d^3 x^3 \operatorname{Sin}[fx] \right) \right) / \left(2 (a-b) (a+b) f (a+b \operatorname{Sin}[e+fx]) \right)
\end{aligned}$$

■ **Problem 169: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^2}{(a+b \operatorname{Sin}[e+fx])^2} dx$$

Optimal (type 4, 671 leaves, 18 steps):

$$\begin{aligned}
& \frac{i (c+dx)^2}{(a^2-b^2)^2 f} - \frac{2d(c+dx) \operatorname{Log}\left[1 - \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^2 f^2} - \frac{ia(c+dx)^2 \operatorname{Log}\left[1 - \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f} - \frac{2d(c+dx) \operatorname{Log}\left[1 - \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^2 f^2} + \\
& \frac{ia(c+dx)^2 \operatorname{Log}\left[1 - \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f} + \frac{2id^2 \operatorname{PolyLog}\left[2, \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^2 f^3} - \frac{2ad(c+dx) \operatorname{PolyLog}\left[2, \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2} + \frac{2id^2 \operatorname{PolyLog}\left[2, \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^2 f^3} + \\
& \frac{2ad(c+dx) \operatorname{PolyLog}\left[2, \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2} - \frac{2iad^2 \operatorname{PolyLog}\left[3, \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3} + \frac{2ia d^2 \operatorname{PolyLog}\left[3, \frac{ibe^{i(e+fx)}}{a\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3} + \frac{b(c+dx)^2 \operatorname{Cos}[e+fx]}{(a^2-b^2) f (a+b\operatorname{Sin}[e+fx])}
\end{aligned}$$

Result (type 4, 8893 leaves):

$$\begin{aligned}
& \frac{1}{(a^2-b^2) f (-1 + \operatorname{Cos}[2e] + i \operatorname{Sin}[2e])} \\
& 2i (\operatorname{Cos}[e] + i \operatorname{Sin}[e]) \left(2cdx \operatorname{Cos}[e] + d^2 x^2 \operatorname{Cos}[e] + \frac{ia c^2 \operatorname{ArcTan}\left[\frac{ia+b\operatorname{Cos}[e+fx]+ib\operatorname{Sin}[e+fx]}{\sqrt{a^2-b^2}}\right] (\operatorname{Cos}[e] - i \operatorname{Sin}[e])}{\sqrt{a^2-b^2}} \right) - \\
& \frac{2acd \operatorname{ArcTan}\left[\frac{ia+b\operatorname{Cos}[e+fx]+ib\operatorname{Sin}[e+fx]}{\sqrt{a^2-b^2}}\right] (\operatorname{Cos}[e] - i \operatorname{Sin}[e])}{\sqrt{a^2-b^2} f} + \frac{1}{2\sqrt{a^2-b^2} f} \\
& cd \left(-4\sqrt{a^2-b^2} f x + 4a \operatorname{ArcTan}\left[\frac{ia+b\operatorname{Cos}[e+fx]+ib\operatorname{Sin}[e+fx]}{\sqrt{a^2-b^2}}\right] + 2\sqrt{a^2-b^2} \operatorname{ArcTan}\left[\frac{2a (\operatorname{Cos}[e+fx] + i \operatorname{Sin}[e+fx])}{b(-1 + \operatorname{Cos}[2e+2fx] + i \operatorname{Sin}[2e+2fx])}\right] \right) - \\
& i\sqrt{a^2-b^2} \operatorname{Log}\left[4a^2 \operatorname{Cos}[2e+2fx] + b^2 (-1 + \operatorname{Cos}[2e+2fx] + i \operatorname{Sin}[2e+2fx])^2 + 4ia^2 \operatorname{Sin}[2e+2fx]\right] (\operatorname{Cos}[e] - i \operatorname{Sin}[e]) - \\
& \frac{ia c^2 \operatorname{ArcTan}\left[\frac{ia+b\operatorname{Cos}[e+fx]+ib\operatorname{Sin}[e+fx]}{\sqrt{a^2-b^2}}\right] (\operatorname{Cos}[e] + i \operatorname{Sin}[e])}{\sqrt{a^2-b^2}} + \frac{2acd \operatorname{ArcTan}\left[\frac{ia+b\operatorname{Cos}[e+fx]+ib\operatorname{Sin}[e+fx]}{\sqrt{a^2-b^2}}\right] (\operatorname{Cos}[e] + i \operatorname{Sin}[e])}{\sqrt{a^2-b^2} f} - \\
& \frac{1}{2f} cd \left(-4fx + \frac{4a \operatorname{ArcTan}\left[\frac{ia+b\operatorname{Cos}[e+fx]+ib\operatorname{Sin}[e+fx]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + 2 \operatorname{ArcTan}\left[\frac{2a (\operatorname{Cos}[e+fx] + i \operatorname{Sin}[e+fx])}{b(-1 + \operatorname{Cos}[2e+2fx] + i \operatorname{Sin}[2e+2fx])}\right] \right) -
\end{aligned}$$

$$\left. \begin{aligned}
& i \operatorname{Log}\left[4 a^2 \operatorname{Cos}[2 e+2 f x]+b^2(-1+\operatorname{Cos}[2 e+2 f x]+i \operatorname{Sin}[2 e+2 f x])^2+4 i a^2 \operatorname{Sin}[2 e+2 f x]\right] \\
& 2 i c d x \operatorname{Sin}[e]+i d^2 x^2 \operatorname{Sin}[e]-2 c d x(\operatorname{Cos}[e]-i \operatorname{Sin}[e])(-1+\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])- \\
& d^2 x^2(\operatorname{Cos}[e]-i \operatorname{Sin}[e])(-1+\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])+2 b d^2(\operatorname{Cos}[e]-i \operatorname{Sin}[e])
\end{aligned} \right) (\operatorname{Cos}[e]+i \operatorname{Sin}[e])+$$

$$\left(- \left(\frac{x^2}{2\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}\right)} + \frac{i x \operatorname{Log}\left[1+\frac{b\left(\operatorname{Cos}[2 e+f x]+i \operatorname{Sin}[2 e+f x]\right)}{i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}}\right]}{f\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}\right)} + \right.$$

$$\left. \frac{\operatorname{PolyLog}\left[2,-\frac{b\left(\operatorname{Cos}[2 e+f x]+i \operatorname{Sin}[2 e+f x]\right)}{i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}}\right]}{f^2\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}\right)} \right) /$$

$$\left(- \frac{2 \operatorname{Cos}[2 e] \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}}{b} + \right.$$

$$\left. \frac{2 i \operatorname{Sin}[2 e] \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}}{b} \right) +$$

$$\left(\frac{x^2}{2\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]+\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}\right)} + \frac{i x \operatorname{Log}\left[1+\frac{b\left(\operatorname{Cos}[2 e+f x]+i \operatorname{Sin}[2 e+f x]\right)}{i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]+\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}}\right]}{f\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]+\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}\right)} + \right.$$

$$\left. \frac{\operatorname{PolyLog}\left[2,-\frac{b\left(\operatorname{Cos}[2 e+f x]+i \operatorname{Sin}[2 e+f x]\right)}{i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]+\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}}\right]}{f^2\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]+\sqrt{\left(-a^2+b^2\right)\left(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]\right)}\right)} \right) /$$

$$\left(- \frac{2 \operatorname{Cos}[2 e] \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}}{b} + \right.$$

$$\left. \frac{2 i \operatorname{Sin}[2 e] \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}}{b} \right) - 2 b d^2(\operatorname{Cos}[e]+i \operatorname{Sin}[e])$$

$$\begin{aligned}
& \left(- \left(\frac{x^2}{2 \left(i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e + f x] + i \sin[2e + f x])}{i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e])} \right]}{f \left(i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e]) \right)} \right) + \right. \\
& \left. \frac{\operatorname{PolyLog} \left[2, - \frac{b (\cos[2e + f x] + i \sin[2e + f x])}{i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e])} \right]}{f^2 \left(i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e]) \right)} \right) / \\
& \left(- \frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \right. \\
& \left. \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) + \\
& \left(\frac{x^2}{2 \left(i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e + f x] + i \sin[2e + f x])}{i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e])} \right]}{f \left(i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e]) \right)} \right) + \\
& \left. \frac{\operatorname{PolyLog} \left[2, - \frac{b (\cos[2e + f x] + i \sin[2e + f x])}{i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e])} \right]}{f^2 \left(i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e]) \right)} \right) / \\
& \left(- \frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \right. \\
& \left. \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) - 2 i a d^2 \\
& \left(\left(\left(\frac{x^2}{2 \left(i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e + f x] + i \sin[2e + f x])}{i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e])} \right]}{f \left(i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e]) \right)} \right) + \right. \\
& \left. \frac{\operatorname{PolyLog} \left[2, - \frac{b (\cos[2e + f x] + i \sin[2e + f x])}{i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e])} \right]}{f^2 \left(i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2)} (\cos[2e] + i \sin[2e]) \right)} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-i a \cos[e] - a \sin[e] - (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) / \\
& \left(b \left(-\frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \right. \right. \\
& \quad \left. \left. \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) - \\
& \left(\left(\frac{x^2}{2 \left(i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])} \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])}} \right]}{f \left(i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])} \right)} + \right. \right. \\
& \quad \left. \left. \frac{\operatorname{PolyLog} \left[2, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])}} \right]}{f^2 \left(i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])} \right)} \right) \right) \\
& \left(-i a \cos[e] - a \sin[e] + (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) / \\
& \left(b \left(-\frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \right. \right. \\
& \quad \left. \left. \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) - 2 a c d f \\
& \left(\left(\left(\frac{x^2}{2 \left(i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])} \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])}} \right]}{f \left(i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])} \right)} + \right. \right. \right. \\
& \quad \left. \left. \frac{\operatorname{PolyLog} \left[2, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])}} \right]}{f^2 \left(i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])} \right)} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-i a \cos[e] - a \sin[e] - (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) \Big/ \\
& \left(b \left(- \frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \right. \right. \\
& \quad \left. \left. \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) - \\
& \left(\left(\frac{x^2}{2 (i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])})} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])}} \right]}{f (i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])})} \right) + \right. \\
& \quad \left. \frac{\operatorname{PolyLog} \left[2, - \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])}} \right]}{f^2 (i a \cos[e] - a \sin[e] - \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])})} \right) \Big/ \\
& \left(-i a \cos[e] - a \sin[e] + (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) \Big/ \\
& \left(b \left(- \frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \right. \right. \\
& \quad \left. \left. \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) - \\
& a d^2 f \left(\left(\left(\frac{x^3}{3 (i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])})} + \frac{i x^2 \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])}} \right]}{f (i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])})} \right) + \right. \right. \\
& \quad \left. \left. \frac{2 x \operatorname{PolyLog} \left[2, - \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])}} \right]}{f^2 (i a \cos[e] - a \sin[e] + \sqrt{(-a^2 + b^2) (\cos[2e] + i \sin[2e])})} \right) \right) +
\end{aligned}$$

$$\left. \frac{2 i \operatorname{PolyLog}\left[3, -\frac{b(\operatorname{Cos}[2 e+f x]+i \operatorname{Sin}[2 e+f x])}{i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]+\sqrt{-a^2+b^2}(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])}\right]}{f^3\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]+\sqrt{-a^2+b^2}(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])\right)}\right)$$

$$\left. \left(-i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\left(\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]\right) \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}\right)\right) /$$

$$\left(b \left(-\frac{2 \operatorname{Cos}[2 e] \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}}{b} + \frac{2 i \operatorname{Sin}[2 e] \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}}{b} \right) \right) -$$

$$\left(\left(\frac{x^3}{3\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{-a^2+b^2}(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])\right)} + \frac{i x^2 \operatorname{Log}\left[1+\frac{b(\operatorname{Cos}[2 e+f x]+i \operatorname{Sin}[2 e+f x])}{i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{-a^2+b^2}(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])}\right]}{f\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{-a^2+b^2}(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])\right)}\right) + \frac{2 x \operatorname{PolyLog}\left[2, -\frac{b(\operatorname{Cos}[2 e+f x]+i \operatorname{Sin}[2 e+f x])}{i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{-a^2+b^2}(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])}\right]}{f^2\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{-a^2+b^2}(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])\right)} + \frac{2 i \operatorname{PolyLog}\left[3, -\frac{b(\operatorname{Cos}[2 e+f x]+i \operatorname{Sin}[2 e+f x])}{i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{-a^2+b^2}(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])}\right]}{f^3\left(i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]-\sqrt{-a^2+b^2}(\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e])\right)}\right) \right)$$

$$\left. \left(-i a \operatorname{Cos}[e]-a \operatorname{Sin}[e]+\left(\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]\right) \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}\right)\right) /$$

$$\left(b \left(-\frac{2 \operatorname{Cos}[2 e] \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}}{b} + \frac{2 i \operatorname{Sin}[2 e] \sqrt{-a^2 \operatorname{Cos}[2 e]+b^2 \operatorname{Cos}[2 e]-i a^2 \operatorname{Sin}[2 e]+i b^2 \operatorname{Sin}[2 e]}}{b} \right) \right) + 2 i a d^2$$

$$\left(\left(\left(\frac{x^2}{2 \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{\operatorname{PolyLog} \left[2, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f^2 \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} \right) (\cos[2e] + i \sin[2e]) \right) / \left(-i a \cos[e] - a \sin[e] - (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) \right) / \left(b \left(-\frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) - \left(\left(\left(\frac{x^2}{2 \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{\operatorname{PolyLog} \left[2, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f^2 \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} \right) (\cos[2e] + i \sin[2e]) \right) / \left(-i a \cos[e] - a \sin[e] + (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) \right) / \left(b \left(-\frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) + 2 a c d f$$

$$\left(\left(\left(\frac{x^2}{2 \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{\operatorname{PolyLog} \left[2, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f^2 \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} \right) (\cos[2e] + i \sin[2e]) \right) / \left(-i a \cos[e] - a \sin[e] - (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) \right) / \left(b \left(-\frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) - \left(\left(\left(\frac{x^2}{2 \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{\operatorname{PolyLog} \left[2, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f^2 \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} \right) (\cos[2e] + i \sin[2e]) \right) / \left(-i a \cos[e] - a \sin[e] + (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) \right) / \left(b \left(-\frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) + \right)$$

$$\begin{aligned}
& a d^2 f \left(\left(\left(\frac{x^3}{3 \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{i x^2 \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \right. \right. \\
& \quad \frac{2 x \operatorname{PolyLog} \left[2, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f^2 \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \\
& \quad \left. \left. \frac{2 i \operatorname{PolyLog} \left[3, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f^3 \left(i a \cos[e] - a \sin[e] + \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} \right) (\cos[2e] + i \sin[2e]) \right) \\
& \quad \left(-i a \cos[e] - a \sin[e] - (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) \Big/ \\
& \quad \left(b \left(-\frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \right. \right. \\
& \quad \left. \left. \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) - \\
& \quad \left(\left(\frac{x^3}{3 \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \frac{i x^2 \operatorname{Log} \left[1 + \frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \right. \right. \\
& \quad \frac{2 x \operatorname{PolyLog} \left[2, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f^2 \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} + \\
& \quad \left. \left. \frac{2 i \operatorname{PolyLog} \left[3, -\frac{b (\cos[2e+fx] + i \sin[2e+fx])}{i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e])} \right]}{f^3 \left(i a \cos[e] - a \sin[e] - \sqrt{-a^2 + b^2} (\cos[2e] + i \sin[2e]) \right)} \right) (\cos[2e] + i \sin[2e]) \right)
\end{aligned}$$

$$\left(-i a \cos[e] - a \sin[e] + (\cos[2e] - i \sin[2e]) \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]} \right) /$$

$$\left(b \left(-\frac{2 \cos[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} + \right. \right.$$

$$\left. \left. \frac{2 i \sin[2e] \sqrt{-a^2 \cos[2e] + b^2 \cos[2e] - i a^2 \sin[2e] + i b^2 \sin[2e]}}{b} \right) \right) +$$

$$\left(\operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] (-a c^2 \cos[e] - 2 a c d x \cos[e] - a d^2 x^2 \cos[e] - b c^2 \sin[f x] - 2 b c d x \sin[f x] - \right.$$

$$\left. b d^2 x^2 \sin[f x]) \right) / (2 (a -$$

$$b) (a +$$

$$b) f (a +$$

$$b \sin[e + f x]))$$

■ **Problem 175: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^m (a + b \sin[e + f x])^2 dx$$

Optimal (type 4, 318 leaves, 10 steps):

$$\frac{a^2 (c + d x)^{1+m}}{d (1+m)} + \frac{b^2 (c + d x)^{1+m}}{2 d (1+m)} - \frac{a b e^{i \left(\frac{e - c f}{d} \right)} (c + d x)^m \left(-\frac{i f (c + d x)}{d} \right)^{-m} \operatorname{Gamma}\left[1 + m, -\frac{i f (c + d x)}{d}\right]}{f} -$$

$$\frac{a b e^{-i \left(\frac{e - c f}{d} \right)} (c + d x)^m \left(\frac{i f (c + d x)}{d} \right)^{-m} \operatorname{Gamma}\left[1 + m, \frac{i f (c + d x)}{d}\right]}{f} + \frac{i 2^{-3-m} b^2 e^{2 i \left(\frac{e - c f}{d} \right)} (c + d x)^m \left(-\frac{i f (c + d x)}{d} \right)^{-m} \operatorname{Gamma}\left[1 + m, -\frac{2 i f (c + d x)}{d}\right]}{f} -$$

$$\frac{i 2^{-3-m} b^2 e^{-2 i \left(\frac{e - c f}{d} \right)} (c + d x)^m \left(\frac{i f (c + d x)}{d} \right)^{-m} \operatorname{Gamma}\left[1 + m, \frac{2 i f (c + d x)}{d}\right]}{f}$$

Result (type 4, 707 leaves):

$$\frac{1}{d f (1+m)} 2^{-3-m} (c+dx)^m \left(\frac{f^2 (c+dx)^2}{d^2} \right)^{-m}$$

$$\left(2^{3+m} a^2 c f \left(\frac{f^2 (c+dx)^2}{d^2} \right)^m + 2^{2+m} b^2 c f \left(\frac{f^2 (c+dx)^2}{d^2} \right)^m + 2^{3+m} a^2 d f x \left(\frac{f^2 (c+dx)^2}{d^2} \right)^m + 2^{2+m} b^2 d f x \left(\frac{f^2 (c+dx)^2}{d^2} \right)^m + \right.$$

$$i b^2 d \left(\frac{i f (c+dx)}{d} \right)^m \operatorname{Cos} \left[2 e - \frac{2 c f}{d} \right] \operatorname{Gamma} \left[1+m, -\frac{2 i f (c+dx)}{d} \right] + i b^2 d m \left(\frac{i f (c+dx)}{d} \right)^m \operatorname{Cos} \left[2 e - \frac{2 c f}{d} \right] \operatorname{Gamma} \left[1+m, -\frac{2 i f (c+dx)}{d} \right] -$$

$$i b^2 d \left(-\frac{i f (c+dx)}{d} \right)^m \operatorname{Cos} \left[2 e - \frac{2 c f}{d} \right] \operatorname{Gamma} \left[1+m, \frac{2 i f (c+dx)}{d} \right] - i b^2 d m \left(-\frac{i f (c+dx)}{d} \right)^m \operatorname{Cos} \left[2 e - \frac{2 c f}{d} \right] \operatorname{Gamma} \left[1+m, \frac{2 i f (c+dx)}{d} \right] -$$

$$b^2 d \left(\frac{i f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[1+m, -\frac{2 i f (c+dx)}{d} \right] \operatorname{Sin} \left[2 e - \frac{2 c f}{d} \right] - b^2 d m \left(\frac{i f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[1+m, -\frac{2 i f (c+dx)}{d} \right] \operatorname{Sin} \left[2 e - \frac{2 c f}{d} \right] -$$

$$b^2 d \left(-\frac{i f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[1+m, \frac{2 i f (c+dx)}{d} \right] \operatorname{Sin} \left[2 e - \frac{2 c f}{d} \right] - b^2 d m \left(-\frac{i f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[1+m, \frac{2 i f (c+dx)}{d} \right] \operatorname{Sin} \left[2 e - \frac{2 c f}{d} \right] -$$

$$2^{3+m} a b d (1+m) \left(-\frac{i f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[1+m, \frac{i f (c+dx)}{d} \right] \left(\operatorname{Cos} \left[e - \frac{c f}{d} \right] - i \operatorname{Sin} \left[e - \frac{c f}{d} \right] \right) -$$

$$2^{3+m} a b d (1+m) \left(\frac{i f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[1+m, -\frac{i f (c+dx)}{d} \right] \left(\operatorname{Cos} \left[e - \frac{c f}{d} \right] + i \operatorname{Sin} \left[e - \frac{c f}{d} \right] \right) \Bigg)$$

■ **Problem 181: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+fx) \operatorname{Sin}[c+dx]}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 76 leaves, 5 steps):

$$\frac{e x}{a} + \frac{f x^2}{2 a} + \frac{(e+fx) \operatorname{Cot} \left[\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right]}{a d} - \frac{2 f \operatorname{Log} \left[\operatorname{Sin} \left[\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2} \right] \right]}{a d^2}$$

Result (type 3, 199 leaves):

$$\left(2 d f x \operatorname{Cos} \left[c + \frac{dx}{2} \right] + \operatorname{Cos} \left[\frac{dx}{2} \right] \left(d^2 x (2 e + f x) - 4 f \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] \right) - 4 d e \operatorname{Sin} \left[\frac{dx}{2} \right] - \right.$$

$$2 d f x \operatorname{Sin} \left[\frac{dx}{2} \right] + 2 d^2 e x \operatorname{Sin} \left[c + \frac{dx}{2} \right] + d^2 f x^2 \operatorname{Sin} \left[c + \frac{dx}{2} \right] - 4 f \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] \operatorname{Sin} \left[c + \frac{dx}{2} \right] \Bigg) /$$

$$\left(2 a d^2 \left(\operatorname{Cos} \left[\frac{c}{2} \right] + \operatorname{Sin} \left[\frac{c}{2} \right] \right) \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right) \right)$$

■ **Problem 182: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c+dx]}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 28 leaves, 2 steps):

$$\frac{x}{a} + \frac{\text{Cos}[c + dx]}{d(a + a \text{Sin}[c + dx])}$$

Result (type 3, 72 leaves):

$$\frac{(\text{Cos}[\frac{1}{2}(c + dx)] + \text{Sin}[\frac{1}{2}(c + dx)])((c + dx) \text{Cos}[\frac{1}{2}(c + dx)] + (-2 + c + dx) \text{Sin}[\frac{1}{2}(c + dx)])}{ad(1 + \text{Sin}[c + dx])}$$

■ **Problem 185: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + fx)^3 \text{Sin}[c + dx]^2}{a + a \text{Sin}[c + dx]} dx$$

Optimal (type 4, 247 leaves, 14 steps):

$$\begin{aligned} & -\frac{i(e + fx)^3}{ad} - \frac{(e + fx)^4}{4af} + \frac{6f^2(e + fx) \text{Cos}[c + dx]}{ad^3} - \frac{(e + fx)^3 \text{Cos}[c + dx]}{ad} - \frac{(e + fx)^3 \text{Cot}[\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}]}{ad} + \frac{6f(e + fx)^2 \text{Log}[1 - i e^{i(c+dx)}]}{ad^2} \\ & - \frac{12if^2(e + fx) \text{PolyLog}[2, i e^{i(c+dx)}]}{ad^3} + \frac{12f^3 \text{PolyLog}[3, i e^{i(c+dx)}]}{ad^4} - \frac{6f^3 \text{Sin}[c + dx]}{ad^4} + \frac{3f(e + fx)^2 \text{Sin}[c + dx]}{ad^2} \end{aligned}$$

Result (type 4, 1378 leaves):

1

$$\begin{aligned}
& 4 a d^4 \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \\
& \left(-6 d^2 e^2 f \operatorname{Cos}\left[\frac{d x}{2}\right] + 12 f^3 \operatorname{Cos}\left[\frac{d x}{2}\right] + 4 d^4 e^3 x \operatorname{Cos}\left[\frac{d x}{2}\right] + 12 i d^3 e^2 f x \operatorname{Cos}\left[\frac{d x}{2}\right] - 12 d^2 e f^2 x \operatorname{Cos}\left[\frac{d x}{2}\right] + \right. \\
& 6 d^4 e^2 f x^2 \operatorname{Cos}\left[\frac{d x}{2}\right] + 12 i d^3 e f^2 x^2 \operatorname{Cos}\left[\frac{d x}{2}\right] - 6 d^2 f^3 x^2 \operatorname{Cos}\left[\frac{d x}{2}\right] + 4 d^4 e f^2 x^3 \operatorname{Cos}\left[\frac{d x}{2}\right] + 4 i d^3 f^3 x^3 \operatorname{Cos}\left[\frac{d x}{2}\right] + \\
& d^4 f^3 x^4 \operatorname{Cos}\left[\frac{d x}{2}\right] + 2 d^3 e^3 \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 12 d e f^2 \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 18 d^3 e^2 f x \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 12 d f^3 x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + \\
& 18 d^3 e f^2 x^2 \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 6 d^3 f^3 x^3 \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 2 d^3 e^3 \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] - 12 d e f^2 \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + 6 d^3 e^2 f x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] - \\
& 12 d f^3 x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + 6 d^3 e f^2 x^2 \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + 2 d^3 f^3 x^3 \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + 6 d^2 e^2 f \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] - 12 f^3 \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] + \\
& 12 d^2 e f^2 x \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] + 6 d^2 f^3 x^2 \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] - 24 d^2 e^2 f \operatorname{Cos}\left[\frac{d x}{2}\right] \operatorname{Log}\left[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]\right] - \\
& 48 d^2 e f^2 x \operatorname{Cos}\left[\frac{d x}{2}\right] \operatorname{Log}\left[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]\right] - 24 d^2 f^3 x^2 \operatorname{Cos}\left[\frac{d x}{2}\right] \operatorname{Log}\left[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]\right] - 10 d^3 e^3 \operatorname{Sin}\left[\frac{d x}{2}\right] + \\
& 12 d e f^2 \operatorname{Sin}\left[\frac{d x}{2}\right] - 18 d^3 e^2 f x \operatorname{Sin}\left[\frac{d x}{2}\right] + 12 d f^3 x \operatorname{Sin}\left[\frac{d x}{2}\right] - 18 d^3 e f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] - 6 d^3 f^3 x^3 \operatorname{Sin}\left[\frac{d x}{2}\right] - 6 d^2 e^2 f \operatorname{Sin}\left[c + \frac{d x}{2}\right] + \\
& 12 f^3 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 4 d^4 e^3 x \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 12 i d^3 e^2 f x \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 12 d^2 e f^2 x \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 6 d^4 e^2 f x^2 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + \\
& 12 i d^3 e f^2 x^2 \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 6 d^2 f^3 x^2 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 4 d^4 e f^2 x^3 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 4 i d^3 f^3 x^3 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + d^4 f^3 x^4 \operatorname{Sin}\left[c + \frac{d x}{2}\right] - \\
& 24 d^2 e^2 f \operatorname{Log}\left[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]\right] \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 48 d^2 e f^2 x \operatorname{Log}\left[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]\right] \operatorname{Sin}\left[c + \frac{d x}{2}\right] - \\
& 24 d^2 f^3 x^2 \operatorname{Log}\left[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]\right] \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 48 f^3 \operatorname{PolyLog}\left[3, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]\right] \left(\operatorname{Cos}\left[\frac{d x}{2}\right] + \operatorname{Sin}\left[c + \frac{d x}{2}\right] \right) + \\
& 48 i d f^2 (e + f x) \operatorname{PolyLog}\left[2, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]\right] \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) - \\
& 6 d^2 e^2 f \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 12 f^3 \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 12 d^2 e f^2 x \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 6 d^2 f^3 x^2 \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 2 d^3 e^3 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - \\
& 12 d e f^2 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 6 d^3 e^2 f x \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 12 d f^3 x \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 6 d^3 e f^2 x^2 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 2 d^3 f^3 x^3 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] \left. \right)
\end{aligned}$$

■ **Problem 187: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x) \operatorname{Sin}[c + d x]^2}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 111 leaves, 8 steps):

$$-\frac{e x}{a} - \frac{f x^2}{2 a} - \frac{(e+f x) \cos [c+d x]}{a d} - \frac{(e+f x) \cot \left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]}{a d} + \frac{2 f \operatorname{Log}\left[\sin \left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]\right]}{a d^2} + \frac{f \sin [c+d x]}{a d^2}$$

Result (type 3, 236 leaves):

$$-\frac{1}{2 a d^2 (1+\sin [c+d x])} \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right] \right) \left(\sin \left[\frac{1}{2}(c+d x)\right] \left(-4 d e + 2 c d e + 2 c f - c^2 f + 2 d^2 e x - 2 d f x + d^2 f x^2 + 2 d (e+f x) \cos [c+d x] - 4 f \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right] - 2 f \sin [c+d x] \right) + \cos \left[\frac{1}{2}(c+d x)\right] \left(2 c d e + 2 c f - c^2 f + 2 d^2 e x + 2 d f x + d^2 f x^2 + 2 d (e+f x) \cos [c+d x] - 4 f \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right] - 2 f \sin [c+d x] \right) \right)$$

■ **Problem 191: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+f x)^3 \sin [c+d x]^3}{a+a \sin [c+d x]} dx$$

Optimal (type 4, 382 leaves, 19 steps):

$$-\frac{3 e f^2 x}{4 a d^2} - \frac{3 f^3 x^2}{8 a d^2} + \frac{i (e+f x)^3}{a d} + \frac{3 (e+f x)^4}{8 a f} - \frac{6 f^2 (e+f x) \cos [c+d x]}{a d^3} + \frac{(e+f x)^3 \cos [c+d x]}{a d} + \frac{(e+f x)^3 \cot \left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]}{a d} - \frac{6 f (e+f x)^2 \operatorname{Log}\left[1-i e^{i(c+d x)}\right]}{a d^2} + \frac{12 i f^2 (e+f x) \operatorname{PolyLog}\left[2, i e^{i(c+d x)}\right]}{a d^3} - \frac{12 f^3 \operatorname{PolyLog}\left[3, i e^{i(c+d x)}\right]}{a d^4} + \frac{6 f^3 \sin [c+d x]}{a d^4} - \frac{3 f (e+f x)^2 \sin [c+d x]}{a d^2} + \frac{3 f^2 (e+f x) \cos [c+d x] \sin [c+d x]}{4 a d^3} - \frac{(e+f x)^3 \cos [c+d x] \sin [c+d x]}{2 a d} - \frac{3 f^3 \sin [c+d x]^2}{8 a d^4} + \frac{3 f (e+f x)^2 \sin [c+d x]^2}{4 a d^2}$$

Result (type 4, 1264 leaves):

$$\begin{aligned}
& \frac{3 e^3 x}{2 a} + \frac{9 e^2 f x^2}{4 a} + \frac{3 e f^2 x^3}{2 a} + \frac{3 f^3 x^4}{8 a} + \frac{1}{a d^4} \\
& 2 f \left(-3 d^2 (e + f x)^2 \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] + 6 i d f (e + f x) \operatorname{PolyLog}[2, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] - \right. \\
& \quad \left. 6 f^2 \operatorname{PolyLog}[3, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] + \frac{i d^3 x (3 e^2 + 3 e f x + f^2 x^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])}{\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])} \right) + \\
& \left(\frac{f^3 x^3 \operatorname{Cos}[c]}{2 a d} - \frac{i f^3 x^3 \operatorname{Sin}[c]}{2 a d} + (d^3 e^3 - 3 i d^2 e^2 f - 6 d e f^2 + 6 i f^3) \left(\frac{\operatorname{Cos}[c]}{2 a d^4} - \frac{i \operatorname{Sin}[c]}{2 a d^4} \right) + \right. \\
& \quad \left. (d^2 e^2 f - 2 i d e f^2 - 2 f^3) \left(\frac{3 x \operatorname{Cos}[c]}{2 a d^3} - \frac{3 i x \operatorname{Sin}[c]}{2 a d^3} \right) + (d e f^2 - i f^3) \left(\frac{3 x^2 \operatorname{Cos}[c]}{2 a d^2} - \frac{3 i x^2 \operatorname{Sin}[c]}{2 a d^2} \right) \right) (\operatorname{Cos}[d x] - i \operatorname{Sin}[d x]) + \\
& \left(\frac{f^3 x^3 \operatorname{Cos}[c]}{2 a d} + \frac{i f^3 x^3 \operatorname{Sin}[c]}{2 a d} + (d^3 e^3 + 3 i d^2 e^2 f - 6 d e f^2 - 6 i f^3) \left(\frac{\operatorname{Cos}[c]}{2 a d^4} + \frac{i \operatorname{Sin}[c]}{2 a d^4} \right) + \right. \\
& \quad \left. \frac{3 x^2 (d e f^2 \operatorname{Cos}[c] + i f^3 \operatorname{Cos}[c] + i d e f^2 \operatorname{Sin}[c] - f^3 \operatorname{Sin}[c])}{2 a d^2} + \right. \\
& \quad \left. \frac{3 x (d^2 e^2 f \operatorname{Cos}[c] + 2 i d e f^2 \operatorname{Cos}[c] - 2 f^3 \operatorname{Cos}[c] + i d^2 e^2 f \operatorname{Sin}[c] - 2 d e f^2 \operatorname{Sin}[c] - 2 i f^3 \operatorname{Sin}[c])}{2 a d^3} \right) (\operatorname{Cos}[d x] + i \operatorname{Sin}[d x]) + \\
& \left(-\frac{i f^3 x^3 \operatorname{Cos}[2 c]}{8 a d} - \frac{f^3 x^3 \operatorname{Sin}[2 c]}{8 a d} + (-4 i d^3 e^3 - 6 d^2 e^2 f + 6 i d e f^2 + 3 f^3) \left(\frac{\operatorname{Cos}[2 c]}{32 a d^4} - \frac{i \operatorname{Sin}[2 c]}{32 a d^4} \right) + \right. \\
& \quad \left. (2 d^2 e^2 f - 2 i d e f^2 - f^3) \left(-\frac{3 i x \operatorname{Cos}[2 c]}{16 a d^3} - \frac{3 x \operatorname{Sin}[2 c]}{16 a d^3} \right) + (2 d e f^2 - i f^3) \left(-\frac{3 i x^2 \operatorname{Cos}[2 c]}{16 a d^2} - \frac{3 x^2 \operatorname{Sin}[2 c]}{16 a d^2} \right) \right) (\operatorname{Cos}[2 d x] - i \operatorname{Sin}[2 d x]) + \\
& \left(\frac{i f^3 x^3 \operatorname{Cos}[2 c]}{8 a d} - \frac{f^3 x^3 \operatorname{Sin}[2 c]}{8 a d} + (4 i d^3 e^3 - 6 d^2 e^2 f - 6 i d e f^2 + 3 f^3) \left(\frac{\operatorname{Cos}[2 c]}{32 a d^4} + \frac{i \operatorname{Sin}[2 c]}{32 a d^4} \right) + \right. \\
& \quad \left. \frac{3 i x^2 (2 d e f^2 \operatorname{Cos}[2 c] + i f^3 \operatorname{Cos}[2 c] + 2 i d e f^2 \operatorname{Sin}[2 c] - f^3 \operatorname{Sin}[2 c])}{16 a d^2} + \frac{1}{16 a d^3} \right. \\
& \quad \left. 3 i x (2 d^2 e^2 f \operatorname{Cos}[2 c] + 2 i d e f^2 \operatorname{Cos}[2 c] - f^3 \operatorname{Cos}[2 c] + 2 i d^2 e^2 f \operatorname{Sin}[2 c] - 2 d e f^2 \operatorname{Sin}[2 c] - i f^3 \operatorname{Sin}[2 c]) \right) \\
& (\operatorname{Cos}[2 d x] + i \operatorname{Sin}[2 d x]) - \frac{2 (e^3 \operatorname{Sin}[\frac{d x}{2}] + 3 e^2 f x \operatorname{Sin}[\frac{d x}{2}] + 3 e f^2 x^2 \operatorname{Sin}[\frac{d x}{2}] + f^3 x^3 \operatorname{Sin}[\frac{d x}{2}])}{a d (\operatorname{Cos}[\frac{c}{2}] + \operatorname{Sin}[\frac{c}{2}]) (\operatorname{Cos}[\frac{c}{2} + \frac{d x}{2}] + \operatorname{Sin}[\frac{c}{2} + \frac{d x}{2}])}
\end{aligned}$$

■ **Problem 192: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \operatorname{Sin}[c + d x]^3}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 278 leaves, 17 steps):

$$\begin{aligned}
& -\frac{f^2 x}{4 a d^2} + \frac{i (e+f x)^2}{a d} + \frac{(e+f x)^3}{2 a f} - \frac{2 f^2 \operatorname{Cos}[c+d x]}{a d^3} + \frac{(e+f x)^2 \operatorname{Cos}[c+d x]}{a d} + \\
& \frac{(e+f x)^2 \operatorname{Cot}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]}{a d} - \frac{4 f (e+f x) \operatorname{Log}\left[1 - i e^{i(c+d x)}\right]}{a d^2} + \frac{4 i f^2 \operatorname{PolyLog}\left[2, i e^{i(c+d x)}\right]}{a d^3} - \\
& \frac{2 f (e+f x) \operatorname{Sin}[c+d x]}{a d^2} + \frac{f^2 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{4 a d^3} - \frac{(e+f x)^2 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 a d} + \frac{f (e+f x) \operatorname{Sin}[c+d x]^2}{2 a d^2}
\end{aligned}$$

Result (type 4, 931 leaves):

$$\begin{aligned}
& \frac{1}{16 a d^3 \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} \\
& \left(8 d^2 e^2 \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 16 f^2 \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 48 d^2 e f x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 24 d^2 f^2 x^2 \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 6 d^2 e^2 \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] - 15 f^2 \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + \right. \\
& 12 d^2 e f x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + 6 d^2 f^2 x^2 \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + 14 d e f \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] + 14 d f^2 x \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] - 2 d e f \operatorname{Cos}\left[2 c + \frac{5 d x}{2}\right] - \\
& 2 d f^2 x \operatorname{Cos}\left[2 c + \frac{5 d x}{2}\right] + 2 d^2 e^2 \operatorname{Cos}\left[3 c + \frac{5 d x}{2}\right] - f^2 \operatorname{Cos}\left[3 c + \frac{5 d x}{2}\right] + 4 d^2 e f x \operatorname{Cos}\left[3 c + \frac{5 d x}{2}\right] + 2 d^2 f^2 x^2 \operatorname{Cos}\left[3 c + \frac{5 d x}{2}\right] + \\
& 8 d \operatorname{Cos}\left[\frac{d x}{2}\right] \left(3 d^2 e^2 x + f^2 x (-2 + 2 i d x + d^2 x^2) + e f (-2 + 4 i d x + 3 d^2 x^2) - 8 f (e+f x) \operatorname{Log}\left[1 - i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]\right]\right) - \\
& 40 d^2 e^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + 16 f^2 \operatorname{Sin}\left[\frac{d x}{2}\right] - 48 d^2 e f x \operatorname{Sin}\left[\frac{d x}{2}\right] - 24 d^2 f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] - 16 d e f \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 24 d^3 e^2 x \operatorname{Sin}\left[c + \frac{d x}{2}\right] + \\
& 32 i d^2 e f x \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 16 d f^2 x \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 24 d^3 e f x^2 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 16 i d^2 f^2 x^2 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 8 d^3 f^2 x^3 \operatorname{Sin}\left[c + \frac{d x}{2}\right] - \\
& 64 d e f \operatorname{Log}\left[1 - i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]\right] \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 64 d f^2 x \operatorname{Log}\left[1 - i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]\right] \operatorname{Sin}\left[c + \frac{d x}{2}\right] + \\
& 64 i f^2 \operatorname{PolyLog}\left[2, i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]\right] \left(\operatorname{Cos}\left[\frac{d x}{2}\right] + \operatorname{Sin}\left[c + \frac{d x}{2}\right]\right) - 14 d e f \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 14 d f^2 x \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + \\
& 6 d^2 e^2 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 15 f^2 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 12 d^2 e f x \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 6 d^2 f^2 x^2 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 2 d^2 e^2 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + \\
& f^2 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 4 d^2 e f x \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 2 d^2 f^2 x^2 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 2 d e f \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] - 2 d f^2 x \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] \left. \right)
\end{aligned}$$

■ **Problem 199: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+f x) \operatorname{Csc}[c+d x]}{a+a \operatorname{Sin}[c+d x]} dx$$

Optimal (type 4, 134 leaves, 9 steps):

$$-\frac{2(e+f x) \operatorname{ArcTanh}\left[e^{i(c+d x)}\right]}{a d} + \frac{(e+f x) \operatorname{Cot}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]}{a d} - \frac{2 f \operatorname{Log}\left[\operatorname{Sin}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]\right]}{a d^2} + \frac{i f \operatorname{PolyLog}\left[2, -e^{i(c+d x)}\right]}{a d^2} - \frac{i f \operatorname{PolyLog}\left[2, e^{i(c+d x)}\right]}{a d^2}$$

Result (type 4, 300 leaves) :

$$\begin{aligned} & \frac{1}{a d^2 (1 + \sin[c + d x])} \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) \left(-2 d (e + f x) \sin\left[\frac{1}{2}(c + d x)\right] + \right. \\ & \quad \left. f (c + d x) \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) - 2 f \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) + \right. \\ & \quad \left. d e \operatorname{Log}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) - c f \operatorname{Log}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) + \right. \\ & \quad \left. f ((c + d x) (\operatorname{Log}[1 - e^{i(c + d x)}] - \operatorname{Log}[1 + e^{i(c + d x)}]) + i (\operatorname{PolyLog}[2, -e^{i(c + d x)}] - \operatorname{PolyLog}[2, e^{i(c + d x)}])) \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) \right) \end{aligned}$$

■ **Problem 200: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c + d x]}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 38 leaves, 3 steps) :

$$-\frac{\operatorname{ArcTanh}[\cos[c + d x]]}{a d} + \frac{\cos[c + d x]}{d (a + a \sin[c + d x])}$$

Result (type 3, 113 leaves) :

$$\begin{aligned} & -\frac{1}{a d (1 + \sin[c + d x])} \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) \\ & \quad \left(\cos\left[\frac{1}{2}(c + d x)\right] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \right) + \left(2 + \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \right) \sin\left[\frac{1}{2}(c + d x)\right] \right) \end{aligned}$$

■ **Problem 203: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \operatorname{Csc}[c + d x]^2}{a + a \sin[c + d x]} dx$$

Optimal (type 4, 463 leaves, 24 steps) :

$$\begin{aligned}
& - \frac{2 i (e+f x)^3}{a d} + \frac{2 (e+f x)^3 \operatorname{ArcTanh}\left[e^{i(c+d x)}\right]}{a d} - \frac{(e+f x)^3 \operatorname{Cot}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]}{a d} - \frac{(e+f x)^3 \operatorname{Cot}[c+d x]}{a d} + \\
& \frac{6 f (e+f x)^2 \operatorname{Log}\left[1-i e^{i(c+d x)}\right]}{a d^2} + \frac{3 f (e+f x)^2 \operatorname{Log}\left[1-e^{2 i(c+d x)}\right]}{a d^2} - \frac{3 i f (e+f x)^2 \operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right]}{a d^2} - \\
& \frac{12 i f^2 (e+f x) \operatorname{PolyLog}\left[2, i e^{i(c+d x)}\right]}{a d^3} + \frac{3 i f (e+f x)^2 \operatorname{PolyLog}\left[2, e^{i(c+d x)}\right]}{a d^2} - \frac{3 i f^2 (e+f x) \operatorname{PolyLog}\left[2, e^{2 i(c+d x)}\right]}{a d^3} + \\
& \frac{6 f^2 (e+f x) \operatorname{PolyLog}\left[3,-e^{i(c+d x)}\right]}{a d^3} + \frac{12 f^3 \operatorname{PolyLog}\left[3, i e^{i(c+d x)}\right]}{a d^4} - \frac{6 f^2 (e+f x) \operatorname{PolyLog}\left[3, e^{i(c+d x)}\right]}{a d^3} + \\
& \frac{3 f^3 \operatorname{PolyLog}\left[3, e^{2 i(c+d x)}\right]}{2 a d^4} + \frac{6 i f^3 \operatorname{PolyLog}\left[4,-e^{i(c+d x)}\right]}{a d^4} - \frac{6 i f^3 \operatorname{PolyLog}\left[4, e^{i(c+d x)}\right]}{a d^4}
\end{aligned}$$

Result (type 4, 1208 leaves):

$$\begin{aligned}
& - \frac{e^3 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]}{ad} - \frac{1}{ad^2} \\
& 3e^2 f \left((c+dx) \left(\operatorname{Log}\left[1 - e^{i(c+dx)}\right] - \operatorname{Log}\left[1 + e^{i(c+dx)}\right] \right) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] + i \left(\operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right] - \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right] \right) \right) - \\
& \frac{1}{4ad^4} e^{-ic} f^3 \operatorname{Csc}[c] \\
& \left(2d^2 x^2 \left(2de^{2ic}x + 3i(-1 + e^{2ic}) \operatorname{Log}\left[1 - e^{2i(c+dx)}\right] \right) + 6d(-1 + e^{2ic})x \operatorname{PolyLog}\left[2, e^{2i(c+dx)}\right] + 3i(-1 + e^{2ic}) \operatorname{PolyLog}\left[3, e^{2i(c+dx)}\right] \right) + \\
& \frac{1}{ad^3} 6ef^2 \left(d^2 x^2 \operatorname{ArcTanh}\left[\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] - idx \operatorname{PolyLog}\left[2, -\operatorname{Cos}[c+dx] - i \operatorname{Sin}[c+dx]\right] \right) + \\
& idx \operatorname{PolyLog}\left[2, \operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] + \operatorname{PolyLog}\left[3, -\operatorname{Cos}[c+dx] - i \operatorname{Sin}[c+dx]\right] - \operatorname{PolyLog}\left[3, \operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] \right) - \\
& \frac{1}{ad^4} f^3 \left(-2d^3 x^3 \operatorname{ArcTanh}\left[\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] + 3id^2 x^2 \operatorname{PolyLog}\left[2, -\operatorname{Cos}[c+dx] - i \operatorname{Sin}[c+dx]\right] - \right. \\
& \left. 3id^2 x^2 \operatorname{PolyLog}\left[2, \operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] - 6dx \operatorname{PolyLog}\left[3, -\operatorname{Cos}[c+dx] - i \operatorname{Sin}[c+dx]\right] + 6dx \right. \\
& \left. \operatorname{PolyLog}\left[3, \operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] - 6i \operatorname{PolyLog}\left[4, -\operatorname{Cos}[c+dx] - i \operatorname{Sin}[c+dx]\right] + 6i \operatorname{PolyLog}\left[4, \operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] \right) + \\
& \frac{3e^2 f \operatorname{Csc}[c] (-dx \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[dx] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[dx]]) \operatorname{Sin}[c]}{ad^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} + \frac{1}{ad^4} \\
& 2f \left(3d^2 (e+fx)^2 \operatorname{Log}\left[1 - i \operatorname{Cos}[c+dx] + \operatorname{Sin}[c+dx]\right] - 6idf(e+fx) \operatorname{PolyLog}\left[2, i \operatorname{Cos}[c+dx] - \operatorname{Sin}[c+dx]\right] + \right. \\
& \left. 6f^2 \operatorname{PolyLog}\left[3, i \operatorname{Cos}[c+dx] - \operatorname{Sin}[c+dx]\right] + \frac{d^3 x (3e^2 + 3efx + f^2 x^2) (-i \operatorname{Cos}[c] + \operatorname{Sin}[c])}{\operatorname{Cos}[c] + i(1 + \operatorname{Sin}[c])} \right) + \\
& \frac{\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(e^3 \operatorname{Sin}\left[\frac{dx}{2}\right] + 3e^2 f x \operatorname{Sin}\left[\frac{dx}{2}\right] + 3ef^2 x^2 \operatorname{Sin}\left[\frac{dx}{2}\right] + f^3 x^3 \operatorname{Sin}\left[\frac{dx}{2}\right] \right)}{2ad} + \\
& \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(e^3 \operatorname{Sin}\left[\frac{dx}{2}\right] + 3e^2 f x \operatorname{Sin}\left[\frac{dx}{2}\right] + 3ef^2 x^2 \operatorname{Sin}\left[\frac{dx}{2}\right] + f^3 x^3 \operatorname{Sin}\left[\frac{dx}{2}\right] \right)}{2ad} + \\
& \frac{2 \left(e^3 \operatorname{Sin}\left[\frac{dx}{2}\right] + 3e^2 f x \operatorname{Sin}\left[\frac{dx}{2}\right] + 3ef^2 x^2 \operatorname{Sin}\left[\frac{dx}{2}\right] + f^3 x^3 \operatorname{Sin}\left[\frac{dx}{2}\right] \right)}{ad \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} - \\
& \left(3ef^2 \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \right) / \left(\sqrt{1 + \operatorname{Tan}[c]^2} \right) \left(idx (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]]) - \pi \operatorname{Log}\left[1 + e^{-2idx}\right] - \right. \right. \\
& \left. \left. 2(dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}\left[1 - e^{2i(dx + \operatorname{ArcTan}[\operatorname{Tan}[c])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[dx]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \right] \right) + \right. \\
& \left. i \operatorname{PolyLog}\left[2, e^{2i(dx + \operatorname{ArcTan}[\operatorname{Tan}[c])}\right] \operatorname{Tan}[c] \right) / \left(ad^3 \sqrt{\operatorname{Sec}[c]^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} \right)
\end{aligned}$$

■ **Problem 204: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+fx)^2 \operatorname{Csc}[c+dx]^2}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 4, 327 leaves, 20 steps):

$$\begin{aligned} & -\frac{2 i (e+f x)^2}{a d} + \frac{2 (e+f x)^2 \operatorname{ArcTanh}\left[e^{i(c+d x)}\right]}{a d} - \frac{(e+f x)^2 \operatorname{Cot}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]}{a d} - \frac{(e+f x)^2 \operatorname{Cot}[c+d x]}{a d} + \\ & \frac{4 f (e+f x) \operatorname{Log}\left[1-i e^{i(c+d x)}\right]}{a d^2} + \frac{2 f (e+f x) \operatorname{Log}\left[1-e^{2 i(c+d x)}\right]}{a d^2} - \frac{2 i f (e+f x) \operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right]}{a d^2} - \frac{4 i f^2 \operatorname{PolyLog}\left[2, i e^{i(c+d x)}\right]}{a d^3} + \\ & \frac{2 i f (e+f x) \operatorname{PolyLog}\left[2, e^{i(c+d x)}\right]}{a d^2} - \frac{i f^2 \operatorname{PolyLog}\left[2, e^{2 i(c+d x)}\right]}{a d^3} + \frac{2 f^2 \operatorname{PolyLog}\left[3,-e^{i(c+d x)}\right]}{a d^3} - \frac{2 f^2 \operatorname{PolyLog}\left[3, e^{i(c+d x)}\right]}{a d^3} \end{aligned}$$

Result (type 4, 663 leaves):

$$\begin{aligned} & \frac{1}{a d^3} \left(-2 i d^2 e f x - i d^2 f^2 x^2 + 2 d^2 e^2 \operatorname{ArcTanh}[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]] + 4 d^2 e f x \operatorname{ArcTanh}[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]] + \right. \\ & \quad 2 d^2 f^2 x^2 \operatorname{ArcTanh}[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]] - 2 d^2 e f x \operatorname{Cot}[c] - d^2 f^2 x^2 \operatorname{Cot}[c] + 2 d e f \operatorname{Log}[1-\operatorname{Cos}[2(c+d x)] - i \operatorname{Sin}[2(c+d x)]] + \\ & \quad 2 d f^2 x \operatorname{Log}[1-\operatorname{Cos}[2(c+d x)] - i \operatorname{Sin}[2(c+d x)]] - 2 i d f (e+f x) \operatorname{PolyLog}[2,-\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]] + \\ & \quad 2 i d f (e+f x) \operatorname{PolyLog}[2,\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]] - i f^2 \operatorname{PolyLog}[2,\operatorname{Cos}[2(c+d x)] + i \operatorname{Sin}[2(c+d x)]] + \\ & \quad \left. 2 f^2 \operatorname{PolyLog}[3,-\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]] - 2 f^2 \operatorname{PolyLog}[3,\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]] \right) - \frac{1}{a d^3} \\ & \quad 2 i f \left(2 i d (e+f x) \operatorname{Log}[1-i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]] + 2 f \operatorname{PolyLog}[2, i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]] + \frac{d^2 x (2 e+f x) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])}{\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])} \right) + \\ & \quad \frac{\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{d x}{2}\right] \left(e^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + 2 e f x \operatorname{Sin}\left[\frac{d x}{2}\right] + f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] \right)}{2 a d} + \\ & \quad \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] \left(e^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + 2 e f x \operatorname{Sin}\left[\frac{d x}{2}\right] + f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] \right)}{2 a d} + \\ & \quad \frac{2 \left(e^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + 2 e f x \operatorname{Sin}\left[\frac{d x}{2}\right] + f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] \right)}{a d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right] \right)} \end{aligned}$$

■ **Problem 205: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+f x) \operatorname{Csc}[c+d x]^2}{a+a \operatorname{Sin}[c+d x]} dx$$

Optimal (type 4, 169 leaves, 12 steps):

$$\begin{aligned} & \frac{2 (e+f x) \operatorname{ArcTanh}\left[e^{i(c+d x)}\right]}{a d} - \frac{(e+f x) \operatorname{Cot}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]}{a d} - \frac{(e+f x) \operatorname{Cot}[c+d x]}{a d} + \\ & \frac{2 f \operatorname{Log}\left[\operatorname{Sin}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]\right]}{a d^2} + \frac{f \operatorname{Log}[\operatorname{Sin}[c+d x]]}{a d^2} - \frac{i f \operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right]}{a d^2} + \frac{i f \operatorname{PolyLog}\left[2, e^{i(c+d x)}\right]}{a d^2} \end{aligned}$$

Result (type 4, 396 leaves):

$$\frac{1}{2 a d^2 (1 + \sin[c + d x])} \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) \left(-d(e + f x) \cos\left[\frac{1}{2}(c + d x)\right] \left(1 + \cot\left[\frac{1}{2}(c + d x)\right] \right) + 4 d(e + f x) \sin\left[\frac{1}{2}(c + d x)\right] - 2 f(c + d x) \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) + 4 f \right. \\ \left. \log\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) + 2 f \log[\sin[c + d x]] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) \right) - \\ 2 d e \log\left[\tan\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) + 2 c f \log\left[\tan\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) - \\ 2 f((c + d x) (\log[1 - e^{i(c + d x)}] - \log[1 + e^{i(c + d x)}]) + i (\text{PolyLog}[2, -e^{i(c + d x)}] - \text{PolyLog}[2, e^{i(c + d x)}])) \\ \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) + d(e + f x) \sin\left[\frac{1}{2}(c + d x)\right] \left(1 + \tan\left[\frac{1}{2}(c + d x)\right] \right) \right)$$

■ **Problem 206: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c + d x]^2}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\frac{\text{ArcTanh}[\cos[c + d x]]}{a d} - \frac{2 \cot[c + d x]}{a d} + \frac{\cot[c + d x]}{d(a + a \sin[c + d x])}$$

Result (type 3, 167 leaves):

$$\frac{1}{2 a d (1 + \sin[c + d x])} \left(-\cos\left[\frac{1}{2}(c + d x)\right]^2 \left(2 + \cot\left[\frac{1}{2}(c + d x)\right] - 2 \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] + 2 \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \right) + \right. \\ \left. 2 \left(\left(3 + \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \right) \sin\left[\frac{1}{2}(c + d x)\right]^2 + \right. \right. \\ \left. \left. \csc[c + d x] \sin\left[\frac{1}{2}(c + d x)\right]^4 + \left(1 + \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \right) \sin[c + d x] \right) \right)$$

■ **Problem 209: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \csc[c + d x]^3}{a + a \sin[c + d x]} dx$$

Optimal (type 4, 600 leaves, 40 steps):

$$\begin{aligned}
& \frac{2 i (e+f x)^3}{a d} - \frac{6 f^2 (e+f x) \operatorname{ArcTanh}\left[e^{i(c+d x)}\right]}{a d^3} - \frac{3 (e+f x)^3 \operatorname{ArcTanh}\left[e^{i(c+d x)}\right]}{a d} + \frac{(e+f x)^3 \operatorname{Cot}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]}{a d} + \\
& \frac{(e+f x)^3 \operatorname{Cot}[c+d x]}{a d} - \frac{3 f (e+f x)^2 \operatorname{Csc}[c+d x]}{2 a d^2} - \frac{(e+f x)^3 \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]}{2 a d} - \frac{6 f (e+f x)^2 \operatorname{Log}\left[1-i e^{i(c+d x)}\right]}{a d^2} - \\
& \frac{3 f (e+f x)^2 \operatorname{Log}\left[1-e^{2 i(c+d x)}\right]}{a d^2} + \frac{3 i f^3 \operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right]}{a d^4} + \frac{9 i f (e+f x)^2 \operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right]}{2 a d^2} + \\
& \frac{12 i f^2 (e+f x) \operatorname{PolyLog}\left[2, i e^{i(c+d x)}\right]}{a d^3} - \frac{3 i f^3 \operatorname{PolyLog}\left[2, e^{i(c+d x)}\right]}{a d^4} - \frac{9 i f (e+f x)^2 \operatorname{PolyLog}\left[2, e^{i(c+d x)}\right]}{2 a d^2} + \\
& \frac{3 i f^2 (e+f x) \operatorname{PolyLog}\left[2, e^{2 i(c+d x)}\right]}{a d^3} - \frac{9 f^2 (e+f x) \operatorname{PolyLog}\left[3,-e^{i(c+d x)}\right]}{a d^3} - \frac{12 f^3 \operatorname{PolyLog}\left[3, i e^{i(c+d x)}\right]}{a d^4} + \\
& \frac{9 f^2 (e+f x) \operatorname{PolyLog}\left[3, e^{i(c+d x)}\right]}{a d^3} - \frac{3 f^3 \operatorname{PolyLog}\left[3, e^{2 i(c+d x)}\right]}{2 a d^4} - \frac{9 i f^3 \operatorname{PolyLog}\left[4,-e^{i(c+d x)}\right]}{a d^4} + \frac{9 i f^3 \operatorname{PolyLog}\left[4, e^{i(c+d x)}\right]}{a d^4}
\end{aligned}$$

Result (type 4, 1370 leaves):

$$\begin{aligned}
& \frac{3 e^3 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]}{2 a d} + \frac{3 e f^2 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]}{a d^3} + \frac{1}{2 a d^2} \\
& 9 e^2 f \left((c+d x) \left(\operatorname{Log}\left[1-e^{i(c+d x)}\right] - \operatorname{Log}\left[1+e^{i(c+d x)}\right] \right) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right] + i \left(\operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right] - \operatorname{PolyLog}\left[2,e^{i(c+d x)}\right] \right) \right) + \\
& \frac{1}{a d^4} 3 f^3 \left((c+d x) \left(\operatorname{Log}\left[1-e^{i(c+d x)}\right] - \operatorname{Log}\left[1+e^{i(c+d x)}\right] \right) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right] + i \left(\operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right] - \operatorname{PolyLog}\left[2,e^{i(c+d x)}\right] \right) \right) + \\
& \frac{1}{4 a d^4} e^{-i c} f^3 \operatorname{Csc}[c] \\
& \left(2 d^2 x^2 \left(2 d e^{2 i c} x + 3 i \left(-1 + e^{2 i c} \right) \operatorname{Log}\left[1-e^{2 i(c+d x)}\right] \right) + 6 d \left(-1 + e^{2 i c} \right) x \operatorname{PolyLog}\left[2,e^{2 i(c+d x)}\right] + 3 i \left(-1 + e^{2 i c} \right) \operatorname{PolyLog}\left[3,e^{2 i(c+d x)}\right] \right) - \\
& \frac{1}{a d^3} 9 e f^2 \left(d^2 x^2 \operatorname{ArcTanh}\left[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] - i d x \operatorname{PolyLog}\left[2,-\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]\right] + \right. \\
& \left. i d x \operatorname{PolyLog}\left[2,\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] + \operatorname{PolyLog}\left[3,-\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]\right] - \operatorname{PolyLog}\left[3,\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] \right) + \\
& \frac{1}{2 a d^4} 3 f^3 \left(-2 d^3 x^3 \operatorname{ArcTanh}\left[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] + 3 i d^2 x^2 \operatorname{PolyLog}\left[2,-\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]\right] - \right. \\
& \left. 3 i d^2 x^2 \operatorname{PolyLog}\left[2,\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] - 6 d x \operatorname{PolyLog}\left[3,-\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]\right] + 6 d x \right. \\
& \left. \operatorname{PolyLog}\left[3,\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] - 6 i \operatorname{PolyLog}\left[4,-\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]\right] + 6 i \operatorname{PolyLog}\left[4,\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] \right) - \\
& \frac{3 e^2 f \operatorname{Csc}[c] \left(-d x \operatorname{Cos}[c] + \operatorname{Log}\left[\operatorname{Cos}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x]\right] \operatorname{Sin}[c] \right)}{a d^2 \left(\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2 \right)} + \frac{1}{a d^4} \\
& 2 f \left(-3 d^2 (e+f x)^2 \operatorname{Log}\left[1-i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]\right] + 6 i d f (e+f x) \operatorname{PolyLog}\left[2,i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]\right] - \right. \\
& \left. 6 f^2 \operatorname{PolyLog}\left[3,i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]\right] + \frac{i d^3 x \left(3 e^2 + 3 e f x + f^2 x^2 \right) \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c] \right)}{\operatorname{Cos}[c] + i \left(1 + \operatorname{Sin}[c] \right)} \right) + \\
& \frac{\operatorname{Csc}[c] \operatorname{Csc}[c+d x]^2 \left(e^3 \operatorname{Sin}[d x] + 3 e^2 f x \operatorname{Sin}[d x] + 3 e f^2 x^2 \operatorname{Sin}[d x] + f^3 x^3 \operatorname{Sin}[d x] \right)}{2 a d} + \frac{1}{2 a d^2} \\
& \operatorname{Csc}[c] \operatorname{Csc}[c+d x] \left(-d e^3 \operatorname{Cos}[c] - 3 d e^2 f x \operatorname{Cos}[c] - 3 d e f^2 x^2 \operatorname{Cos}[c] - d f^3 x^3 \operatorname{Cos}[c] - 3 e^2 f \operatorname{Sin}[c] - \right. \\
& \left. 6 e f^2 x \operatorname{Sin}[c] - 3 f^3 x^2 \operatorname{Sin}[c] - 2 d e^3 \operatorname{Sin}[d x] - 6 d e^2 f x \operatorname{Sin}[d x] - 6 d e f^2 x^2 \operatorname{Sin}[d x] - 2 d f^3 x^3 \operatorname{Sin}[d x] \right) - \\
& \frac{2 \left(e^3 \operatorname{Sin}\left[\frac{d x}{2}\right] + 3 e^2 f x \operatorname{Sin}\left[\frac{d x}{2}\right] + 3 e f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + f^3 x^3 \operatorname{Sin}\left[\frac{d x}{2}\right] \right)}{a d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right] \right)} + \\
& \left(3 e f^2 \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \right) / \left(\sqrt{1 + \operatorname{Tan}[c]^2} \right) \left(i d x \left(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i d x}\right] - \right. \right. \\
& \left. \left. 2 \left(d x + \operatorname{ArcTan}[\operatorname{Tan}[c]] \right) \operatorname{Log}\left[1 - e^{2 i \left(d x + \operatorname{ArcTan}[\operatorname{Tan}[c]] \right)}\right] + \pi \operatorname{Log}\left[\operatorname{Cos}[d x]\right] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}\left[\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]\right] \right) + \right. \\
& \left. i \operatorname{PolyLog}\left[2,e^{2 i \left(d x + \operatorname{ArcTan}[\operatorname{Tan}[c]] \right)}\right] \operatorname{Tan}[c] \right) / \left(a d^3 \sqrt{\operatorname{Sec}[c]^2 \left(\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2 \right)} \right)
\end{aligned}$$

■ **Problem 210: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \operatorname{Csc}[c + d x]^3}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 392 leaves, 30 steps):

$$\begin{aligned} & \frac{2 i (e + f x)^2}{a d} - \frac{3 (e + f x)^2 \operatorname{ArcTanh}\left[e^{i(c+d x)}\right]}{a d} - \frac{f^2 \operatorname{ArcTanh}[\operatorname{Cos}[c + d x]]}{a d^3} + \frac{(e + f x)^2 \operatorname{Cot}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{d x}{2}\right]}{a d} + \\ & \frac{(e + f x)^2 \operatorname{Cot}[c + d x]}{a d} - \frac{f (e + f x) \operatorname{Csc}[c + d x]}{a d^2} - \frac{(e + f x)^2 \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]}{2 a d} - \frac{4 f (e + f x) \operatorname{Log}\left[1 - i e^{i(c+d x)}\right]}{a d^2} - \\ & \frac{2 f (e + f x) \operatorname{Log}\left[1 - e^{2 i(c+d x)}\right]}{a d^2} + \frac{3 i f (e + f x) \operatorname{PolyLog}\left[2, -e^{i(c+d x)}\right]}{a d^2} + \frac{4 i f^2 \operatorname{PolyLog}\left[2, i e^{i(c+d x)}\right]}{a d^3} - \\ & \frac{3 i f (e + f x) \operatorname{PolyLog}\left[2, e^{i(c+d x)}\right]}{a d^2} + \frac{i f^2 \operatorname{PolyLog}\left[2, e^{2 i(c+d x)}\right]}{a d^3} - \frac{3 f^2 \operatorname{PolyLog}\left[3, -e^{i(c+d x)}\right]}{a d^3} + \frac{3 f^2 \operatorname{PolyLog}\left[3, e^{i(c+d x)}\right]}{a d^3} \end{aligned}$$

Result (type 4, 1420 leaves):

$$\begin{aligned}
& \frac{3 e^2 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]}{2 a d} + \frac{f^2 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]}{a d^3} + \frac{1}{a d^2} \\
& 3 e f \left((c+d x) \left(\operatorname{Log}\left[1-e^{i(c+d x)}\right] - \operatorname{Log}\left[1+e^{i(c+d x)}\right] \right) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right] + i \left(\operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right] - \operatorname{PolyLog}\left[2,e^{i(c+d x)}\right] \right) \right) - \\
& \frac{1}{a d^3} 3 f^2 \left(d^2 x^2 \operatorname{ArcTanh}\left[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] - i d x \operatorname{PolyLog}\left[2,-\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]\right] + \right. \\
& \quad \left. i d x \operatorname{PolyLog}\left[2,\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] + \operatorname{PolyLog}\left[3,-\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]\right] - \operatorname{PolyLog}\left[3,\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] \right) - \\
& \frac{2 e f \operatorname{Csc}[c] (-d x \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x]] \operatorname{Sin}[c])}{a d^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} + \frac{1}{a d^3} \\
& 2 i f \left(2 i d (e+f x) \operatorname{Log}\left[1-i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]\right] + 2 f \operatorname{PolyLog}\left[2,i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]\right] + \frac{d^2 x (2 e+f x) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])}{\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])} \right) + \\
& \frac{1}{8 a d^2 \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)} \\
& \operatorname{Csc}[c] \operatorname{Csc}[c+d x]^2 \left(-2 e f \operatorname{Cos}\left[\frac{d x}{2}\right] - 2 f^2 x \operatorname{Cos}\left[\frac{d x}{2}\right] - 2 e f \operatorname{Cos}\left[\frac{3 d x}{2}\right] - 2 f^2 x \operatorname{Cos}\left[\frac{3 d x}{2}\right] - 5 d e^2 \operatorname{Cos}\left[c - \frac{d x}{2}\right] - 10 d e f x \operatorname{Cos}\left[c - \frac{d x}{2}\right] - \right. \\
& \quad 5 d f^2 x^2 \operatorname{Cos}\left[c - \frac{d x}{2}\right] + d e^2 \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 2 d e f x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + d f^2 x^2 \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 2 e f \operatorname{Cos}\left[2 c + \frac{d x}{2}\right] + 2 f^2 x \operatorname{Cos}\left[2 c + \frac{d x}{2}\right] - \\
& \quad d e^2 \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] - 2 d e f x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] - d f^2 x^2 \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + 2 e f \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] + 2 f^2 x \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] + \\
& \quad 3 d e^2 \operatorname{Cos}\left[3 c + \frac{3 d x}{2}\right] + 6 d e f x \operatorname{Cos}\left[3 c + \frac{3 d x}{2}\right] + 3 d f^2 x^2 \operatorname{Cos}\left[3 c + \frac{3 d x}{2}\right] + 4 d e^2 \operatorname{Cos}\left[c + \frac{5 d x}{2}\right] + 8 d e f x \operatorname{Cos}\left[c + \frac{5 d x}{2}\right] + \\
& \quad 4 d f^2 x^2 \operatorname{Cos}\left[c + \frac{5 d x}{2}\right] - 2 d e^2 \operatorname{Cos}\left[3 c + \frac{5 d x}{2}\right] - 4 d e f x \operatorname{Cos}\left[3 c + \frac{5 d x}{2}\right] - 2 d f^2 x^2 \operatorname{Cos}\left[3 c + \frac{5 d x}{2}\right] - d e^2 \operatorname{Sin}\left[\frac{d x}{2}\right] - \\
& \quad 2 d e f x \operatorname{Sin}\left[\frac{d x}{2}\right] - d f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] - d e^2 \operatorname{Sin}\left[\frac{3 d x}{2}\right] - 2 d e f x \operatorname{Sin}\left[\frac{3 d x}{2}\right] - d f^2 x^2 \operatorname{Sin}\left[\frac{3 d x}{2}\right] - 2 e f \operatorname{Sin}\left[c - \frac{d x}{2}\right] - \\
& \quad 2 f^2 x \operatorname{Sin}\left[c - \frac{d x}{2}\right] - 2 e f \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 2 f^2 x \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 3 d e^2 \operatorname{Sin}\left[2 c + \frac{d x}{2}\right] - 6 d e f x \operatorname{Sin}\left[2 c + \frac{d x}{2}\right] - 3 d f^2 x^2 \operatorname{Sin}\left[2 c + \frac{d x}{2}\right] - \\
& \quad 2 e f \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 2 f^2 x \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - d e^2 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 2 d e f x \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - d f^2 x^2 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + \\
& \quad \left. 2 e f \operatorname{Sin}\left[3 c + \frac{3 d x}{2}\right] + 2 f^2 x \operatorname{Sin}\left[3 c + \frac{3 d x}{2}\right] + 2 d e^2 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 4 d e f x \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 2 d f^2 x^2 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] \right) + \\
& \left(f^2 \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \right) / \left(\sqrt{1 + \operatorname{Tan}[c]^2} \right) \left(i d x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]]) - \pi \operatorname{Log}\left[1 + e^{-2 i d x}\right] - \right. \right. \\
& \quad \left. \left. 2 (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}\left[1 - e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[d x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \right] \right) + \right. \\
& \quad \left. i \operatorname{PolyLog}\left[2, e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]])}\right] \operatorname{Tan}[c] \right) / \left(a d^3 \sqrt{\operatorname{Sec}[c]^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} \right)
\end{aligned}$$

■ **Problem 211: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x) \operatorname{Csc}[c + d x]^3}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 216 leaves, 19 steps):

$$\begin{aligned} & - \frac{3 (e + f x) \operatorname{ArcTanh}\left[e^{i(c+dx)}\right]}{a d} + \frac{(e + f x) \operatorname{Cot}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right]}{a d} + \frac{(e + f x) \operatorname{Cot}[c + d x]}{a d} - \frac{f \operatorname{Csc}[c + d x]}{2 a d^2} - \frac{(e + f x) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]}{2 a d} \\ & - \frac{2 f \operatorname{Log}\left[\operatorname{Sin}\left[\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right]\right]}{a d^2} - \frac{f \operatorname{Log}[\operatorname{Sin}[c + d x]]}{a d^2} + \frac{3 i f \operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right]}{2 a d^2} - \frac{3 i f \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right]}{2 a d^2} \end{aligned}$$

Result (type 4, 830 leaves):

$$\begin{aligned} & \frac{f (c + d x) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{d^2 (a + a \operatorname{Sin}[c + d x])} + \frac{1}{4 d^2 (a + a \operatorname{Sin}[c + d x])} \\ & \left(2 d e \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - f \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - 2 c f \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + 2 f (c + d x) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right) \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right] \\ & \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 + \frac{(-d e + c f - f (c + d x)) \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{8 d^2 (a + a \operatorname{Sin}[c + d x])} - \\ & \frac{2 f \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{d^2 (a + a \operatorname{Sin}[c + d x])} - \\ & \frac{f \operatorname{Log}[\operatorname{Sin}[c + d x]] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{d^2 (a + a \operatorname{Sin}[c + d x])} + \frac{3 e \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{2 d (a + a \operatorname{Sin}[c + d x])} - \\ & \frac{3 c f \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{2 d^2 (a + a \operatorname{Sin}[c + d x])} + \frac{1}{2 d^2 (a + a \operatorname{Sin}[c + d x])} \\ & 3 f \left((c + d x) \left(\operatorname{Log}\left[1 - e^{i(c+dx)}\right] - \operatorname{Log}\left[1 + e^{i(c+dx)}\right]\right) + i \left(\operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right] - \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right]\right)\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 + \\ & \frac{(d e - c f + f (c + d x)) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{8 d^2 (a + a \operatorname{Sin}[c + d x])} + \frac{1}{4 d^2 (a + a \operatorname{Sin}[c + d x])} \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \\ & \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 \left(-2 d e \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - f \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 2 c f \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - 2 f (c + d x) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) - \\ & \frac{1}{d^2 (a + a \operatorname{Sin}[c + d x])} 2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) \left(d e \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - c f \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + f (c + d x) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) \end{aligned}$$

■ **Problem 212: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + d x]^3}{a + a \text{Sin}[c + d x]} dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$-\frac{3 \text{ArcTanh}[\text{Cos}[c + d x]]}{2 a d} + \frac{2 \text{Cot}[c + d x]}{a d} - \frac{3 \text{Cot}[c + d x] \text{Csc}[c + d x]}{2 a d} + \frac{\text{Cot}[c + d x] \text{Csc}[c + d x]}{d (a + a \text{Sin}[c + d x])}$$

Result (type 3, 253 leaves):

$$\begin{aligned} & -\frac{1}{8 a d (1 + \text{Sin}[c + d x])} \\ & \left(2 \text{Cot}\left[\frac{1}{2}(c + d x)\right] + \text{Cot}\left[\frac{1}{2}(c + d x)\right]^2 - 4 \text{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \left(2 + \text{Cot}\left[\frac{1}{2}(c + d x)\right] - 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right]\right] + 3 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \right. \\ & \quad \left. + 24 \text{Sin}\left[\frac{1}{2}(c + d x)\right]^2 + 12 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}\left[\frac{1}{2}(c + d x)\right]^2 - 12 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}\left[\frac{1}{2}(c + d x)\right]^2 + \right. \\ & \quad \left. 8 \text{Csc}[c + d x] \text{Sin}\left[\frac{1}{2}(c + d x)\right]^4 + 8 \text{Sin}[c + d x] + 12 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[c + d x] - \right. \\ & \quad \left. 12 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[c + d x] - 2 \text{Tan}\left[\frac{1}{2}(c + d x)\right] - \text{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \end{aligned}$$

■ **Problem 214: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Csc}[c + d x]^3}{(e + f x)^2 (a + a \text{Sin}[c + d x])} dx$$

Optimal (type 9, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\text{Csc}[c + d x]^3}{(e + f x)^2 (a + a \text{Sin}[c + d x])}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 220: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \text{Sin}[c + d x]}{a + b \text{Sin}[c + d x]} dx$$

Optimal (type 4, 544 leaves, 14 steps):

$$\begin{aligned}
& \frac{(e+fx)^4}{4bf} + \frac{ia(e+fx)^3 \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b\sqrt{a^2-b^2}d} - \frac{ia(e+fx)^3 \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b\sqrt{a^2-b^2}d} + \\
& \frac{3af(e+fx)^2 \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b\sqrt{a^2-b^2}d^2} - \frac{3af(e+fx)^2 \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b\sqrt{a^2-b^2}d^2} + \frac{6iaf^2(e+fx) \operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b\sqrt{a^2-b^2}d^3} \\
& \frac{6iaf^2(e+fx) \operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b\sqrt{a^2-b^2}d^3} - \frac{6af^3 \operatorname{PolyLog}\left[4, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b\sqrt{a^2-b^2}d^4} + \frac{6af^3 \operatorname{PolyLog}\left[4, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b\sqrt{a^2-b^2}d^4}
\end{aligned}$$

Result (type 4, 1528 leaves):

$$\begin{aligned}
& x \frac{(4e^3 + 6e^2fx + 4ef^2x^2 + f^3x^3)}{4b} - \frac{1}{b\sqrt{a^2-b^2}d^4 \sqrt{(-a^2+b^2)} (\operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} \\
& ia \left(3i\sqrt{a^2-b^2}d^3 e^2fx \operatorname{Log}\left[1 + \frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& \left. 3i\sqrt{a^2-b^2}d^3 e^2fx^2 \operatorname{Log}\left[1 + \frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& \left. i\sqrt{a^2-b^2}d^3 f^3x^3 \operatorname{Log}\left[1 + \frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& \left. 3\sqrt{a^2-b^2}d^2 f(e+fx)^2 \operatorname{PolyLog}\left[2, -\frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) - \right. \\
& \left. 3\sqrt{a^2-b^2}d^2 f(e+fx)^2 \operatorname{PolyLog}\left[2, \frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{-ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 + a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& \left. 6i\sqrt{a^2-b^2}de f^2 \operatorname{PolyLog}\left[3, -\frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& \left. 6i\sqrt{a^2-b^2}df^3x \operatorname{PolyLog}\left[3, -\frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) - \right. \\
& \left. 6\sqrt{a^2-b^2}f^3 \operatorname{PolyLog}\left[4, -\frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right.
\end{aligned}$$

$$\begin{aligned}
& 6 \sqrt{a^2 - b^2} f^3 \text{PolyLog}\left[4, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} + a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e^2 f x \text{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e f^2 x^2 \text{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& \sqrt{a^2 - b^2} d^3 f^3 x^3 \text{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d e f^2 \text{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d f^3 x \text{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} + a \sin[c]}\right] (-i \cos[c] + \sin[c]) - \\
& 2 i d^3 e^3 \text{ArcTan}\left[\frac{b \cos[c + dx] + i (a + b \sin[c + dx])}{\sqrt{a^2 - b^2}}\right] \sqrt{(-a^2 + b^2) (\cos[2c] + i \sin[2c])}
\end{aligned}$$

■ **Problem 224: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \sin[c + dx]^2}{a + b \sin[c + dx]} dx$$

Optimal (type 4, 643 leaves, 19 steps):

$$\begin{aligned}
& -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cos[c+dx]}{bd^3} - \frac{(e+fx)^3\cos[c+dx]}{bd} - \frac{ia^2(e+fx)^3\operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b^2\sqrt{a^2-b^2}d} + \\
& \frac{ia^2(e+fx)^3\operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b^2\sqrt{a^2-b^2}d} - \frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b^2\sqrt{a^2-b^2}d^2} + \frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b^2\sqrt{a^2-b^2}d^2} - \\
& \frac{6ia^2f^2(e+fx)\operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b^2\sqrt{a^2-b^2}d^3} + \frac{6ia^2f^2(e+fx)\operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b^2\sqrt{a^2-b^2}d^3} + \\
& \frac{6a^2f^3\operatorname{PolyLog}\left[4, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b^2\sqrt{a^2-b^2}d^4} - \frac{6a^2f^3\operatorname{PolyLog}\left[4, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b^2\sqrt{a^2-b^2}d^4} - \frac{6f^3\sin[c+dx]}{bd^4} + \frac{3f(e+fx)^2\sin[c+dx]}{bd^2}
\end{aligned}$$

Result (type 4, 1590 leaves):

$$\begin{aligned}
& \frac{1}{4b^2d^4} \left(-ad^4x(4e^3 + 6e^2fx + 4ef^2x^2 + f^3x^3) - 4bd(e+fx)(-6f^2 + d^2(e+fx)^2)\cos[c+dx] + \frac{1}{\sqrt{a^2-b^2}\sqrt{-(a^2-b^2)}(\cos[c] + i\sin[c])^2} \right. \\
& 4ia^2 \left(3i\sqrt{a^2-b^2}d^3efx\operatorname{Log}\left[1 + \frac{b(\cos[2c+dx] + i\sin[2c+dx])}{ia\cos[c] + \sqrt{-(a^2+b^2)}(\cos[c] + i\sin[c])^2 - a\sin[c]}\right] (\cos[c] + i\sin[c]) + \right. \\
& 3i\sqrt{a^2-b^2}d^3ef^2x^2\operatorname{Log}\left[1 + \frac{b(\cos[2c+dx] + i\sin[2c+dx])}{ia\cos[c] + \sqrt{-(a^2+b^2)}(\cos[c] + i\sin[c])^2 - a\sin[c]}\right] (\cos[c] + i\sin[c]) + \\
& i\sqrt{a^2-b^2}d^3f^3x^3\operatorname{Log}\left[1 + \frac{b(\cos[2c+dx] + i\sin[2c+dx])}{ia\cos[c] + \sqrt{-(a^2+b^2)}(\cos[c] + i\sin[c])^2 - a\sin[c]}\right] (\cos[c] + i\sin[c]) + \\
& 3\sqrt{a^2-b^2}d^2f(e+fx)^2\operatorname{PolyLog}\left[2, -\frac{b(\cos[2c+dx] + i\sin[2c+dx])}{ia\cos[c] + \sqrt{-(a^2+b^2)}(\cos[c] + i\sin[c])^2 - a\sin[c]}\right] (\cos[c] + i\sin[c]) - \\
& 3\sqrt{a^2-b^2}d^2f(e+fx)^2\operatorname{PolyLog}\left[2, \frac{b(\cos[2c+dx] + i\sin[2c+dx])}{-ia\cos[c] + \sqrt{-(a^2+b^2)}(\cos[c] + i\sin[c])^2 + a\sin[c]}\right] (\cos[c] + i\sin[c]) + \\
& 6i\sqrt{a^2-b^2}def^2\operatorname{PolyLog}\left[3, -\frac{b(\cos[2c+dx] + i\sin[2c+dx])}{ia\cos[c] + \sqrt{-(a^2+b^2)}(\cos[c] + i\sin[c])^2 - a\sin[c]}\right] (\cos[c] + i\sin[c]) +
\end{aligned}$$

$$\begin{aligned}
& 6 i \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog}\left[3, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) - \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog}\left[4, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog}\left[4, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (\cos[c] + i \sin[c]) - \\
& 2 i d^3 e^3 \operatorname{ArcTan}\left[\frac{b \cos[c + dx] + i (a + b \sin[c + dx])}{\sqrt{a^2 - b^2}}\right] \sqrt{-(a^2 - b^2)} (\cos[c] + i \sin[c])^2 + \\
& 3 \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d e f^2 \operatorname{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) \Bigg) + 12 b f (-2 \\
& f^2 + d^2 (e + f x)^2) \sin[c + dx] \Bigg)
\end{aligned}$$

■ **Problem 228: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \sin[c + dx]^3}{a + b \sin[c + dx]} dx$$

Optimal (type 4, 802 leaves, 24 steps):

$$\begin{aligned}
& -\frac{3 e f^2 x}{4 b d^2} - \frac{3 f^3 x^2}{8 b d^2} + \frac{a^2 (e+f x)^4}{4 b^3 f} + \frac{(e+f x)^4}{8 b f} - \frac{6 a f^2 (e+f x) \operatorname{Cos}[c+d x]}{b^2 d^3} + \frac{a (e+f x)^3 \operatorname{Cos}[c+d x]}{b^2 d} + \\
& \frac{i a^3 (e+f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b^3 \sqrt{a^2 - b^2} d} - \frac{i a^3 (e+f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b^3 \sqrt{a^2 - b^2} d} + \frac{3 a^3 f (e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b^3 \sqrt{a^2 - b^2} d^2} - \\
& \frac{3 a^3 f (e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b^3 \sqrt{a^2 - b^2} d^2} + \frac{6 i a^3 f^2 (e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b^3 \sqrt{a^2 - b^2} d^3} - \frac{6 i a^3 f^2 (e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b^3 \sqrt{a^2 - b^2} d^3} - \\
& \frac{6 a^3 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b^3 \sqrt{a^2 - b^2} d^4} + \frac{6 a^3 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b^3 \sqrt{a^2 - b^2} d^4} + \frac{6 a f^3 \operatorname{Sin}[c+d x]}{b^2 d^4} - \frac{3 a f (e+f x)^2 \operatorname{Sin}[c+d x]}{b^2 d^2} + \\
& \frac{3 f^2 (e+f x) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{4 b d^3} - \frac{(e+f x)^3 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 b d} - \frac{3 f^3 \operatorname{Sin}[c+d x]^2}{8 b d^4} + \frac{3 f (e+f x)^2 \operatorname{Sin}[c+d x]^2}{4 b d^2}
\end{aligned}$$

Result (type 4, 1851 leaves):

$$\begin{aligned}
& \frac{1}{32 b^3} \left(16 (2 a^2 + b^2) e^3 x + 24 (2 a^2 + b^2) e^2 f x^2 + 16 (2 a^2 + b^2) e f^2 x^3 + 4 (2 a^2 + b^2) f^3 x^4 - \frac{1}{\sqrt{a^2 - b^2} d^4 \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right. \\
& 32 i a^3 \left(3 i \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log}\left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& 3 i \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& i \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 3 \sqrt{a^2 - b^2} d^2 f (e+f x)^2 \operatorname{PolyLog}\left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) - \\
& 3 \sqrt{a^2 - b^2} d^2 f (e+f x)^2 \operatorname{PolyLog}\left[2, \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 + a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& \left. 6 i \sqrt{a^2 - b^2} d e f^2 \operatorname{PolyLog}\left[3, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right.
\end{aligned}$$

$$\begin{aligned}
& 6 i \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog}\left[3, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{-a^2 + b^2} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) - \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog}\left[4, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{-a^2 + b^2} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog}\left[4, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{-a^2 + b^2} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{-a^2 + b^2} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{-a^2 + b^2} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{-a^2 + b^2} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d e f^2 \operatorname{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{-a^2 + b^2} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{-a^2 + b^2} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) - \\
& \left. 2 i d^3 e^3 \operatorname{ArcTan}\left[\frac{b \cos[c + dx] + i (a + b \sin[c + dx])}{\sqrt{a^2 - b^2}}\right] \sqrt{(-a^2 + b^2) (\cos[2c] + i \sin[2c])}\right] + \\
& \frac{16 a b (6 i f^3 - 6 d f^2 (e + f x) - 3 i d^2 f (e + f x)^2 + d^3 (e + f x)^3) (\cos[c + dx] - i \sin[c + dx])}{d^4} + \\
& \frac{16 a b (-6 i f^3 - 6 d f^2 (e + f x) + 3 i d^2 f (e + f x)^2 + d^3 (e + f x)^3) (\cos[c + dx] + i \sin[c + dx])}{d^4} + \\
& \frac{b^2 (3 f^3 + 6 i d f^2 (e + f x) - 6 d^2 f (e + f x)^2 - 4 i d^3 (e + f x)^3) (\cos[2(c + dx)] - i \sin[2(c + dx)])}{d^4} + \\
& \left. \frac{b^2 (3 f^3 - 6 i d f^2 (e + f x) - 6 d^2 f (e + f x)^2 + 4 i d^3 (e + f x)^3) (\cos[2(c + dx)] + i \sin[2(c + dx)])}{d^4}\right)
\end{aligned}$$

■ **Problem 232: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \operatorname{Csc}[c + d x]}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 732 leaves, 22 steps):

$$\begin{aligned} & - \frac{2 (e + f x)^3 \operatorname{ArcTanh}\left[e^{i(c+dx)}\right]}{a d} + \frac{i b (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a \sqrt{a^2 - b^2} d} - \frac{i b (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a \sqrt{a^2 - b^2} d} + \\ & \frac{3 i f (e + f x)^2 \operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right]}{a d^2} - \frac{3 i f (e + f x)^2 \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right]}{a d^2} + \frac{3 b f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a \sqrt{a^2 - b^2} d^2} - \\ & \frac{3 b f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a \sqrt{a^2 - b^2} d^2} - \frac{6 f^2 (e + f x) \operatorname{PolyLog}\left[3, -e^{i(c+dx)}\right]}{a d^3} + \frac{6 f^2 (e + f x) \operatorname{PolyLog}\left[3, e^{i(c+dx)}\right]}{a d^3} + \\ & \frac{6 i b f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a \sqrt{a^2 - b^2} d^3} - \frac{6 i b f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a \sqrt{a^2 - b^2} d^3} - \frac{6 i f^3 \operatorname{PolyLog}\left[4, -e^{i(c+dx)}\right]}{a d^4} + \\ & \frac{6 i f^3 \operatorname{PolyLog}\left[4, e^{i(c+dx)}\right]}{a d^4} - \frac{6 b f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a \sqrt{a^2 - b^2} d^4} + \frac{6 b f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a \sqrt{a^2 - b^2} d^4} \end{aligned}$$

Result (type 4, 2186 leaves):

$$\begin{aligned} & \frac{1}{a d^4} \left(-2 d^3 e^3 \operatorname{ArcTanh}\left[e^{i(c+dx)}\right] + 3 d^3 e^2 f x \operatorname{Log}\left[1 - e^{i(c+dx)}\right] + 3 d^3 e f^2 x^2 \operatorname{Log}\left[1 - e^{i(c+dx)}\right] + d^3 f^3 x^3 \operatorname{Log}\left[1 - e^{i(c+dx)}\right] - \right. \\ & \left. 3 d^3 e^2 f x \operatorname{Log}\left[1 + e^{i(c+dx)}\right] - 3 d^3 e f^2 x^2 \operatorname{Log}\left[1 + e^{i(c+dx)}\right] - d^3 f^3 x^3 \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + 3 i d^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right] - \right. \\ & \left. 3 i d^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right] - 6 d e f^2 \operatorname{PolyLog}\left[3, -e^{i(c+dx)}\right] - 6 d f^3 x \operatorname{PolyLog}\left[3, -e^{i(c+dx)}\right] + 6 d e f^2 \operatorname{PolyLog}\left[3, e^{i(c+dx)}\right] + \right. \\ & \left. 6 d f^3 x \operatorname{PolyLog}\left[3, e^{i(c+dx)}\right] - 6 i f^3 \operatorname{PolyLog}\left[4, -e^{i(c+dx)}\right] + 6 i f^3 \operatorname{PolyLog}\left[4, e^{i(c+dx)}\right] \right) + \frac{1}{a \sqrt{a^2 - b^2} d^4 \sqrt{-(a^2 - b^2)^2 e^{4 i c}}} \\ & b \left(-2 d^3 e^3 \sqrt{-(a^2 - b^2)^2 e^{4 i c}} \operatorname{ArcTan}\left[\frac{i a + b e^{i(c+dx)}}{\sqrt{a^2 - b^2}}\right] + 3 i \sqrt{a^2 - b^2} d^3 e^2 e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f x \operatorname{Log}\left[1 - \frac{i b e^{i(2c+dx)}}{a e^{i c} - \sqrt{(a^2 - b^2) e^{2 i c}}}\right] + \right. \\ & \left. i \sqrt{a^2 - b^2} d^3 e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f^3 x^3 \operatorname{Log}\left[1 - \frac{i b e^{i(2c+dx)}}{a e^{i c} - \sqrt{(a^2 - b^2) e^{2 i c}}}\right] - 3 i \sqrt{a^2 - b^2} d^3 e^2 e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} \right) \end{aligned}$$

$$\begin{aligned}
& f x \operatorname{Log}\left[1 - \frac{i b e^{i(2c+dx)}}{a e^{ic} + \sqrt{(a^2 - b^2) e^{2ic}}}\right] - i \sqrt{a^2 - b^2} d^3 e^{ic} \sqrt{(-a^2 + b^2) e^{2ic}} f^3 x^3 \operatorname{Log}\left[1 - \frac{i b e^{i(2c+dx)}}{a e^{ic} + \sqrt{(a^2 - b^2) e^{2ic}}}\right] - \\
& 3 \sqrt{a^2 - b^2} d^3 e^{ic} \sqrt{(a^2 - b^2) e^{2ic}} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} - \sqrt{(-a^2 + b^2) e^{2ic}}}\right] + 3 \sqrt{a^2 - b^2} d^3 e^{ic} \sqrt{(a^2 - b^2) e^{2ic}} f^2 x^2 \\
& \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2 + b^2) e^{2ic}}}\right] + 3 \sqrt{a^2 - b^2} d^2 e^{ic} \sqrt{(-a^2 + b^2) e^{2ic}} f (e^2 + f^2 x^2) \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} - \sqrt{(a^2 - b^2) e^{2ic}}}\right] - \\
& 3 \sqrt{a^2 - b^2} d^2 e^{ic} \sqrt{(-a^2 + b^2) e^{2ic}} f (e^2 + f^2 x^2) \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + \sqrt{(a^2 - b^2) e^{2ic}}}\right] + 6 i \sqrt{a^2 - b^2} d^2 e^{ic} \sqrt{(a^2 - b^2) e^{2ic}} \\
& f^2 x \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2 + b^2) e^{2ic}}}\right] - 6 i \sqrt{a^2 - b^2} d^2 e^{ic} \sqrt{(a^2 - b^2) e^{2ic}} f^2 x \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2 + b^2) e^{2ic}}}\right] + \\
& 6 i \sqrt{a^2 - b^2} d e^{ic} \sqrt{(-a^2 + b^2) e^{2ic}} f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} - \sqrt{(a^2 - b^2) e^{2ic}}}\right] - 6 i \sqrt{a^2 - b^2} d e^{ic} \sqrt{(-a^2 + b^2) e^{2ic}} f^3 x \\
& \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + \sqrt{(a^2 - b^2) e^{2ic}}}\right] - 6 \sqrt{a^2 - b^2} d e^{ic} \sqrt{(a^2 - b^2) e^{2ic}} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2 + b^2) e^{2ic}}}\right] + \\
& 6 \sqrt{a^2 - b^2} d e^{ic} \sqrt{(a^2 - b^2) e^{2ic}} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2 + b^2) e^{2ic}}}\right] - 6 \sqrt{a^2 - b^2} e^{ic} \sqrt{(-a^2 + b^2) e^{2ic}} f^3 \\
& \operatorname{PolyLog}\left[4, \frac{i b e^{i(2c+dx)}}{a e^{ic} - \sqrt{(a^2 - b^2) e^{2ic}}}\right] + 6 \sqrt{a^2 - b^2} e^{ic} \sqrt{(-a^2 + b^2) e^{2ic}} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2c+dx)}}{a e^{ic} + \sqrt{(a^2 - b^2) e^{2ic}}}\right] \Big)
\end{aligned}$$

■ **Problem 236: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \operatorname{Csc}[c + d x]^2}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 882 leaves, 29 steps):

$$\begin{aligned}
& - \frac{i (e + f x)^3}{a d} + \frac{2 b (e + f x)^3 \operatorname{ArcTanh}\left[e^{i (c+d x)}\right]}{a^2 d} - \frac{(e + f x)^3 \operatorname{Cot}[c + d x]}{a d} - \frac{i b^2 (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i (c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 \sqrt{a^2 - b^2} d} + \\
& \frac{i b^2 (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i (c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 \sqrt{a^2 - b^2} d} + \frac{3 f (e + f x)^2 \operatorname{Log}\left[1 - e^{2 i (c+d x)}\right]}{a d^2} - \frac{3 i b f (e + f x)^2 \operatorname{PolyLog}\left[2, -e^{i (c+d x)}\right]}{a^2 d^2} + \\
& \frac{3 i b f (e + f x)^2 \operatorname{PolyLog}\left[2, e^{i (c+d x)}\right]}{a^2 d^2} - \frac{3 b^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i (c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 \sqrt{a^2 - b^2} d^2} + \frac{3 b^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i (c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 \sqrt{a^2 - b^2} d^2} - \\
& \frac{3 i f^2 (e + f x) \operatorname{PolyLog}\left[2, e^{2 i (c+d x)}\right]}{a d^3} + \frac{6 b f^2 (e + f x) \operatorname{PolyLog}\left[3, -e^{i (c+d x)}\right]}{a^2 d^3} - \frac{6 b f^2 (e + f x) \operatorname{PolyLog}\left[3, e^{i (c+d x)}\right]}{a^2 d^3} - \\
& \frac{6 i b^2 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i (c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 \sqrt{a^2 - b^2} d^3} + \frac{6 i b^2 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i (c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 \sqrt{a^2 - b^2} d^3} + \frac{3 f^3 \operatorname{PolyLog}\left[3, e^{2 i (c+d x)}\right]}{2 a d^4} + \\
& \frac{6 i b f^3 \operatorname{PolyLog}\left[4, -e^{i (c+d x)}\right]}{a^2 d^4} - \frac{6 i b f^3 \operatorname{PolyLog}\left[4, e^{i (c+d x)}\right]}{a^2 d^4} + \frac{6 b^2 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i (c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 \sqrt{a^2 - b^2} d^4} - \frac{6 b^2 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i (c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 \sqrt{a^2 - b^2} d^4}
\end{aligned}$$

Result (type 4, 2452 leaves):

$$\begin{aligned}
& - \frac{b e^3 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right]}{a^2 d} - \frac{1}{a^2 d^2} \\
& 3 b e^2 f \left((c + d x) \left(\operatorname{Log}\left[1 - e^{i (c+d x)}\right] - \operatorname{Log}\left[1 + e^{i (c+d x)}\right] \right) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right] + i \left(\operatorname{PolyLog}\left[2, -e^{i (c+d x)}\right] - \operatorname{PolyLog}\left[2, e^{i (c+d x)}\right] \right) \right) - \\
& \frac{1}{4 a d^4} e^{-i c} f^3 \operatorname{Csc}[c] \\
& \left(2 d^2 x^2 \left(2 d e^{2 i c} x + 3 i \left(-1 + e^{2 i c} \right) \operatorname{Log}\left[1 - e^{2 i (c+d x)}\right] \right) + 6 d \left(-1 + e^{2 i c} \right) x \operatorname{PolyLog}\left[2, e^{2 i (c+d x)}\right] + 3 i \left(-1 + e^{2 i c} \right) \operatorname{PolyLog}\left[3, e^{2 i (c+d x)}\right] \right) + \\
& \frac{1}{a^2 d^3} 6 b e f^2 \left(d^2 x^2 \operatorname{ArcTanh}\left[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] - i d x \operatorname{PolyLog}\left[2, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] \right) + \\
& i d x \operatorname{PolyLog}\left[2, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] + \operatorname{PolyLog}\left[3, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] - \operatorname{PolyLog}\left[3, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] \right) - \\
& \frac{1}{a^2 d^4} b f^3 \left(-2 d^3 x^3 \operatorname{ArcTanh}\left[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] + 3 i d^2 x^2 \operatorname{PolyLog}\left[2, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] - \right. \\
& \left. 3 i d^2 x^2 \operatorname{PolyLog}\left[2, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] - 6 d x \operatorname{PolyLog}\left[3, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] + 6 d x \right. \\
& \left. \operatorname{PolyLog}\left[3, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] - 6 i \operatorname{PolyLog}\left[4, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] + 6 i \operatorname{PolyLog}\left[4, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{3 e^2 f \operatorname{Csc}[c] (-d x \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x]] \operatorname{Sin}[c])}{a d^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} + \frac{1}{a^2 \sqrt{a^2 - b^2} d^4 \sqrt{(-a^2 + b^2) (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])}} \\
& i b^2 \left(3 i \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& 3 i \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& i \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 3 \sqrt{a^2 - b^2} d^2 f (e + f x)^2 \operatorname{PolyLog} \left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) - \\
& 3 \sqrt{a^2 - b^2} d^2 f (e + f x)^2 \operatorname{PolyLog} \left[2, \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 6 i \sqrt{a^2 - b^2} d e f^2 \operatorname{PolyLog} \left[3, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 6 i \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog} \left[3, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) - \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog} \left[4, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog} \left[4, \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log} \left[1 - \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (-i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log} \left[1 - \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (-i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \\
& \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log} \left[1 - \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (-i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \\
& 6 \sqrt{a^2 - b^2} d e f^2 \operatorname{PolyLog} \left[3, \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (-i \operatorname{Cos}[c] + \operatorname{Sin}[c]) +
\end{aligned}$$

$$\begin{aligned}
& 6 \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 + a \sin[c]}}\right] (-i \cos[c] + \sin[c]) - \\
& 2 i d^3 e^3 \operatorname{ArcTan}\left[\frac{b \cos[c + dx] + i (a + b \sin[c + dx])}{\sqrt{a^2 - b^2}}\right] \sqrt{(-a^2 + b^2) (\cos[2c] + i \sin[2c])} \Bigg) + \\
& \frac{\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right] (e^3 \sin\left[\frac{dx}{2}\right] + 3 e^2 f x \sin\left[\frac{dx}{2}\right] + 3 e f^2 x^2 \sin\left[\frac{dx}{2}\right] + f^3 x^3 \sin\left[\frac{dx}{2}\right])}{2 a d} + \\
& \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (e^3 \sin\left[\frac{dx}{2}\right] + 3 e^2 f x \sin\left[\frac{dx}{2}\right] + 3 e f^2 x^2 \sin\left[\frac{dx}{2}\right] + f^3 x^3 \sin\left[\frac{dx}{2}\right])}{2 a d} - \\
& \left(3 e f^2 \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \Big/ \left(\sqrt{1 + \operatorname{Tan}[c]^2}\right) (i dx (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]]) - \pi \operatorname{Log}[1 + e^{-2 i dx}] - \right. \right. \\
& \quad \left. \left. 2 (dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}[1 - e^{2 i (dx + \operatorname{ArcTan}[\operatorname{Tan}[c])}] + \pi \operatorname{Log}[\cos[dx]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]] + \right. \right. \\
& \quad \left. \left. i \operatorname{PolyLog}[2, e^{2 i (dx + \operatorname{ArcTan}[\operatorname{Tan}[c])}] \right) \operatorname{Tan}[c] \right) \Big/ \left(a d^3 \sqrt{\operatorname{Sec}[c]^2 (\cos[c]^2 + \sin[c]^2)}\right)
\end{aligned}$$

■ **Problem 246: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \sin[c + dx]}{(a + b \sin[c + dx])^2} dx$$

Optimal (type 4, 1106 leaves, 30 steps):

$$\begin{aligned}
& - \frac{i a (e + f x)^2}{b (a^2 - b^2) d} + \frac{2 a f (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^2} + \frac{i a^2 (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d} - \frac{i (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d} + \\
& \frac{2 a f (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^2} - \frac{i a^2 (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d} + \frac{i (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d} - \frac{2 i a f^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^3} + \\
& \frac{2 a^2 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^2} - \frac{2 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^2} - \frac{2 i a f^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^3} - \\
& \frac{2 a^2 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^2} + \frac{2 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^2} + \frac{2 i a^2 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^3} - \\
& \frac{2 i f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^3} - \frac{2 i a^2 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^3} + \frac{2 i f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^3} - \frac{a (e + f x)^2 \operatorname{Cos}[c + dx]}{(a^2 - b^2) d (a + b \operatorname{Sin}[c + dx])}
\end{aligned}$$

Result (type 4, 4475 leaves):

$$\begin{aligned}
& \frac{1}{(-a^2 + b^2) d^2} 2 b e f \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} + \right. \\
& \frac{1}{\sqrt{-a^2 + b^2}} \left(2 \left(-c + \frac{\pi}{2} - dx\right) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2 + b^2}}\right] - 2 \left(-c + \operatorname{ArcCos}\left[-\frac{a}{b}\right]\right) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2 + b^2}}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2 + b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2 + b^2} e^{-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx\right)}}{\sqrt{2} \sqrt{b} \sqrt{a + b \operatorname{Sin}[c + dx]}}\right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + \right. \\
& \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2 + b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2 + b^2} e^{\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx\right)}}{\sqrt{2} \sqrt{b} \sqrt{a + b \operatorname{Sin}[c + dx]}}\right] -
\end{aligned}$$

$$\begin{aligned}
& \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] + \\
& \left(-\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] \right) \Bigg] + \\
& \frac{1}{b(-a^2+b^2)d^3} 2a^2 f^2 \operatorname{Cot}[c] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2\left(-c+\frac{\pi}{2}-dx\right) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \right. \\
& \left. \left. 2\left(-c+\operatorname{ArcCos}\left[-\frac{a}{b}\right]\right) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] + \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2}\sqrt{b}\sqrt{a+b \operatorname{Sin}[c+dx]}}\right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + \right. \\
& \left. 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2}\sqrt{b}\sqrt{a+b \operatorname{Sin}[c+dx]}}\right] -
\end{aligned}$$

$$\begin{aligned}
& \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}\right] + \\
& \left(-\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}\right] \right) + \\
& \left(b e^{ic} f^2 \left(d^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} - \sqrt{(-a^2+b^2)} e^{2ic}}\right] - d^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - \right. \right. \\
& \left. \left. 2i dx \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 2i dx \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \right. \right. \\
& \left. \left. 2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] \right) \right) / \\
& \left((-a^2+b^2) d^3 \sqrt{(-a^2+b^2)} e^{2ic} \right) + \frac{2i b e^2 \operatorname{ArcTan}\left[\frac{i b \operatorname{Cos}[c] - i(-a+b \operatorname{Sin}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right]}{\sqrt{-a^2+b^2} \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2}}\right]}{(-a^2+b^2) d \sqrt{-a^2+b^2} \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2} + \\
& \frac{4i a^2 e f \operatorname{ArcTan}\left[\frac{i b \operatorname{Cos}[c] - i(-a+b \operatorname{Sin}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right]}{\sqrt{-a^2+b^2} \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2}\right] \operatorname{Cot}[c]}{b(-a^2+b^2) d^2 \sqrt{-a^2+b^2} \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2} + \\
& \frac{1}{(-a^2+b^2) d} \\
& 2
\end{aligned}$$

a
f²
Csc[c]

$$\left(-\frac{x^2 \operatorname{Cos}[c]}{2b} + \frac{1}{bd} \right.$$

$$x \left(dx \operatorname{Cos}[c] - \frac{2a \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{dx}{2}\right] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (b \operatorname{Cos}\left[c + \frac{dx}{2}\right] + a \operatorname{Sin}\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right]}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right] \operatorname{Cos}[c] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] \operatorname{Sin}[c]} \right) +$$

$$\frac{1}{bd} \left(-\frac{1}{d} a \operatorname{Cos}[c] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b + a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} + \frac{1}{\sqrt{-a^2 + b^2}} \left(2 \left(c - \operatorname{ArcCos}\left[-\frac{a}{b}\right] \right) \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right]} \right) + \right.$$

$$\left. (-2c + \pi - 2dx) \operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \operatorname{Log}\left[\right.$$

$$\left. \frac{(a + b) \left(-a + b - i \sqrt{-a^2 + b^2} \right) \left(1 + i \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right] \right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right] \right)} \right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \right]$$

$$\operatorname{Log}\left[\frac{(a + b) \left(i a - i b + \sqrt{-a^2 + b^2} \right) \left(i + \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right] \right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right] \right)} \right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \left(-\operatorname{ArcTanh}\left[\right. \right. \right.$$

$$\left. \left. \frac{(a - b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}} \right] + \operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{-a^2 + b^2} e^{\frac{1}{2}(-2c + \pi - 2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a + b \operatorname{Sin}[c + dx]}} \right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right)$$

$$\begin{aligned}
& \text{Log} \left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4} i (2c-\pi+2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \sin[c+dx]}} \right] + i \left(\text{PolyLog} \left[2, \frac{(a-i\sqrt{-a^2+b^2}) (a+b+\sqrt{-a^2+b^2} \tan[\frac{1}{4}(2c-\pi+2dx)])}{b (a+b+\sqrt{-a^2+b^2} \cot[\frac{1}{4}(2c+\pi+2dx)])} \right] \right) - \\
& \left. \text{PolyLog} \left[2, \frac{(a+i\sqrt{-a^2+b^2}) (a+b+\sqrt{-a^2+b^2} \tan[\frac{1}{4}(2c-\pi+2dx)])}{b (a+b+\sqrt{-a^2+b^2} \cot[\frac{1}{4}(2c+\pi+2dx)])} \right] \right) \Bigg) + \\
& \frac{2 a x \text{ArcTan} \left[\frac{\text{Sec}[\frac{dx}{2}] (\cos[c]-i \sin[c]) (b \cos[c+\frac{dx}{2}] + a \sin[\frac{dx}{2}])}{\sqrt{a^2-b^2} \sqrt{(\cos[c]-i \sin[c])^2}} \right] \cos[c] (\cos[c]-i \sin[c])}{\sqrt{a^2-b^2} \sqrt{(\cos[c]-i \sin[c])^2}} + \frac{(c+dx) \text{Log}[a+b \sin[c+dx]] \sin[c]}{d} - \\
& \frac{1}{d} b \left(\frac{(c+dx) \text{Log}[a+b \sin[c+dx]]}{b} - 1 / b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx \right)^2 + 4 i \text{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \text{ArcTan} \left[\frac{(a-b) \tan[\frac{1}{2}(-c+\frac{\pi}{2}-dx)]}{\sqrt{a^2-b^2}} \right] \right) + \right. \\
& \left(-c + \frac{\pi}{2} - dx + 2 \text{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \right) \text{Log} \left[1 + \frac{(a-\sqrt{a^2-b^2}) e^{i(-c+\frac{\pi}{2}-dx)}}{b} \right] + \\
& \left(-c + \frac{\pi}{2} - dx - 2 \text{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \right) \text{Log} \left[1 + \frac{(a+\sqrt{a^2-b^2}) e^{i(-c+\frac{\pi}{2}-dx)}}{b} \right] - \left(-c + \frac{\pi}{2} - dx \right) \text{Log}[a+b \sin[c+dx]] - \\
& i \left(\text{PolyLog} \left[2, -\frac{(a-\sqrt{a^2-b^2}) e^{i(-c+\frac{\pi}{2}-dx)}}{b} \right] + \text{PolyLog} \left[2, -\frac{(a+\sqrt{a^2-b^2}) e^{i(-c+\frac{\pi}{2}-dx)}}{b} \right] \right) \Bigg) \text{Sin}[c] \Bigg) - \\
& \left(2 a e f \text{Csc}[c] \left(-b dx \cos[c] + b \text{Log}[a+b \cos[dx]] \sin[c] + b \cos[c] \sin[dx] \right) \text{Sin}[c] + \frac{2 i a b \text{ArcTan} \left[\frac{i b \cos[c]-i(-a+b \sin[c]) \tan[\frac{dx}{2}]}{\sqrt{-a^2+b^2} \cos[c]^2+b^2 \sin[c]^2}} \right] \cos[c]}{\sqrt{-a^2+b^2} \cos[c]^2+b^2 \sin[c]^2}} \right) \Bigg) /
\end{aligned}$$

$$\frac{\left(\frac{(-a^2 + b^2)}{d^2} (b^2 \cos[c]^2 + b^2 \sin[c]^2) + \left(\csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (a^2 e^2 \cos[c] + 2 a^2 e f x \cos[c] + a^2 f^2 x^2 \cos[c] + a b e^2 \sin[d x] + 2 a b e f x \sin[d x] + a b f^2 x^2 \sin[d x]) \right) \right)}{(2 (a - b) b (a + b) d (a + b \sin[c + d x]))}$$

- **Problem 247: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \sin[c + d x]}{(a + b \sin[c + d x])^2} dx$$

Optimal (type 4, 1512 leaves, 36 steps):

$$\begin{aligned}
& - \frac{i a (e + f x)^3}{b (a^2 - b^2) d} + \frac{3 a f (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^2} + \frac{i a^2 (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d} - \frac{i (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d} + \\
& \frac{3 a f (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^2} - \frac{i a^2 (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d} + \frac{i (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d} - \frac{6 i a f^2 (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^3} + \\
& \frac{3 a^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^2} - \frac{3 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^2} - \frac{6 i a f^2 (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^3} - \\
& \frac{3 a^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^2} + \frac{3 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^2} + \frac{6 a f^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^4} + \\
& \frac{6 i a^2 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^3} - \frac{6 i f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^3} + \frac{6 a f^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^4} - \\
& \frac{6 i a^2 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^3} + \frac{6 i f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^3} - \frac{6 a^2 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^4} + \\
& \frac{6 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^4} + \frac{6 a^2 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^4} - \frac{6 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b \sqrt{a^2 - b^2} d^4} - \frac{a (e + f x)^3 \operatorname{Cos}[c + d x]}{(a^2 - b^2) d (a + b \operatorname{Sin}[c + d x])}
\end{aligned}$$

Result (type 4, 7026 leaves):

$$\begin{aligned}
& \frac{1}{(-a^2 + b^2) d^2} 3 b e^2 f \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} + \right. \\
& \left. \frac{1}{\sqrt{-a^2 + b^2}} \left(2 \left(-c + \frac{\pi}{2} - dx\right) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2 + b^2}}\right] - 2 \left(-c + \operatorname{ArcCos}\left[-\frac{a}{b}\right]\right) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2 + b^2}}\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\text{ArcCos}\left[-\frac{a}{b}\right] - 2i \left(\text{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] - \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \\
& \text{Log}\left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \sin[c+dx]}}\right] + \left(\text{ArcCos}\left[-\frac{a}{b}\right] + \right. \\
& \left. 2i \left(\text{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] - \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \sin[c+dx]}}\right] - \\
& \left(\text{ArcCos}\left[-\frac{a}{b}\right] + 2i \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \text{Log}\left[1 - \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] + \\
& \left(-\text{ArcCos}\left[-\frac{a}{b}\right] + 2i \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \text{Log}\left[1 - \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] + \\
& i \left(\text{PolyLog}\left[2, \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] - \right. \\
& \left. \text{PolyLog}\left[2, \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] \right) \right) + \\
& \frac{1}{b(-a^2+b^2)d^3} 6a^2 e f^2 \cot[c] \left(\frac{\pi \text{ArcTan}\left[\frac{b+a \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2(-c+\frac{\pi}{2}-dx) \text{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \right. \\
& \left. \left. 2\left(-c + \text{ArcCos}\left[-\frac{a}{b}\right]\right) \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] + \right. \right. \\
& \left. \left. \left(\text{ArcCos}\left[-\frac{a}{b}\right] - 2i \left(\text{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] - \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Log} \left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \sin[c+dx]}} \right] + \left(\text{ArcCos} \left[-\frac{a}{b} \right] + \right. \\
& \left. 2i \left(\text{ArcTanh} \left[\frac{(a+b) \cot \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right]}{\sqrt{-a^2+b^2}} \right] - \text{ArcTanh} \left[\frac{(-a+b) \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \text{Log} \left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \sin[c+dx]}} \right] - \\
& \left(\text{ArcCos} \left[-\frac{a}{b} \right] + 2i \text{ArcTanh} \left[\frac{(-a+b) \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right]}{\sqrt{-a^2+b^2}} \right] \right) \text{Log} \left[1 - \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right])}{b(a+b+\sqrt{-a^2+b^2} \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right])} \right] + \\
& \left(-\text{ArcCos} \left[-\frac{a}{b} \right] + 2i \text{ArcTanh} \left[\frac{(-a+b) \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right]}{\sqrt{-a^2+b^2}} \right] \right) \text{Log} \left[1 - \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right])}{b(a+b+\sqrt{-a^2+b^2} \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right])} \right] + \\
& i \left(\text{PolyLog} \left[2, \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right])}{b(a+b+\sqrt{-a^2+b^2} \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right])} \right] - \right. \\
& \left. \text{PolyLog} \left[2, \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right])}{b(a+b+\sqrt{-a^2+b^2} \tan \left[\frac{1}{2}(-c+\frac{\pi}{2}-dx) \right])} \right] \right) \right) + \\
& \left(3be^{ic} f^2 \left(d^2 x^2 \text{Log} \left[1 + \frac{be^{i(2c+dx)}}{ia e^{ic} - \sqrt{(-a^2+b^2)} e^{2ic}} \right] - d^2 x^2 \text{Log} \left[1 + \frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}} \right] - \right. \right. \\
& \left. 2ix \text{PolyLog} \left[2, \frac{ibe^{i(2c+dx)}}{ae^{ic} + i\sqrt{(-a^2+b^2)} e^{2ic}} \right] + \right. \\
& \left. 2ix \text{PolyLog} \left[2, -\frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}} \right] + 2 \text{PolyLog} \left[3, \frac{ibe^{i(2c+dx)}}{ae^{ic} + i\sqrt{(-a^2+b^2)} e^{2ic}} \right] - \right. \\
& \left. \left. 2 \text{PolyLog} \left[3, -\frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}} \right] \right) \right) / \left((-a^2+b^2) d^3 \sqrt{(-a^2+b^2)} e^{2ic} \right) +
\end{aligned}$$

$$\left(3 a^2 e^{i c} f^3 \operatorname{Cot}[c] \left(d^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] - d^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] - \right. \right. \\ \left. \left. 2 i d x \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] + 2 i d x \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] + \right. \right. \\ \left. \left. 2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] - 2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] \right) \right) / \\ \left(b (-a^2 + b^2) d^4 \sqrt{(-a^2 + b^2)} e^{2 i c} + \frac{1}{2 b (-a^2 + b^2) d^4 \sqrt{(-a^2 + b^2)} e^{2 i c}} \right)$$

a

 $e^{-i c}$ f^3 $\operatorname{Csc}[c]$

$$\left(2 d^3 e^{2 i c} \sqrt{(-a^2 + b^2)} e^{2 i c} x^3 - 3 a d^2 e^{i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] - \right. \\ \left. 3 a d^2 e^{3 i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] - 3 i d^2 \sqrt{(-a^2 + b^2)} e^{2 i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] + \right. \\ \left. 3 i d^2 e^{2 i c} \sqrt{(-a^2 + b^2)} e^{2 i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] + \right. \\ \left. 3 a d^2 e^{i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] + 3 a d^2 e^{3 i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] - \right. \\ \left. 3 i d^2 \sqrt{(-a^2 + b^2)} e^{2 i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] + \right. \\ \left. 3 i d^2 e^{2 i c} \sqrt{(-a^2 + b^2)} e^{2 i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] + \right. \\ \left. 6 d \left(\sqrt{(-a^2 + b^2)} e^{2 i c} (-1 + e^{2 i c}) + i a e^{i c} (1 + e^{2 i c}) \right) x \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] + \right. \\ \left. 6 d \left(\sqrt{(-a^2 + b^2)} e^{2 i c} (-1 + e^{2 i c}) - i a e^{i c} (1 + e^{2 i c}) \right) x \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2)} e^{2 i c}}\right] - \right.$$

$$\begin{aligned}
& 6 a e^{i c} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c}+i \sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]-6 a e^{3 i c} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c}+i \sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]- \\
& 6 i \sqrt{\left(-a^2+b^2\right) e^{2 i c}} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c}+i \sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]+6 i e^{2 i c} \sqrt{\left(-a^2+b^2\right) e^{2 i c}} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c}+i \sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]+ \\
& 6 a e^{i c} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c}+\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]+6 a e^{3 i c} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c}+\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]- \\
& 6 i \sqrt{\left(-a^2+b^2\right) e^{2 i c}} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c}+\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]+6 i e^{2 i c} \sqrt{\left(-a^2+b^2\right) e^{2 i c}} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c}+\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]\left.\right]+ \\
& \frac{1}{\left(-a^2+b^2\right) d^4 \sqrt{\left(-a^2+b^2\right) e^{2 i c}}} b e^{i c} f^3\left(d^3 x^3 \operatorname{Log}\left[1+\frac{b e^{i(2 c+d x)}}{i a e^{i c}-\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]-d^3 x^3 \operatorname{Log}\left[1+\frac{b e^{i(2 c+d x)}}{i a e^{i c}+\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]-\right. \\
& 3 i d^2 x^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(2 c+d x)}}{a e^{i c}+i \sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]+3 i d^2 x^2 \operatorname{PolyLog}\left[2, -\frac{b e^{i(2 c+d x)}}{i a e^{i c}+\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]+ \\
& 6 d x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c}+i \sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]-6 d x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c}+\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]+ \\
& 6 i \operatorname{PolyLog}\left[4, \frac{i b e^{i(2 c+d x)}}{a e^{i c}+i \sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]-6 i \operatorname{PolyLog}\left[4, -\frac{b e^{i(2 c+d x)}}{i a e^{i c}+\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]\left.\right]+ \\
& \frac{2 i b e^3 \operatorname{ArcTan}\left[\frac{i b \operatorname{Cos}[c]-i(-a+b \operatorname{Sin}[c]) \operatorname{Tan}\left[\frac{d x}{2}\right]}{\sqrt{-a^2+b^2 \operatorname{Cos}[c]^2+b^2 \operatorname{Sin}[c]^2}}\right]}{\left(-a^2+b^2\right) d \sqrt{-a^2+b^2 \operatorname{Cos}[c]^2+b^2 \operatorname{Sin}[c]^2}}+\frac{6 i a^2 e^2 f \operatorname{ArcTan}\left[\frac{i b \operatorname{Cos}[c]-i(-a+b \operatorname{Sin}[c]) \operatorname{Tan}\left[\frac{d x}{2}\right]}{\sqrt{-a^2+b^2 \operatorname{Cos}[c]^2+b^2 \operatorname{Sin}[c]^2}}\right] \operatorname{Cot}[c]}{b\left(-a^2+b^2\right) d^2 \sqrt{-a^2+b^2 \operatorname{Cos}[c]^2+b^2 \operatorname{Sin}[c]^2}}+ \\
& \frac{1}{\left(-a^2+b^2\right) d} \\
& 6 \\
& a \\
& e f^2 \\
& \operatorname{Csc}[c]
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{x^2 \operatorname{Cos}[c]}{2b} + \frac{1}{bd} \right. \\
& x \left(dx \operatorname{Cos}[c] - \frac{2a \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{dx}{2}\right] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (b \operatorname{Cos}\left[c + \frac{dx}{2}\right] + a \operatorname{Sin}\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right]}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right] \operatorname{Cos}[c] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}} - \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] \operatorname{Sin}[c] \right) + \\
& \frac{1}{bd} \left(-\frac{1}{d} a \operatorname{Cos}[c] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} + \frac{1}{\sqrt{-a^2 + b^2}} \left(2 \left(c - \operatorname{ArcCos}\left[-\frac{a}{b}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] + \right. \right. \\
& \left. \left. (-2c + \pi - 2dx) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \operatorname{Log}\left[\right. \right. \\
& \left. \left. \frac{(a+b) \left(-a + b - i \sqrt{-a^2 + b^2} \right) \left(1 + i \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right] \right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right] \right)} \right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \right. \\
& \left. \operatorname{Log}\left[\frac{(a+b) \left(i a - i b + \sqrt{-a^2 + b^2} \right) \left(i + \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right] \right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right] \right)} \right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \left(-\operatorname{ArcTanh}\left[\right. \right. \right. \\
& \left. \left. \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}} \right] + \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2 + b^2} e^{\frac{1}{4}i(-2c + \pi - 2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a + b \operatorname{Sin}[c + dx]}} \right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \\
& \left. \operatorname{Log}\left[\frac{\sqrt{-a^2 + b^2} e^{\frac{1}{4}i(2c - \pi + 2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a + b \operatorname{Sin}[c + dx]}} \right] + i \left(\operatorname{PolyLog}\left[2, \frac{\left(a - i \sqrt{-a^2 + b^2} \right) \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right] \right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right] \right)} \right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. \text{PolyLog}\left[2, \frac{\left(a + i \sqrt{-a^2 + b^2}\right) \left(a + b + \sqrt{-a^2 + b^2} \tan\left[\frac{1}{4} (2c - \pi + 2dx)\right]\right)}{b \left(a + b + \sqrt{-a^2 + b^2} \cot\left[\frac{1}{4} (2c + \pi + 2dx)\right]\right)}\right]\right]\right]\right]\right) + \\
& \frac{2ax \text{ArcTan}\left[\frac{\text{Sec}\left[\frac{dx}{2}\right] (\cos[c] - i \sin[c]) \left(b \cos\left[c + \frac{dx}{2}\right] + a \sin\left[\frac{dx}{2}\right]\right)}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}}\right] \cos[c] (\cos[c] - i \sin[c])}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}} + \frac{(c + dx) \log[a + b \sin[c + dx]] \sin[c]}{d} - \\
& \frac{1}{d} b \left(\frac{(c + dx) \log[a + b \sin[c + dx]]}{b} - 1 / b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx\right)^2 + 4 i \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{(a-b) \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{a^2 - b^2}}\right] \right) + \right. \\
& \left. \left(-c + \frac{\pi}{2} - dx + 2 \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \log\left[1 + \frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i \left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] + \right. \\
& \left. \left(-c + \frac{\pi}{2} - dx - 2 \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \log\left[1 + \frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i \left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] - \left(-c + \frac{\pi}{2} - dx\right) \log[a + b \sin[c + dx]] - \right. \\
& \left. i \left(\text{PolyLog}\left[2, -\frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i \left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] + \text{PolyLog}\left[2, -\frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i \left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] \right) \right) \right) \sin[c] \left. \right) - \\
& \left(3a e^2 f \text{Csc}[c] \left(-bdx \cos[c] + b \log[a + b \cos[dx]] \sin[c] + b \cos[c] \sin[dx] \right) \sin[c] + \frac{2iab \text{ArcTan}\left[\frac{ib \cos[c] - i(-a + b \sin[c]) \tan\left[\frac{dx}{2}\right]}{\sqrt{-a^2 + b^2} \cos[c]^2 + b^2 \sin[c]^2}\right] \cos[c]}{\sqrt{-a^2 + b^2} \cos[c]^2 + b^2 \sin[c]^2} \right) \right) / \\
& \left((-a^2 + b^2) d^2 (b^2 \cos[c]^2 + b^2 \sin[c]^2) + \right. \\
& \left. (\text{Csc}[c] (-a^2 e^3 \cos[c] - 3a^2 e^2 f x \cos[c] - 3a^2 e^2 f^2 x^2 \cos[c] - a^2 f^3 x^3 \cos[c] - a b e^3 \sin[dx] - 3a b e^2 f x \sin[dx] - \right. \\
& \left. 3a b e^2 f^2 x^2 \sin[dx] - a b f^3 x^3 \sin[dx]) \right) / (b (-a^2 + b^2) d (a + b \sin[c + dx]))
\end{aligned}$$

Problem 249: Result more than twice size of optimal antiderivative.

$$\int \frac{(e + f x)^2 \operatorname{Sin}[c + d x]}{(a + b \operatorname{Sin}[c + d x])^3} dx$$

Optimal (type 4, 1584 leaves, 73 steps):

$$\begin{aligned} & -\frac{3 i a^2 (e + f x)^2}{2 b (a^2 - b^2)^2 d} + \frac{i (e + f x)^2}{b (a^2 - b^2) d} + \frac{2 a f^2 \operatorname{ArcTan}\left[\frac{b + a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^3} + \frac{3 a^2 f (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^2 d^2} - \frac{2 f (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^2} + \\ & \frac{3 i a^3 (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{2 b (a^2 - b^2)^{5/2} d} - \frac{3 i a (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{2 b (a^2 - b^2)^{3/2} d} + \frac{3 a^2 f (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^2 d^2} - \frac{2 f (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^2} - \\ & \frac{3 i a^3 (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{2 b (a^2 - b^2)^{5/2} d} + \frac{3 i a (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{2 b (a^2 - b^2)^{3/2} d} - \frac{3 i a^2 f^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^2 d^3} + \frac{2 i f^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^3} + \\ & \frac{3 a^3 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{5/2} d^2} - \frac{3 a f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^2} - \frac{3 i a^2 f^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^2 d^3} + \\ & \frac{2 i f^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2) d^3} - \frac{3 a^3 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{5/2} d^2} + \frac{3 a f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^2} + \\ & \frac{3 i a^3 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{5/2} d^3} - \frac{3 i a f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^3} - \frac{3 i a^3 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{5/2} d^3} + \frac{3 i a f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^3} - \\ & \frac{a (e + f x)^2 \operatorname{Cos}[c + d x]}{2 (a^2 - b^2) d (a + b \operatorname{Sin}[c + d x])^2} - \frac{a f (e + f x)}{b (a^2 - b^2) d^2 (a + b \operatorname{Sin}[c + d x])} - \frac{3 a^2 (e + f x)^2 \operatorname{Cos}[c + d x]}{2 (a^2 - b^2)^2 d (a + b \operatorname{Sin}[c + d x])} + \frac{(e + f x)^2 \operatorname{Cos}[c + d x]}{(a^2 - b^2) d (a + b \operatorname{Sin}[c + d x])} \end{aligned}$$

Result (type 4, 7742 leaves):

$$-\frac{1}{(-a^2 + b^2)^2 d^2} 3 a b e f \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b + a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} + \right.$$

$$\begin{aligned}
& \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(-c + \frac{\pi}{2} - dx \right) \operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - 2 \left(-c + \operatorname{ArcCos} \left[-\frac{a}{b} \right] \right) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) + \\
& \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] - 2i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}} \right] + \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + \right. \\
& \left. 2i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}} \right] - \\
& \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \operatorname{Log} \left[1 - \frac{(a-i\sqrt{-a^2+b^2}) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] + \\
& \left(-\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \operatorname{Log} \left[1 - \frac{(a+i\sqrt{-a^2+b^2}) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] + \\
& i \left(\operatorname{PolyLog} \left[2, \frac{(a-i\sqrt{-a^2+b^2}) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] - \right. \\
& \left. \operatorname{PolyLog} \left[2, \frac{(a+i\sqrt{-a^2+b^2}) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] \right) \right) - \\
& \frac{1}{b(-a^2+b^2)^2 d^3} a^3 f^2 \operatorname{Cot}[c] \left(\frac{\pi \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(-c + \frac{\pi}{2} - dx \right) \operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(-c + \operatorname{ArcCos} \left[-\frac{a}{b} \right] \right) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] + \\
& \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] - 2i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2}i \left(-c + \frac{\pi}{2} - dx \right)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin} [c+dx]}} \right] + \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + \right. \\
& \left. 2i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i \left(-c + \frac{\pi}{2} - dx \right)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin} [c+dx]}} \right] - \\
& \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \operatorname{Log} \left[1 - \frac{(a-i\sqrt{-a^2+b^2}) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] + \\
& \left(-\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \operatorname{Log} \left[1 - \frac{(a+i\sqrt{-a^2+b^2}) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] + \\
& i \left(\operatorname{PolyLog} \left[2, \frac{(a-i\sqrt{-a^2+b^2}) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] - \right. \\
& \left. \operatorname{PolyLog} \left[2, \frac{(a+i\sqrt{-a^2+b^2}) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] \right) \right) - \\
& \frac{1}{(-a^2+b^2)^2 d^3} 2abf^2 \operatorname{Cot} [c] \left(\frac{\pi \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(-c + \frac{\pi}{2} - dx \right) \operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \right. \right. \\
& \left. \left. 2 \left(-c + \operatorname{ArcCos} \left[-\frac{a}{b} \right] \right) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\text{ArcCos}\left[-\frac{a}{b}\right] - 2i \left(\text{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] - \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \\
& \text{Log}\left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \sin[c+dx]}}\right] + \left(\text{ArcCos}\left[-\frac{a}{b}\right] + \right. \\
& \left. 2i \left(\text{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] - \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \sin[c+dx]}}\right] - \\
& \left(\text{ArcCos}\left[-\frac{a}{b}\right] + 2i \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \text{Log}\left[1 - \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] + \\
& \left(-\text{ArcCos}\left[-\frac{a}{b}\right] + 2i \text{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \text{Log}\left[1 - \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] + \\
& i \left(\text{PolyLog}\left[2, \frac{(a-i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] - \right. \\
& \left. \text{PolyLog}\left[2, \frac{(a+i\sqrt{-a^2+b^2})(a+b-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(-c+\frac{\pi}{2}-dx)\right])}\right] \right) \right) - \\
& \left(3ab e^{ic} f^2 \left(d^2 x^2 \text{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} - \sqrt{(-a^2+b^2)} e^{2ic}}\right] - d^2 x^2 \text{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - \right. \right. \\
& \left. 2i dx \text{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 2i dx \text{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \right. \\
& \left. \left. 2 \text{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 2 \text{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(2 (-a^2 + b^2)^2 d^3 \sqrt{(-a^2 + b^2) e^{2ic}} \right) - \frac{3 i a b e^2 \operatorname{ArcTan} \left[\frac{i b \cos[c] - i (-a + b \sin[c]) \operatorname{Tan} \left[\frac{dx}{2} \right]}{\sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} \right]}{(-a^2 + b^2)^2 d \sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} + \\
& \frac{2 i a^3 f^2 \operatorname{ArcTan} \left[\frac{i b \cos[c] - i (-a + b \sin[c]) \operatorname{Tan} \left[\frac{dx}{2} \right]}{\sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} \right]}{b (-a^2 + b^2)^2 d^3 \sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} - \\
& \frac{2 i a b f^2 \operatorname{ArcTan} \left[\frac{i b \cos[c] - i (-a + b \sin[c]) \operatorname{Tan} \left[\frac{dx}{2} \right]}{\sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} \right]}{(-a^2 + b^2)^2 d^3 \sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} - \\
& \frac{2 i a^3 e f \operatorname{ArcTan} \left[\frac{i b \cos[c] - i (-a + b \sin[c]) \operatorname{Tan} \left[\frac{dx}{2} \right]}{\sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} \right] \operatorname{Cot}[c]}{b (-a^2 + b^2)^2 d^2 \sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} - \\
& \frac{4 i a b e f \operatorname{ArcTan} \left[\frac{i b \cos[c] - i (-a + b \sin[c]) \operatorname{Tan} \left[\frac{dx}{2} \right]}{\sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} \right] \operatorname{Cot}[c]}{(-a^2 + b^2)^2 d^2 \sqrt{-a^2 + b^2 \cos[c]^2 + b^2 \sin[c]^2}} - \\
& \frac{1}{(-a^2 + b^2)^2 d} \\
& \frac{a^2}{f^2} \\
& \operatorname{Csc}[c] \\
& \left(-\frac{x^2 \cos[c]}{2b} + \frac{1}{bd} \right) \\
& x \left(d x \cos[c] - \frac{2 a \operatorname{ArcTan} \left[\frac{\sec \left[\frac{dx}{2} \right] (\cos[c] - i \sin[c]) (b \cos \left[c + \frac{dx}{2} \right] + a \sin \left[\frac{dx}{2} \right])}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}} \right]}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}} \right) \cos[c] (\cos[c] - i \sin[c]) - \operatorname{Log}[a + b \sin[c + dx]] \sin[c] \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b d} \left(-\frac{1}{d} a \operatorname{Cos}[c] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(c - \operatorname{ArcCos}\left[-\frac{a}{b}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-2c+\pi-2 d x) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[\right. \right. \right. \\
& \quad \left. \left. \left. \frac{(a+b) \left(-a+b-i \sqrt{-a^2+b^2} \right) \left(1+i \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x)\right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x)\right] \right)} \right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[\frac{(a+b) \left(i a-i b+\sqrt{-a^2+b^2} \right) \left(i+\operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x)\right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x)\right] \right)} \right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \left(-\operatorname{ArcTanh}\left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x)\right]}{\sqrt{-a^2+b^2}} + \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right] \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4} i(-2c+\pi-2 d x)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+d x]}} \right] + \right. \\
& \quad \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right. \\
& \quad \left. \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4} i(2c-\pi+2 d x)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+d x]}} \right] + i \left(\operatorname{PolyLog}\left[2, \frac{\left(a-i \sqrt{-a^2+b^2} \right) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x)\right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x)\right] \right)} \right] - \right. \\
& \quad \left. \left. \left. \operatorname{PolyLog}\left[2, \frac{\left(a+i \sqrt{-a^2+b^2} \right) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x)\right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x)\right] \right)} \right] \right) \right) \right) \right) + \\
& \quad \frac{2 a x \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{d x}{2}\right] \left(\operatorname{Cos}[c]-i \operatorname{Sin}[c] \right) \left(b \operatorname{Cos}\left[c+\frac{d x}{2}\right]+a \operatorname{Sin}\left[\frac{d x}{2}\right] \right)}{\sqrt{a^2-b^2} \sqrt{\left(\operatorname{Cos}[c]-i \operatorname{Sin}[c] \right)^2}}\right] \operatorname{Cos}[c] \left(\operatorname{Cos}[c]-i \operatorname{Sin}[c] \right)}{\sqrt{a^2-b^2} \sqrt{\left(\operatorname{Cos}[c]-i \operatorname{Sin}[c] \right)^2}} + \frac{(c+d x) \operatorname{Log}[a+b \operatorname{Sin}[c+d x]] \operatorname{Sin}[c]}{d} -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{d} \left(\frac{(c+dx) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{b} - 1/b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx \right)^2 + 4 i \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[\frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{a^2 - b^2}} \right] \right) + \right. \\
& \left. \left(-c + \frac{\pi}{2} - dx + 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{\left(a - \sqrt{a^2 - b^2} \right) e^{i \left(-c + \frac{\pi}{2} - dx \right)}}{b} \right] + \right. \\
& \left. \left(-c + \frac{\pi}{2} - dx - 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{\left(a + \sqrt{a^2 - b^2} \right) e^{i \left(-c + \frac{\pi}{2} - dx \right)}}{b} \right] - \left(-c + \frac{\pi}{2} - dx \right) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] - \right. \\
& \left. i \left(\operatorname{PolyLog} \left[2, -\frac{\left(a - \sqrt{a^2 - b^2} \right) e^{i \left(-c + \frac{\pi}{2} - dx \right)}}{b} \right] + \operatorname{PolyLog} \left[2, -\frac{\left(a + \sqrt{a^2 - b^2} \right) e^{i \left(-c + \frac{\pi}{2} - dx \right)}}{b} \right] \right) \operatorname{Sin}[c] \right) - \\
& \frac{1}{(-a^2 + b^2)^2 d} 2 b^2 f^2 \operatorname{Csc}[c] \left(-\frac{x^2 \operatorname{Cos}[c]}{2 b} + \frac{1}{b d} x \left(dx \operatorname{Cos}[c] - \frac{2 a \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{dx}{2} \right] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (b \operatorname{Cos} \left[c + \frac{dx}{2} \right] + a \operatorname{Sin} \left[\frac{dx}{2} \right])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}} \right]}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}} \right) \operatorname{Cos}[c] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) \right. \right. \\
& \left. \left. \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] \operatorname{Sin}[c] \right) + \right. \\
& \left. \frac{1}{b d} \left(-\frac{1}{d} a \operatorname{Cos}[c] \left(\frac{\pi \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2 - b^2}} \right]}{\sqrt{a^2 - b^2}} + \frac{1}{\sqrt{-a^2 + b^2}} \left(2 \left(c - \operatorname{ArcCos} \left[-\frac{a}{b} \right] \right) \operatorname{ArcTanh} \left[\frac{(a-b) \operatorname{Tan} \left[\frac{1}{4} (2c - \pi + 2dx) \right]}{\sqrt{-a^2 + b^2}} \right] \right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (-2c + \pi - 2dx) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right]\right) \operatorname{Log}\left[\right. \\
& \left. \frac{(a+b) \left(-a+b-i\sqrt{-a^2+b^2}\right) \left(1+i \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)} \right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right]\right) \operatorname{Log}\left[\right. \\
& \left. \frac{(a+b) \left(ia-ib+\sqrt{-a^2+b^2}\right) \left(i+\operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)} \right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \left(-\operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] + \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right]\right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4}i(-2c+\pi-2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right]\right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4}i(2c-\pi+2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}}\right] + i \left(\operatorname{PolyLog}\left[2, \frac{(a-i\sqrt{-a^2+b^2}) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(a+i\sqrt{-a^2+b^2}) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)}\right]\right) + \\
& \frac{2ax \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{dx}{2}\right] (\operatorname{Cos}[c]-i \operatorname{Sin}[c]) \left(b \operatorname{Cos}\left[c+\frac{dx}{2}\right]+a \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{\sqrt{a^2-b^2} \sqrt{(\operatorname{Cos}[c]-i \operatorname{Sin}[c])^2}}\right] \operatorname{Cos}[c] (\operatorname{Cos}[c]-i \operatorname{Sin}[c])}{\sqrt{a^2-b^2} \sqrt{(\operatorname{Cos}[c]-i \operatorname{Sin}[c])^2}} + \frac{(c+dx) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] \operatorname{Sin}[c]}{d} - \\
& \frac{1}{d} b \left(\frac{(c+dx) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{b} - 1/b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx\right)^2 + 4i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{a^2-b^2}}\right]\right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-c + \frac{\pi}{2} - dx + 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{\left(a - \sqrt{a^2 - b^2} \right) e^{i \left(-c + \frac{\pi}{2} - dx \right)}}{b} \right] + \\
& \left(-c + \frac{\pi}{2} - dx - 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{\left(a + \sqrt{a^2 - b^2} \right) e^{i \left(-c + \frac{\pi}{2} - dx \right)}}{b} \right] - \left(-c + \frac{\pi}{2} - dx \right) \operatorname{Log} [a + b \operatorname{Sin}[c + dx]] - \\
& i \left(\operatorname{PolyLog} \left[2, -\frac{\left(a - \sqrt{a^2 - b^2} \right) e^{i \left(-c + \frac{\pi}{2} - dx \right)}}{b} \right] + \operatorname{PolyLog} \left[2, -\frac{\left(a + \sqrt{a^2 - b^2} \right) e^{i \left(-c + \frac{\pi}{2} - dx \right)}}{b} \right] \right) \operatorname{Sin}[c] + \\
& \left(a^2 e f \operatorname{Csc}[c] \left(-b dx \operatorname{Cos}[c] + b \operatorname{Log}[a + b \operatorname{Cos}[dx] \operatorname{Sin}[c] + b \operatorname{Cos}[c] \operatorname{Sin}[dx]] \operatorname{Sin}[c] + \right. \right. \\
& \left. \left. \frac{2 i a b \operatorname{ArcTan} \left[\frac{i b \operatorname{Cos}[c] - i (-a + b \operatorname{Sin}[c]) \operatorname{Tan} \left[\frac{dx}{2} \right]}{\sqrt{-a^2 + b^2 \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2}} \right] \operatorname{Cos}[c]}{\sqrt{-a^2 + b^2 \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2}} \right) \right) + \\
& \left((-a^2 + b^2)^2 d^2 (b^2 \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2) + \right. \\
& \left. \left(2 \right. \right. \\
& \left. \left. b^2 \right. \right. \\
& \left. \left. e \right. \right. \\
& \left. \left. f \right. \right. \\
& \left. \left. \operatorname{Csc}[c] \right. \right. \\
& \left. \left. c \right) \right)
\end{aligned}$$

$$\left(\begin{aligned} & -b d x \cos [c] + b \log [a + b \cos [d x] \sin [c] + b \cos [c] \sin [d x]] \sin [c] + \\ & \frac{2 i a b \operatorname{ArcTan}\left[\frac{i b \cos [c] - i (-a + b \sin [c]) \tan \left[\frac{d x}{2}\right]}{\sqrt{-a^2 + b^2 \cos [c]^2 + b^2 \sin [c]^2}}\right] \cos [c]}{\sqrt{-a^2 + b^2 \cos [c]^2 + b^2 \sin [c]^2}} \right) / \left((-a^2 + b^2)^2 d^2 (b^2 \cos [c]^2 + b^2 \sin [c]^2) \right) - \\ & \left(\operatorname{Csc}[c] \left(a^2 e^2 \cos [c] + 2 a^2 e f x \cos [c] + a^2 f^2 x^2 \cos [c] + a b e^2 \sin [d x] + 2 a b e f x \sin [d x] + a b f^2 x^2 \sin [d x] \right) \right) / \\ & \left(2 \right. \\ & \quad b \\ & \quad \left. (-a^2 + b^2) \right. \\ & \quad d \\ & \quad \left. (a + b \sin [c + d x])^2 \right) + \\ & \left(\operatorname{Csc}[c] \left(3 a b^2 d e^2 \cos [c] + 6 a b^2 d e f x \cos [c] + 3 a b^2 d f^2 x^2 \cos [c] - 2 a^3 e f \sin [c] + 2 a b^2 e f \sin [c] - \right. \right. \\ & \quad 2 a^3 f^2 x \sin [c] + 2 a b^2 f^2 x \sin [c] + a^2 b d e^2 \sin [d x] + 2 b^3 d e^2 \sin [d x] + \\ & \quad 2 a^2 b d e f x \sin [d x] + 4 b^3 d e f x \sin [d x] + a^2 b d f^2 x^2 \sin [d x] + \\ & \quad \left. \left. 2 b^3 d f^2 x^2 \sin [d x] \right) \right) / \left(2 b (-a^2 + b^2)^2 d^2 (a + b \sin [c + d x]) \right)
\end{aligned}
\right)$$

■ **Problem 250: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \sin [c + d x]}{(a + b \sin [c + d x])^3} dx$$

Optimal (type 4, 2348 leaves, 92 steps):

$$\begin{aligned}
& - \frac{3 i a^2 (e+f x)^3}{2 b (a^2-b^2)^2 d} + \frac{i (e+f x)^3}{b (a^2-b^2) d} - \frac{3 i a f^2 (e+f x) \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^3} + \frac{9 a^2 f (e+f x)^2 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^2 d^2} - \frac{3 f (e+f x)^2 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^2} + \\
& \frac{3 i a^3 (e+f x)^3 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{5/2} d} - \frac{3 i a (e+f x)^3 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{3/2} d} + \frac{3 i a f^2 (e+f x) \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^3} + \frac{9 a^2 f (e+f x)^2 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^2 d^2} - \\
& \frac{3 f (e+f x)^2 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^2} - \frac{3 i a^3 (e+f x)^3 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{5/2} d} + \frac{3 i a (e+f x)^3 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{3/2} d} - \frac{3 a f^3 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^4} - \\
& \frac{9 i a^2 f^2 (e+f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^2 d^3} + \frac{6 i f^2 (e+f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^3} + \frac{9 a^3 f (e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{5/2} d^2} - \\
& \frac{9 a f (e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{3/2} d^2} + \frac{3 a f^3 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^4} - \frac{9 i a^2 f^2 (e+f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^2 d^3} + \\
& \frac{6 i f^2 (e+f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^3} - \frac{9 a^3 f (e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{5/2} d^2} + \frac{9 a f (e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{3/2} d^2} + \\
& \frac{9 a^2 f^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^2 d^4} - \frac{6 f^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^4} + \frac{9 i a^3 f^2 (e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{5/2} d^3} - \frac{9 i a f^2 (e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^3} + \\
& \frac{9 a^2 f^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^2 d^4} - \frac{6 f^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^4} - \frac{9 i a^3 f^2 (e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{5/2} d^3} + \frac{9 i a f^2 (e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^3} - \\
& \frac{9 a^3 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{5/2} d^4} + \frac{9 a f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^4} + \frac{9 a^3 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{5/2} d^4} - \frac{9 a f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^4} - \\
& \frac{a (e+f x)^3 \operatorname{Cos}[c+d x]}{2 (a^2-b^2) d (a+b \operatorname{Sin}[c+d x])^2} - \frac{3 a f (e+f x)^2}{2 b (a^2-b^2) d^2 (a+b \operatorname{Sin}[c+d x])} - \frac{3 a^2 (e+f x)^3 \operatorname{Cos}[c+d x]}{2 (a^2-b^2)^2 d (a+b \operatorname{Sin}[c+d x])} + \frac{(e+f x)^3 \operatorname{Cos}[c+d x]}{(a^2-b^2) d (a+b \operatorname{Sin}[c+d x])}
\end{aligned}$$

Result (type 4, 14368 leaves):

$$\begin{aligned}
& - \frac{1}{2(-a^2 + b^2)^2 d^2} 9 a b e^2 f \left(\frac{\pi \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{\sqrt{a^2-b^2}} + \right. \\
& \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(-c + \frac{\pi}{2} - dx \right) \operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - 2 \left(-c + \operatorname{ArcCos} \left[-\frac{a}{b} \right] \right) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) + \\
& \left. \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] - 2 i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \right) \\
& \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx \right)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin} [c+dx]}} \right] + \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + \right. \\
& \left. 2 i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx \right)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin} [c+dx]}} \right] - \\
& \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \operatorname{Log} \left[1 - \frac{\left(a - i \sqrt{-a^2+b^2} \right) \left(a + b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a + b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] + \\
& \left(-\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \operatorname{Log} \left[1 - \frac{\left(a + i \sqrt{-a^2+b^2} \right) \left(a + b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a + b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] + \\
& i \left(\operatorname{PolyLog} \left[2, \frac{\left(a - i \sqrt{-a^2+b^2} \right) \left(a + b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a + b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] - \right. \\
& \left. \operatorname{PolyLog} \left[2, \frac{\left(a + i \sqrt{-a^2+b^2} \right) \left(a + b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a + b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b(-a^2+b^2)^2 d^4} 3 a^3 f^3 \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2\left(-c+\frac{\pi}{2}-dx\right) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \right. \\
& 2\left(-c+\operatorname{ArcCos}\left[-\frac{a}{b}\right]\right) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{\sqrt{-a^2+b^2}}\right] + \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}}\right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + \right. \\
& \left. 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i(-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{\left(a-i\sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{b\left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}\right] + \\
& \left(-\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{\left(a+i\sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{b\left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{\left(a-i\sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{b\left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{\left(a+i\sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{b\left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}\right] \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(-a^2 + b^2)^2 d^4} 3 a b f^3 \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(-c + \frac{\pi}{2} - dx\right) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \right. \\
& \left. \left. 2 \left(-c + \operatorname{ArcCos}\left[-\frac{a}{b}\right]\right) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] + \right. \right. \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right) \\
& \left. \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2} i (c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}}\right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + \right. \right. \\
& \left. \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2} i (c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{\left(a - i \sqrt{-a^2+b^2}\right) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]\right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]\right)}\right] + \\
& \left(-\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{\left(a + i \sqrt{-a^2+b^2}\right) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]\right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]\right)}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{\left(a - i \sqrt{-a^2+b^2}\right) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]\right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]\right)}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{\left(a + i \sqrt{-a^2+b^2}\right) \left(a+b - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]\right)}{b \left(a+b + \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]\right)}\right] \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b(-a^2+b^2)^2 d^3} 3 a^3 e f^2 \cot [c] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2\left(-c+\frac{\pi}{2}-d x\right) \operatorname{ArcTanh}\left[\frac{(a+b) \cot \left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \right. \\
& 2\left(-c+\operatorname{ArcCos}\left[-\frac{a}{b}\right]\right) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]}{\sqrt{-a^2+b^2}}\right] + \\
& \left. \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \cot \left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2} i\left(-c+\frac{\pi}{2}-d x\right)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+d x]}}\right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + \right. \\
& \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \cot \left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]}{\sqrt{-a^2+b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2} i\left(-c+\frac{\pi}{2}-d x\right)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+d x]}}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{\left(a-i \sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]\right)}{b\left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]\right)}\right] + \\
& \left(-\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{\left(a+i \sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]\right)}{b\left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]\right)}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{\left(a-i \sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]\right)}{b\left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]\right)}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{\left(a+i \sqrt{-a^2+b^2}\right)\left(a+b-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]\right)}{b\left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right)\right]\right)}\right] \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(-a^2 + b^2)^2 d^3} 6 a b e f^2 \cot [c] \left(\frac{\pi \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(-c + \frac{\pi}{2} - dx \right) \operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \right. \right. \\
& 2 \left(-c + \operatorname{ArcCos} \left[-\frac{a}{b} \right] \right) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] + \\
& \left. \left. \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] - 2 i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \right) \\
& \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2} i (-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}} \right] + \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + \right. \\
& \left. 2 i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2} i (-c+\frac{\pi}{2}-dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}} \right] - \\
& \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \operatorname{Log} \left[1 - \frac{\left(a - i \sqrt{-a^2+b^2} \right) \left(a + b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a + b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] + \\
& \left(-\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{-a^2+b^2}} \right] \right) \operatorname{Log} \left[1 - \frac{\left(a + i \sqrt{-a^2+b^2} \right) \left(a + b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a + b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] + \\
& i \left(\operatorname{PolyLog} \left[2, \frac{\left(a - i \sqrt{-a^2+b^2} \right) \left(a + b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a + b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] - \right. \\
& \left. \operatorname{PolyLog} \left[2, \frac{\left(a + i \sqrt{-a^2+b^2} \right) \left(a + b - \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)}{b \left(a + b + \sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)} \right] \right) \right) - \\
& \left(9 a b e e^{i c} f^2 \left(d^2 x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2c+dx)}}{i a e^{i c} - \sqrt{(-a^2+b^2)} e^{2 i c}} \right] - d^2 x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2c+dx)}}{i a e^{i c} + \sqrt{(-a^2+b^2)} e^{2 i c}} \right] \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Id} x \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{-a^2 + b^2} e^{2ic}}\right] + \\
& 2 \operatorname{Id} x \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{-a^2 + b^2} e^{2ic}}\right] + 2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{-a^2 + b^2} e^{2ic}}\right] - \\
& 2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{-a^2 + b^2} e^{2ic}}\right] \Bigg) / \left(2(-a^2 + b^2)^2 d^3 \sqrt{-a^2 + b^2} e^{2ic}\right) - \\
& \left(3 a^3 e^{ic} f^3 \operatorname{Cot}[c] \left(d^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} - \sqrt{-a^2 + b^2} e^{2ic}}\right] - d^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{-a^2 + b^2} e^{2ic}}\right]\right) - \right. \\
& 2 \operatorname{Id} x \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{-a^2 + b^2} e^{2ic}}\right] + \\
& 2 \operatorname{Id} x \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{-a^2 + b^2} e^{2ic}}\right] + 2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{-a^2 + b^2} e^{2ic}}\right] - \\
& 2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{-a^2 + b^2} e^{2ic}}\right] \Bigg) / \left(2 b (-a^2 + b^2)^2 d^4 \sqrt{-a^2 + b^2} e^{2ic}\right) - \\
& \left(3 a b e^{ic} f^3 \operatorname{Cot}[c] \left(d^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} - \sqrt{-a^2 + b^2} e^{2ic}}\right] - d^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{-a^2 + b^2} e^{2ic}}\right]\right) - \right. \\
& 2 \operatorname{Id} x \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{-a^2 + b^2} e^{2ic}}\right] + 2 \operatorname{Id} x \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{-a^2 + b^2} e^{2ic}}\right] + \\
& 2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{-a^2 + b^2} e^{2ic}}\right] - 2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{-a^2 + b^2} e^{2ic}}\right] \Bigg) / \\
& \left((-a^2 + b^2)^2 d^4 \sqrt{-a^2 + b^2} e^{2ic}\right) - \frac{1}{4 b (-a^2 + b^2)^2 d^4 \sqrt{-a^2 + b^2} e^{2ic}}
\end{aligned}$$

a^2
 e^{-ic}
 f^3
 $\operatorname{Csc}[$
 $c]$

$$\begin{aligned}
& \left(2 d^3 e^{2 i c} \sqrt{(-a^2 + b^2) e^{2 i c}} x^3 - 3 a d^2 e^{i c} x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2 c + d x)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] - \right. \\
& 3 a d^2 e^{3 i c} x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2 c + d x)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] - \\
& 3 i d^2 \sqrt{(-a^2 + b^2) e^{2 i c}} x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2 c + d x)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] + \\
& 3 i d^2 e^{2 i c} \sqrt{(-a^2 + b^2) e^{2 i c}} x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2 c + d x)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] + \\
& 3 a d^2 e^{i c} x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2 c + d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] + \\
& 3 a d^2 e^{3 i c} x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2 c + d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] - \\
& 3 i d^2 \sqrt{(-a^2 + b^2) e^{2 i c}} x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2 c + d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] + \\
& 3 i d^2 e^{2 i c} \sqrt{(-a^2 + b^2) e^{2 i c}} x^2 \operatorname{Log} \left[1 + \frac{b e^{i (2 c + d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] + \\
& 6 d \left(\sqrt{(-a^2 + b^2) e^{2 i c}} (-1 + e^{2 i c}) + i a e^{i c} (1 + e^{2 i c}) \right) x \operatorname{PolyLog} \left[2, \frac{i b e^{i (2 c + d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] + \\
& 6 d \left(\sqrt{(-a^2 + b^2) e^{2 i c}} (-1 + e^{2 i c}) - i a e^{i c} (1 + e^{2 i c}) \right) x \operatorname{PolyLog} \left[2, -\frac{b e^{i (2 c + d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] - \\
& 6 a e^{i c} \operatorname{PolyLog} \left[3, \frac{i b e^{i (2 c + d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] - 6 a e^{3 i c} \operatorname{PolyLog} \left[3, \frac{i b e^{i (2 c + d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] - \\
& 6 i \sqrt{(-a^2 + b^2) e^{2 i c}} \operatorname{PolyLog} \left[3, \frac{i b e^{i (2 c + d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] + \\
& 6 i e^{2 i c} \sqrt{(-a^2 + b^2) e^{2 i c}} \operatorname{PolyLog} \left[3, \frac{i b e^{i (2 c + d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}} \right] +
\end{aligned}$$

$$\begin{aligned}
& 6 a e^{i c} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + 6 a e^{3 i c} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - \\
& 6 i \sqrt{(-a^2+b^2) e^{2 i c}} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 6 i e^{2 i c} \sqrt{(-a^2+b^2) e^{2 i c}} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] \Bigg] - \\
& \frac{1}{2(-a^2+b^2)^2 d^4 \sqrt{(-a^2+b^2) e^{2 i c}}} b e^{-i c} f^3 \operatorname{Csc}[c] \left(2 d^3 e^{2 i c} \sqrt{(-a^2+b^2) e^{2 i c}} x^3 - 3 a d^2 e^{i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} - \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - \right. \\
& 3 a d^2 e^{3 i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} - \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - \\
& 3 i d^2 \sqrt{(-a^2+b^2) e^{2 i c}} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} - \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 3 i d^2 e^{2 i c} \sqrt{(-a^2+b^2) e^{2 i c}} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} - \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 3 a d^2 e^{i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + 3 a d^2 e^{3 i c} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - \\
& 3 i d^2 \sqrt{(-a^2+b^2) e^{2 i c}} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 3 i d^2 e^{2 i c} \sqrt{(-a^2+b^2) e^{2 i c}} x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 6 d \left(\sqrt{(-a^2+b^2) e^{2 i c}} (-1 + e^{2 i c}) + i a e^{i c} (1 + e^{2 i c}) \right) x \operatorname{PolyLog}\left[2, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 6 d \left(\sqrt{(-a^2+b^2) e^{2 i c}} (-1 + e^{2 i c}) - i a e^{i c} (1 + e^{2 i c}) \right) x \operatorname{PolyLog}\left[2, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - \\
& 6 a e^{i c} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - 6 a e^{3 i c} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] -
\end{aligned}$$

$$\begin{aligned}
& 6 i \sqrt{-a^2 + b^2} e^{2 i c} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{-a^2 + b^2} e^{2 i c}}\right] + \\
& 6 i e^{2 i c} \sqrt{-a^2 + b^2} e^{2 i c} \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{-a^2 + b^2} e^{2 i c}}\right] + \\
& 6 a e^{i c} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] + 6 a e^{3 i c} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] - \\
& 6 i \sqrt{-a^2 + b^2} e^{2 i c} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] + \\
& 6 i e^{2 i c} \sqrt{-a^2 + b^2} e^{2 i c} \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] - \\
& \frac{1}{2(-a^2 + b^2)^2 d^4 \sqrt{-a^2 + b^2} e^{2 i c}} 3 a b e^{i c} f^3 \left(d^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{-a^2 + b^2} e^{2 i c}}\right] - d^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] - \right. \\
& 3 i d^2 x^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{-a^2 + b^2} e^{2 i c}}\right] + 3 i d^2 x^2 \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] + \\
& 6 d x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{-a^2 + b^2} e^{2 i c}}\right] - 6 d x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] + \\
& \left. 6 i \operatorname{PolyLog}\left[4, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{-a^2 + b^2} e^{2 i c}}\right] - 6 i \operatorname{PolyLog}\left[4, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] \right) - \\
& \frac{3 i a b e^3 \operatorname{ArcTan}\left[\frac{i b \operatorname{Cos}[c] - i(-a + b \operatorname{Sin}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right]}{\sqrt{-a^2 + b^2 \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2}}\right]}{(-a^2 + b^2)^2 d \sqrt{-a^2 + b^2 \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2}} + \\
& \frac{6 i a^3 e f^2 \operatorname{ArcTan}\left[\frac{i b \operatorname{Cos}[c] - i(-a + b \operatorname{Sin}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right]}{\sqrt{-a^2 + b^2 \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2}}\right]}{b(-a^2 + b^2)^2 d^3 \sqrt{-a^2 + b^2 \operatorname{Cos}[c]^2 + b^2 \operatorname{Sin}[c]^2}} -
\end{aligned}$$

$$\frac{6 i a b e f^2 \operatorname{ArcTan}\left[\frac{i b \cos [c]-i(-a+b \sin [c]) \operatorname{Tan}\left[\frac{d x}{2}\right]}{\sqrt{-a^2+b^2 \cos [c]^2+b^2 \sin [c]^2}}\right]}{(-a^2+b^2)^2 d^3 \sqrt{-a^2+b^2 \cos [c]^2+b^2 \sin [c]^2}} -$$

$$\frac{3 i a^3 e^2 f \operatorname{ArcTan}\left[\frac{i b \cos [c]-i(-a+b \sin [c]) \operatorname{Tan}\left[\frac{d x}{2}\right]}{\sqrt{-a^2+b^2 \cos [c]^2+b^2 \sin [c]^2}}\right] \operatorname{Cot}[c]}{b(-a^2+b^2)^2 d^2 \sqrt{-a^2+b^2 \cos [c]^2+b^2 \sin [c]^2}} -$$

$$\frac{6 i a b e^2 f \operatorname{ArcTan}\left[\frac{i b \cos [c]-i(-a+b \sin [c]) \operatorname{Tan}\left[\frac{d x}{2}\right]}{\sqrt{-a^2+b^2 \cos [c]^2+b^2 \sin [c]^2}}\right] \operatorname{Cot}[c]}{(-a^2+b^2)^2 d^2 \sqrt{-a^2+b^2 \cos [c]^2+b^2 \sin [c]^2}} -$$

$$\frac{1}{(-a^2+b^2)^2 d}$$

$$3 a^2 e f^2 \operatorname{Csc}[c]$$

$$\left(-\frac{x^2 \cos [c]}{2 b} + \frac{1}{b d} \right)$$

$$x \left(d x \cos [c] - \frac{2 a \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{d x}{2}\right](\cos [c]-i \sin [c])\left(b \cos \left[c+\frac{d x}{2}\right]+a \sin \left[\frac{d x}{2}\right]\right)}{\sqrt{a^2-b^2} \sqrt{(\cos [c]-i \sin [c])^2}}\right] \cos [c](\cos [c]-i \sin [c])}{\sqrt{a^2-b^2} \sqrt{(\cos [c]-i \sin [c])^2}} - \operatorname{Log}[a+b \sin [c+d x]] \sin [c] \right) +$$

$$\frac{1}{b d} \left(-\frac{1}{d} a \cos [c] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(c - \operatorname{ArcCos}\left[-\frac{a}{b}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2 c-\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] \right) + \right.$$

$$\left. (-2 c+\pi-2 d x) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2 c+\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2 c-\pi+2 d x)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[\right. \right.$$

$$\begin{aligned}
& \frac{(a+b) \left(-a+b-i\sqrt{-a^2+b^2} \right) \left(1+i \operatorname{Cot} \left[\frac{1}{4} (2c+\pi+2dx) \right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot} \left[\frac{1}{4} (2c+\pi+2dx) \right] \right)} \Bigg] - \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2i \operatorname{ArcTanh} \left[\frac{(a-b) \operatorname{Tan} \left[\frac{1}{4} (2c-\pi+2dx) \right]}{\sqrt{-a^2+b^2}} \right] \right) \\
& \operatorname{Log} \left[\frac{(a+b) \left(ia-ib+\sqrt{-a^2+b^2} \right) \left(i+\operatorname{Cot} \left[\frac{1}{4} (2c+\pi+2dx) \right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot} \left[\frac{1}{4} (2c+\pi+2dx) \right] \right)} \right] + \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2i \left(-\operatorname{ArcTanh} \left[\frac{(a-b) \operatorname{Tan} \left[\frac{1}{4} (2c-\pi+2dx) \right]}{\sqrt{-a^2+b^2}} \right] \right. \right. \\
& \left. \left. + \operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Tan} \left[\frac{1}{4} (2c+\pi+2dx) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4}i(-2c+\pi-2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}} \right] + \\
& \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2i \operatorname{ArcTanh} \left[\frac{(a-b) \operatorname{Tan} \left[\frac{1}{4} (2c-\pi+2dx) \right]}{\sqrt{-a^2+b^2}} \right] - 2i \operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Tan} \left[\frac{1}{4} (2c+\pi+2dx) \right]}{\sqrt{-a^2+b^2}} \right] \right) \\
& \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4}i(2c-\pi+2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}} \right] + i \left(\operatorname{PolyLog} \left[2, \frac{(a-i\sqrt{-a^2+b^2}) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{4} (2c-\pi+2dx) \right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot} \left[\frac{1}{4} (2c+\pi+2dx) \right] \right)} \right] - \right. \\
& \left. \operatorname{PolyLog} \left[2, \frac{(a+i\sqrt{-a^2+b^2}) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{4} (2c-\pi+2dx) \right] \right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot} \left[\frac{1}{4} (2c+\pi+2dx) \right] \right)} \right] \right) \Bigg) \\
& \frac{2ax \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{dx}{2} \right] (\operatorname{Cos}[c]-i \operatorname{Sin}[c]) (b \operatorname{Cos} \left[c+\frac{dx}{2} \right] + a \operatorname{Sin} \left[\frac{dx}{2} \right])}{\sqrt{a^2-b^2} \sqrt{(\operatorname{Cos}[c]-i \operatorname{Sin}[c])^2}} \right] \operatorname{Cos}[c] (\operatorname{Cos}[c]-i \operatorname{Sin}[c])}{\sqrt{a^2-b^2} \sqrt{(\operatorname{Cos}[c]-i \operatorname{Sin}[c])^2}} + \frac{(c+dx) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] \operatorname{Sin}[c]}{d} - \\
& \frac{1}{d} b \left(\frac{(c+dx) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{b} - 1/b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx \right)^2 + 4i \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[\frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]}{\sqrt{a^2-b^2}} \right] \right) + \right. \\
& \left. \left(-c + \frac{\pi}{2} - dx + 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{(a-\sqrt{a^2-b^2}) e^{i(-c+\frac{\pi}{2}-dx)}}{b} \right] \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(-c + \frac{\pi}{2} - dx - 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] - \left(-c + \frac{\pi}{2} - dx\right) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] - \\
& i \left(\operatorname{PolyLog}\left[2, -\frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] \right) \operatorname{Sin}[c] \right) - \\
& \frac{1}{(-a^2 + b^2)^2 d} 6 b^2 e^{f^2} \operatorname{Csc}[c] \left(-\frac{x^2 \operatorname{Cos}[c]}{2b} + \frac{1}{bd} x \left(dx \operatorname{Cos}[c] - \frac{2a \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{dx}{2}\right] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (b \operatorname{Cos}\left[c + \frac{dx}{2}\right] + a \operatorname{Sin}\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right]}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right] \operatorname{Cos}[c] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) \right. \right. \\
& \left. \left. \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] \operatorname{Sin}[c] \right) + \right. \\
& \frac{1}{bd} \left(-\frac{1}{d} a \operatorname{Cos}[c] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} + \frac{1}{\sqrt{-a^2 + b^2}} \left(2 \left(c - \operatorname{ArcCos}\left[-\frac{a}{b}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] + \right. \right. \\
& \left. \left. (-2c + \pi - 2dx) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \operatorname{Log}\left[\right. \right. \\
& \left. \left. \frac{(a+b) \left(-a + b - i \sqrt{-a^2 + b^2}\right) \left(1 + i \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)} \right] - \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c - \pi + 2dx)\right]}{\sqrt{-a^2 + b^2}}\right] \right) \right) \\
& \left. \operatorname{Log}\left[\frac{(a+b) \left(i a - i b + \sqrt{-a^2 + b^2}\right) \left(i + \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)}{b \left(a + b + \sqrt{-a^2 + b^2} \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]\right)} \right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \left(-\operatorname{ArcTanh}\left[\right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx)\right]}{\sqrt{-a^2+b^2}} + \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i(-2c+\pi-2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx)\right]}{\sqrt{-a^2+b^2}}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2dx)\right]}{\sqrt{-a^2+b^2}}\right] \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2}i(2c-\pi+2dx)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx]}}\right] + i \left(\operatorname{PolyLog}\left[2, \frac{(a-i\sqrt{-a^2+b^2})(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2dx)\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(a+i\sqrt{-a^2+b^2})(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx)\right])}{b(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2dx)\right])}\right] \right) \right) + \\
& \frac{2a \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{dx}{2}\right](\operatorname{Cos}[c]-i \operatorname{Sin}[c])(b \operatorname{Cos}\left[c+\frac{dx}{2}\right]+a \operatorname{Sin}\left[\frac{dx}{2}\right])}{\sqrt{a^2-b^2} \sqrt{(\operatorname{Cos}[c]-i \operatorname{Sin}[c])^2}}\right] \operatorname{Cos}[c](\operatorname{Cos}[c]-i \operatorname{Sin}[c])}{\sqrt{a^2-b^2} \sqrt{(\operatorname{Cos}[c]-i \operatorname{Sin}[c])^2}} + \frac{(c+dx) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] \operatorname{Sin}[c]}{d} - \\
& \frac{1}{d} b \left(\frac{(c+dx) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{b} - 1/b \left(-\frac{1}{2}i \left(-c+\frac{\pi}{2}-dx\right)^2 + 4i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{\sqrt{a^2-b^2}}\right] \right) + \right. \\
& \left(-c+\frac{\pi}{2}-dx+2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1+\frac{(a-\sqrt{a^2-b^2}) e^{i(-c+\frac{\pi}{2}-dx)}}{b}\right] + \\
& \left(-c+\frac{\pi}{2}-dx-2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1+\frac{(a+\sqrt{a^2-b^2}) e^{i(-c+\frac{\pi}{2}-dx)}}{b}\right] - \left(-c+\frac{\pi}{2}-dx\right) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] -
\end{aligned}$$

$$i \left(\text{PolyLog} \left[2, -\frac{(a - \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - dx)}}{b} \right] + \text{PolyLog} \left[2, -\frac{(a + \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - dx)}}{b} \right] \right) \text{Sin}[c] +$$

$$\left(3 a^2 e^2 f \text{Csc}[c] \left(-b d x \text{Cos}[c] + b \text{Log}[a + b \text{Cos}[dx] \text{Sin}[c] + b \text{Cos}[c] \text{Sin}[dx]] \text{Sin}[c] + \frac{2 i a b \text{ArcTan} \left[\frac{i b \text{Cos}[c] - i(-a + b \text{Sin}[c]) \text{Tan} \left[\frac{dx}{2} \right]}{\sqrt{-a^2 + b^2 \text{Cos}[c]^2 + b^2 \text{Sin}[c]^2}} \right] \text{Cos}[c]}{\sqrt{-a^2 + b^2 \text{Cos}[c]^2 + b^2 \text{Sin}[c]^2}} \right) \right) /$$

$$\left(\frac{(-a^2 + b^2)^2}{d^2} (b^2 \text{Cos}[c]^2 + b^2 \text{Sin}[c]^2) \right) +$$

$$\left(3 b^2 e^2 f \text{Csc}[c] \left(-b d x \text{Cos}[c] + b \text{Log}[a + b \text{Cos}[dx] \text{Sin}[c] + b \text{Cos}[c] \text{Sin}[dx]] \text{Sin}[c] + \frac{2 i a b \text{ArcTan} \left[\frac{i b \text{Cos}[c] - i(-a + b \text{Sin}[c]) \text{Tan} \left[\frac{dx}{2} \right]}{\sqrt{-a^2 + b^2 \text{Cos}[c]^2 + b^2 \text{Sin}[c]^2}} \right] \text{Cos}[c]}{\sqrt{-a^2 + b^2 \text{Cos}[c]^2 + b^2 \text{Sin}[c]^2}} \right) \right) /$$

$$\left(\frac{(-a^2 + b^2)^2}{d^2} (b^2 \text{Cos}[c]^2 + b^2 \text{Sin}[c]^2) \right) -$$

$$\left(\text{Csc}[c] (a^2 e^3 \text{Cos}[c] + 3 a^2 e^2 f x \text{Cos}[c] + 3 a^2 e f^2 x^2 \text{Cos}[c] + a^2 f^3 x^3 \text{Cos}[c] + a b e^3 \text{Sin}[dx] + 3 a b e^2 f x \text{Sin}[dx] + 3 a b e f^2 x^2 \text{Sin}[dx] + a b f^3 x^3 \text{Sin}[dx]) \right) /$$

$$\left(2 b (-a^2 + b^2) d (a + b \text{Sin}[c + dx])^2 \right) + \frac{1}{2 b (-a^2 + b^2)^2 d^2 (a + b \text{Sin}[c + dx])}$$

Csc[
c]

$$\left(3 a b^2 d e^3 \text{Cos}[c] + 9 a b^2 d e^2 f x \text{Cos}[c] + 9 a b^2 d e f^2 x^2 \text{Cos}[c] + 3 a b^2 d f^3 x^3 \text{Cos}[c] - 3 a^3 e^2 f \text{Sin}[c] + 3 a b^2 e^2 f \text{Sin}[c] - 6 a^3 e f^2 x \text{Sin}[c] + 6 a b^2 e f^2 x \text{Sin}[c] - 3 a^3 f^3 x^2 \text{Sin}[c] + 3 a b^2 f^3 x^2 \text{Sin}[c] + a^2 b d e^3 \text{Sin}[dx] + 2 b^3 d e^3 \text{Sin}[dx] + 3 a^2 b d e^2 f x \text{Sin}[dx] + 6 b^3 d e^2 f x \text{Sin}[dx] + 3 a^2 b d e f^2 x^2 \text{Sin}[dx] + 6 b^3 d e f^2 x^2 \text{Sin}[dx] + a^2 b d f^3 x^3 \text{Sin}[dx] + 2 b^3 d f^3 x^3 \text{Sin}[dx] \right)$$

■ **Problem 253: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x) \text{Cos}[c + dx]}{a + a \text{Sin}[c + dx]} dx$$

Optimal (type 4, 79 leaves, 4 steps):

$$-\frac{i(e+fx)^2}{2af} + \frac{2(e+fx)\operatorname{Log}[1-ie^{i(c+dx)}]}{ad} - \frac{2if\operatorname{PolyLog}[2, ie^{i(c+dx)}]}{ad^2}$$

Result (type 4, 246 leaves):

$$\frac{1}{2ad^2} \left(-ic^2f + icf\pi - 2icdfx + idf\pi x - id^2fx^2 + 4f\pi\operatorname{Log}[1+e^{-i(c+dx)}] + 4cf\operatorname{Log}[1-ie^{i(c+dx)}] + \right. \\ \left. 2f\pi\operatorname{Log}[1-ie^{i(c+dx)}] + 4dfx\operatorname{Log}[1-ie^{i(c+dx)}] - 4f\pi\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] + 4de\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] - \right. \\ \left. 4cf\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] - 2f\pi\operatorname{Log}\left[\sin\left[\frac{1}{4}(2c+\pi+2dx)\right]\right] - 4if\operatorname{PolyLog}[2, ie^{i(c+dx)}] \right)$$

■ **Problem 269: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+fx)^3 \operatorname{Sec}[c+dx]}{a+a\sin[c+dx]} dx$$

Optimal (type 4, 502 leaves, 22 steps):

$$-\frac{3if(e+fx)^2}{2ad^2} - \frac{6if^2(e+fx)\operatorname{ArcTan}[e^{i(c+dx)}]}{ad^3} - \frac{i(e+fx)^3\operatorname{ArcTan}[e^{i(c+dx)}]}{ad} + \frac{3f^2(e+fx)\operatorname{Log}[1+e^{2i(c+dx)}]}{ad^3} + \\ \frac{3if^3\operatorname{PolyLog}[2, -ie^{i(c+dx)}]}{ad^4} + \frac{3if(e+fx)^2\operatorname{PolyLog}[2, -ie^{i(c+dx)}]}{2ad^2} - \frac{3if^3\operatorname{PolyLog}[2, ie^{i(c+dx)}]}{ad^4} - \frac{3if(e+fx)^2\operatorname{PolyLog}[2, ie^{i(c+dx)}]}{2ad^2} - \\ \frac{3if^3\operatorname{PolyLog}[2, -e^{2i(c+dx)}]}{2ad^4} - \frac{3f^2(e+fx)\operatorname{PolyLog}[3, -ie^{i(c+dx)}]}{ad^3} + \frac{3f^2(e+fx)\operatorname{PolyLog}[3, ie^{i(c+dx)}]}{ad^3} - \frac{3if^3\operatorname{PolyLog}[4, -ie^{i(c+dx)}]}{ad^4} + \\ \frac{3if^3\operatorname{PolyLog}[4, ie^{i(c+dx)}]}{ad^4} - \frac{3f(e+fx)^2\operatorname{Sec}[c+dx]}{2ad^2} - \frac{(e+fx)^3\operatorname{Sec}[c+dx]^2}{2ad} + \frac{3f(e+fx)^2\tan[c+dx]}{2ad^2} + \frac{(e+fx)^3\operatorname{Sec}[c+dx]\tan[c+dx]}{2ad}$$

Result (type 4, 1578 leaves):

$$\frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)}{8a(\cos[\frac{c}{2}]-\sin[\frac{c}{2}])(\cos[\frac{c}{2}]+\sin[\frac{c}{2}])} - \\ \frac{1}{2a(\cos[c]+i(-1+\sin[c]))}(\cos[c]+i\sin[c]) \left(-ie^3x - \frac{3}{2}ie^2fx^2 - ie^2fx^2 - \frac{1}{4}if^3x^4 + \frac{e^3\operatorname{Log}[1+i\cos[c+dx]-\sin[c+dx]]}{d} + \right. \\ \left. \frac{3e^2fx\operatorname{Log}[1+i\cos[c+dx]-\sin[c+dx]]}{d} + \frac{3e^2fx^2\operatorname{Log}[1+i\cos[c+dx]-\sin[c+dx]]}{d} + \frac{f^3x^3\operatorname{Log}[1+i\cos[c+dx]-\sin[c+dx]]}{d} + \right. \\ \left. \frac{6if^3\operatorname{PolyLog}[4, -i\cos[c+dx]+\sin[c+dx]]}{d^4} - \frac{ie^3\operatorname{Log}[1+i\cos[c+dx]-\sin[c+dx]](\cos[c]-i\sin[c])}{d} - \right. \\ \left. \frac{3ie^2fx\operatorname{Log}[1+i\cos[c+dx]-\sin[c+dx]](\cos[c]-i\sin[c])}{d} - \frac{3ie^2fx^2\operatorname{Log}[1+i\cos[c+dx]-\sin[c+dx]](\cos[c]-i\sin[c])}{d} - \right. \\ \left. \frac{if^3x^3\operatorname{Log}[1+i\cos[c+dx]-\sin[c+dx]](\cos[c]-i\sin[c])}{d} + \frac{6f^3\operatorname{PolyLog}[4, -i\cos[c+dx]+\sin[c+dx]](\cos[c]-i\sin[c])}{d^4} + \right)$$

$$\begin{aligned}
& \frac{1}{d^3} 6 f^2 (e + f x) \operatorname{PolyLog}[3, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] + i (-1 + \operatorname{Sin}[c])) (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) + \\
& \left. \frac{1}{d^2} 3 f (e + f x)^2 \operatorname{PolyLog}[2, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (-1 - i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \right) - \\
& \frac{1}{2 a d^2 (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c]))} (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) \left(i d^2 e^3 x + 12 i e f^2 x + \frac{3}{2} i d^2 e^2 f x^2 + 6 i f^3 x^2 + i d^2 e f^2 x^3 + \frac{1}{4} i d^2 f^3 x^4 + \right. \\
& i d e^3 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] + \frac{12 i e f^2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]]}{d} - 3 d e^2 f x \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] - \\
& \frac{12 f^3 x \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]]}{d} - 3 d e f^2 x^2 \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] - d f^3 x^3 \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] - \\
& \frac{1}{2} d e^3 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]] - \frac{6 e f^2 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]]}{d} - \\
& \frac{6 i f^3 \operatorname{PolyLog}[4, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]]}{d^2} - d e^3 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \\
& \frac{12 e f^2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d} - 3 i d e^2 f x \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \\
& \frac{12 i f^3 x \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d} - 3 i d e f^2 x^2 \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \\
& i d f^3 x^3 \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \frac{1}{2} i d e^3 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \\
& \frac{6 i e f^2 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d} + \\
& \frac{6 f^3 \operatorname{PolyLog}[4, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d^2} - \frac{1}{d} \\
& 6 f^2 (e + f x) \operatorname{PolyLog}[3, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \frac{1}{d^2} \\
& \left. 3 f (4 f^2 + d^2 (e + f x)^2) \operatorname{PolyLog}[2, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) \right) - \\
& \frac{(e + f x)^3}{2 a d (\operatorname{Cos}[\frac{c}{2} + \frac{d x}{2}] + \operatorname{Sin}[\frac{c}{2} + \frac{d x}{2}])^2} + \frac{3 (e^2 f \operatorname{Sin}[\frac{d x}{2}] + 2 e f^2 x \operatorname{Sin}[\frac{d x}{2}] + f^3 x^2 \operatorname{Sin}[\frac{d x}{2}])}{a d^2 (\operatorname{Cos}[\frac{c}{2}] + \operatorname{Sin}[\frac{c}{2}]) (\operatorname{Cos}[\frac{c}{2} + \frac{d x}{2}] + \operatorname{Sin}[\frac{c}{2} + \frac{d x}{2}])}
\end{aligned}$$

■ **Problem 270: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \operatorname{Sec}[c + d x]}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 278 leaves, 13 steps):

$$\begin{aligned}
& - \frac{i (e + f x)^2 \operatorname{ArcTan}\left[e^{i(c+dx)}\right]}{a d} + \frac{f^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a d^3} + \frac{f^2 \operatorname{Log}[\operatorname{Cos}[c + d x]]}{a d^3} + \frac{i f (e + f x) \operatorname{PolyLog}\left[2, -i e^{i(c+dx)}\right]}{a d^2} - \\
& \frac{i f (e + f x) \operatorname{PolyLog}\left[2, i e^{i(c+dx)}\right]}{a d^2} - \frac{f^2 \operatorname{PolyLog}\left[3, -i e^{i(c+dx)}\right]}{a d^3} + \frac{f^2 \operatorname{PolyLog}\left[3, i e^{i(c+dx)}\right]}{a d^3} - \\
& \frac{f (e + f x) \operatorname{Sec}[c + d x]}{a d^2} - \frac{(e + f x)^2 \operatorname{Sec}[c + d x]^2}{2 a d} + \frac{f (e + f x) \operatorname{Tan}[c + d x]}{a d^2} + \frac{(e + f x)^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a d}
\end{aligned}$$

Result (type 4, 811 leaves):

$$\begin{aligned}
& \frac{x (3 e^2 + 3 e f x + f^2 x^2)}{6 a (\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right])} + \frac{1}{6 a d^3} \\
& \left(-3 d^2 (e + f x)^2 \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] + 6 i d f (e + f x) \operatorname{PolyLog}[2, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] - \right. \\
& \left. 6 f^2 \operatorname{PolyLog}[3, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] + \frac{i d^3 x (3 e^2 + 3 e f x + f^2 x^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])}{\operatorname{Cos}[c] + i (-1 + \operatorname{Sin}[c])} \right) - \\
& \frac{1}{2 a d^2 (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c]))} (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) \left(i d^2 e^2 x + 4 i f^2 x + d^2 e f x^2 \operatorname{Cos}[c] + \right. \\
& \frac{1}{3} d^2 f^2 x^3 (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - i d^2 e f x^2 \operatorname{Sin}[c] + (d^2 e^2 + 4 f^2) x (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (1 - i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \\
& \frac{1}{2} d e^2 (2 d x + 2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] + i \operatorname{Log}[1 + \operatorname{Cos}[2(c + d x)] + i \operatorname{Sin}[2(c + d x)])]) (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \\
& (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \frac{1}{d} f^2 (2 d x + 2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] + i \operatorname{Log}[1 + \operatorname{Cos}[2(c + d x)] + i \operatorname{Sin}[2(c + d x)])]) \\
& (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \\
& e f (d x (d x + 2 i \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]]) + 2 \operatorname{PolyLog}[2, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]]) (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \\
& (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \frac{1}{3 d} f^2 (d^2 x^2 (d x + 3 i \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]]) + 6 d x \operatorname{PolyLog}[2, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]]) + \\
& \left. 6 i \operatorname{PolyLog}[3, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]]) (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) \right) - \\
& \frac{(e + f x)^2}{2 a d (\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \frac{2 (e f \operatorname{Sin}\left[\frac{dx}{2}\right] + f^2 x \operatorname{Sin}\left[\frac{dx}{2}\right])}{a d^2 (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])}
\end{aligned}$$

■ **Problem 271: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x) \operatorname{Sec}[c + d x]}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 172 leaves, 10 steps):

$$-\frac{i(e+fx)\operatorname{ArcTan}\left[e^{i(c+dx)}\right]}{ad} + \frac{if\operatorname{PolyLog}\left[2, -ie^{i(c+dx)}\right]}{2ad^2} - \frac{if\operatorname{PolyLog}\left[2, ie^{i(c+dx)}\right]}{2ad^2} - \frac{f\operatorname{Sec}[c+dx]}{2ad^2} - \frac{(e+fx)\operatorname{Sec}[c+dx]^2}{2ad} + \frac{f\tan[c+dx]}{2ad^2} + \frac{(e+fx)\operatorname{Sec}[c+dx]\tan[c+dx]}{2ad}$$

Result (type 4, 655 leaves):

$$-\frac{1}{4ad^2(1+\sin[c+dx])} \left(2d(e+fx) - 4f\sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) + (c+dx)(cf-d(2e+fx)) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + de\left(c+dx+2\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 - cf\left(c+dx+2\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + de\left(c+dx-2\log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 - cf\left(c+dx-2\log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + \frac{1}{\sqrt{2}} f \left(-(-1)^{3/4}(c+dx)^2 + 1/\sqrt{2} \right) \left(3i\pi(c+dx) + 4\pi\log[1+e^{-i(c+dx)}] - 2(-2c+\pi-2dx)\log[1+ie^{i(c+dx)}] - 4\pi\log\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] + 2\pi\log\left[\sin\left[\frac{1}{4}(2c-\pi+2dx)\right]\right] - 4i\operatorname{PolyLog}\left[2, -ie^{i(c+dx)}\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + \frac{1}{\sqrt{2}} f \left((-1)^{1/4}(c+dx)^2 + 1/\sqrt{2} \right) \left(-i\pi(c+dx) - 4\pi\log[1+e^{-i(c+dx)}] - 2(2c+\pi+2dx)\log[1-ie^{i(c+dx)}] + 4\pi\log\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] + 2\pi\log\left[\sin\left[\frac{1}{4}(2c+\pi+2dx)\right]\right] + 4i\operatorname{PolyLog}\left[2, ie^{i(c+dx)}\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \right)$$

■ **Problem 272: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]}{a+a\sin[c+dx]} dx$$

Optimal (type 3, 37 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sin[c+dx]]}{2ad} - \frac{1}{2d(a+a\sin[c+dx])}$$

Result (type 3, 126 leaves):

$$\frac{1}{2 a d (1 + \sin[c + d x])} \left(-1 - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \right) + \left(-\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \right) \sin[c + d x]$$

■ **Problem 275: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \operatorname{Sec}[c + d x]^2}{a + a \sin[c + d x]} dx$$

Optimal (type 4, 475 leaves, 20 steps):

$$\begin{aligned} & -\frac{2 i (e + f x)^3}{3 a d} - \frac{i f (e + f x)^2 \operatorname{ArcTan}\left[e^{i(c + d x)}\right]}{a d^2} + \frac{f^3 \operatorname{ArcTanh}[\sin[c + d x]]}{a d^4} + \frac{2 f (e + f x)^2 \operatorname{Log}\left[1 + e^{2 i(c + d x)}\right]}{a d^2} + \\ & \frac{f^3 \operatorname{Log}[\cos[c + d x]]}{a d^4} + \frac{i f^2 (e + f x) \operatorname{PolyLog}\left[2, -i e^{i(c + d x)}\right]}{a d^3} - \frac{i f^2 (e + f x) \operatorname{PolyLog}\left[2, i e^{i(c + d x)}\right]}{a d^3} - \\ & \frac{2 i f^2 (e + f x) \operatorname{PolyLog}\left[2, -e^{2 i(c + d x)}\right]}{a d^3} - \frac{f^3 \operatorname{PolyLog}\left[3, -i e^{i(c + d x)}\right]}{a d^4} + \frac{f^3 \operatorname{PolyLog}\left[3, i e^{i(c + d x)}\right]}{a d^4} + \\ & \frac{f^3 \operatorname{PolyLog}\left[3, -e^{2 i(c + d x)}\right]}{a d^4} - \frac{f^2 (e + f x) \operatorname{Sec}[c + d x]}{a d^3} - \frac{f (e + f x)^2 \operatorname{Sec}[c + d x]^2}{2 a d^2} - \frac{(e + f x)^3 \operatorname{Sec}[c + d x]^3}{3 a d} + \\ & \frac{f^2 (e + f x) \operatorname{Tan}[c + d x]}{a d^3} + \frac{2 (e + f x)^3 \operatorname{Tan}[c + d x]}{3 a d} + \frac{f (e + f x)^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a d^2} + \frac{(e + f x)^3 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 a d} \end{aligned}$$

Result (type 4, 1253 leaves):

$$\begin{aligned}
& \frac{1}{2 a d^4} f \left(3 d^2 (e + f x)^2 \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] - 6 i d f (e + f x) \operatorname{PolyLog}[2, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] + \right. \\
& \quad \left. 6 f^2 \operatorname{PolyLog}[3, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] + \frac{d^3 x (3 e^2 + 3 e f x + f^2 x^2) (-i \operatorname{Cos}[c] + \operatorname{Sin}[c])}{\operatorname{Cos}[c] + i (-1 + \operatorname{Sin}[c])} \right) - \\
& \frac{1}{2 a d^3 (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c]))} f (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) \left(5 i d^2 e^2 x + 4 i f^2 x + 5 d^2 e f x^2 \operatorname{Cos}[c] + \frac{5}{3} d^2 f^2 x^3 (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \right. \\
& \quad 5 i d^2 e f x^2 \operatorname{Sin}[c] + (5 d^2 e^2 + 4 f^2) x (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (1 - i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \\
& \quad \frac{5}{2} d e^2 (2 d x + 2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] + i \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]]) (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \\
& \quad (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \frac{1}{d} 2 f^2 (2 d x + 2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] + i \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]]) \\
& \quad (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \\
& \quad 5 e f (d x (d x + 2 i \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]]) + 2 \operatorname{PolyLog}[2, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]]) (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \\
& \quad (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \frac{1}{3 d} 5 f^2 (d^2 x^2 (d x + 3 i \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]]) + 6 d x \operatorname{PolyLog}[2, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]]) + \\
& \quad \left. 6 i \operatorname{PolyLog}[3, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) \right) + \\
& \frac{e^3 \operatorname{Sin}\left[\frac{d x}{2}\right] + 3 e^2 f x \operatorname{Sin}\left[\frac{d x}{2}\right] + 3 e f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + f^3 x^3 \operatorname{Sin}\left[\frac{d x}{2}\right]}{2 a d (\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right])} + \frac{e^3 \operatorname{Sin}\left[\frac{d x}{2}\right] + 3 e^2 f x \operatorname{Sin}\left[\frac{d x}{2}\right] + 3 e f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + f^3 x^3 \operatorname{Sin}\left[\frac{d x}{2}\right]}{3 a d (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right])^3} + \\
& \left(-d e^3 \operatorname{Cos}\left[\frac{c}{2}\right] - 3 e^2 f \operatorname{Cos}\left[\frac{c}{2}\right] - 3 d e^2 f x \operatorname{Cos}\left[\frac{c}{2}\right] - 6 e f^2 x \operatorname{Cos}\left[\frac{c}{2}\right] - 3 d e f^2 x^2 \operatorname{Cos}\left[\frac{c}{2}\right] - 3 f^3 x^2 \operatorname{Cos}\left[\frac{c}{2}\right] - d f^3 x^3 \operatorname{Cos}\left[\frac{c}{2}\right] + \right. \\
& \quad \left. d e^3 \operatorname{Sin}\left[\frac{c}{2}\right] - 3 e^2 f \operatorname{Sin}\left[\frac{c}{2}\right] + 3 d e^2 f x \operatorname{Sin}\left[\frac{c}{2}\right] - 6 e f^2 x \operatorname{Sin}\left[\frac{c}{2}\right] + 3 d e f^2 x^2 \operatorname{Sin}\left[\frac{c}{2}\right] - 3 f^3 x^2 \operatorname{Sin}\left[\frac{c}{2}\right] + d f^3 x^3 \operatorname{Sin}\left[\frac{c}{2}\right] \right) / \\
& \left(6 a d^2 (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right])^2 \right) + \\
& \left(5 d^2 e^3 \operatorname{Sin}\left[\frac{d x}{2}\right] + 12 e f^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + 15 d^2 e^2 f x \operatorname{Sin}\left[\frac{d x}{2}\right] + 12 f^3 x \operatorname{Sin}\left[\frac{d x}{2}\right] + 15 d^2 e f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + 5 d^2 f^3 x^3 \operatorname{Sin}\left[\frac{d x}{2}\right] \right) / \\
& \left(6 a d^3 (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]) \right)
\end{aligned}$$

■ **Problem 278: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^2}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 42 leaves, 3 steps):

$$-\frac{\operatorname{Sec}[c + d x]}{3 d (a + a \operatorname{Sin}[c + d x])} + \frac{2 \operatorname{Tan}[c + d x]}{3 a d}$$

Result (type 3, 103 leaves) :

$$\frac{2 \operatorname{Cos}[c + d x] - 4 \operatorname{Cos}[2(c + d x)] + 8 \operatorname{Sin}[c + d x] + \operatorname{Sin}[2(c + d x)]}{12 a d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) (1 + \operatorname{Sin}[c + d x])}$$

■ **Problem 281: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \operatorname{Sec}[c + d x]^3}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 698 leaves, 32 steps) :

$$\begin{aligned} & - \frac{i f (e + f x)^2}{2 a d^2} - \frac{5 i f^2 (e + f x) \operatorname{ArcTan}\left[e^{i(c + d x)}\right]}{a d^3} - \frac{3 i (e + f x)^3 \operatorname{ArcTan}\left[e^{i(c + d x)}\right]}{4 a d} + \frac{f^2 (e + f x) \operatorname{Log}\left[1 + e^{2 i(c + d x)}\right]}{a d^3} + \\ & \frac{5 i f^3 \operatorname{PolyLog}\left[2, -i e^{i(c + d x)}\right]}{2 a d^4} + \frac{9 i f (e + f x)^2 \operatorname{PolyLog}\left[2, -i e^{i(c + d x)}\right]}{8 a d^2} - \frac{5 i f^3 \operatorname{PolyLog}\left[2, i e^{i(c + d x)}\right]}{2 a d^4} - \frac{9 i f (e + f x)^2 \operatorname{PolyLog}\left[2, i e^{i(c + d x)}\right]}{8 a d^2} - \\ & \frac{i f^3 \operatorname{PolyLog}\left[2, -e^{2 i(c + d x)}\right]}{2 a d^4} - \frac{9 f^2 (e + f x) \operatorname{PolyLog}\left[3, -i e^{i(c + d x)}\right]}{4 a d^3} + \frac{9 f^2 (e + f x) \operatorname{PolyLog}\left[3, i e^{i(c + d x)}\right]}{4 a d^3} - \\ & \frac{9 i f^3 \operatorname{PolyLog}\left[4, -i e^{i(c + d x)}\right]}{4 a d^4} + \frac{9 i f^3 \operatorname{PolyLog}\left[4, i e^{i(c + d x)}\right]}{4 a d^4} - \frac{f^3 \operatorname{Sec}[c + d x]}{4 a d^4} - \frac{9 f (e + f x)^2 \operatorname{Sec}[c + d x]}{8 a d^2} - \frac{f^2 (e + f x) \operatorname{Sec}[c + d x]^2}{4 a d^3} - \\ & \frac{f (e + f x)^2 \operatorname{Sec}[c + d x]^3}{4 a d^2} - \frac{(e + f x)^3 \operatorname{Sec}[c + d x]^4}{4 a d} + \frac{f^3 \operatorname{Tan}[c + d x]}{4 a d^4} + \frac{f (e + f x)^2 \operatorname{Tan}[c + d x]}{2 a d^2} + \frac{f^2 (e + f x) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{4 a d^3} + \\ & \frac{3 (e + f x)^3 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 a d} + \frac{f (e + f x)^2 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{4 a d^2} + \frac{(e + f x)^3 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 a d} \end{aligned}$$

Result (type 4, 2640 leaves) :

$$\begin{aligned} & - \frac{1}{8 a d^2 (\operatorname{Cos}[c] + i (-1 + \operatorname{Sin}[c]))} \\ & 3 (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) \left(-i d^2 e^3 x - 4 i e f^2 x - \frac{3}{2} i d^2 e^2 f x^2 - 2 i f^3 x^2 - i d^2 e f^2 x^3 - \frac{1}{4} i d^2 f^3 x^4 + i d e^3 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] \right) + \\ & \frac{4 i e f^2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]]}{d} + 3 d e^2 f x \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] + \\ & \frac{4 f^3 x \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]]}{d} + 3 d e f^2 x^2 \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] + d f^3 x^3 \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] + \\ & \frac{1}{2} d e^3 \operatorname{Log}[1 + \operatorname{Cos}[2(c + d x)] + i \operatorname{Sin}[2(c + d x)]] + \frac{2 e f^2 \operatorname{Log}[1 + \operatorname{Cos}[2(c + d x)] + i \operatorname{Sin}[2(c + d x)]]}{d} + \\ & \frac{6 i f^3 \operatorname{PolyLog}\left[4, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]\right]}{d^2} + d e^3 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) + \\ & \frac{4 e f^2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d} - 3 i d e^2 f x \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \end{aligned}$$

$$\begin{aligned}
& \frac{4 i f^3 x \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d} - 3 i d e f^2 x^2 \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \\
& i d f^3 x^3 \operatorname{Log}[1 + i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \frac{1}{2} i d e^3 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]] \\
& (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \frac{2 i e f^2 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d} + \\
& \frac{6 f^3 \operatorname{PolyLog}[4, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d^2} + \frac{1}{d} \\
& 6 f^2 (e + f x) \operatorname{PolyLog}[3, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] + i (-1 + \operatorname{Sin}[c])) (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) + \frac{1}{d^2} \\
& f \left(4 f^2 + 3 d^2 (e + f x)^2 \right) \operatorname{PolyLog}[2, -i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (-1 - i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \Big) - \\
& \frac{1}{8 a d^2 (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c]))} (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) \left(3 i d^2 e^3 x + 28 i e f^2 x + \frac{9}{2} i d^2 e^2 f x^2 + 14 i f^3 x^2 + 3 i d^2 e f^2 x^3 + \right. \\
& \left. \frac{3}{4} i d^2 f^3 x^4 + 3 i d e^3 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] + \frac{28 i e f^2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]]}{d} - \right. \\
& \left. 9 d e^2 f x \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] - \frac{28 f^3 x \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]]}{d} - 9 d e f^2 x^2 \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] - \right. \\
& \left. 3 d f^3 x^3 \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] - \frac{3}{2} d e^3 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]] - \right. \\
& \left. \frac{14 e f^2 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]]}{d} - \frac{18 i f^3 \operatorname{PolyLog}[4, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]]}{d^2} - \right. \\
& \left. 3 d e^3 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \frac{28 e f^2 \operatorname{ArcTan}[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d} - \right. \\
& \left. 9 i d e^2 f x \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \frac{28 i f^3 x \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d} - \right. \\
& \left. 9 i d e f^2 x^2 \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - 3 i d f^3 x^3 \operatorname{Log}[1 - i \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \right. \\
& \left. \frac{3}{2} i d e^3 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \right. \\
& \left. \frac{14 i e f^2 \operatorname{Log}[1 + \operatorname{Cos}[2 (c + d x)] + i \operatorname{Sin}[2 (c + d x)]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d} + \right. \\
& \left. \frac{18 f^3 \operatorname{PolyLog}[4, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{d^2} - \frac{1}{d} \right. \\
& \left. 18 f^2 (e + f x) \operatorname{PolyLog}[3, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \frac{1}{d^2} \right. \\
& \left. f \left(28 f^2 + 9 d^2 (e + f x)^2 \right) \operatorname{PolyLog}[2, i \operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) \Big) + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\frac{3 e^3 x \cos[c]}{4 a} + \frac{3 i e^3 x \sin[c]}{4 a}}{1 + \cos[2 c] + i \sin[2 c]} + \frac{\frac{9 e^2 f x^2 \cos[c]}{8 a} + \frac{9 i e^2 f x^2 \sin[c]}{8 a}}{1 + \cos[2 c] + i \sin[2 c]} + \\
& \frac{\frac{3 e f^2 x^3 \cos[c]}{4 a} + \frac{3 i e f^2 x^3 \sin[c]}{4 a}}{1 + \cos[2 c] + i \sin[2 c]} + \\
& \frac{\frac{3 f^3 x^4 \cos[c]}{16 a} + \frac{3 i f^3 x^4 \sin[c]}{16 a}}{1 + \cos[2 c] + i \sin[2 c]} + \\
& \frac{e^3 + 3 e^2 f x + 3 e f^2 x^2 + f^3 x^3}{8 a d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} - \\
& \frac{3 \left(e^2 f \sin\left[\frac{dx}{2}\right] + 2 e f^2 x \sin\left[\frac{dx}{2}\right] + f^3 x^2 \sin\left[\frac{dx}{2}\right] \right)}{4 a d^2 \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)} + \\
& \frac{-e^3 - 3 e^2 f x - 3 e f^2 x^2 - f^3 x^3}{8 a d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^4} + \\
& \frac{e^2 f \sin\left[\frac{dx}{2}\right] + 2 e f^2 x \sin\left[\frac{dx}{2}\right] + f^3 x^2 \sin\left[\frac{dx}{2}\right]}{4 a d^2 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^3} + \\
& \frac{1}{8 a d^3 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} \\
& \left(-2 d^2 e^3 \cos\left[\frac{c}{2}\right] - d e^2 f \cos\left[\frac{c}{2}\right] - 2 e f^2 \cos\left[\frac{c}{2}\right] - 6 d^2 e^2 f x \cos\left[\frac{c}{2}\right] - 2 d e f^2 x \cos\left[\frac{c}{2}\right] - 2 f^3 x \cos\left[\frac{c}{2}\right] - \right. \\
& \quad \left. 6 d^2 e f^2 x^2 \cos\left[\frac{c}{2}\right] - d f^3 x^2 \cos\left[\frac{c}{2}\right] - 2 d^2 f^3 x^3 \cos\left[\frac{c}{2}\right] - 2 d^2 e^3 \sin\left[\frac{c}{2}\right] + d e^2 f \sin\left[\frac{c}{2}\right] - 2 e f^2 \sin\left[\frac{c}{2}\right] - \right. \\
& \quad \left. 6 d^2 e^2 f x \sin\left[\frac{c}{2}\right] + 2 d e f^2 x \sin\left[\frac{c}{2}\right] - 2 f^3 x \sin\left[\frac{c}{2}\right] - 6 d^2 e f^2 x^2 \sin\left[\frac{c}{2}\right] + d f^3 x^2 \sin\left[\frac{c}{2}\right] - 2 d^2 f^3 x^3 \sin\left[\frac{c}{2}\right] \right) + \\
& \frac{7 d^2 e^2 f \sin\left[\frac{dx}{2}\right] + 2 f^3 \sin\left[\frac{dx}{2}\right] + 14 d^2 e f^2 x \sin\left[\frac{dx}{2}\right] + 7 d^2 f^3 x^2 \sin\left[\frac{dx}{2}\right]}{4 a d^4 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)}
\end{aligned}$$

■ **Problem 282: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \operatorname{Sec}[c + d x]^3}{a + a \sin[c + d x]} dx$$

Optimal (type 4, 431 leaves, 17 steps):

$$\begin{aligned}
& - \frac{3 i (e+f x)^2 \operatorname{ArcTan}\left[e^{i(c+d x)}\right]}{4 a d} + \frac{5 f^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{6 a d^3} + \frac{f^2 \operatorname{Log}[\operatorname{Cos}[c+d x]]}{3 a d^3} + \frac{3 i f (e+f x) \operatorname{PolyLog}\left[2, -i e^{i(c+d x)}\right]}{4 a d^2} - \\
& \frac{3 i f (e+f x) \operatorname{PolyLog}\left[2, i e^{i(c+d x)}\right]}{4 a d^2} - \frac{3 f^2 \operatorname{PolyLog}\left[3, -i e^{i(c+d x)}\right]}{4 a d^3} + \frac{3 f^2 \operatorname{PolyLog}\left[3, i e^{i(c+d x)}\right]}{4 a d^3} - \frac{3 f (e+f x) \operatorname{Sec}[c+d x]}{4 a d^2} - \\
& \frac{f^2 \operatorname{Sec}[c+d x]^2}{12 a d^3} - \frac{f (e+f x) \operatorname{Sec}[c+d x]^3}{6 a d^2} - \frac{(e+f x)^2 \operatorname{Sec}[c+d x]^4}{4 a d} + \frac{f (e+f x) \operatorname{Tan}[c+d x]}{3 a d^2} + \frac{f^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{12 a d^3} + \\
& \frac{3 (e+f x)^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{8 a d} + \frac{f (e+f x) \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{6 a d^2} + \frac{(e+f x)^2 \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{4 a d}
\end{aligned}$$

Result (type 4, 1680 leaves):

$$\begin{aligned}
& - \frac{1}{8 a d^2 (\operatorname{Cos}[c] + i (-1 + \operatorname{Sin}[c]))} (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) \\
& \left(-3 i d^2 e^2 x - 4 i f^2 x + 3 d^2 e f x^2 \operatorname{Cos}[c] + d^2 f^2 x^3 (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) + (3 d^2 e^2 + 4 f^2) x (1 + i \operatorname{Cos}[c] - \operatorname{Sin}[c]) (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) - \right. \\
& \left. 3 i d^2 e f x^2 \operatorname{Sin}[c] + \frac{3}{2} d e^2 (2 d x - 2 \operatorname{ArcTan}[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]] + i \operatorname{Log}[1 + \operatorname{Cos}[2(c+d x)] + i \operatorname{Sin}[2(c+d x)])] \right) \\
& (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (-1 - i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \frac{1}{d} 2 f^2 (2 d x - 2 \operatorname{ArcTan}[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]] + \\
& i \operatorname{Log}[1 + \operatorname{Cos}[2(c+d x)] + i \operatorname{Sin}[2(c+d x)])] (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (-1 - i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \\
& 3 e f (d x (d x + 2 i \operatorname{Log}[1 + i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]]) + 2 \operatorname{PolyLog}[2, -i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]]) (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) \\
& (-1 - i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \frac{1}{d} f^2 (d^2 x^2 (d x + 3 i \operatorname{Log}[1 + i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]]) + 6 d x \operatorname{PolyLog}[2, -i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]] + \\
& 6 i \operatorname{PolyLog}[3, -i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]]) (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (-1 - i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \left. \right) - \\
& \frac{1}{24 a d^2 (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c]))} (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) \left(9 i d^2 e^2 x + 28 i f^2 x + 9 d^2 e f x^2 \operatorname{Cos}[c] + 3 d^2 f^2 x^3 \operatorname{Cos}[c] - \right. \\
& \left. 9 i d^2 e f x^2 \operatorname{Sin}[c] - 3 i d^2 f^2 x^3 \operatorname{Sin}[c] + (9 d^2 e^2 + 28 f^2) x (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) (1 - i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \right. \\
& \left. \frac{9}{2} d e^2 (2 d x + 2 \operatorname{ArcTan}[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]] + i \operatorname{Log}[1 + \operatorname{Cos}[2(c+d x)] + i \operatorname{Sin}[2(c+d x)])] (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \right. \\
& \left. (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \frac{1}{d} 14 f^2 (2 d x + 2 \operatorname{ArcTan}[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]] + i \operatorname{Log}[1 + \operatorname{Cos}[2(c+d x)] + i \operatorname{Sin}[2(c+d x)])] \right) \\
& (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \\
& 9 e f (d x (d x + 2 i \operatorname{Log}[1 - i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]]) + 2 \operatorname{PolyLog}[2, i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]]) (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \\
& (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) + \frac{1}{d} 3 f^2 (d^2 x^2 (d x + 3 i \operatorname{Log}[1 - i \operatorname{Cos}[c+d x] + \operatorname{Sin}[c+d x]]) + 6 d x \operatorname{PolyLog}[2, i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]] + \\
& 6 i \operatorname{PolyLog}[3, i \operatorname{Cos}[c+d x] - \operatorname{Sin}[c+d x]]) (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (\operatorname{Cos}[c] + i (1 + \operatorname{Sin}[c])) \left. \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{\frac{3 e^2 x \cos [c]}{4 a} + \frac{3 i e^2 x \sin [c]}{4 a}}{1 + \cos [2 c] + i \sin [2 c]} + \frac{\frac{3 e f x^2 \cos [c]}{4 a} + \frac{3 i e f x^2 \sin [c]}{4 a}}{1 + \cos [2 c] + i \sin [2 c]} + \frac{\frac{f^2 x^3 \cos [c]}{4 a} + \frac{i f^2 x^3 \sin [c]}{4 a}}{1 + \cos [2 c] + i \sin [2 c]} + \\
& \frac{e^2 + 2 e f x + f^2 x^2}{8 a d \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] - \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)^2} + \\
& \frac{-e f \sin \left[\frac{d x}{2} \right] - f^2 x \sin \left[\frac{d x}{2} \right]}{2 a d^2 \left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] - \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)} + \\
& \frac{-e^2 - 2 e f x - f^2 x^2}{8 a d \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)^4} + \\
& \frac{e f \sin \left[\frac{d x}{2} \right] + f^2 x \sin \left[\frac{d x}{2} \right]}{6 a d^2 \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)^3} + \\
& \left(-3 d^2 e^2 \cos \left[\frac{c}{2} \right] - d e f \cos \left[\frac{c}{2} \right] - f^2 \cos \left[\frac{c}{2} \right] - 6 d^2 e f x \cos \left[\frac{c}{2} \right] - d f^2 x \cos \left[\frac{c}{2} \right] - 3 d^2 f^2 x^2 \cos \left[\frac{c}{2} \right] - \right. \\
& \quad \left. 3 d^2 e^2 \sin \left[\frac{c}{2} \right] + d e f \sin \left[\frac{c}{2} \right] - f^2 \sin \left[\frac{c}{2} \right] - 6 d^2 e f x \sin \left[\frac{c}{2} \right] + d f^2 x \sin \left[\frac{c}{2} \right] - 3 d^2 f^2 x^2 \sin \left[\frac{c}{2} \right] \right) / \\
& \left(12 a d^3 \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)^2 \right) + \\
& \frac{7 \left(e f \sin \left[\frac{d x}{2} \right] + f^2 x \sin \left[\frac{d x}{2} \right] \right)}{6 a d^2 \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)}
\end{aligned}$$

■ **Problem 283: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x) \operatorname{Sec}[c + d x]^3}{a + a \sin [c + d x]} dx$$

Optimal (type 4, 241 leaves, 11 steps):

$$\begin{aligned}
& -\frac{3 i (e + f x) \operatorname{ArcTan}\left[e^{i(c+d x)}\right]}{4 a d} + \frac{3 i f \operatorname{PolyLog}\left[2, -i e^{i(c+d x)}\right]}{8 a d^2} - \frac{3 i f \operatorname{PolyLog}\left[2, i e^{i(c+d x)}\right]}{8 a d^2} - \frac{3 f \operatorname{Sec}[c + d x]}{8 a d^2} - \frac{f \operatorname{Sec}[c + d x]^3}{12 a d^2} - \\
& \frac{(e + f x) \operatorname{Sec}[c + d x]^4}{4 a d} + \frac{f \operatorname{Tan}[c + d x]}{4 a d^2} + \frac{3 (e + f x) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 a d} + \frac{(e + f x) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 a d} + \frac{f \operatorname{Tan}[c + d x]^3}{12 a d^2}
\end{aligned}$$

Result (type 4, 1171 leaves):

$$\begin{aligned}
& \frac{-6de - f + 6cf - 6f(c+dx)}{24d^2(a+a\sin[c+dx])} + \frac{-de + cf - f(c+dx)}{8d^2\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2(a+a\sin[c+dx])} + \\
& \frac{f\sin\left[\frac{1}{2}(c+dx)\right]}{12d^2\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)(a+a\sin[c+dx])} + \frac{7f\sin\left[\frac{1}{2}(c+dx)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{12d^2(a+a\sin[c+dx])} + \\
& \frac{3(c+dx)(2de - 2cf + f(c+dx))\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{16d^2(a+a\sin[c+dx])} + \\
& \frac{3e\left(\frac{1}{2}(-c-dx) - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{8d(a+a\sin[c+dx])} - \\
& \frac{3cf\left(\frac{1}{2}(-c-dx) - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{8d^2(a+a\sin[c+dx])} - \\
& \frac{3e\left(\frac{1}{2}(c+dx) - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{8d(a+a\sin[c+dx])} + \\
& \frac{3cf\left(\frac{1}{2}(c+dx) - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{8d^2(a+a\sin[c+dx])} - \\
& \left(3f\left(\frac{1}{4}e^{-\frac{i\pi}{4}}(c+dx)^2 - 1\right)/\left(\sqrt{2}\right)\left(-\frac{3}{4}i\pi(c+dx) - \pi\log[1 + e^{-i(c+dx)}] - 2\left(-\frac{\pi}{4} + \frac{1}{2}(c+dx)\right)\log\left[1 - e^{2i\left(-\frac{\pi}{4} + \frac{1}{2}(c+dx)\right)}\right]\right) + \right. \\
& \left. \pi\log\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] - \frac{1}{2}\pi\log\left[-\sin\left[\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right]\right] + i\operatorname{PolyLog}\left[2, e^{2i\left(-\frac{\pi}{4} + \frac{1}{2}(c+dx)\right)}\right]\right) \\
& \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 / \left(4\sqrt{2}d^2(a+a\sin[c+dx])\right) - \\
& \left(3f\left(\frac{1}{4}e^{\frac{i\pi}{4}}(c+dx)^2 + 1\right)/\left(\sqrt{2}\right)\left(-\frac{1}{4}i\pi(c+dx) - \pi\log[1 + e^{-i(c+dx)}] - 2\left(\frac{\pi}{4} + \frac{1}{2}(c+dx)\right)\log\left[1 - e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(c+dx)\right)}\right]\right) + \pi\log\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] + \right. \\
& \left. \frac{1}{2}\pi\log\left[\sin\left[\frac{\pi}{4} + \frac{1}{2}(c+dx)\right]\right] + i\operatorname{PolyLog}\left[2, e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(c+dx)\right)}\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 / \left(4\sqrt{2}d^2(a+a\sin[c+dx])\right) + \\
& \frac{(de - cf + f(c+dx))\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{8d^2\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2(a+a\sin[c+dx])} - \frac{f\sin\left[\frac{1}{2}(c+dx)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{4d^2\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)(a+a\sin[c+dx])}
\end{aligned}$$

■ **Problem 284: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^3}{a+a\sin[c+dx]} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{3 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 a d} + \frac{1}{8 d (a - a \operatorname{Sin}[c + d x])} - \frac{a}{8 d (a + a \operatorname{Sin}[c + d x])^2} - \frac{1}{4 d (a + a \operatorname{Sin}[c + d x])}$$

Result (type 3, 190 leaves):

$$-\frac{1}{8 d (a + a \operatorname{Sin}[c + d x])} \left(2 + \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2} + 3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 - 3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 - \frac{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2} \right)$$

■ **Problem 294: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \operatorname{Cos}[c + d x]}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 432 leaves, 11 steps):

$$-\frac{i (e + f x)^4}{4 b f} + \frac{(e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b d} + \frac{(e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b d} - \frac{3 i f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b d^2} - \frac{3 i f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b d^2} + \frac{6 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b d^3} + \frac{6 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b d^3} + \frac{6 i f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{b d^4} + \frac{6 i f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{b d^4}$$

Result (type 4, 1068 leaves):

$$\begin{aligned}
& -\frac{1}{4bd^4} i \left(4d^4 e^3 x + 6d^4 e^2 f x^2 + 4d^4 e f^2 x^3 + d^4 f^3 x^4 - 4d^3 e^3 \operatorname{ArcTan}\left[\frac{2ae^{i(c+dx)}}{b(-1+e^{2i(c+dx)})}\right] + 2id^3 e^3 \operatorname{Log}\left[4a^2 e^{2i(c+dx)} + b^2(-1+e^{2i(c+dx)})^2\right] + \right. \\
& 12id^3 e^2 f x \operatorname{Log}\left[1 + \frac{be^{i(2c+dx)}}{ia e^{ic} - \sqrt{(-a^2+b^2)e^{2ic}}}\right] + 12id^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{be^{i(2c+dx)}}{ia e^{ic} - \sqrt{(-a^2+b^2)e^{2ic}}}\right] + \\
& 4id^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{be^{i(2c+dx)}}{ia e^{ic} - \sqrt{(-a^2+b^2)e^{2ic}}}\right] + 12id^3 e^2 f x \operatorname{Log}\left[1 + \frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)e^{2ic}}}\right] + \\
& 12id^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)e^{2ic}}}\right] + 4id^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)e^{2ic}}}\right] + \\
& 12d^2 f (e+fx)^2 \operatorname{PolyLog}\left[2, \frac{ibe^{i(2c+dx)}}{ae^{ic} + i\sqrt{(-a^2+b^2)e^{2ic}}}\right] + 12d^2 f (e+fx)^2 \operatorname{PolyLog}\left[2, -\frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)e^{2ic}}}\right] + \\
& 24idef^2 \operatorname{PolyLog}\left[3, \frac{ibe^{i(2c+dx)}}{ae^{ic} + i\sqrt{(-a^2+b^2)e^{2ic}}}\right] + 24idf^3 x \operatorname{PolyLog}\left[3, \frac{ibe^{i(2c+dx)}}{ae^{ic} + i\sqrt{(-a^2+b^2)e^{2ic}}}\right] + \\
& 24idef^2 \operatorname{PolyLog}\left[3, -\frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)e^{2ic}}}\right] + 24idf^3 x \operatorname{PolyLog}\left[3, -\frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)e^{2ic}}}\right] - \\
& \left. 24f^3 \operatorname{PolyLog}\left[4, \frac{ibe^{i(2c+dx)}}{ae^{ic} + i\sqrt{(-a^2+b^2)e^{2ic}}}\right] - 24f^3 \operatorname{PolyLog}\left[4, -\frac{be^{i(2c+dx)}}{ia e^{ic} + \sqrt{(-a^2+b^2)e^{2ic}}}\right] \right)
\end{aligned}$$

■ **Problem 295: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+fx)^2 \operatorname{Cos}[c+dx]}{a+b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 4, 320 leaves, 9 steps):

$$\begin{aligned}
& -\frac{i(e+fx)^3}{3bf} + \frac{(e+fx)^2 \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{bd} + \frac{(e+fx)^2 \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{bd} - \frac{2if(e+fx) \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{bd^2} \\
& \frac{2if(e+fx) \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{bd^2} + \frac{2f^2 \operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{bd^3} + \frac{2f^2 \operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{bd^3}
\end{aligned}$$

Result (type 4, 647 leaves):

$$\frac{1}{6 b d^3} \left(-6 i d^3 e^2 x - 6 i d^3 e f x^2 - 2 i d^3 f^2 x^3 + 6 i d^2 e^2 \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b(-1+e^{2i(c+dx)})}\right] + 3 d^2 e^2 \operatorname{Log}\left[4 a^2 e^{2i(c+dx)} + b^2(-1+e^{2i(c+dx)})^2\right] + \right.$$

$$12 d^2 e f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2+b^2)} e^{2i c}}\right] + 6 d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2+b^2)} e^{2i c}}\right] +$$

$$12 d^2 e f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2+b^2)} e^{2i c}}\right] + 6 d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2+b^2)} e^{2i c}}\right] -$$

$$12 i d f (e+f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2+b^2)} e^{2i c}}\right] - 12 i d f (e+f x) \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2+b^2)} e^{2i c}}\right] +$$

$$12 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2+b^2)} e^{2i c}}\right] + 12 f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2+b^2)} e^{2i c}}\right] \left. \right)$$

■ **Problem 298: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+f x)^3 \operatorname{Cos}[c+d x]^2}{a+b \operatorname{Sin}[c+d x]} dx$$

Optimal (type 4, 618 leaves, 18 steps):

$$\frac{a(e+f x)^4}{4 b^2 f} - \frac{6 f^2 (e+f x) \operatorname{Cos}[c+d x]}{b d^3} + \frac{(e+f x)^3 \operatorname{Cos}[c+d x]}{b d} + \frac{i \sqrt{a^2-b^2} (e+f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b^2 d} -$$

$$\frac{i \sqrt{a^2-b^2} (e+f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b^2 d} + \frac{3 \sqrt{a^2-b^2} f (e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b^2 d^2} - \frac{3 \sqrt{a^2-b^2} f (e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b^2 d^2} +$$

$$\frac{6 i \sqrt{a^2-b^2} f^2 (e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b^2 d^3} - \frac{6 i \sqrt{a^2-b^2} f^2 (e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b^2 d^3} -$$

$$\frac{6 \sqrt{a^2-b^2} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{b^2 d^4} + \frac{6 \sqrt{a^2-b^2} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{b^2 d^4} + \frac{6 f^3 \operatorname{Sin}[c+d x]}{b d^4} - \frac{3 f (e+f x)^2 \operatorname{Sin}[c+d x]}{b d^2}$$

Result (type 4, 1588 leaves):

$$\frac{1}{4 b^2 d^4} \left(a d^4 x (4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3) + 4 b d (e+f x) (-6 f^2 + d^2 (e+f x)^2) \operatorname{Cos}[c+d x] - \frac{1}{\sqrt{(-a^2+b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right)$$

$$\begin{aligned}
& 4 i \sqrt{a^2 - b^2} \left(3 i \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log} \left[1 + \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 - a \sin[c]}} \right] (\cos[c] + i \sin[c]) + \right. \\
& 3 i \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log} \left[1 + \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 - a \sin[c]}} \right] (\cos[c] + i \sin[c]) + \\
& i \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log} \left[1 + \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 - a \sin[c]}} \right] (\cos[c] + i \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^2 f (e + f x)^2 \operatorname{PolyLog} \left[2, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 - a \sin[c]}} \right] (\cos[c] + i \sin[c]) - \\
& 3 \sqrt{a^2 - b^2} d^2 f (e + f x)^2 \operatorname{PolyLog} \left[2, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 + a \sin[c]}} \right] (\cos[c] + i \sin[c]) + \\
& 6 i \sqrt{a^2 - b^2} d e f^2 \operatorname{PolyLog} \left[3, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 - a \sin[c]}} \right] (\cos[c] + i \sin[c]) + \\
& 6 i \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog} \left[3, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 - a \sin[c]}} \right] (\cos[c] + i \sin[c]) - \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog} \left[4, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 - a \sin[c]}} \right] (\cos[c] + i \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog} \left[4, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 + a \sin[c]}} \right] (\cos[c] + i \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log} \left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 + a \sin[c]}} \right] (-i \cos[c] + \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log} \left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 + a \sin[c]}} \right] (-i \cos[c] + \sin[c]) + \\
& \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log} \left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 + a \sin[c]}} \right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d e f^2 \operatorname{PolyLog} \left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 + a \sin[c]}} \right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog} \left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2 + a \sin[c]}} \right] (-i \cos[c] + \sin[c]) -
\end{aligned}$$

$$2 i d^3 e^3 \operatorname{ArcTan}\left[\frac{b \cos [c+d x]+i(a+b \sin [c+d x])}{\sqrt{a^2-b^2}}\right] \sqrt{(-a^2+b^2)(\cos [2 c]+i \sin [2 c])}-12 b f\left(-2 f^2+d^2(e+f x)^2\right) \sin [c+d x] \Bigg)$$

■ **Problem 302: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+f x)^3 \cos [c+d x]^3}{a+b \sin [c+d x]} dx$$

Optimal (type 4, 737 leaves, 21 steps):

$$\begin{aligned} & -\frac{3 f^3 x}{8 b d^3} + \frac{(e+f x)^3}{4 b d} + \frac{i\left(a^2-b^2\right)(e+f x)^4}{4 b^3 f} - \frac{6 a f^3 \cos [c+d x]}{b^2 d^4} + \frac{3 a f(e+f x)^2 \cos [c+d x]}{b^2 d^2} \\ & - \frac{\left(a^2-b^2\right)(e+f x)^3 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b^3 d} - \frac{\left(a^2-b^2\right)(e+f x)^3 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b^3 d} + \frac{3 i\left(a^2-b^2\right) f(e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b^3 d^2} + \\ & \frac{3 i\left(a^2-b^2\right) f(e+f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b^3 d^2} - \frac{6\left(a^2-b^2\right) f^2(e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b^3 d^3} - \frac{6\left(a^2-b^2\right) f^2(e+f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b^3 d^3} \\ & - \frac{6 i\left(a^2-b^2\right) f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b^3 d^4} - \frac{6 i\left(a^2-b^2\right) f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b^3 d^4} - \frac{6 a f^2(e+f x) \sin [c+d x]}{b^2 d^3} + \frac{a(e+f x)^3 \sin [c+d x]}{b^2 d} + \\ & \frac{3 f^3 \cos [c+d x] \sin [c+d x]}{8 b d^4} - \frac{3 f(e+f x)^2 \cos [c+d x] \sin [c+d x]}{4 b d^2} + \frac{3 f^2(e+f x) \sin [c+d x]^2}{4 b d^3} - \frac{(e+f x)^3 \sin [c+d x]^2}{2 b d} \end{aligned}$$

Result (type 4, 3279 leaves):

$$\begin{aligned} & \frac{1}{2 b^3 d^4\left(-1+e^{2 i c}\right)} \\ & \left(a^2-b^2\right)\left(4 i d^4 e^3 e^{2 i c} x+6 i d^4 e^2 e^{2 i c} f x^2+4 i d^4 e e^{2 i c} f^2 x^3+i d^4 e^2 e^{2 i c} f^3 x^4+2 i d^3 e^3 \operatorname{ArcTan}\left[\frac{2 a e^{i(c+d x)}}{b\left(-1+e^{2 i(c+d x)}\right)}\right]-2 i d^3 e^3 e^{2 i c}\right. \\ & \left.\operatorname{ArcTan}\left[\frac{2 a e^{i(c+d x)}}{b\left(-1+e^{2 i(c+d x)}\right)}\right]+d^3 e^3 \operatorname{Log}\left[4 a^2 e^{2 i(c+d x)}+b^2\left(-1+e^{2 i(c+d x)}\right)^2\right]-d^3 e^3 e^{2 i c} \operatorname{Log}\left[4 a^2 e^{2 i(c+d x)}+b^2\left(-1+e^{2 i(c+d x)}\right)^2\right]+ \right. \\ & \left. 6 d^3 e^2 f x \operatorname{Log}\left[1+\frac{b e^{i(2 c+d x)}}{i a e^{i c}-\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]-6 d^3 e^2 e^{2 i c} f x \operatorname{Log}\left[1+\frac{b e^{i(2 c+d x)}}{i a e^{i c}-\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]+ \right. \\ & \left. 6 d^3 e f^2 x^2 \operatorname{Log}\left[1+\frac{b e^{i(2 c+d x)}}{i a e^{i c}-\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]-6 d^3 e e^{2 i c} f^2 x^2 \operatorname{Log}\left[1+\frac{b e^{i(2 c+d x)}}{i a e^{i c}-\sqrt{\left(-a^2+b^2\right) e^{2 i c}}}\right]+ \right. \end{aligned}$$

$$\begin{aligned}
& 2 d^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} - \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 2 d^3 e^{2ic} f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} - \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \\
& 6 d^3 e^2 f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 6 d^3 e^2 e^{2ic} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \\
& 6 d^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 6 d^3 e e^{2ic} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \\
& 2 d^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 2 d^3 e^{2ic} f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 6 i d^2 (-1 + e^{2ic}) f \\
& (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 6 i d^2 (-1 + e^{2ic}) f (e + f x)^2 \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \\
& 12 d e f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 12 d e e^{2ic} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \\
& 12 d f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 12 d e^{2ic} f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \\
& 12 d e f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 12 d e e^{2ic} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \\
& 12 d f^3 x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 12 d e^{2ic} f^3 x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \\
& 12 i f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 12 i e^{2ic} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] + \\
& 12 i f^3 \operatorname{PolyLog}\left[4, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - 12 i e^{2ic} f^3 \operatorname{PolyLog}\left[4, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] \Bigg) + \\
& \frac{i(-a^2+b^2) e^3 x (1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])}{b^3 (-1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} + \frac{3i(-a^2+b^2) e^2 f x^2 (1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])}{2b^3 (-1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} + \\
& \frac{i(-a^2+b^2) e f^2 x^3 (1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])}{b^3 (-1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} + \\
& \frac{i(-a^2+b^2) f^3 x^4 (1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])}{4b^3 (-1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} +
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{i a f^3 x^3 \cos[c]}{2 b^2 d} + \frac{a f^3 x^3 \sin[c]}{2 b^2 d} + (i d^3 e^3 + 3 d^2 e^2 f - 6 i d e f^2 - 6 f^3) \left(\frac{a \cos[c]}{2 b^2 d^4} - \frac{i a \sin[c]}{2 b^2 d^4} \right) + \right. \\
& \quad \left. (a d^2 e^2 f - 2 i a d e f^2 - 2 a f^3) \left(\frac{3 i x \cos[c]}{2 b^2 d^3} + \frac{3 x \sin[c]}{2 b^2 d^3} \right) + (a d e f^2 - i a f^3) \left(\frac{3 i x^2 \cos[c]}{2 b^2 d^2} + \frac{3 x^2 \sin[c]}{2 b^2 d^2} \right) \right) (\cos[dx] - i \sin[dx]) + \\
& \left(-\frac{i a f^3 x^3 \cos[c]}{2 b^2 d} + \frac{a f^3 x^3 \sin[c]}{2 b^2 d} + (-i d^3 e^3 + 3 d^2 e^2 f + 6 i d e f^2 - 6 f^3) \left(\frac{a \cos[c]}{2 b^2 d^4} + \frac{i a \sin[c]}{2 b^2 d^4} \right) - \right. \\
& \quad \left. \frac{3 i x^2 (a d e f^2 \cos[c] + i a f^3 \cos[c] + i a d e f^2 \sin[c] - a f^3 \sin[c])}{2 b^2 d^2} - \frac{1}{2 b^2 d^3} \right. \\
& \quad \left. 3 i x (a d^2 e^2 f \cos[c] + 2 i a d e f^2 \cos[c] - 2 a f^3 \cos[c] + i a d^2 e^2 f \sin[c] - 2 a d e f^2 \sin[c] - 2 i a f^3 \sin[c]) \right) (\cos[dx] + i \sin[dx]) + \\
& \left(\frac{f^3 x^3 \cos[2c]}{8 b d} - \frac{i f^3 x^3 \sin[2c]}{8 b d} + (4 d^3 e^3 - 6 i d^2 e^2 f - 6 d e f^2 + 3 i f^3) \left(\frac{\cos[2c]}{32 b d^4} - \frac{i \sin[2c]}{32 b d^4} \right) + \right. \\
& \quad \left. (2 d^2 e^2 f - 2 i d e f^2 - f^3) \left(\frac{3 x \cos[2c]}{16 b d^3} - \frac{3 i x \sin[2c]}{16 b d^3} \right) + (2 d e f^2 - i f^3) \left(\frac{3 x^2 \cos[2c]}{16 b d^2} - \frac{3 i x^2 \sin[2c]}{16 b d^2} \right) \right) (\cos[2dx] - i \sin[2dx]) + \\
& \left(\frac{f^3 x^3 \cos[2c]}{8 b d} + \frac{i f^3 x^3 \sin[2c]}{8 b d} + (4 d^3 e^3 + 6 i d^2 e^2 f - 6 d e f^2 - 3 i f^3) \left(\frac{\cos[2c]}{32 b d^4} + \frac{i \sin[2c]}{32 b d^4} \right) + \right. \\
& \quad \left. \frac{3 x^2 (2 d e f^2 \cos[2c] + i f^3 \cos[2c] + 2 i d e f^2 \sin[2c] - f^3 \sin[2c])}{16 b d^2} + \frac{1}{16 b d^3} \right. \\
& \quad \left. 3 x (2 d^2 e^2 f \cos[2c] + 2 i d e f^2 \cos[2c] - f^3 \cos[2c] + 2 i d^2 e^2 f \sin[2c] - 2 d e f^2 \sin[2c] - i f^3 \sin[2c]) \right) (\cos[2dx] + i \sin[2dx])
\end{aligned}$$

■ **Problem 303: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \cos[c + d x]^3}{a + b \sin[c + d x]} dx$$

Optimal (type 4, 548 leaves, 16 steps):

$$\begin{aligned}
& \frac{e f x}{2 b d} + \frac{f^2 x^2}{4 b d} + \frac{i (a^2 - b^2) (e + f x)^3}{3 b^3 f} + \frac{2 a f (e + f x) \operatorname{Cos}[c + d x]}{b^2 d^2} - \frac{(a^2 - b^2) (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b^3 d} \\
& - \frac{(a^2 - b^2) (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b^3 d} + \frac{2 i (a^2 - b^2) f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b^3 d^2} + \frac{2 i (a^2 - b^2) f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b^3 d^2} \\
& - \frac{2 (a^2 - b^2) f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b^3 d^3} - \frac{2 (a^2 - b^2) f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b^3 d^3} - \frac{2 a f^2 \operatorname{Sin}[c + d x]}{b^2 d^3} + \\
& \frac{a (e + f x)^2 \operatorname{Sin}[c + d x]}{b^2 d} - \frac{f (e + f x) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{2 b d^2} + \frac{f^2 \operatorname{Sin}[c + d x]^2}{4 b d^3} - \frac{(e + f x)^2 \operatorname{Sin}[c + d x]^2}{2 b d}
\end{aligned}$$

Result (type 4, 2397 leaves):

$$\begin{aligned}
& \frac{1}{48 b^3 d^3} \\
& e^{-2 i c} \left(48 i a^2 d^3 e^2 e^{2 i c} x - 48 i b^2 d^3 e^2 e^{2 i c} x + 48 i a^2 d^3 e e^{2 i c} f x^2 - 48 i b^2 d^3 e e^{2 i c} f x^2 + 16 i a^2 d^3 e^2 e^{i c} f^2 x^3 - 16 i b^2 d^3 e^2 e^{i c} f^2 x^3 - 48 i a^2 d^2 e^2 \right. \\
& e^{2 i c} \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b(-1 + e^{2 i(c+dx)})}\right] + 48 i b^2 d^2 e^2 e^{2 i c} \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b(-1 + e^{2 i(c+dx)})}\right] + 24 i a b d^2 e^2 e^{i c} \operatorname{Cos}[d x] - 24 i a b d^2 e^2 e^{3 i c} \operatorname{Cos}[d x] + \\
& 48 a b d e e^{i c} f \operatorname{Cos}[d x] + 48 a b d e e^{3 i c} f \operatorname{Cos}[d x] - 48 i a b e^{i c} f^2 \operatorname{Cos}[d x] + 48 i a b e^{3 i c} f^2 \operatorname{Cos}[d x] + 48 i a b d^2 e e^{i c} f x \operatorname{Cos}[d x] - \\
& 48 i a b d^2 e e^{3 i c} f x \operatorname{Cos}[d x] + 48 a b d e e^{i c} f^2 x \operatorname{Cos}[d x] + 48 a b d e e^{3 i c} f^2 x \operatorname{Cos}[d x] + 24 i a b d^2 e^{i c} f^2 x^2 \operatorname{Cos}[d x] - \\
& 24 i a b d^2 e^{3 i c} f^2 x^2 \operatorname{Cos}[d x] + 6 b^2 d^2 e^2 \operatorname{Cos}[2 d x] + 6 b^2 d^2 e^2 e^{4 i c} \operatorname{Cos}[2 d x] - 6 i b^2 d e f \operatorname{Cos}[2 d x] + 6 i b^2 d e e^{4 i c} f \operatorname{Cos}[2 d x] - \\
& 3 b^2 f^2 \operatorname{Cos}[2 d x] - 3 b^2 e^{4 i c} f^2 \operatorname{Cos}[2 d x] + 12 b^2 d^2 e f x \operatorname{Cos}[2 d x] + 12 b^2 d^2 e e^{4 i c} f x \operatorname{Cos}[2 d x] - 6 i b^2 d f^2 x \operatorname{Cos}[2 d x] + \\
& 6 i b^2 d e^{4 i c} f^2 x \operatorname{Cos}[2 d x] + 6 b^2 d^2 f^2 x^2 \operatorname{Cos}[2 d x] + 6 b^2 d^2 e^{4 i c} f^2 x^2 \operatorname{Cos}[2 d x] - 24 a^2 d^2 e^2 e^{2 i c} \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 (-1 + e^{2 i(c+dx)})^2\right] + \\
& 24 b^2 d^2 e^2 e^{2 i c} \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 (-1 + e^{2 i(c+dx)})^2\right] - 96 a^2 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 96 b^2 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 48 a^2 d^2 e^2 e^{i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 48 b^2 d^2 e^2 e^{i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 96 a^2 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 96 b^2 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 48 a^2 d^2 e^2 e^{i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] +
\end{aligned}$$

$$\begin{aligned}
& 48 b^2 d^2 e^{2ic} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2) e^{2ic}}}\right] + 96 i (a^2 - b^2) d e^{2ic} f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2) e^{2ic}}}\right] + \\
& 96 i (a^2 - b^2) d e^{2ic} f (e + f x) \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2) e^{2ic}}}\right] - 96 a^2 e^{2ic} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2) e^{2ic}}}\right] + \\
& 96 b^2 e^{2ic} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2) e^{2ic}}}\right] - 96 a^2 e^{2ic} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2) e^{2ic}}}\right] + \\
& 96 b^2 e^{2ic} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2) e^{2ic}}}\right] + 24 a b d^2 e^2 e^{ic} \operatorname{Sin}[dx] + 24 a b d^2 e^2 e^{3ic} \operatorname{Sin}[dx] - \\
& 48 i a b d e e^{ic} f \operatorname{Sin}[dx] + 48 i a b d e e^{3ic} f \operatorname{Sin}[dx] - 48 a b e^{ic} f^2 \operatorname{Sin}[dx] - 48 a b e^{3ic} f^2 \operatorname{Sin}[dx] + 48 a b d^2 e e^{ic} f x \operatorname{Sin}[dx] + \\
& 48 a b d^2 e e^{3ic} f x \operatorname{Sin}[dx] - 48 i a b d e^{ic} f^2 x \operatorname{Sin}[dx] + 48 i a b d e^{3ic} f^2 x \operatorname{Sin}[dx] + 24 a b d^2 e^{ic} f^2 x^2 \operatorname{Sin}[dx] + \\
& 24 a b d^2 e^{3ic} f^2 x^2 \operatorname{Sin}[dx] - 6 i b^2 d^2 e^2 \operatorname{Sin}[2dx] + 6 i b^2 d^2 e^2 e^{4ic} \operatorname{Sin}[2dx] - 6 b^2 d e f \operatorname{Sin}[2dx] - 6 b^2 d e e^{4ic} f \operatorname{Sin}[2dx] + \\
& 3 i b^2 f^2 \operatorname{Sin}[2dx] - 3 i b^2 e^{4ic} f^2 \operatorname{Sin}[2dx] - 12 i b^2 d^2 e f x \operatorname{Sin}[2dx] + 12 i b^2 d^2 e e^{4ic} f x \operatorname{Sin}[2dx] - \\
& \left. 6 b^2 d f^2 x \operatorname{Sin}[2dx] - 6 b^2 d e^{4ic} f^2 x \operatorname{Sin}[2dx] - 6 i b^2 d^2 f^2 x^2 \operatorname{Sin}[2dx] + 6 i b^2 d^2 e^{4ic} f^2 x^2 \operatorname{Sin}[2dx] \right)
\end{aligned}$$

■ **Problem 304: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x) \operatorname{Cos}[c + dx]^3}{a + b \operatorname{Sin}[c + dx]} dx$$

Optimal (type 4, 351 leaves, 13 steps):

$$\begin{aligned}
& \frac{f x}{4 b d} + \frac{i (a^2 - b^2) (e + f x)^2}{2 b^3 f} + \frac{a f \operatorname{Cos}[c + dx]}{b^2 d^2} - \frac{(a^2 - b^2) (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b^3 d} - \\
& \frac{(a^2 - b^2) (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b^3 d} + \frac{i (a^2 - b^2) f \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b^3 d^2} + \frac{i (a^2 - b^2) f \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b^3 d^2} + \\
& \frac{a (e + f x) \operatorname{Sin}[c + dx]}{b^2 d} - \frac{f \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{4 b d^2} - \frac{(e + f x) \operatorname{Sin}[c + dx]^2}{2 b d}
\end{aligned}$$

Result (type 4, 816 leaves):

$$\frac{1}{8 b^3 d^2} \left(8 a b f \operatorname{Cos}[c + dx] + 2 b^2 d (e + f x) \operatorname{Cos}[2(c + dx)] - 8 a^2 d e \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c + dx]}{a}\right] + \right.$$

$$\begin{aligned}
& 8 b^2 d e \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c + d x]}{a}\right] + 8 a^2 c f \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c + d x]}{a}\right] - 8 b^2 c f \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c + d x]}{a}\right] - a^2 f \left(i (-2 c + \pi - 2 d x)^2 - \right. \\
& 32 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Cot}\left[\frac{1}{4}(2 c + \pi + 2 d x)\right]}{\sqrt{a^2 - b^2}}\right] - 4 \left(-2 c + \pi - 2 d x + 4 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{i(-a + \sqrt{a^2 - b^2}) e^{-i(c+dx)}}{b}\right] - \\
& 4 \left(-2 c + \pi - 2 d x - 4 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{i(a + \sqrt{a^2 - b^2}) e^{-i(c+dx)}}{b}\right] + 4(-2 c + \pi - 2 d x) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]] + \\
& \left. 8(c + d x) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]] + 8 i \left(\operatorname{PolyLog}\left[2, \frac{i(-a + \sqrt{a^2 - b^2}) e^{-i(c+dx)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{i(a + \sqrt{a^2 - b^2}) e^{-i(c+dx)}}{b}\right] \right) \right) + \\
& b^2 f \left(i (-2 c + \pi - 2 d x)^2 - 32 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Cot}\left[\frac{1}{4}(2 c + \pi + 2 d x)\right]}{\sqrt{a^2 - b^2}}\right] - \right. \\
& 4 \left(-2 c + \pi - 2 d x + 4 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{i(-a + \sqrt{a^2 - b^2}) e^{-i(c+dx)}}{b}\right] - \\
& \left. 4 \left(-2 c + \pi - 2 d x - 4 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{i(a + \sqrt{a^2 - b^2}) e^{-i(c+dx)}}{b}\right] + 4(-2 c + \pi - 2 d x) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]] + \right.
\end{aligned}$$

$$8 (c + d x) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]] + 8 i \left(\operatorname{PolyLog}\left[2, \frac{i \left(-a + \sqrt{a^2 - b^2}\right) e^{-i (c + d x)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{i \left(a + \sqrt{a^2 - b^2}\right) e^{-i (c + d x)}}{b}\right] \right) +$$

$$8 a b d (e + f x) \operatorname{Sin}[c + d x] - b^2 f \operatorname{Sin}[2 (c + d x)]$$

■ **Problem 306: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \operatorname{Sec}[c + d x]}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 937 leaves, 29 steps):

$$-\frac{2 i a (e + f x)^3 \operatorname{ArcTan}\left[e^{i (c + d x)}\right]}{(a^2 - b^2) d} - \frac{b (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i (c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d} - \frac{b (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i (c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d} +$$

$$\frac{b (e + f x)^3 \operatorname{Log}\left[1 + e^{2 i (c + d x)}\right]}{(a^2 - b^2) d} + \frac{3 i a f (e + f x)^2 \operatorname{PolyLog}\left[2, -i e^{i (c + d x)}\right]}{(a^2 - b^2) d^2} - \frac{3 i a f (e + f x)^2 \operatorname{PolyLog}\left[2, i e^{i (c + d x)}\right]}{(a^2 - b^2) d^2} +$$

$$\frac{3 i b f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i (c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d^2} + \frac{3 i b f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i (c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d^2} - \frac{3 i b f (e + f x)^2 \operatorname{PolyLog}\left[2, -e^{2 i (c + d x)}\right]}{2 (a^2 - b^2) d^2} -$$

$$\frac{6 a f^2 (e + f x) \operatorname{PolyLog}\left[3, -i e^{i (c + d x)}\right]}{(a^2 - b^2) d^3} + \frac{6 a f^2 (e + f x) \operatorname{PolyLog}\left[3, i e^{i (c + d x)}\right]}{(a^2 - b^2) d^3} - \frac{6 b f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i (c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d^3} -$$

$$\frac{6 b f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i (c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d^3} + \frac{3 b f^2 (e + f x) \operatorname{PolyLog}\left[3, -e^{2 i (c + d x)}\right]}{2 (a^2 - b^2) d^3} - \frac{6 i a f^3 \operatorname{PolyLog}\left[4, -i e^{i (c + d x)}\right]}{(a^2 - b^2) d^4} +$$

$$\frac{6 i a f^3 \operatorname{PolyLog}\left[4, i e^{i (c + d x)}\right]}{(a^2 - b^2) d^4} - \frac{6 i b f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i (c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d^4} - \frac{6 i b f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i (c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d^4} + \frac{3 i b f^3 \operatorname{PolyLog}\left[4, -e^{2 i (c + d x)}\right]}{4 (a^2 - b^2) d^4}$$

Result (type 4, 1977 leaves):

$$-\frac{1}{4 (a - b) (a + b) d^4}$$

$$\begin{aligned}
& \left(8 i a d^3 e^3 \operatorname{ArcTan}\left[e^{i(c+dx)}\right] + 4 i b d^3 e^3 \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b(-1+e^{2i(c+dx)})}\right] - 12 a d^3 e^2 f x \operatorname{Log}\left[1-i e^{i(c+dx)}\right] - 12 a d^3 e f^2 x^2 \operatorname{Log}\left[1-i e^{i(c+dx)}\right] - \right. \\
& 4 a d^3 f^3 x^3 \operatorname{Log}\left[1-i e^{i(c+dx)}\right] + 12 a d^3 e^2 f x \operatorname{Log}\left[1+i e^{i(c+dx)}\right] + 12 a d^3 e f^2 x^2 \operatorname{Log}\left[1+i e^{i(c+dx)}\right] + 4 a d^3 f^3 x^3 \operatorname{Log}\left[1+i e^{i(c+dx)}\right] - \\
& 4 b d^3 e^3 \operatorname{Log}\left[1+e^{2i(c+dx)}\right] - 12 b d^3 e^2 f x \operatorname{Log}\left[1+e^{2i(c+dx)}\right] - 12 b d^3 e f^2 x^2 \operatorname{Log}\left[1+e^{2i(c+dx)}\right] - 4 b d^3 f^3 x^3 \operatorname{Log}\left[1+e^{2i(c+dx)}\right] + \\
& 2 b d^3 e^3 \operatorname{Log}\left[4 a^2 e^{2i(c+dx)} + b^2(-1+e^{2i(c+dx)})^2\right] + 12 b d^3 e^2 f x \operatorname{Log}\left[1+\frac{b e^{i(2c+dx)}}{i a e^{i c}-\sqrt{-a^2+b^2} e^{2 i c}}\right] + \\
& 12 b d^3 e f^2 x^2 \operatorname{Log}\left[1+\frac{b e^{i(2c+dx)}}{i a e^{i c}-\sqrt{-a^2+b^2} e^{2 i c}}\right] + 4 b d^3 f^3 x^3 \operatorname{Log}\left[1+\frac{b e^{i(2c+dx)}}{i a e^{i c}-\sqrt{-a^2+b^2} e^{2 i c}}\right] + \\
& 12 b d^3 e^2 f x \operatorname{Log}\left[1+\frac{b e^{i(2c+dx)}}{i a e^{i c}+\sqrt{-a^2+b^2} e^{2 i c}}\right] + 12 b d^3 e f^2 x^2 \operatorname{Log}\left[1+\frac{b e^{i(2c+dx)}}{i a e^{i c}+\sqrt{-a^2+b^2} e^{2 i c}}\right] + \\
& 4 b d^3 f^3 x^3 \operatorname{Log}\left[1+\frac{b e^{i(2c+dx)}}{i a e^{i c}+\sqrt{-a^2+b^2} e^{2 i c}}\right] - 12 i a d^2 f(e+f x)^2 \operatorname{PolyLog}\left[2,-i e^{i(c+dx)}\right] + 12 i a d^2 f(e+f x)^2 \operatorname{PolyLog}\left[2,i e^{i(c+dx)}\right] + \\
& 6 i b d^2 e^2 f \operatorname{PolyLog}\left[2,-e^{2i(c+dx)}\right] + 12 i b d^2 e f^2 x \operatorname{PolyLog}\left[2,-e^{2i(c+dx)}\right] + 6 i b d^2 f^3 x^2 \operatorname{PolyLog}\left[2,-e^{2i(c+dx)}\right] - \\
& 12 i b d^2 e^2 f \operatorname{PolyLog}\left[2,\frac{i b e^{i(2c+dx)}}{a e^{i c}+i \sqrt{-a^2+b^2} e^{2 i c}}\right] - 24 i b d^2 e f^2 x \operatorname{PolyLog}\left[2,\frac{i b e^{i(2c+dx)}}{a e^{i c}+i \sqrt{-a^2+b^2} e^{2 i c}}\right] - \\
& 12 i b d^2 f^3 x^2 \operatorname{PolyLog}\left[2,\frac{i b e^{i(2c+dx)}}{a e^{i c}+i \sqrt{-a^2+b^2} e^{2 i c}}\right] - 12 i b d^2 e^2 f \operatorname{PolyLog}\left[2,-\frac{b e^{i(2c+dx)}}{i a e^{i c}+\sqrt{-a^2+b^2} e^{2 i c}}\right] - \\
& 24 i b d^2 e f^2 x \operatorname{PolyLog}\left[2,-\frac{b e^{i(2c+dx)}}{i a e^{i c}+\sqrt{-a^2+b^2} e^{2 i c}}\right] - 12 i b d^2 f^3 x^2 \operatorname{PolyLog}\left[2,-\frac{b e^{i(2c+dx)}}{i a e^{i c}+\sqrt{-a^2+b^2} e^{2 i c}}\right] + \\
& 24 a d e f^2 \operatorname{PolyLog}\left[3,-i e^{i(c+dx)}\right] + 24 a d f^3 x \operatorname{PolyLog}\left[3,-i e^{i(c+dx)}\right] - 24 a d e f^2 \operatorname{PolyLog}\left[3,i e^{i(c+dx)}\right] - \\
& 24 a d f^3 x \operatorname{PolyLog}\left[3,i e^{i(c+dx)}\right] - 6 b d e f^2 \operatorname{PolyLog}\left[3,-e^{2i(c+dx)}\right] - 6 b d f^3 x \operatorname{PolyLog}\left[3,-e^{2i(c+dx)}\right] + \\
& 24 b d e f^2 \operatorname{PolyLog}\left[3,\frac{i b e^{i(2c+dx)}}{a e^{i c}+i \sqrt{-a^2+b^2} e^{2 i c}}\right] + 24 b d f^3 x \operatorname{PolyLog}\left[3,\frac{i b e^{i(2c+dx)}}{a e^{i c}+i \sqrt{-a^2+b^2} e^{2 i c}}\right] + \\
& 24 b d e f^2 \operatorname{PolyLog}\left[3,-\frac{b e^{i(2c+dx)}}{i a e^{i c}+\sqrt{-a^2+b^2} e^{2 i c}}\right] + 24 b d f^3 x \operatorname{PolyLog}\left[3,-\frac{b e^{i(2c+dx)}}{i a e^{i c}+\sqrt{-a^2+b^2} e^{2 i c}}\right] + \\
& 24 i a f^3 \operatorname{PolyLog}\left[4,-i e^{i(c+dx)}\right] - 24 i a f^3 \operatorname{PolyLog}\left[4,i e^{i(c+dx)}\right] - 3 i b f^3 \operatorname{PolyLog}\left[4,-e^{2i(c+dx)}\right] + \\
& \left. 24 i b f^3 \operatorname{PolyLog}\left[4,\frac{i b e^{i(2c+dx)}}{a e^{i c}+i \sqrt{-a^2+b^2} e^{2 i c}}\right] + 24 i b f^3 \operatorname{PolyLog}\left[4,-\frac{b e^{i(2c+dx)}}{i a e^{i c}+\sqrt{-a^2+b^2} e^{2 i c}}\right]\right)
\end{aligned}$$

■ **Problem 308:** Result more than twice size of optimal antiderivative.

$$\int \frac{(e + f x) \operatorname{Sec}[c + d x]}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 413 leaves, 19 steps):

$$\begin{aligned} & - \frac{2 i a (e + f x) \operatorname{ArcTan}\left[e^{i(c+dx)}\right]}{(a^2 - b^2) d} - \frac{b (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d} - \frac{b (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d} + \\ & \frac{b (e + f x) \operatorname{Log}\left[1 + e^{2 i(c+dx)}\right]}{(a^2 - b^2) d} + \frac{i a f \operatorname{PolyLog}\left[2, -i e^{i(c+dx)}\right]}{(a^2 - b^2) d^2} - \frac{i a f \operatorname{PolyLog}\left[2, i e^{i(c+dx)}\right]}{(a^2 - b^2) d^2} + \\ & \frac{i b f \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d^2} + \frac{i b f \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2) d^2} - \frac{i b f \operatorname{PolyLog}\left[2, -e^{2 i(c+dx)}\right]}{2 (a^2 - b^2) d^2} \end{aligned}$$

Result (type 4, 2580 leaves):

$$\begin{aligned} & - \frac{b e \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right]}{(a^2 - b^2) d} + \frac{b c f \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right]}{(a^2 - b^2) d^2} - \frac{1}{(a^2 - b^2) d^2} \\ & b^2 f \left(\frac{(c + dx) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]]}{b} - 1 / b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx\right)^2 + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right)\right]}{\sqrt{a^2 - b^2}}\right] \right) + \right. \\ & \left. \left(-c + \frac{\pi}{2} - dx + 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] + \right. \\ & \left. \left(-c + \frac{\pi}{2} - dx - 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] - \left(-c + \frac{\pi}{2} - dx\right) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] - \right. \\ & \left. i \left(\operatorname{PolyLog}\left[2, -\frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - dx\right)}}{b}\right] \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left(2 b (d e - c f) \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \right] + 2 (a - b) (d e - c f) \operatorname{Log} \left[1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] - 2 (a + b) (d e - c f) \operatorname{Log} \left[1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] \right) + \\
& f \left(2 (c + d x) \left(b \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \right] + (a - b) \operatorname{Log} \left[1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] - (a + b) \operatorname{Log} \left[1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] \right) + b \left(-2 (c + d x) \right. \\
& \quad \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \right] + 2 (c + d x) \operatorname{Log} \left[-i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] - 2 i \operatorname{Log} \left[\frac{1}{2} \left(1 - i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] \operatorname{Log} \left[-i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] + \\
& \quad i \operatorname{Log} \left[-i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right]^2 + 2 (c + d x) \operatorname{Log} \left[i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] + 2 i \operatorname{Log} \left[\frac{1}{2} \left(1 + i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] \operatorname{Log} \left[i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] - \\
& \quad i \operatorname{Log} \left[i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right]^2 + 2 i \operatorname{PolyLog} \left[2, \frac{1}{2} \left(1 - i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] - 2 i \operatorname{PolyLog} \left[2, \frac{1}{2} \left(1 + i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] \right) + \\
& 2 i (a - b) \left(\operatorname{Log} \left[1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] \left(\operatorname{Log} \left[\left(\frac{1}{2} + \frac{i}{2} \right) \left(-i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] - \operatorname{Log} \left[\frac{1}{2} \left((1 + i) + (1 - i) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] \right) + \right. \\
& \quad \left. \operatorname{PolyLog} \left[2, \left(-\frac{1}{2} - \frac{i}{2} \right) \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] - \operatorname{PolyLog} \left[2, \left(-\frac{1}{2} + \frac{i}{2} \right) \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] \right) + \\
& 2 i (a + b) \left(\left(-\operatorname{Log} \left[\frac{1}{2} \left((1 + i) - (1 - i) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] + \operatorname{Log} \left[\left(-\frac{1}{2} - \frac{i}{2} \right) \left(i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] \right) \operatorname{Log} \left[1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] - \right. \\
& \quad \left. \operatorname{PolyLog} \left[2, \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] + \operatorname{PolyLog} \left[2, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] \right) \right) \\
& \left(\left(\frac{a e}{a^2 - b^2} - \frac{a c f}{(a^2 - b^2) d} + \frac{a f (c + d x)}{(a^2 - b^2) d} \right) \operatorname{Sec}[c + d x] + \left(-\frac{b e}{a^2 - b^2} + \frac{b c f}{(a^2 - b^2) d} - \frac{b f (c + d x)}{(a^2 - b^2) d} \right) \operatorname{Tan}[c + d x] \right) / \\
& d \left(-\frac{(a - b) (d e - c f) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]} + 2 b (d e - c f) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] - \frac{(a + b) (d e - c f) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]} + \right. \\
& f \left(2 \left(b \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \right] + (a - b) \operatorname{Log} \left[1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] - (a + b) \operatorname{Log} \left[1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] \right) + \right. \\
& b \left(-2 \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \right] + 2 \operatorname{Log} \left[-i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] + 2 \operatorname{Log} \left[i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] - \right. \\
& \quad \frac{\operatorname{Log} \left[1 + \frac{1}{2} (-1 + i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]) \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{1 - i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]} - \frac{\operatorname{Log} \left[-i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{1 - i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]} - \\
& \quad \frac{\operatorname{Log} \left[1 + \frac{1}{2} (-1 - i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]) \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{1 + i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]} - \frac{\operatorname{Log} \left[i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{1 + i \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]} - 2 (c + d x) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] + \left. \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{(c+dx) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{-i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} - \frac{i \operatorname{Log}\left[\frac{1}{2}\left(1 - i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{-i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} + \frac{i \operatorname{Log}\left[-i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{-i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} + \\
& \left. \frac{(c+dx) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} + \frac{i \operatorname{Log}\left[\frac{1}{2}\left(1 + i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} - \frac{i \operatorname{Log}\left[i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right) + \\
& 2(c+dx) \left(-\frac{(a-b) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{(a+b) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} \right) + \\
& 2i(a+b) \left(\left(\left(-\operatorname{Log}\left[\frac{1}{2}\left((1+i) - (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \right) + \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right)\left(i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left(2\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right) \right) - \frac{\operatorname{Log}\left[1 - \left(\frac{1}{2} + \frac{i}{2}\right)\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} + \\
& \frac{\operatorname{Log}\left[1 - \left(\frac{1}{2} - \frac{i}{2}\right)\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} + \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{(1+i) - (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} \right) + 2i(a-b) \left(-\left(\operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right)\left(-i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \right) - \right. \\
& \left. \operatorname{Log}\left[\frac{1}{2}\left((1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right) / \left(2\left(1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right) \right) + \\
& \frac{\operatorname{Log}\left[1 + \left(\frac{1}{2} - \frac{i}{2}\right)\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} - \frac{\operatorname{Log}\left[1 + \left(\frac{1}{2} + \frac{i}{2}\right)\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} + \\
& \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(-i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{(1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 310: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+fx)^3 \operatorname{Sec}[c+dx]^2}{a+b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 4, 923 leaves, 29 steps):

$$\begin{aligned}
& - \frac{i a (e + f x)^3}{(a^2 - b^2) d} - \frac{6 i b f (e + f x)^2 \operatorname{ArcTan}\left[e^{i(c+dx)}\right]}{(a^2 - b^2) d^2} + \frac{i b^2 (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d} - \\
& \frac{i b^2 (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d} + \frac{3 a f (e + f x)^2 \operatorname{Log}\left[1 + e^{2i(c+dx)}\right]}{(a^2 - b^2) d^2} + \frac{6 i b f^2 (e + f x) \operatorname{PolyLog}\left[2, -i e^{i(c+dx)}\right]}{(a^2 - b^2) d^3} - \\
& \frac{6 i b f^2 (e + f x) \operatorname{PolyLog}\left[2, i e^{i(c+dx)}\right]}{(a^2 - b^2) d^3} + \frac{3 b^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^2} - \frac{3 b^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^2} - \\
& \frac{3 i a f^2 (e + f x) \operatorname{PolyLog}\left[2, -e^{2i(c+dx)}\right]}{(a^2 - b^2) d^3} - \frac{6 b f^3 \operatorname{PolyLog}\left[3, -i e^{i(c+dx)}\right]}{(a^2 - b^2) d^4} + \frac{6 b f^3 \operatorname{PolyLog}\left[3, i e^{i(c+dx)}\right]}{(a^2 - b^2) d^4} + \\
& \frac{6 i b^2 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^3} - \frac{6 i b^2 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^3} + \frac{3 a f^3 \operatorname{PolyLog}\left[3, -e^{2i(c+dx)}\right]}{2 (a^2 - b^2) d^4} - \\
& \frac{6 b^2 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^4} + \frac{6 b^2 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^4} - \frac{b (e + f x)^3 \operatorname{Sec}[c + dx]}{(a^2 - b^2) d} + \frac{a (e + f x)^3 \operatorname{Tan}[c + dx]}{(a^2 - b^2) d}
\end{aligned}$$

Result (type 4, 2241 leaves):

$$\begin{aligned}
& \frac{b (e + f x)^3 \operatorname{Sec}[c]}{(-a^2 + b^2) d} - \frac{1}{(a^2 - b^2)^{3/2} d^4 \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} \\
& i b^2 \left(3 i \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log}\left[1 + \frac{b (\operatorname{Cos}[2c + dx] + i \operatorname{Sin}[2c + dx])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& \left. 3 i \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{b (\operatorname{Cos}[2c + dx] + i \operatorname{Sin}[2c + dx])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& \left. i \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{b (\operatorname{Cos}[2c + dx] + i \operatorname{Sin}[2c + dx])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& \left. 3 \sqrt{a^2 - b^2} d^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, -\frac{b (\operatorname{Cos}[2c + dx] + i \operatorname{Sin}[2c + dx])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) - \right. \\
& \left. 3 \sqrt{a^2 - b^2} d^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{b (\operatorname{Cos}[2c + dx] + i \operatorname{Sin}[2c + dx])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 + a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right.
\end{aligned}$$

$$\begin{aligned}
& 6 i \sqrt{a^2 - b^2} d e f^2 \text{PolyLog}\left[3, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 6 i \sqrt{a^2 - b^2} d f^3 x \text{PolyLog}\left[3, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) - \\
& 6 \sqrt{a^2 - b^2} f^3 \text{PolyLog}\left[4, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} f^3 \text{PolyLog}\left[4, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e^2 f x \text{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e f^2 x^2 \text{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& \sqrt{a^2 - b^2} d^3 f^3 x^3 \text{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d e f^2 \text{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d f^3 x \text{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) - \\
& 2 i d^3 e^3 \text{ArcTan}\left[\frac{b \cos[c + dx] + i (a + b \sin[c + dx])}{\sqrt{a^2 - b^2}}\right] \sqrt{(-a^2 + b^2)} (\cos[2c] + i \sin[2c]) \Bigg) + \\
& \frac{e^3 \sin\left[\frac{dx}{2}\right] + 3 e^2 f x \sin\left[\frac{dx}{2}\right] + 3 e f^2 x^2 \sin\left[\frac{dx}{2}\right] + f^3 x^3 \sin\left[\frac{dx}{2}\right]}{(a + b) d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\
& \frac{e^3 \sin\left[\frac{dx}{2}\right] + 3 e^2 f x \sin\left[\frac{dx}{2}\right] + 3 e f^2 x^2 \sin\left[\frac{dx}{2}\right] + f^3 x^3 \sin\left[\frac{dx}{2}\right]}{(a - b) d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\
& \frac{1}{2 (a^2 - b^2) d^4} \\
& f (-6 i a d^3 e^2 x - 6 i a d^3 e f x^2 - 2 i a d^3 f^2 x^3 - 12 i b d^2 e^2 \text{ArcTan}[\cos[c + dx] + i \sin[c + dx]] - \\
& 24 i b d^2 e f x \text{ArcTan}[\cos[c + dx] + i \sin[c + dx]] - 12 i b d^2 f^2 x^2 \text{ArcTan}[\cos[c + dx] + i \sin[c + dx]] +
\end{aligned}$$

$$\begin{aligned}
& 6 a d^2 e^2 \operatorname{Log}[1 + \operatorname{Cos}[2(c + dx)] + i \operatorname{Sin}[2(c + dx)]] + 12 a d^2 e f x \operatorname{Log}[1 + \operatorname{Cos}[2(c + dx)] + i \operatorname{Sin}[2(c + dx)]] + \\
& 6 a d^2 f^2 x^2 \operatorname{Log}[1 + \operatorname{Cos}[2(c + dx)] + i \operatorname{Sin}[2(c + dx)]] - 12 i b d f (e + f x) \operatorname{PolyLog}[2, i \operatorname{Cos}[c + dx] - \operatorname{Sin}[c + dx]] + \\
& 12 i b d f (e + f x) \operatorname{PolyLog}[2, -i \operatorname{Cos}[c + dx] + \operatorname{Sin}[c + dx]] - 6 i a d e f \operatorname{PolyLog}[2, -\operatorname{Cos}[2(c + dx)] - i \operatorname{Sin}[2(c + dx)]] - \\
& 6 i a d f^2 x \operatorname{PolyLog}[2, -\operatorname{Cos}[2(c + dx)] - i \operatorname{Sin}[2(c + dx)]] + \\
& 12 b f^2 \operatorname{PolyLog}[3, i \operatorname{Cos}[c + dx] - \operatorname{Sin}[c + dx]] - 12 b f^2 \operatorname{PolyLog}[3, -i \operatorname{Cos}[c + dx] + \operatorname{Sin}[c + dx]] + \\
& 3 a f^2 \operatorname{PolyLog}[3, -\operatorname{Cos}[2(c + dx)] - i \operatorname{Sin}[2(c + dx)]] + 6 a d^3 e^2 x \operatorname{Tan}[c] + 6 a d^3 e f x^2 \operatorname{Tan}[c] + 2 a d^3 f^2 x^3 \operatorname{Tan}[c]
\end{aligned}$$

■ **Problem 311: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \operatorname{Sec}[c + dx]^2}{a + b \operatorname{Sin}[c + dx]} dx$$

Optimal (type 4, 659 leaves, 24 steps):

$$\begin{aligned}
& -\frac{i a (e + f x)^2}{(a^2 - b^2) d} - \frac{4 i b f (e + f x) \operatorname{ArcTan}\left[e^{i(c+dx)}\right]}{(a^2 - b^2) d^2} + \frac{i b^2 (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d} - \frac{i b^2 (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d} + \\
& \frac{2 a f (e + f x) \operatorname{Log}\left[1 + e^{2i(c+dx)}\right]}{(a^2 - b^2) d^2} + \frac{2 i b f^2 \operatorname{PolyLog}[2, -i e^{i(c+dx)}]}{(a^2 - b^2) d^3} - \frac{2 i b f^2 \operatorname{PolyLog}[2, i e^{i(c+dx)}]}{(a^2 - b^2) d^3} + \\
& \frac{2 b^2 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^2} - \frac{2 b^2 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^2} - \frac{i a f^2 \operatorname{PolyLog}[2, -e^{2i(c+dx)}]}{(a^2 - b^2) d^3} + \\
& \frac{2 i b^2 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^3} - \frac{2 i b^2 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{3/2} d^3} - \frac{b (e + f x)^2 \operatorname{Sec}[c + dx]}{(a^2 - b^2) d} + \frac{a (e + f x)^2 \operatorname{Tan}[c + dx]}{(a^2 - b^2) d}
\end{aligned}$$

Result (type 4, 1368 leaves):

$$\begin{aligned}
& \frac{b (e + f x)^2 \operatorname{Sec}[c]}{(-a^2 + b^2) d} + \frac{2 a e f \operatorname{Sec}[c] (\operatorname{Cos}[c] \operatorname{Log}[\operatorname{Cos}[c] \operatorname{Cos}[dx] - \operatorname{Sin}[c] \operatorname{Sin}[dx]] + dx \operatorname{Sin}[c])}{(a^2 - b^2) d^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} + \\
& \frac{4 i b e f \operatorname{ArcTan}\left[\frac{-i \operatorname{Sin}[c] - i \operatorname{Cos}[c] \operatorname{Tan}\left[\frac{dx}{2}\right]}{\sqrt{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}\right]}{(a^2 - b^2) d^2 \sqrt{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}} + \left(a f^2 \operatorname{Csc}[c] \left(d^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[c]]} x^2 - \right. \right. \\
& \left. \left. 1 / \left(\sqrt{1 + \operatorname{Cot}[c]^2} \right) \operatorname{Cot}[c] \left(i d x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[c]]) - \pi \operatorname{Log}[1 + e^{-2i dx}] - 2 (dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]) \operatorname{Log}[1 - e^{2i (dx - \operatorname{ArcTan}[\operatorname{Cot}[c]])}] \right) + \right. \right. \\
& \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[dx]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[c]] \operatorname{Log}[\operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] + i \operatorname{PolyLog}[2, e^{2i (dx - \operatorname{ArcTan}[\operatorname{Cot}[c]])}] \right) \right) \operatorname{Sec}[c] \Big) /
\end{aligned}$$

$$\begin{aligned}
& \left((a^2 - b^2) d^3 \sqrt{\operatorname{Csc}[c]^2 (\cos[c]^2 + \sin[c]^2)} \right) + \frac{1}{(a^2 - b^2) d^3} 2 b f^2 \left(-1 / \left(\sqrt{1 + \operatorname{Cot}[c]^2} \right) \operatorname{Csc}[c] \left(d x - \operatorname{ArcTan}[\operatorname{Cot}[c]] \right) \right. \\
& \quad \left. \left(\operatorname{Log}\left[1 - e^{i(d x - \operatorname{ArcTan}[\operatorname{Cot}[c]])}\right] - \operatorname{Log}\left[1 + e^{i(d x - \operatorname{ArcTan}[\operatorname{Cot}[c]])}\right] \right) + i \left(\operatorname{PolyLog}\left[2, -e^{i(d x - \operatorname{ArcTan}[\operatorname{Cot}[c]])}\right] - \operatorname{PolyLog}\left[2, e^{i(d x - \operatorname{ArcTan}[\operatorname{Cot}[c]])}\right] \right) \right) + \\
& \quad \left. \frac{2 \operatorname{ArcTan}[\operatorname{Cot}[c]] \operatorname{ArcTanh}\left[\frac{\sin[c] + \cos[c] \operatorname{Tan}\left[\frac{d x}{2}\right]}{\sqrt{\cos[c]^2 + \sin[c]^2}}\right]}{\sqrt{\cos[c]^2 + \sin[c]^2}} \right) - \frac{1}{(a^2 - b^2)^{3/2} d^3 \sqrt{(-a^2 + b^2) (\cos[2 c] + i \sin[2 c])}} \\
& i b^2 \left(2 \sqrt{a^2 - b^2} d f (e + f x) \operatorname{PolyLog}\left[2, -\frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} - a \sin[c]} \right] (\cos[c] + i \sin[c]) - \right. \\
& \quad \left. 2 \sqrt{a^2 - b^2} d f (e + f x) \operatorname{PolyLog}\left[2, \frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} + a \sin[c]} \right] (\cos[c] + i \sin[c]) - \right. \\
& \quad \left. i \left(-2 \sqrt{a^2 - b^2} f^2 \operatorname{PolyLog}\left[3, -\frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} - a \sin[c]} \right] (\cos[c] + i \sin[c]) + \right. \right. \\
& \quad \left. \left. 2 \sqrt{a^2 - b^2} f^2 \operatorname{PolyLog}\left[3, \frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} + a \sin[c]} \right] (\cos[c] + i \sin[c]) + \right. \right. \\
& \quad \left. \left. d^2 \left(\sqrt{a^2 - b^2} f x (2 e + f x) \left(-\operatorname{Log}\left[1 + \frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} - a \sin[c]} \right] \right) + \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[1 - \frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{-i a \cos[c] + \sqrt{(-a^2 + b^2) (\cos[c] + i \sin[c])^2} + a \sin[c]} \right] \right) (\cos[c] + i \sin[c]) + \right. \\
& \quad \left. \left. 2 e^2 \operatorname{ArcTan}\left[\frac{b \cos[c + d x] + i (a + b \sin[c + d x])}{\sqrt{a^2 - b^2}}\right] \sqrt{(-a^2 + b^2) (\cos[2 c] + i \sin[2 c])} \right) \right) \right) + \\
& \frac{e^2 \sin\left[\frac{d x}{2}\right] + 2 e f x \sin\left[\frac{d x}{2}\right] + f^2 x^2 \sin\left[\frac{d x}{2}\right]}{(a + b) d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right)} + \frac{e^2 \sin\left[\frac{d x}{2}\right] + 2 e f x \sin\left[\frac{d x}{2}\right] + f^2 x^2 \sin\left[\frac{d x}{2}\right]}{(a - b) d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right)}
\end{aligned}$$

■ Problem 317: Attempted integration timed out after 120 seconds.

$$\int \frac{(e + f x)^m \operatorname{Sec}[c + d x]}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 9, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(e + f x)^m \operatorname{Sec}[c + d x]}{a + b \operatorname{Sin}[c + d x]}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 323: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \operatorname{Cos}[c + d x]}{(a + b \operatorname{Sin}[c + d x])^3} dx$$

Optimal (type 4, 357 leaves, 12 steps):

$$\begin{aligned} & - \frac{i a f (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^2} + \frac{i a f (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^2} - \frac{f^2 \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{b (a^2 - b^2) d^3} - \\ & \frac{a f^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^3} + \frac{a f^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{b (a^2 - b^2)^{3/2} d^3} - \frac{(e + f x)^2}{2 b d (a + b \operatorname{Sin}[c + d x])^2} + \frac{f (e + f x) \operatorname{Cos}[c + d x]}{(a^2 - b^2) d^2 (a + b \operatorname{Sin}[c + d x])} \end{aligned}$$

Result (type 4, 1104 leaves):

$$\begin{aligned}
& \frac{f^2 x \operatorname{Cot}[c]}{b(-a^2 + b^2) d^2} - \frac{1}{2b(-a^2 + b^2) d^2 (-1 + e^{2ic})} i e^{ic} f \\
& \left(4 e^{ic} f x + \frac{4 i a e^{-ic} \operatorname{ArcTan}\left[\frac{i a + b e^{i(c+dx)}}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} - \frac{4 i a e^{ic} \operatorname{ArcTan}\left[\frac{i a + b e^{i(c+dx)}}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} + \frac{2 e^{-ic} f \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b(-1 + e^{2i(c+dx)})}\right]}{d} - \frac{2 e^{ic} f \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b(-1 + e^{2i(c+dx)})}\right]}{d} \right. \\
& \frac{i e^{-ic} f \operatorname{Log}\left[4 a^2 e^{2i(c+dx)} + b^2 (-1 + e^{2i(c+dx)})^2\right]}{d} + \frac{i e^{ic} f \operatorname{Log}\left[4 a^2 e^{2i(c+dx)} + b^2 (-1 + e^{2i(c+dx)})^2\right]}{d} + \frac{2 i a f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} - \sqrt{(-a^2 + b^2) e^{2ic}}}\right]}{\sqrt{(-a^2 + b^2) e^{2ic}}} - \\
& \frac{2 i a e^{2ic} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} - \sqrt{(-a^2 + b^2) e^{2ic}}}\right]}{\sqrt{(-a^2 + b^2) e^{2ic}}} - \frac{2 i a f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2 + b^2) e^{2ic}}}\right]}{\sqrt{(-a^2 + b^2) e^{2ic}}} + \frac{2 i a e^{2ic} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2 + b^2) e^{2ic}}}\right]}{\sqrt{(-a^2 + b^2) e^{2ic}}} - \\
& \left. \frac{2 a (-1 + e^{2ic}) f \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2 + b^2) e^{2ic}}}\right]}{d \sqrt{(-a^2 + b^2) e^{2ic}}} + \frac{2 a (-1 + e^{2ic}) f \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2 + b^2) e^{2ic}}}\right]}{d \sqrt{(-a^2 + b^2) e^{2ic}}} \right) - \\
& \frac{f^2 x \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right)}{2b(-a+b)(a+b)d^2} - \frac{(e+fx)^2}{2bd(a+b \operatorname{Sin}[c+dx])^2} + \\
& \frac{\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (-a e f \operatorname{Cos}[c] - a f^2 x \operatorname{Cos}[c] - b e f \operatorname{Sin}[dx] - b f^2 x \operatorname{Sin}[dx])}{2(a-b)b(a+b)d^2(a+b \operatorname{Sin}[c+dx])}
\end{aligned}$$

■ **Problem 324: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+fx)^3 \operatorname{Cos}[c+dx]}{(a+b \operatorname{Sin}[c+dx])^3} dx$$

Optimal (type 4, 753 leaves, 19 steps):

$$\begin{aligned}
& \frac{3 i f (e+f x)^2}{2 b (a^2-b^2) d^2} - \frac{3 f^2 (e+f x) \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^3} - \frac{3 i a f (e+f x)^2 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{3/2} d^2} - \\
& \frac{3 f^2 (e+f x) \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^3} + \frac{3 i a f (e+f x)^2 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{2 b (a^2-b^2)^{3/2} d^2} + \frac{3 i f^3 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^4} - \\
& \frac{3 a f^2 (e+f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^3} + \frac{3 i f^3 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2) d^4} + \frac{3 a f^2 (e+f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^3} - \\
& \frac{3 i a f^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^4} + \frac{3 i a f^3 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{b (a^2-b^2)^{3/2} d^4} - \frac{(e+f x)^3}{2 b d (a+b \operatorname{Sin}[c+d x])^2} + \frac{3 f (e+f x)^2 \operatorname{Cos}[c+d x]}{2 (a^2-b^2) d^2 (a+b \operatorname{Sin}[c+d x])}
\end{aligned}$$

Result (type 4, 8931 leaves):

$$\begin{aligned}
& - \frac{1}{b (-a^2+b^2) d^2 (-1+\operatorname{Cos}[2 c]+i \operatorname{Sin}[2 c])} \\
& 3 i f (\operatorname{Cos}[c]+i \operatorname{Sin}[c]) \left(2 e f x \operatorname{Cos}[c]+f^2 x^2 \operatorname{Cos}[c]+ \frac{i a e^2 \operatorname{ArcTan}\left[\frac{i a+b \operatorname{Cos}[c+d x]+i b \operatorname{Sin}[c+d x]}{\sqrt{a^2-b^2}}\right] (\operatorname{Cos}[c]-i \operatorname{Sin}[c])}{\sqrt{a^2-b^2}} \right) - \\
& \frac{2 a e f \operatorname{ArcTan}\left[\frac{i a+b \operatorname{Cos}[c+d x]+i b \operatorname{Sin}[c+d x]}{\sqrt{a^2-b^2}}\right] (\operatorname{Cos}[c]-i \operatorname{Sin}[c])}{\sqrt{a^2-b^2} d} + \frac{1}{2 \sqrt{a^2-b^2} d} \\
& e f \left(-4 \sqrt{a^2-b^2} d x+4 a \operatorname{ArcTan}\left[\frac{i a+b \operatorname{Cos}[c+d x]+i b \operatorname{Sin}[c+d x]}{\sqrt{a^2-b^2}}\right]+2 \sqrt{a^2-b^2} \operatorname{ArcTan}\left[\frac{2 a (\operatorname{Cos}[c+d x]+i \operatorname{Sin}[c+d x])}{b (-1+\operatorname{Cos}[2 c+2 d x]+i \operatorname{Sin}[2 c+2 d x])}\right] \right) - \\
& \left. i \sqrt{a^2-b^2} \operatorname{Log}\left[4 a^2 \operatorname{Cos}[2 c+2 d x]+b^2 (-1+\operatorname{Cos}[2 c+2 d x]+i \operatorname{Sin}[2 c+2 d x])^2+4 i a^2 \operatorname{Sin}[2 c+2 d x]\right] \right) (\operatorname{Cos}[c]-i \operatorname{Sin}[c]) - \\
& \frac{i a e^2 \operatorname{ArcTan}\left[\frac{i a+b \operatorname{Cos}[c+d x]+i b \operatorname{Sin}[c+d x]}{\sqrt{a^2-b^2}}\right] (\operatorname{Cos}[c]+i \operatorname{Sin}[c])}{\sqrt{a^2-b^2}} + \frac{2 a e f \operatorname{ArcTan}\left[\frac{i a+b \operatorname{Cos}[c+d x]+i b \operatorname{Sin}[c+d x]}{\sqrt{a^2-b^2}}\right] (\operatorname{Cos}[c]+i \operatorname{Sin}[c])}{\sqrt{a^2-b^2} d} -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2d} e^f \left(-4dx + \frac{4a \operatorname{ArcTan}\left[\frac{i a + b \cos[c+dx] + i b \sin[c+dx]}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} + 2 \operatorname{ArcTan}\left[\frac{2a (\cos[c+dx] + i \sin[c+dx])}{b (-1 + \cos[2c+2dx] + i \sin[2c+2dx])}\right] - \right. \\
& \left. i \operatorname{Log}\left[4a^2 \cos[2c+2dx] + b^2 (-1 + \cos[2c+2dx] + i \sin[2c+2dx])^2 + 4ia^2 \sin[2c+2dx]\right] (\cos[c] + i \sin[c]) + \right. \\
& 2iefx \sin[c] + if^2 x^2 \sin[c] - 2efx (\cos[c] - i \sin[c]) (-1 + \cos[2c] + i \sin[2c]) - \\
& f^2 x^2 (\cos[c] - i \sin[c]) (-1 + \cos[2c] + i \sin[2c]) + 2bf^2 (\cos[c] - i \sin[c]) \\
& \left. \left(- \left(\frac{x^2}{2 (i a \cos[c] - a \sin[c] - \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c]))} + \frac{i x \operatorname{Log}\left[1 + \frac{b (\cos[2c+dx] + i \sin[2c+dx])}{i a \cos[c] - a \sin[c] - \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c])}\right]}{d (i a \cos[c] - a \sin[c] - \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c]))}\right) + \right. \right. \\
& \left. \left. \frac{\operatorname{PolyLog}\left[2, -\frac{b (\cos[2c+dx] + i \sin[2c+dx])}{i a \cos[c] - a \sin[c] - \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c])}\right]}{d^2 (i a \cos[c] - a \sin[c] - \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c]))}\right) \right) / \\
& \left(- \frac{2 \cos[2c] \sqrt{-a^2 \cos[2c] + b^2 \cos[2c] - i a^2 \sin[2c] + i b^2 \sin[2c]}}{b} + \right. \\
& \left. \frac{2 i \sin[2c] \sqrt{-a^2 \cos[2c] + b^2 \cos[2c] - i a^2 \sin[2c] + i b^2 \sin[2c]}}{b} \right) + \\
& \left(\frac{x^2}{2 (i a \cos[c] - a \sin[c] + \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c]))} + \frac{i x \operatorname{Log}\left[1 + \frac{b (\cos[2c+dx] + i \sin[2c+dx])}{i a \cos[c] - a \sin[c] + \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c])}\right]}{d (i a \cos[c] - a \sin[c] + \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c]))}\right) + \\
& \left. \frac{\operatorname{PolyLog}\left[2, -\frac{b (\cos[2c+dx] + i \sin[2c+dx])}{i a \cos[c] - a \sin[c] + \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c])}\right]}{d^2 (i a \cos[c] - a \sin[c] + \sqrt{-a^2 + b^2} (\cos[2c] + i \sin[2c]))}\right) \right) / \\
& \left(- \frac{2 \cos[2c] \sqrt{-a^2 \cos[2c] + b^2 \cos[2c] - i a^2 \sin[2c] + i b^2 \sin[2c]}}{b} + \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \frac{2 i \operatorname{Sin}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} \right) \right) - 2 b f^2 (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) \\
& \left(\left(\frac{x^2}{2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right) + \right. \\
& \left. \frac{\operatorname{PolyLog} \left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d^2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right)}{\left. \right) / \\
& \left(-\frac{2 \operatorname{Cos}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} + \right. \\
& \left. \frac{2 i \operatorname{Sin}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} \right) + \\
& \left(\frac{x^2}{2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right) + \\
& \left. \frac{\operatorname{PolyLog} \left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d^2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right)}{\left. \right) / \\
& \left(-\frac{2 \operatorname{Cos}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} + \right. \\
& \left. \frac{2 i \operatorname{Sin}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} \right) \left. \right) - 2 a d e f \\
& \left(\left(\left(\frac{x^2}{2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} + \frac{i x \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right) + \right. \right. \\
& \left. \left. \frac{\operatorname{PolyLog} \left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d^2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right)}{\left. \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\text{PolyLog}\left[2, -\frac{b(\cos[2c+dx]+i\sin[2c+dx])}{ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])}\right]}{d^2\left(ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)}\right) \\
& \left. \left(-ia\cos[c]-a\sin[c]-(\cos[2c]-i\sin[2c])\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}\right)\right) / \\
& \left(b \left(-\frac{2\cos[2c]\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}}{b} + \right. \right. \\
& \left. \left. \frac{2i\sin[2c]\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}}{b} \right) \right) - \\
& \left(\left(\frac{x^2}{2\left(ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)} + \frac{ix\text{Log}\left[1+\frac{b(\cos[2c+dx]+i\sin[2c+dx])}{ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])}\right]}{d\left(ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)} \right) + \right. \\
& \left. \frac{\text{PolyLog}\left[2, -\frac{b(\cos[2c+dx]+i\sin[2c+dx])}{ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])}\right]}{d^2\left(ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)}\right) \right) \\
& \left. \left(-ia\cos[c]-a\sin[c]+(\cos[2c]-i\sin[2c])\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}\right)\right) / \\
& \left(b \left(-\frac{2\cos[2c]\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}}{b} + \right. \right. \\
& \left. \left. \frac{2i\sin[2c]\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}}{b} \right) \right) - 2iaf^2 \\
& \left(\left(\left(\frac{x^2}{2\left(ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)} + \frac{ix\text{Log}\left[1+\frac{b(\cos[2c+dx]+i\sin[2c+dx])}{ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])}\right]}{d\left(ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)} \right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\text{PolyLog}\left[2, -\frac{b(\cos[2c+dx]+i\sin[2c+dx])}{ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])}\right]}{d^2\left(ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)}\right) \\
& \left. \left(-ia\cos[c]-a\sin[c]-(\cos[2c]-i\sin[2c])\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}\right)\right) / \\
& \left(b \left(-\frac{2\cos[2c]\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}}{b} + \right. \right. \\
& \left. \left. \frac{2i\sin[2c]\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}}{b} \right) \right) - \\
& \left(\left(\frac{x^2}{2\left(ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)} + \frac{ix\text{Log}\left[1+\frac{b(\cos[2c+dx]+i\sin[2c+dx])}{ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])}\right]}{d\left(ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)} \right) + \right. \\
& \left. \frac{\text{PolyLog}\left[2, -\frac{b(\cos[2c+dx]+i\sin[2c+dx])}{ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])}\right]}{d^2\left(ia\cos[c]-a\sin[c]-\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)} \right) \\
& \left. \left(-ia\cos[c]-a\sin[c]+(\cos[2c]-i\sin[2c])\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}\right)\right) / \\
& \left(b \left(-\frac{2\cos[2c]\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}}{b} + \right. \right. \\
& \left. \left. \frac{2i\sin[2c]\sqrt{-a^2\cos[2c]+b^2\cos[2c]-ia^2\sin[2c]+ib^2\sin[2c]}}{b} \right) \right) - adf^2 \\
& \left(\left(\left(\frac{x^3}{3\left(ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)} + \frac{ix^2\text{Log}\left[1+\frac{b(\cos[2c+dx]+i\sin[2c+dx])}{ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])}\right]}{d\left(ia\cos[c]-a\sin[c]+\sqrt{-a^2+b^2}(\cos[2c]+i\sin[2c])\right)} \right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 x \operatorname{PolyLog}\left[2, -\frac{b(\cos[2 c+d x]+i \sin[2 c+d x])}{i a \cos[c]-a \sin[c]+\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])}\right]}{d^2\left(i a \cos[c]-a \sin[c]+\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])\right)} + \\
& \left. \frac{2 i \operatorname{PolyLog}\left[3, -\frac{b(\cos[2 c+d x]+i \sin[2 c+d x])}{i a \cos[c]-a \sin[c]+\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])}\right]}{d^3\left(i a \cos[c]-a \sin[c]+\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])\right)}\right) \\
& \left. \left(-i a \cos[c]-a \sin[c]-\left(\cos[2 c]-i \sin[2 c]\right) \sqrt{-a^2 \cos[2 c]+b^2 \cos[2 c]-i a^2 \sin[2 c]+i b^2 \sin[2 c]}\right)\right) / \\
& \left(b\left(-\frac{2 \cos[2 c] \sqrt{-a^2 \cos[2 c]+b^2 \cos[2 c]-i a^2 \sin[2 c]+i b^2 \sin[2 c]}}{b} + \right.\right. \\
& \left.\left.\frac{2 i \sin[2 c] \sqrt{-a^2 \cos[2 c]+b^2 \cos[2 c]-i a^2 \sin[2 c]+i b^2 \sin[2 c]}}{b}\right)\right) - \\
& \left(\left(\frac{x^3}{3\left(i a \cos[c]-a \sin[c]-\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])\right)} + \frac{i x^2 \operatorname{Log}\left[1+\frac{b(\cos[2 c+d x]+i \sin[2 c+d x])}{i a \cos[c]-a \sin[c]-\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])}\right]}{d\left(i a \cos[c]-a \sin[c]-\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])\right)}\right) + \right. \\
& \left. \frac{2 x \operatorname{PolyLog}\left[2, -\frac{b(\cos[2 c+d x]+i \sin[2 c+d x])}{i a \cos[c]-a \sin[c]-\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])}\right]}{d^2\left(i a \cos[c]-a \sin[c]-\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])\right)} + \right. \\
& \left. \frac{2 i \operatorname{PolyLog}\left[3, -\frac{b(\cos[2 c+d x]+i \sin[2 c+d x])}{i a \cos[c]-a \sin[c]-\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])}\right]}{d^3\left(i a \cos[c]-a \sin[c]-\sqrt{-a^2+b^2}(\cos[2 c]+i \sin[2 c])\right)}\right) \\
& \left. \left(-i a \cos[c]-a \sin[c]+\left(\cos[2 c]-i \sin[2 c]\right) \sqrt{-a^2 \cos[2 c]+b^2 \cos[2 c]-i a^2 \sin[2 c]+i b^2 \sin[2 c]}\right)\right) / \\
& \left(b\left(-\frac{2 \cos[2 c] \sqrt{-a^2 \cos[2 c]+b^2 \cos[2 c]-i a^2 \sin[2 c]+i b^2 \sin[2 c]}}{b} + \right.\right.
\end{aligned}$$

$$\left. \left. \left. \frac{2 i \sin[2 c] \sqrt{-a^2 \cos[2 c] + b^2 \cos[2 c] - i a^2 \sin[2 c] + i b^2 \sin[2 c]}}{b} \right) \right) \right) + 2 a d e f$$

$$\left(\left(\left(\frac{x^2}{2 \left(i a \cos[c] - a \sin[c] + \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c]) \right)} + \frac{i x \operatorname{Log}\left[1 + \frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{i a \cos[c] - a \sin[c] + \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c])}\right]}{d \left(i a \cos[c] - a \sin[c] + \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c]) \right)} + \right. \right.$$

$$\left. \left. \frac{\operatorname{PolyLog}\left[2, -\frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{i a \cos[c] - a \sin[c] + \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c])}\right]}{d^2 \left(i a \cos[c] - a \sin[c] + \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c]) \right)} \right) (\cos[2 c] + i \sin[2 c]) \right) \right)$$

$$\left. \left(-i a \cos[c] - a \sin[c] - (\cos[2 c] - i \sin[2 c]) \sqrt{-a^2 \cos[2 c] + b^2 \cos[2 c] - i a^2 \sin[2 c] + i b^2 \sin[2 c]} \right) \right) /$$

$$\left(b \left(-\frac{2 \cos[2 c] \sqrt{-a^2 \cos[2 c] + b^2 \cos[2 c] - i a^2 \sin[2 c] + i b^2 \sin[2 c]}}{b} + \right. \right.$$

$$\left. \left. \frac{2 i \sin[2 c] \sqrt{-a^2 \cos[2 c] + b^2 \cos[2 c] - i a^2 \sin[2 c] + i b^2 \sin[2 c]}}{b} \right) \right) -$$

$$\left(\left(\left(\frac{x^2}{2 \left(i a \cos[c] - a \sin[c] - \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c]) \right)} + \frac{i x \operatorname{Log}\left[1 + \frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{i a \cos[c] - a \sin[c] - \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c])}\right]}{d \left(i a \cos[c] - a \sin[c] - \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c]) \right)} + \right. \right.$$

$$\left. \left. \frac{\operatorname{PolyLog}\left[2, -\frac{b (\cos[2 c + d x] + i \sin[2 c + d x])}{i a \cos[c] - a \sin[c] - \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c])}\right]}{d^2 \left(i a \cos[c] - a \sin[c] - \sqrt{(-a^2 + b^2)} (\cos[2 c] + i \sin[2 c]) \right)} \right) (\cos[2 c] + i \sin[2 c]) \right) \right)$$

$$\left. \left(-i a \cos[c] - a \sin[c] + (\cos[2 c] - i \sin[2 c]) \sqrt{-a^2 \cos[2 c] + b^2 \cos[2 c] - i a^2 \sin[2 c] + i b^2 \sin[2 c]} \right) \right) /$$

$$\left(b \left(-\frac{2 \cos[2 c] \sqrt{-a^2 \cos[2 c] + b^2 \cos[2 c] - i a^2 \sin[2 c] + i b^2 \sin[2 c]}}{b} + \right. \right.$$

$$\begin{aligned}
& \left. \left. \left. \frac{2 i \operatorname{Sin}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} \right) \right) \right) + 2 i a f^2 \\
& \left(\left(\left(\frac{x^2}{2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} + \frac{i x \operatorname{Log}\left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])}\right]}{d \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} + \right. \right. \right. \\
& \left. \left. \left. \frac{\operatorname{PolyLog}\left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])}\right]}{d^2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right]}{(\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right) \right) \right) \\
& \left. \left. \left. \left(-i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - (\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]) \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]} \right) \right) \right) \right) / \\
& \left(b \left(-\frac{2 \operatorname{Cos}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} + \right. \right. \\
& \left. \left. \left. \frac{2 i \operatorname{Sin}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} \right) \right) \right) - \\
& \left(\left(\left(\frac{x^2}{2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} + \frac{i x \operatorname{Log}\left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])}\right]}{d \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} + \right. \right. \right. \\
& \left. \left. \left. \frac{\operatorname{PolyLog}\left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])}\right]}{d^2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right]}{(\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right) \right) \right) \\
& \left. \left. \left. \left(-i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + (\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]) \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]} \right) \right) \right) \right) / \\
& \left(b \left(-\frac{2 \operatorname{Cos}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{2 i \operatorname{Sin}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} \right) \right) \right) + a d f^2 \\
& \left(\left(\left(\frac{x^3}{3 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} + \frac{i x^2 \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right)} + \right. \\
& \left. \frac{2 x \operatorname{PolyLog} \left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d^2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right)} + \\
& \left. \frac{2 i \operatorname{PolyLog} \left[3, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d^3 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] + \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right) \right) (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \\
& \left. \left. \left. \left(-i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - (\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]) \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]} \right) \right) \right) \right) / \\
& \left(b \left(-\frac{2 \operatorname{Cos}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} + \right. \right. \\
& \left. \left. \frac{2 i \operatorname{Sin}[2 c] \sqrt{-a^2 \operatorname{Cos}[2 c] + b^2 \operatorname{Cos}[2 c] - i a^2 \operatorname{Sin}[2 c] + i b^2 \operatorname{Sin}[2 c]}}{b} \right) \right) - \\
& \left(\left(\left(\frac{x^3}{3 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} + \frac{i x^2 \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right)} + \right. \\
& \left. \frac{2 x \operatorname{PolyLog} \left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \right]}{d^2 \left(i a \operatorname{Cos}[c] - a \operatorname{Sin}[c] - \sqrt{(-a^2 + b^2)} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c]) \right)} \right)} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 i \text{PolyLog}\left[3, -\frac{b(\text{Cos}[2 c+d x]+i \text{Sin}[2 c+d x])}{i a \text{Cos}[c]-a \text{Sin}[c]-\sqrt{-a^2+b^2}(\text{Cos}[2 c]+i \text{Sin}[2 c])}\right]}{d^3\left(i a \text{Cos}[c]-a \text{Sin}[c]-\sqrt{-a^2+b^2}(\text{Cos}[2 c]+i \text{Sin}[2 c])\right)}\left(\text{Cos}[2 c]+i \text{Sin}[2 c]\right)}{\left(-i a \text{Cos}[c]-a \text{Sin}[c]+(\text{Cos}[2 c]-i \text{Sin}[2 c])\sqrt{-a^2 \text{Cos}[2 c]+b^2 \text{Cos}[2 c]-i a^2 \text{Sin}[2 c]+i b^2 \text{Sin}[2 c]}\right)}\Bigg/ \\
& \left(b\left(-\frac{2 \text{Cos}[2 c]\sqrt{-a^2 \text{Cos}[2 c]+b^2 \text{Cos}[2 c]-i a^2 \text{Sin}[2 c]+i b^2 \text{Sin}[2 c]}}{b}+\right.\right. \\
& \left.\left.\frac{2 i \text{Sin}[2 c]\sqrt{-a^2 \text{Cos}[2 c]+b^2 \text{Cos}[2 c]-i a^2 \text{Sin}[2 c]+i b^2 \text{Sin}[2 c]}}{b}\right)\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg) - \\
& \frac{(e+f x)^3}{2 b d(a+b \text{Sin}[c+d x])^2} - \left(3 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]\left(a e^2 f \text{Cos}[c]+2 a e f^2 x \text{Cos}[c]+a f^3 x^2 \text{Cos}[c]+b\right.\right. \\
& e^2 \\
& f \\
& \text{Sin}[d x]+2 \\
& b \\
& e \\
& f^2 \\
& x \\
& \text{Sin}[d x]+b \\
& f^3 \\
& x^2 \\
& \left.\left.\left.\left.\left.\left.\left.\text{Sin}[d x]\right)\right)\right)\right)\right)\right)\Bigg)\Bigg/ \left(4(a-b) b(a+b) d^2(a+b \text{Sin}[c+d x])\right)
\end{aligned}$$

■ **Problem 325: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+f x)^3 \text{Cos}[c+d x] \text{Cot}[c+d x]}{a+b \text{Sin}[c+d x]} dx$$

Optimal (type 4, 765 leaves, 33 steps):

$$\begin{aligned}
& - \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3 \operatorname{ArcTanh}\left[e^{i(c+dx)}\right]}{ad} - \frac{i\sqrt{a^2-b^2}(e+fx)^3 \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{abd} + \frac{i\sqrt{a^2-b^2}(e+fx)^3 \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{abd} + \\
& \frac{3if(e+fx)^2 \operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right]}{ad^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right]}{ad^2} - \frac{3\sqrt{a^2-b^2}f(e+fx)^2 \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{abd^2} + \\
& \frac{3\sqrt{a^2-b^2}f(e+fx)^2 \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{abd^2} - \frac{6f^2(e+fx) \operatorname{PolyLog}\left[3, -e^{i(c+dx)}\right]}{ad^3} + \frac{6f^2(e+fx) \operatorname{PolyLog}\left[3, e^{i(c+dx)}\right]}{ad^3} - \\
& \frac{6i\sqrt{a^2-b^2}f^2(e+fx) \operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{abd^3} + \frac{6i\sqrt{a^2-b^2}f^2(e+fx) \operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{abd^3} - \frac{6if^3 \operatorname{PolyLog}\left[4, -e^{i(c+dx)}\right]}{ad^4} + \\
& \frac{6if^3 \operatorname{PolyLog}\left[4, e^{i(c+dx)}\right]}{ad^4} + \frac{6\sqrt{a^2-b^2}f^3 \operatorname{PolyLog}\left[4, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{abd^4} - \frac{6\sqrt{a^2-b^2}f^3 \operatorname{PolyLog}\left[4, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{abd^4}
\end{aligned}$$

Result (type 4, 1897 leaves):

$$\begin{aligned}
& - \frac{x(4e^3 + 6e^2fx + 4ef^2x^2 + f^3x^3)}{4b} + \\
& \frac{1}{ad^4} \left(-2d^3e^3 \operatorname{ArcTanh}\left[e^{i(c+dx)}\right] + 3d^3e^2fx \operatorname{Log}\left[1 - e^{i(c+dx)}\right] + 3d^3e^2fx \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + 3d^3ef^2x^2 \operatorname{Log}\left[1 - e^{i(c+dx)}\right] + d^3f^3x^3 \operatorname{Log}\left[1 - e^{i(c+dx)}\right] - \right. \\
& \quad 3d^3e^2fx \operatorname{Log}\left[1 + e^{i(c+dx)}\right] - 3d^3ef^2x^2 \operatorname{Log}\left[1 + e^{i(c+dx)}\right] - d^3f^3x^3 \operatorname{Log}\left[1 + e^{i(c+dx)}\right] + 3id^2f(e+fx)^2 \operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right] - \\
& \quad 3id^2f(e+fx)^2 \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right] - 6def^2 \operatorname{PolyLog}\left[3, -e^{i(c+dx)}\right] - 6df^3x \operatorname{PolyLog}\left[3, -e^{i(c+dx)}\right] + 6def^2 \operatorname{PolyLog}\left[3, e^{i(c+dx)}\right] + \\
& \quad \left. 6df^3x \operatorname{PolyLog}\left[3, e^{i(c+dx)}\right] - 6if^3 \operatorname{PolyLog}\left[4, -e^{i(c+dx)}\right] + 6if^3 \operatorname{PolyLog}\left[4, e^{i(c+dx)}\right] \right) + \frac{1}{abd^4 \sqrt{(-a^2+b^2)} (\operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} \\
& i\sqrt{a^2-b^2} \left(3i\sqrt{a^2-b^2} d^3e^2fx \operatorname{Log}\left[1 + \frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& \quad 3i\sqrt{a^2-b^2} d^3ef^2x^2 \operatorname{Log}\left[1 + \frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& \quad i\sqrt{a^2-b^2} d^3f^3x^3 \operatorname{Log}\left[1 + \frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& \quad \left. 3\sqrt{a^2-b^2} d^2f(e+fx)^2 \operatorname{PolyLog}\left[2, -\frac{b(\operatorname{Cos}[2c+dx] + i \operatorname{Sin}[2c+dx])}{ia \operatorname{Cos}[c] + \sqrt{(-a^2+b^2)} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 - a \operatorname{Sin}[c]}\right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) - \right.
\end{aligned}$$

$$\begin{aligned}
& 3 \sqrt{a^2 - b^2} d^2 f (e + f x)^2 \text{PolyLog}\left[2, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 6 i \sqrt{a^2 - b^2} d e f^2 \text{PolyLog}\left[3, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 6 i \sqrt{a^2 - b^2} d f^3 x \text{PolyLog}\left[3, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) - \\
& 6 \sqrt{a^2 - b^2} f^3 \text{PolyLog}\left[4, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} f^3 \text{PolyLog}\left[4, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (\cos[c] + i \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e^2 f x \text{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e f^2 x^2 \text{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& \sqrt{a^2 - b^2} d^3 f^3 x^3 \text{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d e f^2 \text{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) + \\
& 6 \sqrt{a^2 - b^2} d f^3 x \text{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (-i \cos[c] + \sin[c]) - \\
& 2 i d^3 e^3 \text{ArcTan}\left[\frac{b \cos[c + dx] + i (a + b \sin[c + dx])}{\sqrt{a^2 - b^2}}\right] \sqrt{(-a^2 + b^2)} (\cos[2c] + i \sin[2c])
\end{aligned}$$

■ **Problem 329: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \cos[c + dx]^2 \cot[c + dx]}{a + b \sin[c + dx]} dx$$

Optimal (type 4, 763 leaves, 34 steps):

$$\begin{aligned}
& -\frac{i(e+fx)^4}{4af} - \frac{i(a^2-b^2)(e+fx)^4}{4ab^2f} + \frac{6f^3\cos[c+dx]}{bd^4} - \frac{3f(e+fx)^2\cos[c+dx]}{bd^2} + \frac{(a^2-b^2)(e+fx)^3\operatorname{Log}\left[1-\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{ab^2d} + \\
& \frac{(a^2-b^2)(e+fx)^3\operatorname{Log}\left[1-\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{ab^2d} + \frac{(e+fx)^3\operatorname{Log}\left[1-e^{2i(c+dx)}\right]}{ad} - \frac{3i(a^2-b^2)f(e+fx)^2\operatorname{PolyLog}\left[2,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{ab^2d^2} - \\
& \frac{3i(a^2-b^2)f(e+fx)^2\operatorname{PolyLog}\left[2,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{ab^2d^2} - \frac{3if(e+fx)^2\operatorname{PolyLog}\left[2,e^{2i(c+dx)}\right]}{2ad^2} + \frac{6(a^2-b^2)f^2(e+fx)\operatorname{PolyLog}\left[3,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{ab^2d^3} + \\
& \frac{6(a^2-b^2)f^2(e+fx)\operatorname{PolyLog}\left[3,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{ab^2d^3} + \frac{3f^2(e+fx)\operatorname{PolyLog}\left[3,e^{2i(c+dx)}\right]}{2ad^3} + \frac{6i(a^2-b^2)f^3\operatorname{PolyLog}\left[4,\frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{ab^2d^4} + \\
& \frac{6i(a^2-b^2)f^3\operatorname{PolyLog}\left[4,\frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{ab^2d^4} + \frac{3if^3\operatorname{PolyLog}\left[4,e^{2i(c+dx)}\right]}{4ad^4} + \frac{6f^2(e+fx)\sin[c+dx]}{bd^3} - \frac{(e+fx)^3\sin[c+dx]}{bd}
\end{aligned}$$

Result (type 4, 3808 leaves):

$$\begin{aligned}
& -\frac{1}{4ad^3}e^{-ic}f^2\operatorname{Csc}[c] \\
& \quad (2d^2x^2(2de^{2ic}x+3i(-1+e^{2ic})\operatorname{Log}[1-e^{2i(c+dx)}]) + 6d(-1+e^{2ic})x\operatorname{PolyLog}[2,e^{2i(c+dx)}] + 3i(-1+e^{2ic})\operatorname{PolyLog}[3,e^{2i(c+dx)}]) - \\
& \frac{1}{4a}e^{ic}f^3\operatorname{Csc}[c](x^4+(-1+e^{-2ic})x^4+1/(2d^4)e^{-2ic}(-1+e^{2ic})) \\
& \quad (2d^4x^4+4id^3x^3\operatorname{Log}[1-e^{2i(c+dx)}] + 6d^2x^2\operatorname{PolyLog}[2,e^{2i(c+dx)}] + 6idx\operatorname{PolyLog}[3,e^{2i(c+dx)}] - 3\operatorname{PolyLog}[4,e^{2i(c+dx)}]) + \\
& \frac{1}{2ab^2d^4(-1+e^{2ic})}(a^2-b^2)\left(-4id^4e^3e^{2ic}x-6id^4e^2e^{2ic}fx^2-4id^4e^{2ic}f^2x^3-id^4e^{2ic}f^3x^4- \right. \\
& \quad \left. 2id^3e^3\operatorname{ArcTan}\left[\frac{2ae^{i(c+dx)}}{b(-1+e^{2i(c+dx)})}\right] + 2id^3e^3e^{2ic}\operatorname{ArcTan}\left[\frac{2ae^{i(c+dx)}}{b(-1+e^{2i(c+dx)})}\right] - d^3e^3\operatorname{Log}[4a^2e^{2i(c+dx)}+b^2(-1+e^{2i(c+dx)})^2] + \right. \\
& \quad \left. d^3e^3e^{2ic}\operatorname{Log}[4a^2e^{2i(c+dx)}+b^2(-1+e^{2i(c+dx)})^2] - 6d^3e^2fx\operatorname{Log}\left[1+\frac{be^{i(2c+dx)}}{iae^{ic}-\sqrt{(-a^2+b^2)}e^{2ic}}\right] + \right. \\
& \quad \left. 6d^3e^2e^{2ic}fx\operatorname{Log}\left[1+\frac{be^{i(2c+dx)}}{iae^{ic}-\sqrt{(-a^2+b^2)}e^{2ic}}\right] - 6d^3e^2fx^2\operatorname{Log}\left[1+\frac{be^{i(2c+dx)}}{iae^{ic}-\sqrt{(-a^2+b^2)}e^{2ic}}\right] + \right. \\
& \quad \left. 6d^3e^{2ic}f^2x^2\operatorname{Log}\left[1+\frac{be^{i(2c+dx)}}{iae^{ic}-\sqrt{(-a^2+b^2)}e^{2ic}}\right] - 2d^3f^3x^3\operatorname{Log}\left[1+\frac{be^{i(2c+dx)}}{iae^{ic}-\sqrt{(-a^2+b^2)}e^{2ic}}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 d^3 e^{2 i c} f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 6 d^3 e^2 f x \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 6 d^3 e^2 e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 6 d^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 6 d^3 e e^{2 i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 2 d^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 2 d^3 e^{2 i c} f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 6 i d^2 (-1 + e^{2 i c}) f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \\
& 6 i d^2 (-1 + e^{2 i c}) f (e + f x)^2 \operatorname{PolyLog}\left[2, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \\
& 12 d e f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + 12 d e e^{2 i c} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \\
& 12 d f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + 12 d e^{2 i c} f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \\
& 12 d e f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + 12 d e e^{2 i c} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \\
& 12 d f^3 x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + 12 d e^{2 i c} f^3 x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \\
& 12 i f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + 12 i e^{2 i c} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \\
& 12 i f^3 \operatorname{PolyLog}\left[4, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + 12 i e^{2 i c} f^3 \operatorname{PolyLog}\left[4, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] \Bigg) + \\
& \frac{e^3 \operatorname{Csc}[c] (-d x \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x]] \operatorname{Sin}[c])}{a d (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} + \\
& \operatorname{Csc}[c] \\
& \left(\frac{\operatorname{Cos}[c + d x]}{8 b^2 d^4} - \frac{i \operatorname{Sin}[c + d x]}{8 b^2 d^4} \right) \\
& (4 a d^4 e^3 x \operatorname{Cos}[d x] + 6 a d^4 e^2 f x^2 \operatorname{Cos}[d x] + 4 a d^4 e f^2 x^3 \operatorname{Cos}[d x] + a d^4 f^3 x^4 \operatorname{Cos}[d x] + 4 a d^4 e^3 x \operatorname{Cos}[2 c + d x] + \\
& 6 a d^4 e^2 f x^2 \operatorname{Cos}[2 c + d x] + 4 a d^4 e f^2 x^3 \operatorname{Cos}[2 c + d x] + a d^4 f^3 x^4 \operatorname{Cos}[2 c + d x] - 2 b d^3 e^3 \operatorname{Cos}[c + 2 d x] - 6 i b d^2 e^2 f \operatorname{Cos}[c + 2 d x] +
\end{aligned}$$

$$\begin{aligned}
& 12 b d e f^2 \cos [c+2 d x]+12 i b f^3 \cos [c+2 d x]-6 b d^3 e^2 f x \cos [c+2 d x]-12 i b d^2 e f^2 x \cos [c+2 d x]+12 b d f^3 x \cos [c+2 d x]- \\
& 6 b d^3 e f^2 x^2 \cos [c+2 d x]-6 i b d^2 f^3 x^2 \cos [c+2 d x]-2 b d^3 f^3 x^3 \cos [c+2 d x]+2 b d^3 e^3 \cos [3 c+2 d x]+6 i b d^2 e^2 f \cos [3 c+2 d x]- \\
& 12 b d e f^2 \cos [3 c+2 d x]-12 i b f^3 \cos [3 c+2 d x]+6 b d^3 e^2 f x \cos [3 c+2 d x]+12 i b d^2 e f^2 x \cos [3 c+2 d x]- \\
& 12 b d f^3 x \cos [3 c+2 d x]+6 b d^3 e f^2 x^2 \cos [3 c+2 d x]+6 i b d^2 f^3 x^2 \cos [3 c+2 d x]+2 b d^3 f^3 x^3 \cos [3 c+2 d x]-4 i b d^3 e^3 \sin [c]- \\
& 12 b d^2 e^2 f \sin [c]+24 i b d e f^2 \sin [c]+24 b f^3 \sin [c]-12 i b d^3 e^2 f x \sin [c]-24 b d^2 e f^2 x \sin [c]+24 i b d f^3 x \sin [c]- \\
& 12 i b d^3 e f^2 x^2 \sin [c]-12 b d^2 f^3 x^2 \sin [c]-4 i b d^3 f^3 x^3 \sin [c]+4 i a d^4 e^3 x \sin [d x]+6 i a d^4 e^2 f x^2 \sin [d x]+ \\
& 4 i a d^4 e f^2 x^3 \sin [d x]+i a d^4 f^3 x^4 \sin [d x]+4 i a d^4 e^3 x \sin [2 c+d x]+6 i a d^4 e^2 f x^2 \sin [2 c+d x]+4 i a d^4 e f^2 x^3 \sin [2 c+d x]+ \\
& i a d^4 f^3 x^4 \sin [2 c+d x]-2 i b d^3 e^3 \sin [c+2 d x]+6 b d^2 e^2 f \sin [c+2 d x]+12 i b d e f^2 \sin [c+2 d x]-12 b f^3 \sin [c+2 d x]- \\
& 6 i b d^3 e^2 f x \sin [c+2 d x]+12 b d^2 e f^2 x \sin [c+2 d x]+12 i b d f^3 x \sin [c+2 d x]-6 i b d^3 e f^2 x^2 \sin [c+2 d x]+ \\
& 6 b d^2 f^3 x^2 \sin [c+2 d x]-2 i b d^3 f^3 x^3 \sin [c+2 d x]+2 i b d^3 e^3 \sin [3 c+2 d x]-6 b d^2 e^2 f \sin [3 c+2 d x]- \\
& 12 i b d e f^2 \sin [3 c+2 d x]+12 b f^3 \sin [3 c+2 d x]+6 i b d^3 e^2 f x \sin [3 c+2 d x]-12 b d^2 e f^2 x \sin [3 c+2 d x]- \\
& 12 i b d f^3 x \sin [3 c+2 d x]+6 i b d^3 e f^2 x^2 \sin [3 c+2 d x]-6 b d^2 f^3 x^2 \sin [3 c+2 d x]+2 i b d^3 f^3 x^3 \sin [3 c+2 d x]) - \\
& \left(3 e^2 f \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2+1\right) / \left(\sqrt{1+\operatorname{Tan}[c]^2}\right)\left(i d x(-\pi+2 \operatorname{ArcTan}[\operatorname{Tan}[c]])-\pi \operatorname{Log}\left[1+e^{-2 i d x}\right]-\right. \right. \\
& \quad 2(d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}\left[1-e^{2 i(d x+\operatorname{ArcTan}[\operatorname{Tan}[c]])}\right]+\pi \operatorname{Log}[\operatorname{Cos}[d x]]+2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\operatorname{Sin}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]]+ \\
& \quad \left. i \operatorname{PolyLog}\left[2, e^{2 i(d x+\operatorname{ArcTan}[\operatorname{Tan}[c]])}\right]\right) \operatorname{Tan}[c] \Big) / \left(2 a d^2 \sqrt{\operatorname{Sec}[c]^2(\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2)}\right)
\end{aligned}$$

■ **Problem 330: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+f x)^2 \cos [c+d x]^2 \cot [c+d x]}{a+b \sin [c+d x]} d x$$

Optimal (type 4, 566 leaves, 26 steps):

$$\begin{aligned}
& -\frac{i(e+f x)^3}{3 a f}-\frac{i\left(a^2-b^2\right)(e+f x)^3}{3 a b^2 f}-\frac{2 f(e+f x) \cos [c+d x]}{b d^2}+\frac{\left(a^2-b^2\right)(e+f x)^2 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{a b^2 d}+ \\
& \frac{\left(a^2-b^2\right)(e+f x)^2 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{a b^2 d}+\frac{(e+f x)^2 \operatorname{Log}\left[1-e^{2 i(c+d x)}\right]}{a d}-\frac{2 i\left(a^2-b^2\right) f(e+f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{a b^2 d^2}- \\
& \frac{2 i\left(a^2-b^2\right) f(e+f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{a b^2 d^2}-\frac{i f(e+f x) \operatorname{PolyLog}\left[2, e^{2 i(c+d x)}\right]}{a d^2}+\frac{2\left(a^2-b^2\right) f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{a b^2 d^3}+ \\
& \frac{2\left(a^2-b^2\right) f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{a b^2 d^3}+\frac{f^2 \operatorname{PolyLog}\left[3, e^{2 i(c+d x)}\right]}{2 a d^3}+\frac{2 f^2 \sin [c+d x]}{b d^3}-\frac{(e+f x)^2 \sin [c+d x]}{b d}
\end{aligned}$$

Result (type 4, 1740 leaves):

$$\begin{aligned}
& -\frac{1}{12 a d^3} e^{-i c} f^2 \operatorname{Csc}[c] \\
& \quad \left(2 d^2 x^2\left(2 d e^{2 i c} x+3 i\left(-1+e^{2 i c}\right) \operatorname{Log}\left[1-e^{2 i(c+d x)}\right]\right)+6 d\left(-1+e^{2 i c}\right) x \operatorname{PolyLog}\left[2, e^{2 i(c+d x)}\right]+3 i\left(-1+e^{2 i c}\right) \operatorname{PolyLog}\left[3, e^{2 i(c+d x)}\right]\right)+
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{6 a b^2 d^3 (-1 + e^{2 i c})} (a^2 - b^2) \left(-12 i d^3 e^2 e^{2 i c} x - 12 i d^3 e e^{2 i c} f x^2 - 4 i d^3 e^{2 i c} f^2 x^3 - 6 i d^2 e^2 \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b(-1 + e^{2 i(c+dx)})}\right] + \right. \\
& 6 i d^2 e^2 e^{2 i c} \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b(-1 + e^{2 i(c+dx)})}\right] - 3 d^2 e^2 \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 (-1 + e^{2 i(c+dx)})^2\right] + \\
& 3 d^2 e^2 e^{2 i c} \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 (-1 + e^{2 i(c+dx)})^2\right] - 12 d^2 e f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 12 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 6 d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 6 d^2 e^{2 i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 12 d^2 e f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 12 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 6 d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 6 d^2 e^{2 i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 12 i d (-1 + e^{2 i c}) f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \\
& 12 i d (-1 + e^{2 i c}) f (e + f x) \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 12 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 12 e^{2 i c} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 12 f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& \left. 12 e^{2 i c} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right]\right) + \frac{a x (3 e^2 + 3 e f x + f^2 x^2) \operatorname{Cos}[c] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{6 b^2} - \\
& \frac{\operatorname{Cos}[d x] (2 d e f \operatorname{Cos}[c] + 2 d f^2 x \operatorname{Cos}[c] + d^2 e^2 \operatorname{Sin}[c] - 2 f^2 \operatorname{Sin}[c] + 2 d^2 e f x \operatorname{Sin}[c] + d^2 f^2 x^2 \operatorname{Sin}[c])}{b d^3} + \\
& \frac{e^2 \operatorname{Csc}[c] (-d x \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x]]) \operatorname{Sin}[c]}{a d (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} - \\
& \frac{(d^2 e^2 \operatorname{Cos}[c] - 2 f^2 \operatorname{Cos}[c] + 2 d^2 e f x \operatorname{Cos}[c] + d^2 f^2 x^2 \operatorname{Cos}[c] - 2 d e f \operatorname{Sin}[c] - 2 d f^2 x \operatorname{Sin}[c]) \operatorname{Sin}[d x]}{b d^3} - \\
& \left(e f \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \right) / \left(\sqrt{1 + \operatorname{Tan}[c]^2} \right) \left(i d x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]]) - \pi \operatorname{Log}[1 + e^{-2 i d x}] - \right. \right. \\
& \left. \left. 2 (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}[1 - e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]])}] + \pi \operatorname{Log}[\operatorname{Cos}[d x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \right] \right) +
\end{aligned}$$

$$i \operatorname{PolyLog}\left[2, e^{2i(dx + \operatorname{ArcTan}[\operatorname{Tan}[c]])} \operatorname{Tan}[c]\right) \Big/ \left(a d^2 \sqrt{\operatorname{Sec}[c]^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)}\right)$$

■ **Problem 331: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x) \operatorname{Cos}[c + d x]^2 \operatorname{Cot}[c + d x]}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 379 leaves, 22 steps):

$$\begin{aligned} & -\frac{i(e+fx)^2}{2af} - \frac{i(a^2-b^2)(e+fx)^2}{2ab^2f} - \frac{f \operatorname{Cos}[c+dx]}{bd^2} + \frac{(a^2-b^2)(e+fx) \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{ab^2d} + \\ & \frac{(a^2-b^2)(e+fx) \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{ab^2d} + \frac{(e+fx) \operatorname{Log}[1 - e^{2i(c+dx)}]}{ad} - \frac{i(a^2-b^2)f \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{ab^2d^2} - \\ & \frac{i(a^2-b^2)f \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{ab^2d^2} - \frac{if \operatorname{PolyLog}\left[2, e^{2i(c+dx)}\right]}{2ad^2} - \frac{(e+fx) \operatorname{Sin}[c+dx]}{bd} \end{aligned}$$

Result (type 4, 868 leaves):

$$\begin{aligned} & \frac{1}{ab^2d^2} \left(-abf \operatorname{Cos}[c+dx] + b^2de \operatorname{Log}[\operatorname{Sin}[c+dx]] - b^2cf \operatorname{Log}[\operatorname{Sin}[c+dx]] + a^2de \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right] - \right. \\ & \left. b^2de \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right] - a^2cf \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right] + b^2cf \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right] + \frac{1}{8}a^2f \left(i(-2c+\pi-2dx)^2 - \right. \right. \\ & \left. \left. 32i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2dx)\right]}{\sqrt{a^2-b^2}}\right] - 4 \left(-2c+\pi-2dx + 4 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{i(-a+\sqrt{a^2-b^2})e^{-i(c+dx)}}{b}\right] - \right. \right. \\ & \left. \left. 4 \left(-2c+\pi-2dx - 4 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{i(a+\sqrt{a^2-b^2})e^{-i(c+dx)}}{b}\right] + 4(-2c+\pi-2dx) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] + \right. \right. \end{aligned}$$

$$\begin{aligned}
& 8 (c + dx) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] + 8 i \left(\operatorname{PolyLog}\left[2, \frac{i \left(-a + \sqrt{a^2 - b^2}\right) e^{-i(c+dx)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{i \left(a + \sqrt{a^2 - b^2}\right) e^{-i(c+dx)}}{b}\right] \right) \\
& \frac{1}{8} b^2 f \left(i (-2c + \pi - 2dx)^2 - 32 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Cot}\left[\frac{1}{4}(2c + \pi + 2dx)\right]}{\sqrt{a^2 - b^2}}\right] - \right. \\
& 4 \left(-2c + \pi - 2dx + 4 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{i \left(-a + \sqrt{a^2 - b^2}\right) e^{-i(c+dx)}}{b}\right] - \\
& 4 \left(-2c + \pi - 2dx - 4 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{i \left(a + \sqrt{a^2 - b^2}\right) e^{-i(c+dx)}}{b}\right] + 4 (-2c + \pi - 2dx) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] + \\
& 8 (c + dx) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] + 8 i \left(\operatorname{PolyLog}\left[2, \frac{i \left(-a + \sqrt{a^2 - b^2}\right) e^{-i(c+dx)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{i \left(a + \sqrt{a^2 - b^2}\right) e^{-i(c+dx)}}{b}\right] \right) + \\
& \left. b^2 f \left((c + dx) \operatorname{Log}\left[1 - e^{2i(c+dx)}\right] - \frac{1}{2} i \left((c + dx)^2 + \operatorname{PolyLog}\left[2, e^{2i(c+dx)}\right] \right) \right) - a b d (e + f x) \operatorname{Sin}[c + dx] \right)
\end{aligned}$$

■ **Problem 333: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \operatorname{Cos}[c + dx]^3 \operatorname{Cot}[c + dx]}{a + b \operatorname{Sin}[c + dx]} dx$$

Optimal (type 4, 1138 leaves, 53 steps):

$$\begin{aligned}
& \frac{3 e f^2 x}{4 b d^2} + \frac{3 f^3 x^2}{8 b d^2} - \frac{(e+f x)^4}{8 b f} + \frac{(a^2-b^2)(e+f x)^4}{4 b^3 f} - \frac{2(e+f x)^3 \operatorname{ArcTanh}\left[e^{i(c+d x)}\right]}{a d} - \frac{6 f^2(e+f x) \operatorname{Cos}[c+d x]}{a d^3} - \\
& \frac{6(a^2-b^2) f^2(e+f x) \operatorname{Cos}[c+d x]}{a b^2 d^3} + \frac{(e+f x)^3 \operatorname{Cos}[c+d x]}{a d} + \frac{(a^2-b^2)(e+f x)^3 \operatorname{Cos}[c+d x]}{a b^2 d} + \frac{3 f^3 \operatorname{Cos}[c+d x]^2}{8 b d^4} - \frac{3 f(e+f x)^2 \operatorname{Cos}[c+d x]^2}{4 b d^2} + \\
& \frac{i(a^2-b^2)^{3/2}(e+f x)^3 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{a b^3 d} - \frac{i(a^2-b^2)^{3/2}(e+f x)^3 \operatorname{Log}\left[1-\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{a b^3 d} + \frac{3 i f(e+f x)^2 \operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right]}{a d^2} - \\
& \frac{3 i f(e+f x)^2 \operatorname{PolyLog}\left[2,e^{i(c+d x)}\right]}{a d^2} + \frac{3(a^2-b^2)^{3/2} f(e+f x)^2 \operatorname{PolyLog}\left[2,\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{a b^3 d^2} - \frac{3(a^2-b^2)^{3/2} f(e+f x)^2 \operatorname{PolyLog}\left[2,\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{a b^3 d^2} - \\
& \frac{6 f^2(e+f x) \operatorname{PolyLog}\left[3,-e^{i(c+d x)}\right]}{a d^3} + \frac{6 f^2(e+f x) \operatorname{PolyLog}\left[3,e^{i(c+d x)}\right]}{a d^3} + \frac{6 i(a^2-b^2)^{3/2} f^2(e+f x) \operatorname{PolyLog}\left[3,\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{a b^3 d^3} - \\
& \frac{6 i(a^2-b^2)^{3/2} f^2(e+f x) \operatorname{PolyLog}\left[3,\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{a b^3 d^3} - \frac{6 i f^3 \operatorname{PolyLog}\left[4,-e^{i(c+d x)}\right]}{a d^4} + \frac{6 i f^3 \operatorname{PolyLog}\left[4,e^{i(c+d x)}\right]}{a d^4} - \\
& \frac{6(a^2-b^2)^{3/2} f^3 \operatorname{PolyLog}\left[4,\frac{i b e^{i(c+d x)}}{a-\sqrt{a^2-b^2}}\right]}{a b^3 d^4} + \frac{6(a^2-b^2)^{3/2} f^3 \operatorname{PolyLog}\left[4,\frac{i b e^{i(c+d x)}}{a+\sqrt{a^2-b^2}}\right]}{a b^3 d^4} + \frac{6 f^3 \operatorname{Sin}[c+d x]}{a d^4} + \frac{6(a^2-b^2) f^3 \operatorname{Sin}[c+d x]}{a b^2 d^4} - \\
& \frac{3 f(e+f x)^2 \operatorname{Sin}[c+d x]}{a d^2} - \frac{3(a^2-b^2) f(e+f x)^2 \operatorname{Sin}[c+d x]}{a b^2 d^2} + \frac{3 f^2(e+f x) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{4 b d^3} - \frac{(e+f x)^3 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 b d}
\end{aligned}$$

Result (type 4, 3263 leaves):

$$\begin{aligned}
& -\frac{(-2 a^2+3 b^2) e^3 x}{2 b^3} - \frac{3(-2 a^2+3 b^2) e^2 f x^2}{4 b^3} - \frac{(-2 a^2+3 b^2) e f^2 x^3}{2 b^3} - \frac{(-2 a^2+3 b^2) f^3 x^4}{8 b^3} + \\
& \frac{1}{a d^4} \left(-2 d^3 e^3 \operatorname{ArcTanh}\left[e^{i(c+d x)}\right] + 3 d^3 e^2 f x \operatorname{Log}\left[1-e^{i(c+d x)}\right] + 3 d^3 e f^2 x^2 \operatorname{Log}\left[1-e^{i(c+d x)}\right] + d^3 f^3 x^3 \operatorname{Log}\left[1-e^{i(c+d x)}\right] - \right. \\
& \left. 3 d^3 e^2 f x \operatorname{Log}\left[1+e^{i(c+d x)}\right] - 3 d^3 e f^2 x^2 \operatorname{Log}\left[1+e^{i(c+d x)}\right] - d^3 f^3 x^3 \operatorname{Log}\left[1+e^{i(c+d x)}\right] + 3 i d^2 f(e+f x)^2 \operatorname{PolyLog}\left[2,-e^{i(c+d x)}\right] - \right. \\
& \left. 3 i d^2 f(e+f x)^2 \operatorname{PolyLog}\left[2,e^{i(c+d x)}\right] - 6 d e f^2 \operatorname{PolyLog}\left[3,-e^{i(c+d x)}\right] - 6 d f^3 x \operatorname{PolyLog}\left[3,-e^{i(c+d x)}\right] + 6 d e f^2 \operatorname{PolyLog}\left[3,e^{i(c+d x)}\right] + \right. \\
& \left. 6 d f^3 x \operatorname{PolyLog}\left[3,e^{i(c+d x)}\right] - 6 i f^3 \operatorname{PolyLog}\left[4,-e^{i(c+d x)}\right] + 6 i f^3 \operatorname{PolyLog}\left[4,e^{i(c+d x)}\right] \right) + \frac{1}{a b^3 d^4 \sqrt{-(a^2-b^2)^2 e^{4 i c}}} \\
& (a^2-b^2)^{3/2} \left(-2 d^3 e^3 \sqrt{-(a^2-b^2)^2 e^{4 i c}} \operatorname{ArcTan}\left[\frac{i a+b e^{i(c+d x)}}{\sqrt{a^2-b^2}}\right] + 3 i \sqrt{a^2-b^2} d^3 e^2 e^{i c} \sqrt{(-a^2+b^2) e^{2 i c}} f x \operatorname{Log}\left[1-\frac{i b e^{i(2 c+d x)}}{a e^{i c}-\sqrt{(a^2-b^2) e^{2 i c}}}\right] \right) +
\end{aligned}$$

$$\begin{aligned}
& i \sqrt{a^2 - b^2} d^3 e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f^3 x^3 \operatorname{Log}\left[1 - \frac{i b e^{i(2c+dx)}}{a e^{i c} - \sqrt{(a^2 - b^2) e^{2 i c}}}\right] - 3 i \sqrt{a^2 - b^2} d^3 e^2 e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} \\
& f x \operatorname{Log}\left[1 - \frac{i b e^{i(2c+dx)}}{a e^{i c} + \sqrt{(a^2 - b^2) e^{2 i c}}}\right] - i \sqrt{a^2 - b^2} d^3 e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f^3 x^3 \operatorname{Log}\left[1 - \frac{i b e^{i(2c+dx)}}{a e^{i c} + \sqrt{(a^2 - b^2) e^{2 i c}}}\right] - \\
& 3 \sqrt{a^2 - b^2} d^3 e e^{i c} \sqrt{(a^2 - b^2) e^{2 i c}} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + 3 \sqrt{a^2 - b^2} d^3 e e^{i c} \sqrt{(a^2 - b^2) e^{2 i c}} f^2 x^2 \\
& \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + 3 \sqrt{a^2 - b^2} d^2 e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f (e^2 + f^2 x^2) \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{i c} - \sqrt{(a^2 - b^2) e^{2 i c}}}\right] - \\
& 3 \sqrt{a^2 - b^2} d^2 e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f (e^2 + f^2 x^2) \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{i c} + \sqrt{(a^2 - b^2) e^{2 i c}}}\right] + 6 i \sqrt{a^2 - b^2} d^2 e e^{i c} \sqrt{(a^2 - b^2) e^{2 i c}} \\
& f^2 x \operatorname{PolyLog}\left[2, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 6 i \sqrt{a^2 - b^2} d^2 e e^{i c} \sqrt{(a^2 - b^2) e^{2 i c}} f^2 x \operatorname{PolyLog}\left[2, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 6 i \sqrt{a^2 - b^2} d e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} - \sqrt{(a^2 - b^2) e^{2 i c}}}\right] - 6 i \sqrt{a^2 - b^2} d e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f^3 x \\
& \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} + \sqrt{(a^2 - b^2) e^{2 i c}}}\right] - 6 \sqrt{a^2 - b^2} d e e^{i c} \sqrt{(a^2 - b^2) e^{2 i c}} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{i c} + i \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + \\
& 6 \sqrt{a^2 - b^2} d e e^{i c} \sqrt{(a^2 - b^2) e^{2 i c}} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - 6 \sqrt{a^2 - b^2} e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f^3 \\
& \operatorname{PolyLog}\left[4, \frac{i b e^{i(2c+dx)}}{a e^{i c} - \sqrt{(a^2 - b^2) e^{2 i c}}}\right] + 6 \sqrt{a^2 - b^2} e^{i c} \sqrt{(-a^2 + b^2) e^{2 i c}} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2c+dx)}}{a e^{i c} + \sqrt{(a^2 - b^2) e^{2 i c}}}\right] \Bigg) + \\
& \left(\frac{a f^3 x^3 \operatorname{Cos}[c]}{2 b^2 d} - \frac{i a f^3 x^3 \operatorname{Sin}[c]}{2 b^2 d} + (d^3 e^3 - 3 i d^2 e^2 f - 6 d e f^2 + 6 i f^3) \left(\frac{a \operatorname{Cos}[c]}{2 b^2 d^4} - \frac{i a \operatorname{Sin}[c]}{2 b^2 d^4} \right) + \right. \\
& \left. (a d^2 e^2 f - 2 i a d e f^2 - 2 a f^3) \left(\frac{3 x \operatorname{Cos}[c]}{2 b^2 d^3} - \frac{3 i x \operatorname{Sin}[c]}{2 b^2 d^3} \right) + (a d e f^2 - i a f^3) \left(\frac{3 x^2 \operatorname{Cos}[c]}{2 b^2 d^2} - \frac{3 i x^2 \operatorname{Sin}[c]}{2 b^2 d^2} \right) \right) (\operatorname{Cos}[dx] - i \operatorname{Sin}[dx]) + \\
& \left(\frac{a f^3 x^3 \operatorname{Cos}[c]}{2 b^2 d} + \frac{i a f^3 x^3 \operatorname{Sin}[c]}{2 b^2 d} + (d^3 e^3 + 3 i d^2 e^2 f - 6 d e f^2 - 6 i f^3) \left(\frac{a \operatorname{Cos}[c]}{2 b^2 d^4} + \frac{i a \operatorname{Sin}[c]}{2 b^2 d^4} \right) + \right. \\
& \left. \frac{3 x^2 (a d e f^2 \operatorname{Cos}[c] + i a f^3 \operatorname{Cos}[c] + i a d e f^2 \operatorname{Sin}[c] - a f^3 \operatorname{Sin}[c])}{2 b^2 d^2} + \frac{1}{2 b^2 d^3} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. 3 x \left(a d^2 e^2 f \cos[c] + 2 i a d e f^2 \cos[c] - 2 a f^3 \cos[c] + i a d^2 e^2 f \sin[c] - 2 a d e f^2 \sin[c] - 2 i a f^3 \sin[c] \right) \right) (\cos[dx] + i \sin[dx]) + \\
& \left(-\frac{i f^3 x^3 \cos[2c]}{8bd} - \frac{f^3 x^3 \sin[2c]}{8bd} + (-4 i d^3 e^3 - 6 d^2 e^2 f + 6 i d e f^2 + 3 f^3) \left(\frac{\cos[2c]}{32bd^4} - \frac{i \sin[2c]}{32bd^4} \right) + \right. \\
& \left. (2 d^2 e^2 f - 2 i d e f^2 - f^3) \left(-\frac{3 i x \cos[2c]}{16bd^3} - \frac{3 x \sin[2c]}{16bd^3} \right) + (2 d e f^2 - i f^3) \left(-\frac{3 i x^2 \cos[2c]}{16bd^2} - \frac{3 x^2 \sin[2c]}{16bd^2} \right) \right) (\cos[2dx] - i \sin[2dx]) + \\
& \left(\frac{i f^3 x^3 \cos[2c]}{8bd} - \frac{f^3 x^3 \sin[2c]}{8bd} + (4 i d^3 e^3 - 6 d^2 e^2 f - 6 i d e f^2 + 3 f^3) \left(\frac{\cos[2c]}{32bd^4} + \frac{i \sin[2c]}{32bd^4} \right) + \right. \\
& \left. \frac{3 i x^2 (2 d e f^2 \cos[2c] + i f^3 \cos[2c] + 2 i d e f^2 \sin[2c] - f^3 \sin[2c])}{16bd^2} + \frac{1}{16bd^3} \right) \\
& \left. 3 i x (2 d^2 e^2 f \cos[2c] + 2 i d e f^2 \cos[2c] - f^3 \cos[2c] + 2 i d^2 e^2 f \sin[2c] - 2 d e f^2 \sin[2c] - i f^3 \sin[2c]) \right) (\cos[2dx] + i \sin[2dx])
\end{aligned}$$

■ **Problem 337: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \cos[c + dx] \cot[c + dx]^2}{a + b \sin[c + dx]} dx$$

Optimal (type 4, 852 leaves, 48 steps):

$$\begin{aligned}
& \frac{i b (e + f x)^4}{4 a^2 f} + \frac{i (a^2 - b^2) (e + f x)^4}{4 a^2 b f} - \frac{6 f (e + f x)^2 \operatorname{ArcTanh}\left[e^{i(c+dx)}\right]}{a d^2} - \frac{(e + f x)^3 \operatorname{Csc}[c + dx]}{a d} - \frac{(a^2 - b^2) (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b d} - \\
& \frac{(a^2 - b^2) (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b d} - \frac{b (e + f x)^3 \operatorname{Log}\left[1 - e^{i(c+dx)}\right]}{a^2 d} + \frac{6 i f^2 (e + f x) \operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right]}{a d^3} - \\
& \frac{6 i f^2 (e + f x) \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right]}{a d^3} + \frac{3 i (a^2 - b^2) f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b d^2} + \frac{3 i (a^2 - b^2) f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b d^2} + \\
& \frac{3 i b f (e + f x)^2 \operatorname{PolyLog}\left[2, e^{2i(c+dx)}\right]}{2 a^2 d^2} - \frac{6 f^3 \operatorname{PolyLog}\left[3, -e^{i(c+dx)}\right]}{a d^4} + \frac{6 f^3 \operatorname{PolyLog}\left[3, e^{i(c+dx)}\right]}{a d^4} - \\
& \frac{6 (a^2 - b^2) f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b d^3} - \frac{6 (a^2 - b^2) f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b d^3} - \frac{3 b f^2 (e + f x) \operatorname{PolyLog}\left[3, e^{2i(c+dx)}\right]}{2 a^2 d^3} - \\
& \frac{6 i (a^2 - b^2) f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b d^4} - \frac{6 i (a^2 - b^2) f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b d^4} - \frac{3 i b f^3 \operatorname{PolyLog}\left[4, e^{2i(c+dx)}\right]}{4 a^2 d^4}
\end{aligned}$$

Result (type 4, 3114 leaves):

$$\begin{aligned}
& \frac{3 e^2 f \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]}{a d^2} + \frac{1}{a d^3} \\
& 6 e f^2 \left((c+dx) \left(\operatorname{Log}\left[1 - e^{i(c+dx)}\right] - \operatorname{Log}\left[1 + e^{i(c+dx)}\right] \right) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] + i \left(\operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right] - \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right] \right) \right) + \\
& \frac{1}{4 a^2 d^3} b e^{-i c} f^2 \operatorname{Csc}[c] \\
& \left(2 d^2 x^2 \left(2 d e^{2 i c} x + 3 i \left(-1 + e^{2 i c} \right) \operatorname{Log}\left[1 - e^{2 i(c+dx)}\right] \right) + 6 d \left(-1 + e^{2 i c} \right) x \operatorname{PolyLog}\left[2, e^{2 i(c+dx)}\right] + 3 i \left(-1 + e^{2 i c} \right) \operatorname{PolyLog}\left[3, e^{2 i(c+dx)}\right] \right) - \\
& \frac{1}{a d^4} 6 f^3 \left(d^2 x^2 \operatorname{ArcTanh}\left[\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] - i d x \operatorname{PolyLog}\left[2, -\operatorname{Cos}[c+dx] - i \operatorname{Sin}[c+dx]\right] + \right. \\
& \left. i d x \operatorname{PolyLog}\left[2, \operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] + \operatorname{PolyLog}\left[3, -\operatorname{Cos}[c+dx] - i \operatorname{Sin}[c+dx]\right] - \operatorname{PolyLog}\left[3, \operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]\right] \right) + \\
& \frac{1}{4 a^2} b e^{i c} f^3 \operatorname{Csc}[c] \left(x^4 + \left(-1 + e^{-2 i c} \right) x^4 + 1 / \left(2 d^4 \right) e^{-2 i c} \left(-1 + e^{2 i c} \right) \right. \\
& \left. \left(2 d^4 x^4 + 4 i d^3 x^3 \operatorname{Log}\left[1 - e^{2 i(c+dx)}\right] + 6 d^2 x^2 \operatorname{PolyLog}\left[2, e^{2 i(c+dx)}\right] + 6 i d x \operatorname{PolyLog}\left[3, e^{2 i(c+dx)}\right] - 3 \operatorname{PolyLog}\left[4, e^{2 i(c+dx)}\right] \right) \right) + \\
& \frac{1}{2 a^2 b d^4 \left(-1 + e^{2 i c} \right)} \left(a^2 - b^2 \right) \left(4 i d^4 e^3 e^{2 i c} x + 6 i d^4 e^2 e^{2 i c} f x^2 + 4 i d^4 e e^{2 i c} f^2 x^3 + i d^4 e^2 e^{2 i c} f^3 x^4 + \right. \\
& \left. 2 i d^3 e^3 \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b \left(-1 + e^{2 i(c+dx)} \right)}\right] - 2 i d^3 e^3 e^{2 i c} \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b \left(-1 + e^{2 i(c+dx)} \right)}\right] + d^3 e^3 \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 \left(-1 + e^{2 i(c+dx)} \right)^2 \right] - \right. \\
& \left. d^3 e^3 e^{2 i c} \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 \left(-1 + e^{2 i(c+dx)} \right)^2 \right] + 6 d^3 e^2 f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - \right. \\
& \left. 6 d^3 e^2 e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + 6 d^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - \right. \\
& \left. 6 d^3 e e^{2 i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + 2 d^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - \right. \\
& \left. 2 d^3 e^2 e^{2 i c} f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + 6 d^3 e^2 f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - \right. \\
& \left. 6 d^3 e^2 e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + 6 d^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - \right. \\
& \left. 6 d^3 e e^{2 i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + 2 d^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 d^3 e^{2 i c} f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + 6 i d^2 (-1 + e^{2 i c}) f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 6 i d^2 (-1 + e^{2 i c}) f (e + f x)^2 \operatorname{PolyLog}\left[2, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 12 d e f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - 12 d e e^{2 i c} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 12 d f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - 12 d e^{2 i c} f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 12 d e f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - 12 d e e^{2 i c} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 12 d f^3 x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - 12 d e^{2 i c} f^3 x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 12 i f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - 12 i e^{2 i c} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 12 i f^3 \operatorname{PolyLog}\left[4, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - 12 i e^{2 i c} f^3 \operatorname{PolyLog}\left[4, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] \Bigg) + \frac{1}{8 a b d} \\
& (-4 b e^3 - 12 b e^2 f x - 12 b e f^2 x^2 - 4 b f^3 x^3 - 4 a d e^3 x \operatorname{Cos}[c] - 6 a d e^2 f x^2 \operatorname{Cos}[c] - 4 a d e f^2 x^3 \operatorname{Cos}[c] - a d f^3 x^4 \operatorname{Cos}[c]) \\
& \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] - \\
& \frac{b e^3 \operatorname{Csc}[c] (-d x \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x]]) \operatorname{Sin}[c]}{a^2 d (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} + \\
& \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (-e^3 \operatorname{Sin}\left[\frac{d x}{2}\right] - 3 e^2 f x \operatorname{Sin}\left[\frac{d x}{2}\right] - 3 e f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] - f^3 x^3 \operatorname{Sin}\left[\frac{d x}{2}\right])}{2 a d} + \\
& \frac{\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{d x}{2}\right] (e^3 \operatorname{Sin}\left[\frac{d x}{2}\right] + 3 e^2 f x \operatorname{Sin}\left[\frac{d x}{2}\right] + 3 e f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + f^3 x^3 \operatorname{Sin}\left[\frac{d x}{2}\right])}{2 a d} + \\
& \left(3 b e^2 f \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \Big/ \left(\sqrt{1 + \operatorname{Tan}[c]^2}\right) \left(i d x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]]) - \pi \operatorname{Log}[1 + e^{-2 i d x}] - \right. \right. \\
& \quad \left. \left. 2 (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}[1 - e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]])}] + \pi \operatorname{Log}[\operatorname{Cos}[d x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]] + \right. \right. \\
& \quad \left. \left. i \operatorname{PolyLog}[2, e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]])}] \operatorname{Tan}[c]\right) \Big/ \left(2 a^2 d^2 \sqrt{\operatorname{Sec}[c]^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)}\right)
\end{aligned}$$

■ **Problem 338: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \cos[c + d x] \cot[c + d x]^2}{a + b \sin[c + d x]} dx$$

Optimal (type 4, 616 leaves, 37 steps):

$$\begin{aligned} & \frac{i b (e + f x)^3}{3 a^2 f} + \frac{i (a^2 - b^2) (e + f x)^3}{3 a^2 b f} - \frac{4 f (e + f x) \operatorname{ArcTanh}\left[e^{i(c+dx)}\right]}{a d^2} - \frac{(e + f x)^2 \operatorname{Csc}[c + d x]}{a d} - \frac{(a^2 - b^2) (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b d} \\ & \frac{(a^2 - b^2) (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b d} - \frac{b (e + f x)^2 \operatorname{Log}\left[1 - e^{2 i(c+dx)}\right]}{a^2 d} + \frac{2 i f^2 \operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right]}{a d^3} - \frac{2 i f^2 \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right]}{a d^3} \\ & \frac{2 i (a^2 - b^2) f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b d^2} + \frac{2 i (a^2 - b^2) f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b d^2} + \frac{i b f (e + f x) \operatorname{PolyLog}\left[2, e^{2 i(c+dx)}\right]}{a^2 d^2} \\ & \frac{2 (a^2 - b^2) f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b d^3} - \frac{2 (a^2 - b^2) f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b d^3} - \frac{b f^2 \operatorname{PolyLog}\left[3, e^{2 i(c+dx)}\right]}{2 a^2 d^3} \end{aligned}$$

Result (type 4, 1905 leaves):

$$\begin{aligned} & \frac{2 e f \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right]}{a d^2} + \frac{1}{a d^3} \\ & 2 f^2 \left((c + d x) (\operatorname{Log}[1 - e^{i(c+dx)}] - \operatorname{Log}[1 + e^{i(c+dx)}]) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right] + i (\operatorname{PolyLog}[2, -e^{i(c+dx)}] - \operatorname{PolyLog}[2, e^{i(c+dx)}]) \right) + \\ & \frac{1}{12 a^2 d^3} b e^{-i c} f^2 \operatorname{Csc}[c] \\ & (2 d^2 x^2 (2 d e^{2 i c} x + 3 i (-1 + e^{2 i c}) \operatorname{Log}[1 - e^{2 i(c+dx)}]) + 6 d (-1 + e^{2 i c}) x \operatorname{PolyLog}[2, e^{2 i(c+dx)}] + 3 i (-1 + e^{2 i c}) \operatorname{PolyLog}[3, e^{2 i(c+dx)}]) + \\ & \frac{1}{6 a^2 b d^3 (-1 + e^{2 i c})} (a^2 - b^2) \left(12 i d^3 e^2 e^{2 i c} x + 12 i d^3 e e^{2 i c} f x^2 + 4 i d^3 e^{2 i c} f^2 x^3 + 6 i d^2 e^2 \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b (-1 + e^{2 i(c+dx)})}\right] - \right. \\ & \left. 6 i d^2 e^2 e^{2 i c} \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b (-1 + e^{2 i(c+dx)})}\right] + 3 d^2 e^2 \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 (-1 + e^{2 i(c+dx)})^2\right] - \right. \\ & \left. 3 d^2 e^2 e^{2 i c} \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 (-1 + e^{2 i(c+dx)})^2\right] + 12 d^2 e f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \right. \\ & \left. 12 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] + 6 d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] - \right. \end{aligned}$$

$$\begin{aligned}
& 6 d^2 e^{2 i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} - \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + 12 d^2 e f x \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - \\
& 12 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + 6 d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - \\
& 6 d^2 e^{2 i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + 12 i d (-1 + e^{2 i c}) f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 12 i d (-1 + e^{2 i c}) f (e + f x) \operatorname{PolyLog}\left[2, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 12 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - 12 e^{2 i c} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{(-a^2+b^2) e^{2 i c}}}\right] + \\
& 12 f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] - 12 e^{2 i c} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{(-a^2+b^2) e^{2 i c}}}\right] \Bigg) + \\
& \frac{(-3 b e^2 - 6 b e f x - 3 b f^2 x^2 - 3 a d e^2 x \operatorname{Cos}[c] - 3 a d e f x^2 \operatorname{Cos}[c] - a d f^2 x^3 \operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{6 a b d} - \\
& \frac{b e^2 \operatorname{Csc}[c] (-d x \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x]]) \operatorname{Sin}[c]}{a^2 d (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} + \\
& \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (-e^2 \operatorname{Sin}\left[\frac{d x}{2}\right] - 2 e f x \operatorname{Sin}\left[\frac{d x}{2}\right] - f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right])}{2 a d} + \\
& \frac{\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{d x}{2}\right] (e^2 \operatorname{Sin}\left[\frac{d x}{2}\right] + 2 e f x \operatorname{Sin}\left[\frac{d x}{2}\right] + f^2 x^2 \operatorname{Sin}\left[\frac{d x}{2}\right])}{2 a d} + \\
& \left(b e f \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \Big/ \left(\sqrt{1 + \operatorname{Tan}[c]^2} \right) \left(i d x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]]) - \pi \operatorname{Log}[1 + e^{-2 i d x}] - \right. \right. \right. \\
& \quad \left. \left. 2 (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}[1 - e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]])}] + \pi \operatorname{Log}[\operatorname{Cos}[d x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]] + \right. \right. \\
& \quad \left. \left. i \operatorname{PolyLog}[2, e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]])}] \operatorname{Tan}[c] \right) \Big/ \left(a^2 d^2 \sqrt{\operatorname{Sec}[c]^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} \right) \Big)
\end{aligned}$$

■ **Problem 339: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x) \operatorname{Cos}[c + d x] \operatorname{Cot}[c + d x]^2}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 386 leaves, 28 steps):

$$\frac{i b (e + f x)^2}{2 a^2 f} + \frac{i (a^2 - b^2) (e + f x)^2}{2 a^2 b f} - \frac{f \operatorname{ArcTanh}[\operatorname{Cos}[c + d x]]}{a d^2} - \frac{(e + f x) \operatorname{Csc}[c + d x]}{a d} -$$

$$\frac{(a^2 - b^2) (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b d} - \frac{(a^2 - b^2) (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b d} - \frac{b (e + f x) \operatorname{Log}\left[1 - e^{2 i(c+dx)}\right]}{a^2 d} +$$

$$\frac{i (a^2 - b^2) f \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b d^2} + \frac{i (a^2 - b^2) f \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b d^2} + \frac{i b f \operatorname{PolyLog}\left[2, e^{2 i(c+dx)}\right]}{2 a^2 d^2}$$

Result (type 4, 1107 leaves):

$$\frac{(-d e \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + c f \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - f (c + d x) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]) \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]}{2 a d^2} -$$

$$\frac{b e \operatorname{Log}[\operatorname{Sin}[c + d x]]}{a^2 d} + \frac{b c f \operatorname{Log}[\operatorname{Sin}[c + d x]]}{a^2 d^2} - \frac{e \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right]}{b d} + \frac{b e \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right]}{a^2 d} +$$

$$\frac{c f \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right]}{b d^2} - \frac{b c f \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right]}{a^2 d^2} + \frac{f \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right]}{a d^2} - \frac{1}{d^2} f \left(\frac{(c + d x) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{b} - \right.$$

$$\left. 1 / b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - d x\right)^2 + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x\right)\right]}{\sqrt{a^2 - b^2}}\right] + \left(-c + \frac{\pi}{2} - d x + 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right]\right) \right.$$

$$\left. \operatorname{Log}\left[1 + \frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - d x\right)}}{b}\right] + \left(-c + \frac{\pi}{2} - d x - 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right]\right) \operatorname{Log}\left[1 + \frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - d x\right)}}{b}\right] - \right.$$

$$\left. \left(-c + \frac{\pi}{2} - d x\right) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]] - i \left(\operatorname{PolyLog}\left[2, -\frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - d x\right)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i\left(-c + \frac{\pi}{2} - d x\right)}}{b}\right] \right) \right) \right) +$$

$$\frac{1}{a^2 d^2} b^2 f \left(\frac{(c + d x) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{b} - 1 / b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - d x \right)^2 + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right)\right]}{\sqrt{a^2 - b^2}}\right] \right) + \right. \\ \left. \left(-c + \frac{\pi}{2} - d x + 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i \left(-c + \frac{\pi}{2} - d x\right)}}{b}\right] + \right. \\ \left. \left(-c + \frac{\pi}{2} - d x - 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i \left(-c + \frac{\pi}{2} - d x\right)}}{b}\right] - \left(-c + \frac{\pi}{2} - d x\right) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]] - \right. \\ \left. i \left(\operatorname{PolyLog}\left[2, -\frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i \left(-c + \frac{\pi}{2} - d x\right)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i \left(-c + \frac{\pi}{2} - d x\right)}}{b}\right] \right) \right) - \\ \frac{b f \left((c + d x) \operatorname{Log}\left[1 - e^{2 i (c + d x)}\right] - \frac{1}{2} i \left((c + d x)^2 + \operatorname{PolyLog}\left[2, e^{2 i (c + d x)}\right] \right) \right)}{a^2 d^2} + \\ \frac{\operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \left(-d e \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + c f \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] - f (c + d x) \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right)}{2 a d^2}$$

■ **Problem 341: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \operatorname{Cos}[c + d x]^2 \operatorname{Cot}[c + d x]^2}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 1144 leaves, 66 steps):

$$\begin{aligned}
& - \frac{i (e + f x)^3}{a d} - \frac{(e + f x)^4}{4 a f} - \frac{(a^2 - b^2) (e + f x)^4}{4 a b^2 f} + \frac{2 b (e + f x)^3 \operatorname{ArcTanh}\left[e^{i (c + d x)}\right]}{a^2 d} + \frac{6 b f^2 (e + f x) \operatorname{Cos}[c + d x]}{a^2 d^3} + \frac{6 (a^2 - b^2) f^2 (e + f x) \operatorname{Cos}[c + d x]}{a^2 b d^3} \\
& \frac{b (e + f x)^3 \operatorname{Cos}[c + d x]}{a^2 d} - \frac{(a^2 - b^2) (e + f x)^3 \operatorname{Cos}[c + d x]}{a^2 b d} - \frac{(e + f x)^3 \operatorname{Cot}[c + d x]}{a d} - \frac{i (a^2 - b^2)^{3/2} (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i (c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^2 d} + \\
& \frac{i (a^2 - b^2)^{3/2} (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i (c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^2 d} + \frac{3 f (e + f x)^2 \operatorname{Log}\left[1 - e^{2 i (c + d x)}\right]}{a d^2} - \frac{3 i b f (e + f x)^2 \operatorname{PolyLog}\left[2, -e^{i (c + d x)}\right]}{a^2 d^2} + \\
& \frac{3 i b f (e + f x)^2 \operatorname{PolyLog}\left[2, e^{i (c + d x)}\right]}{a^2 d^2} - \frac{3 (a^2 - b^2)^{3/2} f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i (c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^2 d^2} + \frac{3 (a^2 - b^2)^{3/2} f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i (c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^2 d^2} - \\
& \frac{3 i f^2 (e + f x) \operatorname{PolyLog}\left[2, e^{2 i (c + d x)}\right]}{a d^3} + \frac{6 b f^2 (e + f x) \operatorname{PolyLog}\left[3, -e^{i (c + d x)}\right]}{a^2 d^3} - \frac{6 b f^2 (e + f x) \operatorname{PolyLog}\left[3, e^{i (c + d x)}\right]}{a^2 d^3} - \\
& \frac{6 i (a^2 - b^2)^{3/2} f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i (c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^2 d^3} + \frac{6 i (a^2 - b^2)^{3/2} f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i (c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^2 d^3} + \frac{3 f^3 \operatorname{PolyLog}\left[3, e^{2 i (c + d x)}\right]}{2 a d^4} + \\
& \frac{6 i b f^3 \operatorname{PolyLog}\left[4, -e^{i (c + d x)}\right]}{a^2 d^4} - \frac{6 i b f^3 \operatorname{PolyLog}\left[4, e^{i (c + d x)}\right]}{a^2 d^4} + \frac{6 (a^2 - b^2)^{3/2} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i (c + d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^2 d^4} - \frac{6 (a^2 - b^2)^{3/2} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i (c + d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^2 d^4} - \\
& \frac{6 b f^3 \operatorname{Sin}[c + d x]}{a^2 d^4} - \frac{6 (a^2 - b^2) f^3 \operatorname{Sin}[c + d x]}{a^2 b d^4} + \frac{3 b f (e + f x)^2 \operatorname{Sin}[c + d x]}{a^2 d^2} + \frac{3 (a^2 - b^2) f (e + f x)^2 \operatorname{Sin}[c + d x]}{a^2 b d^2}
\end{aligned}$$

Result (type 4, 4632 leaves):

$$\begin{aligned}
& - \frac{b e^3 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right]}{a^2 d} - \frac{1}{a^2 d^2} \\
& 3 b e^2 f \left((c + d x) \left(\operatorname{Log}\left[1 - e^{i (c + d x)}\right] - \operatorname{Log}\left[1 + e^{i (c + d x)}\right] \right) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right] + i \left(\operatorname{PolyLog}\left[2, -e^{i (c + d x)}\right] - \operatorname{PolyLog}\left[2, e^{i (c + d x)}\right] \right) \right) - \\
& \frac{1}{4 a d^4} e^{-i c} f^3 \operatorname{Csc}[c] \\
& \left(2 d^2 x^2 \left(2 d e^{2 i c} x + 3 i (-1 + e^{2 i c}) \operatorname{Log}\left[1 - e^{2 i (c + d x)}\right] \right) + 6 d (-1 + e^{2 i c}) x \operatorname{PolyLog}\left[2, e^{2 i (c + d x)}\right] + 3 i (-1 + e^{2 i c}) \operatorname{PolyLog}\left[3, e^{2 i (c + d x)}\right] \right) + \\
& \frac{1}{a^2 d^3} 6 b e f^2 \left(d^2 x^2 \operatorname{ArcTanh}\left[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] - i d x \operatorname{PolyLog}\left[2, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] + \right. \\
& \left. i d x \operatorname{PolyLog}\left[2, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] + \operatorname{PolyLog}\left[3, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] - \operatorname{PolyLog}\left[3, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] \right) - \\
& \frac{1}{a^2 d^4} b f^3 \left(-2 d^3 x^3 \operatorname{ArcTanh}\left[\operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] + 3 i d^2 x^2 \operatorname{PolyLog}\left[2, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] - \right. \\
& \left. 3 i d^2 x^2 \operatorname{PolyLog}\left[2, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] - 6 d x \operatorname{PolyLog}\left[3, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] + 6 d x \right. \\
& \left. \operatorname{PolyLog}\left[3, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] - 6 i \operatorname{PolyLog}\left[4, -\operatorname{Cos}[c + d x] - i \operatorname{Sin}[c + d x]\right] + 6 i \operatorname{PolyLog}\left[4, \operatorname{Cos}[c + d x] + i \operatorname{Sin}[c + d x]\right] \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{3 e^2 f \operatorname{Csc}[c] (-d x \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x]] \operatorname{Sin}[c])}{a d^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} + \frac{1}{a^2 b^2 d^4 \sqrt{-a^2 + b^2} (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} \\
& i (a^2 - b^2)^{3/2} \left(3 i \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \right. \\
& 3 i \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& i \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log} \left[1 + \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 3 \sqrt{a^2 - b^2} d^2 f (e + f x)^2 \operatorname{PolyLog} \left[2, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) - \\
& 3 \sqrt{a^2 - b^2} d^2 f (e + f x)^2 \operatorname{PolyLog} \left[2, \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 6 i \sqrt{a^2 - b^2} d e f^2 \operatorname{PolyLog} \left[3, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 6 i \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog} \left[3, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) - \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog} \left[4, -\frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} - a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 6 \sqrt{a^2 - b^2} f^3 \operatorname{PolyLog} \left[4, \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (\operatorname{Cos}[c] + i \operatorname{Sin}[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e^2 f x \operatorname{Log} \left[1 - \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (-i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \\
& 3 \sqrt{a^2 - b^2} d^3 e f^2 x^2 \operatorname{Log} \left[1 - \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (-i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \\
& \sqrt{a^2 - b^2} d^3 f^3 x^3 \operatorname{Log} \left[1 - \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (-i \operatorname{Cos}[c] + \operatorname{Sin}[c]) + \\
& 6 \sqrt{a^2 - b^2} d e f^2 \operatorname{PolyLog} \left[3, \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]} \right] (-i \operatorname{Cos}[c] + \operatorname{Sin}[c]) +
\end{aligned}$$

$$\begin{aligned}
& 6 \sqrt{a^2 - b^2} d f^3 x \operatorname{PolyLog}\left[3, \frac{b (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])}{-i a \operatorname{Cos}[c] + \sqrt{(-a^2 + b^2) (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2} + a \operatorname{Sin}[c]}\right] (-i \operatorname{Cos}[c] + \operatorname{Sin}[c]) - \\
& 2 i d^3 e^3 \operatorname{ArcTan}\left[\frac{b \operatorname{Cos}[c + d x] + i (a + b \operatorname{Sin}[c + d x])}{\sqrt{a^2 - b^2}}\right] \sqrt{(-a^2 + b^2) (\operatorname{Cos}[2 c] + i \operatorname{Sin}[2 c])} + \\
& \operatorname{Csc}[c] \operatorname{Csc}[c + d x] \left(\frac{\operatorname{Cos}[c + d x]}{16 a b^2 d^4} - \frac{i \operatorname{Sin}[c + d x]}{16 a b^2 d^4}\right) (8 i b^2 d^3 e^3 \operatorname{Cos}[c] + 24 i b^2 d^3 e^2 f x \operatorname{Cos}[c] + 24 i b^2 d^3 e f^2 x^2 \operatorname{Cos}[c] + \\
& 8 i b^2 d^3 f^3 x^3 \operatorname{Cos}[c] - 2 a b d^3 e^3 \operatorname{Cos}[d x] + 18 i a b d^2 e^2 f \operatorname{Cos}[d x] + 12 a b d e f^2 \operatorname{Cos}[d x] - 36 i a b f^3 \operatorname{Cos}[d x] - 6 a b d^3 e^2 f x \operatorname{Cos}[d x] + \\
& 36 i a b d^2 e f^2 x \operatorname{Cos}[d x] + 12 a b d f^3 x \operatorname{Cos}[d x] - 6 a b d^3 e f^2 x^2 \operatorname{Cos}[d x] + 18 i a b d^2 f^3 x^2 \operatorname{Cos}[d x] - 2 a b d^3 f^3 x^3 \operatorname{Cos}[d x] + \\
& 2 a b d^3 e^3 \operatorname{Cos}[2 c + d x] - 18 i a b d^2 e^2 f \operatorname{Cos}[2 c + d x] - 12 a b d e f^2 \operatorname{Cos}[2 c + d x] + 36 i a b f^3 \operatorname{Cos}[2 c + d x] + 6 a b d^3 e^2 f x \operatorname{Cos}[2 c + d x] - \\
& 36 i a b d^2 e f^2 x \operatorname{Cos}[2 c + d x] - 12 a b d f^3 x \operatorname{Cos}[2 c + d x] + 6 a b d^3 e f^2 x^2 \operatorname{Cos}[2 c + d x] - 18 i a b d^2 f^3 x^2 \operatorname{Cos}[2 c + d x] + \\
& 2 a b d^3 f^3 x^3 \operatorname{Cos}[2 c + d x] - 8 i b^2 d^3 e^3 \operatorname{Cos}[c + 2 d x] - 4 a^2 d^4 e^3 x \operatorname{Cos}[c + 2 d x] - 24 i b^2 d^3 e^2 f x \operatorname{Cos}[c + 2 d x] - 6 a^2 d^4 e^2 f x^2 \operatorname{Cos}[c + 2 d x] - \\
& 24 i b^2 d^3 e f^2 x^2 \operatorname{Cos}[c + 2 d x] - 4 a^2 d^4 e f^2 x^3 \operatorname{Cos}[c + 2 d x] - 8 i b^2 d^3 f^3 x^3 \operatorname{Cos}[c + 2 d x] - a^2 d^4 f^3 x^4 \operatorname{Cos}[c + 2 d x] + \\
& 4 a^2 d^4 e^3 x \operatorname{Cos}[3 c + 2 d x] + 6 a^2 d^4 e^2 f x^2 \operatorname{Cos}[3 c + 2 d x] + 4 a^2 d^4 e f^2 x^3 \operatorname{Cos}[3 c + 2 d x] + a^2 d^4 f^3 x^4 \operatorname{Cos}[3 c + 2 d x] - \\
& 2 a b d^3 e^3 \operatorname{Cos}[2 c + 3 d x] - 6 i a b d^2 e^2 f \operatorname{Cos}[2 c + 3 d x] + 12 a b d e f^2 \operatorname{Cos}[2 c + 3 d x] + 12 i a b f^3 \operatorname{Cos}[2 c + 3 d x] - \\
& 6 a b d^3 e^2 f x \operatorname{Cos}[2 c + 3 d x] - 12 i a b d^2 e f^2 x \operatorname{Cos}[2 c + 3 d x] + 12 a b d f^3 x \operatorname{Cos}[2 c + 3 d x] - 6 a b d^3 e f^2 x^2 \operatorname{Cos}[2 c + 3 d x] - \\
& 6 i a b d^2 f^3 x^2 \operatorname{Cos}[2 c + 3 d x] - 2 a b d^3 f^3 x^3 \operatorname{Cos}[2 c + 3 d x] + 2 a b d^3 e^3 \operatorname{Cos}[4 c + 3 d x] + 6 i a b d^2 e^2 f \operatorname{Cos}[4 c + 3 d x] - \\
& 12 a b d e f^2 \operatorname{Cos}[4 c + 3 d x] - 12 i a b f^3 \operatorname{Cos}[4 c + 3 d x] + 6 a b d^3 e^2 f x \operatorname{Cos}[4 c + 3 d x] + 12 i a b d^2 e f^2 x \operatorname{Cos}[4 c + 3 d x] - \\
& 12 a b d f^3 x \operatorname{Cos}[4 c + 3 d x] + 6 a b d^3 e f^2 x^2 \operatorname{Cos}[4 c + 3 d x] + 6 i a b d^2 f^3 x^2 \operatorname{Cos}[4 c + 3 d x] + 2 a b d^3 f^3 x^3 \operatorname{Cos}[4 c + 3 d x] - 8 b^2 d^3 e^3 \operatorname{Sin}[c] - \\
& 8 i a^2 d^4 e^3 x \operatorname{Sin}[c] - 24 b^2 d^3 e^2 f x \operatorname{Sin}[c] - 12 i a^2 d^4 e^2 f x^2 \operatorname{Sin}[c] - 24 b^2 d^3 e f^2 x^2 \operatorname{Sin}[c] - 8 i a^2 d^4 e f^2 x^3 \operatorname{Sin}[c] - 8 b^2 d^3 f^3 x^3 \operatorname{Sin}[c] - \\
& 2 i a^2 d^4 f^3 x^4 \operatorname{Sin}[c] + 2 i a b d^3 e^3 \operatorname{Sin}[d x] - 6 a b d^2 e^2 f \operatorname{Sin}[d x] - 12 i a b d e f^2 \operatorname{Sin}[d x] + 12 a b f^3 \operatorname{Sin}[d x] + 6 i a b d^3 e^2 f x \operatorname{Sin}[d x] - \\
& 12 a b d^2 e f^2 x \operatorname{Sin}[d x] - 12 i a b d f^3 x \operatorname{Sin}[d x] + 6 i a b d^3 e f^2 x^2 \operatorname{Sin}[d x] - 6 a b d^2 f^3 x^2 \operatorname{Sin}[d x] + 2 i a b d^3 f^3 x^3 \operatorname{Sin}[d x] - \\
& 2 i a b d^3 e^3 \operatorname{Sin}[2 c + d x] + 6 a b d^2 e^2 f \operatorname{Sin}[2 c + d x] + 12 i a b d e f^2 \operatorname{Sin}[2 c + d x] - 12 a b f^3 \operatorname{Sin}[2 c + d x] - 6 i a b d^3 e^2 f x \operatorname{Sin}[2 c + d x] + \\
& 12 a b d^2 e f^2 x \operatorname{Sin}[2 c + d x] + 12 i a b d f^3 x \operatorname{Sin}[2 c + d x] - 6 i a b d^3 e f^2 x^2 \operatorname{Sin}[2 c + d x] + 6 a b d^2 f^3 x^2 \operatorname{Sin}[2 c + d x] - \\
& 2 i a b d^3 f^3 x^3 \operatorname{Sin}[2 c + d x] + 8 b^2 d^3 e^3 \operatorname{Sin}[c + 2 d x] - 4 i a^2 d^4 e^3 x \operatorname{Sin}[c + 2 d x] + 24 b^2 d^3 e^2 f x \operatorname{Sin}[c + 2 d x] - 6 i a^2 d^4 e^2 f x^2 \operatorname{Sin}[c + 2 d x] + \\
& 24 b^2 d^3 e f^2 x^2 \operatorname{Sin}[c + 2 d x] - 4 i a^2 d^4 e f^2 x^3 \operatorname{Sin}[c + 2 d x] + 8 b^2 d^3 f^3 x^3 \operatorname{Sin}[c + 2 d x] - i a^2 d^4 f^3 x^4 \operatorname{Sin}[c + 2 d x] + \\
& 4 i a^2 d^4 e^3 x \operatorname{Sin}[3 c + 2 d x] + 6 i a^2 d^4 e^2 f x^2 \operatorname{Sin}[3 c + 2 d x] + 4 i a^2 d^4 e f^2 x^3 \operatorname{Sin}[3 c + 2 d x] + i a^2 d^4 f^3 x^4 \operatorname{Sin}[3 c + 2 d x] - \\
& 2 i a b d^3 e^3 \operatorname{Sin}[2 c + 3 d x] + 6 a b d^2 e^2 f \operatorname{Sin}[2 c + 3 d x] + 12 i a b d e f^2 \operatorname{Sin}[2 c + 3 d x] - 12 a b f^3 \operatorname{Sin}[2 c + 3 d x] - \\
& 6 i a b d^3 e^2 f x \operatorname{Sin}[2 c + 3 d x] + 12 a b d^2 e f^2 x \operatorname{Sin}[2 c + 3 d x] + 12 i a b d f^3 x \operatorname{Sin}[2 c + 3 d x] - 6 i a b d^3 e f^2 x^2 \operatorname{Sin}[2 c + 3 d x] + \\
& 6 a b d^2 f^3 x^2 \operatorname{Sin}[2 c + 3 d x] - 2 i a b d^3 f^3 x^3 \operatorname{Sin}[2 c + 3 d x] + 2 i a b d^3 e^3 \operatorname{Sin}[4 c + 3 d x] - 6 a b d^2 e^2 f \operatorname{Sin}[4 c + 3 d x] - \\
& 12 i a b d e f^2 \operatorname{Sin}[4 c + 3 d x] + 12 a b f^3 \operatorname{Sin}[4 c + 3 d x] + 6 i a b d^3 e^2 f x \operatorname{Sin}[4 c + 3 d x] - 12 a b d^2 e f^2 x \operatorname{Sin}[4 c + 3 d x] - \\
& 12 i a b d f^3 x \operatorname{Sin}[4 c + 3 d x] + 6 i a b d^3 e f^2 x^2 \operatorname{Sin}[4 c + 3 d x] - 6 a b d^2 f^3 x^2 \operatorname{Sin}[4 c + 3 d x] + 2 i a b d^3 f^3 x^3 \operatorname{Sin}[4 c + 3 d x]) - \\
& (3 e f^2 \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 / \left(\sqrt{1 + \operatorname{Tan}[c]^2}\right) (i d x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]]) - \pi \operatorname{Log}[1 + e^{-i d x}] - \right. \\
& 2 (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}[1 - e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c])}] + \pi \operatorname{Log}[\operatorname{Cos}[d x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]) + \\
& i \operatorname{PolyLog}[2, e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c])}] \operatorname{Tan}[c]) \left. \right) / \left(a d^3 \sqrt{\operatorname{Sec}[c]^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)}\right)
\end{aligned}$$

■ **Problem 342: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \operatorname{Cos}[c + d x]^2 \operatorname{Cot}[c + d x]^2}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 4, 840 leaves, 53 steps):

$$\begin{aligned}
& -\frac{i(e+fx)^2}{ad} - \frac{(e+fx)^3}{3af} - \frac{(a^2-b^2)(e+fx)^3}{3ab^2f} + \frac{2b(e+fx)^2 \operatorname{ArcTanh}[e^{i(c+dx)}]}{a^2d} + \frac{2bf^2 \cos[c+dx]}{a^2d^3} + \frac{2(a^2-b^2)f^2 \cos[c+dx]}{a^2bd^3} \\
& - \frac{b(e+fx)^2 \cos[c+dx]}{a^2d} - \frac{(a^2-b^2)(e+fx)^2 \cos[c+dx]}{a^2bd} - \frac{(e+fx)^2 \cot[c+dx]}{ad} - \frac{i(a^2-b^2)^{3/2}(e+fx)^2 \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{a^2bd^2} + \\
& \frac{i(a^2-b^2)^{3/2}(e+fx)^2 \operatorname{Log}\left[1 - \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{a^2bd^2} + \frac{2f(e+fx) \operatorname{Log}[1 - e^{2i(c+dx)}]}{a^2d^2} - \frac{2ibf(e+fx) \operatorname{PolyLog}[2, -e^{i(c+dx)}]}{a^2d^2} + \\
& \frac{2ibf(e+fx) \operatorname{PolyLog}[2, e^{i(c+dx)}]}{a^2d^2} - \frac{2(a^2-b^2)^{3/2}f(e+fx) \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{a^2bd^2} + \frac{2(a^2-b^2)^{3/2}f(e+fx) \operatorname{PolyLog}\left[2, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{a^2bd^2} - \\
& \frac{if^2 \operatorname{PolyLog}[2, e^{2i(c+dx)}]}{ad^3} + \frac{2bf^2 \operatorname{PolyLog}[3, -e^{i(c+dx)}]}{a^2d^3} - \frac{2bf^2 \operatorname{PolyLog}[3, e^{i(c+dx)}]}{a^2d^3} - \frac{2i(a^2-b^2)^{3/2}f^2 \operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right]}{a^2bd^3} + \\
& \frac{2i(a^2-b^2)^{3/2}f^2 \operatorname{PolyLog}\left[3, \frac{ibe^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right]}{a^2bd^3} + \frac{2bf(e+fx) \sin[c+dx]}{a^2d^2} + \frac{2(a^2-b^2)f(e+fx) \sin[c+dx]}{a^2bd^2}
\end{aligned}$$

Result (type 4, 2574 leaves):

$$\begin{aligned}
& -\frac{be^2 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]}{a^2d} - \frac{1}{a^2d^2} \\
& + 2bef \left((c+dx) (\operatorname{Log}[1 - e^{i(c+dx)}] - \operatorname{Log}[1 + e^{i(c+dx)}]) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] + i (\operatorname{PolyLog}[2, -e^{i(c+dx)}] - \operatorname{PolyLog}[2, e^{i(c+dx)}]) \right) + \\
& \frac{1}{a^2d^3} 2bf^2 (d^2x^2 \operatorname{ArcTanh}[\cos[c+dx] + i \sin[c+dx]] - idx \operatorname{PolyLog}[2, -\cos[c+dx] - i \sin[c+dx]] + \\
& \quad i dx \operatorname{PolyLog}[2, \cos[c+dx] + i \sin[c+dx]] + \operatorname{PolyLog}[3, -\cos[c+dx] - i \sin[c+dx]] - \operatorname{PolyLog}[3, \cos[c+dx] + i \sin[c+dx]]) + \\
& \frac{2ef \operatorname{Csc}[c] (-dx \cos[c] + \operatorname{Log}[\cos[dx] \sin[c] + \cos[c] \sin[dx]] \sin[c])}{ad^2 (\cos[c]^2 + \sin[c]^2)} + \frac{1}{a^2b^2d^3 \sqrt{(-a^2+b^2)} (\cos[2c] + i \sin[2c])} \\
& + i(a^2-b^2)^{3/2} \left(2\sqrt{a^2-b^2} df(e+fx) \operatorname{PolyLog}\left[2, -\frac{b(\cos[2c+dx] + i \sin[2c+dx])}{ia \cos[c] + \sqrt{(-a^2+b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]}\right] (\cos[c] + i \sin[c]) - \right. \\
& \quad \left. 2\sqrt{a^2-b^2} df(e+fx) \operatorname{PolyLog}\left[2, \frac{b(\cos[2c+dx] + i \sin[2c+dx])}{-ia \cos[c] + \sqrt{(-a^2+b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]}\right] (\cos[c] + i \sin[c]) - \right.
\end{aligned}$$

$$\begin{aligned}
& i \left(-2 \sqrt{a^2 - b^2} f^2 \operatorname{PolyLog}\left[3, -\frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]} \right] (\cos[c] + i \sin[c]) + \right. \\
& \quad \left. 2 \sqrt{a^2 - b^2} f^2 \operatorname{PolyLog}\left[3, \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]} \right] (\cos[c] + i \sin[c]) + \right. \\
& \quad \left. d^2 \left(\sqrt{a^2 - b^2} f x (2e + f x) \left(-\operatorname{Log}\left[1 + \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 - a \sin[c]} \right] + \right. \right. \\
& \quad \quad \left. \left. \operatorname{Log}\left[1 - \frac{b (\cos[2c + dx] + i \sin[2c + dx])}{-i a \cos[c] + \sqrt{(-a^2 + b^2)} (\cos[c] + i \sin[c])^2 + a \sin[c]} \right] \right) (\cos[c] + i \sin[c]) + \right. \\
& \quad \quad \left. \left. 2 e^2 \operatorname{ArcTan}\left[\frac{b \cos[c + dx] + i (a + b \sin[c + dx])}{\sqrt{a^2 - b^2}}\right] \sqrt{(-a^2 + b^2)} (\cos[2c] + i \sin[2c]) \right) \right) + \\
& \operatorname{Csc}[c] \operatorname{Csc}[c + dx] \left(\frac{\cos[c + dx]}{24 a b^2 d^3} - \frac{i \sin[c + dx]}{24 a b^2 d^3} \right) (12 i b^2 d^2 e^2 \cos[c] + 24 i b^2 d^2 e f x \cos[c] + 12 i b^2 d^2 f^2 x^2 \cos[c] - \\
& \quad 3 a b d^2 e^2 \cos[dx] + 18 i a b d e f \cos[dx] + 6 a b f^2 \cos[dx] - 6 a b d^2 e f x \cos[dx] + 18 i a b d f^2 x \cos[dx] - \\
& \quad 3 a b d^2 f^2 x^2 \cos[dx] + 3 a b d^2 e^2 \cos[2c + dx] - 18 i a b d e f \cos[2c + dx] - 6 a b f^2 \cos[2c + dx] + \\
& \quad 6 a b d^2 e f x \cos[2c + dx] - 18 i a b d f^2 x \cos[2c + dx] + 3 a b d^2 f^2 x^2 \cos[2c + dx] - 12 i b^2 d^2 e^2 \cos[c + 2 dx] - \\
& \quad 6 a^2 d^3 e^2 x \cos[c + 2 dx] - 24 i b^2 d^2 e f x \cos[c + 2 dx] - 6 a^2 d^3 e f x^2 \cos[c + 2 dx] - 12 i b^2 d^2 f^2 x^2 \cos[c + 2 dx] - \\
& \quad 2 a^2 d^3 f^2 x^3 \cos[c + 2 dx] + 6 a^2 d^3 e^2 x \cos[3c + 2 dx] + 6 a^2 d^3 e f x^2 \cos[3c + 2 dx] + 2 a^2 d^3 f^2 x^3 \cos[3c + 2 dx] - \\
& \quad 3 a b d^2 e^2 \cos[2c + 3 dx] - 6 i a b d e f \cos[2c + 3 dx] + 6 a b f^2 \cos[2c + 3 dx] - 6 a b d^2 e f x \cos[2c + 3 dx] - \\
& \quad 6 i a b d f^2 x \cos[2c + 3 dx] - 3 a b d^2 f^2 x^2 \cos[2c + 3 dx] + 3 a b d^2 e^2 \cos[4c + 3 dx] + 6 i a b d e f \cos[4c + 3 dx] - \\
& \quad 6 a b f^2 \cos[4c + 3 dx] + 6 a b d^2 e f x \cos[4c + 3 dx] + 6 i a b d f^2 x \cos[4c + 3 dx] + 3 a b d^2 f^2 x^2 \cos[4c + 3 dx] - \\
& \quad 12 b^2 d^2 e^2 \sin[c] - 12 i a^2 d^3 e^2 x \sin[c] - 24 b^2 d^2 e f x \sin[c] - 12 i a^2 d^3 e f x^2 \sin[c] - 12 b^2 d^2 f^2 x^2 \sin[c] - \\
& \quad 4 i a^2 d^3 f^2 x^3 \sin[c] + 3 i a b d^2 e^2 \sin[dx] - 6 a b d e f \sin[dx] - 6 i a b f^2 \sin[dx] + 6 i a b d^2 e f x \sin[dx] - \\
& \quad 6 a b d f^2 x \sin[dx] + 3 i a b d^2 f^2 x^2 \sin[dx] - 3 i a b d^2 e^2 \sin[2c + dx] + 6 a b d e f \sin[2c + dx] + 6 i a b f^2 \sin[2c + dx] - \\
& \quad 6 i a b d^2 e f x \sin[2c + dx] + 6 a b d f^2 x \sin[2c + dx] - 3 i a b d^2 f^2 x^2 \sin[2c + dx] + 12 b^2 d^2 e^2 \sin[c + 2 dx] - \\
& \quad 6 i a^2 d^3 e^2 x \sin[c + 2 dx] + 24 b^2 d^2 e f x \sin[c + 2 dx] - 6 i a^2 d^3 e f x^2 \sin[c + 2 dx] + 12 b^2 d^2 f^2 x^2 \sin[c + 2 dx] - \\
& \quad 2 i a^2 d^3 f^2 x^3 \sin[c + 2 dx] + 6 i a^2 d^3 e^2 x \sin[3c + 2 dx] + 6 i a^2 d^3 e f x^2 \sin[3c + 2 dx] + 2 i a^2 d^3 f^2 x^3 \sin[3c + 2 dx] - \\
& \quad 3 i a b d^2 e^2 \sin[2c + 3 dx] + 6 a b d e f \sin[2c + 3 dx] + 6 i a b f^2 \sin[2c + 3 dx] - 6 i a b d^2 e f x \sin[2c + 3 dx] + \\
& \quad 6 a b d f^2 x \sin[2c + 3 dx] - 3 i a b d^2 f^2 x^2 \sin[2c + 3 dx] + 3 i a b d^2 e^2 \sin[4c + 3 dx] - 6 a b d e f \sin[4c + 3 dx] - \\
& \quad 6 i a b f^2 \sin[4c + 3 dx] + 6 i a b d^2 e f x \sin[4c + 3 dx] - 6 a b d f^2 x \sin[4c + 3 dx] + 3 i a b d^2 f^2 x^2 \sin[4c + 3 dx]) - \\
& \left(f^2 \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \right) / \left(\sqrt{1 + \operatorname{Tan}[c]^2} \right) \left(i dx (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]]) - \pi \operatorname{Log}[1 + e^{-2 i dx}] - \right. \right. \\
& \quad \left. \left. 2 (dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}[1 - e^{2 i (dx + \operatorname{ArcTan}[\operatorname{Tan}[c]])}] + \pi \operatorname{Log}[\cos[dx]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]] + \right. \right. \\
& \quad \left. \left. i \operatorname{PolyLog}[2, e^{2 i (dx + \operatorname{ArcTan}[\operatorname{Tan}[c]])}] \operatorname{Tan}[c] \right) \right) / \left(a d^3 \sqrt{\operatorname{Sec}[c]^2 (\cos[c]^2 + \sin[c]^2)} \right)
\end{aligned}$$

■ **Problem 345: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^3 \cos[c + d x]^3 \cot[c + d x]^2}{a + b \sin[c + d x]} dx$$

Optimal (type 4, 1432 leaves, 85 steps):

$$\begin{aligned} & \frac{3 b f^3 x}{8 a^2 d^3} + \frac{3 (a^2 - b^2) f^3 x}{8 a^2 b d^3} - \frac{b (e + f x)^3}{4 a^2 d} - \frac{(a^2 - b^2) (e + f x)^3}{4 a^2 b d} + \frac{i b (e + f x)^4}{4 a^2 f} - \frac{i (a^2 - b^2)^2 (e + f x)^4}{4 a^2 b^3 f} - \frac{6 f (e + f x)^2 \operatorname{ArcTanh}\left[e^{i(c+dx)}\right]}{a d^2} + \\ & \frac{6 f^3 \cos[c + d x]}{a d^4} + \frac{6 (a^2 - b^2) f^3 \cos[c + d x]}{a b^2 d^4} - \frac{3 f (e + f x)^2 \cos[c + d x]}{a d^2} - \frac{3 (a^2 - b^2) f (e + f x)^2 \cos[c + d x]}{a b^2 d^2} - \frac{(e + f x)^3 \operatorname{Csc}[c + d x]}{a d} + \\ & \frac{(a^2 - b^2)^2 (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d} + \frac{(a^2 - b^2)^2 (e + f x)^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d} - \frac{b (e + f x)^3 \operatorname{Log}\left[1 - e^{2i(c+dx)}\right]}{a^2 d} + \\ & \frac{6 i f^2 (e + f x) \operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right]}{a d^3} - \frac{6 i f^2 (e + f x) \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right]}{a d^3} - \frac{3 i (a^2 - b^2)^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^2} - \\ & \frac{3 i (a^2 - b^2)^2 f (e + f x)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^2} + \frac{3 i b f (e + f x)^2 \operatorname{PolyLog}\left[2, e^{2i(c+dx)}\right]}{2 a^2 d^2} - \frac{6 f^3 \operatorname{PolyLog}\left[3, -e^{i(c+dx)}\right]}{a d^4} + \\ & \frac{6 f^3 \operatorname{PolyLog}\left[3, e^{i(c+dx)}\right]}{a d^4} + \frac{6 (a^2 - b^2)^2 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^3} + \frac{6 (a^2 - b^2)^2 f^2 (e + f x) \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^3} - \\ & \frac{3 b f^2 (e + f x) \operatorname{PolyLog}\left[3, e^{2i(c+dx)}\right]}{2 a^2 d^3} + \frac{6 i (a^2 - b^2)^2 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^4} + \frac{6 i (a^2 - b^2)^2 f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^4} - \\ & \frac{3 i b f^3 \operatorname{PolyLog}\left[4, e^{2i(c+dx)}\right]}{4 a^2 d^4} + \frac{6 f^2 (e + f x) \sin[c + d x]}{a d^3} + \frac{6 (a^2 - b^2) f^2 (e + f x) \sin[c + d x]}{a b^2 d^3} - \frac{(e + f x)^3 \sin[c + d x]}{a d} - \\ & \frac{(a^2 - b^2) (e + f x)^3 \sin[c + d x]}{a b^2 d} - \frac{3 b f^3 \cos[c + d x] \sin[c + d x]}{8 a^2 d^4} - \frac{3 (a^2 - b^2) f^3 \cos[c + d x] \sin[c + d x]}{8 a^2 b d^4} + \\ & \frac{3 b f (e + f x)^2 \cos[c + d x] \sin[c + d x]}{4 a^2 d^2} + \frac{3 (a^2 - b^2) f (e + f x)^2 \cos[c + d x] \sin[c + d x]}{4 a^2 b d^2} - \frac{3 b f^2 (e + f x) \sin[c + d x]^2}{4 a^2 d^3} - \\ & \frac{3 (a^2 - b^2) f^2 (e + f x) \sin[c + d x]^2}{4 a^2 b d^3} + \frac{b (e + f x)^3 \sin[c + d x]^2}{2 a^2 d} + \frac{(a^2 - b^2) (e + f x)^3 \sin[c + d x]^2}{2 a^2 b d} \end{aligned}$$

Result (type 4, 4084 leaves):

$$\begin{aligned} & \frac{(-e^3 - 3 e^2 f x - 3 e f^2 x^2 - f^3 x^3) \operatorname{Csc}[c + d x]}{a d} + \frac{3 e^2 f \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right]}{a d^2} + \frac{1}{a d^3} \\ & \left. + 6 e f^2 \left((c + d x) (\operatorname{Log}[1 - e^{i(c+dx)}] - \operatorname{Log}[1 + e^{i(c+dx)}]) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right] + i (\operatorname{PolyLog}[2, -e^{i(c+dx)}] - \operatorname{PolyLog}[2, e^{i(c+dx)}]) \right) \right) + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{4 a^2 d^3} b e^{-i c} f^2 \operatorname{Csc}[c] \\
& \left(2 d^2 x^2 \left(2 d e^{2 i c} x + 3 i \left(-1 + e^{2 i c} \right) \operatorname{Log}\left[1 - e^{2 i (c+d x)}\right]\right) + 6 d \left(-1 + e^{2 i c} \right) x \operatorname{PolyLog}\left[2, e^{2 i (c+d x)}\right] + 3 i \left(-1 + e^{2 i c} \right) \operatorname{PolyLog}\left[3, e^{2 i (c+d x)}\right] \right) - \\
& \frac{1}{a d^4} 6 f^3 \left(d^2 x^2 \operatorname{ArcTanh}\left[\operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] - i d x \operatorname{PolyLog}\left[2, -\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]\right] + \right. \\
& \left. i d x \operatorname{PolyLog}\left[2, \operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] + \operatorname{PolyLog}\left[3, -\operatorname{Cos}[c+d x] - i \operatorname{Sin}[c+d x]\right] - \operatorname{PolyLog}\left[3, \operatorname{Cos}[c+d x] + i \operatorname{Sin}[c+d x]\right] \right) + \\
& \frac{1}{4 a^2} b e^{i c} f^3 \operatorname{Csc}[c] \left(x^4 + \left(-1 + e^{-2 i c} \right) x^4 + 1 / \left(2 d^4 \right) e^{-2 i c} \left(-1 + e^{2 i c} \right) \right. \\
& \left. \left(2 d^4 x^4 + 4 i d^3 x^3 \operatorname{Log}\left[1 - e^{2 i (c+d x)}\right] + 6 d^2 x^2 \operatorname{PolyLog}\left[2, e^{2 i (c+d x)}\right] + 6 i d x \operatorname{PolyLog}\left[3, e^{2 i (c+d x)}\right] - 3 \operatorname{PolyLog}\left[4, e^{2 i (c+d x)}\right] \right) \right) + \\
& \frac{1}{2 a^2 b^3 d^4 \left(-1 + e^{2 i c} \right)} \left(a^2 - b^2 \right)^2 \left(-4 i d^4 e^3 e^{2 i c} x - 6 i d^4 e^2 e^{2 i c} f x^2 - 4 i d^4 e e^{2 i c} f^2 x^3 - i d^4 e^2 e^{2 i c} f^3 x^4 - \right. \\
& \left. 2 i d^3 e^3 \operatorname{ArcTan}\left[\frac{2 a e^{i (c+d x)}}{b \left(-1 + e^{2 i (c+d x)} \right)}\right] + 2 i d^3 e^3 e^{2 i c} \operatorname{ArcTan}\left[\frac{2 a e^{i (c+d x)}}{b \left(-1 + e^{2 i (c+d x)} \right)}\right] - d^3 e^3 \operatorname{Log}\left[4 a^2 e^{2 i (c+d x)} + b^2 \left(-1 + e^{2 i (c+d x)} \right)^2\right] + \right. \\
& \left. d^3 e^3 e^{2 i c} \operatorname{Log}\left[4 a^2 e^{2 i (c+d x)} + b^2 \left(-1 + e^{2 i (c+d x)} \right)^2\right] - 6 d^3 e^2 f x \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + \right. \\
& \left. 6 d^3 e^2 e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - 6 d^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + \right. \\
& \left. 6 d^3 e e^{2 i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - 2 d^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + \right. \\
& \left. 2 d^3 e^2 e^{2 i c} f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} - \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - 6 d^3 e^2 f x \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + \right. \\
& \left. 6 d^3 e^2 e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - 6 d^3 e f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + \right. \\
& \left. 6 d^3 e e^{2 i c} f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - 2 d^3 f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] + \right. \\
& \left. 2 d^3 e^2 e^{2 i c} f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{i (2 c+d x)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - 6 i d^2 \left(-1 + e^{2 i c} \right) f \left(e + f x \right)^2 \operatorname{PolyLog}\left[2, \frac{i b e^{i (2 c+d x)}}{a e^{i c} + i \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - \right. \\
& \left. 6 i d^2 \left(-1 + e^{2 i c} \right) f \left(e + f x \right)^2 \operatorname{PolyLog}\left[2, -\frac{b e^{i (2 c+d x)}}{i a e^{i c} + \sqrt{\left(-a^2 + b^2 \right) e^{2 i c}}}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 12 d e f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 12 d e e^{2ic} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] - \\
& 12 d f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 12 d e^{2ic} f^3 x \operatorname{PolyLog}\left[3, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] - \\
& 12 d e f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 12 d e e^{2ic} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - \\
& 12 d f^3 x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 12 d e^{2ic} f^3 x \operatorname{PolyLog}\left[3, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] - \\
& 12 i f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 12 i e^{2ic} f^3 \operatorname{PolyLog}\left[4, \frac{i b e^{i(2c+dx)}}{a e^{ic} + i \sqrt{(-a^2+b^2)} e^{2ic}}\right] - \\
& 12 i f^3 \operatorname{PolyLog}\left[4, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] + 12 i e^{2ic} f^3 \operatorname{PolyLog}\left[4, -\frac{b e^{i(2c+dx)}}{i a e^{ic} + \sqrt{(-a^2+b^2)} e^{2ic}}\right] \Bigg) - \\
& \frac{b e^3 \operatorname{Csc}[c] (-dx \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[dx] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[dx]]) \operatorname{Sin}[c]}{a^2 d (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} - \\
& \frac{i (-a^2 + 2b^2) e^3 x (1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])}{b^3 (-1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} - \\
& \frac{3 i (-a^2 + 2b^2) e^2 f x^2 (1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])}{2 b^3 (-1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} - \\
& \frac{i (-a^2 + 2b^2) e f^2 x^3 (1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])}{b^3 (-1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} - \\
& \frac{i (-a^2 + 2b^2) f^3 x^4 (1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])}{4 b^3 (-1 + \operatorname{Cos}[2c] + i \operatorname{Sin}[2c])} + \\
& \left(-\frac{i a f^3 x^3 \operatorname{Cos}[c]}{2 b^2 d} - \frac{a f^3 x^3 \operatorname{Sin}[c]}{2 b^2 d} + (-i d^3 e^3 - 3 d^2 e^2 f + 6 i d e f^2 + 6 f^3) \left(\frac{a \operatorname{Cos}[c]}{2 b^2 d^4} - \frac{i a \operatorname{Sin}[c]}{2 b^2 d^4} \right) + \right. \\
& \left. (a d^2 e^2 f - 2 i a d e f^2 - 2 a f^3) \left(-\frac{3 i x \operatorname{Cos}[c]}{2 b^2 d^3} - \frac{3 x \operatorname{Sin}[c]}{2 b^2 d^3} \right) + (a d e f^2 - i a f^3) \left(-\frac{3 i x^2 \operatorname{Cos}[c]}{2 b^2 d^2} - \frac{3 x^2 \operatorname{Sin}[c]}{2 b^2 d^2} \right) \right) (\operatorname{Cos}[dx] - i \operatorname{Sin}[dx]) + \\
& \left(\frac{i a f^3 x^3 \operatorname{Cos}[c]}{2 b^2 d} - \frac{a f^3 x^3 \operatorname{Sin}[c]}{2 b^2 d} + (i d^3 e^3 - 3 d^2 e^2 f - 6 i d e f^2 + 6 f^3) \left(\frac{a \operatorname{Cos}[c]}{2 b^2 d^4} + \frac{i a \operatorname{Sin}[c]}{2 b^2 d^4} \right) + \right. \\
& \left. \frac{3 i x^2 (a d e f^2 \operatorname{Cos}[c] + i a f^3 \operatorname{Cos}[c] + i a d e f^2 \operatorname{Sin}[c] - a f^3 \operatorname{Sin}[c])}{2 b^2 d^2} + \frac{1}{2 b^2 d^3} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. 3 i x \left(a d^2 e^2 f \cos [c] + 2 i a d e f^2 \cos [c] - 2 a f^3 \cos [c] + i a d^2 e^2 f \sin [c] - 2 a d e f^2 \sin [c] - 2 i a f^3 \sin [c] \right) \right) (\cos [d x] + i \sin [d x]) + \\
& \left(-\frac{f^3 x^3 \cos [2 c]}{8 b d} + \frac{i f^3 x^3 \sin [2 c]}{8 b d} + (4 d^3 e^3 - 6 i d^2 e^2 f - 6 d e f^2 + 3 i f^3) \left(-\frac{\cos [2 c]}{32 b d^4} + \frac{i \sin [2 c]}{32 b d^4} \right) + \right. \\
& \left. (2 i d^2 e^2 f + 2 d e f^2 - i f^3) \left(\frac{3 i x \cos [2 c]}{16 b d^3} + \frac{3 x \sin [2 c]}{16 b d^3} \right) + (2 i d e f^2 + f^3) \left(\frac{3 i x^2 \cos [2 c]}{16 b d^2} + \frac{3 x^2 \sin [2 c]}{16 b d^2} \right) \right) (\cos [2 d x] - i \sin [2 d x]) + \\
& \left(-\frac{f^3 x^3 \cos [2 c]}{8 b d} - \frac{i f^3 x^3 \sin [2 c]}{8 b d} + (4 d^3 e^3 + 6 i d^2 e^2 f - 6 d e f^2 - 3 i f^3) \left(-\frac{\cos [2 c]}{32 b d^4} - \frac{i \sin [2 c]}{32 b d^4} \right) - \frac{1}{16 b d^3} \right. \\
& \left. 3 i x \left(-2 i d^2 e^2 f \cos [2 c] + 2 d e f^2 \cos [2 c] + i f^3 \cos [2 c] + 2 d^2 e^2 f \sin [2 c] + 2 i d e f^2 \sin [2 c] - f^3 \sin [2 c] \right) - \right. \\
& \left. \frac{3 i x^2 \left(-2 i d e f^2 \cos [2 c] + f^3 \cos [2 c] + 2 d e f^2 \sin [2 c] + i f^3 \sin [2 c] \right)}{16 b d^2} \right) (\cos [2 d x] + i \sin [2 d x]) + \\
& \left(3 b e^2 f \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \right) / \left(\sqrt{1 + \operatorname{Tan}[c]^2} \right) \left(i d x \left(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i d x}\right] - \right. \right. \\
& \left. \left. 2 (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}\left[1 - e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c])}\right] + \pi \operatorname{Log}[\cos [d x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]] \right) + \right. \\
& \left. i \operatorname{PolyLog}\left[2, e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c])}\right] \right) \operatorname{Tan}[c] \right) / \left(2 a^2 d^2 \sqrt{\operatorname{Sec}[c]^2 (\cos [c]^2 + \sin [c]^2)} \right)
\end{aligned}$$

■ **Problem 346: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x)^2 \cos [c + d x]^3 \cot [c + d x]^2}{a + b \sin [c + d x]} dx$$

Optimal (type 4, 1051 leaves, 60 steps):

$$\begin{aligned}
& -\frac{b e f x}{2 a^2 d} - \frac{(a^2 - b^2) e f x}{2 a^2 b d} - \frac{b f^2 x^2}{4 a^2 d} - \frac{(a^2 - b^2) f^2 x^2}{4 a^2 b d} + \frac{i b (e + f x)^3}{3 a^2 f} - \frac{i (a^2 - b^2)^2 (e + f x)^3}{3 a^2 b^3 f} - \frac{4 f (e + f x) \operatorname{ArcTanh}\left[e^{i(c+dx)}\right]}{a d^2} \\
& - \frac{2 f (e + f x) \operatorname{Cos}[c + dx]}{a d^2} - \frac{2 (a^2 - b^2) f (e + f x) \operatorname{Cos}[c + dx]}{a b^2 d^2} - \frac{(e + f x)^2 \operatorname{Csc}[c + dx]}{a d} + \frac{(a^2 - b^2)^2 (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d} + \\
& - \frac{(a^2 - b^2)^2 (e + f x)^2 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d} - \frac{b (e + f x)^2 \operatorname{Log}\left[1 - e^{2i(c+dx)}\right]}{a^2 d} + \frac{2 i f^2 \operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right]}{a d^3} - \\
& - \frac{2 i f^2 \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right]}{a d^3} - \frac{2 i (a^2 - b^2)^2 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^2} - \frac{2 i (a^2 - b^2)^2 f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^2} + \\
& - \frac{i b f (e + f x) \operatorname{PolyLog}\left[2, e^{2i(c+dx)}\right]}{a^2 d^2} + \frac{2 (a^2 - b^2)^2 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^3} + \frac{2 (a^2 - b^2)^2 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^3} - \\
& - \frac{b f^2 \operatorname{PolyLog}\left[3, e^{2i(c+dx)}\right]}{2 a^2 d^3} + \frac{2 f^2 \operatorname{Sin}[c + dx]}{a d^3} + \frac{2 (a^2 - b^2) f^2 \operatorname{Sin}[c + dx]}{a b^2 d^3} - \frac{(e + f x)^2 \operatorname{Sin}[c + dx]}{a d} - \\
& - \frac{(a^2 - b^2) (e + f x)^2 \operatorname{Sin}[c + dx]}{a b^2 d} + \frac{b f (e + f x) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{2 a^2 d^2} + \frac{(a^2 - b^2) f (e + f x) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{2 a^2 b d^2} - \\
& - \frac{b f^2 \operatorname{Sin}[c + dx]^2}{4 a^2 d^3} - \frac{(a^2 - b^2) f^2 \operatorname{Sin}[c + dx]^2}{4 a^2 b d^3} + \frac{b (e + f x)^2 \operatorname{Sin}[c + dx]^2}{2 a^2 d} + \frac{(a^2 - b^2) (e + f x)^2 \operatorname{Sin}[c + dx]^2}{2 a^2 b d}
\end{aligned}$$

Result (type 4, 5228 leaves):

$$\begin{aligned}
& \frac{2 e f \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right]}{a d^2} + \frac{1}{a d^3} \\
& + 2 f^2 \left((c + dx) \left(\operatorname{Log}\left[1 - e^{i(c+dx)}\right] - \operatorname{Log}\left[1 + e^{i(c+dx)}\right] \right) - c \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right] + i \left(\operatorname{PolyLog}\left[2, -e^{i(c+dx)}\right] - \operatorname{PolyLog}\left[2, e^{i(c+dx)}\right] \right) \right) + \\
& - \frac{1}{12 a^2 d^3} b e^{-i c} f^2 \operatorname{Csc}[c] \\
& + \left(2 d^2 x^2 \left(2 d e^{2 i c} x + 3 i (-1 + e^{2 i c}) \operatorname{Log}\left[1 - e^{2 i(c+dx)}\right] \right) + 6 d (-1 + e^{2 i c}) x \operatorname{PolyLog}\left[2, e^{2 i(c+dx)}\right] + 3 i (-1 + e^{2 i c}) \operatorname{PolyLog}\left[3, e^{2 i(c+dx)}\right] \right) + \\
& - \frac{1}{6 a^2 b^3 d^3 (-1 + e^{2 i c})} (a^2 - b^2)^2 \left(-12 i d^3 e^2 e^{2 i c} x - 12 i d^3 e e^{2 i c} f x^2 - 4 i d^3 e^{2 i c} f^2 x^3 - 6 i d^2 e^2 \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b (-1 + e^{2 i(c+dx)})}\right] \right) + \\
& + 6 i d^2 e^2 e^{2 i c} \operatorname{ArcTan}\left[\frac{2 a e^{i(c+dx)}}{b (-1 + e^{2 i(c+dx)})}\right] - 3 d^2 e^2 \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 (-1 + e^{2 i(c+dx)})^2\right] + \\
& + 3 d^2 e^2 e^{2 i c} \operatorname{Log}\left[4 a^2 e^{2 i(c+dx)} + b^2 (-1 + e^{2 i(c+dx)})^2\right] - 12 d^2 e f x \operatorname{Log}\left[1 + \frac{b e^{i(2c+dx)}}{i a e^{i c} - \sqrt{(-a^2 + b^2) e^{2 i c}}}\right] +
\end{aligned}$$

$$\begin{aligned}
& 12 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} - \sqrt{-a^2 + b^2} e^{2 i c}}\right] - 6 d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} - \sqrt{-a^2 + b^2} e^{2 i c}}\right] + \\
& 6 d^2 e^2 i c f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} - \sqrt{-a^2 + b^2} e^{2 i c}}\right] - 12 d^2 e f x \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] + \\
& 12 d^2 e e^{2 i c} f x \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] - 6 d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] + \\
& 6 d^2 e^2 i c f^2 x^2 \operatorname{Log}\left[1 + \frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] - 12 i d (-1 + e^{2 i c}) f (e + f x) \operatorname{PolyLog}\left[2, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{-a^2 + b^2} e^{2 i c}}\right] - \\
& 12 i d (-1 + e^{2 i c}) f (e + f x) \operatorname{PolyLog}\left[2, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] - \\
& 12 f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{-a^2 + b^2} e^{2 i c}}\right] + 12 e^{2 i c} f^2 \operatorname{PolyLog}\left[3, \frac{i b e^{i(2 c+d x)}}{a e^{i c} + i \sqrt{-a^2 + b^2} e^{2 i c}}\right] - \\
& 12 f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] + 12 e^{2 i c} f^2 \operatorname{PolyLog}\left[3, -\frac{b e^{i(2 c+d x)}}{i a e^{i c} + \sqrt{-a^2 + b^2} e^{2 i c}}\right] \Bigg) - \\
& \frac{b e^2 \operatorname{Csc}[c] (-d x \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x]] \operatorname{Sin}[c])}{a^2 d (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} + \\
& \operatorname{Csc}[c] \operatorname{Csc}[c + d x] \left(\frac{\operatorname{Cos}[2 c + 2 d x]}{192 a b^3 d^3} - \frac{i \operatorname{Sin}[2 c + 2 d x]}{192 a b^3 d^3} \right) \\
& (-12 a b^2 d^2 e^2 \operatorname{Cos}[d x] + 12 i a b^2 d e f \operatorname{Cos}[d x] + 6 a b^2 f^2 \operatorname{Cos}[d x] + 48 i a^3 d^3 e^2 x \operatorname{Cos}[d x] - 96 i a b^2 d^3 e^2 x \operatorname{Cos}[d x] - 24 a b^2 d^2 e f x \operatorname{Cos}[d x] + \\
& 12 i a b^2 d f^2 x \operatorname{Cos}[d x] + 48 i a^3 d^3 e f x^2 \operatorname{Cos}[d x] - 96 i a b^2 d^3 e f x^2 \operatorname{Cos}[d x] - 12 a b^2 d^2 f^2 x^2 \operatorname{Cos}[d x] + 16 i a^3 d^3 f^2 x^3 \operatorname{Cos}[d x] - \\
& 32 i a b^2 d^3 f^2 x^3 \operatorname{Cos}[d x] + 12 a b^2 d^2 e^2 \operatorname{Cos}[2 c + d x] - 12 i a b^2 d e f \operatorname{Cos}[2 c + d x] - 6 a b^2 f^2 \operatorname{Cos}[2 c + d x] + 48 i a^3 d^3 e^2 x \operatorname{Cos}[2 c + d x] - \\
& 96 i a b^2 d^3 e^2 x \operatorname{Cos}[2 c + d x] + 24 a b^2 d^2 e f x \operatorname{Cos}[2 c + d x] - 12 i a b^2 d f^2 x \operatorname{Cos}[2 c + d x] + 48 i a^3 d^3 e f x^2 \operatorname{Cos}[2 c + d x] - \\
& 96 i a b^2 d^3 e f x^2 \operatorname{Cos}[2 c + d x] + 12 a b^2 d^2 f^2 x^2 \operatorname{Cos}[2 c + d x] + 16 i a^3 d^3 f^2 x^3 \operatorname{Cos}[2 c + d x] - 32 i a b^2 d^3 f^2 x^3 \operatorname{Cos}[2 c + d x] - \\
& 48 i a^2 b d^2 e^2 \operatorname{Cos}[c + 2 d x] - 96 i b^3 d^2 e^2 \operatorname{Cos}[c + 2 d x] + 96 i a^2 b f^2 \operatorname{Cos}[c + 2 d x] - 96 i a^2 b d^2 e f x \operatorname{Cos}[c + 2 d x] - \\
& 192 i b^3 d^2 e f x \operatorname{Cos}[c + 2 d x] - 48 i a^2 b d^2 f^2 x^2 \operatorname{Cos}[c + 2 d x] - 96 i b^3 d^2 f^2 x^2 \operatorname{Cos}[c + 2 d x] + 48 i a^2 b d^2 e^2 \operatorname{Cos}[3 c + 2 d x] + \\
& 96 i b^3 d^2 e^2 \operatorname{Cos}[3 c + 2 d x] - 96 i a^2 b f^2 \operatorname{Cos}[3 c + 2 d x] + 96 i a^2 b d^2 e f x \operatorname{Cos}[3 c + 2 d x] + 192 i b^3 d^2 e f x \operatorname{Cos}[3 c + 2 d x] + \\
& 48 i a^2 b d^2 f^2 x^2 \operatorname{Cos}[3 c + 2 d x] + 96 i b^3 d^2 f^2 x^2 \operatorname{Cos}[3 c + 2 d x] + 6 a b^2 d^2 e^2 \operatorname{Cos}[2 c + 3 d x] + 6 i a b^2 d e f \operatorname{Cos}[2 c + 3 d x] - \\
& 3 a b^2 f^2 \operatorname{Cos}[2 c + 3 d x] - 48 i a^3 d^3 e^2 x \operatorname{Cos}[2 c + 3 d x] + 96 i a b^2 d^3 e^2 x \operatorname{Cos}[2 c + 3 d x] + 12 a b^2 d^2 e f x \operatorname{Cos}[2 c + 3 d x] + \\
& 6 i a b^2 d f^2 x \operatorname{Cos}[2 c + 3 d x] - 48 i a^3 d^3 e f x^2 \operatorname{Cos}[2 c + 3 d x] + 96 i a b^2 d^3 e f x^2 \operatorname{Cos}[2 c + 3 d x] + 6 a b^2 d^2 f^2 x^2 \operatorname{Cos}[2 c + 3 d x] - \\
& 16 i a^3 d^3 f^2 x^3 \operatorname{Cos}[2 c + 3 d x] + 32 i a b^2 d^3 f^2 x^3 \operatorname{Cos}[2 c + 3 d x] - 6 a b^2 d^2 e^2 \operatorname{Cos}[4 c + 3 d x] - 6 i a b^2 d e f \operatorname{Cos}[4 c + 3 d x] + \\
& 3 a b^2 f^2 \operatorname{Cos}[4 c + 3 d x] - 48 i a^3 d^3 e^2 x \operatorname{Cos}[4 c + 3 d x] + 96 i a b^2 d^3 e^2 x \operatorname{Cos}[4 c + 3 d x] - 12 a b^2 d^2 e f x \operatorname{Cos}[4 c + 3 d x] - \\
& 6 i a b^2 d f^2 x \operatorname{Cos}[4 c + 3 d x] - 48 i a^3 d^3 e f x^2 \operatorname{Cos}[4 c + 3 d x] + 96 i a b^2 d^3 e f x^2 \operatorname{Cos}[4 c + 3 d x] - 6 a b^2 d^2 f^2 x^2 \operatorname{Cos}[4 c + 3 d x] - \\
& 16 i a^3 d^3 f^2 x^3 \operatorname{Cos}[4 c + 3 d x] + 32 i a b^2 d^3 f^2 x^3 \operatorname{Cos}[4 c + 3 d x] + 24 i a^2 b d^2 e^2 \operatorname{Cos}[3 c + 4 d x] - 48 a^2 b d e f \operatorname{Cos}[3 c + 4 d x] -
\end{aligned}$$

$$\begin{aligned}
& 48 i a^2 b f^2 \cos[3 c + 4 d x] + 48 i a^2 b d^2 e f x \cos[3 c + 4 d x] - 48 a^2 b d f^2 x \cos[3 c + 4 d x] + 24 i a^2 b d^2 f^2 x^2 \cos[3 c + 4 d x] - \\
& 24 i a^2 b d^2 e^2 \cos[5 c + 4 d x] + 48 a^2 b d e f \cos[5 c + 4 d x] + 48 i a^2 b f^2 \cos[5 c + 4 d x] - 48 i a^2 b d^2 e f x \cos[5 c + 4 d x] + \\
& 48 a^2 b d f^2 x \cos[5 c + 4 d x] - 24 i a^2 b d^2 f^2 x^2 \cos[5 c + 4 d x] - 6 a b^2 d^2 e^2 \cos[4 c + 5 d x] - 6 i a b^2 d e f \cos[4 c + 5 d x] + \\
& 3 a b^2 f^2 \cos[4 c + 5 d x] - 12 a b^2 d^2 e f x \cos[4 c + 5 d x] - 6 i a b^2 d f^2 x \cos[4 c + 5 d x] - 6 a b^2 d^2 f^2 x^2 \cos[4 c + 5 d x] + \\
& 6 a b^2 d^2 e^2 \cos[6 c + 5 d x] + 6 i a b^2 d e f \cos[6 c + 5 d x] - 3 a b^2 f^2 \cos[6 c + 5 d x] + 12 a b^2 d^2 e f x \cos[6 c + 5 d x] + \\
& 6 i a b^2 d f^2 x \cos[6 c + 5 d x] + 6 a b^2 d^2 f^2 x^2 \cos[6 c + 5 d x] + 48 a^2 b d^2 e^2 \sin[c] - 96 i a^2 b d e f \sin[c] - 96 a^2 b f^2 \sin[c] + \\
& 96 a^2 b d^2 e f x \sin[c] - 96 i a^2 b d f^2 x \sin[c] + 48 a^2 b d^2 f^2 x^2 \sin[c] - 48 a^3 d^3 e^2 x \sin[d x] + 96 a b^2 d^3 e^2 x \sin[d x] - 48 a^3 d^3 e f x^2 \sin[d x] + \\
& 96 a b^2 d^3 e f x^2 \sin[d x] - 16 a^3 d^3 f^2 x^3 \sin[d x] + 32 a b^2 d^3 f^2 x^3 \sin[d x] - 48 a^3 d^3 e^2 x \sin[2 c + d x] + 96 a b^2 d^3 e^2 x \sin[2 c + d x] - \\
& 48 a^3 d^3 e f x^2 \sin[2 c + d x] + 96 a b^2 d^3 e f x^2 \sin[2 c + d x] - 16 a^3 d^3 f^2 x^3 \sin[2 c + d x] + 32 a b^2 d^3 f^2 x^3 \sin[2 c + d x] + \\
& 48 a^2 b d^2 e^2 \sin[c + 2 d x] + 96 b^3 d^2 e^2 \sin[c + 2 d x] - 96 a^2 b f^2 \sin[c + 2 d x] + 96 a^2 b d^2 e f x \sin[c + 2 d x] + 192 b^3 d^2 e f x \sin[c + 2 d x] + \\
& 48 a^2 b d^2 f^2 x^2 \sin[c + 2 d x] + 96 b^3 d^2 f^2 x^2 \sin[c + 2 d x] - 48 a^2 b d^2 e^2 \sin[3 c + 2 d x] - 96 b^3 d^2 e^2 \sin[3 c + 2 d x] + 96 a^2 b f^2 \sin[3 c + 2 d x] - \\
& 96 a^2 b d^2 e f x \sin[3 c + 2 d x] - 192 b^3 d^2 e f x \sin[3 c + 2 d x] - 48 a^2 b d^2 f^2 x^2 \sin[3 c + 2 d x] - 96 b^3 d^2 f^2 x^2 \sin[3 c + 2 d x] + \\
& 6 i a b^2 d^2 e^2 \sin[2 c + 3 d x] - 6 a b^2 d e f \sin[2 c + 3 d x] - 3 i a b^2 f^2 \sin[2 c + 3 d x] + 48 a^3 d^3 e^2 x \sin[2 c + 3 d x] - \\
& 96 a b^2 d^3 e^2 x \sin[2 c + 3 d x] + 12 i a b^2 d^2 e f x \sin[2 c + 3 d x] - 6 a b^2 d f^2 x \sin[2 c + 3 d x] + 48 a^3 d^3 e f x^2 \sin[2 c + 3 d x] - \\
& 96 a b^2 d^3 e f x^2 \sin[2 c + 3 d x] + 6 i a b^2 d^2 f^2 x^2 \sin[2 c + 3 d x] + 16 a^3 d^3 f^2 x^3 \sin[2 c + 3 d x] - 32 a b^2 d^3 f^2 x^3 \sin[2 c + 3 d x] - \\
& 6 i a b^2 d^2 e^2 \sin[4 c + 3 d x] + 6 a b^2 d e f \sin[4 c + 3 d x] + 3 i a b^2 f^2 \sin[4 c + 3 d x] + 48 a^3 d^3 e^2 x \sin[4 c + 3 d x] - \\
& 96 a b^2 d^3 e^2 x \sin[4 c + 3 d x] - 12 i a b^2 d^2 e f x \sin[4 c + 3 d x] + 6 a b^2 d f^2 x \sin[4 c + 3 d x] + 48 a^3 d^3 e f x^2 \sin[4 c + 3 d x] - \\
& 96 a b^2 d^3 e f x^2 \sin[4 c + 3 d x] - 6 i a b^2 d^2 f^2 x^2 \sin[4 c + 3 d x] + 16 a^3 d^3 f^2 x^3 \sin[4 c + 3 d x] - 32 a b^2 d^3 f^2 x^3 \sin[4 c + 3 d x] - \\
& 24 a^2 b d^2 e^2 \sin[3 c + 4 d x] - 48 i a^2 b d e f \sin[3 c + 4 d x] + 48 a^2 b f^2 \sin[3 c + 4 d x] - 48 a^2 b d^2 e f x \sin[3 c + 4 d x] - \\
& 48 i a^2 b d f^2 x \sin[3 c + 4 d x] - 24 a^2 b d^2 f^2 x^2 \sin[3 c + 4 d x] + 24 a^2 b d^2 e^2 \sin[5 c + 4 d x] + 48 i a^2 b d e f \sin[5 c + 4 d x] - \\
& 48 a^2 b f^2 \sin[5 c + 4 d x] + 48 a^2 b d^2 e f x \sin[5 c + 4 d x] + 48 i a^2 b d f^2 x \sin[5 c + 4 d x] + 24 a^2 b d^2 f^2 x^2 \sin[5 c + 4 d x] - \\
& 6 i a b^2 d^2 e^2 \sin[4 c + 5 d x] + 6 a b^2 d e f \sin[4 c + 5 d x] + 3 i a b^2 f^2 \sin[4 c + 5 d x] - 12 i a b^2 d^2 e f x \sin[4 c + 5 d x] + \\
& 6 a b^2 d f^2 x \sin[4 c + 5 d x] - 6 i a b^2 d^2 f^2 x^2 \sin[4 c + 5 d x] + 6 i a b^2 d^2 e^2 \sin[6 c + 5 d x] - 6 a b^2 d e f \sin[6 c + 5 d x] - \\
& 3 i a b^2 f^2 \sin[6 c + 5 d x] + 12 i a b^2 d^2 e f x \sin[6 c + 5 d x] - 6 a b^2 d f^2 x \sin[6 c + 5 d x] + 6 i a b^2 d^2 f^2 x^2 \sin[6 c + 5 d x] \Big) + \\
& \left(b e f \operatorname{Csc}[c] \operatorname{Sec}[c] \left(d^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[c]]} x^2 + 1 \Big/ \left(\sqrt{1 + \operatorname{Tan}[c]^2} \right) \left(i d x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]]) - \pi \operatorname{Log}[1 + e^{-2 i d x}] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 (d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]) \operatorname{Log}[1 - e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c])}] \right] + \pi \operatorname{Log}[\operatorname{Cos}[d x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{Log}[\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \right] \right) + \right. \\
& \quad \left. i \operatorname{PolyLog}[2, e^{2 i (d x + \operatorname{ArcTan}[\operatorname{Tan}[c])}] \right] \Big) \Big/ \left(a^2 d^2 \sqrt{\operatorname{Sec}[c]^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2)} \right)
\end{aligned}$$

■ **Problem 347: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e + f x) \cos[c + d x]^3 \cot[c + d x]^2}{a + b \sin[c + d x]} dx$$

Optimal (type 4, 641 leaves, 45 steps):

$$\begin{aligned}
& -\frac{b f x}{4 a^2 d} - \frac{(a^2 - b^2) f x}{4 a^2 b d} + \frac{i b (e + f x)^2}{2 a^2 f} - \frac{i (a^2 - b^2)^2 (e + f x)^2}{2 a^2 b^3 f} - \frac{f \operatorname{ArcTanh}[\operatorname{Cos}[c + d x]]}{a d^2} \\
& \frac{f \operatorname{Cos}[c + d x]}{a d^2} - \frac{(a^2 - b^2) f \operatorname{Cos}[c + d x]}{a b^2 d^2} - \frac{(e + f x) \operatorname{Csc}[c + d x]}{a d} + \frac{(a^2 - b^2)^2 (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d} + \\
& \frac{(a^2 - b^2)^2 (e + f x) \operatorname{Log}\left[1 - \frac{i b e^{i(c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d} - \frac{b (e + f x) \operatorname{Log}\left[1 - e^{2 i(c+d x)}\right]}{a^2 d} - \frac{i (a^2 - b^2)^2 f \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a - \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^2} \\
& \frac{i (a^2 - b^2)^2 f \operatorname{PolyLog}\left[2, \frac{i b e^{i(c+d x)}}{a + \sqrt{a^2 - b^2}}\right]}{a^2 b^3 d^2} + \frac{i b f \operatorname{PolyLog}\left[2, e^{2 i(c+d x)}\right]}{2 a^2 d^2} - \frac{(e + f x) \operatorname{Sin}[c + d x]}{a d} - \frac{(a^2 - b^2) (e + f x) \operatorname{Sin}[c + d x]}{a b^2 d} + \\
& \frac{b f \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{4 a^2 d^2} + \frac{(a^2 - b^2) f \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{4 a^2 b d^2} + \frac{b (e + f x) \operatorname{Sin}[c + d x]^2}{2 a^2 d} + \frac{(a^2 - b^2) (e + f x) \operatorname{Sin}[c + d x]^2}{2 a^2 b d}
\end{aligned}$$

Result (type 4, 1644 leaves):

$$\begin{aligned}
& -\frac{a f \operatorname{Cos}[c + d x]}{b^2 d^2} - \frac{(d e - c f + f (c + d x)) \operatorname{Cos}[2 (c + d x)]}{4 b d^2} + \\
& \frac{(-d e \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + c f \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - f (c + d x) \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]) \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]}{2 a d^2} - \frac{b e \operatorname{Log}[\operatorname{Sin}[c + d x]]}{a^2 d} + \\
& \frac{b c f \operatorname{Log}[\operatorname{Sin}[c + d x]]}{a^2 d^2} + \frac{a^2 e \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+d x]}{a}\right]}{b^3 d} - \frac{2 e \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+d x]}{a}\right]}{b d} + \frac{b e \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+d x]}{a}\right]}{a^2 d} - \\
& \frac{a^2 c f \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+d x]}{a}\right]}{b^3 d^2} + \frac{2 c f \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+d x]}{a}\right]}{b d^2} - \frac{b c f \operatorname{Log}\left[1 + \frac{b \operatorname{Sin}[c+d x]}{a}\right]}{a^2 d^2} + \frac{f \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right]}{a d^2} - \frac{1}{d^2} \\
& 2 f \left(\frac{(c + d x) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{b} - 1 / b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - d x \right)^2 + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right)\right]}{\sqrt{a^2 - b^2}}\right] \right) + \left(-c + \frac{\pi}{2} - d x + \right. \right. \\
& \left. \left. 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(a - \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - d x)}}{b}\right] + \left(-c + \frac{\pi}{2} - d x - 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(a + \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - d x)}}{b}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-c + \frac{\pi}{2} - dx \right) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] - i \left(\operatorname{PolyLog}\left[2, -\frac{(a - \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{(a + \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] \right) \Bigg) \Bigg) + \\
& \frac{1}{b^2 d^2} a^2 f \left(\frac{(c + dx) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]]}{b} - 1/b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx \right)^2 + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right)\right]}{\sqrt{a^2 - b^2}}\right] \right) + \right. \\
& \left. \left(-c + \frac{\pi}{2} - dx + 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(a - \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] + \right. \\
& \left. \left(-c + \frac{\pi}{2} - dx - 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(a + \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] - \left(-c + \frac{\pi}{2} - dx \right) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] - \right. \\
& \left. i \left(\operatorname{PolyLog}\left[2, -\frac{(a - \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{(a + \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] \right) \right) \Bigg) \Bigg) + \frac{1}{a^2 d^2} \\
& b^2 f \left(\frac{(c + dx) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]]}{b} - 1/b \left(-\frac{1}{2} i \left(-c + \frac{\pi}{2} - dx \right)^2 + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right)\right]}{\sqrt{a^2 - b^2}}\right] \right) + \right. \\
& \left. \left(-c + \frac{\pi}{2} - dx + 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(a - \sqrt{a^2 - b^2}) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] + \right.
\end{aligned}$$

$$\left(-c + \frac{\pi}{2} - dx - 2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] - \left(-c + \frac{\pi}{2} - dx\right) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]] -$$

$$i \left(\operatorname{PolyLog}\left[2, -\frac{\left(a - \sqrt{a^2 - b^2}\right) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] + \operatorname{PolyLog}\left[2, -\frac{\left(a + \sqrt{a^2 - b^2}\right) e^{i(-c + \frac{\pi}{2} - dx)}}{b}\right] \right) \Bigg) -$$

$$\frac{bf \left((c + dx) \operatorname{Log}[1 - e^{2i(c+dx)}] - \frac{1}{2} i \left((c + dx)^2 + \operatorname{PolyLog}[2, e^{2i(c+dx)}] \right) \right)}{a^2 d^2} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \left(-de \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + cf \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] - f(c + dx) \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)}{2 a d^2} -$$

$$\frac{a(de - cf + f(c + dx)) \operatorname{Sin}[c + dx]}{b^2 d^2} +$$

$$\frac{f \operatorname{Sin}[2(c + dx)]}{8 b d^2}$$

Test results for the 72 problems in "4.1.1.1 (a+b sin)^n.m"

- **Problem 4: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 26 leaves, 1 step):

$$\frac{2 a \operatorname{Cos}[c + dx]}{d \sqrt{a + a \operatorname{Sin}[c + dx]}}$$

Result (type 3, 65 leaves):

$$\frac{2 \left(-\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)}$$

- **Problem 5: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + a \operatorname{Sin}[c + dx]}} dx$$

Optimal (type 3, 47 leaves, 2 steps) :

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 73 leaves) :

$$\frac{(2+2i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)}{d \sqrt{a} (1+\sin[c+dx])}$$

■ **Problem 6: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a \sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 77 leaves, 3 steps) :

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\cos[c+dx]}{2 d (a+a \sin[c+dx])^{3/2}}$$

Result (type 3, 108 leaves) :

$$\frac{\left(\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)\left(-\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]+(1+i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right]\right)(1+\sin[c+dx])\right)}{d (a(1+\sin[c+dx]))^{3/2}} \Bigg/ (2$$

■ **Problem 7: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 107 leaves, 4 steps) :

$$-\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\cos[c+dx]}{4 d (a+a \sin[c+dx])^{5/2}} - \frac{3 \cos[c+dx]}{16 a d (a+a \sin[c+dx])^{3/2}}$$

Result (type 3, 196 leaves) :

$$\frac{1}{16 d (a (1 + \sin[c + d x]))^{5/2}} \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) \left(8 \sin\left[\frac{1}{2}(c + d x)\right] - 4 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) \right) +$$

$$6 \sin\left[\frac{1}{2}(c + d x)\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2 - 3 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^3 +$$

$$(3 + 3 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c + d x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^4$$

- **Problem 8: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[c + d x])^{4/3} dx$$

Optimal (type 5, 67 leaves, 2 steps):

$$\frac{2 \times 2^{5/6} a \cos[c + d x] \operatorname{Hypergeometric2F1}\left[-\frac{5}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \sin[c + d x])\right] (a + a \sin[c + d x])^{1/3}}{d (1 + \sin[c + d x])^{5/6}}$$

Result (type 5, 314 leaves):

$$\frac{1}{2 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^3}$$

$$\left(-\frac{3}{2} (-5 + \cos[c + d x]) \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right) - \frac{1}{8 (1 + i e^{-i(c+d x)})^{2/3} \sqrt{1 - \sin[c + d x]}} 3 (-1)^{3/4} \right.$$

$$\left. e^{-\frac{3}{2} i(c+d x)} (i + e^{i(c+d x)}) \left(-20 e^{i(c+d x)} \sqrt{\cos\left[\frac{1}{4}(2c + \pi + 2dx)\right]^2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -i e^{-i(c+d x)}\right] + \right.$$

$$2 (1 + i e^{-i(c+d x)})^{2/3} (1 + e^{2i(c+d x)}) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \sin\left[\frac{1}{4}(2c + \pi + 2dx)\right]^2\right] - \right.$$

$$\left. \left. 5 i \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -i e^{-i(c+d x)}\right] \sqrt{2 - 2 \sin[c + d x]} \right) \right) (a (1 + \sin[c + d x]))^{4/3}$$

- **Problem 10: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[c + d x])^{1/3} dx$$

Optimal (type 5, 66 leaves, 2 steps):

$$\frac{2^{5/6} \cos[c + d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \sin[c + d x])\right] (a + a \sin[c + d x])^{1/3}}{d (1 + \sin[c + d x])^{5/6}}$$

Result (type 5, 546 leaves):

$$\frac{3 (a (1 + \sin[c + dx]))^{1/3}}{d} + \frac{1}{d (\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)])} 2\sqrt{2} (1 + \sin[c + dx])^{1/6} (a (1 + \sin[c + dx]))^{1/3}$$

$$\left(- \left[i \cos\left[\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right] \right]^{1/3} \left(- \frac{3 i \left(e^{-i\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)} + e^{i\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)} \right)^{2/3} \text{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)}\right]}{2^{2/3} \left(1 + e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)} \right)^{2/3}} - \right. \right.$$

$$\left. \frac{3 i e^{i\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)} \left(1 + e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)}\right]}{2 \times 2^{2/3} \left(e^{-i\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)} + e^{i\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)} \right)^{1/3}} \right) \Bigg/$$

$$\left(2 \left(1 + \cos\left[2\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)\right] \right)^{1/6} \right) + \left(3 \cos\left[\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right] \right)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos\left[\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right]\right]^2$$

$$\sin\left[\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right] \Bigg/ \left(5 \left(1 + \cos\left[2\left(\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right)\right] \right)^{1/6} \sqrt{\sin\left[\frac{\pi}{4} + \frac{1}{2}(-c - dx)\right]^2} \right)$$

■ **Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \sin[c + dx])^{2/3}} dx$$

Optimal (type 5, 66 leaves, 2 steps):

$$\frac{\cos[c + dx] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{6}, \frac{3}{2}, \frac{1}{2}(1 - \sin[c + dx])\right] (1 + \sin[c + dx])^{1/6}}{2^{1/6} d (a + a \sin[c + dx])^{2/3}}$$

Result (type 5, 604 leaves):

$$\begin{aligned}
& \frac{2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \left(-3 + \frac{3 \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} \right)}{d (a (1 + \sin[c+dx]))^{2/3}} \\
& \frac{1}{d (a (1 + \sin[c+dx]))^{2/3}} 2\sqrt{2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) (1 + \sin[c+dx])^{1/6} \\
& \left(- \left(i \cos\left[\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right] \right)^{1/3} \left(- \frac{3 i \left(e^{-i\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)} + e^{i\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)} \right)^{2/3} \text{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)}\right]}{2^{2/3} \left(1 + e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)} \right)^{2/3}} \right. \right. \\
& \left. \left. \frac{3 i e^{i\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)} \left(1 + e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)}\right]}{2 \times 2^{2/3} \left(e^{-i\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)} + e^{i\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)} \right)^{1/3}} \right) \right) / \\
& \left(2 \left(1 + \cos\left[2\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)\right] \right)^{1/6} \right) + \left(3 \cos\left[\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right] \right)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos\left[\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right]^2\right] \\
& \left. \sin\left[\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right] \right) / \left(5 \left(1 + \cos\left[2\left(\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right)\right] \right)^{1/6} \sqrt{\sin\left[\frac{\pi}{4} + \frac{1}{2}(-c-dx)\right]^2} \right)
\end{aligned}$$

■ **Problem 58: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sin[c + dx])^{4/3} dx$$

Optimal (type 6, 109 leaves, 3 steps):

$$\begin{aligned}
& - \left(\sqrt{2} (a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2} (1 - \sin[c+dx]), \frac{b(1 - \sin[c+dx])}{a+b}\right] \cos[c+dx] (a+b \sin[c+dx])^{1/3} \right) / \\
& \left(d \sqrt{1 + \sin[c+dx]} \left(\frac{a+b \sin[c+dx]}{a+b} \right)^{1/3} \right)
\end{aligned}$$

Result (type 6, 244 leaves):

$$\begin{aligned}
& - \frac{1}{16 b d} 3 \operatorname{Sec}[c+d x] (a+b \operatorname{Sin}[c+d x])^{1/3} \\
& \left(4 b^2 \operatorname{Cos}[c+d x]^2 + 4 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{a+b \operatorname{Sin}[c+d x]}{a-b}, \frac{a+b \operatorname{Sin}[c+d x]}{a+b}\right] \sqrt{-\frac{b(-1+\operatorname{Sin}[c+d x])}{a+b}} \sqrt{\frac{b(1+\operatorname{Sin}[c+d x])}{-a+b}} \right. \\
& \left. 5 a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, \frac{a+b \operatorname{Sin}[c+d x]}{a-b}, \frac{a+b \operatorname{Sin}[c+d x]}{a+b}\right] \sqrt{-\frac{b(-1+\operatorname{Sin}[c+d x])}{a+b}} \sqrt{\frac{b(1+\operatorname{Sin}[c+d x])}{-a+b}} (a+b \operatorname{Sin}[c+d x]) \right)
\end{aligned}$$

■ **Problem 63: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \operatorname{Sin}[c+d x])^{4/3}} dx$$

Optimal (type 6, 111 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{3}{2}, \frac{1}{2}, (1 - \operatorname{Sin}[c+d x])\right], \frac{b(1 - \operatorname{Sin}[c+d x])}{a+b} \operatorname{Cos}[c+d x] \left(\frac{a+b \operatorname{Sin}[c+d x]}{a+b}\right)^{1/3}}{(a+b) d \sqrt{1 + \operatorname{Sin}[c+d x]} (a+b \operatorname{Sin}[c+d x])^{1/3}}$$

Result (type 6, 262 leaves):

$$\begin{aligned}
& - \left(3 \operatorname{Sec}[c+d x] \right. \\
& \left(5 a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, \frac{a+b \operatorname{Sin}[c+d x]}{a-b}, \frac{a+b \operatorname{Sin}[c+d x]}{a+b}\right] \sqrt{-\frac{b(-1+\operatorname{Sin}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sin}[c+d x])}{a-b}} (a+b \operatorname{Sin}[c+d x]) \right. \\
& \left. 2 \left(5 b^2 \operatorname{Cos}[c+d x]^2 + 2 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, \frac{1}{2}, \frac{8}{3}, \frac{a+b \operatorname{Sin}[c+d x]}{a-b}, \frac{a+b \operatorname{Sin}[c+d x]}{a+b}\right] \sqrt{-\frac{b(-1+\operatorname{Sin}[c+d x])}{a+b}} \right. \right. \\
& \left. \left. \sqrt{\frac{b(1+\operatorname{Sin}[c+d x])}{-a+b}} (a+b \operatorname{Sin}[c+d x])^2 \right) \right) / (10 b (a^2 - b^2) d (a+b \operatorname{Sin}[c+d x])^{1/3})
\end{aligned}$$

Test results for the 113 problems in "4.1.11 (e x)^m (a+b x^n)^p sin.m"

- Problem 31: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sin[c + dx]}{x (a + bx)^2} dx$$

Optimal (type 4, 149 leaves, 12 steps):

$$\begin{aligned} & -\frac{d \cos\left[c - \frac{ad}{b}\right] \operatorname{CosIntegral}\left[\frac{ad}{b} + dx\right]}{ab} + \frac{\operatorname{CosIntegral}[dx] \sin[c]}{a^2} - \frac{\operatorname{CosIntegral}\left[\frac{ad}{b} + dx\right] \sin\left[c - \frac{ad}{b}\right]}{a^2} + \\ & \frac{\sin[c + dx]}{a(a + bx)} + \frac{\cos[c] \operatorname{SinIntegral}[dx]}{a^2} - \frac{\cos\left[c - \frac{ad}{b}\right] \operatorname{SinIntegral}\left[\frac{ad}{b} + dx\right]}{a^2} + \frac{d \sin\left[c - \frac{ad}{b}\right] \operatorname{SinIntegral}\left[\frac{ad}{b} + dx\right]}{ab} \end{aligned}$$

Result (type 4, 641 leaves):

$$\begin{aligned} & \frac{1}{2a^2b(a + bx)} e^{-\frac{id(2a+bx)}{b}} \left(iab e^{\frac{2iad}{b}} \cos[c] - iab e^{\frac{2id(a+bx)}{b}} \cos[c] - a^2 d e^{\frac{id(3a+bx)}{b}} \cos[c] \operatorname{ExpIntegralEi}\left[-\frac{id(a + bx)}{b}\right] - \right. \\ & ab d e^{\frac{id(3a+bx)}{b}} x \cos[c] \operatorname{ExpIntegralEi}\left[-\frac{id(a + bx)}{b}\right] - a^2 d e^{\frac{id(a+bx)}{b}} \cos[c] \operatorname{ExpIntegralEi}\left[\frac{id(a + bx)}{b}\right] - \\ & ab d e^{\frac{id(a+bx)}{b}} x \cos[c] \operatorname{ExpIntegralEi}\left[\frac{id(a + bx)}{b}\right] + a b e^{\frac{2iad}{b}} \sin[c] + a b e^{\frac{2id(a+bx)}{b}} \sin[c] + 2 b e^{\frac{id(2a+bx)}{b}} (a + bx) \operatorname{CosIntegral}[dx] \sin[c] + \\ & i a^2 d e^{\frac{id(3a+bx)}{b}} \operatorname{ExpIntegralEi}\left[-\frac{id(a + bx)}{b}\right] \sin[c] + i a b d e^{\frac{id(3a+bx)}{b}} x \operatorname{ExpIntegralEi}\left[-\frac{id(a + bx)}{b}\right] \sin[c] - \\ & i a^2 d e^{\frac{id(a+bx)}{b}} \operatorname{ExpIntegralEi}\left[\frac{id(a + bx)}{b}\right] \sin[c] - i a b d e^{\frac{id(a+bx)}{b}} x \operatorname{ExpIntegralEi}\left[\frac{id(a + bx)}{b}\right] \sin[c] - \\ & 2 b e^{\frac{id(2a+bx)}{b}} (a + bx) \operatorname{CosIntegral}\left[d\left(\frac{a}{b} + x\right)\right] \sin\left[c - \frac{ad}{b}\right] + 2 a b e^{\frac{id(2a+bx)}{b}} \cos[c] \operatorname{SinIntegral}[dx] + 2 b^2 e^{\frac{id(2a+bx)}{b}} x \cos[c] \operatorname{SinIntegral}[dx] - \\ & \left. 2 a b e^{\frac{id(2a+bx)}{b}} \cos\left[c - \frac{ad}{b}\right] \operatorname{SinIntegral}\left[d\left(\frac{a}{b} + x\right)\right] - 2 b^2 e^{\frac{id(2a+bx)}{b}} x \cos\left[c - \frac{ad}{b}\right] \operatorname{SinIntegral}\left[d\left(\frac{a}{b} + x\right)\right] \right) \end{aligned}$$

- Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sin[c + dx]}{x (a + bx)^3} dx$$

Optimal (type 4, 261 leaves, 17 steps):

$$\frac{d \cos[c + dx]}{2ab(a+bx)} - \frac{d \cos\left[c - \frac{ad}{b}\right] \operatorname{CosIntegral}\left[\frac{ad}{b} + dx\right]}{a^2 b} + \frac{\operatorname{CosIntegral}[dx] \sin[c]}{a^3} - \frac{\operatorname{CosIntegral}\left[\frac{ad}{b} + dx\right] \sin\left[c - \frac{ad}{b}\right]}{a^3} +$$

$$\frac{d^2 \operatorname{CosIntegral}\left[\frac{ad}{b} + dx\right] \sin\left[c - \frac{ad}{b}\right]}{2ab^2} + \frac{\sin[c + dx]}{2a(a+bx)^2} + \frac{\sin[c + dx]}{a^2(a+bx)} + \frac{\cos[c] \operatorname{SinIntegral}[dx]}{a^3} -$$

$$\frac{\cos\left[c - \frac{ad}{b}\right] \operatorname{SinIntegral}\left[\frac{ad}{b} + dx\right]}{a^3} + \frac{d^2 \cos\left[c - \frac{ad}{b}\right] \operatorname{SinIntegral}\left[\frac{ad}{b} + dx\right]}{2ab^2} + \frac{d \sin\left[c - \frac{ad}{b}\right] \operatorname{SinIntegral}\left[\frac{ad}{b} + dx\right]}{a^2 b}$$

Result (type 4, 2093 leaves):

$$-\frac{1}{a} 2b \cos[c] \left(\frac{1}{8b^3(a+bx)^2} i e^{-\frac{id(2a+bx)}{b}} \left(-1 + e^{\frac{2iad}{b}}\right) \left(-b \left(b \left(1 + e^{\frac{2id(a+bx)}{b}}\right) + id \left(-1 + e^{\frac{2id(a+bx)}{b}}\right) (a+bx)\right) - \right.$$

$$d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi}\left[-\frac{id(a+bx)}{b}\right] - d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi}\left[\frac{id(a+bx)}{b}\right] \Big) +$$

$$\frac{1}{8b^3(a+bx)^2} i e^{-\frac{id(2a+bx)}{b}} \left(1 + e^{\frac{2iad}{b}}\right) \left(b \left(b \left(-1 + e^{\frac{2id(a+bx)}{b}}\right) + id \left(1 + e^{\frac{2id(a+bx)}{b}}\right) (a+bx)\right) - \right.$$

$$d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi}\left[-\frac{id(a+bx)}{b}\right] + d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi}\left[\frac{id(a+bx)}{b}\right] \Big) \Big) -$$

$$\frac{1}{a^2} b^2 \cos[c] \left(\frac{1}{4b^3(a+bx)} e^{-\frac{id(2a+bx)}{b}} \left(1 + e^{\frac{2iad}{b}}\right) \left(ib \left(-1 + e^{\frac{2id(a+bx)}{b}}\right) + d e^{\frac{id(a+bx)}{b}} (a+bx) \operatorname{ExpIntegralEi}\left[-\frac{id(a+bx)}{b}\right] + \right.$$

$$d e^{\frac{id(a+bx)}{b}} (a+bx) \operatorname{ExpIntegralEi}\left[\frac{id(a+bx)}{b}\right] \Big) + \frac{1}{8b^4(a+bx)^2}$$

$$ia e^{-\frac{id(2a+bx)}{b}} \left(1 + e^{\frac{2iad}{b}}\right) \left(-b \left(b \left(-1 + e^{\frac{2id(a+bx)}{b}}\right) + id \left(1 + e^{\frac{2id(a+bx)}{b}}\right) (a+bx)\right) + d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi}\left[-\frac{id(a+bx)}{b}\right] - \right.$$

$$d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi}\left[\frac{id(a+bx)}{b}\right] \Big) + \frac{1}{8b^4(a+bx)^2}$$

$$a e^{-\frac{id(2a+bx)}{b}} \left(-1 + e^{\frac{2iad}{b}}\right) \left(b \left(ib \left(1 + e^{\frac{2id(a+bx)}{b}}\right) - d \left(-1 + e^{\frac{2id(a+bx)}{b}}\right) (a+bx)\right) + id^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi}\left[-\frac{id(a+bx)}{b}\right] + \right.$$

$$id^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi}\left[\frac{id(a+bx)}{b}\right] \Big) + \frac{1}{4b^3(a+bx)} e^{-\frac{id(2a+bx)}{b}} \left(-1 + e^{\frac{2iad}{b}}\right)$$

$$\left(d e^{\frac{id(a+bx)}{b}} (a+bx) \operatorname{ExpIntegralEi}\left[-\frac{id(a+bx)}{b}\right] - i \left(b + b e^{\frac{2id(a+bx)}{b}} - id e^{\frac{id(a+bx)}{b}} (a+bx) \operatorname{ExpIntegralEi}\left[\frac{id(a+bx)}{b}\right] \right) \right) \Big) -$$

$$\frac{1}{a^2} b^2 \left(-\frac{1}{4b^3(a+bx)} e^{-\frac{id(2a+bx)}{b}} \left(1 + e^{\frac{2iad}{b}}\right) \left(b + b e^{\frac{2id(a+bx)}{b}} + id e^{\frac{id(a+bx)}{b}} (a+bx) \operatorname{ExpIntegralEi}\left[-\frac{id(a+bx)}{b}\right] - \right.$$

$$id e^{\frac{id(a+bx)}{b}} (a+bx) \operatorname{ExpIntegralEi}\left[\frac{id(a+bx)}{b}\right] \Big) - \frac{1}{4b^3(a+bx)} e^{-\frac{id(2a+bx)}{b}} \left(-1 + e^{\frac{2iad}{b}}\right)$$

$$\begin{aligned}
& \left(b - b e^{\frac{2id(a+bx)}{b}} + id e^{\frac{id(a+bx)}{b}} (a+bx) \operatorname{ExpIntegralEi} \left[-\frac{id(a+bx)}{b} \right] + id e^{\frac{id(a+bx)}{b}} (a+bx) \operatorname{ExpIntegralEi} \left[\frac{id(a+bx)}{b} \right] \right) - \\
& \frac{1}{8b^4(a+bx)^2} a e^{-\frac{id(2a+bx)}{b}} \left(-1 + e^{\frac{2iad}{b}} \right) \left(b \left(b \left(-1 + e^{\frac{2id(a+bx)}{b}} \right) + id \left(1 + e^{\frac{2id(a+bx)}{b}} \right) (a+bx) \right) - \right. \\
& \quad \left. d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi} \left[-\frac{id(a+bx)}{b} \right] + d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi} \left[\frac{id(a+bx)}{b} \right] \right) + \frac{1}{8b^4(a+bx)^2} \\
& a e^{-\frac{id(2a+bx)}{b}} \left(1 + e^{\frac{2iad}{b}} \right) \left(b \left(b \left(1 + e^{\frac{2id(a+bx)}{b}} \right) + id \left(-1 + e^{\frac{2id(a+bx)}{b}} \right) (a+bx) \right) + d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi} \left[-\frac{id(a+bx)}{b} \right] + \right. \\
& \quad \left. d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi} \left[\frac{id(a+bx)}{b} \right] \right) \left. \right) \operatorname{Sin}[c] - \\
& \frac{1}{a} 2b \left(\frac{1}{8b^3(a+bx)^2} e^{-\frac{id(2a+bx)}{b}} \left(-1 + e^{\frac{2iad}{b}} \right) \left(b \left(b \left(-1 + e^{\frac{2id(a+bx)}{b}} \right) + id \left(1 + e^{\frac{2id(a+bx)}{b}} \right) (a+bx) \right) - \right. \right. \\
& \quad \left. \left. d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi} \left[-\frac{id(a+bx)}{b} \right] + d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi} \left[\frac{id(a+bx)}{b} \right] \right) - \frac{1}{8b^3(a+bx)^2} \right. \\
& \quad \left. e^{-\frac{id(2a+bx)}{b}} \left(1 + e^{\frac{2iad}{b}} \right) \left(b \left(b \left(1 + e^{\frac{2id(a+bx)}{b}} \right) + id \left(-1 + e^{\frac{2id(a+bx)}{b}} \right) (a+bx) \right) + d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi} \left[-\frac{id(a+bx)}{b} \right] + \right. \right. \\
& \quad \left. \left. d^2 e^{\frac{id(a+bx)}{b}} (a+bx)^2 \operatorname{ExpIntegralEi} \left[\frac{id(a+bx)}{b} \right] \right) \right) \left. \right) \operatorname{Sin}[c] + \\
& \frac{1}{2a^3} \left(2 \operatorname{CosIntegral}[dx] \operatorname{Sin}[c] - 2 \operatorname{CosIntegral} \left[\frac{ad}{b} + dx \right] \operatorname{Sin} \left[c - \frac{ad}{b} \right] + 2 \operatorname{Cos}[c] \operatorname{SinIntegral}[dx] - \right. \\
& \quad \left. 2 \operatorname{Cos} \left[c - \frac{ad}{b} \right] \operatorname{SinIntegral} \left[\frac{ad}{b} + dx \right] \right)
\end{aligned}$$

■ **Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c+dx]}{x^2(a+bx)^3} dx$$

Optimal (type 4, 299 leaves, 21 steps):

$$\begin{aligned}
& -\frac{d \operatorname{Cos}[c+dx]}{2a^2(a+bx)} + \frac{d \operatorname{Cos}[c] \operatorname{CosIntegral}[dx]}{a^3} + \frac{2d \operatorname{Cos} \left[c - \frac{ad}{b} \right] \operatorname{CosIntegral} \left[\frac{ad}{b} + dx \right]}{a^3} - \\
& \frac{3b \operatorname{CosIntegral}[dx] \operatorname{Sin}[c]}{a^4} + \frac{3b \operatorname{CosIntegral} \left[\frac{ad}{b} + dx \right] \operatorname{Sin} \left[c - \frac{ad}{b} \right]}{a^4} - \frac{d^2 \operatorname{CosIntegral} \left[\frac{ad}{b} + dx \right] \operatorname{Sin} \left[c - \frac{ad}{b} \right]}{2a^2b} - \\
& \frac{\operatorname{Sin}[c+dx]}{a^3x} - \frac{b \operatorname{Sin}[c+dx]}{2a^2(a+bx)^2} - \frac{2b \operatorname{Sin}[c+dx]}{a^3(a+bx)} - \frac{3b \operatorname{Cos}[c] \operatorname{SinIntegral}[dx]}{a^4} - \frac{d \operatorname{Sin}[c] \operatorname{SinIntegral}[dx]}{a^3} + \\
& \frac{3b \operatorname{Cos} \left[c - \frac{ad}{b} \right] \operatorname{SinIntegral} \left[\frac{ad}{b} + dx \right]}{a^4} - \frac{d^2 \operatorname{Cos} \left[c - \frac{ad}{b} \right] \operatorname{SinIntegral} \left[\frac{ad}{b} + dx \right]}{2a^2b} - \frac{2d \operatorname{Sin} \left[c - \frac{ad}{b} \right] \operatorname{SinIntegral} \left[\frac{ad}{b} + dx \right]}{a^3}
\end{aligned}$$

$$\begin{aligned}
& d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[\frac{i d (a+b x)}{b}\right] + \frac{1}{4 b^2 (a+b x)} e^{-\frac{i d (2 a+b x)}{b}} \left(-1+e^{\frac{2 i a d}{b}}\right) \\
& \left(d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[-\frac{i d (a+b x)}{b}\right] - i\left(b+b e^{\frac{2 i d (a+b x)}{b}} - i d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[\frac{i d (a+b x)}{b}\right]\right) \right) \operatorname{Sin}[c] + \\
& \frac{1}{4 a^3} (4 i b^2+a b d) \left(\operatorname{Cos}[c] \left(-\frac{1}{4 b^2 (a+b x)} e^{-\frac{i d (2 a+b x)}{b}} \left(1+e^{\frac{2 i a d}{b}}\right) \left(b+b e^{\frac{2 i d (a+b x)}{b}} + i d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[-\frac{i d (a+b x)}{b}\right] - \right.\right.\right. \\
& \quad \left.\left.\left. i d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[\frac{i d (a+b x)}{b}\right]\right) - \frac{1}{4 b^2 (a+b x)} e^{-\frac{i d (2 a+b x)}{b}} \left(-1+e^{\frac{2 i a d}{b}}\right) \right.\right. \\
& \quad \left.\left.\left(b-b e^{\frac{2 i d (a+b x)}{b}} + i d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[-\frac{i d (a+b x)}{b}\right] + i d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[\frac{i d (a+b x)}{b}\right]\right)\right) - \right. \\
& \quad \left.\left(\frac{1}{4 b^2 (a+b x)} e^{-\frac{i d (2 a+b x)}{b}} \left(1+e^{\frac{2 i a d}{b}}\right) \left(i b\left(-1+e^{\frac{2 i d (a+b x)}{b}}\right) + d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[-\frac{i d (a+b x)}{b}\right] + \right.\right.\right. \\
& \quad \left.\left.\left. d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[\frac{i d (a+b x)}{b}\right]\right) + \frac{1}{4 b^2 (a+b x)} e^{-\frac{i d (2 a+b x)}{b}} \left(-1+e^{\frac{2 i a d}{b}}\right) \right.\right. \\
& \quad \left.\left.\left(d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[-\frac{i d (a+b x)}{b}\right] - i\left(b+b e^{\frac{2 i d (a+b x)}{b}} - i d e^{\frac{i d (a+b x)}{b}} (a+b x) \operatorname{ExpIntegralEi}\left[\frac{i d (a+b x)}{b}\right]\right)\right) \right) \right) \operatorname{Sin}[c] - \\
& \frac{(2 a^2+5 a b x+2 b^2 x^2) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{2 a^3 x (a+b x)^2} + \frac{1}{2 a^4} \left(2 a d \operatorname{Cos}[c] \operatorname{CosIntegral}[d x] - \right.
\end{aligned}$$

6

b

 $\operatorname{CosIntegral}[d x]$ $\operatorname{Sin}[c] + 6$

b

 $\operatorname{CosIntegral}\left[\frac{a d}{b} + d x\right]$ $\operatorname{Sin}\left[c - \frac{a d}{b}\right] - 6$

b

 $\operatorname{Cos}[c]$ $\operatorname{SinIntegral}[d x] - 2$

a

 $d \operatorname{Sin}[c]$ $\operatorname{SinIntegral}[d x] + 6$ $b \operatorname{Cos}\left[c - \frac{a d}{b}\right]$ $\operatorname{SinIntegral}\left[\frac{a d}{b} + d x\right]$

■ **Problem 57:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \sin[c + dx]}{a + bx^2} dx$$

Optimal (type 4, 273 leaves, 14 steps):

$$\frac{2 \cos[c + dx]}{bd^3} + \frac{a \cos[c + dx]}{b^2 d} - \frac{x^2 \cos[c + dx]}{bd} - \frac{(-a)^{3/2} \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right] \sin\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2b^{5/2}} +$$

$$\frac{(-a)^{3/2} \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right] \sin\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2b^{5/2}} + \frac{2x \sin[c + dx]}{bd^2} -$$

$$\frac{(-a)^{3/2} \cos\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right]}{2b^{5/2}} - \frac{(-a)^{3/2} \cos\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right]}{2b^{5/2}}$$

Result (type 4, 275 leaves):

$$\frac{1}{2b^{5/2}d^3} \left(4b^{3/2} \cos[c + dx] + 2a\sqrt{b}d^2 \cos[c + dx] - 2b^{3/2}d^2x^2 \cos[c + dx] + \right.$$

$$\left. ia^{3/2}d^3 \operatorname{CosIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] \sin\left[c - \frac{i\sqrt{a}d}{\sqrt{b}}\right] - ia^{3/2}d^3 \operatorname{CosIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] \sin\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] + \right.$$

$$\left. 4b^{3/2}dx \sin[c + dx] + ia^{3/2}d^3 \cos\left[c - \frac{i\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] + ia^{3/2}d^3 \cos\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}} - dx\right] \right)$$

■ **Problem 58: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^3 \sin[c + dx]}{a + bx^2} dx$$

Optimal (type 4, 209 leaves, 12 steps):

$$\frac{x \cos[c + dx]}{bd} - \frac{a \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right] \sin\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2b^2} - \frac{a \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right] \sin\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2b^2} +$$

$$\frac{\sin[c + dx]}{bd^2} + \frac{a \cos\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right]}{2b^2} - \frac{a \cos\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right]}{2b^2}$$

Result (type 4, 202 leaves):

$$-\frac{1}{2b^2d^2} \left(2bdx \cos[c + dx] + ad^2 \operatorname{CosIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] \sin\left[c - \frac{i\sqrt{a}d}{\sqrt{b}}\right] + ad^2 \operatorname{CosIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] \sin\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] - \right.$$

$$\left. 2b \sin[c + dx] + ad^2 \cos\left[c - \frac{i\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] - ad^2 \cos\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}} - dx\right] \right)$$

■ **Problem 59: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 \operatorname{Sin}[c + d x]}{a + b x^2} dx$$

Optimal (type 4, 227 leaves, 11 steps):

$$\frac{\operatorname{Cos}[c + d x]}{b d} - \frac{\sqrt{-a} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 b^{3/2}} + \frac{\sqrt{-a} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 b^{3/2}} - \frac{\sqrt{-a} \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 b^{3/2}} - \frac{\sqrt{-a} \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 b^{3/2}}$$

Result (type 4, 216 leaves):

$$-\frac{1}{2 b^{3/2} d} \left(2 \sqrt{b} \operatorname{Cos}[c + d x] + i \sqrt{a} d \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] - i \sqrt{a} d \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + i \sqrt{a} d \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + i \sqrt{a} d \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right)$$

■ **Problem 60: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x \operatorname{Sin}[c + d x]}{a + b x^2} dx$$

Optimal (type 4, 177 leaves, 8 steps):

$$\frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 b} + \frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 b} - \frac{\operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 b} + \frac{\operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 b}$$

Result (type 4, 163 leaves):

$$\frac{1}{2 b} \left(\operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] + \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right)$$

■ **Problem 61: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[c + d x]}{a + b x^2} dx$$

Optimal (type 4, 213 leaves, 8 steps) :

$$\frac{\text{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right] \text{Sin}\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2\sqrt{-a}\sqrt{b}} + \frac{\text{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right] \text{Sin}\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2\sqrt{-a}\sqrt{b}} - \frac{\text{Cos}\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right]}{2\sqrt{-a}\sqrt{b}} - \frac{\text{Cos}\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right]}{2\sqrt{-a}\sqrt{b}}$$

Result (type 4, 172 leaves) :

$$\frac{1}{2\sqrt{a}\sqrt{b}} i \left(\text{CosIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] \text{Sin}\left[c - \frac{i\sqrt{a}d}{\sqrt{b}}\right] - \text{CosIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] \text{Sin}\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] + \text{Cos}\left[c - \frac{i\sqrt{a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] + \text{Cos}\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}} - dx\right] \right)$$

■ **Problem 62: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sin}[c + dx]}{x(a + bx^2)} dx$$

Optimal (type 4, 197 leaves, 13 steps) :

$$\frac{\text{CosIntegral}[dx] \text{Sin}[c]}{a} - \frac{\text{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right] \text{Sin}\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2a} - \frac{\text{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right] \text{Sin}\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2a} + \frac{\text{Cos}[c] \text{SinIntegral}[dx]}{a} + \frac{\text{Cos}\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right]}{2a} - \frac{\text{Cos}\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right]}{2a}$$

Result (type 4, 179 leaves) :

$$-\frac{1}{2a} \left(-2 \text{CosIntegral}[dx] \text{Sin}[c] + \text{CosIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] \text{Sin}\left[c - \frac{i\sqrt{a}d}{\sqrt{b}}\right] + \text{CosIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] \text{Sin}\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] - 2 \text{Cos}[c] \text{SinIntegral}[dx] + \text{Cos}\left[c - \frac{i\sqrt{a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}} + x\right)\right] - \text{Cos}\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}} - dx\right] \right)$$

■ **Problem 63: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sin}[c + dx]}{x^2(a + bx^2)} dx$$

Optimal (type 4, 250 leaves, 14 steps) :

$$\frac{d \operatorname{Cos}[c] \operatorname{CosIntegral}[d x]}{a} - \frac{\sqrt{b} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2(-a)^{3/2}} + \frac{\sqrt{b} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2(-a)^{3/2}} - \frac{\operatorname{Sin}[c + d x]}{a x} -$$

$$\frac{d \operatorname{Sin}[c] \operatorname{SinIntegral}[d x]}{a} - \frac{\sqrt{b} \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2(-a)^{3/2}} - \frac{\sqrt{b} \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2(-a)^{3/2}}$$

Result (type 4, 238 leaves):

$$\frac{d \operatorname{Cos}[c] \operatorname{CosIntegral}[d x]}{a} - \frac{1}{2 a^{3/2} x}$$

$$i \left(\sqrt{b} x \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] - \sqrt{b} x \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] - 2 i \sqrt{a} \operatorname{Sin}[c + d x] - 2 i \sqrt{a} d x \right.$$

$$\left. \operatorname{Sin}[c] \operatorname{SinIntegral}[d x] + \sqrt{b} x \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \sqrt{b} x \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right)$$

■ **Problem 64: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[c + d x]}{x^3 (a + b x^2)} dx$$

Optimal (type 4, 270 leaves, 18 steps):

$$-\frac{d \operatorname{Cos}[c + d x]}{2 a x} - \frac{b \operatorname{CosIntegral}[d x] \operatorname{Sin}[c]}{a^2} - \frac{d^2 \operatorname{CosIntegral}[d x] \operatorname{Sin}[c]}{2 a} + \frac{b \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 a^2} +$$

$$\frac{b \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 a^2} - \frac{\operatorname{Sin}[c + d x]}{2 a x^2} - \frac{b \operatorname{Cos}[c] \operatorname{SinIntegral}[d x]}{a^2} -$$

$$\frac{d^2 \operatorname{Cos}[c] \operatorname{SinIntegral}[d x]}{2 a} - \frac{b \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 a^2} + \frac{b \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 a^2}$$

Result (type 4, 247 leaves):

$$-\frac{1}{2 a^2 x^2} \left(a d x \operatorname{Cos}[c + d x] + (2 b + a d^2) x^2 \operatorname{CosIntegral}[d x] \operatorname{Sin}[c] - b x^2 \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] -$$

$$b x^2 \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + a \operatorname{Sin}[c + d x] + 2 b x^2 \operatorname{Cos}[c] \operatorname{SinIntegral}[d x] + a d^2 x^2 \operatorname{Cos}[c] \operatorname{SinIntegral}[d x] -$$

$$b x^2 \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + b x^2 \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right)$$

■ **Problem 65: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^4 \operatorname{Sin}[c + d x]}{(a + b x^2)^2} dx$$

Optimal (type 4, 450 leaves, 24 steps):

$$\begin{aligned} & - \frac{\operatorname{Cos}[c + d x]}{b^2 d} - \frac{a d \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{4 b^3} - \frac{a d \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{4 b^3} \\ & - \frac{3 \sqrt{-a} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 b^{5/2}} + \frac{3 \sqrt{-a} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 b^{5/2}} + \frac{x \operatorname{Sin}[c + d x]}{2 b^2} \\ & - \frac{x^3 \operatorname{Sin}[c + d x]}{2 b (a + b x^2)} - \frac{3 \sqrt{-a} \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{4 b^{5/2}} - \frac{a d \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{4 b^3} \\ & - \frac{3 \sqrt{-a} \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{4 b^{5/2}} + \frac{a d \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{4 b^3} \end{aligned}$$

Result (type 4, 632 leaves):

$$\begin{aligned} & - \frac{1}{4 b^3 d (a + b x^2)} \\ & \left(4 a b \operatorname{Cos}[c + d x] + 4 b^2 x^2 \operatorname{Cos}[c + d x] + \sqrt{a} d (a + b x^2) \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \left(\sqrt{a} d \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] + 3 i \sqrt{b} \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) + \right. \\ & \left. \sqrt{a} d (a + b x^2) \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \left(\sqrt{a} d \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] - 3 i \sqrt{b} \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) - 2 a b d x \operatorname{Sin}[c + d x] + \right. \\ & \left. 3 i a^{3/2} \sqrt{b} d \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + 3 i \sqrt{a} b^{3/2} d x^2 \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \right. \\ & \left. a^2 d^2 \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - a b d^2 x^2 \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \right. \\ & \left. 3 i a^{3/2} \sqrt{b} d \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + 3 i \sqrt{a} b^{3/2} d x^2 \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + \right. \\ & \left. a^2 d^2 \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + a b d^2 x^2 \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \end{aligned}$$

■ **Problem 66: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^3 \operatorname{Sin}[c + dx]}{(a + bx^2)^2} dx$$

Optimal (type 4, 431 leaves, 20 steps):

$$\frac{\sqrt{-a} d \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{4 b^{5/2}} - \frac{\sqrt{-a} d \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{4 b^{5/2}} + \frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 b^2} +$$

$$\frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 b^2} + \frac{\operatorname{Sin}[c + dx]}{2 b^2} - \frac{x^2 \operatorname{Sin}[c + dx]}{2 b (a + bx^2)} - \frac{\operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{2 b^2} +$$

$$\frac{\sqrt{-a} d \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{4 b^{5/2}} + \frac{\operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{2 b^2} + \frac{\sqrt{-a} d \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{4 b^{5/2}}$$

Result (type 4, 583 leaves):

$$\frac{1}{4 b^{5/2} (a + bx^2)} \left((a + bx^2) \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \left(-i \sqrt{a} d \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] + 2 \sqrt{b} \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) + \right.$$

$$(a + bx^2) \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \left(i \sqrt{a} d \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + 2 \sqrt{b} \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) + 2 a \sqrt{b} \operatorname{Sin}[c + dx] +$$

$$2 a \sqrt{b} \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + 2 b^{3/2} x^2 \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] +$$

$$i a^{3/2} d \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + i \sqrt{a} b d x^2 \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] -$$

$$2 a \sqrt{b} \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx\right] - 2 b^{3/2} x^2 \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx\right] +$$

$$i a^{3/2} d \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx\right] + i \sqrt{a} b d x^2 \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx\right] \left. \right)$$

■ **Problem 67: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 \operatorname{Sin}[c + dx]}{(a + bx^2)^2} dx$$

Optimal (type 4, 416 leaves, 17 steps):

$$\frac{d \cos\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{4 b^2} + \frac{d \cos\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{4 b^2} - \frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right] \sin\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 \sqrt{-a} b^{3/2}} +$$

$$\frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right] \sin\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 \sqrt{-a} b^{3/2}} - \frac{x \sin[c + dx]}{2 b (a + b x^2)} - \frac{\cos\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{4 \sqrt{-a} b^{3/2}} +$$

$$\frac{d \sin\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{4 b^2} - \frac{\cos\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{4 \sqrt{-a} b^{3/2}} - \frac{d \sin\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{4 b^2}$$

Result (type 4, 583 leaves):

$$\frac{1}{4 \sqrt{a} b^2 (a + b x^2)} \left((a + b x^2) \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \left(\sqrt{a} d \cos\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] + i \sqrt{b} \sin\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) + \right.$$

$$\left. (a + b x^2) \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \left(\sqrt{a} d \cos\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] - i \sqrt{b} \sin\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) - 2 \sqrt{a} b x \sin[c + dx] + \right.$$

$$i a \sqrt{b} \cos\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + i b^{3/2} x^2 \cos\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] -$$

$$a^{3/2} d \sin\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \sqrt{a} b d x^2 \sin\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] +$$

$$i a \sqrt{b} \cos\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx\right] + i b^{3/2} x^2 \cos\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx\right] +$$

$$a^{3/2} d \sin\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx\right] + \sqrt{a} b d x^2 \sin\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx\right] \left. \right)$$

■ **Problem 68: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x \sin[c + dx]}{(a + b x^2)^2} dx$$

Optimal (type 4, 239 leaves, 9 steps):

$$\frac{d \cos\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{4 \sqrt{-a} b^{3/2}} - \frac{d \cos\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{4 \sqrt{-a} b^{3/2}} -$$

$$\frac{\sin[c + dx]}{2 b (a + b x^2)} + \frac{d \sin\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{4 \sqrt{-a} b^{3/2}} + \frac{d \sin\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{4 \sqrt{-a} b^{3/2}}$$

Result (type 4, 309 leaves) :

$$\begin{aligned}
 & - \frac{1}{4 \sqrt{a} b^{3/2} (a + b x^2)} i \left(d (a + b x^2) \cos \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] - d (a + b x^2) \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] - \right. \\
 & \quad \left. 2 i \sqrt{a} \sqrt{b} \sin [c + d x] + a d \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + b d x^2 \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + \right. \\
 & \quad \left. a d \sin \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x \right] + b d x^2 \sin \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x \right] \right)
 \end{aligned}$$

■ **Problem 69: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin [c + d x]}{(a + b x^2)^2} dx$$

Optimal (type 4, 476 leaves, 18 steps) :

$$\begin{aligned}
 & - \frac{d \cos \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right]}{4 a b} - \frac{d \cos \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right]}{4 a b} + \frac{\operatorname{CosIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right] \sin \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right]}{4 (-a)^{3/2} \sqrt{b}} - \\
 & \frac{\operatorname{CosIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right] \sin \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right]}{4 (-a)^{3/2} \sqrt{b}} - \frac{\sin [c + d x]}{4 a \sqrt{b} (\sqrt{-a} - \sqrt{b} x)} + \frac{\sin [c + d x]}{4 a \sqrt{b} (\sqrt{-a} + \sqrt{b} x)} + \frac{\cos \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right]}{4 (-a)^{3/2} \sqrt{b}} - \\
 & \frac{d \sin \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right]}{4 a b} + \frac{\cos \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right]}{4 (-a)^{3/2} \sqrt{b}} + \frac{d \sin \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right]}{4 a b}
 \end{aligned}$$

Result (type 4, 585 leaves) :

$$\begin{aligned} & \frac{1}{4 a^{3/2} b (a + b x^2)} \left(- (a + b x^2) \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] \left(\sqrt{a} d \operatorname{Cos} \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] - i \sqrt{b} \operatorname{Sin} \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \right) - \right. \\ & (a + b x^2) \operatorname{CosIntegral} \left[d \left(- \frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] \left(\sqrt{a} d \operatorname{Cos} \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] + i \sqrt{b} \operatorname{Sin} \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \right) + 2 \sqrt{a} b x \operatorname{Sin} [c + d x] + \\ & i a \sqrt{b} \operatorname{Cos} \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + i b^{3/2} x^2 \operatorname{Cos} \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + \\ & a^{3/2} d \operatorname{Sin} \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + \sqrt{a} b d x^2 \operatorname{Sin} \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + \\ & i a \sqrt{b} \operatorname{Cos} \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x \right] + i b^{3/2} x^2 \operatorname{Cos} \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x \right] - \\ & a^{3/2} d \operatorname{Sin} \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x \right] - \sqrt{a} b d x^2 \operatorname{Sin} \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x \right] \left. \right) \end{aligned}$$

■ **Problem 70: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin} [c + d x]}{x (a + b x^2)^2} dx$$

Optimal (type 4, 435 leaves, 22 steps):

$$\begin{aligned} & \frac{d \operatorname{Cos} \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right]}{4 (-a)^{3/2} \sqrt{b}} - \frac{d \operatorname{Cos} \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right]}{4 (-a)^{3/2} \sqrt{b}} + \frac{\operatorname{CosIntegral} [d x] \operatorname{Sin} [c]}{a^2} - \\ & \frac{\operatorname{CosIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right] \operatorname{Sin} \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right]}{2 a^2} - \frac{\operatorname{CosIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right] \operatorname{Sin} \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right]}{2 a^2} + \frac{\operatorname{Sin} [c + d x]}{2 a (a + b x^2)} + \\ & \frac{\operatorname{Cos} [c] \operatorname{SinIntegral} [d x]}{a^2} + \frac{\operatorname{Cos} \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right]}{2 a^2} + \frac{d \operatorname{Sin} \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right]}{4 (-a)^{3/2} \sqrt{b}} - \\ & \frac{\operatorname{Cos} \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right]}{2 a^2} + \frac{d \operatorname{Sin} \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right]}{4 (-a)^{3/2} \sqrt{b}} \end{aligned}$$

Result (type 4, 650 leaves):

$$\frac{1}{4 a^2 \sqrt{b} (a + b x^2)} \left(4 a \sqrt{b} \operatorname{CosIntegral}[d x] \operatorname{Sin}[c] + 4 b^{3/2} x^2 \operatorname{CosIntegral}[d x] \operatorname{Sin}[c] - \right.$$

$$i (a + b x^2) \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \left(\sqrt{a} d \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] - 2 i \sqrt{b} \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right]\right) +$$

$$i (a + b x^2) \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \left(\sqrt{a} d \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + 2 i \sqrt{b} \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right]\right) +$$

$$2 a \sqrt{b} \operatorname{Sin}[c + d x] + 4 a \sqrt{b} \operatorname{Cos}[c] \operatorname{SinIntegral}[d x] + 4 b^{3/2} x^2 \operatorname{Cos}[c] \operatorname{SinIntegral}[d x] -$$

$$2 a \sqrt{b} \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - 2 b^{3/2} x^2 \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] +$$

$$i a^{3/2} d \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + i \sqrt{a} b d x^2 \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] +$$

$$2 a \sqrt{b} \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + 2 b^{3/2} x^2 \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] +$$

$$i a^{3/2} d \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + i \sqrt{a} b d x^2 \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \Big)$$

■ **Problem 71: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[c + d x]}{x^2 (a + b x^2)^2} dx$$

Optimal (type 4, 501 leaves, 32 steps):

$$\frac{d \operatorname{Cos}[c] \operatorname{CosIntegral}[d x]}{a^2} + \frac{d \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{4 a^2} + \frac{d \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{4 a^2} +$$

$$\frac{3 \sqrt{b} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 (-a)^{5/2}} - \frac{3 \sqrt{b} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{4 (-a)^{5/2}} - \frac{\operatorname{Sin}[c + d x]}{a^2 x} +$$

$$\frac{\sqrt{b} \operatorname{Sin}[c + d x]}{4 a^2 (\sqrt{-a} - \sqrt{b} x)} - \frac{\sqrt{b} \operatorname{Sin}[c + d x]}{4 a^2 (\sqrt{-a} + \sqrt{b} x)} - \frac{d \operatorname{Sin}[c] \operatorname{SinIntegral}[d x]}{a^2} + \frac{3 \sqrt{b} \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{4 (-a)^{5/2}} +$$

$$\frac{d \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{4 a^2} + \frac{3 \sqrt{b} \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{4 (-a)^{5/2}} - \frac{d \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{4 a^2}$$

Result (type 4, 768 leaves):

$$\begin{aligned}
& \frac{1}{4 a^{5/2} x (a + b x^2)} \left(4 \sqrt{a} d x (a + b x^2) \operatorname{Cos}[c] \operatorname{CosIntegral}[d x] + \right. \\
& a^{3/2} d x \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \sqrt{a} b d x^3 \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 3 i a \sqrt{b} x \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] - 3 i b^{3/2} x^3 \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] + \\
& x (a + b x^2) \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \left(\sqrt{a} d \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + 3 i \sqrt{b} \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) - 4 a^{3/2} \operatorname{Sin}[c + d x] - \\
& 6 \sqrt{a} b x^2 \operatorname{Sin}[c + d x] - 4 a^{3/2} d x \operatorname{Sin}[c] \operatorname{SinIntegral}[d x] - 4 \sqrt{a} b d x^3 \operatorname{Sin}[c] \operatorname{SinIntegral}[d x] - \\
& 3 i a \sqrt{b} x \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - 3 i b^{3/2} x^3 \operatorname{Cos}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& a^{3/2} d x \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \sqrt{a} b d x^3 \operatorname{Sin}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 3 i a \sqrt{b} x \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] - 3 i b^{3/2} x^3 \operatorname{Cos}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + \\
& a^{3/2} d x \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + \sqrt{a} b d x^3 \operatorname{Sin}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \left. \right)
\end{aligned}$$

■ **Problem 72: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^3 \operatorname{Sin}[c + d x]}{(a + b x^2)^3} dx$$

Optimal (type 4, 476 leaves, 27 steps):

$$\begin{aligned}
& \frac{d x \cos [c+d x]}{8 b^2 (a+b x^2)} + \frac{3 d \cos \left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16 \sqrt{-a} b^{5 / 2}} - \frac{3 d \cos \left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16 \sqrt{-a} b^{5 / 2}} \\
& \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right] \sin \left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 b^3} - \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right] \sin \left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 b^3} - \frac{x^2 \sin [c+d x]}{4 b (a+b x^2)^2} \\
& \frac{\sin [c+d x]}{4 b^2 (a+b x^2)} + \frac{d^2 \cos \left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16 b^3} + \frac{3 d \sin \left[c+\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}-d x\right]}{16 \sqrt{-a} b^{5 / 2}} \\
& \frac{d^2 \cos \left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16 b^3} + \frac{3 d \sin \left[c-\frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}}+d x\right]}{16 \sqrt{-a} b^{5 / 2}}
\end{aligned}$$

Result (type 4, 647 leaves):

$$\begin{aligned}
& \frac{1}{16 b^2} \left(-\frac{2 \cos [d x] (d x (a+b x^2) \cos [c]+2 (a+2 b x^2) \sin [c])}{(a+b x^2)^2} + \right. \\
& \frac{2 (-2 (a+2 b x^2) \cos [c]+d x (a+b x^2) \sin [c]) \sin [d x]}{(a+b x^2)^2} + \frac{1}{b} d^2 \cos [c] \left(-i \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] + \right. \\
& \left. i \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] + \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \left(-\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x \right] \right) \right) \right) + \\
& \frac{1}{\sqrt{a} \sqrt{b}} 3 d \cos [c] \left(-i \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] + i \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] + \right. \\
& \left. \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \left(-\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x \right] \right) \right) - \\
& \frac{1}{\sqrt{a} \sqrt{b}} 3 d \sin [c] \left(\operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] + \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] + \right. \\
& \left. i \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x \right] \right) \right) - \\
& \frac{1}{b} d^2 \sin [c] \left(\operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] + \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] + \right. \\
& \left. i \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}}+x \right) \right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x \right] \right) \right) \right)
\end{aligned}$$

■ **Problem 73: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 \operatorname{Sin}[c + d x]}{(a + b x^2)^3} dx$$

Optimal (type 4, 746 leaves, 28 steps):

$$\begin{aligned} & -\frac{d \operatorname{Cos}[c + d x]}{8 b^2 (a + b x^2)} - \frac{d \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 a b^2} - \frac{d \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 a b^2} + \\ & \frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{3/2} b^{3/2}} + \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 \sqrt{-a} b^{5/2}} - \frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{3/2} b^{3/2}} - \\ & \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 \sqrt{-a} b^{5/2}} - \frac{\operatorname{Sin}[c + d x]}{16 a b^{3/2} (\sqrt{-a} - \sqrt{b} x)} + \frac{\operatorname{Sin}[c + d x]}{16 a b^{3/2} (\sqrt{-a} + \sqrt{b} x)} - \frac{x \operatorname{Sin}[c + d x]}{4 b (a + b x^2)^2} + \\ & \frac{\operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 (-a)^{3/2} b^{3/2}} + \frac{d^2 \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 \sqrt{-a} b^{5/2}} - \frac{d \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 a b^2} + \\ & \frac{\operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 (-a)^{3/2} b^{3/2}} + \frac{d^2 \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 \sqrt{-a} b^{5/2}} + \frac{d \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 a b^2} \end{aligned}$$

Result (type 4, 927 leaves):

$$\begin{aligned}
& \frac{1}{16 a^{3/2} b^2} \left(-\frac{2 a^{5/2} d \cos[c] \cos[dx]}{(a+bx^2)^2} - \frac{2 a^{3/2} b d x^2 \cos[c] \cos[dx]}{(a+bx^2)^2} - \frac{2 a^{3/2} b x \cos[dx] \sin[c]}{(a+bx^2)^2} + \right. \\
& \frac{2 \sqrt{a} b^2 x^3 \cos[dx] \sin[c]}{(a+bx^2)^2} + \frac{\operatorname{CosIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] \left(-\sqrt{a} \sqrt{b} d \cos\left[c-\frac{i\sqrt{a} d}{\sqrt{b}}\right] + i(b-a d^2) \sin\left[c-\frac{i\sqrt{a} d}{\sqrt{b}}\right]\right)}{\sqrt{b}} + \\
& \frac{i \operatorname{CosIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] \left(i \sqrt{a} \sqrt{b} d \cos\left[c+\frac{i\sqrt{a} d}{\sqrt{b}}\right] + (-b+a d^2) \sin\left[c+\frac{i\sqrt{a} d}{\sqrt{b}}\right]\right)}{\sqrt{b}} - \\
& \frac{2 a^{3/2} b x \cos[c] \sin[dx]}{(a+bx^2)^2} + \frac{2 \sqrt{a} b^2 x^3 \cos[c] \sin[dx]}{(a+bx^2)^2} + \frac{2 a^{5/2} d \sin[c] \sin[dx]}{(a+bx^2)^2} + \frac{2 a^{3/2} b d x^2 \sin[c] \sin[dx]}{(a+bx^2)^2} + \\
& i \sqrt{b} \cos[c] \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] - \frac{i a d^2 \cos[c] \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]}{\sqrt{b}} + \\
& \sqrt{a} d \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \sin[c] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] - i \sqrt{a} d \cos[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] - \\
& \sqrt{b} \sin[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] + \frac{a d^2 \sin[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right]}{\sqrt{b}} + \\
& i \sqrt{b} \cos[c] \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a} d}{\sqrt{b}}-dx\right] - \frac{i a d^2 \cos[c] \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a} d}{\sqrt{b}}-dx\right]}{\sqrt{b}} - \\
& \sqrt{a} d \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \sin[c] \operatorname{SinIntegral}\left[\frac{i\sqrt{a} d}{\sqrt{b}}-dx\right] - i \sqrt{a} d \cos[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a} d}{\sqrt{b}}-dx\right] + \\
& \left. \sqrt{b} \sin[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a} d}{\sqrt{b}}-dx\right] - \frac{a d^2 \sin[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a} d}{\sqrt{b}}-dx\right]}{\sqrt{b}} \right)
\end{aligned}$$

■ **Problem 74: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x \sin[c+dx]}{(a+bx^2)^3} dx$$

Optimal (type 4, 512 leaves, 19 steps):

$$\begin{aligned}
& - \frac{d \operatorname{Cos}[c + d x]}{16 a b^{3/2} (\sqrt{-a} - \sqrt{b} x)} + \frac{d \operatorname{Cos}[c + d x]}{16 a b^{3/2} (\sqrt{-a} + \sqrt{b} x)} - \frac{d \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 (-a)^{3/2} b^{3/2}} + \\
& \frac{d \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 (-a)^{3/2} b^{3/2}} + \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 a b^2} + \\
& \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 a b^2} - \frac{\operatorname{Sin}[c + d x]}{4 b (a + b x^2)^2} - \frac{d^2 \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 a b^2} - \\
& \frac{d \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 (-a)^{3/2} b^{3/2}} + \frac{d^2 \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 a b^2} - \frac{d \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 (-a)^{3/2} b^{3/2}}
\end{aligned}$$

Result (type 4, 634 leaves):

$$\begin{aligned}
& \frac{1}{16 a b} \left(\frac{2 \operatorname{Cos}[d x] (d x (a + b x^2) \operatorname{Cos}[c] - 2 a \operatorname{Sin}[c])}{(a + b x^2)^2} - \right. \\
& \frac{2 (2 a \operatorname{Cos}[c] + d x (a + b x^2) \operatorname{Sin}[c]) \operatorname{Sin}[d x]}{(a + b x^2)^2} + \frac{1}{b} d^2 \operatorname{Cos}[c] \left(i \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] - \right. \\
& \left. i \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) \right) + \\
& \frac{1}{\sqrt{a} \sqrt{b}} d \operatorname{Cos}[c] \left(-i \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + i \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \right. \\
& \left. \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(-\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) - \\
& \frac{1}{\sqrt{a} \sqrt{b}} d \operatorname{Sin}[c] \left(\operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \right. \\
& \left. i \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) + \\
& \frac{1}{b} d^2 \operatorname{Sin}[c] \left(\operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \right. \\
& \left. i \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) \right)
\end{aligned}$$

■ **Problem 75: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]}{(a + bx^2)^3} dx$$

Optimal (type 4, 856 leaves, 28 steps):

$$\begin{aligned} & \frac{d \cos[c + dx]}{16 (-a)^{3/2} b (\sqrt{-a} - \sqrt{b} x)} + \frac{d \cos[c + dx]}{16 (-a)^{3/2} b (\sqrt{-a} + \sqrt{b} x)} - \frac{3 d \cos\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{16 a^2 b} - \\ & \frac{3 d \cos\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{16 a^2 b} - \frac{3 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right] \sin\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{5/2} \sqrt{b}} + \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right] \sin\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{3/2} b^{3/2}} + \\ & \frac{3 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right] \sin\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{5/2} \sqrt{b}} - \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right] \sin\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{3/2} b^{3/2}} - \frac{\sin[c + dx]}{16 (-a)^{3/2} \sqrt{b} (\sqrt{-a} - \sqrt{b} x)^2} - \\ & \frac{3 \sin[c + dx]}{16 a^2 \sqrt{b} (\sqrt{-a} - \sqrt{b} x)} + \frac{\sin[c + dx]}{16 (-a)^{3/2} \sqrt{b} (\sqrt{-a} + \sqrt{b} x)^2} + \frac{3 \sin[c + dx]}{16 a^2 \sqrt{b} (\sqrt{-a} + \sqrt{b} x)} - \frac{3 \cos\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{16 (-a)^{5/2} \sqrt{b}} + \\ & \frac{d^2 \cos\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{16 (-a)^{3/2} b^{3/2}} - \frac{3 d \sin\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - dx\right]}{16 a^2 b} - \\ & \frac{3 \cos\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{16 (-a)^{5/2} \sqrt{b}} + \frac{d^2 \cos\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{16 (-a)^{3/2} b^{3/2}} + \frac{3 d \sin\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + dx\right]}{16 a^2 b} \end{aligned}$$

Result (type 4, 932 leaves):

$$\begin{aligned}
& \frac{1}{16 a^2 b^{3/2}} \left(\frac{2 a^2 \sqrt{b} d \operatorname{Cos}[c] \operatorname{Cos}[d x]}{(a+b x^2)^2} + \frac{2 a b^{3/2} d x^2 \operatorname{Cos}[c] \operatorname{Cos}[d x]}{(a+b x^2)^2} + \frac{10 a b^{3/2} x \operatorname{Cos}[d x] \operatorname{Sin}[c]}{(a+b x^2)^2} + \right. \\
& \frac{6 b^{5/2} x^3 \operatorname{Cos}[d x] \operatorname{Sin}[c]}{(a+b x^2)^2} + \frac{i \operatorname{CosIntegral}\left[d\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]\left(3 i \sqrt{a} \sqrt{b} d \operatorname{Cos}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right]+(3 b+a d^2) \operatorname{Sin}\left[c-\frac{i \sqrt{a} d}{\sqrt{b}}\right]\right)}{\sqrt{a}} - \\
& \frac{i \operatorname{CosIntegral}\left[d\left(-\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]\left(-3 i \sqrt{a} \sqrt{b} d \operatorname{Cos}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right]+(3 b+a d^2) \operatorname{Sin}\left[c+\frac{i \sqrt{a} d}{\sqrt{b}}\right]\right)}{\sqrt{a}} + \\
& \frac{10 a b^{3/2} x \operatorname{Cos}[c] \operatorname{Sin}[d x]}{(a+b x^2)^2} + \frac{6 b^{5/2} x^3 \operatorname{Cos}[c] \operatorname{Sin}[d x]}{(a+b x^2)^2} - \frac{2 a^2 \sqrt{b} d \operatorname{Sin}[c] \operatorname{Sin}[d x]}{(a+b x^2)^2} - \frac{2 a b^{3/2} d x^2 \operatorname{Sin}[c] \operatorname{Sin}[d x]}{(a+b x^2)^2} + \\
& \frac{3 i b \operatorname{Cos}[c] \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]}{\sqrt{a}} + i \sqrt{a} d^2 \operatorname{Cos}[c] \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] + \\
& 3 \sqrt{b} d \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sin}[c] \operatorname{SinIntegral}\left[d\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] - 3 i \sqrt{b} d \operatorname{Cos}[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] - \\
& \frac{3 b \operatorname{Sin}[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]}{\sqrt{a}} - \sqrt{a} d^2 \operatorname{Sin}[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] + \\
& \frac{3 i b \operatorname{Cos}[c] \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right]}{\sqrt{a}} + i \sqrt{a} d^2 \operatorname{Cos}[c] \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right] - \\
& 3 \sqrt{b} d \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sin}[c] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right] - 3 i \sqrt{b} d \operatorname{Cos}[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right] + \\
& \left. \frac{3 b \operatorname{Sin}[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right]}{\sqrt{a}} + \sqrt{a} d^2 \operatorname{Sin}[c] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right] \right)
\end{aligned}$$

■ **Problem 76: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[c+d x]}{x(a+b x^2)^3} dx$$

Optimal (type 4, 730 leaves, 41 steps):

$$\begin{aligned}
& \frac{d \operatorname{Cos}[c + d x]}{16 a^2 \sqrt{b} (\sqrt{-a} - \sqrt{b} x)} - \frac{d \operatorname{Cos}[c + d x]}{16 a^2 \sqrt{b} (\sqrt{-a} + \sqrt{b} x)} - \frac{5 d \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 (-a)^{5/2} \sqrt{b}} + \\
& \frac{5 d \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 (-a)^{5/2} \sqrt{b}} + \frac{\operatorname{CosIntegral}[d x] \operatorname{Sin}[c]}{a^3} - \frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 a^3} - \\
& \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 a^2 b} - \frac{\operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{2 a^3} - \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 a^2 b} + \\
& \frac{\operatorname{Sin}[c + d x]}{4 a (a + b x^2)^2} + \frac{\operatorname{Sin}[c + d x]}{2 a^2 (a + b x^2)} + \frac{\operatorname{Cos}[c] \operatorname{SinIntegral}[d x]}{a^3} + \frac{\operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 a^3} + \\
& \frac{d^2 \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 a^2 b} - \frac{5 d \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 (-a)^{5/2} \sqrt{b}} - \\
& \frac{\operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 a^3} - \frac{d^2 \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 a^2 b} - \frac{5 d \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 (-a)^{5/2} \sqrt{b}}
\end{aligned}$$

Result (type 4, 1384 leaves):

$$\begin{aligned}
& \operatorname{Cos}[c] \left(\frac{\operatorname{SinIntegral}[d x]}{a^3} + \frac{1}{16 a^2 b} \right. \\
& \left. \frac{\left(\frac{(i \sqrt{a} \sqrt{b} d + b d x) \operatorname{Cos}[d x] + b \operatorname{Sin}[d x]}{(\sqrt{a} - i \sqrt{b} x)^2} + i d^2 \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] - d^2 \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \right)}{16 a^{5/2}} \right) - \\
& \frac{5 i \sqrt{b} \left(-\frac{\operatorname{Sin}[d x]}{i \sqrt{a} \sqrt{b} + b x} + \frac{d \left(\operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + i \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \right)}{b} \right)}{16 a^{5/2}}
\end{aligned}$$

$$\begin{aligned}
& \frac{i \operatorname{CosIntegral}\left[-\frac{i\sqrt{a}d}{\sqrt{b}}+dx\right] \operatorname{Sinh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] - \operatorname{Cosh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right]}{2a^3} + \frac{1}{16a^2b} \\
& \left(\frac{(-i\sqrt{a}\sqrt{b}d+bdx)\operatorname{Cos}[dx] + b\operatorname{Sin}[dx]}{(\sqrt{a}+i\sqrt{b}x)^2} - id^2 \operatorname{CosIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] + d^2 \operatorname{Cosh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right] \right) + \\
& \frac{5i\sqrt{b} \left(-\frac{\operatorname{Sin}[dx]}{-i\sqrt{a}\sqrt{b}+bx} + \frac{d \left(\operatorname{Cosh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] + i \operatorname{Sinh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right] \right)}{b} \right)}{16a^{5/2}} - \\
& \left. \frac{-i \operatorname{CosIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}}+dx\right] \operatorname{Sinh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] + \operatorname{Cosh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}}+dx\right]}{2a^3} \right) + \\
& \operatorname{Sin}[c] \left(\frac{\operatorname{CosIntegral}[dx]}{a^3} + \frac{1}{16a^2b} \left(-d^2 \operatorname{Cosh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] + \frac{b\operatorname{Cos}[dx] + (-i\sqrt{a}\sqrt{b}d-bdx)\operatorname{Sin}[dx]}{(\sqrt{a}-i\sqrt{b}x)^2} - \right. \right. \\
& \left. \left. i d^2 \operatorname{Sinh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] \right) - \frac{5i\sqrt{b} \left(-\frac{\operatorname{Cos}[dx]}{i\sqrt{a}\sqrt{b}+bx} + \frac{id \left(\operatorname{CosIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] + i \operatorname{Cosh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] \right)}{b} \right)}{16a^{5/2}} - \\
& \frac{\operatorname{Cosh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[-\frac{i\sqrt{a}d}{\sqrt{b}}+dx\right] + i \operatorname{Sinh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right]}{2a^3} + \frac{1}{16a^2b} \\
& \left(-d^2 \operatorname{Cosh}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right] + \frac{b\operatorname{Cos}[dx] + i\sqrt{a}\sqrt{b}d\operatorname{Sin}[dx] - bdx\operatorname{Sin}[dx]}{(\sqrt{a}+i\sqrt{b}x)^2} - \right.
\end{aligned}$$

$$i d^2 \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + \frac{5 i \sqrt{b} \left(-\frac{\operatorname{Cos}[d x]}{-i \sqrt{a} \sqrt{b} + b x} - \frac{d \left(i \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] - \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right)}{b} \right)}{16 a^{5/2}} - \frac{\operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} + d x\right] + i \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} + d x\right]}{2 a^3}$$

■ **Problem 77: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[c + d x]}{x^2 (a + b x^2)^3} dx$$

Optimal (type 4, 875 leaves, 60 steps):

$$\frac{d \operatorname{Cos}[c + d x]}{16 (-a)^{5/2} (\sqrt{-a} - \sqrt{b} x)} + \frac{d \operatorname{Cos}[c + d x]}{16 (-a)^{5/2} (\sqrt{-a} + \sqrt{b} x)} + \frac{d \operatorname{Cos}[c] \operatorname{CosIntegral}[d x]}{a^3} + \frac{7 d \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 a^3} + \frac{7 d \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 a^3} - \frac{15 \sqrt{b} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{7/2}} + \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{5/2} \sqrt{b}} + \frac{15 \sqrt{b} \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{7/2}} - \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{5/2} \sqrt{b}} - \frac{\operatorname{Sin}[c + d x]}{a^3 x} - \frac{\sqrt{b} \operatorname{Sin}[c + d x]}{16 (-a)^{5/2} (\sqrt{-a} - \sqrt{b} x)^2} + \frac{7 \sqrt{b} \operatorname{Sin}[c + d x]}{16 a^3 (\sqrt{-a} - \sqrt{b} x)} + \frac{\sqrt{b} \operatorname{Sin}[c + d x]}{16 (-a)^{5/2} (\sqrt{-a} + \sqrt{b} x)^2} - \frac{7 \sqrt{b} \operatorname{Sin}[c + d x]}{16 a^3 (\sqrt{-a} + \sqrt{b} x)} - \frac{d \operatorname{Sin}[c] \operatorname{SinIntegral}[d x]}{a^3} - \frac{15 \sqrt{b} \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 (-a)^{7/2}} + \frac{d^2 \operatorname{Cos}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 (-a)^{5/2} \sqrt{b}} + \frac{7 d \operatorname{Sin}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 a^3} - \frac{15 \sqrt{b} \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 (-a)^{7/2}} + \frac{d^2 \operatorname{Cos}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 (-a)^{5/2} \sqrt{b}} - \frac{7 d \operatorname{Sin}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 a^3}$$

Result (type 4, 1673 leaves):

$$\begin{aligned}
& - \frac{1}{16 a^{7/2} \sqrt{b} x (a + b x^2)^2} i \left(-2 i a^{5/2} \sqrt{b} dx \cos [c + dx] - 2 i a^{3/2} b^{3/2} dx^3 \cos [c + dx] + \right. \\
& 16 i \sqrt{a} \sqrt{b} dx (a + b x^2)^2 \cos [c] \operatorname{CosIntegral}[dx] + 7 i a^{5/2} \sqrt{b} dx \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + \\
& 14 i a^{3/2} b^{3/2} dx^3 \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + 7 i \sqrt{a} b^{5/2} dx^5 \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + \\
& 15 a^2 b x \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] + a^3 d^2 x \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] + \\
& 30 a b^2 x^3 \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] + 2 a^2 b d^2 x^3 \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] + \\
& 15 b^3 x^5 \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] + a b^2 d^2 x^5 \operatorname{CosIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] - \\
& x (a + b x^2)^2 \operatorname{CosIntegral} \left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] \left(-7 i \sqrt{a} \sqrt{b} d \cos \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] + (15 b + a d^2) \sin \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \right) - \\
& 16 i a^{5/2} \sqrt{b} \sin [c + dx] - 50 i a^{3/2} b^{3/2} x^2 \sin [c + dx] - 30 i \sqrt{a} b^{5/2} x^4 \sin [c + dx] - 16 i a^{5/2} \sqrt{b} dx \sin [c] \operatorname{SinIntegral}[dx] - \\
& 32 i a^{3/2} b^{3/2} dx^3 \sin [c] \operatorname{SinIntegral}[dx] - 16 i \sqrt{a} b^{5/2} dx^5 \sin [c] \operatorname{SinIntegral}[dx] + \\
& 15 a^2 b x \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + a^3 d^2 x \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + \\
& 30 a b^2 x^3 \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + 2 a^2 b d^2 x^3 \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + \\
& 15 b^3 x^5 \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + a b^2 d^2 x^5 \cos \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] - \\
& 7 i a^{5/2} \sqrt{b} dx \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] - 14 i a^{3/2} b^{3/2} dx^3 \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] - \\
& 7 i \sqrt{a} b^{5/2} dx^5 \sin \left[c - \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right) \right] + 15 a^2 b x \cos \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx \right] + \\
& a^3 d^2 x \cos \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx \right] + 30 a b^2 x^3 \cos \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx \right] + \\
& 2 a^2 b d^2 x^3 \cos \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx \right] + 15 b^3 x^5 \cos \left[c + \frac{i \sqrt{a} d}{\sqrt{b}} \right] \operatorname{SinIntegral} \left[\frac{i \sqrt{a} d}{\sqrt{b}} - dx \right] +
\end{aligned}$$

$$a b^2 d^2 x^5 \cos\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}} - dx\right] + 7 i a^{5/2} \sqrt{b} dx \sin\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}} - dx\right] +$$

$$14 i a^{3/2} b^{3/2} dx^3 \sin\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}} - dx\right] + 7 i \sqrt{a} b^{5/2} dx^5 \sin\left[c + \frac{i\sqrt{a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{i\sqrt{a}d}{\sqrt{b}} - dx\right]$$

■ **Problem 78: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c+dx]}{x^3 (a+bx^2)^3} dx$$

Optimal (type 4, 791 leaves, 46 steps):

$$-\frac{d \cos[c+dx]}{2a^3x} - \frac{\sqrt{b}d \cos[c+dx]}{16a^3(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{b}d \cos[c+dx]}{16a^3(\sqrt{-a}+\sqrt{b}x)} - \frac{9\sqrt{b}d \cos\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right]}{16(-a)^{7/2}} +$$

$$\frac{9\sqrt{b}d \cos\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right]}{16(-a)^{7/2}} - \frac{3b \operatorname{CosIntegral}[dx] \sin[c]}{a^4} - \frac{d^2 \operatorname{CosIntegral}[dx] \sin[c]}{2a^3} +$$

$$\frac{3b \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right] \sin\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2a^4} + \frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right] \sin\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{16a^3} + \frac{3b \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right] \sin\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{2a^4} +$$

$$\frac{d^2 \operatorname{CosIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right] \sin\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right]}{16a^3} - \frac{\sin[c+dx]}{2a^3x^2} - \frac{b \sin[c+dx]}{4a^2(a+bx^2)^2} - \frac{b \sin[c+dx]}{a^3(a+bx^2)} - \frac{3b \cos[c] \operatorname{SinIntegral}[dx]}{a^4} -$$

$$\frac{d^2 \cos[c] \operatorname{SinIntegral}[dx]}{2a^3} - \frac{3b \cos\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right]}{2a^4} - \frac{d^2 \cos\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right]}{16a^3} -$$

$$\frac{9\sqrt{b}d \sin\left[c + \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} - dx\right]}{16(-a)^{7/2}} + \frac{3b \cos\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right]}{2a^4} +$$

$$\frac{d^2 \cos\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right]}{16a^3} - \frac{9\sqrt{b}d \sin\left[c - \frac{\sqrt{-a}d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{-a}d}{\sqrt{b}} + dx\right]}{16(-a)^{7/2}}$$

Result (type 4, 995 leaves):

$$\begin{aligned}
& \frac{1}{16 a^4} \left(- \frac{2 a \operatorname{Cos}[d x] \left(d x \left(4 a^2 + 7 a b x^2 + 3 b^2 x^4 \right) \operatorname{Cos}[c] + 2 \left(2 a^2 + 9 a b x^2 + 6 b^2 x^4 \right) \operatorname{Sin}[c] \right)}{x^2 (a + b x^2)^2} + \right. \\
& \left. \frac{2 a \left(-2 \left(2 a^2 + 9 a b x^2 + 6 b^2 x^4 \right) \operatorname{Cos}[c] + d x \left(4 a^2 + 7 a b x^2 + 3 b^2 x^4 \right) \operatorname{Sin}[c] \right) \operatorname{Sin}[d x]}{x^2 (a + b x^2)^2} - \right. \\
& 8 (6 b + a d^2) (\operatorname{CosIntegral}[d x] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{SinIntegral}[d x]) + \\
& 24 b \operatorname{Cos}[c] \left(i \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] - i \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \right. \\
& \left. \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] - \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) + a d^2 \operatorname{Cos}[c] \left(i \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] - \right. \\
& \left. i \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] - \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) + \\
& 9 \sqrt{a} \sqrt{b} d \operatorname{Cos}[c] \left(-i \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + i \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + \right. \\
& \left. \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(-\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) - \\
& 9 \sqrt{a} \sqrt{b} d \operatorname{Sin}[c] \left(\operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \right. \\
& \left. i \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) + \\
& 24 b \operatorname{Sin}[c] \left(\operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + \right. \\
& \left. i \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) + \\
& a d^2 \operatorname{Sin}[c] \left(\operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + \operatorname{Cosh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + \right. \\
& \left. i \operatorname{Sinh}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)\right] + \operatorname{SinIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] \right) \right) \right)
\end{aligned}$$

■ **Problem 94: Result is not expressed in closed-form.**

$$\int \frac{x^4 \operatorname{Sin}[c + d x]}{a + b x^3} dx$$

Optimal (type 4, 371 leaves, 15 steps):

$$\begin{aligned} & -\frac{x \operatorname{Cos}[c + d x]}{b d} + \frac{a^{2/3} \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{3 b^{5/3}} + \frac{(-1)^{2/3} a^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{3 b^{5/3}} \\ & - \frac{(-1)^{1/3} a^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{3 b^{5/3}} + \frac{\operatorname{Sin}[c + d x]}{b d^2} - \frac{(-1)^{2/3} a^{2/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 b^{5/3}} \\ & - \frac{a^{2/3} \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 b^{5/3}} - \frac{(-1)^{1/3} a^{2/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 b^{5/3}} \end{aligned}$$

Result (type 7, 231 leaves):

$$\begin{aligned} & \frac{1}{6 b^2 d^2} \left(-i a d^2 \operatorname{RootSum}\left[a + b \#1^3 \&, \frac{1}{\#1} (\operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - \right. \right. \\ & \quad \left. \left. i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)]) \& \right] + \right. \\ & \quad \left. i a d^2 \operatorname{RootSum}\left[a + b \#1^3 \&, \frac{1}{\#1} (\operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] + i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] + \right. \right. \\ & \quad \left. \left. i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)]) \& \right] + 6 b (-d x \operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x]) \right) \end{aligned}$$

■ **Problem 95: Result is not expressed in closed-form.**

$$\int \frac{x^3 \operatorname{Sin}[c + d x]}{a + b x^3} dx$$

Optimal (type 4, 357 leaves, 14 steps):

$$\begin{aligned} & -\frac{\operatorname{Cos}[c + d x]}{b d} - \frac{a^{1/3} \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{3 b^{4/3}} + \frac{(-1)^{1/3} a^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{3 b^{4/3}} \\ & - \frac{(-1)^{2/3} a^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{3 b^{4/3}} - \frac{(-1)^{1/3} a^{1/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 b^{4/3}} \\ & - \frac{a^{1/3} \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 b^{4/3}} - \frac{(-1)^{2/3} a^{1/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 b^{4/3}} \end{aligned}$$

Result (type 7, 216 leaves):

$$\begin{aligned}
& - \frac{1}{6 b^2 d} \left(6 b \operatorname{Cos}[c + d x] + i a d \operatorname{RootSum}\left[a + b \#1^3 \&, \frac{1}{\#1^2} (\operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - \right. \right. \\
& \quad \left. \left. i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \right) \& \right] - \\
& \quad i a d \operatorname{RootSum}\left[a + b \#1^3 \&, \frac{1}{\#1^2} (\operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] + i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] + \right. \\
& \quad \left. i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \right) \& \left. \right] \Big)
\end{aligned}$$

■ **Problem 96: Result is not expressed in closed-form.**

$$\int \frac{x^2 \operatorname{Sin}[c + d x]}{a + b x^3} dx$$

Optimal (type 4, 281 leaves, 11 steps):

$$\begin{aligned}
& \frac{\operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{3 b} + \frac{\operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{3 b} + \frac{\operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{3 b} - \\
& \frac{\operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 b} + \frac{\operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 b} + \frac{\operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 b}
\end{aligned}$$

Result (type 7, 186 leaves):

$$\begin{aligned}
& \frac{1}{6 b} \\
& i \left(\operatorname{RootSum}\left[a + b \#1^3 \&, \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \right. \right. \\
& \quad \left. \left. \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \right) \& \right] - \operatorname{RootSum}\left[a + b \#1^3 \&, \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] + \right. \\
& \quad \left. i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] + i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \right) \& \left. \right)
\end{aligned}$$

■ **Problem 97: Result is not expressed in closed-form.**

$$\int \frac{x \operatorname{Sin}[c + d x]}{a + b x^3} dx$$

Optimal (type 4, 343 leaves, 11 steps):

$$\begin{aligned}
& - \frac{\operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{3 a^{1/3} b^{2/3}} - \frac{(-1)^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^{1/3} b^{2/3}} + \\
& \frac{(-1)^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^{1/3} b^{2/3}} + \frac{(-1)^{2/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 a^{1/3} b^{2/3}} - \\
& \frac{\operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^{1/3} b^{2/3}} + \frac{(-1)^{1/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^{1/3} b^{2/3}}
\end{aligned}$$

Result (type 7, 196 leaves):

$$\frac{1}{6b} i \left(\text{RootSum}\left[a + b \#1^3 \ \&, 1 / \#1 \left(\text{Cos}[c + d \#1] \text{CosIntegral}[d(x - \#1)] - \right. \right. \right. \\ \left. \left. \left. i \text{CosIntegral}[d(x - \#1)] \text{Sin}[c + d \#1] - i \text{Cos}[c + d \#1] \text{SinIntegral}[d(x - \#1)] - \text{Sin}[c + d \#1] \text{SinIntegral}[d(x - \#1)] \right) \right] - \right. \\ \left. \text{RootSum}\left[a + b \#1^3 \ \&, 1 / \#1 \left(\text{Cos}[c + d \#1] \text{CosIntegral}[d(x - \#1)] + i \text{CosIntegral}[d(x - \#1)] \text{Sin}[c + d \#1] + \right. \right. \right. \\ \left. \left. \left. i \text{Cos}[c + d \#1] \text{SinIntegral}[d(x - \#1)] - \text{Sin}[c + d \#1] \text{SinIntegral}[d(x - \#1)] \right) \right] \right) \right)$$

■ **Problem 98: Result is not expressed in closed-form.**

$$\int \frac{\text{Sin}[c + d x]}{a + b x^3} dx$$

Optimal (type 4, 343 leaves, 11 steps):

$$\frac{\text{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \text{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{3 a^{2/3} b^{1/3}} - \frac{(-1)^{1/3} \text{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \text{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^{2/3} b^{1/3}} + \\ \frac{(-1)^{2/3} \text{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \text{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^{2/3} b^{1/3}} + \frac{(-1)^{1/3} \text{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 a^{2/3} b^{1/3}} + \\ \frac{\text{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^{2/3} b^{1/3}} + \frac{(-1)^{2/3} \text{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^{2/3} b^{1/3}}$$

Result (type 7, 196 leaves):

$$\frac{1}{6b} i \left(\text{RootSum}\left[a + b \#1^3 \ \&, 1 / \#1^2 \left(\text{Cos}[c + d \#1] \text{CosIntegral}[d(x - \#1)] - \right. \right. \right. \\ \left. \left. \left. i \text{CosIntegral}[d(x - \#1)] \text{Sin}[c + d \#1] - i \text{Cos}[c + d \#1] \text{SinIntegral}[d(x - \#1)] - \text{Sin}[c + d \#1] \text{SinIntegral}[d(x - \#1)] \right) \right] - \right. \\ \left. \text{RootSum}\left[a + b \#1^3 \ \&, 1 / \#1^2 \left(\text{Cos}[c + d \#1] \text{CosIntegral}[d(x - \#1)] + i \text{CosIntegral}[d(x - \#1)] \text{Sin}[c + d \#1] + \right. \right. \right. \\ \left. \left. \left. i \text{Cos}[c + d \#1] \text{SinIntegral}[d(x - \#1)] - \text{Sin}[c + d \#1] \text{SinIntegral}[d(x - \#1)] \right) \right] \right) \right)$$

■ **Problem 99: Result is not expressed in closed-form.**

$$\int \frac{\text{Sin}[c + d x]}{x(a + b x^3)} dx$$

Optimal (type 4, 301 leaves, 16 steps):

$$\frac{\text{CosIntegral}[d x] \text{Sin}[c]}{a} - \frac{\text{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \text{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{3 a} - \frac{\text{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \text{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{3 a} - \\ \frac{\text{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \text{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{3 a} + \frac{\text{Cos}[c] \text{SinIntegral}[d x]}{a} + \frac{\text{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 a} - \\ \frac{\text{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 a} - \frac{\text{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 a}$$

Result (type 7, 206 leaves):

$$\frac{1}{6 a} \left(-i \operatorname{RootSum}\left[a + b \#1^3 \&, \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - \right. \right. \\ \left. \left. i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \&\right] + \right. \\ \left. i \operatorname{RootSum}\left[a + b \#1^3 \&, \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] + i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] + i \operatorname{Cos}[c + d \#1] \right. \right. \\ \left. \left. \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \&\right] + 6 \operatorname{CosIntegral}[d x] \operatorname{Sin}[c] + 6 \operatorname{Cos}[c] \operatorname{SinIntegral}[d x] \right)$$

■ **Problem 100: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c + d x]}{x^2 (a + b x^3)} dx$$

Optimal (type 4, 380 leaves, 17 steps):

$$\frac{d \operatorname{Cos}[c] \operatorname{CosIntegral}[d x]}{a} + \frac{b^{1/3} \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{3 a^{4/3}} + \\ \frac{(-1)^{2/3} b^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^{4/3}} - \frac{(-1)^{1/3} b^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^{4/3}} - \\ \frac{\operatorname{Sin}[c + d x]}{a x} - \frac{d \operatorname{Sin}[c] \operatorname{SinIntegral}[d x]}{a} - \frac{(-1)^{2/3} b^{1/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 a^{4/3}} + \\ \frac{b^{1/3} \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^{4/3}} - \frac{(-1)^{1/3} b^{1/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^{4/3}}$$

Result (type 7, 233 leaves):

$$\frac{1}{6 a x} \left(6 d x \operatorname{Cos}[c] \operatorname{CosIntegral}[d x] - i x \operatorname{RootSum}\left[a + b \#1^3 \&, \frac{1}{\#1} (\operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - \right. \right. \\ \left. \left. i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)]) \&\right] + \right. \\ \left. i x \operatorname{RootSum}\left[a + b \#1^3 \&, \frac{1}{\#1} (\operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] + i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] + \right. \right. \\ \left. \left. i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)]) \&\right] - 6 \operatorname{Sin}[c + d x] - 6 d x \operatorname{Sin}[c] \operatorname{SinIntegral}[d x] \right)$$

■ **Problem 101: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c + d x]}{x^3 (a + b x^3)} dx$$

Optimal (type 4, 408 leaves, 18 steps):

$$\begin{aligned}
& - \frac{d \cos[c + dx]}{2ax} - \frac{d^2 \operatorname{CosIntegral}[dx] \sin[c]}{2a} - \frac{b^{2/3} \operatorname{CosIntegral}\left[\frac{a^{1/3}d}{b^{1/3}} + dx\right] \sin\left[c - \frac{a^{1/3}d}{b^{1/3}}\right]}{3a^{5/3}} + \\
& \frac{(-1)^{1/3} b^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}} - dx\right] \sin\left[c + \frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}}\right]}{3a^{5/3}} - \frac{(-1)^{2/3} b^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}} + dx\right] \sin\left[c - \frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}}\right]}{3a^{5/3}} - \\
& \frac{\sin[c + dx]}{2ax^2} - \frac{d^2 \cos[c] \operatorname{SinIntegral}[dx]}{2a} - \frac{(-1)^{1/3} b^{2/3} \cos\left[c + \frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}} - dx\right]}{3a^{5/3}} - \\
& \frac{b^{2/3} \cos\left[c - \frac{a^{1/3}d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3}d}{b^{1/3}} + dx\right]}{3a^{5/3}} - \frac{(-1)^{2/3} b^{2/3} \cos\left[c - \frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}} + dx\right]}{3a^{5/3}}
\end{aligned}$$

Result (type 7, 253 leaves):

$$\begin{aligned}
& \frac{1}{6ax^2} \left(-i x^2 \operatorname{RootSum}\left[a + b\#1^3 \&, \frac{1}{\#1^2} (\cos[c + d\#1] \operatorname{CosIntegral}[d(x - \#1)] - \right. \right. \\
& \quad \left. \left. i \operatorname{CosIntegral}[d(x - \#1)] \sin[c + d\#1] - i \cos[c + d\#1] \operatorname{SinIntegral}[d(x - \#1)] - \sin[c + d\#1] \operatorname{SinIntegral}[d(x - \#1)]) \& \right] + \right. \\
& \quad \left. i x^2 \operatorname{RootSum}\left[a + b\#1^3 \&, \frac{1}{\#1^2} (\cos[c + d\#1] \operatorname{CosIntegral}[d(x - \#1)] + i \operatorname{CosIntegral}[d(x - \#1)] \sin[c + d\#1] + \right. \right. \\
& \quad \left. \left. i \cos[c + d\#1] \operatorname{SinIntegral}[d(x - \#1)] - \sin[c + d\#1] \operatorname{SinIntegral}[d(x - \#1)]) \& \right] - \right. \\
& \quad \left. \left. 3(d x \cos[c + dx] + d^2 x^2 \operatorname{CosIntegral}[dx] \sin[c] + \sin[c + dx] + d^2 x^2 \cos[c] \operatorname{SinIntegral}[dx]) \right) \right)
\end{aligned}$$

■ **Problem 102: Result is not expressed in closed-form.**

$$\int \frac{x^3 \sin[c + dx]}{(a + bx^3)^2} dx$$

Optimal (type 4, 714 leaves, 23 steps):

$$\begin{aligned}
& - \frac{(-1)^{2/3} d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{1/3} b^{5/3}} - \frac{d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{1/3} b^{5/3}} + \\
& \frac{(-1)^{1/3} d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{1/3} b^{5/3}} + \frac{\operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{9 a^{2/3} b^{4/3}} - \\
& \frac{(-1)^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{2/3} b^{4/3}} + \frac{(-1)^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{2/3} b^{4/3}} - \frac{x \operatorname{Sin}[c + d x]}{3 b (a + b x^3)} + \\
& \frac{(-1)^{1/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{2/3} b^{4/3}} - \frac{(-1)^{2/3} d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{1/3} b^{5/3}} + \\
& \frac{\operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{2/3} b^{4/3}} + \frac{d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{1/3} b^{5/3}} + \\
& \frac{(-1)^{2/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{2/3} b^{4/3}} - \frac{(-1)^{1/3} d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{1/3} b^{5/3}}
\end{aligned}$$

Result (type 7, 383 leaves):

$$\begin{aligned}
& \frac{1}{18 b^2} \left(\operatorname{RootSum}\left[a + b \#1^3 \&, \right. \right. \\
& \quad \frac{1}{\#1^2} \left(i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] + \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] + \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \right. \\
& \quad \left. i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 - i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 - \right. \\
& \quad \left. i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \left. + \operatorname{RootSum}\left[a + b \#1^3 \&, \right. \right. \\
& \quad \frac{1}{\#1^2} \left(-i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] + \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] + \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + \right. \\
& \quad \left. i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 + i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 + \right. \\
& \quad \left. i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \left. - \frac{6 b x \operatorname{Sin}[c + d x]}{a + b x^3} \right)
\end{aligned}$$

■ **Problem 103: Result is not expressed in closed-form.**

$$\int \frac{x^2 \operatorname{Sin}[c + d x]}{(a + b x^3)^2} dx$$

Optimal (type 4, 371 leaves, 12 steps):

$$\begin{aligned}
& - \frac{(-1)^{1/3} d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{2/3} b^{4/3}} + \frac{d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{2/3} b^{4/3}} + \\
& \frac{(-1)^{2/3} d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{2/3} b^{4/3}} - \frac{\operatorname{Sin}[c + d x]}{3 b (a + b x^3)} - \frac{(-1)^{1/3} d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{2/3} b^{4/3}} - \\
& \frac{d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{2/3} b^{4/3}} - \frac{(-1)^{2/3} d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{2/3} b^{4/3}}
\end{aligned}$$

Result (type 7, 214 leaves):

$$\begin{aligned}
& \frac{1}{18 b^2} \left(d \operatorname{RootSum}\left[a + b \#1^3 \&, 1 / \#1^2 (\operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - \right. \right. \\
& \quad \left. \left. i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)]) \& \right] + \right. \\
& \quad \left. d \operatorname{RootSum}\left[a + b \#1^3 \&, 1 / \#1^2 (\operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] + i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] + \right. \right. \\
& \quad \left. \left. i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)]) \& \right] - \frac{6 b \operatorname{Sin}[c + d x]}{a + b x^3} \right)
\end{aligned}$$

■ **Problem 104: Result is not expressed in closed-form.**

$$\int \frac{x \operatorname{Sin}[c + d x]}{(a + b x^3)^2} dx$$

Optimal (type 4, 691 leaves, 34 steps):

$$\begin{aligned}
& - \frac{d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a b} - \frac{d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a b} - \\
& \frac{d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a b} - \frac{\operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{9 a^{4/3} b^{2/3}} - \\
& \frac{(-1)^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{4/3} b^{2/3}} + \frac{(-1)^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{4/3} b^{2/3}} + \\
& \frac{\operatorname{Sin}[c + d x]}{3 a b x} - \frac{\operatorname{Sin}[c + d x]}{3 b x (a + b x^3)} + \frac{(-1)^{2/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{4/3} b^{2/3}} - \\
& \frac{d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a b} - \frac{\operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{4/3} b^{2/3}} + \frac{d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a b} + \\
& \frac{(-1)^{1/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{4/3} b^{2/3}} + \frac{d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a b}
\end{aligned}$$

Result (type 7, 408 leaves):

$$\begin{aligned}
& - \frac{1}{18 a b (a + b x^3)} \left((a + b x^3) \operatorname{RootSum}\left[a + b \#1^3 \&, \right. \right. \\
& \quad \frac{1}{\#1} \left(-i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + \right. \\
& \quad \quad i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 - i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 - \\
& \quad \quad \left. \left. i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \right] + (a + b x^3) \operatorname{RootSum}\left[a + b \#1^3 \&, \right. \\
& \quad \frac{1}{\#1} \left(i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \right. \\
& \quad \quad i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 + i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 + \\
& \quad \quad \left. \left. i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \right] - 6 b x^2 \operatorname{Sin}[c + d x] \Big)
\end{aligned}$$

■ **Problem 105: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c + d x]}{(a + b x^3)^2} dx$$

Optimal (type 4, 735 leaves, 36 steps):

$$\begin{aligned}
& \frac{(-1)^{2/3} d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{4/3} b^{2/3}} + \frac{d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{4/3} b^{2/3}} - \\
& \frac{(-1)^{1/3} d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{4/3} b^{2/3}} + \frac{2 \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{9 a^{5/3} b^{1/3}} - \\
& \frac{2 (-1)^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{5/3} b^{1/3}} + \frac{2 (-1)^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{9 a^{5/3} b^{1/3}} + \frac{\operatorname{Sin}[c + d x]}{3 a b x^2} - \\
& \frac{\operatorname{Sin}[c + d x]}{3 b x^2 (a + b x^3)} + \frac{2 (-1)^{1/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{5/3} b^{1/3}} + \frac{(-1)^{2/3} d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{4/3} b^{2/3}} + \\
& \frac{2 \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{5/3} b^{1/3}} - \frac{d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{4/3} b^{2/3}} + \\
& \frac{2 (-1)^{2/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{5/3} b^{1/3}} + \frac{(-1)^{1/3} d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{4/3} b^{2/3}}
\end{aligned}$$

Result (type 7, 406 leaves):

$$\begin{aligned}
& - \frac{1}{18 a b (a + b x^3)} \left((a + b x^3) \operatorname{RootSum}\left[a + b \#1^3, \right. \right. \\
& \quad \frac{1}{\#1^2} \left(-2 i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - 2 \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - 2 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + \right. \\
& \quad \left. \left. 2 i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 - i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 - \right. \right. \\
& \quad \left. \left. i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \right) + (a + b x^3) \operatorname{RootSum}\left[a + b \#1^3, \& \right. \\
& \quad \frac{1}{\#1^2} \left(2 i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - 2 \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - 2 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \right. \\
& \quad \left. \left. 2 i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 + i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 + \right. \right. \\
& \quad \left. \left. i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \right) - 6 b x \operatorname{Sin}[c + d x] \left. \right)
\end{aligned}$$

■ **Problem 106: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c + d x]}{x (a + b x^3)^2} dx$$

Optimal (type 4, 693 leaves, 41 steps):

$$\begin{aligned}
& \frac{(-1)^{1/3} d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{5/3} b^{1/3}} - \frac{d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{5/3} b^{1/3}} - \\
& \frac{(-1)^{2/3} d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{5/3} b^{1/3}} + \frac{\operatorname{CosIntegral}[d x] \operatorname{Sin}[c]}{a^2} - \frac{\operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{3 a^2} - \\
& \frac{\operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^2} - \frac{\operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^2} + \\
& \frac{\operatorname{Sin}[c + d x]}{3 a b x^3} - \frac{\operatorname{Sin}[c + d x]}{3 b x^3 (a + b x^3)} + \frac{\operatorname{Cos}[c] \operatorname{SinIntegral}[d x]}{a^2} + \frac{\operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 a^2} + \\
& \frac{(-1)^{1/3} d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{5/3} b^{1/3}} - \frac{\operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^2} + \frac{d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{5/3} b^{1/3}} - \\
& \frac{\operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^2} + \frac{(-1)^{2/3} d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{5/3} b^{1/3}}
\end{aligned}$$

Result (type 4, 1819 leaves):

$$\operatorname{Sin}[c] \left(\frac{\operatorname{CosIntegral}[d x]}{a^2} - \frac{1}{(1 + (-1)^{1/3})^2 a^2 b^{1/3}} (3 b^{1/3} - 2 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3}) \right)$$

$$\begin{aligned}
& \left(\cos \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral} \left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx \right] + \sin \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx \right] \right) + \\
& \left((21 - 22 (-1)^{1/3} + 21 (-1)^{2/3}) b^{1/3} \left(-\frac{\cos [dx]}{b^{1/3} (-(-1)^{1/3} a^{1/3} + b^{1/3} x)} + 1/b^{2/3} d \left(-\operatorname{CosIntegral} \left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx \right] \sin \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. \cos \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx \right] \right) \right) \right) / \left(3 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^2 a^{5/3} - \right. \\
& \left. \left((2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3}) \left(\cos \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] + \sin \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] \right) \right) \right) / \\
& \left((-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^2 a^2 b^{1/3} \right) + \\
& \left((22 - 21 (-1)^{1/3} + 21 (-1)^{2/3}) b^{1/3} \left(-\frac{\cos [dx]}{b^{1/3} (a^{1/3} + b^{1/3} x)} + \frac{d \left(\operatorname{CosIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] \sin \left[\frac{a^{1/3} d}{b^{1/3}} \right] - \cos \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] \right)}{b^{2/3}} \right) \right) \right) / \\
& \left(3 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^2 a^{5/3} - \left((2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3}) \right. \right. \\
& \quad \left. \left. \left(\cos \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx \right] + \sin \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx \right] \right) \right) \right) / \\
& \left((-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^2 a^2 b^{1/3} \right) + \frac{1}{3 (1 + (-1)^{1/3})^2 a^{5/3}} \left((22 b^{1/3} - 21 (-1)^{1/3} b^{1/3} + 21 (-1)^{2/3} b^{1/3}) \left(-\frac{\cos [dx]}{b^{1/3} ((-1)^{2/3} a^{1/3} + b^{1/3} x)} + \right. \right. \\
& \quad \left. \left. 1/b^{2/3} d \left(\operatorname{CosIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx \right] \sin \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] - \cos \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx \right] \right) \right) \right) \right) + \\
\cos [c] & \left(\frac{\operatorname{SinIntegral} [dx]}{a^2} - \frac{1}{(1 + (-1)^{1/3})^2 a^2 b^{1/3}} (3 b^{1/3} - 2 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3}) \right. \\
& \left(\operatorname{CosIntegral} \left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx \right] \sin \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] - \cos \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx \right] \right) + \\
& \left((21 - 22 (-1)^{1/3} + 21 (-1)^{2/3}) b^{1/3} \left(-\frac{\sin [dx]}{b^{1/3} (-(-1)^{1/3} a^{1/3} + b^{1/3} x)} + 1/b^{2/3} d \left(\cos \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral} \left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx \right] + \right. \right. \right. \\
& \quad \left. \left. \left. \sin \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx \right] \right) \right) \right) \right) / \left(3 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^2 a^{5/3} - \right. \\
& \left. \left((2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3}) \left(-\operatorname{CosIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] \sin \left[\frac{a^{1/3} d}{b^{1/3}} \right] + \cos \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] \right) \right) \right) / \\
& \left((-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^2 a^2 b^{1/3} \right) +
\end{aligned}$$

$$\left(\left(22 - 21 (-1)^{1/3} + 21 (-1)^{2/3} \right) b^{1/3} \left(-\frac{\text{Sin}[d x]}{b^{1/3} (a^{1/3} + b^{1/3} x)} + \frac{d \left(\text{Cos} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \text{CosIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] + \text{Sin} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] \right)}{b^{2/3}} \right) \right) /$$

$$(3 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^2 a^{5/3}) - \left((2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3}) \right.$$

$$\left. \left(-\text{CosIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \text{Sin} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] + \text{Cos} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \right) \right) /$$

$$\left((-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^2 a^2 b^{1/3} \right) + \frac{1}{3 (1 + (-1)^{1/3})^2 a^{5/3}} \left(22 b^{1/3} - 21 (-1)^{1/3} b^{1/3} + 21 (-1)^{2/3} b^{1/3} \right) \left(-\frac{\text{Sin}[d x]}{b^{1/3} ((-1)^{2/3} a^{1/3} + b^{1/3} x)} + \right.$$

$$\left. \left. 1 / b^{2/3} d \left(\text{Cos} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{CosIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] + \text{Sin} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \right) \right) \right)$$

■ **Problem 107: Result is not expressed in closed-form.**

$$\int \frac{\text{Sin}[c + d x]}{x^2 (a + b x^3)^2} dx$$

Optimal (type 4, 712 leaves, 47 steps):

$$\frac{d \text{Cos}[c] \text{CosIntegral}[d x]}{a^2} + \frac{d \text{Cos} \left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \text{CosIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right]}{9 a^2} + \frac{d \text{Cos} \left[c - \frac{a^{1/3} d}{b^{1/3}} \right] \text{CosIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^2} +$$

$$\frac{d \text{Cos} \left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{CosIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^2} + \frac{4 b^{1/3} \text{CosIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] \text{Sin} \left[c - \frac{a^{1/3} d}{b^{1/3}} \right]}{9 a^{7/3}} +$$

$$\frac{4 (-1)^{2/3} b^{1/3} \text{CosIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right] \text{Sin} \left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right]}{9 a^{7/3}} - \frac{4 (-1)^{1/3} b^{1/3} \text{CosIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \text{Sin} \left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right]}{9 a^{7/3}} +$$

$$\frac{\text{Sin}[c + d x]}{3 a b x^4} - \frac{4 \text{Sin}[c + d x]}{3 a^2 x} - \frac{\text{Sin}[c + d x]}{3 b x^4 (a + b x^3)} - \frac{d \text{Sin}[c] \text{SinIntegral}[d x]}{a^2} - \frac{4 (-1)^{2/3} b^{1/3} \text{Cos} \left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right]}{9 a^{7/3}} +$$

$$\frac{d \text{Sin} \left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right]}{9 a^2} + \frac{4 b^{1/3} \text{Cos} \left[c - \frac{a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^{7/3}} - \frac{d \text{Sin} \left[c - \frac{a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^2} -$$

$$\frac{4 (-1)^{1/3} b^{1/3} \text{Cos} \left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^{7/3}} - \frac{d \text{Sin} \left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^2}$$

Result (type 7, 445 leaves):

$$\begin{aligned}
& - \frac{1}{3 a^2 x (a + b x^3)} \\
& \left((3 a + 4 b x^3) \operatorname{Cos}[d x] \operatorname{Sin}[c] + (3 a + 4 b x^3) \operatorname{Cos}[c] \operatorname{Sin}[d x] - \frac{1}{6} x (a + b x^3) \left(18 d \operatorname{Cos}[c] \operatorname{CosIntegral}[d x] + \operatorname{RootSum}\left[a + b \#1^3 \&, \right. \right. \right. \\
& \left. \left. \frac{1}{\#1} (-4 i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - 4 \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - 4 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + \right. \right. \\
& \left. \left. 4 i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 - i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \right. \right. \\
& \left. \left. \#1 - i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \right) + \operatorname{RootSum}\left[a + b \#1^3 \&, \right. \\
& \left. \frac{1}{\#1} (4 i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - 4 \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - 4 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \right. \\
& \left. \left. 4 i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 + i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \right. \right. \\
& \left. \left. \#1 + i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \right) - 18 d \operatorname{Sin}[c] \operatorname{SinIntegral}[d x] \left. \right) \left. \right)
\end{aligned}$$

■ **Problem 108: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c + d x]}{x^3 (a + b x^3)^2} dx$$

Optimal (type 4, 800 leaves, 51 steps):

$$\begin{aligned}
& - \frac{d \operatorname{Cos}[c + d x]}{2 a^2 x} - \frac{(-1)^{2/3} b^{1/3} d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right]}{9 a^{7/3}} - \frac{b^{1/3} d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^{7/3}} + \\
& \frac{(-1)^{1/3} b^{1/3} d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^{7/3}} - \frac{d^2 \operatorname{CosIntegral}[d x] \operatorname{Sin}[c]}{2 a^2} - \\
& \frac{5 b^{2/3} \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}} \right]}{9 a^{8/3}} + \frac{5 (-1)^{1/3} b^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right]}{9 a^{8/3}} - \\
& \frac{5 (-1)^{2/3} b^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right]}{9 a^{8/3}} + \frac{\operatorname{Sin}[c + d x]}{3 a b x^5} - \frac{5 \operatorname{Sin}[c + d x]}{6 a^2 x^2} - \frac{\operatorname{Sin}[c + d x]}{3 b x^5 (a + b x^3)} - \frac{d^2 \operatorname{Cos}[c] \operatorname{SinIntegral}[d x]}{2 a^2} - \\
& \frac{5 (-1)^{1/3} b^{2/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right]}{9 a^{8/3}} - \frac{(-1)^{2/3} b^{1/3} d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right]}{9 a^{7/3}} - \\
& \frac{5 b^{2/3} \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^{8/3}} + \frac{b^{1/3} d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^{7/3}} - \\
& \frac{5 (-1)^{2/3} b^{2/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^{8/3}} - \frac{(-1)^{1/3} b^{1/3} d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right]}{9 a^{7/3}}
\end{aligned}$$

Result (type 7, 470 leaves):

$$\frac{1}{18 a^2} \left(\text{RootSum} \left[a + b \#1^3 \ \&, \right. \right. \\ \frac{1}{\#1^2} \left(-5 i \text{Cos} [c + d \#1] \text{CosIntegral} [d (x - \#1)] - 5 \text{CosIntegral} [d (x - \#1)] \text{Sin} [c + d \#1] - 5 \text{Cos} [c + d \#1] \text{SinIntegral} [d (x - \#1)] + \right. \\ \left. 5 i \text{Sin} [c + d \#1] \text{SinIntegral} [d (x - \#1)] + d \text{Cos} [c + d \#1] \text{CosIntegral} [d (x - \#1)] \#1 - i d \text{CosIntegral} [d (x - \#1)] \text{Sin} [c + d \#1] \#1 - \right. \\ \left. i d \text{Cos} [c + d \#1] \text{SinIntegral} [d (x - \#1)] \#1 - d \text{Sin} [c + d \#1] \text{SinIntegral} [d (x - \#1)] \#1 \right) \& \left. \right] + \text{RootSum} \left[a + b \#1^3 \ \&, \right. \\ \frac{1}{\#1^2} \left(5 i \text{Cos} [c + d \#1] \text{CosIntegral} [d (x - \#1)] - 5 \text{CosIntegral} [d (x - \#1)] \text{Sin} [c + d \#1] - 5 \text{Cos} [c + d \#1] \text{SinIntegral} [d (x - \#1)] - \right. \\ \left. 5 i \text{Sin} [c + d \#1] \text{SinIntegral} [d (x - \#1)] + d \text{Cos} [c + d \#1] \text{CosIntegral} [d (x - \#1)] \#1 + i d \text{CosIntegral} [d (x - \#1)] \text{Sin} [c + d \#1] \#1 + \right. \\ \left. i d \text{Cos} [c + d \#1] \text{SinIntegral} [d (x - \#1)] \#1 - d \text{Sin} [c + d \#1] \text{SinIntegral} [d (x - \#1)] \#1 \right) \& \left. \right] - \\ \frac{1}{x^2 (a + b x^3)} \left(3 a d x \text{Cos} [c + d x] + 3 b d x^4 \text{Cos} [c + d x] + 3 d^2 x^2 (a + b x^3) \text{CosIntegral} [d x] \text{Sin} [c] + 3 a \text{Sin} [c + d x] + \right. \\ \left. 5 b x^3 \text{Sin} [c + d x] + 3 d^2 x^2 (a + b x^3) \text{Cos} [c] \text{SinIntegral} [d x] \right) \left. \right)$$

■ **Problem 109: Result is not expressed in closed-form.**

$$\int \frac{x^3 \text{Sin} [c + d x]}{(a + b x^3)^3} dx$$

Optimal (type 4, 772 leaves, 71 steps):

$$\frac{d \text{Cos} [c + d x]}{18 a b^2 x} - \frac{d \text{Cos} [c + d x]}{18 b^2 x (a + b x^3)} + \frac{\text{CosIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] \text{Sin} \left[c - \frac{a^{1/3} d}{b^{1/3}} \right]}{27 a^{5/3} b^{4/3}} + \frac{d^2 \text{CosIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] \text{Sin} \left[c - \frac{a^{1/3} d}{b^{1/3}} \right]}{54 a b^2} - \\ \frac{(-1)^{1/3} \text{CosIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right] \text{Sin} \left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right]}{27 a^{5/3} b^{4/3}} + \frac{d^2 \text{CosIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right] \text{Sin} \left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right]}{54 a b^2} + \\ \frac{(-1)^{2/3} \text{CosIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \text{Sin} \left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right]}{27 a^{5/3} b^{4/3}} + \frac{d^2 \text{CosIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \text{Sin} \left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right]}{54 a b^2} + \\ \frac{\text{Sin} [c + d x]}{18 a b^2 x^2} - \frac{x \text{Sin} [c + d x]}{6 b (a + b x^3)^2} - \frac{\text{Sin} [c + d x]}{18 b^2 x^2 (a + b x^3)} + \frac{(-1)^{1/3} \text{Cos} \left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right]}{27 a^{5/3} b^{4/3}} - \\ \frac{d^2 \text{Cos} \left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right]}{54 a b^2} + \frac{\text{Cos} \left[c - \frac{a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right]}{27 a^{5/3} b^{4/3}} + \frac{d^2 \text{Cos} \left[c - \frac{a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right]}{54 a b^2} + \\ \frac{(-1)^{2/3} \text{Cos} \left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right]}{27 a^{5/3} b^{4/3}} + \frac{d^2 \text{Cos} \left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right]}{54 a b^2}$$

Result (type 7, 457 leaves):

$$\frac{1}{108 a b^2} \left(i \operatorname{RootSum}\left[a + b \#1^3 \ \&, \frac{1}{\#1^2} \left(2 \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d(x - \#1)] - 2 i \operatorname{CosIntegral}[d(x - \#1)] \operatorname{Sin}[c + d \#1] - 2 i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d(x - \#1)] - 2 \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d(x - \#1)] + d^2 \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d(x - \#1)] \#1^2 - i d^2 \operatorname{CosIntegral}[d(x - \#1)] \operatorname{Sin}[c + d \#1] \#1^2 - i d^2 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d(x - \#1)] \#1^2 - d^2 \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d(x - \#1)] \#1^2 \right) \ \& \right] - i \operatorname{RootSum}\left[a + b \#1^3 \ \&, \frac{1}{\#1^2} \left(2 \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d(x - \#1)] + 2 i \operatorname{CosIntegral}[d(x - \#1)] \operatorname{Sin}[c + d \#1] + 2 i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d(x - \#1)] - 2 \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d(x - \#1)] + d^2 \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d(x - \#1)] \#1^2 + i d^2 \operatorname{CosIntegral}[d(x - \#1)] \operatorname{Sin}[c + d \#1] \#1^2 + i d^2 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d(x - \#1)] \#1^2 - d^2 \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d(x - \#1)] \#1^2 \right) \ \& \right] + \frac{6 b x (d x (a + b x^3) \operatorname{Cos}[c + d x] + (-2 a + b x^3) \operatorname{Sin}[c + d x])}{(a + b x^3)^2} \right)$$

■ **Problem 110: Result is not expressed in closed-form.**

$$\int \frac{x^2 \operatorname{Sin}[c + d x]}{(a + b x^3)^3} dx$$

Optimal (type 4, 777 leaves, 37 steps):

$$\begin{aligned} & \frac{d \operatorname{Cos}[c + d x]}{18 a b^2 x^2} - \frac{d \operatorname{Cos}[c + d x]}{18 b^2 x^2 (a + b x^3)} - \frac{(-1)^{1/3} d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{27 a^{5/3} b^{4/3}} + \frac{d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{5/3} b^{4/3}} + \\ & \frac{(-1)^{2/3} d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{5/3} b^{4/3}} - \frac{d^2 \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{54 a^{4/3} b^{5/3}} - \\ & \frac{(-1)^{2/3} d^2 \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{54 a^{4/3} b^{5/3}} + \frac{(-1)^{1/3} d^2 \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{54 a^{4/3} b^{5/3}} - \\ & \frac{\operatorname{Sin}[c + d x]}{6 b (a + b x^3)^2} + \frac{(-1)^{2/3} d^2 \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{54 a^{4/3} b^{5/3}} - \frac{(-1)^{1/3} d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{27 a^{5/3} b^{4/3}} - \\ & \frac{d^2 \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{54 a^{4/3} b^{5/3}} - \frac{d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{5/3} b^{4/3}} + \\ & \frac{(-1)^{1/3} d^2 \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{54 a^{4/3} b^{5/3}} - \frac{(-1)^{2/3} d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{5/3} b^{4/3}} \end{aligned}$$

Result (type 7, 449 leaves):

$$\frac{1}{108 a b^2} \left(i d \operatorname{RootSum}\left[a + b \#1^3 \&, \right. \right. \\ \left. \frac{1}{\#1^2} \left(-2 i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - 2 \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - 2 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + \right. \right. \\ \left. \left. 2 i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 - i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 - \right. \right. \\ \left. \left. i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \right] - i d \operatorname{RootSum}\left[a + b \#1^3 \&, \right. \\ \left. \frac{1}{\#1^2} \left(2 i \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - 2 \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - 2 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - \right. \right. \\ \left. \left. 2 i \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 + i d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 + \right. \right. \\ \left. \left. i d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 \right) \& \right] + \\ \left. \frac{6 b \operatorname{Cos}[d x] \left(d x \left(a + b x^3 \right) \operatorname{Cos}[c] - 3 a \operatorname{Sin}[c] \right)}{\left(a + b x^3 \right)^2} - \frac{6 b \left(3 a \operatorname{Cos}[c] + d x \left(a + b x^3 \right) \operatorname{Sin}[c] \right) \operatorname{Sin}[d x]}{\left(a + b x^3 \right)^2} \right)$$

■ **Problem 111: Result is not expressed in closed-form.**

$$\int \frac{x \operatorname{Sin}[c + d x]}{\left(a + b x^3 \right)^3} dx$$

Optimal (type 4, 1141 leaves, 89 steps):

$$\begin{aligned}
& \frac{d \operatorname{Cos}[c + d x]}{18 a b^2 x^3} - \frac{d \operatorname{Cos}[c + d x]}{18 b^2 x^3 (a + b x^3)} - \frac{2 d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{27 a^2 b} - \frac{2 d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^2 b} \\
& \frac{2 d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^2 b} - \frac{2 \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{27 a^{7/3} b^{2/3}} + \\
& \frac{d^2 \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{54 a^{5/3} b^{4/3}} - \frac{2 (-1)^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^{7/3} b^{2/3}} - \\
& \frac{(-1)^{1/3} d^2 \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{54 a^{5/3} b^{4/3}} + \frac{2 (-1)^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^{7/3} b^{2/3}} + \\
& \frac{(-1)^{2/3} d^2 \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{54 a^{5/3} b^{4/3}} - \frac{\operatorname{Sin}[c + d x]}{18 a b^2 x^4} + \frac{2 \operatorname{Sin}[c + d x]}{9 a^2 b x} - \frac{\operatorname{Sin}[c + d x]}{6 b x (a + b x^3)^2} + \frac{\operatorname{Sin}[c + d x]}{18 b^2 x^4 (a + b x^3)} + \\
& \frac{2 (-1)^{2/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{27 a^{7/3} b^{2/3}} + \frac{(-1)^{1/3} d^2 \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{54 a^{5/3} b^{4/3}} - \\
& \frac{2 d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{27 a^2 b} - \frac{2 \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{7/3} b^{2/3}} + \frac{d^2 \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{54 a^{5/3} b^{4/3}} + \\
& \frac{2 d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^2 b} + \frac{2 (-1)^{1/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{7/3} b^{2/3}} + \\
& \frac{(-1)^{2/3} d^2 \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{54 a^{5/3} b^{4/3}} + \frac{2 d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^2 b}
\end{aligned}$$

Result (type 7, 698 leaves):

$$\begin{aligned}
& - \frac{1}{108 a^2 b^2} \\
& \left(\text{RootSum}\left[a + b \#1^3 \ \&, \frac{1}{\#1^2} \left(-i a d^2 \text{Cos}[c + d \#1] \text{CosIntegral}[d (x - \#1)] - a d^2 \text{CosIntegral}[d (x - \#1)] \text{Sin}[c + d \#1] - a d^2 \text{Cos}[c + d \#1] \right. \right. \\
& \quad \text{SinIntegral}[d (x - \#1)] + i a d^2 \text{Sin}[c + d \#1] \text{SinIntegral}[d (x - \#1)] - 4 i b \text{Cos}[c + d \#1] \text{CosIntegral}[d (x - \#1)] \#1 - \\
& \quad 4 b \text{CosIntegral}[d (x - \#1)] \text{Sin}[c + d \#1] \#1 - 4 b \text{Cos}[c + d \#1] \text{SinIntegral}[d (x - \#1)] \#1 + 4 i b \text{Sin}[c + d \#1] \\
& \quad \text{SinIntegral}[d (x - \#1)] \#1 + 4 b d \text{Cos}[c + d \#1] \text{CosIntegral}[d (x - \#1)] \#1^2 - 4 i b d \text{CosIntegral}[d (x - \#1)] \text{Sin}[c + d \#1] \#1^2 - \\
& \quad \left. \left. 4 i b d \text{Cos}[c + d \#1] \text{SinIntegral}[d (x - \#1)] \#1^2 - 4 b d \text{Sin}[c + d \#1] \text{SinIntegral}[d (x - \#1)] \#1^2 \right) \ \& \right] + \\
& \text{RootSum}\left[a + b \#1^3 \ \&, \frac{1}{\#1^2} \left(i a d^2 \text{Cos}[c + d \#1] \text{CosIntegral}[d (x - \#1)] - a d^2 \text{CosIntegral}[d (x - \#1)] \text{Sin}[c + d \#1] - \right. \\
& \quad a d^2 \text{Cos}[c + d \#1] \text{SinIntegral}[d (x - \#1)] - i a d^2 \text{Sin}[c + d \#1] \text{SinIntegral}[d (x - \#1)] + 4 i b \text{Cos}[c + d \#1] \\
& \quad \text{CosIntegral}[d (x - \#1)] \#1 - 4 b \text{CosIntegral}[d (x - \#1)] \text{Sin}[c + d \#1] \#1 - 4 b \text{Cos}[c + d \#1] \text{SinIntegral}[d (x - \#1)] \#1 - \\
& \quad \left. 4 i b \text{Sin}[c + d \#1] \text{SinIntegral}[d (x - \#1)] \#1 + 4 b d \text{Cos}[c + d \#1] \text{CosIntegral}[d (x - \#1)] \#1^2 + 4 i b d \text{CosIntegral}[d (x - \#1)] \right. \\
& \quad \left. \text{Sin}[c + d \#1] \#1^2 + 4 i b d \text{Cos}[c + d \#1] \text{SinIntegral}[d (x - \#1)] \#1^2 - 4 b d \text{Sin}[c + d \#1] \text{SinIntegral}[d (x - \#1)] \#1^2 \right) \ \& \right] - \\
& \left. \frac{6 b \text{Cos}[d x] (a d (a + b x^3) \text{Cos}[c] + b x^2 (7 a + 4 b x^3) \text{Sin}[c])}{(a + b x^3)^2} - \frac{6 b (b x^2 (7 a + 4 b x^3) \text{Cos}[c] - a d (a + b x^3) \text{Sin}[c]) \text{Sin}[d x]}{(a + b x^3)^2} \right)
\end{aligned}$$

■ **Problem 112: Result is not expressed in closed-form.**

$$\int \frac{\text{Sin}[c + d x]}{(a + b x^3)^3} dx$$

Optimal (type 4, 1161 leaves, 99 steps):

$$\begin{aligned}
& \frac{d \operatorname{Cos}[c + d x]}{18 a b^2 x^4} - \frac{d \operatorname{Cos}[c + d x]}{18 a^2 b x} - \frac{d \operatorname{Cos}[c + d x]}{18 b^2 x^4 (a + b x^3)} + \frac{(-1)^{2/3} d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{7/3} b^{2/3}} + \\
& \frac{d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{7/3} b^{2/3}} - \frac{(-1)^{1/3} d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{7/3} b^{2/3}} + \\
& \frac{5 \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{27 a^{8/3} b^{1/3}} - \frac{d^2 \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{54 a^2 b} - \\
& \frac{5 (-1)^{1/3} \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^{8/3} b^{1/3}} - \frac{d^2 \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{54 a^2 b} + \\
& \frac{5 (-1)^{2/3} \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^{8/3} b^{1/3}} - \frac{d^2 \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{54 a^2 b} - \\
& \frac{\operatorname{Sin}[c + d x]}{9 a b^2 x^5} + \frac{5 \operatorname{Sin}[c + d x]}{18 a^2 b x^2} - \frac{\operatorname{Sin}[c + d x]}{6 b x^2 (a + b x^3)^2} + \frac{\operatorname{Sin}[c + d x]}{9 b^2 x^5 (a + b x^3)} + \frac{5 (-1)^{1/3} \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{27 a^{8/3} b^{1/3}} + \\
& \frac{d^2 \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{54 a^2 b} + \frac{(-1)^{2/3} d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{9 a^{7/3} b^{2/3}} + \\
& \frac{5 \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{8/3} b^{1/3}} - \frac{d^2 \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{54 a^2 b} - \\
& \frac{d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{7/3} b^{2/3}} + \frac{5 (-1)^{2/3} \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{8/3} b^{1/3}} - \\
& \frac{d^2 \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{54 a^2 b} + \frac{(-1)^{1/3} d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{9 a^{7/3} b^{2/3}}
\end{aligned}$$

Result (type 7, 675 leaves):

$$\frac{1}{108 a^2} \left(-\frac{1}{b} i \operatorname{RootSum}\left[a + b \#1^3 \&, \frac{1}{\#1^2} \left(-10 \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] + 10 i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] + 10 i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + 10 \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] - 6 i d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 - 6 d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 - 6 d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 + 6 i d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 + d^2 \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1^2 - i d^2 \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1^2 - i d^2 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1^2 - d^2 \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1^2 \right) \& \right] + \frac{1}{b} i \operatorname{RootSum}\left[a + b \#1^3 \&, \frac{1}{\#1^2} \left(-10 \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] - 10 i \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] - 10 i \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + 10 \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] + 6 i d \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1 - 6 d \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1 - 6 d \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 - 6 i d \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1 + d^2 \operatorname{Cos}[c + d \#1] \operatorname{CosIntegral}[d (x - \#1)] \#1^2 + i d^2 \operatorname{CosIntegral}[d (x - \#1)] \operatorname{Sin}[c + d \#1] \#1^2 + i d^2 \operatorname{Cos}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1^2 - d^2 \operatorname{Sin}[c + d \#1] \operatorname{SinIntegral}[d (x - \#1)] \#1^2 \right) \& \right] - \frac{6 x \operatorname{Cos}[d x] (d x (a + b x^3) \operatorname{Cos}[c] - (8 a + 5 b x^3) \operatorname{Sin}[c])}{(a + b x^3)^2} + \frac{6 x ((8 a + 5 b x^3) \operatorname{Cos}[c] + d x (a + b x^3) \operatorname{Sin}[c]) \operatorname{Sin}[d x]}{(a + b x^3)^2} \right)$$

■ **Problem 113: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c + d x]}{x (a + b x^3)^3} dx$$

Optimal (type 4, 1163 leaves, 110 steps):

$$\begin{aligned}
& \frac{d \operatorname{Cos}[c + d x]}{18 a b^2 x^5} - \frac{d \operatorname{Cos}[c + d x]}{18 a^2 b x^2} - \frac{d \operatorname{Cos}[c + d x]}{18 b^2 x^5 (a + b x^3)} + \frac{4 (-1)^{1/3} d \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{27 a^{8/3} b^{1/3}} - \\
& \frac{4 d \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{8/3} b^{1/3}} - \frac{4 (-1)^{2/3} d \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{8/3} b^{1/3}} + \frac{\operatorname{CosIntegral}[d x] \operatorname{Sin}[c]}{a^3} - \\
& \frac{\operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{3 a^3} + \frac{d^2 \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right]}{54 a^{7/3} b^{2/3}} - \frac{\operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^3} + \\
& \frac{(-1)^{2/3} d^2 \operatorname{CosIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{54 a^{7/3} b^{2/3}} - \frac{\operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{3 a^3} - \\
& \frac{(-1)^{1/3} d^2 \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{54 a^{7/3} b^{2/3}} - \frac{\operatorname{Sin}[c + d x]}{6 a b^2 x^6} + \frac{\operatorname{Sin}[c + d x]}{3 a^2 b x^3} - \frac{\operatorname{Sin}[c + d x]}{6 b x^3 (a + b x^3)^2} + \frac{\operatorname{Sin}[c + d x]}{6 b^2 x^6 (a + b x^3)} + \\
& \frac{\operatorname{Cos}[c] \operatorname{SinIntegral}[d x]}{a^3} + \frac{\operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 a^3} - \frac{(-1)^{2/3} d^2 \operatorname{Cos}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{54 a^{7/3} b^{2/3}} + \\
& \frac{4 (-1)^{1/3} d \operatorname{Sin}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{27 a^{8/3} b^{1/3}} - \frac{\operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^3} + \\
& \frac{d^2 \operatorname{Cos}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{54 a^{7/3} b^{2/3}} + \frac{4 d \operatorname{Sin}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{8/3} b^{1/3}} - \frac{\operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^3} - \\
& \frac{(-1)^{1/3} d^2 \operatorname{Cos}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{54 a^{7/3} b^{2/3}} + \frac{4 (-1)^{2/3} d \operatorname{Sin}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{27 a^{8/3} b^{1/3}}
\end{aligned}$$

Result (type 4, 2929 leaves):

$$\begin{aligned}
& \operatorname{Sin}[c] \left(\frac{\operatorname{CosIntegral}[d x]}{a^3} - \right. \\
& \left. \left((-1)^{2/3} (63 - 64 (-1)^{1/3} + 62 (-1)^{2/3}) \left(d^2 \operatorname{Cos}\left[\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[d \left(\frac{a^{1/3}}{b^{1/3}} + x\right)\right] + \frac{b^{1/3} (b^{1/3} \operatorname{Cos}[d x] - d (a^{1/3} + b^{1/3} x) \operatorname{Sin}[d x])}{(a^{1/3} + b^{1/3} x)^2} + \right. \right. \right. \\
& \left. \left. \left. d^2 \operatorname{Sin}\left[\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[d \left(\frac{a^{1/3}}{b^{1/3}} + x\right)\right] \right) \right) / (18 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^3 a^{7/3} b^{2/3}) - \right. \\
& \left. \left((-1)^{2/3} (64 - 62 (-1)^{1/3} + 63 (-1)^{2/3}) \left(d^2 \operatorname{Cos}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[d \left(\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}} + x\right)\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{b^{1/3} (b^{1/3} \cos[dx] - d((-1)^{2/3} a^{1/3} + b^{1/3} x) \sin[dx])}{((-1)^{2/3} a^{1/3} + b^{1/3} x)^2} + d^2 \sin\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[d\left(\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}} + x\right)\right] \Bigg) \Bigg) / \\
(18 (1 + (-1)^{1/3})^3 a^{7/3} b^{2/3}) & + \frac{1}{(1 + (-1)^{1/3})^2 a^3} (2 - 3(-1)^{1/3} + 2(-1)^{2/3}) \\
\left(\cos\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx\right] + \sin\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]\right) & - \\
\left((-1)^{2/3} (64 - 62(-1)^{1/3} + 63(-1)^{2/3}) \left(d^2 \cos\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[d\left(-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}} + x\right)\right] + \right. \right. & \\
\left. \frac{b^{2/3} \cos[dx] + b^{1/3} d((-1)^{1/3} a^{1/3} - b^{1/3} x) \sin[dx]}{((-1)^{1/3} a^{1/3} - b^{1/3} x)^2} + d^2 \sin\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]\right) & \Bigg) \Bigg) / \\
(18 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^3 a^{7/3} b^{2/3}) - \frac{1}{9 (1 + (-1)^{1/3})^3 a^{8/3}} (-1)^{2/3} (59 - 67(-1)^{1/3} + 63(-1)^{2/3}) b^{1/3} \left(-\frac{\cos[dx]}{b^{1/3} (-(-1)^{1/3} a^{1/3} + b^{1/3} x)} + \right. & \\
\left. 1/b^{2/3} d \left(-\operatorname{CosIntegral}\left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx\right] \sin\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] + \cos\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]\right) & \Bigg) - \\
\frac{1}{(1 + (-1)^{1/3})^2 a^3 b^{1/3}} (-1)^{2/3} (5 b^{1/3} - 5(-1)^{1/3} b^{1/3} + 4(-1)^{2/3} b^{1/3}) & \\
\left(\cos\left[\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right] + \sin\left[\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]\right) - & \\
\left((59 - 67(-1)^{1/3} + 63(-1)^{2/3}) b^{1/3} \left(-\frac{\cos[dx]}{b^{1/3} (a^{1/3} + b^{1/3} x)} + \frac{d \left(\operatorname{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right] \sin\left[\frac{a^{1/3} d}{b^{1/3}}\right] - \cos\left[\frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]\right)}{b^{2/3}}\right) \right) & \Bigg) \Bigg) / \\
(9 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^3 a^{8/3}) + \frac{1}{(1 + (-1)^{1/3})^2 a^3 b^{1/3}} (-1)^{2/3} (2 b^{1/3} - 2(-1)^{1/3} b^{1/3} + 3(-1)^{2/3} b^{1/3}) & \\
\left(\cos\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right] + \sin\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]\right) - & \\
\left((-1)^{2/3} (59 b^{1/3} - 67(-1)^{1/3} b^{1/3} + 63(-1)^{2/3} b^{1/3}) \left(-\frac{\cos[dx]}{b^{1/3} ((-1)^{2/3} a^{1/3} + b^{1/3} x)} + 1/b^{2/3} d \left(\operatorname{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right] \right. \right. & \\
\left. \left. \sin\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] - \cos\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]\right) \right) & \Bigg) \Bigg) / (9 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^3 a^{8/3}) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \cos[c] \left(\frac{\text{SinIntegral}[d x]}{a^3} - \left((-1)^{2/3} (63 - 64 (-1)^{1/3} + 62 (-1)^{2/3}) \left(-d^2 \text{CosIntegral}\left[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right)\right] \text{Sin}\left[\frac{a^{1/3} d}{b^{1/3}}\right] + \right. \right. \right. \\
& \quad \left. \left. \frac{b^{1/3} (d (a^{1/3} + b^{1/3} x) \text{Cos}[d x] + b^{1/3} \text{Sin}[d x])}{(a^{1/3} + b^{1/3} x)^2} + d^2 \text{Cos}\left[\frac{a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right)\right] \right) \right) / \\
& (18 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^3 a^{7/3} b^{2/3}) - \left((-1)^{2/3} (64 - 62 (-1)^{1/3} + 63 (-1)^{2/3}) \right. \\
& \quad \left. \left(-d^2 \text{CosIntegral}\left[d \left(\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}} + x \right)\right] \text{Sin}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] + \frac{b^{1/3} (d ((-1)^{2/3} a^{1/3} + b^{1/3} x) \text{Cos}[d x] + b^{1/3} \text{Sin}[d x])}{((-1)^{2/3} a^{1/3} + b^{1/3} x)^2} + \right. \right. \\
& \quad \left. \left. d^2 \text{Cos}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[d \left(\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}} + x \right)\right] \right) \right) / (18 (1 + (-1)^{1/3})^3 a^{7/3} b^{2/3}) + \frac{1}{(1 + (-1)^{1/3})^2 a^3} \\
& (2 - 3 (-1)^{1/3} + 2 (-1)^{2/3}) \left(\text{CosIntegral}\left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + d x\right] \text{Sin}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] - \text{Cos}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \right) + \\
& \left((-1)^{2/3} (64 - 62 (-1)^{1/3} + 63 (-1)^{2/3}) \left(-d^2 \text{CosIntegral}\left[d \left(-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}} + x \right)\right] \text{Sin}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] + \right. \right. \\
& \quad \left. \left. \frac{b^{1/3} d ((-1)^{1/3} a^{1/3} - b^{1/3} x) \text{Cos}[d x] - b^{2/3} \text{Sin}[d x]}{((-1)^{1/3} a^{1/3} - b^{1/3} x)^2} + d^2 \text{Cos}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \right) \right) / \\
& (18 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^3 a^{7/3} b^{2/3}) - \frac{1}{9 (1 + (-1)^{1/3})^3 a^{8/3}} (-1)^{2/3} (59 - 67 (-1)^{1/3} + 63 (-1)^{2/3}) b^{1/3} \left(-\frac{\text{Sin}[d x]}{b^{1/3} (-(-1)^{1/3} a^{1/3} + b^{1/3} x)} + \right. \\
& \quad \left. 1/b^{2/3} d \left(\text{Cos}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{CosIntegral}\left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + d x\right] + \text{Sin}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \right) \right) - \\
& \frac{1}{(1 + (-1)^{1/3})^2 a^3 b^{1/3}} (-1)^{2/3} (5 b^{1/3} - 5 (-1)^{1/3} b^{1/3} + 4 (-1)^{2/3} b^{1/3}) \\
& \left(-\text{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \text{Sin}\left[\frac{a^{1/3} d}{b^{1/3}}\right] + \text{Cos}\left[\frac{a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \right) - \\
& \left((59 - 67 (-1)^{1/3} + 63 (-1)^{2/3}) b^{1/3} \left(-\frac{\text{Sin}[d x]}{b^{1/3} (a^{1/3} + b^{1/3} x)} + \frac{d \left(\text{Cos}\left[\frac{a^{1/3} d}{b^{1/3}}\right] \text{CosIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] + \text{Sin}\left[\frac{a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \right)}{b^{2/3}} \right) \right) / \\
& (9 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^3 a^{8/3}) + \frac{1}{(1 + (-1)^{1/3})^2 a^3 b^{1/3}} (-1)^{2/3} (2 b^{1/3} - 2 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3})
\end{aligned}$$

$$\left(-\text{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right] \text{Sin}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] + \text{Cos}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right] \right) -$$

$$\left((-1)^{2/3} (59 b^{1/3} - 67 (-1)^{1/3} b^{1/3} + 63 (-1)^{2/3} b^{1/3}) \left(-\frac{\text{Sin}[dx]}{b^{1/3} ((-1)^{2/3} a^{1/3} + b^{1/3} x)} + 1/b^{2/3} d \left(\text{Cos}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{CosIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right] + \text{Sin}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right] \right) \right) \right) / \left(9 (-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^3 a^{8/3} \right)$$

Test results for the 357 problems in "4.1.12 (e x)^m (a+b sin(c+d x^n))^p.m"

- **Problem 36: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{a + b \text{Sin}[c + dx^2]} dx$$

Optimal (type 4, 245 leaves, 9 steps):

$$-\frac{i x^2 \text{Log}\left[1 - \frac{i b e^{i(c+dx^2)}}{a - \sqrt{a^2 - b^2}}\right]}{2 \sqrt{a^2 - b^2} d} + \frac{i x^2 \text{Log}\left[1 - \frac{i b e^{i(c+dx^2)}}{a + \sqrt{a^2 - b^2}}\right]}{2 \sqrt{a^2 - b^2} d} - \frac{\text{PolyLog}\left[2, \frac{i b e^{i(c+dx^2)}}{a - \sqrt{a^2 - b^2}}\right]}{2 \sqrt{a^2 - b^2} d^2} + \frac{\text{PolyLog}\left[2, \frac{i b e^{i(c+dx^2)}}{a + \sqrt{a^2 - b^2}}\right]}{2 \sqrt{a^2 - b^2} d^2}$$

Result (type 4, 952 leaves):

$$\begin{aligned}
& \frac{1}{2d^2} \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx^2)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \right. \\
& \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(c - \operatorname{ArcCos}\left[-\frac{a}{b}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx^2)\right]}{\sqrt{-a^2+b^2}}\right] + (-2c+\pi-2dx^2) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2dx^2)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx^2)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[\frac{(a+b) \left(-a+b-i\sqrt{-a^2+b^2}\right) \left(1+i \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2dx^2)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2dx^2)\right]\right)}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \left(-\operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx^2)\right]}{\sqrt{-a^2+b^2}}\right] + \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2dx^2)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right. \\
& \left. \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4}i(-2c+\pi-2dx^2)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx^2]}}\right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx^2)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \right. \\
& \left. \left. 2i \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2dx^2)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4}i(2c-\pi+2dx^2)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+dx^2]}}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx^2)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 + \frac{i \left(i a + \sqrt{-a^2+b^2}\right) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx^2)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2dx^2)\right]\right)}\right] + \right. \\
& \left. i \left(\operatorname{PolyLog}\left[2, \frac{\left(a-i\sqrt{-a^2+b^2}\right) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx^2)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2dx^2)\right]\right)}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{\left(a+i\sqrt{-a^2+b^2}\right) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2dx^2)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2dx^2)\right]\right)}\right] \right) \right) \right)
\end{aligned}$$

■ **Problem 81: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^5}{a + b \sin[c + d x^3]} dx$$

Optimal (type 4, 245 leaves, 9 steps) :

$$-\frac{i x^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx^3)}}{a - \sqrt{a^2 - b^2}}\right]}{3 \sqrt{a^2 - b^2} d} + \frac{i x^3 \operatorname{Log}\left[1 - \frac{i b e^{i(c+dx^3)}}{a + \sqrt{a^2 - b^2}}\right]}{3 \sqrt{a^2 - b^2} d} - \frac{\operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx^3)}}{a - \sqrt{a^2 - b^2}}\right]}{3 \sqrt{a^2 - b^2} d^2} + \frac{\operatorname{PolyLog}\left[2, \frac{i b e^{i(c+dx^3)}}{a + \sqrt{a^2 - b^2}}\right]}{3 \sqrt{a^2 - b^2} d^2}$$

Result (type 4, 952 leaves) :

$$\begin{aligned}
& \frac{1}{3 d^2} \left(\frac{\pi \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x^3)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \right. \\
& \frac{1}{\sqrt{-a^2+b^2}} \left(2 \left(c - \operatorname{ArcCos}\left[-\frac{a}{b}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x^3)\right]}{\sqrt{-a^2+b^2}}\right] + (-2c+\pi-2 d x^3) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2 d x^3)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x^3)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[\frac{(a+b) \left(-a+b-i \sqrt{-a^2+b^2}\right) \left(1+i \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x^3)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x^3)\right]\right)}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \left(-\operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x^3)\right]}{\sqrt{-a^2+b^2}}\right] + \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2 d x^3)\right]}{\sqrt{-a^2+b^2}}\right] \right) \right) \right. \\
& \left. \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4} i(-2c+\pi-2 d x^3)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+d x^3]}}\right] + \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x^3)\right]}{\sqrt{-a^2+b^2}}\right] - \right. \\
& \left. 2 i \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Tan}\left[\frac{1}{4}(2c+\pi+2 d x^3)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{4} i(2c-\pi+2 d x^3)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Sin}[c+d x^3]}}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a}{b}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x^3)\right]}{\sqrt{-a^2+b^2}}\right] \right) \operatorname{Log}\left[1 + \frac{i \left(i a + \sqrt{-a^2+b^2}\right) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x^3)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x^3)\right]\right)}\right] + \right. \\
& \left. i \left(\operatorname{PolyLog}\left[2, \frac{\left(a-i \sqrt{-a^2+b^2}\right) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x^3)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x^3)\right]\right)}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{\left(a+i \sqrt{-a^2+b^2}\right) \left(a+b+\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{4}(2c-\pi+2 d x^3)\right]\right)}{b \left(a+b+\sqrt{-a^2+b^2} \operatorname{Cot}\left[\frac{1}{4}(2c+\pi+2 d x^3)\right]\right)}\right] \right) \right) \right)
\end{aligned}$$

■ **Problem 165: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e + f x)^3 \sin[a + b (c + d x)^2] dx$$

Optimal (type 4, 341 leaves, 14 steps):

$$\begin{aligned} & - \frac{3 f (d e - c f)^2 \cos[a + b (c + d x)^2]}{2 b d^4} - \frac{3 f^2 (d e - c f) (c + d x) \cos[a + b (c + d x)^2]}{2 b d^4} - \frac{f^3 (c + d x)^2 \cos[a + b (c + d x)^2]}{2 b d^4} + \\ & \frac{3 f^2 (d e - c f) \sqrt{\frac{\pi}{2}} \cos[a] \operatorname{FresnelC}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right]}{2 b^{3/2} d^4} + \frac{(d e - c f)^3 \sqrt{\frac{\pi}{2}} \cos[a] \operatorname{FresnelS}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right]}{\sqrt{b} d^4} + \\ & \frac{(d e - c f)^3 \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right] \sin[a]}{\sqrt{b} d^4} - \frac{3 f^2 (d e - c f) \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right] \sin[a]}{2 b^{3/2} d^4} + \frac{f^3 \sin[a + b (c + d x)^2]}{2 b^2 d^4} \end{aligned}$$

Result (type 4, 283 leaves):

$$\begin{aligned} & - \frac{1}{2 \sqrt{2} b^2 d^4} \left(\cos[a + b (c + d x)^2] - i \sin[a + b (c + d x)^2] \right) \left(\cos[a + b (c + d x)^2] + i \sin[a + b (c + d x)^2] \right) \\ & \left(-\sqrt{b} (d e - c f) \sqrt{\pi} \operatorname{FresnelS}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right] \left(2 b (d e - c f)^2 \cos[a] - 3 f^2 \sin[a] \right) - \right. \\ & \left. \sqrt{b} (d e - c f) \sqrt{\pi} \operatorname{FresnelC}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right] \left(3 f^2 \cos[a] + 2 b (d e - c f)^2 \sin[a] \right) + \right. \\ & \left. \sqrt{2} f \left(b (c^2 f^2 - c d f (3 e + f x) + d^2 (3 e^2 + 3 e f x + f^2 x^2)) \cos[a + b (c + d x)^2] - f^2 \sin[a + b (c + d x)^2] \right) \right) \end{aligned}$$

■ **Problem 166: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e + f x)^2 \sin[a + b (c + d x)^2] dx$$

Optimal (type 4, 256 leaves, 11 steps):

$$\begin{aligned}
& - \frac{f (d e - c f) \operatorname{Cos}[a + b (c + d x)^2]}{b d^3} - \frac{f^2 (c + d x) \operatorname{Cos}[a + b (c + d x)^2]}{2 b d^3} + \\
& \frac{f^2 \sqrt{\frac{\pi}{2}} \operatorname{Cos}[a] \operatorname{FresnelC}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right]}{2 b^{3/2} d^3} + \frac{(d e - c f)^2 \sqrt{\frac{\pi}{2}} \operatorname{Cos}[a] \operatorname{FresnelS}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right]}{\sqrt{b} d^3} + \\
& \frac{(d e - c f)^2 \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right] \operatorname{Sin}[a]}{\sqrt{b} d^3} - \frac{f^2 \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right] \operatorname{Sin}[a]}{2 b^{3/2} d^3}
\end{aligned}$$

Result (type 4, 214 leaves):

$$\begin{aligned}
& - \frac{1}{2 \sqrt{2} b^{3/2} d^3} \left(\sqrt{2} \sqrt{b} f (2 d e - c f + d f x) \operatorname{Cos}[a + b (c + d x)^2] - \sqrt{\pi} \operatorname{FresnelS}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right] (2 b (d e - c f)^2 \operatorname{Cos}[a] - f^2 \operatorname{Sin}[a]) - \right. \\
& \left. \sqrt{\pi} \operatorname{FresnelC}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)\right] (f^2 \operatorname{Cos}[a] + 2 b (d e - c f)^2 \operatorname{Sin}[a]) \right) \\
& (\operatorname{Cos}[a + b (c + d x)^2] - i \operatorname{Sin}[a + b (c + d x)^2]) (\operatorname{Cos}[a + b (c + d x)^2] + i \operatorname{Sin}[a + b (c + d x)^2])
\end{aligned}$$

■ **Problem 171: Attempted integration timed out after 120 seconds.**

$$\int (e + f x)^3 \operatorname{Sin}[a + b (c + d x)^3] dx$$

Optimal (type 4, 434 leaves, 14 steps):

$$\begin{aligned}
& - \frac{f^2 (d e - c f) \operatorname{Cos}[a + b (c + d x)^3]}{b d^4} - \frac{f^3 (c + d x) \operatorname{Cos}[a + b (c + d x)^3]}{3 b d^4} - \\
& \frac{e^{i a} f^3 (c + d x) \operatorname{Gamma}\left[\frac{1}{3}, -i b (c + d x)^3\right]}{18 b d^4 (-i b (c + d x)^3)^{1/3}} + \frac{i e^{i a} (d e - c f)^3 (c + d x) \operatorname{Gamma}\left[\frac{1}{3}, -i b (c + d x)^3\right]}{6 d^4 (-i b (c + d x)^3)^{1/3}} - \\
& \frac{e^{-i a} f^3 (c + d x) \operatorname{Gamma}\left[\frac{1}{3}, i b (c + d x)^3\right]}{18 b d^4 (i b (c + d x)^3)^{1/3}} - \frac{i e^{-i a} (d e - c f)^3 (c + d x) \operatorname{Gamma}\left[\frac{1}{3}, i b (c + d x)^3\right]}{6 d^4 (i b (c + d x)^3)^{1/3}} + \\
& \frac{i e^{i a} f (d e - c f)^2 (c + d x)^2 \operatorname{Gamma}\left[\frac{2}{3}, -i b (c + d x)^3\right]}{2 d^4 (-i b (c + d x)^3)^{2/3}} - \frac{i e^{-i a} f (d e - c f)^2 (c + d x)^2 \operatorname{Gamma}\left[\frac{2}{3}, i b (c + d x)^3\right]}{2 d^4 (i b (c + d x)^3)^{2/3}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 172: Attempted integration timed out after 120 seconds.**

$$\int (e + f x)^2 \sin[a + b (c + d x)^3] dx$$

Optimal (type 4, 280 leaves, 10 steps):

$$\begin{aligned} & - \frac{f^2 \cos[a + b (c + d x)^3]}{3 b d^3} + \frac{i e^{i a} (d e - c f)^2 (c + d x) \operatorname{Gamma}\left[\frac{1}{3}, -i b (c + d x)^3\right]}{6 d^3 (-i b (c + d x)^3)^{1/3}} - \frac{i e^{-i a} (d e - c f)^2 (c + d x) \operatorname{Gamma}\left[\frac{1}{3}, i b (c + d x)^3\right]}{6 d^3 (i b (c + d x)^3)^{1/3}} + \\ & \frac{i e^{i a} f (d e - c f) (c + d x)^2 \operatorname{Gamma}\left[\frac{2}{3}, -i b (c + d x)^3\right]}{3 d^3 (-i b (c + d x)^3)^{2/3}} - \frac{i e^{-i a} f (d e - c f) (c + d x)^2 \operatorname{Gamma}\left[\frac{2}{3}, i b (c + d x)^3\right]}{3 d^3 (i b (c + d x)^3)^{2/3}} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 173: Attempted integration timed out after 120 seconds.**

$$\int (e + f x) \sin[a + b (c + d x)^3] dx$$

Optimal (type 4, 235 leaves, 8 steps):

$$\begin{aligned} & \frac{i e^{i a} (d e - c f) (c + d x) \operatorname{Gamma}\left[\frac{1}{3}, -i b (c + d x)^3\right]}{6 d^2 (-i b (c + d x)^3)^{1/3}} - \frac{i e^{-i a} (d e - c f) (c + d x) \operatorname{Gamma}\left[\frac{1}{3}, i b (c + d x)^3\right]}{6 d^2 (i b (c + d x)^3)^{1/3}} + \\ & \frac{i e^{i a} f (c + d x)^2 \operatorname{Gamma}\left[\frac{2}{3}, -i b (c + d x)^3\right]}{6 d^2 (-i b (c + d x)^3)^{2/3}} - \frac{i e^{-i a} f (c + d x)^2 \operatorname{Gamma}\left[\frac{2}{3}, i b (c + d x)^3\right]}{6 d^2 (i b (c + d x)^3)^{2/3}} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 175: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sin[a + b (c + d x)^3]}{e + f x} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sin[a + b (c + d x)^3]}{e + f x}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 176: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sin}[a + b (c + d x)^3]}{(e + f x)^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\text{Sin}[a + b (c + d x)^3]}{(e + f x)^2}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 183: Result more than twice size of optimal antiderivative.**

$$\int (e + f x) \text{Sin}\left[a + \frac{b}{(c + d x)^3}\right] dx$$

Optimal (type 4, 235 leaves, 8 steps):

$$\begin{aligned} & - \frac{i e^{i a} f \left(-\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2 \text{Gamma}\left[-\frac{2}{3}, -\frac{i b}{(c+d x)^3}\right]}{6 d^2} + \frac{i e^{-i a} f \left(\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2 \text{Gamma}\left[-\frac{2}{3}, \frac{i b}{(c+d x)^3}\right]}{6 d^2} \\ & - \frac{i e^{i a} (d e - c f) \left(-\frac{i b}{(c+d x)^3}\right)^{1/3} (c+d x) \text{Gamma}\left[-\frac{1}{3}, -\frac{i b}{(c+d x)^3}\right]}{6 d^2} + \frac{i e^{-i a} (d e - c f) \left(\frac{i b}{(c+d x)^3}\right)^{1/3} (c+d x) \text{Gamma}\left[-\frac{1}{3}, \frac{i b}{(c+d x)^3}\right]}{6 d^2} \end{aligned}$$

Result (type 4, 700 leaves):

$$\begin{aligned} & \frac{e (c + d x) \text{Cos}\left[\frac{b}{(c+d x)^3}\right] \text{Sin}[a]}{d} + \frac{f (-c + d x) (c + d x) \text{Cos}\left[\frac{b}{(c+d x)^3}\right] \text{Sin}[a]}{2 d^2} + \frac{1}{2 d^2} \\ & 3 b f \left(\frac{1}{2} \text{Cos}[a] \left(\frac{\text{Gamma}\left[\frac{1}{3}, -\frac{i b}{(c+d x)^3}\right]}{3 \left(-\frac{i b}{(c+d x)^3}\right)^{1/3} (c+d x)} + \frac{\text{Gamma}\left[\frac{1}{3}, \frac{i b}{(c+d x)^3}\right]}{3 \left(\frac{i b}{(c+d x)^3}\right)^{1/3} (c+d x)} \right) + \frac{1}{2} i \left(\frac{\text{Gamma}\left[\frac{1}{3}, -\frac{i b}{(c+d x)^3}\right]}{3 \left(-\frac{i b}{(c+d x)^3}\right)^{1/3} (c+d x)} - \frac{\text{Gamma}\left[\frac{1}{3}, \frac{i b}{(c+d x)^3}\right]}{3 \left(\frac{i b}{(c+d x)^3}\right)^{1/3} (c+d x)} \right) \text{Sin}[a] \right) + \frac{1}{d} \\ & 3 b e \left(\frac{1}{2} \text{Cos}[a] \left(\frac{\text{Gamma}\left[\frac{2}{3}, -\frac{i b}{(c+d x)^3}\right]}{3 \left(-\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2} + \frac{\text{Gamma}\left[\frac{2}{3}, \frac{i b}{(c+d x)^3}\right]}{3 \left(\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2} \right) + \frac{1}{2} i \left(\frac{\text{Gamma}\left[\frac{2}{3}, -\frac{i b}{(c+d x)^3}\right]}{3 \left(-\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2} - \frac{\text{Gamma}\left[\frac{2}{3}, \frac{i b}{(c+d x)^3}\right]}{3 \left(\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2} \right) \text{Sin}[a] \right) - \\ & \frac{1}{d^2} 3 b c f \left(\frac{1}{2} \text{Cos}[a] \left(\frac{\text{Gamma}\left[\frac{2}{3}, -\frac{i b}{(c+d x)^3}\right]}{3 \left(-\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2} + \frac{\text{Gamma}\left[\frac{2}{3}, \frac{i b}{(c+d x)^3}\right]}{3 \left(\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2} \right) + \frac{1}{2} i \left(\frac{\text{Gamma}\left[\frac{2}{3}, -\frac{i b}{(c+d x)^3}\right]}{3 \left(-\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2} - \frac{\text{Gamma}\left[\frac{2}{3}, \frac{i b}{(c+d x)^3}\right]}{3 \left(\frac{i b}{(c+d x)^3}\right)^{2/3} (c+d x)^2} \right) \text{Sin}[a] \right) + \\ & \frac{e (c + d x) \text{Cos}[a] \text{Sin}\left[\frac{b}{(c+d x)^3}\right]}{d} + \frac{f (-c + d x) (c + d x) \text{Cos}[a] \text{Sin}\left[\frac{b}{(c+d x)^3}\right]}{2 d^2} \end{aligned}$$

■ **Problem 190: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin\left[a + b\sqrt{c + dx}\right]}{e + fx} dx$$

Optimal (type 4, 238 leaves, 8 steps):

$$\frac{\operatorname{CosIntegral}\left[\frac{b\sqrt{-de+cf}}{\sqrt{f}} + b\sqrt{c+dx}\right] \operatorname{Sin}\left[a - \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right]}{f} + \frac{\operatorname{CosIntegral}\left[\frac{b\sqrt{-de+cf}}{\sqrt{f}} - b\sqrt{c+dx}\right] \operatorname{Sin}\left[a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right]}{f} -$$

$$\frac{\operatorname{Cos}\left[a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right] \operatorname{SinIntegral}\left[\frac{b\sqrt{-de+cf}}{\sqrt{f}} - b\sqrt{c+dx}\right]}{f} + \frac{\operatorname{Cos}\left[a - \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right] \operatorname{SinIntegral}\left[\frac{b\sqrt{-de+cf}}{\sqrt{f}} + b\sqrt{c+dx}\right]}{f}$$

Result (type 4, 238 leaves):

$$\frac{1}{2f} i e^{-i\left(a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)} \left(\operatorname{ExpIntegralEi}\left[-i b \left(-\frac{\sqrt{-de+cf}}{\sqrt{f}} + \sqrt{c+dx}\right)\right] - e^{2i\left(a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)} \operatorname{ExpIntegralEi}\left[i b \left(-\frac{\sqrt{-de+cf}}{\sqrt{f}} + \sqrt{c+dx}\right)\right] \right) +$$

$$e^{\frac{2ib\sqrt{-de+cf}}{\sqrt{f}}} \operatorname{ExpIntegralEi}\left[-i b \left(\frac{\sqrt{-de+cf}}{\sqrt{f}} + \sqrt{c+dx}\right)\right] - e^{2ia} \operatorname{ExpIntegralEi}\left[i b \left(\frac{\sqrt{-de+cf}}{\sqrt{f}} + \sqrt{c+dx}\right)\right]$$

■ **Problem 191: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin\left[a + b\sqrt{c + dx}\right]}{(e + fx)^2} dx$$

Optimal (type 4, 339 leaves, 10 steps):

$$\frac{bd \operatorname{Cos}\left[a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right] \operatorname{CosIntegral}\left[\frac{b\sqrt{-de+cf}}{\sqrt{f}} - b\sqrt{c+dx}\right]}{2f^{3/2}\sqrt{-de+cf}} - \frac{bd \operatorname{Cos}\left[a - \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right] \operatorname{CosIntegral}\left[\frac{b\sqrt{-de+cf}}{\sqrt{f}} + b\sqrt{c+dx}\right]}{2f^{3/2}\sqrt{-de+cf}} - \frac{\operatorname{Sin}\left[a + b\sqrt{c+dx}\right]}{f(e+fx)} +$$

$$\frac{bd \operatorname{Sin}\left[a + \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right] \operatorname{SinIntegral}\left[\frac{b\sqrt{-de+cf}}{\sqrt{f}} - b\sqrt{c+dx}\right]}{2f^{3/2}\sqrt{-de+cf}} + \frac{bd \operatorname{Sin}\left[a - \frac{b\sqrt{-de+cf}}{\sqrt{f}}\right] \operatorname{SinIntegral}\left[\frac{b\sqrt{-de+cf}}{\sqrt{f}} + b\sqrt{c+dx}\right]}{2f^{3/2}\sqrt{-de+cf}}$$

Result (type 4, 397 leaves):

$$\frac{1}{4 f^{3/2}} i d e^{-i a}$$

$$\left(\frac{2 e^{-i b \sqrt{c+d x}} \sqrt{f}}{d e+d f x} - \frac{i b e^{-\frac{i b \sqrt{-d e+c f}}{\sqrt{f}}} \text{ExpIntegralEi}\left[-i b \left(-\frac{\sqrt{-d e+c f}}{\sqrt{f}} + \sqrt{c+d x}\right)\right]}{\sqrt{-d e+c f}} \right) + \frac{i b e^{\frac{i b \sqrt{-d e+c f}}{\sqrt{f}}} \text{ExpIntegralEi}\left[-i b \left(\frac{\sqrt{-d e+c f}}{\sqrt{f}} + \sqrt{c+d x}\right)\right]}{\sqrt{-d e+c f}} +$$

$$e^{2 i a} \left(\frac{2 e^{i b \sqrt{c+d x}} \sqrt{f}}{d e+d f x} - \frac{i b e^{\frac{i b \sqrt{-d e+c f}}{\sqrt{f}}} \text{ExpIntegralEi}\left[i b \left(-\frac{\sqrt{-d e+c f}}{\sqrt{f}} + \sqrt{c+d x}\right)\right]}{\sqrt{-d e+c f}} \right) + \frac{i b e^{-\frac{i b \sqrt{-d e+c f}}{\sqrt{f}}} \text{ExpIntegralEi}\left[i b \left(\frac{\sqrt{-d e+c f}}{\sqrt{f}} + \sqrt{c+d x}\right)\right]}{\sqrt{-d e+c f}} \right)$$

■ **Problem 193: Result more than twice size of optimal antiderivative.**

$$\int (e+f x) \sin[a+b(c+d x)^{3/2}] dx$$

Optimal (type 4, 291 leaves, 9 steps):

$$\frac{2 f \sqrt{c+d x} \cos[a+b(c+d x)^{3/2}]}{3 b d^2} - \frac{e^{i a} f \sqrt{c+d x} \text{Gamma}\left[\frac{1}{3}, -i b(c+d x)^{3/2}\right]}{9 b d^2 (-i b(c+d x)^{3/2})^{1/3}} - \frac{e^{-i a} f \sqrt{c+d x} \text{Gamma}\left[\frac{1}{3}, i b(c+d x)^{3/2}\right]}{9 b d^2 (i b(c+d x)^{3/2})^{1/3}} +$$

$$\frac{i e^{i a} (d e-c f)(c+d x) \text{Gamma}\left[\frac{2}{3}, -i b(c+d x)^{3/2}\right]}{3 d^2 (-i b(c+d x)^{3/2})^{2/3}} - \frac{i e^{-i a} (d e-c f)(c+d x) \text{Gamma}\left[\frac{2}{3}, i b(c+d x)^{3/2}\right]}{3 d^2 (i b(c+d x)^{3/2})^{2/3}}$$

Result (type 4, 705 leaves):

$$-\frac{2 f \sqrt{c+d x} \cos[a] \cos[b(c+d x)^{3/2}]}{3 b d^2} + \frac{f \cos[a] \left(-\frac{2 \sqrt{c+d x} \text{Gamma}\left[\frac{1}{3}, -i b(c+d x)^{3/2}\right]}{3 (-i b(c+d x)^{3/2})^{1/3}} - \frac{2 \sqrt{c+d x} \text{Gamma}\left[\frac{1}{3}, i b(c+d x)^{3/2}\right]}{3 (i b(c+d x)^{3/2})^{1/3}} \right)}{6 b d^2} -$$

$$\frac{i e \cos[a] \left(-\frac{2 (c+d x) \text{Gamma}\left[\frac{2}{3}, -i b(c+d x)^{3/2}\right]}{3 (-i b(c+d x)^{3/2})^{2/3}} + \frac{2 (c+d x) \text{Gamma}\left[\frac{2}{3}, i b(c+d x)^{3/2}\right]}{3 (i b(c+d x)^{3/2})^{2/3}} \right)}{2 d} + \frac{i c f \cos[a] \left(-\frac{2 (c+d x) \text{Gamma}\left[\frac{2}{3}, -i b(c+d x)^{3/2}\right]}{3 (-i b(c+d x)^{3/2})^{2/3}} + \frac{2 (c+d x) \text{Gamma}\left[\frac{2}{3}, i b(c+d x)^{3/2}\right]}{3 (i b(c+d x)^{3/2})^{2/3}} \right)}{2 d^2} +$$

$$\frac{i f \left(-\frac{2 \sqrt{c+d x} \text{Gamma}\left[\frac{1}{3}, -i b(c+d x)^{3/2}\right]}{3 (-i b(c+d x)^{3/2})^{1/3}} + \frac{2 \sqrt{c+d x} \text{Gamma}\left[\frac{1}{3}, i b(c+d x)^{3/2}\right]}{3 (i b(c+d x)^{3/2})^{1/3}} \right) \sin[a]}{6 b d^2} + \frac{e \left(-\frac{2 (c+d x) \text{Gamma}\left[\frac{2}{3}, -i b(c+d x)^{3/2}\right]}{3 (-i b(c+d x)^{3/2})^{2/3}} - \frac{2 (c+d x) \text{Gamma}\left[\frac{2}{3}, i b(c+d x)^{3/2}\right]}{3 (i b(c+d x)^{3/2})^{2/3}} \right) \sin[a]}{2 d} -$$

$$\frac{c f \left(-\frac{2 (c+d x) \text{Gamma}\left[\frac{2}{3}, -i b(c+d x)^{3/2}\right]}{3 (-i b(c+d x)^{3/2})^{2/3}} - \frac{2 (c+d x) \text{Gamma}\left[\frac{2}{3}, i b(c+d x)^{3/2}\right]}{3 (i b(c+d x)^{3/2})^{2/3}} \right) \sin[a]}{2 d^2} + \frac{2 f \sqrt{c+d x} \sin[a] \sin[b(c+d x)^{3/2}]}{3 b d^2}$$

■ **Problem 197: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e + f x)^2 \operatorname{Sin}\left[a + \frac{b}{\sqrt{c + d x}}\right] dx$$

Optimal (type 4, 611 leaves, 23 steps):

$$\begin{aligned} & \frac{b^5 f^2 \sqrt{c + d x} \operatorname{Cos}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{360 d^3} - \frac{b^3 f (d e - c f) \sqrt{c + d x} \operatorname{Cos}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{6 d^3} + \frac{b (d e - c f)^2 \sqrt{c + d x} \operatorname{Cos}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{d^3} - \\ & \frac{b^3 f^2 (c + d x)^{3/2} \operatorname{Cos}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{180 d^3} + \frac{b f (d e - c f) (c + d x)^{3/2} \operatorname{Cos}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{3 d^3} + \frac{b f^2 (c + d x)^{5/2} \operatorname{Cos}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{15 d^3} + \\ & \frac{b^6 f^2 \operatorname{CosIntegral}\left[\frac{b}{\sqrt{c + d x}}\right] \operatorname{Sin}[a]}{360 d^3} - \frac{b^4 f (d e - c f) \operatorname{CosIntegral}\left[\frac{b}{\sqrt{c + d x}}\right] \operatorname{Sin}[a]}{6 d^3} + \frac{b^2 (d e - c f)^2 \operatorname{CosIntegral}\left[\frac{b}{\sqrt{c + d x}}\right] \operatorname{Sin}[a]}{d^3} + \\ & \frac{b^4 f^2 (c + d x) \operatorname{Sin}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{360 d^3} - \frac{b^2 f (d e - c f) (c + d x) \operatorname{Sin}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{6 d^3} + \frac{(d e - c f)^2 (c + d x) \operatorname{Sin}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{d^3} - \\ & \frac{b^2 f^2 (c + d x)^2 \operatorname{Sin}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{60 d^3} + \frac{f (d e - c f) (c + d x)^2 \operatorname{Sin}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{d^3} + \frac{f^2 (c + d x)^3 \operatorname{Sin}\left[a + \frac{b}{\sqrt{c + d x}}\right]}{3 d^3} + \\ & \frac{b^6 f^2 \operatorname{Cos}[a] \operatorname{SinIntegral}\left[\frac{b}{\sqrt{c + d x}}\right]}{360 d^3} - \frac{b^4 f (d e - c f) \operatorname{Cos}[a] \operatorname{SinIntegral}\left[\frac{b}{\sqrt{c + d x}}\right]}{6 d^3} + \frac{b^2 (d e - c f)^2 \operatorname{Cos}[a] \operatorname{SinIntegral}\left[\frac{b}{\sqrt{c + d x}}\right]}{d^3} \end{aligned}$$

Result (type 4, 557 leaves):

$$\begin{aligned} & \frac{1}{720 d^3} i e^{-i a} \left(e^{-\frac{i b}{\sqrt{c + d x}}} \sqrt{c + d x} \left(-i b^5 f^2 + b^4 f^2 \sqrt{c + d x} + 2 i b^3 f (30 d e - 29 c f + d f x) - 6 b^2 f \sqrt{c + d x} (10 d e - 9 c f + d f x) + \right. \right. \\ & \quad \left. \left. 120 \sqrt{c + d x} (c^2 f^2 - c d f (3 e + f x) + d^2 (3 e^2 + 3 e f x + f^2 x^2)) - 24 i b (11 c^2 f^2 - c d f (25 e + 3 f x) + d^2 (15 e^2 + 5 e f x + f^2 x^2)) \right) \right) - \\ & \quad e^{i \left(2 a + \frac{b}{\sqrt{c + d x}} \right)} \sqrt{c + d x} \left(i b^5 f^2 + b^4 f^2 \sqrt{c + d x} - 2 i b^3 f (30 d e - 29 c f + d f x) - 6 b^2 f \sqrt{c + d x} (10 d e - 9 c f + d f x) + \right. \\ & \quad \left. 120 \sqrt{c + d x} (c^2 f^2 - c d f (3 e + f x) + d^2 (3 e^2 + 3 e f x + f^2 x^2)) + 24 i b (11 c^2 f^2 - c d f (25 e + 3 f x) + d^2 (15 e^2 + 5 e f x + f^2 x^2)) \right) \right) + \\ & \quad b^2 (360 d^2 e^2 - 60 (b^2 + 12 c) d e f + (b^4 + 60 b^2 c + 360 c^2) f^2) \operatorname{ExpIntegralEi}\left[-\frac{i b}{\sqrt{c + d x}}\right] - \\ & \quad \left. b^2 e^{2 i a} (360 d^2 e^2 - 60 (b^2 + 12 c) d e f + (b^4 + 60 b^2 c + 360 c^2) f^2) \operatorname{ExpIntegralEi}\left[\frac{i b}{\sqrt{c + d x}}\right] \right) \end{aligned}$$

■ **Problem 200: Unable to integrate problem.**

$$\int \frac{\sin\left[a + \frac{b}{\sqrt{c+dx}}\right]}{e+fx} dx$$

Optimal (type 4, 276 leaves, 13 steps):

$$\begin{aligned} & -\frac{2 \operatorname{CosIntegral}\left[\frac{b}{\sqrt{c+dx}}\right] \operatorname{Sin}[a]}{f} + \frac{\operatorname{CosIntegral}\left[\frac{b\sqrt{f}}{\sqrt{-de+cf}} + \frac{b}{\sqrt{c+dx}}\right] \operatorname{Sin}\left[a - \frac{b\sqrt{f}}{\sqrt{-de+cf}}\right]}{f} + \frac{\operatorname{CosIntegral}\left[\frac{b\sqrt{f}}{\sqrt{-de+cf}} - \frac{b}{\sqrt{c+dx}}\right] \operatorname{Sin}\left[a + \frac{b\sqrt{f}}{\sqrt{-de+cf}}\right]}{f} \\ & \frac{2 \operatorname{Cos}[a] \operatorname{SinIntegral}\left[\frac{b}{\sqrt{c+dx}}\right]}{f} - \frac{\operatorname{Cos}\left[a + \frac{b\sqrt{f}}{\sqrt{-de+cf}}\right] \operatorname{SinIntegral}\left[\frac{b\sqrt{f}}{\sqrt{-de+cf}} - \frac{b}{\sqrt{c+dx}}\right]}{f} + \frac{\operatorname{Cos}\left[a - \frac{b\sqrt{f}}{\sqrt{-de+cf}}\right] \operatorname{SinIntegral}\left[\frac{b\sqrt{f}}{\sqrt{-de+cf}} + \frac{b}{\sqrt{c+dx}}\right]}{f} \end{aligned}$$

Result (type 8, 24 leaves):

$$\int \frac{\sin\left[a + \frac{b}{\sqrt{c+dx}}\right]}{e+fx} dx$$

■ **Problem 201: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sin\left[a + \frac{b}{\sqrt{c+dx}}\right]}{(e+fx)^2} dx$$

Optimal (type 4, 350 leaves, 10 steps):

$$\begin{aligned} & -\frac{bd \operatorname{Cos}\left[a + \frac{b\sqrt{f}}{\sqrt{-de+cf}}\right] \operatorname{CosIntegral}\left[\frac{b\sqrt{f}}{\sqrt{-de+cf}} - \frac{b}{\sqrt{c+dx}}\right]}{2\sqrt{f}(-de+cf)^{3/2}} + \frac{bd \operatorname{Cos}\left[a - \frac{b\sqrt{f}}{\sqrt{-de+cf}}\right] \operatorname{CosIntegral}\left[\frac{b\sqrt{f}}{\sqrt{-de+cf}} + \frac{b}{\sqrt{c+dx}}\right]}{2\sqrt{f}(-de+cf)^{3/2}} + \\ & \frac{(c+dx) \operatorname{Sin}\left[a + \frac{b}{\sqrt{c+dx}}\right]}{(de-cf)(e+fx)} - \frac{bd \operatorname{Sin}\left[a + \frac{b\sqrt{f}}{\sqrt{-de+cf}}\right] \operatorname{SinIntegral}\left[\frac{b\sqrt{f}}{\sqrt{-de+cf}} - \frac{b}{\sqrt{c+dx}}\right]}{2\sqrt{f}(-de+cf)^{3/2}} - \frac{bd \operatorname{Sin}\left[a - \frac{b\sqrt{f}}{\sqrt{-de+cf}}\right] \operatorname{SinIntegral}\left[\frac{b\sqrt{f}}{\sqrt{-de+cf}} + \frac{b}{\sqrt{c+dx}}\right]}{2\sqrt{f}(-de+cf)^{3/2}} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 203: Result more than twice size of optimal antiderivative.**

$$\int (e+fx) \sin\left[a + \frac{b}{(c+dx)^{3/2}}\right] dx$$

Optimal (type 4, 251 leaves, 8 steps):

$$\begin{aligned}
& - \frac{i e^{i a} f \left(-\frac{i b}{(c+d x)^{3/2}}\right)^{4/3} (c+d x)^2 \operatorname{Gamma}\left[-\frac{4}{3}, -\frac{i b}{(c+d x)^{3/2}}\right]}{3 d^2} + \frac{i e^{-i a} f \left(\frac{i b}{(c+d x)^{3/2}}\right)^{4/3} (c+d x)^2 \operatorname{Gamma}\left[-\frac{4}{3}, \frac{i b}{(c+d x)^{3/2}}\right]}{3 d^2} \\
& - \frac{i e^{i a} (d e - c f) \left(-\frac{i b}{(c+d x)^{3/2}}\right)^{2/3} (c+d x) \operatorname{Gamma}\left[-\frac{2}{3}, -\frac{i b}{(c+d x)^{3/2}}\right]}{3 d^2} + \frac{i e^{-i a} (d e - c f) \left(\frac{i b}{(c+d x)^{3/2}}\right)^{2/3} (c+d x) \operatorname{Gamma}\left[-\frac{2}{3}, \frac{i b}{(c+d x)^{3/2}}\right]}{3 d^2}
\end{aligned}$$

Result (type 4, 835 leaves):

$$\begin{aligned}
& \frac{3 b e \operatorname{Cos}[a] \left(\frac{2 \operatorname{Gamma}\left[\frac{1}{3}, -\frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(-\frac{i b}{(c+d x)^{3/2}}\right)^{1/3} \sqrt{c+d x}} + \frac{2 \operatorname{Gamma}\left[\frac{1}{3}, \frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(\frac{i b}{(c+d x)^{3/2}}\right)^{1/3} \sqrt{c+d x}}\right)}{4 d} - \frac{3 b c f \operatorname{Cos}[a] \left(\frac{2 \operatorname{Gamma}\left[\frac{1}{3}, -\frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(-\frac{i b}{(c+d x)^{3/2}}\right)^{1/3} \sqrt{c+d x}} + \frac{2 \operatorname{Gamma}\left[\frac{1}{3}, \frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(\frac{i b}{(c+d x)^{3/2}}\right)^{1/3} \sqrt{c+d x}}\right)}{4 d^2} + \\
& \frac{9 i b^2 f \operatorname{Cos}[a] \left(\frac{2 \operatorname{Gamma}\left[\frac{2}{3}, -\frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(-\frac{i b}{(c+d x)^{3/2}}\right)^{2/3} (c+d x)} - \frac{2 \operatorname{Gamma}\left[\frac{2}{3}, \frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(\frac{i b}{(c+d x)^{3/2}}\right)^{2/3} (c+d x)}\right)}{8 d^2} + \frac{e (c+d x) \operatorname{Cos}\left[\frac{b}{(c+d x)^{3/2}}\right] \operatorname{Sin}[a]}{d} + \\
& \frac{3 i b e \left(\frac{2 \operatorname{Gamma}\left[\frac{1}{3}, -\frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(-\frac{i b}{(c+d x)^{3/2}}\right)^{1/3} \sqrt{c+d x}} - \frac{2 \operatorname{Gamma}\left[\frac{1}{3}, \frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(\frac{i b}{(c+d x)^{3/2}}\right)^{1/3} \sqrt{c+d x}}\right) \operatorname{Sin}[a]}{4 d} - \frac{3 i b c f \left(\frac{2 \operatorname{Gamma}\left[\frac{1}{3}, -\frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(-\frac{i b}{(c+d x)^{3/2}}\right)^{1/3} \sqrt{c+d x}} - \frac{2 \operatorname{Gamma}\left[\frac{1}{3}, \frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(\frac{i b}{(c+d x)^{3/2}}\right)^{1/3} \sqrt{c+d x}}\right) \operatorname{Sin}[a]}{4 d^2} - \\
& \frac{9 b^2 f \left(\frac{2 \operatorname{Gamma}\left[\frac{2}{3}, -\frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(-\frac{i b}{(c+d x)^{3/2}}\right)^{2/3} (c+d x)} + \frac{2 \operatorname{Gamma}\left[\frac{2}{3}, \frac{i b}{(c+d x)^{3/2}}\right]}{3 \left(\frac{i b}{(c+d x)^{3/2}}\right)^{2/3} (c+d x)}\right) \operatorname{Sin}[a]}{8 d^2} + \frac{f \sqrt{c+d x} \operatorname{Cos}\left[\frac{b}{(c+d x)^{3/2}}\right] \left(3 b \operatorname{Cos}[a] - 2 c \sqrt{c+d x} \operatorname{Sin}[a] + (c+d x)^{3/2} \operatorname{Sin}[a]\right)}{2 d^2} + \\
& \frac{e (c+d x) \operatorname{Cos}[a] \operatorname{Sin}\left[\frac{b}{(c+d x)^{3/2}}\right]}{d} + \frac{f \sqrt{c+d x} \left(-2 c \sqrt{c+d x} \operatorname{Cos}[a] + (c+d x)^{3/2} \operatorname{Cos}[a] - 3 b \operatorname{Sin}[a]\right) \operatorname{Sin}\left[\frac{b}{(c+d x)^{3/2}}\right]}{2 d^2}
\end{aligned}$$

■ **Problem 210: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}\left[a + b (c+d x)^{1/3}\right]}{e + f x} dx$$

Optimal (type 4, 396 leaves, 11 steps):

$$\begin{aligned}
& \frac{\operatorname{CosIntegral}\left[\frac{b (d e - c f)^{1/3}}{f^{1/3}} + b (c+d x)^{1/3}\right] \operatorname{Sin}\left[a - \frac{b (d e - c f)^{1/3}}{f^{1/3}}\right]}{f} + \frac{\operatorname{CosIntegral}\left[\frac{(-1)^{1/3} b (d e - c f)^{1/3}}{f^{1/3}} - b (c+d x)^{1/3}\right] \operatorname{Sin}\left[a + \frac{(-1)^{1/3} b (d e - c f)^{1/3}}{f^{1/3}}\right]}{f} + \\
& \frac{\operatorname{CosIntegral}\left[\frac{(-1)^{2/3} b (d e - c f)^{1/3}}{f^{1/3}} + b (c+d x)^{1/3}\right] \operatorname{Sin}\left[a - \frac{(-1)^{2/3} b (d e - c f)^{1/3}}{f^{1/3}}\right]}{f} - \frac{\operatorname{Cos}\left[a + \frac{(-1)^{1/3} b (d e - c f)^{1/3}}{f^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/3} b (d e - c f)^{1/3}}{f^{1/3}} - b (c+d x)^{1/3}\right]}{f} + \\
& \frac{\operatorname{Cos}\left[a - \frac{b (d e - c f)^{1/3}}{f^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{b (d e - c f)^{1/3}}{f^{1/3}} + b (c+d x)^{1/3}\right]}{f} + \frac{\operatorname{Cos}\left[a - \frac{(-1)^{2/3} b (d e - c f)^{1/3}}{f^{1/3}}\right] \operatorname{SinIntegral}\left[\frac{(-1)^{2/3} b (d e - c f)^{1/3}}{f^{1/3}} + b (c+d x)^{1/3}\right]}{f}
\end{aligned}$$

Result (type 7, 118 leaves) :

$$\frac{1}{2f} i \left(\text{RootSum}\left[de - cf + f \#1^3 \ \&, e^{-i a - i b \#1} \text{ExpIntegralEi}\left[-i b \left((c + dx)^{1/3} - \#1\right)\right] \ \&\right] - \text{RootSum}\left[de - cf + f \#1^3 \ \&, e^{i a + i b \#1} \text{ExpIntegralEi}\left[i b \left((c + dx)^{1/3} - \#1\right)\right] \ \&\right] \right)$$

■ **Problem 211: Result is not expressed in closed-form.**

$$\int \frac{\text{Sin}\left[a + b \left(c + dx\right)^{1/3}\right]}{\left(e + fx\right)^2} dx$$

Optimal (type 4, 555 leaves, 13 steps) :

$$\begin{aligned} & - \frac{(-1)^{1/3} b d \text{Cos}\left[a + \frac{(-1)^{1/3} b (de - cf)^{1/3}}{f^{1/3}}\right] \text{CosIntegral}\left[\frac{(-1)^{1/3} b (de - cf)^{1/3}}{f^{1/3}} - b \left(c + dx\right)^{1/3}\right]}{3 f^{4/3} (de - cf)^{2/3}} + \\ & \frac{b d \text{Cos}\left[a - \frac{b (de - cf)^{1/3}}{f^{1/3}}\right] \text{CosIntegral}\left[\frac{b (de - cf)^{1/3}}{f^{1/3}} + b \left(c + dx\right)^{1/3}\right]}{3 f^{4/3} (de - cf)^{2/3}} + \\ & \frac{(-1)^{2/3} b d \text{Cos}\left[a - \frac{(-1)^{2/3} b (de - cf)^{1/3}}{f^{1/3}}\right] \text{CosIntegral}\left[\frac{(-1)^{2/3} b (de - cf)^{1/3}}{f^{1/3}} + b \left(c + dx\right)^{1/3}\right]}{3 f^{4/3} (de - cf)^{2/3}} - \\ & \frac{\text{Sin}\left[a + b \left(c + dx\right)^{1/3}\right]}{f \left(e + fx\right)} - \frac{(-1)^{1/3} b d \text{Sin}\left[a + \frac{(-1)^{1/3} b (de - cf)^{1/3}}{f^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{1/3} b (de - cf)^{1/3}}{f^{1/3}} - b \left(c + dx\right)^{1/3}\right]}{3 f^{4/3} (de - cf)^{2/3}} - \\ & \frac{b d \text{Sin}\left[a - \frac{b (de - cf)^{1/3}}{f^{1/3}}\right] \text{SinIntegral}\left[\frac{b (de - cf)^{1/3}}{f^{1/3}} + b \left(c + dx\right)^{1/3}\right]}{3 f^{4/3} (de - cf)^{2/3}} - \\ & \frac{(-1)^{2/3} b d \text{Sin}\left[a - \frac{(-1)^{2/3} b (de - cf)^{1/3}}{f^{1/3}}\right] \text{SinIntegral}\left[\frac{(-1)^{2/3} b (de - cf)^{1/3}}{f^{1/3}} + b \left(c + dx\right)^{1/3}\right]}{3 f^{4/3} (de - cf)^{2/3}} \end{aligned}$$

Result (type 7, 180 leaves) :

$$\frac{1}{6 f^2} \left(\frac{3 i e^{-i \left(a + b \left(c + dx\right)^{1/3}\right)} \left(-1 + e^{2 i \left(a + b \left(c + dx\right)^{1/3}\right)}\right) f}{e + fx} + b d \text{RootSum}\left[de - cf + f \#1^3 \ \&, \frac{e^{-i a - i b \#1} \text{ExpIntegralEi}\left[-i b \left((c + dx)^{1/3} - \#1\right)\right]}{\#1^2}\right] \ \&\right) + b d \text{RootSum}\left[de - cf + f \#1^3 \ \&, \frac{e^{i a + i b \#1} \text{ExpIntegralEi}\left[i b \left((c + dx)^{1/3} - \#1\right)\right]}{\#1^2}\right] \ \&\right)$$

■ **Problem 212: Result unnecessarily involves imaginary or complex numbers.**

$$\int \left(e + fx\right)^2 \text{Sin}\left[a + b \left(c + dx\right)^{2/3}\right] dx$$

Optimal (type 4, 513 leaves, 17 steps) :

$$\begin{aligned}
& \frac{6 f (d e - c f) \operatorname{Cos}\left[a + b (c + d x)^{2/3}\right]}{b^3 d^3} - \frac{3 (d e - c f)^2 (c + d x)^{1/3} \operatorname{Cos}\left[a + b (c + d x)^{2/3}\right]}{2 b d^3} + \\
& \frac{105 f^2 (c + d x) \operatorname{Cos}\left[a + b (c + d x)^{2/3}\right]}{8 b^3 d^3} - \frac{3 f (d e - c f) (c + d x)^{4/3} \operatorname{Cos}\left[a + b (c + d x)^{2/3}\right]}{b d^3} - \frac{3 f^2 (c + d x)^{7/3} \operatorname{Cos}\left[a + b (c + d x)^{2/3}\right]}{2 b d^3} + \\
& \frac{3 (d e - c f)^2 \sqrt{\frac{\pi}{2}} \operatorname{Cos}[a] \operatorname{FresnelC}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)^{1/3}\right]}{2 b^{3/2} d^3} + \frac{315 f^2 \sqrt{\frac{\pi}{2}} \operatorname{Cos}[a] \operatorname{FresnelS}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)^{1/3}\right]}{16 b^{9/2} d^3} + \\
& \frac{315 f^2 \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)^{1/3}\right] \operatorname{Sin}[a]}{16 b^{9/2} d^3} - \frac{3 (d e - c f)^2 \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{b} \sqrt{\frac{2}{\pi}} (c + d x)^{1/3}\right] \operatorname{Sin}[a]}{2 b^{3/2} d^3} - \\
& \frac{315 f^2 (c + d x)^{1/3} \operatorname{Sin}\left[a + b (c + d x)^{2/3}\right]}{16 b^4 d^3} + \frac{6 f (d e - c f) (c + d x)^{2/3} \operatorname{Sin}\left[a + b (c + d x)^{2/3}\right]}{b^2 d^3} + \frac{21 f^2 (c + d x)^{5/3} \operatorname{Sin}\left[a + b (c + d x)^{2/3}\right]}{4 b^2 d^3}
\end{aligned}$$

Result (type 4, 510 leaves):

$$\begin{aligned}
& -\frac{1}{64 b^{9/2} d^3} 3 i \left((\operatorname{Cos}[a] + i \operatorname{Sin}[a]) \right. \\
& \left. \left((1 + i) (-105 i f^2 + 8 b^3 (d e - c f)^2) \sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left[\frac{(1 + i) \sqrt{b} (c + d x)^{1/3}}{\sqrt{2}}\right] + 2 \sqrt{b} (-105 f^2 (c + d x)^{1/3} - 8 i b^3 d^2 (c + d x)^{1/3} (e + f x)^2 + \right. \right. \\
& \left. \left. 4 b^2 f (c + d x)^{2/3} (8 d e - c f + 7 d f x) + 2 i b f (16 d e + 19 c f + 35 d f x) \right) (\operatorname{Cos}[b (c + d x)^{2/3}] + i \operatorname{Sin}[b (c + d x)^{2/3}]) \right) - \\
& \left(2 \sqrt{b} (-105 f^2 (c + d x)^{1/3} + 8 i b^3 d^2 (c + d x)^{1/3} (e + f x)^2 + 4 b^2 f (c + d x)^{2/3} (8 d e - c f + 7 d f x) - 2 i b f (16 d e + 19 c f + 35 d f x)) - \right. \\
& \left. (1 + i) (105 i f^2 + 8 b^3 (d^2 e^2 + c^2 f^2)) \sqrt{\frac{\pi}{2}} \operatorname{Erf}\left[\frac{(1 + i) \sqrt{b} (c + d x)^{1/3}}{\sqrt{2}}\right] (\operatorname{Cos}[b (c + d x)^{2/3}] + i \operatorname{Sin}[b (c + d x)^{2/3}]) + (8 + 8 i) b^3 c d e \right. \\
& \left. f \sqrt{2 \pi} \operatorname{Erf}\left[\frac{(1 + i) \sqrt{b} (c + d x)^{1/3}}{\sqrt{2}}\right] (\operatorname{Cos}[b (c + d x)^{2/3}] + i \operatorname{Sin}[b (c + d x)^{2/3}]) \right) \left. (\operatorname{Cos}[a + b (c + d x)^{2/3}] - i \operatorname{Sin}[a + b (c + d x)^{2/3}]) \right)
\end{aligned}$$

■ **Problem 217: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e + f x)^2 \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right] dx$$

Optimal (type 4, 855 leaves, 29 steps):

$$\begin{aligned}
& \frac{b^5 f (d e - c f) (c + d x)^{1/3} \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{120 d^3} - \frac{b^7 f^2 (c + d x)^{2/3} \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{120960 d^3} + \frac{b (d e - c f)^2 (c + d x)^{2/3} \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{2 d^3} - \\
& \frac{b^3 f (d e - c f) (c + d x) \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{60 d^3} + \frac{b^5 f^2 (c + d x)^{4/3} \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{20160 d^3} + \frac{b f (d e - c f) (c + d x)^{5/3} \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{5 d^3} - \\
& \frac{b^3 f^2 (c + d x)^2 \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{1008 d^3} + \frac{b f^2 (c + d x)^{8/3} \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{24 d^3} - \frac{b^9 f^2 \operatorname{Cos}[a] \operatorname{CosIntegral}\left[\frac{b}{(c + d x)^{1/3}}\right]}{120960 d^3} + \\
& \frac{b^3 (d e - c f)^2 \operatorname{Cos}[a] \operatorname{CosIntegral}\left[\frac{b}{(c + d x)^{1/3}}\right]}{2 d^3} + \frac{b^6 f (d e - c f) \operatorname{CosIntegral}\left[\frac{b}{(c + d x)^{1/3}}\right] \operatorname{Sin}[a]}{120 d^3} + \frac{b^8 f^2 (c + d x)^{1/3} \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{120960 d^3} - \\
& \frac{b^2 (d e - c f)^2 (c + d x)^{1/3} \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{2 d^3} + \frac{b^4 f (d e - c f) (c + d x)^{2/3} \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{120 d^3} - \frac{b^6 f^2 (c + d x) \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{60480 d^3} + \\
& \frac{(d e - c f)^2 (c + d x) \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{d^3} - \frac{b^2 f (d e - c f) (c + d x)^{4/3} \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{20 d^3} + \frac{b^4 f^2 (c + d x)^{5/3} \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{5040 d^3} + \\
& \frac{f (d e - c f) (c + d x)^2 \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{d^3} - \frac{b^2 f^2 (c + d x)^{7/3} \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{168 d^3} + \frac{f^2 (c + d x)^3 \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{1/3}}\right]}{3 d^3} + \\
& \frac{b^6 f (d e - c f) \operatorname{Cos}[a] \operatorname{SinIntegral}\left[\frac{b}{(c + d x)^{1/3}}\right]}{120 d^3} + \frac{b^9 f^2 \operatorname{Sin}[a] \operatorname{SinIntegral}\left[\frac{b}{(c + d x)^{1/3}}\right]}{120960 d^3} - \frac{b^3 (d e - c f)^2 \operatorname{Sin}[a] \operatorname{SinIntegral}\left[\frac{b}{(c + d x)^{1/3}}\right]}{2 d^3}
\end{aligned}$$

Result (type 4, 929 leaves):

$$\begin{aligned}
& -\frac{1}{241920d^3} i \left((\cos[a] + i \sin[a]) \left(60480 i b^3 d^2 e^2 \operatorname{ExpIntegralEi} \left[\frac{i b}{(c+dx)^{1/3}} \right] + \right. \right. \\
& 1008 b^6 d e f \operatorname{ExpIntegralEi} \left[\frac{i b}{(c+dx)^{1/3}} \right] - 120960 i b^3 c d e f \operatorname{ExpIntegralEi} \left[\frac{i b}{(c+dx)^{1/3}} \right] - i b^9 f^2 \operatorname{ExpIntegralEi} \left[\frac{i b}{(c+dx)^{1/3}} \right] - \\
& 1008 b^6 c f^2 \operatorname{ExpIntegralEi} \left[\frac{i b}{(c+dx)^{1/3}} \right] + 60480 i b^3 c^2 f^2 \operatorname{ExpIntegralEi} \left[\frac{i b}{(c+dx)^{1/3}} \right] + (c+dx)^{1/3} \\
& (b^8 f^2 - i b^7 f^2 (c+dx)^{1/3} - 2 b^6 f^2 (c+dx)^{2/3} + 24 i b^3 f (c+dx)^{2/3} (-84 d e + 79 c f - 5 d f x) + 6 i b^5 f (168 d e - 167 c f + d f x) + \\
& 24 b^4 f (c+dx)^{1/3} (42 d e - 41 c f + d f x) + 40320 (c+dx)^{2/3} (c^2 f^2 - c d f (3 e + f x) + d^2 (3 e^2 + 3 e f x + f^2 x^2)) + \\
& 1008 i b (c+dx)^{1/3} (41 c^2 f^2 - 2 c d f (48 e + 7 f x) + d^2 (60 e^2 + 24 e f x + 5 f^2 x^2)) - \\
& 144 b^2 (383 c^2 f^2 - 2 c d f (399 e + 16 f x) + d^2 (420 e^2 + 42 e f x + 5 f^2 x^2))) \left(\cos \left[\frac{b}{(c+dx)^{1/3}} \right] + i \sin \left[\frac{b}{(c+dx)^{1/3}} \right] \right) \Bigg) - \\
& \left((c+dx)^{1/3} (b^8 f^2 + i b^7 f^2 (c+dx)^{1/3} - 2 b^6 f^2 (c+dx)^{2/3} - 6 i b^5 f (168 d e - 167 c f + d f x) + 24 b^4 f (c+dx)^{1/3} (42 d e - 41 c f + d f x) + \right. \\
& 24 i b^3 f (c+dx)^{2/3} (84 d e - 79 c f + 5 d f x) + 40320 (c+dx)^{2/3} (c^2 f^2 - c d f (3 e + f x) + d^2 (3 e^2 + 3 e f x + f^2 x^2)) - \\
& 1008 i b (c+dx)^{1/3} (41 c^2 f^2 - 2 c d f (48 e + 7 f x) + d^2 (60 e^2 + 24 e f x + 5 f^2 x^2)) - \\
& 144 b^2 (383 c^2 f^2 - 2 c d f (399 e + 16 f x) + d^2 (420 e^2 + 42 e f x + 5 f^2 x^2))) + \\
& i b^3 (-60480 d^2 e^2 + 1008 (-i b^3 + 120 c) d e f + (b^6 + 1008 i b^3 c - 60480 c^2) f^2) \operatorname{ExpIntegralEi} \left[-\frac{i b}{(c+dx)^{1/3}} \right] \\
& \left. \left(\cos \left[\frac{b}{(c+dx)^{1/3}} \right] + i \sin \left[\frac{b}{(c+dx)^{1/3}} \right] \right) \right) \left(\cos \left[a + \frac{b}{(c+dx)^{1/3}} \right] - i \sin \left[a + \frac{b}{(c+dx)^{1/3}} \right] \right) \Bigg)
\end{aligned}$$

■ **Problem 220: Result is not expressed in closed-form.**

$$\int \frac{\sin \left[a + \frac{b}{(c+dx)^{1/3}} \right]}{e+fx} dx$$

Optimal (type 4, 434 leaves, 16 steps):

$$\begin{aligned}
& -\frac{3 \operatorname{CosIntegral} \left[\frac{b}{(c+dx)^{1/3}} \right] \sin[a] + \operatorname{CosIntegral} \left[\frac{b f^{1/3}}{(de-cf)^{1/3}} + \frac{b}{(c+dx)^{1/3}} \right] \sin \left[a - \frac{b f^{1/3}}{(de-cf)^{1/3}} \right]}{f} + \\
& \frac{\operatorname{CosIntegral} \left[\frac{(-1)^{1/3} b f^{1/3}}{(de-cf)^{1/3}} - \frac{b}{(c+dx)^{1/3}} \right] \sin \left[a + \frac{(-1)^{1/3} b f^{1/3}}{(de-cf)^{1/3}} \right] + \operatorname{CosIntegral} \left[\frac{(-1)^{2/3} b f^{1/3}}{(de-cf)^{1/3}} + \frac{b}{(c+dx)^{1/3}} \right] \sin \left[a - \frac{(-1)^{2/3} b f^{1/3}}{(de-cf)^{1/3}} \right]}{f} - \\
& \frac{3 \cos[a] \operatorname{SinIntegral} \left[\frac{b}{(c+dx)^{1/3}} \right] - \operatorname{Cos} \left[a + \frac{(-1)^{1/3} b f^{1/3}}{(de-cf)^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{1/3} b f^{1/3}}{(de-cf)^{1/3}} - \frac{b}{(c+dx)^{1/3}} \right]}{f} + \\
& \frac{\operatorname{Cos} \left[a - \frac{b f^{1/3}}{(de-cf)^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{b f^{1/3}}{(de-cf)^{1/3}} + \frac{b}{(c+dx)^{1/3}} \right] + \operatorname{Cos} \left[a - \frac{(-1)^{2/3} b f^{1/3}}{(de-cf)^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{2/3} b f^{1/3}}{(de-cf)^{1/3}} + \frac{b}{(c+dx)^{1/3}} \right]}{f}
\end{aligned}$$

Result (type 7, 170 leaves):

$$\frac{1}{2f} i \left(\left(-3 \operatorname{ExpIntegralEi} \left[-\frac{ib}{(c+dx)^{1/3}} \right] + \operatorname{RootSum} \left[de - cf + f \#1^3 \&, e^{-\frac{ib}{\#1}} \operatorname{ExpIntegralEi} \left[-ib \left(\frac{1}{(c+dx)^{1/3}} - \frac{1}{\#1} \right) \right] \& \right] \right) (\operatorname{Cos}[a] - i \operatorname{Sin}[a]) + \right. \\ \left. \left(3 \operatorname{ExpIntegralEi} \left[\frac{ib}{(c+dx)^{1/3}} \right] - \operatorname{RootSum} \left[de - cf + f \#1^3 \&, e^{\frac{ib}{\#1}} \operatorname{ExpIntegralEi} \left[ib \left(\frac{1}{(c+dx)^{1/3}} - \frac{1}{\#1} \right) \right] \& \right] \right) (\operatorname{Cos}[a] + i \operatorname{Sin}[a]) \right)$$

■ **Problem 221: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin} \left[a + \frac{b}{(c+dx)^{1/3}} \right]}{(e+fx)^2} dx$$

Optimal (type 4, 566 leaves, 13 steps):

$$\frac{bd \operatorname{Cos} \left[a + \frac{bf^{1/3}}{(-de+cf)^{1/3}} \right] \operatorname{CosIntegral} \left[\frac{bf^{1/3}}{(-de+cf)^{1/3}} - \frac{b}{(c+dx)^{1/3}} \right]}{3f^{2/3} (-de+cf)^{4/3}} - \\ \frac{(-1)^{2/3} bd \operatorname{Cos} \left[a + \frac{(-1)^{2/3} bf^{1/3}}{(-de+cf)^{1/3}} \right] \operatorname{CosIntegral} \left[\frac{(-1)^{2/3} bf^{1/3}}{(-de+cf)^{1/3}} - \frac{b}{(c+dx)^{1/3}} \right]}{3f^{2/3} (-de+cf)^{4/3}} + \frac{(-1)^{1/3} bd \operatorname{Cos} \left[a - \frac{(-1)^{1/3} bf^{1/3}}{(-de+cf)^{1/3}} \right] \operatorname{CosIntegral} \left[\frac{(-1)^{1/3} bf^{1/3}}{(-de+cf)^{1/3}} + \frac{b}{(c+dx)^{1/3}} \right]}{3f^{2/3} (-de+cf)^{4/3}} + \\ \frac{(c+dx) \operatorname{Sin} \left[a + \frac{b}{(c+dx)^{1/3}} \right]}{(de-cf)(e+fx)} - \frac{bd \operatorname{Sin} \left[a + \frac{bf^{1/3}}{(-de+cf)^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{bf^{1/3}}{(-de+cf)^{1/3}} - \frac{b}{(c+dx)^{1/3}} \right]}{3f^{2/3} (-de+cf)^{4/3}} - \\ \frac{(-1)^{2/3} bd \operatorname{Sin} \left[a + \frac{(-1)^{2/3} bf^{1/3}}{(-de+cf)^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{2/3} bf^{1/3}}{(-de+cf)^{1/3}} - \frac{b}{(c+dx)^{1/3}} \right]}{3f^{2/3} (-de+cf)^{4/3}} - \frac{(-1)^{1/3} bd \operatorname{Sin} \left[a - \frac{(-1)^{1/3} bf^{1/3}}{(-de+cf)^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{1/3} bf^{1/3}}{(-de+cf)^{1/3}} + \frac{b}{(c+dx)^{1/3}} \right]}{3f^{2/3} (-de+cf)^{4/3}}$$

Result (type 7, 313 leaves):

$$\frac{1}{6 f (-d e + c f) (e + f x)}$$

$$\left((\cos[a] + i \sin[a]) \left(b d (e + f x) \operatorname{RootSum}\left[d e - c f + f \#^3 \&, \frac{\operatorname{ExpIntegralEi}\left[\frac{i b}{(c+d x)^{1/3}}\right] - e^{\frac{i b}{\#1}} \operatorname{ExpIntegralEi}\left[i b \left(\frac{1}{(c+d x)^{1/3}} - \frac{1}{\#1}\right)\right]}{\#1}\right] \& \right) + \right.$$

$$\left. (c + d x) \left(3 i f \cos\left[\frac{b}{(c+d x)^{1/3}}\right] - 3 f \sin\left[\frac{b}{(c+d x)^{1/3}}\right] \right) \right) +$$

$$i \left(-3 c f - 3 d f x + b d (e + f x) \operatorname{RootSum}\left[d e - c f + f \#^3 \&, \frac{\operatorname{ExpIntegralEi}\left[-\frac{i b}{(c+d x)^{1/3}}\right] - e^{-\frac{i b}{\#1}} \operatorname{ExpIntegralEi}\left[-i b \left(\frac{1}{(c+d x)^{1/3}} - \frac{1}{\#1}\right)\right]}{\#1}\right] \& \right)$$

$$\left(-i \cos\left[\frac{b}{(c+d x)^{1/3}}\right] + \sin\left[\frac{b}{(c+d x)^{1/3}}\right] \right) \left(\cos\left[a + \frac{b}{(c+d x)^{1/3}}\right] - i \sin\left[a + \frac{b}{(c+d x)^{1/3}}\right] \right)$$

■ **Problem 222: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e + f x)^2 \sin\left[a + \frac{b}{(c + d x)^{2/3}}\right] dx$$

Optimal (type 4, 630 leaves, 24 steps):

$$\begin{aligned}
& \frac{2 b (d e - c f)^2 (c + d x)^{1/3} \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{d^3} - \frac{8 b^3 f^2 (c + d x) \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{315 d^3} + \\
& \frac{b f (d e - c f) (c + d x)^{4/3} \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{2 d^3} + \frac{2 b f^2 (c + d x)^{7/3} \operatorname{Cos}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{21 d^3} + \frac{b^3 f (d e - c f) \operatorname{Cos}[a] \operatorname{CosIntegral}\left[\frac{b}{(c + d x)^{2/3}}\right]}{2 d^3} - \\
& \frac{16 b^{9/2} f^2 \sqrt{2 \pi} \operatorname{Cos}[a] \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{(c + d x)^{1/3}}\right]}{315 d^3} + \frac{2 b^{3/2} (d e - c f)^2 \sqrt{2 \pi} \operatorname{Cos}[a] \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{(c + d x)^{1/3}}\right]}{d^3} + \\
& \frac{2 b^{3/2} (d e - c f)^2 \sqrt{2 \pi} \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{(c + d x)^{1/3}}\right] \operatorname{Sin}[a]}{d^3} + \frac{16 b^{9/2} f^2 \sqrt{2 \pi} \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{(c + d x)^{1/3}}\right] \operatorname{Sin}[a]}{315 d^3} + \frac{16 b^4 f^2 (c + d x)^{1/3} \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{315 d^3} - \\
& \frac{b^2 f (d e - c f) (c + d x)^{2/3} \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{2 d^3} + \frac{(d e - c f)^2 (c + d x) \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{d^3} - \frac{4 b^2 f^2 (c + d x)^{5/3} \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{105 d^3} + \\
& \frac{f (d e - c f) (c + d x)^2 \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{d^3} + \frac{f^2 (c + d x)^3 \operatorname{Sin}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{3 d^3} - \frac{b^3 f (d e - c f) \operatorname{Sin}[a] \operatorname{SinIntegral}\left[\frac{b}{(c + d x)^{2/3}}\right]}{2 d^3}
\end{aligned}$$

Result (type 4, 613 leaves):

$$\begin{aligned}
& \frac{1}{1260 d^3} i e^{-i a} \left(e^{-\frac{i b}{(c + d x)^{2/3}} (c + d x)^{1/3} \left(32 b^4 f^2 + 16 i b^3 f^2 (c + d x)^{2/3} + 3 b^2 f (c + d x)^{1/3} (-105 d e + 97 c f - 8 d f x) - \right. \right. \\
& \quad \left. \left. 15 i b (84 d^2 e^2 + 21 d e f (-7 c + d x) + f^2 (67 c^2 - 13 c d x + 4 d^2 x^2)) + 210 (c + d x)^{2/3} (c^2 f^2 - c d f (3 e + f x) + d^2 (3 e^2 + 3 e f x + f^2 x^2)) \right) \right) - \\
& \quad e^{i \left(2 a + \frac{b}{(c + d x)^{2/3}} \right) (c + d x)^{1/3} \left(32 b^4 f^2 - 16 i b^3 f^2 (c + d x)^{2/3} + 3 b^2 f (c + d x)^{1/3} (-105 d e + 97 c f - 8 d f x) + \right. \\
& \quad \left. 15 i b (84 d^2 e^2 + 21 d e f (-7 c + d x) + f^2 (67 c^2 - 13 c d x + 4 d^2 x^2)) + 210 (c + d x)^{2/3} (c^2 f^2 - c d f (3 e + f x) + d^2 (3 e^2 + 3 e f x + f^2 x^2)) \right) + \\
& \quad 4 (-1)^{1/4} b^{3/2} e^{2 i a} \left(315 i d^2 e^2 - 630 i c d e f + (8 b^3 + 315 i c^2) f^2 \right) \sqrt{\pi} \operatorname{Erfi}\left[\frac{(-1)^{1/4} \sqrt{b}}{(c + d x)^{1/3}}\right] - \\
& \quad 4 (-1)^{1/4} b^{3/2} \left(315 d^2 e^2 - 630 c d e f + (8 i b^3 + 315 c^2) f^2 \right) \sqrt{\pi} \operatorname{Erfi}\left[\frac{(-1)^{3/4} \sqrt{b}}{(c + d x)^{1/3}}\right] + \\
& \quad \left. 315 i b^3 f (-d e + c f) \operatorname{ExpIntegralEi}\left[-\frac{i b}{(c + d x)^{2/3}}\right] + 315 i b^3 e^{2 i a} f (-d e + c f) \operatorname{ExpIntegralEi}\left[\frac{i b}{(c + d x)^{2/3}}\right] \right)
\end{aligned}$$

■ **Problem 226:** Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Sin}\left[a + \frac{b}{(c + d x)^{2/3}}\right]}{(e + f x)^2} dx$$

Optimal (type 9, 24 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\text{Sin}\left[a + \frac{b}{(c+dx)^{2/3}}\right]}{(e+fx)^2}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 260: Unable to integrate problem.**

$$\int x^3 \text{Sin}[a + b (c + dx)^n] dx$$

Optimal (type 4, 503 leaves, 14 steps):

$$\begin{aligned} & -\frac{i c^3 e^{i a} (c+dx)^{-i b (c+dx)^n} \Gamma\left[\frac{1}{n}, -i b (c+dx)^n\right]}{2 d^4 n} + \frac{i c^3 e^{-i a} (c+dx) (i b (c+dx)^n)^{-1/n} \Gamma\left[\frac{1}{n}, i b (c+dx)^n\right]}{2 d^4 n} + \\ & \frac{3 i c^2 e^{i a} (c+dx)^2 (-i b (c+dx)^n)^{-2/n} \Gamma\left[\frac{2}{n}, -i b (c+dx)^n\right]}{2 d^4 n} - \frac{3 i c^2 e^{-i a} (c+dx)^2 (i b (c+dx)^n)^{-2/n} \Gamma\left[\frac{2}{n}, i b (c+dx)^n\right]}{2 d^4 n} - \\ & \frac{3 i c e^{i a} (c+dx)^3 (-i b (c+dx)^n)^{-3/n} \Gamma\left[\frac{3}{n}, -i b (c+dx)^n\right]}{2 d^4 n} + \frac{3 i c e^{-i a} (c+dx)^3 (i b (c+dx)^n)^{-3/n} \Gamma\left[\frac{3}{n}, i b (c+dx)^n\right]}{2 d^4 n} + \\ & \frac{i e^{i a} (c+dx)^4 (-i b (c+dx)^n)^{-4/n} \Gamma\left[\frac{4}{n}, -i b (c+dx)^n\right]}{2 d^4 n} - \frac{i e^{-i a} (c+dx)^4 (i b (c+dx)^n)^{-4/n} \Gamma\left[\frac{4}{n}, i b (c+dx)^n\right]}{2 d^4 n} \end{aligned}$$

Result (type 8, 18 leaves):

$$\int x^3 \text{Sin}[a + b (c + dx)^n] dx$$

■ **Problem 261: Unable to integrate problem.**

$$\int x^2 \text{Sin}[a + b (c + dx)^n] dx$$

Optimal (type 4, 369 leaves, 11 steps):

$$\begin{aligned} & \frac{i c^2 e^{i a} (c+dx)^{-i b (c+dx)^n} \Gamma\left[\frac{1}{n}, -i b (c+dx)^n\right]}{2 d^3 n} - \frac{i c^2 e^{-i a} (c+dx) (i b (c+dx)^n)^{-1/n} \Gamma\left[\frac{1}{n}, i b (c+dx)^n\right]}{2 d^3 n} - \\ & \frac{i c e^{i a} (c+dx)^2 (-i b (c+dx)^n)^{-2/n} \Gamma\left[\frac{2}{n}, -i b (c+dx)^n\right]}{d^3 n} + \frac{i c e^{-i a} (c+dx)^2 (i b (c+dx)^n)^{-2/n} \Gamma\left[\frac{2}{n}, i b (c+dx)^n\right]}{d^3 n} + \\ & \frac{i e^{i a} (c+dx)^3 (-i b (c+dx)^n)^{-3/n} \Gamma\left[\frac{3}{n}, -i b (c+dx)^n\right]}{2 d^3 n} - \frac{i e^{-i a} (c+dx)^3 (i b (c+dx)^n)^{-3/n} \Gamma\left[\frac{3}{n}, i b (c+dx)^n\right]}{2 d^3 n} \end{aligned}$$

Result (type 8, 18 leaves):

$$\int x^2 \text{Sin}[a + b (c + dx)^n] dx$$

■ **Problem 266: Unable to integrate problem.**

$$\int x^3 (a + b \sin[c + d (f + g x)^n]) dx$$

Optimal (type 4, 519 leaves, 16 steps):

$$\begin{aligned} & \frac{a x^4}{4} - \frac{i b e^{i c} f^3 (f + g x) (-i d (f + g x)^n)^{-1/n} \Gamma\left[\frac{1}{n}, -i d (f + g x)^n\right]}{2 g^4 n} + \frac{i b e^{-i c} f^3 (f + g x) (i d (f + g x)^n)^{-1/n} \Gamma\left[\frac{1}{n}, i d (f + g x)^n\right]}{2 g^4 n} + \\ & \frac{3 i b e^{i c} f^2 (f + g x)^2 (-i d (f + g x)^n)^{-2/n} \Gamma\left[\frac{2}{n}, -i d (f + g x)^n\right]}{2 g^4 n} - \frac{3 i b e^{-i c} f^2 (f + g x)^2 (i d (f + g x)^n)^{-2/n} \Gamma\left[\frac{2}{n}, i d (f + g x)^n\right]}{2 g^4 n} - \\ & \frac{3 i b e^{i c} f (f + g x)^3 (-i d (f + g x)^n)^{-3/n} \Gamma\left[\frac{3}{n}, -i d (f + g x)^n\right]}{2 g^4 n} + \frac{3 i b e^{-i c} f (f + g x)^3 (i d (f + g x)^n)^{-3/n} \Gamma\left[\frac{3}{n}, i d (f + g x)^n\right]}{2 g^4 n} + \\ & \frac{i b e^{i c} (f + g x)^4 (-i d (f + g x)^n)^{-4/n} \Gamma\left[\frac{4}{n}, -i d (f + g x)^n\right]}{2 g^4 n} - \frac{i b e^{-i c} (f + g x)^4 (i d (f + g x)^n)^{-4/n} \Gamma\left[\frac{4}{n}, i d (f + g x)^n\right]}{2 g^4 n} \end{aligned}$$

Result (type 8, 22 leaves):

$$\int x^3 (a + b \sin[c + d (f + g x)^n]) dx$$

■ **Problem 267: Unable to integrate problem.**

$$\int x^2 (a + b \sin[c + d (f + g x)^n]) dx$$

Optimal (type 4, 383 leaves, 13 steps):

$$\begin{aligned} & \frac{a x^3}{3} + \frac{i b e^{i c} f^2 (f + g x) (-i d (f + g x)^n)^{-1/n} \Gamma\left[\frac{1}{n}, -i d (f + g x)^n\right]}{2 g^3 n} - \frac{i b e^{-i c} f^2 (f + g x) (i d (f + g x)^n)^{-1/n} \Gamma\left[\frac{1}{n}, i d (f + g x)^n\right]}{2 g^3 n} - \\ & \frac{i b e^{i c} f (f + g x)^2 (-i d (f + g x)^n)^{-2/n} \Gamma\left[\frac{2}{n}, -i d (f + g x)^n\right]}{g^3 n} + \frac{i b e^{-i c} f (f + g x)^2 (i d (f + g x)^n)^{-2/n} \Gamma\left[\frac{2}{n}, i d (f + g x)^n\right]}{g^3 n} + \\ & \frac{i b e^{i c} (f + g x)^3 (-i d (f + g x)^n)^{-3/n} \Gamma\left[\frac{3}{n}, -i d (f + g x)^n\right]}{2 g^3 n} - \frac{i b e^{-i c} (f + g x)^3 (i d (f + g x)^n)^{-3/n} \Gamma\left[\frac{3}{n}, i d (f + g x)^n\right]}{2 g^3 n} \end{aligned}$$

Result (type 8, 22 leaves):

$$\int x^2 (a + b \sin[c + d (f + g x)^n]) dx$$

■ **Problem 272: Unable to integrate problem.**

$$\int x^2 (a + b \sin[c + d (f + g x)^n])^2 dx$$

Optimal (type 4, 856 leaves, 28 steps):

$$\begin{aligned}
& \frac{(2a^2 + b^2) f^2 x}{2g^2} - \frac{(2a^2 + b^2) f (f + gx)^2}{2g^3} + \frac{(2a^2 + b^2) (f + gx)^3}{6g^3} + \\
& \frac{iab e^{ic} f^2 (f + gx) (-id (f + gx)^n)^{-1/n} \text{Gamma}\left[\frac{1}{n}, -id (f + gx)^n\right]}{g^3 n} - \frac{iab e^{-ic} f^2 (f + gx) (id (f + gx)^n)^{-1/n} \text{Gamma}\left[\frac{1}{n}, id (f + gx)^n\right]}{g^3 n} + \\
& \frac{2^{-2-\frac{1}{n}} b^2 e^{2ic} f^2 (f + gx) (-id (f + gx)^n)^{-1/n} \text{Gamma}\left[\frac{1}{n}, -2id (f + gx)^n\right]}{g^3 n} + \frac{2^{-2-\frac{1}{n}} b^2 e^{-2ic} f^2 (f + gx) (id (f + gx)^n)^{-1/n} \text{Gamma}\left[\frac{1}{n}, 2id (f + gx)^n\right]}{g^3 n} - \\
& \frac{2iab e^{ic} f (f + gx)^2 (-id (f + gx)^n)^{-2/n} \text{Gamma}\left[\frac{2}{n}, -id (f + gx)^n\right]}{g^3 n} + \frac{2iab e^{-ic} f (f + gx)^2 (id (f + gx)^n)^{-2/n} \text{Gamma}\left[\frac{2}{n}, id (f + gx)^n\right]}{g^3 n} - \\
& \frac{2^{-1-\frac{2}{n}} b^2 e^{2ic} f (f + gx)^2 (-id (f + gx)^n)^{-2/n} \text{Gamma}\left[\frac{2}{n}, -2id (f + gx)^n\right]}{g^3 n} - \frac{2^{-1-\frac{2}{n}} b^2 e^{-2ic} f (f + gx)^2 (id (f + gx)^n)^{-2/n} \text{Gamma}\left[\frac{2}{n}, 2id (f + gx)^n\right]}{g^3 n} + \\
& \frac{iab e^{ic} (f + gx)^3 (-id (f + gx)^n)^{-3/n} \text{Gamma}\left[\frac{3}{n}, -id (f + gx)^n\right]}{g^3 n} - \frac{iab e^{-ic} (f + gx)^3 (id (f + gx)^n)^{-3/n} \text{Gamma}\left[\frac{3}{n}, id (f + gx)^n\right]}{g^3 n} + \\
& \frac{2^{-2-\frac{3}{n}} b^2 e^{2ic} (f + gx)^3 (-id (f + gx)^n)^{-3/n} \text{Gamma}\left[\frac{3}{n}, -2id (f + gx)^n\right]}{g^3 n} + \frac{2^{-2-\frac{3}{n}} b^2 e^{-2ic} (f + gx)^3 (id (f + gx)^n)^{-3/n} \text{Gamma}\left[\frac{3}{n}, 2id (f + gx)^n\right]}{g^3 n}
\end{aligned}$$

Result (type 8, 24 leaves):

$$\int x^2 (a + b \sin[c + d (f + gx)^n])^2 dx$$

■ **Problem 273: Unable to integrate problem.**

$$\int x (a + b \sin[c + d (f + gx)^n])^2 dx$$

Optimal (type 4, 556 leaves, 19 steps):

$$\begin{aligned}
& - \frac{(2a^2 + b^2)fx}{2g} + \frac{(2a^2 + b^2)(f+gx)^2}{4g^2} - \frac{ia be^{ic} f(f+gx)(-id(f+gx)^n)^{-1/n} \text{Gamma}\left[\frac{1}{n}, -id(f+gx)^n\right]}{g^2 n} + \\
& \frac{ia be^{-ic} f(f+gx)(id(f+gx)^n)^{-1/n} \text{Gamma}\left[\frac{1}{n}, id(f+gx)^n\right]}{g^2 n} - \frac{2^{-2-\frac{1}{n}} b^2 e^{2ic} f(f+gx)(-id(f+gx)^n)^{-1/n} \text{Gamma}\left[\frac{1}{n}, -2id(f+gx)^n\right]}{g^2 n} - \\
& \frac{2^{-2-\frac{1}{n}} b^2 e^{-2ic} f(f+gx)(id(f+gx)^n)^{-1/n} \text{Gamma}\left[\frac{1}{n}, 2id(f+gx)^n\right]}{g^2 n} + \\
& \frac{ia be^{ic} (f+gx)^2 (-id(f+gx)^n)^{-2/n} \text{Gamma}\left[\frac{2}{n}, -id(f+gx)^n\right]}{g^2 n} - \frac{ia be^{-ic} (f+gx)^2 (id(f+gx)^n)^{-2/n} \text{Gamma}\left[\frac{2}{n}, id(f+gx)^n\right]}{g^2 n} + \\
& \frac{4^{-1-\frac{1}{n}} b^2 e^{2ic} (f+gx)^2 (-id(f+gx)^n)^{-2/n} \text{Gamma}\left[\frac{2}{n}, -2id(f+gx)^n\right]}{g^2 n} + \frac{4^{-1-\frac{1}{n}} b^2 e^{-2ic} (f+gx)^2 (id(f+gx)^n)^{-2/n} \text{Gamma}\left[\frac{2}{n}, 2id(f+gx)^n\right]}{g^2 n}
\end{aligned}$$

Result (type 8, 22 leaves):

$$\int x (a + b \sin[c + d(f + gx)^n])^2 dx$$

■ **Problem 282: Attempted integration timed out after 120 seconds.**

$$\int \frac{x^2}{(a + b \sin[c + d(f + gx)^n])^2} dx$$

Optimal (type 9, 24 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2}{(a + b \sin[c + d(f + gx)^n])^2}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 283: Attempted integration timed out after 120 seconds.**

$$\int \frac{x}{(a + b \sin[c + d(f + gx)^n])^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{(a + b \sin[c + d(f + gx)^n])^2}, x\right]$$

Result (type 1, 1 leaves):

???

- **Problem 285: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{x (a + b \sin[c + d (f + g x)^n])^2} dx$$

Optimal (type 9, 24 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x (a + b \sin[c + d (f + g x)^n])^2}, x\right]$$

Result (type 1, 1 leaves):

???

- **Problem 286: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{x^2 (a + b \sin[c + d (f + g x)^n])^2} dx$$

Optimal (type 9, 24 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2 (a + b \sin[c + d (f + g x)^n])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Test results for the 653 problems in "4.1.1.2 (g cos)^p (a+b sin)^m.m"

- **Problem 8: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + d x] (a + a \sin[c + d x]) dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$-\frac{a \operatorname{Log}[1 - \sin[c + d x]]}{d}$$

Result (type 3, 83 leaves):

$$-\frac{a \operatorname{Log}[\cos[c + d x]]}{d} - \frac{a \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d}$$

- **Problem 10: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + d x]^3 (a + a \sin[c + d x]) dx$$

Optimal (type 3, 39 leaves, 4 steps):

$$\frac{a \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{a^2}{2d(a - a \sin[c + dx])}$$

Result (type 3, 143 leaves):

$$-\frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} +$$

$$\frac{a \operatorname{Sec}[c + dx]^2}{2d} + \frac{a}{4d\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{a}{4d\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}$$

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^5 (a + a \sin[c + dx]) dx$$

Optimal (type 3, 84 leaves, 4 steps):

$$\frac{3a \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a^3}{8d(a - a \sin[c + dx])^2} + \frac{a^2}{4d(a - a \sin[c + dx])} - \frac{a^2}{8d(a + a \sin[c + dx])}$$

Result (type 3, 207 leaves):

$$-\frac{3a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} + \frac{3a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} +$$

$$\frac{a \operatorname{Sec}[c + dx]^4}{4d} + \frac{a}{16d\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{3a}{16d\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} -$$

$$\frac{a}{16d\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} - \frac{3a}{16d\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}$$

■ **Problem 20: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^2 (a + a \sin[c + dx])^2 dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$-a^2 x + \frac{2a^4 \cos[c + dx]}{d(a^2 - a^2 \sin[c + dx])}$$

Result (type 3, 101 leaves):

$$-\frac{a^2 \left((c + dx) \cos\left[\frac{1}{2}(c + dx)\right] - (4 + c + dx) \sin\left[\frac{1}{2}(c + dx)\right] \right) (1 + \sin[c + dx])^2}{d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4}$$

■ **Problem 25: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^7 (a + a \text{Sin}[c + d x])^2 dx$$

Optimal (type 3, 109 leaves, 4 steps):

$$\frac{a^2 \text{ArcTanh}[\text{Sin}[c + d x]]}{4 d} + \frac{a^5}{12 d (a - a \text{Sin}[c + d x])^3} + \frac{a^4}{8 d (a - a \text{Sin}[c + d x])^2} + \frac{3 a^3}{16 d (a - a \text{Sin}[c + d x])} - \frac{a^3}{16 d (a + a \text{Sin}[c + d x])}$$

Result (type 3, 290 leaves):

$$\left(\left(-3 - 12 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 + \right. \\ \left. 12 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 + \right. \\ \left. \frac{4 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^6} + \frac{6 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{9 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} \right) \\ \left. (a + a \text{Sin}[c + d x])^2 \right) / \left(48 d \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^6 \right)$$

■ **Problem 32: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x] (a + a \text{Sin}[c + d x])^3 dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{(a + a \text{Sin}[c + d x])^4}{4 a d}$$

Result (type 3, 47 leaves):

$$\frac{a^3 (-28 \text{Cos}[2(c + d x)] + \text{Cos}[4(c + d x)] + 56 \text{Sin}[c + d x] - 8 \text{Sin}[3(c + d x)])}{32 d}$$

■ **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^3 (a + a \text{Sin}[c + d x])^3 dx$$

Optimal (type 3, 40 leaves, 3 steps):

$$\frac{a^3 \text{Log}[1 - \text{Sin}[c + d x]]}{d} + \frac{2 a^4}{d (a - a \text{Sin}[c + d x])}$$

Result (type 3, 92 leaves):

$$- \left(2 a^3 \left(-1 - \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] + \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] \sin [c + d x] \right) \right) / \left(d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^2 \right)$$

■ **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^5 (a + a \sin [c + d x])^8 dx$$

Optimal (type 3, 67 leaves, 3 steps):

$$\frac{4 (a + a \sin [c + d x])^{11}}{11 a^3 d} - \frac{(a + a \sin [c + d x])^{12}}{3 a^4 d} + \frac{(a + a \sin [c + d x])^{13}}{13 a^5 d}$$

Result (type 3, 139 leaves):

$$- \frac{1}{1757184 d} a^8 (4434144 \cos [2 (c + d x)] + 815100 \cos [4 (c + d x)] - 354640 \cos [6 (c + d x)] - 92664 \cos [8 (c + d x)] + 20592 \cos [10 (c + d x)] - 572 \cos [12 (c + d x)] - 8314020 \sin [c + d x] + 877591 \sin [3 (c + d x)] + 872157 \sin [5 (c + d x)] + 6006 \sin [7 (c + d x)] - 58630 \sin [9 (c + d x)] + 4485 \sin [11 (c + d x)] - 33 \sin [13 (c + d x)])$$

■ **Problem 43: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^3 (a + a \sin [c + d x])^8 dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$\frac{(a + a \sin [c + d x])^{10}}{5 a^2 d} - \frac{(a + a \sin [c + d x])^{11}}{11 a^3 d}$$

Result (type 3, 109 leaves):

$$\frac{1}{56320 d} a^8 (-284240 \cos [2 (c + d x)] + 25080 \cos [6 (c + d x)] - 3520 \cos [8 (c + d x)] + 88 \cos [10 (c + d x)] + 461890 \sin [c + d x] - 106590 \sin [3 (c + d x)] - 31977 \sin [5 (c + d x)] + 11495 \sin [7 (c + d x)] - 715 \sin [9 (c + d x)] + 5 \sin [11 (c + d x)])$$

■ **Problem 45: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x] (a + a \sin [c + d x])^8 dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{(a + a \sin [c + d x])^9}{9 a d}$$

Result (type 3, 97 leaves):

$$\frac{1}{2304 d} a^8 (-31824 \operatorname{Cos}[2(c+dx)] + 8568 \operatorname{Cos}[4(c+dx)] - 816 \operatorname{Cos}[6(c+dx)] + 18 \operatorname{Cos}[8(c+dx)] + 43758 \operatorname{Sin}[c+dx] - 18564 \operatorname{Sin}[3(c+dx)] + 3060 \operatorname{Sin}[5(c+dx)] - 153 \operatorname{Sin}[7(c+dx)] + \operatorname{Sin}[9(c+dx)])$$

■ **Problem 49: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^4 (a+a \operatorname{Sin}[c+dx])^8 dx$$

Optimal (type 3, 179 leaves, 8 steps):

$$\frac{1155 a^8 x}{8} - \frac{385 a^8 \operatorname{Cos}[c+dx]^3}{4 d} + \frac{1155 a^8 \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{8 d} + \frac{2 a^{15} \operatorname{Cos}[c+dx]^{11}}{3 d (a-a \operatorname{Sin}[c+dx])^7} - \frac{22 a^{13} \operatorname{Cos}[c+dx]^9}{3 d (a-a \operatorname{Sin}[c+dx])^5} - \frac{66 a^{14} \operatorname{Cos}[c+dx]^7}{d (a^2 - a^2 \operatorname{Sin}[c+dx])^3} - \frac{231 a^{16} \operatorname{Cos}[c+dx]^5}{4 d (a^8 - a^8 \operatorname{Sin}[c+dx])}$$

Result (type 3, 465 leaves):

$$\frac{1155 (c+dx) (a+a \operatorname{Sin}[c+dx])^8}{8 d (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^{16}} - \frac{78 \operatorname{Cos}[c+dx] (a+a \operatorname{Sin}[c+dx])^8}{d (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^{16}} + \frac{2 \operatorname{Cos}[3(c+dx)] (a+a \operatorname{Sin}[c+dx])^8}{3 d (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^{16}} + \frac{64 (a+a \operatorname{Sin}[c+dx])^8}{3 d (\operatorname{Cos}[\frac{1}{2}(c+dx)] - \operatorname{Sin}[\frac{1}{2}(c+dx)])^2 (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^{16}} + \frac{128 \operatorname{Sin}[\frac{1}{2}(c+dx)] (a+a \operatorname{Sin}[c+dx])^8}{3 d (\operatorname{Cos}[\frac{1}{2}(c+dx)] - \operatorname{Sin}[\frac{1}{2}(c+dx)])^3 (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^{16}} - \frac{1024 \operatorname{Sin}[\frac{1}{2}(c+dx)] (a+a \operatorname{Sin}[c+dx])^8}{3 d (\operatorname{Cos}[\frac{1}{2}(c+dx)] - \operatorname{Sin}[\frac{1}{2}(c+dx)]) (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^{16}} - \frac{31 (a+a \operatorname{Sin}[c+dx])^8 \operatorname{Sin}[2(c+dx)]}{4 d (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^{16}} + \frac{(a+a \operatorname{Sin}[c+dx])^8 \operatorname{Sin}[4(c+dx)]}{32 d (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^{16}}$$

■ **Problem 50: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^5 (a+a \operatorname{Sin}[c+dx])^8 dx$$

Optimal (type 3, 110 leaves, 3 steps):

$$-\frac{80 a^8 \operatorname{Log}[1 - \operatorname{Sin}[c+dx]]}{d} - \frac{31 a^8 \operatorname{Sin}[c+dx]}{d} - \frac{4 a^8 \operatorname{Sin}[c+dx]^2}{d} - \frac{a^8 \operatorname{Sin}[c+dx]^3}{3 d} + \frac{16 a^{10}}{d (a-a \operatorname{Sin}[c+dx])^2} - \frac{80 a^9}{d (a-a \operatorname{Sin}[c+dx])}$$

Result (type 3, 341 leaves):

$$\frac{2 \operatorname{Cos}[2(c+dx)](a+a \operatorname{Sin}[c+dx])^8}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^{16}} - \frac{160 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right](a+a \operatorname{Sin}[c+dx])^8}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^{16}} +$$

$$\frac{16(a+a \operatorname{Sin}[c+dx])^8}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^{16}} -$$

$$\frac{80(a+a \operatorname{Sin}[c+dx])^8}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^{16}} -$$

$$\frac{125 \operatorname{Sin}[c+dx](a+a \operatorname{Sin}[c+dx])^8}{4d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^{16}} + \frac{(a+a \operatorname{Sin}[c+dx])^8 \operatorname{Sin}[3(c+dx)]}{12d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^{16}}$$

■ **Problem 57: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 37 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{2ad} - \frac{1}{2d(a+a \operatorname{Sin}[c+dx])}$$

Result (type 3, 126 leaves):

$$\frac{1}{2ad(1+\operatorname{Sin}[c+dx])} \left(-1 - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \right.$$

$$\left. \left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) \operatorname{Sin}[c+dx] \right)$$

■ **Problem 58: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^2}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 42 leaves, 3 steps):

$$-\frac{\operatorname{Sec}[c+dx]}{3d(a+a \operatorname{Sin}[c+dx])} + \frac{2 \operatorname{Tan}[c+dx]}{3ad}$$

Result (type 3, 103 leaves):

$$\frac{2 \operatorname{Cos}[c+dx] - 4 \operatorname{Cos}[2(c+dx)] + 8 \operatorname{Sin}[c+dx] + \operatorname{Sin}[2(c+dx)]}{12ad \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right) (1+\operatorname{Sin}[c+dx])}$$

■ **Problem 59: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^3}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 77 leaves, 4 steps) :

$$\frac{3 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 a d} + \frac{1}{8 d (a - a \operatorname{Sin}[c + d x])} - \frac{a}{8 d (a + a \operatorname{Sin}[c + d x])^2} - \frac{1}{4 d (a + a \operatorname{Sin}[c + d x])}$$

Result (type 3, 190 leaves) :

$$-\frac{1}{8 d (a + a \operatorname{Sin}[c + d x])} \left(2 + \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + 3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 - 3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 - \frac{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2} \right)$$

■ **Problem 61: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^5}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 120 leaves, 4 steps) :

$$\frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{16 a d} + \frac{a}{32 d (a - a \operatorname{Sin}[c + d x])^2} + \frac{1}{8 d (a - a \operatorname{Sin}[c + d x])} - \frac{a^2}{24 d (a + a \operatorname{Sin}[c + d x])^3} - \frac{3 a}{32 d (a + a \operatorname{Sin}[c + d x])^2} - \frac{3}{16 d (a + a \operatorname{Sin}[c + d x])}$$

Result (type 3, 267 leaves) :

$$\frac{1}{96 d (a + a \operatorname{Sin}[c + d x])} \left(-18 - \frac{4}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{9}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - 30 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 + 30 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 + \frac{3 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^4} + \frac{12 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2} \right)$$

■ **Problem 68: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^2}{(a + a \operatorname{Sin}[c + d x])^2} dx$$

Optimal (type 3, 34 leaves, 2 steps) :

$$-\frac{x}{a^2} - \frac{2 \operatorname{Cos}[c + dx]}{d (a^2 + a^2 \operatorname{Sin}[c + dx])}$$

Result (type 3, 75 leaves) :

$$-\frac{\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3 \left((c + dx) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + (-4 + c + dx) \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)}{a^2 d (1 + \operatorname{Sin}[c + dx])^2}$$

■ **Problem 70: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]}{(a + a \operatorname{Sin}[c + dx])^2} dx$$

Optimal (type 3, 60 leaves, 4 steps) :

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{4 a^2 d} - \frac{1}{4 d (a + a \operatorname{Sin}[c + dx])^2} - \frac{1}{4 d (a^2 + a^2 \operatorname{Sin}[c + dx])}$$

Result (type 3, 139 leaves) :

$$-\frac{1}{4 d (a + a \operatorname{Sin}[c + dx])^2} \left(1 + \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2 + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4 - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4\right)$$

■ **Problem 72: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^3}{(a + a \operatorname{Sin}[c + dx])^2} dx$$

Optimal (type 3, 104 leaves, 4 steps) :

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{4 a^2 d} - \frac{a}{12 d (a + a \operatorname{Sin}[c + dx])^3} - \frac{1}{8 d (a + a \operatorname{Sin}[c + dx])^2} + \frac{1}{16 d (a^2 - a^2 \operatorname{Sin}[c + dx])} - \frac{3}{16 d (a^2 + a^2 \operatorname{Sin}[c + dx])}$$

Result (type 3, 217 leaves) :

$$\begin{aligned}
& - \frac{1}{48 d (a + a \sin[c + d x])^2} \left(6 + \frac{4}{\left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2} + \right. \\
& 9 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2 + 12 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^4 - \\
& \left. 12 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^4 - \frac{3 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^4}{\left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right] \right)^2} \right)
\end{aligned}$$

■ **Problem 76: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^7}{(a + a \sin[c + d x])^3} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$-\frac{(a - a \sin[c + d x])^4}{4 a^7 d}$$

Result (type 3, 48 leaves):

$$-\frac{-28 \cos[2(c + d x)] + \cos[4(c + d x)] + 8(-7 \sin[c + d x] + \sin[3(c + d x)])}{32 a^3 d}$$

■ **Problem 80: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^3}{(a + a \sin[c + d x])^3} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$-\frac{\operatorname{Log}[1 + \sin[c + d x]]}{a^3 d} - \frac{2}{d(a^3 + a^3 \sin[c + d x])}$$

Result (type 3, 89 leaves):

$$\begin{aligned}
& \frac{1}{d(a + a \sin[c + d x])^3} \\
& 2 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^4 \left(-1 - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \right) \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2
\end{aligned}$$

■ **Problem 81: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^2}{(a + a \sin[c + d x])^3} dx$$

Optimal (type 3, 27 leaves, 1 step):

$$-\frac{\text{Cos}[c + d x]^3}{3 d (a + a \text{Sin}[c + d x])^3}$$

Result (type 3, 66 leaves):

$$\frac{(-3 \text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Cos}\left[\frac{3}{2}(c + d x)\right]) (\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right])^3}{3 a^3 d (1 + \text{Sin}[c + d x])^3}$$

■ **Problem 83: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]}{(a + a \text{Sin}[c + d x])^3} dx$$

Optimal (type 3, 82 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[c + d x]]}{8 a^3 d} - \frac{1}{6 d (a + a \text{Sin}[c + d x])^3} - \frac{1}{8 a d (a + a \text{Sin}[c + d x])^2} - \frac{1}{8 d (a^3 + a^3 \text{Sin}[c + d x])}$$

Result (type 3, 167 leaves):

$$-\frac{1}{24 d (a + a \text{Sin}[c + d x])^3} \left(4 + 3 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 + 3 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^4 + 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right] \right) \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^6 - 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right] \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^6$$

■ **Problem 88: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[c + d x]^8}{(a + a \text{Sin}[c + d x])^8} dx$$

Optimal (type 3, 127 leaves, 5 steps):

$$\frac{x}{a^8} - \frac{2 \text{Cos}[c + d x]^7}{7 a d (a + a \text{Sin}[c + d x])^7} + \frac{2 \text{Cos}[c + d x]^5}{5 a^3 d (a + a \text{Sin}[c + d x])^5} - \frac{2 \text{Cos}[c + d x]^3}{3 a^2 d (a^2 + a^2 \text{Sin}[c + d x])^3} + \frac{2 \text{Cos}[c + d x]}{d (a^8 + a^8 \text{Sin}[c + d x])}$$

Result (type 3, 263 leaves):

$$\frac{1}{105 d (a + a \sin[c + dx])^8} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^9$$

$$\left(480 \sin\left[\frac{1}{2}(c + dx)\right] - 240 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) - 1056 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 + \right.$$

$$528 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3 + 976 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 -$$

$$488 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 - 704 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 +$$

$$\left. 105 (c + dx) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^7 \right)$$

■ **Problem 90: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^6}{(a + a \sin[c + dx])^8} dx$$

Optimal (type 3, 58 leaves, 2 steps):

$$-\frac{\cos[c + dx]^7}{9 d (a + a \sin[c + dx])^8} - \frac{\cos[c + dx]^7}{63 a d (a + a \sin[c + dx])^7}$$

Result (type 3, 128 leaves):

$$-\left(315 \cos\left[\frac{1}{2}(c + dx)\right] - 189 \cos\left[\frac{3}{2}(c + dx)\right] - 63 \cos\left[\frac{5}{2}(c + dx)\right] + 9 \cos\left[\frac{7}{2}(c + dx)\right] - 189 \sin\left[\frac{1}{2}(c + dx)\right] - \right.$$

$$\left. 105 \sin\left[\frac{3}{2}(c + dx)\right] + 27 \sin\left[\frac{5}{2}(c + dx)\right] + \sin\left[\frac{9}{2}(c + dx)\right] \right) / \left(504 a^8 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^9 \right)$$

■ **Problem 101: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^7 \sqrt{a + a \sin[c + dx]} dx$$

Optimal (type 3, 97 leaves, 3 steps):

$$\frac{16 (a + a \sin[c + dx])^{9/2}}{9 a^4 d} - \frac{24 (a + a \sin[c + dx])^{11/2}}{11 a^5 d} + \frac{12 (a + a \sin[c + dx])^{13/2}}{13 a^6 d} - \frac{2 (a + a \sin[c + dx])^{15/2}}{15 a^7 d}$$

Result (type 3, 1137 leaves):

$$\begin{aligned}
& \frac{35 \operatorname{Cos}\left[\frac{dx}{2}\right] \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{64 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \frac{35 \operatorname{Cos}\left[\frac{3dx}{2}\right] \left(\operatorname{Cos}\left[\frac{3c}{2}\right] - \operatorname{Sin}\left[\frac{3c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{192 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{21 \operatorname{Cos}\left[\frac{5dx}{2}\right] \left(\operatorname{Cos}\left[\frac{5c}{2}\right] + \operatorname{Sin}\left[\frac{5c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{320 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \frac{3 \operatorname{Cos}\left[\frac{7dx}{2}\right] \left(\operatorname{Cos}\left[\frac{7c}{2}\right] - \operatorname{Sin}\left[\frac{7c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{64 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{7 \operatorname{Cos}\left[\frac{9dx}{2}\right] \left(\operatorname{Cos}\left[\frac{9c}{2}\right] + \operatorname{Sin}\left[\frac{9c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{576 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \frac{7 \operatorname{Cos}\left[\frac{11dx}{2}\right] \left(\operatorname{Cos}\left[\frac{11c}{2}\right] - \operatorname{Sin}\left[\frac{11c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{704 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{\operatorname{Cos}\left[\frac{13dx}{2}\right] \left(\operatorname{Cos}\left[\frac{13c}{2}\right] + \operatorname{Sin}\left[\frac{13c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{832 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \frac{\operatorname{Cos}\left[\frac{15dx}{2}\right] \left(\operatorname{Cos}\left[\frac{15c}{2}\right] - \operatorname{Sin}\left[\frac{15c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{960 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{35 \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \operatorname{Sin}\left[\frac{dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{64 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{35 \left(\operatorname{Cos}\left[\frac{3c}{2}\right] + \operatorname{Sin}\left[\frac{3c}{2}\right]\right) \operatorname{Sin}\left[\frac{3dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{192 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{21 \left(\operatorname{Cos}\left[\frac{5c}{2}\right] - \operatorname{Sin}\left[\frac{5c}{2}\right]\right) \operatorname{Sin}\left[\frac{5dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{320 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{3 \left(\operatorname{Cos}\left[\frac{7c}{2}\right] + \operatorname{Sin}\left[\frac{7c}{2}\right]\right) \operatorname{Sin}\left[\frac{7dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{64 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{7 \left(\operatorname{Cos}\left[\frac{9c}{2}\right] - \operatorname{Sin}\left[\frac{9c}{2}\right]\right) \operatorname{Sin}\left[\frac{9dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{576 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{7 \left(\operatorname{Cos}\left[\frac{11c}{2}\right] + \operatorname{Sin}\left[\frac{11c}{2}\right]\right) \operatorname{Sin}\left[\frac{11dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{704 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{\left(\operatorname{Cos}\left[\frac{13c}{2}\right] - \operatorname{Sin}\left[\frac{13c}{2}\right]\right) \operatorname{Sin}\left[\frac{13dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{832 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{\left(\operatorname{Cos}\left[\frac{15c}{2}\right] + \operatorname{Sin}\left[\frac{15c}{2}\right]\right) \operatorname{Sin}\left[\frac{15dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{960 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 102: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx]^6 \sqrt{a + a \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{256 a^4 \operatorname{Cos}[c + dx]^7}{3003 d (a + a \operatorname{Sin}[c + dx])^{7/2}} - \frac{64 a^3 \operatorname{Cos}[c + dx]^7}{429 d (a + a \operatorname{Sin}[c + dx])^{5/2}} - \frac{24 a^2 \operatorname{Cos}[c + dx]^7}{143 d (a + a \operatorname{Sin}[c + dx])^{3/2}} - \frac{2 a \operatorname{Cos}[c + dx]^7}{13 d \sqrt{a + a \operatorname{Sin}[c + dx]}}$$

Result (type 3, 995 leaves):

$$\begin{aligned}
& - \frac{5 \operatorname{Cos}\left[\frac{dx}{2}\right] \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{8d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{5 \operatorname{Cos}\left[\frac{3dx}{2}\right] \left(\operatorname{Cos}\left[\frac{3c}{2}\right] + \operatorname{Sin}\left[\frac{3c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{32d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{3 \operatorname{Cos}\left[\frac{5dx}{2}\right] \left(\operatorname{Cos}\left[\frac{5c}{2}\right] - \operatorname{Sin}\left[\frac{5c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{32d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{3 \operatorname{Cos}\left[\frac{7dx}{2}\right] \left(\operatorname{Cos}\left[\frac{7c}{2}\right] + \operatorname{Sin}\left[\frac{7c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{112d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{\operatorname{Cos}\left[\frac{9dx}{2}\right] \left(\operatorname{Cos}\left[\frac{9c}{2}\right] - \operatorname{Sin}\left[\frac{9c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{48d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{\operatorname{Cos}\left[\frac{11dx}{2}\right] \left(\operatorname{Cos}\left[\frac{11c}{2}\right] + \operatorname{Sin}\left[\frac{11c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{352d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{\operatorname{Cos}\left[\frac{13dx}{2}\right] \left(\operatorname{Cos}\left[\frac{13c}{2}\right] - \operatorname{Sin}\left[\frac{13c}{2}\right]\right) \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{416d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{5 \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \operatorname{Sin}\left[\frac{dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{8d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{5 \left(\operatorname{Cos}\left[\frac{3c}{2}\right] - \operatorname{Sin}\left[\frac{3c}{2}\right]\right) \operatorname{Sin}\left[\frac{3dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{32d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{3 \left(\operatorname{Cos}\left[\frac{5c}{2}\right] + \operatorname{Sin}\left[\frac{5c}{2}\right]\right) \operatorname{Sin}\left[\frac{5dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{32d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{3 \left(\operatorname{Cos}\left[\frac{7c}{2}\right] - \operatorname{Sin}\left[\frac{7c}{2}\right]\right) \operatorname{Sin}\left[\frac{7dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{112d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{\left(\operatorname{Cos}\left[\frac{9c}{2}\right] + \operatorname{Sin}\left[\frac{9c}{2}\right]\right) \operatorname{Sin}\left[\frac{9dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{48d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{\left(\operatorname{Cos}\left[\frac{11c}{2}\right] - \operatorname{Sin}\left[\frac{11c}{2}\right]\right) \operatorname{Sin}\left[\frac{11dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{352d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{\left(\operatorname{Cos}\left[\frac{13c}{2}\right] + \operatorname{Sin}\left[\frac{13c}{2}\right]\right) \operatorname{Sin}\left[\frac{13dx}{2}\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])}}{416d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

- **Problem 108: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx] \sqrt{a + a \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 40 leaves, 3 steps):

$$\frac{\sqrt{2} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sin}[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{d}$$

Result (type 3, 95 leaves):

$$\begin{aligned}
& - \left((2 - 2i) (-1)^{1/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{dx}{4}\right] \left(\operatorname{Cos}\left[\frac{1}{4}(2c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(2c + dx)\right]\right)\right] \sqrt{a(1 + \operatorname{Sin}[c + dx])} \right) / \\
& \left(d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right)
\end{aligned}$$

- **Problem 109: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c + dx]^2 \sqrt{a + a \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 72 leaves, 3 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{\sqrt{2} d} + \frac{\operatorname{Sec}[c+dx] \sqrt{a+a \sin[c+dx]}}{d}$$

Result (type 3, 106 leaves):

$$\frac{1}{d} \operatorname{Sec}[c+dx] \sqrt{a(1+\sin[c+dx])} \left(1 - (1+i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \operatorname{Sec}\left[\frac{dx}{4}\right] \left(\cos\left[\frac{1}{4}(2c+dx)\right] - \sin\left[\frac{1}{4}(2c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)\right)$$

- **Problem 110: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^3 \sqrt{a+a \sin[c+dx]} dx$$

Optimal (type 3, 95 leaves, 5 steps):

$$\frac{3\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sin[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{4\sqrt{2} d} - \frac{3a}{4d \sqrt{a+a \sin[c+dx]}} + \frac{\operatorname{Sec}[c+dx]^2 \sqrt{a+a \sin[c+dx]}}{2d}$$

Result (type 3, 271 leaves):

$$\left(\left(-2 - (3-3i)(-1)^{1/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \operatorname{Sec}\left[\frac{dx}{4}\right] \left(\cos\left[\frac{1}{4}(2c+dx)\right] + \sin\left[\frac{1}{4}(2c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right) \right) + \frac{2 \sin\left[\frac{dx}{2}\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} \right) \sqrt{a(1+\sin[c+dx])} \Big/ \left(4d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right)^2$$

- **Problem 111: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^4 \sqrt{a+a \sin[c+dx]} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$-\frac{5\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{8\sqrt{2} d} - \frac{5a^2 \cos[c+dx]}{8d (a+a \sin[c+dx])^{3/2}} + \frac{5a \operatorname{Sec}[c+dx]}{6d \sqrt{a+a \sin[c+dx]}} + \frac{\operatorname{Sec}[c+dx]^3 \sqrt{a+a \sin[c+dx]}}{3d}$$

Result (type 3, 302 leaves):

$$\left(\left(\frac{6 \operatorname{Sin}\left[\frac{dx}{2}\right]}{\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]} - \frac{3 \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)}{\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]} - \right. \right. \\ \left. \left. (15 + 15i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{dx}{4}\right] \left(\operatorname{Cos}\left[\frac{1}{4}(2c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(2c+dx)\right] \right)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^2 + \right. \\ \left. \frac{4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3} + \frac{12 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^2}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} \right) \\ \left. \sqrt{a(1 + \operatorname{Sin}[c+dx])} \right) / \left(24d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 \right)$$

■ **Problem 112: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c+dx]^5 \sqrt{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 149 leaves, 7 steps):

$$\frac{35 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sin}[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{64 \sqrt{2} d} - \frac{35 a^2}{96 d (a+a \operatorname{Sin}[c+dx])^{3/2}} - \\ \frac{35 a}{64 d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \frac{7 a \operatorname{Sec}[c+dx]^2}{16 d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \frac{\operatorname{Sec}[c+dx]^4 \sqrt{a+a \operatorname{Sin}[c+dx]}}{4 d}$$

Result (type 3, 179 leaves):

$$\left(\sqrt{a(1 + \operatorname{Sin}[c+dx])} \right. \\ \left. \left((-420 + 420i) (-1)^{1/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{dx}{4}\right] \left(\operatorname{Cos}\left[\frac{1}{4}(2c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(2c+dx)\right] \right)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 + \right. \\ \left. \frac{-102 - 70 \operatorname{Cos}[2(c+dx)] + 329 \operatorname{Sin}[c+dx] + 105 \operatorname{Sin}[3(c+dx)]}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^4} \right) / \left(768 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^4 \right)$$

■ **Problem 113: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c+dx]^6 \sqrt{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\begin{aligned}
& - \frac{63 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{128 \sqrt{2} d} - \frac{63 a^2 \cos[c+dx]}{128 d (a+a \sin[c+dx])^{3/2}} - \frac{21 a^2 \sec[c+dx]}{80 d (a+a \sin[c+dx])^{3/2}} + \\
& \frac{21 a \sec[c+dx]}{32 d \sqrt{a+a \sin[c+dx]}} + \frac{3 a \sec[c+dx]^3}{10 d \sqrt{a+a \sin[c+dx]}} + \frac{\sec[c+dx]^5 \sqrt{a+a \sin[c+dx]}}{5 d}
\end{aligned}$$

Result (type 3, 191 leaves):

$$\begin{aligned}
& \left(\sqrt{a (1 + \sin[c+dx])} \right. \\
& \left. \left((-2520 - 2520 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\frac{1}{2} + \frac{i}{2}\right] (-1)^{3/4} \sec\left[\frac{dx}{4}\right] \left(\cos\left[\frac{1}{4}(2c+dx)\right] - \sin\left[\frac{1}{4}(2c+dx)\right] \right) \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 + \right. \\
& \left. \left. \frac{649 + 1092 \cos[2(c+dx)] + 315 \cos[4(c+dx)] + 1572 \sin[c+dx] + 420 \sin[3(c+dx)]}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^5} \right) \right) / \\
& \left(5120 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 \right)
\end{aligned}$$

■ **Problem 114: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^7 (a+a \sin[c+dx])^{3/2} dx$$

Optimal (type 3, 97 leaves, 3 steps):

$$\frac{16 (a+a \sin[c+dx])^{11/2}}{11 a^4 d} - \frac{24 (a+a \sin[c+dx])^{13/2}}{13 a^5 d} + \frac{4 (a+a \sin[c+dx])^{15/2}}{5 a^6 d} - \frac{2 (a+a \sin[c+dx])^{17/2}}{17 a^7 d}$$

Result (type 3, 865 leaves):

$$\begin{aligned}
& \frac{35 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{7 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{32 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{7 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{160 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{\operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{5 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{352 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{\operatorname{Cos}\left[\frac{13}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{416 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{\operatorname{Cos}\left[\frac{15}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{640 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{\operatorname{Cos}\left[\frac{17}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{2176 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{35 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]}{32 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]}{160 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{(a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{5 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right]}{352 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{(a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right]}{416 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{(a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{15}{2}(c+dx)\right]}{640 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{(a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{17}{2}(c+dx)\right]}{2176 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3}
\end{aligned}$$

■ **Problem 115: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^6 (a+a \operatorname{Sin}[c+dx])^{3/2} dx$$

Optimal (type 3, 159 leaves, 5 steps):

$$\begin{aligned}
& - \frac{4096 a^5 \operatorname{Cos}[c+dx]^7}{45045 d (a+a \operatorname{Sin}[c+dx])^{7/2}} - \frac{1024 a^4 \operatorname{Cos}[c+dx]^7}{6435 d (a+a \operatorname{Sin}[c+dx])^{5/2}} - \\
& \frac{128 a^3 \operatorname{Cos}[c+dx]^7}{715 d (a+a \operatorname{Sin}[c+dx])^{3/2}} - \frac{32 a^2 \operatorname{Cos}[c+dx]^7}{195 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \frac{2 a \operatorname{Cos}[c+dx]^7 \sqrt{a+a \operatorname{Sin}[c+dx]}}{15 d}
\end{aligned}$$

Result (type 3, 865 leaves):

$$\begin{aligned}
& \frac{45 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{25 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{39 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{320 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{3 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{448 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{17 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{576 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{3 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{704 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{3 \operatorname{Cos}\left[\frac{13}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{832 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{\operatorname{Cos}\left[\frac{15}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{960 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{45 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{25 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{39 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]}{320 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{3 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]}{448 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{17 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right]}{576 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{3 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right]}{704 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{3 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right]}{832 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{(a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Sin}\left[\frac{15}{2}(c+dx)\right]}{960 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3}
\end{aligned}$$

■ **Problem 121: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c+dx] (a+a \operatorname{Sin}[c+dx])^{3/2} dx$$

Optimal (type 3, 62 leaves, 4 steps):

$$\frac{2 \sqrt{2} a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sin}[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{d} - \frac{2 a \sqrt{a+a \operatorname{Sin}[c+dx]}}{d}$$

Result (type 3, 98 leaves):

$$\begin{aligned}
& - \left(2 \left((2+2i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right] + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) (a(1+\operatorname{Sin}[c+dx]))^{3/2} \right) / \\
& \left(d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 \right)
\end{aligned}$$

■ **Problem 122: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^2 (a+a \operatorname{Sin}[c+dx])^{3/2} dx$$

Optimal (type 3, 26 leaves, 1 step):

$$\frac{2 a \operatorname{Sec}[c+dx] \sqrt{a+a \operatorname{Sin}[c+dx]}}{d}$$

Result (type 3, 67 leaves) :

$$\frac{2 (a (1 + \sin [c + d x]))^{3/2}}{d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^3}$$

■ **Problem 123: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec [c + d x]^3 (a + a \sin [c + d x])^{3/2} dx$$

Optimal (type 3, 73 leaves, 4 steps) :

$$\frac{a^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{a+a \sin [c+d x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} d} + \frac{\sec [c + d x]^2 (a + a \sin [c + d x])^{3/2}}{2 d}$$

Result (type 3, 134 leaves) :

$$\left(a \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] + (1 + i) (-1)^{1/4} \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4} (c + d x) \right] \right) \right] \right) (-1 + \sin [c + d x]) \right) \sqrt{a (1 + \sin [c + d x])} \Big/ \left(2 d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^2 \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right) \right)$$

■ **Problem 124: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec [c + d x]^4 (a + a \sin [c + d x])^{3/2} dx$$

Optimal (type 3, 107 leaves, 4 steps) :

$$-\frac{a^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{2} \sqrt{a+a \sin [c+d x]}} \right]}{2 \sqrt{2} d} + \frac{a \sec [c + d x] \sqrt{a + a \sin [c + d x]}}{2 d} + \frac{\sec [c + d x]^3 (a + a \sin [c + d x])^{3/2}}{3 d}$$

Result (type 3, 130 leaves) :

$$\frac{1}{d} \left(\frac{1}{12} + \frac{i}{12} \right) a \sec [c + d x]^3 \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2 \sqrt{a (1 + \sin [c + d x])} \left(6 (-1)^{3/4} \operatorname{ArcTanh} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \left(-1 + \tan \left[\frac{1}{4} (c + d x) \right] \right) \right] \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^3 - (1 - i) (-5 + 3 \sin [c + d x]) \right)$$

■ **Problem 125: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec [c + d x]^5 (a + a \sin [c + d x])^{3/2} dx$$

Optimal (type 3, 127 leaves, 6 steps) :

$$\frac{15 a^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{a+a \sin [c+d x]}}{\sqrt{2} \sqrt{a}} \right]}{32 \sqrt{2} d} - \frac{15 a^2}{32 d \sqrt{a + a \sin [c + d x]}} + \frac{5 a \sec [c + d x]^2 \sqrt{a + a \sin [c + d x]}}{16 d} + \frac{\sec [c + d x]^4 (a + a \sin [c + d x])^{3/2}}{4 d}$$

Result (type 3, 161 leaves) :

$$\frac{1}{d} \left(\frac{1}{128} + \frac{i}{128} \right) a \operatorname{Sec}[c + dx]^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 \sqrt{a(1 + \operatorname{Sin}[c + dx])} \\ \left(-60(-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4} \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]\right)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right)^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) + \\ (1 - i)(-9 + 15 \operatorname{Cos}[2(c + dx)] + 40 \operatorname{Sin}[c + dx]) \right)$$

■ **Problem 126: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c + dx]^6 (a + a \operatorname{Sin}[c + dx])^{3/2} dx$$

Optimal (type 3, 169 leaves, 6 steps) :

$$-\frac{7 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c + dx]}{\sqrt{2} \sqrt{a + \operatorname{Sin}[c + dx]}}\right]}{16 \sqrt{2} d} - \frac{7 a^3 \operatorname{Cos}[c + dx]}{16 d (a + a \operatorname{Sin}[c + dx])^{3/2}} + \\ \frac{7 a^2 \operatorname{Sec}[c + dx]}{12 d \sqrt{a + a \operatorname{Sin}[c + dx]}} + \frac{7 a \operatorname{Sec}[c + dx]^3 \sqrt{a + a \operatorname{Sin}[c + dx]}}{30 d} + \frac{\operatorname{Sec}[c + dx]^5 (a + a \operatorname{Sin}[c + dx])^{3/2}}{5 d}$$

Result (type 3, 288 leaves) :

$$\frac{1}{240 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^5} \\ \left(30 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] - 15 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) + (105 + 105 i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]\right)\right] \right) \\ \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 + \frac{24 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^5} + \\ \frac{40 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^3} + \frac{90 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2}{\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]} \right) (a(1 + \operatorname{Sin}[c + dx]))^{3/2}$$

■ **Problem 127: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx]^5 (a + a \operatorname{Sin}[c + dx])^{5/2} dx$$

Optimal (type 3, 73 leaves, 3 steps) :

$$\frac{8(a + a \operatorname{Sin}[c + dx])^{11/2}}{11 a^3 d} - \frac{8(a + a \operatorname{Sin}[c + dx])^{13/2}}{13 a^4 d} + \frac{2(a + a \operatorname{Sin}[c + dx])^{15/2}}{15 a^5 d}$$

Result (type 3, 865 leaves) :

$$\begin{aligned}
 & \frac{45 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{65 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \\
 & \frac{\operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{320 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{5 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{5 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \\
 & \frac{5 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{704 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{5 \operatorname{Cos}\left[\frac{13}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{832 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \frac{\operatorname{Cos}\left[\frac{15}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{960 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \\
 & \frac{45 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \frac{65 (a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \\
 & \frac{(a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]}{320 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \frac{5 (a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{5 (a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right]}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \\
 & \frac{5 (a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right]}{704 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{5 (a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right]}{832 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{(a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{15}{2}(c+dx)\right]}{960 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5}
 \end{aligned}$$

■ **Problem 128: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^4 (a+a \operatorname{Sin}[c+dx])^{5/2} dx$$

Optimal (type 3, 159 leaves, 5 steps) :

$$\begin{aligned}
 & - \frac{4096 a^5 \operatorname{Cos}[c+dx]^5}{15015 d (a+a \operatorname{Sin}[c+dx])^{5/2}} - \frac{1024 a^4 \operatorname{Cos}[c+dx]^5}{3003 d (a+a \operatorname{Sin}[c+dx])^{3/2}} - \\
 & \frac{128 a^3 \operatorname{Cos}[c+dx]^5}{429 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \frac{32 a^2 \operatorname{Cos}[c+dx]^5 \sqrt{a+a \operatorname{Sin}[c+dx]}}{143 d} - \frac{2 a \operatorname{Cos}[c+dx]^5 (a+a \operatorname{Sin}[c+dx])^{3/2}}{13 d}
 \end{aligned}$$

Result (type 3, 757 leaves) :

$$\begin{aligned}
& - \frac{9 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{8d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \frac{3 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{32d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} \\
& - \frac{29 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{160d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{5 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{112d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{\operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{48d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} \\
& + \frac{5 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{352d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \frac{\operatorname{Cos}\left[\frac{13}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{416d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \frac{9 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{5/2}}{8d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \\
& + \frac{3(a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]}{32d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \frac{29(a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]}{160d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{5(a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]}{112d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \\
& - \frac{(a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right]}{48d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{5(a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right]}{352d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{(a(1+\operatorname{Sin}[c+dx]))^{5/2} \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right]}{416d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5}
\end{aligned}$$

■ **Problem 132: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c+dx] (a+a \operatorname{Sin}[c+dx])^{5/2} dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$\frac{4\sqrt{2} a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sin}[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{d} - \frac{4a^2 \sqrt{a+a \operatorname{Sin}[c+dx]}}{d} - \frac{2a(a+a \operatorname{Sin}[c+dx])^{3/2}}{3d}$$

Result (type 3, 126 leaves):

$$\begin{aligned}
& - \left((a(1+\operatorname{Sin}[c+dx]))^{5/2} \left((24+24i)(-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4} \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right] + 15 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \right. \right. \\
& \left. \left. \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] + 15 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] \right) \right) / \left(3d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5 \right)
\end{aligned}$$

■ **Problem 134: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c+dx]^3 (a+a \operatorname{Sin}[c+dx])^{5/2} dx$$

Optimal (type 3, 69 leaves, 4 steps):

$$-\frac{a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sin}[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{\sqrt{2}d} + \frac{a \operatorname{Sec}[c+dx]^2 (a+a \operatorname{Sin}[c+dx])^{3/2}}{d}$$

Result (type 3, 138 leaves):

$$-\left(a^2 \left(-\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] + (1+i)(-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right](-1 + \sin[c+dx])\right) \sqrt{a(1 + \sin[c+dx])}\right) / \left(d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right)$$

■ **Problem 135: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^4 (a + a \sin[c+dx])^{5/2} dx$$

Optimal (type 3, 30 leaves, 1 step):

$$\frac{2 a \sec[c+dx]^3 (a + a \sin[c+dx])^{3/2}}{3 d}$$

Result (type 3, 69 leaves):

$$\frac{2 (a (1 + \sin[c+dx]))^{5/2}}{3 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}$$

■ **Problem 136: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec[c+dx]^5 (a + a \sin[c+dx])^{5/2} dx$$

Optimal (type 3, 103 leaves, 5 steps):

$$\frac{3 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sin[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{16 \sqrt{2} d} + \frac{3 a \sec[c+dx]^2 (a + a \sin[c+dx])^{3/2}}{16 d} + \frac{\sec[c+dx]^4 (a + a \sin[c+dx])^{5/2}}{4 d}$$

Result (type 3, 174 leaves):

$$\left(a^2 \sqrt{a(1 + \sin[c+dx])} \left(11 \cos\left[\frac{1}{2}(c+dx)\right] + 3 \cos\left[\frac{3}{2}(c+dx)\right] + 11 \sin\left[\frac{1}{2}(c+dx)\right] + (3+3i)(-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right](-3 + \cos[2(c+dx)] + 4 \sin[c+dx]) - 3 \sin\left[\frac{3}{2}(c+dx)\right]\right)\right) / \left(32 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right)$$

■ **Problem 137: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec[c+dx]^6 (a + a \sin[c+dx])^{5/2} dx$$

Optimal (type 3, 139 leaves, 5 steps):

$$-\frac{a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{4 \sqrt{2} d} + \frac{a^2 \sec[c+dx] \sqrt{a+a \sin[c+dx]}}{4 d} + \frac{a \sec[c+dx]^3 (a + a \sin[c+dx])^{3/2}}{6 d} + \frac{\sec[c+dx]^5 (a + a \sin[c+dx])^{5/2}}{5 d}$$

Result (type 3, 129 leaves) :

$$\left(\left((15 + 15i) (-1)^{3/4} \operatorname{ArcTanh} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \left(-1 + \operatorname{Tan} \left[\frac{1}{4} (c + dx) \right] \right) \right] + \frac{89 - 15 \operatorname{Cos}[2(c + dx)] - 80 \operatorname{Sin}[c + dx]}{2 \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^5} \right) (a (1 + \operatorname{Sin}[c + dx]))^{5/2} \right) / \left(60 d \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^5 \right)$$

■ **Problem 138: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c + dx]^7 (a + a \operatorname{Sin}[c + dx])^{5/2} dx$$

Optimal (type 3, 159 leaves, 7 steps) :

$$\frac{35 a^{5/2} \operatorname{ArcTanh} \left[\frac{\sqrt{a + a \operatorname{Sin}[c + dx]}}{\sqrt{2} \sqrt{a}} \right]}{128 \sqrt{2} d} - \frac{35 a^3}{128 d \sqrt{a + a \operatorname{Sin}[c + dx]}} + \frac{35 a^2 \operatorname{Sec}[c + dx]^2 \sqrt{a + a \operatorname{Sin}[c + dx]}}{192 d} + \frac{7 a \operatorname{Sec}[c + dx]^4 (a + a \operatorname{Sin}[c + dx])^{3/2}}{48 d} + \frac{\operatorname{Sec}[c + dx]^6 (a + a \operatorname{Sin}[c + dx])^{5/2}}{6 d}$$

Result (type 3, 176 leaves) :

$$\frac{1}{d} \left(\frac{1}{3072} + \frac{i}{3072} \right) a^2 \operatorname{Sec}[c + dx]^6 \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^4 \sqrt{a (1 + \operatorname{Sin}[c + dx])} \left(-840 (-1)^{1/4} \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \operatorname{Tan} \left[\frac{1}{4} (c + dx) \right] \right) \right] \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^6 \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right) + (1 - i) (490 \operatorname{Cos}[2(c + dx)] + 791 \operatorname{Sin}[c + dx] - 15 (10 + 7 \operatorname{Sin}[3(c + dx)])) \right)$$

■ **Problem 139: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx]^7 (a + a \operatorname{Sin}[c + dx])^{7/2} dx$$

Optimal (type 3, 97 leaves, 3 steps) :

$$\frac{16 (a + a \operatorname{Sin}[c + dx])^{15/2}}{15 a^4 d} - \frac{24 (a + a \operatorname{Sin}[c + dx])^{17/2}}{17 a^5 d} + \frac{12 (a + a \operatorname{Sin}[c + dx])^{19/2}}{19 a^6 d} - \frac{2 (a + a \operatorname{Sin}[c + dx])^{21/2}}{21 a^7 d}$$

Result (type 3, 1189 leaves) :

$$\begin{aligned}
& \frac{91 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{91 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{256 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{7 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{1280 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{43 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{448 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{7 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{512 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 \operatorname{Cos}\left[\frac{13}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{512 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{7 \operatorname{Cos}\left[\frac{15}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{3840 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{7 \operatorname{Cos}\left[\frac{17}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{4352 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{7 \operatorname{Cos}\left[\frac{19}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{9728 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \\
& \frac{\operatorname{Cos}\left[\frac{21}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{10752 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{91 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{91 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]}{256 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]}{1280 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{43 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]}{448 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right]}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \\
& \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right]}{512 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right]}{512 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{15}{2}(c+dx)\right]}{3840 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{17}{2}(c+dx)\right]}{4352 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{19}{2}(c+dx)\right]}{9728 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{(a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{21}{2}(c+dx)\right]}{10752 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7}
\end{aligned}$$

■ **Problem 140: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^6 (a+a \operatorname{Sin}[c+dx])^{7/2} dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$\begin{aligned}
& - \frac{131072 a^7 \operatorname{Cos}[c+dx]^7}{969969 d (a+a \operatorname{Sin}[c+dx])^{7/2}} - \frac{32768 a^6 \operatorname{Cos}[c+dx]^7}{138567 d (a+a \operatorname{Sin}[c+dx])^{5/2}} - \frac{12288 a^5 \operatorname{Cos}[c+dx]^7}{46189 d (a+a \operatorname{Sin}[c+dx])^{3/2}} - \frac{1024 a^4 \operatorname{Cos}[c+dx]^7}{4199 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \\
& \frac{64 a^3 \operatorname{Cos}[c+dx]^7 \sqrt{a+a \operatorname{Sin}[c+dx]}}{323 d} - \frac{48 a^2 \operatorname{Cos}[c+dx]^7 (a+a \operatorname{Sin}[c+dx])^{3/2}}{323 d} - \frac{2 a \operatorname{Cos}[c+dx]^7 (a+a \operatorname{Sin}[c+dx])^{5/2}}{19 d}
\end{aligned}$$

Result (type 3, 1081 leaves):

$$\begin{aligned}
& - \frac{143 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{13 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{13 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{23 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{448 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{19 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{704 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{7 \operatorname{Cos}\left[\frac{13}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{3328 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{\operatorname{Cos}\left[\frac{15}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{256 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \\
& \frac{7 \operatorname{Cos}\left[\frac{17}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{4352 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{\operatorname{Cos}\left[\frac{19}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{4864 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{143 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \\
& \frac{13 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{13 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{23 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]}{448 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \\
& \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right]}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{19 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right]}{704 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right]}{3328 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{(a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{15}{2}(c+dx)\right]}{256 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{17}{2}(c+dx)\right]}{4352 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{(a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{19}{2}(c+dx)\right]}{4864 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7}
\end{aligned}$$

■ **Problem 141: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^5 (a+a \operatorname{Sin}[c+dx])^{7/2} dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$\frac{8 (a+a \operatorname{Sin}[c+dx])^{13/2}}{13 a^3 d} - \frac{8 (a+a \operatorname{Sin}[c+dx])^{15/2}}{15 a^4 d} + \frac{2 (a+a \operatorname{Sin}[c+dx])^{17/2}}{17 a^5 d}$$

Result (type 3, 865 leaves):

$$\begin{aligned}
& \frac{55 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{11 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{24 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{\operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{20 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{3 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{32 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{5 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{96 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{\operatorname{Cos}\left[\frac{13}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{104 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{7 \operatorname{Cos}\left[\frac{15}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{1920 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \\
& \frac{\operatorname{Cos}\left[\frac{17}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{2176 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{55 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{11 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]}{24 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{(a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]}{20 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{3 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]}{32 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{5 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right]}{96 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{(a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right]}{104 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{15}{2}(c+dx)\right]}{1920 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{(a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{17}{2}(c+dx)\right]}{2176 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7}
\end{aligned}$$

■ **Problem 142: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^4 (a+a \operatorname{Sin}[c+dx])^{7/2} dx$$

Optimal (type 3, 191 leaves, 6 steps):

$$\begin{aligned}
& - \frac{16384 a^6 \operatorname{Cos}[c+dx]^5}{45045 d (a+a \operatorname{Sin}[c+dx])^{5/2}} - \frac{4096 a^5 \operatorname{Cos}[c+dx]^5}{9009 d (a+a \operatorname{Sin}[c+dx])^{3/2}} - \frac{512 a^4 \operatorname{Cos}[c+dx]^5}{1287 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \\
& \frac{128 a^3 \operatorname{Cos}[c+dx]^5 \sqrt{a+a \operatorname{Sin}[c+dx]}}{429 d} - \frac{8 a^2 \operatorname{Cos}[c+dx]^5 (a+a \operatorname{Sin}[c+dx])^{3/2}}{39 d} - \frac{2 a \operatorname{Cos}[c+dx]^5 (a+a \operatorname{Sin}[c+dx])^{5/2}}{15 d}
\end{aligned}$$

Result (type 3, 865 leaves):

$$\begin{aligned}
& - \frac{99 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{11 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{77 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{320 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{43 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{448 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{576 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{17 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{704 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{7 \operatorname{Cos}\left[\frac{13}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{832 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \\
& \frac{\operatorname{Cos}\left[\frac{15}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{960 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{99 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{7/2}}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{11 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \\
& \frac{77 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]}{320 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{43 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]}{448 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right]}{576 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \\
& \frac{17 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right]}{704 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{7 (a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right]}{832 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} + \frac{(a(1+\operatorname{Sin}[c+dx]))^{7/2} \operatorname{Sin}\left[\frac{15}{2}(c+dx)\right]}{960 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7}
\end{aligned}$$

■ **Problem 143: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^3 (a+a \operatorname{Sin}[c+dx])^{7/2} dx$$

Optimal (type 3, 49 leaves, 3 steps):

$$\frac{4 (a+a \operatorname{Sin}[c+dx])^{11/2}}{11 a^2 d} - \frac{2 (a+a \operatorname{Sin}[c+dx])^{13/2}}{13 a^3 d}$$

Result (type 3, 757 leaves):

■ **Problem 146: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec[c + dx] (a + a \sin[c + dx])^{7/2} dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$\frac{8\sqrt{2} a^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a\sin[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{d} - \frac{8a^3 \sqrt{a+a\sin[c+dx]}}{d} - \frac{4a^2 (a+a\sin[c+dx])^{3/2}}{3d} - \frac{2a (a+a\sin[c+dx])^{5/2}}{5d}$$

Result (type 3, 165 leaves):

$$-\left(a^3 (1 + \sin[c + dx])^3 \sqrt{a (1 + \sin[c + dx])}\right) \left((480 + 480i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c + dx)\right]\right)\right] + 330 \cos\left[\frac{1}{2}(c + dx)\right] - 35 \cos\left[\frac{3}{2}(c + dx)\right] - 3 \cos\left[\frac{5}{2}(c + dx)\right] + 330 \sin\left[\frac{1}{2}(c + dx)\right] + 35 \sin\left[\frac{3}{2}(c + dx)\right] - 3 \sin\left[\frac{5}{2}(c + dx)\right] \right) / \left(30d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7\right)$$

■ **Problem 148: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec[c + dx]^3 (a + a \sin[c + dx])^{7/2} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$-\frac{3\sqrt{2} a^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a\sin[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{d} + \frac{3a^3 \sqrt{a+a\sin[c+dx]}}{d} + \frac{a \sec[c+dx]^2 (a+a\sin[c+dx])^{5/2}}{d}$$

Result (type 3, 159 leaves):

$$\left(a^3 \sqrt{a (1 + \sin[c + dx])}\right) \left(3 \cos\left[\frac{1}{2}(c + dx)\right] + \cos\left[\frac{3}{2}(c + dx)\right] + 3 \sin\left[\frac{1}{2}(c + dx)\right] - (6 + 6i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c + dx)\right]\right)\right] (-1 + \sin[c + dx]) - \sin\left[\frac{3}{2}(c + dx)\right]\right) / \left(d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)\right)$$

■ **Problem 150: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec[c + dx]^5 (a + a \sin[c + dx])^{7/2} dx$$

Optimal (type 3, 106 leaves, 5 steps):

$$-\frac{a^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a\sin[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{8\sqrt{2}d} - \frac{a^2 \sec[c+dx]^2 (a+a\sin[c+dx])^{3/2}}{8d} + \frac{a \sec[c+dx]^4 (a+a\sin[c+dx])^{5/2}}{2d}$$

Result (type 3, 172 leaves) :

$$\begin{aligned}
 & - \left(a^3 \sqrt{a (1 + \sin[c + dx])} \left(-7 \cos\left[\frac{1}{2}(c + dx)\right] + \cos\left[\frac{3}{2}(c + dx)\right] - 7 \sin\left[\frac{1}{2}(c + dx)\right] + \right. \right. \\
 & \quad \left. \left. (1 + i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c + dx)\right]\right)\right] \right) (-3 + \cos[2(c + dx)] + 4 \sin[c + dx]) - \sin\left[\frac{3}{2}(c + dx)\right] \right) / \\
 & \left(16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)
 \end{aligned}$$

■ **Problem 151: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^6 (a + a \sin[c + dx])^{7/2} dx$$

Optimal (type 3, 30 leaves, 1 step) :

$$\frac{2 a \sec[c + dx]^5 (a + a \sin[c + dx])^{5/2}}{5 d}$$

Result (type 3, 69 leaves) :

$$\frac{2 (a (1 + \sin[c + dx]))^{7/2}}{5 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^7}$$

■ **Problem 152: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec[c + dx]^7 (a + a \sin[c + dx])^{7/2} dx$$

Optimal (type 3, 135 leaves, 6 steps) :

$$\begin{aligned}
 & \frac{5 a^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + a \sin[c + dx]}}{\sqrt{2} \sqrt{a}}\right]}{64 \sqrt{2} d} + \frac{5 a^2 \sec[c + dx]^2 (a + a \sin[c + dx])^{3/2}}{64 d} + \\
 & \frac{5 a \sec[c + dx]^4 (a + a \sin[c + dx])^{5/2}}{48 d} + \frac{\sec[c + dx]^6 (a + a \sin[c + dx])^{7/2}}{6 d}
 \end{aligned}$$

Result (type 3, 205 leaves) :

$$\begin{aligned}
 & \left(a^3 \sqrt{a (1 + \sin[c + dx])} \right. \\
 & \quad \left(198 \cos\left[\frac{1}{2}(c + dx)\right] + 85 \cos\left[\frac{3}{2}(c + dx)\right] - 15 \cos\left[\frac{5}{2}(c + dx)\right] - (60 + 60 i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c + dx)\right]\right)\right] \right) \\
 & \quad \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 + 198 \sin\left[\frac{1}{2}(c + dx)\right] - 85 \sin\left[\frac{3}{2}(c + dx)\right] - 15 \sin\left[\frac{5}{2}(c + dx)\right] \right) / \\
 & \left(768 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)
 \end{aligned}$$

■ **Problem 153: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec[c + dx]^8 (a + a \sin[c + dx])^{7/2} dx$$

Optimal (type 3, 171 leaves, 6 steps):

$$-\frac{a^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{8 \sqrt{2} d} + \frac{a^3 \sec[c+dx] \sqrt{a+a \sin[c+dx]}}{8 d} +$$

$$\frac{a^2 \sec[c+dx]^3 (a+a \sin[c+dx])^{3/2}}{12 d} + \frac{a \sec[c+dx]^5 (a+a \sin[c+dx])^{5/2}}{10 d} + \frac{\sec[c+dx]^7 (a+a \sin[c+dx])^{7/2}}{7 d}$$

Result (type 3, 139 leaves):

$$\left((a (1 + \sin[c + dx]))^{7/2} \left((105 + 105 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + dx)\right]\right)\right] \right) + \right.$$

$$\left. \frac{2286 - 770 \cos[2 (c + dx)] - 2471 \sin[c + dx] + 105 \sin[3 (c + dx)]}{4 \left(\cos\left[\frac{1}{2} (c + dx)\right] - \sin\left[\frac{1}{2} (c + dx)\right]\right)^7} \right) / \left(840 d \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right]\right)^7 \right)$$

■ **Problem 154: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^9 (a + a \sin[c + dx])^{7/2} dx$$

Optimal (type 3, 191 leaves, 8 steps):

$$\frac{315 a^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sin[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{2048 \sqrt{2} d} - \frac{315 a^4}{2048 d \sqrt{a+a \sin[c+dx]}} + \frac{105 a^3 \sec[c+dx]^2 \sqrt{a+a \sin[c+dx]}}{1024 d} +$$

$$\frac{21 a^2 \sec[c+dx]^4 (a+a \sin[c+dx])^{3/2}}{256 d} + \frac{3 a \sec[c+dx]^6 (a+a \sin[c+dx])^{5/2}}{32 d} + \frac{\sec[c+dx]^8 (a+a \sin[c+dx])^{7/2}}{8 d}$$

Result (type 3, 735 leaves):

$$\begin{aligned}
& - \frac{(a(1 + \sin[c + dx]))^{7/2}}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^8} - \\
& \left(\left(\frac{315}{2048} + \frac{315i}{2048} \right) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right] \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right)\right] (a(1 + \sin[c + dx]))^{7/2} \right) / \\
& \left(d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7 \right) + \frac{(a(1 + \sin[c + dx]))^{7/2}}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7} + \\
& \frac{5(a(1 + \sin[c + dx]))^{7/2}}{64d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^5 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7} + \\
& \frac{41(a(1 + \sin[c + dx]))^{7/2}}{512d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7} + \\
& \frac{187(a(1 + \sin[c + dx]))^{7/2}}{2048d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7} + \\
& \frac{\sin\left[\frac{1}{2}(c + dx)\right] (a(1 + \sin[c + dx]))^{7/2}}{8d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^8 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7} + \\
& \frac{5\sin\left[\frac{1}{2}(c + dx)\right] (a(1 + \sin[c + dx]))^{7/2}}{32d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^6 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7} + \\
& \frac{41\sin\left[\frac{1}{2}(c + dx)\right] (a(1 + \sin[c + dx]))^{7/2}}{256d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7} + \\
& \frac{187\sin\left[\frac{1}{2}(c + dx)\right] (a(1 + \sin[c + dx]))^{7/2}}{1024d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^7}
\end{aligned}$$

■ **Problem 155: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c + dx]^{10} (a + a \sin[c + dx])^{7/2} dx$$

Optimal (type 3, 233 leaves, 8 steps):

$$\begin{aligned}
& - \frac{11 a^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + dx]}{\sqrt{2} \sqrt{a + a \sin[c + dx]}}\right]}{64 \sqrt{2} d} - \frac{11 a^5 \cos[c + dx]}{64 d (a + a \sin[c + dx])^{3/2}} + \frac{11 a^4 \operatorname{Sec}[c + dx]}{48 d \sqrt{a + a \sin[c + dx]}} + \frac{11 a^3 \operatorname{Sec}[c + dx]^3 \sqrt{a + a \sin[c + dx]}}{120 d} + \\
& \frac{11 a^2 \operatorname{Sec}[c + dx]^5 (a + a \sin[c + dx])^{3/2}}{140 d} + \frac{11 a \operatorname{Sec}[c + dx]^7 (a + a \sin[c + dx])^{5/2}}{126 d} + \frac{\operatorname{Sec}[c + dx]^9 (a + a \sin[c + dx])^{7/2}}{9 d}
\end{aligned}$$

Result (type 3, 388 leaves) :

$$\frac{1}{20160 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^9} \left(630 \sin\left[\frac{1}{2}(c+dx)\right] - 315 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) + \right. \\ \left. (3465 + 3465i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + \right. \\ \left. \frac{1120 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^9} + \frac{1440 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^7} + \frac{1512 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^5} + \right. \\ \left. \frac{1680 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^3} + \frac{3150 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2}{\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]} \right) (a(1 + \sin[c+dx]))^{7/2}$$

■ **Problem 161: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2}{\sqrt{a+a\sin[c+dx]}} dx$$

Optimal (type 3, 30 leaves, 1 step) :

$$-\frac{2a\cos[c+dx]^3}{3d(a+a\sin[c+dx])^{3/2}}$$

Result (type 3, 67 leaves) :

$$-\frac{2\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{3d\sqrt{a(1+\sin[c+dx])}}$$

■ **Problem 163: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]}{\sqrt{a+a\sin[c+dx]}} dx$$

Optimal (type 3, 60 leaves, 4 steps) :

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+a\sin[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{\sqrt{2}\sqrt{a}d} - \frac{1}{d\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 76 leaves) :

$$\frac{-1 - (1+i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{d\sqrt{a(1+\sin[c+dx])}}$$

■ **Problem 164: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[c + d x]^2}{\sqrt{a + a \text{Sin}[c + d x]}} dx$$

Optimal (type 3, 102 leaves, 4 steps):

$$-\frac{3 \text{ArcTanh}\left[\frac{\sqrt{a} \text{Cos}[c + d x]}{\sqrt{2} \sqrt{a + a \text{Sin}[c + d x]}}\right]}{4 \sqrt{2} \sqrt{a} d} - \frac{3 a \text{Cos}[c + d x]}{4 d (a + a \text{Sin}[c + d x])^{3/2}} + \frac{\text{Sec}[c + d x]}{d \sqrt{a + a \text{Sin}[c + d x]}}$$

Result (type 3, 118 leaves):

$$-\frac{1}{4 d \sqrt{a} (1 + \text{Sin}[c + d x])} \\ \text{Sec}[c + d x] \left(-1 - (3 + 3 i) (-1)^{3/4} \text{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \text{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] \right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \\ \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right] \right)^2 - 3 \text{Sin}[c + d x]$$

■ **Problem 165: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[c + d x]^3}{\sqrt{a + a \text{Sin}[c + d x]}} dx$$

Optimal (type 3, 116 leaves, 6 steps):

$$\frac{5 \text{ArcTanh}\left[\frac{\sqrt{a + a \text{Sin}[c + d x]}}{\sqrt{2} \sqrt{a}}\right]}{8 \sqrt{2} \sqrt{a} d} - \frac{5 a}{12 d (a + a \text{Sin}[c + d x])^{3/2}} - \frac{5}{8 d \sqrt{a + a \text{Sin}[c + d x]}} + \frac{\text{Sec}[c + d x]^2}{2 d \sqrt{a + a \text{Sin}[c + d x]}}$$

Result (type 3, 108 leaves):

$$\left((-30 - 30 i) (-1)^{1/4} \text{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \text{Tan}\left[\frac{1}{4} (c + d x)\right]\right)\right] \right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right] \right) - \\ \text{Sec}[c + d x]^2 (11 + 15 \text{Cos}[2 (c + d x)] - 20 \text{Sin}[c + d x]) \Big/ (48 d \sqrt{a} (1 + \text{Sin}[c + d x]))$$

■ **Problem 166: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[c + d x]^4}{\sqrt{a + a \text{Sin}[c + d x]}} dx$$

Optimal (type 3, 162 leaves, 6 steps):

$$-\frac{35 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{2} \sqrt{a+a \sin [c+d x]}}\right]}{64 \sqrt{2} \sqrt{a} d}-\frac{35 a \cos [c+d x]}{64 d(a+a \sin [c+d x])^{3 / 2}}-\frac{7 a \sec [c+d x]}{24 d(a+a \sin [c+d x])^{3 / 2}}+\frac{35 \sec [c+d x]}{48 d \sqrt{a+a \sin [c+d x]}}+\frac{\sec [c+d x]^3}{3 d \sqrt{a+a \sin [c+d x]}}$$

Result (type 3, 117 leaves):

$$\left(\left(420+420 i\right)(-1)^{3 / 4} \operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3 / 4}\left(-1+\tan \left[\frac{1}{4}(c+d x)\right]\right)\right]\right)\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)+\frac{\sec [c+d x]^3(102+70 \cos [2(c+d x)]+329 \sin [c+d x]+105 \sin [3(c+d x)])}{\left(768 d \sqrt{a(1+\sin [c+d x])}\right)}$$

■ **Problem 167: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec [c+d x]^5}{\sqrt{a+a \sin [c+d x]}} d x$$

Optimal (type 3, 175 leaves, 8 steps):

$$\frac{63 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sin [c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{128 \sqrt{2} \sqrt{a} d}-\frac{21 a}{64 d(a+a \sin [c+d x])^{3 / 2}}-\frac{9 a \sec [c+d x]^2}{40 d(a+a \sin [c+d x])^{3 / 2}}-\frac{63}{128 d \sqrt{a+a \sin [c+d x]}}+\frac{63 \sec [c+d x]^2}{160 d \sqrt{a+a \sin [c+d x]}}+\frac{\sec [c+d x]^4}{4 d \sqrt{a+a \sin [c+d x]}}$$

Result (type 3, 130 leaves):

$$\left(\left(-315-315 i\right)(-1)^{1 / 4} \operatorname{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{1 / 4}\left(1+\tan \left[\frac{1}{4}(c+d x)\right]\right)\right]\right)\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)-\frac{1}{8} \sec [c+d x]^4(649+1092 \cos [2(c+d x)]+315 \cos [4(c+d x)]-1572 \sin [c+d x]-420 \sin [3(c+d x)])}{\left(640 d \sqrt{a(1+\sin [c+d x])}\right)}$$

■ **Problem 168: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec [c+d x]^6}{\sqrt{a+a \sin [c+d x]}} d x$$

Optimal (type 3, 221 leaves, 8 steps):

$$-\frac{231 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{2} \sqrt{a+a \sin [c+d x]}}\right]}{512 \sqrt{2} \sqrt{a} d}-\frac{231 a \cos [c+d x]}{512 d(a+a \sin [c+d x])^{3 / 2}}-\frac{77 a \sec [c+d x]}{320 d(a+a \sin [c+d x])^{3 / 2}}-\frac{11 a \sec [c+d x]^3}{60 d(a+a \sin [c+d x])^{3 / 2}}+\frac{77 \sec [c+d x]}{128 d \sqrt{a+a \sin [c+d x]}}+\frac{11 \sec [c+d x]^3}{40 d \sqrt{a+a \sin [c+d x]}}+\frac{\sec [c+d x]^5}{5 d \sqrt{a+a \sin [c+d x]}}$$

Result (type 3, 140 leaves):

$$\left((3465 + 3465 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]\right)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) + \frac{1}{16} \operatorname{Sec}[c + dx]^5 (11090 + 11352 \operatorname{Cos}[2(c + dx)] + 2310 \operatorname{Cos}[4(c + dx)] + 36850 \operatorname{Sin}[c + dx] + 17787 \operatorname{Sin}[3(c + dx)] + 3465 \operatorname{Sin}[5(c + dx)]) \Big/ (7680 d \sqrt{a(1 + \operatorname{Sin}[c + dx])})$$

- **Problem 172: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^4}{(a + a \operatorname{Sin}[c + dx])^{3/2}} dx$$

Optimal (type 3, 30 leaves, 1 step):

$$-\frac{2 a \operatorname{Cos}[c + dx]^5}{5 d (a + a \operatorname{Sin}[c + dx])^{5/2}}$$

Result (type 3, 69 leaves):

$$-\frac{2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^5 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^3}{5 d (a (1 + \operatorname{Sin}[c + dx]))^{3/2}}$$

- **Problem 174: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c + dx]^2}{(a + a \operatorname{Sin}[c + dx])^{3/2}} dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$-\frac{2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c + dx]}{\sqrt{2} \sqrt{a + a \operatorname{Sin}[c + dx]}}\right]}{a^{3/2} d} + \frac{2 \operatorname{Cos}[c + dx]}{a d \sqrt{a + a \operatorname{Sin}[c + dx]}}$$

Result (type 3, 100 leaves):

$$\frac{1}{d (a (1 + \operatorname{Sin}[c + dx]))^{3/2}} + 2 \left((2 + 2 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]\right)\right] + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^3$$

- **Problem 176: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + dx]}{(a + a \operatorname{Sin}[c + dx])^{3/2}} dx$$

Optimal (type 3, 89 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+a\sin[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}a^{3/2}d} - \frac{1}{3d(a+a\sin[c+dx])^{3/2}} - \frac{1}{2ad\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 106 leaves):

$$\left(-2-3\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)\right)^2 - (3+3i)(-1)^{1/4}\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{1/4}\left(1+\tan\left[\frac{1}{4}(c+dx)\right]\right)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^3\right)/\left(6d(a(1+\sin[c+dx]))^{3/2}\right)$$

■ **Problem 177: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[c+dx]^2}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 134 leaves, 5 steps):

$$-\frac{15\text{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{2}\sqrt{a+a\sin[c+dx]}}\right]}{32\sqrt{2}a^{3/2}d} - \frac{15\cos[c+dx]}{32d(a+a\sin[c+dx])^{3/2}} - \frac{\text{Sec}[c+dx]}{4d(a+a\sin[c+dx])^{3/2}} + \frac{5\text{Sec}[c+dx]}{8ad\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 224 leaves):

$$\left(-4+\frac{8\sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]}+14\sin\left[\frac{1}{2}(c+dx)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)-7\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)\right)^2 + (15+15i)(-1)^{3/4}\text{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left[\frac{1}{4}(c+dx)\right]\right)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^3 + \frac{8\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]}\right)/\left(32d(a(1+\sin[c+dx]))^{3/2}\right)$$

■ **Problem 178: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[c+dx]^3}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 150 leaves, 7 steps):

$$\frac{7\text{ArcTanh}\left[\frac{\sqrt{a+a\sin[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{16\sqrt{2}a^{3/2}d} - \frac{7}{24d(a+a\sin[c+dx])^{3/2}} - \frac{\text{Sec}[c+dx]^2}{5d(a+a\sin[c+dx])^{3/2}} - \frac{7}{16ad\sqrt{a+a\sin[c+dx]}} + \frac{7\text{Sec}[c+dx]^2}{20ad\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 241 leaves):

$$\left(-40 - \frac{24}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} - 90 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)^2 -$$

$$(105 + 105i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 +$$

$$\frac{15 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3}{\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]} + \frac{30 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} \Big/ (240 d (a (1 + \sin[c+dx]))^{3/2})$$

■ **Problem 179: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^4}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 195 leaves, 7 steps):

$$-\frac{105 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a\sin[c+dx]}}\right]}{256 \sqrt{2} a^{3/2} d} - \frac{105 \cos[c+dx]}{256 d (a+a\sin[c+dx])^{3/2}} -$$

$$\frac{7 \sec[c+dx]}{32 d (a+a\sin[c+dx])^{3/2}} - \frac{\sec[c+dx]^3}{6 d (a+a\sin[c+dx])^{3/2}} + \frac{35 \sec[c+dx]}{64 a d \sqrt{a+a\sin[c+dx]}} + \frac{\sec[c+dx]^3}{4 a d \sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 334 leaves):

$$\frac{1}{768 d (a (1 + \sin[c+dx]))^{3/2}}$$

$$\left(-68 + \frac{64 \sin\left[\frac{1}{2}(c+dx)\right]}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3} - \frac{32}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} + \frac{136 \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} + \right.$$

$$246 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) - 123 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 +$$

$$(315 + 315i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 +$$

$$\left. \frac{32 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^3} + \frac{192 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3}{\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]} \right)$$

■ **Problem 180: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^5}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 211 leaves, 9 steps):

$$\frac{99 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sin [c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{256 \sqrt{2} a^{3/2} d} - \frac{33}{128 d (a+a \sin [c+d x])^{3/2}} - \frac{99 \operatorname{Sec}[c+d x]^2}{560 d (a+a \sin [c+d x])^{3/2}} -$$

$$\frac{\operatorname{Sec}[c+d x]^4}{7 d (a+a \sin [c+d x])^{3/2}} - \frac{99}{256 a d \sqrt{a+a \sin [c+d x]}} + \frac{99 \operatorname{Sec}[c+d x]^2}{320 a d \sqrt{a+a \sin [c+d x]}} + \frac{11 \operatorname{Sec}[c+d x]^4}{56 a d \sqrt{a+a \sin [c+d x]}}$$

Result (type 3, 376 leaves):

$$\frac{1}{8960 d (a (1 + \sin [c+d x]))^{3/2}}$$

$$\left(-1120 - \frac{320}{\left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{672}{\left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - 2800 \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^2 - \right.$$

$$\left. (3465 + 3465 i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4}(c+d x)\right]\right)\right] \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^3 + \right.$$

$$\left. \frac{140 \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^3}{\left(\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right)^3} + \frac{665 \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^3}{\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]} + \right.$$

$$\left. \frac{280 \sin \left[\frac{1}{2}(c+d x)\right] \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^3}{\left(\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{1330 \sin \left[\frac{1}{2}(c+d x)\right] \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^3}{\left(\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right)^2} \right)$$

■ **Problem 181: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+d x]^6}{(a+a \sin [c+d x])^{3/2}} dx$$

Optimal (type 3, 256 leaves, 9 steps):

$$-\frac{3003 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{2} \sqrt{a+a \sin [c+d x]}}\right]}{8192 \sqrt{2} a^{3/2} d} - \frac{3003 \cos [c+d x]}{8192 d (a+a \sin [c+d x])^{3/2}} - \frac{1001 \operatorname{Sec}[c+d x]}{5120 d (a+a \sin [c+d x])^{3/2}} - \frac{143 \operatorname{Sec}[c+d x]^3}{960 d (a+a \sin [c+d x])^{3/2}} -$$

$$\frac{\operatorname{Sec}[c+d x]^5}{8 d (a+a \sin [c+d x])^{3/2}} + \frac{1001 \operatorname{Sec}[c+d x]}{2048 a d \sqrt{a+a \sin [c+d x]}} + \frac{143 \operatorname{Sec}[c+d x]^3}{640 a d \sqrt{a+a \sin [c+d x]}} + \frac{13 \operatorname{Sec}[c+d x]^5}{80 a d \sqrt{a+a \sin [c+d x]}}$$

Result (type 3, 444 leaves):

$$\frac{1}{122880 d (a (1 + \sin[c + dx]))^{3/2}} \left(-8860 + \frac{3840 \sin\left[\frac{1}{2}(c + dx)\right]}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^5} - \frac{1920}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \right. \\ \left. \frac{9920 \sin\left[\frac{1}{2}(c + dx)\right]}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} - \frac{4960}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{17720 \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]} + \right. \\ \left. 32490 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) - 16245 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 + \right. \\ \left. (45045 + 45045 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c + dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3 + \right. \\ \left. \frac{1536 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^5} + \frac{6400 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{28800 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3}{\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]} \right)$$

■ **Problem 182: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{10}}{(a + a \sin[c + dx])^{5/2}} dx$$

Optimal (type 3, 95 leaves, 3 steps):

$$-\frac{64 a^3 \cos[c + dx]^{11}}{2145 d (a + a \sin[c + dx])^{11/2}} - \frac{16 a^2 \cos[c + dx]^{11}}{195 d (a + a \sin[c + dx])^{9/2}} - \frac{2 a \cos[c + dx]^{11}}{15 d (a + a \sin[c + dx])^{7/2}}$$

Result (type 3, 865 leaves):

$$\begin{aligned}
& - \frac{45 \cos\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{64 d (a(1+\sin[c+dx]))^{5/2}} + \frac{65 \cos\left[\frac{3}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{192 d (a(1+\sin[c+dx]))^{5/2}} + \\
& \frac{\cos\left[\frac{5}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{320 d (a(1+\sin[c+dx]))^{5/2}} + \frac{5 \cos\left[\frac{7}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{64 d (a(1+\sin[c+dx]))^{5/2}} + \\
& \frac{5 \cos\left[\frac{9}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{192 d (a(1+\sin[c+dx]))^{5/2}} + \frac{5 \cos\left[\frac{11}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{704 d (a(1+\sin[c+dx]))^{5/2}} + \\
& \frac{5 \cos\left[\frac{13}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{832 d (a(1+\sin[c+dx]))^{5/2}} - \frac{\cos\left[\frac{15}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{960 d (a(1+\sin[c+dx]))^{5/2}} + \\
& \frac{45 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{64 d (a(1+\sin[c+dx]))^{5/2}} + \frac{65 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{3}{2}(c+dx)\right]}{192 d (a(1+\sin[c+dx]))^{5/2}} - \\
& \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{5}{2}(c+dx)\right]}{320 d (a(1+\sin[c+dx]))^{5/2}} + \frac{5 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{7}{2}(c+dx)\right]}{64 d (a(1+\sin[c+dx]))^{5/2}} - \\
& \frac{5 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{9}{2}(c+dx)\right]}{192 d (a(1+\sin[c+dx]))^{5/2}} + \frac{5 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{11}{2}(c+dx)\right]}{704 d (a(1+\sin[c+dx]))^{5/2}} - \\
& \frac{5 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{13}{2}(c+dx)\right]}{832 d (a(1+\sin[c+dx]))^{5/2}} - \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{15}{2}(c+dx)\right]}{960 d (a(1+\sin[c+dx]))^{5/2}}
\end{aligned}$$

■ **Problem 183: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^9}{(a+a\sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 121 leaves, 3 steps):

$$\frac{32 (a+a\sin[c+dx])^{5/2}}{5 a^5 d} - \frac{64 (a+a\sin[c+dx])^{7/2}}{7 a^6 d} + \frac{16 (a+a\sin[c+dx])^{9/2}}{3 a^7 d} - \frac{16 (a+a\sin[c+dx])^{11/2}}{11 a^8 d} + \frac{2 (a+a\sin[c+dx])^{13/2}}{13 a^9 d}$$

Result (type 3, 757 leaves):

$$\begin{aligned}
& \frac{9 \cos\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{8d(a(1+\sin[c+dx]))^{5/2}} - \frac{3 \cos\left[\frac{3}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{32d(a(1+\sin[c+dx]))^{5/2}} + \\
& \frac{29 \cos\left[\frac{5}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{160d(a(1+\sin[c+dx]))^{5/2}} + \frac{5 \cos\left[\frac{7}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{112d(a(1+\sin[c+dx]))^{5/2}} + \\
& \frac{\cos\left[\frac{9}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{48d(a(1+\sin[c+dx]))^{5/2}} + \frac{5 \cos\left[\frac{11}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{352d(a(1+\sin[c+dx]))^{5/2}} - \\
& \frac{\cos\left[\frac{13}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{416d(a(1+\sin[c+dx]))^{5/2}} + \frac{9 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{8d(a(1+\sin[c+dx]))^{5/2}} + \\
& \frac{3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{3}{2}(c+dx)\right]}{32d(a(1+\sin[c+dx]))^{5/2}} + \frac{29 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{5}{2}(c+dx)\right]}{160d(a(1+\sin[c+dx]))^{5/2}} - \\
& \frac{5 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{7}{2}(c+dx)\right]}{112d(a(1+\sin[c+dx]))^{5/2}} + \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{9}{2}(c+dx)\right]}{48d(a(1+\sin[c+dx]))^{5/2}} - \\
& \frac{5 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{11}{2}(c+dx)\right]}{352d(a(1+\sin[c+dx]))^{5/2}} - \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \sin\left[\frac{13}{2}(c+dx)\right]}{416d(a(1+\sin[c+dx]))^{5/2}}
\end{aligned}$$

■ **Problem 186: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^6}{(a+a\sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 30 leaves, 1 step):

$$-\frac{2a \cos[c+dx]^7}{7d(a+a\sin[c+dx])^{7/2}}$$

Result (type 3, 69 leaves):

$$-\frac{2 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{7d(a(1+\sin[c+dx]))^{5/2}}$$

■ **Problem 188: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^4}{(a+a\sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$-\frac{4\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a\sin[c+dx]}}\right]}{a^{5/2}d} + \frac{2 \cos[c+dx]^3}{3ad(a+a\sin[c+dx])^{3/2}} + \frac{4 \cos[c+dx]}{a^2d\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 128 leaves) :

$$\left(\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 \left((24+24i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \right) + 15 \cos\left[\frac{1}{2}(c+dx)\right] - \cos\left[\frac{3}{2}(c+dx)\right] - 15 \sin\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{3}{2}(c+dx)\right] \right) / (3d(a(1+\sin[c+dx]))^{5/2})$$

■ **Problem 190: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^2}{(a+a\sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 75 leaves, 3 steps) :

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{2}\sqrt{a+a\sin[c+dx]}}\right]}{\sqrt{2}a^{5/2}d} - \frac{\cos[c+dx]}{ad(a+a\sin[c+dx])^{3/2}}$$

Result (type 3, 108 leaves) :

$$-\frac{1}{d(a(1+\sin[c+dx]))^{5/2}} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] + (1+i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] (1+\sin[c+dx]) \right)$$

■ **Problem 192: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]}{(a+a\sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 113 leaves, 6 steps) :

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+a\sin[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{4\sqrt{2}a^{5/2}d} - \frac{1}{5d(a+a\sin[c+dx])^{5/2}} - \frac{1}{6ad(a+a\sin[c+dx])^{3/2}} - \frac{1}{4a^2d\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 131 leaves) :

$$\left(-12 - 10 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 - 15 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 - (15+15i)(-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 \right) / (60d(a(1+\sin[c+dx]))^{5/2})$$

■ **Problem 193: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^2}{(a+a\sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 167 leaves, 6 steps) :

$$\begin{aligned}
& - \frac{35 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{2} \sqrt{a+a \sin [c+d x]}}\right]}{128 \sqrt{2} a^{5/2} d} - \frac{\operatorname{Sec}[c+d x]}{6 d (a+a \sin [c+d x])^{5/2}} \\
& \frac{35 \cos [c+d x]}{128 a d (a+a \sin [c+d x])^{3/2}} - \frac{7 \operatorname{Sec}[c+d x]}{48 a d (a+a \sin [c+d x])^{3/2}} + \frac{35 \operatorname{Sec}[c+d x]}{96 a^2 d \sqrt{a+a \sin [c+d x]}}
\end{aligned}$$

Result (type 3, 284 leaves):

$$\begin{aligned}
& \frac{1}{384 d (a (1 + \sin [c+d x]))^{5/2}} \\
& \left(-32 + \frac{64 \sin \left[\frac{1}{2} (c+d x)\right]}{\cos \left[\frac{1}{2} (c+d x)\right] + \sin \left[\frac{1}{2} (c+d x)\right]} + 88 \sin \left[\frac{1}{2} (c+d x)\right] \left(\cos \left[\frac{1}{2} (c+d x)\right] + \sin \left[\frac{1}{2} (c+d x)\right] \right) - 44 \left(\cos \left[\frac{1}{2} (c+d x)\right] + \sin \left[\frac{1}{2} (c+d x)\right] \right)^2 + \right. \\
& \left. 114 \sin \left[\frac{1}{2} (c+d x)\right] \left(\cos \left[\frac{1}{2} (c+d x)\right] + \sin \left[\frac{1}{2} (c+d x)\right] \right)^3 - 57 \left(\cos \left[\frac{1}{2} (c+d x)\right] + \sin \left[\frac{1}{2} (c+d x)\right] \right)^4 + (105 + 105 i) (-1)^{3/4} \right. \\
& \left. \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan \left[\frac{1}{4} (c+d x)\right]\right)\right] \left(\cos \left[\frac{1}{2} (c+d x)\right] + \sin \left[\frac{1}{2} (c+d x)\right] \right)^5 + \frac{48 \left(\cos \left[\frac{1}{2} (c+d x)\right] + \sin \left[\frac{1}{2} (c+d x)\right] \right)^5}{\cos \left[\frac{1}{2} (c+d x)\right] - \sin \left[\frac{1}{2} (c+d x)\right]} \right)
\end{aligned}$$

■ **Problem 194: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+d x]^3}{(a+a \sin [c+d x])^{5/2}} dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$\begin{aligned}
& \frac{9 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sin [c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{32 \sqrt{2} a^{5/2} d} - \frac{\operatorname{Sec}[c+d x]^2}{7 d (a+a \sin [c+d x])^{5/2}} - \frac{3}{16 a d (a+a \sin [c+d x])^{3/2}} \\
& \frac{9 \operatorname{Sec}[c+d x]^2}{70 a d (a+a \sin [c+d x])^{3/2}} - \frac{9}{32 a^2 d \sqrt{a+a \sin [c+d x]}} + \frac{9 \operatorname{Sec}[c+d x]^2}{40 a^2 d \sqrt{a+a \sin [c+d x]}}
\end{aligned}$$

Result (type 3, 266 leaves):

$$\frac{1}{1120 d (a (1 + \sin[c + dx]))^{5/2}} \left(-112 - \frac{80}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - 140 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 - 280 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 - \right. \\ \left. (315 + 315i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 + \right. \\ \left. \frac{35 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]} + \frac{70 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 195: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^4}{(a+a\sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 233 leaves, 8 steps):

$$-\frac{1155 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a\sin[c+dx]}}\right]}{4096 \sqrt{2} a^{5/2} d} - \frac{\sec[c+dx]^3}{8 d (a+a\sin[c+dx])^{5/2}} - \frac{1155 \cos[c+dx]}{4096 a d (a+a\sin[c+dx])^{3/2}} - \\ \frac{77 \sec[c+dx]}{512 a d (a+a\sin[c+dx])^{3/2}} - \frac{11 \sec[c+dx]^3}{96 a d (a+a\sin[c+dx])^{3/2}} + \frac{385 \sec[c+dx]}{1024 a^2 d \sqrt{a+a\sin[c+dx]}} + \frac{11 \sec[c+dx]^3}{64 a^2 d \sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 394 leaves):

$$\frac{1}{12288 d (a (1 + \sin[c + dx]))^{5/2}} \left(-736 + \frac{768 \sin\left[\frac{1}{2}(c+dx)\right]}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{384}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{1472 \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} + \right. \\ \left. 2072 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right) - 1036 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 + \right. \\ \left. 3090 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3 - 1545 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 + \right. \\ \left. (3465 + 3465i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 + \right. \\ \left. \frac{256 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{1920 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]} \right)$$

- **Problem 197: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (e \cos[c + dx])^{5/2} (a + a \sin[c + dx]) dx$$

Optimal (type 4, 95 leaves, 4 steps):

$$-\frac{2 a (e \cos[c + dx])^{7/2}}{7 d e} + \frac{6 a e^2 \sqrt{e \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5 d \sqrt{\cos[c + dx]}} + \frac{2 a e (e \cos[c + dx])^{3/2} \sin[c + dx]}{5 d}$$

Result (type 5, 264 leaves):

$$\frac{1}{560 d \sqrt{e \cos[c + dx]}} a e^3 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left(-154 \cos[dx] - 182 \cos[2c + dx] + 14 \cos[2c + 3dx] - 14 \cos[4c + 3dx] - 30 \sin[c] + \right. \\ \left. 168 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] (\cos[dx] - i \sin[dx]) \sqrt{1 + \cos[2(c + dx)] + i \sin[2(c + dx)]} + \right. \\ \left. 56 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] (\cos[dx] + i \sin[dx]) \sqrt{1 + \cos[2(c + dx)] + i \sin[2(c + dx)]} + \right. \\ \left. 20 \sin[c + 2dx] - 20 \sin[3c + 2dx] + 5 \sin[3c + 4dx] - 5 \sin[5c + 4dx] \right)$$

- **Problem 199: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{e \cos[c + dx]} (a + a \sin[c + dx]) dx$$

Optimal (type 4, 63 leaves, 3 steps):

$$-\frac{2 a (e \cos[c + dx])^{3/2}}{3 d e} + \frac{2 a \sqrt{e \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{d \sqrt{\cos[c + dx]}}$$

Result (type 5, 260 leaves):

$$\frac{1}{6 d \left((1 + e^{2i dx}) \cos[c] + i (-1 + e^{2i dx}) \sin[c] \right)} \\ a \sqrt{e \cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (\cos[dx] + i \sin[dx]) \left(-6 \cos[dx] - 6 \cos[2c + dx] - 2 \sin[c] + \right. \\ \left. 6 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] (\cos[dx] - i \sin[dx]) \sqrt{1 + \cos[2(c + dx)] + i \sin[2(c + dx)]} + \right. \\ \left. 2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] (\cos[dx] + i \sin[dx]) \sqrt{1 + \cos[2(c + dx)] + i \sin[2(c + dx)]} + \right. \\ \left. \sin[c + 2dx] - \sin[3c + 2dx] \right)$$

- **Problem 201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[c + dx]}{(e \cos[c + dx])^{3/2}} dx$$

Optimal (type 4, 91 leaves, 4 steps):

$$\frac{2a}{de\sqrt{e\cos[c+dx]}} - \frac{2a\sqrt{e\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{de^2\sqrt{\cos[c+dx]}} + \frac{2a\sin[c+dx]}{de\sqrt{e\cos[c+dx]}}$$

Result (type 5, 188 leaves):

$$-\frac{1}{6de\sqrt{e\cos[c+dx]}} a \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left(-6(\cos[dx] + \sin[c]) + \right. \\ \left. 3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i\sin[c])^2\right] (\cos[dx] - i\sin[dx]) \sqrt{1 + \cos[2(c+dx)] + i\sin[2(c+dx)]} + \right. \\ \left. \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i\sin[c])^2\right] (\cos[dx] + i\sin[dx]) \sqrt{1 + \cos[2(c+dx)] + i\sin[2(c+dx)]} \right)$$

- **Problem 203: Result unnecessarily involves higher level functions.**

$$\int \frac{a + a \sin[c + dx]}{(e \cos[c + dx])^{7/2}} dx$$

Optimal (type 4, 126 leaves, 5 steps):

$$\frac{2a}{5de(e\cos[c+dx])^{5/2}} - \frac{6a\sqrt{e\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5de^4\sqrt{\cos[c+dx]}} + \frac{2a\sin[c+dx]}{5de(e\cos[c+dx])^{5/2}} + \frac{6a\sin[c+dx]}{5de^3\sqrt{e\cos[c+dx]}}$$

Result (type 5, 160 leaves):

$$\left(2a\sqrt{e\cos[c+dx]} (\cos[c+dx] - i\sin[c+dx]) \right. \\ \left. \left(3i + \cos[c+dx] + 3i\sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] (-1 + \sin[c+dx]) - 2i\sin[c+dx] \right) \right) / \\ \left(5de^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

- **Problem 210: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[c + dx])^2}{(e \cos[c + dx])^{5/2}} dx$$

Optimal (type 4, 89 leaves, 4 steps):

$$-\frac{2 a^2 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d e^2 \sqrt{e \cos [c+d x]}}+\frac{4 a^4 \sqrt{e \cos [c+d x]}}{3 d e^3\left(a^2-a^2 \sin [c+d x]\right)}$$

Result (type 4, 1198 leaves):

$$\frac{\cos [c+d x]^3\left(-\frac{2}{3}+\frac{4}{3\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2}\right)\left(a+a \sin [c+d x]\right)^2}{d\left(e \cos [c+d x]\right)^{5 / 2}\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4}+$$

$$\left(2 \cos [c+d x]^2\left(-\frac{\cos \left[\frac{1}{2}(c+d x)\right] \sqrt{\cos [c+d x]}}{3\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)}-\frac{\sqrt{\cos [c+d x]} \sin \left[\frac{1}{2}(c+d x)\right]}{3\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)}\right)\right.$$

$$\left.\left(a+a \sin [c+d x]\right)^2\left(\cos [c+d x]-2 \sqrt{2} \cos \left[\frac{1}{4}(c+d x)\right]\right)^2 \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}}\right.$$

$$\left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x)\right]^2}\right]\right) /$$

$$\left(3 d\left(e \cos [c+d x]\right)^{5 / 2}\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4\left(\frac{1}{3 \cos [c+d x]^{3 / 2}} \sin [c+d x]\left(\cos [c+d x]-2 \sqrt{2} \cos \left[\frac{1}{4}(c+d x)\right]\right)^2\right.\right.$$

$$\left.\left.\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x)\right]^2}\right)\right) +$$

$$\frac{1}{3 \sqrt{\cos [c+d x]}} 2\left(-\sin [c+d x]+\frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{2} \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x)\right]^2}}\right) +$$

$$\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]$$

$$\sin\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} - \left(\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right]\right)^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]$$

$$\left(\frac{(-2+\sqrt{2})\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])} + \frac{(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right])\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])^2}\right) \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} /$$

$$\left(\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} - \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{2(3-2\sqrt{2})} \sqrt{1-\frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}}\right)$$

■ **Problem 212: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\sin[c+dx])^2}{(e\cos[c+dx])^{9/2}} dx$$

Optimal (type 4, 114 leaves, 4 steps):

$$\frac{2a^2\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{7de^4\sqrt{e\cos[c+dx]}} + \frac{2a^2\sin[c+dx]}{7de^3(e\cos[c+dx])^{3/2}} + \frac{4(a^2+a^2\sin[c+dx])}{7de(e\cos[c+dx])^{7/2}}$$

Result (type 4, 1227 leaves):

$$\frac{\cos[c+dx]^5 \left(\frac{2}{7} + \frac{2}{7(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])^4} + \frac{2}{7(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])^2}\right) (a+a\sin[c+dx])^2}{d(e\cos[c+dx])^{9/2}(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right])^4} -$$

$$\left(2\cos[c+dx]^4 \left(\frac{\cos\left[\frac{1}{2}(c+dx)\right]\sqrt{\cos[c+dx]}}{7(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])} + \frac{\sqrt{\cos[c+dx]}\sin\left[\frac{1}{2}(c+dx)\right]}{7(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])}\right)\right)$$

$$(a+a\sin[c+dx])^2 \left(\cos[c+dx]-2\sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right]\right)^2 \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right) \Big/ \\
& \left(7d(e\cos[c+dx])^{9/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 \left(-\frac{1}{7\cos[c+dx]^{3/2}} \sin[c+dx] \left(\cos[c+dx] - 2\sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right]^2\right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right) - \right. \right. \\
& \left. \frac{1}{7\sqrt{\cos[c+dx]}} 2 \left(-\sin[c+dx] + \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2}\sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2}}\right) + \right. \\
& \left. \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \\
& \left. \sin\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \left(\sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right]^2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \right. \\
& \left. \left. \left(\frac{(-2+\sqrt{2})\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])} + \frac{(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right])\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])^2}\right) \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right) \Big/
\end{aligned}$$

$$\left(\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} - \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}}{\sqrt{2(3 - 2\sqrt{2})} \sqrt{1 - \frac{\tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}}} \right) \Bigg|$$

■ **Problem 222: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[c + dx])^3}{(e \cos[c + dx])^{9/2}} dx$$

Optimal (type 4, 127 leaves, 5 steps):

$$-\frac{2a^3 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21de^4 \sqrt{e \cos[c + dx]}} + \frac{4a^5 \sqrt{e \cos[c + dx]}}{7de^5 (a - a \sin[c + dx])^2} - \frac{2a^6 \sqrt{e \cos[c + dx]}}{21de^5 (a^3 - a^3 \sin[c + dx])}$$

Result (type 4, 1227 leaves):

$$\frac{\cos[c + dx]^5 \left(-\frac{2}{21} + \frac{4}{7 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^4} - \frac{2}{21 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2} \right) (a + a \sin[c + dx])^3}{d (e \cos[c + dx])^{9/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6} +$$

$$\left(2 \cos[c + dx]^4 \left(-\frac{\cos\left[\frac{1}{2}(c + dx)\right] \sqrt{\cos[c + dx]}}{21 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)} - \frac{\sqrt{\cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{21 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)} \right) \right)$$

$$(a + a \sin[c + dx])^3 \left(\cos[c + dx] - 2\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] \right)^2 \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}}$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \Bigg| \Bigg| /$$

$$\left(21d (e \cos[c + dx])^{9/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 \left(\frac{1}{21 \cos[c + dx]^{3/2}} \sin[c + dx] \left(\cos[c + dx] - 2\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] \right)^2 \right. \right.$$

$$\begin{aligned}
& \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} + \\
& \frac{1}{21\sqrt{\cos[c + dx]}} 2 \left(-\sin[c + dx] + \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}} \right) + \\
& \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \\
& \sin\left[\frac{1}{4}(c + dx)\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} - \left(\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] \right)^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \\
& \left(\frac{(-2 + \sqrt{2}) \sin\left[\frac{1}{2}(c + dx)\right]}{2(1 + \cos\left[\frac{1}{2}(c + dx)\right])} + \frac{(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]) \sin\left[\frac{1}{2}(c + dx)\right]}{2(1 + \cos\left[\frac{1}{2}(c + dx)\right])^2} \right) \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \Big/ \\
& \left(\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} - \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}}{\sqrt{2(3 - 2\sqrt{2})} \sqrt{1 - \frac{\tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}}} \right) \Big)
\end{aligned}$$

■ **Problem 230: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[c + dx])^4}{(e \cos[c + dx])^{9/2}} dx$$

Optimal (type 4, 127 leaves, 5 steps):

$$\frac{10 a^4 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21 d e^4 \sqrt{e \cos[c + dx]}} + \frac{4 a^7 (e \cos[c + dx])^{5/2}}{7 d e^7 (a - a \sin[c + dx])^3} - \frac{20 a^8 \sqrt{e \cos[c + dx]}}{21 d e^5 (a^4 - a^4 \sin[c + dx])}$$

Result (type 4, 1227 leaves):

$$\begin{aligned}
& \frac{\cos[c+dx]^5 \left(\frac{10}{21} + \frac{8}{7 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^4} - \frac{32}{21 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} \right) (a + a \sin[c+dx])^4}{d (e \cos[c+dx])^{9/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8} - \\
& \left(10 \cos[c+dx]^4 \left(\frac{5 \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\cos[c+dx]}}{21 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)} + \frac{5 \sqrt{\cos[c+dx]} \sin\left[\frac{1}{2}(c+dx)\right]}{21 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)} \right) \right. \\
& (a + a \sin[c+dx])^4 \left(\cos[c+dx] - 2\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right] \right) / \\
& \left(21 d (e \cos[c+dx])^{9/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 \left(-\frac{1}{21 \cos[c+dx]^{3/2}} 5 \sin[c+dx] \left(\cos[c+dx] - 2\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \right. \right. \\
& \left. \left. \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right)^2 - \right. \\
& \left. \frac{1}{21 \sqrt{\cos[c+dx]}} 10 \left(-\sin[c+dx] + \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2}} \right) + \right. \\
& \left. \sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)
\end{aligned}$$

$$\begin{aligned} & \sin\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} - \left(\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \\ & \left. \left(\frac{(-2+\sqrt{2}) \sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])} + \frac{(-1+\sqrt{2}-(-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]) \sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])^2} \right) \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) / \\ & \left(\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right. \\ & \left. - \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{2(3-2\sqrt{2})} \sqrt{1-\frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}} \right) \right) \end{aligned}$$

■ **Problem 232: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \sin[c+dx])^4}{(e \cos[c+dx])^{13/2}} dx$$

Optimal (type 4, 169 leaves, 6 steps):

$$-\frac{2a^4 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{77de^6 \sqrt{e \cos[c+dx]}} + \frac{4a^7 \sqrt{e \cos[c+dx]}}{11de^7 (a-a \sin[c+dx])^3} - \frac{2a^8 \sqrt{e \cos[c+dx]}}{77de^7 (a^2-a^2 \sin[c+dx])^2} - \frac{2a^8 \sqrt{e \cos[c+dx]}}{77de^7 (a^4-a^4 \sin[c+dx])}$$

Result (type 4, 1256 leaves):

$$\begin{aligned} & \left(\cos[c+dx]^7 \right. \\ & \left. \left(-\frac{2}{77} + \frac{4}{11(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])^6} - \frac{2}{77(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])^4} - \frac{2}{77(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])^2} \right) \right. \\ & \left. (a+a \sin[c+dx])^4 \right) / \left(d(e \cos[c+dx])^{13/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 \right) + \\ & \left(2 \cos[c+dx]^6 \left(-\frac{\cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\cos[c+dx]}}{77(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])} - \frac{\sqrt{\cos[c+dx]} \sin\left[\frac{1}{2}(c+dx)\right]}{77(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])} \right) \right) \end{aligned}$$

$$\begin{aligned}
& (a + a \sin[c + dx])^4 \left(\cos[c + dx] - 2\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] \right)^2 \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \right) / \\
& \left(77d (e \cos[c + dx])^{13/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^8 \left(\frac{1}{77 \cos[c + dx]^{3/2}} \sin[c + dx] \left(\cos[c + dx] - 2\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] \right)^2 \right. \right. \\
& \left. \left. \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \right) + \right. \\
& \left. \frac{1}{77\sqrt{\cos[c + dx]}} \left(-\sin[c + dx] + \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}} \right) + \right. \\
& \left. \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \left. \sin\left[\frac{1}{4}(c + dx)\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} - \left(\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] \right)^2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \left. \left(\frac{(-2 + \sqrt{2}) \sin\left[\frac{1}{2}(c + dx)\right]}{2(1 + \cos\left[\frac{1}{2}(c + dx)\right])} + \frac{(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]) \sin\left[\frac{1}{2}(c + dx)\right]}{2(1 + \cos\left[\frac{1}{2}(c + dx)\right])^2} \right) \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \right) /
\end{aligned}$$

$$\left(\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} - \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}}{\sqrt{2(3 - 2\sqrt{2})} \sqrt{1 - \frac{\tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}}} \right) \Bigg|$$

■ **Problem 237: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{3/2}}{a + a \sin[c + dx]} dx$$

Optimal (type 4, 66 leaves, 3 steps):

$$\frac{2e\sqrt{e\cos[c+dx]}}{ad} + \frac{2e^2\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{ad\sqrt{e\cos[c+dx]}}$$

Result (type 4, 1089 leaves):

$$\left(2(e \cos[c + dx])^{3/2} \operatorname{Sec}[c + dx]^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \right. \\ \left. \left(\frac{\cos\left[\frac{1}{2}(c + dx)\right] \sqrt{\cos[c + dx]}}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]} - \frac{\sqrt{\cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]} \right) \left(\cos[c + dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right]^2 \right) \right. \\ \left. \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}} \right) \Bigg| \\ \left(d(a + a \sin[c + dx]) \left(\frac{1}{\cos[c + dx]^{3/2}} \sin[c + dx] \left(\cos[c + dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right]^2 \right) \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \right) \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}} \right) + \right)$$

$$\frac{1}{\sqrt{\cos[c+dx]}} \left(-\sin[c+dx] - \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2}\sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}} \right.$$

$$\left. \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right.$$

$$\left. \sin\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} + \left(\sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right.$$

$$\left. \left(\frac{(-2+\sqrt{2})\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])} + \frac{(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right])\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])^2} \right) \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) /$$

$$\left(\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} + \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{2(3-2\sqrt{2})}} \sqrt{1-\frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \right) \right)$$

■ **Problem 239: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{e \cos[c+dx]} (a + a \sin[c+dx])} dx$$

Optimal (type 4, 78 leaves, 3 steps):

$$\frac{2\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3ad\sqrt{e \cos[c+dx]}} - \frac{2\sqrt{e \cos[c+dx]}}{3de(a + a \sin[c+dx])}$$

Result (type 4, 1182 leaves):

$$\frac{\cos[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \left(-\frac{2}{3} - \frac{2}{3(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right])^2} \right)}{d\sqrt{e \cos[c+dx]} (a + a \sin[c+dx])} +$$

$$\left(2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \left(\frac{\cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\cos[c+dx]}}{3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)} - \frac{\sqrt{\cos[c+dx]} \sin\left[\frac{1}{2}(c+dx)\right]}{3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)} \right) \right)$$

$$\left(\cos[c+dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) /$$

$$\left(3d\sqrt{e\cos[c+dx]}(a+a\sin[c+dx]) \left(\frac{1}{3\cos[c+dx]^{3/2}} \sin[c+dx] \left(\cos[c+dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \right) \right)$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) +$$

$$\frac{1}{3\sqrt{\cos[c+dx]}} 2 \left(-\sin[c+dx] - \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2} \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2}} \right)$$

$$\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]$$

$$\sin\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} + \left(\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]$$

$$\left(\frac{(-2 + \sqrt{2}) \sin\left[\frac{1}{2}(c + dx)\right]}{2(1 + \cos\left[\frac{1}{2}(c + dx)\right])} + \frac{(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]) \sin\left[\frac{1}{2}(c + dx)\right]}{2(1 + \cos\left[\frac{1}{2}(c + dx)\right])^2} \right) \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \Bigg/$$

$$\left(\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} + \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}}{\sqrt{2(3 - 2\sqrt{2})} \sqrt{1 - \frac{\tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}}} \right) \Bigg)$$

■ **Problem 247: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{3/2}}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 4, 83 leaves, 3 steps):

$$-\frac{2e^2 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d \sqrt{e \cos[c + dx]}} - \frac{4e \sqrt{e \cos[c + dx]}}{3d(a^2 + a^2 \sin[c + dx])}$$

Result (type 4, 1190 leaves):

$$\frac{(e \cos[c + dx])^{3/2} \sec[c + dx] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4 \left(\frac{2}{3} - \frac{4}{3(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])^2}\right)}{d(a + a \sin[c + dx])^2}$$

$$\left(2(e \cos[c + dx])^{3/2} \sec[c + dx]^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4 \right.$$

$$\left. \left(-\frac{\cos\left[\frac{1}{2}(c + dx)\right] \sqrt{\cos[c + dx]}}{3(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} + \frac{\sqrt{\cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{3(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} \right) \left(\cos[c + dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right]^2 \right)^2 \right.$$

$$\left. \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \right) \Bigg/$$

$$\begin{aligned}
& \left(3 d (a + a \sin[c + d x])^2 \left(-\frac{1}{3 \cos[c + d x]^{3/2}} \sin[c + d x] \left(\cos[c + d x] + 2 \sqrt{2} \cos\left[\frac{1}{4}(c + d x)\right]^2 \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]}{1 + \cos\left[\frac{1}{2}(c + d x)\right]}} \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + d x)\right]^2} \right) - \right. \right. \\
& \left. \frac{1}{3 \sqrt{\cos[c + d x]}} \left(2 \left(-\sin[c + d x] - \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]}{1 + \cos\left[\frac{1}{2}(c + d x)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \tan\left[\frac{1}{4}(c + d x)\right]}{\sqrt{2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + d x)\right]^2}} \right. \right. \right. \\
& \left. \sqrt{2} \cos\left[\frac{1}{4}(c + d x)\right] \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]}{1 + \cos\left[\frac{1}{2}(c + d x)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \left. \sin\left[\frac{1}{4}(c + d x)\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + d x)\right]^2} + \left(\sqrt{2} \cos\left[\frac{1}{4}(c + d x)\right]^2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \right. \\
& \left. \left. \left(\frac{(-2 + \sqrt{2}) \sin\left[\frac{1}{2}(c + d x)\right]}{2(1 + \cos\left[\frac{1}{2}(c + d x)\right])} + \frac{(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]) \sin\left[\frac{1}{2}(c + d x)\right]}{2(1 + \cos\left[\frac{1}{2}(c + d x)\right])^2} \right) \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + d x)\right]^2} \right) / \right. \\
& \left. \left. \left(\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]}{1 + \cos\left[\frac{1}{2}(c + d x)\right]}} \right) + \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]}{1 + \cos\left[\frac{1}{2}(c + d x)\right]}} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + d x)\right]^2}}{\sqrt{2(3 - 2\sqrt{2})} \sqrt{1 - \frac{\tan\left[\frac{1}{4}(c + d x)\right]^2}{3 - 2\sqrt{2}}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \tan\left[\frac{1}{4}(c + d x)\right]^2}{3 - 2\sqrt{2}}}} \right) \right) \right)
\end{aligned}$$

■ **Problem 249: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{e \cos[c + d x]} (a + a \sin[c + d x])^2} dx$$

Optimal (type 4, 116 leaves, 4 steps):

$$\frac{2\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{7a^2d\sqrt{e\cos[c+dx]}} - \frac{2\sqrt{e\cos[c+dx]}}{7de(a+a\sin[c+dx])^2} - \frac{2\sqrt{e\cos[c+dx]}}{7de(a^2+a^2\sin[c+dx])}$$

Result (type 4, 1209 leaves):

$$\left(\cos[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 \left(-\frac{2}{7} - \frac{2}{7\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{2}{7\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right) \right) /$$

$$\left(d\sqrt{e\cos[c+dx]} (a+a\sin[c+dx])^2 \right) +$$

$$\left(2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 \left(\frac{\cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\cos[c+dx]}}{7\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} - \frac{\sqrt{\cos[c+dx]} \sin\left[\frac{1}{2}(c+dx)\right]}{7\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} \right) \right)$$

$$\left(\cos[c+dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \sqrt{\frac{-1+\sqrt{2} - (-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}}$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) /$$

$$\left(7d\sqrt{e\cos[c+dx]} (a+a\sin[c+dx])^2 \left(\frac{1}{7\cos[c+dx]^{3/2}} \sin[c+dx] \left(\cos[c+dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \right. \right.$$

$$\left. \left. \sqrt{\frac{-1+\sqrt{2} - (-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) \right) +$$

$$\begin{aligned}
& \frac{1}{7 \sqrt{\cos[c+dx]}} \left(-\sin[c+dx] - \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2}\sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}} \right) \\
& \sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \\
& \sin\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} + \left(\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \\
& \left(\frac{(-2+\sqrt{2})\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])} + \frac{(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right])\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])^2} \right) \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \Big/ \\
& \left(\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \right) + \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{2(3-2\sqrt{2})} \sqrt{1-\frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}} \right)
\end{aligned}$$

■ **Problem 259: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^{3/2}}{(a+a \sin[c+dx])^3} dx$$

Optimal (type 4, 118 leaves, 4 steps):

$$-\frac{2e^2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21a^3 d \sqrt{e \cos[c+dx]}} - \frac{4e \sqrt{e \cos[c+dx]}}{7ad(a+a \sin[c+dx])^2} + \frac{2e \sqrt{e \cos[c+dx]}}{21d(a^3+a^3 \sin[c+dx])}$$

Result (type 4, 1217 leaves):

$$\frac{1}{d(a+a \sin[c+dx])^3} (e \cos[c+dx])^{3/2} \operatorname{Sec}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 \\
\left(\frac{2}{21} - \frac{4}{7(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right])^4} + \frac{2}{21(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right])^2} \right) -$$

$$\begin{aligned}
& \left(2 (e \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right)^6 \\
& \left(-\frac{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \sqrt{\operatorname{Cos}[c + d x]}}{21 (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])} + \frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{21 (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])} \right) \left(\operatorname{Cos}[c + d x] + 2\sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right]^2 \right. \\
& \left. \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]^2} \right) \Big/ \\
& \left(21 d (a + a \operatorname{Sin}[c + d x])^3 \left(-\frac{1}{21 \operatorname{Cos}[c + d x]^{3/2}} \operatorname{Sin}[c + d x] \left(\operatorname{Cos}[c + d x] + 2\sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right]^2 \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \right. \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]^2} \right) - \right. \\
& \left. \frac{1}{21 \sqrt{\operatorname{Cos}[c + d x]}} \left(-\operatorname{Sin}[c + d x] - \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{2} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]^2}} \right) - \right. \\
& \left. \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]^2} + \left(\sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \right)
\end{aligned}$$

$$\left(\frac{(-2 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{2(1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right])} + \frac{(-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]) \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{2(1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right])^2} \right) \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \Bigg/$$

$$\left(\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}} + \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}}{\sqrt{2(3 - 2\sqrt{2})} \sqrt{1 - \frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}}} \right) \Bigg)$$

■ **Problem 261: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{e \operatorname{Cos}[c + dx]} (a + a \operatorname{Sin}[c + dx])^3} dx$$

Optimal (type 4, 153 leaves, 5 steps):

$$\frac{10 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{77 a^3 d \sqrt{e \operatorname{Cos}[c + dx]}} - \frac{2 \sqrt{e \operatorname{Cos}[c + dx]}}{11 d e (a + a \operatorname{Sin}[c + dx])^3} - \frac{10 \sqrt{e \operatorname{Cos}[c + dx]}}{77 a d e (a + a \operatorname{Sin}[c + dx])^2} - \frac{10 \sqrt{e \operatorname{Cos}[c + dx]}}{77 d e (a^3 + a^3 \operatorname{Sin}[c + dx])}$$

Result (type 4, 1236 leaves):

$$\left(\operatorname{Cos}[c + dx] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right)^6$$

$$\left(-\frac{10}{77} - \frac{2}{11 (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])^6} - \frac{10}{77 (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])^4} - \frac{10}{77 (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])^2} \right) \Bigg/$$

$$\left(d \sqrt{e \operatorname{Cos}[c + dx]} (a + a \operatorname{Sin}[c + dx])^3 + 10 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6 \right)$$

$$\left(\frac{5 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \sqrt{\operatorname{Cos}[c + dx]}}{77 (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])} - \frac{5 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{77 (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])} \right) \left(\operatorname{Cos}[c + dx] + 2\sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right]^2 \right)$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \Bigg/$$

$$\begin{aligned}
& \left(77 d \sqrt{e \cos [c+d x]} (a+a \sin [c+d x])^3 \left(\frac{1}{77 \cos [c+d x]^{3/2}} 5 \sin [c+d x] \left(\cos [c+d x]+2 \sqrt{2} \cos \left[\frac{1}{4}(c+d x) \right]^2 \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x) \right]}{1+\cos \left[\frac{1}{2}(c+d x) \right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x) \right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x) \right]^2} \right) \right. \\
& \left. \left. \left. \frac{1}{77 \sqrt{\cos [c+d x]}} 10 \left(-\sin [c+d x]-\frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x) \right]}{1+\cos \left[\frac{1}{2}(c+d x) \right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x) \right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \tan \left[\frac{1}{4}(c+d x) \right]}{\sqrt{2} \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x) \right]^2}} \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \cos \left[\frac{1}{4}(c+d x) \right] \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x) \right]}{1+\cos \left[\frac{1}{2}(c+d x) \right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x) \right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \right. \right. \\
& \left. \left. \sin \left[\frac{1}{4}(c+d x) \right] \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x) \right]^2} + \left(\sqrt{2} \cos \left[\frac{1}{4}(c+d x) \right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x) \right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \right. \right. \right. \\
& \left. \left. \left. \left(\frac{(-2+\sqrt{2}) \sin \left[\frac{1}{2}(c+d x) \right]}{2\left(1+\cos \left[\frac{1}{2}(c+d x) \right]\right)} + \frac{(-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x) \right]) \sin \left[\frac{1}{2}(c+d x) \right]}{2\left(1+\cos \left[\frac{1}{2}(c+d x) \right]\right)^2} \right) \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x) \right]^2} \right) \right) \right) \\
& \left. \left. \left. \left(\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x) \right]}{1+\cos \left[\frac{1}{2}(c+d x) \right]}} \right) + \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x) \right]}{1+\cos \left[\frac{1}{2}(c+d x) \right]}} \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x) \right]^2}}{\sqrt{2\left(3-2 \sqrt{2}\right)} \sqrt{1-\frac{\tan \left[\frac{1}{4}(c+d x) \right]^2}{3-2 \sqrt{2}}}} \sqrt{1-\frac{(17-12 \sqrt{2}) \tan \left[\frac{1}{4}(c+d x) \right]^2}{3-2 \sqrt{2}}}} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 267: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos [c+d x])^{7/2}}{(a+a \sin [c+d x])^4} dx$$

Optimal (type 4, 120 leaves, 4 steps) :

$$\frac{10 e^4 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21 a^4 d \sqrt{e \cos[c+dx]}} - \frac{4 e (e \cos[c+dx])^{5/2}}{7 a d (a + a \sin[c+dx])^3} + \frac{20 e^3 \sqrt{e \cos[c+dx]}}{21 d (a^4 + a^4 \sin[c+dx])}$$

Result (type 4, 1219 leaves) :

$$\frac{1}{d (a + a \sin[c+dx])^4} (e \cos[c+dx])^{7/2} \sec[c+dx]^3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8$$

$$\left(-\frac{10}{21} - \frac{8}{7 (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^4} + \frac{32}{21 (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^2} \right) +$$

$$\left(10 (e \cos[c+dx])^{7/2} \sec[c+dx]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 \right.$$

$$\left. \left(\frac{5 \cos[\frac{1}{2}(c+dx)] \sqrt{\cos[c+dx]}}{21 (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])} - \frac{5 \sqrt{\cos[c+dx]} \sin[\frac{1}{2}(c+dx)]}{21 (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])} \right) \left(\cos[c+dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \right.$$

$$\left. \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos[\frac{1}{2}(c+dx)]}{1 + \cos[\frac{1}{2}(c+dx)]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan[\frac{1}{4}(c+dx)]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) \Bigg/$$

$$\left(21 d (a + a \sin[c+dx])^4 \left(\frac{1}{21 \cos[c+dx]^{3/2}} 5 \sin[c+dx] \left(\cos[c+dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos[\frac{1}{2}(c+dx)]}{1 + \cos[\frac{1}{2}(c+dx)]}} \right. \right.$$

$$\left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan[\frac{1}{4}(c+dx)]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) \right) +$$

$$\frac{1}{21 \sqrt{\cos[c+dx]}} 10 \left(-\sin[c+dx] - \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2} \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2}} \right. -$$

$$\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]$$

$$\sin\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} + \left(\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] \right)^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]$$

$$\left. \left(\frac{(-2+\sqrt{2})\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])} + \frac{(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right])\sin\left[\frac{1}{2}(c+dx)\right]}{2(1+\cos\left[\frac{1}{2}(c+dx)\right])^2} \right) \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) /$$

$$\left(\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} + \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{2(3-2\sqrt{2})} \sqrt{1-\frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}} \right) \right)$$

■ **Problem 269: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^{3/2}}{(a+a \sin[c+dx])^4} dx$$

Optimal (type 4, 154 leaves, 5 steps):

$$-\frac{2e^2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{77a^4 d \sqrt{e \cos[c+dx]}} - \frac{4e \sqrt{e \cos[c+dx]}}{11ad(a+a \sin[c+dx])^3} + \frac{2e \sqrt{e \cos[c+dx]}}{77d(a^2+a^2 \sin[c+dx])^2} + \frac{2e \sqrt{e \cos[c+dx]}}{77d(a^4+a^4 \sin[c+dx])}$$

Result (type 4, 1244 leaves):

$$\frac{1}{d(a+a \sin[c+dx])^4} (e \cos[c+dx])^{3/2} \operatorname{Sec}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8$$

$$\left(\frac{2}{77} - \frac{4}{11(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right])^6} + \frac{2}{77(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right])^4} + \frac{2}{77(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right])^2} \right) -$$

$$\begin{aligned}
& \left(2 (e \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right)^8 \\
& \left(-\frac{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \sqrt{\operatorname{Cos}[c + d x]}}{77 (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])} + \frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{77 (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])} \right) \left(\operatorname{Cos}[c + d x] + 2\sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right]^2 \right. \\
& \left. \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]^2} \right) \Big/ \\
& \left(77 d (a + a \operatorname{Sin}[c + d x])^4 \left(-\frac{1}{77 \operatorname{Cos}[c + d x]^{3/2}} \operatorname{Sin}[c + d x] \left(\operatorname{Cos}[c + d x] + 2\sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right]^2 \right) \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]^2} \right) - \right. \\
& \left. \frac{1}{77 \sqrt{\operatorname{Cos}[c + d x]}} 2 \left(-\operatorname{Sin}[c + d x] - \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{2} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]^2}} \right) - \right. \\
& \left. \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]^2} + \left(\sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \right)
\end{aligned}$$

$$\left(\frac{(-2 + \sqrt{2}) \sin\left[\frac{1}{2}(c + dx)\right]}{2(1 + \cos\left[\frac{1}{2}(c + dx)\right])} + \frac{(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]) \sin\left[\frac{1}{2}(c + dx)\right]}{2(1 + \cos\left[\frac{1}{2}(c + dx)\right])^2} \right) \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \Bigg/$$

$$\left(\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} + \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}}{\sqrt{2(3 - 2\sqrt{2})} \sqrt{1 - \frac{\tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2}{3 - 2\sqrt{2}}}} \right) \Bigg)$$

■ **Problem 271: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{e \cos[c + dx]} (a + a \sin[c + dx])^4} dx$$

Optimal (type 4, 191 leaves, 6 steps):

$$\frac{2\sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{33 a^4 d \sqrt{e \cos[c + dx]}} - \frac{2\sqrt{e \cos[c + dx]}}{15 d e (a + a \sin[c + dx])^4} -$$

$$\frac{14\sqrt{e \cos[c + dx]}}{165 a d e (a + a \sin[c + dx])^3} - \frac{2\sqrt{e \cos[c + dx]}}{33 d e (a^2 + a^2 \sin[c + dx])^2} - \frac{2\sqrt{e \cos[c + dx]}}{33 d e (a^4 + a^4 \sin[c + dx])}$$

Result (type 4, 1263 leaves):

$$\left(\cos[c + dx] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)^8 \left(-\frac{2}{33} - \frac{2}{15 (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])^8} - \frac{14}{165 (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])^6} - \right.$$

$$\left. \frac{2}{33 (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])^4} - \frac{2}{33 (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])^2} \right) \Bigg/ \left(d \sqrt{e \cos[c + dx]} (a + a \sin[c + dx])^4 \right) +$$

$$\left(2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)^8 \left(\frac{\cos\left[\frac{1}{2}(c + dx)\right] \sqrt{\cos[c + dx]}}{33 (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} - \frac{\sqrt{\cos[c + dx]} \sin\left[\frac{1}{2}(c + dx)\right]}{33 (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} \right)$$

$$\left(\cos[c + dx] + 2\sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] \right)^2 \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}}$$

$$\begin{aligned}
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right) \Big/ \\
& \left(33 d \sqrt{e \text{Cos}[c+dx]} (a+a \text{Sin}[c+dx])^4 \left(\frac{1}{33 \text{Cos}[c+dx]^{3/2}} \text{Sin}[c+dx] \left(\text{Cos}[c+dx] + 2\sqrt{2} \text{Cos}\left[\frac{1}{4}(c+dx)\right]^2 \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \text{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\text{Cos}\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) + \right. \right. \\
& \left. \left. \frac{1}{33 \sqrt{\text{Cos}[c+dx]}} 2 \left(-\text{Sin}[c+dx] - \frac{\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \text{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\text{Cos}\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2} \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2}} \right) - \right. \right. \\
& \left. \left. \sqrt{2} \text{Cos}\left[\frac{1}{4}(c+dx)\right] \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \text{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\text{Cos}\left[\frac{1}{2}(c+dx)\right]}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \right. \\
& \left. \left. \text{Sin}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \left(\sqrt{2} \text{Cos}\left[\frac{1}{4}(c+dx)\right]^2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \right. \right. \\
& \left. \left. \left. \left(\frac{(-2+\sqrt{2}) \text{Sin}\left[\frac{1}{2}(c+dx)\right]}{2(1+\text{Cos}\left[\frac{1}{2}(c+dx)\right])} + \frac{(-1+\sqrt{2}-(-2+\sqrt{2}) \text{Cos}\left[\frac{1}{2}(c+dx)\right]) \text{Sin}\left[\frac{1}{2}(c+dx)\right]}{2(1+\text{Cos}\left[\frac{1}{2}(c+dx)\right])^2} \right) \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) \Big/ \right. \right.
\end{aligned}$$

$$\left(\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} + \frac{\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{2(3 - 2\sqrt{2})} \sqrt{1 - \frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3 - 2\sqrt{2}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2}{3 - 2\sqrt{2}}}} \right)$$

■ **Problem 273: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e \cos[c+dx])^{3/2} \sqrt{a + a \sin[c+dx]} \, dx$$

Optimal (type 3, 236 leaves, 8 steps):

$$-\frac{a (e \cos[c+dx])^{5/2}}{2 d e \sqrt{a + a \sin[c+dx]}} + \frac{3 e \sqrt{e \cos[c+dx]} \sqrt{a + a \sin[c+dx]}}{4 d} - \frac{3 e^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c+dx]}}{\sqrt{e}}\right] \sqrt{1 + \cos[c+dx]} \sqrt{a + a \sin[c+dx]}}{4 d (1 + \cos[c+dx] + \sin[c+dx])} + \frac{3 e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c+dx]}{\sqrt{e \cos[c+dx]} \sqrt{1 + \cos[c+dx]}}\right] \sqrt{1 + \cos[c+dx]} \sqrt{a + a \sin[c+dx]}}{4 d (1 + \cos[c+dx] + \sin[c+dx])}$$

Result (type 3, 269 leaves):

$$-\left(i e^{-i(c+dx)} \sqrt{e \cos[c+dx]} \right. \\ \left. - i \sqrt{1 + e^{2i(c+dx)}} - 2 e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} + 2 i e^{2i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} + e^{3i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} - 3 d e^{2i(c+dx)} x + \right. \\ \left. 3 e^{2i(c+dx)} \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - 3 i e^{2i(c+dx)} \operatorname{Log}\left[1 + \sqrt{1 + e^{2i(c+dx)}}\right] \right) \sqrt{a(1 + \sin[c+dx])} / \left(4 d (i + e^{i(c+dx)}) \sqrt{1 + e^{2i(c+dx)}} \right)$$

■ **Problem 274: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{e \cos[c+dx]} \sqrt{a + a \sin[c+dx]} \, dx$$

Optimal (type 3, 194 leaves, 7 steps):

$$-\frac{a (e \cos[c+dx])^{3/2}}{d e \sqrt{a + a \sin[c+dx]}} + \frac{\sqrt{e} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c+dx]}}{\sqrt{e}}\right] \sqrt{1 + \cos[c+dx]} \sqrt{a + a \sin[c+dx]}}{d (1 + \cos[c+dx] + \sin[c+dx])} + \frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c+dx]}{\sqrt{e \cos[c+dx]} \sqrt{1 + \cos[c+dx]}}\right] \sqrt{1 + \cos[c+dx]} \sqrt{a + a \sin[c+dx]}}{d (1 + \cos[c+dx] + \sin[c+dx])}$$

Result (type 3, 195 leaves):

$$- \left(i \sqrt{e \cos[c+dx]} \left(-i \sqrt{1+e^{2i(c+dx)}} + e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} + i d e^{i(c+dx)} x + i e^{i(c+dx)} \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] - e^{i(c+dx)} \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] \right) \right. \\ \left. \sqrt{a(1+\sin[c+dx])} \right) / \left(d \left(i + e^{i(c+dx)} \right) \sqrt{1+e^{2i(c+dx)}} \right)$$

- **Problem 275: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+a \sin[c+dx]}}{\sqrt{e \cos[c+dx]}} dx$$

Optimal (type 3, 161 leaves, 6 steps):

$$- \frac{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c+dx]}}{\sqrt{e}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{d \sqrt{e} (1+\cos[c+dx]+\sin[c+dx])} + \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c+dx]}{\sqrt{e \cos[c+dx]} \sqrt{1+\cos[c+dx]}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{d \sqrt{e} (1+\cos[c+dx]+\sin[c+dx])}$$

Result (type 3, 108 leaves):

$$\frac{\sqrt{1+e^{2i(c+dx)}} \left(dx - \operatorname{ArcSinh}\left[e^{i(c+dx)}\right] + i \operatorname{Log}\left[1+\sqrt{1+e^{2i(c+dx)}}\right] \right) \sqrt{a(1+\sin[c+dx])}}{d (1-i e^{i(c+dx)}) \sqrt{e \cos[c+dx]}}$$

- **Problem 280: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (e \cos[c+dx])^{5/2} (a+a \sin[c+dx])^{3/2} dx$$

Optimal (type 3, 319 leaves, 10 steps):

$$- \frac{15 a^3 (e \cos[c+dx])^{7/2}}{32 d e (a+a \sin[c+dx])^{3/2}} + \frac{15 a^2 e (e \cos[c+dx])^{3/2}}{64 d \sqrt{a+a \sin[c+dx]}} - \frac{3 a^2 (e \cos[c+dx])^{7/2}}{8 d e \sqrt{a+a \sin[c+dx]}} - \\ \frac{a (e \cos[c+dx])^{7/2} \sqrt{a+a \sin[c+dx]}}{4 d e} + \frac{45 a e^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c+dx]}}{\sqrt{e}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{64 d (1+\cos[c+dx]+\sin[c+dx])} + \\ \frac{45 a e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c+dx]}{\sqrt{e \cos[c+dx]} \sqrt{1+\cos[c+dx]}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{64 d (1+\cos[c+dx]+\sin[c+dx])}$$

Result (type 4, 2816 leaves):

$$\left((e \cos[c+dx])^{5/2} \operatorname{Sec}[c+dx]^2 (a(1+\sin[c+dx]))^{3/2} \left(-\frac{3}{4} \cos\left[\frac{1}{2}(c+dx)\right] + \frac{3}{64} \cos\left[\frac{3}{2}(c+dx)\right] - \frac{1}{8} \cos\left[\frac{5}{2}(c+dx)\right] - \frac{1}{32} \cos\left[\frac{7}{2}(c+dx)\right] \right) + \right. \\ \left. \frac{3}{4} \sin\left[\frac{1}{2}(c+dx)\right] + \frac{3}{64} \sin\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8} \sin\left[\frac{5}{2}(c+dx)\right] - \frac{1}{32} \sin\left[\frac{7}{2}(c+dx)\right] \right) / \left(d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \right) +$$

$$\begin{aligned}
& \left(45 \sqrt{3-2\sqrt{2}} (e \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sec}[c+dx]^2 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2\right) \right. \\
& \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \left. \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \left(2-2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\sqrt{2}\sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4}\right) \right. \\
& \left. \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right. \right. \\
& \left. \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \\
& \left. \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \right. \\
& \left. \sqrt{2} \left(\operatorname{Log}\left[1+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2\right] - \operatorname{Log}\left[2-2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\sqrt{2}\sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4}\right] \right) \right. \\
& \left. \left. \left(1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4\right) \right) \right) / \\
& \left(128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right)^4 \left(4 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 - 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 - 52 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \right. \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 200 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 - 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 - \\
& 200 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 + 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 + 52 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 - \\
& \left. 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 - 4 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} + 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12 \sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2 \sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 32\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 56\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} -
\end{aligned}$$

$$\begin{aligned}
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \left. \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \right)
\end{aligned}$$

- **Problem 281: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (e \cos[c+dx])^{3/2} (a+a \sin[c+dx])^{3/2} dx$$

Optimal (type 3, 278 leaves, 9 steps):

$$\begin{aligned}
& -\frac{7 a^2 (e \cos [c+d x])^{5 / 2}}{12 d e \sqrt{a+a \sin [c+d x]}} + \frac{7 a e \sqrt{e \cos [c+d x]} \sqrt{a+a \sin [c+d x]}}{8 d} - \\
& \frac{a (e \cos [c+d x])^{5 / 2} \sqrt{a+a \sin [c+d x]}}{3 d e} - \frac{7 a e^{3 / 2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos [c+d x]}}{\sqrt{e}}\right] \sqrt{1+\cos [c+d x]} \sqrt{a+a \sin [c+d x]}}{8 d (1+\cos [c+d x]+\sin [c+d x])} + \\
& \frac{7 a e^{3 / 2} \operatorname{ArcTan}\left[\frac{\sqrt{e \sin [c+d x]}}{\sqrt{e \cos [c+d x]} \sqrt{1+\cos [c+d x]}}\right] \sqrt{1+\cos [c+d x]} \sqrt{a+a \sin [c+d x]}}{8 d (1+\cos [c+d x]+\sin [c+d x])}
\end{aligned}$$

Result (type 4, 2810 leaves):

$$\begin{aligned}
& \left((e \cos[c + dx])^{3/2} \sec[c + dx] (a(1 + \sin[c + dx]))^{3/2} \right. \\
& \quad \left. \left(\frac{5}{12} \cos\left[\frac{1}{2}(c + dx)\right] - \frac{3}{8} \cos\left[\frac{3}{2}(c + dx)\right] - \frac{1}{12} \cos\left[\frac{5}{2}(c + dx)\right] + \frac{5}{12} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{3}{8} \sin\left[\frac{3}{2}(c + dx)\right] - \frac{1}{12} \sin\left[\frac{5}{2}(c + dx)\right] \right) \right) / \\
& \quad \left(d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)^3 - \\
& \quad \left(7 \sqrt{3 - 2\sqrt{2}} (e \cos[c + dx])^{3/2} \sec[c + dx] (a(1 + \sin[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \right. \\
& \quad \left(1 + \tan\left[\frac{1}{4}(c + dx)\right] \right)^2 \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \right. \\
& \quad \left. \sqrt{1 - 3 \tan\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c + dx)\right]^2} \left(2 - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4} \right) \right) \\
& \quad \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} \right. \\
& \quad \left. \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \quad \left. \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4} - \right. \\
& \quad \left. \sqrt{2} \left(\log\left[1 + \tan\left[\frac{1}{4}(c + dx)\right]^2\right] - \log\left[2 - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4}\right] \right) \right) \\
& \quad \left. \left(1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4 \right) \right) \right) / \\
& \quad \left(16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)^3
\end{aligned}$$

$$\begin{aligned}
& \left(-4 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 + 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 + 52 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \right)^2 - \\
& 200 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 + 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 + 200 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 - \\
& 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 - 52 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 + \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 + 4 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} - 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} + \\
& 2\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} +
\end{aligned}$$

$$\begin{aligned}
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 56 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \left. \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 282: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{e \cos [c+dx]} (a+a \sin [c+dx])^{3/2} dx$$

Optimal (type 3, 243 leaves, 8 steps):

$$\begin{aligned}
& - \frac{5 a^2 (e \operatorname{Cos}[c+d x])^{3/2}}{4 d e \sqrt{a+a \operatorname{Sin}[c+d x]}} - \frac{a (e \operatorname{Cos}[c+d x])^{3/2} \sqrt{a+a \operatorname{Sin}[c+d x]}}{2 d e} + \frac{5 a \sqrt{e} \operatorname{ArcSinh}\left[\frac{\sqrt{e \operatorname{Cos}[c+d x]}}{\sqrt{e}}\right] \sqrt{1+\operatorname{Cos}[c+d x]} \sqrt{a+a \operatorname{Sin}[c+d x]}}{4 d (1+\operatorname{Cos}[c+d x]+\operatorname{Sin}[c+d x])} + \\
& \frac{5 a \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} \operatorname{Sin}[c+d x]}{\sqrt{e \operatorname{Cos}[c+d x]} \sqrt{1+\operatorname{Cos}[c+d x]}}\right] \sqrt{1+\operatorname{Cos}[c+d x]} \sqrt{a+a \operatorname{Sin}[c+d x]}}{4 d (1+\operatorname{Cos}[c+d x]+\operatorname{Sin}[c+d x])}
\end{aligned}$$

Result (type 4, 2322 leaves):

$$\begin{aligned}
& \left(\sqrt{e \operatorname{Cos}[c+d x]} (a (1+\operatorname{Sin}[c+d x]))^{3/2} \left(-\frac{3}{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \frac{1}{4} \operatorname{Cos}\left[\frac{3}{2}(c+d x)\right] + \frac{3}{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] - \frac{1}{4} \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] \right) \right) / \\
& \left(d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] \right)^3 \right) - \left(25 \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^2 \sqrt{e \operatorname{Cos}[c+d x]} (a (1+\operatorname{Sin}[c+d x]))^{3/2} \right. \\
& \left. \left(\sqrt{2} \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[2+\sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] \right) \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} + \right. \right. \\
& \left. \left. 4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2} \sqrt{1+(-3+2\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2} + 8 \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2} \sqrt{1+(-3+2\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2} \right) \right) / \\
& \left(64 d \sqrt{\operatorname{Cos}[c+d x]} \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] \right)^4 \left(\frac{1}{16 \sqrt{\operatorname{Cos}[c+d x]}} 5 \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right] \right. \right. \\
& \left. \left. \left(\sqrt{2} \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[2+\sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] \right) \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} + \right. \right. \right. \\
& \left. \left. \left. 4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2} \sqrt{1+(-3+2\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2} + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 8 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \frac{1}{16\operatorname{Cos}[c+dx]^{3/2}} 5\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sin}[c+dx] \\
& \left(\sqrt{2}\left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\right] - \operatorname{Log}\left[2+\sqrt{2}\sqrt{\operatorname{Cos}[c+dx]\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4-2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right]\right)\sqrt{\operatorname{Cos}[c+dx]\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4} + \right. \\
& 4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 8 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \\
& \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& \frac{1}{8\sqrt{\operatorname{Cos}[c+dx]}} 5\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^2 \left(\left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\right] - \operatorname{Log}\left[2+\sqrt{2}\sqrt{\operatorname{Cos}[c+dx]\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4-2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right]\right) \right. \\
& \left. \left(-\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 \operatorname{Sin}[c+dx] + \operatorname{Cos}[c+dx]\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right) / \left(\sqrt{2}\sqrt{\operatorname{Cos}[c+dx]\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4}\right) + \\
& \left((-3+2\sqrt{2})\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \right. \\
& \left.\sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right) / \left(\sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right) +
\end{aligned}$$

$$\begin{aligned}
& \left(2 \left(-3 + 2\sqrt{2} \right) \text{EllipticPi} \left[-3 + 2\sqrt{2}, -\text{ArcSin} \left[\frac{\text{Tan} \left[\frac{1}{4} (c + dx) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \right. \\
& \quad \left. \text{Tan} \left[\frac{1}{4} (c + dx) \right] \sqrt{3 - 2\sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right) / \left(\sqrt{1 + \left(-3 + 2\sqrt{2} \right) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right) - \\
& \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\text{Tan} \left[\frac{1}{4} (c + dx) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \text{Tan} \left[\frac{1}{4} (c + dx) \right] \sqrt{1 + \left(-3 + 2\sqrt{2} \right) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right) / \\
& \left(\sqrt{3 - 2\sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right) - \left(2 \text{EllipticPi} \left[-3 + 2\sqrt{2}, -\text{ArcSin} \left[\frac{\text{Tan} \left[\frac{1}{4} (c + dx) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \right. \\
& \quad \left. \text{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \text{Tan} \left[\frac{1}{4} (c + dx) \right] \sqrt{1 + \left(-3 + 2\sqrt{2} \right) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right) / \left(\sqrt{3 - 2\sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right) + \\
& \frac{\text{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \sqrt{3 - 2\sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \sqrt{1 + \left(-3 + 2\sqrt{2} \right) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2}}{\sqrt{3 - 2\sqrt{2}} \sqrt{1 - \frac{\text{Tan} \left[\frac{1}{4} (c + dx) \right]^2}{3 - 2\sqrt{2}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2}{3 - 2\sqrt{2}}}} \\
& \frac{2 \text{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \sqrt{3 - 2\sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \sqrt{1 + \left(-3 + 2\sqrt{2} \right) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2}}{\sqrt{3 - 2\sqrt{2}} \sqrt{1 - \frac{\text{Tan} \left[\frac{1}{4} (c + dx) \right]^2}{3 - 2\sqrt{2}}} \sqrt{1 - \frac{(17 - 12\sqrt{2}) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2}{3 - 2\sqrt{2}}} \left(1 - \frac{(-3 + 2\sqrt{2}) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2}{3 - 2\sqrt{2}} \right)} + \\
& \sqrt{2} \sqrt{\text{Cos} [c + dx] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^4} \left(\frac{1}{2} \text{Tan} \left[\frac{1}{4} (c + dx) \right] - \right.
\end{aligned}$$

$$\left(-\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] + \frac{-\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 \operatorname{Sin}[c+dx] + \operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4}} \right) /$$

$$\left(2 + \sqrt{2} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 - 2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) \Bigg|$$

- **Problem 283: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[c + dx])^{3/2}}{\sqrt{e \operatorname{Cos}[c + dx]}} dx$$

Optimal (type 3, 198 leaves, 7 steps):

$$\frac{a \sqrt{e \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]} - 3 a \operatorname{ArcSinh}\left[\frac{\sqrt{e \operatorname{Cos}[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]}}{d e} +$$

$$\frac{3 a \operatorname{ArcTan}\left[\frac{\sqrt{e} \operatorname{Sin}[c + dx]}{\sqrt{e \operatorname{Cos}[c + dx]} \sqrt{1 + \operatorname{Cos}[c + dx]}}\right] \sqrt{1 + \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]}}{d \sqrt{e} (1 + \operatorname{Cos}[c + dx] + \operatorname{Sin}[c + dx])}$$

Result (type 4, 2750 leaves):

$$\frac{\operatorname{Cos}[c + dx] \left(-\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right) (a (1 + \operatorname{Sin}[c + dx]))^{3/2}}{d \sqrt{e \operatorname{Cos}[c + dx]} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}$$

$$\left(3 \sqrt{3 - 2\sqrt{2}} \operatorname{Cos}[c + dx] (a (1 + \operatorname{Sin}[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]\right)^2 \right)$$

$$\sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}}$$

$$\sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \left(2 - 2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} \right)$$

$$\begin{aligned}
& \left(4 \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \sqrt{3 - 2 \sqrt{2} - \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2} \sqrt{1 + (-3 + 2 \sqrt{2}) \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2} \right. \\
& \sqrt{1 - 6 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 + \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^4} + 8 \operatorname{EllipticPi} \left[-3 + 2 \sqrt{2}, -\operatorname{ArcSin} \left[\frac{\operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \\
& \sqrt{3 - 2 \sqrt{2} - \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2} \sqrt{1 + (-3 + 2 \sqrt{2}) \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2} \sqrt{1 - 6 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 + \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^4} - \\
& \left. \sqrt{2} \left(\operatorname{Log} \left[1 + \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 \right] - \operatorname{Log} \left[2 - 2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 \right] + \sqrt{2} \sqrt{1 - 6 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 + \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^4} \right) \right) \\
& \left. \left(1 - 6 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 + \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^4 \right) \right) \Bigg) / \\
& \left(2 d \sqrt{e \operatorname{Cos}[c + d x]} \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right) \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right) \right)^3 \\
& \left(-4 \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 + 3 \sqrt{2} \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 + 52 \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 - \right. \\
& 39 \sqrt{2} \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 - 200 \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^4 + 150 \sqrt{2} \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^4 + \\
& 200 \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^6 - 150 \sqrt{2} \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^6 - 52 \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^8 + \\
& 39 \sqrt{2} \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^8 + 4 \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^{10} - 3 \sqrt{2} \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^{10} + \\
& 2 \sqrt{2 (3 - 2 \sqrt{2})} \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right] \sqrt{3 - 2 \sqrt{2} - \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2} \sqrt{\frac{-3 + 2 \sqrt{2} + \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2}{-3 + 2 \sqrt{2}}} \\
& \left. \sqrt{\frac{-3 + 2 \sqrt{2} + 17 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 - 12 \sqrt{2} \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2}{-3 + 2 \sqrt{2}}} \sqrt{1 - 3 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2 + 2 \sqrt{2} \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2} - \right.
\end{aligned}$$

$$\begin{aligned}
& 4 \sqrt{3 - 2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \\
& \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} - 4 \sqrt{3 - 2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^3 \\
& \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \left. \left. \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} \right) \right)
\end{aligned}$$

- **Problem 284: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[c + dx])^{3/2}}{(e \cos[c + dx])^{3/2}} dx$$

Optimal (type 3, 210 leaves, 7 steps):

$$\begin{aligned}
& \frac{4 a \sqrt{a + a \sin[c + dx]}}{d e \sqrt{e \cos[c + dx]}} - \frac{2 a^2 \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{d e^{3/2} (a + a \cos[c + dx] + a \sin[c + dx])} - \\
& \frac{2 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{e \sin[c + dx]}}{\sqrt{e \cos[c + dx]} \sqrt{1 + \cos[c + dx]}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{d e^{3/2} (a + a \cos[c + dx] + a \sin[c + dx])}
\end{aligned}$$

Result (type 4, 2727 leaves):

$$\begin{aligned}
& \frac{4 \cos[c + dx]^2 (a (1 + \sin[c + dx]))^{3/2}}{d (e \cos[c + dx])^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} - \\
& \left(\sqrt{3 - 2\sqrt{2}} \cos[c + dx]^2 (a (1 + \sin[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{1 - 3 \tan\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c + dx)\right]^2} \left(2 - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4} \right) \\
& \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} \right. \\
& \left. \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \left. \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4} + \right. \\
& \left. \sqrt{2} \left(\operatorname{Log}\left[1 + \tan\left[\frac{1}{4}(c + dx)\right]^2\right] - \operatorname{Log}\left[2 - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4}\right] \right) \right) \\
& \left. \left(1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4 \right) \right) \Bigg/ \left(d (e \cos[c + dx])^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 \right) \\
& \left(4 \sec\left[\frac{1}{4}(c + dx)\right]^2 - 3\sqrt{2} \sec\left[\frac{1}{4}(c + dx)\right]^2 - 52 \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^2 + 39\sqrt{2} \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^2 \right)^2 + \\
& 200 \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^4 - 150\sqrt{2} \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^4 - \\
& 200 \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^6 + 150\sqrt{2} \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^6 + 52 \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^8 - \\
& 39\sqrt{2} \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^8 - 4 \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^{10} + 3\sqrt{2} \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right]^{10} + \\
& 2 \sqrt{2(3 - 2\sqrt{2})} \sec\left[\frac{1}{4}(c + dx)\right]^2 \tan\left[\frac{1}{4}(c + dx)\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12 \sqrt{2(3 - 2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2 \sqrt{2(3 - 2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 6 \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + 4\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 48 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 32\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 84 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + 56\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + 48 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 32\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - 6 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8 -
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} + 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^8 \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} + \\
& 4\sqrt{3 - 2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \\
& \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} - 4\sqrt{3 - 2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^3 \\
& \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \left. \left. \left. \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 289: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (e \operatorname{Cos}[c + dx])^{3/2} (a + a \operatorname{Sin}[c + dx])^{5/2} dx$$

Optimal (type 3, 323 leaves, 10 steps):

$$\begin{aligned}
& -\frac{77 a^3 (e \operatorname{Cos}[c + dx])^{5/2}}{96 d e \sqrt{a + a \operatorname{Sin}[c + dx]}} + \frac{77 a^2 e \sqrt{e \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]}}{64 d} - \\
& \frac{11 a^2 (e \operatorname{Cos}[c + dx])^{5/2} \sqrt{a + a \operatorname{Sin}[c + dx]}}{24 d e} - \frac{77 a^2 e^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \operatorname{Cos}[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]}}{64 d (1 + \operatorname{Cos}[c + dx] + \operatorname{Sin}[c + dx])} + \\
& \frac{77 a^2 e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} \operatorname{Sin}[c + dx]}{\sqrt{e \operatorname{Cos}[c + dx]} \sqrt{1 + \operatorname{Cos}[c + dx]}}\right] \sqrt{1 + \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]}}{64 d (1 + \operatorname{Cos}[c + dx] + \operatorname{Sin}[c + dx])} - \frac{a (e \operatorname{Cos}[c + dx])^{5/2} (a + a \operatorname{Sin}[c + dx])^{3/2}}{4 d e}
\end{aligned}$$

Result (type 4, 2838 leaves):

$$\left((e \operatorname{Cos}[c + dx])^{3/2} \operatorname{Sec}[c + dx] (a (1 + \operatorname{Sin}[c + dx]))^{5/2} \left(\frac{5}{12} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \frac{35}{64} \operatorname{Cos}\left[\frac{3}{2}(c + dx)\right] - \frac{5}{24} \operatorname{Cos}\left[\frac{5}{2}(c + dx)\right] + \frac{1}{32} \operatorname{Cos}\left[\frac{7}{2}(c + dx)\right] \right) + \right.$$

$$\begin{aligned}
& \left. \left. \left. \left. \frac{5}{12} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{35}{64} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] - \frac{5}{24} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] - \frac{1}{32} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] \right) \right) \right) / \left(d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right)^5 - \\
& \left(77 \sqrt{3-2\sqrt{2}} \left(e \operatorname{Cos}[c+dx] \right)^{3/2} \operatorname{Sec}[c+dx] \left(a \left(1 + \operatorname{Sin}[c+dx] \right) \right)^{5/2} \sqrt{3-2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \right) \right. \\
& \sqrt{\frac{-3+2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{\frac{-3+2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \left(2-2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \sqrt{2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \\
& \left. \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right. \right. \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \\
& \sqrt{3-2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& \left. \sqrt{2} \left(\operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2\right] - \operatorname{Log}\left[2-2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \sqrt{2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4}\right] \right) \right) \\
& \left. \left. \left. \left. \left(1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \right) \right) \right) \right) \right) / \\
& \left(128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right)^5 \\
& \left(-4 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 + 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 + 52 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - \right.
\end{aligned}$$

$$\begin{aligned}
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 200 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 + 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 + \\
& 200 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 - 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 - 52 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 + \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 + 4 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} - 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} + \\
& 2\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} +
\end{aligned}$$

$$\begin{aligned}
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 56 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \left. \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \right)
\end{aligned}$$

- **Problem 290: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{e \operatorname{Cos}[c+dx]} (a+a \operatorname{Sin}[c+dx])^{5/2} dx$$

Optimal (type 3, 286 leaves, 9 steps):

$$\begin{aligned}
& - \frac{15 a^3 (e \cos [c+d x])^{3/2}}{8 d e \sqrt{a+a \sin [c+d x]}} - \frac{3 a^2 (e \cos [c+d x])^{3/2} \sqrt{a+a \sin [c+d x]}}{4 d e} + \frac{15 a^2 \sqrt{e} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos [c+d x]}}{\sqrt{e}}\right] \sqrt{1+\cos [c+d x]} \sqrt{a+a \sin [c+d x]}}{8 d (1+\cos [c+d x]+\sin [c+d x])} + \\
& \frac{15 a^2 \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin [c+d x]}{\sqrt{e \cos [c+d x]} \sqrt{1+\cos [c+d x]}}\right] \sqrt{1+\cos [c+d x]} \sqrt{a+a \sin [c+d x]}}{8 d (1+\cos [c+d x]+\sin [c+d x])} - \frac{a (e \cos [c+d x])^{3/2} (a+a \sin [c+d x])^{3/2}}{3 d e}
\end{aligned}$$

Result (type 4, 2350 leaves):

$$\begin{aligned}
& \left(\sqrt{e \cos [c+d x]} (a (1+\sin [c+d x]))^{5/2} \right. \\
& \left. \left(-\frac{29}{12} \cos \left[\frac{1}{2} (c+d x) \right] - \frac{5}{8} \cos \left[\frac{3}{2} (c+d x) \right] + \frac{1}{12} \cos \left[\frac{5}{2} (c+d x) \right] + \frac{29}{12} \sin \left[\frac{1}{2} (c+d x) \right] - \frac{5}{8} \sin \left[\frac{3}{2} (c+d x) \right] - \frac{1}{12} \sin \left[\frac{5}{2} (c+d x) \right] \right) / \\
& \left(d \left(\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right)^5 \right) - \left(225 \cos \left[\frac{1}{4} (c+d x) \right]^2 \sqrt{e \cos [c+d x]} (a (1+\sin [c+d x]))^{5/2} \right. \\
& \left. \left(\sqrt{2} \left(\log \left[\sec \left[\frac{1}{4} (c+d x) \right]^2 \right] - \log \left[2 + \sqrt{2} \sqrt{\cos [c+d x] \sec \left[\frac{1}{4} (c+d x) \right]^4} - 2 \tan \left[\frac{1}{4} (c+d x) \right]^2 \right) \sqrt{\cos [c+d x] \sec \left[\frac{1}{4} (c+d x) \right]^4} + \right. \right. \\
& \left. \left. 4 \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c+d x) \right]}{\sqrt{3-2 \sqrt{2}}} \right], 17-12 \sqrt{2} \right] \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4} (c+d x) \right]^2} \sqrt{1+(-3+2 \sqrt{2}) \tan \left[\frac{1}{4} (c+d x) \right]^2} + 8 \right. \right. \\
& \left. \left. \operatorname{EllipticPi} \left[-3+2 \sqrt{2}, -\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c+d x) \right]}{\sqrt{3-2 \sqrt{2}}} \right], 17-12 \sqrt{2} \right] \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4} (c+d x) \right]^2} \sqrt{1+(-3+2 \sqrt{2}) \tan \left[\frac{1}{4} (c+d x) \right]^2} \right) \right) / \\
& \left(256 d \sqrt{\cos [c+d x]} \left(\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right)^6 \left(\frac{1}{32 \sqrt{\cos [c+d x]}} 15 \cos \left[\frac{1}{4} (c+d x) \right] \sin \left[\frac{1}{4} (c+d x) \right] \right. \right. \\
& \left. \left. \left(\sqrt{2} \left(\log \left[\sec \left[\frac{1}{4} (c+d x) \right]^2 \right] - \log \left[2 + \sqrt{2} \sqrt{\cos [c+d x] \sec \left[\frac{1}{4} (c+d x) \right]^4} - 2 \tan \left[\frac{1}{4} (c+d x) \right]^2 \right) \sqrt{\cos [c+d x] \sec \left[\frac{1}{4} (c+d x) \right]^4} + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 8 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \left. \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) - \frac{1}{32 \operatorname{Cos}[c+dx]^{3/2}} 15 \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sin}[c+dx] \\
& \left(\sqrt{2} \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\right] - \operatorname{Log}\left[2+\sqrt{2} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 - 2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right] \right) \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4} + \right. \\
& 4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 8 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \\
& \left. \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) - \\
& \frac{1}{16 \sqrt{\operatorname{Cos}[c+dx]}} 15 \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^2 \left(\left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\right] - \operatorname{Log}\left[2+\sqrt{2} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 - 2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right] \right) \right. \\
& \left. \left(-\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 \operatorname{Sin}[c+dx] + \operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \right) \right) / \left(\sqrt{2} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4} \right) + \\
& \left((-3+2\sqrt{2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) / \left(\sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) + \\
& \left(2(-3+2\sqrt{2}) \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \right. \\
& \left. \tan\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) / \left(\sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) - \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] \sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) / \\
& \left(\sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) - \left(2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] \sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) / \left(\sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right) + \\
& \frac{\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{3-2\sqrt{2}} \sqrt{1-\frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}} - \\
& \frac{2 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{3-2\sqrt{2}} \sqrt{1-\frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}} + \\
& \sqrt{2} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4} \left(\frac{1}{2} \tan\left[\frac{1}{4}(c+dx)\right] - \right.
\end{aligned}$$

$$\left(-\sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] + \frac{-\sec\left[\frac{1}{4}(c+dx)\right]^4 \sin[c+dx] + \cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 \tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4}} \right) /$$

$$\left(2 + \sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 - 2 \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) \Bigg|$$

- **Problem 291: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[c + dx])^{5/2}}{\sqrt{e \cos[c + dx]}} dx$$

Optimal (type 3, 247 leaves, 8 steps):

$$-\frac{7 a^2 \sqrt{e \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 d e} - \frac{21 a^2 \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 d \sqrt{e} (1 + \cos[c + dx] + \sin[c + dx])} +$$

$$\frac{21 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c + dx]}{\sqrt{e \cos[c + dx]} \sqrt{1 + \cos[c + dx]}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 d \sqrt{e} (1 + \cos[c + dx] + \sin[c + dx])} - \frac{a \sqrt{e \cos[c + dx]} (a + a \sin[c + dx])^{3/2}}{2 d e}$$

Result (type 4, 2782 leaves):

$$\left(\cos[c + dx] (a (1 + \sin[c + dx]))^{5/2} \left(-\frac{5}{2} \cos\left[\frac{1}{2}(c + dx)\right] + \frac{1}{4} \cos\left[\frac{3}{2}(c + dx)\right] - \frac{5}{2} \sin\left[\frac{1}{2}(c + dx)\right] - \frac{1}{4} \sin\left[\frac{3}{2}(c + dx)\right] \right) \right) /$$

$$\left(d \sqrt{e \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 \right) -$$

$$\left(21 \sqrt{3 - 2\sqrt{2}} \cos[c + dx] (a (1 + \sin[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \left(1 + \tan\left[\frac{1}{4}(c + dx)\right] \right)^2 \right)$$

$$\sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}}$$

$$\begin{aligned}
& \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \left(2 - 2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} \right) \\
& \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \right. \\
& \left. \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \left. \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} - \right. \\
& \left. \sqrt{2} \left(\operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2\right] - \operatorname{Log}\left[2 - 2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4}\right] \right) \right. \\
& \left. \left. \left(1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4 \right) \right) \right) / \\
& \left(8 d \sqrt{e \operatorname{Cos}[c + dx]} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right)^5 \\
& \left(-4 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 + 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 + 52 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - \right. \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 200 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4 + 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4 + \\
& 200 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^6 - 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^6 - 52 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^8 + \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^8 + 4 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^{10} - 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^{10} + \\
& \left. 2\sqrt{2(3 - 2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12 \sqrt{2(3 - 2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2 \sqrt{2(3 - 2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 6 \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - 4\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 48 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 84 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - 56\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - 48 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + 6 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} - 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^8 \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} + \\
& 4\sqrt{3 - 2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \\
& \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} - 4\sqrt{3 - 2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^3 \\
& \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \left. \left. \left. \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 292: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[c + dx])^{5/2}}{(e \operatorname{Cos}[c + dx])^{3/2}} dx$$

Optimal (type 3, 239 leaves, 8 steps):

$$\begin{aligned}
& \frac{5 a^3 (e \operatorname{Cos}[c + dx])^{3/2}}{d e^3 \sqrt{a + a \operatorname{Sin}[c + dx]}} - \frac{5 a^2 \operatorname{ArcSinh}\left[\frac{\sqrt{e \operatorname{Cos}[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]}}{d e^{3/2} (1 + \operatorname{Cos}[c + dx] + \operatorname{Sin}[c + dx])} - \\
& \frac{5 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{e} \operatorname{Sin}[c + dx]}{\sqrt{e \operatorname{Cos}[c + dx]} \sqrt{1 + \operatorname{Cos}[c + dx]}}\right] \sqrt{1 + \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]}}{d e^{3/2} (1 + \operatorname{Cos}[c + dx] + \operatorname{Sin}[c + dx])} + \frac{4 a (a + a \operatorname{Sin}[c + dx])^{3/2}}{d e \sqrt{e \operatorname{Cos}[c + dx]}}
\end{aligned}$$

Result (type 4, 2753 leaves):

$$\frac{\operatorname{Cos}[c + dx]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \frac{8}{\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]} - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) (a (1 + \operatorname{Sin}[c + dx]))^{5/2}}{d (e \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^5}$$

$$\begin{aligned}
& \left(5 \sqrt{3 - 2\sqrt{2}} \operatorname{Cos}[c + dx]^2 (a (1 + \operatorname{Sin}[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \right. \\
& \left. \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 \right) \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \right. \\
& \left. \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \left(2 - 2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} \right) \right) \\
& \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \right. \\
& \left. \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \left. \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} + \right. \\
& \left. \sqrt{2} \left[\operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2\right] - \operatorname{Log}\left[2 - 2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4}\right] \right] \right) \\
& \left. \left(1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4 \right) \right) \Bigg/ \left(2d (e \operatorname{Cos}[c + dx])^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6 \right) \\
& \left(4 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 - 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 - 52 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \right. \\
& 200 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4 - 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4 - \\
& 200 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^6 + 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^6 + 52 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^8 - \\
& \left. 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^8 - 4 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^{10} + 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^{10} + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12 \sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2 \sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 32\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 56\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} -
\end{aligned}$$

$$\begin{aligned}
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right)
\end{aligned}$$

- **Problem 293: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[c + dx])^{5/2}}{(e \cos[c + dx])^{5/2}} dx$$

Optimal (type 3, 204 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 a^2 \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c+dx]}}{\sqrt{e}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{d e^{5/2} (1+\cos[c+dx] + \sin[c+dx])} \\
& \frac{2 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{e \sin[c+dx]}}{\sqrt{e \cos[c+dx]} \sqrt{1+\cos[c+dx]}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{d e^{5/2} (1+\cos[c+dx] + \sin[c+dx])} + \frac{4 a (a + a \sin[c + dx])^{3/2}}{3 d e (e \cos[c + dx])^{3/2}}
\end{aligned}$$

Result (type 4, 2795 leaves):

$$\begin{aligned}
& \frac{\cos[c+dx]^3 \left(\frac{4}{3 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)} + \frac{8 \sin\left[\frac{1}{2}(c+dx)\right]}{3 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} \right) (a(1+\sin[c+dx]))^{5/2}}{d (e \cos[c+dx])^{5/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5} + \\
& \left(\sqrt{3-2\sqrt{2}} \cos[c+dx]^3 (a(1+\sin[c+dx]))^{5/2} \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{4}(c+dx)\right]^2 \right) \right. \\
& \sqrt{\frac{-3+2\sqrt{2} + \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{\frac{-3+2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \left. \sqrt{1-3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} \left(2-2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \sqrt{2} \sqrt{1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \\
& \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \right. \\
& \left. \sqrt{1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \\
& \left. \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - \right. \\
& \left. \sqrt{2} \left(\log\left[1 + \tan\left[\frac{1}{4}(c+dx)\right]^2\right] - \log\left[2-2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \sqrt{2} \sqrt{1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4}\right] \right) \right) \\
& \left. \left(1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4 \right) \right) \Bigg) / \\
& \left(d (e \cos[c+dx])^{5/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-4 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 + 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 + 52 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - \right. \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 200 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 + 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 + \\
& 200 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 - 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 - 52 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 + \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 + 4 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} - 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} + \\
& 2\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} +
\end{aligned}$$

$$\begin{aligned}
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 56 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \left. \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 294: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[c + dx])^{5/2}}{(e \cos[c + dx])^{7/2}} dx$$

Optimal (type 3, 36 leaves, 1 step):

$$\frac{2 (a + a \sin[c + dx])^{5/2}}{5 d e (e \cos[c + dx])^{5/2}}$$

Result (type 3, 87 leaves):

$$\frac{2 a^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \sqrt{a(1 + \sin[c + dx])}}{5 d e^3 \sqrt{e \cos[c + dx]} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}$$

■ **Problem 298: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{5/2}}{\sqrt{a + a \sin[c + dx]}} dx$$

Optimal (type 3, 244 leaves, 8 steps):

$$-\frac{a (e \cos[c + dx])^{7/2}}{2 d e (a + a \sin[c + dx])^{3/2}} + \frac{e (e \cos[c + dx])^{3/2}}{4 d \sqrt{a + a \sin[c + dx]}} + \frac{3 e^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 d (a + a \cos[c + dx] + a \sin[c + dx])} +$$

$$\frac{3 e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c + dx]}{\sqrt{e \cos[c + dx]} \sqrt{1 + \cos[c + dx]}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 d (a + a \cos[c + dx] + a \sin[c + dx])}$$

Result (type 4, 2305 leaves):

$$\frac{1}{d \sqrt{a(1 + \sin[c + dx])}} (e \cos[c + dx])^{5/2} \sec[c + dx]^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)$$

$$\left(-\frac{1}{2} \cos\left[\frac{1}{2}(c + dx)\right] + \frac{1}{4} \cos\left[\frac{3}{2}(c + dx)\right] + \frac{1}{2} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{4} \sin\left[\frac{3}{2}(c + dx)\right] \right) - \left(9 \cos\left[\frac{1}{4}(c + dx)\right]^2 (e \cos[c + dx])^{5/2} \right.$$

$$\left. \left(\sqrt{2} \left(\log\left[\sec\left[\frac{1}{4}(c + dx)\right]^2\right] - \log\left[2 + \sqrt{2} \sqrt{\cos[c + dx] \sec\left[\frac{1}{4}(c + dx)\right]^4 - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2}\right] \right) \sqrt{\cos[c + dx] \sec\left[\frac{1}{4}(c + dx)\right]^4} + \right.$$

$$4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} + 8$$

$$\left. \left. \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} \right) \right) /$$

$$\left(64 d \operatorname{Cos}[c + d x]^{5/2} \sqrt{a (1 + \operatorname{Sin}[c + d x])} \right) \left(\frac{1}{16 \sqrt{\operatorname{Cos}[c + d x]}} 3 \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right] \right.$$

$$\left. \left(\sqrt{2} \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^2\right] - \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4 - 2 \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2}\right] \right) \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4} + \right.$$

$$4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]}{\sqrt{3 - 2 \sqrt{2}}}\right], 17 - 12 \sqrt{2}\right] \sqrt{3 - 2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2} \sqrt{1 + (-3 + 2 \sqrt{2}) \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2} +$$

$$8 \operatorname{EllipticPi}\left[-3 + 2 \sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]}{\sqrt{3 - 2 \sqrt{2}}}\right], 17 - 12 \sqrt{2}\right] \sqrt{3 - 2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2}$$

$$\left. \sqrt{1 + (-3 + 2 \sqrt{2}) \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2} \right) - \frac{1}{16 \operatorname{Cos}[c + d x]^{3/2}} 3 \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right]^2 \operatorname{Sin}[c + d x]$$

$$\left(\sqrt{2} \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^2\right] - \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4 - 2 \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2}\right] \right) \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4} + \right.$$

$$4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]}{\sqrt{3 - 2 \sqrt{2}}}\right], 17 - 12 \sqrt{2}\right] \sqrt{3 - 2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2} \sqrt{1 + (-3 + 2 \sqrt{2}) \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2} +$$

$$8 \operatorname{EllipticPi}\left[-3 + 2 \sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]}{\sqrt{3 - 2 \sqrt{2}}}\right], 17 - 12 \sqrt{2}\right]$$

$$\left. \sqrt{3 - 2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2} \sqrt{1 + (-3 + 2 \sqrt{2}) \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2} \right) -$$

$$\begin{aligned}
& \frac{1}{8 \sqrt{\cos[c+dx]}} 3 \cos\left[\frac{1}{4}(c+dx)\right]^2 \left(\left(\left(\log\left[\sec\left[\frac{1}{4}(c+dx)\right]\right]^2 - \log\left[2 + \sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 - 2 \tan\left[\frac{1}{4}(c+dx)\right]^2}\right]\right) \right. \right. \\
& \left. \left. \left(-\sec\left[\frac{1}{4}(c+dx)\right]^4 \sin[c+dx] + \cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 \tan\left[\frac{1}{4}(c+dx)\right] \right) \right) / \left(\sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4} \right) + \right. \\
& \left((-3 + 2\sqrt{2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] \right. \\
& \left. \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) / \left(\sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) + \\
& \left(2(-3 + 2\sqrt{2}) \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sec\left[\frac{1}{4}(c+dx)\right]^2 \right. \\
& \left. \tan\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) / \left(\sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) - \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) / \\
& \left(\sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) - \left(2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \\
& \left. \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) / \left(\sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{\sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{3-2\sqrt{2}} \sqrt{1-\frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}} \\
& + \frac{2 \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{3-2\sqrt{2}} \sqrt{1-\frac{\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}} \left(1-\frac{(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}\right) \\
& \sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4} \left(\frac{1}{2} \tan\left[\frac{1}{4}(c+dx)\right] - \right. \\
& \left. \left(-\sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] + \frac{-\sec\left[\frac{1}{4}(c+dx)\right]^4 \sin[c+dx] + \cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 \tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4}} \right) \right) \\
& \left(2 + \sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4} - 2 \tan\left[\frac{1}{4}(c+dx)\right]^2 \right)
\end{aligned}$$

- **Problem 299: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^{3/2}}{\sqrt{a+a \sin[c+dx]}} dx$$

Optimal (type 3, 200 leaves, 7 steps):

$$\frac{e \sqrt{e \cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{ad} - \frac{e^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c+dx]}}{\sqrt{e}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{ad(1+\cos[c+dx]+\sin[c+dx])} +$$

$$\frac{e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c+dx]}{\sqrt{e \cos[c+dx]} \sqrt{1+\cos[c+dx]}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{ad(1+\cos[c+dx]+\sin[c+dx])}$$

Result (type 4, 2723 leaves):

$$\frac{(e \cos[c+dx])^{3/2} \sec[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{d \sqrt{a(1+\sin[c+dx])}} -$$

$$\left(\sqrt{3-2\sqrt{2}} (e \cos[c+dx])^{3/2} \sec[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right) \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right.$$

$$\left. \left(1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)^2 \sqrt{\frac{-3+2\sqrt{2} + \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{\frac{-3+2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]}{-3+2\sqrt{2}}} \right.$$

$$\left. \sqrt{1-3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} \left(2-2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \sqrt{2} \sqrt{1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4}\right) \right.$$

$$\left. \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \right.$$

$$\left. \sqrt{1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right.$$

$$\left. \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - \right.$$

$$\left. \sqrt{2} \left(\log\left[1 + \tan\left[\frac{1}{4}(c+dx)\right]\right]^2 - \log\left[2-2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \sqrt{2} \sqrt{1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4}\right] \right) \right)$$

$$\left(1-6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4\right) \Bigg) \Bigg/ \left(2d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)\right)$$

$$\begin{aligned}
& \sqrt{a(1 + \sin[c + dx])} \left(-4 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 + 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 + 52 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - \right. \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 200 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4 + 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4 + \\
& 200 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^6 - 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^6 - 52 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^8 + \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^8 + 4 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^{10} - 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^{10} + \\
& 2\sqrt{2(3 - 2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} - \\
& 12\sqrt{2(3 - 2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^3 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} + \\
& 2\sqrt{2(3 - 2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^5 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} + \\
& 6 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} - 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} - \\
& 48 \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} +
\end{aligned}$$

$$\begin{aligned}
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 56 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \left. \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 300: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{e \cos [c+dx]}}{\sqrt{a+a \sin [c+dx]}} dx$$

Optimal (type 3, 169 leaves, 6 steps):

$$\frac{2\sqrt{e} \operatorname{ArcSinh}\left[\frac{\sqrt{e}\cos[c+dx]}{\sqrt{e}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a\sin[c+dx]} + 2\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e}\sin[c+dx]}{\sqrt{e}\cos[c+dx]}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a\sin[c+dx]}}{d(a+a\cos[c+dx]+a\sin[c+dx])}$$

Result (type 4, 2607 leaves) :

$$\left(\sqrt{3-2\sqrt{2}} \sqrt{e\cos[c+dx]} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{4}(c+dx)\right]^2\right) \right. \\ \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\ \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} \left(2-2\tan\left[\frac{1}{4}(c+dx)\right]^2+\sqrt{2}\sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4}\right) \\ \left(4\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2} \right. \\ \left. \sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4} + 8\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \\ \left. \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4} + \right. \\ \left. \sqrt{2} \left(\operatorname{Log}\left[1+\tan\left[\frac{1}{4}(c+dx)\right]^2\right] - \operatorname{Log}\left[2-2\tan\left[\frac{1}{4}(c+dx)\right]^2+\sqrt{2}\sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4}\right] \right) \right. \\ \left. \left. \left(1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4\right) \right) \right) / \\ \left(d\sqrt{a(1+\sin[c+dx])} \left(4\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 - 3\sqrt{2}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 - 52\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \right. \right. \\ \left. \left. 39\sqrt{2}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 200\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 - 150\sqrt{2}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 - \right. \right.$$

$$\begin{aligned}
& 200 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 + 150\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 + 52 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 - \\
& 39\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 - 4 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} + 3\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10} + \\
& 2\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2\sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 32\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} +
\end{aligned}$$

$$\begin{aligned}
& 56 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4}-4\sqrt{3-2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{\frac{-3+2\sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \left. \left. \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 305: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^{7/2}}{(a+a \sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 247 leaves, 8 steps):

$$\frac{e (e \cos[c + dx])^{5/2}}{2 a d \sqrt{a + a \sin[c + dx]}} + \frac{5 e^3 \sqrt{e \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 a^2 d} -$$

$$\frac{5 e^{7/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 a^2 d (1 + \cos[c + dx] + \sin[c + dx])} + \frac{5 e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{e \sin[c + dx]}}{\sqrt{e \cos[c + dx]} \sqrt{1 + \cos[c + dx]}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 a^2 d (1 + \cos[c + dx] + \sin[c + dx])}$$

Result (type 4, 2786 leaves):

$$\frac{1}{d (a (1 + \sin[c + dx]))^{3/2}} (e \cos[c + dx])^{7/2} \sec[c + dx]^3 \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right] \right)^3$$

$$\left(\frac{3}{2} \cos\left[\frac{1}{2} (c + dx)\right] + \frac{1}{4} \cos\left[\frac{3}{2} (c + dx)\right] + \frac{3}{2} \sin\left[\frac{1}{2} (c + dx)\right] - \frac{1}{4} \sin\left[\frac{3}{2} (c + dx)\right] \right) -$$

$$\left(5 \sqrt{3 - 2\sqrt{2}} (e \cos[c + dx])^{7/2} \sec[c + dx]^3 \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right] \right)^3 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4} (c + dx)\right]^2} \right.$$

$$\left. \left(1 + \tan\left[\frac{1}{4} (c + dx)\right] \right)^2 \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4} (c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4} (c + dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4} (c + dx)\right]^2}{-3 + 2\sqrt{2}}} \right.$$

$$\left. \sqrt{1 - 3 \tan\left[\frac{1}{4} (c + dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4} (c + dx)\right]^2} \left(2 - 2 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \tan\left[\frac{1}{4} (c + dx)\right]^4} \right)^4 \right)$$

$$\left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4} (c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4} (c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4} (c + dx)\right]^2} \right.$$

$$\left. \sqrt{1 - 6 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \tan\left[\frac{1}{4} (c + dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4} (c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right.$$

$$\left. \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4} (c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4} (c + dx)\right]^2} \sqrt{1 - 6 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \tan\left[\frac{1}{4} (c + dx)\right]^4} - \right.$$

$$\left. \sqrt{2} \left(\log\left[1 + \tan\left[\frac{1}{4} (c + dx)\right]\right]^2 - \log\left[2 - 2 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \tan\left[\frac{1}{4} (c + dx)\right]^4}\right] \right) \right)$$

$$\begin{aligned}
& \left(1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4 \right) \Bigg) \Bigg) / \left(8d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) \\
(a (1 + \sin[c+dx]))^{3/2} & \left(-4 \sec\left[\frac{1}{4}(c+dx)\right]^2 + 3\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 + 52 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 - \right. \\
& 39\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 200 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 + 150\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 + \\
& 200 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 - 150\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 - 52 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8 + \\
& 39\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8 + 4 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^{10} - 3\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^{10} + \\
& 2\sqrt{2(3-2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12\sqrt{2(3-2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2\sqrt{2(3-2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 6 \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4} - 4\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4} -
\end{aligned}$$

$$\begin{aligned}
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 56 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \left. \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 306: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{5/2}}{(a + a \sin[c + dx])^{3/2}} dx$$

Optimal (type 3, 215 leaves, 7 steps):

$$\frac{e (e \cos[c + dx])^{3/2}}{a d \sqrt{a + a \sin[c + dx]}} + \frac{3 e^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{d (a^2 + a^2 \cos[c + dx] + a^2 \sin[c + dx])} +$$

$$\frac{3 e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{e \sin[c + dx]}}{\sqrt{e \cos[c + dx]}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{d (a^2 + a^2 \cos[c + dx] + a^2 \sin[c + dx])}$$

Result (type 4, 2726 leaves):

$$\frac{1}{d (a (1 + \sin[c + dx]))^{3/2}} (e \cos[c + dx])^{5/2} \sec[c + dx]^2 \left(\cos\left[\frac{1}{2} (c + dx)\right] - \sin\left[\frac{1}{2} (c + dx)\right] \right) \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right] \right)^3 +$$

$$\left(3 \sqrt{3 - 2\sqrt{2}} (e \cos[c + dx])^{5/2} \sec[c + dx]^2 \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right] \right)^2 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4} (c + dx)\right]^2} \right.$$

$$\left. \left(1 + \tan\left[\frac{1}{4} (c + dx)\right]^2 \right) \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4} (c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4} (c + dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4} (c + dx)\right]^2}{-3 + 2\sqrt{2}}} \right.$$

$$\left. \sqrt{1 - 3 \tan\left[\frac{1}{4} (c + dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4} (c + dx)\right]^2} \left(2 - 2 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \tan\left[\frac{1}{4} (c + dx)\right]^4} \right) \right.$$

$$\left. \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4} (c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4} (c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4} (c + dx)\right]^2} \right.$$

$$\left. \sqrt{1 - 6 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \tan\left[\frac{1}{4} (c + dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4} (c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right.$$

$$\left. \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4} (c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4} (c + dx)\right]^2} \sqrt{1 - 6 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \tan\left[\frac{1}{4} (c + dx)\right]^4} + \right.$$

$$\left. \sqrt{2} \left(\log\left[1 + \tan\left[\frac{1}{4} (c + dx)\right]^2\right] - \log\left[2 - 2 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4} (c + dx)\right]^2 + \tan\left[\frac{1}{4} (c + dx)\right]^4}\right] \right) \right)$$

$$\left. \left(1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4 \right) \right) \Bigg) /$$

$$\left(2d(a(1+\sin[c+dx]))^{3/2} \left(4 \sec\left[\frac{1}{4}(c+dx)\right]^2 - 3\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 - 52 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \right. \right.$$

$$39\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 200 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 - 150\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 -$$

$$200 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 + 150\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 + 52 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8 -$$

$$39\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8 - 4 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^{10} + 3\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^{10} +$$

$$2\sqrt{2(3-2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}}$$

$$\sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} -$$

$$12\sqrt{2(3-2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}}$$

$$\sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} +$$

$$2\sqrt{2(3-2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}}$$

$$\sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} -$$

$$6 \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4} + 4\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4} +$$

$$\begin{aligned}
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 56 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \left. \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 307: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{3/2}}{(a + a \sin[c + dx])^{3/2}} dx$$

Optimal (type 3, 236 leaves, 8 steps):

$$\begin{aligned} & - \frac{2 (e \cos[c + dx])^{5/2}}{d e (a + a \sin[c + dx])^{3/2}} - \frac{2 e \sqrt{e \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{a^2 d} + \\ & \frac{2 e^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]} + 2 e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c + dx]}{\sqrt{e \cos[c + dx]} \sqrt{1 + \cos[c + dx]}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{a^2 d (1 + \cos[c + dx] + \sin[c + dx])} \end{aligned}$$

Result (type 4, 2723 leaves):

$$\begin{aligned} & - \frac{4 (e \cos[c + dx])^{3/2} \operatorname{Sec}[c + dx] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{d (a (1 + \sin[c + dx]))^{3/2}} + \\ & \left(\sqrt{3 - 2\sqrt{2}} (e \cos[c + dx])^{3/2} \operatorname{Sec}[c + dx] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \right. \\ & \left. \left(1 + \tan\left[\frac{1}{4}(c + dx)\right]\right)^2 \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c + dx)\right]}{-3 + 2\sqrt{2}}} \right. \\ & \left. \sqrt{1 - 3 \tan\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c + dx)\right]^2} \left(2 - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4}\right) \right. \\ & \left. \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} \right. \right. \\ & \left. \left. \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \right. \\ & \left. \left. \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4} - \right. \right. \\ & \left. \left. \sqrt{2} \left(\log\left[1 + \tan\left[\frac{1}{4}(c + dx)\right]^2\right] - \log\left[2 - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c + dx)\right]^2 + \tan\left[\frac{1}{4}(c + dx)\right]^4}\right]\right) \right) \end{aligned}$$

$$\left(1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4\right) \Bigg) \Bigg) / \left(d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

$$\begin{aligned}
& (a(1 + \sin[c+dx]))^{3/2} \left(-4 \sec\left[\frac{1}{4}(c+dx)\right]^2 + 3\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 + 52 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 - \right. \\
& 39\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 200 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 + 150\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 + \\
& 200 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 - 150\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 - 52 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8 + \\
& 39\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8 + 4 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^{10} - 3\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^{10} + \\
& 2\sqrt{2(3-2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12\sqrt{2(3-2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2\sqrt{2(3-2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\tan\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\tan\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\tan\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 6 \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4} - 4\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\tan\left[\frac{1}{4}(c+dx)\right]^2+\tan\left[\frac{1}{4}(c+dx)\right]^4} -
\end{aligned}$$

$$\begin{aligned}
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 56 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 - 12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \left. \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + 2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 313: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{9/2}}{(a + a \sin[c + dx])^{5/2}} dx$$

Optimal (type 3, 261 leaves, 9 steps):

$$\frac{e (e \cos[c + dx])^{7/2}}{2 a d (a + a \sin[c + dx])^{3/2}} + \frac{7 e^3 (e \cos[c + dx])^{3/2}}{4 a^2 d \sqrt{a + a \sin[c + dx]}} + \frac{21 e^{9/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 d (a^3 + a^3 \cos[c + dx] + a^3 \sin[c + dx])} +$$

$$\frac{21 e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c + dx]}{\sqrt{e \cos[c + dx]} \sqrt{1 + \cos[c + dx]}}\right] \sqrt{1 + \cos[c + dx]} \sqrt{a + a \sin[c + dx]}}{4 d (a^3 + a^3 \cos[c + dx] + a^3 \sin[c + dx])}$$

Result (type 4, 2330 leaves):

$$\frac{1}{d (a (1 + \sin[c + dx]))^{5/2}} (e \cos[c + dx])^{9/2} \operatorname{Sec}[c + dx]^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5$$

$$\left(\frac{5}{2} \cos\left[\frac{1}{2}(c + dx)\right] - \frac{1}{4} \cos\left[\frac{3}{2}(c + dx)\right] - \frac{5}{2} \sin\left[\frac{1}{2}(c + dx)\right] - \frac{1}{4} \sin\left[\frac{3}{2}(c + dx)\right] \right) -$$

$$\left(441 \cos\left[\frac{1}{4}(c + dx)\right]^2 (e \cos[c + dx])^{9/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 \right.$$

$$\left. \left(\sqrt{2} \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2\right] - \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\cos[c + dx] \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^4 - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2}\right] \right) \sqrt{\cos[c + dx] \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^4} + \right.$$

$$4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} + 8$$

$$\left. \left. \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \tan\left[\frac{1}{4}(c + dx)\right]^2} \right) \right) /$$

$$\left(64 d \cos[c + dx]^{9/2} (a (1 + \sin[c + dx]))^{5/2} \left(\frac{1}{16 \sqrt{\cos[c + dx]}} 21 \cos\left[\frac{1}{4}(c + dx)\right] \sin\left[\frac{1}{4}(c + dx)\right] \right) \right)$$

$$\begin{aligned}
& \left(\sqrt{2} \left(\text{Log} \left[\text{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \right] - \text{Log} \left[2 + \sqrt{2} \sqrt{\text{Cos}[c + dx] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^4 - 2 \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right] \right) \sqrt{\text{Cos}[c + dx] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^4} + \right. \\
& 4 \text{EllipticF} \left[\text{ArcSin} \left[\frac{\text{Tan} \left[\frac{1}{4} (c + dx) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \sqrt{3 - 2\sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} + \\
& 8 \text{EllipticPi} \left[-3 + 2\sqrt{2}, -\text{ArcSin} \left[\frac{\text{Tan} \left[\frac{1}{4} (c + dx) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \sqrt{3 - 2\sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \\
& \left. \sqrt{1 + (-3 + 2\sqrt{2}) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right) - \frac{1}{16 \text{Cos}[c + dx]^{3/2}} 21 \text{Cos} \left[\frac{1}{4} (c + dx) \right]^2 \text{Sin}[c + dx] \\
& \left(\sqrt{2} \left(\text{Log} \left[\text{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \right] - \text{Log} \left[2 + \sqrt{2} \sqrt{\text{Cos}[c + dx] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^4 - 2 \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right] \right) \sqrt{\text{Cos}[c + dx] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^4} + \right. \\
& 4 \text{EllipticF} \left[\text{ArcSin} \left[\frac{\text{Tan} \left[\frac{1}{4} (c + dx) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \sqrt{3 - 2\sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} + \\
& 8 \text{EllipticPi} \left[-3 + 2\sqrt{2}, -\text{ArcSin} \left[\frac{\text{Tan} \left[\frac{1}{4} (c + dx) \right]}{\sqrt{3 - 2\sqrt{2}}} \right], 17 - 12\sqrt{2} \right] \\
& \left. \sqrt{3 - 2\sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \sqrt{1 + (-3 + 2\sqrt{2}) \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right) - \\
& \frac{1}{8 \sqrt{\text{Cos}[c + dx]}} 21 \text{Cos} \left[\frac{1}{4} (c + dx) \right]^2 \left(\left(\left(\text{Log} \left[\text{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \right] - \text{Log} \left[2 + \sqrt{2} \sqrt{\text{Cos}[c + dx] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^4 - 2 \text{Tan} \left[\frac{1}{4} (c + dx) \right]^2} \right] \right) \right. \right. \\
& \left. \left. \left(-\text{Sec} \left[\frac{1}{4} (c + dx) \right]^4 \text{Sin}[c + dx] + \text{Cos}[c + dx] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^4 \text{Tan} \left[\frac{1}{4} (c + dx) \right] \right) \right) \right) / \left(\sqrt{2} \sqrt{\text{Cos}[c + dx] \text{Sec} \left[\frac{1}{4} (c + dx) \right]^4} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left((-3 + 2\sqrt{2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \right. \\
& \quad \left. \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right) / \left(\sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) + \\
& \left(2(-3+2\sqrt{2}) \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \right. \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right) / \left(\sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) - \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) / \\
& \left(\sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) - \left(2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) / \left(\sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) + \\
& \frac{\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{3-2\sqrt{2}} \sqrt{1-\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}} - \\
& \frac{2 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}}{\sqrt{3-2\sqrt{2}} \sqrt{1-\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}} \sqrt{1-\frac{(17-12\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}}}} \left(1 - \frac{(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{3-2\sqrt{2}} \right) +
\end{aligned}$$

$$\sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4} \left(\frac{1}{2} \tan\left[\frac{1}{4}(c+dx)\right] - \left(-\sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right] + \frac{-\sec\left[\frac{1}{4}(c+dx)\right]^4 \sin[c+dx] + \cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 \tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4}} \right) \right)$$

$$\left(2 + \sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4} - 2 \tan\left[\frac{1}{4}(c+dx)\right]^2 \right)$$

- **Problem 314: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^{7/2}}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 239 leaves, 8 steps):

$$-\frac{4 e (e \cos[c+dx])^{5/2}}{a d (a+a \sin[c+dx])^{3/2}} - \frac{5 e^3 \sqrt{e \cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{a^3 d} + \frac{5 e^{7/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \cos[c+dx]}}{\sqrt{e}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{a^3 d (1+\cos[c+dx] + \sin[c+dx])} - \frac{5 e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sin[c+dx]}{\sqrt{e \cos[c+dx]} \sqrt{1+\cos[c+dx]}}\right] \sqrt{1+\cos[c+dx]} \sqrt{a+a \sin[c+dx]}}{a^3 d (1+\cos[c+dx] + \sin[c+dx])}$$

Result (type 4, 2779 leaves):

$$\frac{1}{d (a (1+\sin[c+dx]))^{5/2}} (e \cos[c+dx])^{7/2} \sec[c+dx]^3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 \left(-\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] - \frac{8}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} \right) + \left(5 \sqrt{3-2\sqrt{2}} (e \cos[c+dx])^{7/2} \sec[c+dx]^3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right)$$

$$\begin{aligned}
& \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \right) \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \left(2-2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\sqrt{2}\sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right) \\
& \left(4\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right. \\
& \left. \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 8\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right. \\
& \left. \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1+(-3+2\sqrt{2})\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \right. \\
& \left. \sqrt{2} \left(\operatorname{Log}\left[1+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2\right] - \operatorname{Log}\left[2-2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\sqrt{2}\sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4}\right] \right) \right) \\
& \left. \left(1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \right) \right) \Bigg/ \left(2d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) \\
& (a(1+\operatorname{Sin}[c+dx]))^{5/2} \left(-4\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2+3\sqrt{2}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2+52\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2- \right. \\
& 39\sqrt{2}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-200\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4+150\sqrt{2}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4+ \\
& 200\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6-150\sqrt{2}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6-52\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8+ \\
& 39\sqrt{2}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8+4\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10}-3\sqrt{2}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^{10}+ \\
& \left. 2\sqrt{2(3-2\sqrt{2})}\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12 \sqrt{2(3 - 2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2 \sqrt{2(3 - 2\sqrt{2})} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 6 \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - 4\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 48 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 84 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - 56\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} - 48 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 32\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + 6 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} - 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^8 \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} + \\
& 4\sqrt{3 - 2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right] \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \\
& \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} - 4\sqrt{3 - 2\sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^3 \\
& \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\frac{-3 + 2\sqrt{2} + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \sqrt{\frac{-3 + 2\sqrt{2} + 17 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 - 12\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}{-3 + 2\sqrt{2}}} \\
& \left. \left. \left. \sqrt{1 - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + 2\sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{1 - 6 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^4} \right) \right) \right)
\end{aligned}$$

■ **Problem 315: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \operatorname{Cos}[c + dx])^{5/2}}{(a + a \operatorname{Sin}[c + dx])^{5/2}} dx$$

Optimal (type 3, 218 leaves, 7 steps):

$$\begin{aligned}
& -\frac{4e(e \operatorname{Cos}[c + dx])^{3/2}}{3ad(a + a \operatorname{Sin}[c + dx])^{3/2}} - \frac{2e^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{e \operatorname{Cos}[c + dx]}}{\sqrt{e}}\right] \sqrt{1 + \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]}}{d(a^3 + a^3 \operatorname{Cos}[c + dx] + a^3 \operatorname{Sin}[c + dx])} \\
& \frac{2e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} \operatorname{Sin}[c + dx]}{\sqrt{e \operatorname{Cos}[c + dx]} \sqrt{1 + \operatorname{Cos}[c + dx]}}\right] \sqrt{1 + \operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sin}[c + dx]}}{d(a^3 + a^3 \operatorname{Cos}[c + dx] + a^3 \operatorname{Sin}[c + dx])}
\end{aligned}$$

Result (type 4, 2766 leaves):

$$\begin{aligned}
& \frac{1}{d(a(1 + \operatorname{Sin}[c + dx]))^{5/2}} (e \operatorname{Cos}[c + dx])^{5/2} \operatorname{Sec}[c + dx]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^5 \\
& \left(\frac{8 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{3 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2} - \frac{4}{3 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{3-2\sqrt{2}} (e \cos[c+dx])^{5/2} \sec[c+dx]^2 \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^4 \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right. \\
& \left. \left(1 + \tan\left[\frac{1}{4}(c+dx)\right]^2 \right) \sqrt{\frac{-3+2\sqrt{2} + \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{\frac{-3+2\sqrt{2} + 17 \tan\left[\frac{1}{4}(c+dx)\right]^2 - 12\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \right. \right. \\
& \left. \sqrt{1 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 2\sqrt{2} \tan\left[\frac{1}{4}(c+dx)\right]^2} \left(2 - 2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} \right) \right. \\
& \left. \left(4 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1 + (-3+2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \right. \right. \\
& \left. \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + 8 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right. \\
& \left. \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1 + (-3+2\sqrt{2}) \tan\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4} + \right. \\
& \left. \sqrt{2} \left(\log\left[1 + \tan\left[\frac{1}{4}(c+dx)\right]^2\right] - \log\left[2 - 2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \sqrt{2} \sqrt{1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4}\right] \right) \right. \\
& \left. \left. \left. \left(1 - 6 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \tan\left[\frac{1}{4}(c+dx)\right]^4 \right) \right) \right) \right) / \\
& \left(d (a (1 + \sin[c+dx]))^{5/2} \left(4 \sec\left[\frac{1}{4}(c+dx)\right]^2 - 3\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 - 52 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + \right. \right. \\
& 39\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^2 + 200 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 - 150\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^4 - \\
& 200 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 + 150\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^6 + 52 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8 - \\
& \left. \left. 39\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^8 - 4 \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^{10} + 3\sqrt{2} \sec\left[\frac{1}{4}(c+dx)\right]^2 \tan\left[\frac{1}{4}(c+dx)\right]^{10} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 12 \sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& 2 \sqrt{2(3-2\sqrt{2})} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^5 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2\sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \\
& \sqrt{\frac{-3+2\sqrt{2}+17\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2\sqrt{2}}} \sqrt{1-3\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2\sqrt{2}\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} - \\
& 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 4\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 32\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - \\
& 84 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 56\sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4 \\
& \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 48 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} -
\end{aligned}$$

$$\begin{aligned}
& 32 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^6 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 6 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + 4 \sqrt{2} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^8 \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} + \\
& 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right] \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} - 4 \sqrt{3-2 \sqrt{2}} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^3 \\
& \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\frac{-3+2 \sqrt{2}+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \sqrt{\frac{-3+2 \sqrt{2}+17 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2-12 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}{-3+2 \sqrt{2}}} \\
& \left. \sqrt{1-3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2+2 \sqrt{2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{1-6 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^4} \right)
\end{aligned}$$

■ **Problem 327: Attempted integration timed out after 120 seconds.**

$$\int (e \operatorname{Cos}[c+dx])^p (a+a \operatorname{Sin}[c+dx])^8 dx$$

Optimal (type 5, 95 leaves, 2 steps):

$$-\frac{1}{d e (1+p)} 2^{\frac{17+p}{2}} a^8 (e \operatorname{Cos}[c+dx])^{1+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-15-p), \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1-\operatorname{Sin}[c+dx])\right] (1+\operatorname{Sin}[c+dx])^{\frac{1}{2}(-1-p)}$$

Result (type 1, 1 leaves):

???

■ **Problem 328: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (e \operatorname{Cos}[c+dx])^p (a+a \operatorname{Sin}[c+dx])^3 dx$$

Optimal (type 5, 95 leaves, 2 steps):

$$-\frac{1}{d e (1+p)} 2^{\frac{7+p}{2}} a^3 (e \cos [c+d x])^{1+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-5-p), \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1-\sin [c+d x])\right] (1+\sin [c+d x])^{\frac{1}{2}(-1-p)}$$

Result (type 5, 462 leaves):

$$\frac{1}{d \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^6} i 2^{-3-p} \left(e^{-i(c+d x)} + e^{i(c+d x)}\right)^p \left(1 + e^{2i(c+d x)}\right)^{-p} \cos [c+d x]^{-p} (e \cos [c+d x])^p \left(-\frac{i e^{-3i(c+d x)} \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-\frac{p}{2}, -p, -\frac{1}{2}-\frac{p}{2}, -e^{2i(c+d x)}\right]}{3+p} - \frac{6 e^{-2i(c+d x)} \operatorname{Hypergeometric2F1}\left[-1-\frac{p}{2}, -p, -\frac{p}{2}, -e^{2i(c+d x)}\right]}{2+p} + \frac{15 i e^{-i(c+d x)} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}-\frac{p}{2}, -p, \frac{1}{2}-\frac{p}{2}, -e^{2i(c+d x)}\right]}{1+p} - \frac{15 i e^{i(c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}-\frac{p}{2}, -p, \frac{3}{2}-\frac{p}{2}, -e^{2i(c+d x)}\right]}{-1+p} - \frac{6 e^{2i(c+d x)} \operatorname{Hypergeometric2F1}\left[1-\frac{p}{2}, -p, 2-\frac{p}{2}, -e^{2i(c+d x)}\right]}{-2+p} + \frac{i e^{3i(c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}-\frac{p}{2}, -p, \frac{5}{2}-\frac{p}{2}, -e^{2i(c+d x)}\right]}{-3+p} + \frac{20 \operatorname{Hypergeometric2F1}\left[-p, -\frac{p}{2}, 1-\frac{p}{2}, -e^{2i(c+d x)}\right]}{p} \right) (a + a \sin [c+d x])^3$$

■ **Problem 329: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (e \cos [c+d x])^p (a + a \sin [c+d x])^2 dx$$

Optimal (type 5, 95 leaves, 2 steps):

$$-\frac{1}{d e (1+p)} 2^{\frac{5+p}{2}} a^2 (e \cos [c+d x])^{1+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-3-p), \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1-\sin [c+d x])\right] (1+\sin [c+d x])^{\frac{1}{2}(-1-p)}$$

Result (type 5, 351 leaves):

$$\frac{1}{d \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^4} 2^{-2-p} \left(e^{-i(c+d x)} + e^{i(c+d x)}\right)^p \left(1 + e^{2i(c+d x)}\right)^{-p} \cos [c+d x]^{-p} (e \cos [c+d x])^p \left(-\frac{i e^{-2i(c+d x)} \operatorname{Hypergeometric2F1}\left[-1-\frac{p}{2}, -p, -\frac{p}{2}, -e^{2i(c+d x)}\right]}{2+p} - \frac{4 e^{-i(c+d x)} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}-\frac{p}{2}, -p, \frac{1}{2}-\frac{p}{2}, -e^{2i(c+d x)}\right]}{1+p} + \frac{4 e^{i(c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}-\frac{p}{2}, -p, \frac{3}{2}-\frac{p}{2}, -e^{2i(c+d x)}\right]}{-1+p} - \frac{i e^{2i(c+d x)} \operatorname{Hypergeometric2F1}\left[1-\frac{p}{2}, -p, 2-\frac{p}{2}, -e^{2i(c+d x)}\right]}{-2+p} + \frac{6 i \operatorname{Hypergeometric2F1}\left[-p, -\frac{p}{2}, 1-\frac{p}{2}, -e^{2i(c+d x)}\right]}{p} \right) (a + a \sin [c+d x])^2$$

- **Problem 330: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (e \cos [c + d x])^p (a + a \sin [c + d x]) dx$$

Optimal (type 5, 93 leaves, 2 steps):

$$-\frac{1}{d e (1+p)} 2^{\frac{3}{2}+\frac{p}{2}} a (e \cos [c + d x])^{1+p} \text{Hypergeometric2F1}\left[\frac{1}{2}(-1-p), \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1-\sin [c + d x])\right] (1 + \sin [c + d x])^{\frac{1}{2}(-1-p)}$$

Result (type 5, 266 leaves):

$$\frac{1}{d(-1+p)p(1+p)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} 2^{-1-p} a \left(1 + e^{2i(c+dx)}\right)^{-1-p} \left(e^{-i(c+dx)} \left(1 + e^{2i(c+dx)}\right)\right)^{1+p} \\ \cos [c + d x]^{-p} (e \cos [c + d x])^p \left(-(-1+p)p \text{Hypergeometric2F1}\left[\frac{1}{2}(-1-p), -p, \frac{1-p}{2}, -e^{2i(c+dx)}\right] + \right. \\ \left. e^{i(c+dx)}(1+p) \left(e^{i(c+dx)}p \text{Hypergeometric2F1}\left[\frac{1-p}{2}, -p, \frac{3-p}{2}, -e^{2i(c+dx)}\right] + \right. \right. \\ \left. \left. 2i(-1+p) \text{Hypergeometric2F1}\left[-p, -\frac{p}{2}, 1-\frac{p}{2}, -e^{2i(c+dx)}\right]\right)\right) (1 + \sin [c + d x])$$

- **Problem 331: Unable to integrate problem.**

$$\int \frac{(e \cos [c + d x])^p}{a + a \sin [c + d x]} dx$$

Optimal (type 5, 95 leaves, 2 steps):

$$-\frac{1}{a d e (1+p)} 2^{\frac{1}{2}+\frac{p}{2}} (e \cos [c + d x])^{1+p} \text{Hypergeometric2F1}\left[\frac{3-p}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1-\sin [c + d x])\right] (1 + \sin [c + d x])^{\frac{1}{2}(-1-p)}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \cos [c + d x])^p}{a + a \sin [c + d x]} dx$$

- **Problem 332: Unable to integrate problem.**

$$\int \frac{(e \cos [c + d x])^p}{(a + a \sin [c + d x])^2} dx$$

Optimal (type 5, 93 leaves, 2 steps):

$$-\frac{1}{a^2 d e (1+p)} 2^{\frac{1}{2}(-3+p)} (e \cos [c + d x])^{1+p} \text{Hypergeometric2F1}\left[\frac{5-p}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1-\sin [c + d x])\right] (1 + \sin [c + d x])^{\frac{1}{2}(-1-p)}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \cos [c + d x])^p}{(a + a \sin [c + d x])^2} dx$$

■ **Problem 333: Unable to integrate problem.**

$$\int \frac{(e \cos [c + d x])^p}{(a + a \sin [c + d x])^3} dx$$

Optimal (type 5, 93 leaves, 2 steps):

$$-\frac{1}{a^3 d e (1+p)} 2^{\frac{1}{2}(-5+p)} (e \cos [c + d x])^{1+p} \text{Hypergeometric2F1}\left[\frac{7-p}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1 - \sin [c + d x])\right] (1 + \sin [c + d x])^{\frac{1}{2}(-1-p)}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \cos [c + d x])^p}{(a + a \sin [c + d x])^3} dx$$

■ **Problem 334: Unable to integrate problem.**

$$\int \frac{(e \cos [c + d x])^p}{(a + a \sin [c + d x])^8} dx$$

Optimal (type 5, 93 leaves, 2 steps):

$$-\frac{1}{a^8 d e (1+p)} 2^{\frac{1}{2}(-15+p)} (e \cos [c + d x])^{1+p} \text{Hypergeometric2F1}\left[\frac{17-p}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1 - \sin [c + d x])\right] (1 + \sin [c + d x])^{\frac{1}{2}(-1-p)}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \cos [c + d x])^p}{(a + a \sin [c + d x])^8} dx$$

■ **Problem 335: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (e \cos [c + d x])^p (a + a \sin [c + d x])^{7/2} dx$$

Optimal (type 5, 103 leaves, 3 steps):

$$-\left(2^{4+\frac{p}{2}} a^4 (e \cos [c + d x])^{1+p} \text{Hypergeometric2F1}\left[\frac{1}{2}(-6-p), \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1 - \sin [c + d x])\right] (1 + \sin [c + d x])^{-p/2}\right) / \left(d e (1+p) \sqrt{a + a \sin [c + d x]}\right)$$

Result (type 5, 629 leaves):

$$\frac{1}{d \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^7} (1 + i) 2^{-3-p} e^{ip(c+dx)} \left(e^{-i(c+dx)} + e^{i(c+dx)} \right)^p \left(1 + e^{2i(c+dx)} \right)^{-p}$$

$$\cos [c + dx]^{-p} \left(e \cos [c + dx] \right)^p \left(\frac{e^{-\frac{1}{2}i(7+2p)(c+dx)} \text{Hypergeometric2F1} \left[-\frac{7}{4} - \frac{p}{2}, -p, -\frac{3}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{7 + 2p} - \right.$$

$$\frac{7i e^{-\frac{1}{2}i(5+2p)(c+dx)} \text{Hypergeometric2F1} \left[-\frac{5}{4} - \frac{p}{2}, -p, -\frac{1}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{5 + 2p} -$$

$$\frac{21 e^{-\frac{1}{2}i(3+2p)(c+dx)} \text{Hypergeometric2F1} \left[-\frac{3}{4} - \frac{p}{2}, -p, \frac{1}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{3 + 2p} +$$

$$\frac{35i e^{-\frac{1}{2}i(1+2p)(c+dx)} \text{Hypergeometric2F1} \left[-\frac{1}{4} - \frac{p}{2}, -p, \frac{3}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{1 + 2p} +$$

$$\frac{35 e^{-\frac{1}{2}i(-1+2p)(c+dx)} \text{Hypergeometric2F1} \left[\frac{1}{4} - \frac{p}{2}, -p, \frac{5}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{-1 + 2p} -$$

$$\frac{21i e^{\frac{1}{2}i(3-2p)(c+dx)} \text{Hypergeometric2F1} \left[\frac{3}{4} - \frac{p}{2}, -p, \frac{7}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{-3 + 2p} - \frac{7 e^{\frac{1}{2}i(5-2p)(c+dx)} \text{Hypergeometric2F1} \left[\frac{5}{4} - \frac{p}{2}, -p, \frac{9}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{-5 + 2p} +$$

$$\left. \frac{i e^{-\frac{1}{2}i(-7+2p)(c+dx)} \text{Hypergeometric2F1} \left[\frac{7}{4} - \frac{p}{2}, -p, \frac{11}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{-7 + 2p} \right) (a(1 + \sin [c + dx]))^{7/2}$$

■ **Problem 336: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (e \cos [c + dx])^p (a + a \sin [c + dx])^{5/2} dx$$

Optimal (type 5, 103 leaves, 3 steps):

$$- \left(2^{3+\frac{p}{2}} a^3 (e \cos [c + dx])^{1+p} \text{Hypergeometric2F1} \left[\frac{1}{2} (-4 - p), \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2} (1 - \sin [c + dx]) \right] (1 + \sin [c + dx])^{-p/2} \right) /$$

$$\left(d e (1 + p) \sqrt{a + a \sin [c + dx]} \right)$$

Result (type 5, 504 leaves):

$$\begin{aligned}
& - \frac{1}{d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5} (1-i) 2^{-3-p} e^{ip(c+dx)} \left(e^{-i(c+dx)} + e^{i(c+dx)} \right)^p \left(1 + e^{2i(c+dx)} \right)^{-p} \\
& \cos[c+dx]^{-p} \left(e \cos[c+dx] \right)^p \left(- \frac{2 e^{-\frac{1}{2}i(5+2p)(c+dx)} \operatorname{Hypergeometric2F1}\left[-\frac{5}{4} - \frac{p}{2}, -p, -\frac{1}{4} - \frac{p}{2}, -e^{2i(c+dx)}\right]}{5+2p} + \right. \\
& \frac{10 i e^{-\frac{1}{2}i(3+2p)(c+dx)} \operatorname{Hypergeometric2F1}\left[-\frac{3}{4} - \frac{p}{2}, -p, \frac{1}{4} - \frac{p}{2}, -e^{2i(c+dx)}\right]}{3+2p} + \\
& \frac{20 e^{-\frac{1}{2}i(1+2p)(c+dx)} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4} - \frac{p}{2}, -p, \frac{3}{4} - \frac{p}{2}, -e^{2i(c+dx)}\right]}{1+2p} - \\
& \frac{20 i e^{\frac{1}{2}i(1-2p)(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4} - \frac{p}{2}, -p, \frac{5}{4} - \frac{p}{2}, -e^{2i(c+dx)}\right]}{-1+2p} - \\
& \frac{10 e^{\frac{1}{2}i(3-2p)(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4} - \frac{p}{2}, -p, \frac{7}{4} - \frac{p}{2}, -e^{2i(c+dx)}\right]}{-3+2p} + \\
& \left. \frac{2 i e^{-\frac{1}{2}i(-5+2p)(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{5}{4} - \frac{p}{2}, -p, \frac{9}{4} - \frac{p}{2}, -e^{2i(c+dx)}\right]}{-5+2p} \right) (a(1+\sin[c+dx]))^{5/2}
\end{aligned}$$

- **Problem 337: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (e \cos[c+dx])^p (a + a \sin[c+dx])^{3/2} dx$$

Optimal (type 5, 103 leaves, 3 steps):

$$\begin{aligned}
& - \left(2^{2+\frac{p}{2}} a^2 (e \cos[c+dx])^{1+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-2-p), \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1-\sin[c+dx])\right] (1+\sin[c+dx])^{-p/2} \right) / \\
& \left(d e (1+p) \sqrt{a + a \sin[c+dx]} \right)
\end{aligned}$$

Result (type 5, 378 leaves):

$$\begin{aligned}
& - \frac{1}{d \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^3} (1 + i) 2^{-2-p} e^{ip(c+dx)} \left(e^{-i(c+dx)} + e^{i(c+dx)} \right)^p \\
& \left(1 + e^{2i(c+dx)} \right)^{-p} \cos [c + dx]^{-p} \left(e \cos [c + dx] \right)^p \left(\frac{2 e^{-\frac{1}{2}i(3+2p)(c+dx)} \text{Hypergeometric2F1} \left[-\frac{3}{4} - \frac{p}{2}, -p, \frac{1}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{3 + 2p} \right. \\
& \frac{6 i e^{-\frac{1}{2}i(1+2p)(c+dx)} \text{Hypergeometric2F1} \left[-\frac{1}{4} - \frac{p}{2}, -p, \frac{3}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{1 + 2p} - \\
& \frac{6 e^{\frac{1}{2}i(1-2p)(c+dx)} \text{Hypergeometric2F1} \left[\frac{1}{4} - \frac{p}{2}, -p, \frac{5}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{-1 + 2p} + \\
& \left. \frac{2 i e^{-\frac{1}{2}i(-3+2p)(c+dx)} \text{Hypergeometric2F1} \left[\frac{3}{4} - \frac{p}{2}, -p, \frac{7}{4} - \frac{p}{2}, -e^{2i(c+dx)} \right]}{-3 + 2p} \right) (a(1 + \sin [c + dx]))^{3/2}
\end{aligned}$$

- **Problem 338: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (e \cos [c + dx])^p \sqrt{a + a \sin [c + dx]} dx$$

Optimal (type 5, 97 leaves, 3 steps):

$$\frac{2^{1+\frac{p}{2}} a (e \cos [c + dx])^{1+p} \text{Hypergeometric2F1} \left[-\frac{p}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2} (1 - \sin [c + dx]) \right] (1 + \sin [c + dx])^{-p/2}}{d e (1+p) \sqrt{a + a \sin [c + dx]}}$$

Result (type 5, 310 leaves):

$$\begin{aligned}
& \frac{1}{d (-1 + 2p) (1 + 2p) \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)} (1 + i) 2^{-p} e^{-\frac{1}{2}i dx} \cos [c + dx]^{-p} (e \cos [c + dx])^p \\
& \left(e^{i dx} (1 + 2p) \text{Hypergeometric2F1} \left[\frac{1}{4} (1 - 2p), -p, \frac{1}{4} (5 - 2p), -e^{2i dx} (\cos [c] + i \sin [c])^2 \right] \left(\cos \left[\frac{c}{2} \right] + i \sin \left[\frac{c}{2} \right] \right) + \right. \\
& (-1 + 2p) \text{Hypergeometric2F1} \left[\frac{1}{4} (-1 - 2p), -p, \frac{1}{4} (3 - 2p), -e^{2i dx} (\cos [c] + i \sin [c])^2 \right] \left(i \cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left. \right) \\
& \left(e^{-i dx} \left((1 + e^{2i dx}) \cos [c] + i (-1 + e^{2i dx}) \sin [c] \right) \right)^p (1 + e^{2i dx} \cos [2c] + i e^{2i dx} \sin [2c])^{-p} \sqrt{a (1 + \sin [c + dx])}
\end{aligned}$$

- **Problem 340: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos [c + dx])^p}{(a + a \sin [c + dx])^{3/2}} dx$$

Optimal (type 5, 102 leaves, 3 steps):

$$\frac{2^{-1+\frac{p}{2}} (e \cos[c+dx])^{1+p} \operatorname{Hypergeometric2F1}\left[\frac{4-p}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1-\sin[c+dx])\right] (1+\sin[c+dx])^{1-\frac{p}{2}}}{d e (1+p) (a+a \sin[c+dx])^{3/2}}$$

Result (type 5, 228 leaves):

$$\frac{1}{a d p (-4+p^2) \sqrt{2-2 \sin[c+dx]} (a(1+\sin[c+dx]))^{3/2}} 2^{-1+\frac{p}{2}} \cos[c+dx] (e \cos[c+dx])^p (1-\sin[c+dx])^{-p/2} \left(4 a p (2+p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-p), \frac{1}{2}(-2+p), \frac{p}{2}, \frac{1}{2}(1+\sin[c+dx])\right] + (-2+p)(1+\sin[c+dx]) \left(2 a (2+p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-p), \frac{p}{2}, \frac{2+p}{2}, \frac{1}{2}(1+\sin[c+dx])\right] + a p \operatorname{Hypergeometric2F1}\left[\frac{1-p}{2}, \frac{2+p}{2}, \frac{4+p}{2}, \frac{1}{2}(1+\sin[c+dx])\right] (1+\sin[c+dx]) \right) \right)$$

■ **Problem 341: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^p}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 5, 105 leaves, 3 steps):

$$\frac{2^{-2+\frac{p}{2}} (e \cos[c+dx])^{1+p} \operatorname{Hypergeometric2F1}\left[\frac{6-p}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1-\sin[c+dx])\right] (1+\sin[c+dx])^{1-\frac{p}{2}}}{a d e (1+p) (a+a \sin[c+dx])^{3/2}}$$

Result (type 5, 304 leaves):

$$\frac{1}{a^4 d (-4+p) (-2+p) p (2+p) \sqrt{2-2 \sin[c+dx]} (1+\sin[c+dx])^3} 2^{-2+\frac{p}{2}} \cos[c+dx] (e \cos[c+dx])^p (1-\sin[c+dx])^{-p/2} \sqrt{a(1+\sin[c+dx])} \left(8 a p (-4+p^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-p), \frac{1}{2}(-4+p), \frac{1}{2}(-2+p), \frac{1}{2}(1+\sin[c+dx])\right] + (-4+p)(1+\sin[c+dx]) \left(4 a p (2+p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-p), \frac{1}{2}(-2+p), \frac{p}{2}, \frac{1}{2}(1+\sin[c+dx])\right] + (-2+p)(1+\sin[c+dx]) \left(2 a (2+p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-p), \frac{p}{2}, \frac{2+p}{2}, \frac{1}{2}(1+\sin[c+dx])\right] + a p \operatorname{Hypergeometric2F1}\left[\frac{1-p}{2}, \frac{2+p}{2}, \frac{4+p}{2}, \frac{1}{2}(1+\sin[c+dx])\right] (1+\sin[c+dx]) \right) \right) \right)$$

■ **Problem 343: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^7 (a+a \sin[c+dx])^m dx$$

Optimal (type 3, 109 leaves, 3 steps):

$$\frac{8 (a + a \sin[c + dx])^{4+m}}{a^4 d (4+m)} - \frac{12 (a + a \sin[c + dx])^{5+m}}{a^5 d (5+m)} + \frac{6 (a + a \sin[c + dx])^{6+m}}{a^6 d (6+m)} - \frac{(a + a \sin[c + dx])^{7+m}}{a^7 d (7+m)}$$

Result (type 3, 796 leaves):

$$\frac{1}{d} (a (1 + \sin[c + dx]))^m \left(\frac{6144 + 1084 m + 117 m^2 + 5 m^3}{16 (4+m) (5+m) (6+m) (7+m)} + \frac{(29400 + 2578 m + 171 m^2 + 5 m^3) \left(-\frac{1}{128} i \cos[c + dx] + \frac{1}{128} \sin[c + dx]\right)}{(4+m) (5+m) (6+m) (7+m)} + \frac{(29400 + 2578 m + 171 m^2 + 5 m^3) \left(\frac{1}{128} i \cos[c + dx] + \frac{1}{128} \sin[c + dx]\right)}{(4+m) (5+m) (6+m) (7+m)} + \frac{(804 m + 109 m^2 + 5 m^3) \left(\frac{3}{64} \cos[2(c + dx)] - \frac{3}{64} i \sin[2(c + dx)]\right)}{(4+m) (5+m) (6+m) (7+m)} + \frac{(804 m + 109 m^2 + 5 m^3) \left(\frac{3}{64} \cos[2(c + dx)] + \frac{3}{64} i \sin[2(c + dx)]\right)}{(4+m) (5+m) (6+m) (7+m)} + \frac{(1960 + 1070 m + 93 m^2 + 3 m^3) \left(-\frac{3}{128} i \cos[3(c + dx)] + \frac{3}{128} \sin[3(c + dx)]\right)}{(4+m) (5+m) (6+m) (7+m)} + \frac{(1960 + 1070 m + 93 m^2 + 3 m^3) \left(\frac{3}{128} i \cos[3(c + dx)] + \frac{3}{128} \sin[3(c + dx)]\right)}{(4+m) (5+m) (6+m) (7+m)} + \frac{(44 m + 17 m^2 + m^3) \left(\frac{3}{32} \cos[4(c + dx)] - \frac{3}{32} i \sin[4(c + dx)]\right)}{(4+m) (5+m) (6+m) (7+m)} + \frac{(44 m + 17 m^2 + m^3) \left(\frac{3}{32} \cos[4(c + dx)] + \frac{3}{32} i \sin[4(c + dx)]\right)}{(4+m) (5+m) (6+m) (7+m)} + \frac{(294 + 103 m + 5 m^2) \left(-\frac{1}{128} i \cos[5(c + dx)] + \frac{1}{128} \sin[5(c + dx)]\right)}{(5+m) (6+m) (7+m)} + \frac{(294 + 103 m + 5 m^2) \left(\frac{1}{128} i \cos[5(c + dx)] + \frac{1}{128} \sin[5(c + dx)]\right)}{(5+m) (6+m) (7+m)} + \frac{\frac{1}{64} m \cos[6(c + dx)] - \frac{1}{64} i m \sin[6(c + dx)]}{(6+m) (7+m)} + \frac{\frac{1}{64} m \cos[6(c + dx)] + \frac{1}{64} i m \sin[6(c + dx)]}{(6+m) (7+m)} + \frac{-\frac{1}{128} i \cos[7(c + dx)] + \frac{1}{128} \sin[7(c + dx)]}{7+m} + \frac{\frac{1}{128} i \cos[7(c + dx)] + \frac{1}{128} \sin[7(c + dx)]}{7+m} \right)$$

■ **Problem 347: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sec[c + dx] (a + a \sin[c + dx])^m dx$$

Optimal (type 5, 40 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}\left[1, m, 1+m, \frac{1}{2}(1 + \sin[c + dx])\right] (a + a \sin[c + dx])^m}{2 d m}$$

Result (type 6, 6933 leaves):

$$-\left(\cos\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^{2m} \cot\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 (a + a \sin[c + dx])^m \right. \\ \left. \left(\left(2 \text{AppellF1}\left[1, 1-2m, 2m, 2, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^4 \right) \right) / \right)$$

$$\begin{aligned}
& \left(2 \operatorname{AppellF1}\left[1, 1-2m, 2m, 2, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \\
& \quad \left(-2m \operatorname{AppellF1}\left[2, 1-2m, 1+2m, 3, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \\
& \quad \left. (1-2m) \operatorname{AppellF1}\left[2, 2-2m, 2m, 3, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) + \\
& \quad \left((1+m) \operatorname{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2}\left(1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right), 1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \left(-1 + \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^2 \right) / \\
& \quad \left((1+2m) \left(-2(1+m) \operatorname{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2}\left(1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right), 1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \\
& \quad \left. \left(\operatorname{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2}\left(1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right), 1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + m \operatorname{AppellF1}\left[2+2m, 1+2m, \right. \right. \right. \\
& \quad \left. \left. \left. 1, 3+2m, \frac{1}{2}\left(1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right), 1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \left(-1 + \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right) \right) \right) \right) / \\
& \quad \left(d \left(\cos\left[\frac{\pi}{4} + \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right)\right] \right) \left(\cos\left[\frac{\pi}{4} + \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right)\right] + \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right)\right] \right) \right) \\
& \quad \left(1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) \\
& \quad \left(\frac{1}{2\left(1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^2} \cos\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^{2m} \operatorname{Csc}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right] \operatorname{Sec}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right] \right) \\
& \quad \left(\left(2 \operatorname{AppellF1}\left[1, 1-2m, 2m, 2, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^4 \right) / \right. \\
& \quad \left(2 \operatorname{AppellF1}\left[1, 1-2m, 2m, 2, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \\
& \quad \left(-2m \operatorname{AppellF1}\left[2, 1-2m, 1+2m, 3, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \\
& \quad \left. (1-2m) \operatorname{AppellF1}\left[2, 2-2m, 2m, 3, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) + \left((1+m) \right. \\
& \quad \left. \operatorname{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2}\left(1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right), 1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \left(-1 + \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^2 \right) / \\
& \quad \left((1+2m) \left(-2(1+m) \operatorname{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2}\left(1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right), 1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right.
\end{aligned}$$

$$1, 3 + 2 m, \frac{1}{2} \left(1 - \operatorname{Tan} \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right), 1 - \operatorname{Tan} \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right) \left(-1 + \operatorname{Tan} \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right) \right) \right) \right) \right) \right) \right) \right) \right)$$

- **Problem 348: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^3 (a + a \operatorname{Sin}[c + d x])^m dx$$

Optimal (type 5, 47 leaves, 2 steps):

$$\frac{a \operatorname{Hypergeometric2F1} \left[2, -1 + m, m, \frac{1}{2} (1 + \operatorname{Sin}[c + d x]) \right] (a + a \operatorname{Sin}[c + d x])^{-1+m}}{4 d (1 - m)}$$

Result (type 6, 27 160 leaves): Display of huge result suppressed!

- **Problem 349: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^5 (a + a \operatorname{Sin}[c + d x])^m dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{a^2 \operatorname{Hypergeometric2F1} \left[3, -2 + m, -1 + m, \frac{1}{2} (1 + \operatorname{Sin}[c + d x]) \right] (a + a \operatorname{Sin}[c + d x])^{-2+m}}{8 d (2 - m)}$$

Result (type 5, 443 leaves):

$$\begin{aligned} & -\frac{1}{32 d} \left(\operatorname{Sec} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right)^m (a + a \operatorname{Sin}[c + d x])^m \left(1 + \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right)^{-m} \\ & \left(-\frac{6 \left(1 + \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right)^m \operatorname{Hypergeometric2F1} \left[m, m, 1 + m, -\operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right]}{m} - \frac{1}{1 + m} \right. \\ & 4 \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \left(1 + \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right)^m \operatorname{Hypergeometric2F1} \left[m, 1 + m, 2 + m, -\operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right] - \\ & \frac{1}{2 + m} \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^4 \left(1 + \operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right)^m \operatorname{Hypergeometric2F1} \left[m, 2 + m, 3 + m, -\operatorname{Cot} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right] + \\ & \left. \frac{4 \left(-1 - \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 + \left(1 + \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right)^m \right)}{-1 + m} + \frac{1}{(-2 + m)(-1 + m)} \right) \\ & \left(-1 + \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^4 + \left(1 + \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 \right)^m - m \left(\operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^2 + \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x \right) \right]^4 \right) \right) \end{aligned}$$

■ **Problem 350: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^4 (a+a \sin [c+d x])^m d x$$

Optimal (type 5, 83 leaves, 3 steps):

$$-\frac{1}{5 d} 2^{\frac{5}{2}+m} a^2 \cos [c+d x]^5 \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, -\frac{3}{2}-m, \frac{7}{2}, \frac{1}{2}(1-\sin [c+d x])\right] (1+\sin [c+d x])^{-\frac{1}{2}-m} (a+a \sin [c+d x])^{-2+m}$$

Result (type 6, 9362 leaves):

$$\begin{aligned} & -\left(\left(3072 \cos [c+d x]^4 (a+a \sin [c+d x])^m \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right] \left(1-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]\right)^{2(2+m)} \left(\frac{1}{1+\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2}\right)^{9+2 m}\right.\right. \\ & \left.\left(\left(\operatorname{AppellF1}\left[\frac{1}{2},-2(2+m), 7+2 m, \frac{3}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right] \left(1+\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right)^2\right) / \right. \\ & \left.\left(3 \operatorname{AppellF1}\left[\frac{1}{2},-2(2+m), 7+2 m, \frac{3}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right]-\right.\right. \\ & \left.\left.2\left(2(2+m) \operatorname{AppellF1}\left[\frac{3}{2},-3-2 m, 7+2 m, \frac{5}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right]+ \right.\right. \\ & \left.\left.(7+2 m) \operatorname{AppellF1}\left[\frac{3}{2},-2(2+m), 2(4+m), \frac{5}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right] \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right)+ \right. \\ & \left.\operatorname{AppellF1}\left[\frac{1}{2},-2(2+m), 9+2 m, \frac{3}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right] / \right. \\ & \left.\left(3 \operatorname{AppellF1}\left[\frac{1}{2},-2(2+m), 9+2 m, \frac{3}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right]-\right.\right. \\ & \left.\left.2\left(2(2+m) \operatorname{AppellF1}\left[\frac{3}{2},-3-2 m, 9+2 m, \frac{5}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right]+ \right.\right. \\ & \left.\left.(9+2 m) \operatorname{AppellF1}\left[\frac{3}{2},-2(2+m), 2(5+m), \frac{5}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right] \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right)+ \right. \\ & \left.\left(2 \operatorname{AppellF1}\left[\frac{1}{2},-2(2+m), 2(4+m), \frac{3}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right] \left(1+\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right)\right) / \right. \\ & \left.\left(-3 \operatorname{AppellF1}\left[\frac{1}{2},-2(2+m), 2(4+m), \frac{3}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right]+ \right.\right. \\ & \left.\left.4\left((2+m) \operatorname{AppellF1}\left[\frac{3}{2},-3-2 m, 8+2 m, \frac{5}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right]+ \right.\right. \\ & \left.\left.(4+m) \operatorname{AppellF1}\left[\frac{3}{2},-2(2+m), 9+2 m, \frac{5}{2}, \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2,-\tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right] \tan \left[\frac{1}{4}\left(-c+\frac{\pi}{2}-d x\right)\right]^2\right)\right) / \end{aligned}$$

$$\left((1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 4+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \right. \\ \left. (2+m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 5+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right)^2 \right)$$

■ **Problem 352: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^2 (a+a\sin[c+dx])^m dx$$

Optimal (type 5, 73 leaves, 3 steps):

$$\frac{1}{d} 2^{-\frac{1}{2}+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{3}{2}-m, \frac{1}{2}, \frac{1}{2}(1-\sin[c+dx])\right] \sec[c+dx] (1+\sin[c+dx])^{\frac{1}{2}-m} (a+a\sin[c+dx])^m$$

Result (type 6, 12061 leaves):

$$-\left(\left(\cot\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] (a+a\sin[c+dx])^m \left(1-\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{2(-1+m)} \right. \right. \\ \left. \left(\frac{1}{1+\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2} \right)^{2m} \left(-\left(\left(5 \operatorname{AppellF1}\left[-\frac{1}{2}, 2-2m, 2m, \frac{1}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right) \right) \right) \right. \\ \left(\operatorname{AppellF1}\left[-\frac{1}{2}, 2-2m, 2m, \frac{1}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] - \right. \\ \left. 4 \left(m \operatorname{AppellF1}\left[\frac{1}{2}, 2-2m, 1+2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + (-1+m) \right. \right. \\ \left. \left. \operatorname{AppellF1}\left[\frac{1}{2}, 3-2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right) \right) + \\ \left(15 \operatorname{AppellF1}\left[\frac{1}{2}, 2-2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right) \left. \right) / \\ \left(\operatorname{AppellF1}\left[\frac{1}{2}, 2-2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] - \right. \\ \left. \frac{4}{3} \left(m \operatorname{AppellF1}\left[\frac{3}{2}, 2-2m, 1+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \right. \right. \\ \left. \left. (-1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 3-2m, 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right) + \\ \left(25 \operatorname{AppellF1}\left[\frac{3}{2}, 2-2m, 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^4 \right) \left. \right) / \\ \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, 2-2m, 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] - \right.$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \operatorname{Sec}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] - 4 \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \\
& \left(m\left(-\frac{7}{18}(1+2m) \operatorname{AppellF1}\left[\frac{9}{2}, 2-2m, 2+2m, \frac{11}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right]\right.\right. \\
& \quad \operatorname{Sec}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] + \frac{7}{18}(2-2m) \operatorname{AppellF1}\left[\frac{9}{2}, 3-2m, 1+2m, \frac{11}{2},\right. \\
& \quad \left.\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] \left.\right) + \\
& (-1+m) \left(-\frac{7}{9}m \operatorname{AppellF1}\left[\frac{9}{2}, 3-2m, 1+2m, \frac{11}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right]\right. \\
& \quad \operatorname{Sec}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] + \frac{7}{18}(3-2m) \operatorname{AppellF1}\left[\frac{9}{2}, 4-2m, 2m, \frac{11}{2},\right. \\
& \quad \left.\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] \left.\right) \left.\right) / \\
& \left(7 \operatorname{AppellF1}\left[\frac{5}{2}, 2-2m, 2m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] - 4\left(m \operatorname{AppellF1}\left[\frac{7}{2}, 2-2m,\right.\right.\right. \\
& \quad \left.\left.1+2m, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + (-1+m) \right. \\
& \quad \left.\operatorname{AppellF1}\left[\frac{7}{2}, 3-2m, 2m, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^2 \left.\right) \left.\right) \left.\right)
\end{aligned}$$

■ **Problem 353: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^4 (a+a \operatorname{Sin}[c+dx])^m dx$$

Optimal (type 5, 83 leaves, 3 steps):

$$\frac{1}{3ad} 2^{-\frac{3}{2}+m} \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{5}{2}-m, -\frac{1}{2}, \frac{1}{2}(1-\operatorname{Sin}[c+dx])\right] \operatorname{Sec}[c+dx]^3 (1+\operatorname{Sin}[c+dx])^{\frac{1}{2}-m} (a+a \operatorname{Sin}[c+dx])^{1+m}$$

Result (type 6, 3545 leaves):

$$\begin{aligned}
& \left(\operatorname{AppellF1}\left[-\frac{3}{2}, 4-2m, -7+2m, -\frac{1}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right]\right. \\
& \quad \left.\operatorname{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^{2m} \operatorname{Csc}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^{13} \operatorname{Sec}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] (a+a \operatorname{Sin}[c+dx])^m\right) / \\
& \left(192d\left(-1+\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^4\left(2(-7+2m) \operatorname{AppellF1}\left[-\frac{1}{2}, 4-2m, -6+2m, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right]\right.\right. \\
& \quad \left.\left.4(-2+m) \operatorname{AppellF1}\left[-\frac{1}{2}, 5-2m, -7+2m, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] +\right.\right.
\end{aligned}$$

$$\left(192 \left(-1 + \cot \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right] \right)^2 \right)^4 \left(2 (-7 + 2m) \operatorname{AppellF1} \left[-\frac{1}{2}, 4 - 2m, -6 + 2m, \frac{1}{2}, \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right] + \right. \\ \left. 4 (-2 + m) \operatorname{AppellF1} \left[-\frac{1}{2}, 5 - 2m, -7 + 2m, \frac{1}{2}, \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right] + \right. \\ \left. \operatorname{AppellF1} \left[-\frac{3}{2}, 4 - 2m, -7 + 2m, -\frac{1}{2}, \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right] \cot \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right) \right)$$

- **Problem 354: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (e \cos [c + dx])^{5/2} (a + a \sin [c + dx])^m dx$$

Optimal (type 5, 88 leaves, 3 steps):

$$-\frac{1}{7de} 2^{\frac{11}{4}+m} a (e \cos [c + dx])^{7/2} \operatorname{Hypergeometric2F1} \left[\frac{7}{4}, -\frac{3}{4} - m, \frac{11}{4}, \frac{1}{2} (1 - \sin [c + dx]) \right] (1 + \sin [c + dx])^{-\frac{3}{4}-m} (a + a \sin [c + dx])^{-1+m}$$

Result (type 6, 32821 leaves): Display of huge result suppressed!

- **Problem 355: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (e \cos [c + dx])^{3/2} (a + a \sin [c + dx])^m dx$$

Optimal (type 5, 88 leaves, 3 steps):

$$-\frac{1}{5de} 2^{\frac{9}{4}+m} a (e \cos [c + dx])^{5/2} \operatorname{Hypergeometric2F1} \left[\frac{5}{4}, -\frac{1}{4} - m, \frac{9}{4}, \frac{1}{2} (1 - \sin [c + dx]) \right] (1 + \sin [c + dx])^{-\frac{1}{4}-m} (a + a \sin [c + dx])^{-1+m}$$

Result (type 6, 13703 leaves):

$$\left(64 \cos \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]^{-2m} (e \cos [c + dx])^{3/2} (a + a \sin [c + dx])^m \right. \\ \left(\cos \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]^{2m} \cos [c + dx]^{7/2} + \cos \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx \right) \right]^{2m} \cos [c + dx]^{3/2} \sin [c + dx]^2 \right) \\ \left(1 - \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right)^{2m} \left(\frac{1}{1 + \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2} \right)^{3+2m} \sqrt{\frac{\tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right] - \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^3}{\left(1 + \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right)^2}} \\ \left(\left(25 \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2} - 2m, 3 + 2m, \frac{5}{4}, \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right] \left(1 + \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right) \right) / \right. \\ \left(-5 \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2} - 2m, 3 + 2m, \frac{5}{4}, \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right] + \right. \\ \left. 2 \left((6 + 4m) \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2} - 2m, 4 + 2m, \frac{9}{4}, \tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx \right) \right]^2 \right] + \right. \right)$$

$$\begin{aligned}
 & (1 + 4 m) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2} - 2 m, 3 + 2 m, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2 \Big) + \\
 & \left(25 \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2} - 2 m, 4 + 2 m, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \right) / \\
 & \left(5 \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2} - 2 m, 4 + 2 m, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] - \right. \\
 & 2 \left(4 (2 + m) \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2 m, 5 + 2 m, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \right) + \\
 & (1 + 4 m) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2} - 2 m, 4 + 2 m, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2 \Big) + \\
 & 9 \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2 \left(- \left(\left(\operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2 m, 3 + 2 m, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \right. \right. \right. \\
 & \left. \left. \left(1 + \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2 \right) \right) \right) / \left(-9 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2 m, 3 + 2 m, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \right) \Big) + \\
 & 2 \left((6 + 4 m) \operatorname{AppellF1}\left[\frac{9}{4}, -\frac{1}{2} - 2 m, 4 + 2 m, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \right) + \\
 & (1 + 4 m) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2} - 2 m, 3 + 2 m, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2 \Big) \Big) + \\
 & \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2 m, 4 + 2 m, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] / \\
 & \left(-9 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2 m, 4 + 2 m, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \right) + \\
 & 2 \left(4 (2 + m) \operatorname{AppellF1}\left[\frac{9}{4}, -\frac{1}{2} - 2 m, 5 + 2 m, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \right) + \\
 & (1 + 4 m) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2} - 2 m, 4 + 2 m, \frac{13}{4}, \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2 \Big) \Big) \Big) \Big) \Big) / \\
 & \left(5 d \operatorname{Cos}[c + d x]^{3/2} \left(\frac{64}{5} m \operatorname{Sec}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right] \left(1 - \operatorname{Tan}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - d x\right)\right]^2 \right)^{-1+2 m} \right) \right) \Big) \Big) \Big) \Big) \Big) /
 \end{aligned}$$

$$\left. \left. \left. \left. \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2} - 2m, 4 + 2m, \frac{13}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^2\right)^2\right)^2\right)$$

■ **Problem 356: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{e \cos[c + dx]} (a + a \sin[c + dx])^m dx$$

Optimal (type 5, 88 leaves, 3 steps):

$$-\frac{1}{3de} 2^{7+m} a (e \cos[c + dx])^{3/2} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{1}{4} - m, \frac{7}{4}, \frac{1}{2} (1 - \sin[c + dx])\right] (1 + \sin[c + dx])^{\frac{1}{4}-m} (a + a \sin[c + dx])^{-1+m}$$

Result (type 6, 3061 leaves):

$$\begin{aligned} & - \left(\left(28 \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right. \right. \\ & \quad \left. \left. \cos\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^{2m} \sqrt{\cos[c + dx]} \sqrt{e \cos[c + dx]} \sin\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] (a + a \sin[c + dx])^m \right) \right) / \\ & \left(d \left(21 \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - 6 \right. \right. \\ & \quad \left(4 (1 + m) \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2} - 2m, 3 + 2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \\ & \quad \left. (1 + 4m) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2} - 2m, 2 + 2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) \\ & \left. \left(\left(14 \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \cos\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^{1+2m} \sqrt{\cos[c + dx]} \right) \right) / \right. \\ & \left(21 \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - \right. \\ & \quad \left. 6 \left(4 (1 + m) \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2} - 2m, 3 + 2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (1 + 4m) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2} - 2m, 2 + 2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) \right) - \\ & \left(28m \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \cos\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^{-1+2m} \right. \\ & \quad \left. \sqrt{\cos[c + dx]} \sin\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) / \left(21 \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - \right. \end{aligned}$$

$$\begin{aligned}
& 6 \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \left(4(1+m) \left(-\frac{7}{22}(3+2m) \operatorname{AppellF1}\left[\frac{11}{4}, -\frac{1}{2}-2m, 4+2m, \frac{15}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, \right.\right.\right. \\
& \quad \left. - \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right) \operatorname{Sec}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right] + \frac{7}{22}\left(-\frac{1}{2}-2m\right) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}-2m, \right. \\
& \quad \left. 3+2m, \frac{15}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right) \operatorname{Sec}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right] \Bigg) + \\
& (1+4m) \left(-\frac{7}{22}(2+2m) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}-2m, 3+2m, \frac{15}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right] + \frac{7}{22}\left(\frac{1}{2}-2m\right) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}-2m, 2+2m, \frac{15}{4}, \right.\right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right) \operatorname{Sec}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right] \Bigg) \Bigg) \Bigg) / \\
& \left(21 \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}-2m, 2+2m, \frac{7}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - \right. \\
& \quad \left. 6 \left(4(1+m) \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}-2m, 3+2m, \frac{11}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + (1+4m) \right.\right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}-2m, 2+2m, \frac{11}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 357: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[c + dx])^m}{\sqrt{e \cos[c + dx]}} dx$$

Optimal (type 5, 86 leaves, 3 steps):

$$-\frac{1}{de} 2^{\frac{5}{4}+m} a \sqrt{e \cos[c + dx]} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}-m, \frac{5}{4}, \frac{1}{2}(1 - \sin[c + dx])\right] (1 + \sin[c + dx])^{\frac{3}{4}-m} (a + a \sin[c + dx])^{-1+m}$$

Result (type 6, 3947 leaves):

$$\begin{aligned}
& - \left(\left(10 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}-2m, 2m, \frac{5}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right. \right. \\
& \quad \left. \left. \cos\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^{2m} \sqrt{\cos[c + dx]} \operatorname{Sec}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 (a + a \sin[c + dx])^m \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) \Bigg) / \\
& \left(d \sqrt{e \cos[c + dx]} \left(-1 + \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right) \left(-8m \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}-2m, 1+2m, \frac{9}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \\
& \quad \left. (2-8m) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}-2m, 2m, \frac{9}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \\
& \quad \left. 5 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}-2m, 2m, \frac{5}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] \operatorname{Csc}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 - \frac{5}{18}\left(\frac{3}{2}-2m\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{2}-2m, \right. \\
& \left. 2m, \frac{13}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] \operatorname{Csc}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right]^2 + \\
& \frac{5}{2} \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}-2m, 2m, \frac{5}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] + \\
& 5\left(\frac{1}{5} m \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}-2m, 1+2m, \frac{9}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] \right. \\
& \left. \operatorname{Csc}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 - \frac{1}{10}\left(\frac{1}{2}-2m\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}-2m, 2m, \frac{9}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right. \\
& \left. \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] \operatorname{Csc}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) / \\
& \left(\left(-1+\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2\right) \left(-8 m \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}-2m, 1+2m, \frac{9}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \right. \\
& \left.(2-8 m) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}-2m, 2m, \frac{9}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \right. \\
& \left. 5 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}-2m, 2m, \frac{5}{4}, \operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^2\right) \left.\right) \left.\right) \left.\right) \left.\right) \left.\right)
\end{aligned}$$

■ **Problem 358: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[c + dx])^m}{(e \cos[c + dx])^{3/2}} dx$$

Optimal (type 5, 82 leaves, 3 steps):

$$\frac{2^{\frac{3}{4}+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{5}{4}-m, \frac{3}{4}, \frac{1}{2}(1-\sin[c+dx])\right] (1+\sin[c+dx])^{\frac{1}{4}-m} (a+a \sin[c+dx])^m}{d e \sqrt{e \cos[c+dx]}}$$

Result (type 6, 10902 leaves):

$$-\left(\left(\operatorname{Cot}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right] (a+a \sin[c+dx])^m \left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^{2(-1+m)} \left(\frac{1}{1+\operatorname{Tan}\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}\right)^{-1+2m}\right)\right)$$

$$\begin{aligned}
& \sqrt{\frac{\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right] - \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^3}{\left(1 + \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]\right)^2}} \left(\left(63 \operatorname{AppellF1}\left[-\frac{1}{4}, \frac{3}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] \right) \right) / \\
& \left(-3 \operatorname{AppellF1}\left[-\frac{1}{4}, \frac{3}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] + \right. \\
& \quad 2 \left(4m \operatorname{AppellF1}\left[\frac{3}{4}, \frac{3}{2} - 2m, 1 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] + \right. \\
& \quad \quad \left. (-3 + 4m) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{5}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] \right) \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2 \Big) + \\
& \left(98 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{3}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2 \right) / \\
& \left(7 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{3}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] - \right. \\
& \quad 2 \left(4m \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2} - 2m, 1 + 2m, \frac{11}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] + \right. \\
& \quad \quad \left. (-3 + 4m) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{5}{2} - 2m, 2m, \frac{11}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] \right) \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2 \Big) + \\
& \left(33 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2} - 2m, 2m, \frac{11}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^4 \right) / \\
& \left(11 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2} - 2m, 2m, \frac{11}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] - \right. \\
& \quad 2 \left(4m \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2} - 2m, 1 + 2m, \frac{15}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] + \right. \\
& \quad \quad \left. (-3 + 4m) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{5}{2} - 2m, 2m, \frac{15}{4}, \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2, -\tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2\right] \right) \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2 \Big) \Big) / \\
& \left(21 d (e \operatorname{Cos}[c + dx])^{3/2} \left(-\frac{1}{42} (-1 + 2m) \operatorname{Sec}\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2 \left(1 - \tan\left[\frac{1}{4}(-c + \frac{\pi}{2} - dx)\right]^2 \right)^{2(-1+m)} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2} \right)^{2m} \sqrt{\frac{\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right] - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^3}{\left(1 + \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^2}} \\
& \left(\left(63 \operatorname{AppellF1}\left[-\frac{1}{4}, \frac{3}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right) / \right. \\
& \quad \left(-3 \operatorname{AppellF1}\left[-\frac{1}{4}, \frac{3}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \\
& \quad \left. 2 \left(4m \operatorname{AppellF1}\left[\frac{3}{4}, \frac{3}{2} - 2m, 1 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (-3 + 4m) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{5}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) + \\
& \quad \left(98 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{3}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) / \\
& \quad \left(7 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{3}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - \right. \\
& \quad \left. 2 \left(4m \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2} - 2m, 1 + 2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (-3 + 4m) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{5}{2} - 2m, 2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) + \\
& \quad \left(33 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2} - 2m, 2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^4 \right) / \\
& \quad \left(11 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2} - 2m, 2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - \right. \\
& \quad \left. 2 \left(4m \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2} - 2m, 1 + 2m, \frac{15}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (-3 + 4m) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{5}{2} - 2m, 2m, \frac{15}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) \Big) - \\
& \frac{1}{21} (-1+m) \operatorname{Sec}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \left(1 - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^{-1+2(-1+m)} \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2}\right)^{-1+2m}
\end{aligned}$$

$$\sqrt{\frac{\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right] - \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^3}{\left(1 + \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^2}}$$

$$\begin{aligned}
& - \left((a + a \sin[c + dx])^m \left(1 - \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right)^{-3+2m} \left(\frac{1}{1 + \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2} \right)^{-1+2m} \sqrt{\frac{\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right] - \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^3}{\left(1 + \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^2}} \right. \\
& \left(- \left(\left(195 \operatorname{AppellF1}\left[-\frac{3}{4}, \frac{5}{2} - 2m, 2m, \frac{1}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \operatorname{Cot}\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) / \right. \\
& \left(\operatorname{AppellF1}\left[-\frac{3}{4}, \frac{5}{2} - 2m, 2m, \frac{1}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - \right. \\
& \left. 2 \left(4m \operatorname{AppellF1}\left[\frac{1}{4}, \frac{5}{2} - 2m, 1 + 2m, \frac{5}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + (-5 + 4m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{7}{2} - 2m, 2m, \frac{5}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) \right) + \\
& \left(11700 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{5}{2} - 2m, 2m, \frac{5}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right) / \\
& \left(5 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{5}{2} - 2m, 2m, \frac{5}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - \right. \\
& \left. 2 \left(4m \operatorname{AppellF1}\left[\frac{5}{4}, \frac{5}{2} - 2m, 1 + 2m, \frac{9}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \right. \\
& \left. \left. (-5 + 4m) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{2} - 2m, 2m, \frac{9}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) + \\
& \left(6318 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{5}{2} - 2m, 2m, \frac{9}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) / \\
& \left(9 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{5}{2} - 2m, 2m, \frac{9}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - \right. \\
& \left. 2 \left(4m \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{2} - 2m, 1 + 2m, \frac{13}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \right. \\
& \left. \left. (-5 + 4m) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{7}{2} - 2m, 2m, \frac{13}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) + \\
& \left(3380 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{2} - 2m, 2m, \frac{13}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^4 \right) / \\
& \left(13 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{2} - 2m, 2m, \frac{13}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] - \right. \\
& \left. 2 \left(4m \operatorname{AppellF1}\left[\frac{13}{4}, \frac{5}{2} - 2m, 1 + 2m, \frac{17}{4}, \tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4} \left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (-5 + 4 m) \operatorname{AppellF1}\left[\frac{13}{4}, \frac{7}{2} - 2 m, 2 m, \frac{17}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2 + \\
& \left(765 \operatorname{AppellF1}\left[\frac{13}{4}, \frac{5}{2} - 2 m, 2 m, \frac{17}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^6\right) / \\
& \left(17 \operatorname{AppellF1}\left[\frac{13}{4}, \frac{5}{2} - 2 m, 2 m, \frac{17}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] - \right. \\
& \left. 2\left(4 m \operatorname{AppellF1}\left[\frac{17}{4}, \frac{5}{2} - 2 m, 1 + 2 m, \frac{21}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] + \right. \\
& \left. (-5 + 4 m) \operatorname{AppellF1}\left[\frac{17}{4}, \frac{7}{2} - 2 m, 2 m, \frac{21}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right) \right) / \\
& \left(2340 d \left(e \operatorname{Cos}[c + d x]\right)^{5/2} - \frac{1}{4680} (-1 + 2 m) \operatorname{Sec}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right] \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right)^{-3+2 m} \right. \\
& \left. \left(\frac{1}{1 + \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2}\right)^{2 m} \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right] - \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^3}{\left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right)^2}} \right. \\
& \left. - \left(\left(195 \operatorname{AppellF1}\left[-\frac{3}{4}, \frac{5}{2} - 2 m, 2 m, \frac{1}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Cot}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right) / \right. \\
& \left. \left(\operatorname{AppellF1}\left[-\frac{3}{4}, \frac{5}{2} - 2 m, 2 m, \frac{1}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] - \right. \right. \\
& \left. 2\left(4 m \operatorname{AppellF1}\left[\frac{1}{4}, \frac{5}{2} - 2 m, 1 + 2 m, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] + (-5 + 4 m) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{7}{2} - 2 m, \right. \right. \\
& \left. \left. 2 m, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right) + \left(11700 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{5}{2} - 2 m, \right. \right. \\
& \left. \left. 2 m, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \right) / \left(5 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{5}{2} - 2 m, 2 m, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, \right. \right. \\
& \left. \left. -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] - 2\left(4 m \operatorname{AppellF1}\left[\frac{5}{4}, \frac{5}{2} - 2 m, 1 + 2 m, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] + \right. \\
& \left. (-5 + 4 m) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{2} - 2 m, 2 m, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - d x\right)\right]^2\right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]-\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^3}{\left(1+\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^2}}} \left(1-\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-3+2m} \left(\frac{1}{1+\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}\right)^{-1+2m} \\
4680 & \left(-\left(\left(195 \operatorname{AppellF1}\left[-\frac{3}{4}, \frac{5}{2}-2m, 2m, \frac{1}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \cot\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) / \right. \\
& \left(\operatorname{AppellF1}\left[-\frac{3}{4}, \frac{5}{2}-2m, 2m, \frac{1}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] - \right. \\
& \left. 2\left(4m \operatorname{AppellF1}\left[\frac{1}{4}, \frac{5}{2}-2m, 1+2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \right. \\
& \left. \left(-5+4m\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{7}{2}-2m, 2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) + \\
& \left(11700 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{5}{2}-2m, 2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] / \right. \\
& \left(5 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{5}{2}-2m, 2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] - \right. \\
& \left. 2\left(4m \operatorname{AppellF1}\left[\frac{5}{4}, \frac{5}{2}-2m, 1+2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \right. \\
& \left. \left(-5+4m\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{2}-2m, 2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) + \\
& \left(6318 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{5}{2}-2m, 2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) / \\
& \left(9 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{5}{2}-2m, 2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] - \right. \\
& \left. 2\left(4m \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{2}-2m, 1+2m, \frac{13}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \right. \\
& \left. \left(-5+4m\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{7}{2}-2m, 2m, \frac{13}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) + \\
& \left(3380 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{2}-2m, 2m, \frac{13}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^4\right) / \\
& \left(13 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{2}-2m, 2m, \frac{13}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] - \right. \\
& \left. 2\left(4m \operatorname{AppellF1}\left[\frac{13}{4}, \frac{5}{2}-2m, 1+2m, \frac{17}{4}, \tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \right.
\end{aligned}$$

$$\text{AppellF1}\left[\frac{17}{4}, \frac{7}{2} - 2m, 2m, \frac{21}{4}, \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^2\right)$$

■ **Problem 371: Result more than twice size of optimal antiderivative.**

$$\int (e \cos[c + dx])^{-3-2m} (a + a \sin[c + dx])^m dx$$

Optimal (type 5, 70 leaves, 3 steps):

$$\frac{(e \cos[c + dx])^{-2(1+m)} \text{Hypergeometric2F1}\left[2, -1-m, -m, \frac{1}{2}(1 - \sin[c + dx])\right] (a + a \sin[c + dx])^{1+m}}{4 a d e (1+m)}$$

Result (type 5, 206 leaves):

$$\frac{1}{8 d e^3 m (-1 + m^2)} (e \cos[c + dx])^{-2m} \left(\sec\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2\right)^{-m} (a(1 + \sin[c + dx]))^m$$

$$\left(2(-1 + m^2) \text{Hypergeometric2F1}\left[-m, -m, 1-m, -\tan\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2\right] + (-1 + m) m \text{Csc}\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2 \left(\sec\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2\right)^m + m(1 + m) \text{Hypergeometric2F1}\left[1-m, -m, 2-m, -\tan\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2\right] \tan\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2\right)$$

■ **Problem 372: Result more than twice size of optimal antiderivative.**

$$\int (e \cos[c + dx])^{4-2m} (a + a \sin[c + dx])^m dx$$

Optimal (type 5, 89 leaves, 4 steps):

$$\frac{1}{5 d e} 2^{\frac{5}{2}-m} (e \cos[c + dx])^{5-2m} \text{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1}{2}(-3 + 2m), \frac{7}{2}, \frac{1}{2}(1 + \sin[c + dx])\right] (1 - \sin[c + dx])^{-\frac{5}{2}+m} (a + a \sin[c + dx])^m$$

Result (type 5, 200 leaves):

$$-\frac{1}{d(-1+2m)} 32 e^4 (e \cos[c + dx])^{-2m} \left(\text{Hypergeometric2F1}\left[\frac{1}{2}-m, 3-m, \frac{3}{2}-m, -\tan\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2\right] - 2 \text{Hypergeometric2F1}\left[\frac{1}{2}-m, 4-m, \frac{3}{2}-m, -\tan\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2\right] + \text{Hypergeometric2F1}\left[\frac{1}{2}-m, 5-m, \frac{3}{2}-m, -\tan\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2\right]\right)$$

$$\left(\sec\left[\frac{1}{4}(2c - \pi + 2dx)\right]^2\right)^{-m} (a(1 + \sin[c + dx]))^m \tan\left[\frac{1}{4}(2c - \pi + 2dx)\right]$$

■ **Problem 375: Result more than twice size of optimal antiderivative.**

$$\int (e \cos [c + d x])^{-2-2 m} (a + a \sin [c + d x])^m dx$$

Optimal (type 5, 87 leaves, 4 steps):

$$-\frac{1}{d e} 2^{-\frac{1}{2}-m} (e \cos [c + d x])^{-1-2 m} \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2}(3+2 m), \frac{1}{2}, \frac{1}{2}(1+\sin [c + d x])\right] (1-\sin [c + d x])^{\frac{1}{2}+m} (a + a \sin [c + d x])^m$$

Result (type 5, 186 leaves):

$$\frac{1}{2 d e^2 (-1 + 4 m^2)} (e \cos [c + d x])^{-2 m} \cot \left[\frac{1}{4} (2 c + \pi + 2 d x) \right] \\ \left((-1 + 2 m) \cot \left[\frac{1}{4} (2 c - \pi + 2 d x) \right]^2 \text{Hypergeometric2F1}\left[-\frac{1}{2} - m, -m, \frac{1}{2} - m, -\tan \left[\frac{1}{4} (2 c - \pi + 2 d x) \right]^2\right] + \right. \\ \left. (1 + 2 m) \text{Hypergeometric2F1}\left[\frac{1}{2} - m, -m, \frac{3}{2} - m, -\tan \left[\frac{1}{4} (2 c - \pi + 2 d x) \right]^2\right] \right) \left(\sec \left[\frac{1}{4} (2 c - \pi + 2 d x) \right]^2 \right)^{-m} (a (1 + \sin [c + d x]))^m$$

■ **Problem 380: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + d x]^3 (a + b \sin [c + d x]) dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{a \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{\sec [c + d x]^2 (b + a \sin [c + d x])}{2 d}$$

Result (type 3, 83 leaves):

$$\frac{1}{2 d} \left(a \left(-\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] + \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) + b \sec [c + d x]^2 + a \sec [c + d x] \tan [c + d x] \right)$$

■ **Problem 381: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + d x]^5 (a + b \sin [c + d x]) dx$$

Optimal (type 3, 61 leaves, 4 steps):

$$\frac{3 a \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{\sec [c + d x]^4 (b + a \sin [c + d x])}{4 d} + \frac{3 a \sec [c + d x] \tan [c + d x]}{8 d}$$

Result (type 3, 207 leaves):

$$\begin{aligned}
& - \frac{3 a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\
& \frac{b \operatorname{Sec}[c+d x]^4}{4 d} + \frac{a}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{3 a}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} - \\
& \frac{a}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{3 a}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}
\end{aligned}$$

- **Problem 389: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+d x] (a+b \sin[c+d x])^2 dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{(a+b \sin[c+d x])^3}{3 b d}$$

Result (type 3, 46 leaves):

$$\frac{a^2 \sin[c+d x]}{d} + \frac{a b \sin^2[c+d x]}{d} + \frac{b^2 \sin^3[c+d x]}{3 d}$$

- **Problem 391: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+d x]^3 (a+b \sin[c+d x])^2 dx$$

Optimal (type 3, 59 leaves, 3 steps):

$$\frac{(a^2-b^2) \operatorname{ArcTanh}[\sin[c+d x]]}{2 d} + \frac{\sec[c+d x]^2 (b+a \sin[c+d x]) (a+b \sin[c+d x])}{2 d}$$

Result (type 3, 139 leaves):

$$\begin{aligned}
& \frac{1}{4 d} \left(2 \left(-a^2 + b^2 \right) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right] + \right. \\
& \left. 2 \left(a^2 - b^2 \right) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right] + \frac{(a+b)^2}{\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{(a-b)^2}{\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2} \right)
\end{aligned}$$

- **Problem 392: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+d x]^5 (a+b \sin[c+d x])^2 dx$$

Optimal (type 3, 99 leaves, 4 steps):

$$\frac{(3a^2 - b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{8d} + \frac{\operatorname{Sec}[c + dx]^4 (b + a \operatorname{Sin}[c + dx]) (a + b \operatorname{Sin}[c + dx])}{4d} + \frac{\operatorname{Sec}[c + dx]^2 (2ab + (3a^2 - b^2) \operatorname{Sin}[c + dx])}{8d}$$

Result (type 3, 219 leaves):

$$\frac{1}{16d} \left(2(-3a^2 + b^2) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right] + \right. \\ \left. 2(3a^2 - b^2) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right] + \frac{(a + b)^2}{\left(\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^4} + \right. \\ \left. \frac{3a^2 + 2ab - b^2}{\left(\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^2} - \frac{(a - b)^2}{\left(\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^4} + \frac{-3a^2 + 2ab + b^2}{\left(\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^2} \right)$$

■ **Problem 402: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx] (a + b \operatorname{Sin}[c + dx])^3 dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{(a + b \operatorname{Sin}[c + dx])^4}{4bd}$$

Result (type 3, 67 leaves):

$$\frac{a^3 \operatorname{Sin}[c + dx]}{d} + \frac{3a^2 b \operatorname{Sin}[c + dx]^2}{2d} + \frac{ab^2 \operatorname{Sin}[c + dx]^3}{d} + \frac{b^3 \operatorname{Sin}[c + dx]^4}{4d}$$

■ **Problem 405: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^5 (a + b \operatorname{Sin}[c + dx])^3 dx$$

Optimal (type 3, 94 leaves, 4 steps):

$$\frac{3a(a^2 - b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{8d} + \frac{3a \operatorname{Sec}[c + dx]^2 (b + a \operatorname{Sin}[c + dx]) (a + b \operatorname{Sin}[c + dx])}{8d} + \frac{\operatorname{Sec}[c + dx]^3 (a + b \operatorname{Sin}[c + dx])^3 \operatorname{Tan}[c + dx]}{4d}$$

Result (type 3, 213 leaves):

$$\frac{1}{16d} \left(-6a(a^2 - b^2) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right] + \right. \\ \left. 6a(a^2 - b^2) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right] + \frac{(a+b)^3}{\left(\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^4} + \right. \\ \left. \frac{3(a-b)(a+b)^2}{\left(\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^2} - \frac{(a-b)^3}{\left(\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^4} - \frac{3(a-b)^2(a+b)}{\left(\operatorname{Cos} \left[\frac{1}{2}(c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^2} \right)$$

■ **Problem 413: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx]^5 (a + b \operatorname{Sin}[c + dx])^8 dx$$

Optimal (type 3, 144 leaves, 3 steps):

$$\frac{(a^2 - b^2)^2 (a + b \operatorname{Sin}[c + dx])^9}{9b^5d} - \frac{2a(a^2 - b^2)(a + b \operatorname{Sin}[c + dx])^{10}}{5b^5d} + \\ \frac{2(3a^2 - b^2)(a + b \operatorname{Sin}[c + dx])^{11}}{11b^5d} - \frac{a(a + b \operatorname{Sin}[c + dx])^{12}}{3b^5d} + \frac{(a + b \operatorname{Sin}[c + dx])^{13}}{13b^5d}$$

Result (type 3, 572 leaves):

$$\frac{1}{26357760d} \left(-205920(80a^7b + 168a^5b^3 + 70a^3b^5 + 5ab^7) \operatorname{Cos}[2(c + dx)] - 25740(256a^7b + 224a^5b^3 - 5ab^7) \operatorname{Cos}[4(c + dx)] - \right. \\ \left. 1098240a^7b \operatorname{Cos}[6(c + dx)] + 3843840a^5b^3 \operatorname{Cos}[6(c + dx)] + 2402400a^3b^5 \operatorname{Cos}[6(c + dx)] + 171600ab^7 \operatorname{Cos}[6(c + dx)] + \right. \\ \left. 1441440a^5b^3 \operatorname{Cos}[8(c + dx)] - 51480ab^7 \operatorname{Cos}[8(c + dx)] - 288288a^3b^5 \operatorname{Cos}[10(c + dx)] - 20592ab^7 \operatorname{Cos}[10(c + dx)] + \right. \\ \left. 8580ab^7 \operatorname{Cos}[12(c + dx)] + 16473600a^8 \operatorname{Sin}[c + dx] + 57657600a^6b^2 \operatorname{Sin}[c + dx] + 43243200a^4b^4 \operatorname{Sin}[c + dx] + 7207200a^2b^6 \operatorname{Sin}[c + dx] + \right. \\ \left. 128700b^8 \operatorname{Sin}[c + dx] + 2745600a^8 \operatorname{Sin}[3(c + dx)] - 3843840a^6b^2 \operatorname{Sin}[3(c + dx)] - 9609600a^4b^4 \operatorname{Sin}[3(c + dx)] - \right. \\ \left. 2402400a^2b^6 \operatorname{Sin}[3(c + dx)] - 53625b^8 \operatorname{Sin}[3(c + dx)] + 329472a^8 \operatorname{Sin}[5(c + dx)] - 6918912a^6b^2 \operatorname{Sin}[5(c + dx)] - \right. \\ \left. 5765760a^4b^4 \operatorname{Sin}[5(c + dx)] - 720720a^2b^6 \operatorname{Sin}[5(c + dx)] - 6435b^8 \operatorname{Sin}[5(c + dx)] - 1647360a^6b^2 \operatorname{Sin}[7(c + dx)] + \right. \\ \left. 1029600a^4b^4 \operatorname{Sin}[7(c + dx)] + 514800a^2b^6 \operatorname{Sin}[7(c + dx)] + 12870b^8 \operatorname{Sin}[7(c + dx)] + 800800a^4b^4 \operatorname{Sin}[9(c + dx)] + \right. \\ \left. 80080a^2b^6 \operatorname{Sin}[9(c + dx)] - 1430b^8 \operatorname{Sin}[9(c + dx)] - 65520a^2b^6 \operatorname{Sin}[11(c + dx)] - 1755b^8 \operatorname{Sin}[11(c + dx)] + 495b^8 \operatorname{Sin}[13(c + dx)] \right)$$

■ **Problem 414: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx]^3 (a + b \operatorname{Sin}[c + dx])^8 dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{(a^2 - b^2)(a + b \operatorname{Sin}[c + dx])^9}{9b^3d} + \frac{a(a + b \operatorname{Sin}[c + dx])^{10}}{5b^3d} - \frac{(a + b \operatorname{Sin}[c + dx])^{11}}{11b^3d}$$

Result (type 3, 438 leaves):

$$\frac{1}{506880d} \left(-7920 (64a^7b + 168a^5b^3 + 84a^3b^5 + 7ab^7) \cos[2(c+dx)] - 15840 (8a^7b - 7a^3b^5 - ab^7) \cos[4(c+dx)] + 147840a^5b^3 \cos[6(c+dx)] + 73920a^3b^5 \cos[6(c+dx)] + 3960ab^7 \cos[6(c+dx)] - 27720a^3b^5 \cos[8(c+dx)] - 3960ab^7 \cos[8(c+dx)] + 792a^7b^7 \cos[10(c+dx)] + 380160a^8 \sin[c+dx] + 1774080a^6b^2 \sin[c+dx] + 1663200a^4b^4 \sin[c+dx] + 332640a^2b^6 \sin[c+dx] + 6930b^8 \sin[c+dx] + 42240a^8 \sin[3(c+dx)] - 295680a^6b^2 \sin[3(c+dx)] - 554400a^4b^4 \sin[3(c+dx)] - 147840a^2b^6 \sin[3(c+dx)] - 3630b^8 \sin[3(c+dx)] - 177408a^6b^2 \sin[5(c+dx)] - 110880a^4b^4 \sin[5(c+dx)] + 495b^8 \sin[5(c+dx)] + 79200a^4b^4 \sin[7(c+dx)] + 23760a^2b^6 \sin[7(c+dx)] + 495b^8 \sin[7(c+dx)] - 6160a^2b^6 \sin[9(c+dx)] - 275b^8 \sin[9(c+dx)] + 45b^8 \sin[11(c+dx)] \right)$$

■ **Problem 465: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]}{(a+b\sin[c+dx])^8} dx$$

Optimal (type 3, 385 leaves, 4 steps):

$$\begin{aligned} & -\frac{\operatorname{Log}[1-\sin[c+dx]]}{2(a+b)^8d} + \frac{\operatorname{Log}[1+\sin[c+dx]]}{2(a-b)^8d} - \frac{8ab(a^2+b^2)(a^4+6a^2b^2+b^4)\operatorname{Log}[a+b\sin[c+dx]]}{(a^2-b^2)^8d} + \\ & \frac{b}{7(a^2-b^2)d(a+b\sin[c+dx])^7} + \frac{ab}{3(a^2-b^2)^2d(a+b\sin[c+dx])^6} + \frac{b(3a^2+b^2)}{5(a^2-b^2)^3d(a+b\sin[c+dx])^5} + \\ & \frac{ab(a^2+b^2)}{(a^2-b^2)^4d(a+b\sin[c+dx])^4} + \frac{b(5a^4+10a^2b^2+b^4)}{3(a^2-b^2)^5d(a+b\sin[c+dx])^3} + \frac{ab(3a^2+b^2)(a^2+3b^2)}{(a^2-b^2)^6d(a+b\sin[c+dx])^2} + \frac{b(7a^6+35a^4b^2+21a^2b^4+b^6)}{(a^2-b^2)^7d(a+b\sin[c+dx])} \end{aligned}$$

Result (type 3, 847 leaves):

$$\begin{aligned}
& \frac{16 i \left(a^7 b + 7 a^5 b^3 + 7 a^3 b^5 + a b^7 \right) (c + d x)}{(a - b)^8 (a + b)^8 d} + \frac{i \operatorname{ArcTan}\left[\operatorname{Csc}[c + d x] \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right] \right) \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)\right]}{(a - b)^8 d} \\
& \frac{i \operatorname{ArcTan}\left[\operatorname{Csc}[c + d x] \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right] \right) \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)\right]}{(a + b)^8 d} - \frac{\operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right] \right)^2\right]}{2 (a + b)^8 d} + \\
& \frac{\operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2\right]}{2 (a - b)^8 d} - \frac{8 \left(a^7 b + 7 a^5 b^3 + 7 a^3 b^5 + a b^7 \right) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{(a^2 - b^2)^8 d} + \\
& \frac{1}{3360 (a^2 - b^2)^7 d (a + b \operatorname{Sin}[c + d x])^7} \left(46176 a^{12} b + 368176 a^{10} b^3 + 1198292 a^8 b^5 + 1066342 a^6 b^7 + 403302 a^4 b^9 + 20066 a^2 b^{11} + 2286 b^{13} - \right. \\
& 249648 a^{10} b^3 \operatorname{Cos}[2(c + d x)] - 1190224 a^8 b^5 \operatorname{Cos}[2(c + d x)] - 1321089 a^6 b^7 \operatorname{Cos}[2(c + d x)] - 527429 a^4 b^9 \operatorname{Cos}[2(c + d x)] - \\
& 35539 a^2 b^{11} \operatorname{Cos}[2(c + d x)] - 2471 b^{13} \operatorname{Cos}[2(c + d x)] + 51100 a^8 b^5 \operatorname{Cos}[4(c + d x)] + 239610 a^6 b^7 \operatorname{Cos}[4(c + d x)] + \\
& 137690 a^4 b^9 \operatorname{Cos}[4(c + d x)] + 14350 a^2 b^{11} \operatorname{Cos}[4(c + d x)] + 770 b^{13} \operatorname{Cos}[4(c + d x)] - 735 a^6 b^7 \operatorname{Cos}[6(c + d x)] - \\
& 3675 a^4 b^9 \operatorname{Cos}[6(c + d x)] - 2205 a^2 b^{11} \operatorname{Cos}[6(c + d x)] - 105 b^{13} \operatorname{Cos}[6(c + d x)] + 229152 a^{11} b^2 \operatorname{Sin}[c + d x] + 1230376 a^9 b^4 \operatorname{Sin}[c + d x] + \\
& 2302916 a^7 b^6 \operatorname{Sin}[c + d x] + 1297156 a^5 b^8 \operatorname{Sin}[c + d x] + 255276 a^3 b^{10} \operatorname{Sin}[c + d x] + 7364 a b^{12} \operatorname{Sin}[c + d x] - 149240 a^9 b^4 \operatorname{Sin}[3(c + d x)] - \\
& 692370 a^7 b^6 \operatorname{Sin}[3(c + d x)] - 506170 a^5 b^8 \operatorname{Sin}[3(c + d x)] - 127190 a^3 b^{10} \operatorname{Sin}[3(c + d x)] - 3430 a b^{12} \operatorname{Sin}[3(c + d x)] + \\
& \left. 9450 a^7 b^6 \operatorname{Sin}[5(c + d x)] + 45570 a^5 b^8 \operatorname{Sin}[5(c + d x)] + 24990 a^3 b^{10} \operatorname{Sin}[5(c + d x)] + 630 a b^{12} \operatorname{Sin}[5(c + d x)] \right)
\end{aligned}$$

■ **Problem 466: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^3}{(a + b \operatorname{Sin}[c + d x])^8} dx$$

Optimal (type 3, 527 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(a + 9 b) \operatorname{Log}[1 - \operatorname{Sin}[c + d x]]}{4 (a + b)^9 d} + \frac{(a - 9 b) \operatorname{Log}[1 + \operatorname{Sin}[c + d x]]}{4 (a - b)^9 d} + \frac{8 a b^3 (15 a^6 + 63 a^4 b^2 + 45 a^2 b^4 + 5 b^6) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{(a^2 - b^2)^9 d} \\
& \frac{b (7 a^2 + 9 b^2)}{14 (a^2 - b^2)^2 d (a + b \operatorname{Sin}[c + d x])^7} - \frac{\operatorname{Sec}[c + d x]^2 (b - a \operatorname{Sin}[c + d x])}{2 (a^2 - b^2) d (a + b \operatorname{Sin}[c + d x])^7} - \frac{a b (3 a^2 + 13 b^2)}{6 (a^2 - b^2)^3 d (a + b \operatorname{Sin}[c + d x])^6} \\
& \frac{b (5 a^4 + 50 a^2 b^2 + 9 b^4)}{10 (a^2 - b^2)^4 d (a + b \operatorname{Sin}[c + d x])^5} - \frac{a b (a^4 + 20 a^2 b^2 + 11 b^4)}{2 (a^2 - b^2)^5 d (a + b \operatorname{Sin}[c + d x])^4} - \frac{b (3 a^6 + 115 a^4 b^2 + 129 a^2 b^4 + 9 b^6)}{6 (a^2 - b^2)^6 d (a + b \operatorname{Sin}[c + d x])^3} \\
& \frac{a b (a^6 + 77 a^4 b^2 + 147 a^2 b^4 + 31 b^6)}{2 (a^2 - b^2)^7 d (a + b \operatorname{Sin}[c + d x])^2} - \frac{b (a^8 + 196 a^6 b^2 + 574 a^4 b^4 + 244 a^2 b^6 + 9 b^8)}{2 (a^2 - b^2)^8 d (a + b \operatorname{Sin}[c + d x])}
\end{aligned}$$

Result (type 3, 1237 leaves):

$$\begin{aligned}
& - \frac{16 i (15 a^7 b^3 + 63 a^5 b^5 + 45 a^3 b^7 + 5 a b^9) (c + d x)}{(a - b)^9 (a + b)^9 d} + \frac{1}{2 (a - b)^9 d} \\
& i (a - 9 b) \operatorname{ArcTan}\left[\operatorname{Csc}[c + d x] \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right) + \frac{1}{2 (a + b)^9 d} \\
& i (-a - 9 b) \operatorname{ArcTan}\left[\operatorname{Csc}[c + d x] \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right) + \\
& \frac{(-a - 9 b) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right)^2\right]}{4 (a + b)^9 d} + \frac{(a - 9 b) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right)^2\right]}{4 (a - b)^9 d} + \\
& \frac{8 (15 a^7 b^3 + 63 a^5 b^5 + 45 a^3 b^7 + 5 a b^9) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{(a^2 - b^2)^9 d} + \\
& \frac{1}{26880 (a^2 - b^2)^8 d (a + b \operatorname{Sin}[c + d x])^7} \operatorname{Sec}[c + d x]^2 \left(-60480 a^{14} b - 2155120 a^{12} b^3 - 10531096 a^{10} b^5 - 18656885 a^8 b^7 - 11704100 a^6 b^9 - \right. \\
& 2859110 a^4 b^{11} - 153820 a^2 b^{13} - 5469 b^{15} - 47040 a^{14} b \operatorname{Cos}[2 (c + d x)] - 2190400 a^{12} b^3 \operatorname{Cos}[2 (c + d x)] - 3544396 a^{10} b^5 \operatorname{Cos}[2 (c + d x)] + \\
& 128224 a^8 b^7 \operatorname{Cos}[2 (c + d x)] + 4162744 a^6 b^9 \operatorname{Cos}[2 (c + d x)] + 1322704 a^4 b^{11} \operatorname{Cos}[2 (c + d x)] + 171764 a^2 b^{13} \operatorname{Cos}[2 (c + d x)] - \\
& 3600 b^{15} \operatorname{Cos}[2 (c + d x)] + 58800 a^{12} b^3 \operatorname{Cos}[4 (c + d x)] + 6695640 a^{10} b^5 \operatorname{Cos}[4 (c + d x)] + 17845324 a^8 b^7 \operatorname{Cos}[4 (c + d x)] + \\
& 11544064 a^6 b^9 \operatorname{Cos}[4 (c + d x)] + 2887864 a^4 b^{11} \operatorname{Cos}[4 (c + d x)] + 96264 a^2 b^{13} \operatorname{Cos}[4 (c + d x)] + 9324 b^{15} \operatorname{Cos}[4 (c + d x)] - \\
& 8820 a^{10} b^5 \operatorname{Cos}[6 (c + d x)] - 1410080 a^8 b^7 \operatorname{Cos}[6 (c + d x)] - 3831800 a^6 b^9 \operatorname{Cos}[6 (c + d x)] - 1515920 a^4 b^{11} \operatorname{Cos}[6 (c + d x)] - \\
& 109620 a^2 b^{13} \operatorname{Cos}[6 (c + d x)] - 5040 b^{15} \operatorname{Cos}[6 (c + d x)] + 105 a^8 b^7 \operatorname{Cos}[8 (c + d x)] + 20580 a^6 b^9 \operatorname{Cos}[8 (c + d x)] + \\
& 60270 a^4 b^{11} \operatorname{Cos}[8 (c + d x)] + 25620 a^2 b^{13} \operatorname{Cos}[8 (c + d x)] + 945 b^{15} \operatorname{Cos}[8 (c + d x)] + 13440 a^{15} \operatorname{Sin}[c + d x] - 164640 a^{13} b^2 \operatorname{Sin}[c + d x] - \\
& 5702480 a^{11} b^4 \operatorname{Sin}[c + d x] - 20202406 a^9 b^6 \operatorname{Sin}[c + d x] - 24081736 a^7 b^8 \operatorname{Sin}[c + d x] - 9935716 a^5 b^{10} \operatorname{Sin}[c + d x] - \\
& 1391096 a^3 b^{12} \operatorname{Sin}[c + d x] - 36806 a b^{14} \operatorname{Sin}[c + d x] - 70560 a^{13} b^2 \operatorname{Sin}[3 (c + d x)] - 5955320 a^{11} b^4 \operatorname{Sin}[3 (c + d x)] - \\
& 15658566 a^9 b^6 \operatorname{Sin}[3 (c + d x)] - 13417656 a^7 b^8 \operatorname{Sin}[3 (c + d x)] - 3705156 a^5 b^{10} \operatorname{Sin}[3 (c + d x)] - 326816 a^3 b^{12} \operatorname{Sin}[3 (c + d x)] - \\
& 3206 a b^{14} \operatorname{Sin}[3 (c + d x)] + 29400 a^{11} b^4 \operatorname{Sin}[5 (c + d x)] + 4071970 a^9 b^6 \operatorname{Sin}[5 (c + d x)] + 10871560 a^7 b^8 \operatorname{Sin}[5 (c + d x)] + \\
& 5210380 a^5 b^{10} \operatorname{Sin}[5 (c + d x)] + 875280 a^3 b^{12} \operatorname{Sin}[5 (c + d x)] + 15330 a b^{14} \operatorname{Sin}[5 (c + d x)] - 1470 a^9 b^6 \operatorname{Sin}[7 (c + d x)] - \\
& 262920 a^7 b^8 \operatorname{Sin}[7 (c + d x)] - 737940 a^5 b^{10} \operatorname{Sin}[7 (c + d x)] - 283080 a^3 b^{12} \operatorname{Sin}[7 (c + d x)] - 4830 a b^{14} \operatorname{Sin}[7 (c + d x)] \left. \right)
\end{aligned}$$

■ **Problem 495: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^3 (a + b \operatorname{Sin}[c + d x])^{5/2} dx$$

Optimal (type 3, 83 leaves, 3 steps):

$$- \frac{2 (a^2 - b^2) (a + b \operatorname{Sin}[c + d x])^{7/2}}{7 b^3 d} + \frac{4 a (a + b \operatorname{Sin}[c + d x])^{9/2}}{9 b^3 d} - \frac{2 (a + b \operatorname{Sin}[c + d x])^{11/2}}{11 b^3 d}$$

Result (type 3, 198 leaves):

$$\left(-2 a^2 (64 a^4 - 780 a^2 b^2 - 705 b^4) \sqrt{1 + \frac{b \sin[c + d x]}{a}} \left(-1 + \sqrt{1 + \frac{b \sin[c + d x]}{a}} \right) + \right. \\ \left. b (a + b \sin[c + d x]) (8 a b (3 a^2 - 136 b^2) \cos[2 (c + d x)] - 322 a b^3 \cos[4 (c + d x)] + 2 (32 a^4 + 1698 a^2 b^2 + 279 b^4) \sin[c + d x] + \right. \\ \left. b^2 (452 a^2 - 81 b^2) \sin[3 (c + d x)] - 63 b^4 \sin[5 (c + d x)] \right) / \left(5544 b^3 d \sqrt{a + b \sin[c + d x]} \right)$$

■ **Problem 497: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + d x] (a + b \sin[c + d x])^{5/2} dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$-\frac{(a-b)^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sin[c+d x]}}{\sqrt{a-b}}\right]}{d} + \frac{(a+b)^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sin[c+d x]}}{\sqrt{a+b}}\right]}{d} - \frac{4 a b \sqrt{a+b \sin[c+d x]}}{d} - \frac{2 b (a+b \sin[c+d x])^{3/2}}{3 d}$$

Result (type 3, 286 leaves):

$$\frac{1}{6 d} \left(-\frac{(6 a^3 - 18 a^2 b + 11 a b^2 - 6 b^3) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sin[c+d x]}}{\sqrt{a-b}}\right]}{\sqrt{a-b}} + 1 / \left(\sqrt{-a+b} \sqrt{-(a+b)^2} \right) \right. \\ \left. \left(\sqrt{-a-b} \sqrt{-a+b} (6 a^3 + 18 a^2 b + 11 a b^2 + 6 b^3) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sin[c+d x]}}{\sqrt{a+b}}\right] - b \left(7 a b \sqrt{-a+b} \sqrt{a+b} \operatorname{ArcTan}\left[\frac{\sqrt{a+b \sin[c+d x]}}{\sqrt{-a-b}}\right] + \right. \right. \right. \\ \left. \left. \left. \sqrt{-(a+b)^2} \left(-7 a b \operatorname{ArcTan}\left[\frac{\sqrt{a+b \sin[c+d x]}}{\sqrt{-a+b}}\right] + 4 \sqrt{-a+b} \sqrt{a+b \sin[c+d x]} (7 a + b \sin[c+d x]) \right) \right) \right) \right)$$

■ **Problem 574: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + d x])^{11/2}}{a + b \sin[c + d x]} dx$$

Optimal (type 4, 531 leaves, 15 steps):

$$\begin{aligned}
& - \frac{(-a^2 + b^2)^{9/4} e^{11/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{11/2} d} - \frac{(-a^2 + b^2)^{9/4} e^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{11/2} d} + \\
& \frac{2 e (e \cos[c+dx])^{9/2}}{9 b d} + \frac{2 a (21 a^4 - 49 a^2 b^2 + 33 b^4) e^6 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21 b^6 d \sqrt{e \cos[c+dx]}} - \\
& \frac{a (a^2 - b^2)^3 e^6 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{b^6 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos[c+dx]}} - \frac{a (a^2 - b^2)^3 e^6 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{b^6 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos[c+dx]}} - \\
& \frac{2 e^3 (e \cos[c+dx])^{5/2} (7 (a^2 - b^2) - 5 a b \sin[c+dx])}{35 b^3 d} + \frac{2 e^5 \sqrt{e \cos[c+dx]} (21 (a^2 - b^2)^2 - a b (7 a^2 - 12 b^2) \sin[c+dx])}{21 b^5 d}
\end{aligned}$$

Result (type 6, 2235 leaves):

$$\begin{aligned}
& \frac{1}{1680 b^4 d \cos[c+dx]^{11/2}} \\
& (e \cos[c+dx])^{11/2} \left(- \frac{1}{\sqrt{1 - \cos[c+dx]^2} (a + b \sin[c+dx])} - 2 (280 a^4 - 636 a^2 b^2 + 721 b^4) (a + b \sqrt{1 - \cos[c+dx]^2}) \left(\left(5 a (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \sqrt{\cos[c+dx]}\right) / \left(\sqrt{1 - \cos[c+dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{5}{4}, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + (-a^2 + b^2) \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) \right) - \\
& \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] \right) \left. \right) \sin[c+dx] + \\
& \frac{1}{\sqrt{1 - \cos[c+dx]^2} (-1 + 2 \cos[c+dx]^2) (a + b \sin[c+dx])} (840 a^4 - 1764 a^2 b^2 + 959 b^4) (a + b \sqrt{1 - \cos[c+dx]^2}) \\
& \cos[2(c+dx)]
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2}\right) (-2a^2 + b^2) \operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2}(-a^2+b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right) (-2a^2 + b^2) \operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2}(-a^2+b^2)^{3/4}} + \frac{4\sqrt{\cos[c+dx]}}{b} + \right. \\
& \left(10a(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \sqrt{\cos[c+dx]} \right) / \left(\sqrt{1 - \cos[c+dx]^2} \left(5(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) \right) - \\
& \left(36a(a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \cos[c+dx]^{5/2} \right) / \left(5\sqrt{1 - \cos[c+dx]^2} \left(9(a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) \right) + \\
& \frac{1}{b^{3/2}(-a^2+b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2a^2 + b^2) \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\cos[c+dx]} + ib\cos[c+dx]\right] - \\
& \frac{1}{b^{3/2}(-a^2+b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2a^2 + b^2) \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\cos[c+dx]} + ib\cos[c+dx]\right] \Bigg) \sin[c+dx] - \\
& \frac{1}{(1 - \cos[c+dx]^2)(a + b\sin[c+dx])} 2(-392a^3b + 722ab^3) \left(a + b\sqrt{1 - \cos[c+dx]^2} \right) \\
& \left(\left(5b(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \sqrt{\cos[c+dx]} \sqrt{1 - \cos[c+dx]^2} \right) / \right. \\
& \left(\left(-5(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[c+dx]^2}{-a^2+b^2} \right) + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) + \frac{1}{4\sqrt{2}\sqrt{b}(a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + \right.
\end{aligned}$$

$$2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c+dx]}}{(a^2 - b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c+dx]} + b \cos[c+dx]\right] +$$

$$\left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c+dx]} + b \cos[c+dx]\right]\right) \operatorname{Sin}[c+dx]^2 +$$

$$\frac{1}{d} (e \cos[c+dx])^{11/2} \operatorname{Sec}[c+dx]^5 \left(\frac{(-9a^2 + 14b^2) \cos[2(c+dx)]}{45b^3} + \frac{\cos[4(c+dx)]}{36b} - \frac{a(28a^2 - 51b^2) \sin[c+dx]}{42b^4} + \frac{a \sin[3(c+dx)]}{14b^2} \right)$$

■ **Problem 575: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^{9/2}}{a+b \sin[c+dx]} dx$$

Optimal (type 4, 446 leaves, 14 steps):

$$\frac{(-a^2 + b^2)^{7/4} e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right] - (-a^2 + b^2)^{7/4} e^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right] + \frac{2e (e \cos[c+dx])^{7/2}}{7bd} - \frac{2a(5a^2 - 8b^2) e^4 \sqrt{e \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] + a(a^2 - b^2)^2 e^5 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2}(c+dx), 2\right]}{5b^4 d \sqrt{\cos[c+dx]}} + \frac{b^5 (b - \sqrt{-a^2 + b^2}) d \sqrt{e \cos[c+dx]}}{b^5 (b + \sqrt{-a^2 + b^2}) d \sqrt{e \cos[c+dx]}} - \frac{2e^3 (e \cos[c+dx])^{3/2} (5(a^2 - b^2) - 3ab \sin[c+dx])}{15b^3 d}$$

Result (type 6, 1228 leaves):

$$-\frac{1}{5b^3 d \cos[c+dx]^{9/2}} (e \cos[c+dx])^{9/2}$$

$$\begin{aligned}
& \left(\frac{1}{12 \sqrt{1 - \cos[c + dx]^2} (a + b \sin[c + dx])} (2a^2b - 5b^3) (a + b \sqrt{1 - \cos[c + dx]^2}) \left(- \left(56a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2 \right], \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{3/2} \right) / \left(\sqrt{1 - \cos[c + dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2 \right], \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) - \right. \\
& \quad \left. 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[c + dx]^2 \right], \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \\
& \quad \left. \left. \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 (a^2 + b^2 (-1 + \cos[c + dx]^2)) \Big) - \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} \\
& (3 + 3i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \\
& \quad \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] \right) \Big) \\
& \sin[c + dx] - \frac{1}{(1 - \cos[c + dx]^2) (a + b \sin[c + dx])} 2 (5a^3 - 8ab^2) (a + b \sqrt{1 - \cos[c + dx]^2}) \\
& \left(\left(7b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2 \right], \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{3/2} \sqrt{1 - \cos[c + dx]^2} \right) / \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2 \right], \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) + 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[c + dx]^2 \right], \right. \right. \\
& \quad \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[c + dx]^2 \right], \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) \cos[c + dx]^2 \Big) \\
& (a^2 + b^2 (-1 + \cos[c + dx]^2)) \Big) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \quad \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] - \right. \\
& \quad \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \Big) \sin[c + dx]^2 + \\
& \frac{(e \cos[c + dx])^{9/2} \operatorname{Sec}[c + dx]^4 \left(\frac{(-28a^2 + 37b^2) \cos[c + dx]}{42b^3} + \frac{\cos[3(c + dx)]}{14b} + \frac{a \sin[2(c + dx)]}{5b^2} \right)}{d}
\end{aligned}$$

■ **Problem 576: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{7/2}}{a + b \sin[c + dx]} dx$$

Optimal (type 4, 461 leaves, 14 steps):

$$\begin{aligned} & - \frac{(-a^2 + b^2)^{5/4} e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{7/2} d} - \frac{(-a^2 + b^2)^{5/4} e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{7/2} d} + \frac{2 e (e \cos[c + dx])^{5/2}}{5 b d} \\ & + \frac{2 a (3 a^2 - 4 b^2) e^4 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{3 b^4 d \sqrt{e \cos[c + dx]}} + \frac{a (a^2 - b^2)^2 e^4 \sqrt{\cos[c + dx]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2\right]}{b^4 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \cos[c + dx]}} + \\ & - \frac{a (a^2 - b^2)^2 e^4 \sqrt{\cos[c + dx]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2\right]}{b^4 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \cos[c + dx]}} - \frac{2 e^3 \sqrt{e \cos[c + dx]} (3 (a^2 - b^2) - a b \sin[c + dx])}{3 b^3 d} \end{aligned}$$

Result (type 6, 2155 leaves):

$$\begin{aligned} & \frac{(e \cos[c + dx])^{7/2} \sec[c + dx]^3 \left(\frac{\cos[2(c+dx)]}{5b} + \frac{2a \sin[c+dx]}{3b^2}\right)}{d} - \\ & \frac{1}{60 b^2 d \cos[c + dx]^{7/2}} (e \cos[c + dx])^{7/2} \left(- \frac{1}{\sqrt{1 - \cos[c + dx]^2} (a + b \sin[c + dx])} - 2 (10 a^2 - 27 b^2) \left(a + b \sqrt{1 - \cos[c + dx]^2}\right) \right. \\ & \left. \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] \sqrt{\cos[c + dx]}\right) / \left(\sqrt{1 - \cos[c + dx]^2} \left(5 (a^2 - b^2) \right. \right. \right. \right. \\ & \left. \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] + \right. \right. \right. \right. \\ & \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] \cos[c + dx]^2\right) (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) \right) - \\ & \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8}\right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] + \right. \\ & \left. \log\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx]\right] - \right. \\ & \left. \log\left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx]\right] \right) \left. \right) \sin[c + dx] + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{1 - \cos[c + dx]^2} (-1 + 2 \cos[c + dx]^2) (a + b \sin[c + dx])} (30 a^2 - 33 b^2) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \cos[2(c + dx)] \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} \right) + \\
& \frac{4 \sqrt{\cos[c + dx]}}{b} + \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \right) / \\
& \left(\sqrt{1 - \cos[c + dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \quad \left. \left. \left. \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) - \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{5/2} \right) / \\
& \left(5 \sqrt{1 - \cos[c + dx]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] - \right. \right. \\
& \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] \right) \sin[c + dx] + \\
& \frac{1}{(1 - \cos[c + dx]^2) (a + b \sin[c + dx])} 28 a b \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\
& \quad \left. \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \right. \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + \left. (a^2-b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \right) \cos [c+d x]^2 \\ & \left. (a^2+b^2(-1+\cos [c+d x]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{(a^2-b^2)^{1/4}} \right] + \right. \\ & \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{(a^2-b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] + \right. \\ & \left. \left. \operatorname{Log} \left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] \right) \right) \sin [c+d x]^2 \end{aligned}$$

- **Problem 577: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos [c+d x])^{5/2}}{a+b \sin [c+d x]} dx$$

Optimal (type 4, 384 leaves, 13 steps):

$$\begin{aligned} & \frac{(-a^2+b^2)^{3/4} e^{5/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}} \right]}{b^{5/2} d} - \frac{(-a^2+b^2)^{3/4} e^{5/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}} \right]}{b^{5/2} d} + \\ & \frac{2 e (e \cos [c+d x])^{3/2}}{3 b d} + \frac{2 a e^2 \sqrt{e \cos [c+d x]} \operatorname{EllipticE} \left[\frac{1}{2} (c+d x), 2 \right]}{b^2 d \sqrt{\cos [c+d x]}} - \\ & \frac{a (a^2-b^2) e^3 \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+d x), 2 \right]}{b^3 (b-\sqrt{-a^2+b^2}) d \sqrt{e \cos [c+d x]}} - \frac{a (a^2-b^2) e^3 \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+d x), 2 \right]}{b^3 (b+\sqrt{-a^2+b^2}) d \sqrt{e \cos [c+d x]}} \end{aligned}$$

Result (type 6, 1151 leaves):

$$\begin{aligned}
& \frac{2 (e \cos [c+d x])^{5/2} \operatorname{Sec}[c+d x]}{3 b d} + \\
& \frac{1}{b d \cos [c+d x]^{5/2}} (e \cos [c+d x])^{5/2} \left(\frac{1}{12 \sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])} b \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \right) \left(- \left(56 a \left(a^2-b^2 \right) \right. \right. \\
& \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3/2} \Big) / \left(\sqrt{1-\cos [c+d x]^2} \left(7 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + \left(-a^2+b^2 \right) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \cos [c+d x]^2 \right) \left(a^2+b^2 \left(-1+\cos [c+d x]^2 \right) \right) \Big) - \\
& \frac{1}{\sqrt{b} \left(-a^2+b^2 \right)^{1/4}} (3+3 i) \left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2 \right)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2 \right)^{1/4}}\right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right] + \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right] \right) \Big) \sin [c+d x] - \frac{1}{\left(1-\cos [c+d x]^2 \right) (a+b \sin [c+d x])} \\
& 2 a \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \left(\left(7 b \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3/2} \sqrt{1-\cos [c+d x]^2} \right) / \right. \\
& \left(3 \left(-7 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \cos [c+d x]^2 \right) \right. \\
& \left. \left(a^2+b^2 \left(-1+\cos [c+d x]^2 \right) \right) \right) + \frac{1}{4 \sqrt{2} b^{3/2} \left(a^2-b^2 \right)^{1/4}} a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2 \right)^{1/4}}\right] + \right. \\
& \left. 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2 \right)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{1/4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{1/4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right] \right) \Big) \sin [c+d x]^2 \Big)
\end{aligned}$$

■ **Problem 578: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \cos [c + d x])^{3/2}}{a + b \sin [c + d x]} dx$$

Optimal (type 4, 397 leaves, 13 steps):

$$\frac{(-a^2 + b^2)^{1/4} e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right] - (-a^2 + b^2)^{1/4} e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right] + \frac{2 e \sqrt{e \cos [c + d x]}}{b d}}{b^{3/2} d} - \frac{2 a e^2 \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] - a (a^2 - b^2) e^2 \sqrt{\cos [c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + d x), 2\right]}{b^2 d \sqrt{e \cos [c + d x]}} - \frac{a (a^2 - b^2) e^2 \sqrt{\cos [c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + d x), 2\right]}{b^2 (a^2 - b (b - \sqrt{-a^2 + b^2})) d \sqrt{e \cos [c + d x]}} - \frac{a (a^2 - b^2) e^2 \sqrt{\cos [c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + d x), 2\right]}{b^2 (a^2 - b (b + \sqrt{-a^2 + b^2})) d \sqrt{e \cos [c + d x]}}$$

Result (type 6, 624 leaves):

$$\frac{1}{20 d \cos [c + d x]^{3/2} \sqrt{1 - \cos [c + d x]^2} (a + b \sin [c + d x])} (e \cos [c + d x])^{3/2} \left(a + b \sqrt{1 - \cos [c + d x]^2} \right) - \left(- \left(72 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2}\right] \cos [c + d x]^{5/2} \right) / \left(\sqrt{1 - \cos [c + d x]^2} \right) \right. \\ \left. \left(9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2}\right] \right) \cos [c + d x]^2 \right) (a^2 + b^2 (-1 + \cos [c + d x]^2)) \right) + \\ \frac{1}{b^{3/2}} (5 - 5 i) \left(2 (-a^2 + b^2)^{1/4} \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos [c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 (-a^2 + b^2)^{1/4} \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos [c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + \right. \\ \left. (4 + 4 i) \sqrt{b} \sqrt{\cos [c + d x]} + (-a^2 + b^2)^{1/4} \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos [c + d x]} + i b \cos [c + d x]\right] - \right. \\ \left. (-a^2 + b^2)^{1/4} \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos [c + d x]} + i b \cos [c + d x]\right] \right) \left. \right) \sin [c + d x]$$

■ **Problem 579: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{e \cos [c + d x]}}{a + b \sin [c + d x]} dx$$

Optimal (type 4, 292 leaves, 9 steps) :

$$\frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{\sqrt{b} (-a^2+b^2)^{1/4} d} - \frac{\sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{\sqrt{b} (-a^2+b^2)^{1/4} d} +$$

$$\frac{a e \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{b \left(b - \sqrt{-a^2+b^2}\right) d \sqrt{e \cos[c+dx]}} + \frac{a e \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{b \left(b + \sqrt{-a^2+b^2}\right) d \sqrt{e \cos[c+dx]}}$$

Result (type 6, 556 leaves) :

$$\frac{1}{12 d \sqrt{\cos[c+dx]} \sqrt{\sin[c+dx]^2 (a+b \sin[c+dx])}} \sqrt{e \cos[c+dx]} \sin[c+dx]$$

$$\left(-\frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \right.$$

$$\left. \left. (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] \right) -$$

$$\left(56 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \cos[c+dx]^{3/2} \right) / \left((a^2-b^2+b^2 \cos[c+dx]^2) \right.$$

$$\left. \left(7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right.$$

$$\left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 \sqrt{\sin[c+dx]^2} \right) \left(a+b \sqrt{\sin[c+dx]^2} \right)$$

■ **Problem 580: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{e \cos[c+dx]} (a+b \sin[c+dx])} dx$$

Optimal (type 4, 299 leaves, 9 steps) :

$$-\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{3/4} d \sqrt{e}} - \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{3/4} d \sqrt{e}} +$$

$$\frac{a \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{(a^2-b \left(b - \sqrt{-a^2+b^2}\right)) d \sqrt{e \cos[c+dx]}} + \frac{a \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{(a^2-b \left(b + \sqrt{-a^2+b^2}\right)) d \sqrt{e \cos[c+dx]}}$$

Result (type 6, 567 leaves) :

$$\begin{aligned}
 & - \frac{1}{d \sqrt{e \cos[c+dx]} \sqrt{1 - \cos[c+dx]^2} (a + b \sin[c+dx])} - 2 \sqrt{\cos[c+dx]} \left(a + b \sqrt{1 - \cos[c+dx]^2} \right) \\
 & \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c+dx]} \right) / \left(\sqrt{1 - \cos[c+dx]^2} \right. \right. \\
 & \quad \left. \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2} \right] \right) \cos[c+dx]^2 \right) (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) - \\
 & \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2 + b^2)^{1/4}} \right] \right) + \\
 & \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx] \right] - \\
 & \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx] \right] \right) \sin[c+dx]
 \end{aligned}$$

■ **Problem 581: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \cos[c+dx])^{3/2} (a + b \sin[c+dx])} dx$$

Optimal (type 4, 411 leaves, 13 steps) :

$$\begin{aligned}
 & \frac{b^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{(-a^2 + b^2)^{5/4} d e^{3/2}} - \frac{b^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{(-a^2 + b^2)^{5/4} d e^{3/2}} - \\
 & \frac{2 a \sqrt{e \cos[c+dx]} \operatorname{EllipticE} \left[\frac{1}{2} (c + dx), 2 \right]}{(a^2 - b^2) d e^2 \sqrt{\cos[c+dx]}} - \frac{a b \sqrt{\cos[c+dx]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{(a^2 - b^2) \left(b - \sqrt{-a^2 + b^2} \right) d e \sqrt{e \cos[c+dx]}} - \\
 & \frac{a b \sqrt{\cos[c+dx]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{(a^2 - b^2) \left(b + \sqrt{-a^2 + b^2} \right) d e \sqrt{e \cos[c+dx]}} - \frac{2 (b - a \sin[c+dx])}{(a^2 - b^2) d e \sqrt{e \cos[c+dx]}}
 \end{aligned}$$

Result (type 6, 1186 leaves) :

$$\begin{aligned}
& \frac{2 \operatorname{Cos}[c+d x] (-b+a \operatorname{Sin}[c+d x])}{(a^2-b^2) d (e \operatorname{Cos}[c+d x])^{3/2}} - \\
& \frac{1}{(a-b)(a+b) d (e \operatorname{Cos}[c+d x])^{3/2}} \operatorname{Cos}[c+d x]^{3/2} \left(\frac{1}{12 \sqrt{1-\operatorname{Cos}[c+d x]^2} (a+b \operatorname{Sin}[c+d x])} (a^2+b^2) (a+b \sqrt{1-\operatorname{Cos}[c+d x]^2}) \right) \\
& \left(- \left(56 a (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2} \right] \operatorname{Cos}[c+d x]^{3/2} \right) / \left(\sqrt{1-\operatorname{Cos}[c+d x]^2} \left(7 (a^2-b^2) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2} \right] \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2} \right] \operatorname{Cos}[c+d x]^2 \right) (a^2+b^2 (-1+\operatorname{Cos}[c+d x]^2)) \right) \right) - \\
& \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{(-a^2+b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{(-a^2+b^2)^{1/4}} \right] - \right. \\
& \quad \left. \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + i b \operatorname{Cos}[c+d x] \right] + \right. \\
& \quad \left. \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + i b \operatorname{Cos}[c+d x] \right] \right) \left. \right) \operatorname{Sin}[c+d x] - \\
& \frac{1}{(1-\operatorname{Cos}[c+d x]^2) (a+b \operatorname{Sin}[c+d x])} 2 a b \left(a+b \sqrt{1-\operatorname{Cos}[c+d x]^2} \right) \\
& \left(\left(7 b (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2} \right] \operatorname{Cos}[c+d x]^{3/2} \sqrt{1-\operatorname{Cos}[c+d x]^2} \right) / \right. \\
& \quad \left(3 \left(-7 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[c+d x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2} \right] \operatorname{Cos}[c+d x]^2 \right) \right) \right) \\
& \quad \left. (a^2+b^2 (-1+\operatorname{Cos}[c+d x]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{(a^2-b^2)^{1/4}} \right] + \right. \\
& \quad \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{(a^2-b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + b \operatorname{Cos}[c+d x] \right] - \right. \\
& \quad \left. \left. \operatorname{Log} \left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + b \operatorname{Cos}[c+d x] \right] \right) \right) \left. \right) \operatorname{Sin}[c+d x]^2 \left. \right)
\end{aligned}$$

- **Problem 582: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \cos[c + dx])^{5/2} (a + b \sin[c + dx])} dx$$

Optimal (type 4, 434 leaves, 13 steps):

$$\begin{aligned} & - \frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{7/4} d e^{5/2}} - \frac{b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{7/4} d e^{5/2}} + \\ & \frac{2 a \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3(a^2-b^2) d e^2 \sqrt{e \cos[c+dx]}} - \frac{a b^2 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{(a^2-b^2) \left(a^2-b \left(b-\sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \cos[c+dx]}} - \\ & \frac{a b^2 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{(a^2-b^2) \left(a^2-b \left(b+\sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \cos[c+dx]}} - \frac{2(b-a \sin[c+dx])}{3(a^2-b^2) d e (e \cos[c+dx])^{3/2}} \end{aligned}$$

Result (type 6, 1192 leaves):

$$\begin{aligned}
& \frac{2 \operatorname{Cos}[c+d x] (-b+a \operatorname{Sin}[c+d x])}{3 (a^2-b^2) d (e \operatorname{Cos}[c+d x])^{5/2}} + \\
& \frac{1}{3 (a-b) (a+b) d (e \operatorname{Cos}[c+d x])^{5/2}} \operatorname{Cos}[c+d x]^{5/2} \left(-\frac{1}{\sqrt{1-\operatorname{Cos}[c+d x]^2} (a+b \operatorname{Sin}[c+d x])} 2 (a^2-3 b^2) (a+b \sqrt{1-\operatorname{Cos}[c+d x]^2}) \right. \\
& \left. \left(\left(5 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Cos}[c+d x]} \right) / \left(\sqrt{1-\operatorname{Cos}[c+d x]^2} \left(5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] + (-a^2+b^2) \right. \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \right) \operatorname{Cos}[c+d x]^2 (a^2+b^2 (-1+\operatorname{Cos}[c+d x]^2)) \right) \right) - \frac{1}{(-a^2+b^2)^{3/4}} \right. \\
& \left. \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + i b \operatorname{Cos}[c+d x] \right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + i b \operatorname{Cos}[c+d x] \right] \right) \right) \right) \\
& \operatorname{Sin}[c+d x] - \frac{1}{(1-\operatorname{Cos}[c+d x]^2) (a+b \operatorname{Sin}[c+d x])} 2 a b (a+b \sqrt{1-\operatorname{Cos}[c+d x]^2}) \\
& \left(\left(5 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{1-\operatorname{Cos}[c+d x]^2} \right) / \right. \\
& \left. \left(\left(-5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[c+d x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \right) \operatorname{Cos}[c+d x]^2 \right) \right) \\
& \left. \left(a^2+b^2 (-1+\operatorname{Cos}[c+d x]^2) \right) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4}} a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{(a^2-b^2)^{1/4}}\right] + \right. \\
& \left. 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + b \operatorname{Cos}[c+d x] \right] + \right. \\
& \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + b \operatorname{Cos}[c+d x] \right] \right) \right) \operatorname{Sin}[c+d x]^2 \Big)
\end{aligned}$$

■ **Problem 583: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \cos[c + dx])^{7/2} (a + b \sin[c + dx])} dx$$

Optimal (type 4, 486 leaves, 14 steps):

$$\frac{b^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{9/4} d e^{7/2}} - \frac{b^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{9/4} d e^{7/2}} -$$

$$\frac{2a(3a^2 - 8b^2) \sqrt{e \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5(a^2 - b^2)^2 d e^4 \sqrt{\cos[c+dx]}} + \frac{a b^3 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{(a^2 - b^2)^2 (b - \sqrt{-a^2+b^2}) d e^3 \sqrt{e \cos[c+dx]}} +$$

$$\frac{a b^3 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{(a^2 - b^2)^2 (b + \sqrt{-a^2+b^2}) d e^3 \sqrt{e \cos[c+dx]}} - \frac{2(b - a \sin[c+dx])}{5(a^2 - b^2) d e (e \cos[c+dx])^{5/2}} + \frac{2(5b^3 + a(3a^2 - 8b^2) \sin[c+dx])}{5(a^2 - b^2)^2 d e^3 \sqrt{e \cos[c+dx]}}$$

Result (type 6, 1275 leaves):

$$\frac{1}{5(a-b)^2 (a+b)^2 d (e \cos[c+dx])^{7/2}}$$

$$\cos[c+dx]^{7/2} \left(\frac{1}{12 \sqrt{1 - \cos[c+dx]^2} (a+b \sin[c+dx])} (3a^4 - 8a^2 b^2 - 5b^4) (a+b \sqrt{1 - \cos[c+dx]^2}) \left(- \left(56a(a^2 - b^2) \right. \right. \right.$$

$$\left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \cos[c+dx]^{3/2} \right) / \left(\sqrt{1 - \cos[c+dx]^2} \left(7(a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \right. \right. \right.$$

$$\left. \left. \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2} \right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\right. \right. \right.$$

$$\left. \left. \frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2} \right] \right) \cos[c+dx]^2 (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) - \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}}$$

$$(3 + 3i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right.$$

$$\left. \left. (-a^2 + b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx] \right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx] \right] \right) \right)$$

$$\sin[c+dx] - \frac{1}{(1 - \cos[c+dx]^2) (a+b \sin[c+dx])} 2(3a^3 b - 8a b^3) (a+b \sqrt{1 - \cos[c+dx]^2})$$

$$\begin{aligned}
& \left(\left(7b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{3/2} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] - \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \sin[c + dx]^2 + \\
& \frac{\cos[c + dx]^4 \left(\frac{2 \sec[c + dx]^3 (-b + a \sin[c + dx])}{5 (a^2 - b^2)} + \frac{2 \sec[c + dx] (5b^3 + 3a^3 \sin[c + dx] - 8ab^2 \sin[c + dx])}{5 (a^2 - b^2)^2} \right)}{d (e \cos[c + dx])^{7/2}}
\end{aligned}$$

■ **Problem 584: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{11/2}}{(a + b \sin[c + dx])^2} dx$$

Optimal (type 4, 543 leaves, 15 steps):

$$\begin{aligned}
& \frac{9a (-a^2 + b^2)^{5/4} e^{11/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2b^{11/2} d} - \frac{9a (-a^2 + b^2)^{5/4} e^{11/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2b^{11/2} d} - \\
& \frac{3(21a^4 - 28a^2b^2 + 5b^4) e^6 \sqrt{\cos[c + dx]} \operatorname{EllipticF} \left[\frac{1}{2} (c + dx), 2 \right]}{7b^6 d \sqrt{e \cos[c + dx]}} + \frac{9a^2 (a^2 - b^2)^2 e^6 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{2b^6 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \cos[c + dx]}} + \\
& \frac{9a^2 (a^2 - b^2)^2 e^6 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{2b^6 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \cos[c + dx]}} + \frac{9e^3 (e \cos[c + dx])^{5/2} (7a - 5b \sin[c + dx])}{35b^3 d} - \\
& \frac{e (e \cos[c + dx])^{9/2}}{bd (a + b \sin[c + dx])} - \frac{3e^5 \sqrt{e \cos[c + dx]} (21a (a^2 - b^2) - b (7a^2 - 5b^2) \sin[c + dx])}{7b^5 d}
\end{aligned}$$

Result (type 6, 2230 leaves) :

$$\begin{aligned}
& - \frac{1}{70 b^5 d \operatorname{Cos}[c + d x]^{11/2}} (e \operatorname{Cos}[c + d x])^{11/2} \\
& \left(- \frac{1}{\sqrt{1 - \operatorname{Cos}[c + d x]^2} (a + b \operatorname{Sin}[c + d x])} 2 (70 a^3 b - 93 a b^3) (a + b \sqrt{1 - \operatorname{Cos}[c + d x]^2}) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c + d x]^2\right], \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2} \right) \sqrt{\operatorname{Cos}[c + d x]} \right) / \left(\sqrt{1 - \operatorname{Cos}[c + d x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2}\right] - \right. \right. \\
& \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2} \right] \right) \operatorname{Cos}[c + d x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Cos}[c + d x]^2)) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \\
& \quad \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \right. \right. \\
& \quad \left. \left. \sqrt{\operatorname{Cos}[c + d x]} + i b \operatorname{Cos}[c + d x]\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[c + d x]} + i b \operatorname{Cos}[c + d x]\right] \right) \right) \operatorname{Sin}[c + d x] + \\
& \frac{1}{\sqrt{1 - \operatorname{Cos}[c + d x]^2} (-1 + 2 \operatorname{Cos}[c + d x]^2) (a + b \operatorname{Sin}[c + d x])} (140 a^3 b - 147 a b^3) (a + b \sqrt{1 - \operatorname{Cos}[c + d x]^2}) \operatorname{Cos}[2 (c + d x)] \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right]}{b^{3/2} (-a^2 + b^2)^{3/4}} + \frac{4 \sqrt{\operatorname{Cos}[c + d x]}}{b} + \right. \\
& \quad \left(10 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2}\right] \sqrt{\operatorname{Cos}[c + d x]} \right) / \left(\sqrt{1 - \operatorname{Cos}[c + d x]^2} \left(5 (a^2 - b^2) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2}\right] \right) \operatorname{Cos}[c + d x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Cos}[c + d x]^2)) \right) - \\
& \quad \left(36 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2}\right] \operatorname{Cos}[c + d x]^{5/2} \right) / \left(5 \sqrt{1 - \operatorname{Cos}[c + d x]^2} \left(9 (a^2 - b^2) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \operatorname{Cos}[c + d x]^2, \frac{b^2 \operatorname{Cos}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left((-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^2 \right) (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] \right) \operatorname{Sin}[c + dx] - \\
& \frac{1}{(1 - \cos[c + dx]^2) (a + b \operatorname{Sin}[c + dx])} 2 (35a^4 - 126a^2b^2 + 75b^4) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \\
& \left(\left(5b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \frac{1}{4\sqrt{2}\sqrt{b}(a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] + \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \operatorname{Sin}[c + dx]^2 + \\
& \frac{(e \cos[c + dx])^{11/2} \operatorname{Sec}[c + dx]^5 \left(\frac{2a \cos[2(c + dx)]}{5b^3} - \frac{(-28a^2 + 17b^2) \operatorname{Sin}[c + dx]}{14b^4} - \frac{(-a^2 + b^2)^2}{b^5 (a + b \operatorname{Sin}[c + dx])} - \frac{\operatorname{Sin}[3(c + dx)]}{14b^2} \right)}{d}
\end{aligned}$$

- **Problem 585:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \cos[c + dx])^{9/2}}{(a + b \operatorname{Sin}[c + dx])^2} dx$$

Optimal (type 4, 459 leaves, 14 steps):

$$\begin{aligned}
& \frac{7 a (-a^2 + b^2)^{3/4} e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{9/2} d} - \frac{7 a (-a^2 + b^2)^{3/4} e^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{9/2} d} + \\
& \frac{7 (5 a^2 - 3 b^2) e^4 \sqrt{e \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2} (c+dx), 2\right]}{5 b^4 d \sqrt{\cos[c+dx]}} - \frac{7 a^2 (a^2 - b^2) e^5 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{2 b^5 \left(b - \sqrt{-a^2 + b^2}\right) d \sqrt{e \cos[c+dx]}} - \\
& \frac{7 a^2 (a^2 - b^2) e^5 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{2 b^5 \left(b + \sqrt{-a^2 + b^2}\right) d \sqrt{e \cos[c+dx]}} + \frac{7 e^3 (e \cos[c+dx])^{3/2} (5 a - 3 b \sin[c+dx])}{15 b^3 d} - \frac{e (e \cos[c+dx])^{7/2}}{b d (a + b \sin[c+dx])}
\end{aligned}$$

Result (type 6, 1229 leaves):

$$\begin{aligned}
& \frac{1}{10 b^3 d \cos[c+dx]^{9/2}} 7 (e \cos[c+dx])^{9/2} \left(\frac{1}{6 \sqrt{1 - \cos[c+dx]^2} (a + b \sin[c+dx])} \right. \\
& a b \left(a + b \sqrt{1 - \cos[c+dx]^2} \right) \left(- \left(56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2}\right] \cos[c+dx]^{3/2} \right) / \right. \\
& \left(\sqrt{1 - \cos[c+dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2}\right] \right) \cos[c+dx]^2 \right) \\
& \left. \left. (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) - \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3 i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] - \right. \right. \\
& \left. \left. 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] + \right. \right. \\
& \left. \left. \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] \right) \right) \sin[c+dx] - \\
& \frac{1}{(1 - \cos[c+dx]^2) (a + b \sin[c+dx])} 2 (5 a^2 - 3 b^2) \left(a + b \sqrt{1 - \cos[c+dx]^2} \right) \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2}\right] \cos[c+dx]^{3/2} \sqrt{1 - \cos[c+dx]^2} \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right. \right. \\
& \quad \left. \left. + \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \\
& \quad \left. \left. + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \right. \\
& \quad \left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \sin[c + dx]^2 + \\
& \quad \frac{(e \cos[c + dx])^{9/2} \operatorname{Sec}[c + dx]^4 \left(\frac{4 a \cos[c + dx]}{3 b^3} + \frac{a^2 \cos[c + dx] - b^2 \cos[c + dx]}{b^3 (a + b \sin[c + dx])} - \frac{\sin[2(c + dx)]}{5 b^2} \right)}{d}
\end{aligned}$$

■ **Problem 586: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{7/2}}{(a + b \sin[c + dx])^2} dx$$

Optimal (type 4, 473 leaves, 14 steps):

$$\begin{aligned}
& \frac{5 a (-a^2 + b^2)^{1/4} e^{7/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 b^{7/2} d} - \frac{5 a (-a^2 + b^2)^{1/4} e^{7/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 b^{7/2} d} + \\
& \frac{5 (3 a^2 - b^2) e^4 \sqrt{\cos[c + dx]} \operatorname{EllipticF} \left[\frac{1}{2} (c + dx), 2 \right]}{3 b^4 d \sqrt{e \cos[c + dx]}} - \frac{5 a^2 (a^2 - b^2) e^4 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{2 b^4 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \cos[c + dx]}} - \\
& \frac{5 a^2 (a^2 - b^2) e^4 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{2 b^4 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \cos[c + dx]}} + \frac{5 e^3 \sqrt{e \cos[c + dx]} (3 a - b \sin[c + dx])}{3 b^3 d} - \frac{e (e \cos[c + dx])^{5/2}}{b d (a + b \sin[c + dx])}
\end{aligned}$$

Result (type 6, 2156 leaves):

$$\frac{(e \cos[c + dx])^{7/2} \operatorname{Sec}[c + dx]^3 \left(-\frac{2 \sin[c + dx]}{3 b^2} + \frac{a^2 - b^2}{b^3 (a + b \sin[c + dx])} \right)}{d} +$$

$$\begin{aligned}
& \frac{1}{6 b^3 d \operatorname{Cos}[c+d x]^{7/2}} \left(e \operatorname{Cos}[c+d x] \right)^{7/2} \left(-\frac{1}{\sqrt{1-\operatorname{Cos}[c+d x]^2} (a+b \operatorname{Sin}[c+d x])} 8 a b \left(a+b \sqrt{1-\operatorname{Cos}[c+d x]^2} \right) \right. \\
& \left(\left(5 a \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Cos}[c+d x]} \right) / \left(\sqrt{1-\operatorname{Cos}[c+d x]^2} \left(5 \left(a^2-b^2 \right) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \right) + \right. \right. \right. \\
& \left. \left. \left. \left(-a^2+b^2 \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \right) \operatorname{Cos}[c+d x]^2 \right) \left(a^2+b^2 \left(-1+\operatorname{Cos}[c+d x]^2 \right) \right) \right) - \\
& \frac{1}{\left(-a^2+b^2 \right)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] \right) + \\
& \left. \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + i b \operatorname{Cos}[c+d x] \right] - \right. \right. \right. \\
& \left. \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\operatorname{Cos}[c+d x]} + i b \operatorname{Cos}[c+d x] \right] \right) \right) \right) \operatorname{Sin}[c+d x] + \\
& \frac{1}{\sqrt{1-\operatorname{Cos}[c+d x]^2} \left(-1+2 \operatorname{Cos}[c+d x]^2 \right) (a+b \operatorname{Sin}[c+d x])} 6 a b \left(a+b \sqrt{1-\operatorname{Cos}[c+d x]^2} \right) \operatorname{Cos}[2(c+d x)] \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) \left(-2 a^2+b^2 \right) \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right]}{b^{3/2} \left(-a^2+b^2 \right)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) \left(-2 a^2+b^2 \right) \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right]}{b^{3/2} \left(-a^2+b^2 \right)^{3/4}} \right) + \\
& \frac{4 \sqrt{\operatorname{Cos}[c+d x]}}{b} + \left(10 a \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Cos}[c+d x]} \right) / \\
& \left(\sqrt{1-\operatorname{Cos}[c+d x]^2} \left(5 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] + \left(-a^2+b^2 \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \right) \operatorname{Cos}[c+d x]^2 \right) \\
& \left. \left(a^2+b^2 \left(-1+\operatorname{Cos}[c+d x]^2 \right) \right) \right) - \left(36 a \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] \operatorname{Cos}[c+d x]^{5/2} \right) / \\
& \left(5 \sqrt{1-\operatorname{Cos}[c+d x]^2} \left(9 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Cos}[c+d x]^2, \frac{b^2 \operatorname{Cos}[c+d x]^2}{-a^2+b^2}\right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + \right. \\
& \quad \left. (-a^2+b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \cos [c+d x]^2 \right) (a^2+b^2 (-1+\cos [c+d x]^2)) \Bigg) + \\
& \frac{1}{b^{3/2} (-a^2+b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2+b^2) \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] - \\
& \frac{1}{b^{3/2} (-a^2+b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2+b^2) \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] \Bigg) \sin [c+d x] - \\
& \frac{1}{(1-\cos [c+d x]^2) (a+b \sin [c+d x])} 2 (3 a^2-5 b^2) \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \\
& \left(\left(5 b (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \sqrt{\cos [c+d x]} \sqrt{1-\cos [c+d x]^2} \right) / \right. \\
& \left(\left(-5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos [c+d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \right) \cos [c+d x]^2 \right) \\
& \quad \left. (a^2+b^2 (-1+\cos [c+d x]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{(a^2-b^2)^{1/4}} \right] + \right. \\
& \quad \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{(a^2-b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] + \right. \\
& \quad \left. \left. \operatorname{Log} \left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] \right) \right) \sin [c+d x]^2 \Bigg)
\end{aligned}$$

■ **Problem 587: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \cos [c+d x])^{5/2}}{(a+b \sin [c+d x])^2} dx$$

Optimal (type 4, 390 leaves, 13 steps):

$$\frac{3 a e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{\left(-a^2+b^2\right)^{1/4} \sqrt{e}}\right]}{2 b^{5/2} \left(-a^2+b^2\right)^{1/4} d} - \frac{3 a e^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{\left(-a^2+b^2\right)^{1/4} \sqrt{e}}\right]}{2 b^{5/2} \left(-a^2+b^2\right)^{1/4} d} -$$

$$\frac{3 e^2 \sqrt{e \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{b^2 d \sqrt{\cos [c+d x]}} + \frac{3 a^2 e^3 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{2 b^3 \left(b-\sqrt{-a^2+b^2}\right) d \sqrt{e \cos [c+d x]}} +$$

$$\frac{3 a^2 e^3 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{2 b^3 \left(b+\sqrt{-a^2+b^2}\right) d \sqrt{e \cos [c+d x]}} - \frac{e \left(e \cos [c+d x]\right)^{3/2}}{b d (a+b \sin [c+d x])}$$

Result (type 6, 617 leaves):

$$-\frac{\left(e \cos [c+d x]\right)^{5/2} \operatorname{Sec}[c+d x]}{b d (a+b \sin [c+d x])} + \frac{1}{b d \cos [c+d x]^{5/2} \left(1-\cos [c+d x]^2\right) (a+b \sin [c+d x])} 3 \left(e \cos [c+d x]\right)^{5/2} \left(a+b \sqrt{1-\cos [c+d x]^2}\right)$$

$$\left(\left(7 b \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3/2} \sqrt{1-\cos [c+d x]^2}\right) / \right.$$

$$\left(3\left(-7\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]+2\right.\right.$$

$$\left.\left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4},-\frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]+\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right)\right.$$

$$\left.\left.\cos [c+d x]^2\right)\left(a^2+b^2\left(-1+\cos [c+d x]^2\right)\right)\right) + \frac{1}{4 \sqrt{2} b^{3/2} \left(a^2-b^2\right)^{1/4}}$$

$$a\left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1/4}}\right]+2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1/4}}\right]+\operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1/4} \sqrt{\cos [c+d x]}\right]+\right.$$

$$\left.\left.b \cos [c+d x]\right]-\operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1/4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right]\right) \left.\right) \sin [c+d x]^2$$

■ **Problem 588: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(e \cos [c+d x]\right)^{3/2}}{\left(a+b \sin [c+d x]\right)^2} d x$$

Optimal (type 4, 404 leaves, 13 steps):

$$\begin{aligned}
& - \frac{a e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{\left(-a^2+b^2\right)^{1/4} \sqrt{e}}\right]}{2 b^{3/2} \left(-a^2+b^2\right)^{3/4} d} - \frac{a e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{\left(-a^2+b^2\right)^{1/4} \sqrt{e}}\right]}{2 b^{3/2} \left(-a^2+b^2\right)^{3/4} d} - \frac{e^2 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{b^2 d \sqrt{e \cos [c+d x]}} + \\
& \frac{a^2 e^2 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{2 b^2 \left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos [c+d x]}} + \frac{a^2 e^2 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{2 b^2 \left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos [c+d x]}} - \frac{e \sqrt{e \cos [c+d x]}}{b d (a+b \sin [c+d x])}
\end{aligned}$$

Result (type 6, 614 leaves):

$$\begin{aligned}
& - \frac{\left(e \cos [c+d x]\right)^{3/2} \operatorname{Sec}[c+d x]}{b d (a+b \sin [c+d x])} + \frac{1}{b d \cos [c+d x]^{3/2} \left(1-\cos [c+d x]\right)^2 (a+b \sin [c+d x])} \left(e \cos [c+d x]\right)^{3/2} \left(a+b \sqrt{1-\cos [c+d x]^2}\right) \\
& \left(\left(5 b \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \sqrt{\cos [c+d x]} \sqrt{1-\cos [c+d x]^2}\right) / \right. \\
& \left(\left(-5 \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4},-\frac{1}{2}, 2, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \cos [c+d x]^2 \left(a^2+b^2\left(-1+\cos [c+d x]^2\right)\right) \right) \right) + \\
& \frac{1}{4 \sqrt{2} \sqrt{b} \left(a^2-b^2\right)^{3/4}} a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1/4}}\right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b} \left(a^2-b^2\right)^{1/4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right] + \right. \\
& \left. \operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b} \left(a^2-b^2\right)^{1/4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right] \right) \right) \sin [c+d x]^2
\end{aligned}$$

■ **Problem 589: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{e \cos [c+d x]}}{(a+b \sin [c+d x])^2} dx$$

Optimal (type 4, 422 leaves, 13 steps):

$$\begin{aligned}
& - \frac{a \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 \sqrt{b} (-a^2+b^2)^{5/4} d} + \frac{a \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 \sqrt{b} (-a^2+b^2)^{5/4} d} + \\
& \frac{\sqrt{e \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{(a^2-b^2) d \sqrt{\cos[c+dx]}} + \frac{a^2 e \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{2b(a^2-b^2)(b-\sqrt{-a^2+b^2}) d \sqrt{e \cos[c+dx]}} + \\
& \frac{a^2 e \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{2b(a^2-b^2)(b+\sqrt{-a^2+b^2}) d \sqrt{e \cos[c+dx]}} + \frac{b(e \cos[c+dx])^{3/2}}{(a^2-b^2) d e (a+b \sin[c+dx])}
\end{aligned}$$

Result (type 6, 1182 leaves):

$$\begin{aligned}
& - \frac{b \cos [c+d x] \sqrt{e \cos [c+d x]}}{\left(-a^2+b^2\right) d(a+b \sin [c+d x])} + \\
& \frac{1}{2(a-b)(a+b) d \sqrt{\cos [c+d x]}} \sqrt{e \cos [c+d x]} \left(\frac{1}{6 \sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])} a \left(a+b \sqrt{1-\cos [c+d x]^2}\right) \right. \\
& \left. \left(- \left(56 a \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3/2} \right) / \left(\sqrt{1-\cos [c+d x]^2} \left(7 \left(a^2-b^2\right) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \right) \right) + \right. \\
& \left. \left. \left. \left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \cos [c+d x]^2 \left(a^2+b^2(-1+\cos [c+d x]^2)\right) \right) \right) - \\
& \frac{1}{\sqrt{b} \left(-a^2+b^2\right)^{1/4}} (3+3 i) \left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1/4}}\right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b} \left(-a^2+b^2\right)^{1/4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right] + \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b} \left(-a^2+b^2\right)^{1/4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right] \right) \left. \right) \sin [c+d x] - \frac{1}{\left(1-\cos [c+d x]^2\right)(a+b \sin [c+d x])} \\
& 2 b \left(a+b \sqrt{1-\cos [c+d x]^2}\right) \left(\left(7 b \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3/2} \sqrt{1-\cos [c+d x]^2} \right) / \right. \\
& \left. \left(3 \left(-7 \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4},-\frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \cos [c+d x]^2 \right) \right) \\
& \left. \left. \left. \left(a^2+b^2(-1+\cos [c+d x]^2)\right) \right) + \frac{1}{4 \sqrt{2} b^{3/2} \left(a^2-b^2\right)^{1/4}} a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1/4}}\right] + \right. \right. \\
& \left. \left. 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b} \left(a^2-b^2\right)^{1/4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right] - \right. \right. \\
& \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b} \left(a^2-b^2\right)^{1/4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right] \right) \right) \left. \right) \sin [c+d x]^2 \left. \right)
\end{aligned}$$

- **Problem 590: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{e \cos[c + dx]} (a + b \sin[c + dx])^2} dx$$

Optimal (type 4, 429 leaves, 13 steps):

$$\frac{3 a \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 (-a^2+b^2)^{7/4} d \sqrt{e}} + \frac{3 a \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 (-a^2+b^2)^{7/4} d \sqrt{e}} -$$

$$\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2} (c+dx), 2\right]}{(a^2-b^2) d \sqrt{e \cos[c+dx]}} + \frac{3 a^2 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{2 (a^2-b^2) \left(a^2-b \left(b-\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos[c+dx]}} +$$

$$\frac{3 a^2 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{2 (a^2-b^2) \left(a^2-b \left(b+\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos[c+dx]}} + \frac{b \sqrt{e \cos[c+dx]}}{(a^2-b^2) d e (a+b \sin[c+dx])}$$

Result (type 6, 1181 leaves):

$$\begin{aligned}
& \frac{b \cos [c+d x]}{\left(a^2-b^2\right) d \sqrt{e \cos [c+d x]}(a+b \sin [c+d x])} + \\
& \frac{1}{2(a-b)(a+b) d \sqrt{e \cos [c+d x]}} \sqrt{\cos [c+d x]} \left(-\frac{1}{\sqrt{1-\cos [c+d x]^2}(a+b \sin [c+d x])} 4 a\left(a+b \sqrt{1-\cos [c+d x]^2}\right) \right. \\
& \left. \left(\left(5 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \sqrt{\cos [c+d x]} \right) / \left(\sqrt{1-\cos [c+d x]^2}\left(5\left(a^2-b^2\right) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] - 2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right) \cos [c+d x]^2\right)\left(a^2+b^2(-1+\cos [c+d x]^2)\right) \right) - \\
& \frac{1}{\left(-a^2+b^2\right)^{3 / 4}}\left(\frac{1}{8}-\frac{i}{8}\right) \sqrt{b}\left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]+ \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right]- \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right]\right) \left. \right) \sin [c+d x]+\frac{1}{(1-\cos [c+d x]^2)(a+b \sin [c+d x])} \\
& 2 b\left(a+b \sqrt{1-\cos [c+d x]^2}\right)\left(\left(5 b\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \sqrt{\cos [c+d x]} \sqrt{1-\cos [c+d x]^2}\right) / \right. \\
& \left. \left(\left(-5\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]+2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4},-\frac{1}{2}, 2, \frac{9}{4}, \cos [c+d x]^2, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right)+\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right) \cos [c+d x]^2\right) \right. \\
& \left. \left(a^2+b^2(-1+\cos [c+d x]^2)\right)\right)+\frac{1}{4 \sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{3 / 4}} a\left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]+ \right. \\
& \left. 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]-\operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right]+ \right. \\
& \left. \operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right]\right) \left. \right) \sin [c+d x]^2\left. \right)
\end{aligned}$$

- **Problem 591: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \cos[c + dx])^{3/2} (a + b \sin[c + dx])^2} dx$$

Optimal (type 4, 492 leaves, 14 steps):

$$\begin{aligned} & - \frac{5 a b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 (-a^2+b^2)^{9/4} d e^{3/2}} + \frac{5 a b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 (-a^2+b^2)^{9/4} d e^{3/2}} - \frac{(2 a^2+3 b^2) \sqrt{e \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{(a^2-b^2)^2 d e^2 \sqrt{\cos[c+dx]}} \\ & - \frac{5 a^2 b \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{2 (a^2-b^2)^2 (b-\sqrt{-a^2+b^2}) d e \sqrt{e \cos[c+dx]}} - \frac{5 a^2 b \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{2 (a^2-b^2)^2 (b+\sqrt{-a^2+b^2}) d e \sqrt{e \cos[c+dx]}} + \\ & \frac{b}{(a^2-b^2) d e \sqrt{e \cos[c+dx]} (a+b \sin[c+dx])} - \frac{5 a b - (2 a^2+3 b^2) \sin[c+dx]}{(a^2-b^2)^2 d e \sqrt{e \cos[c+dx]}} \end{aligned}$$

Result (type 6, 1260 leaves):

$$\begin{aligned} & - \frac{1}{2 (a-b)^2 (a+b)^2 d (e \cos[c+dx])^{3/2}} \\ & \cos[c+dx]^{3/2} \left(\frac{1}{12 \sqrt{1-\cos[c+dx]^2} (a+b \sin[c+dx])} (2 a^3+8 a b^2) (a+b \sqrt{1-\cos[c+dx]^2}) \left(- \left(56 a (a^2-b^2) \right. \right. \right. \\ & \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \cos[c+dx]^{3/2} \right) / \left(\sqrt{1-\cos[c+dx]^2} \left(7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \right. \right. \right. \right. \\ & \left. \left. \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\right. \right. \right. \right. \\ & \left. \left. \frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 (a^2+b^2 (-1+\cos[c+dx]^2)) \right) - \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} \\ & \left. \left. \left. \left. (3+3 i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \right. \right. \right. \\ & \left. \left. \left. \left. (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx] \right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx] \right] \right) \right) \right) \right) \\ & \sin[c+dx] - \frac{1}{(1-\cos[c+dx]^2) (a+b \sin[c+dx])} 2 (2 a^2 b+3 b^3) (a+b \sqrt{1-\cos[c+dx]^2}) \end{aligned}$$

$$\left(\left(7b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{3/2} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\ \left. \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \right) \cos[c + dx]^2 \right. \right. \\ \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \left. \right) \\ \left(a^2 + b^2 (-1 + \cos[c + dx]^2) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\ \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] - \right. \\ \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \left. \right) \sin[c + dx]^2 + \\ \frac{\cos[c + dx]^2 \left(-\frac{b^3 \cos[c + dx]}{(a^2 - b^2)^2 (a + b \sin[c + dx])} + \frac{2 \operatorname{Sec}[c + dx] (-2ab + a^2 \sin[c + dx] + b^2 \sin[c + dx])}{(a^2 - b^2)^2} \right)}{d (e \cos[c + dx])^{3/2}}$$

- **Problem 592: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \cos[c + dx])^{5/2} (a + b \sin[c + dx])^2} dx$$

Optimal (type 4, 514 leaves, 14 steps):

$$\frac{7ab^{5/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2(-a^2 + b^2)^{11/4} d e^{5/2}} + \frac{7ab^{5/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2(-a^2 + b^2)^{11/4} d e^{5/2}} + \frac{(2a^2 + 5b^2) \sqrt{\cos[c + dx]} \operatorname{EllipticF} \left[\frac{1}{2}(c + dx), 2 \right]}{3(a^2 - b^2)^2 d e^2 \sqrt{e \cos[c + dx]}} - \\ \frac{7a^2 b^2 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2}(c + dx), 2 \right]}{2(a^2 - b^2)^2 (a^2 - b(b - \sqrt{-a^2 + b^2})) d e^2 \sqrt{e \cos[c + dx]}} - \frac{7a^2 b^2 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2}(c + dx), 2 \right]}{2(a^2 - b^2)^2 (a^2 - b(b + \sqrt{-a^2 + b^2})) d e^2 \sqrt{e \cos[c + dx]}} + \\ \frac{b}{(a^2 - b^2) d e (e \cos[c + dx])^{3/2} (a + b \sin[c + dx])} - \frac{7ab - (2a^2 + 5b^2) \sin[c + dx]}{3(a^2 - b^2)^2 d e (e \cos[c + dx])^{3/2}}$$

Result (type 6, 1258 leaves):

$$\frac{1}{6(a - b)^2 (a + b)^2 d (e \cos[c + dx])^{5/2}}$$

$$\begin{aligned}
& \cos [c+d x]^{5 / 2} \left(-\frac{1}{\sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])} 2\left(2 a^3-16 a b^2\right)\left(a+b \sqrt{1-\cos [c+d x]^2}\right)\left(\left(5 a\left(a^2-b^2\right)\right.\right.\right. \\
& \quad \left.\left.\left.\text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \sqrt{\cos [c+d x]}\right) / \left(\sqrt{1-\cos [c+d x]^2}\left(5\left(a^2-b^2\right)\text{AppellF1}\left[\frac{1}{4}, \frac{1}{2},\right.\right.\right.\right. \\
& \quad \left.\left.\left.1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]+ \right.\right.\right. \\
& \quad \left.\left.\left.(-a^2+b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right) \cos [c+d x]^2\right)\left(a^2+b^2(-1+\cos [c+d x]^2)\right)\right) - \\
& \quad \frac{1}{(-a^2+b^2)^{3 / 4}}\left(\frac{1}{8}-\frac{i}{8}\right) \sqrt{b}\left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]+ \right. \\
& \quad \left.\log \left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right]- \right. \\
& \quad \left.\log \left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right]\right) \left.\right) \sin [c+d x]- \\
& \quad \frac{1}{(1-\cos [c+d x]^2)(a+b \sin [c+d x])} 2\left(2 a^2 b+5 b^3\right)\left(a+b \sqrt{1-\cos [c+d x]^2}\right) \\
& \quad \left(\left(5 b\left(a^2-b^2\right) \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \sqrt{\cos [c+d x]} \sqrt{1-\cos [c+d x]^2}\right) / \right. \\
& \quad \left(\left(-5\left(a^2-b^2\right) \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]+2\left(2 b^2 \text{AppellF1}\left[\frac{5}{4},-\frac{1}{2}, 2, \frac{9}{4}, \cos [c+d x]^2, \right.\right.\right. \\
& \quad \left.\left.\left.\frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]+\left(a^2-b^2\right) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right) \cos [c+d x]^2\right) \\
& \quad \left.\left(a^2+b^2(-1+\cos [c+d x]^2)\right)\right)+\frac{1}{4 \sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{3 / 4}} a\left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]+ \right. \\
& \quad \left.2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]-\log \left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right]+ \right. \\
& \quad \left.\log \left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right]\right) \left.\right) \sin [c+d x]^2)+
\end{aligned}$$

$$\frac{\cos [c+d x]^3 \left(-\frac{b^3}{\left(a^2-b^2\right)^2 (a+b \sin [c+d x])} + \frac{2 \operatorname{Sec}[c+d x]^2 (-2 a b+a^2 \sin [c+d x]+b^2 \sin [c+d x])}{3\left(a^2-b^2\right)^2} \right)}{d\left(e \cos [c+d x]\right)^{5 / 2}}$$

- **Problem 593: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(e \cos [c+d x]\right)^{7 / 2}(a+b \sin [c+d x])^2} d x$$

Optimal (type 4, 574 leaves, 15 steps):

$$\begin{aligned} & -\frac{9 a b^{7 / 2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{13 / 4} d e^{7 / 2}}+\frac{9 a b^{7 / 2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{13 / 4} d e^{7 / 2}}-\frac{3\left(2 a^4-10 a^2 b^2-7 b^4\right) \sqrt{e \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5\left(a^2-b^2\right)^3 d e^4 \sqrt{\cos [c+d x]}}+ \\ & \frac{9 a^2 b^3 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{2\left(a^2-b^2\right)^3\left(b-\sqrt{-a^2+b^2}\right) d e^3 \sqrt{e \cos [c+d x]}}+\frac{9 a^2 b^3 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{2\left(a^2-b^2\right)^3\left(b+\sqrt{-a^2+b^2}\right) d e^3 \sqrt{e \cos [c+d x]}}+ \\ & \frac{b}{\left(a^2-b^2\right) d e\left(e \cos [c+d x]\right)^{5 / 2}(a+b \sin [c+d x])}-\frac{9 a b-\left(2 a^2+7 b^2\right) \sin [c+d x]}{5\left(a^2-b^2\right)^2 d e\left(e \cos [c+d x]\right)^{5 / 2}}+\frac{3\left(15 a b^3+\left(2 a^4-10 a^2 b^2-7 b^4\right) \sin [c+d x]\right)}{5\left(a^2-b^2\right)^3 d e^3 \sqrt{e \cos [c+d x]}} \end{aligned}$$

Result (type 6, 1343 leaves):

$$\begin{aligned} & -\frac{1}{10(a-b)^3(a+b)^3 d\left(e \cos [c+d x]\right)^{7 / 2}} \\ & 3 \cos [c+d x]^{7 / 2}\left(\frac{1}{12 \sqrt{1-\cos [c+d x]^2}(a+b \sin [c+d x])}\left(2 a^5-10 a^3 b^2-22 a b^4\right)\left(a+b \sqrt{1-\cos [c+d x]^2}\right)\right. \\ & \left(-\left(56 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3 / 2}\right) / \left(\sqrt{1-\cos [c+d x]^2}\left(7\left(a^2-b^2\right)\right.\right.\right. \\ & \left.\left.\left.\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right)+\right.\right.\right. \\ & \left.\left.\left.\left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right) \cos [c+d x]^2\right)\left(a^2+b^2(-1+\cos [c+d x]^2)\right)\right) - \\ & \frac{1}{\sqrt{b}\left(-a^2+b^2\right)^{1 / 4}}(3+3 i)\left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-\right. \\ & \left.\operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right]+ \right. \end{aligned}$$

$$\begin{aligned}
& - \frac{11 (9 a^4 - 11 a^2 b^2 + 2 b^4) e^{13/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{13/2} (-a^2+b^2)^{1/4} d} + \\
& \frac{11 (9 a^4 - 11 a^2 b^2 + 2 b^4) e^{13/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{13/2} (-a^2+b^2)^{1/4} d} + \frac{11 a (45 a^2 - 37 b^2) e^6 \sqrt{e \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2} (c+dx), 2\right]}{20 b^6 d \sqrt{\cos[c+dx]}} - \\
& \frac{11 a (9 a^4 - 11 a^2 b^2 + 2 b^4) e^7 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{8 b^7 \left(b - \sqrt{-a^2+b^2}\right) d \sqrt{e \cos[c+dx]}} - \\
& \frac{11 a (9 a^4 - 11 a^2 b^2 + 2 b^4) e^7 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{8 b^7 \left(b + \sqrt{-a^2+b^2}\right) d \sqrt{e \cos[c+dx]}} - \frac{e (e \cos[c+dx])^{11/2}}{2 b d (a+b \sin[c+dx])^2} - \\
& \frac{11 e^3 (e \cos[c+dx])^{7/2} (9 a + 2 b \sin[c+dx])}{28 b^3 d (a+b \sin[c+dx])} + \frac{11 e^5 (e \cos[c+dx])^{3/2} (5 (9 a^2 - 2 b^2) - 27 a b \sin[c+dx])}{60 b^5 d}
\end{aligned}$$

Result (type 6, 1326 leaves):

$$\begin{aligned}
& \frac{1}{40 b^5 d \cos[c+dx]^{13/2}} 11 (e \cos[c+dx])^{13/2} \left(\frac{1}{12 \sqrt{1 - \cos[c+dx]^2} (a+b \sin[c+dx])} \right. \\
& \left. (18 a^2 b - 10 b^3) \left(a + b \sqrt{1 - \cos[c+dx]^2} \right) \left(- \left(56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \cos[c+dx]^{3/2} \right) / \right. \right. \\
& \left. \left(\sqrt{1 - \cos[c+dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 \right) \right. \\
& \left. (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) - \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \right. \\
& \left. 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] + \right. \\
& \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] \right) \right) \left. \right) \sin[c+dx] -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(1 - \cos[c + dx])^2 (a + b \sin[c + dx])} 2 (45 a^3 - 37 a b^2) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{3/2} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[c + dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] - \right. \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \sin[c + dx]^2 + \\
& \frac{1}{d} (e \cos[c + dx])^{13/2} \sec[c + dx]^6 \left(-\frac{(-168 a^2 + 65 b^2) \cos[c + dx]}{42 b^5} - \right. \\
& \frac{\cos[3(c + dx)]}{14 b^3} + \\
& \frac{-a^4 \cos[c + dx] + 2 a^2 b^2 \cos[c + dx] - b^4 \cos[c + dx]}{2 b^5 (a + b \sin[c + dx])^2} + \\
& \frac{19 (a^3 \cos[c + dx] - a b^2 \cos[c + dx])}{4 b^5 (a + b \sin[c + dx])} - \\
& \left. \frac{3 a \sin[2(c + dx)]}{5 b^4} \right)
\end{aligned}$$

■ **Problem 595: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{11/2}}{(a + b \sin[c + dx])^3} dx$$

Optimal (type 4, 589 leaves, 15 steps):

$$\begin{aligned}
& \frac{9 (7 a^4 - 9 a^2 b^2 + 2 b^4) e^{11/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{11/2} (-a^2+b^2)^{3/4} d} + \frac{9 (7 a^4 - 9 a^2 b^2 + 2 b^4) e^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{11/2} (-a^2+b^2)^{3/4} d} + \\
& \frac{3 a (21 a^2 - 13 b^2) e^6 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), 2\right]}{4 b^6 d \sqrt{e \cos [c+d x]}} - \frac{9 a (7 a^4 - 9 a^2 b^2 + 2 b^4) e^6 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+d x), 2\right]}{8 b^6 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos [c+d x]}} - \\
& \frac{9 a (7 a^4 - 9 a^2 b^2 + 2 b^4) e^6 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+d x), 2\right]}{8 b^6 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos [c+d x]}} - \frac{e (e \cos [c+d x])^{9/2}}{2 b d (a+b \sin [c+d x])^2} - \\
& \frac{9 e^3 (e \cos [c+d x])^{5/2} (7 a+2 b \sin [c+d x])}{20 b^3 d (a+b \sin [c+d x])} + \frac{3 e^5 \sqrt{e \cos [c+d x]} (3 (7 a^2 - 2 b^2) - 7 a b \sin [c+d x])}{4 b^5 d}
\end{aligned}$$

Result (type 6, 2224 leaves):

$$\begin{aligned}
& \frac{(e \cos [c+d x])^{11/2} \operatorname{Sec}[c+d x]^5 \left(-\frac{\cos [2(c+d x)]}{5 b^3} - \frac{2 a \sin [c+d x]}{b^4} - \frac{(-a^2+b^2)^2}{2 b^5 (a+b \sin [c+d x])^2} + \frac{17 a (a^2-b^2)}{4 b^5 (a+b \sin [c+d x])}\right)}{d} + \\
& \frac{1}{40 b^5 d \cos [c+d x]^{11/2}} 3 (e \cos [c+d x])^{11/2} \left(-\frac{1}{\sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])}\right. \\
& 2 (30 a^2 b - 16 b^3) \left(a+b \sqrt{1-\cos [c+d x]^2}\right) \left(\left(5 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \sqrt{\cos [c+d x]}\right) / \right. \\
& \left.\left(\sqrt{1-\cos [c+d x]^2} \left(5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right.\right.\right.\right. \right. \\
& \left.\left.\left.\cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right) \cos [c+d x]^2\right) \\
& \left.\left(a^2+b^2 (-1+\cos [c+d x]^2)\right)\right) - \frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8}\right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{(-a^2+b^2)^{1/4}}\right] - \right. \\
& \left. 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x]\right] - \right. \\
& \left. \left.\operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x]\right]\right)\right) \left.\right) \sin [c+d x] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{1 - \cos[c + dx]^2} (-1 + 2 \cos[c + dx]^2) (a + b \sin[c + dx])} (40 a^2 b - 14 b^3) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \cos[2(c + dx)] \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} \right) + \\
& \frac{4 \sqrt{\cos[c + dx]}}{b} + \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \right) / \\
& \left(\sqrt{1 - \cos[c + dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) - \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{5/2} \right) / \\
& \left(5 \sqrt{1 - \cos[c + dx]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] - \right. \right. \\
& \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] \right) \sin[c + dx] - \\
& \frac{1}{(1 - \cos[c + dx]^2) (a + b \sin[c + dx])} 2 (25 a^3 - 37 a b^2) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\
& \left. \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \right. \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} + (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right) \cos [c+d x]^2 \\ & \left. \left(a^2+b^2(-1+\cos [c+d x]^2) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4}} a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{(a^2-b^2)^{1/4}}\right] + \right. \right. \\ & \left. \left. 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right] + \right. \right. \\ & \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right] \right) \right) \sin [c+d x]^2 \end{aligned}$$

■ **Problem 596: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos [c+d x])^{9/2}}{(a+b \sin [c+d x])^3} dx$$

Optimal (type 4, 483 leaves, 14 steps):

$$\begin{aligned} & \frac{7(5 a^2-2 b^2) e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{9/2}(-a^2+b^2)^{1/4} d} - \frac{7(5 a^2-2 b^2) e^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{9/2}(-a^2+b^2)^{1/4} d} \\ & + \frac{35 a e^4 \sqrt{e \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{4 b^4 d \sqrt{\cos [c+d x]}} + \frac{7 a(5 a^2-2 b^2) e^5 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{8 b^5\left(b-\sqrt{-a^2+b^2}\right) d \sqrt{e \cos [c+d x]}} \\ & - \frac{7 a(5 a^2-2 b^2) e^5 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{8 b^5\left(b+\sqrt{-a^2+b^2}\right) d \sqrt{e \cos [c+d x]}} \\ & - \frac{e(e \cos [c+d x])^{7/2}}{2 b d(a+b \sin [c+d x])^2} - \frac{7 e^3(e \cos [c+d x])^{3/2}(5 a+2 b \sin [c+d x])}{12 b^3 d(a+b \sin [c+d x])} \end{aligned}$$

Result (type 6, 1231 leaves):

$$\begin{aligned}
& \frac{(e \cos [c+d x])^{9/2} \operatorname{Sec}[c+d x]^4 \left(-\frac{2 \cos [c+d x]}{3 b^3} + \frac{a^2 \cos [c+d x]-b^2 \cos [c+d x]}{2 b^3 (a+b \sin [c+d x])^2} - \frac{11 a \cos [c+d x]}{4 b^3 (a+b \sin [c+d x])}\right)}{d} \\
& \frac{1}{8 b^3 d \cos [c+d x]^{9/2}} \left(\frac{1}{6 \sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])} \right. \\
& b \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \left(-\left(56 a \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3/2} \right) / \right. \\
& \left(\sqrt{1-\cos [c+d x]^2} \left(7 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + \left(-a^2+b^2 \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \cos [c+d x]^2 \right) \right. \\
& \left. \left(a^2+b^2 \left(-1+\cos [c+d x]^2 \right) \right) \right) - \frac{1}{\sqrt{b} \left(-a^2+b^2 \right)^{1/4}} \left(3+3 i \right) \left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] - \right. \\
& \left. 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] + \right. \\
& \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] \right) \right) \left. \right) \left. \right) \sin [c+d x] - \\
& \frac{1}{\left(1-\cos [c+d x]^2 \right) (a+b \sin [c+d x])} 10 a \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \\
& \left(\left(7 b \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3/2} \sqrt{1-\cos [c+d x]^2} \right) / \left(3 \left(-7 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\right. \right. \right. \right. \\
& \left. \left. \left. \frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \cos [c+d x]^2 \right) \left(a^2+b^2 \left(-1+\cos [c+d x]^2 \right) \right) \right) \right) + \\
& \frac{1}{4 \sqrt{2} b^{3/2} \left(a^2-b^2 \right)^{1/4}} a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2 \right)^{1/4}} \right] + 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2 \right)^{1/4}} \right] + \operatorname{Log}\left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} \right. \right. \\
& \left. \left. \left(a^2-b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] - \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] \right) \right) \left. \right) \left. \right) \sin [c+d x]^2 \left. \right)
\end{aligned}$$

■ **Problem 597: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{7/2}}{(a + b \sin[c + dx])^3} dx$$

Optimal (type 4, 497 leaves, 14 steps):

$$\frac{5(3a^2 - 2b^2)e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{e\cos[c+dx]}}{(-a^2+b^2)^{1/4}\sqrt{e}}\right] - 5(3a^2 - 2b^2)e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b}\sqrt{e\cos[c+dx]}}{(-a^2+b^2)^{1/4}\sqrt{e}}\right]}{8b^{7/2}(-a^2+b^2)^{3/4}d} + \frac{15ae^4\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - 5a(3a^2 - 2b^2)e^4\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{4b^4d\sqrt{e\cos[c+dx]}} + \frac{5a(3a^2 - 2b^2)e^4\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{8b^4\left(a^2 - b\left(b - \sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cos[c+dx]}} - \frac{e(e\cos[c+dx])^{5/2}}{2bd(a+b\sin[c+dx])^2} - \frac{5e^3\sqrt{e\cos[c+dx]}(3a+2b\sin[c+dx])}{4b^3d(a+b\sin[c+dx])}$$

Result (type 6, 2154 leaves):

$$\frac{(e \cos[c + dx])^{7/2} \operatorname{Sec}[c + dx]^3 \left(\frac{a^2 - b^2}{2b^3(a+b\sin[c+dx])^2} - \frac{9a}{4b^3(a+b\sin[c+dx])} \right)}{d} - \frac{1}{8b^3d\cos[c+dx]^{7/2}} (e \cos[c + dx])^{7/2} \left(- \frac{1}{\sqrt{1 - \cos[c + dx]^2} (a + b \sin[c + dx])} \right. \\ \left. 12b \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \left(\left(5a(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] \sqrt{\cos[c + dx]} \right) / \right. \\ \left. \left(\sqrt{1 - \cos[c + dx]^2} \left(5(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] \right) \cos[c + dx]^2 \right) \right) \right. \\ \left. (a^2 + b^2(-1 + \cos[c + dx]^2)) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - \right. \\ \left. 2 \operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\cos[c+dx]} + ib\cos[c+dx] \right] - \right.$$

$$\left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right. \\ \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\ \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] + \right. \\ \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \left. \right) \sin[c + dx]^2$$

- **Problem 598: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{5/2}}{(a + b \sin[c + dx])^3} dx$$

Optimal (type 4, 505 leaves, 14 steps):

$$\frac{3 (a^2 - 2 b^2) e^{5/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{8 b^{5/2} (-a^2 + b^2)^{5/4} d} - \frac{3 (a^2 - 2 b^2) e^{5/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{8 b^{5/2} (-a^2 + b^2)^{5/4} d} + \\ \frac{3 a e^2 \sqrt{e \cos[c + dx]} \operatorname{EllipticE} \left[\frac{1}{2} (c + dx), 2 \right]}{4 b^2 (a^2 - b^2) d \sqrt{\cos[c + dx]}} - \frac{3 a (a^2 - 2 b^2) e^3 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{8 b^3 (a^2 - b^2) (b - \sqrt{-a^2 + b^2}) d \sqrt{e \cos[c + dx]}} - \\ \frac{3 a (a^2 - 2 b^2) e^3 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{8 b^3 (a^2 - b^2) (b + \sqrt{-a^2 + b^2}) d \sqrt{e \cos[c + dx]}} - \frac{e (e \cos[c + dx])^{3/2}}{2 b d (a + b \sin[c + dx])^2} + \frac{3 a e (e \cos[c + dx])^{3/2}}{4 b (a^2 - b^2) d (a + b \sin[c + dx])}$$

Result (type 6, 1225 leaves):

$$\begin{aligned}
& \frac{(e \cos [c+d x])^{5/2} \operatorname{Sec}[c+d x]^2 \left(-\frac{\cos [c+d x]}{2 b(a+b \sin [c+d x])^2} - \frac{3 a \cos [c+d x]}{4 b(-a^2+b^2)(a+b \sin [c+d x])} \right)}{d} + \\
& \frac{1}{8(a-b) b(a+b) d \cos [c+d x]^{5/2}} \left(\frac{1}{6 \sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])} \right. \\
& \left. b \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \left(-\left(56 a \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \cos [c+d x]^{3/2} \right) \right) / \\
& \left(\sqrt{1-\cos [c+d x]^2} \left(7 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + \left(-a^2+b^2 \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \right) \cos [c+d x]^2 \right) \\
& \left. \left(a^2+b^2 \left(-1+\cos [c+d x]^2 \right) \right) \right) - \frac{1}{\sqrt{b} \left(-a^2+b^2 \right)^{1/4}} \left(3+3 i \right) \left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] - \right. \\
& \left. 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] + \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] \right) \left. \right) \left. \right) \sin [c+d x] - \frac{1}{\left(1-\cos [c+d x]^2 \right) (a+b \sin [c+d x])} \\
& 2 a \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \left(\left(7 b \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \cos [c+d x]^{3/2} \sqrt{1-\cos [c+d x]^2} \right) / \right. \\
& \left(3 \left(-7 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \right) \cos [c+d x]^2 \right) \\
& \left. \left(a^2+b^2 \left(-1+\cos [c+d x]^2 \right) \right) \right) + \frac{1}{4 \sqrt{2} b^{3/2} \left(a^2-b^2 \right)^{1/4}} \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2 \right)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2 \right)^{1/4}} \right] + \operatorname{Log}\left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] \right) \left. \right) \left. \right) \sin [c+d x]^2 \left. \right)
\end{aligned}$$

- **Problem 599: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos [c + d x])^{3/2}}{(a + b \sin [c + d x])^3} dx$$

Optimal (type 4, 519 leaves, 14 steps):

$$\frac{(a^2 + 2 b^2) e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{8 b^{3/2} (-a^2 + b^2)^{7/4} d} + \frac{(a^2 + 2 b^2) e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{8 b^{3/2} (-a^2 + b^2)^{7/4} d} -$$

$$\frac{a e^2 \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{4 b^2 (a^2 - b^2) d \sqrt{e \cos [c + d x]}} + \frac{a (a^2 + 2 b^2) e^2 \sqrt{\cos [c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + d x), 2\right]}{8 b^2 (a^2 - b^2) \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \cos [c + d x]}} +$$

$$\frac{a (a^2 + 2 b^2) e^2 \sqrt{\cos [c + d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + d x), 2\right]}{8 b^2 (a^2 - b^2) \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \cos [c + d x]}} - \frac{e \sqrt{e \cos [c + d x]}}{2 b d (a + b \sin [c + d x])^2} + \frac{a e \sqrt{e \cos [c + d x]}}{4 b (a^2 - b^2) d (a + b \sin [c + d x])}$$

Result (type 6, 1211 leaves):

$$\begin{aligned}
& \frac{(e \cos [c+d x])^{3/2} \operatorname{Sec}[c+d x] \left(-\frac{1}{2 b(a+b \sin [c+d x])^2} - \frac{a}{4 b(-a^2+b^2)(a+b \sin [c+d x])} \right)}{d} \\
& \frac{1}{8(a-b) b(a+b) d \cos [c+d x]^{3/2}} (e \cos [c+d x])^{3/2} \left(\frac{1}{\sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])} \right. \\
& 4 b \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \left(\left(5 a \left(a^2-b^2 \right) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \sqrt{\cos [c+d x]} \right) / \right. \\
& \left. \left(\sqrt{1-\cos [c+d x]^2} \left(5 \left(a^2-b^2 \right) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + \left(-a^2+b^2 \right) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \right) \cos [c+d x]^2 \right) \right. \\
& \left. \left(a^2+b^2 \left(-1+\cos [c+d x]^2 \right) \right) \right) - \frac{1}{\left(-a^2+b^2 \right)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] - \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] \right) \left. \right) \sin [c+d x] - \frac{1}{\left(1-\cos [c+d x]^2 \right) (a+b \sin [c+d x])} \\
& 2 a \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \left(\left(5 b \left(a^2-b^2 \right) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \sqrt{\cos [c+d x]} \sqrt{1-\cos [c+d x]^2} \right) / \right. \\
& \left(\left(-5 \left(a^2-b^2 \right) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos [c+d x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + \left(a^2-b^2 \right) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \right) \cos [c+d x]^2 \right) \right. \\
& \left. \left(a^2+b^2 \left(-1+\cos [c+d x]^2 \right) \right) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2 \right)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2 \right)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] + \right. \\
& \left. \operatorname{Log} \left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] \right) \left. \right) \sin [c+d x]^2 \left. \right)
\end{aligned}$$

- **Problem 600: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{e \cos[c + dx]}}{(a + b \sin[c + dx])^3} dx$$

Optimal (type 4, 514 leaves, 14 steps):

$$\frac{(3a^2 + 2b^2) \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right] - (3a^2 + 2b^2) \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{8 \sqrt{b} (-a^2 + b^2)^{9/4} d} +$$

$$\frac{5a \sqrt{e \cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] - a(3a^2 + 2b^2) e \sqrt{\cos[c + dx]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2}(c + dx), 2\right]}{4(a^2 - b^2)^2 d \sqrt{\cos[c + dx]} + 8b(a^2 - b^2)^2 (b - \sqrt{-a^2 + b^2}) d \sqrt{e \cos[c + dx]}} +$$

$$\frac{a(3a^2 + 2b^2) e \sqrt{\cos[c + dx]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2}(c + dx), 2\right]}{8b(a^2 - b^2)^2 (b + \sqrt{-a^2 + b^2}) d \sqrt{e \cos[c + dx]}} + \frac{b(e \cos[c + dx])^{3/2}}{2(a^2 - b^2) d e (a + b \sin[c + dx])^2} + \frac{5ab(e \cos[c + dx])^{3/2}}{4(a^2 - b^2)^2 d e (a + b \sin[c + dx])}$$

Result (type 6, 1232 leaves):

$$\frac{\sqrt{e \cos[c + dx]} \left(\frac{b \cos[c + dx]}{2(a^2 - b^2)(a + b \sin[c + dx])^2} + \frac{5ab \cos[c + dx]}{4(a^2 - b^2)^2(a + b \sin[c + dx])} \right)}{d} +$$

$$\frac{1}{8(a - b)^2(a + b)^2 d \sqrt{\cos[c + dx]}} \sqrt{e \cos[c + dx]} \left(\frac{1}{12 \sqrt{1 - \cos[c + dx]^2} (a + b \sin[c + dx])} \right.$$

$$\left. (8a^2 + 2b^2) (a + b \sqrt{1 - \cos[c + dx]^2}) \left(- \left(56a(a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] \cos[c + dx]^{3/2} \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[c + dx]^2} \left(7(a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right.$$

$$\left. \left. \left. \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right)$$

$$\left. (a^2 + b^2(-1 + \cos[c + dx]^2)) \right) - \frac{1}{\sqrt{b}(-a^2 + b^2)^{1/4}} (3 + 3i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - \right.$$

$$\left. 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx]\right] + \right.$$

Result (type 6, 1226 leaves) :

$$\begin{aligned}
 & \frac{\cos [c+d x] \left(\frac{b}{2\left(a^2-b^2\right)\left(a+b \sin [c+d x]\right)^2} + \frac{7 a b}{4\left(a^2-b^2\right)^2\left(a+b \sin [c+d x]\right)} \right)}{d \sqrt{e \cos [c+d x]}} + \\
 & \frac{1}{8(a-b)^2(a+b)^2 d \sqrt{e \cos [c+d x]}} \sqrt{\cos [c+d x]} \left(-\frac{1}{\sqrt{1-\cos [c+d x]^2}(a+b \sin [c+d x])} \right. \\
 & \quad \left. 2\left(8 a^2+6 b^2\right)\left(a+b \sqrt{1-\cos [c+d x]^2}\right)\left(\left(5 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \sqrt{\cos [c+d x]}\right) / \right. \right. \\
 & \quad \left. \left(\sqrt{1-\cos [c+d x]^2}\left(5\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]+(-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right)\right) \cos [c+d x]^2\right) \\
 & \quad \left.\left.\left(a^2+b^2(-1+\cos [c+d x]^2)\right)\right)\right) -\frac{1}{\left(-a^2+b^2\right)^{3 / 4}}\left(\frac{1}{8}-\frac{i}{8}\right) \sqrt{b}\left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-\right. \\
 & \quad \left. 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]+\operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right]-\right. \\
 & \quad \left.\left.\operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right]\right)\right)\right) \sin [c+d x]+ \\
 & \frac{1}{\left(1-\cos [c+d x]^2\right)\left(a+b \sin [c+d x]\right)} 14 a b\left(a+b \sqrt{1-\cos [c+d x]^2}\right) \\
 & \quad \left(\left(5 b\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \sqrt{\cos [c+d x]} \sqrt{1-\cos [c+d x]^2}\right) / \right. \\
 & \quad \left(\left(-5\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]+2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4},-\frac{1}{2}, 2, \frac{9}{4}, \cos [c+d x]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right)+\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right)\right) \cos [c+d x]^2\right) \\
 & \quad \left.\left.\left(a^2+b^2(-1+\cos [c+d x]^2)\right)\right)\right) +\frac{1}{4 \sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{3 / 4}} a\left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]+\right. \\
 & \quad \left. 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]-\operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right]+\right.
 \end{aligned}$$

$$\begin{aligned}
& (3 + 3i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i)\sqrt{b}\sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i)\sqrt{b}\sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b} \right. \right. \\
& \quad \left. \left. (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + ib \cos[c+dx] \right] + \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + ib \cos[c+dx] \right] \right) \\
& \sin[c+dx] - \frac{1}{(1-\cos[c+dx])^2 (a+b\sin[c+dx])} 2(8a^3b+37ab^3) \left(a+b\sqrt{1-\cos[c+dx]^2} \right) \\
& \left(\left(7b(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2} \right] \cos[c+dx]^{3/2} \sqrt{1-\cos[c+dx]^2} \right) / \right. \\
& \quad \left(3 \left(-7(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2} \right] + 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[c+dx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \cos[c+dx]^2}{-a^2+b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2} \right] \right) \cos[c+dx]^2 \right) \\
& \quad \left. (a^2+b^2(-1+\cos[c+dx]^2)) \right) + \frac{1}{4\sqrt{2}b^{3/2}(a^2-b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[c+dx]}}{(a^2-b^2)^{1/4}} \right] + \right. \\
& \quad 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[c+dx]}}{(a^2-b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2-b^2} - \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4} \sqrt{\cos[c+dx]} + b \cos[c+dx] \right] - \\
& \quad \left. \operatorname{Log} \left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4} \sqrt{\cos[c+dx]} + b \cos[c+dx] \right] \right) \sin[c+dx]^2 + \\
& \frac{1}{d(e\cos[c+dx])^{3/2}} \cos[c+dx]^2 \left(-\frac{b^3 \cos[c+dx]}{2(a^2-b^2)^2(a+b\sin[c+dx])^2} - \frac{13ab^3 \cos[c+dx]}{4(a^2-b^2)^3(a+b\sin[c+dx])} + \right. \\
& \quad \left. \frac{2 \operatorname{Sec}[c+dx](-3a^2b-b^3+a^3\sin[c+dx]+3ab^2\sin[c+dx])}{(a^2-b^2)^3} \right)
\end{aligned}$$

- **Problem 603: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e\cos[c+dx])^{5/2} (a+b\sin[c+dx])^3} dx$$

Optimal (type 4, 614 leaves, 15 steps):

$$\begin{aligned}
& - \frac{7 b^{5/2} (9 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{15/4} d e^{5/2}} - \frac{7 b^{5/2} (9 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{15/4} d e^{5/2}} + \\
& \frac{a (8 a^2 + 69 b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2} (c+dx), 2\right]}{12 (a^2-b^2)^3 d e^2 \sqrt{e \cos[c+dx]}} - \frac{7 a b^2 (9 a^2 + 2 b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{8 (a^2-b^2)^3 \left(a^2-b \left(b-\sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \cos[c+dx]}} - \\
& \frac{7 a b^2 (9 a^2 + 2 b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{8 (a^2-b^2)^3 \left(a^2-b \left(b+\sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \cos[c+dx]}} + \frac{b}{2 (a^2-b^2) d e (e \cos[c+dx])^{3/2} (a+b \sin[c+dx])^2} + \\
& \frac{11 a b}{4 (a^2-b^2)^2 d e (e \cos[c+dx])^{3/2} (a+b \sin[c+dx])} - \frac{7 b (9 a^2 + 2 b^2) - a (8 a^2 + 69 b^2) \sin[c+dx]}{12 (a^2-b^2)^3 d e (e \cos[c+dx])^{3/2}}
\end{aligned}$$

Result (type 6, 1308 leaves):

$$\begin{aligned}
& \frac{1}{24 (a-b)^3 (a+b)^3 d (e \cos[c+dx])^{5/2}} \\
& \cos[c+dx]^{5/2} \left(- \frac{1}{\sqrt{1-\cos[c+dx]^2} (a+b \sin[c+dx])} - 2 (8 a^4 - 120 a^2 b^2 - 42 b^4) \left(a+b \sqrt{1-\cos[c+dx]^2} \right) \right. \\
& \left(\left(5 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \sqrt{\cos[c+dx]} \right) / \left(\sqrt{1-\cos[c+dx]^2} \left(5 (a^2-b^2) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 \right) \left(a^2+b^2 (-1+\cos[c+dx]^2) \right) \right) - \\
& \frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] \right) \left. \right) \sin[c+dx] - \\
& \frac{1}{(1-\cos[c+dx]^2) (a+b \sin[c+dx])} - 2 (8 a^3 b + 69 a b^3) \left(a+b \sqrt{1-\cos[c+dx]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] + \right. \\
& \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \left. \right) \sin[c + dx]^2 + \\
& \frac{1}{d (e \cos[c + dx])^{5/2}} \cos[c + dx]^3 \left(-\frac{b^3}{2 (a^2 - b^2)^2 (a + b \sin[c + dx])^2} - \frac{15 a b^3}{4 (a^2 - b^2)^3 (a + b \sin[c + dx])} + \right. \\
& \left. \frac{2 \operatorname{Sec}[c + dx]^2 (-3 a^2 b - b^3 + a^3 \sin[c + dx] + 3 a b^2 \sin[c + dx])}{3 (a^2 - b^2)^3} \right)
\end{aligned}$$

- **Problem 604: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \cos[c + dx])^{7/2} (a + b \sin[c + dx])^3} dx$$

Optimal (type 4, 685 leaves, 16 steps):

$$\begin{aligned}
& \frac{9 b^{7/2} (11 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right] - 9 b^{7/2} (11 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2 + b^2)^{17/4} d e^{7/2}} - \frac{9 b^{7/2} (11 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2 + b^2)^{17/4} d e^{7/2}} - \\
& \frac{3 a (8 a^4 - 64 a^2 b^2 - 139 b^4) \sqrt{e \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2} (c+dx), 2\right]}{20 (a^2 - b^2)^4 d e^4 \sqrt{\cos[c+dx]}} + \\
& \frac{9 a b^3 (11 a^2 + 2 b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{8 (a^2 - b^2)^4 (b - \sqrt{-a^2 + b^2}) d e^3 \sqrt{e \cos[c+dx]}} + \\
& \frac{9 a b^3 (11 a^2 + 2 b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{8 (a^2 - b^2)^4 (b + \sqrt{-a^2 + b^2}) d e^3 \sqrt{e \cos[c+dx]}} + \\
& \frac{b}{2 (a^2 - b^2) d e (e \cos[c+dx])^{5/2} (a + b \sin[c+dx])^2} + \frac{13 a b}{4 (a^2 - b^2)^2 d e (e \cos[c+dx])^{5/2} (a + b \sin[c+dx])} - \\
& \frac{9 b (11 a^2 + 2 b^2) - a (8 a^2 + 109 b^2) \sin[c+dx]}{20 (a^2 - b^2)^3 d e (e \cos[c+dx])^{5/2}} + \frac{3 (15 b^3 (11 a^2 + 2 b^2) + a (8 a^4 - 64 a^2 b^2 - 139 b^4) \sin[c+dx])}{20 (a^2 - b^2)^4 d e^3 \sqrt{e \cos[c+dx]}}
\end{aligned}$$

Result (type 6, 1408 leaves):

$$\begin{aligned}
& - \frac{1}{40 (a-b)^4 (a+b)^4 d (e \cos[c+dx])^{7/2}} \\
& 3 \cos[c+dx]^{7/2} \left(\frac{1}{12 \sqrt{1 - \cos[c+dx]^2} (a + b \sin[c+dx])} (8 a^6 - 64 a^4 b^2 - 304 a^2 b^4 - 30 b^6) (a + b \sqrt{1 - \cos[c+dx]^2}) \right. \\
& \left(- \left(56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2}\right] \cos[c+dx]^{3/2} \right) / \left(\sqrt{1 - \cos[c+dx]^2} \left(7 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2 + b^2}\right] \right) \cos[c+dx]^2 \right) (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) \Big) - \\
& \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3 i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{39 a (11 a^4 - 17 a^2 b^2 + 6 b^4) e^{15/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{15/2} (-a^2+b^2)^{3/4} d} + \frac{39 a (11 a^4 - 17 a^2 b^2 + 6 b^4) e^{15/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{15/2} (-a^2+b^2)^{3/4} d} + \\
& \frac{13 (231 a^4 - 203 a^2 b^2 + 20 b^4) e^8 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2} (c+dx), 2\right]}{56 b^8 d \sqrt{e \cos[c+dx]}} - \\
& \frac{39 a^2 (11 a^4 - 17 a^2 b^2 + 6 b^4) e^8 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{16 b^8 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos[c+dx]}} - \\
& \frac{39 a^2 (11 a^4 - 17 a^2 b^2 + 6 b^4) e^8 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{16 b^8 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos[c+dx]}} - \frac{e (e \cos[c+dx])^{13/2}}{3 b d (a+b \sin[c+dx])^3} - \\
& \frac{13 e^3 (e \cos[c+dx])^{9/2} (11 a + 4 b \sin[c+dx])}{84 b^3 d (a+b \sin[c+dx])^2} - \frac{39 e^5 (e \cos[c+dx])^{5/2} (77 a^2 - 20 b^2 + 22 a b \sin[c+dx])}{280 b^5 d (a+b \sin[c+dx])} + \\
& \frac{13 e^7 \sqrt{e \cos[c+dx]} (21 a (11 a^2 - 6 b^2) - b (77 a^2 - 20 b^2) \sin[c+dx])}{56 b^7 d}
\end{aligned}$$

Result (type 6, 2302 leaves):

$$\begin{aligned}
& \frac{1}{560 b^7 d \cos[c+dx]^{15/2}} \\
& (e \cos[c+dx])^{15/2} \left(-\frac{1}{\sqrt{1-\cos[c+dx]^2} (a+b \sin[c+dx])} - 2 (4410 a^3 b - 3418 a b^3) \left(a+b \sqrt{1-\cos[c+dx]^2}\right) \left(\left(5 a (a^2-b^2) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \sqrt{\cos[c+dx]}\right) / \left(\sqrt{1-\cos[c+dx]^2} \left(5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 (a^2+b^2 (-1+\cos[c+dx]^2)) \right) \right) - \\
& \frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8}\right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] \right) + \\
& \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] -
\end{aligned}$$

$$\begin{aligned}
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left(a^2 + b^2 (-1 + \cos[c + dx]^2) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] + \right. \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \sin[c + dx]^2 + \\
& \frac{1}{d} (e \cos[c + dx])^{15/2} \sec[c + dx]^7 \left(-\frac{4 a \cos[2(c + dx)]}{5 b^5} + \frac{(-280 a^2 + 79 b^2) \sin[c + dx]}{42 b^6} - \right. \\
& \frac{(-a^2 + b^2)^3}{3 b^7 (a + b \sin[c + dx])^3} - \\
& \frac{37 a (a^2 - b^2)^2}{12 b^7 (a + b \sin[c + dx])^2} + \\
& \frac{(-a^2 + b^2) (-393 a^2 + 76 b^2)}{24 b^7 (a + b \sin[c + dx])} + \\
& \left. \frac{\sin[3(c + dx)]}{14 b^4} \right)
\end{aligned}$$

■ **Problem 606: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{13/2}}{(a + b \sin[c + dx])^4} dx$$

Optimal (type 4, 557 leaves, 15 steps):

$$\begin{aligned}
& \frac{77 a (3 a^2 - 2 b^2) e^{13/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{13/2} (-a^2+b^2)^{1/4} d} - \frac{77 a (3 a^2 - 2 b^2) e^{13/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{13/2} (-a^2+b^2)^{1/4} d} \\
& \frac{77 (15 a^2 - 4 b^2) e^6 \sqrt{e \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{40 b^6 d \sqrt{\cos [c+d x]}} + \frac{77 a^2 (3 a^2 - 2 b^2) e^7 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{16 b^7 \left(b-\sqrt{-a^2+b^2}\right) d \sqrt{e \cos [c+d x]}} + \\
& \frac{77 a^2 (3 a^2 - 2 b^2) e^7 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{16 b^7 \left(b+\sqrt{-a^2+b^2}\right) d \sqrt{e \cos [c+d x]}} - \frac{e (e \cos [c+d x])^{11/2}}{3 b d (a+b \sin [c+d x])^3} - \\
& \frac{11 e^3 (e \cos [c+d x])^{7/2} (9 a+4 b \sin [c+d x])}{60 b^3 d (a+b \sin [c+d x])^2} - \frac{77 e^5 (e \cos [c+d x])^{3/2} (15 a^2-4 b^2+6 a b \sin [c+d x])}{120 b^5 d (a+b \sin [c+d x])}
\end{aligned}$$

Result(type 6, 1331 leaves):

$$\begin{aligned}
& -\frac{1}{80 b^5 d \cos [c+d x]^{13/2}} 77 (e \cos [c+d x])^{13/2} \\
& \left(\frac{1}{2 \sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])} a b \left(a+b \sqrt{1-\cos [c+d x]^2}\right) \left(-\left(56 a \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3/2}\right) / \left(\sqrt{1-\cos [c+d x]^2} \left(7 \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] - \right. \right. \right. \\
& \left. \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + \left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \cos [c+d x]^2 \left(a^2+b^2(-1+\cos [c+d x]^2)\right) \right) - \frac{1}{\sqrt{b}(-a^2+b^2)^{1/4}} \\
& (3+3 i) \left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b} \right. \right. \\
& \left. \left. (-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}(-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right] \right) \Big) \\
& \sin [c+d x] - \frac{1}{(1-\cos [c+d x]^2)(a+b \sin [c+d x])} 2(15 a^2-4 b^2) \left(a+b \sqrt{1-\cos [c+d x]^2}\right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(7b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{3/2} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] - \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \left. \right) \sin[c + dx]^2 + \\
& \frac{1}{d} (e \cos[c + dx])^{13/2} \sec[c + dx]^6 \left(-\frac{8 a \cos[c + dx]}{3 b^5} + \frac{-a^4 \cos[c + dx] + 2 a^2 b^2 \cos[c + dx] - b^4 \cos[c + dx]}{3 b^5 (a + b \sin[c + dx])^3} + \right. \\
& \frac{9 (a^3 \cos[c + dx] - a b^2 \cos[c + dx])}{4 b^5 (a + b \sin[c + dx])^2} + \\
& \frac{-71 a^2 \cos[c + dx] + 20 b^2 \cos[c + dx]}{8 b^5 (a + b \sin[c + dx])} + \\
& \left. \frac{\sin[2(c + dx)]}{5 b^4} \right)
\end{aligned}$$

■ **Problem 607: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{11/2}}{(a + b \sin[c + dx])^4} dx$$

Optimal (type 4, 571 leaves, 15 steps):

$$\begin{aligned}
& \frac{15 a (7 a^2 - 6 b^2) e^{11/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{11/2} (-a^2+b^2)^{3/4} d} - \frac{15 a (7 a^2 - 6 b^2) e^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{11/2} (-a^2+b^2)^{3/4} d} \\
& + \frac{5 (21 a^2 - 4 b^2) e^6 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), 2\right]}{8 b^6 d \sqrt{e \cos [c+d x]}} + \frac{15 a^2 (7 a^2 - 6 b^2) e^6 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+d x), 2\right]}{16 b^6 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos [c+d x]}} + \\
& \frac{15 a^2 (7 a^2 - 6 b^2) e^6 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+d x), 2\right]}{16 b^6 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos [c+d x]}} - \frac{e (e \cos [c+d x])^{9/2}}{3 b d (a+b \sin [c+d x])^3} - \\
& \frac{e^3 (e \cos [c+d x])^{5/2} (7 a + 4 b \sin [c+d x])}{4 b^3 d (a+b \sin [c+d x])^2} - \frac{5 e^5 \sqrt{e \cos [c+d x]} (21 a^2 - 4 b^2 + 14 a b \sin [c+d x])}{8 b^5 d (a+b \sin [c+d x])}
\end{aligned}$$

Result (type 6, 2220 leaves):

$$\begin{aligned}
& \frac{(e \cos [c+d x])^{11/2} \operatorname{Sec}[c+d x]^5 \left(\frac{2 \sin [c+d x]}{3 b^4} - \frac{(-a^2+b^2)^2}{3 b^5 (a+b \sin [c+d x])^3} + \frac{25 a (a^2-b^2)}{12 b^5 (a+b \sin [c+d x])^2} + \frac{-165 a^2+52 b^2}{24 b^5 (a+b \sin [c+d x])} \right)}{d} \\
& \frac{1}{16 b^5 d \cos [c+d x]^{11/2}} (e \cos [c+d x])^{11/2} \left(-\frac{1}{\sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])} \right. \\
& 76 a b \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \left(\left(5 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \sqrt{\cos [c+d x]} \right) / \right. \\
& \left(\sqrt{1-\cos [c+d x]^2} \left(5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \right) \cos [c+d x]^2 \right) \\
& \left. (a^2+b^2 (-1+\cos [c+d x]^2)) \right) - \frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{(-a^2+b^2)^{1/4}}\right] - \right. \\
& \left. 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x]\right] - \right. \\
& \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x]\right] \right) \right) \sin [c+d x] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{1 - \cos[c + dx]^2} (-1 + 2 \cos[c + dx]^2) (a + b \sin[c + dx])} - 32 a b \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \cos[2(c + dx)] \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} \right) + \\
& \frac{4 \sqrt{\cos[c + dx]}}{b} + \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \right) / \\
& \left(\sqrt{1 - \cos[c + dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \quad \left. \left. \left. \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) - \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{5/2} \right) / \\
& \left(5 \sqrt{1 - \cos[c + dx]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] - \right. \right. \\
& \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx] \right] \left. \right) \sin[c + dx] - \\
& \frac{1}{(1 - \cos[c + dx]^2) (a + b \sin[c + dx])} - 2 (41 a^2 - 20 b^2) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\
& \left. \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \right. \right. \right. \right. \right. \right.
\end{aligned}$$

$$\left. \begin{aligned} & \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} + \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^2 \\ & \left(a^2+b^2(-1+\cos [c+d x]^2)\right) + \frac{1}{4 \sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{3 / 4}} a\left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]+ \right. \\ & \left. 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]-\operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right]+ \right. \\ & \left. \operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right]\right) \sin [c+d x]^2 \end{aligned} \right)$$

■ **Problem 608: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(e \cos [c+d x]\right)^{9 / 2}}{\left(a+b \sin [c+d x]\right)^4} d x$$

Optimal (type 4, 591 leaves, 15 steps):

$$\begin{aligned} & \frac{7 a\left(5 a^2-6 b^2\right) e^{9 / 2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4} \sqrt{e}}\right]}{16 b^{9 / 2}\left(-a^2+b^2\right)^{5 / 4} d}-\frac{7 a\left(5 a^2-6 b^2\right) e^{9 / 2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4} \sqrt{e}}\right]}{16 b^{9 / 2}\left(-a^2+b^2\right)^{5 / 4} d}+ \\ & \frac{7\left(5 a^2-4 b^2\right) e^4 \sqrt{e \cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{8 b^4\left(a^2-b^2\right) d \sqrt{\cos [c+d x]}}-\frac{7 a^2\left(5 a^2-6 b^2\right) e^5 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{16 b^5\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right) d \sqrt{e \cos [c+d x]}}- \\ & \frac{7 a^2\left(5 a^2-6 b^2\right) e^5 \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+d x), 2\right]}{16 b^5\left(a^2-b^2\right)\left(b+\sqrt{-a^2+b^2}\right) d \sqrt{e \cos [c+d x]}}-\frac{e\left(e \cos [c+d x]\right)^{7 / 2}}{3 b d\left(a+b \sin [c+d x]\right)^3}+ \\ & \frac{7\left(5 a^2-4 b^2\right) e^3\left(e \cos [c+d x]\right)^{3 / 2}}{8 b^3\left(a^2-b^2\right) d\left(a+b \sin [c+d x]\right)}-\frac{7 e^3\left(e \cos [c+d x]\right)^{3 / 2}\left(5 a+4 b \sin [c+d x]\right)}{12 b^3 d\left(a+b \sin [c+d x]\right)^2} \end{aligned}$$

Result (type 6, 1294 leaves):

$$\begin{aligned} & \frac{\left(e \cos [c+d x]\right)^{9 / 2} \operatorname{Sec}[c+d x]^4\left(\frac{a^2 \cos [c+d x]-b^2 \cos [c+d x]}{3 b^3\left(a+b \sin [c+d x]\right)^3}-\frac{5 a \cos [c+d x]}{4 b^3\left(a+b \sin [c+d x]\right)^2}+\frac{-19 a^2 \cos [c+d x]+12 b^2 \cos [c+d x]}{8 b^3\left(-a^2+b^2\right)\left(a+b \sin [c+d x]\right)}\right)}{d}+ \\ & \frac{1}{16(a-b) b^3(a+b) d \cos [c+d x]^{9 / 2}} 7\left(e \cos [c+d x]\right)^{9 / 2}\left(\frac{1}{6 \sqrt{1-\cos [c+d x]^2}(a+b \sin [c+d x])}\right) \end{aligned}$$

$$\begin{aligned}
& a b \left(a + b \sqrt{1 - \cos [c + d x]^2} \right) \left(- \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2} \right] \cos [c + d x]^{3/2} \right) / \right. \\
& \left(\sqrt{1 - \cos [c + d x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2} \right] \right) \cos [c + d x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos [c + d x]^2)) \right) - \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos [c + d x]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos [c + d x]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos [c + d x]} + i b \cos [c + d x] \right] + \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos [c + d x]} + i b \cos [c + d x] \right] \right) \left. \right) \sin [c + d x] - \\
& \frac{1}{(1 - \cos [c + d x]^2) (a + b \sin [c + d x])} 2 (5 a^2 - 4 b^2) \left(a + b \sqrt{1 - \cos [c + d x]^2} \right) \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2} \right] \cos [c + d x]^{3/2} \sqrt{1 - \cos [c + d x]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos [c + d x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos [c + d x]^2, \frac{b^2 \cos [c + d x]^2}{-a^2 + b^2} \right] \right) \cos [c + d x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos [c + d x]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c + d x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c + d x]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos [c + d x]} + b \cos [c + d x] \right] - \\
& \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos [c + d x]} + b \cos [c + d x] \right] \right) \left. \right) \sin [c + d x]^2 \left. \right)
\end{aligned}$$

■ **Problem 609: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos [c + d x])^{7/2}}{(a + b \sin [c + d x])^4} dx$$

Optimal (type 4, 597 leaves, 15 steps):

$$\begin{aligned}
 & - \frac{5 a (a^2 - 2 b^2) e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{7/2} (-a^2+b^2)^{7/4} d} - \frac{5 a (a^2 - 2 b^2) e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{7/2} (-a^2+b^2)^{7/4} d} + \\
 & \frac{5 (3 a^2 - 4 b^2) e^4 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2} (c+dx), 2\right]}{24 b^4 (a^2 - b^2) d \sqrt{e \cos[c+dx]}} - \frac{5 a^2 (a^2 - 2 b^2) e^4 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{16 b^4 (a^2 - b^2) \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos[c+dx]}} - \\
 & \frac{5 a^2 (a^2 - 2 b^2) e^4 \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{16 b^4 (a^2 - b^2) \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \cos[c+dx]}} - \frac{e (e \cos[c+dx])^{5/2}}{3 b d (a + b \sin[c+dx])^3} - \\
 & \frac{5 (3 a^2 - 4 b^2) e^3 \sqrt{e \cos[c+dx]}}{24 b^3 (a^2 - b^2) d (a + b \sin[c+dx])} + \frac{5 e^3 \sqrt{e \cos[c+dx]} (3 a + 4 b \sin[c+dx])}{12 b^3 d (a + b \sin[c+dx])^2}
 \end{aligned}$$

Result (type 6, 1263 leaves):

$$\begin{aligned}
 & \frac{(e \cos[c+dx])^{7/2} \operatorname{Sec}[c+dx]^3 \left(\frac{a^2-b^2}{3 b^3 (a+b \sin[c+dx])^3} - \frac{13 a}{12 b^3 (a+b \sin[c+dx])^2} + \frac{-33 a^2+28 b^2}{24 b^3 (-a^2+b^2) (a+b \sin[c+dx])} \right)}{d} + \\
 & \frac{1}{48 (a-b) b^3 (a+b) d \cos[c+dx]^{7/2}} 5 (e \cos[c+dx])^{7/2} \left(- \frac{1}{\sqrt{1-\cos[c+dx]^2} (a+b \sin[c+dx])} \right. \\
 & \left. 4 a b \left(a + b \sqrt{1-\cos[c+dx]^2} \right) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \sqrt{\cos[c+dx]} \right) / \right. \\
 & \left. \left(\sqrt{1-\cos[c+dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) \cos[c+dx]^2 \right) \right) \\
 & \left. (a^2 + b^2 (-1 + \cos[c+dx]^2)) \right) - \frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \right. \\
 & \left. 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] - \right. \\
 & \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] \right) \right) \sin[c+dx] -
 \end{aligned}$$

$$\frac{1}{(1 - \cos[c + dx])^2 (a + b \sin[c + dx])} 2 (3a^2 - 4b^2) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right)$$

$$\left(\left(5b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \sqrt{1 - \cos[c + dx]^2} \right) / \right.$$

$$\left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \right. \right.$$

$$\left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2$$

$$\left(a^2 + b^2 (-1 + \cos[c + dx]^2) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right.$$

$$2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] +$$

$$\left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \sin[c + dx]^2$$

■ **Problem 610: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^{5/2}}{(a + b \sin[c + dx])^4} dx$$

Optimal (type 4, 574 leaves, 15 steps):

$$-\frac{a (a^2 - 6b^2) e^{5/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{16 b^{5/2} (-a^2 + b^2)^{9/4} d} + \frac{a (a^2 - 6b^2) e^{5/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{16 b^{5/2} (-a^2 + b^2)^{9/4} d} + \frac{(a^2 + 4b^2) e^2 \sqrt{e \cos[c + dx]} \operatorname{EllipticE} \left[\frac{1}{2} (c + dx), 2 \right]}{8 b^2 (a^2 - b^2)^2 d \sqrt{\cos[c + dx]}}$$

$$\frac{a^2 (a^2 - 6b^2) e^3 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{-2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{16 b^3 (a^2 - b^2)^2 (b - \sqrt{-a^2 + b^2}) d \sqrt{e \cos[c + dx]}} - \frac{a^2 (a^2 - 6b^2) e^3 \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{-2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2 \right]}{16 b^3 (a^2 - b^2)^2 (b + \sqrt{-a^2 + b^2}) d \sqrt{e \cos[c + dx]}}$$

$$\frac{e (e \cos[c + dx])^{3/2}}{3 b d (a + b \sin[c + dx])^3} + \frac{a e (e \cos[c + dx])^{3/2}}{4 b (a^2 - b^2) d (a + b \sin[c + dx])^2} + \frac{(a^2 + 4b^2) e (e \cos[c + dx])^{3/2}}{8 b (a^2 - b^2)^2 d (a + b \sin[c + dx])}$$

Result (type 6, 1286 leaves):

$$\frac{(e \cos[c + dx])^{5/2} \operatorname{Sec}[c + dx]^2 \left(-\frac{\cos[c + dx]}{3 b (a + b \sin[c + dx])^3} - \frac{a \cos[c + dx]}{4 b (-a^2 + b^2) (a + b \sin[c + dx])^2} + \frac{a^2 \cos[c + dx] + 4 b^2 \cos[c + dx]}{8 b (-a^2 + b^2)^2 (a + b \sin[c + dx])} \right)}{d} +$$

$$\begin{aligned}
& \frac{1}{16 (a-b)^2 b (a+b)^2 d \cos [c+d x]^{5/2}} (e \cos [c+d x])^{5/2} \left(\frac{1}{6 \sqrt{1-\cos [c+d x]^2} (a+b \sin [c+d x])} \right. \\
& 5 a b \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \left(- \left(56 a (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \cos [c+d x]^{3/2} \right) / \right. \\
& \left. \left(\sqrt{1-\cos [c+d x]^2} \left(7 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \right) \cos [c+d x]^2 \right) \right. \\
& \left. \left. (a^2+b^2 (-1+\cos [c+d x]^2)) \right) \right) - \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{(-a^2+b^2)^{1/4}} \right] - \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{(-a^2+b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] + \right. \\
& \left. \left. \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos [c+d x]} + i b \cos [c+d x] \right] \right) \right) \left. \right) \sin [c+d x] - \\
& \frac{1}{(1-\cos [c+d x]^2) (a+b \sin [c+d x])} 2 (a^2+4 b^2) \left(a+b \sqrt{1-\cos [c+d x]^2} \right) \\
& \left(\left(7 b (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \cos [c+d x]^{3/2} \sqrt{1-\cos [c+d x]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2} \right] \right) \cos [c+d x]^2 \right) \right. \\
& \left. \left. (a^2+b^2 (-1+\cos [c+d x]^2)) \right) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{(a^2-b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{(a^2-b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] - \right. \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos [c+d x]} + b \cos [c+d x] \right] \right) \right) \left. \right) \sin [c+d x]^2 \left. \right)
\end{aligned}$$

■ Problem 611: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \cos[c + dx])^{3/2}}{(a + b \sin[c + dx])^4} dx$$

Optimal (type 4, 592 leaves, 15 steps):

$$\begin{aligned} & - \frac{a (a^2 + 6 b^2) e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{3/2} (-a^2 + b^2)^{11/4} d} - \frac{a (a^2 + 6 b^2) e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 b^{3/2} (-a^2 + b^2)^{11/4} d} - \frac{(3 a^2 + 4 b^2) e^2 \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{24 b^2 (a^2 - b^2)^2 d \sqrt{e \cos[c + dx]}} + \\ & \frac{a^2 (a^2 + 6 b^2) e^2 \sqrt{\cos[c + dx]} \operatorname{EllipticPi}\left[\frac{-2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2\right]}{16 b^2 (a^2 - b^2)^2 (a^2 - b (b - \sqrt{-a^2 + b^2})) d \sqrt{e \cos[c + dx]}} + \frac{a^2 (a^2 + 6 b^2) e^2 \sqrt{\cos[c + dx]} \operatorname{EllipticPi}\left[\frac{-2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2\right]}{16 b^2 (a^2 - b^2)^2 (a^2 - b (b + \sqrt{-a^2 + b^2})) d \sqrt{e \cos[c + dx]}} - \\ & \frac{e \sqrt{e \cos[c + dx]}}{3 b d (a + b \sin[c + dx])^3} + \frac{a e \sqrt{e \cos[c + dx]}}{12 b (a^2 - b^2) d (a + b \sin[c + dx])^2} + \frac{(3 a^2 + 4 b^2) e \sqrt{e \cos[c + dx]}}{24 b (a^2 - b^2)^2 d (a + b \sin[c + dx])} \end{aligned}$$

Result (type 6, 1263 leaves):

$$\begin{aligned} & \frac{(e \cos[c + dx])^{3/2} \operatorname{Sec}[c + dx] \left(-\frac{1}{3 b (a + b \sin[c + dx])^3} - \frac{a}{12 b (-a^2 + b^2) (a + b \sin[c + dx])^2} + \frac{3 a^2 + 4 b^2}{24 b (-a^2 + b^2)^2 (a + b \sin[c + dx])} \right)}{d} - \\ & \frac{1}{48 (a - b)^2 b (a + b)^2 d \cos[c + dx]^{3/2}} (e \cos[c + dx])^{3/2} \left(\frac{1}{\sqrt{1 - \cos[c + dx]^2} (a + b \sin[c + dx])} \right. \\ & \left. 28 a b (a + b \sqrt{1 - \cos[c + dx]^2}) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] \sqrt{\cos[c + dx]} \right) / \right. \right. \\ & \left. \left(\sqrt{1 - \cos[c + dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \right. \\ & \left. \left. \left. \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] \right) \cos[c + dx]^2 \right) \right) \\ & \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - \right. \\ & \left. 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx]\right] - \right. \\ & \left. \left. \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[c + dx]} + i b \cos[c + dx]\right] \right) \right) \sin[c + dx] - \end{aligned}$$

$$\frac{1}{(1 - \cos[c + dx])^2 (a + b \sin[c + dx])} 2 (3a^2 + 4b^2) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right)$$

$$\left(\left(5b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \sqrt{1 - \cos[c + dx]^2} \right) / \right.$$

$$\left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \right. \right.$$

$$\left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right) + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2$$

$$(a^2 + b^2 (-1 + \cos[c + dx]^2)) \left. \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right.$$

$$2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] +$$

$$\left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \sin[c + dx]^2$$

■ **Problem 612: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{e \cos[c + dx]}}{(a + b \sin[c + dx])^4} dx$$

Optimal (type 4, 579 leaves, 15 steps):

$$-\frac{5a(a^2 + 2b^2)\sqrt{e}\operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{e\cos[c+dx]}}{(-a^2+b^2)^{1/4}\sqrt{e}}\right]}{16\sqrt{b}(-a^2+b^2)^{13/4}d} + \frac{5a(a^2 + 2b^2)\sqrt{e}\operatorname{ArcTanh}\left[\frac{\sqrt{b}\sqrt{e\cos[c+dx]}}{(-a^2+b^2)^{1/4}\sqrt{e}}\right]}{16\sqrt{b}(-a^2+b^2)^{13/4}d} +$$

$$\frac{(11a^2 + 4b^2)\sqrt{e\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{8(a^2 - b^2)^3 d \sqrt{\cos[c+dx]}} + \frac{5a^2(a^2 + 2b^2)e\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{16b(a^2 - b^2)^3 (b - \sqrt{-a^2 + b^2}) d \sqrt{e\cos[c+dx]}} +$$

$$\frac{5a^2(a^2 + 2b^2)e\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(c+dx), 2\right]}{16b(a^2 - b^2)^3 (b + \sqrt{-a^2 + b^2}) d \sqrt{e\cos[c+dx]}} +$$

$$\frac{b(e\cos[c+dx])^{3/2}}{3(a^2 - b^2)d e (a + b \sin[c + dx])^3} + \frac{3ab(e\cos[c+dx])^{3/2}}{4(a^2 - b^2)^2 d e (a + b \sin[c + dx])^2} + \frac{b(11a^2 + 4b^2)(e\cos[c+dx])^{3/2}}{8(a^2 - b^2)^3 d e (a + b \sin[c + dx])}$$

Result (type 6, 1294 leaves) :

$$\begin{aligned}
& \frac{\sqrt{e \cos [c+d x]} \left(\frac{b \cos [c+d x]}{3\left(a^2-b^2\right)\left(a+b \sin [c+d x]\right)^3} + \frac{3 a b \cos [c+d x]}{4\left(a^2-b^2\right)^2\left(a+b \sin [c+d x]\right)^2} - \frac{-11 a^2 b \cos [c+d x]-4 b^3 \cos [c+d x]}{8\left(a^2-b^2\right)^3\left(a+b \sin [c+d x]\right)} \right)}{d} \\
& \frac{1}{16(a-b)^3(a+b)^3 d \sqrt{\cos [c+d x]}} \sqrt{e \cos [c+d x]} \left(\frac{1}{12 \sqrt{1-\cos [c+d x]^2}\left(a+b \sin [c+d x]\right)} \right. \\
& \left. (16 a^3+14 a b^2)\left(a+b \sqrt{1-\cos [c+d x]^2}\right)\left(-\left(56 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3 / 2}\right) / \right. \right. \\
& \left. \left(\sqrt{1-\cos [c+d x]^2}\left(7\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right)+\left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right) \cos [c+d x]^2\right) \right. \\
& \left. \left(a^2+b^2\left(-1+\cos [c+d x]^2\right)\right)\right)-\frac{1}{\sqrt{b}\left(-a^2+b^2\right)^{1 / 4}}(3+3 i)\left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-\right. \\
& \left. 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\cos [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-\log \left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right]+\right. \\
& \left. \left. \log \left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+i b \cos [c+d x]\right]\right)\right) \sin [c+d x]- \\
& \frac{1}{\left(1-\cos [c+d x]^2\right)\left(a+b \sin [c+d x]\right)} 2\left(11 a^2 b+4 b^3\right)\left(a+b \sqrt{1-\cos [c+d x]^2}\right) \\
& \left(\left(7 b\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right] \cos [c+d x]^{3 / 2} \sqrt{1-\cos [c+d x]^2}\right) / \right. \\
& \left(3\left(-7\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]+2\left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4},-\frac{1}{2}, 2, \frac{11}{4}, \cos [c+d x]^2, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right)+\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos [c+d x]^2, \frac{b^2 \cos [c+d x]^2}{-a^2+b^2}\right]\right) \cos [c+d x]^2\right) \right. \\
& \left. \left(a^2+b^2\left(-1+\cos [c+d x]^2\right)\right)\right)+\frac{1}{4 \sqrt{2} b^{3 / 2}\left(a^2-b^2\right)^{1 / 4}} a\left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]+\right. \\
& \left. 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\cos [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]+\log \left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\cos [c+d x]}+b \cos [c+d x]\right]-\right.
\end{aligned}$$

$$\left. \log \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \sin^2[c + dx]$$

■ **Problem 613: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{e \cos[c + dx]} (a + b \sin[c + dx])^4} dx$$

Optimal (type 4, 593 leaves, 15 steps):

$$\frac{7 a \sqrt{b} (5 a^2 + 6 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right] + 7 a \sqrt{b} (5 a^2 + 6 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{16 (-a^2 + b^2)^{15/4} d \sqrt{e}} + \frac{(57 a^2 + 20 b^2) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right] + 7 a^2 (5 a^2 + 6 b^2) \sqrt{\cos[c + dx]} \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2\right]}{24 (a^2 - b^2)^3 d \sqrt{e \cos[c + dx]}} + \frac{7 a^2 (5 a^2 + 6 b^2) \sqrt{\cos[c + dx]} \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (c + dx), 2\right] + \frac{b \sqrt{e \cos[c + dx]}}{3 (a^2 - b^2) d e (a + b \sin[c + dx])^3}}{16 (a^2 - b^2)^3 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \cos[c + dx]}} + \frac{11 a b \sqrt{e \cos[c + dx]}}{12 (a^2 - b^2)^2 d e (a + b \sin[c + dx])^2} + \frac{b (57 a^2 + 20 b^2) \sqrt{e \cos[c + dx]}}{24 (a^2 - b^2)^3 d e (a + b \sin[c + dx])}$$

Result (type 6, 1276 leaves):

$$\frac{\cos[c + dx] \left(\frac{b}{3 (a^2 - b^2) (a + b \sin[c + dx])^3} + \frac{11 a b}{12 (a^2 - b^2)^2 (a + b \sin[c + dx])^2} + \frac{b (57 a^2 + 20 b^2)}{24 (a^2 - b^2)^3 (a + b \sin[c + dx])} \right)}{d \sqrt{e \cos[c + dx]}} + \frac{1}{48 (a - b)^3 (a + b)^3 d \sqrt{e \cos[c + dx]}} \sqrt{\cos[c + dx]} \left(-\frac{1}{\sqrt{1 - \cos[c + dx]^2} (a + b \sin[c + dx])} + 2 (48 a^3 + 106 a b^2) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] \sqrt{\cos[c + dx]} \right) / \left(\sqrt{1 - \cos[c + dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2}\right] \right) \cos[c + dx]^2 \right) \right)$$

$$\begin{aligned} & \left. \left((a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i)\sqrt{b}\sqrt{\cos[c+dx]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \right. \\ & \left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{(1+i)\sqrt{b}\sqrt{\cos[c+dx]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i)\sqrt{b}(-a^2 + b^2)^{1/4} \sqrt{\cos[c+dx]} + ib \cos[c+dx] \right] - \right. \right. \\ & \left. \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i)\sqrt{b}(-a^2 + b^2)^{1/4} \sqrt{\cos[c+dx]} + ib \cos[c+dx] \right] \right) \right) \sin[c + dx] - \\ & \frac{1}{(1 - \cos[c + dx]^2) (a + b \sin[c + dx])} 2 (-57 a^2 b - 20 b^3) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \\ & \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\cos[c + dx]} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\ & \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[c + dx]^2, \right. \right. \right. \right. \\ & \left. \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\ & \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\ & \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] + \right. \\ & \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \sin[c + dx]^2 \end{aligned}$$

■ **Problem 614: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \cos[c + dx])^{3/2} (a + b \sin[c + dx])^4} dx$$

Optimal (type 4, 674 leaves, 16 steps):

$$\begin{aligned}
& - \frac{15 a b^{3/2} (7 a^2 + 6 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 (-a^2+b^2)^{17/4} d e^{3/2}} + \frac{15 a b^{3/2} (7 a^2 + 6 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \cos[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{16 (-a^2+b^2)^{17/4} d e^{3/2}} - \\
& \frac{(16 a^4 + 151 a^2 b^2 + 28 b^4) \sqrt{e \cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2} (c+dx), 2\right]}{8 (a^2-b^2)^4 d e^2 \sqrt{\cos[c+dx]}} - \frac{15 a^2 b (7 a^2 + 6 b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{16 (a^2-b^2)^4 \left(b-\sqrt{-a^2+b^2}\right) d e \sqrt{e \cos[c+dx]}} - \\
& \frac{15 a^2 b (7 a^2 + 6 b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (c+dx), 2\right]}{16 (a^2-b^2)^4 \left(b+\sqrt{-a^2+b^2}\right) d e \sqrt{e \cos[c+dx]}} + \\
& \frac{b}{3 (a^2-b^2) d e \sqrt{e \cos[c+dx]} (a+b \sin[c+dx])^3} + \frac{13 a b}{12 (a^2-b^2)^2 d e \sqrt{e \cos[c+dx]} (a+b \sin[c+dx])^2} + \\
& \frac{b (89 a^2 + 28 b^2)}{24 (a^2-b^2)^3 d e \sqrt{e \cos[c+dx]} (a+b \sin[c+dx])} - \frac{15 a b (7 a^2 + 6 b^2) - (16 a^4 + 151 a^2 b^2 + 28 b^4) \sin[c+dx]}{8 (a^2-b^2)^4 d e \sqrt{e \cos[c+dx]}}
\end{aligned}$$

Result (type 6, 1390 leaves):

$$\begin{aligned}
& - \frac{1}{16 (a-b)^4 (a+b)^4 d (e \cos[c+dx])^{3/2}} \\
& \cos[c+dx]^{3/2} \left(\frac{1}{12 \sqrt{1-\cos[c+dx]^2} (a+b \sin[c+dx])} (16 a^5 + 256 a^3 b^2 + 118 a b^4) (a+b \sqrt{1-\cos[c+dx]^2}) \right. \\
& \left(- \left(56 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \cos[c+dx]^{3/2} \right) / \left(\sqrt{1-\cos[c+dx]^2} \left(7 (a^2-b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \right) + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[c+dx]^2, \frac{b^2 \cos[c+dx]^2}{-a^2+b^2}\right] \cos[c+dx]^2 \right) (a^2+b^2 (-1+\cos[c+dx]^2)) \right) \right) - \\
& \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \right. \\
& \left. \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] + \right. \right. \\
& \left. \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[c+dx]} + i b \cos[c+dx]\right] \right) \right) \right) \sin[c+dx] -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(1 - \cos[c + dx])^2 (a + b \sin[c + dx])} 2 (16 a^4 b + 151 a^2 b^3 + 28 b^5) \left(a + b \sqrt{1 - \cos[c + dx]^2} \right) \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \cos[c + dx]^{3/2} \sqrt{1 - \cos[c + dx]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[c + dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[c + dx]^2, \frac{b^2 \cos[c + dx]^2}{-a^2 + b^2} \right] \right) \cos[c + dx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[c + dx]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] - \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[c + dx]} + b \cos[c + dx] \right] \right) \right) \sin[c + dx]^2 + \\
& \frac{1}{d (e \cos[c + dx])^{3/2}} \cos[c + dx]^2 \left(-\frac{b^3 \cos[c + dx]}{3 (a^2 - b^2)^2 (a + b \sin[c + dx])^3} - \frac{7 a b^3 \cos[c + dx]}{4 (a^2 - b^2)^3 (a + b \sin[c + dx])^2} + \right. \\
& \frac{-55 a^2 b^3 \cos[c + dx] - 12 b^5 \cos[c + dx]}{8 (a^2 - b^2)^4 (a + b \sin[c + dx])} + \\
& \left. \frac{2 \operatorname{Sec}[c + dx] (-4 a^3 b - 4 a b^3 + a^4 \sin[c + dx] + 6 a^2 b^2 \sin[c + dx] + b^4 \sin[c + dx])}{(a^2 - b^2)^4} \right)
\end{aligned}$$

■ **Problem 615: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{c \cos[e + fx]} \sqrt{a + b \sin[e + fx]}} dx$$

Optimal (type 4, 183 leaves, 2 steps):

$$\left(2\sqrt{2} (-a+b)^{1/4} \sqrt{c \cos[e+fx]} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(a+b)^{1/4} \sqrt{\frac{1+\cos[e+fx]+\sin[e+fx]}{1+\cos[e+fx]-\sin[e+fx]}}}{(-a+b)^{1/4}}\right], -1\right] \sqrt{\frac{a+b \sin[e+fx]}{(a-b)(1-\sin[e+fx])}} \right) /$$

$$\left((a+b)^{1/4} c f \sqrt{\frac{1+\cos[e+fx]+\sin[e+fx]}{1+\cos[e+fx]-\sin[e+fx]}} \sqrt{a+b \sin[e+fx]} \right)$$

Result (type 4, 4001 leaves):

$$- \left(\left(4 \cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}\right], -1\right] \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \right. \right.$$

$$\left. \left. \left(1+\tan\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{\frac{b-\sqrt{-a^2+b^2}+a \tan[\frac{1}{2}(e+fx)]}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \sqrt{-\frac{b+\sqrt{-a^2+b^2}+a \tan[\frac{1}{2}(e+fx)]}{(-a+b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right) \right. /$$

$$\left. \left(f \sqrt{\cos[e+fx]} \sqrt{c \cos[e+fx]} (a+b \sin[e+fx]) \sqrt{\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right. \right.$$

$$\left. \left. - \left(2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}\right], -1\right] \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \right) \right. /$$

$$\left. \sqrt{\frac{b-\sqrt{-a^2+b^2}+a \tan[\frac{1}{2}(e+fx)]}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \sqrt{-\frac{b+\sqrt{-a^2+b^2}+a \tan[\frac{1}{2}(e+fx)]}{(-a+b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right) /$$

$$\begin{aligned}
& \left(\frac{\sqrt{\cos[e+fx]} \sqrt{a+b \sin[e+fx]}}{\sqrt{\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}}} \right) - \\
& \left(2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}}\right]}, -1\right] \left(1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \right) \\
& \left(\frac{b-\sqrt{-a^2+b^2}+a \tan[\frac{1}{2}(e+fx)]}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])} \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \tan[\frac{1}{2}(e+fx)]}{(-a+b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right) / \\
& \left(\frac{\sqrt{\cos[e+fx]} \sqrt{a+b \sin[e+fx]}}{\sqrt{\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}}} \right) + \\
& \left(2b \cos\left[\frac{1}{2}(e+fx)\right]^2 \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}}\right]}, -1\right] \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \right) \\
& \left(\left(1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{\frac{b-\sqrt{-a^2+b^2}+a \tan[\frac{1}{2}(e+fx)]}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \sqrt{-\frac{b+\sqrt{-a^2+b^2}+a \tan[\frac{1}{2}(e+fx)]}{(-a+b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right) / \\
& \left((a+b \sin[e+fx])^{3/2} \sqrt{\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(4 \cos\left[\frac{1}{2}(e+fx)\right] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}\right], -1\right] \sin\left[\frac{1}{2}(e+fx)\right] \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \right. \\
& \left. \left(1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{\frac{b-\sqrt{-a^2+b^2}+a\tan[\frac{1}{2}(e+fx)]}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \sqrt{-\frac{b+\sqrt{-a^2+b^2}+a\tan[\frac{1}{2}(e+fx)]}{(-a+b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right) / \\
& \left(\sqrt{\cos[e+fx]} \sqrt{a+b\sin[e+fx]} \sqrt{\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right) - \\
& \left(2 \cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}\right], -1\right] \sin[e+fx] \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \right. \\
& \left. \left(1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{\frac{b-\sqrt{-a^2+b^2}+a\tan[\frac{1}{2}(e+fx)]}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \sqrt{-\frac{b+\sqrt{-a^2+b^2}+a\tan[\frac{1}{2}(e+fx)]}{(-a+b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right) / \\
& \left(\cos[e+fx]^{3/2} \sqrt{a+b\sin[e+fx]} \sqrt{\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}} \right) + \\
& \left(2 \cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(-a-b+\sqrt{-a^2+b^2})(1+\tan[\frac{1}{2}(e+fx)])}{(a-b+\sqrt{-a^2+b^2})(-1+\tan[\frac{1}{2}(e+fx)])}\right], -1\right] \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{\frac{b - \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \sqrt{-\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{(-a+b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \\
& \left(\frac{(-a-b + \sqrt{-a^2 + b^2}) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])} - \frac{(-a-b + \sqrt{-a^2 + b^2}) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 (1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{2(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])^2} \right) \Bigg/ \\
& \left(\sqrt{\operatorname{Cos}[e+fx]} \sqrt{a+b \operatorname{Sin}[e+fx]} \left(\frac{(-a-b + \sqrt{-a^2 + b^2})(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])} \right)^{3/2} \right) - \left(2(a-b + \sqrt{-a^2 + b^2}) \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] \right)^2 \\
& \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)^2 \sqrt{\frac{b - \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \sqrt{-\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{(-a+b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \\
& \left(\frac{(-a-b + \sqrt{-a^2 + b^2}) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])} - \frac{(-a-b + \sqrt{-a^2 + b^2}) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 (1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{2(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])^2} \right) \Bigg/ \\
& \left((-a-b + \sqrt{-a^2 + b^2}) \sqrt{\operatorname{Cos}[e+fx]} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{1 - \frac{(-a-b + \sqrt{-a^2 + b^2})(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \right. \\
& \left. \sqrt{1 + \frac{(-a-b + \sqrt{-a^2 + b^2})(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \right) - \\
& \left(2 \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(-a-b + \sqrt{-a^2 + b^2})(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{(a-b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}\right], -1\right] \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{-\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{(-a + b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \\
& \left(\frac{a \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2(a - b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])} - \frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 (b - \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{2(a - b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])^2} \right) \Big/ \\
& \left(\sqrt{\operatorname{Cos}[e+fx]} \sqrt{a + b \operatorname{Sin}[e+fx]} \sqrt{\frac{(-a - b + \sqrt{-a^2 + b^2})(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{(a - b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \sqrt{\frac{b - \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{(a - b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \right) - \\
& \left(2 \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(-a - b + \sqrt{-a^2 + b^2})(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{(a - b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}\right], -1 \right] \right. \\
& \left. (-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]) \right) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{\frac{b - \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{(a - b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \\
& \left(-\frac{a \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2(-a + b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])} + \frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 (b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{2(-a + b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])^2} \right) \Big/ \\
& \left(\sqrt{\operatorname{Cos}[e+fx]} \sqrt{a + b \operatorname{Sin}[e+fx]} \sqrt{\frac{(-a - b + \sqrt{-a^2 + b^2})(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}{(a - b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \sqrt{-\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{(-a + b + \sqrt{-a^2 + b^2})(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])}} \right) \Big) \Big) \Big)
\end{aligned}$$

■ **Problem 617: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e \operatorname{Cos}[c+dx])^p (a + b \operatorname{Sin}[c+dx])^2 dx$$

Optimal (type 5, 157 leaves, 3 steps):

$$\frac{a b (3+p) (e \cos [c+d x])^{1+p}}{d e (1+p) (2+p)} - \frac{(b^2 + a^2 (2+p)) (e \cos [c+d x])^{1+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{d e (1+p) (2+p) \sqrt{\sin [c+d x]^2}} - \frac{b (e \cos [c+d x])^{1+p} (a+b \sin [c+d x])}{d e (2+p)}$$

Result (type 5, 288 leaves):

$$\frac{1}{d (1+p) \sqrt{\sin [c+d x]^2}} (e \cos [c+d x])^p \left(\frac{1}{-1+p} 2^{-p} a b (1+e^{2 i (c+d x)})^{-1-p} (e^{-i (c+d x)} (1+e^{2 i (c+d x)}))^{1+p} \cos [c+d x]^{-p} \right. \\ \left. \left(-(-1+p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-p), -p, \frac{1-p}{2}, -e^{2 i (c+d x)}\right] + e^{2 i (c+d x)} (1+p) \operatorname{Hypergeometric2F1}\left[\frac{1-p}{2}, -p, \frac{3-p}{2}, -e^{2 i (c+d x)}\right] \right) \right. \\ \left. \sqrt{\sin [c+d x]^2} - \frac{1}{2} b^2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cos [c+d x]^2\right] \sin [2(c+d x)] - \right. \\ \left. \frac{1}{2} a^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cos [c+d x]^2\right] \sin [2(c+d x)] \right)$$

■ **Problem 618: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (e \cos [c+d x])^p (a+b \sin [c+d x]) dx$$

Optimal (type 5, 97 leaves, 2 steps):

$$\frac{b (e \cos [c+d x])^{1+p}}{d e (1+p)} - \frac{a (e \cos [c+d x])^{1+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cos [c+d x]^2\right] \sin [c+d x]}{d e (1+p) \sqrt{\sin [c+d x]^2}}$$

Result (type 5, 233 leaves):

$$\frac{1}{2 d (1+p)} (e \cos [c+d x])^p \left(1 / (-1+p) 2^{-p} b (1+e^{2 i (c+d x)})^{-1-p} (e^{-i (c+d x)} (1+e^{2 i (c+d x)}))^{1+p} \cos [c+d x]^{-p} \right. \\ \left. \left(-(-1+p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-p), -p, \frac{1-p}{2}, -e^{2 i (c+d x)}\right] + e^{2 i (c+d x)} (1+p) \operatorname{Hypergeometric2F1}\left[\frac{1-p}{2}, -p, \frac{3-p}{2}, -e^{2 i (c+d x)}\right] \right) \right. \\ \left. \frac{a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cos [c+d x]^2\right] \sin [2(c+d x)]}{\sqrt{\sin [c+d x]^2}} \right)$$

■ **Problem 619: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos [c+d x])^p}{a+b \sin [c+d x]} dx$$

Optimal (type 6, 158 leaves, 1 step):

$$-\frac{1}{bd(1-p)}$$

$$e \operatorname{AppellF1}\left[1-p, \frac{1-p}{2}, \frac{1-p}{2}, 2-p, \frac{a+b}{a+b\sin[c+dx]}, \frac{a-b}{a+b\sin[c+dx]}\right] (e \cos[c+dx])^{-1+p} \left(-\frac{b(1-\sin[c+dx])}{a+b\sin[c+dx]}\right)^{\frac{1-p}{2}} \left(\frac{b(1+\sin[c+dx])}{a+b\sin[c+dx]}\right)^{\frac{1-p}{2}}$$

Result (type 6, 6000 leaves):

$$\begin{aligned} & \left(a^2 (e \cos[c+dx])^p \tan[c+dx] (1+\tan[c+dx]^2)^{-1-\frac{p}{2}} \left(b \tan[c+dx] + a \sqrt{1+\tan[c+dx]^2} \right) \right. \\ & \left(- \left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[c+dx]^2\right] \sqrt{1+\tan[c+dx]^2} \right) / \right. \\ & \left(-3 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[c+dx]^2\right] + \left(2 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\ & \left. \left. \left(-1+\frac{b^2}{a^2}\right) \tan[c+dx]^2\right] + a^2 p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[c+dx]^2\right] \right) \tan[c+dx]^2 \right) + \\ & \left(2 b \operatorname{AppellF1}\left[1, \frac{1+p}{2}, 1, 2, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2}\right] \tan[c+dx] \right) / \\ & \left(-4 a^2 \operatorname{AppellF1}\left[1, \frac{1+p}{2}, 1, 2, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[c+dx]^2\right] + \left(2 (a^2-b^2) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\tan[c+dx]^2, \right. \right. \right. \\ & \left. \left. \left(-1+\frac{b^2}{a^2}\right) \tan[c+dx]^2\right] + a^2 (1+p) \operatorname{AppellF1}\left[2, \frac{3+p}{2}, 1, 3, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[c+dx]^2\right] \right) \tan[c+dx]^2 \right) \left. \right) / \\ & \left(d (a+b\sin[c+dx]) \left(a + \frac{b \tan[c+dx]}{\sqrt{1+\tan[c+dx]^2}} \right) (-b^2 \tan[c+dx]^2 + a^2 (1+\tan[c+dx]^2)) \right) \\ & \left(- \frac{1}{\left(a + \frac{b \tan[c+dx]}{\sqrt{1+\tan[c+dx]^2}} \right) (-b^2 \tan[c+dx]^2 + a^2 (1+\tan[c+dx]^2))^2} \right. \\ & a^2 \tan[c+dx] (2 a^2 \sec[c+dx]^2 \tan[c+dx] - 2 b^2 \sec[c+dx]^2 \tan[c+dx]) (1+\tan[c+dx]^2)^{-1-\frac{p}{2}} \\ & \left(b \tan[c+dx] + a \sqrt{1+\tan[c+dx]^2} \right) \left(- \left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[c+dx]^2\right] \sqrt{1+\tan[c+dx]^2} \right) / \right. \\ & \left(-3 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[c+dx]^2\right] + \left(2 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Tan}[c + d x]^2 \right) + \\
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \operatorname{Tan}[c + d x] \right) / \\
& \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \right) + \\
& \frac{1}{\left(a + \frac{b \operatorname{Tan}[c + d x]}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} \right) (-b^2 \operatorname{Tan}[c + d x]^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2))} a^2 \operatorname{Tan}[c + d x] (1 + \operatorname{Tan}[c + d x]^2)^{-1 - \frac{p}{2}} \\
& \left(b \operatorname{Sec}[c + d x]^2 + \frac{a \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} \right) \\
& \left(- \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) / \right. \\
& \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \right) + \\
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \operatorname{Tan}[c + d x] \right) / \\
& \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \right) \right) - \\
& \frac{1}{\left(a + \frac{b \operatorname{Tan}[c + d x]}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} \right)^2 (-b^2 \operatorname{Tan}[c + d x]^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2))} a^2 \operatorname{Tan}[c + d x] (1 + \operatorname{Tan}[c + d x]^2)^{-1 - \frac{p}{2}} \\
& \left(- \frac{b \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]^2}{(1 + \operatorname{Tan}[c + d x]^2)^{3/2}} + \frac{b \operatorname{Sec}[c + d x]^2}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} \right) \left(b \operatorname{Tan}[c + d x] + a \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(- \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] \sqrt{1 + \operatorname{Tan}[c+d x]^2} \right) / \right. \\
& \quad \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] \right) \operatorname{Tan}[c+d x]^2 \right) + \\
& \quad \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c+d x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+d x]^2}{a^2} \right] \operatorname{Tan}[c+d x] \right) / \\
& \quad \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c+d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] \right) \operatorname{Tan}[c+d x]^2 \right) + \\
& \quad \frac{1}{\left(a + \frac{b \operatorname{Tan}[c+d x]}{\sqrt{1 + \operatorname{Tan}[c+d x]^2}} \right) \left(-b^2 \operatorname{Tan}[c+d x]^2 + a^2 (1 + \operatorname{Tan}[c+d x]^2) \right)} 2 a^2 \left(-1 - \frac{p}{2} \right) \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]^2 \\
& \quad (1 + \operatorname{Tan}[c+d x]^2)^{-2-\frac{p}{2}} \left(b \operatorname{Tan}[c+d x] + a \sqrt{1 + \operatorname{Tan}[c+d x]^2} \right) \\
& \quad \left(- \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] \sqrt{1 + \operatorname{Tan}[c+d x]^2} \right) / \right. \\
& \quad \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] \right) \operatorname{Tan}[c+d x]^2 \right) + \\
& \quad \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c+d x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+d x]^2}{a^2} \right] \operatorname{Tan}[c+d x] \right) / \\
& \quad \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c+d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[c+d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+d x]^2 \right] \right) \operatorname{Tan}[c+d x]^2 \right) + \\
& \quad \frac{1}{\left(a + \frac{b \operatorname{Tan}[c+d x]}{\sqrt{1 + \operatorname{Tan}[c+d x]^2}} \right) \left(-b^2 \operatorname{Tan}[c+d x]^2 + a^2 (1 + \operatorname{Tan}[c+d x]^2) \right)} a^2 \operatorname{Sec}[c+d x]^2 (1 + \operatorname{Tan}[c+d x]^2)^{-1-\frac{p}{2}}
\end{aligned}$$

$$\begin{aligned}
& \left(b \operatorname{Tan}[c + d x] + a \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) \\
& \left(- \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) / \right. \\
& \quad \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \right) + \\
& \quad \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \operatorname{Tan}[c + d x] \right) / \\
& \quad \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \right) \Bigg) + \\
& \frac{1}{\left(a + \frac{b \operatorname{Tan}[c + d x]}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} \right) (-b^2 \operatorname{Tan}[c + d x]^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2))} a^2 \operatorname{Tan}[c + d x] (1 + \operatorname{Tan}[c + d x]^2)^{-1 - \frac{p}{2}} \\
& \left(b \operatorname{Tan}[c + d x] + a \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) \\
& \left(- \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) / \left(\sqrt{1 + \operatorname{Tan}[c + d x]^2} \right. \right. \\
& \quad \left. \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \right) \Bigg) - \\
& \quad \left(3 a \left(-\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 + \frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \right. \right. \\
& \quad \left. \frac{2}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) / \\
& \quad \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \right) / \\
& \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 + \\
& \left(2 b \tan[c+dx] \left(1/a^2 (-a^2+b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{1}{2} (1+p) \operatorname{AppellF1} \left[2, 1 + \frac{1+p}{2}, 1, 3, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) / \\
& \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 + \\
& \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \sqrt{1 + \tan[c+dx]^2} \left(2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. 2, \frac{5}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \right) \\
& \operatorname{Sec}[c+dx]^2 \tan[c+dx] - 3 a^2 \left(-\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 + \frac{p}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \right. \\
& \quad \left. \tan[c+dx] + \frac{2}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \\
& \tan[c+dx]^2 \left(2 (a^2 - b^2) \left(-\frac{3}{5} p \operatorname{AppellF1} \left[\frac{5}{2}, 1 + \frac{p}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \right. \right. \\
& \quad \left. \left. \tan[c+dx] + \frac{12}{5} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{p}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) + \\
& a^2 p \left(\frac{6}{5} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{2+p}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. \frac{3}{5} (2+p) \operatorname{AppellF1} \left[\frac{5}{2}, 1 + \frac{2+p}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) / \\
& \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] \tan^2[c + dx] \right)^2 - \\
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan^2[c + dx], \frac{(-a^2 + b^2) \tan^2[c + dx]}{a^2} \right] \tan[c + dx] \left(2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. 3, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] \right) \right. \\
& \quad \left. \sec^2[c + dx] \tan[c + dx] - 4 a^2 \left(\left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] \sec^2[c + dx] \right)^2 \right. \\
& \quad \left. \tan[c + dx] - \frac{1}{2} (1+p) \operatorname{AppellF1} \left[2, 1 + \frac{1+p}{2}, 1, 3, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] \sec^2[c + dx] \tan[c + dx] \right) + \\
& \quad \tan^2[c + dx] \left(2 (a^2 - b^2) \left(\frac{8}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[3, \frac{1+p}{2}, 3, 4, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] \sec^2[c + dx] \right)^2 \right. \\
& \quad \left. \tan[c + dx] - \frac{2}{3} (1+p) \operatorname{AppellF1} \left[3, 1 + \frac{1+p}{2}, 2, 4, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] \sec^2[c + dx] \tan[c + dx] \right) + \\
& \quad a^2 (1+p) \left(\frac{4}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[3, \frac{3+p}{2}, 2, 4, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] \sec^2[c + dx] \tan[c + dx] - \right. \\
& \quad \left. \frac{2}{3} (3+p) \operatorname{AppellF1} \left[3, 1 + \frac{3+p}{2}, 1, 4, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] \sec^2[c + dx] \tan[c + dx] \right) \right) \Bigg) / \\
& \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan^2[c + dx], \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan^2[c + dx], \left(-1 + \frac{b^2}{a^2} \right) \tan^2[c + dx] \right] \right) \tan^2[c + dx] \right)^2 \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 620: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c + dx])^p}{(a + b \sin[c + dx])^2} dx$$

Optimal (type 6, 170 leaves, 1 step):

$$- \left[e \operatorname{AppellF1} \left[2 - p, \frac{1-p}{2}, \frac{1-p}{2}, 3 - p, \frac{a+b}{a+b \sin[c + dx]}, \frac{a-b}{a+b \sin[c + dx]} \right] \right]$$

$$(e \cos[c + dx])^{-1+p} \left(- \frac{b(1 - \sin[c + dx])}{a + b \sin[c + dx]} \right)^{\frac{1-p}{2}} \left(\frac{b(1 + \sin[c + dx])}{a + b \sin[c + dx]} \right)^{\frac{1-p}{2}} \Bigg) / (bd(2-p)(a + b \sin[c + dx]))$$

Result (type 6, 6875 leaves):

$$\begin{aligned}
 & \left(a^2 (e \cos[c + dx])^p \tan[c + dx] (1 + \tan[c + dx]^2)^{-p/2} \right. \\
 & \left(\left(3 (a^2 + b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{(-a^2 + b^2) \tan[c + dx]^2}{a^2} \right] \right) / \left(\left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \\
 & \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] + a^2 p \right. \right. \right. \right. \\
 & \left. \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] \right) \tan[c + dx]^2 \right) \left(-b^2 \tan[c + dx]^2 + a^2 (1 + \tan[c + dx]^2) \right) \right) + \\
 & \frac{1}{(b^2 \tan[c + dx]^2 - a^2 (1 + \tan[c + dx]^2))^2} 2 a b \left(- \left(3 a b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, \frac{(-a^2 + b^2) \tan[c + dx]^2}{a^2} \right] \right) / \right. \\
 & \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\tan[c + dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] + \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 3, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \\
 & \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] \right) \tan[c + dx]^2 \right) + \\
 & \left(2 (-a^2 + b^2) \operatorname{AppellF1} \left[1, \frac{1}{2} (-1 + p), 2, 2, -\tan[c + dx]^2, \frac{(-a^2 + b^2) \tan[c + dx]^2}{a^2} \right] \tan[c + dx] \sqrt{1 + \tan[c + dx]^2} \right) / \\
 & \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1}{2} (-1 + p), 2, 2, -\tan[c + dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] + \right. \\
 & \left. \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1}{2} (-1 + p), 3, 3, -\tan[c + dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] + a^2 (-1 + p) \right. \right. \\
 & \left. \left. \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[c + dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] \right) \tan[c + dx]^2 \right) \right) \right) / \\
 & \left((-a^2 + b^2) d (a + b \sin[c + dx])^2 \left(-\frac{1}{-a^2 + b^2} a^2 p \sec[c + dx]^2 \tan[c + dx]^2 (1 + \tan[c + dx]^2)^{-1 - \frac{p}{2}} \right. \right. \\
 & \left. \left(\left(3 (a^2 + b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{(-a^2 + b^2) \tan[c + dx]^2}{a^2} \right] \right) / \left(\left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[c + dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \right. \right. \\
 & \left. \left. \left. \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[c + dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c + dx]^2 \right] \right) \tan[c + dx]^2 \right) \left(-b^2 \tan[c + dx]^2 + a^2 (1 + \tan[c + dx]^2) \right) \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{(b^2 \tan[c+dx]^2 - a^2 (1 + \tan[c+dx]^2))^2} 2ab \left(- \left(3ab \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2)\tan[c+dx]^2}{a^2} \right] \right) / \right. \\
& \quad \left(-3a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \left(4(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \right) + \\
& \quad \left(2(-a^2+b^2) \operatorname{AppellF1} \left[1, \frac{1}{2}(-1+p), 2, 2, -\tan[c+dx]^2, \frac{(-a^2+b^2)\tan[c+dx]^2}{a^2} \right] \tan[c+dx] \right. \\
& \quad \left. \sqrt{1 + \tan[c+dx]^2} \right) / \left(-4a^2 \operatorname{AppellF1} \left[1, \frac{1}{2}(-1+p), 2, 2, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right. \\
& \quad \left. \left(4(a^2-b^2) \operatorname{AppellF1} \left[2, \frac{1}{2}(-1+p), 3, 3, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left. a^2(-1+p) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \right) \right) \Bigg) + \\
& \frac{1}{-a^2+b^2} a^2 \operatorname{Sec}[c+dx]^2 (1 + \tan[c+dx]^2)^{-p/2} \left(\left(3(a^2+b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2)\tan[c+dx]^2}{a^2} \right] \right) / \right. \\
& \quad \left(\left(-3a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right. \right. \\
& \quad \left(2(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \right) (-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2)) \Bigg) + \\
& \frac{1}{(b^2 \tan[c+dx]^2 - a^2 (1 + \tan[c+dx]^2))^2} 2ab \left(- \left(3ab \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2)\tan[c+dx]^2}{a^2} \right] \right) / \right. \\
& \quad \left(-3a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \left(4(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \right) + \\
& \quad \left(2(-a^2+b^2) \operatorname{AppellF1} \left[1, \frac{1}{2}(-1+p), 2, 2, -\tan[c+dx]^2, \frac{(-a^2+b^2)\tan[c+dx]^2}{a^2} \right] \tan[c+dx] \sqrt{1 + \tan[c+dx]^2} \right) / \right. \\
& \quad \left(-4a^2 \operatorname{AppellF1} \left[1, \frac{1}{2}(-1+p), 2, 2, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1}{2} (-1 + p), 3, 3, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \right. \\
& \quad \left. a^2 (-1 + p) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Tan}[c + d x]^2 \right) \Bigg) + \\
& \frac{1}{-a^2 + b^2} a^2 \operatorname{Tan}[c + d x] (1 + \operatorname{Tan}[c + d x]^2)^{-p/2} \left(- \left(\left(3 (a^2 + b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \right. \right. \right. \\
& \quad \left. \left. \left(2 a^2 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] - 2 b^2 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) \right) / \left(\left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right) + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Tan}[c + d x]^2 \right) (-b^2 \operatorname{Tan}[c + d x]^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2))^2 \right) \Bigg) + \\
& \left(3 (a^2 + b^2) \left(-\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 + \frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \right. \right. \\
& \quad \left. \left. \frac{1}{3 a^2} 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) \right) / \\
& \left(\left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right) \operatorname{Tan}[c + d x]^2 \right) (-b^2 \operatorname{Tan}[c + d x]^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2)) \right) \Bigg) - \\
& \frac{1}{(b^2 \operatorname{Tan}[c + d x]^2 - a^2 (1 + \operatorname{Tan}[c + d x]^2))^3} 4 a b (-2 a^2 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + 2 b^2 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]) \\
& \left(- \left(3 a b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \right) / \right. \\
& \quad \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \right) + \\
& \left. \left(2 (-a^2 + b^2) \operatorname{AppellF1} \left[1, \frac{1}{2} (-1 + p), 2, 2, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \operatorname{Tan}[c + d x] \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(-4 a^2 \operatorname{AppellF1}\left[1, \frac{1}{2}(-1+p), 2, 2, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] + \right. \\
& \left. \left(4(a^2-b^2) \operatorname{AppellF1}\left[2, \frac{1}{2}(-1+p), 3, 3, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] + \right. \right. \\
& \left. \left. a^2(-1+p) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right]\right)\tan[c+dx]^2 \right) - \\
& \left(3(a^2+b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2)\tan[c+dx]^2}{a^2}\right] \left(2 \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] + a^2 p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] \right) \right) \\
& \operatorname{Sec}[c+dx]^2 \tan[c+dx] - 3 a^2 \left(-\frac{1}{3} p \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{p}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \right. \\
& \left. \tan[c+dx] + \frac{2}{3} \left(-1+\frac{b^2}{a^2}\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \\
& \tan[c+dx]^2 \left(2(a^2-b^2) \left(-\frac{3}{5} p \operatorname{AppellF1}\left[\frac{5}{2}, 1+\frac{p}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \right. \right. \\
& \left. \left. \tan[c+dx] + \frac{12}{5} \left(-1+\frac{b^2}{a^2}\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{p}{2}, 3, \frac{7}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \right. \\
& \left. a^2 p \left(\frac{6}{5} \left(-1+\frac{b^2}{a^2}\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2+p}{2}, 2, \frac{7}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \left. \left. \frac{3}{5} (2+p) \operatorname{AppellF1}\left[\frac{5}{2}, 1+\frac{2+p}{2}, 1, \frac{7}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \right) \Big/ \\
& \left(\left(-3 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] + \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] + a^2 p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\tan[c+dx]^2\right] \right) \tan[c+dx]^2 \right)^2 \\
& \left. (-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2)) \right) + \frac{1}{(b^2 \tan[c+dx]^2 - a^2 (1 + \tan[c+dx]^2))^2} \\
& 2 a b \left(- \left(3 a b \left(-\frac{1}{3} p \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{p}{2}, 2, \frac{5}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2)\tan[c+dx]^2}{a^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] + \right. \right. \right. \\
& \left. \left. \left. 1 / (3 a^2) 4 (-a^2+b^2) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 3, \frac{5}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2)\tan[c+dx]^2}{a^2}\right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(-3 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \left(4\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + a^2 p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right]\right) \operatorname{Tan}[c+d x]^2 \Bigg) + \\
& \left(2\left(-a^2+b^2\right) \operatorname{AppellF1}\left[1, \frac{1}{2}(-1+p), 2, 2, -\operatorname{Tan}[c+d x]^2, \frac{\left(-a^2+b^2\right) \operatorname{Tan}[c+d x]^2}{a^2}\right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]^2\right) / \\
& \left(\sqrt{1+\operatorname{Tan}[c+d x]^2}\left(-4 a^2 \operatorname{AppellF1}\left[1, \frac{1}{2}(-1+p), 2, 2, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \right. \right. \\
& \quad \left. \left(4\left(a^2-b^2\right) \operatorname{AppellF1}\left[2, \frac{1}{2}(-1+p), 3, 3, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \right. \right. \\
& \quad \left. \left. a^2(-1+p) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right]\right) \operatorname{Tan}[c+d x]^2\right) \Bigg) + \\
& \left(2\left(-a^2+b^2\right) \operatorname{AppellF1}\left[1, \frac{1}{2}(-1+p), 2, 2, -\operatorname{Tan}[c+d x]^2, \frac{\left(-a^2+b^2\right) \operatorname{Tan}[c+d x]^2}{a^2}\right] \operatorname{Sec}[c+d x]^2 \sqrt{1+\operatorname{Tan}[c+d x]^2}\right) / \\
& \left(-4 a^2 \operatorname{AppellF1}\left[1, \frac{1}{2}(-1+p), 2, 2, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \right. \\
& \quad \left(4\left(a^2-b^2\right) \operatorname{AppellF1}\left[2, \frac{1}{2}(-1+p), 3, 3, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \right. \\
& \quad \left. a^2(-1+p) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right]\right) \operatorname{Tan}[c+d x]^2 \Bigg) + \left(2\left(-a^2+b^2\right) \operatorname{Tan}[c+d x] \right. \\
& \quad \left. \left(-\frac{1}{2}(-1+p) \operatorname{AppellF1}\left[2, 1+\frac{1}{2}(-1+p), 2, 3, -\operatorname{Tan}[c+d x]^2, \frac{\left(-a^2+b^2\right) \operatorname{Tan}[c+d x]^2}{a^2}\right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] + \right. \right. \\
& \quad \left. \left. 1 / a^2 2\left(-a^2+b^2\right) \operatorname{AppellF1}\left[2, \frac{1}{2}(-1+p), 3, 3, -\operatorname{Tan}[c+d x]^2, \frac{\left(-a^2+b^2\right) \operatorname{Tan}[c+d x]^2}{a^2}\right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]\right) \right) \\
& \quad \left. \sqrt{1+\operatorname{Tan}[c+d x]^2}\right) / \left(-4 a^2 \operatorname{AppellF1}\left[1, \frac{1}{2}(-1+p), 2, 2, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \right. \\
& \quad \left(4\left(a^2-b^2\right) \operatorname{AppellF1}\left[2, \frac{1}{2}(-1+p), 3, 3, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \right. \\
& \quad \left. a^2(-1+p) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right]\right) \operatorname{Tan}[c+d x]^2 \Bigg) + \\
& \left(3 a b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c+d x]^2, \frac{\left(-a^2+b^2\right) \operatorname{Tan}[c+d x]^2}{a^2}\right] \left(2\left(4\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c+d x]^2, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Sec}[c + d x]^2 \\
& \operatorname{Tan}[c + d x] - 3 a^2 \left(-\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 + \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \right. \\
& \left. \frac{4}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) + \operatorname{Tan}[c + d x]^2 \\
& \left(4 (a^2 - b^2) \left(-\frac{3}{5} p \operatorname{AppellF1} \left[\frac{5}{2}, 1 + \frac{p}{2}, 3, \frac{7}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \right. \right. \\
& \left. \left. \frac{18}{5} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{p}{2}, 4, \frac{7}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) + \right. \\
& \left. a^2 p \left(\frac{12}{5} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{2+p}{2}, 3, \frac{7}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] - \right. \right. \\
& \left. \left. \frac{3}{5} (2+p) \operatorname{AppellF1} \left[\frac{5}{2}, 1 + \frac{2+p}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) \right) \right) \Big/ \\
& \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \right)^2 - \\
& \left(2 (-a^2 + b^2) \operatorname{AppellF1} \left[1, \frac{1}{2} (-1+p), 2, 2, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \operatorname{Tan}[c + d x] \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right. \\
& \left. \left(2 \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1}{2} (-1+p), 3, 3, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 (-1+p) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. 2, 3, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] - 4 a^2 \left(-\frac{1}{2} (-1+p) \operatorname{AppellF1} \left[2, \right. \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{1}{2} (-1+p), 2, 3, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + 2 \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} (-1+p), 3, 3, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) + \operatorname{Tan}[c + d x]^2 \left(4 (a^2 - b^2) \right. \right. \\
& \left. \left. \left(-\frac{2}{3} (-1+p) \operatorname{AppellF1} \left[3, 1 + \frac{1}{2} (-1+p), 3, 4, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \right. \right. \\
& \left. \left. 4 \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[3, \frac{1}{2} (-1+p), 4, 4, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) + \right. \\
& \left. a^2 (-1+p) \left(\frac{8}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[3, \frac{1+p}{2}, 3, 4, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] - \right. \right.
\end{aligned}$$

$$\frac{2}{3} (1+p) \operatorname{AppellF1}\left[3, 1 + \frac{1+p}{2}, 2, 4, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]\right] \right) \right) \right) \right) \right) /$$

$$\left(-4 a^2 \operatorname{AppellF1}\left[1, \frac{1}{2} (-1+p), 2, 2, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] + \left(4 (a^2 - b^2) \operatorname{AppellF1}\left[2, \frac{1}{2} (-1+p), 3, 3, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] + a^2 (-1+p) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \right) \operatorname{Tan}[c+dx]^2 \right) \right) \right) \right) \right)$$

■ **Problem 621: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^p}{(a+b \sin[c+dx])^3} dx$$

Optimal (type 6, 170 leaves, 1 step):

$$- \left(e \operatorname{AppellF1}\left[3-p, \frac{1-p}{2}, \frac{1-p}{2}, 4-p, \frac{a+b}{a+b \sin[c+dx]}, \frac{a-b}{a+b \sin[c+dx]}\right] (e \cos[c+dx])^{-1+p} \left(-\frac{b(1-\sin[c+dx])}{a+b \sin[c+dx]} \right)^{\frac{1-p}{2}} \left(\frac{b(1+\sin[c+dx])}{a+b \sin[c+dx]} \right)^{\frac{1-p}{2}} \right) / (bd(3-p)(a+b \sin[c+dx])^2)$$

Result (type 6, 20626 leaves): Display of huge result suppressed!

■ **Problem 622: Unable to integrate problem.**

$$\int \frac{(e \cos[c+dx])^p}{(a+b \sin[c+dx])^8} dx$$

Optimal (type 6, 170 leaves, 1 step):

$$- \left(e \operatorname{AppellF1}\left[8-p, \frac{1-p}{2}, \frac{1-p}{2}, 9-p, \frac{a+b}{a+b \sin[c+dx]}, \frac{a-b}{a+b \sin[c+dx]}\right] (e \cos[c+dx])^{-1+p} \left(-\frac{b(1-\sin[c+dx])}{a+b \sin[c+dx]} \right)^{\frac{1-p}{2}} \left(\frac{b(1+\sin[c+dx])}{a+b \sin[c+dx]} \right)^{\frac{1-p}{2}} \right) / (bd(8-p)(a+b \sin[c+dx])^7)$$

Result (type 8, 25 leaves):

$$\int \frac{(e \cos[c+dx])^p}{(a+b \sin[c+dx])^8} dx$$

■ **Problem 623: Result more than twice size of optimal antiderivative.**

$$\int (e \cos[c+dx])^p (a+b \sin[c+dx])^{5/2} dx$$

Optimal (type 6, 156 leaves, 2 steps) :

$$\frac{1}{7bd} 2 e \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{9}{2}, \frac{a+b \sin[c+dx]}{a-b}, \frac{a+b \sin[c+dx]}{a+b} \right]$$

$$(e \cos[c+dx])^{-1+p} (a+b \sin[c+dx])^{7/2} \left(1 - \frac{a+b \sin[c+dx]}{a-b} \right)^{\frac{1-p}{2}} \left(1 - \frac{a+b \sin[c+dx]}{a+b} \right)^{\frac{1-p}{2}}$$

Result (type 6, 2612 leaves) :

$$\frac{1}{2d} \cos[c+dx]^{-p} (e \cos[c+dx])^p$$

$$\left(- \left(10 (a^2 - b^2)^2 (2a^2 + b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{5}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \cos[c+dx]^{1+p} (a+b \sin[c+dx])^{3/2} \right. \right.$$

$$\left. \left(-\sqrt{b^2} + b \sin[c+dx] \right) \left(\sqrt{b^2} + b \sin[c+dx] \right) (1 - \sin^2[c+dx])^{-\frac{1-p}{2}} \left(-\frac{a^2 - b^2 - 2a(a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2} \right)^{\frac{1}{2}(-3+p)} \right) /$$

$$\left(3b^3 (a - \sqrt{b^2}) (a + \sqrt{b^2}) \left(5(a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{5}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] + \right. \right.$$

$$\left. (-1+p) \left((-a + \sqrt{b^2}) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{7}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] - \right. \right.$$

$$\left. \left. (a + \sqrt{b^2}) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{3-p}{2}, \frac{1-p}{2}, \frac{7}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right) (a+b \sin[c+dx]) \right) \right) -$$

$$\frac{1}{15b^3} 8a (a^2 - b^2) \cos[c+dx]^{1+p} (a+b \sin[c+dx])^{3/2} \left(-\sqrt{b^2} + b \sin[c+dx] \right) \left(\sqrt{b^2} + b \sin[c+dx] \right) (1 - \sin^2[c+dx])^{-\frac{1-p}{2}}$$

$$\left(-\frac{a^2 - b^2 - 2a(a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2} \right)^{\frac{1}{2}(-3+p)}$$

$$\left(- \left(25a \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{5}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right) / \left(5(a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{5}{2}, \right. \right. \right.$$

$$\left. \left. \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] + (-1+p) \left((-a + \sqrt{b^2}) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{7}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \right. \right. \right.$$

$$\begin{aligned}
& \left. \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] - \left(a+\sqrt{b^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{3}{2} - \frac{p}{2}, \frac{1}{2} - \frac{p}{2}, \frac{7}{2}, \frac{a+b\sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] (a+b\sin[c+dx]) \Bigg) + \\
& \left(21 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2} - \frac{p}{2}, \frac{1}{2} - \frac{p}{2}, \frac{7}{2}, \frac{a+b\sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] (a+b\sin[c+dx]) \right) / \\
& \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2} - \frac{p}{2}, \frac{1}{2} - \frac{p}{2}, \frac{7}{2}, \frac{a+b\sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] + \right. \\
& (-1+p) \left(\left(-a+\sqrt{b^2} \right) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1}{2} - \frac{p}{2}, \frac{3}{2} - \frac{p}{2}, \frac{9}{2}, \frac{a+b\sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] - \right. \\
& \left. \left. \left(a+\sqrt{b^2} \right) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{3}{2} - \frac{p}{2}, \frac{1}{2} - \frac{p}{2}, \frac{9}{2}, \frac{a+b\sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] \right) (a+b\sin[c+dx]) \right) \Bigg) + \\
& \frac{1}{105 b (-2 a^2 + b^2 + 4 a (a+b\sin[c+dx]) - 2 (a+b\sin[c+dx])^2)} 2 (a^2 - b^2) \cos[c+dx]^{1+p} \cos[2(c+dx)] \\
& (a+b\sin[c+dx])^{3/2} \left(-\sqrt{b^2} + b\sin[c+dx] \right) \left(\sqrt{b^2} + b\sin[c+dx] \right) \\
& (1 - \sin[c+dx]^2)^{-\frac{1}{2} - \frac{p}{2}} \left(-\frac{a^2 - b^2 - 2 a (a+b\sin[c+dx]) + (a+b\sin[c+dx])^2}{b^2} \right)^{\frac{1}{2} (-3+p)} \\
& \left(- \left(175 (2 a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{p}{2}, \frac{1}{2} - \frac{p}{2}, \frac{5}{2}, \frac{a+b\sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] \right) / \right. \\
& \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{p}{2}, \frac{1}{2} - \frac{p}{2}, \frac{5}{2}, \frac{a+b\sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] + \right. \\
& (-1+p) \left(\left(-a+\sqrt{b^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2} - \frac{p}{2}, \frac{3}{2} - \frac{p}{2}, \frac{7}{2}, \frac{a+b\sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] - \left(a+\sqrt{b^2} \right) \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{5}{2}, \frac{3}{2} - \frac{p}{2}, \frac{1}{2} - \frac{p}{2}, \frac{7}{2}, \frac{a+b\sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b\sin[c+dx]}{a+\sqrt{b^2}} \right] \right) (a+b\sin[c+dx]) \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \left(588 a \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{7}{2}, \frac{a+b \sin [c+d x]}{a-\sqrt{b^2}}, \frac{a+b \sin [c+d x]}{a+\sqrt{b^2}} \right] (a+b \sin [c+d x]) \right) / \\
& \left(7 \left(a^2-b^2 \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{7}{2}, \frac{a+b \sin [c+d x]}{a-\sqrt{b^2}}, \frac{a+b \sin [c+d x]}{a+\sqrt{b^2}} \right] + \right. \\
& (-1+p) \left(\left(-a+\sqrt{b^2} \right) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{9}{2}, \frac{a+b \sin [c+d x]}{a-\sqrt{b^2}}, \frac{a+b \sin [c+d x]}{a+\sqrt{b^2}} \right] - \right. \\
& \left. \left. \left(a+\sqrt{b^2} \right) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{3-p}{2}, \frac{1-p}{2}, \frac{9}{2}, \frac{a+b \sin [c+d x]}{a-\sqrt{b^2}}, \frac{a+b \sin [c+d x]}{a+\sqrt{b^2}} \right] \right) (a+b \sin [c+d x]) \right) - \\
& \left(270 \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{9}{2}, \frac{a+b \sin [c+d x]}{a-\sqrt{b^2}}, \frac{a+b \sin [c+d x]}{a+\sqrt{b^2}} \right] (a+b \sin [c+d x])^2 \right) / \\
& \left(9 \left(a^2-b^2 \right) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{9}{2}, \frac{a+b \sin [c+d x]}{a-\sqrt{b^2}}, \frac{a+b \sin [c+d x]}{a+\sqrt{b^2}} \right] + \right. \\
& (-1+p) \left(\left(-a+\sqrt{b^2} \right) \operatorname{AppellF1} \left[\frac{9}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{11}{2}, \frac{a+b \sin [c+d x]}{a-\sqrt{b^2}}, \frac{a+b \sin [c+d x]}{a+\sqrt{b^2}} \right] - \right. \\
& \left. \left. \left(a+\sqrt{b^2} \right) \operatorname{AppellF1} \left[\frac{9}{2}, \frac{3-p}{2}, \frac{1-p}{2}, \frac{11}{2}, \frac{a+b \sin [c+d x]}{a-\sqrt{b^2}}, \frac{a+b \sin [c+d x]}{a+\sqrt{b^2}} \right] \right) (a+b \sin [c+d x]) \right) \left. \right) \left. \right) \left. \right)
\end{aligned}$$

■ **Problem 624: Result more than twice size of optimal antiderivative.**

$$\int (e \cos [c+d x])^p (a+b \sin [c+d x])^{3/2} dx$$

Optimal (type 6, 156 leaves, 2 steps):

$$\begin{aligned}
& \frac{1}{5 b d} 2 e \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{7}{2}, \frac{a+b \sin [c+d x]}{a-b}, \frac{a+b \sin [c+d x]}{a+b} \right] \\
& (e \cos [c+d x])^{-1+p} (a+b \sin [c+d x])^{5/2} \left(1 - \frac{a+b \sin [c+d x]}{a-b} \right)^{\frac{1-p}{2}} \left(1 - \frac{a+b \sin [c+d x]}{a+b} \right)^{\frac{1-p}{2}}
\end{aligned}$$

Result (type 6, 447 leaves):

$$\begin{aligned}
& - \left(14 (a^2 - b^2)^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{7}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right. \\
& \quad \left. (e \cos[c+dx])^p \sec[c+dx]^3 (a+b \sin[c+dx])^{5/2} \left(-\sqrt{b^2} + b \sin[c+dx] \right) \left(\sqrt{b^2} + b \sin[c+dx] \right) \right) / \\
& \left(5 b^3 (a-\sqrt{b^2}) (a+\sqrt{b^2}) d \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{7}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] + \right. \right. \\
& \quad (-1+p) \left((-a+\sqrt{b^2}) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{9}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] - \right. \\
& \quad \left. \left. (a+\sqrt{b^2}) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{3-p}{2}, \frac{1-p}{2}, \frac{9}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right) (a+b \sin[c+dx]) \right) \Big)
\end{aligned}$$

■ **Problem 625: Result more than twice size of optimal antiderivative.**

$$\int (e \cos[c+dx])^p \sqrt{a+b \sin[c+dx]} dx$$

Optimal (type 6, 156 leaves, 2 steps):

$$\begin{aligned}
& \frac{1}{3bd} 2 e \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{5}{2}, \frac{a+b \sin[c+dx]}{a-b}, \frac{a+b \sin[c+dx]}{a+b} \right] \\
& (e \cos[c+dx])^{-1+p} (a+b \sin[c+dx])^{3/2} \left(1 - \frac{a+b \sin[c+dx]}{a-b} \right)^{\frac{1-p}{2}} \left(1 - \frac{a+b \sin[c+dx]}{a+b} \right)^{\frac{1-p}{2}}
\end{aligned}$$

Result (type 6, 447 leaves):

$$\begin{aligned}
& - \left(10 (a^2 - b^2)^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{5}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right. \\
& \quad \left. (e \cos[c+dx])^p \sec[c+dx]^3 (a+b \sin[c+dx])^{3/2} \left(-\sqrt{b^2} + b \sin[c+dx] \right) \left(\sqrt{b^2} + b \sin[c+dx] \right) \right) / \\
& \quad \left(3 b^3 (a-\sqrt{b^2}) (a+\sqrt{b^2}) d \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{5}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] + \right. \right. \\
& \quad \left. (-1+p) \left((-a+\sqrt{b^2}) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{7}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] - \right. \right. \\
& \quad \left. \left. (a+\sqrt{b^2}) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{3-p}{2}, \frac{1-p}{2}, \frac{7}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right) (a+b \sin[c+dx]) \right) \Big)
\end{aligned}$$

■ **Problem 626: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^p}{\sqrt{a+b \sin[c+dx]}} dx$$

Optimal (type 6, 154 leaves, 2 steps):

$$\begin{aligned}
& \frac{1}{bd} 2 e \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{3}{2}, \frac{a+b \sin[c+dx]}{a-b}, \frac{a+b \sin[c+dx]}{a+b} \right] \\
& (e \cos[c+dx])^{-1+p} \sqrt{a+b \sin[c+dx]} \left(1 - \frac{a+b \sin[c+dx]}{a-b} \right)^{\frac{1-p}{2}} \left(1 - \frac{a+b \sin[c+dx]}{a+b} \right)^{\frac{1-p}{2}}
\end{aligned}$$

Result (type 6, 445 leaves):

$$\begin{aligned}
& - \left(6 (a^2 - b^2)^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{3}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right. \\
& \quad \left. (e \cos[c+dx])^p \sec[c+dx]^3 \sqrt{a+b \sin[c+dx]} \left(-\sqrt{b^2} + b \sin[c+dx] \right) \left(\sqrt{b^2} + b \sin[c+dx] \right) \right) / \\
& \left(b^3 (a - \sqrt{b^2}) (a + \sqrt{b^2}) d \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{3}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] + \right. \right. \\
& \quad (-1+p) \left((-a + \sqrt{b^2}) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{5}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] - \right. \\
& \quad \left. \left. (a + \sqrt{b^2}) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3-p}{2}, \frac{1-p}{2}, \frac{5}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right) (a+b \sin[c+dx]) \right) \Big)
\end{aligned}$$

■ **Problem 627: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^p}{(a+b \sin[c+dx])^{3/2}} dx$$

Optimal (type 6, 154 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{bd \sqrt{a+b \sin[c+dx]}} \\
& 2 e \operatorname{AppellF1} \left[-\frac{1}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{1}{2}, \frac{a+b \sin[c+dx]}{a-b}, \frac{a+b \sin[c+dx]}{a+b} \right] (e \cos[c+dx])^{-1+p} \left(1 - \frac{a+b \sin[c+dx]}{a-b} \right)^{\frac{1-p}{2}} \left(1 - \frac{a+b \sin[c+dx]}{a+b} \right)^{\frac{1-p}{2}}
\end{aligned}$$

Result (type 6, 444 leaves):

$$\begin{aligned}
& \left(2 (a^2 - b^2)^2 \operatorname{AppellF1} \left[-\frac{1}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{1}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right. \\
& \quad \left. (e \cos[c+dx])^p \sec[c+dx]^3 \left(\sqrt{b^2} - b \sin[c+dx] \right) \left(\sqrt{b^2} + b \sin[c+dx] \right) \right) / \\
& \left(b^3 (a - \sqrt{b^2}) (a + \sqrt{b^2}) d \sqrt{a+b \sin[c+dx]} \left((-a^2 + b^2) \operatorname{AppellF1} \left[-\frac{1}{2}, \frac{1-p}{2}, \frac{1-p}{2}, \frac{1}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] - \right. \right. \\
& \quad (-1+p) \left((-a + \sqrt{b^2}) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{3}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] - \right. \\
& \quad \left. \left. (a + \sqrt{b^2}) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3-p}{2}, \frac{1-p}{2}, \frac{3}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] (a+b \sin[c+dx]) \right) \right) \Bigg)
\end{aligned}$$

■ **Problem 628: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \cos[c+dx])^p}{(a+b \sin[c+dx])^{5/2}} dx$$

Optimal (type 6, 156 leaves, 2 steps):

$$\begin{aligned}
& - \left(2 e \operatorname{AppellF1} \left[-\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, -\frac{1}{2}, \frac{a+b \sin[c+dx]}{a-b}, \frac{a+b \sin[c+dx]}{a+b} \right] \right. \\
& \quad \left. (e \cos[c+dx])^{-1+p} \left(1 - \frac{a+b \sin[c+dx]}{a-b} \right)^{\frac{1-p}{2}} \left(1 - \frac{a+b \sin[c+dx]}{a+b} \right)^{\frac{1-p}{2}} \right) / (3 b d (a+b \sin[c+dx])^{3/2})
\end{aligned}$$

Result (type 6, 446 leaves):

$$\begin{aligned}
& - \left(2 (a^2 - b^2)^2 \operatorname{AppellF1} \left[-\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, -\frac{1}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right. \\
& \quad \left. (e \cos[c+dx])^p \sec[c+dx]^3 \left(\sqrt{b^2} - b \sin[c+dx] \right) \left(\sqrt{b^2} + b \sin[c+dx] \right) \right) / \\
& \left(3 b^3 (a - \sqrt{b^2}) (a + \sqrt{b^2}) d (a + b \sin[c+dx])^{3/2} \left((a^2 - b^2) \operatorname{AppellF1} \left[-\frac{3}{2}, \frac{1-p}{2}, \frac{1-p}{2}, -\frac{1}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] - \right. \right. \\
& \quad (-1+p) \left((-a + \sqrt{b^2}) \operatorname{AppellF1} \left[-\frac{1}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{1}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] - \right. \\
& \quad \left. \left. (a + \sqrt{b^2}) \operatorname{AppellF1} \left[-\frac{1}{2}, \frac{3-p}{2}, \frac{1-p}{2}, \frac{1}{2}, \frac{a+b \sin[c+dx]}{a-\sqrt{b^2}}, \frac{a+b \sin[c+dx]}{a+\sqrt{b^2}} \right] \right) (a + b \sin[c+dx]) \right) \Bigg)
\end{aligned}$$

■ **Problem 629: Unable to integrate problem.**

$$\int (e \cos[c+dx])^p (a+b \sin[c+dx])^m dx$$

Optimal (type 6, 158 leaves, 2 steps):

$$\begin{aligned}
& \frac{1}{bd(1+m)} e \operatorname{AppellF1} \left[1+m, \frac{1-p}{2}, \frac{1-p}{2}, 2+m, \frac{a+b \sin[c+dx]}{a-b}, \frac{a+b \sin[c+dx]}{a+b} \right] \\
& (e \cos[c+dx])^{-1+p} (a+b \sin[c+dx])^{1+m} \left(1 - \frac{a+b \sin[c+dx]}{a-b} \right)^{\frac{1-p}{2}} \left(1 - \frac{a+b \sin[c+dx]}{a+b} \right)^{\frac{1-p}{2}}
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int (e \cos[c+dx])^p (a+b \sin[c+dx])^m dx$$

■ **Problem 630: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^7 (a+b \sin[c+dx])^m dx$$

Optimal (type 3, 254 leaves, 3 steps):

$$\begin{aligned}
& - \frac{(a^2 - b^2)^3 (a+b \sin[c+dx])^{1+m}}{b^7 d (1+m)} + \frac{6a (a^2 - b^2)^2 (a+b \sin[c+dx])^{2+m}}{b^7 d (2+m)} - \frac{3 (5a^4 - 6a^2 b^2 + b^4) (a+b \sin[c+dx])^{3+m}}{b^7 d (3+m)} + \\
& \frac{4a (5a^2 - 3b^2) (a+b \sin[c+dx])^{4+m}}{b^7 d (4+m)} - \frac{3 (5a^2 - b^2) (a+b \sin[c+dx])^{5+m}}{b^7 d (5+m)} + \frac{6a (a+b \sin[c+dx])^{6+m}}{b^7 d (6+m)} - \frac{(a+b \sin[c+dx])^{7+m}}{b^7 d (7+m)}
\end{aligned}$$

Result (type 3, 1639 leaves) :

$$\frac{1}{d} (a + b \sin[c + dx])^m$$

$$\left(- \left(a \left(11520 a^6 - 48384 a^4 b^2 + 80640 a^2 b^4 - 80640 b^6 - 12096 a^4 b^2 m + 50232 a^2 b^4 m - 112224 b^6 m + 1728 a^4 b^2 m^2 + 3324 a^2 b^4 m^2 - 54542 b^6 m^2 - \right. \right. \right.$$

$$\left. \left. 840 a^2 b^4 m^3 - 13125 b^6 m^3 - 12 a^2 b^4 m^4 - 1829 b^6 m^4 - 147 b^6 m^5 - 5 b^6 m^6 \right) \right) / \left(16 b^7 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m) \right) +$$

$$\left(\left(176400 b^6 + 46080 a^6 m - 182016 a^4 b^2 m + 279936 a^2 b^4 m + 194868 b^6 m - 42624 a^4 b^2 m^2 + 169440 a^2 b^4 m^2 + 78968 b^6 m^2 + \right. \right.$$

$$\left. 1152 a^4 b^2 m^3 + 29328 a^2 b^4 m^3 + 16299 b^6 m^3 + 1632 a^2 b^4 m^4 + 2027 b^6 m^4 + 48 a^2 b^4 m^5 + 153 b^6 m^5 + 5 b^6 m^6 \right)$$

$$\left(- \frac{i \cos[c + dx]}{128 b^6} + \frac{\sin[c + dx]}{128 b^6} \right) \Bigg) / \left((1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m) \right) +$$

$$\left(\left(176400 b^6 + 46080 a^6 m - 182016 a^4 b^2 m + 279936 a^2 b^4 m + 194868 b^6 m - 42624 a^4 b^2 m^2 + 169440 a^2 b^4 m^2 + 78968 b^6 m^2 + \right. \right.$$

$$\left. 1152 a^4 b^2 m^3 + 29328 a^2 b^4 m^3 + 16299 b^6 m^3 + 1632 a^2 b^4 m^4 + 2027 b^6 m^4 + 48 a^2 b^4 m^5 + 153 b^6 m^5 + 5 b^6 m^6 \right)$$

$$\left(\frac{i \cos[c + dx]}{128 b^6} + \frac{\sin[c + dx]}{128 b^6} \right) \Bigg) / \left((1+m) (2+m) (3+m) (4+m) (5+m) (6+m) (7+m) \right) +$$

$$\left(\left(1920 a^4 m - 7104 a^2 b^2 m + 10008 b^4 m - 1696 a^2 b^2 m^2 + 6370 b^4 m^2 - 32 a^2 b^2 m^3 + 1411 b^4 m^3 + 134 b^4 m^4 + 5 b^4 m^5 \right) \right.$$

$$\left. \left(\frac{3 a \cos[2(c + dx)]}{64 b^5} - \frac{3 i a \sin[2(c + dx)]}{64 b^5} \right) \right) \Bigg) / \left((2+m) (3+m) (4+m) (5+m) (6+m) (7+m) \right) +$$

$$\left(\left(1920 a^4 m - 7104 a^2 b^2 m + 10008 b^4 m - 1696 a^2 b^2 m^2 + 6370 b^4 m^2 - 32 a^2 b^2 m^3 + 1411 b^4 m^3 + 134 b^4 m^4 + 5 b^4 m^5 \right) \right.$$

$$\left. \left(\frac{3 a \cos[2(c + dx)]}{64 b^5} + \frac{3 i a \sin[2(c + dx)]}{64 b^5} \right) \right) \Bigg) / \left((2+m) (3+m) (4+m) (5+m) (6+m) (7+m) \right) +$$

$$\left(5880 b^4 - 640 a^4 m + 2208 a^2 b^2 m + 3602 b^4 m + 552 a^2 b^2 m^2 + 797 b^4 m^2 + 24 a^2 b^2 m^3 + 78 b^4 m^3 + 3 b^4 m^4 \right)$$

$$\left(- \frac{3 i \cos[3(c + dx)]}{128 b^4} + \frac{3 \sin[3(c + dx)]}{128 b^4} \right) \Bigg) / \left((3+m) (4+m) (5+m) (6+m) (7+m) \right) +$$

$$\left(5880 b^4 - 640 a^4 m + 2208 a^2 b^2 m + 3602 b^4 m + 552 a^2 b^2 m^2 + 797 b^4 m^2 + 24 a^2 b^2 m^3 + 78 b^4 m^3 + 3 b^4 m^4 \right)$$

$$\left(\frac{3 i \cos[3(c + dx)]}{128 b^4} + \frac{3 \sin[3(c + dx)]}{128 b^4} \right) \Bigg) / \left((3+m) (4+m) (5+m) (6+m) (7+m) \right) +$$

$$\frac{\left(20 a^2 m - 64 b^2 m - 17 b^2 m^2 - b^2 m^3 \right) \left(- \frac{3 a \cos[4(c + dx)]}{32 b^3} - \frac{3 i a \sin[4(c + dx)]}{32 b^3} \right)}{(4+m) (5+m) (6+m) (7+m)} + \frac{\left(20 a^2 m - 64 b^2 m - 17 b^2 m^2 - b^2 m^3 \right) \left(- \frac{3 a \cos[4(c + dx)]}{32 b^3} + \frac{3 i a \sin[4(c + dx)]}{32 b^3} \right)}{(4+m) (5+m) (6+m) (7+m)} +$$

$$\left(\frac{(294 b^2 + 24 a^2 m + 79 b^2 m + 5 b^2 m^2) \left(-\frac{i \cos[5(c+dx)]}{128 b^2} + \frac{\sin[5(c+dx)]}{128 b^2} \right)}{(5+m)(6+m)(7+m)} + \frac{(294 b^2 + 24 a^2 m + 79 b^2 m + 5 b^2 m^2) \left(\frac{i \cos[5(c+dx)]}{128 b^2} + \frac{\sin[5(c+dx)]}{128 b^2} \right)}{(5+m)(6+m)(7+m)} + \frac{\frac{a m \cos[6(c+dx)]}{64 b} - \frac{i a m \sin[6(c+dx)]}{64 b}}{(6+m)(7+m)} + \frac{\frac{a m \cos[6(c+dx)]}{64 b} + \frac{i a m \sin[6(c+dx)]}{64 b}}{(6+m)(7+m)} + \frac{-\frac{1}{128} i \cos[7(c+dx)] + \frac{1}{128} \sin[7(c+dx)]}{7+m} + \frac{\frac{1}{128} i \cos[7(c+dx)] + \frac{1}{128} \sin[7(c+dx)]}{7+m} \right)$$

■ **Problem 634: Unable to integrate problem.**

$$\int \sec[c+dx] (a+b \sin[c+dx])^m dx$$

Optimal (type 5, 115 leaves, 5 steps):

$$-\frac{\text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{a+b \sin[c+dx]}{a-b}\right] (a+b \sin[c+dx])^{1+m}}{2(a-b)d(1+m)} + \frac{\text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{a+b \sin[c+dx]}{a+b}\right] (a+b \sin[c+dx])^{1+m}}{2(a+b)d(1+m)}$$

Result (type 8, 21 leaves):

$$\int \sec[c+dx] (a+b \sin[c+dx])^m dx$$

■ **Problem 635: Unable to integrate problem.**

$$\int \sec[c+dx]^3 (a+b \sin[c+dx])^m dx$$

Optimal (type 5, 183 leaves, 6 steps):

$$-\frac{(a-b(1-m)) \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{a+b \sin[c+dx]}{a-b}\right] (a+b \sin[c+dx])^{1+m}}{4(a-b)^2 d(1+m)} + \frac{(a+b-bm) \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{a+b \sin[c+dx]}{a+b}\right] (a+b \sin[c+dx])^{1+m}}{4(a+b)^2 d(1+m)} - \frac{\sec[c+dx]^2 (b-a \sin[c+dx]) (a+b \sin[c+dx])^{1+m}}{2(a^2-b^2)d}$$

Result (type 8, 23 leaves):

$$\int \sec[c+dx]^3 (a+b \sin[c+dx])^m dx$$

■ **Problem 636: Unable to integrate problem.**

$$\int \sec[c+dx]^5 (a+b \sin[c+dx])^m dx$$

Optimal (type 5, 305 leaves, 7 steps):

$$\begin{aligned}
& - \frac{1}{16 (a-b)^3 d (1+m)} (3 a^2 - 3 a b (2-m) + b^2 (3-4 m+m^2)) \operatorname{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{a+b \operatorname{Sin}[c+d x]}{a-b}\right] (a+b \operatorname{Sin}[c+d x])^{1+m} + \\
& \frac{1}{16 (a+b)^3 d (1+m)} (3 a^2 + 3 a b (2-m) + b^2 (3-4 m+m^2)) \operatorname{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{a+b \operatorname{Sin}[c+d x]}{a+b}\right] (a+b \operatorname{Sin}[c+d x])^{1+m} - \\
& \frac{\operatorname{Sec}[c+d x]^4 (b-a \operatorname{Sin}[c+d x]) (a+b \operatorname{Sin}[c+d x])^{1+m}}{4 (a^2-b^2) d} + \\
& \frac{\operatorname{Sec}[c+d x]^2 (a+b \operatorname{Sin}[c+d x])^{1+m} (b (b^2 (3-m) - a^2 (1+m)) + a (3 a^2 - b^2 (5-2 m)) \operatorname{Sin}[c+d x])}{8 (a^2-b^2)^2 d}
\end{aligned}$$

Result (type 8, 23 leaves):

$$\int \operatorname{Sec}[c+d x]^5 (a+b \operatorname{Sin}[c+d x])^m dx$$

■ **Problem 637: Unable to integrate problem.**

$$\int \operatorname{Cos}[c+d x]^4 (a+b \operatorname{Sin}[c+d x])^m dx$$

Optimal (type 6, 129 leaves, 2 steps):

$$\frac{\operatorname{AppellF1}\left[1+m, -\frac{3}{2}, -\frac{3}{2}, 2+m, \frac{a+b \operatorname{Sin}[c+d x]}{a-b}, \frac{a+b \operatorname{Sin}[c+d x]}{a+b}\right] \operatorname{Cos}[c+d x]^3 (a+b \operatorname{Sin}[c+d x])^{1+m}}{b d (1+m) \left(1 - \frac{a+b \operatorname{Sin}[c+d x]}{a-b}\right)^{3/2} \left(1 - \frac{a+b \operatorname{Sin}[c+d x]}{a+b}\right)^{3/2}}$$

Result (type 8, 23 leaves):

$$\int \operatorname{Cos}[c+d x]^4 (a+b \operatorname{Sin}[c+d x])^m dx$$

■ **Problem 638: Unable to integrate problem.**

$$\int \operatorname{Cos}[c+d x]^2 (a+b \operatorname{Sin}[c+d x])^m dx$$

Optimal (type 6, 127 leaves, 2 steps):

$$\frac{\operatorname{AppellF1}\left[1+m, -\frac{1}{2}, -\frac{1}{2}, 2+m, \frac{a+b \operatorname{Sin}[c+d x]}{a-b}, \frac{a+b \operatorname{Sin}[c+d x]}{a+b}\right] \operatorname{Cos}[c+d x] (a+b \operatorname{Sin}[c+d x])^{1+m}}{b d (1+m) \sqrt{1 - \frac{a+b \operatorname{Sin}[c+d x]}{a-b}} \sqrt{1 - \frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}$$

Result (type 8, 23 leaves):

$$\int \operatorname{Cos}[c+d x]^2 (a+b \operatorname{Sin}[c+d x])^m dx$$

■ **Problem 639: Unable to integrate problem.**

$$\int \operatorname{Sec}[c+d x]^2 (a+b \operatorname{Sin}[c+d x])^m dx$$

Optimal (type 6, 129 leaves, 2 steps) :

$$\frac{1}{bd(1+m)} \text{AppellF1}\left[1+m, \frac{3}{2}, \frac{3}{2}, 2+m, \frac{a+b\sin[c+dx]}{a-b}, \frac{a+b\sin[c+dx]}{a+b}\right]$$

$$\sec[c+dx]^3 (a+b\sin[c+dx])^{1+m} \left(1 - \frac{a+b\sin[c+dx]}{a-b}\right)^{3/2} \left(1 - \frac{a+b\sin[c+dx]}{a+b}\right)^{3/2}$$

Result (type 8, 23 leaves) :

$$\int \sec[c+dx]^2 (a+b\sin[c+dx])^m dx$$

■ **Problem 640: Unable to integrate problem.**

$$\int \sec[c+dx]^4 (a+b\sin[c+dx])^m dx$$

Optimal (type 6, 129 leaves, 2 steps) :

$$\frac{1}{bd(1+m)} \text{AppellF1}\left[1+m, \frac{5}{2}, \frac{5}{2}, 2+m, \frac{a+b\sin[c+dx]}{a-b}, \frac{a+b\sin[c+dx]}{a+b}\right]$$

$$\sec[c+dx]^5 (a+b\sin[c+dx])^{1+m} \left(1 - \frac{a+b\sin[c+dx]}{a-b}\right)^{5/2} \left(1 - \frac{a+b\sin[c+dx]}{a+b}\right)^{5/2}$$

Result (type 8, 23 leaves) :

$$\int \sec[c+dx]^4 (a+b\sin[c+dx])^m dx$$

■ **Problem 641: Unable to integrate problem.**

$$\int (e \cos[c+dx])^{5/2} (a+b\sin[c+dx])^m dx$$

Optimal (type 6, 134 leaves, 2 steps) :

$$\frac{e \text{AppellF1}\left[1+m, -\frac{3}{4}, -\frac{3}{4}, 2+m, \frac{a+b\sin[c+dx]}{a-b}, \frac{a+b\sin[c+dx]}{a+b}\right] (e \cos[c+dx])^{3/2} (a+b\sin[c+dx])^{1+m}}{bd(1+m) \left(1 - \frac{a+b\sin[c+dx]}{a-b}\right)^{3/4} \left(1 - \frac{a+b\sin[c+dx]}{a+b}\right)^{3/4}}$$

Result (type 8, 27 leaves) :

$$\int (e \cos[c+dx])^{5/2} (a+b\sin[c+dx])^m dx$$

■ **Problem 642: Unable to integrate problem.**

$$\int (e \cos[c+dx])^{3/2} (a+b\sin[c+dx])^m dx$$

Optimal (type 6, 134 leaves, 2 steps) :

$$\frac{e \operatorname{AppellF1}\left[1+m, -\frac{1}{4}, -\frac{1}{4}, 2+m, \frac{a+b\sin[c+dx]}{a-b}, \frac{a+b\sin[c+dx]}{a+b}\right] \sqrt{e \cos[c+dx]} (a+b\sin[c+dx])^{1+m}}{bd(1+m) \left(1 - \frac{a+b\sin[c+dx]}{a-b}\right)^{1/4} \left(1 - \frac{a+b\sin[c+dx]}{a+b}\right)^{1/4}}$$

Result (type 8, 27 leaves):

$$\int (e \cos[c+dx])^{3/2} (a+b\sin[c+dx])^m dx$$

■ **Problem 643: Unable to integrate problem.**

$$\int \sqrt{e \cos[c+dx]} (a+b\sin[c+dx])^m dx$$

Optimal (type 6, 134 leaves, 2 steps):

$$\left(e \operatorname{AppellF1}\left[1+m, \frac{1}{4}, \frac{1}{4}, 2+m, \frac{a+b\sin[c+dx]}{a-b}, \frac{a+b\sin[c+dx]}{a+b}\right] (a+b\sin[c+dx])^{1+m} \left(1 - \frac{a+b\sin[c+dx]}{a-b}\right)^{1/4} \left(1 - \frac{a+b\sin[c+dx]}{a+b}\right)^{1/4} \right) / (bd(1+m) \sqrt{e \cos[c+dx]})$$

Result (type 8, 27 leaves):

$$\int \sqrt{e \cos[c+dx]} (a+b\sin[c+dx])^m dx$$

■ **Problem 644: Unable to integrate problem.**

$$\int \frac{(a+b\sin[c+dx])^m}{\sqrt{e \cos[c+dx]}} dx$$

Optimal (type 6, 134 leaves, 2 steps):

$$\left(e \operatorname{AppellF1}\left[1+m, \frac{3}{4}, \frac{3}{4}, 2+m, \frac{a+b\sin[c+dx]}{a-b}, \frac{a+b\sin[c+dx]}{a+b}\right] (a+b\sin[c+dx])^{1+m} \left(1 - \frac{a+b\sin[c+dx]}{a-b}\right)^{3/4} \left(1 - \frac{a+b\sin[c+dx]}{a+b}\right)^{3/4} \right) / (bd(1+m) (e \cos[c+dx])^{3/2})$$

Result (type 8, 27 leaves):

$$\int \frac{(a+b\sin[c+dx])^m}{\sqrt{e \cos[c+dx]}} dx$$

■ **Problem 645: Unable to integrate problem.**

$$\int \frac{(a+b\sin[c+dx])^m}{(e \cos[c+dx])^{3/2}} dx$$

Optimal (type 6, 134 leaves, 2 steps):

$$\left(e \operatorname{AppellF1} \left[1+m, \frac{5}{4}, \frac{5}{4}, 2+m, \frac{a+b \sin [c+d x]}{a-b}, \frac{a+b \sin [c+d x]}{a+b} \right] \right. \\ \left. (a+b \sin [c+d x])^{1+m} \left(1 - \frac{a+b \sin [c+d x]}{a-b} \right)^{5/4} \left(1 - \frac{a+b \sin [c+d x]}{a+b} \right)^{5/4} \right) / (b d (1+m) (e \cos [c+d x])^{5/2})$$

Result (type 8, 27 leaves):

$$\int \frac{(a+b \sin [c+d x])^m}{(e \cos [c+d x])^{3/2}} dx$$

■ **Problem 646: Unable to integrate problem.**

$$\int \frac{(a+b \sin [c+d x])^m}{(e \cos [c+d x])^{5/2}} dx$$

Optimal (type 6, 134 leaves, 2 steps):

$$\left(e \operatorname{AppellF1} \left[1+m, \frac{7}{4}, \frac{7}{4}, 2+m, \frac{a+b \sin [c+d x]}{a-b}, \frac{a+b \sin [c+d x]}{a+b} \right] \right. \\ \left. (a+b \sin [c+d x])^{1+m} \left(1 - \frac{a+b \sin [c+d x]}{a-b} \right)^{7/4} \left(1 - \frac{a+b \sin [c+d x]}{a+b} \right)^{7/4} \right) / (b d (1+m) (e \cos [c+d x])^{7/2})$$

Result (type 8, 27 leaves):

$$\int \frac{(a+b \sin [c+d x])^m}{(e \cos [c+d x])^{5/2}} dx$$

■ **Problem 647: Unable to integrate problem.**

$$\int (e \cos [c+d x])^{-4-m} (a+b \sin [c+d x])^m dx$$

Optimal (type 5, 598 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(e \cos[c + dx])^{-3-m} (a + b \sin[c + dx])^{1+m}}{(a-b) d e^{3+m}} + \\
& \frac{2b (e \cos[c + dx])^{-1-m} (a + b \sin[c + dx])^{1+m}}{(a-b)^2 d e^3 (1+m) (3+m)} + \frac{a (e \cos[c + dx])^{-3-m} (1 + \sin[c + dx]) (a + b \sin[c + dx])^{1+m}}{(a^2 - b^2) d e^{3+m}} + \\
& \frac{a (3b + a(2+m)) (e \cos[c + dx])^{-3-m} (1 - \sin[c + dx]) (1 + \sin[c + dx]) (a + b \sin[c + dx])^{1+m}}{(a-b) (a+b)^2 d e^{3+m}} - \\
& \left(2^{\frac{3-m}{2}} a b (e \cos[c + dx])^{-1-m} \operatorname{Hypergeometric2F1} \left[\frac{1}{2} (-1-m), \frac{1+m}{2}, \frac{1-m}{2}, \frac{(a-b)(1 - \sin[c + dx])}{2(a+b \sin[c + dx])} \right] \right. \\
& \quad \left. \left(\frac{(a+b)(1 + \sin[c + dx])}{a + b \sin[c + dx]} \right)^{\frac{1+m}{2}} (a + b \sin[c + dx])^{1+m} \right) / \left((a-b)^2 (a+b) d e^3 (1+m) (3+m) \right) - \\
& \left(2^{-\frac{1-m}{2}} a (2ab - b^2 + a^2(2+m)) (e \cos[c + dx])^{-3-m} \operatorname{Hypergeometric2F1} \left[\frac{1-m}{2}, \frac{3+m}{2}, \frac{3-m}{2}, \frac{(a-b)(1 - \sin[c + dx])}{2(a+b \sin[c + dx])} \right] \right. \\
& \quad \left. (1 - \sin[c + dx])^2 \left(\frac{(a+b)(1 + \sin[c + dx])}{a + b \sin[c + dx]} \right)^{\frac{3+m}{2}} (a + b \sin[c + dx])^{1+m} \right) / \left((a-b) (a+b)^3 d e^{3+m} (1-m) (3+m) \right)
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int (e \cos[c + dx])^{-4-m} (a + b \sin[c + dx])^m dx$$

■ **Problem 648: Unable to integrate problem.**

$$\int (e \cos[c + dx])^{-3-m} (a + b \sin[c + dx])^m dx$$

Optimal (type 5, 311 leaves, ? steps):

$$\begin{aligned}
& \frac{(e \cos[c + dx])^{-m} \operatorname{Sec}[c + dx]^4 (-1 + \sin[c + dx]) (1 + \sin[c + dx]) (a + b \sin[c + dx])^{1+m}}{(a-b) d e^3 (2+m)} + \frac{1}{(a-b)^2 d e^3 m (2+m)} \\
& \frac{(-2b + a(2+m)) (e \cos[c + dx])^{-m} \operatorname{Sec}[c + dx]^4 (-1 + \sin[c + dx]) (1 + \sin[c + dx])^2 (a + b \sin[c + dx])^{1+m}}{(a-b)^3 d e^3 m (1+m)} - \\
& \frac{1}{(a-b)^3 d e^3 m (1+m)} (-b^2 + a^2(1+m)) (e \cos[c + dx])^{-m} \operatorname{Hypergeometric2F1} \left[\frac{m}{2}, 1+m, 2+m, -\frac{2(a+b \sin[c + dx])}{(a-b)(-1 + \sin[c + dx])} \right] \\
& \operatorname{Sec}[c + dx]^4 (1 + \sin[c + dx])^3 \left(\frac{(a+b)(1 + \sin[c + dx])}{(a-b)(-1 + \sin[c + dx])} \right)^{\frac{1}{2}(-2+m)} (a + b \sin[c + dx])^{1+m}
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int (e \cos[c + dx])^{-3-m} (a + b \sin[c + dx])^m dx$$

■ **Problem 649: Unable to integrate problem.**

$$\int (e \cos [c + d x])^{-2-m} (a + b \sin [c + d x])^m dx$$

Optimal (type 5, 201 leaves, 3 steps):

$$-\frac{(e \cos [c + d x])^{-1-m} (a + b \sin [c + d x])^{1+m}}{(a-b) d e (1+m)} + \frac{1}{(a^2 - b^2) d e (1+m)} 2^{\frac{1}{2}-\frac{m}{2}} a (e \cos [c + d x])^{-1-m}$$

$$\text{Hypergeometric2F1}\left[\frac{1}{2}(-1-m), \frac{1+m}{2}, \frac{1-m}{2}, \frac{(a-b)(1-\sin[c+dx])}{2(a+b\sin[c+dx])}\right] \left(\frac{(a+b)(1+\sin[c+dx])}{a+b\sin[c+dx]}\right)^{\frac{1+m}{2}} (a+b\sin[c+dx])^{1+m}$$

Result (type 8, 29 leaves):

$$\int (e \cos [c + d x])^{-2-m} (a + b \sin [c + d x])^m dx$$

■ **Problem 650: Unable to integrate problem.**

$$\int (e \cos [c + d x])^{-1-m} (a + b \sin [c + d x])^m dx$$

Optimal (type 5, 132 leaves, 1 step):

$$\frac{1}{(a+b) d (1+m)} e (e \cos [c + d x])^{-2-m} \text{Hypergeometric2F1}\left[1+m, \frac{2+m}{2}, 2+m, \frac{2(a+b\sin[c+dx])}{(a+b)(1+\sin[c+dx])}\right]$$

$$(1-\sin[c+dx]) \left(-\frac{(a-b)(1-\sin[c+dx])}{(a+b)(1+\sin[c+dx])}\right)^{m/2} (a+b\sin[c+dx])^{1+m}$$

Result (type 8, 29 leaves):

$$\int (e \cos [c + d x])^{-1-m} (a + b \sin [c + d x])^m dx$$

■ **Problem 651: Unable to integrate problem.**

$$\int (e \cos [c + d x])^{-m} (a + b \sin [c + d x])^m dx$$

Optimal (type 6, 152 leaves, 2 steps):

$$\frac{1}{b d (1+m)} e \text{AppellF1}\left[1+m, \frac{1+m}{2}, \frac{1+m}{2}, 2+m, \frac{a+b\sin[c+dx]}{a-b}, \frac{a+b\sin[c+dx]}{a+b}\right]$$

$$(e \cos [c + d x])^{-1-m} (a + b \sin [c + d x])^{1+m} \left(1 - \frac{a + b \sin [c + d x]}{a - b}\right)^{\frac{1+m}{2}} \left(1 - \frac{a + b \sin [c + d x]}{a + b}\right)^{\frac{1+m}{2}}$$

Result (type 8, 27 leaves):

$$\int (e \cos [c + d x])^{-m} (a + b \sin [c + d x])^m dx$$

■ **Problem 652: Unable to integrate problem.**

$$\int (e \cos [c + d x])^{1-m} (a + b \sin [c + d x])^m dx$$

Optimal (type 6, 142 leaves, 2 steps):

$$\frac{1}{b d (1+m)} e \operatorname{AppellF1} \left[1+m, \frac{m}{2}, \frac{m}{2}, 2+m, \frac{a+b \sin [c+d x]}{a-b}, \frac{a+b \sin [c+d x]}{a+b} \right] \\ (e \cos [c+d x])^{-m} (a+b \sin [c+d x])^{1+m} \left(1 - \frac{a+b \sin [c+d x]}{a-b} \right)^{m/2} \left(1 - \frac{a+b \sin [c+d x]}{a+b} \right)^{m/2}$$

Result (type 8, 29 leaves):

$$\int (e \cos [c + d x])^{1-m} (a + b \sin [c + d x])^m dx$$

■ **Problem 653: Unable to integrate problem.**

$$\int (e \cos [c + d x])^{2-m} (a + b \sin [c + d x])^m dx$$

Optimal (type 6, 152 leaves, 2 steps):

$$\frac{1}{b d (1+m)} e \operatorname{AppellF1} \left[1+m, \frac{1}{2} (-1+m), \frac{1}{2} (-1+m), 2+m, \frac{a+b \sin [c+d x]}{a-b}, \frac{a+b \sin [c+d x]}{a+b} \right] \\ (e \cos [c+d x])^{1-m} (a+b \sin [c+d x])^{1+m} \left(1 - \frac{a+b \sin [c+d x]}{a-b} \right)^{\frac{1}{2} (-1+m)} \left(1 - \frac{a+b \sin [c+d x]}{a+b} \right)^{\frac{1}{2} (-1+m)}$$

Result (type 8, 29 leaves):

$$\int (e \cos [c + d x])^{2-m} (a + b \sin [c + d x])^m dx$$

Test results for the 36 problems in "4.1.13 (d+e x)^m sin(a+b x+c x^2)^n.m"

Test results for the 208 problems in "4.1.1.3 (g tan)^p (a+b sin)^m.m"

■ **Problem 1: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin [c + d x]) \tan [c + d x]^5 dx$$

Optimal (type 3, 115 leaves, 3 steps):

$$-\frac{23 a \operatorname{Log}[1 - \sin [c + d x]]}{16 d} + \frac{7 a \operatorname{Log}[1 + \sin [c + d x]]}{16 d} - \frac{a \sin [c + d x]}{d} + \frac{a^3}{8 d (a - a \sin [c + d x])^2} - \frac{a^2}{d (a - a \sin [c + d x])} + \frac{a^2}{8 d (a + a \sin [c + d x])}$$

Result (type 3, 246 leaves) :

$$\begin{aligned} & - \frac{a \operatorname{Log}[\operatorname{Cos}[c + d x]]}{d} - \frac{15 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{15 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} - \\ & \frac{a \operatorname{Sec}[c + d x]^2}{d} + \frac{a \operatorname{Sec}[c + d x]^4}{4 d} + \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{9 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \\ & \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{9 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a \operatorname{Sin}[c + d x]}{d} \end{aligned}$$

■ **Problem 2: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sin}[c + d x]) \operatorname{Tan}[c + d x]^3 dx$$

Optimal (type 3, 71 leaves, 3 steps) :

$$\frac{5 a \operatorname{Log}[1 - \operatorname{Sin}[c + d x]]}{4 d} - \frac{a \operatorname{Log}[1 + \operatorname{Sin}[c + d x]]}{4 d} + \frac{a \operatorname{Sin}[c + d x]}{d} + \frac{a^2}{2 d (a - a \operatorname{Sin}[c + d x])}$$

Result (type 3, 166 leaves) :

$$\begin{aligned} & \frac{a \operatorname{Log}[\operatorname{Cos}[c + d x]]}{d} + \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} - \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \\ & \frac{a \operatorname{Sec}[c + d x]^2}{2 d} + \frac{a}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{a \operatorname{Sin}[c + d x]}{d} \end{aligned}$$

■ **Problem 3: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sin}[c + d x]) \operatorname{Tan}[c + d x] dx$$

Optimal (type 3, 30 leaves, 3 steps) :

$$\frac{a \operatorname{Log}[1 - \operatorname{Sin}[c + d x]]}{d} - \frac{a \operatorname{Sin}[c + d x]}{d}$$

Result (type 3, 83 leaves) :

$$\frac{a \operatorname{Log}[\operatorname{Cos}[c + d x]]}{d} - \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} - \frac{a \operatorname{Sin}[c + d x]}{d}$$

■ **Problem 21: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sin}[c + d x])^2 \operatorname{Tan}[c + d x]^2 dx$$

Optimal (type 3, 71 leaves, 6 steps) :

$$-\frac{5a^2x}{2} + \frac{2a^2 \cos[c+dx]}{d} + \frac{2a^2 \cos[c+dx]}{d(1-\sin[c+dx])} + \frac{a^2 \cos[c+dx] \sin[c+dx]}{2d}$$

Result (type 3, 145 leaves):

$$-\left(a^2 (1 + \sin[c+dx])^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] (10(c+dx) - 8\cos[c+dx] - \sin[2(c+dx)]) + \sin\left[\frac{1}{2}(c+dx)\right] (-2(8+5c+5dx) + 8\cos[c+dx] + \sin[2(c+dx)]) \right) \right) / \left(4d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 \right)$$

■ **Problem 25: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[c+dx])^3 \tan[c+dx]^7 dx$$

Optimal (type 3, 160 leaves, 3 steps):

$$\frac{209a^3 \log[1-\sin[c+dx]]}{16d} - \frac{a^3 \log[1+\sin[c+dx]]}{16d} + \frac{7a^3 \sin[c+dx]}{d} + \frac{3a^3 \sin^2[c+dx]}{2d} + \frac{a^3 \sin^3[c+dx]}{3d} + \frac{a^6}{6d(a-a\sin[c+dx])^3} - \frac{13a^5}{8d(a-a\sin[c+dx])^2} + \frac{71a^4}{8d(a-a\sin[c+dx])}$$

Result (type 3, 480 leaves):

$$-\frac{3\cos[2(c+dx)](a+a\sin[c+dx])^3}{4d(\cos[\frac{1}{2}(c+dx)]+\sin[\frac{1}{2}(c+dx)])^6} + \frac{209\log[\cos[\frac{1}{2}(c+dx)]-\sin[\frac{1}{2}(c+dx)]](a+a\sin[c+dx])^3}{8d(\cos[\frac{1}{2}(c+dx)]+\sin[\frac{1}{2}(c+dx)])^6} - \frac{\log[\cos[\frac{1}{2}(c+dx)]+\sin[\frac{1}{2}(c+dx)]](a+a\sin[c+dx])^3}{8d(\cos[\frac{1}{2}(c+dx)]+\sin[\frac{1}{2}(c+dx)])^6} + \frac{(a+a\sin[c+dx])^3}{6d(\cos[\frac{1}{2}(c+dx)]-\sin[\frac{1}{2}(c+dx)])^6(\cos[\frac{1}{2}(c+dx)]+\sin[\frac{1}{2}(c+dx)])^6} - \frac{13(a+a\sin[c+dx])^3}{8d(\cos[\frac{1}{2}(c+dx)]-\sin[\frac{1}{2}(c+dx)])^4(\cos[\frac{1}{2}(c+dx)]+\sin[\frac{1}{2}(c+dx)])^6} + \frac{71(a+a\sin[c+dx])^3}{8d(\cos[\frac{1}{2}(c+dx)]-\sin[\frac{1}{2}(c+dx)])^2(\cos[\frac{1}{2}(c+dx)]+\sin[\frac{1}{2}(c+dx)])^6} + \frac{29\sin[c+dx](a+a\sin[c+dx])^3}{4d(\cos[\frac{1}{2}(c+dx)]+\sin[\frac{1}{2}(c+dx)])^6} - \frac{(a+a\sin[c+dx])^3 \sin[3(c+dx)]}{12d(\cos[\frac{1}{2}(c+dx)]+\sin[\frac{1}{2}(c+dx)])^6}$$

■ **Problem 29: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[c+dx])^3 \tan[c+dx]^6 dx$$

Optimal (type 3, 180 leaves, 9 steps) :

$$-\frac{23 a^3 x}{2} + \frac{136 a^3 \operatorname{Cos}[c+d x]}{5 d} - \frac{136 a^3 \operatorname{Cos}[c+d x]^3}{15 d} + \frac{23 a^3 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 d} +$$

$$\frac{a^6 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^5}{5 d (a-a \operatorname{Sin}[c+d x])^3} - \frac{13 a^5 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^4}{15 d (a-a \operatorname{Sin}[c+d x])^2} + \frac{23 a^6 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^3}{3 d (a^3-a^3 \operatorname{Sin}[c+d x])}$$

Result (type 3, 561 leaves) :

$$-\frac{23 (c+d x) (a+a \operatorname{Sin}[c+d x])^3}{2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \frac{27 \operatorname{Cos}[c+d x] (a+a \operatorname{Sin}[c+d x])^3}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} -$$

$$\frac{\operatorname{Cos}[3(c+d x)] (a+a \operatorname{Sin}[c+d x])^3}{12 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \frac{(a+a \operatorname{Sin}[c+d x])^3}{5 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} -$$

$$\frac{28 (a+a \operatorname{Sin}[c+d x])^3}{15 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} +$$

$$\frac{2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+a \operatorname{Sin}[c+d x])^3}{5 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^5 \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} -$$

$$\frac{56 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+a \operatorname{Sin}[c+d x])^3}{15 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3 \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} +$$

$$\frac{394 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+a \operatorname{Sin}[c+d x])^3}{15 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \frac{3 (a+a \operatorname{Sin}[c+d x])^3 \operatorname{Sin}[2(c+d x)]}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6}$$

■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sin}[c+d x])^4 \operatorname{Tan}[c+d x]^5 dx$$

Optimal (type 3, 129 leaves, 3 steps) :

$$-\frac{25 a^4 \operatorname{Log}[1-\operatorname{Sin}[c+d x]]}{d} - \frac{16 a^4 \operatorname{Sin}[c+d x]}{d} - \frac{9 a^4 \operatorname{Sin}[c+d x]^2}{2 d} -$$

$$\frac{4 a^4 \operatorname{Sin}[c+d x]^3}{3 d} - \frac{a^4 \operatorname{Sin}[c+d x]^4}{4 d} + \frac{a^6}{d (a-a \operatorname{Sin}[c+d x])^2} - \frac{11 a^5}{d (a-a \operatorname{Sin}[c+d x])}$$

Result (type 3, 390 leaves) :

$$\frac{19 \operatorname{Cos}[2(c+dx)](a+a \operatorname{Sin}[c+dx])^4}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8} - \frac{\operatorname{Cos}[4(c+dx)](a+a \operatorname{Sin}[c+dx])^4}{32 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8} -$$

$$\frac{50 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right](a+a \operatorname{Sin}[c+dx])^4}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8} + \frac{(a+a \operatorname{Sin}[c+dx])^4}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8} -$$

$$\frac{11(a+a \operatorname{Sin}[c+dx])^4}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8} -$$

$$\frac{17 \operatorname{Sin}[c+dx](a+a \operatorname{Sin}[c+dx])^4}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8} + \frac{(a+a \operatorname{Sin}[c+dx])^4 \operatorname{Sin}[3(c+dx)]}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}$$

■ **Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+dx]^7}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 130 leaves, 8 steps):

$$-\frac{35 \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{128 a d} + \frac{35 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{128 a d} -$$

$$\frac{35 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^3}{192 a d} + \frac{7 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^5}{48 a d} - \frac{\operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^7}{8 a d} + \frac{\operatorname{Tan}[c+dx]^8}{8 a d}$$

Result (type 3, 342 leaves):

$$\frac{1}{384 d (a+a \operatorname{Sin}[c+dx])} \left(-192 + \frac{6}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6} - \frac{40}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \right.$$

$$\frac{114}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + 105 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 -$$

$$105 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 + \frac{4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6} -$$

$$\left. \frac{27 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{87 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 45: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+dx]^5}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 106 leaves, 7 steps):

$$\frac{5 \operatorname{ArcTanh}[\sin[c + dx]]}{16 ad} - \frac{5 \sec[c + dx] \tan[c + dx]}{16 ad} + \frac{5 \sec[c + dx] \tan[c + dx]^3}{24 ad} - \frac{\sec[c + dx] \tan[c + dx]^5}{6 ad} + \frac{\tan[c + dx]^6}{6 ad}$$

Result (type 3, 267 leaves):

$$\frac{1}{96 d (a + a \sin[c + dx])} \left(48 + \frac{4}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4} - \frac{21}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2} - 30 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right] \right. \\ \left. \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 + 30 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 + \right. \\ \left. \frac{3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^4} - \frac{18 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2} \right)$$

■ **Problem 46: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c + dx]^3}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$-\frac{3 \operatorname{ArcTanh}[\sin[c + dx]]}{8 ad} + \frac{3 \sec[c + dx] \tan[c + dx]}{8 ad} - \frac{\sec[c + dx] \tan[c + dx]^3}{4 ad} + \frac{\tan[c + dx]^4}{4 ad}$$

Result (type 3, 189 leaves):

$$\frac{1}{8 d (a + a \sin[c + dx])} \left(-4 + \frac{1}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2} + 3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 - \right. \\ \left. 3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 + \frac{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2} \right)$$

■ **Problem 47: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c + dx]}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 37 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}[\sin[c + dx]]}{2 ad} + \frac{1}{2 d (a + a \sin[c + dx])}$$

Result (type 3, 126 leaves) :

$$\frac{1}{2 a d (1 + \sin[c + d x])} \left(1 - \log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] + \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) + \left(-\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] + \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) \sin[c + d x]$$

■ **Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c + d x]^2}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 50 leaves, 5 steps) :

$$\frac{\sec[c + d x]}{a d} - \frac{\sec[c + d x]^3}{3 a d} + \frac{\tan[c + d x]^3}{3 a d}$$

Result (type 3, 106 leaves) :

$$\frac{6 - 10 \cos[c + d x] + 2 \cos[2(c + d x)] + 8 \sin[c + d x] - 5 \sin[2(c + d x)]}{12 a d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right) (1 + \sin[c + d x])}$$

■ **Problem 56: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 23 leaves, 1 step) :

$$-\frac{\cos[c + d x]}{d (a + a \sin[c + d x])}$$

Result (type 3, 48 leaves) :

$$\frac{2 \sin \left[\frac{1}{2} (c + d x) \right] \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)}{d (a + a \sin[c + d x])}$$

■ **Problem 57: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + d x]^2}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 29 leaves, 4 steps) :

$$\frac{\operatorname{ArcTanh}[\cos[c + d x]]}{a d} - \frac{\cot[c + d x]}{a d}$$

Result (type 3, 69 leaves) :

$$-\frac{1}{2 a d} \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right] \left(\cos[c + d x] + \left(-\log \left[\cos \left[\frac{1}{2} (c + d x) \right] \right] + \log \left[\sin \left[\frac{1}{2} (c + d x) \right] \right] \right) \sin[c + d x] \right)$$

■ **Problem 58: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + dx]^4}{a + a \text{Sin}[c + dx]} dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$-\frac{\text{ArcTanh}[\text{Cos}[c + dx]]}{2ad} - \frac{\text{Cot}[c + dx]^3}{3ad} + \frac{\text{Cot}[c + dx] \text{Csc}[c + dx]}{2ad}$$

Result (type 3, 124 leaves):

$$-\frac{1}{96ad(1 + \text{Sin}[c + dx])} \text{Csc}\left[\frac{1}{2}(c + dx)\right] \text{Sec}\left[\frac{1}{2}(c + dx)\right] \left(\text{Csc}\left[\frac{1}{2}(c + dx)\right] + \text{Sec}\left[\frac{1}{2}(c + dx)\right] \right)^2 \\ \left(\text{Cos}[3(c + dx)] + \text{Cos}[c + dx] (3 - 6 \text{Sin}[c + dx]) + 6 \left(\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] - \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) \text{Sin}[c + dx]^3 \right)$$

■ **Problem 59: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + dx]^6}{a + a \text{Sin}[c + dx]} dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$\frac{3 \text{ArcTanh}[\text{Cos}[c + dx]]}{8ad} - \frac{\text{Cot}[c + dx]^5}{5ad} - \frac{3 \text{Cot}[c + dx] \text{Csc}[c + dx]}{8ad} + \frac{\text{Cot}[c + dx]^3 \text{Csc}[c + dx]}{4ad}$$

Result (type 3, 189 leaves):

$$-\frac{1}{640ad} \text{Csc}[c + dx]^5 \\ \left(80 \text{Cos}[c + dx] + 40 \text{Cos}[3(c + dx)] + 8 \text{Cos}[5(c + dx)] - 150 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[c + dx] + 150 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[c + dx] + \right. \\ \left. 20 \text{Sin}[2(c + dx)] + 75 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[3(c + dx)] - 75 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[3(c + dx)] - \right. \\ \left. 50 \text{Sin}[4(c + dx)] - 15 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[5(c + dx)] + 15 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[5(c + dx)] \right)$$

■ **Problem 60: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + dx]^8}{a + a \text{Sin}[c + dx]} dx$$

Optimal (type 3, 106 leaves, 7 steps):

$$-\frac{5 \text{ArcTanh}[\text{Cos}[c + dx]]}{16ad} - \frac{\text{Cot}[c + dx]^7}{7ad} + \frac{5 \text{Cot}[c + dx] \text{Csc}[c + dx]}{16ad} - \frac{5 \text{Cot}[c + dx]^3 \text{Csc}[c + dx]}{24ad} + \frac{\text{Cot}[c + dx]^5 \text{Csc}[c + dx]}{6ad}$$

Result (type 3, 284 leaves):

$$\begin{aligned}
& - \frac{1}{86\,016\,a\,d\,(1+\sin[c+dx])} \\
& \operatorname{Csc}[c+dx]^5 \left(\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \right)^2 \left(1680 \cos[c+dx] + 1008 \cos[3(c+dx)] + 336 \cos[5(c+dx)] + 48 \cos[7(c+dx)] + \right. \\
& 3675 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] \sin[c+dx] - 3675 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] \sin[c+dx] - 1190 \sin[2(c+dx)] - \\
& 2205 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] \sin[3(c+dx)] + 2205 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] \sin[3(c+dx)] + \\
& 392 \sin[4(c+dx)] + 735 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] \sin[5(c+dx)] - 735 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] \sin[5(c+dx)] - \\
& \left. 462 \sin[6(c+dx)] - 105 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] \sin[7(c+dx)] + 105 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] \sin[7(c+dx)] \right)
\end{aligned}$$

■ **Problem 63: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c+dx]^3}{(a+a\sin[c+dx])^2} dx$$

Optimal (type 3, 104 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}[\sin[c+dx]]}{8a^2d} + \frac{a}{12d(a+a\sin[c+dx])^3} - \frac{1}{4d(a+a\sin[c+dx])^2} + \frac{1}{16d(a^2-a^2\sin[c+dx])} + \frac{3}{16d(a^2+a^2\sin[c+dx])}$$

Result (type 3, 217 leaves):

$$\begin{aligned}
& \frac{1}{48d(a+a\sin[c+dx])^2} \left(-12 + \frac{4}{(\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^2} + \right. \\
& 9 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + 6 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 - \\
& \left. 6 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 + \frac{3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4}{(\cos[\frac{1}{2}(c+dx)] - \sin[\frac{1}{2}(c+dx)])^2} \right)
\end{aligned}$$

■ **Problem 64: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c+dx]}{(a+a\sin[c+dx])^2} dx$$

Optimal (type 3, 60 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sin[c+dx]]}{4a^2d} + \frac{1}{4d(a+a\sin[c+dx])^2} - \frac{1}{4d(a^2+a^2\sin[c+dx])}$$

Result (type 3, 139 leaves):

$$-\frac{1}{4d(a+a\sin[c+dx])^2} \left(-1 + \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 \right)$$

■ **Problem 65: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx]}{(a+a\sin[c+dx])^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$\frac{\log[\sin[c+dx]]}{a^2 d} - \frac{\log[1+\sin[c+dx]]}{a^2 d} + \frac{1}{d(a^2+a^2\sin[c+dx])}$$

Result (type 3, 112 leaves):

$$\frac{1}{a^2 d (1+\sin[c+dx])^2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \left(1 - 2 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + \log[\sin[c+dx]] + \left(-2 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + \log[\sin[c+dx]] \right) \sin[c+dx]$$

■ **Problem 74: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c+dx]}{(a+a\sin[c+dx])^3} dx$$

Optimal (type 3, 82 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sin[c+dx]]}{8a^3 d} + \frac{1}{6d(a+a\sin[c+dx])^3} - \frac{1}{8ad(a+a\sin[c+dx])^2} - \frac{1}{8d(a^3+a^3\sin[c+dx])}$$

Result (type 3, 167 leaves):

$$\frac{1}{24d(a+a\sin[c+dx])^3} \left(4 - 3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 - 3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 - 3 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 + 3 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 \right)$$

■ **Problem 76: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + d x]^3}{(a + a \text{Sin}[c + d x])^3} dx$$

Optimal (type 3, 86 leaves, 3 steps):

$$\frac{3 \text{Csc}[c + d x]}{a^3 d} - \frac{\text{Csc}[c + d x]^2}{2 a^3 d} + \frac{5 \text{Log}[\text{Sin}[c + d x]]}{a^3 d} - \frac{5 \text{Log}[1 + \text{Sin}[c + d x]]}{a^3 d} + \frac{2}{d (a^3 + a^3 \text{Sin}[c + d x])}$$

Result (type 3, 226 leaves):

$$\frac{1}{8 a^3 d (1 + \text{Sin}[c + d x])^3} \left(\cos\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^4 \left(16 - \left(1 + \text{Cot}\left[\frac{1}{2}(c + d x)\right] \right)^2 + 12 \text{Cot}\left[\frac{1}{2}(c + d x)\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right)^2 - 80 \text{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 + 40 \text{Log}[\text{Sin}[c + d x]] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 + 12 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right] - \left(1 + \text{Tan}\left[\frac{1}{2}(c + d x)\right] \right)^2$$

■ **Problem 77: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + d x]^5}{(a + a \text{Sin}[c + d x])^3} dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$\frac{4 \text{Csc}[c + d x]}{a^3 d} - \frac{2 \text{Csc}[c + d x]^2}{a^3 d} + \frac{\text{Csc}[c + d x]^3}{a^3 d} - \frac{\text{Csc}[c + d x]^4}{4 a^3 d} + \frac{4 \text{Log}[\text{Sin}[c + d x]]}{a^3 d} - \frac{4 \text{Log}[1 + \text{Sin}[c + d x]]}{a^3 d}$$

Result (type 3, 558 leaves):

$$\begin{aligned}
& \frac{9 \cot\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{4d(a+a\sin[c+dx])^3} - \frac{17 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{32d(a+a\sin[c+dx])^3} + \\
& \frac{\cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a\sin[c+dx])^3} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{64d(a+a\sin[c+dx])^3} - \\
& \frac{8 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{d(a+a\sin[c+dx])^3} + \frac{4 \operatorname{Log}[\sin[c+dx]] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{d(a+a\sin[c+dx])^3} - \\
& \frac{17 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{32d(a+a\sin[c+dx])^3} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{64d(a+a\sin[c+dx])^3} + \\
& \frac{9 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6 \tan\left[\frac{1}{2}(c+dx)\right]}{4d(a+a\sin[c+dx])^3} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6 \tan\left[\frac{1}{2}(c+dx)\right]}{8d(a+a\sin[c+dx])^3}
\end{aligned}$$

■ **Problem 85: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx]^3}{(a+a\sin[c+dx])^4} dx$$

Optimal (type 3, 106 leaves, 3 steps):

$$\frac{4 \operatorname{Csc}[c+dx]}{a^4 d} - \frac{\operatorname{Csc}[c+dx]^2}{2a^4 d} + \frac{9 \operatorname{Log}[\sin[c+dx]]}{a^4 d} - \frac{9 \operatorname{Log}[1+\sin[c+dx]]}{a^4 d} + \frac{1}{d(a^2+a^2\sin[c+dx])^2} + \frac{5}{d(a^4+a^4\sin[c+dx])}$$

Result (type 3, 275 leaves):

$$\begin{aligned}
& \frac{1}{8a^4 d (1+\sin[c+dx])^4} \\
& \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 \left(8 - \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)^4 \sin\left[\frac{1}{2}(c+dx)\right]^2 + 40 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 + \right. \\
& \quad \left. 16 \cot\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 - 144 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \\
& \quad \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 + 72 \operatorname{Log}[\sin[c+dx]] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 + \\
& \quad \left. 16 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 \tan\left[\frac{1}{2}(c+dx)\right] - \cos\left[\frac{1}{2}(c+dx)\right]^2 \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^4 \right)
\end{aligned}$$

■ **Problem 88: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx]^2}{(a+a\sin[c+dx])^4} dx$$

Optimal (type 3, 108 leaves, 14 steps):

$$\frac{4 \operatorname{ArcTanh}[\cos[c + dx]]}{a^4 d} - \frac{\cot[c + dx]}{a^4 d} - \frac{2 \cot[c + dx]}{5 a^4 d (1 + \csc[c + dx])^3} + \frac{31 \cot[c + dx]}{15 a^4 d (1 + \csc[c + dx])^2} - \frac{104 \cot[c + dx]}{15 a^4 d (1 + \csc[c + dx])}$$

Result (type 3, 315 leaves):

$$\frac{1}{30 d (a + a \sin[c + dx])^4} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3$$

$$\left(24 \sin\left[\frac{1}{2}(c + dx)\right] - 12 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) + 76 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 - \right.$$

$$38 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3 + 316 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 -$$

$$15 \cot\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 + 120 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 -$$

$$120 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 + 15 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 \tan\left[\frac{1}{2}(c + dx)\right] \left. \right)$$

■ **Problem 89: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx]^4}{(a + a \sin[c + dx])^4} dx$$

Optimal (type 3, 120 leaves, 14 steps):

$$\frac{14 \operatorname{ArcTanh}[\cos[c + dx]]}{a^4 d} - \frac{9 \cot[c + dx]}{a^4 d} - \frac{\cot[c + dx]^3}{3 a^4 d} + \frac{2 \cot[c + dx] \csc[c + dx]}{a^4 d} + \frac{4 \cot[c + dx]}{3 a^4 d (1 + \csc[c + dx])^2} - \frac{44 \cot[c + dx]}{3 a^4 d (1 + \csc[c + dx])}$$

Result (type 3, 589 leaves):

$$\begin{aligned}
& \frac{8 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5}{3d(a+a\operatorname{Sin}[c+dx])^4} - \frac{4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{3d(a+a\operatorname{Sin}[c+dx])^4} + \\
& \frac{80 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7}{3d(a+a\operatorname{Sin}[c+dx])^4} - \frac{13 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{3d(a+a\operatorname{Sin}[c+dx])^4} + \\
& \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{2d(a+a\operatorname{Sin}[c+dx])^4} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{24d(a+a\operatorname{Sin}[c+dx])^4} + \\
& \frac{14 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{d(a+a\operatorname{Sin}[c+dx])^4} - \frac{14 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{d(a+a\operatorname{Sin}[c+dx])^4} - \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{2d(a+a\operatorname{Sin}[c+dx])^4} + \frac{13 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{3d(a+a\operatorname{Sin}[c+dx])^4} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24d(a+a\operatorname{Sin}[c+dx])^4}
\end{aligned}$$

■ **Problem 90: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^6}{(a+a\operatorname{Sin}[c+dx])^4} dx$$

Optimal (type 3, 133 leaves, 16 steps):

$$\begin{aligned}
& \frac{27 \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{2a^4d} - \frac{16 \operatorname{Cot}[c+dx]}{a^4d} - \frac{3 \operatorname{Cot}[c+dx]^3}{a^4d} - \\
& \frac{\operatorname{Cot}[c+dx]^5}{5a^4d} + \frac{11 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{2a^4d} + \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{a^4d} - \frac{8 \operatorname{Cot}[c+dx]}{a^4d(1+\operatorname{Csc}[c+dx])}
\end{aligned}$$

Result (type 3, 733 leaves):

$$\begin{aligned}
& \frac{16 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7}{d(a+a \operatorname{Sin}[c+dx])^4} - \frac{33 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{5d(a+a \operatorname{Sin}[c+dx])^4} + \\
& \frac{11 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{8d(a+a \operatorname{Sin}[c+dx])^4} - \frac{53 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{160d(a+a \operatorname{Sin}[c+dx])^4} + \\
& \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{16d(a+a \operatorname{Sin}[c+dx])^4} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{160d(a+a \operatorname{Sin}[c+dx])^4} + \\
& \frac{27 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{2d(a+a \operatorname{Sin}[c+dx])^4} - \frac{27 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{2d(a+a \operatorname{Sin}[c+dx])^4} - \\
& \frac{11 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{8d(a+a \operatorname{Sin}[c+dx])^4} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{16d(a+a \operatorname{Sin}[c+dx])^4} + \\
& \frac{33 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{5d(a+a \operatorname{Sin}[c+dx])^4} + \frac{53 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{160d(a+a \operatorname{Sin}[c+dx])^4} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{160d(a+a \operatorname{Sin}[c+dx])^4}
\end{aligned}$$

- **Problem 91: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a \operatorname{Sin}[e+fx]} \operatorname{Tan}[e+fx]^4 dx$$

Optimal (type 3, 162 leaves, 15 steps):

$$\begin{aligned}
& \frac{11 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[e+fx]}{\sqrt{2} \sqrt{a+a \operatorname{Sin}[e+fx]}}\right]}{8 \sqrt{2} f} - \frac{27 \operatorname{Sec}[e+fx] \sqrt{a(1+\operatorname{Sin}[e+fx])}}{8 f} - \\
& \frac{\operatorname{Sec}[e+fx]^3 \sqrt{a(1+\operatorname{Sin}[e+fx])}}{12 f} + \frac{29 \sqrt{a+a \operatorname{Sin}[e+fx]} \operatorname{Tan}[e+fx]}{12 f} + \frac{5 \sqrt{a(1+\operatorname{Sin}[e+fx])} \operatorname{Tan}[e+fx]^3}{12 f}
\end{aligned}$$

Result (type 3, 394 leaves):

$$\frac{1}{24 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3} \left(\frac{6 \sin\left[\frac{fx}{2}\right]}{\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]} - \frac{3 \left(\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)}{\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]} + \right. \\ \left. (33 + 33i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{fx}{4}\right] \left(\cos\left[\frac{1}{4}(2e+fx)\right] - \sin\left[\frac{1}{4}(2e+fx)\right] \right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 - \right. \\ \left. 48 \cos\left[\frac{fx}{2}\right] \left(\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 + \right. \\ \left. 48 \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \sin\left[\frac{fx}{2}\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 + \right. \\ \left. \frac{4 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2}{\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3} - \frac{36 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2}{\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]} \right) \sqrt{a(1 + \sin[e+fx])}$$

■ **Problem 92: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a + a \sin[e+fx]} \tan[e+fx]^2 dx$$

Optimal (type 3, 101 leaves, 4 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{2} f} + \frac{5 \operatorname{Sec}[e+fx] \sqrt{a+a \sin[e+fx]}}{f} - \frac{2 \operatorname{Sec}[e+fx] (a+a \sin[e+fx])^{3/2}}{a f}$$

Result (type 3, 114 leaves):

$$\frac{1}{f} \operatorname{Sec}[e+fx] \\ \left(3 + (1-i) (-1)^{1/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{fx}{4}\right] \left(\cos\left[\frac{1}{4}(2e+fx)\right] - \sin\left[\frac{1}{4}(2e+fx)\right] \right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) - \\ 2 \sin[e+fx] \sqrt{a(1 + \sin[e+fx])}$$

■ **Problem 93: Result more than twice size of optimal antiderivative.**

$$\int \cot[e+fx]^2 \sqrt{a+a \sin[e+fx]} dx$$

Optimal (type 3, 89 leaves, 4 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{a+a \sin[e+fx]}}\right]}{f} + \frac{3 a \cos[e+fx]}{f \sqrt{a+a \sin[e+fx]}} - \frac{\cot[e+fx] \sqrt{a+a \sin[e+fx]}}{f}$$

Result (type 3, 206 leaves):

$$\left(\operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^4 \sqrt{a(1+\sin[e+fx])} \left(-4 \cos\left[\frac{1}{2}(e+fx)\right] + 2 \cos\left[\frac{3}{2}(e+fx)\right] + 4 \sin\left[\frac{1}{2}(e+fx)\right] - \right. \right. \\ \left. \left. \operatorname{Log}\left[1 + \cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right] \sin[e+fx] + \operatorname{Log}\left[1 - \cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right] \sin[e+fx] + 2 \sin\left[\frac{3}{2}(e+fx)\right] \right) \right) / \\ \left(f \left(1 + \cot\left[\frac{1}{2}(e+fx)\right] \right) \left(\operatorname{Csc}\left[\frac{1}{4}(e+fx)\right] - \operatorname{Sec}\left[\frac{1}{4}(e+fx)\right] \right) \left(\operatorname{Csc}\left[\frac{1}{4}(e+fx)\right] + \operatorname{Sec}\left[\frac{1}{4}(e+fx)\right] \right) \right)$$

■ **Problem 95: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \sin[e+fx])^{3/2} \tan[e+fx]^4 dx$$

Optimal (type 3, 167 leaves, 14 steps):

$$-\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{2 \sqrt{2} f} + \frac{2 a^3 \cos[e+fx]^3}{3 f (a + a \sin[e+fx])^{3/2}} - \\ \frac{4 a^2 \cos[e+fx]}{f \sqrt{a + a \sin[e+fx]}} - \frac{7 a \operatorname{Sec}[e+fx] \sqrt{a + a \sin[e+fx]}}{2 f} + \frac{\operatorname{Sec}[e+fx]^3 (a + a \sin[e+fx])^{3/2}}{3 f}$$

Result (type 3, 141 leaves):

$$\frac{1}{6 f} a \operatorname{Sec}[e+fx]^3 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sqrt{a(1+\sin[e+fx])} \left(-45 + 6 \cos[2(e+fx)] + \right. \\ \left. (3 + 3i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 + 54 \sin[e+fx] + \sin[3(e+fx)] \right)$$

■ **Problem 103: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan[e+fx]^4}{\sqrt{a + a \sin[e+fx]}} dx$$

Optimal (type 3, 150 leaves, 17 steps):

$$-\frac{67 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{64 \sqrt{2} \sqrt{a} f} - \frac{\operatorname{Sec}[e+fx] (53 + 127 \sin[e+fx])}{192 f \sqrt{a + a \sin[e+fx]}} + \frac{a \sin[e+fx] \tan[e+fx]}{24 f (a + a \sin[e+fx])^{3/2}} + \frac{\tan[e+fx]^3}{3 f \sqrt{a + a \sin[e+fx]}}$$

Result (type 3, 118 leaves):

$$\left((804 + 804i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) - \right. \\ \left. \operatorname{Sec}[e+fx]^3 (90 + 122 \cos[2(e+fx)] - 41 \sin[e+fx] + 183 \sin[3(e+fx)]) \right) / \left(768 f \sqrt{a(1+\sin[e+fx])} \right)$$

■ **Problem 104: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan[e + f x]^2}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$\frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{4 \sqrt{2} \sqrt{a} f} - \frac{\sec[e + f x]}{2 f \sqrt{a + a \sin[e + f x]}} + \frac{3 \sec[e + f x] \sqrt{a + a \sin[e + f x]}}{4 a f}$$

Result (type 3, 118 leaves):

$$-\frac{1}{4 f \sqrt{a} (1 + \sin[e + f x])} \sec[e + f x] \left(-1 + (5 + 5 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - 3 \sin[e + f x]$$

■ **Problem 105: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + f x]^2}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 62 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} f} - \frac{\cot[e + f x]}{f \sqrt{a + a \sin[e + f x]}}$$

Result (type 3, 138 leaves):

$$\frac{1}{8 f \sqrt{a} (1 + \sin[e + f x])} \operatorname{Csc}\left[\frac{1}{4} (e + f x)\right] \sec\left[\frac{1}{4} (e + f x)\right] \left(-2 \cos\left[\frac{1}{2} (e + f x)\right] + 2 \sin\left[\frac{1}{2} (e + f x)\right] \right) + \left(\log\left[1 + \cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right]\right] - \log\left[1 - \cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right] \right) \sin[e + f x] \left(1 + \tan\left[\frac{1}{2} (e + f x)\right] \right)$$

■ **Problem 106: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + f x]^4}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 135 leaves, 11 steps):

$$-\frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{a+a \sin[e+fx]}}\right]}{8 \sqrt{a} f} + \frac{9 \cot[e+fx]}{8 f \sqrt{a+a \sin[e+fx]}} + \frac{\cot[e+fx] \csc[e+fx]}{12 f \sqrt{a+a \sin[e+fx]}} - \frac{\cot[e+fx] \csc[e+fx]^2}{3 f \sqrt{a+a \sin[e+fx]}}$$

Result (type 3, 292 leaves):

$$\frac{1}{24 f \left(\csc\left[\frac{1}{4}(e+fx)\right]^2 - \sec\left[\frac{1}{4}(e+fx)\right]^2 \right)^3 \sqrt{a(1+\sin[e+fx])}}$$

$$\csc\left[\frac{1}{2}(e+fx)\right]^9 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(36 \cos\left[\frac{1}{2}(e+fx)\right] - 46 \cos\left[\frac{3}{2}(e+fx)\right] - \right.$$

$$54 \cos\left[\frac{5}{2}(e+fx)\right] - 36 \sin\left[\frac{1}{2}(e+fx)\right] - 63 \log\left[1 + \cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right] \sin[e+fx] +$$

$$63 \log\left[1 - \cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right] \sin[e+fx] - 46 \sin\left[\frac{3}{2}(e+fx)\right] + 54 \sin\left[\frac{5}{2}(e+fx)\right] +$$

$$\left. 21 \log\left[1 + \cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right] \sin[3(e+fx)] - 21 \log\left[1 - \cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right] \sin[3(e+fx)] \right)$$

■ **Problem 107: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan[e+fx]^4}{(a+a \sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 177 leaves, 20 steps):

$$\frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{256 \sqrt{2} a^{3/2} f} + \frac{7 \cos[e+fx]}{256 f (a+a \sin[e+fx])^{3/2}} -$$

$$\frac{\sec[e+fx] (65 + 87 \sin[e+fx])}{192 f (a+a \sin[e+fx])^{3/2}} + \frac{a \sin[e+fx] \tan[e+fx]}{12 f (a+a \sin[e+fx])^{5/2}} + \frac{\tan[e+fx]^3}{3 f (a+a \sin[e+fx])^{3/2}}$$

Result (type 3, 334 leaves):

$$\frac{1}{768 f (a (1 + \sin[e + f x]))^{3/2}} \left(124 + \frac{64 \sin\left[\frac{1}{2}(e + f x)\right]}{\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3} - \frac{32}{\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^2} - \frac{248 \sin\left[\frac{1}{2}(e + f x)\right]}{\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]} + 342 \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right) - 171 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^2 - (21 + 21i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 + \frac{32 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3}{\left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3} - \frac{192 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3}{\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]} \right)$$

■ **Problem 108: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan[e + f x]^2}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 134 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{32 \sqrt{2} a^{3/2} f} + \frac{\cos[e + f x]}{32 f (a + a \sin[e + f x])^{3/2}} - \frac{\sec[e + f x]}{4 f (a + a \sin[e + f x])^{3/2}} + \frac{5 \sec[e + f x]}{8 a f \sqrt{a + a \sin[e + f x]}}$$

Result (type 3, 128 leaves):

$$-\left(\sec[e + f x] \left(-25 - \cos[2(e + f x)] + (2 + 2i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right]\right) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4 - 40 \sin[e + f x]\right) / (64 f (a (1 + \sin[e + f x]))^{3/2})$$

■ **Problem 109: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cot[e + f x]^2}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 113 leaves, 6 steps):

$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{a + a \sin[e + f x]}}\right]}{a^{3/2} f} - \frac{2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{a^{3/2} f} - \frac{\cot[e + f x]}{a f \sqrt{a + a \sin[e + f x]}}$$

Result (type 3, 206 leaves):

$$\frac{1}{4 f (a (1 + \sin[e + f x]))^{3/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \left((16 + 16i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] - \cot\left[\frac{1}{4}(e + f x)\right] + 2 \left(3 \log\left[1 + \cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] - 3 \log\left[1 - \cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] + \sec\left[\frac{1}{2}(e + f x)\right] + \csc[e + f x] \sin\left[\frac{1}{4}(e + f x)\right]^2 - \csc[e + f x] \sin\left[\frac{1}{4}(e + f x)\right] \sin\left[\frac{3}{4}(e + f x)\right] \right) \right)$$

■ **Problem 110: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + f x]^4}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 144 leaves, 10 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{a + a \sin[e + f x]}}\right]}{8 a^{3/2} f} - \frac{\cot[e + f x]}{8 a f \sqrt{a + a \sin[e + f x]}} + \frac{11 \cot[e + f x] \csc[e + f x]}{12 a f \sqrt{a + a \sin[e + f x]}} - \frac{\cot[e + f x] \csc[e + f x]^2 \sqrt{a + a \sin[e + f x]}}{3 a^2 f}$$

Result (type 3, 294 leaves):

$$\frac{1}{24 f \left(\csc\left[\frac{1}{4}(e + f x)\right]^2 - \sec\left[\frac{1}{4}(e + f x)\right]^2 \right)^3 (a (1 + \sin[e + f x]))^{3/2}} \left(\csc\left[\frac{1}{2}(e + f x)\right]^9 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \left(-132 \cos\left[\frac{1}{2}(e + f x)\right] + 62 \cos\left[\frac{3}{2}(e + f x)\right] + 6 \cos\left[\frac{5}{2}(e + f x)\right] + 132 \sin\left[\frac{1}{2}(e + f x)\right] - 9 \log\left[1 + \cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] \sin[e + f x] + 9 \log\left[1 - \cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \sin[e + f x] + 62 \sin\left[\frac{3}{2}(e + f x)\right] - 6 \sin\left[\frac{5}{2}(e + f x)\right] + 3 \log\left[1 + \cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] \sin[3(e + f x)] - 3 \log\left[1 - \cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \sin[3(e + f x)] \right) \right)$$

■ **Problem 111: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan[e + f x]^4}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 207 leaves, 23 steps):

$$\frac{317 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{4096 \sqrt{2} a^{5/2} f} + \frac{317 \cos[e+fx]}{3072 f (a+a \sin[e+fx])^{5/2}} - \frac{\sec[e+fx] (115+129 \sin[e+fx])}{384 f (a+a \sin[e+fx])^{5/2}} +$$

$$\frac{317 \cos[e+fx]}{4096 a f (a+a \sin[e+fx])^{3/2}} + \frac{5 a \sin[e+fx] \tan[e+fx]}{48 f (a+a \sin[e+fx])^{7/2}} + \frac{\tan[e+fx]^3}{3 f (a+a \sin[e+fx])^{5/2}}$$

Result (type 3, 394 leaves):

$$\frac{1}{12288 f (a(1+\sin[e+fx]))^{5/2}}$$

$$\left(\frac{1312 + \frac{768 \sin\left[\frac{1}{2}(e+fx)\right]}{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^3} - \frac{384}{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2} - \frac{2624 \sin\left[\frac{1}{2}(e+fx)\right]}{\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]} + \right.$$

$$2584 \sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right) - 1292 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2 +$$

$$402 \sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^3 - 201 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4 -$$

$$(951 + 951 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5 +$$

$$\left. \frac{256 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5}{\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^3} - \frac{1152 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5}{\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]} \right)$$

■ **Problem 112: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan[e+fx]^2}{(a+a \sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 167 leaves, 6 steps):

$$-\frac{11 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{128 \sqrt{2} a^{5/2} f} - \frac{\sec[e+fx]}{6 f (a+a \sin[e+fx])^{5/2}} -$$

$$\frac{11 \cos[e+fx]}{128 a f (a+a \sin[e+fx])^{3/2}} + \frac{17 \sec[e+fx]}{48 a f (a+a \sin[e+fx])^{3/2}} + \frac{11 \sec[e+fx]}{96 a^2 f \sqrt{a+a \sin[e+fx]}}$$

Result (type 3, 284 leaves):

$$\frac{1}{384 f (a (1 + \sin[e + f x]))^{5/2}} \left(-32 + \frac{64 \sin\left[\frac{1}{2}(e + f x)\right]}{\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]} - \right. \\ \left. 104 \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) + 52 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 - \right. \\ \left. 30 \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 + 15 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 + (33 + 33i) (-1)^{3/4} \right. \\ \left. \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 + \frac{48 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5}{\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]} \right)$$

■ **Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + f x]^2}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 141 leaves, 7 steps):

$$\frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{a + a \sin[e + f x]}}\right]}{a^{5/2} f} - \frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{2} a^{5/2} f} - \frac{2 \cos[e + f x]}{a f (a + a \sin[e + f x])^{3/2}} - \frac{\cot[e + f x]}{a f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 451 leaves):

$$\frac{1}{4 f (a (1 + \sin[e + f x]))^{5/2}} \\ \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \left(8 \sin\left[\frac{1}{2}(e + f x)\right] - 4 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) + 2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + \right. \\ \left. (28 + 28i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 - \right. \\ \left. \cot\left[\frac{1}{4}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + 10 \log\left[1 + \cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] \right. \\ \left. \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 - 10 \log\left[1 - \cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + \right. \\ \left. \frac{2 \sin\left[\frac{1}{4}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2}{\cos\left[\frac{1}{4}(e + f x)\right] - \sin\left[\frac{1}{4}(e + f x)\right]} - \frac{2 \sin\left[\frac{1}{4}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2}{\cos\left[\frac{1}{4}(e + f x)\right] + \sin\left[\frac{1}{4}(e + f x)\right]} - \right. \\ \left. \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \tan\left[\frac{1}{4}(e + f x)\right] \right)$$

■ **Problem 114: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cot}[e + f x]^4}{(a + a \text{Sin}[e + f x])^{5/2}} dx$$

Optimal (type 3, 191 leaves, 16 steps):

$$\frac{45 \text{ArcTanh}\left[\frac{\sqrt{a} \text{Cos}[e+fx]}{\sqrt{a+a \text{Sin}[e+fx]}}\right]}{8 a^{5/2} f} - \frac{4 \sqrt{2} \text{ArcTanh}\left[\frac{\sqrt{a} \text{Cos}[e+fx]}{\sqrt{2} \sqrt{a+a \text{Sin}[e+fx]}}\right]}{a^{5/2} f} - \frac{19 \text{Cot}[e + f x]}{8 a^2 f \sqrt{a + a \text{Sin}[e + f x]}} + \frac{13 \text{Cot}[e + f x] \text{Csc}[e + f x]}{12 a^2 f \sqrt{a + a \text{Sin}[e + f x]}} - \frac{\text{Cot}[e + f x] \text{Csc}[e + f x]^2}{3 a^2 f \sqrt{a + a \text{Sin}[e + f x]}}$$

Result (type 3, 332 leaves):

$$\frac{1}{192 f (a (1 + \text{Sin}[e + f x]))^{5/2}} \left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right] \right)^5 - \frac{1}{\left(\text{Csc}\left[\frac{1}{4} (e + f x)\right]^2 - \text{Sec}\left[\frac{1}{4} (e + f x)\right]^2\right)^3} 8 \text{Csc}\left[\frac{1}{2} (e + f x)\right]^9 - \left((1536 + 1536 i) (-1)^{3/4} \text{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \text{Tan}\left[\frac{1}{4} (e + f x)\right]\right)\right] - \left(396 \text{Cos}\left[\frac{1}{2} (e + f x)\right] - 218 \text{Cos}\left[\frac{3}{2} (e + f x)\right] - 114 \text{Cos}\left[\frac{5}{2} (e + f x)\right] - 396 \text{Sin}\left[\frac{1}{2} (e + f x)\right] - 405 \text{Log}\left[1 + \text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \right) \text{Sin}[e + f x] + 405 \text{Log}\left[1 - \text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \text{Sin}[e + f x] - 218 \text{Sin}\left[\frac{3}{2} (e + f x)\right] + 114 \text{Sin}\left[\frac{5}{2} (e + f x)\right] + 135 \text{Log}\left[1 + \text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \text{Sin}[3 (e + f x)] - 135 \text{Log}\left[1 - \text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \text{Sin}[3 (e + f x)] \right)$$

■ **Problem 115: Result unnecessarily involves higher level functions.**

$$\int (a + a \text{Sin}[e + f x])^{1/3} \text{Tan}[e + f x]^4 dx$$

Optimal (type 4, 982 leaves, 10 steps):

$$\begin{aligned}
& - \frac{361 \operatorname{Sec}[e + f x] (a + a \operatorname{Sin}[e + f x])^{1/3}}{126 f} + \frac{361 \operatorname{Sec}[e + f x] (1 - \operatorname{Sin}[e + f x]) (a + a \operatorname{Sin}[e + f x])^{1/3}}{63 f} - \\
& \frac{\operatorname{Sec}[e + f x] (65 a^2 - 142 a^2 \operatorname{Sin}[e + f x])}{42 f (a - a \operatorname{Sin}[e + f x]) (a + a \operatorname{Sin}[e + f x])^{2/3}} + \frac{361 (1 + \sqrt{3}) \operatorname{Sec}[e + f x] (1 - \operatorname{Sin}[e + f x]) (a + a \operatorname{Sin}[e + f x])^{2/3}}{63 f (2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \operatorname{Sin}[e + f x])^{1/3})} - \\
& \left(361 \times 2^{1/3} \operatorname{EllipticE} \left[\operatorname{ArcCos} \left[\frac{2^{1/3} a^{1/3} - (1 - \sqrt{3}) (a + a \operatorname{Sin}[e + f x])^{1/3}}{2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \operatorname{Sin}[e + f x])^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \operatorname{Sec}[e + f x] (a + a \operatorname{Sin}[e + f x])^{2/3} \right. \\
& \left. (2^{1/3} a^{1/3} - (a + a \operatorname{Sin}[e + f x])^{1/3}) \sqrt{\frac{2^{2/3} a^{2/3} + 2^{1/3} a^{1/3} (a + a \operatorname{Sin}[e + f x])^{1/3} + (a + a \operatorname{Sin}[e + f x])^{2/3}}{(2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \operatorname{Sin}[e + f x])^{1/3})^2}} \right] / \\
& \left(21 \times 3^{3/4} a^{2/3} f \sqrt{-\frac{(a + a \operatorname{Sin}[e + f x])^{1/3} (2^{1/3} a^{1/3} - (a + a \operatorname{Sin}[e + f x])^{1/3})}{(2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \operatorname{Sin}[e + f x])^{1/3})^2}} \right) - \\
& \left(361 (1 - \sqrt{3}) \operatorname{EllipticF} \left[\operatorname{ArcCos} \left[\frac{2^{1/3} a^{1/3} - (1 - \sqrt{3}) (a + a \operatorname{Sin}[e + f x])^{1/3}}{2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \operatorname{Sin}[e + f x])^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \operatorname{Sec}[e + f x] (a + a \operatorname{Sin}[e + f x])^{2/3} \right. \\
& \left. (2^{1/3} a^{1/3} - (a + a \operatorname{Sin}[e + f x])^{1/3}) \sqrt{\frac{2^{2/3} a^{2/3} + 2^{1/3} a^{1/3} (a + a \operatorname{Sin}[e + f x])^{1/3} + (a + a \operatorname{Sin}[e + f x])^{2/3}}{(2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \operatorname{Sin}[e + f x])^{1/3})^2}} \right] / \\
& \left(63 \times 2^{2/3} 3^{1/4} a^{2/3} f \sqrt{-\frac{(a + a \operatorname{Sin}[e + f x])^{1/3} (2^{1/3} a^{1/3} - (a + a \operatorname{Sin}[e + f x])^{1/3})}{(2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \operatorname{Sin}[e + f x])^{1/3})^2}} \right) + \\
& \frac{3 a^2 \operatorname{Sin}[e + f x] \operatorname{Tan}[e + f x]}{2 f (a - a \operatorname{Sin}[e + f x]) (a + a \operatorname{Sin}[e + f x])^{2/3}} - \frac{3 a^2 \operatorname{Sin}[e + f x]^2 \operatorname{Tan}[e + f x]}{f (a - a \operatorname{Sin}[e + f x]) (a + a \operatorname{Sin}[e + f x])^{2/3}}
\end{aligned}$$

Result (type 5, 593 leaves):

$$\frac{1}{f} (a (1 + \sin[e + f x]))^{1/3} \left(\frac{361}{63} - \frac{86}{63} \sec[e + f x] (-1 + 2 \sin[e + f x]) + \frac{1}{21} \sec[e + f x]^3 (-1 + 8 \sin[e + f x]) \right) +$$

$$\frac{1}{189 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)} 722 \sqrt{2} (1 + \sin[e + f x])^{1/6} (a (1 + \sin[e + f x]))^{1/3}$$

$$\left(- \left(i \cos\left[\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right] \right)^{1/3} \left(- \frac{3 i \left(e^{-i\left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)} + e^{i\left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)} \right)^{2/3} \text{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2 i\left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)}\right]}{2^{2/3} \left(1 + e^{2 i\left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)} \right)^{2/3}} - \right.$$

$$\left. \frac{3 i e^{i\left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)} \left(1 + e^{2 i\left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -e^{2 i\left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)}\right]}{2 \times 2^{2/3} \left(e^{-i\left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)} + e^{i\left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)} \right)^{1/3}} \right) \Bigg/$$

$$\left(2 \left(1 + \cos\left[2 \left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)\right] \right)^{1/6} \right) + \left(3 \cos\left[\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right] \right)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos\left[\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right]^2\right]$$

$$\sin\left[\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right] \Bigg/ \left(5 \left(1 + \cos\left[2 \left(\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right)\right] \right)^{1/6} \sqrt{\sin\left[\frac{\pi}{4} + \frac{1}{2}(-e - f x)\right]^2} \right)$$

- **Problem 116: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^{1/3} \tan[e + f x]^2 dx$$

Optimal (type 5, 123 leaves, 4 steps):

$$\frac{5 a \cos[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{7}{6}, \frac{3}{2}, \frac{1}{2}(1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{1/6}}{3 \times 2^{1/6} f (a + a \sin[e + f x])^{2/3}} +$$

$$\frac{7 \sec[e + f x] (a + a \sin[e + f x])^{1/3}}{f} - \frac{3 \sec[e + f x] (a + a \sin[e + f x])^{4/3}}{a f}$$

Result (type 5, 566 leaves):

$$\frac{(a(1 + \sin[ex + fx]))^{1/3} (-5 + \sec[ex + fx] (-1 + 2 \sin[ex + fx]))}{f} -$$

$$\frac{1}{3f \left(\cos\left[\frac{1}{2}(ex + fx)\right] + \sin\left[\frac{1}{2}(ex + fx)\right] \right)} 10\sqrt{2} (1 + \sin[ex + fx])^{1/6} (a(1 + \sin[ex + fx]))^{1/3}$$

$$\left(- \left(i \cos\left[\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right] \right)^{1/3} \left(- \frac{3i \left(e^{-i\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)} + e^{i\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)} \right)^{2/3} \text{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)}\right]}{2^{2/3} \left(1 + e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)} \right)^{2/3}} - \right. \right.$$

$$\left. \frac{3i e^{i\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)} \left(1 + e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -e^{2i\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)}\right]}{2 \times 2^{2/3} \left(e^{-i\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)} + e^{i\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)} \right)^{1/3}} \right) \Bigg/$$

$$\left(2 \left(1 + \cos\left[2\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)\right] \right)^{1/6} \right) + \left(3 \cos\left[\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right] \right)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos\left[\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right]^2\right]$$

$$\sin\left[\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right] \Bigg/ \left(5 \left(1 + \cos\left[2\left(\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right)\right] \right)^{1/6} \sqrt{\sin\left[\frac{\pi}{4} + \frac{1}{2}(ex - fx)\right]^2} \right)$$

■ **Problem 117: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[ex + fx]^2 (a + a \sin[ex + fx])^{1/3} dx$$

Optimal (type 6, 80 leaves, 3 steps):

$$\frac{1}{11a^2f} 6\sqrt{2} \text{AppellF1}\left[\frac{11}{6}, -\frac{1}{2}, 2, \frac{17}{6}, \frac{1}{2} (1 + \sin[ex + fx]), 1 + \sin[ex + fx]\right] \sec[ex + fx] \sqrt{1 - \sin[ex + fx]} (a + a \sin[ex + fx])^{7/3}$$

Result (type 6, 10034 leaves):

$$\frac{(-4 - \cot[ex + fx]) (a(1 + \sin[ex + fx]))^{1/3}}{f} +$$

$$\left((60 + 60i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(ex + fx)\right]\right)\right], \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(ex + fx)\right]\right) \right] \cos\left[\frac{1}{2}(ex + fx)\right]^2$$

$$\sin\left[\frac{1}{2}(ex + fx)\right] (a(1 + \sin[ex + fx]))^{1/3} \left(1 + \tan\left[\frac{1}{2}(ex + fx)\right]\right)$$

$$\left((5 + 5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(ex + fx)\right]\right)\right], \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(ex + fx)\right]\right) \right] \tan\left[\frac{1}{2}(ex + fx)\right] +$$

$$\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(ex + fx)\right]\right)\right], \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(ex + fx)\right]\right) \right] \left(1 + \tan\left[\frac{1}{2}(ex + fx)\right]\right) +$$

$$\begin{aligned}
& i \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right) \Bigg) / \\
& \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \left(-400 i \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] \right)^2 \right. \\
& \quad \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]^3 + 8 \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \right. \right. \\
& \quad \left. \left. \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] + i \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] \right)^2 \\
& \quad \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right)^2 + 5 \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] \Bigg) \\
& \quad \left(-5 \left(2 \operatorname{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, \frac{7}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] + \right. \right. \\
& \quad \left. \left. i \operatorname{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] \right) - \right. \\
& \quad \left. 2 \operatorname{AppellF1}\left[\frac{8}{3}, \frac{7}{3}, \frac{1}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] \right) \\
& \quad \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right)^2 + (2 + 2 i) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \right. \\
& \quad \left. \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] \left(-2 + \operatorname{Cos}[e + f x] + \operatorname{Cos}[2(e + f x)] - 3 \operatorname{Sin}[e + f x] \right) - (2 - 2 i) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \right. \\
& \quad \left. \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right)\right] \left(-2 + \operatorname{Cos}[e + f x] + \operatorname{Cos}[2(e + f x)] - 3 \operatorname{Sin}[e + f x] \right) \Bigg) \Bigg) + \\
& \left(\left(\frac{5}{2} + \frac{5 i}{2} \right) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right)\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right] \\
& (a (1 + \operatorname{Sin}[e + f x]))^{1/3} \\
& \left(\frac{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right)^{2/3}}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2}} \right) \Bigg) /
\end{aligned}$$

$$\left. \left. \left. \left. \left. (5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right] \right] \right] \right] \right] /$$

$$\left(3 f \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(- \frac{1}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)^2} \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \left(\frac{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}} \right)^{2/3} \right) \right)$$

$$\left(3 + 3 \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 - 3 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 + \left(\frac{3 + 3i}{4} \right) \left(2^{2/3} \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \right) \right. \right.$$

$$\left. \left. + i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \right) \right)$$

$$\operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right] \left(i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(\frac{(1+i) (-i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right)^{1/3}$$

$$\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) + (5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right]$$

$$\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \left(2^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right] \right.$$

$$\left. \left. \left(i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(\frac{(1+i) (-i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right)^{1/3} - (1-i) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) \right) \right) /$$

$$\left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \right) + \right.$$

$$i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] + \frac{1}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}$$

$$\left. \left. \left. \left. (5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right] \right] \right] \right] +$$

$$\begin{aligned}
& \frac{1}{3 \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]\right) \left(\frac{1 + \tan\left[\frac{1}{2}(e + f x)\right]}{\sqrt{\sec\left[\frac{1}{2}(e + f x)\right]^2}}\right)^{1/3}} 4 \left(\frac{1}{2} \sqrt{\sec\left[\frac{1}{2}(e + f x)\right]^2} - \frac{\tan\left[\frac{1}{2}(e + f x)\right] \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]\right)}{2 \sqrt{\sec\left[\frac{1}{2}(e + f x)\right]^2}} \right) \\
& \left(3 + 3 \sec\left[\frac{1}{2}(e + f x)\right]^2 - 3 \tan\left[\frac{1}{2}(e + f x)\right]^2 + \left(\frac{3}{4} + \frac{3i}{4}\right) \left(2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right], \left(\frac{1}{2} - \frac{i}{2}\right) \right. \right. \right. \\
& \left. \left. \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right) + i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \right) \\
& \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e + f x)\right]}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]}\right] \left(i + \tan\left[\frac{1}{2}(e + f x)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e + f x)\right])}{1 + \tan\left[\frac{1}{2}(e + f x)\right]}\right)^{1/3} \\
& \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]\right) + (5 + 5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \\
& \tan\left[\frac{1}{2}(e + f x)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e + f x)\right]}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]}\right] \right. \\
& \left. \left(i + \tan\left[\frac{1}{2}(e + f x)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e + f x)\right])}{1 + \tan\left[\frac{1}{2}(e + f x)\right]}\right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]\right) \right) \right) / \\
& \left(\left(1 + \tan\left[\frac{1}{2}(e + f x)\right]\right) \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] + i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \right. \right. \\
& \left. \left. + \frac{1}{1 + \tan\left[\frac{1}{2}(e + f x)\right]} (5 + 5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \right) \right) \\
& \left. \left. + \frac{1}{1 + \tan\left[\frac{1}{2}(e + f x)\right]} \right) \right) \\
& 2 \left(\frac{1 + \tan\left[\frac{1}{2}(e + f x)\right]}{\sqrt{\sec\left[\frac{1}{2}(e + f x)\right]^2}} \right)^{2/3} \left(- \left(\left(\left(\frac{3}{8} + \frac{3i}{8}\right) \sec\left[\frac{1}{2}(e + f x)\right]^2 \right) 2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right], \left(\frac{1}{2} - \frac{i}{2}\right) \right. \right. \right. \right. \\
& \left. \left. \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right) + i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]}\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{1 + \tan\left[\frac{1}{2}(e+fx)\right]}\right)^{1/3} \\
& \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right) + (5+5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \\
& \tan\left[\frac{1}{2}(e+fx)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]}\right] \right. \\
& \left. \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{1 + \tan\left[\frac{1}{2}(e+fx)\right]}\right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) \Bigg) \Bigg) / \\
& \left(\left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^2 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] + \right. \right. \\
& \left. \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] + \frac{1}{1 + \tan\left[\frac{1}{2}(e+fx)\right]}\right) \right. \\
& \left. (5+5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \tan\left[\frac{1}{2}(e+fx)\right]\right) \Bigg) \Bigg) - \\
& \left(\left(\frac{3}{4} + \frac{3i}{4}\right) \left(\left(-\frac{5}{24} + \frac{5i}{24}\right) \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, \frac{7}{3}, \frac{11}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 - \right. \right. \\
& \left. \left(\frac{5}{96} + \frac{5i}{96}\right) \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 + i \right. \\
& \left.\left(-\frac{5}{96} + \frac{5i}{96}\right) \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 - \right. \\
& \left.\left(\frac{5}{24} + \frac{5i}{24}\right) \text{AppellF1}\left[\frac{8}{3}, \frac{7}{3}, \frac{1}{3}, \frac{11}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2\right) - \\
& \left.\left(\frac{5}{2} + \frac{5i}{2}\right) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \right. \\
& \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) / \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^2 + \frac{1}{1 + \tan\left[\frac{1}{2}(e+fx)\right]} \left(\frac{5}{2} + \frac{5i}{2}\right) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \right. \\
& \left.\left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{1 + \tan\left[\frac{1}{2}(e+fx)\right]} (5+5i) \right. \\
& \left.\left(-\frac{1}{30} + \frac{i}{30}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1-i}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2^{1/3}} \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right) \right] + \right. \\
& \quad \left. i \text{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right) \right] \right) \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \right. \\
& \quad \left. \frac{(1+i) + (1-i) \text{Tan} \left[\frac{1}{2} (e+fx) \right]}{2 + 2 \text{Tan} \left[\frac{1}{2} (e+fx) \right]} \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \left(\frac{(1+i) (-i + \text{Tan} \left[\frac{1}{2} (e+fx) \right])}{1 + \text{Tan} \left[\frac{1}{2} (e+fx) \right]} \right)^{1/3} \left(1 + \text{Tan} \left[\frac{1}{2} (e+fx) \right] \right) + 2^{2/3} \\
& \quad \left(\left(-\frac{5}{24} + \frac{5i}{24} \right) \text{AppellF1} \left[\frac{8}{3}, \frac{1}{3}, \frac{7}{3}, \frac{11}{3}, \left(\frac{1+i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right) \right] \text{Csc} \left[\frac{1}{2} (e+fx) \right]^2 - \right. \\
& \quad \left(\frac{5}{96} + \frac{5i}{96} \right) \text{AppellF1} \left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1+i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right) \right] \text{Csc} \left[\frac{1}{2} (e+fx) \right]^2 + i \\
& \quad \left(\left(-\frac{5}{96} + \frac{5i}{96} \right) \text{AppellF1} \left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1+i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right) \right] \text{Csc} \left[\frac{1}{2} (e+fx) \right]^2 - \right. \\
& \quad \left. \left(\frac{5}{24} + \frac{5i}{24} \right) \text{AppellF1} \left[\frac{8}{3}, \frac{7}{3}, \frac{1}{3}, \frac{11}{3}, \left(\frac{1+i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right) \right] \right) \\
& \quad \left. \text{Csc} \left[\frac{1}{2} (e+fx) \right]^2 \right) \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \text{Tan} \left[\frac{1}{2} (e+fx) \right]}{2 + 2 \text{Tan} \left[\frac{1}{2} (e+fx) \right]} \right] \\
& \quad \left(i + \text{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \left(\frac{(1+i) (-i + \text{Tan} \left[\frac{1}{2} (e+fx) \right])}{1 + \text{Tan} \left[\frac{1}{2} (e+fx) \right]} \right)^{1/3} \left(1 + \text{Tan} \left[\frac{1}{2} (e+fx) \right] \right) + \frac{1}{3 \left(\frac{(1+i) (-i + \text{Tan} \left[\frac{1}{2} (e+fx) \right])}{1 + \text{Tan} \left[\frac{1}{2} (e+fx) \right]} \right)^{2/3}} \\
& \quad 2^{2/3} \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right) \right] + \right. \\
& \quad \left. i \text{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right) \right] \right) \\
& \quad \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \text{Tan} \left[\frac{1}{2} (e+fx) \right]}{2 + 2 \text{Tan} \left[\frac{1}{2} (e+fx) \right]} \right] \left(i + \text{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \left(1 + \text{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \\
& \quad \left(-\frac{\left(\frac{1}{2} + \frac{i}{2} \right) \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 (-i + \text{Tan} \left[\frac{1}{2} (e+fx) \right])}{\left(1 + \text{Tan} \left[\frac{1}{2} (e+fx) \right] \right)^2} + \frac{\left(\frac{1}{2} + \frac{i}{2} \right) \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2}{1 + \text{Tan} \left[\frac{1}{2} (e+fx) \right]} \right) + \left(\frac{5}{2} + \frac{5i}{2} \right) \\
& \quad \text{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \text{Cot} \left[\frac{1}{2} (e+fx) \right] \right) \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \\
& \quad \left(2^{2/3} \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \text{Tan} \left[\frac{1}{2} (e+fx) \right]}{2 + 2 \text{Tan} \left[\frac{1}{2} (e+fx) \right]} \right] \left(i + \text{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{(1+i)(-i + \tan[\frac{1}{2}(e+fx)])}{1 + \tan[\frac{1}{2}(e+fx)]} \right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right] \right) + (5+5i) \\
& \left(\left(-\frac{1}{30} + \frac{i}{30} \right) \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \text{Csc}\left[\frac{1}{2}(e+fx)\right]^2 - \right. \\
& \left. \left(\frac{1}{30} + \frac{i}{30}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \text{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \right) \\
& \tan\left[\frac{1}{2}(e+fx)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i)\tan[\frac{1}{2}(e+fx)]}{2 + 2\tan[\frac{1}{2}(e+fx)]}\right] \left(i + \tan\left[\frac{1}{2}(e+fx)\right]\right) \right. \\
& \left. \left(\frac{(1+i)(-i + \tan[\frac{1}{2}(e+fx)])}{1 + \tan[\frac{1}{2}(e+fx)]} \right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + \frac{1}{3((1+i) + (1-i)\tan[\frac{1}{2}(e+fx)])} \\
& 2 \times 2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] + \right. \\
& \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \right) \\
& \left(i + \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(\frac{(1+i)(-i + \tan[\frac{1}{2}(e+fx)])}{1 + \tan[\frac{1}{2}(e+fx)]} \right)^{1/3} \left(1 + \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(2 + 2\tan\left[\frac{1}{2}(e+fx)\right] \right) \\
& \left(-\frac{\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2((1+i) + (1-i)\tan[\frac{1}{2}(e+fx)])}{(2 + 2\tan[\frac{1}{2}(e+fx)])^2} + \frac{(\frac{1}{2} - \frac{i}{2})\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2 + 2\tan[\frac{1}{2}(e+fx)]} \right) \\
& \left(-\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i)\tan[\frac{1}{2}(e+fx)]}{2 + 2\tan[\frac{1}{2}(e+fx)]}\right] + \frac{1}{\left(1 - \frac{(1+i) + (1-i)\tan[\frac{1}{2}(e+fx)]}{2 + 2\tan[\frac{1}{2}(e+fx)]}\right)^{1/3}} \right) + (5+5i) \\
& \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \tan\left[\frac{1}{2}(e+fx)\right] \\
& \left(\left(-\frac{1}{2} + \frac{i}{2} \right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{2^{1/3}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i)\tan[\frac{1}{2}(e+fx)]}{2 + 2\tan[\frac{1}{2}(e+fx)]}\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)
\end{aligned}$$

$$\int \text{Cot}[e + f x]^4 (a + a \text{Sin}[e + f x])^{1/3} dx$$

Optimal (type 6, 80 leaves, 3 steps):

$$\frac{1}{17 a^3 f} 12 \sqrt{2} \text{AppellF1}\left[\frac{17}{6}, -\frac{3}{2}, 4, \frac{23}{6}, \frac{1}{2} (1 + \text{Sin}[e + f x]), 1 + \text{Sin}[e + f x]\right] \text{Sec}[e + f x] \sqrt{1 - \text{Sin}[e + f x]} (a + a \text{Sin}[e + f x])^{10/3}$$

Result (type 6, 9225 leaves):

$$\frac{\left(\frac{239}{54} + \frac{77}{54} \text{Cot}[e + f x] - \frac{1}{18} \text{Cot}[e + f x] \text{Csc}[e + f x] - \frac{1}{3} \text{Cot}[e + f x] \text{Csc}[e + f x]^2\right) (a (1 + \text{Sin}[e + f x]))^{1/3}}{f}$$

$$\left(\left(\frac{560}{9} + \frac{560 i}{9}\right) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right]\right.$$

$$\left.\text{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \text{Sin}\left[\frac{1}{2} (e + f x)\right] (a (1 + \text{Sin}[e + f x]))^{1/3} \left(1 + \text{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right.$$

$$\left.\left((5 + 5 i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right] \text{Tan}\left[\frac{1}{2} (e + f x)\right] +\right.$$

$$\left.\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right] \left(1 + \text{Tan}\left[\frac{1}{2} (e + f x)\right]\right) +\right.$$

$$\left.\left.i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right] \left(1 + \text{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right)\right) /$$

$$\left(f \left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right) \left(-400 i \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right]\right)^2$$

$$\left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right) \text{Sin}\left[\frac{1}{2} (e + f x)\right]^3 + 8 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right]\right)^2$$

$$\left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right) + i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right]\right)^2$$

$$\left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right)^2 + 5 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right]$$

$$\left(-5 \left(2 \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, \frac{7}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right] +\right.$$

$$\left.i \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right] -\right.$$

$$\left.2 \text{AppellF1}\left[\frac{8}{3}, \frac{7}{3}, \frac{1}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right]\right)$$

$$\left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right)^2 + (2 + 2 i) \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2} (e + f x)\right]\right)\right],$$

$$\begin{aligned}
& 50 i \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right]^2 \\
& \left(3 + 4 \operatorname{Cos}[e + f x] + \operatorname{Cos}[2(e + f x)] - 2 \operatorname{Sin}[e + f x] - \operatorname{Sin}[2(e + f x)] \right) \Bigg) - \\
& \left(\left(\frac{239}{216} + \frac{239 i}{216} \right) \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] \operatorname{Csc}[e + f x] (a (1 + \operatorname{Sin}[e + f x]))^{1/3} \left(\frac{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}} \right)^{2/3} \right. \\
& \left. \left((2 - 2 i) \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right] \right)^2 + (2 - 2 i) \operatorname{Cos}[e + f x] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right] \right)^2 + \\
& \left(2^{2/3} \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \right) + \right. \\
& \left. i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \right) \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \right. \\
& \left. \frac{(1 + i) + (1 - i) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right] \left(i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(\frac{(1 + i) (-i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right)^{1/3} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) + \\
& (5 + 5 i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \Bigg) \\
& \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \left(2^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1 + i) + (1 - i) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right] \right. \\
& \left. \left(i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(\frac{(1 + i) (-i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right)^{1/3} - (1 - i) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) \Bigg) / \\
& \left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \right) + \right. \\
& \left. i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \right) + \frac{1}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left((5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) \Bigg) / \\
& \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(- \frac{1}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)^2} \left(\frac{3}{8} + \frac{3i}{8} \right) \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right. \\
& \left. \left(\frac{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}} \right)^{2/3} \left((2 - 2i) \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 + (2 - 2i) \operatorname{Cos} [e + f x] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 + 2^{2/3} \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \right. \right. \right. \right. \\
& \left. \left. \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right) + i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \right. \right. \\
& \left. \left. \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right) \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right] \left(i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) \\
& \left(\frac{(1+i) (-i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right)^{1/3} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) + (5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \right. \\
& \left. \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \left(2^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right] \right) \\
& \left. \left(i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(\frac{(1+i) (-i + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right)^{1/3} - (1-i) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) \Bigg) / \\
& \left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] + \right. \right. \\
& \left. \left. i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \right] \right) + \frac{1}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. (5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right], \left(\frac{1-i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right] \tan \left[\frac{1}{2} (e + f x) \right] \right) \right) + \\
& \frac{1}{\left(1 + \tan \left[\frac{1}{2} (e + f x) \right] \right) \left(\frac{1 + \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\sec \left[\frac{1}{2} (e + f x) \right]^2}} \right)^{1/3} \left(\frac{1+i}{2} \right) \left(\frac{1}{2} \sqrt{\sec \left[\frac{1}{2} (e + f x) \right]^2} - \frac{\tan \left[\frac{1}{2} (e + f x) \right] \left(1 + \tan \left[\frac{1}{2} (e + f x) \right] \right)}{2 \sqrt{\sec \left[\frac{1}{2} (e + f x) \right]^2}} \right)} \\
& \left((2 - 2i) \sec \left[\frac{1}{2} (e + f x) \right]^2 + (2 - 2i) \cos [e + f x] \sec \left[\frac{1}{2} (e + f x) \right]^2 + 2^{2/3} \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right], \right. \right. \\
& \quad \left. \left. \left(\frac{1-i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right] + i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right], \left(\frac{1-i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right] \right) \\
& \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan \left[\frac{1}{2} (e + f x) \right]}{2 + 2 \tan \left[\frac{1}{2} (e + f x) \right]} \right] \left(i + \tan \left[\frac{1}{2} (e + f x) \right] \right) \left(\frac{(1+i) (-i + \tan \left[\frac{1}{2} (e + f x) \right])}{1 + \tan \left[\frac{1}{2} (e + f x) \right]} \right)^{1/3} \\
& \left(1 + \tan \left[\frac{1}{2} (e + f x) \right] \right) + (5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right], \left(\frac{1-i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right] \\
& \tan \left[\frac{1}{2} (e + f x) \right] \left(2^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan \left[\frac{1}{2} (e + f x) \right]}{2 + 2 \tan \left[\frac{1}{2} (e + f x) \right]} \right] \right. \\
& \quad \left. \left. \left. \left(i + \tan \left[\frac{1}{2} (e + f x) \right] \right) \left(\frac{(1+i) (-i + \tan \left[\frac{1}{2} (e + f x) \right])}{1 + \tan \left[\frac{1}{2} (e + f x) \right]} \right)^{1/3} - (1-i) \left(1 + \tan \left[\frac{1}{2} (e + f x) \right] \right) \right) \right) \right) / \\
& \left(\left(1 + \tan \left[\frac{1}{2} (e + f x) \right] \right) \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right], \left(\frac{1-i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right] + i \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right], \left(\frac{1-i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right] + \frac{1}{1 + \tan \left[\frac{1}{2} (e + f x) \right]} (5 + 5i) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right], \left(\frac{1-i}{2} \right) \left(1 + \cot \left[\frac{1}{2} (e + f x) \right] \right) \right] \tan \left[\frac{1}{2} (e + f x) \right] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{1 + \tan\left[\frac{1}{2}(e + f x)\right]} \left(\frac{3}{4} + \frac{3i}{4}\right) \left(\frac{1 + \tan\left[\frac{1}{2}(e + f x)\right]}{\sqrt{\sec\left[\frac{1}{2}(e + f x)\right]^2}}\right)^{2/3} \left[(-2 + 2i) \sec\left[\frac{1}{2}(e + f x)\right]^2 \sin[e + f x] + \right. \\
& (2 - 2i) \sec\left[\frac{1}{2}(e + f x)\right]^2 \tan\left[\frac{1}{2}(e + f x)\right] + (2 - 2i) \cos[e + f x] \sec\left[\frac{1}{2}(e + f x)\right]^2 \tan\left[\frac{1}{2}(e + f x)\right] - \\
& \left. \left(\sec\left[\frac{1}{2}(e + f x)\right]\right)^2 \left(2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] + \right. \right. \\
& \quad \left. \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right]\right) \right. \\
& \quad \left. \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e + f x)\right]}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]}\right] \left(i + \tan\left[\frac{1}{2}(e + f x)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e + f x)\right])}{1 + \tan\left[\frac{1}{2}(e + f x)\right]}\right)^{1/3} \right. \\
& \quad \left. \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]\right) + (5 + 5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \right) \\
& \quad \left. \tan\left[\frac{1}{2}(e + f x)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e + f x)\right]}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]}\right] \right. \right. \\
& \quad \left. \left. \left(i + \tan\left[\frac{1}{2}(e + f x)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e + f x)\right])}{1 + \tan\left[\frac{1}{2}(e + f x)\right]}\right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]\right)\right)\right) \right] / \\
& \left(2 \left(1 + \tan\left[\frac{1}{2}(e + f x)\right]\right)\right)^2 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] + \right. \\
& \quad \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] + \frac{1}{1 + \tan\left[\frac{1}{2}(e + f x)\right]} (5 + 5i) \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \tan\left[\frac{1}{2}(e + f x)\right]\right) \right) - \\
& \left(\left(-\frac{5}{24} + \frac{5i}{24}\right) \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, \frac{7}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \csc\left[\frac{1}{2}(e + f x)\right]^2 - \right. \\
& \quad \left. \left(\frac{5}{96} + \frac{5i}{96}\right) \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \csc\left[\frac{1}{2}(e + f x)\right]^2 + i \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]}\right] \left(i + \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right) \\
& \left(-\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sec\left[\frac{1}{2}(e+fx)\right]^2 (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{\left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^2} + \frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sec\left[\frac{1}{2}(e+fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e+fx)\right]}\right) + \\
& \left(\frac{5}{2} + \frac{5i}{2}\right) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \\
& \sec\left[\frac{1}{2}(e+fx)\right]^2 \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]}\right] \right. \\
& \left. \left(i + \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{1 + \tan\left[\frac{1}{2}(e+fx)\right]}\right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) + \\
& (5+5i) \left(\left(-\frac{1}{30} + \frac{i}{30}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2 - \right. \\
& \left.\left(\frac{1}{30} + \frac{i}{30}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] \csc\left[\frac{1}{2}(e+fx)\right]^2\right) \\
& \tan\left[\frac{1}{2}(e+fx)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]}\right] \left(i + \tan\left[\frac{1}{2}(e+fx)\right]\right) \right. \\
& \left.\left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{1 + \tan\left[\frac{1}{2}(e+fx)\right]}\right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) + \frac{1}{3((1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right])} \\
& 2 \times 2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right] + \right. \\
& \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right]\right)\right]\right) \\
& \left(i + \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{1 + \tan\left[\frac{1}{2}(e+fx)\right]}\right)^{1/3} \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(2 + 2 \tan\left[\frac{1}{2}(e+fx)\right]\right) \\
& \left(-\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2 ((1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right])}{\left(2 + 2 \tan\left[\frac{1}{2}(e+fx)\right]\right)^2} + \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sec\left[\frac{1}{2}(e+fx)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(e+fx)\right]}\right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]} \right] + \frac{1}{\left(1 - \frac{(1+i)+(1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]} \right)^{1/3}} \right) + \\
(5+5i) & \text{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} + \frac{i}{2} \right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right] \right), \left(\frac{1-i}{2} - \frac{i}{2} \right) \left(1 + \cot\left[\frac{1}{2}(e+fx)\right] \right) \right] \tan\left[\frac{1}{2}(e+fx)\right] \\
& \left(\left(-\frac{1}{2} + \frac{i}{2} \right) \sec\left[\frac{1}{2}(e+fx)\right] \right)^2 + \frac{1}{2^{1/3}} \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]} \right] \sec\left[\frac{1}{2}(e+fx)\right] \right)^2 \\
& \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{1 + \tan\left[\frac{1}{2}(e+fx)\right]} \right)^{1/3} + \left(2^{2/3} \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]} \right] \right. \\
& \left. \left(i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(-\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sec\left[\frac{1}{2}(e+fx)\right]^2 (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{\left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^2} + \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sec\left[\frac{1}{2}(e+fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e+fx)\right]} \right) \right) / \\
& \left(3 \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{1 + \tan\left[\frac{1}{2}(e+fx)\right]} \right)^{2/3} \right) + \left(2 \times 2^{2/3} \left(i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(e+fx)\right])}{1 + \tan\left[\frac{1}{2}(e+fx)\right]} \right)^{1/3} \right. \\
& \left. \left(2 + 2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(-\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2 ((1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right])}{\left(2 + 2 \tan\left[\frac{1}{2}(e+fx)\right]\right)^2} + \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sec\left[\frac{1}{2}(e+fx)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(e+fx)\right]} \right) \right) \\
& \left(-\text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]} \right] + \right. \\
& \left. \frac{1}{\left(1 - \frac{(1+i)+(1-i) \tan\left[\frac{1}{2}(e+fx)\right]}{2+2 \tan\left[\frac{1}{2}(e+fx)\right]} \right)^{1/3}} \right) \right) / \left(3 \left((1+i) + (1-i) \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) /
\end{aligned}$$

$$\left(\left(\left(1 + \tan\left[\frac{1}{2}(e + f x)\right] \right) \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] + i \right. \right. \right. \\ \left. \left. \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] + \frac{1}{1 + \tan\left[\frac{1}{2}(e + f x)\right]} (5 + 5 i) \right. \right. \\ \left. \left. \left. \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(e + f x)\right]\right)\right] \tan\left[\frac{1}{2}(e + f x)\right] \right) \right) \right)$$

■ **Problem 119: Result unnecessarily involves higher level functions.**

$$\int \frac{\tan[e + f x]^4}{(a + a \sin[e + f x])^{1/3}} dx$$

Optimal (type 4, 551 leaves, 8 steps):

$$\frac{973 \operatorname{Sec}[e + f x]}{396 f (a + a \sin[e + f x])^{1/3}} - \frac{973 \operatorname{Sec}[e + f x] (1 - \sin[e + f x])}{495 f (a + a \sin[e + f x])^{1/3}} - \frac{\operatorname{Sec}[e + f x] (95 a + 356 a \sin[e + f x])}{132 f (1 - \sin[e + f x]) (a + a \sin[e + f x])^{4/3}} + \\ \left(973 \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} a^{1/3} - (1 - \sqrt{3}) (a + a \sin[e + f x])^{1/3}}{2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \sin[e + f x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \operatorname{Sec}[e + f x] (a + a \sin[e + f x])^{2/3} \right. \\ \left. (2^{1/3} a^{1/3} - (a + a \sin[e + f x])^{1/3}) \sqrt{\frac{2^{2/3} a^{2/3} + 2^{1/3} a^{1/3} (a + a \sin[e + f x])^{1/3} + (a + a \sin[e + f x])^{2/3}}{(2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \sin[e + f x])^{1/3})^2}} \right) / \\ \left(495 \times 2^{1/3} 3^{1/4} a^{4/3} f \sqrt{-\frac{(a + a \sin[e + f x])^{1/3} (2^{1/3} a^{1/3} - (a + a \sin[e + f x])^{1/3})}{(2^{1/3} a^{1/3} - (1 + \sqrt{3}) (a + a \sin[e + f x])^{1/3})^2}} \right) + \\ \frac{3 a^2 \sin[e + f x] \tan[e + f x]}{4 f (a - a \sin[e + f x]) (a + a \sin[e + f x])^{4/3}} + \frac{3 a^2 \sin[e + f x]^2 \tan[e + f x]}{f (a - a \sin[e + f x]) (a + a \sin[e + f x])^{4/3}}$$

Result (type 5, 128 leaves):

$$\left(973 \sqrt{2} \operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \operatorname{Sin}\left[\frac{1}{4} (2 e + \pi + 2 f x)\right]^2\right] + \right. \\ \left. \operatorname{Sec}[e + f x]^3 \sqrt{1 - \operatorname{Sin}[e + f x]} (-49 - 64 \operatorname{Cos}[2 (e + f x)] + 22 \operatorname{Sin}[e + f x] - 128 \operatorname{Sin}[3 (e + f x)]) \right] \Big) / \\ \left(495 f \sqrt{1 - \operatorname{Sin}[e + f x]} (a (1 + \operatorname{Sin}[e + f x]))^{1/3} \right)$$

■ **Problem 121: Unable to integrate problem.**

$$\int \frac{\operatorname{Cot}[e + f x]^2}{(a + a \operatorname{Sin}[e + f x])^{1/3}} dx$$

Optimal (type 6, 80 leaves, 3 steps):

$$\frac{1}{7 a^2 f} 6 \sqrt{2} \operatorname{AppellF1}\left[\frac{7}{6}, -\frac{1}{2}, 2, \frac{13}{6}, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x]\right] \operatorname{Sec}[e + f x] \sqrt{1 - \operatorname{Sin}[e + f x]} (a + a \operatorname{Sin}[e + f x])^{5/3}$$

Result (type 8, 25 leaves):

$$\int \frac{\operatorname{Cot}[e + f x]^2}{(a + a \operatorname{Sin}[e + f x])^{1/3}} dx$$

■ **Problem 122: Unable to integrate problem.**

$$\int \frac{\operatorname{Cot}[e + f x]^4}{(a + a \operatorname{Sin}[e + f x])^{1/3}} dx$$

Optimal (type 6, 80 leaves, 3 steps):

$$\frac{1}{13 a^3 f} 12 \sqrt{2} \operatorname{AppellF1}\left[\frac{13}{6}, -\frac{3}{2}, 4, \frac{19}{6}, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x]\right] \operatorname{Sec}[e + f x] \sqrt{1 - \operatorname{Sin}[e + f x]} (a + a \operatorname{Sin}[e + f x])^{8/3}$$

Result (type 8, 25 leaves):

$$\int \frac{\operatorname{Cot}[e + f x]^4}{(a + a \operatorname{Sin}[e + f x])^{1/3}} dx$$

■ **Problem 123: Attempted integration timed out after 120 seconds.**

$$\int (a + a \operatorname{Sin}[e + f x])^3 (g \operatorname{Tan}[e + f x])^p dx$$

Optimal (type 5, 269 leaves, 10 steps):

$$\frac{a^3 \operatorname{Hypergeometric2F1}\left[1, \frac{1+p}{2}, \frac{3+p}{2}, -\operatorname{Tan}[e+fx]^2\right] (g \operatorname{Tan}[e+fx])^{1+p}}{f g (1+p)} +$$

$$\frac{3 a^3 (\operatorname{Cos}[e+fx]^2)^{\frac{1+p}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+p}{2}, \frac{2+p}{2}, \frac{4+p}{2}, \operatorname{Sin}[e+fx]^2\right] \operatorname{Sin}[e+fx] (g \operatorname{Tan}[e+fx])^{1+p}}{f g (2+p)} +$$

$$\frac{a^3 (\operatorname{Cos}[e+fx]^2)^{\frac{1+p}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+p}{2}, \frac{4+p}{2}, \frac{6+p}{2}, \operatorname{Sin}[e+fx]^2\right] \operatorname{Sin}[e+fx]^3 (g \operatorname{Tan}[e+fx])^{1+p}}{f g (4+p)} +$$

$$\frac{3 a^3 \operatorname{Hypergeometric2F1}\left[2, \frac{3+p}{2}, \frac{5+p}{2}, -\operatorname{Tan}[e+fx]^2\right] (g \operatorname{Tan}[e+fx])^{3+p}}{f g^3 (3+p)}$$

Result(type 1, 1 leaves):

???

■ **Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sin}[e+fx])^2 (g \operatorname{Tan}[e+fx])^p dx$$

Optimal (type 5, 187 leaves, 8 steps):

$$\frac{a^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1+p}{2}, \frac{3+p}{2}, -\operatorname{Tan}[e+fx]^2\right] (g \operatorname{Tan}[e+fx])^{1+p}}{f g (1+p)} +$$

$$\frac{2 a^2 (\operatorname{Cos}[e+fx]^2)^{\frac{1+p}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+p}{2}, \frac{2+p}{2}, \frac{4+p}{2}, \operatorname{Sin}[e+fx]^2\right] \operatorname{Sin}[e+fx] (g \operatorname{Tan}[e+fx])^{1+p}}{f g (2+p)} +$$

$$\frac{a^2 \operatorname{Hypergeometric2F1}\left[2, \frac{3+p}{2}, \frac{5+p}{2}, -\operatorname{Tan}[e+fx]^2\right] (g \operatorname{Tan}[e+fx])^{3+p}}{f g^3 (3+p)}$$

Result(type 6, 9890 leaves):

$$\left(2^{1+p} (a + a \operatorname{Sin}[e+fx])^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\right)^p\right.$$

$$\left.\left(\left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2\right) / \right.$$

$$\left.\left((1+p) \left(\left(3+p\right) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)\right) +$$

$$\begin{aligned}
& \left(4 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) / \left((1+p) \right. \\
& \quad \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + 2 \left(-2 \operatorname{AppellF1} \left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) - \\
& \left(4 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) / \left((1+p) \right. \\
& \quad \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + 2 \left(-3 \operatorname{AppellF1} \left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) + \\
& \left(4 (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \tan \left[\frac{1}{2} (e+fx) \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) / \\
& \quad \left((2+p) \left((4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \left(-2 \operatorname{AppellF1} \left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. p \operatorname{AppellF1} \left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \right) \\
& \tan [e+fx]^{-p} (g \tan [e+fx])^p \left(\cos \left[\frac{1}{2} (e+fx) \right]^4 \tan [e+fx]^p + 4 \cos \left[\frac{1}{2} (e+fx) \right]^3 \sin \left[\frac{1}{2} (e+fx) \right] \tan [e+fx]^p + \right. \\
& \quad 6 \cos \left[\frac{1}{2} (e+fx) \right]^2 \sin \left[\frac{1}{2} (e+fx) \right]^2 \tan [e+fx]^p + \\
& \quad 4 \cos \left[\frac{1}{2} (e+fx) \right] \sin \left[\frac{1}{2} (e+fx) \right]^3 \tan [e+fx]^p + \\
& \quad \left. \left. \sin \left[\frac{1}{2} (e+fx) \right]^4 \tan [e+fx]^p \right) \right) / \\
& \left(f \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right) \right)^4 \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^3 \\
& \left(-\frac{1}{\left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^4} 3 \times 2^{1+p} \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right]^2 \left(-\frac{\tan \left[\frac{1}{2} (e+fx) \right]}{-1 + \tan \left[\frac{1}{2} (e+fx) \right]^2} \right)^p \right)
\end{aligned}$$

$$\begin{aligned}
& \left((1+p) \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad 2 \left(-2 \operatorname{AppellF1} \left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \quad \left. \left. p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) - \\
& \left(4 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) / \left((1+p) \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, \right. \right. \right. \\
& \quad \left. \left. 3, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + 2 \left(-3 \operatorname{AppellF1} \left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad \quad \left. \left. p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) + \\
& \left(4 (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \tan \left[\frac{1}{2} (e+fx) \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) / \\
& \left((2+p) \left((4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad 2 \left(-2 \operatorname{AppellF1} \left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \quad \left. \left. p \operatorname{AppellF1} \left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \right) + \frac{1}{\left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^3} \\
& 2^{1+p} p \tan \left[\frac{1}{2} (e+fx) \right] \left(-\frac{\tan \left[\frac{1}{2} (e+fx) \right]}{-1 + \tan \left[\frac{1}{2} (e+fx) \right]^2} \right)^{-1+p} \left(\frac{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right]^2}{\left(-1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^2} - \frac{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}{2 \left(-1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)} \right) \\
& \left(\left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 \right) \right) / \left((1+p) \right. \\
& \quad \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \quad \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) + \\
& \left(4 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) / \\
& \left((1+p) \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \left(-2 \operatorname{AppellF1} \left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \Bigg) - \\
& \left(4(3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) / \left((1+p) \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, \right. \right. \right. \\
& \left. \left. \left. 3, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left(-3 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left(4(4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) / \\
& \left((2+p) \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. 2 \left(-2 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \Bigg) + \\
& \frac{1}{\left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^3} 2^{1+p} \tan\left[\frac{1}{2}(e+fx)\right] \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \left(\left(2(3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) / \left((1+p) \right. \\
& \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Bigg) + \\
& \left((3+p) \left(-1 / (3+p)(1+p) \operatorname{AppellF1}\left[1 + \frac{1+p}{2}, p, 2, 1 + \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right] + 1 / (3+p)p(1+p) \operatorname{AppellF1}\left[1 + \frac{1+p}{2}, 1+p, 1, 1 + \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) / \left((1+p) \right. \\
& \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-2-p} \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^2 \tan[e+fx]^p (g \tan[e+fx])^p \Big/ \\
& \left(f(1+p)(a+a\sin[e+fx]) \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] + \right. \right. \\
& \quad \left. \left(-(2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] + \right. \right. \\
& \quad \left. \left. p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \tan\left[\frac{1}{2}(e+fx)\right] \right) \\
& \left(\left(2p(2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}[e+fx]^2 \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} \right. \right. \\
& \quad \left. \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-2-p} \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^2 \tan[e+fx]^{-1+p} \right) \Big/ \\
& \left((1+p) \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] + \right. \right. \\
& \quad \left. \left(-(2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] + \right. \right. \\
& \quad \left. \left. p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \tan\left[\frac{1}{2}(e+fx)\right] \right) \Big) + \\
& \left(2p(2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} \right. \\
& \quad \left. \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-2-p} \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-1+p} \tan[e+fx]^p \right) \Big/ \\
& \left((1+p) \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] + \right. \right. \\
& \quad \left. \left(-(2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] + \right. \right. \\
& \quad \left. \left. p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \tan\left[\frac{1}{2}(e+fx)\right] \right) \Big) + \\
& \left((-2-p)(2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-3-p} \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^2 \tan[e+fx]^p \right) \Big/ \\
& \left((1+p) \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 (2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-p} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right. \\
& \left. \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-2-p} \left(\frac{1}{2} \left(- (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right) + \right. \right. \\
& \left. \left. p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& (2+p) \left(-\frac{1}{2}(1+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \left. \frac{1}{2(2+p)} p(1+p) \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(- (2+p) \left(-\frac{1}{2}(2+p) \operatorname{AppellF1}\left[3+p, p, 4+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right. \\
& \left. \left. \frac{1}{2(3+p)} p(2+p) \operatorname{AppellF1}\left[3+p, 1+p, 3+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \right. \\
& \left. p \left(-\frac{1}{2(3+p)} (2+p)^2 \operatorname{AppellF1}\left[3+p, 1+p, 3+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right. \\
& \left. \left. \frac{1}{2(3+p)} (1+p)(2+p) \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{2}+\frac{p}{2}, 2+p\right\}, \left\{\frac{5}{2}+\frac{p}{2}\right\}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right) \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \\
& \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{2p} \operatorname{Tan}[e+fx]^p \Big/ \left((1+p) \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right) + \right. \\
& \left. \left(- (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right) + p \right. \\
& \left. \left. \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big) \Big) \Big) \Big)
\end{aligned}$$

■ **Problem 127: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \operatorname{Tan}[e+fx])^p}{(a+a \operatorname{Sin}[e+fx])^2} dx$$

Optimal (type 5, 138 leaves, 10 steps):

$$\frac{(g \operatorname{Tan}[e+fx])^{1+p}}{a^2 f g (1+p)} - \frac{2 (\operatorname{Cos}[e+fx]^2)^{\frac{5+p}{2}} \operatorname{Hypergeometric2F1}\left[\frac{2+p}{2}, \frac{5+p}{2}, \frac{4+p}{2}, \operatorname{Sin}[e+fx]^2\right] \operatorname{Sec}[e+fx]^3 (g \operatorname{Tan}[e+fx])^{2+p}}{a^2 f g^2 (2+p)} + \frac{2 (g \operatorname{Tan}[e+fx])^{3+p}}{a^2 f g^3 (3+p)}$$

Result (type 6, 7283 leaves):

$$\left(2^{1+p} (2+p) \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-p} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-4-p} \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\right)^p \right)$$

$$\begin{aligned}
& \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)^2 \Bigg)^p \left(\left(\operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^2 \right) / \right. \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], \right. \right. \\
& \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - \\
& \left(2 \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) \right) \Bigg) / \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - (3+p) \operatorname{AppellF1}\left[2+p, p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], \right. \right. \\
& \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + \\
& \left(2 \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right) / \left((2+p) \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - (4+p) \operatorname{AppellF1}\left[2+p, p, 5+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. p \operatorname{AppellF1}\left[2+p, 1+p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Bigg) (g \operatorname{Tan}[e+fx])^p \Bigg) / \\
& \left(f(1+p)(a+a \operatorname{Sin}[e+fx])^2 \left(\frac{1}{1+p} 2^{1+p} p(2+p) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)^{-p} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
& \left. \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right)^{-4+p} \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)^{-1+p} \\
& \left(\left(\operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^2 \right) / \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, \right. \right. \\
& \left. \left. 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right. \\
& \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - \\
& \left(2 \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) \right) \Bigg) / \left((2+p) \operatorname{AppellF1}\left[1+p, p, \right. \right. \\
& \left. \left. 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - (3+p) \operatorname{AppellF1}\left[2+p, p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right)
\end{aligned}$$

$$\begin{aligned}
& p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \\
& \left(2 \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)\right) / \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& (3+p) \operatorname{AppellF1}\left[2+p, p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& p \operatorname{AppellF1}\left[2+p, 1+p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left. \right) + \\
& \left(2 \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]\right) / \left((2+p) \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], \right. \right. \\
& \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - (4+p) \operatorname{AppellF1}\left[2+p, p, 5+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. p \operatorname{AppellF1}\left[2+p, 1+p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) \left. \right) + \\
& \frac{1}{1+p} 2^{1+p} (2+p) \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-p} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-4-p} \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\right)^p \\
& \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{2p} \\
& \left(\left(\operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)\right)\right) / \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left. \right) + \\
& \left(\left(-\frac{1}{2}(1+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right. \\
& \left. \left. 1/(2(2+p))^p (1+p) \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right) \right. \\
& \left.\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^2\right) / \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\text{AppellF1} \left[1+p, p, 2+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^2 \right) / \right. \\
& \quad \left((2+p) \text{AppellF1} \left[1+p, p, 2+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad \quad (2+p) \text{AppellF1} \left[2+p, p, 3+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \quad \left. p \text{AppellF1} \left[2+p, 1+p, 2+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) - \\
& \quad \left(2 \text{AppellF1} \left[1+p, p, 3+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right] \right) \right) / \\
& \quad \left((2+p) \text{AppellF1} \left[1+p, p, 3+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad \quad (3+p) \text{AppellF1} \left[2+p, p, 4+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \quad \left. p \text{AppellF1} \left[2+p, 1+p, 3+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) + \\
& \quad \left(2 \text{AppellF1} \left[1+p, p, 4+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \right) / \\
& \quad \left((2+p) \text{AppellF1} \left[1+p, p, 4+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad \quad (4+p) \text{AppellF1} \left[2+p, p, 5+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \quad \left. p \text{AppellF1} \left[2+p, 1+p, 4+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) \left. \right) \\
& \quad \left(\frac{\text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right]^2}{\left(-1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^2} - \frac{\text{Sec} \left[\frac{1}{2} (e+fx) \right]^2}{2 \left(-1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^2} \right) \left. \right) \left. \right)
\end{aligned}$$

■ **Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \tan[e+fx])^p}{(a+a \sin[e+fx])^3} dx$$

Optimal (type 5, 248 leaves, 13 steps):

$$\frac{(g \operatorname{Tan}[e + f x])^{1+p}}{a^3 f g (1+p)} - \frac{3 (\operatorname{Cos}[e + f x]^2)^{\frac{7+p}{2}} \operatorname{Hypergeometric2F1}\left[\frac{2+p}{2}, \frac{7+p}{2}, \frac{4+p}{2}, \operatorname{Sin}[e + f x]^2\right] \operatorname{Sec}[e + f x]^5 (g \operatorname{Tan}[e + f x])^{2+p}}{a^3 f g^2 (2+p)} + \frac{5 (g \operatorname{Tan}[e + f x])^{3+p}}{a^3 f g^3 (3+p)} -$$

$$\frac{(\operatorname{Cos}[e + f x]^2)^{\frac{7+p}{2}} \operatorname{Hypergeometric2F1}\left[\frac{4+p}{2}, \frac{7+p}{2}, \frac{6+p}{2}, \operatorname{Sin}[e + f x]^2\right] \operatorname{Sec}[e + f x]^3 (g \operatorname{Tan}[e + f x])^{4+p}}{a^3 f g^4 (4+p)} + \frac{4 (g \operatorname{Tan}[e + f x])^{5+p}}{a^3 f g^5 (5+p)}$$

Result (type 6, 11802 leaves):

$$\left(2^{1+p} (2+p) \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)^{-p} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)^{-6-p} \left(-\frac{\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}\right)^p$$

$$\left(-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)^2 \left(\operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)^4\right) /$$

$$\left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] - (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right) -$$

$$\left(4 \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)^3\right) /$$

$$\left((2+p) \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] - (3+p) \operatorname{AppellF1}\left[2+p, p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right) +$$

$$\left(8 \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)^2\right) /$$

$$\left((2+p) \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] - (4+p) \operatorname{AppellF1}\left[2+p, p, 5+p, 3+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right) -$$

$$\left(8 \operatorname{AppellF1}\left[1+p, p, 5+p, 2+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right) /$$

$$\left((2+p) \operatorname{AppellF1}\left[1+p, p, 5+p, 2+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] - (5+p) \operatorname{AppellF1}\left[2+p, p, 6+p, 3+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 5+p, 3+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right) +$$

$$\left(4 \operatorname{AppellF1}\left[1+p, p, 6+p, 2+p, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right], -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right) / \left((2+p) \operatorname{AppellF1}\left[1+p, p, 6+p, 2+p,\right.\right.$$

$$\begin{aligned}
& \left. \left(\tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right] \right) - (6+p) \operatorname{AppellF1}\left[2+p, p, 7+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. p \operatorname{AppellF1}\left[2+p, 1+p, 6+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(g \tan[e+fx] \right)^p \Big/ \\
& \left(f(1+p)(a+a \sin[e+fx])^3 \left(\frac{1}{1+p} 2^{1+p} p(2+p) \sec\left[\frac{1}{2}(e+fx)\right]^2 \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right] \right)^{-p} \tan\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(e+fx)\right] \right)^{-6-p} \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+p} \right) \right. \\
& \left(\left(\operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right] \right)^4 \right) \Big/ \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, \right. \right. \right. \\
& \left. \left. \left. 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] - (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \right. \\
& \left. \tan\left[\frac{1}{2}(e+fx)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \tan\left[\frac{1}{2}(e+fx)\right] \right) - \\
& \left(4 \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right] \right)^3 \right) \Big/ \left((2+p) \operatorname{AppellF1}\left[1+p, p, \right. \right. \\
& \left. \left. 3+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] - (3+p) \operatorname{AppellF1}\left[2+p, p, 4+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \\
& \tan\left[\frac{1}{2}(e+fx)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 3+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \tan\left[\frac{1}{2}(e+fx)\right] \Big) + \\
& \left(8 \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right] \right)^2 \right) \Big/ \left((2+p) \operatorname{AppellF1}\left[1+p, p, \right. \right. \\
& \left. \left. 4+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] - (4+p) \operatorname{AppellF1}\left[2+p, p, 5+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \\
& \tan\left[\frac{1}{2}(e+fx)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 4+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \tan\left[\frac{1}{2}(e+fx)\right] \Big) - \\
& \left(8 \operatorname{AppellF1}\left[1+p, p, 5+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big/ \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 5+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& \left. (5+p) \operatorname{AppellF1}\left[2+p, p, 6+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \tan\left[\frac{1}{2}(e+fx)\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& p \operatorname{AppellF1}\left[2+p, 1+p, 5+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \left(4 \operatorname{AppellF1}\left[1+p, p, 6+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]\right) / \left((2+p) \operatorname{AppellF1}\left[1+p, p, 6+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - (6+p) \operatorname{AppellF1}\left[2+p, p, 7+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. p \operatorname{AppellF1}\left[2+p, 1+p, 6+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) + \\
& \frac{1}{1+p} 2^p (-6-p)(2+p) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-p} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-7-p} \\
& \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\right)^p \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^2 \Bigg) \\
& \left(\left(\operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^4\right) / \right. \\
& \quad \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& \quad (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \quad \left. p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) - \\
& \left(4 \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^3\right) / \\
& \quad \left((2+p) \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& \quad (3+p) \operatorname{AppellF1}\left[2+p, p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \quad \left. p \operatorname{AppellF1}\left[2+p, 1+p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) + \\
& \left(8 \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^2\right) / \\
& \quad \left((2+p) \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& \quad \left. (4+p) \operatorname{AppellF1}\left[2+p, p, 5+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& p \operatorname{AppellF1}\left[2+p, 1+p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \\
& \left(8 \operatorname{AppellF1}\left[1+p, p, 5+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)\right) / \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 5+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& (5+p) \operatorname{AppellF1}\left[2+p, p, 6+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \left. p \operatorname{AppellF1}\left[2+p, 1+p, 5+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) + \\
& \left(4 \operatorname{AppellF1}\left[1+p, p, 6+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]\right) / \left((2+p) \operatorname{AppellF1}\left[1+p, p, 6+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], \right. \right. \\
& \left. \left.-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - (6+p) \operatorname{AppellF1}\left[2+p, p, 7+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. p \operatorname{AppellF1}\left[2+p, 1+p, 6+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) + \\
& \frac{1}{1+p} 2^p (2+p) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-p} \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{-6-p} \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\right)^p \\
& \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^{2p} \left(\left(\operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^4\right) / \right. \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \left. p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) - \\
& \left(4 \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^3\right) / \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& (3+p) \operatorname{AppellF1}\left[2+p, p, 4+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \left. p \operatorname{AppellF1}\left[2+p, 1+p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) +
\end{aligned}$$

$$\begin{aligned}
& \left(8 \operatorname{AppellF1} \left[1+p, p, 4+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^2 \right) / \\
& \left((2+p) \operatorname{AppellF1} \left[1+p, p, 4+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad (4+p) \operatorname{AppellF1} \left[2+p, p, 5+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \left. p \operatorname{AppellF1} \left[2+p, 1+p, 4+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) - \\
& \left(8 \operatorname{AppellF1} \left[1+p, p, 5+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right] \right) \right) / \\
& \left((2+p) \operatorname{AppellF1} \left[1+p, p, 5+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad (5+p) \operatorname{AppellF1} \left[2+p, p, 6+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \left. p \operatorname{AppellF1} \left[2+p, 1+p, 5+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) + \\
& \left(4 \operatorname{AppellF1} \left[1+p, p, 6+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \right) / \left((2+p) \operatorname{AppellF1} \left[1+p, p, 6+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right] \right] - (6+p) \operatorname{AppellF1} \left[2+p, p, 7+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \right. \\
& \quad \left. p \operatorname{AppellF1} \left[2+p, 1+p, 6+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) - \\
& \frac{1}{1+p} 2^p p (2+p) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \left(-1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^{-1-p} \tan \left[\frac{1}{2} (e+fx) \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^{-6-p} \\
& \left(-\frac{\tan \left[\frac{1}{2} (e+fx) \right]}{-1 + \tan \left[\frac{1}{2} (e+fx) \right]^2} \right)^p \left(-1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^p \\
& \left(\left(\operatorname{AppellF1} \left[1+p, p, 2+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^4 \right) / \right. \\
& \quad \left((2+p) \operatorname{AppellF1} \left[1+p, p, 2+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad (2+p) \operatorname{AppellF1} \left[2+p, p, 3+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \left. \left. p \operatorname{AppellF1} \left[2+p, 1+p, 2+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{1}{2} (4+p) \operatorname{AppellF1}\left[2+p, p, 5+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \quad \frac{1}{2} p \operatorname{AppellF1}\left[2+p, 1+p, 4+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \\
& \quad (2+p) \left(-\frac{1}{2(2+p)} (1+p) (4+p) \operatorname{AppellF1}\left[2+p, p, 5+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \quad \quad \left. \frac{1}{2(2+p)} p (1+p) \operatorname{AppellF1}\left[2+p, 1+p, 4+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) - (4+p) \\
& \quad \left(-\frac{1}{2(3+p)} (2+p) (5+p) \operatorname{AppellF1}\left[3+p, p, 6+p, 4+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{2(3+p)} \right. \\
& \quad \quad p (2+p) \operatorname{AppellF1}\left[3+p, 1+p, 5+p, 4+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left. \right) \tan\left[\frac{1}{2}(e+fx)\right] + \\
& \quad p \left(-\frac{1}{2(3+p)} (2+p) (4+p) \operatorname{AppellF1}\left[3+p, 1+p, 5+p, 4+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \quad \quad \left. \frac{1}{2(3+p)} (1+p) (2+p) \operatorname{AppellF1}\left[3+p, 2+p, 4+p, 4+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) \\
& \quad \left. \tan\left[\frac{1}{2}(e+fx)\right] \right) \Bigg/ \left((2+p) \operatorname{AppellF1}\left[1+p, p, 4+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& \quad (4+p) \operatorname{AppellF1}\left[2+p, p, 5+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \tan\left[\frac{1}{2}(e+fx)\right] + \\
& \quad \left. p \operatorname{AppellF1}\left[2+p, 1+p, 4+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \tan\left[\frac{1}{2}(e+fx)\right] \right)^2 + \\
& \quad \left(8 \operatorname{AppellF1}\left[1+p, p, 5+p, 2+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \\
& \quad \left(-\frac{1}{2} (5+p) \operatorname{AppellF1}\left[2+p, p, 6+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \quad \quad \frac{1}{2} p \operatorname{AppellF1}\left[2+p, 1+p, 5+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + (2+p) \\
& \quad \quad \left(-\frac{1}{2(2+p)} (1+p) (5+p) \operatorname{AppellF1}\left[2+p, p, 6+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{2(2+p)} \right. \\
& \quad \quad \quad \left. p (1+p) \operatorname{AppellF1}\left[2+p, 1+p, 5+p, 3+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) - (5+p) \left(-\frac{1}{2(3+p)} \right. \\
& \quad \quad \quad \left. (2+p) (6+p) \operatorname{AppellF1}\left[3+p, p, 7+p, 4+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{2(3+p)} p (2+p) \right. \\
& \quad \quad \quad \left. \operatorname{AppellF1}\left[3+p, 1+p, 6+p, 4+p, \tan\left[\frac{1}{2}(e+fx)\right], -\tan\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) \tan\left[\frac{1}{2}(e+fx)\right] + p \left(-\frac{1}{2(3+p)} \right.
\end{aligned}$$

$$\begin{aligned}
& (2+p)(5+p) \operatorname{AppellF1}\left[3+p, 1+p, 6+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{2(3+p)}(1+p) \\
& (2+p) \operatorname{AppellF1}\left[3+p, 2+p, 5+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \Big) \Big) / \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 5+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - (5+p) \operatorname{AppellF1}\left[2+p, p, 6+p, \right. \right. \\
& \quad \left. \left. 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + p \operatorname{AppellF1}\left[2+p, 1+p, 5+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)^2 - \left(4 \operatorname{AppellF1}\left[1+p, p, 6+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right) \\
& \left(-\frac{1}{2}(6+p) \operatorname{AppellF1}\left[2+p, p, 7+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \quad \left. \frac{1}{2} p \operatorname{AppellF1}\left[2+p, 1+p, 6+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \quad (2+p) \left(-\frac{1}{2(2+p)}(1+p)(6+p) \operatorname{AppellF1}\left[2+p, p, 7+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \quad \quad \left. \frac{1}{2(2+p)} p(1+p) \operatorname{AppellF1}\left[2+p, 1+p, 6+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) - (6+p) \\
& \quad \left(-\frac{1}{2(3+p)}(2+p)(7+p) \operatorname{AppellF1}\left[3+p, p, 8+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{2(3+p)} \right. \\
& \quad \left. p(2+p) \operatorname{AppellF1}\left[3+p, 1+p, 7+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \quad p \left(-\frac{1}{2(3+p)}(2+p)(6+p) \operatorname{AppellF1}\left[3+p, 1+p, 7+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \\
& \quad \quad \left. \frac{1}{2(3+p)}(1+p)(2+p) \operatorname{AppellF1}\left[3+p, 2+p, 6+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Big) / \left((2+p) \operatorname{AppellF1}\left[1+p, p, 6+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \\
& \quad (6+p) \operatorname{AppellF1}\left[2+p, p, 7+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \quad \left. p \operatorname{AppellF1}\left[2+p, 1+p, 6+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)^2 - \\
& \left(\operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)^4 \right. \\
& \quad \left. \left(-\frac{1}{2}(2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 + \\
& (2+p) \left(-\frac{1}{2}(1+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 + \right. \\
& \quad \left. \frac{1}{2(2+p)} p(1+p) \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right) - \\
& (2+p) \left(-\frac{1}{2}(2+p) \operatorname{AppellF1}\left[3+p, p, 4+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 + \frac{1}{2(3+p)} \right. \\
& \quad \left. p(2+p) \operatorname{AppellF1}\left[3+p, 1+p, 3+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] + \\
& p \left(-\frac{1}{2(3+p)}(2+p)^2 \operatorname{AppellF1}\left[3+p, 1+p, 3+p, 4+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 + \frac{1}{2(3+p)} \right. \\
& \quad \left. (1+p)(2+p) \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{2}+\frac{p}{2}, 2+p\right\}, \left\{\frac{5}{2}+\frac{p}{2}\right\}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right) / \\
& \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] - (2+p) \operatorname{AppellF1}\left[2+p, p, \right. \right. \\
& \quad \left. \left. 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \quad \left. p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right)^2 + \\
& \frac{1}{1+p} 2^{1+p} p(2+p) \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right)^{-p} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right)^{-6-p} \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}\right)^{-1+p} \\
& \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right)^{2p} \\
& \left(\left(\operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right)^4\right) / \right. \\
& \quad \left((2+p) \operatorname{AppellF1}\left[1+p, p, 2+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] - \right. \\
& \quad \left. (2+p) \operatorname{AppellF1}\left[2+p, p, 3+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \quad \left. p \operatorname{AppellF1}\left[2+p, 1+p, 2+p, 3+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right) - \\
& \quad \left. \left(4 \operatorname{AppellF1}\left[1+p, p, 3+p, 2+p, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right], -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right)^3\right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left((2+p) \operatorname{AppellF1} \left[1+p, p, 3+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad (3+p) \operatorname{AppellF1} \left[2+p, p, 4+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \left. p \operatorname{AppellF1} \left[2+p, 1+p, 3+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) + \\
& \left(8 \operatorname{AppellF1} \left[1+p, p, 4+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^2 \right) / \\
& \left((2+p) \operatorname{AppellF1} \left[1+p, p, 4+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad (4+p) \operatorname{AppellF1} \left[2+p, p, 5+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \left. p \operatorname{AppellF1} \left[2+p, 1+p, 4+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) - \\
& \left(8 \operatorname{AppellF1} \left[1+p, p, 5+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right] \right) \right) / \\
& \left((2+p) \operatorname{AppellF1} \left[1+p, p, 5+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad (5+p) \operatorname{AppellF1} \left[2+p, p, 6+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \left. p \operatorname{AppellF1} \left[2+p, 1+p, 5+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) + \\
& \left(4 \operatorname{AppellF1} \left[1+p, p, 6+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \right) / \\
& \left((2+p) \operatorname{AppellF1} \left[1+p, p, 6+p, 2+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] - \right. \\
& \quad (6+p) \operatorname{AppellF1} \left[2+p, p, 7+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] + \\
& \quad \left. p \operatorname{AppellF1} \left[2+p, 1+p, 6+p, 3+p, \tan \left[\frac{1}{2} (e+fx) \right], -\tan \left[\frac{1}{2} (e+fx) \right] \right] \tan \left[\frac{1}{2} (e+fx) \right] \right) \Bigg) \\
& \left(\frac{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right]^2}{\left(-1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^2} - \frac{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}{2 \left(-1 + \tan \left[\frac{1}{2} (e+fx) \right] \right)^2} \right) \Bigg)
\end{aligned}$$

■ **Problem 129: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e+fx])^m (g \tan[e+fx])^p dx$$

Optimal (type 6, 111 leaves, 4 steps) :

$$\frac{1}{f g (1+p)} \text{AppellF1}\left[1+p, \frac{1+p}{2}, \frac{1}{2}(1-2m+p), 2+p, \text{Sin}[e+f x], -\text{Sin}[e+f x]\right]$$

$$(1-\text{Sin}[e+f x])^{\frac{1+p}{2}} (1+\text{Sin}[e+f x])^{\frac{1}{2}(1-2m+p)} (a+a \text{Sin}[e+f x])^m (g \text{Tan}[e+f x])^{1+p}$$

Result (type 6, 3773 leaves) :

$$\begin{aligned} & \left(2(-3+p) \text{AppellF1}\left[\frac{1-p}{2}, -p, 1+m, \frac{3-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \right. \\ & \quad \left. \text{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-m} (a+a \text{Sin}[e+f x])^m \text{Tan}[e+f x]^p (g \text{Tan}[e+f x])^p \right) / \\ & \left(f(-1+p) \left(2p \text{AppellF1}\left[\frac{3-p}{2}, 1-p, 1+m, \frac{5-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] + \right. \right. \\ & \quad \left. \left. 2(1+m) \text{AppellF1}\left[\frac{3-p}{2}, -p, 2+m, \frac{5-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (-3+p) \text{AppellF1}\left[\frac{1-p}{2}, -p, 1+m, \frac{3-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \text{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) \right) \\ & \left(\left(2(-3+p) p \text{AppellF1}\left[\frac{1-p}{2}, -p, 1+m, \frac{3-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \right. \right. \\ & \quad \left. \left. \text{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-m} \text{Sec}[e+f x]^2 \text{Tan}[e+f x]^{-1+p} \right) / \right. \\ & \quad \left((-1+p) \left(2p \text{AppellF1}\left[\frac{3-p}{2}, 1-p, 1+m, \frac{5-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] + \right. \right. \\ & \quad \left. \left. 2(1+m) \text{AppellF1}\left[\frac{3-p}{2}, -p, 2+m, \frac{5-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (-3+p) \text{AppellF1}\left[\frac{1-p}{2}, -p, 1+m, \frac{3-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \text{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) \right) \right) + \\ & \left(2(-3+p) \text{AppellF1}\left[\frac{1-p}{2}, -p, 1+m, \frac{3-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \right. \\ & \quad \left. \text{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \left(\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-m} \text{Tan}[e+f x]^p \right) / \\ & \quad \left((-1+p) \left(2p \text{AppellF1}\left[\frac{3-p}{2}, 1-p, 1+m, \frac{5-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] + \right. \right. \\ & \quad \left. \left. 2(1+m) \text{AppellF1}\left[\frac{3-p}{2}, -p, 2+m, \frac{5-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (-3+p) \text{AppellF1}\left[\frac{1-p}{2}, -p, 1+m, \frac{3-p}{2}, \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \text{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) \right) \right) + \end{aligned}$$

$$\begin{aligned}
& \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-m} \left(-(-3+p) \operatorname{AppellF1}\left[\frac{1-p}{2}, -p, 1+m, \frac{3-p}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right. \\
& \quad \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 + (-3+p) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(-\frac{1}{3-p} (1-p) {}_p\operatorname{AppellF1}\left[1 + \frac{1-p}{2}, 1-p, 1+m, 1 + \frac{3-p}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{3-p} (1+m) (1-p) \operatorname{AppellF1}\left[1 + \frac{1-p}{2}, \right. \right. \\
& \quad \left. \left. 1 + \frac{1-p}{2}, -p, 2+m, 1 + \frac{3-p}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + \\
& \quad 2p \left(-\frac{1}{5-p} (1+m) (3-p) \operatorname{AppellF1}\left[1 + \frac{3-p}{2}, 1-p, 2+m, 1 + \frac{5-p}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{5-p} (1-p) (3-p) \operatorname{AppellF1}\left[1 + \frac{3-p}{2}, 2-p, 1+m, 1 + \frac{5-p}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + \\
& \quad 2(1+m) \left(-\frac{1}{5-p} (3-p) {}_p\operatorname{AppellF1}\left[1 + \frac{3-p}{2}, 1-p, 2+m, 1 + \frac{5-p}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{5-p} (2+m) (3-p) \operatorname{AppellF1}\left[1 + \frac{3-p}{2}, -p, 3+m, 1 + \frac{5-p}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \operatorname{Tan}[e + f x]^p \Big/ \\
& \quad \left((-1+p) \left(2p \operatorname{AppellF1}\left[\frac{3-p}{2}, 1-p, 1+m, \frac{5-p}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \quad \left. 2(1+m) \operatorname{AppellF1}\left[\frac{3-p}{2}, -p, 2+m, \frac{5-p}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \quad \left. (-3+p) \operatorname{AppellF1}\left[\frac{1-p}{2}, -p, 1+m, \frac{3-p}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \Big) \Big)
\end{aligned}$$

■ **Problem 130: Unable to integrate problem.**

$$\int (a + a \operatorname{Sin}[e + f x])^m \operatorname{Tan}[e + f x]^3 dx$$

Optimal (type 5, 163 leaves, 4 steps):

$$\frac{a(4+m) \operatorname{Hypergeometric2F1}\left[1, -1+m, m, \frac{1}{2}(1+\operatorname{Sin}[e+fx])\right] (a+a \operatorname{Sin}[e+fx])^{-1+m}}{4 f (1-m)} + \frac{a^2 \operatorname{Sin}[e+fx]^2 (a+a \operatorname{Sin}[e+fx])^{-1+m}}{f m (a-a \operatorname{Sin}[e+fx])} + \frac{(a+a \operatorname{Sin}[e+fx])^{-1+m} (a(2-3m-m^2) + 2am \operatorname{Sin}[e+fx])}{2 f (1-m) m (1-\operatorname{Sin}[e+fx])}$$

Result (type 8, 23 leaves) :

$$\int (a + a \sin[e + f x])^m \tan[e + f x]^3 dx$$

■ **Problem 131: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m \tan[e + f x] dx$$

Optimal (type 5, 72 leaves, 3 steps) :

$$\frac{(a + a \sin[e + f x])^m \operatorname{Hypergeometric2F1}\left[1, 1 + m, 2 + m, \frac{1}{2}(1 + \sin[e + f x])\right] (a + a \sin[e + f x])^{1+m}}{2 f m} + \frac{4 a f (1 + m)}{2 f m}$$

Result (type 6, 9890 leaves) :

$$\begin{aligned} & - \left(\left(\cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sin[e + f x] (a + a \sin[e + f x])^m \left(\frac{1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2m} \right. \right. \\ & \left(\left(\left(2 \operatorname{AppellF1}\left[1, 1 - 2m, 2m, 2, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^4 \right) / \right. \right. \\ & \left(\left(-1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left(-2 \operatorname{AppellF1}\left[1, 1 - 2m, 2m, 2, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) + \right. \\ & \left(2m \operatorname{AppellF1}\left[2, 1 - 2m, 1 + 2m, 3, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\ & \left. \left. \left. (-1 + 2m) \operatorname{AppellF1}\left[2, 2 - 2m, 2m, 3, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) + \\ & \left(4 \operatorname{AppellF1}\left[1, -2m, 1 + 2m, 2, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^4 \right) / \\ & \left(\left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left(-2 \operatorname{AppellF1}\left[1, -2m, 1 + 2m, 2, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\ & \left(2m \operatorname{AppellF1}\left[2, 1 - 2m, 1 + 2m, 3, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + 2m) \right. \\ & \left. \left. \operatorname{AppellF1}\left[2, -2m, 2 + 2m, 3, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) + \\ & \left((1 + m) \operatorname{AppellF1}\left[1 + 2m, 2m, 1, 2 + 2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(-1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) / \\ & \left((1 + 2m) \left(2 (1 + m) \operatorname{AppellF1}\left[1 + 2m, 2m, 1, 2 + 2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\ & \left. \left. \operatorname{AppellF1}\left[2 + 2m, 2m, 2, 3 + 2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + m \operatorname{AppellF1}\left[2 + 2m, \right. \right. \end{aligned}$$

$$\begin{aligned} & 1, 3+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \\ & \left(-1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right) \left(-\frac{1}{3+2m} (2+2m) \operatorname{AppellF1}\left[3+2m, 2m, 3, 4+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \\ & \quad \left. \left. 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{1}{2(3+2m)} m(2+2m) \operatorname{AppellF1}\left[3+2m, 1+2m, \right. \right. \\ & \quad \left. \left. 2, 4+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \right. \\ & \quad \left. m\left(-\frac{1}{2(3+2m)} (2+2m) \operatorname{AppellF1}\left[3+2m, 1+2m, 2, 4+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \right. \\ & \quad \left. \left. \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{1}{4(3+2m)} (1+2m)(2+2m) \operatorname{AppellF1}\left[3+2m, 2+2m, 1, 4+2m, \right. \right. \right. \\ & \quad \left. \left. \left. \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)\right)\right) / \\ & \left((1+2m) \left(2(1+m) \operatorname{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \right. \right. \\ & \quad \left. \left. \left(\operatorname{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + m \operatorname{AppellF1}\left[2+2m, 1+2m, \right. \right. \right. \right. \\ & \quad \left. \left. \left. 1, 3+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right)\right)\right)\right)\right) \end{aligned}$$

■ **Problem 132: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx] (a+a \sin[e+fx])^m dx$$

Optimal (type 5, 43 leaves, 2 steps):

$$\frac{\operatorname{Hypergeometric2F1}\left[1, 1+m, 2+m, 1+\sin[e+fx]\right] (a+a \sin[e+fx])^{1+m}}{af(1+m)}$$

Result (type 6, 12204 leaves):

$$\begin{aligned} & -\frac{1}{f} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} (a+a \sin[e+fx])^m \\ & \left(\frac{1}{2^m} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2m} \left(-1 + (-\operatorname{Csc}[e+fx])^m \operatorname{Hypergeometric2F1}\left[m, m, 1+m, 2 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Csc}[e+fx]\right]\right) + \right. \\ & \quad \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2+2m} \operatorname{Csc}[e+fx] \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{-m} \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\left(i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^2}+\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)}+2 m\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{1+m} \right. \\
& \left. \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2},-\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{-1-m}\right)\right) \Bigg) - \\
& \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{2 m} \operatorname{Csc}[e+f x]\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{-m} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \\
& \left(4 m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2},-\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^m \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]-\right. \\
& \operatorname{AppellF1}\left[2 m, m, m, 1+2 m,-\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]},-\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right]\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m \\
& \left.\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m+\operatorname{AppellF1}\left[2 m, m, m, 1+2 m,\frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]},\frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right] \right. \\
& \left.\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m\right) \Bigg) / \\
& \left(16 m\left(\frac{1}{8}\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^2\right)^{-m} \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\left(4 m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2},-\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \right. \\
& \left.\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^m \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]-\operatorname{AppellF1}\left[2 m, m, m, 1+2 m,-\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]},\right. \right. \\
& \left. \left.-\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right]\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m+\operatorname{AppellF1}\left[2 m, m, m, 1+2 m,\right. \right. \\
& \left. \left.\frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]},\frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right]\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m\right) - \\
& \frac{1}{8 m}\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{-m}\left(2 m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2},-\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{1+m} \right. \\
& \left. 4 m^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2},-\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^m \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2-\left(\left((1-i) m^2\right.\right.\right.
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\left(-i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^2}+\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)}+m \operatorname{AppellF1}\left[2 m, m, m, \right. \right. \\
& \left. \left. 1+2 m, \frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}, \frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right]\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^{-1+m} \right. \\
& \left. \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\left(i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^2}+\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)}+2 m\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{1+m} \right. \right. \\
& \left. \left. \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-1-m}\right)\right)\right)\right) \Bigg) + \\
& \frac{1}{f} \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{-2 m}\left(a+a \operatorname{Sin}[e+f x]\right)^m\left(-\frac{1}{2 m} \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{2 m}\left(-1+(-\operatorname{Csc}[e+f x])^m\right. \right. \\
& \left. \left. \operatorname{Hypergeometric2F1}\left[m, m, 1+m, 2 \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Csc}[e+f x]\right]\right)\right) + \\
& \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{2+2 m}\left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-m}\left(4 m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \right. \right. \\
& \left. \left. \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^m \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]+ \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[2 m, m, m, 1+2 m, -\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}, -\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right] \right. \right. \\
& \left. \left. \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m- \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[2 m, m, m, 1+2 m, \frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}, \frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right] \right. \right. \\
& \left. \left. \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}\right)^m\right)\right) / \\
& \left(16 m\left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]-\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]+\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right] \\
& \left(\frac{-i+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right)^m + \\
& \text{AppellF1}\left[2m, m, m, 1+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right] \\
& \left(\frac{-i+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right)^m \Big/ \\
& \left(16m \left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) \left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)\right) \\
& \left(\frac{1}{8} \left(\sec\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right)^{-m} \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \\
& \left(4m \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \left(\sec\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right)^m \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \\
& \text{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right)^m \\
& \left(\frac{i+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right)^m + \text{AppellF1}\left[2m, m, m, 1+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right] \\
& \left(\frac{-i+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right)^m - \\
& \frac{1}{8m} \left(\sec\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right)^{-m} \left(2m \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \left(\sec\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right)^{1+m} + \\
& 4m^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \left(\sec\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right)^m \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - \\
& \left(\left((1-i)m^2 \text{AppellF1}\left[1+2m, m, 1+m, 2+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}\right]\right)\right)
\end{aligned}$$

$$\begin{aligned}
& 1 + 2m, \frac{1 - i}{1 + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}, \frac{1 + i}{1 + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]} \left(\frac{-i + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{1 + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]} \right)^m \left(\frac{i + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{1 + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]} \right)^{-1+m} \\
& \left(-\frac{\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \left(i + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)}{2 \left(1 + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^2} + \frac{\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{2 \left(1 + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)} + 2m \left(\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^2 \right)^{1+m} \\
& \left(-\text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \left(1 + \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{-1-m} \right) \right) \right)
\end{aligned}$$

■ **Problem 133: Unable to integrate problem.**

$$\int \cot[e + fx]^3 (a + a \sin[e + fx])^m dx$$

Optimal (type 5, 83 leaves, 3 steps):

$$-\frac{\csc[e + fx]^2 (a + a \sin[e + fx])^{2+m}}{2 a^2 f} - \frac{(2 - m) \text{Hypergeometric2F1}[2, 2 + m, 3 + m, 1 + \sin[e + fx]] (a + a \sin[e + fx])^{2+m}}{2 a^2 f (2 + m)}$$

Result (type 8, 23 leaves):

$$\int \cot[e + fx]^3 (a + a \sin[e + fx])^m dx$$

■ **Problem 134: Unable to integrate problem.**

$$\int \cot[e + fx]^5 (a + a \sin[e + fx])^m dx$$

Optimal (type 5, 123 leaves, 4 steps):

$$\begin{aligned}
& \frac{(9 - m) \csc[e + fx]^3 (a + a \sin[e + fx])^{3+m}}{12 a^3 f} - \frac{\csc[e + fx]^4 (a + a \sin[e + fx])^{3+m}}{4 a^3 f} - \\
& \frac{(12 - 9m + m^2) \text{Hypergeometric2F1}[3, 3 + m, 4 + m, 1 + \sin[e + fx]] (a + a \sin[e + fx])^{3+m}}{12 a^3 f (3 + m)}
\end{aligned}$$

Result (type 8, 23 leaves):

$$\int \cot[e + fx]^5 (a + a \sin[e + fx])^m dx$$

■ **Problem 135: Unable to integrate problem.**

$$\int (a + a \sin[e + fx])^m \tan[e + fx]^4 dx$$

Optimal (type 5, 311 leaves, 6 steps):

$$\frac{1}{3 f (1-m) m} 2^{-\frac{3}{2}+m} (9-12 m-7 m^2+6 m^3+m^4) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{2}-m, \frac{3}{2}, \frac{1}{2}(1-\sin[e+f x])\right] \sec[e+f x] (1-\sin[e+f x])$$

$$(1+\sin[e+f x])^{\frac{1}{2}-m} (a+a \sin[e+f x])^m - \frac{\sec[e+f x] (a+a \sin[e+f x])^{-1+m} (a(6-m-7 m^2-m^3)-a(9-6 m-8 m^2-m^3) \sin[e+f x])}{3 f (1-m) m (1-\sin[e+f x])} +$$

$$\frac{a^2 \sin[e+f x] (a+a \sin[e+f x])^{-1+m} \tan[e+f x]}{f (1-m) (a-a \sin[e+f x])} - \frac{a^2 \sin[e+f x]^2 (a+a \sin[e+f x])^{-1+m} \tan[e+f x]}{f m (a-a \sin[e+f x])}$$

Result (type 8, 23 leaves):

$$\int (a+a \sin[e+f x])^m \tan[e+f x]^4 dx$$

- **Problem 136: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a \sin[e+f x])^m \tan[e+f x]^2 dx$$

Optimal (type 5, 157 leaves, 5 steps):

$$\frac{\sec[e+f x] (a+a \sin[e+f x])^m}{f (1-m) m} + \frac{1}{f (1-m) m}$$

$$2^{-\frac{1}{2}+m} (1-m-m^2) \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{3}{2}-m, \frac{1}{2}, \frac{1}{2}(1-\sin[e+f x])\right] \sec[e+f x] (1+\sin[e+f x])^{\frac{1}{2}-m} (a+a \sin[e+f x])^m -$$

$$\frac{\sec[e+f x] (a+a \sin[e+f x])^{1+m}}{a f m}$$

Result (type 6, 25720 leaves): Display of huge result suppressed!

- **Problem 138: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+f x]^2 (a+a \sin[e+f x])^m dx$$

Optimal (type 6, 89 leaves, 3 steps):

$$\frac{1}{a^2 f (3+2 m)} 2 \sqrt{2} \text{AppellF1}\left[\frac{3}{2}+m, -\frac{1}{2}, 2, \frac{5}{2}+m, \frac{1}{2}(1+\sin[e+f x]), 1+\sin[e+f x]\right] \sec[e+f x] \sqrt{1-\sin[e+f x]} (a+a \sin[e+f x])^{2+m}$$

Result (type 6, 47775 leaves): Display of huge result suppressed!

- **Problem 139: Unable to integrate problem.**

$$\int \cot[e+f x]^4 (a+a \sin[e+f x])^m dx$$

Optimal (type 6, 89 leaves, 3 steps):

$$\frac{1}{a^3 f (5+2 m)} 4 \sqrt{2} \text{AppellF1}\left[\frac{5}{2}+m, -\frac{3}{2}, 4, \frac{7}{2}+m, \frac{1}{2}(1+\sin[e+f x]), 1+\sin[e+f x]\right] \sec[e+f x] \sqrt{1-\sin[e+f x]} (a+a \sin[e+f x])^{3+m}$$

Result (type 8, 23 leaves):

$$\int \cot [e + f x]^4 (a + a \sin [e + f x])^m dx$$

■ **Problem 158: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^4 (a + b \sin [c + d x])^2 dx$$

Optimal (type 3, 133 leaves, 13 steps):

$$a^2 x - \frac{3 b^2 x}{2} + \frac{3 a b \operatorname{ArcTanh}[\cos [c + d x]]}{d} - \frac{3 a b \cos [c + d x]}{d} + \frac{a^2 \cot [c + d x]}{d} - \frac{3 b^2 \cot [c + d x]}{2 d} + \frac{b^2 \cos [c + d x]^2 \cot [c + d x]}{2 d} - \frac{a b \cos [c + d x] \cot [c + d x]^2}{d} - \frac{a^2 \cot [c + d x]^3}{3 d}$$

Result (type 3, 293 leaves):

$$\frac{(2 a^2 - 3 b^2) (c + d x)}{2 d} - \frac{2 a b \cos [c + d x]}{d} + \frac{(4 a^2 \cos [\frac{1}{2} (c + d x)] - 3 b^2 \cos [\frac{1}{2} (c + d x)]) \operatorname{Csc}[\frac{1}{2} (c + d x)]}{6 d} - \frac{a b \operatorname{Csc}[\frac{1}{2} (c + d x)]^2}{4 d} - \frac{a^2 \cot [\frac{1}{2} (c + d x)] \operatorname{Csc}[\frac{1}{2} (c + d x)]^2}{24 d} + \frac{3 a b \operatorname{Log}[\cos [\frac{1}{2} (c + d x)]]}{d} - \frac{3 a b \operatorname{Log}[\sin [\frac{1}{2} (c + d x)]]}{d} + \frac{a b \operatorname{Sec}[\frac{1}{2} (c + d x)]^2}{4 d} + \frac{\operatorname{Sec}[\frac{1}{2} (c + d x)] (-4 a^2 \sin [\frac{1}{2} (c + d x)] + 3 b^2 \sin [\frac{1}{2} (c + d x)])}{6 d} - \frac{b^2 \sin [2 (c + d x)]}{4 d} + \frac{a^2 \operatorname{Sec}[\frac{1}{2} (c + d x)]^2 \operatorname{Tan}[\frac{1}{2} (c + d x)]}{24 d}$$

■ **Problem 175: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot [c + d x]^5}{a + b \sin [c + d x]} dx$$

Optimal (type 3, 148 leaves, 3 steps):

$$-\frac{b (2 a^2 - b^2) \operatorname{Csc}[c + d x]}{a^4 d} + \frac{(2 a^2 - b^2) \operatorname{Csc}[c + d x]^2}{2 a^3 d} + \frac{b \operatorname{Csc}[c + d x]^3}{3 a^2 d} - \frac{\operatorname{Csc}[c + d x]^4}{4 a d} + \frac{(a^2 - b^2)^2 \operatorname{Log}[\sin [c + d x]]}{a^5 d} - \frac{(a^2 - b^2)^2 \operatorname{Log}[a + b \sin [c + d x]]}{a^5 d}$$

Result (type 3, 347 leaves):

$$\frac{(-11 a^2 b \cos [\frac{1}{2} (c + d x)] + 6 b^3 \cos [\frac{1}{2} (c + d x)]) \operatorname{Csc}[\frac{1}{2} (c + d x)]}{12 a^4 d} + \frac{(7 a^2 - 4 b^2) \operatorname{Csc}[\frac{1}{2} (c + d x)]^2}{32 a^3 d} + \frac{b \cot [\frac{1}{2} (c + d x)] \operatorname{Csc}[\frac{1}{2} (c + d x)]^2}{24 a^2 d} - \frac{\operatorname{Csc}[\frac{1}{2} (c + d x)]^4}{64 a d} + \frac{(a^4 - 2 a^2 b^2 + b^4) \operatorname{Log}[\sin [c + d x]]}{a^5 d} + \frac{(-a^4 + 2 a^2 b^2 - b^4) \operatorname{Log}[a + b \sin [c + d x]]}{a^5 d} + \frac{(7 a^2 - 4 b^2) \operatorname{Sec}[\frac{1}{2} (c + d x)]^2}{32 a^3 d} - \frac{\operatorname{Sec}[\frac{1}{2} (c + d x)]^4}{64 a d} + \frac{\operatorname{Sec}[\frac{1}{2} (c + d x)] (-11 a^2 b \sin [\frac{1}{2} (c + d x)] + 6 b^3 \sin [\frac{1}{2} (c + d x)])}{12 a^4 d} + \frac{b \operatorname{Sec}[\frac{1}{2} (c + d x)]^2 \operatorname{Tan}[\frac{1}{2} (c + d x)]}{24 a^2 d}$$

■ **Problem 179: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + d x]^4}{a + b \text{Sin}[c + d x]} dx$$

Optimal (type 3, 154 leaves, 7 steps):

$$\frac{2 (a^2 - b^2)^{3/2} \text{ArcTan}\left[\frac{b + a \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^2 - b^2}}\right]}{a^4 d} - \frac{b (3 a^2 - 2 b^2) \text{ArcTanh}[\text{Cos}[c + d x]]}{2 a^4 d} +$$

$$\frac{(4 a^2 - 3 b^2) \text{Cot}[c + d x]}{3 a^3 d} + \frac{b \text{Cot}[c + d x] \text{Csc}[c + d x]}{2 a^2 d} - \frac{\text{Cot}[c + d x] \text{Csc}[c + d x]^2}{3 a d}$$

Result (type 3, 350 leaves):

$$\frac{2 (a^2 - b^2)^{3/2} \text{ArcTan}\left[\frac{\text{Sec}\left[\frac{1}{2}(c + d x)\right] (b \text{Cos}\left[\frac{1}{2}(c + d x)\right] + a \text{Sin}\left[\frac{1}{2}(c + d x)\right])}{\sqrt{a^2 - b^2}}\right]}{a^4 d} + \frac{(4 a^2 \text{Cos}\left[\frac{1}{2}(c + d x)\right] - 3 b^2 \text{Cos}\left[\frac{1}{2}(c + d x)\right]) \text{Csc}\left[\frac{1}{2}(c + d x)\right]}{6 a^3 d} +$$

$$\frac{b \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{8 a^2 d} - \frac{\text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{24 a d} + \frac{(-3 a^2 b + 2 b^3) \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right]\right]}{2 a^4 d} + \frac{(3 a^2 b - 2 b^3) \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 a^4 d} -$$

$$\frac{b \text{Sec}\left[\frac{1}{2}(c + d x)\right]^2}{8 a^2 d} + \frac{\text{Sec}\left[\frac{1}{2}(c + d x)\right] (-4 a^2 \text{Sin}\left[\frac{1}{2}(c + d x)\right] + 3 b^2 \text{Sin}\left[\frac{1}{2}(c + d x)\right])}{6 a^3 d} + \frac{\text{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{24 a d}$$

■ **Problem 186: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + d x]^5}{(a + b \text{Sin}[c + d x])^2} dx$$

Optimal (type 3, 188 leaves, 3 steps):

$$-\frac{4 b (a^2 - b^2) \text{Csc}[c + d x]}{a^5 d} + \frac{(2 a^2 - 3 b^2) \text{Csc}[c + d x]^2}{2 a^4 d} + \frac{2 b \text{Csc}[c + d x]^3}{3 a^3 d} - \frac{\text{Csc}[c + d x]^4}{4 a^2 d} +$$

$$\frac{(a^4 - 6 a^2 b^2 + 5 b^4) \text{Log}[\text{Sin}[c + d x]]}{a^6 d} - \frac{(a^4 - 6 a^2 b^2 + 5 b^4) \text{Log}[a + b \text{Sin}[c + d x]]}{a^6 d} + \frac{(a^2 - b^2)^2}{a^5 d (a + b \text{Sin}[c + d x])}$$

Result (type 3, 380 leaves):

$$\begin{aligned}
& \frac{(-11 a^2 b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 12 b^3 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] - (7 a^2 - 12 b^2) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{6 a^5 d} + \frac{(7 a^2 - 12 b^2) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{32 a^4 d} + \\
& \frac{b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{12 a^3 d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{64 a^2 d} + \frac{(a^4 - 6 a^2 b^2 + 5 b^4) \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^6 d} + \\
& \frac{(-a^4 + 6 a^2 b^2 - 5 b^4) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{a^6 d} + \frac{(7 a^2 - 12 b^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{32 a^4 d} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}{64 a^2 d} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] (-11 a^2 b \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 12 b^3 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{6 a^5 d} + \frac{(a-b)^2 (a+b)^2}{a^5 d (a+b \operatorname{Sin}[c+dx])} + \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{12 a^3 d}
\end{aligned}$$

■ **Problem 192: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Tan}[c+dx]^5}{(a+b \operatorname{Sin}[c+dx])^3} dx$$

Optimal (type 3, 321 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(8 a^2 - 5 a b - b^2) \operatorname{Log}[1 - \operatorname{Sin}[c+dx]]}{16 (a+b)^5 d} - \frac{(8 a^2 + 5 a b - b^2) \operatorname{Log}[1 + \operatorname{Sin}[c+dx]]}{16 (a-b)^5 d} + \\
& \frac{a^3 (a^4 + 13 a^2 b^2 + 10 b^4) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{(a^2 - b^2)^5 d} - \frac{a^5}{2 (a^2 - b^2)^3 d (a+b \operatorname{Sin}[c+dx])^2} - \frac{a^4 (a^2 + 5 b^2)}{(a^2 - b^2)^4 d (a+b \operatorname{Sin}[c+dx])} + \\
& \frac{\operatorname{Sec}[c+dx]^4 (a (a^2 + 3 b^2) - b (3 a^2 + b^2) \operatorname{Sin}[c+dx])}{4 (a^2 - b^2)^3 d} - \frac{\operatorname{Sec}[c+dx]^2 (8 a^3 (a^2 + 5 b^2) - b (27 a^4 + 22 a^2 b^2 - b^4) \operatorname{Sin}[c+dx])}{8 (a^2 - b^2)^4 d}
\end{aligned}$$

Result (type 3, 588 leaves):

$$\begin{aligned}
& - \frac{2i(a^7 + 13a^5b^2 + 10a^3b^4)(c+dx)}{(a-b)^5(a+b)^5d} + \frac{1}{8(a-b)^5d} \\
& i(-8a^2 - 5ab + b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \\
& \frac{1}{8(a+b)^5d} i(-8a^2 + 5ab + b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \\
& \frac{(-8a^2 + 5ab + b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a+b)^5d} + \frac{(-8a^2 - 5ab + b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a-b)^5d} + \\
& \frac{(a^7 + 13a^5b^2 + 10a^3b^4) \operatorname{Log}[a+b\sin[c+dx]]}{(a^2 - b^2)^5d} + \frac{1}{16(a+b)^3d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \\
& \frac{-7a-b}{16(a+b)^4d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{1}{16(a-b)^3d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \\
& \frac{-7a+b}{16(a-b)^4d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a^5}{2(a-b)^3(a+b)^3d(a+b\sin[c+dx])^2} - \frac{a^4(a^2 + 5b^2)}{(a-b)^4(a+b)^4d(a+b\sin[c+dx])}
\end{aligned}$$

■ **Problem 193: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+dx]^3}{(a+b\sin[c+dx])^3} dx$$

Optimal (type 3, 232 leaves, 4 steps):

$$\begin{aligned}
& \frac{(2a-b) \operatorname{Log}[1-\sin[c+dx]]}{4(a+b)^4d} + \frac{(2a+b) \operatorname{Log}[1+\sin[c+dx]]}{4(a-b)^4d} - \frac{a(a^4 + 8a^2b^2 + 3b^4) \operatorname{Log}[a+b\sin[c+dx]]}{(a^2 - b^2)^4d} + \\
& \frac{a^3}{2(a^2 - b^2)^2d(a+b\sin[c+dx])^2} + \frac{a^2(a^2 + 3b^2)}{(a^2 - b^2)^3d(a+b\sin[c+dx])} + \frac{\operatorname{Sec}[c+dx]^2(a(a^2 + 3b^2) - b(3a^2 + b^2)\sin[c+dx])}{2(a^2 - b^2)^3d}
\end{aligned}$$

Result (type 3, 471 leaves):

$$\frac{2i(a^5 + 8a^3b^2 + 3ab^4)(c+dx)}{(a-b)^4(a+b)^4d} + \frac{1}{2(a+b)^4d}$$

$$+ i(2a-b)\text{ArcTan}\left[\text{Csc}[c+dx]\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \frac{1}{2(a-b)^4d}$$

$$+ i(2a+b)\text{ArcTan}\left[\text{Csc}[c+dx]\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] +$$

$$\frac{(2a-b)\text{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{4(a+b)^4d} + \frac{(2a+b)\text{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{4(a-b)^4d} +$$

$$\frac{(-a^5 - 8a^3b^2 - 3ab^4)\text{Log}[a+b\sin[c+dx]]}{(a^2-b^2)^4d} + \frac{1}{4(a+b)^3d\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{1}{4(a-b)^3d\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{a^3}{2(a-b)^2(a+b)^2d(a+b\sin[c+dx])^2} + \frac{a^2(a^2+3b^2)}{(a-b)^3(a+b)^3d(a+b\sin[c+dx])}$$

■ **Problem 203: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+b\sin[ex+fx])^3 (g\tan[ex+fx])^p dx$$

Optimal (type 5, 271 leaves, 10 steps):

$$\frac{a^3 \text{Hypergeometric2F1}\left[1, \frac{1+p}{2}, \frac{3+p}{2}, -\tan[ex+fx]^2\right] (g\tan[ex+fx])^{1+p}}{fg(1+p)} + \frac{1}{fg(2+p)}$$

$$+ \frac{3a^2b(\cos[ex+fx]^2)^{\frac{1+p}{2}} \text{Hypergeometric2F1}\left[\frac{1+p}{2}, \frac{2+p}{2}, \frac{4+p}{2}, \sin[ex+fx]^2\right] \sin[ex+fx] (g\tan[ex+fx])^{1+p}}{fg(4+p)}$$

$$+ \frac{b^3(\cos[ex+fx]^2)^{\frac{1+p}{2}} \text{Hypergeometric2F1}\left[\frac{1+p}{2}, \frac{4+p}{2}, \frac{6+p}{2}, \sin[ex+fx]^2\right] \sin[ex+fx]^3 (g\tan[ex+fx])^{1+p}}{fg(4+p)}$$

$$+ \frac{3ab^2 \text{Hypergeometric2F1}\left[2, \frac{3+p}{2}, \frac{5+p}{2}, -\tan[ex+fx]^2\right] (g\tan[ex+fx])^{3+p}}{fg^3(3+p)}$$

Result (type 6, 16820 leaves):

$$\left(\left(\left(a^3(3+p)\text{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(ex+fx)\right]^2, -\tan\left[\frac{1}{2}(ex+fx)\right]^2\right] \tan\left[\frac{1}{2}(ex+fx)\right] \left(-\frac{\tan\left[\frac{1}{2}(ex+fx)\right]}{-1+\tan\left[\frac{1}{2}(ex+fx)\right]^2}\right)^p\right)\right)\right)/$$

$$\left((1+p)\left(1+\tan\left[\frac{1}{2}(ex+fx)\right]^2\right)\left((3+p)\text{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(ex+fx)\right]^2, -\tan\left[\frac{1}{2}(ex+fx)\right]^2\right] -\right.\right.$$

$$\left.\left.2\left(\text{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(ex+fx)\right]^2, -\tan\left[\frac{1}{2}(ex+fx)\right]^2\right] -\right.\right.\right.$$

$$\begin{aligned}
& p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \Bigg) - \\
& \left(8b^3(4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 4, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1+\tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right) / \\
& \left((2+p) \left(1+\tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^4 \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 4, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left(-4 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 5, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. \left. p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 4, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \Bigg) \\
& \tan[e+fx]^{-p} (g \tan[e+fx])^p \left(-\frac{1}{8} b^3 \sin[3(e+fx)] \tan[e+fx]^p - a^3 \sin[e+fx]^3 \sin[3(e+fx)] \tan[e+fx]^p + \right. \\
& \quad \frac{3}{8} i b^3 \sin[2(e+fx)] \sin[3(e+fx)] \tan[e+fx]^p + \\
& \quad \frac{3}{8} b^3 \sin[2(e+fx)]^2 \sin[3(e+fx)] \tan[e+fx]^p - \\
& \quad \frac{1}{8} i b^3 \sin[2(e+fx)]^3 \sin[3(e+fx)] \tan[e+fx]^p + \\
& \quad \cos[e+fx]^3 (a^3 \cos[3(e+fx)] \tan[e+fx]^p - i a^3 \sin[3(e+fx)] \tan[e+fx]^p) + \\
& \quad \cos[2(e+fx)]^3 \left(\frac{1}{8} i b^3 \cos[3(e+fx)] \tan[e+fx]^p + \frac{1}{8} b^3 \sin[3(e+fx)] \tan[e+fx]^p \right) + \\
& \quad \sin[e+fx]^2 \left(-\frac{3}{2} a^2 b \sin[3(e+fx)] \tan[e+fx]^p + \frac{3}{2} i a^2 b \sin[2(e+fx)] \sin[3(e+fx)] \tan[e+fx]^p \right) + \\
& \quad \sin[e+fx] \left(-\frac{3}{4} a b^2 \sin[3(e+fx)] \tan[e+fx]^p + \frac{3}{2} i a b^2 \sin[2(e+fx)] \sin[3(e+fx)] \tan[e+fx]^p + \right. \\
& \quad \left. \frac{3}{4} a b^2 \sin[2(e+fx)]^2 \sin[3(e+fx)] \tan[e+fx]^p \right) + \cos[2(e+fx)]^2 \\
& \quad \left(-\frac{3}{8} b^3 \sin[3(e+fx)] \tan[e+fx]^p - \frac{3}{4} a b^2 \sin[e+fx] \sin[3(e+fx)] \tan[e+fx]^p + \frac{3}{8} i b^3 \sin[2(e+fx)] \sin[3(e+fx)] \right. \\
& \quad \left. \tan[e+fx]^p + \cos[3(e+fx)] \left(-\frac{3}{8} i b^3 \tan[e+fx]^p - \frac{3}{4} i a b^2 \sin[e+fx] \tan[e+fx]^p - \frac{3}{8} b^3 \sin[2(e+fx)] \tan[e+fx]^p \right) \right) + \\
& \quad \cos[3(e+fx)] \left(-\frac{1}{8} i b^3 \tan[e+fx]^p - i a^3 \sin[e+fx]^3 \tan[e+fx]^p - \frac{3}{8} b^3 \sin[2(e+fx)] \tan[e+fx]^p + \frac{3}{8} i b^3 \sin[2(e+fx)]^2 \right. \\
& \quad \left. \tan[e+fx]^p + \frac{1}{8} b^3 \sin[2(e+fx)]^3 \tan[e+fx]^p + \sin[e+fx]^2 \left(-\frac{3}{2} i a^2 b \tan[e+fx]^p - \frac{3}{2} a^2 b \sin[2(e+fx)] \tan[e+fx]^p \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \sin[e + f x] \left(-\frac{3}{4} i a b^2 \tan[e + f x]^p - \frac{3}{2} a b^2 \sin[2(e + f x)] \tan[e + f x]^p + \frac{3}{4} i a b^2 \sin[2(e + f x)]^2 \tan[e + f x]^p \right) \right] \right) + \right. \\
& \cos[e + f x]^2 \left(\frac{3}{2} a^2 b \sin[3(e + f x)] \tan[e + f x]^p + 3 a^3 \sin[e + f x] \sin[3(e + f x)] \tan[e + f x]^p - \frac{3}{2} i a^2 b \sin[2(e + f x)] \sin[3(e + f x)] \right. \\
& \quad \left. \tan[e + f x]^p + \cos[3(e + f x)] \left(\frac{3}{2} i a^2 b \tan[e + f x]^p + 3 i a^3 \sin[e + f x] \tan[e + f x]^p + \frac{3}{2} a^2 b \sin[2(e + f x)] \tan[e + f x]^p \right) + \right. \\
& \quad \left. \left. \cos[2(e + f x)] \left(-\frac{3}{2} i a^2 b \cos[3(e + f x)] \tan[e + f x]^p - \frac{3}{2} a^2 b \sin[3(e + f x)] \tan[e + f x]^p \right) \right] \right) + \\
& \cos[e + f x] \left(\frac{3}{4} i a b^2 \sin[3(e + f x)] \tan[e + f x]^p + 3 i a^3 \sin[e + f x]^2 \sin[3(e + f x)] \tan[e + f x]^p + \right. \\
& \quad \frac{3}{2} a b^2 \sin[2(e + f x)] \sin[3(e + f x)] \tan[e + f x]^p - \frac{3}{4} i a b^2 \sin[2(e + f x)]^2 \sin[3(e + f x)] \tan[e + f x]^p + \\
& \quad \cos[2(e + f x)]^2 \left(-\frac{3}{4} a b^2 \cos[3(e + f x)] \tan[e + f x]^p + \frac{3}{4} i a b^2 \sin[3(e + f x)] \tan[e + f x]^p \right) + \\
& \quad \sin[e + f x] \left(3 i a^2 b \sin[3(e + f x)] \tan[e + f x]^p + 3 a^2 b \sin[2(e + f x)] \sin[3(e + f x)] \tan[e + f x]^p \right) + \\
& \quad \cos[3(e + f x)] \left(-\frac{3}{4} a b^2 \tan[e + f x]^p - 3 a^3 \sin[e + f x]^2 \tan[e + f x]^p + \frac{3}{2} i a b^2 \sin[2(e + f x)] \tan[e + f x]^p + \right. \\
& \quad \left. \frac{3}{4} a b^2 \sin[2(e + f x)]^2 \tan[e + f x]^p + \sin[e + f x] \left(-3 a^2 b \tan[e + f x]^p + 3 i a^2 b \sin[2(e + f x)] \tan[e + f x]^p \right) \right) + \cos[2(e + f x)] \\
& \quad \left(-\frac{3}{2} i a b^2 \sin[3(e + f x)] \tan[e + f x]^p - 3 i a^2 b \sin[e + f x] \sin[3(e + f x)] \tan[e + f x]^p - \frac{3}{2} a b^2 \sin[2(e + f x)] \sin[3(e + f x)] \right. \\
& \quad \left. \tan[e + f x]^p + \cos[3(e + f x)] \left(\frac{3}{2} a b^2 \tan[e + f x]^p + 3 a^2 b \sin[e + f x] \tan[e + f x]^p - \frac{3}{2} i a b^2 \sin[2(e + f x)] \tan[e + f x]^p \right) \right) \Bigg) + \\
& \cos[2(e + f x)] \left(\frac{3}{8} b^3 \sin[3(e + f x)] \tan[e + f x]^p + \frac{3}{2} a^2 b \sin[e + f x]^2 \sin[3(e + f x)] \tan[e + f x]^p - \right. \\
& \quad \frac{3}{4} i b^3 \sin[2(e + f x)] \sin[3(e + f x)] \tan[e + f x]^p - \frac{3}{8} b^3 \sin[2(e + f x)]^2 \sin[3(e + f x)] \tan[e + f x]^p + \\
& \quad \sin[e + f x] \left(\frac{3}{2} a b^2 \sin[3(e + f x)] \tan[e + f x]^p - \frac{3}{2} i a b^2 \sin[2(e + f x)] \sin[3(e + f x)] \tan[e + f x]^p \right) + \\
& \quad \cos[3(e + f x)] \left(\frac{3}{8} i b^3 \tan[e + f x]^p + \frac{3}{2} i a^2 b \sin[e + f x]^2 \tan[e + f x]^p + \frac{3}{4} b^3 \sin[2(e + f x)] \tan[e + f x]^p - \right. \\
& \quad \left. \frac{3}{8} i b^3 \sin[2(e + f x)]^2 \tan[e + f x]^p + \sin[e + f x] \left(\frac{3}{2} i a b^2 \tan[e + f x]^p + \frac{3}{2} a b^2 \sin[2(e + f x)] \tan[e + f x]^p \right) \right) \Bigg) \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \Big/ \left((1+p) \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^3 \right. \\
& \left. \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left(-2 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left(6 a b^2 (3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right) \Big/ \\
& \left((1+p) \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad 2 \left(-2 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left(12 a b^2 (3+p) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(-1 / (3+p)^2 (1+p) \operatorname{AppellF1}\left[1+\frac{1+p}{2}, p, 3, 1+\frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + 1 / (3+p)p(1+p) \operatorname{AppellF1}\left[1+\frac{1+p}{2}, 1+p, 2, 1+\frac{3+p}{2}, \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right) \Big/ \\
& \left((1+p) \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad 2 \left(-2 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left(36 a b^2 (3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left((1+p) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^4 \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad 2 \left(-3 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \quad \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(6 a b^2 (3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right) / \\
& \left((1+p) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^3 \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad 2 \left(-3 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \quad \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(12 a b^2 (3+p) \tan\left[\frac{1}{2}(e+fx)\right] \left(-1 / (3+p) 3 (1+p) \operatorname{AppellF1}\left[1 + \frac{1+p}{2}, p, 4, 1 + \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + 1 / (3+p) p (1+p) \operatorname{AppellF1}\left[1 + \frac{1+p}{2}, 1+p, 3, 1 + \frac{3+p}{2}, \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right) / \\
& \left((1+p) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^3 \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad 2 \left(-3 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \quad \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(12 a^2 b (4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{2}(e+fx)\right]^3 \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right) /
\end{aligned}$$

$$\begin{aligned}
& \left. \tan\left[\frac{1}{2}(e+fx)\right]^3 \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1+\tan\left[\frac{1}{2}(e+fx)\right]^2}\right)^p\right] / \\
& \left((2+p) \left(1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right)^4 \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 3, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad 2 \left(-3 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 4, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left(8b^3 (4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 3, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{2}(e+fx)\right] \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1+\tan\left[\frac{1}{2}(e+fx)\right]^2}\right)^p\right] / \\
& \left((2+p) \left(1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right)^3 \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 3, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad 2 \left(-3 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 4, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left(8b^3 (4+p) \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-1/(4+p) 3(2+p) \operatorname{AppellF1}\left[1+\frac{2+p}{2}, p, 4, 1+\frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + 1/(4+p)p(2+p) \operatorname{AppellF1}\left[1+\frac{2+p}{2}, 1+p, 3, 1+\frac{4+p}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1+\tan\left[\frac{1}{2}(e+fx)\right]^2}\right)^p \right) / \\
& \left((2+p) \left(1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right)^3 \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 3, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad 2 \left(-3 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 4, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 4, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \Bigg) + \\
& \left(a^3 p (3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right] \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1+\tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^{-1+p} \right. \\
& \quad \left. \left(\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]^2}{(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2)^2} - \frac{\sec\left[\frac{1}{2}(e+fx)\right]^2}{2(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2)} \right) \right) / \\
& \quad \left((1+p) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left(\operatorname{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \quad \left(12 a b^2 p (3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left. \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1+\tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^{-1+p} \left(\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]^2}{(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2)^2} - \frac{\sec\left[\frac{1}{2}(e+fx)\right]^2}{2(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2)} \right) \right) / \\
& \quad \left((1+p) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left(-2 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \quad \left(12 a b^2 p (3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left. \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1+\tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^{-1+p} \left(\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]^2}{(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2)^2} - \frac{\sec\left[\frac{1}{2}(e+fx)\right]^2}{2(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2)} \right) \right) / \\
& \quad \left((1+p) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right)^3 \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left((2+p) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right)^4 \left((4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, p, 4, \frac{4+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad 2 \left(-4 \operatorname{AppellF1} \left[\frac{4+p}{2}, p, 5, \frac{6+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. \left. p \operatorname{AppellF1} \left[\frac{4+p}{2}, 1+p, 4, \frac{6+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) - \\
& \left(a^3 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}{-1 + \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2} \right)^p \right. \\
& \quad \left(-2 \left(\operatorname{AppellF1} \left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \\
& \quad (3+p) \left(-\frac{1}{3+p} (1+p) \operatorname{AppellF1} \left[1 + \frac{1+p}{2}, p, 2, 1 + \frac{3+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \\
& \quad \left. \frac{1}{3+p} p (1+p) \operatorname{AppellF1} \left[1 + \frac{1+p}{2}, 1+p, 1, 1 + \frac{3+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) - \\
& \quad 2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \left(-\frac{1}{5+p} 2 (3+p) \operatorname{AppellF1} \left[1 + \frac{3+p}{2}, p, 3, 1 + \frac{5+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\right. \right. \\
& \quad \left. \left. \frac{1}{2} (e+fx) \right] + \frac{1}{5+p} p (3+p) \operatorname{AppellF1} \left[1 + \frac{3+p}{2}, 1+p, 2, 1 + \frac{5+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] - p \left(-\frac{1}{5+p} (3+p) \operatorname{AppellF1} \left[1 + \frac{3+p}{2}, 1+p, 2, 1 + \frac{5+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{1}{5+p} (1+p) (3+p) \operatorname{AppellF1} \left[1 + \frac{3+p}{2}, 2+p, 1, 1 + \frac{5+p}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right) \Big/ \left((1+p) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \right. \\
& \quad \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] - p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 - \\
& \left(12 a b^2 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}{-1 + \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2} \right)^p \right)
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-3 \left(-\frac{1}{5+p} 4(3+p) \operatorname{AppellF1}\left[1+\frac{3+p}{2}, p, 5, 1+\frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+p} p(3+p) \operatorname{AppellF1}\left[1+\frac{3+p}{2}, 1+p, 4, 1+\frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + p \left(-\frac{1}{5+p} 3(3+p) \operatorname{AppellF1}\left[1+\frac{3+p}{2}, 1+p, 4, \right. \right. \\
& \quad \left. \left. 1+\frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+p} (1+p)(3+p) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[1+\frac{3+p}{2}, 2+p, 3, 1+\frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) \Bigg) / \\
& \left((1+p) \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^3 \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left(-3 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + p \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right) - \right. \\
& \left. \left(6a^2b(4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right. \right. \\
& \quad \left. \left(-2 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + (4+p) \right. \\
& \quad \left(-\frac{1}{4+p} 2(2+p) \operatorname{AppellF1}\left[1+\frac{2+p}{2}, p, 3, 1+\frac{4+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \left. \frac{1}{4+p} p(2+p) \operatorname{AppellF1}\left[1+\frac{2+p}{2}, 1+p, 2, 1+\frac{4+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-2 \left(-\frac{1}{6+p} 3(4+p) \operatorname{AppellF1}\left[1+\frac{4+p}{2}, p, 4, 1+\frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} p(4+p) \operatorname{AppellF1}\left[1+\frac{4+p}{2}, 1+p, 3, 1+\frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + p \left(-\frac{1}{6+p} 2(4+p) \operatorname{AppellF1}\left[1+\frac{4+p}{2}, 1+p, 3, \right. \right. \\
& \quad \left. \left. 1+\frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} (1+p)(4+p) \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \text{AppellF1}\left[1 + \frac{4+p}{2}, 2+p, 2, 1 + \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right]\right)\right)\right)\right) / \\
& \left((2+p) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \left((4+p) \text{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \quad 2 \left(-2 \text{AppellF1}\left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + p \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right) - \\
& \left(8b^3 (4+p) \text{AppellF1}\left[\frac{2+p}{2}, p, 3, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right. \\
& \quad \left(2 \left(-3 \text{AppellF1}\left[\frac{4+p}{2}, p, 4, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. p \text{AppellF1}\left[\frac{4+p}{2}, 1+p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + (4+p) \right. \\
& \quad \left(-\frac{1}{4+p} 3(2+p) \text{AppellF1}\left[1 + \frac{2+p}{2}, p, 4, 1 + \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \left. \frac{1}{4+p} p(2+p) \text{AppellF1}\left[1 + \frac{2+p}{2}, 1+p, 3, 1 + \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& \quad 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-3 \left(-\frac{1}{6+p} 4(4+p) \text{AppellF1}\left[1 + \frac{4+p}{2}, p, 5, 1 + \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} p(4+p) \text{AppellF1}\left[1 + \frac{4+p}{2}, 1+p, 4, 1 + \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + p \left(-\frac{1}{6+p} 3(4+p) \text{AppellF1}\left[1 + \frac{4+p}{2}, 1+p, 4, \right. \right. \\
& \quad \left. \left. 1 + \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} (1+p)(4+p) \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[1 + \frac{4+p}{2}, 2+p, 3, 1 + \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right) / \\
& \left((2+p) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^3 \left((4+p) \text{AppellF1}\left[\frac{2+p}{2}, p, 3, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left(-3 \text{AppellF1}\left[\frac{4+p}{2}, p, 4, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + p \right) \right) \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{4+p}{2}, 1+p, 3, \frac{6+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \Big) + \\
& \left(8 b^3 (4+p) \text{AppellF1}\left[\frac{2+p}{2}, p, 4, \frac{4+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{\text{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^p \right. \\
& \left. \left(2 \left(-4 \text{AppellF1}\left[\frac{4+p}{2}, p, 5, \frac{6+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) + \right. \right. \\
& \quad p \text{AppellF1}\left[\frac{4+p}{2}, 1+p, 4, \frac{6+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + (4+p) \\
& \quad \left. \left(-\frac{1}{4+p} 4(2+p) \text{AppellF1}\left[1+\frac{2+p}{2}, p, 5, 1+\frac{4+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \left. \frac{1}{4+p} p(2+p) \text{AppellF1}\left[1+\frac{2+p}{2}, 1+p, 4, 1+\frac{4+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& 2 \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-4 \left(-\frac{1}{6+p} 5(4+p) \text{AppellF1}\left[1+\frac{4+p}{2}, p, 6, 1+\frac{6+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} p(4+p) \text{AppellF1}\left[1+\frac{4+p}{2}, 1+p, 5, 1+\frac{6+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) + p \left(-\frac{1}{6+p} 4(4+p) \text{AppellF1}\left[1+\frac{4+p}{2}, 1+p, 5, \right. \right. \\
& \quad \left. \left. 1+\frac{6+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} (1+p)(4+p) \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \text{AppellF1}\left[1+\frac{4+p}{2}, 2+p, 4, 1+\frac{6+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right) \right) \right) \right) / \\
& \left. \left((2+p) \left(1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^4 \left((4+p) \text{AppellF1}\left[\frac{2+p}{2}, p, 4, \frac{4+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) + \right. \right. \\
& \quad \left. \left. 2 \left(-4 \text{AppellF1}\left[\frac{4+p}{2}, p, 5, \frac{6+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) + p \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \text{AppellF1}\left[\frac{4+p}{2}, 1+p, 4, \frac{6+p}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \right) \right) \right) \right) \Big)
\end{aligned}$$

▪ **Problem 204: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \sin[e + fx])^2 (g \tan[e + fx])^p dx$$

Optimal (type 5, 186 leaves, 8 steps):

$$\frac{a^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1+p}{2}, \frac{3+p}{2}, -\operatorname{Tan}[e+fx]^2\right] (g \operatorname{Tan}[e+fx])^{1+p}}{fg(1+p)} + \frac{1}{fg(2+p)}$$

$$2ab (\operatorname{Cos}[e+fx]^2)^{\frac{1+p}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+p}{2}, \frac{2+p}{2}, \frac{4+p}{2}, \operatorname{Sin}[e+fx]^2\right] \operatorname{Sin}[e+fx] (g \operatorname{Tan}[e+fx])^{1+p} +$$

$$\frac{b^2 \operatorname{Hypergeometric2F1}\left[2, \frac{3+p}{2}, \frac{5+p}{2}, -\operatorname{Tan}[e+fx]^2\right] (g \operatorname{Tan}[e+fx])^{3+p}}{fg^3(3+p)}$$

Result (type 6, 10333 leaves):

$$\left(2^{1+p} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\right)^p\right.$$

$$\left(\left(a^2(3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2\right) / \right.$$

$$\left((1+p) \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right) +$$

$$\left(4b^2(3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)\right) /$$

$$\left((1+p) \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] +$$

$$2 \left(-2 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] +$$

$$p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right) -$$

$$\left(4b^2(3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) / \left((1+p) \right.$$

$$\left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left(-3 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] +$$

$$p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right) +$$

$$\left(4ab(4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)\right) /$$

$$\left((2+p) \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] +$$

$$\begin{aligned}
& 2 \left(-2 \operatorname{AppellF1} \left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. p \operatorname{AppellF1} \left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) \\
& \tan[e+fx]^{-p} (g \tan[e+fx])^p \left(-\frac{1}{4} b^2 \cos[2(e+fx)]^3 \tan[e+fx]^p + \frac{1}{4} i b^2 \sin[2(e+fx)] \tan[e+fx]^p + \right. \\
& \quad i a^2 \sin[e+fx]^2 \sin[2(e+fx)] \tan[e+fx]^p + \\
& \quad \frac{1}{2} b^2 \sin[2(e+fx)]^2 \tan[e+fx]^p - \\
& \quad \frac{1}{4} i b^2 \sin[2(e+fx)]^3 \tan[e+fx]^p + \\
& \quad \cos[e+fx]^2 (a^2 \cos[2(e+fx)] \tan[e+fx]^p - i a^2 \sin[2(e+fx)] \tan[e+fx]^p) + \\
& \quad \cos[2(e+fx)]^2 \left(\frac{1}{2} b^2 \tan[e+fx]^p + a b \sin[e+fx] \tan[e+fx]^p - \frac{1}{4} i b^2 \sin[2(e+fx)] \tan[e+fx]^p \right) + \\
& \quad \sin[e+fx] (i a b \sin[2(e+fx)] \tan[e+fx]^p + a b \sin[2(e+fx)]^2 \tan[e+fx]^p) + \\
& \quad \cos[2(e+fx)] \left(-\frac{1}{4} b^2 \tan[e+fx]^p - a b \sin[e+fx] \tan[e+fx]^p - a^2 \sin[e+fx]^2 \tan[e+fx]^p - \frac{1}{4} b^2 \sin[2(e+fx)]^2 \tan[e+fx]^p \right) + \\
& \quad \cos[e+fx] (-i a b \cos[2(e+fx)]^2 \tan[e+fx]^p + a b \sin[2(e+fx)] \tan[e+fx]^p + 2 a^2 \sin[e+fx] \sin[2(e+fx)] \tan[e+fx]^p - \\
& \quad i a b \sin[2(e+fx)]^2 \tan[e+fx]^p + \cos[2(e+fx)] (i a b \tan[e+fx]^p + 2 i a^2 \sin[e+fx] \tan[e+fx]^p) \Bigg) \Bigg) / \\
& \left(f \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right)^3 \left(-\frac{1}{\left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^4} 3 \times 2^{1+p} \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right]^2 \left(-\frac{\tan \left[\frac{1}{2} (e+fx) \right]}{-1 + \tan \left[\frac{1}{2} (e+fx) \right]^2} \right)^p \right. \\
& \quad \left(\left(a^2 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \left((1+p) \right. \right. \\
& \quad \left. \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) + \\
& \quad \left(4 b^2 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) / \left((1+p) \left((3+p) \right. \right. \\
& \quad \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + 2 \left(-2 \operatorname{AppellF1} \left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \Bigg) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left(4 a b (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \tan \left[\frac{1}{2} (e+f x) \right] \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left((2+p) \left((4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \right. \\
& \quad 2 \left(-2 \operatorname{AppellF1} \left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. \left. p \operatorname{AppellF1} \left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) + \frac{1}{\left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^3} \\
& 2^{1+p} p \tan \left[\frac{1}{2} (e+f x) \right] \left(-\frac{\tan \left[\frac{1}{2} (e+f x) \right]}{-1 + \tan \left[\frac{1}{2} (e+f x) \right]^2} \right)^{-1+p} \left(\frac{\operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right]^2}{\left(-1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^2} - \frac{\operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2}{2 \left(-1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)} \right) \\
& \left(\left(a^2 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) / \left((1+p) \right. \right. \\
& \quad \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) + \\
& \left(4 b^2 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left((1+p) \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \right. \\
& \quad 2 \left(-2 \operatorname{AppellF1} \left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. \left. p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) - \\
& \left(4 b^2 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) / \left((1+p) \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, \right. \right. \right. \\
& \quad \left. \left. \left. 3, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + 2 \left(-3 \operatorname{AppellF1} \left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) + \\
& \left(4 a b (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \tan \left[\frac{1}{2} (e+f x) \right] \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) / \\
& \left((2+p) \left((4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(-2 \operatorname{AppellF1} \left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. p \operatorname{AppellF1} \left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) + \\
& \frac{1}{\left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^3} 2^{1+p} \tan \left[\frac{1}{2} (e+fx) \right] \left(-\frac{\tan \left[\frac{1}{2} (e+fx) \right]}{-1 + \tan \left[\frac{1}{2} (e+fx) \right]^2} \right)^p \left(\left(2 a^2 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \right) / \left((1+p) \right. \\
& \quad \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) + \\
& \left(a^2 (3+p) \left(-1 / (3+p) (1+p) \operatorname{AppellF1} \left[1 + \frac{1+p}{2}, p, 2, 1 + \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + 1 / (3+p) p (1+p) \operatorname{AppellF1} \left[1 + \frac{1+p}{2}, 1+p, 1, 1 + \frac{3+p}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) \left(1 + \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 \right) / \left((1+p) \right. \\
& \quad \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) + \\
& \left(4 b^2 (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) / \\
& \left((1+p) \left((3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \left(-2 \operatorname{AppellF1} \left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. p \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) + \\
& \left(4 b^2 (3+p) \left(-1 / (3+p) 2 (1+p) \operatorname{AppellF1} \left[1 + \frac{1+p}{2}, p, 3, 1 + \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + 1 / (3+p) p (1+p) \operatorname{AppellF1} \left[1 + \frac{1+p}{2}, 1+p, 2, 1 + \frac{3+p}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \Big/ \left((1+p) \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \right.\right.\right. \\
& \left.\left.\left.\frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left(-2 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right.\right. \\
& \left.\left.\left.p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) - \\
& \left(4b^2(3+p) \left(-1/(3+p)3(1+p) \operatorname{AppellF1}\left[1 + \frac{1+p}{2}, p, 4, 1 + \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right.\right. \\
& \left.\left.\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + 1/(3+p)p(1+p) \operatorname{AppellF1}\left[1 + \frac{1+p}{2}, 1+p, 3, 1 + \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right.\right.\right. \\
& \left.\left.\left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) \Big/ \left((1+p) \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \right.\right.\right. \\
& \left.\left.\left.\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left(-3 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 4, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right.\right. \\
& \left.\left.\left.p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) + \\
& \left(4ab(4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \Big/ \\
& \left((2+p) \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right. \\
& \left.\left.2 \left(-2 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right.\right. \\
& \left.\left.\left.p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) + \\
& \left(2ab(4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)\right) \Big/ \\
& \left((2+p) \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right. \\
& \left.\left.2 \left(-2 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right.\right. \\
& \left.\left.\left.p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) + \\
& \left(4ab(4+p) \tan\left[\frac{1}{2}(e+fx)\right] \left(-1/(4+p)2(2+p) \operatorname{AppellF1}\left[1 + \frac{2+p}{2}, p, 3, 1 + \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right.\right. \\
& \left.\left.\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + 1/(4+p)p(2+p) \operatorname{AppellF1}\left[1 + \frac{2+p}{2}, 1+p, 2, 1 + \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right.\right.\right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \Big/ \left((2+p) \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \right. \right. \right. \\
& \left. \left. \left. \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left(-2 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) - \right. \\
& \left(a^2 (3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right. \\
& \left. \left(-2 \left(\operatorname{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 1, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + (3+p) \left(-\frac{1}{3+p} \right. \right. \\
& \left. \left. (1+p) \operatorname{AppellF1}\left[1 + \frac{1+p}{2}, p, 2, 1 + \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3+p} \right. \right. \\
& \left. \left. p (1+p) \operatorname{AppellF1}\left[1 + \frac{1+p}{2}, 1+p, 1, 1 + \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) - \\
& 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{1}{5+p} 2 (3+p) \operatorname{AppellF1}\left[1 + \frac{3+p}{2}, p, 3, 1 + \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+p} p (3+p) \operatorname{AppellF1}\left[1 + \frac{3+p}{2}, 1+p, 2, 1 + \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - p \left(-\frac{1}{5+p} (3+p) \operatorname{AppellF1}\left[1 + \frac{3+p}{2}, 1+p, 2, 1 + \frac{5+p}{2}, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+p} (1+p) (3+p) \operatorname{AppellF1}\left[1 + \frac{3+p}{2}, \right. \right. \\
& \left. \left. 2+p, 1, 1 + \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big/ \left((1+p) \right. \\
& \left. \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(4 b^2 (3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 2, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right. \\
& \left. \left(2 \left(-2 \operatorname{AppellF1}\left[\frac{3+p}{2}, p, 3, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 2, \frac{5+p}{2}, \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + p \left(-\frac{1}{5+p} 3(3+p) \operatorname{AppellF1}\left[1 + \frac{3+p}{2}, 1+p, 4, 1 + \frac{5+p}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+p} (1+p)(3+p) \operatorname{AppellF1}\left[1 + \frac{3+p}{2}, \right. \right. \\
& \left. \left. 2+p, 3, 1 + \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big/ \\
& \left((1+p) \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 3, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + 2 \left(-3 \operatorname{AppellF1}\left[\frac{3+p}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. p, 4, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + \right. \right. \right. \\
& \left. \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 3, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(4ab(4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \tan\left[\frac{1}{2}(e+fx)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \\
& \left(2 \left(-2 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + (4+p) \left(-\frac{1}{4+p} 2(2+p) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[1 + \frac{2+p}{2}, p, 3, 1 + \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{4+p} \right. \right. \\
& \left. \left. p(2+p) \operatorname{AppellF1}\left[1 + \frac{2+p}{2}, 1+p, 2, 1 + \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-2 \left(-\frac{1}{6+p} 3(4+p) \operatorname{AppellF1}\left[1 + \frac{4+p}{2}, p, 4, 1 + \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} p(4+p) \operatorname{AppellF1}\left[1 + \frac{4+p}{2}, 1+p, 3, 1 + \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + p \left(-\frac{1}{6+p} 2(4+p) \operatorname{AppellF1}\left[1 + \frac{4+p}{2}, 1+p, 3, 1 + \frac{6+p}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} (1+p)(4+p) \operatorname{AppellF1}\left[1 + \frac{4+p}{2}, \right. \right. \right. \\
& \left. \left. \left. 2+p, 2, 1 + \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \Big/ \\
& \left((2+p) \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + 2 \left(-2 \operatorname{AppellF1}\left[\frac{4+p}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 + p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) - \\
& \left(a (3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left(-2 \left(\operatorname{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \left. \left. p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + (3+p) \left(-\frac{1}{3+p} \right. \right. \\
& \left. \left. (1+p) \operatorname{AppellF1}\left[1+\frac{1+p}{2}, p, 2, 1+\frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3+p} \right. \right. \\
& \left. \left. p (1+p) \operatorname{AppellF1}\left[1+\frac{1+p}{2}, 1+p, 1, 1+\frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - \right. \\
& \left. 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{1}{5+p} 2 (3+p) \operatorname{AppellF1}\left[1+\frac{3+p}{2}, p, 3, 1+\frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+p} p (3+p) \operatorname{AppellF1}\left[1+\frac{3+p}{2}, 1+p, 2, 1+\frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - p \left(-\frac{1}{5+p} (3+p) \operatorname{AppellF1}\left[1+\frac{3+p}{2}, 1+p, 2, 1+\frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+p} (1+p) (3+p) \operatorname{AppellF1}\left[1+\frac{3+p}{2}, 2+p, \right. \right. \\
& \left. \left. 1, 1+\frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big) \Big) / \left((1+p) \right. \\
& \left((3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, p, 1, \frac{3+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+p}{2}, p, 2, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - p \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+p, 1, \frac{5+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big) - \\
& \left(2 b (4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right] \right. \\
& \left(2 \left(-2 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + (4+p) \left(-\frac{1}{4+p} \right. \right. \\
& \left. \left. 2 (2+p) \operatorname{AppellF1}\left[1+\frac{2+p}{2}, p, 3, 1+\frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{4+p} \right. \right. \\
& \left. \left. p (2+p) \operatorname{AppellF1}\left[1+\frac{2+p}{2}, 1+p, 2, 1+\frac{4+p}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-2 \left(-\frac{1}{6+p} 3(4+p) \operatorname{AppellF1}\left[1+\frac{4+p}{2}, p, 4, 1+\frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} p(4+p) \operatorname{AppellF1}\left[1+\frac{4+p}{2}, 1+p, 3, 1+\frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + p \left(-\frac{1}{6+p} 2(4+p) \operatorname{AppellF1}\left[1+\frac{4+p}{2}, 1+p, 3, \right. \right. \\
& \quad \left. \left. 1+\frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{6+p} (1+p)(4+p) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[1+\frac{4+p}{2}, 2+p, 2, 1+\frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left((2+p) \left((4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, p, 2, \frac{4+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \right. \right. \\
& \quad \left. \left. \left(-2 \operatorname{AppellF1}\left[\frac{4+p}{2}, p, 3, \frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. \left. p \operatorname{AppellF1}\left[\frac{4+p}{2}, 1+p, 2, \frac{6+p}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \operatorname{Tan}[e+fx]^p \right) \Bigg)
\end{aligned}$$

■ **Problem 206: Result more than twice size of optimal antiderivative.**

$$\int \frac{(g \operatorname{Tan}[e+fx])^p}{a+b \operatorname{Sin}[e+fx]} dx$$

Optimal (type 6, 284 leaves, 0 steps):

$$\begin{aligned}
& \frac{1}{(a^2-b^2) f(-1+p)} \\
& a g \left(1 - \frac{b^2 \operatorname{Cos}[e+fx]^2}{-a^2+b^2} \right)^{\frac{1}{2}(-1+p)} \operatorname{Hypergeometric2F1}\left[\frac{1-p}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \frac{\operatorname{Cos}[e+fx]^2 - \frac{b^2 \operatorname{Cos}[e+fx]^2}{-a^2+b^2}}{1 - \frac{b^2 \operatorname{Cos}[e+fx]^2}{-a^2+b^2}}\right] (\operatorname{Sin}[e+fx]^2)^{\frac{1-p}{2}} (g \operatorname{Tan}[e+fx])^{-1+p} + \\
& \frac{1}{(-a^2+b^2) f(-1+p)} b \operatorname{AppellF1}\left[\frac{1-p}{2}, -\frac{p}{2}, 1, \frac{3-p}{2}, \operatorname{Cos}[e+fx]^2, \frac{b^2 \operatorname{Cos}[e+fx]^2}{-a^2+b^2}\right] \operatorname{Cos}[e+fx] (\operatorname{Sin}[e+fx]^2)^{-p/2} (g \operatorname{Tan}[e+fx])^p
\end{aligned}$$

Result (type 6, 3354 leaves):

$$\left(\operatorname{Tan}[e+fx]^{1+p} (g \operatorname{Tan}[e+fx])^p \right. \\
\left. \left(\frac{\operatorname{Hypergeometric2F1}\left[1, \frac{1+p}{2}, \frac{3+p}{2}, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right]}{a(1+p)} - \frac{\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+\frac{p}{2}, 2+\frac{p}{2}, -\operatorname{Tan}[e+fx]^2\right] \operatorname{Tan}[e+fx]}{2b+bp} \right) \right)$$

$$\begin{aligned}
& \left(a^2 (a^2 - b^2) (4 + p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 1, \frac{4+p}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x] \sqrt{1 + \tan[e + f x]^2} \right) / \\
& \left(b (2 + p) \left(a^2 (4 + p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 1, \frac{4+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 2, \frac{6+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{6+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2) \right) \right) \right) / \\
& \left(f (a + b \sin[e + f x]) \left((1 + p) \sec[e + f x]^2 \tan[e + f x]^p \left(\frac{\operatorname{Hypergeometric2F1} \left[1, \frac{1+p}{2}, \frac{3+p}{2}, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right]}{a (1 + p)} - \right. \right. \right. \\
& \quad \left. \left. \frac{\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 + \frac{p}{2}, 2 + \frac{p}{2}, -\tan[e + f x]^2 \right] \tan[e + f x]}{2 b + b p} - \right. \right. \\
& \quad \left. \left(a^2 (a^2 - b^2) (4 + p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 1, \frac{4+p}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x] \sqrt{1 + \tan[e + f x]^2} \right) / \right. \\
& \quad \left(b (2 + p) \left(a^2 (4 + p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 1, \frac{4+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 2, \frac{6+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{6+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2) \right) \right) \right) \right) + \\
& \tan[e + f x]^{1+p} \left(- \frac{\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 + \frac{p}{2}, 2 + \frac{p}{2}, -\tan[e + f x]^2 \right] \sec[e + f x]^2}{2 b + b p} - \right. \\
& \quad \left. \frac{2 \left(1 + \frac{p}{2} \right) \sec[e + f x]^2 \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 + \frac{p}{2}, 2 + \frac{p}{2}, -\tan[e + f x]^2 \right] + \frac{1}{\sqrt{1 + \tan[e + f x]^2}} \right)}{2 b + b p} + \right. \\
& \quad \left. \left(a^2 (a^2 - b^2) (4 + p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 1, \frac{4+p}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \tan[e + f x] \left(-2 a^2 \sec[e + f x]^2 \tan[e + f x] + 2 b^2 \sec[e + f x]^2 \tan[e + f x] \sqrt{1 + \tan[e + f x]^2} \right) \right] / \\
& \left(b (2 + p) \left(a^2 (4 + p) \operatorname{AppellF1} \left[\frac{2 + p}{2}, -\frac{1}{2}, 1, \frac{4 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4 + p}{2}, -\frac{1}{2}, 2, \frac{6 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4 + p}{2}, \frac{1}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \frac{6 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2) \right)^2 \right) - \\
& \left(a^2 (a^2 - b^2) (4 + p) \operatorname{AppellF1} \left[\frac{2 + p}{2}, -\frac{1}{2}, 1, \frac{4 + p}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \sec[e + f x]^2 \tan[e + f x]^2 \right) / \\
& \left(b (2 + p) \sqrt{1 + \tan[e + f x]^2} \left(a^2 (4 + p) \operatorname{AppellF1} \left[\frac{2 + p}{2}, -\frac{1}{2}, 1, \frac{4 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4 + p}{2}, -\frac{1}{2}, 2, \frac{6 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4 + p}{2}, \frac{1}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \frac{6 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2) \right) \right) - \\
& \left(a^2 (a^2 - b^2) (4 + p) \operatorname{AppellF1} \left[\frac{2 + p}{2}, -\frac{1}{2}, 1, \frac{4 + p}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \sec[e + f x]^2 \sqrt{1 + \tan[e + f x]^2} \right) / \\
& \left(b (2 + p) \left(a^2 (4 + p) \operatorname{AppellF1} \left[\frac{2 + p}{2}, -\frac{1}{2}, 1, \frac{4 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4 + p}{2}, -\frac{1}{2}, 2, \frac{6 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4 + p}{2}, \frac{1}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \frac{6 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2) \right) \right) - \\
& \left(a^2 (a^2 - b^2) (4 + p) \tan[e + f x] \left(\frac{1}{a^2 (4 + p)} - 2 (-a^2 + b^2) (2 + p) \operatorname{AppellF1} \left[1 + \frac{2 + p}{2}, -\frac{1}{2}, 2, 1 + \frac{4 + p}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \sec[e + f x]^2 \tan[e + f x] + \frac{1}{4 + p} (2 + p) \operatorname{AppellF1} \left[1 + \frac{2 + p}{2}, \frac{1}{2}, 1, 1 + \right. \right. \\
& \quad \left. \left. \frac{4 + p}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) \sqrt{1 + \tan[e + f x]^2} \right) / \\
& \left(b (2 + p) \left(a^2 (4 + p) \operatorname{AppellF1} \left[\frac{2 + p}{2}, -\frac{1}{2}, 1, \frac{4 + p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 2, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 1, \right. \right. \\
& \quad \left. \left. \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \left(b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2) \right) + \frac{1}{a} \\
& \operatorname{Csc}[e+fx] \operatorname{Sec}[e+fx] \left(-\operatorname{Hypergeometric2F1} \left[1, \frac{1+p}{2}, \frac{3+p}{2}, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \frac{1}{1 - \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2} \right) + \\
& \left(a^2 (a^2 - b^2) (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 1, \frac{4+p}{2}, -\tan[e+fx]^2, \frac{(-a^2+b^2) \tan[e+fx]^2}{a^2} \right] \tan[e+fx] \sqrt{1 + \tan[e+fx]^2} \right. \\
& \quad \left(2 \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 2, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 1, \frac{6+p}{2}, \right. \right. \right. \\
& \quad \left. \left. -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + a^2 (4+p) \left(\frac{1}{4+p} 2 \left(-1 + \frac{b^2}{a^2} \right) (2+p) \operatorname{AppellF1} \left[1 + \frac{2+p}{2}, \right. \right. \right. \\
& \quad \left. \left. -\frac{1}{2}, 2, 1 + \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{1}{4+p} (2+p) \operatorname{AppellF1} \left[1 + \frac{2+p}{2}, \frac{1}{2}, 1, \right. \right. \\
& \quad \left. \left. 1 + \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \tan[e+fx]^2 \left(-2 (a^2 - b^2) \left(\frac{1}{6+p} 4 \left(-1 + \frac{b^2}{a^2} \right) \right. \right. \\
& \quad \left. \left. (4+p) \operatorname{AppellF1} \left[1 + \frac{4+p}{2}, -\frac{1}{2}, 3, 1 + \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{1}{6+p} \right. \right. \\
& \quad \left. \left. (4+p) \operatorname{AppellF1} \left[1 + \frac{4+p}{2}, \frac{1}{2}, 2, 1 + \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + a^2 \left(\frac{1}{6+p} \right. \right. \\
& \quad \left. \left. 2 \left(-1 + \frac{b^2}{a^2} \right) (4+p) \operatorname{AppellF1} \left[1 + \frac{4+p}{2}, \frac{1}{2}, 2, 1 + \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \right. \right. \\
& \quad \left. \left. \frac{1}{6+p} (4+p) \operatorname{AppellF1} \left[1 + \frac{4+p}{2}, \frac{3}{2}, 1, 1 + \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \right) \Bigg) / \\
& \left(b (2+p) \left(a^2 (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 1, \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 2, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \left(b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2) \right) \right) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ Problem 207: Result more than twice size of optimal antiderivative.

$$\int \frac{(g \operatorname{Tan}[e + f x])^p}{(a + b \operatorname{Sin}[e + f x])^2} dx$$

Optimal (type 6, 737 leaves, 0 steps):

$$\begin{aligned} & \frac{1}{2(a^2 - b^2)^2(-a^2 + b^2)f} a^2 \operatorname{Cos}[e + f x] (1 - \operatorname{Cos}[e + f x]^2)^{\frac{1}{2}(-1+q)} \\ & \left(1 - \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right)^{-2 + \frac{3-q}{2} + \frac{1}{2}(-1+q)} \left((2(a^2 - b^2) + b^2(1+q) \operatorname{Cos}[e + f x]^2) \operatorname{HurwitzLerchPhi}\left[-\frac{a^2 \operatorname{Cot}[e + f x]^2}{a^2 - b^2}, 1, \frac{1-q}{2}\right] - \right. \\ & \left. b^2(-1+q) \operatorname{Cos}[e + f x]^2 \operatorname{HurwitzLerchPhi}\left[-\frac{a^2 \operatorname{Cot}[e + f x]^2}{a^2 - b^2}, 1, \frac{3-q}{2}\right] \right) \operatorname{Sin}[e + f x] (\operatorname{Sin}[e + f x]^2)^{\frac{1}{2}(-1-q)} (g \operatorname{Tan}[e + f x])^q - \\ & \frac{1}{(a^2 - b^2)^2 f (-1+q)} a^2 \operatorname{Cos}[e + f x] \left(1 - \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right)^{\frac{1}{2}(-1+q)} \operatorname{Hypergeometric2F1}\left[\frac{1-q}{2}, \frac{1-q}{2}, \frac{3-q}{2}, \frac{\operatorname{Cos}[e + f x]^2 - \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}}{1 - \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}}\right] \\ & \operatorname{Sin}[e + f x] (\operatorname{Sin}[e + f x]^2)^{\frac{1}{2}(-1-q)} (g \operatorname{Tan}[e + f x])^q + \frac{1}{(a^2 - b^2)^2 f (-1+q)} \\ & b^2 \operatorname{Cos}[e + f x] \left(1 - \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right)^{\frac{1}{2}(-1+q)} \operatorname{Hypergeometric2F1}\left[\frac{1-q}{2}, \frac{1-q}{2}, \frac{3-q}{2}, \frac{\operatorname{Cos}[e + f x]^2 - \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}}{1 - \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}}\right] \\ & \operatorname{Sin}[e + f x] (\operatorname{Sin}[e + f x]^2)^{\frac{1}{2}(-1-q)} (g \operatorname{Tan}[e + f x])^q - \frac{1}{(a^2 - b^2)^2 f (-1+q)} \\ & 2 a b \operatorname{AppellF1}\left[\frac{1-q}{2}, -\frac{q}{2}, 2, \frac{3-q}{2}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right] \operatorname{Cos}[e + f x] (\operatorname{Sin}[e + f x]^2)^{-q/2} (g \operatorname{Tan}[e + f x])^q \end{aligned}$$

Result (type 6, 3387 leaves):

$$\begin{aligned} & \left(\operatorname{Tan}[e + f x]^{1+p} (g \operatorname{Tan}[e + f x])^p \left(\frac{1}{a^2(1+p)} \left(-(a^2 + b^2) \operatorname{Hypergeometric2F1}\left[1, \frac{1+p}{2}, \frac{3+p}{2}, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] + \right. \right. \\ & \left. \left. 2 b^2 \operatorname{Hypergeometric2F1}\left[2, \frac{1+p}{2}, \frac{3+p}{2}, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \right) + \right. \\ & \left. \left(2 a^3 b (a^2 - b^2) (4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \operatorname{Tan}[e + f x] \sqrt{1 + \operatorname{Tan}[e + f x]^2} \right) / \right. \\ & \left. \left((2+p) \left(a^2 (4+p) \operatorname{AppellF1}\left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + \right. \right. \right. \\ & \left. \left. \left(-4(a^2 - b^2) \operatorname{AppellF1}\left[\frac{4+p}{2}, -\frac{1}{2}, 3, \frac{6+p}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{4+p}{2}, \frac{1}{2}, 2, \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right) \tan[e+fx]^2 \left(b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2) \right)^2 \right) \right) \Big/ \\
& \left((-a^2 + b^2) f (a + b \sin[e+fx])^2 \left(\frac{1}{-a^2 + b^2} (1+p) \sec[e+fx]^2 \tan[e+fx]^p \left(\frac{1}{a^2 (1+p)} \left(-(a^2 + b^2) \operatorname{Hypergeometric2F1} \left[1, \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{1+p}{2}, \frac{3+p}{2}, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] + 2 b^2 \operatorname{Hypergeometric2F1} \left[2, \frac{1+p}{2}, \frac{3+p}{2}, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \right) + \right. \\
& \left. \left(2 a^3 b (a^2 - b^2) (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \tan[e+fx] \sqrt{1 + \tan[e+fx]^2} \right) \right) \Big/ \\
& \left((2+p) \left(a^2 (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 3, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 2, \right. \right. \right. \\
& \left. \left. \left. \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right) \tan[e+fx]^2 \left(b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2) \right)^2 \right) \right) + \\
& \frac{1}{-a^2 + b^2} \tan[e+fx]^{1+p} \left(- \left(\left(4 a^3 b (a^2 - b^2) (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \right. \right. \right. \\
& \left. \left. \left. \tan[e+fx] \left(-2 a^2 \sec[e+fx]^2 \tan[e+fx] + 2 b^2 \sec[e+fx]^2 \tan[e+fx] \right) \sqrt{1 + \tan[e+fx]^2} \right) \right) \right) \Big/ \\
& \left((2+p) \left(a^2 (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 3, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 2, \right. \right. \right. \\
& \left. \left. \left. \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right) \tan[e+fx]^2 \left(b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2) \right)^3 \right) \right) + \\
& \left(2 a^3 b (a^2 - b^2) (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \sec[e+fx]^2 \tan[e+fx]^2 \right) \Big/ \\
& \left((2+p) \sqrt{1 + \tan[e+fx]^2} \left(a^2 (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 3, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right) \tan[e+fx]^2 \left(b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2) \right)^2 \Bigg) + \\
& \left(2a^3 b (a^2 - b^2) (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \frac{(-a^2+b^2) \tan[e+fx]^2}{a^2} \right] \operatorname{Sec}[e+fx]^2 \sqrt{1 + \tan[e+fx]^2} \right) \Bigg) / \\
& \left((2+p) \left(a^2 (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 3, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 2, \right. \right. \right. \\
& \left. \left. \left. \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right) \right) \tan[e+fx]^2 \left(b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2) \right)^2 \right) \Bigg) + \\
& \left(2a^3 b (a^2 - b^2) (4+p) \tan[e+fx] \left(\frac{1}{a^2 (4+p)} 4 (-a^2 + b^2) (2+p) \operatorname{AppellF1} \left[1 + \frac{2+p}{2}, -\frac{1}{2}, 3, 1 + \frac{4+p}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{(-a^2+b^2) \tan[e+fx]^2}{a^2} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{1}{4+p} (2+p) \operatorname{AppellF1} \left[1 + \frac{2+p}{2}, \frac{1}{2}, 2, \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{4+p}{2}, -\tan[e+fx]^2, \frac{(-a^2+b^2) \tan[e+fx]^2}{a^2} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \sqrt{1 + \tan[e+fx]^2} \right) \Bigg) / \\
& \left((2+p) \left(a^2 (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 3, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 2, \right. \right. \right. \\
& \left. \left. \left. \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right) \right) \tan[e+fx]^2 \left(b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2) \right)^2 \right) \Bigg) - \\
& \left(2a^3 b (a^2 - b^2) (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e+fx]^2, \frac{(-a^2+b^2) \tan[e+fx]^2}{a^2} \right] \tan[e+fx] \right. \\
& \left. \sqrt{1 + \tan[e+fx]^2} \left(2 \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 3, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \right. \\
& \left. \left. \left. a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 2, \frac{6+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right) \operatorname{Sec}[e+fx]^2 \tan[e+fx] + a^2 (4+p) \left(\frac{1}{4+p} \right. \right. \right. \\
& \left. \left. \left. 4 \left(-1 + \frac{b^2}{a^2} \right) (2+p) \operatorname{AppellF1} \left[1 + \frac{2+p}{2}, -\frac{1}{2}, 3, 1 + \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \right. \right. \right. \\
& \left. \left. \left. \frac{1}{4+p} (2+p) \operatorname{AppellF1} \left[1 + \frac{2+p}{2}, \frac{1}{2}, 2, 1 + \frac{4+p}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \tan[e + f x]^2 \left(-4 (a^2 - b^2) \left(\frac{1}{6+p} 6 \left(-1 + \frac{b^2}{a^2} \right) (4+p) \operatorname{AppellF1} \left[1 + \frac{4+p}{2}, -\frac{1}{2}, 4, 1 + \frac{6+p}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \frac{1}{6+p} (4+p) \operatorname{AppellF1} \left[1 + \frac{4+p}{2}, \frac{1}{2}, 3, 1 + \frac{6+p}{2}, \right. \right. \\
& \quad \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) + a^2 \left(\frac{1}{6+p} 4 \left(-1 + \frac{b^2}{a^2} \right) (4+p) \right. \\
& \quad \left. \operatorname{AppellF1} \left[1 + \frac{4+p}{2}, \frac{1}{2}, 3, 1 + \frac{6+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] - \frac{1}{6+p} \right. \\
& \quad \left. (4+p) \operatorname{AppellF1} \left[1 + \frac{4+p}{2}, \frac{3}{2}, 2, 1 + \frac{6+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \Bigg) \Bigg) / \\
& \left((2+p) \left(a^2 (4+p) \operatorname{AppellF1} \left[\frac{2+p}{2}, -\frac{1}{2}, 2, \frac{4+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+p}{2}, -\frac{1}{2}, 3, \frac{6+p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{4+p}{2}, \frac{1}{2}, 2, \frac{6+p}{2}, \right. \right. \right. \\
& \quad \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right)^2 (b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2))^2 \Bigg) + \frac{1}{a^2 (1+p)} \\
& \left(2 b^2 (1+p) \operatorname{Csc}[e + f x] \operatorname{Sec}[e + f x] \left(-\operatorname{Hypergeometric2F1} \left[2, \frac{1+p}{2}, \frac{3+p}{2}, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] + \frac{1}{\left(1 - \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right)^2} \right) - \right. \\
& \quad \left. (a^2 + b^2) (1+p) \operatorname{Csc}[e + f x] \operatorname{Sec}[e + f x] \left(-\operatorname{Hypergeometric2F1} \left[1, \frac{1+p}{2}, \frac{3+p}{2}, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] + \frac{1}{1 - \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2}} \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

Test results for the 837 problems in "4.1.2.1 (a+b sin)^m (c+d sin)^n.m"

- Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{\sin[x]^3}{a + a \sin[x]} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{3x}{2a} + \frac{2 \cos[x]}{a} - \frac{3 \cos[x] \sin[x]}{2a} + \frac{\cos[x] \sin[x]^2}{a + a \sin[x]}$$

Result (type 3, 87 leaves) :

$$\frac{1}{8 a (1 + \sin[x])} \left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right) \left(4 (1 + 3 x) \cos\left[\frac{x}{2}\right] + 3 \cos\left[\frac{3 x}{2}\right] + \cos\left[\frac{5 x}{2}\right] - 20 \sin\left[\frac{x}{2}\right] + 12 x \sin\left[\frac{x}{2}\right] + 3 \sin\left[\frac{3 x}{2}\right] - \sin\left[\frac{5 x}{2}\right] \right)$$

■ **Problem 6: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin[x]}{a + a \sin[x]} dx$$

Optimal (type 3, 17 leaves, 2 steps) :

$$\frac{x}{a} + \frac{\cos[x]}{a + a \sin[x]}$$

Result (type 3, 42 leaves) :

$$\frac{\left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right) \left(x \cos\left[\frac{x}{2}\right] + (-2 + x) \sin\left[\frac{x}{2}\right] \right)}{a (1 + \sin[x])}$$

■ **Problem 7: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{a + a \sin[x]} dx$$

Optimal (type 3, 12 leaves, 1 step) :

$$-\frac{\cos[x]}{a + a \sin[x]}$$

Result (type 3, 29 leaves) :

$$\frac{2 \sin\left[\frac{x}{2}\right] \left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right)}{a + a \sin[x]}$$

■ **Problem 8: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[x]}{a + a \sin[x]} dx$$

Optimal (type 3, 20 leaves, 3 steps) :

$$-\frac{\operatorname{ArcTanh}[\cos[x]]}{a} + \frac{\cos[x]}{a + a \sin[x]}$$

Result (type 3, 74 leaves) :

$$-\frac{\left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right) \left(\cos\left[\frac{x}{2}\right] \left(\log\left[\cos\left[\frac{x}{2}\right]\right] - \log\left[\sin\left[\frac{x}{2}\right]\right] \right) + \left(2 + \log\left[\cos\left[\frac{x}{2}\right]\right] - \log\left[\sin\left[\frac{x}{2}\right]\right] \right) \sin\left[\frac{x}{2}\right] \right)}{a (1 + \sin[x])}$$

■ **Problem 9: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]^2}{a + a \text{Sin}[x]} dx$$

Optimal (type 3, 26 leaves, 5 steps):

$$\frac{\text{ArcTanh}[\text{Cos}[x]]}{a} - \frac{2 \text{Cot}[x]}{a} + \frac{\text{Cot}[x]}{a + a \text{Sin}[x]}$$

Result (type 3, 63 leaves):

$$\frac{-\text{Cot}\left[\frac{x}{2}\right] + 2 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] - 2 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] + \frac{4 \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]} + \text{Tan}\left[\frac{x}{2}\right]}{2 a}$$

■ **Problem 11: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]^4}{a + a \text{Sin}[x]} dx$$

Optimal (type 3, 55 leaves, 6 steps):

$$\frac{3 \text{ArcTanh}[\text{Cos}[x]]}{2 a} - \frac{4 \text{Cot}[x]}{a} - \frac{4 \text{Cot}[x]^3}{3 a} + \frac{3 \text{Cot}[x] \text{Csc}[x]}{2 a} + \frac{\text{Cot}[x] \text{Csc}[x]^2}{a + a \text{Sin}[x]}$$

Result (type 3, 113 leaves):

$$\frac{1}{24 a} \left(-20 \text{Cot}\left[\frac{x}{2}\right] + 3 \text{Csc}\left[\frac{x}{2}\right]^2 + 36 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] - 36 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] - 3 \text{Sec}\left[\frac{x}{2}\right]^2 + 8 \text{Csc}[x]^3 \text{Sin}\left[\frac{x}{2}\right]^4 + \frac{48 \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]} - \frac{1}{2} \text{Csc}\left[\frac{x}{2}\right]^4 \text{Sin}[x] + 20 \text{Tan}\left[\frac{x}{2}\right] \right)$$

■ **Problem 17: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]}{(a + a \text{Sin}[x])^2} dx$$

Optimal (type 3, 38 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[\text{Cos}[x]]}{a^2} + \frac{4 \text{Cos}[x]}{3 a^2 (1 + \text{Sin}[x])} + \frac{\text{Cos}[x]}{3 (a + a \text{Sin}[x])^2}$$

Result (type 3, 129 leaves):

$$\frac{1}{6 a^2 (1 + \text{Sin}[x])^2} \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right) \left(\text{Cos}\left[\frac{3 x}{2}\right] \left(8 + 3 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] - 3 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \right) + \text{Cos}\left[\frac{x}{2}\right] \left(-6 - 9 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + 9 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \right) - 6 \left(3 + 2 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Cos}[x] \left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \right) - 2 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \right) \text{Sin}\left[\frac{x}{2}\right] \right)$$

■ **Problem 18: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]^2}{(a + a \text{Sin}[x])^2} dx$$

Optimal (type 3, 45 leaves, 6 steps):

$$\frac{2 \text{ArcTanh}[\text{Cos}[x]]}{a^2} - \frac{10 \text{Cot}[x]}{3 a^2} + \frac{2 \text{Cot}[x]}{a^2 (1 + \text{Sin}[x])} + \frac{\text{Cot}[x]}{3 (a + a \text{Sin}[x])^2}$$

Result (type 3, 166 leaves):

$$\frac{1}{6 (a + a \text{Sin}[x])^2} \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right) \left(4 \text{Sin}\left[\frac{x}{2}\right] - 2 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right) + 28 \text{Sin}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^2 - 3 \text{Cot}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^3 + \right. \\ \left. 12 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^3 - 12 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^3 + 3 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^3 \text{Tan}\left[\frac{x}{2}\right] \right)$$

■ **Problem 19: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]^3}{(a + a \text{Sin}[x])^2} dx$$

Optimal (type 3, 64 leaves, 7 steps):

$$-\frac{7 \text{ArcTanh}[\text{Cos}[x]]}{2 a^2} + \frac{16 \text{Cot}[x]}{3 a^2} - \frac{7 \text{Cot}[x] \text{Csc}[x]}{2 a^2} + \frac{8 \text{Cot}[x] \text{Csc}[x]}{3 a^2 (1 + \text{Sin}[x])} + \frac{\text{Cot}[x] \text{Csc}[x]}{3 (a + a \text{Sin}[x])^2}$$

Result (type 3, 203 leaves):

$$\frac{1}{24 a^2 (1 + \text{Sin}[x])^2} \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right) \left(-16 \text{Sin}\left[\frac{x}{2}\right] - 3 \left(1 + \text{Cot}\left[\frac{x}{2}\right] \right)^3 \text{Sin}\left[\frac{x}{2}\right] + 8 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right) - 160 \text{Sin}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^2 + 24 \text{Cot}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^3 - \right. \\ \left. 84 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^3 + 84 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^3 - 24 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right)^3 \text{Tan}\left[\frac{x}{2}\right] + 3 \text{Cos}\left[\frac{x}{2}\right] \left(1 + \text{Tan}\left[\frac{x}{2}\right] \right)^3 \right)$$

■ **Problem 20: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]^4}{(a + a \text{Sin}[x])^2} dx$$

Optimal (type 3, 65 leaves, 7 steps):

$$\frac{5 \text{ArcTanh}[\text{Cos}[x]]}{a^2} - \frac{4 \text{Cot}[x]}{a^2} - \frac{\text{Cot}[x]^3}{3 a^2} + \frac{\text{Cot}[x] \text{Csc}[x]}{a^2} - \frac{\text{Cos}[x]}{3 a^2 (1 + \text{Sin}[x])^2} - \frac{13 \text{Cos}[x]}{3 a^2 (1 + \text{Sin}[x])}$$

Result (type 3, 374 leaves):

$$\begin{aligned} & \frac{2 \operatorname{Sin}\left[\frac{x}{2}\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)}{3 (a + a \operatorname{Sin}[x])^2} - \frac{\left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^2}{3 (a + a \operatorname{Sin}[x])^2} + \frac{26 \operatorname{Sin}\left[\frac{x}{2}\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^3}{3 (a + a \operatorname{Sin}[x])^2} - \frac{11 \operatorname{Cot}\left[\frac{x}{2}\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4}{6 (a + a \operatorname{Sin}[x])^2} + \\ & \frac{\operatorname{Csc}\left[\frac{x}{2}\right]^2 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4}{4 (a + a \operatorname{Sin}[x])^2} - \frac{\operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^2 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4}{24 (a + a \operatorname{Sin}[x])^2} + \frac{5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4}{(a + a \operatorname{Sin}[x])^2} - \\ & \frac{5 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4}{(a + a \operatorname{Sin}[x])^2} - \frac{\operatorname{Sec}\left[\frac{x}{2}\right]^2 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4}{4 (a + a \operatorname{Sin}[x])^2} + \frac{11 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4 \operatorname{Tan}\left[\frac{x}{2}\right]}{6 (a + a \operatorname{Sin}[x])^2} + \frac{\operatorname{Sec}\left[\frac{x}{2}\right]^2 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4 \operatorname{Tan}\left[\frac{x}{2}\right]}{24 (a + a \operatorname{Sin}[x])^2} \end{aligned}$$

■ **Problem 28: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[x]}{(a + a \operatorname{Sin}[x])^3} dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cos}[x]]}{a^3} + \frac{\operatorname{Cos}[x]}{5 (a + a \operatorname{Sin}[x])^3} + \frac{7 \operatorname{Cos}[x]}{15 a (a + a \operatorname{Sin}[x])^2} + \frac{22 \operatorname{Cos}[x]}{15 (a^3 + a^3 \operatorname{Sin}[x])}$$

Result (type 3, 160 leaves):

$$\begin{aligned} & \frac{1}{15 (a + a \operatorname{Sin}[x])^3} \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right) \left(-6 \operatorname{Sin}\left[\frac{x}{2}\right] + 3 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right) - 14 \operatorname{Sin}\left[\frac{x}{2}\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^2 + 7 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^3 - \right. \\ & \left. 44 \operatorname{Sin}\left[\frac{x}{2}\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4 - 15 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^5 + 15 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^5\right) \end{aligned}$$

■ **Problem 29: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[x]^2}{(a + a \operatorname{Sin}[x])^3} dx$$

Optimal (type 3, 65 leaves, 7 steps):

$$\frac{3 \operatorname{ArcTanh}[\operatorname{Cos}[x]]}{a^3} - \frac{24 \operatorname{Cot}[x]}{5 a^3} + \frac{\operatorname{Cot}[x]}{5 (a + a \operatorname{Sin}[x])^3} + \frac{3 \operatorname{Cot}[x]}{5 a (a + a \operatorname{Sin}[x])^2} + \frac{3 \operatorname{Cot}[x]}{a^3 + a^3 \operatorname{Sin}[x]}$$

Result (type 3, 206 leaves):

$$\begin{aligned} & \frac{1}{10 (a + a \operatorname{Sin}[x])^3} \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right) \left(4 \operatorname{Sin}\left[\frac{x}{2}\right] - 2 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right) + 16 \operatorname{Sin}\left[\frac{x}{2}\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^2 - 8 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^3 + 76 \operatorname{Sin}\left[\frac{x}{2}\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^4 - \right. \\ & \left. 5 \operatorname{Cot}\left[\frac{x}{2}\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^5 + 30 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^5 - 30 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right] \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^5 + 5 \left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^5 \operatorname{Tan}\left[\frac{x}{2}\right]\right) \end{aligned}$$

■ **Problem 30: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]^3}{(a + a \text{Sin}[x])^3} dx$$

Optimal (type 3, 86 leaves, 8 steps):

$$-\frac{13 \text{ArcTanh}[\text{Cos}[x]]}{2 a^3} + \frac{152 \text{Cot}[x]}{15 a^3} - \frac{13 \text{Cot}[x] \text{Csc}[x]}{2 a^3} + \frac{\text{Cot}[x] \text{Csc}[x]}{5 (a + a \text{Sin}[x])^3} + \frac{11 \text{Cot}[x] \text{Csc}[x]}{15 a (a + a \text{Sin}[x])^2} + \frac{76 \text{Cot}[x] \text{Csc}[x]}{15 (a^3 + a^3 \text{Sin}[x])}$$

Result (type 3, 247 leaves):

$$\frac{1}{120 a^3 (1 + \text{Sin}[x])^3} \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right) \left(-48 \text{Sin}\left[\frac{x}{2}\right] - 15 \left(1 + \text{Cot}\left[\frac{x}{2}\right]\right)^5 \text{Sin}\left[\frac{x}{2}\right]^3 + 24 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right) - 272 \text{Sin}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^2 + 136 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^3 - 1712 \text{Sin}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^4 + 180 \text{Cot}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^5 - 780 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^5 + 780 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^5 - 180 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^5 \text{Tan}\left[\frac{x}{2}\right] + 15 \text{Cos}\left[\frac{x}{2}\right]^3 \left(1 + \text{Tan}\left[\frac{x}{2}\right]\right)^5 \right)$$

■ **Problem 31: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[x]^4}{(a + a \text{Sin}[x])^3} dx$$

Optimal (type 3, 103 leaves, 8 steps):

$$\frac{23 \text{ArcTanh}[\text{Cos}[x]]}{2 a^3} - \frac{136 \text{Cot}[x]}{5 a^3} - \frac{136 \text{Cot}[x]^3}{15 a^3} + \frac{23 \text{Cot}[x] \text{Csc}[x]}{2 a^3} + \frac{\text{Cot}[x] \text{Csc}[x]^2}{5 (a + a \text{Sin}[x])^3} + \frac{13 \text{Cot}[x] \text{Csc}[x]^2}{15 a (a + a \text{Sin}[x])^2} + \frac{23 \text{Cot}[x] \text{Csc}[x]^2}{3 (a^3 + a^3 \text{Sin}[x])}$$

Result (type 3, 299 leaves):

$$\frac{1}{120 a^3 (1 + \text{Sin}[x])^3} \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right) \left(48 \text{Sin}\left[\frac{x}{2}\right] - 5 \text{Cos}\left[\frac{x}{2}\right] \left(1 + \text{Cot}\left[\frac{x}{2}\right]\right)^5 \text{Sin}\left[\frac{x}{2}\right]^2 + 45 \left(1 + \text{Cot}\left[\frac{x}{2}\right]\right)^5 \text{Sin}\left[\frac{x}{2}\right]^3 - 24 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right) + 352 \text{Sin}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^2 - 176 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^3 + 2752 \text{Sin}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^4 - 400 \text{Cot}\left[\frac{x}{2}\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^5 + 1380 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^5 - 1380 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^5 + 400 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^5 \text{Tan}\left[\frac{x}{2}\right] - 45 \text{Cos}\left[\frac{x}{2}\right]^3 \left(1 + \text{Tan}\left[\frac{x}{2}\right]\right)^5 + 5 \text{Cos}\left[\frac{x}{2}\right]^2 \text{Sin}\left[\frac{x}{2}\right] \left(1 + \text{Tan}\left[\frac{x}{2}\right]\right)^5 \right)$$

■ **Problem 36: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \text{Sin}[c + d x]} dx$$

Optimal (type 3, 26 leaves, 1 step):

$$\frac{2 a \cos [c+d x]}{d \sqrt{a+a \sin [c+d x]}}$$

Result (type 3, 65 leaves):

$$\frac{2 \left(-\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right) \sqrt{a (1+\sin [c+d x])}}{d \left(\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right)}$$

■ **Problem 37: Result more than twice size of optimal antiderivative.**

$$\int \csc [c+d x] \sqrt{a+a \sin [c+d x]} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcTanh} \left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{a+a \sin [c+d x]}} \right]}{d}$$

Result (type 3, 94 leaves):

$$\left(\left(-\log \left[1 + \cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right] + \log \left[1 - \cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right] \right) \sqrt{a (1+\sin [c+d x])} \right) / \left(d \left(\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right) \right)$$

■ **Problem 38: Result more than twice size of optimal antiderivative.**

$$\int \csc [c+d x]^2 \sqrt{a+a \sin [c+d x]} dx$$

Optimal (type 3, 64 leaves, 3 steps):

$$\frac{\sqrt{a} \operatorname{ArcTanh} \left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{a+a \sin [c+d x]}} \right]}{d} - \frac{a \cot [c+d x]}{d \sqrt{a+a \sin [c+d x]}}$$

Result (type 3, 178 leaves):

$$\begin{aligned} & - \left(\csc \left[\frac{1}{2} (c+d x) \right] \right)^4 \sqrt{a (1+\sin [c+d x])} \left(2 \cos \left[\frac{1}{2} (c+d x) \right] - 2 \sin \left[\frac{1}{2} (c+d x) \right] + \right. \\ & \quad \left. \left(\log \left[1 + \cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right] - \log \left[1 - \cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right] \right) \sin [c+d x] \right) / \\ & \quad \left(d \left(1 + \cot \left[\frac{1}{2} (c+d x) \right] \right) \left(\csc \left[\frac{1}{4} (c+d x) \right] - \sec \left[\frac{1}{4} (c+d x) \right] \right) \left(\csc \left[\frac{1}{4} (c+d x) \right] + \sec \left[\frac{1}{4} (c+d x) \right] \right) \right) \end{aligned}$$

■ **Problem 39: Result more than twice size of optimal antiderivative.**

$$\int \csc [c+d x]^3 \sqrt{a+a \sin [c+d x]} dx$$

Optimal (type 3, 102 leaves, 4 steps) :

$$-\frac{3\sqrt{a}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{a+a\sin[c+dx]}}\right]}{4d} - \frac{3a\cot[c+dx]}{4d\sqrt{a+a\sin[c+dx]}} - \frac{a\cot[c+dx]\csc[c+dx]}{2d\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 249 leaves) :

$$\frac{1}{4d\left(1+\cot\left[\frac{1}{2}(c+dx)\right]\right)\left(\csc\left[\frac{1}{4}(c+dx)\right]^2-\sec\left[\frac{1}{4}(c+dx)\right]^2\right)^2} \csc\left[\frac{1}{2}(c+dx)\right]^7\sqrt{a(1+\sin[c+dx])}\left(-2\cos\left[\frac{1}{2}(c+dx)\right]-6\cos\left[\frac{3}{2}(c+dx)\right]-3\log\left[1+\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right]\right)+3\cos[2(c+dx)]\log\left[1+\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right]+3\log\left[1-\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right]-3\cos[2(c+dx)]\log\left[1-\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right]+2\sin\left[\frac{1}{2}(c+dx)\right]-6\sin\left[\frac{3}{2}(c+dx)\right]\right)$$

■ **Problem 40: Result more than twice size of optimal antiderivative.**

$$\int \csc[c+dx]^4 \sqrt{a+a\sin[c+dx]} dx$$

Optimal (type 3, 138 leaves, 5 steps) :

$$-\frac{5\sqrt{a}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{a+a\sin[c+dx]}}\right]}{8d} - \frac{5a\cot[c+dx]}{8d\sqrt{a+a\sin[c+dx]}} - \frac{5a\cot[c+dx]\csc[c+dx]}{12d\sqrt{a+a\sin[c+dx]}} - \frac{a\cot[c+dx]\csc[c+dx]^2}{3d\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 285 leaves) :

$$\frac{1}{24d\left(1+\cot\left[\frac{1}{2}(c+dx)\right]\right)\left(\csc\left[\frac{1}{4}(c+dx)\right]^2-\sec\left[\frac{1}{4}(c+dx)\right]^2\right)^3} \csc\left[\frac{1}{2}(c+dx)\right]^{10}\sqrt{a(1+\sin[c+dx])}\left(-84\cos\left[\frac{1}{2}(c+dx)\right]-10\cos\left[\frac{3}{2}(c+dx)\right]+30\cos\left[\frac{5}{2}(c+dx)\right]+84\sin\left[\frac{1}{2}(c+dx)\right]-45\log\left[1+\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right]\sin[c+dx]+45\log\left[1-\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right]\sin[c+dx]-10\sin\left[\frac{3}{2}(c+dx)\right]-30\sin\left[\frac{5}{2}(c+dx)\right]+15\log\left[1+\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right]\sin[3(c+dx)]-15\log\left[1-\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right]\sin[3(c+dx)]\right)$$

■ **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \csc[c+dx]\sqrt{a-a\sin[c+dx]} dx$$

Optimal (type 3, 38 leaves, 2 steps) :

$$\frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{a-a \sin [c+d x]}}\right]}{d}$$

Result (type 3, 97 leaves) :

$$\left(\left(\operatorname{Log}\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \sqrt{a - a \sin [c+d x]} \right) / \left(d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

■ **Problem 42: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx] \sqrt{-a+a \sin [c+d x]} dx$$

Optimal (type 3, 39 leaves, 2 steps) :

$$\frac{2 \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{-a+a \sin [c+d x]}}\right]}{d}$$

Result (type 3, 96 leaves) :

$$\left(\left(\operatorname{Log}\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \sqrt{a(-1 + \sin [c+d x])} \right) / \left(d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

■ **Problem 43: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx] \sqrt{-a-a \sin [c+d x]} dx$$

Optimal (type 3, 40 leaves, 2 steps) :

$$\frac{2 \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{-a-a \sin [c+d x]}}\right]}{d}$$

Result (type 3, 95 leaves) :

$$\left(\left(-\operatorname{Log}\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \sqrt{-a(1 + \sin [c+d x])} \right) / \left(d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

■ **Problem 49: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c + dx]^2 (a + a \text{Sin}[c + dx])^{3/2} dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$-\frac{3 a^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a} \text{Cos}[c+dx]}{\sqrt{a+a \text{Sin}[c+dx]}}\right]}{d} - \frac{a^2 \text{Cot}[c + dx]}{d \sqrt{a + a \text{Sin}[c + dx]}}$$

Result (type 3, 180 leaves):

$$-\left(a \text{Csc}\left[\frac{1}{2}(c + dx)\right]^4 \sqrt{a(1 + \text{Sin}[c + dx])} \left(2 \text{Cos}\left[\frac{1}{2}(c + dx)\right] - 2 \text{Sin}\left[\frac{1}{2}(c + dx)\right] \right) + \right. \\ \left. 3 \left(\text{Log}\left[1 + \text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \text{Log}\left[1 - \text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) \text{Sin}[c + dx] \right) / \\ \left(d \left(1 + \text{Cot}\left[\frac{1}{2}(c + dx)\right] \right) \left(\text{Csc}\left[\frac{1}{4}(c + dx)\right] - \text{Sec}\left[\frac{1}{4}(c + dx)\right] \right) \left(\text{Csc}\left[\frac{1}{4}(c + dx)\right] + \text{Sec}\left[\frac{1}{4}(c + dx)\right] \right) \right)$$

■ **Problem 50: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c + dx]^3 (a + a \text{Sin}[c + dx])^{3/2} dx$$

Optimal (type 3, 106 leaves, 5 steps):

$$-\frac{7 a^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a} \text{Cos}[c+dx]}{\sqrt{a+a \text{Sin}[c+dx]}}\right]}{4 d} - \frac{7 a^2 \text{Cot}[c + dx]}{4 d \sqrt{a + a \text{Sin}[c + dx]}} - \frac{a^2 \text{Cot}[c + dx] \text{Csc}[c + dx]}{2 d \sqrt{a + a \text{Sin}[c + dx]}}$$

Result (type 3, 250 leaves):

$$\frac{1}{4 d \left(1 + \text{Cot}\left[\frac{1}{2}(c + dx)\right] \right) \left(\text{Csc}\left[\frac{1}{4}(c + dx)\right]^2 - \text{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \right)^2} \\ a \text{Csc}\left[\frac{1}{2}(c + dx)\right]^7 \sqrt{a(1 + \text{Sin}[c + dx])} \left(6 \text{Cos}\left[\frac{1}{2}(c + dx)\right] - 14 \text{Cos}\left[\frac{3}{2}(c + dx)\right] - 7 \text{Log}\left[1 + \text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \right. \\ \left. 7 \text{Cos}[2(c + dx)] \text{Log}\left[1 + \text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + 7 \text{Log}\left[1 - \text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \right. \\ \left. 7 \text{Cos}[2(c + dx)] \text{Log}\left[1 - \text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - 6 \text{Sin}\left[\frac{1}{2}(c + dx)\right] - 14 \text{Sin}\left[\frac{3}{2}(c + dx)\right] \right)$$

■ **Problem 58: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c + dx]^3 (a + a \text{Sin}[c + dx])^{5/2} dx$$

Optimal (type 3, 106 leaves, 4 steps):

$$-\frac{19 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{a+a \sin [c+d x]}}\right]}{4 d}-\frac{9 a^3 \cot [c+d x]}{4 d \sqrt{a+a \sin [c+d x]}}-\frac{a^2 \cot [c+d x] \csc [c+d x] \sqrt{a+a \sin [c+d x]}}{2 d}$$

Result (type 3, 252 leaves):

$$\frac{1}{4 d\left(1+\cot \left[\frac{1}{2}(c+d x)\right]\right)\left(\csc \left[\frac{1}{4}(c+d x)\right]^2-\sec \left[\frac{1}{4}(c+d x)\right]^2\right)^2} a^2 \csc \left[\frac{1}{2}(c+d x)\right]^7 \sqrt{a(1+\sin [c+d x])}\left(14 \cos \left[\frac{1}{2}(c+d x)\right]-22 \cos \left[\frac{3}{2}(c+d x)\right]-19 \log \left[1+\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]+19 \cos [2(c+d x)] \log \left[1+\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]+19 \log \left[1-\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]-19 \cos [2(c+d x)] \log \left[1-\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]-14 \sin \left[\frac{1}{2}(c+d x)\right]-22 \sin \left[\frac{3}{2}(c+d x)\right]\right)$$

■ **Problem 60: Result more than twice size of optimal antiderivative.**

$$\int \csc [c+d x]^5(a+a \sin [c+d x])^{5/2} d x$$

Optimal (type 3, 182 leaves, 6 steps):

$$-\frac{163 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{a+a \sin [c+d x]}}\right]}{64 d}-\frac{163 a^3 \cot [c+d x]}{64 d \sqrt{a+a \sin [c+d x]}}-\frac{163 a^3 \cot [c+d x] \csc [c+d x]}{96 d \sqrt{a+a \sin [c+d x]}}-\frac{17 a^3 \cot [c+d x] \csc [c+d x]^2}{24 d \sqrt{a+a \sin [c+d x]}}-\frac{a^2 \cot [c+d x] \csc [c+d x]^3 \sqrt{a+a \sin [c+d x]}}{4 d}$$

Result (type 3, 370 leaves):

$$\frac{1}{192 d\left(1+\cot \left[\frac{1}{2}(c+d x)\right]\right)\left(\csc \left[\frac{1}{4}(c+d x)\right]^2-\sec \left[\frac{1}{4}(c+d x)\right]^2\right)^4} a^2 \csc \left[\frac{1}{2}(c+d x)\right]^{13} \sqrt{a(1+\sin [c+d x])}\left(-1030 \cos \left[\frac{1}{2}(c+d x)\right]+3102 \cos \left[\frac{3}{2}(c+d x)\right]-326 \cos \left[\frac{5}{2}(c+d x)\right]-978 \cos \left[\frac{7}{2}(c+d x)\right]+1467 \log \left[1+\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]-1956 \cos [2(c+d x)] \log \left[1+\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]+489 \cos [4(c+d x)] \log \left[1+\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]-1467 \log \left[1-\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]+1956 \cos [2(c+d x)] \log \left[1-\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]-489 \cos [4(c+d x)] \log \left[1-\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]+1030 \sin \left[\frac{1}{2}(c+d x)\right]+3102 \sin \left[\frac{3}{2}(c+d x)\right]+326 \sin \left[\frac{5}{2}(c+d x)\right]-978 \sin \left[\frac{7}{2}(c+d x)\right]\right)$$

■ **Problem 61: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]^3}{\sqrt{a + a \sin[c + dx]}} dx$$

Optimal (type 3, 139 leaves, 6 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{\sqrt{a} d} - \frac{28 \cos[c+dx]}{15 d \sqrt{a+a \sin[c+dx]}} - \frac{2 \cos[c+dx] \sin[c+dx]^2}{5 d \sqrt{a+a \sin[c+dx]}} + \frac{2 \cos[c+dx] \sqrt{a+a \sin[c+dx]}}{15 a d}$$

Result (type 3, 150 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \left((-60 - 60i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] - 60 \cos\left[\frac{1}{2}(c+dx)\right] + 5 \cos\left[\frac{3}{2}(c+dx)\right] + 3 \cos\left[\frac{5}{2}(c+dx)\right] + 60 \sin\left[\frac{1}{2}(c+dx)\right] + 5 \sin\left[\frac{3}{2}(c+dx)\right] - 3 \sin\left[\frac{5}{2}(c+dx)\right] \right) \right) / \left(30 d \sqrt{a(1 + \sin[c+dx])} \right)$$

■ **Problem 62: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]^2}{\sqrt{a + a \sin[c + dx]}} dx$$

Optimal (type 3, 105 leaves, 4 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{\sqrt{a} d} + \frac{4 \cos[c+dx]}{3 d \sqrt{a+a \sin[c+dx]}} - \frac{2 \cos[c+dx] \sqrt{a+a \sin[c+dx]}}{3 a d}$$

Result (type 3, 105 leaves):

$$-\frac{1}{3 d \sqrt{a(1 + \sin[c+dx])}} \left((-6 - 6i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] - 2 \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 63: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]}{\sqrt{a + a \sin[c + dx]}} dx$$

Optimal (type 3, 72 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{\sqrt{a} d} - \frac{2 \cos[c+dx]}{d \sqrt{a+a \sin[c+dx]}}$$

Result (type 3, 98 leaves):

$$-\frac{1}{d\sqrt{a(1+\sin[c+dx])}}$$

$$2\left((1+i)(-1)^{3/4}\operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right]+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)$$

■ **Problem 64: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+a\sin[c+dx]}} dx$$

Optimal (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{2}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\operatorname{Cos}[c+dx]}{\sqrt{2}\sqrt{a+a\sin[c+dx]}}\right]}{\sqrt{a}d}$$

Result (type 3, 73 leaves):

$$\frac{(2+2i)(-1)^{3/4}\operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right]\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}{d\sqrt{a(1+\sin[c+dx])}}$$

■ **Problem 65: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Csc}[c+dx]}{\sqrt{a+a\sin[c+dx]}} dx$$

Optimal (type 3, 84 leaves, 5 steps):

$$-\frac{2\operatorname{ArcTanh}\left[\frac{\sqrt{a}\operatorname{Cos}[c+dx]}{\sqrt{a+a\sin[c+dx]}}\right]}{\sqrt{a}d} + \frac{\sqrt{2}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\operatorname{Cos}[c+dx]}{\sqrt{2}\sqrt{a+a\sin[c+dx]}}\right]}{\sqrt{a}d}$$

Result (type 3, 128 leaves):

$$-\frac{1}{d\sqrt{a(1+\sin[c+dx])}}\left((2+2i)(-1)^{3/4}\operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right]+\right.$$

$$\left.\operatorname{Log}\left[1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]-\operatorname{Log}\left[1-\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]\right)\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)$$

■ **Problem 66: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Csc}[c+dx]^2}{\sqrt{a+a\sin[c+dx]}} dx$$

Optimal (type 3, 109 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \text{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{\sqrt{a} d} - \frac{\text{Cot}[c+dx]}{d \sqrt{a+a \sin[c+dx]}}$$

Result (type 3, 168 leaves):

$$\frac{1}{4 d \sqrt{a} (1 + \sin[c + dx])} \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right] \right) \left((8 + 8 i) (-1)^{3/4} \text{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + dx)\right]\right)\right] - \text{Cot}\left[\frac{1}{4} (c + dx)\right] + 2 \text{Log}\left[1 + \cos\left[\frac{1}{2} (c + dx)\right] - \sin\left[\frac{1}{2} (c + dx)\right]\right] - 2 \text{Log}\left[1 - \cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right]\right] + 2 \text{Sec}\left[\frac{1}{2} (c + dx)\right] - \tan\left[\frac{1}{4} (c + dx)\right] \right)$$

■ **Problem 67: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c+dx]^3}{\sqrt{a+a \sin[c+dx]}} dx$$

Optimal (type 3, 146 leaves, 7 steps):

$$-\frac{7 \text{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{4 \sqrt{a} d} + \frac{\sqrt{2} \text{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{\sqrt{a} d} + \frac{\text{Cot}[c+dx]}{4 d \sqrt{a+a \sin[c+dx]}} - \frac{\text{Cot}[c+dx] \text{Csc}[c+dx]}{2 d \sqrt{a+a \sin[c+dx]}}$$

Result (type 3, 767 leaves):

$$\begin{aligned}
& - \frac{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]}{4d\sqrt{a(1+\sin[c+dx])}} + \frac{1}{d\sqrt{a(1+\sin[c+dx])}} \\
& (2+2i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right) + \\
& \frac{\operatorname{Cot}\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{8d\sqrt{a(1+\sin[c+dx])}} - \frac{\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{32d\sqrt{a(1+\sin[c+dx])}} - \\
& \frac{7 \operatorname{Log}\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{8d\sqrt{a(1+\sin[c+dx])}} + \\
& \frac{7 \operatorname{Log}\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{8d\sqrt{a(1+\sin[c+dx])}} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{32d\sqrt{a(1+\sin[c+dx])}} + \frac{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]}{16d \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)^2 \sqrt{a(1+\sin[c+dx])}} - \\
& \frac{\sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{4d \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right) \sqrt{a(1+\sin[c+dx])}} - \frac{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]}{16d \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)^2 \sqrt{a(1+\sin[c+dx])}} + \\
& \frac{\sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{4d \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right) \sqrt{a(1+\sin[c+dx])}} + \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{8d\sqrt{a(1+\sin[c+dx])}}
\end{aligned}$$

■ **Problem 68: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c+dx]^4}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$\begin{aligned}
& \frac{15 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a\sin[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} + \frac{\cos[c+dx] \sin[c+dx]^3}{2d(a+a\sin[c+dx])^{3/2}} - \\
& \frac{31 \cos[c+dx]}{5ad\sqrt{a+a\sin[c+dx]}} - \frac{9 \cos[c+dx] \sin[c+dx]^2}{10ad\sqrt{a+a\sin[c+dx]}} + \frac{13 \cos[c+dx] \sqrt{a+a\sin[c+dx]}}{10a^2 d}
\end{aligned}$$

Result (type 3, 178 leaves):

$$\frac{1}{20 d (a (1 + \sin[c + d x]))^{3/2}} \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) \\ \left(-55 \cos\left[\frac{1}{2} (c + d x)\right] - 41 \cos\left[\frac{3}{2} (c + d x)\right] - 3 \cos\left[\frac{5}{2} (c + d x)\right] + \cos\left[\frac{7}{2} (c + d x)\right] + 55 \sin\left[\frac{1}{2} (c + d x)\right] - (150 + 150 i) (-1)^{3/4} \right. \\ \left. \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + d x)\right]\right)\right] \right) (1 + \sin[c + d x]) - 41 \sin\left[\frac{3}{2} (c + d x)\right] + 3 \sin\left[\frac{5}{2} (c + d x)\right] + \sin\left[\frac{7}{2} (c + d x)\right] \right)$$

■ **Problem 69: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + d x]^3}{(a + a \sin[c + d x])^{3/2}} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$-\frac{11 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{2} \sqrt{a + a \sin[c + d x]}}\right]}{2 \sqrt{2} a^{3/2} d} + \frac{\cos[c + d x] \sin[c + d x]^2}{2 d (a + a \sin[c + d x])^{3/2}} + \frac{13 \cos[c + d x]}{3 a d \sqrt{a + a \sin[c + d x]}} - \frac{7 \cos[c + d x] \sqrt{a + a \sin[c + d x]}}{6 a^2 d}$$

Result (type 3, 156 leaves):

$$\left(\left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) \left(11 \cos\left[\frac{1}{2} (c + d x)\right] + 7 \cos\left[\frac{3}{2} (c + d x)\right] + \cos\left[\frac{5}{2} (c + d x)\right] - \right. \right. \\ \left. 11 \sin\left[\frac{1}{2} (c + d x)\right] + (33 + 33 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + d x)\right]\right)\right] \right) (1 + \sin[c + d x]) + \\ \left. 7 \sin\left[\frac{3}{2} (c + d x)\right] - \sin\left[\frac{5}{2} (c + d x)\right] \right) / (6 d (a (1 + \sin[c + d x]))^{3/2})$$

■ **Problem 70: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + d x]^2}{(a + a \sin[c + d x])^{3/2}} dx$$

Optimal (type 3, 105 leaves, 4 steps):

$$\frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{2} \sqrt{a + a \sin[c + d x]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\cos[c + d x]}{2 d (a + a \sin[c + d x])^{3/2}} - \frac{2 \cos[c + d x]}{a d \sqrt{a + a \sin[c + d x]}}$$

Result (type 3, 134 leaves):

$$-\left(\left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) \left(3 \cos\left[\frac{1}{2} (c + d x)\right] + 2 \cos\left[\frac{3}{2} (c + d x)\right] - 3 \sin\left[\frac{1}{2} (c + d x)\right] + \right. \right. \\ \left. (7 + 7 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + d x)\right]\right)\right] \right) (1 + \sin[c + d x]) + 2 \sin\left[\frac{3}{2} (c + d x)\right] \right) / (2 d (a (1 + \sin[c + d x]))^{3/2})$$

■ **Problem 71: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]}{(a + a \sin[c + dx])^{3/2}} dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} + \frac{\cos[c + dx]}{2 d (a + a \sin[c + dx])^{3/2}}$$

Result (type 3, 108 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] + (3+3i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] (1 + \sin[c+dx]) \right) \right) / (2 d (a (1 + \sin[c+dx]))^{3/2})$$

■ **Problem 72: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \sin[c + dx])^{3/2}} dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\cos[c + dx]}{2 d (a + a \sin[c + dx])^{3/2}}$$

Result (type 3, 108 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(-\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] + (1+i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] (1 + \sin[c+dx]) \right) \right) / (2 d (a (1 + \sin[c+dx]))^{3/2})$$

■ **Problem 73: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\csc[c + dx]}{(a + a \sin[c + dx])^{3/2}} dx$$

Optimal (type 3, 114 leaves, 6 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{a^{3/2} d} + \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} + \frac{\cos[c + dx]}{2 d (a + a \sin[c + dx])^{3/2}}$$

Result (type 3, 223 leaves) :

$$\frac{1}{2 d (a (1 + \sin[c + d x]))^{3/2}} \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right] - (5 + 5 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + d x)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 - 2 \operatorname{Log}\left[1 + \cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 + 2 \operatorname{Log}\left[1 - \cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2$$

■ **Problem 74: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c + d x]^2}{(a + a \sin[c + d x])^{3/2}} dx$$

Optimal (type 3, 144 leaves, 7 steps) :

$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{a + a \sin[c + d x]}}\right]}{a^{3/2} d} - \frac{9 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{2} \sqrt{a + a \sin[c + d x]}}\right]}{2 \sqrt{2} a^{3/2} d} + \frac{\operatorname{Cot}[c + d x]}{2 d (a + a \sin[c + d x])^{3/2}} - \frac{3 \operatorname{Cot}[c + d x]}{2 a d \sqrt{a + a \sin[c + d x]}}$$

Result (type 3, 449 leaves) :

$$\frac{1}{4 d (a (1 + \sin[c + d x]))^{3/2}} \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) \left(4 \sin\left[\frac{1}{2} (c + d x)\right] - 2 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) + 2 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 + (18 + 18 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + d x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 - \operatorname{Cot}\left[\frac{1}{4} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 + 6 \operatorname{Log}\left[1 + \cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 - 6 \operatorname{Log}\left[1 - \cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 + \frac{2 \sin\left[\frac{1}{4} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2}{\cos\left[\frac{1}{4} (c + d x)\right] - \sin\left[\frac{1}{4} (c + d x)\right]} - \frac{2 \sin\left[\frac{1}{4} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2}{\cos\left[\frac{1}{4} (c + d x)\right] + \sin\left[\frac{1}{4} (c + d x)\right]} - \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right] \right)$$

■ **Problem 75: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + dx]^3}{(a + a \text{Sin}[c + dx])^{3/2}} dx$$

Optimal (type 3, 186 leaves, 8 steps):

$$-\frac{19 \text{ArcTanh}\left[\frac{\sqrt{a} \text{Cos}[c+dx]}{\sqrt{a+a \text{Sin}[c+dx]}}\right]}{4 a^{3/2} d} + \frac{13 \text{ArcTanh}\left[\frac{\sqrt{a} \text{Cos}[c+dx]}{\sqrt{2} \sqrt{a+a \text{Sin}[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} + \frac{\text{Cot}[c + dx] \text{Csc}[c + dx]}{2 d (a + a \text{Sin}[c + dx])^{3/2}} + \frac{7 \text{Cot}[c + dx]}{4 a d \sqrt{a + a \text{Sin}[c + dx]}} - \frac{\text{Cot}[c + dx] \text{Csc}[c + dx]}{a d \sqrt{a + a \text{Sin}[c + dx]}}$$

Result (type 3, 889 leaves):

$$\begin{aligned} & -\frac{\text{Sin}\left[\frac{1}{2}(c + dx)\right] \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)}{d (a (1 + \text{Sin}[c + dx]))^{3/2}} + \\ & \frac{\left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2}{2 d (a (1 + \text{Sin}[c + dx]))^{3/2}} - \frac{3 \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{4 d (a (1 + \text{Sin}[c + dx]))^{3/2}} + \frac{1}{d (a (1 + \text{Sin}[c + dx]))^{3/2}} \\ & \left(\frac{13}{2} + \frac{13 i}{2}\right) (-1)^{3/4} \text{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \text{Sec}\left[\frac{1}{4}(c + dx)\right] \left(\text{Cos}\left[\frac{1}{4}(c + dx)\right] - \text{Sin}\left[\frac{1}{4}(c + dx)\right]\right)\right] \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3 + \\ & \frac{3 \text{Cot}\left[\frac{1}{4}(c + dx)\right] \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{8 d (a (1 + \text{Sin}[c + dx]))^{3/2}} - \frac{\text{Csc}\left[\frac{1}{4}(c + dx)\right]^2 \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{32 d (a (1 + \text{Sin}[c + dx]))^{3/2}} - \\ & \frac{19 \text{Log}\left[1 + \text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{8 d (a (1 + \text{Sin}[c + dx]))^{3/2}} + \\ & \frac{19 \text{Log}\left[1 - \text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{8 d (a (1 + \text{Sin}[c + dx]))^{3/2}} + \\ & \frac{\text{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{32 d (a (1 + \text{Sin}[c + dx]))^{3/2}} + \frac{\left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{16 d \left(\text{Cos}\left[\frac{1}{4}(c + dx)\right] - \text{Sin}\left[\frac{1}{4}(c + dx)\right]\right)^2 (a (1 + \text{Sin}[c + dx]))^{3/2}} - \\ & \frac{3 \text{Sin}\left[\frac{1}{4}(c + dx)\right] \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{4 d \left(\text{Cos}\left[\frac{1}{4}(c + dx)\right] - \text{Sin}\left[\frac{1}{4}(c + dx)\right]\right) (a (1 + \text{Sin}[c + dx]))^{3/2}} - \frac{\left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{16 d \left(\text{Cos}\left[\frac{1}{4}(c + dx)\right] + \text{Sin}\left[\frac{1}{4}(c + dx)\right]\right)^2 (a (1 + \text{Sin}[c + dx]))^{3/2}} + \\ & \frac{3 \text{Sin}\left[\frac{1}{4}(c + dx)\right] \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3}{4 d \left(\text{Cos}\left[\frac{1}{4}(c + dx)\right] + \text{Sin}\left[\frac{1}{4}(c + dx)\right]\right) (a (1 + \text{Sin}[c + dx]))^{3/2}} + \frac{3 \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3 \text{Tan}\left[\frac{1}{4}(c + dx)\right]}{8 d (a (1 + \text{Sin}[c + dx]))^{3/2}} \end{aligned}$$

■ **Problem 76: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sin}[c + dx]^5}{(a + a \text{Sin}[c + dx])^{5/2}} dx$$

Optimal (type 3, 221 leaves, 8 steps) :

$$\frac{283 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{\cos[c+dx] \sin[c+dx]^4}{4 d (a+a \sin[c+dx])^{5/2}} + \frac{21 \cos[c+dx] \sin[c+dx]^3}{16 a d (a+a \sin[c+dx])^{3/2}} -$$

$$\frac{1729 \cos[c+dx]}{120 a^2 d \sqrt{a+a \sin[c+dx]}} - \frac{157 \cos[c+dx] \sin[c+dx]^2}{80 a^2 d \sqrt{a+a \sin[c+dx]}} + \frac{787 \cos[c+dx] \sqrt{a+a \sin[c+dx]}}{240 a^3 d}$$

Result (type 3, 221 leaves) :

$$-\frac{1}{480 d (a (1 + \sin[c+dx]))^{5/2}} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)$$

$$\left(2547 \cos\left[\frac{1}{2}(c+dx)\right] + 3603 \cos\left[\frac{3}{2}(c+dx)\right] - 872 \cos\left[\frac{5}{2}(c+dx)\right] + 52 \cos\left[\frac{7}{2}(c+dx)\right] + 12 \cos\left[\frac{9}{2}(c+dx)\right] - 2547 \sin\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$\left. (8490 + 8490 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 +$$

$$3603 \sin\left[\frac{3}{2}(c+dx)\right] + 872 \sin\left[\frac{5}{2}(c+dx)\right] + 52 \sin\left[\frac{7}{2}(c+dx)\right] - 12 \sin\left[\frac{9}{2}(c+dx)\right] \left. \right)$$

■ **Problem 77: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c+dx]^4}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 183 leaves, 7 steps) :

$$-\frac{163 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{\cos[c+dx] \sin[c+dx]^3}{4 d (a+a \sin[c+dx])^{5/2}} +$$

$$\frac{17 \cos[c+dx] \sin[c+dx]^2}{16 a d (a+a \sin[c+dx])^{3/2}} + \frac{197 \cos[c+dx]}{24 a^2 d \sqrt{a+a \sin[c+dx]}} - \frac{95 \cos[c+dx] \sqrt{a+a \sin[c+dx]}}{48 a^3 d}$$

Result (type 3, 197 leaves) :

$$\frac{1}{96 d (a (1 + \sin[c+dx]))^{5/2}} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(279 \cos\left[\frac{1}{2}(c+dx)\right] + 399 \cos\left[\frac{3}{2}(c+dx)\right] - 88 \cos\left[\frac{5}{2}(c+dx)\right] + 8 \cos\left[\frac{7}{2}(c+dx)\right] - \right.$$

$$279 \sin\left[\frac{1}{2}(c+dx)\right] + (978 + 978 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 +$$

$$399 \sin\left[\frac{3}{2}(c+dx)\right] + 88 \sin\left[\frac{5}{2}(c+dx)\right] + 8 \sin\left[\frac{7}{2}(c+dx)\right] \left. \right)$$

■ **Problem 78: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]^3}{(a + a \sin[c + dx])^{5/2}} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$\frac{75 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{\cos[c+dx] \sin[c+dx]^2}{4 d (a + a \sin[c+dx])^{5/2}} - \frac{13 \cos[c+dx]}{16 a d (a + a \sin[c+dx])^{3/2}} - \frac{9 \cos[c+dx]}{4 a^2 d \sqrt{a + a \sin[c+dx]}}$$

Result (type 3, 173 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(-45 \cos\left[\frac{1}{2}(c+dx)\right] - 69 \cos\left[\frac{3}{2}(c+dx)\right] + 16 \cos\left[\frac{5}{2}(c+dx)\right] + 45 \sin\left[\frac{1}{2}(c+dx)\right] \right) - (150 + 150i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 - 69 \sin\left[\frac{3}{2}(c+dx)\right] - 16 \sin\left[\frac{5}{2}(c+dx)\right] \right) / (32 d (a (1 + \sin[c+dx]))^{5/2})$$

■ **Problem 79: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]^2}{(a + a \sin[c + dx])^{5/2}} dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$-\frac{19 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\cos[c+dx]}{4 d (a + a \sin[c+dx])^{5/2}} + \frac{13 \cos[c+dx]}{16 a d (a + a \sin[c+dx])^{3/2}}$$

Result (type 3, 196 leaves):

$$\frac{1}{16 d (a (1 + \sin[c+dx]))^{5/2}} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(8 \sin\left[\frac{1}{2}(c+dx)\right] - 4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) - 26 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + 13 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 + (19 + 19i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4$$

■ **Problem 80: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]}{(a + a \sin[c + dx])^{5/2}} dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$-\frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{\cos[c+dx]}{4 d (a+a \sin[c+dx])^{5/2}} - \frac{5 \cos[c+dx]}{16 a d (a+a \sin[c+dx])^{3/2}}$$

Result (type 3, 196 leaves):

$$\frac{1}{16 d (a (1 + \sin[c+dx]))^{5/2}} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(-8 \sin\left[\frac{1}{2}(c+dx)\right] + 4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) +$$

$$10 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 - 5 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 +$$

$$(5 + 5i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4$$

■ **Problem 81: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$-\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\cos[c+dx]}{4 d (a+a \sin[c+dx])^{5/2}} - \frac{3 \cos[c+dx]}{16 a d (a+a \sin[c+dx])^{3/2}}$$

Result (type 3, 196 leaves):

$$\frac{1}{16 d (a (1 + \sin[c+dx]))^{5/2}} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(8 \sin\left[\frac{1}{2}(c+dx)\right] - 4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) +$$

$$6 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 - 3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 +$$

$$(3 + 3i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4$$

■ **Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c+dx]}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 144 leaves, 7 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{a^{5/2} d} + \frac{43 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{\cos[c+dx]}{4 d (a+a \sin[c+dx])^{5/2}} + \frac{11 \cos[c+dx]}{16 a d (a+a \sin[c+dx])^{3/2}}$$

Result (type 3, 296 leaves):

$$\frac{1}{16 d (a (1 + \sin[c + d x]))^{5/2}} \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) \left(-8 \sin\left[\frac{1}{2} (c + d x)\right] + 4 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) - 22 \sin\left[\frac{1}{2} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 + 11 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^3 - (43 + 43 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + d x)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 - 16 \operatorname{Log}\left[1 + \cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 + 16 \operatorname{Log}\left[1 - \cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4$$

- **Problem 83: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c + d x]^2}{(a + a \sin[c + d x])^{5/2}} dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$\frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{a + a \sin[c + d x]}}\right]}{a^{5/2} d} - \frac{115 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{2} \sqrt{a + a \sin[c + d x]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{\operatorname{Cot}[c + d x]}{4 d (a + a \sin[c + d x])^{5/2}} + \frac{15 \operatorname{Cot}[c + d x]}{16 a d (a + a \sin[c + d x])^{3/2}} - \frac{35 \operatorname{Cot}[c + d x]}{16 a^2 d \sqrt{a + a \sin[c + d x]}}$$

Result (type 3, 509 leaves):

$$\begin{aligned}
& \frac{1}{16 d (a (1 + \sin[c + d x]))^{5/2}} \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) \\
& \left(8 \sin\left[\frac{1}{2} (c + d x)\right] - 4 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) + 38 \sin\left[\frac{1}{2} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 - \right. \\
& 19 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^3 + 8 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 + \\
& (115 + 115 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + d x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 - \\
& 4 \cot\left[\frac{1}{4} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 + 40 \log\left[1 + \cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right] \\
& \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 - 40 \log\left[1 - \cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 + \\
& \frac{8 \sin\left[\frac{1}{4} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4}{\cos\left[\frac{1}{4} (c + d x)\right] - \sin\left[\frac{1}{4} (c + d x)\right]} - \frac{8 \sin\left[\frac{1}{4} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4}{\cos\left[\frac{1}{4} (c + d x)\right] + \sin\left[\frac{1}{4} (c + d x)\right]} - \\
& \left. 4 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 \tan\left[\frac{1}{4} (c + d x)\right] \right)
\end{aligned}$$

■ **Problem 84: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c + d x]^3}{(a + a \sin[c + d x])^{5/2}} dx$$

Optimal (type 3, 224 leaves, 9 steps):

$$\begin{aligned}
& -\frac{39 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{a + a \sin[c + d x]}}\right]}{4 a^{5/2} d} + \frac{219 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{2} \sqrt{a + a \sin[c + d x]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{\cot[c + d x] \operatorname{Csc}[c + d x]}{4 d (a + a \sin[c + d x])^{5/2}} + \\
& \frac{19 \cot[c + d x] \operatorname{Csc}[c + d x]}{16 a d (a + a \sin[c + d x])^{3/2}} + \frac{63 \cot[c + d x]}{16 a^2 d \sqrt{a + a \sin[c + d x]}} - \frac{31 \cot[c + d x] \operatorname{Csc}[c + d x]}{16 a^2 d \sqrt{a + a \sin[c + d x]}}
\end{aligned}$$

Result (type 3, 680 leaves):

$$\begin{aligned}
& \frac{1}{32 d (a (1 + \sin[c + d x]))^{5/2}} \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) \\
& \left(-16 \sin\left[\frac{1}{2} (c + d x)\right] + 8 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) - 108 \sin\left[\frac{1}{2} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^2 \right. \\
& 54 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^3 - 40 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 - \\
& (438 + 438 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + d x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 + \\
& 20 \operatorname{Cot}\left[\frac{1}{4} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 - \operatorname{Csc}\left[\frac{1}{4} (c + d x)\right]^2 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 - \\
& 156 \operatorname{Log}\left[1 + \cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right]\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 + \\
& 156 \operatorname{Log}\left[1 - \cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right]\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 + \\
& \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^2 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 + \frac{2 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4}{\left(\cos\left[\frac{1}{4} (c + d x)\right] - \sin\left[\frac{1}{4} (c + d x)\right] \right)^2} - \\
& \frac{40 \sin\left[\frac{1}{4} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4}{\cos\left[\frac{1}{4} (c + d x)\right] - \sin\left[\frac{1}{4} (c + d x)\right]} - \frac{2 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4}{\left(\cos\left[\frac{1}{4} (c + d x)\right] + \sin\left[\frac{1}{4} (c + d x)\right] \right)^2} + \\
& \left. \frac{40 \sin\left[\frac{1}{4} (c + d x)\right] \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4}{\cos\left[\frac{1}{4} (c + d x)\right] + \sin\left[\frac{1}{4} (c + d x)\right]} + 20 \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^4 \tan\left[\frac{1}{4} (c + d x)\right] \right)
\end{aligned}$$

- **Problem 85: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \sin[e + f x]}}{\sqrt{\sin[e + f x]}} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$-\frac{2 \sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{a + a \sin[e + f x]}}\right]}{f}$$

Result (type 3, 143 leaves):

$$-\left(\sqrt{2} \sqrt{-1 + e^{2i(e+fx)}} \left(\text{ArcTan}\left[\frac{1}{\sqrt{-1 + e^{2i(e+fx)}}}\right] + i \text{Log}\left[e^{i(e+fx)} + \sqrt{-1 + e^{2i(e+fx)}}\right] \right) \sqrt{a(1 + \text{Sin}[e+fx])}\right) /$$

$$\left((i + e^{i(e+fx)}) \sqrt{-i e^{-i(e+fx)} (-1 + e^{2i(e+fx)})} f \right)$$

- **Problem 86: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a - a \text{Sin}[e+fx]}}{\sqrt{-\text{Sin}[e+fx]}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{2\sqrt{a} \text{ArcSin}\left[\frac{\sqrt{a} \text{Cos}[e+fx]}{\sqrt{a-a \text{Sin}[e+fx]}}\right]}{f}$$

Result (type 3, 143 leaves):

$$\frac{\sqrt{2} \sqrt{-1 + e^{2i(e+fx)}} \left(\text{ArcTan}\left[\frac{1}{\sqrt{-1 + e^{2i(e+fx)}}}\right] - i \text{Log}\left[e^{i(e+fx)} + \sqrt{-1 + e^{2i(e+fx)}}\right] \right) \sqrt{a - a \text{Sin}[e+fx]}}{(-i + e^{i(e+fx)}) \sqrt{i e^{-i(e+fx)} (-1 + e^{2i(e+fx)})} f}$$

- **Problem 87: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\text{Sin}[x]} \sqrt{1 + \text{Sin}[x]}} dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$-\sqrt{2} \text{ArcSin}\left[\frac{\text{Cos}[x]}{1 + \text{Sin}[x]}\right]$$

Result (type 4, 123 leaves):

$$\left(2 \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{x}{4}\right]}}\right], -1\right] + \text{EllipticPi}\left[1 - \sqrt{2}, -\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{x}{4}\right]}}\right], -1\right] + \text{EllipticPi}\left[1 + \sqrt{2}, -\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{x}{4}\right]}}\right], -1\right] \right) \right.$$

$$\left. \text{Sec}\left[\frac{x}{4}\right]^2 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right] \right) \sqrt{\text{Sin}[x]} \right) / \left(\sqrt{1 - \text{Cot}\left[\frac{x}{4}\right]^2} \sqrt{1 + \text{Sin}[x]} \text{Tan}\left[\frac{x}{4}\right]^{3/2} \right)$$

- **Problem 88: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\sin[x]} \sqrt{a + a \sin[x]}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[x]}{\sqrt{2} \sqrt{\sin[x]} \sqrt{a + a \sin[x]}}\right]}{\sqrt{a}}$$

Result (type 4, 125 leaves):

$$\left(2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{4}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[1 - \sqrt{2}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{4}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[1 + \sqrt{2}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{4}\right]}}\right], -1\right] \right) \right. \\ \left. \operatorname{Sec}\left[\frac{x}{4}\right]^2 \left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right) \sqrt{\sin[x]} \right) / \left(\sqrt{1 - \cot\left[\frac{x}{4}\right]^2} \sqrt{a(1 + \sin[x])} \tan\left[\frac{x}{4}\right]^{3/2} \right)$$

- **Problem 89: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1 - \sin[x]} \sqrt{\sin[x]}} dx$$

Optimal (type 3, 31 leaves, 2 steps):

$$\sqrt{2} \operatorname{ArcTanh}\left[\frac{\cos[x]}{\sqrt{2} \sqrt{1 - \sin[x]} \sqrt{\sin[x]}}\right]$$

Result (type 4, 125 leaves):

$$\left(2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{4}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[-1 - \sqrt{2}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{4}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[-1 + \sqrt{2}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{4}\right]}}\right], -1\right] \right) \right. \\ \left. \operatorname{Sec}\left[\frac{x}{4}\right]^2 \left(\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right) \sin[x] \right) / \left(\sqrt{1 - \cot\left[\frac{x}{4}\right]^2} \sqrt{-(-1 + \sin[x]) \sin[x]} \tan\left[\frac{x}{4}\right]^{3/2} \right)$$

- **Problem 90: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\sin[x]} \sqrt{a - a \sin[x]}} dx$$

Optimal (type 3, 42 leaves, 2 steps) :

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[x]}{\sqrt{2} \sqrt{\sin[x]} \sqrt{a - a \sin[x]}}\right]}{\sqrt{a}}$$

Result (type 4, 128 leaves) :

$$\left(2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{4}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[-1 - \sqrt{2}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{4}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[-1 + \sqrt{2}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{4}\right]}}\right], -1\right] \right) \right. \\ \left. \operatorname{Sec}\left[\frac{x}{4}\right]^2 \left(\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right) \sqrt{\sin[x]} \right) / \left(\sqrt{1 - \cot\left[\frac{x}{4}\right]^2} \sqrt{a - a \sin[x]} \tan\left[\frac{x}{4}\right]^{3/2} \right)$$

- **Problem 91: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[c + d x]^{1/3}}{(a + a \sin[c + d x])^2} dx$$

Optimal (type 5, 184 leaves, 5 steps) :

$$\frac{4 \cos[c + d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \sin[c + d x]^2\right] \sin[c + d x]^{1/3}}{9 a^2 d \sqrt{\cos[c + d x]^2}} - \frac{\cos[c + d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \sin[c + d x]^2\right] \sin[c + d x]^{4/3}}{36 a^2 d \sqrt{\cos[c + d x]^2}} - \frac{\cos[c + d x] \sin[c + d x]^{1/3}}{9 a^2 d (1 + \sin[c + d x])} - \frac{\cos[c + d x] \sin[c + d x]^{1/3}}{3 d (a + a \sin[c + d x])^2}$$

Result (type 5, 598 leaves) :

$$\frac{1}{d (a + a \sin[c + d x])^2} \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^4$$

$$\left(\frac{2 \sin\left[\frac{1}{2}(c + d x)\right]}{3 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^3} - \frac{1}{3 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{2 \sin\left[\frac{1}{2}(c + d x)\right]}{9 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)} \right) \sin[c + d x]^{1/3} +$$

$$\left(\cos[c + d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, \cos[c + d x]^2\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^4 \sin[c + d x]^{4/3} \right) /$$

$$(27 d (\sin[c + d x]^2)^{2/3} (a + a \sin[c + d x])^2) + \left((-4 + \cos[c + d x]) \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^4 \right.$$

$$\left. \left(-\frac{1}{9} \sin[c + d x]^{1/3} - \left(2 (-1)^{2/3} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{1 + (-1)^{1/3} (1 - \sqrt{3}) \sin[c + d x]^{2/3}}{1 + (-1)^{1/3} (1 + \sqrt{3}) \sin[c + d x]^{2/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right) (1 + (-1)^{1/3} (1 + \sqrt{3}) \sin[c + d x]^{2/3})^2 \right. \right.$$

$$\left. \sqrt{\frac{(-1)^{1/3} (1 + (-1)^{1/3} \sin[c + d x]^{2/3}) \sin[c + d x]^{2/3}}{(1 + (-1)^{1/3} (1 + \sqrt{3}) \sin[c + d x]^{2/3})^2}} \sqrt{\frac{1 - (-1)^{1/3} \sin[c + d x]^{2/3} + (-1)^{2/3} \sin[c + d x]^{4/3}}{(1 + (-1)^{1/3} (1 + \sqrt{3}) \sin[c + d x]^{2/3})^2}} \right) /$$

$$\left. \left. \left(9 \times 3^{1/4} \sin[c + d x]^{1/3} \sqrt{1 - \sin[c + d x]^2} \right) \right) \right) / \left(d \left(\cos[c + d x] - 4 \sqrt{\cos[c + d x]^2} \sec[c + d x] \right) (a + a \sin[c + d x])^2 \right)$$

■ **Problem 96: Unable to integrate problem.**

$$\int \csc[c + d x] (a + a \sin[c + d x])^{2/3} dx$$

Optimal (type 6, 77 leaves, 4 steps):

$$\frac{2 \times 2^{1/6} \operatorname{AppellF1}\left[\frac{1}{2}, 1, -\frac{1}{6}, \frac{3}{2}, 1 - \sin[c + d x], \frac{1}{2} (1 - \sin[c + d x])\right] \cos[c + d x] (a + a \sin[c + d x])^{2/3}}{d (1 + \sin[c + d x])^{7/6}}$$

Result (type 8, 23 leaves):

$$\int \csc[c + d x] (a + a \sin[c + d x])^{2/3} dx$$

■ **Problem 97: Mathematica result simpler than optimal antiderivative, IF it can be verified!**

$$\int \csc[c + d x]^2 (a + a \sin[c + d x])^{2/3} dx$$

Optimal (type 6, 77 leaves, 4 steps):

$$\frac{2 \times 2^{1/6} \operatorname{AppellF1}\left[\frac{1}{2}, 2, -\frac{1}{6}, \frac{3}{2}, 1 - \sin[c+dx], \frac{1}{2}(1 - \sin[c+dx])\right] \cos[c+dx] (a + a \sin[c+dx])^{2/3}}{d (1 + \sin[c+dx])^{7/6}}$$

Result (type 5, 143 leaves):

$$-\left(2 e^{i(c+dx)} \left(-i - e^{i(c+dx)} + (1 + i e^{-i(c+dx)})^{2/3} (-i + e^{i(c+dx)}) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -i e^{-i(c+dx)}\right]\right) (a (1 + \sin[c+dx]))^{2/3}\right) / (d (-i + e^{i(c+dx)}) (i + e^{i(c+dx)})^2)$$

■ **Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin[c+dx]^3 (a + a \sin[c+dx])^{4/3} dx$$

Optimal (type 5, 162 leaves, 6 steps):

$$\frac{388 \times 2^{5/6} a \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \sin[c+dx])\right] (a + a \sin[c+dx])^{1/3}}{455 d (1 + \sin[c+dx])^{5/6}} - \frac{72 \cos[c+dx] (a + a \sin[c+dx])^{4/3}}{455 d} - \frac{3 \cos[c+dx] \sin[c+dx]^2 (a + a \sin[c+dx])^{4/3}}{13 d} - \frac{6 \cos[c+dx] (a + a \sin[c+dx])^{7/3}}{65 a d}$$

Result (type 5, 346 leaves):

$$\frac{1}{91 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} (a (1 + \sin[c+dx]))^{4/3} \left(-\frac{1}{40 (1 + i e^{-i(c+dx)})^{2/3} \sqrt{1 - \sin[c+dx]}} 291 (-1)^{3/4} e^{-\frac{3}{2}i(c+dx)} (i + e^{i(c+dx)}) \left(-20 e^{i(c+dx)} \sqrt{\cos\left[\frac{1}{4}(2c + \pi + 2dx)\right]^2} \right. \right. \\ \left. \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -i e^{-i(c+dx)}\right] + 2 (1 + i e^{-i(c+dx)})^{2/3} (1 + e^{2i(c+dx)}) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \sin\left[\frac{1}{4}(2c + \pi + 2dx)\right]^2\right] - 5i \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -i e^{-i(c+dx)}\right] \sqrt{2 - 2 \sin[c+dx]}\right) - \right. \\ \left. \frac{3}{40} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right) (-1940 + 790 \cos[c+dx] - 98 \cos[3(c+dx)] + 278 \sin[2(c+dx)] - 35 \sin[4(c+dx)]) \right)$$

■ **Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin[c+dx]^2 (a + a \sin[c+dx])^{4/3} dx$$

Optimal (type 5, 127 leaves, 4 steps):

$$\frac{37 \times 2^{5/6} a \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1-\sin[c+dx])\right] (a+a\sin[c+dx])^{1/3}}{35 d (1+\sin[c+dx])^{5/6}} + \frac{9 \cos[c+dx] (a+a\sin[c+dx])^{4/3}}{70 d} - \frac{3 \cos[c+dx] (a+a\sin[c+dx])^{7/3}}{10 a d}$$

Result (type 5, 336 leaves):

$$\frac{1}{28 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} (a(1+\sin[c+dx]))^{4/3} \left(-\frac{1}{40 (1+i e^{-i(c+dx)})^{2/3} \sqrt{1-\sin[c+dx]}} 111 (-1)^{3/4} e^{-\frac{3}{2}i(c+dx)} (i+e^{i(c+dx)}) \left(-20 e^{i(c+dx)} \sqrt{\cos\left[\frac{1}{4}(2c+\pi+2dx)\right]^2} \right. \right. \right. \\ \left. \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -i e^{-i(c+dx)}\right] + 2 (1+i e^{-i(c+dx)})^{2/3} (1+e^{2i(c+dx)}) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \sin\left[\frac{1}{4}(2c+\pi+2dx)\right]^2\right] - 5i \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -i e^{-i(c+dx)}\right] \sqrt{2-2\sin[c+dx]}\right) - \right. \\ \left. \frac{3}{10} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right) (-185 + 60 \cos[c+dx] - 7 \cos[3(c+dx)] + 22 \sin[2(c+dx)]) \right)$$

■ **Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin[c+dx] (a+a\sin[c+dx])^{4/3} dx$$

Optimal (type 5, 97 leaves, 3 steps):

$$\frac{8 \times 2^{5/6} a \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1-\sin[c+dx])\right] (a+a\sin[c+dx])^{1/3}}{7 d (1+\sin[c+dx])^{5/6}} - \frac{3 \cos[c+dx] (a+a\sin[c+dx])^{4/3}}{7 d}$$

Result (type 5, 324 leaves):

$$\frac{1}{7 d \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^3}$$

$$(a (1 + \sin [c + d x]))^{4/3} \left(-\frac{1}{4 (1 + i e^{-i (c+d x)})^{2/3} \sqrt{1 - \sin [c + d x]}} 3 (-1)^{3/4} e^{-\frac{3}{2} i (c+d x)} (i + e^{i (c+d x)}) \left(-20 e^{i (c+d x)} \sqrt{\cos \left[\frac{1}{4} (2 c + \pi + 2 d x) \right]^2} \right. \right.$$

$$\text{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -i e^{-i (c+d x)} \right] + 2 (1 + i e^{-i (c+d x)})^{2/3} (1 + e^{2 i (c+d x)}) \text{Hypergeometric2F1} \left[\frac{1}{2}, \right.$$

$$\left. \frac{5}{6}, \frac{11}{6}, \sin \left[\frac{1}{4} (2 c + \pi + 2 d x) \right]^2 \right] - 5 i \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -i e^{-i (c+d x)} \right] \sqrt{2 - 2 \sin [c + d x]} \left. \right) -$$

$$\left. \frac{3}{2} \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right) (-10 + 4 \cos [c + d x] + \sin [2 (c + d x)]) \right)$$

■ **Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin [c + d x])^{4/3} dx$$

Optimal (type 5, 67 leaves, 2 steps):

$$\frac{2 \times 2^{5/6} a \cos [c + d x] \text{Hypergeometric2F1} \left[-\frac{5}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2} (1 - \sin [c + d x]) \right] (a + a \sin [c + d x])^{1/3}}{d (1 + \sin [c + d x])^{5/6}}$$

Result (type 5, 314 leaves):

$$\frac{1}{2 d \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^3}$$

$$\left(-\frac{3}{2} (-5 + \cos [c + d x]) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right) - \frac{1}{8 (1 + i e^{-i (c+d x)})^{2/3} \sqrt{1 - \sin [c + d x]}} 3 (-1)^{3/4} \right.$$

$$e^{-\frac{3}{2} i (c+d x)} (i + e^{i (c+d x)}) \left(-20 e^{i (c+d x)} \sqrt{\cos \left[\frac{1}{4} (2 c + \pi + 2 d x) \right]^2} \text{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -i e^{-i (c+d x)} \right] + \right.$$

$$2 (1 + i e^{-i (c+d x)})^{2/3} (1 + e^{2 i (c+d x)}) \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \sin \left[\frac{1}{4} (2 c + \pi + 2 d x) \right]^2 \right] -$$

$$\left. \left. 5 i \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -i e^{-i (c+d x)} \right] \sqrt{2 - 2 \sin [c + d x]} \right) \right) (a (1 + \sin [c + d x]))^{4/3}$$

■ **Problem 102: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + d x] (a + a \operatorname{Sin}[c + d x])^{4/3} dx$$

Optimal (type 6, 78 leaves, 4 steps):

$$-\frac{1}{d (1 + \operatorname{Sin}[c + d x])^{5/6}} 2 \times 2^{5/6} a \operatorname{AppellF1}\left[\frac{1}{2}, 1, -\frac{5}{6}, \frac{3}{2}, 1 - \operatorname{Sin}[c + d x], \frac{1}{2} (1 - \operatorname{Sin}[c + d x])\right] \operatorname{Cos}[c + d x] (a + a \operatorname{Sin}[c + d x])^{1/3}$$

Result (type 6, 9193 leaves):

$$\frac{3 (a (1 + \operatorname{Sin}[c + d x]))^{4/3}}{d \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^2} -$$

$$\left((120 + 120 i) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \right.$$

$$\operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] (a (1 + \operatorname{Sin}[c + d x]))^{4/3} \left(1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)$$

$$\left((5 + 5 i) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \right.$$

$$\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right) +$$

$$\left. i \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right) \right) /$$

$$\left(d \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^3 \left(-400 i \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] \right)^2$$

$$\left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]^3 + 8 \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \right.$$

$$\left. \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] + i \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] \right)^2$$

$$\left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right)^2 + 5 \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right]$$

$$\left(-5 \left(2 \operatorname{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, \frac{7}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] + \right.$$

$$i \operatorname{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] -$$

$$\left. 2 \operatorname{AppellF1}\left[\frac{8}{3}, \frac{7}{3}, \frac{1}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right)\right] \right)$$

$$\left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right)^2 + (2 + 2 i) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right]\right), \right.$$

$$\begin{aligned}
& 50 i \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right)\right]^2 \\
& \left. (3 + 4 \operatorname{Cos}[c + dx] + \operatorname{Cos}[2(c + dx)] - 2 \operatorname{Sin}[c + dx] - \operatorname{Sin}[2(c + dx)]) \right) \Bigg] - \\
& \left(\left(\frac{3}{4} + \frac{3i}{4} \right) \operatorname{Cos}\left[\frac{3}{2}(c + dx)\right] \operatorname{Csc}[c + dx] (a(1 + \operatorname{Sin}[c + dx]))^{4/3} \left(\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2}} \right)^{2/3} \right. \\
& \left. \left((2 - 2i) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 + (2 - 2i) \operatorname{Cos}[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 + \right. \right. \\
& \left. \left. 2^{2/3} \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right)\right] \right) + \right. \right. \\
& \left. \left. i \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right)\right] \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \right. \right. \\
& \left. \left. \frac{(1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{2 + 2 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]} \right] \left(i + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) \left(\frac{(1+i) (-i + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right])}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]} \right)^{1/3} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) \right) + \\
& (5 + 5i) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right)\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \left(2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{2 + 2 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]} \right] \right. \\
& \left. \left(i + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) \left(\frac{(1+i) (-i + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right])}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]} \right)^{1/3} - (1-i) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) \right) \Bigg] / \\
& \left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right)\right] \right) + \right. \\
& \left. i \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]\right)\right] + \frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]} \right)
\end{aligned}$$

$$\left. \left((5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right) \right] \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \right) \right) \Bigg/$$

$$\left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^3 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \right) \left(- \frac{1}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \right)^2} \left(\frac{3}{8} + \frac{3i}{8} \right) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \right.$$

$$\left. \left(\frac{1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2}} \right)^{2/3} \left((2 - 2i) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 + (2 - 2i) \operatorname{Cos} [c + dx] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 + 2^{2/3} \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \right. \right. \right.$$

$$\left. \left. \left. \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right) \right) + i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right), \right. \right. \right.$$

$$\left. \left. \left. \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right) \right) \right) \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]} \right] \left(i + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \right) \right)$$

$$\left(\frac{(1+i) (-i + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]} \right)^{1/3} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \right) + (5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right), \right.$$

$$\left. \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right) \right] \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \left(2^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]} \right] \right)$$

$$\left. \left(i + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \right) \left(\frac{(1+i) (-i + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]} \right)^{1/3} - (1-i) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \right) \right) \Bigg/$$

$$\left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \right) \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right) \right] + \right.$$

$$\left. i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + dx) \right] \right) \right] + \frac{1}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]} \right)$$

$$\begin{aligned}
& \left. \left. \left. (5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right) \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) \right) + \\
& \frac{1}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \left(\frac{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}} \right)^{1/3} \left(\frac{1+i}{2} \right) \left(\frac{1}{2} \sqrt{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2} - \frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right)}{2 \sqrt{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}} \right)} \\
& \left((2 - 2i) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 + (2 - 2i) \operatorname{Cos} [c+dx] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 + 2^{2/3} \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right), \right. \right. \right. \right. \\
& \left. \left. \left. \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right) \right] \right) + i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right) \right] \right) \\
& \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]} \right] \left(i + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \left(\frac{(1+i) (-i + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]} \right)^{1/3} \\
& \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) + (5 + 5i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right) \right] \\
& \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(2^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]} \right] \right. \\
& \left. \left. \left. \left(i + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \left(\frac{(1+i) (-i + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]} \right)^{1/3} - (1-i) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) \right) \right) / \\
& \left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right) \right] \right) + i \right. \\
& \left. \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right) \right] + \frac{1}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]} (5 + 5i) \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right), \left(\frac{1-i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right] \right) \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{1 + \tan\left[\frac{1}{2}(c + dx)\right]} \left(\frac{3}{4} + \frac{3i}{4}\right) \left(\frac{1 + \tan\left[\frac{1}{2}(c + dx)\right]}{\sqrt{\sec\left[\frac{1}{2}(c + dx)\right]^2}}\right)^{2/3} \left[(-2 + 2i) \sec\left[\frac{1}{2}(c + dx)\right]^2 \sin[c + dx] + \right. \\
& (2 - 2i) \sec\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] + (2 - 2i) \cos[c + dx] \sec\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] - \\
& \left. \left(\sec\left[\frac{1}{2}(c + dx)\right]\right)^2 \left(2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right)\right] + \right. \right. \\
& \quad \left. \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right)\right]\right) \right. \\
& \quad \left. \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c + dx)\right]}{2 + 2 \tan\left[\frac{1}{2}(c + dx)\right]}\right] \left(i + \tan\left[\frac{1}{2}(c + dx)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(c + dx)\right])}{1 + \tan\left[\frac{1}{2}(c + dx)\right]}\right)^{1/3} \right. \\
& \quad \left. \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]\right) + (5 + 5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right)\right] \right) \\
& \quad \left. \tan\left[\frac{1}{2}(c + dx)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c + dx)\right]}{2 + 2 \tan\left[\frac{1}{2}(c + dx)\right]}\right] \right. \right. \\
& \quad \left. \left. \left(i + \tan\left[\frac{1}{2}(c + dx)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(c + dx)\right])}{1 + \tan\left[\frac{1}{2}(c + dx)\right]}\right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]\right)\right)\right) \right) / \\
& \left(2 \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]\right)\right)^2 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right)\right] + \right. \\
& \quad \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right)\right] + \frac{1}{1 + \tan\left[\frac{1}{2}(c + dx)\right]} (5 + 5i) \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right)\right] \tan\left[\frac{1}{2}(c + dx)\right]\right) \right) - \\
& \left(\left(-\frac{5}{24} + \frac{5i}{24}\right) \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, \frac{7}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right)\right] \csc\left[\frac{1}{2}(c + dx)\right]^2 - \right. \\
& \quad \left. \left(\frac{5}{96} + \frac{5i}{96}\right) \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right)\right] \csc\left[\frac{1}{2}(c + dx)\right]^2 + i \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2 + 2 \tan\left[\frac{1}{2}(c+dx)\right]}\right] \left(i + \tan\left[\frac{1}{2}(c+dx)\right]\right) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left(-\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sec\left[\frac{1}{2}(c+dx)\right]^2 (-i + \tan\left[\frac{1}{2}(c+dx)\right])}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sec\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right) + \\
& \left(\frac{5}{2} + \frac{5i}{2}\right) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] \\
& \sec\left[\frac{1}{2}(c+dx)\right]^2 \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2 + 2 \tan\left[\frac{1}{2}(c+dx)\right]}\right] \right. \\
& \left. \left(i + \tan\left[\frac{1}{2}(c+dx)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(c+dx)\right])}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) + \\
& (5 + 5i) \left(\left(-\frac{1}{30} + \frac{i}{30}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2 - \right. \\
& \left.\left(\frac{1}{30} + \frac{i}{30}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \tan\left[\frac{1}{2}(c+dx)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2 + 2 \tan\left[\frac{1}{2}(c+dx)\right]}\right] \left(i + \tan\left[\frac{1}{2}(c+dx)\right]\right) \right. \\
& \left.\left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(c+dx)\right])}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) + \frac{1}{3((1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right])} \\
& 2 \times 2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] + \right. \\
& \left.i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right]\right) \\
& \left(i + \tan\left[\frac{1}{2}(c+dx)\right]\right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(c+dx)\right])}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right) \left(2 + 2 \tan\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left(-\frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 ((1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right])}{\left(2 + 2 \tan\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sec\left[\frac{1}{2}(c+dx)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(c+dx)\right]}\right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2+2 \tan\left[\frac{1}{2}(c+dx)\right]} \right] + \frac{1}{\left(1 - \frac{(1+i)+(1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2+2 \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3}} \right) + \\
(5+5i) & \text{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2} + \frac{i}{2} \right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2} - \frac{i}{2} \right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] \tan\left[\frac{1}{2}(c+dx)\right] \\
& \left(\left(-\frac{1}{2} + \frac{i}{2} \right) \sec\left[\frac{1}{2}(c+dx)\right] \right)^2 + \frac{1}{2^{1/3}} \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2+2 \tan\left[\frac{1}{2}(c+dx)\right]} \right] \sec\left[\frac{1}{2}(c+dx)\right] \right)^2 \\
& \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(c+dx)\right])}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} + \left(2^{2/3} \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2+2 \tan\left[\frac{1}{2}(c+dx)\right]} \right] \right. \\
& \left. \left(i + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(-\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sec\left[\frac{1}{2}(c+dx)\right]^2 (-i + \tan\left[\frac{1}{2}(c+dx)\right])}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sec\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right) \right) / \\
& \left(3 \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(c+dx)\right])}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{2/3} \right) + \left(2 \times 2^{2/3} \left(i + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{(1+i) (-i + \tan\left[\frac{1}{2}(c+dx)\right])}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \right. \\
& \left. \left(2 + 2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(-\frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \left((1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right] \right)}{\left(2 + 2 \tan\left[\frac{1}{2}(c+dx)\right] \right)^2} + \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sec\left[\frac{1}{2}(c+dx)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(c+dx)\right]} \right) \right) \\
& \left(-\text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2+2 \tan\left[\frac{1}{2}(c+dx)\right]} \right] + \right. \\
& \left. \frac{1}{\left(1 - \frac{(1+i)+(1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2+2 \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3}} \right) \right) / \left(3 \left((1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) /
\end{aligned}$$

$$\left(\left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] + i \right. \right. \\ \left. \left. \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] + \frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}(5 + 5i) \right. \right. \\ \left. \left. \left. \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right)$$

■ **Problem 103: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \csc[c+dx]^2 (a + a \sin[c+dx])^{4/3} dx$$

Optimal (type 6, 78 leaves, 4 steps):

$$-\frac{1}{d(1 + \sin[c+dx])^{5/6}} 2 \times 2^{5/6} a \text{AppellF1}\left[\frac{1}{2}, 2, -\frac{5}{6}, \frac{3}{2}, 1 - \sin[c+dx], \frac{1}{2}(1 - \sin[c+dx])\right] \cos[c+dx] (a + a \sin[c+dx])^{1/3}$$

Result (type 6, 9202 leaves):

$$\frac{(-1 - \cot[c+dx]) (a(1 + \sin[c+dx]))^{4/3}}{d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \\ \left((60 + 60i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] \cos\left[\frac{1}{2}(c+dx)\right]^2 \right. \\ \left. \sin\left[\frac{1}{2}(c+dx)\right] (a(1 + \sin[c+dx]))^{4/3} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right) \right. \\ \left((5 + 5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\ \left. \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right) + \right. \\ \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right) \right) \right) / \\ \left(d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3 \left(-400i \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right]^2 \right. \right. \\ \left. \left. \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \sin\left[\frac{1}{2}(c+dx)\right]^3 + 8 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right)\left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right], \right. \right. \right.$$

$$\begin{aligned}
& \left(5 \left(2 \operatorname{AppellF1} \left[\frac{8}{3}, \frac{1}{3}, \frac{7}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] + \right. \\
& \quad i \operatorname{AppellF1} \left[\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] - \\
& \quad \left. 2 \operatorname{AppellF1} \left[\frac{8}{3}, \frac{7}{3}, \frac{1}{3}, \frac{11}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] \right) \\
& \quad \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^2 + (2 + 2 i) \operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right), \right. \\
& \quad \left. \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] (2 + \operatorname{Cos}[c + d x] - \operatorname{Cos}[2 (c + d x)] + 3 \operatorname{Sin}[c + d x]) - (2 - 2 i) \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \right. \\
& \quad \left. \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right] (2 + \operatorname{Cos}[c + d x] - \operatorname{Cos}[2 (c + d x)] + 3 \operatorname{Sin}[c + d x]) \right) - \\
& \quad 50 i \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right]^2 \\
& \quad \left. (3 + 4 \operatorname{Cos}[c + d x] + \operatorname{Cos}[2 (c + d x)] - 2 \operatorname{Sin}[c + d x] - \operatorname{Sin}[2 (c + d x)]) \right) \Bigg] + \\
& \left(\left(\frac{1}{4} + \frac{i}{4} \right) \operatorname{Cos} \left[\frac{3}{2} (c + d x) \right] \operatorname{Csc}[c + d x] (a (1 + \operatorname{Sin}[c + d x]))^{4/3} \left(\frac{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}} \right)^{2/3} \right. \\
& \quad \left. (2 - 2 i) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right] \right)^2 + (2 - 2 i) \operatorname{Cos}[c + d x] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 + \\
& \quad \left(2^{2/3} \left(\operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + d x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + d x) \right] \right) \right] + \right. \\
& \quad \left. i \operatorname{AppellF1} \left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + d x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + d x) \right] \right) \right] \right) \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \right. \\
& \quad \left. \frac{(1 + i) + (1 - i) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]} \right] \left(i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \left(\frac{(1 + i) (-i + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right])}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]} \right)^{1/3} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) + \\
& \quad (5 + 5 i) \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2} + \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + d x) \right] \right), \left(\frac{1}{2} - \frac{i}{2} \right) \left(1 + \operatorname{Cot} \left[\frac{1}{2} (c + d x) \right] \right) \right] \right) \\
& \quad \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \left(2^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1 + i) + (1 - i) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{2 + 2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(i + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{(1+i)(-i+\tan[\frac{1}{2}(c+dx)])}{1+\tan[\frac{1}{2}(c+dx)]} \right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \Bigg) / \\
& \left(\left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] + \right. \right. \\
& \quad \left. \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] + \frac{1}{1+\tan[\frac{1}{2}(c+dx)]} \right. \right. \\
& \quad \left. \left. (5+5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \Bigg) / \\
& \left(d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(-\frac{1}{(1+\tan[\frac{1}{2}(c+dx)])^2} \left(\frac{3}{8} + \frac{3i}{8} \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \right. \\
& \quad \left. \left. \left(\frac{1+\tan[\frac{1}{2}(c+dx)]}{\sqrt{\sec[\frac{1}{2}(c+dx)]^2}} \right)^{2/3} \left((2-2i) \sec\left[\frac{1}{2}(c+dx)\right]^2 + (2-2i) \cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^2 + 2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] + i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] \right) \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i)+(1-i)\tan[\frac{1}{2}(c+dx)]}{2+2\tan[\frac{1}{2}(c+dx)]} \right] \left(i + \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \\
& \quad \left(\frac{(1+i)(-i+\tan[\frac{1}{2}(c+dx)])}{1+\tan[\frac{1}{2}(c+dx)]} \right)^{1/3} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) + (5+5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \right. \\
& \quad \left. \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] \tan\left[\frac{1}{2}(c+dx)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i)+(1-i)\tan[\frac{1}{2}(c+dx)]}{2+2\tan[\frac{1}{2}(c+dx)]} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(i + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{(1+i)(-i + \tan\left[\frac{1}{2}(c+dx)\right])}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \Bigg) / \\
& \left(\left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] \right) + \right. \\
& \quad i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] + \frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \\
& \quad \left. \left((5+5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \Bigg) + \\
& \frac{1}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\sec\left[\frac{1}{2}(c+dx)\right]^2}} \right)^{1/3}} \left(\frac{1+i}{2} \right) \left(\frac{1}{2} \sqrt{\sec\left[\frac{1}{2}(c+dx)\right]^2} - \frac{\tan\left[\frac{1}{2}(c+dx)\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right)}{2 \sqrt{\sec\left[\frac{1}{2}(c+dx)\right]^2}} \right) \\
& \left((2-2i) \sec\left[\frac{1}{2}(c+dx)\right]^2 + (2-2i) \cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^2 + \left(2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \right. \right. \\
& \quad \left. \left. \left(\frac{1-i}{2} \right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) + i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] \Bigg) \\
& \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2 + 2 \tan\left[\frac{1}{2}(c+dx)\right]} \right] \left(i + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{(1+i)(-i + \tan\left[\frac{1}{2}(c+dx)\right])}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \\
& \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) + (5+5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] \\
& \tan\left[\frac{1}{2}(c+dx)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \tan\left[\frac{1}{2}(c+dx)\right]}{2 + 2 \tan\left[\frac{1}{2}(c+dx)\right]} \right] \right) \\
& \left(i + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{(1+i)(-i + \tan\left[\frac{1}{2}(c+dx)\right])}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} - (1-i) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \Bigg) / \\
& \left(\left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right] \right) \right] \right) + i \right.
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] + \frac{1}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]} (5 + 5i) \\
& \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right] \Bigg) + \\
& \frac{1}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]} \left(\frac{3}{4} + \frac{3i}{4}\right) \left(\frac{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}}\right)^{2/3} \left((-2 + 2i) \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Sin}[c+dx] + \right. \\
& (2 - 2i) \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] + (2 - 2i) \text{Cos}[c+dx] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] - \\
& \left. \left(\text{Sec}\left[\frac{1}{2}(c+dx)\right]\right)^2 \left(2^{2/3} \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] + \right. \right. \\
& \left. \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right]\right) \right) \\
& \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{2 + 2 \text{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \left(i + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right) \left(\frac{(1+i) (-i + \text{Tan}\left[\frac{1}{2}(c+dx)\right])}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} \\
& \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right) + (5 + 5i) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right] \left(2^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{2 + 2 \text{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \right. \\
& \left. \left(i + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right) \left(\frac{(1+i) (-i + \text{Tan}\left[\frac{1}{2}(c+dx)\right])}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} - (1-i) \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right) \Bigg) / \\
& \left(2 \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right)^2 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] + \right. \\
& \left. i \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] + \frac{1}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]} (5 + 5i) \right. \\
& \left. \left.\text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \text{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) + \frac{1}{3 \left(\frac{(1+i)(-i+\operatorname{Tan}[\frac{1}{2}(c+dx)])}{1+\operatorname{Tan}[\frac{1}{2}(c+dx)]} \right)^{2/3}} 2^{2/3} \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right], \left(\frac{1-i}{2}\right) \right. \\
& \quad \left. \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] + i \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] \right) \\
& \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan}[\frac{1}{2}(c+dx)]}{2 + 2 \operatorname{Tan}[\frac{1}{2}(c+dx)]}\right] \left(i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \\
& \quad \left(-\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (-i + \operatorname{Tan}[\frac{1}{2}(c+dx)])}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\left(\frac{1}{2} + \frac{i}{2}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right) + \\
& \left(\frac{5}{2} + \frac{5i}{2}\right) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan}[\frac{1}{2}(c+dx)]}{2 + 2 \operatorname{Tan}[\frac{1}{2}(c+dx)]}\right] \right. \\
& \quad \left. \left(i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{(1+i)(-i+\operatorname{Tan}[\frac{1}{2}(c+dx)])}{1+\operatorname{Tan}[\frac{1}{2}(c+dx)]} \right)^{1/3} - (1-i) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) + \\
& (5+5i) \left(\left(-\frac{1}{30} + \frac{i}{30} \right) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 - \right. \\
& \quad \left. \left(\frac{1}{30} + \frac{i}{30} \right) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \right) \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan}[\frac{1}{2}(c+dx)]}{2 + 2 \operatorname{Tan}[\frac{1}{2}(c+dx)]}\right] \left(i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right. \\
& \quad \left. \left(\frac{(1+i)(-i+\operatorname{Tan}[\frac{1}{2}(c+dx)])}{1+\operatorname{Tan}[\frac{1}{2}(c+dx)]} \right)^{1/3} - (1-i) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) + \frac{1}{3((1+i) + (1-i) \operatorname{Tan}[\frac{1}{2}(c+dx)])} \\
& 2 \times 2^{2/3} \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] + \right. \\
& \quad \left. i \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] \right) \\
& \left(i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{(1+i)(-i+\operatorname{Tan}[\frac{1}{2}(c+dx)])}{1+\operatorname{Tan}[\frac{1}{2}(c+dx)]} \right)^{1/3} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(2 + 2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left((1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)}{\left(2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\left(\frac{1}{2}-\frac{i}{2}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right) \\
& \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] + \frac{1}{\left(1-\frac{(1+i)+(1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3}} \right) + \\
(5+5i) & \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1}{2}+\frac{i}{2}\right) \left(1+\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1}{2}-\frac{i}{2}\right) \left(1+\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \\
& \left(\left(-\frac{1}{2}+\frac{i}{2}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + \frac{1}{2^{1/3}} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \left(\frac{(1+i) \left(-i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} + \left(2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \right. \right. \\
& \left. \left. \left(i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(-i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\left(\frac{1}{2}+\frac{i}{2}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right) \right) \right) / \\
& \left(3 \left(\frac{(1+i) \left(-i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)^{2/3} \right) + \left(2 \times 2^{2/3} \left(i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{(1+i) \left(-i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \right. \\
& \left. \left(2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left((1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)}{\left(2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\left(\frac{1}{2}-\frac{i}{2}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right) \\
& \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2+2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] + \right.
\end{aligned}$$

$$\left. \left. \left. \left. \left. \frac{1}{\left(1 - \frac{(1+i)+(1-i)\tan\left[\frac{1}{2}(c+dx)\right]}{2+2\tan\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3}} \right) \right) \right) \right) \right) \left/ \left(3 \left((1+i) + (1-i)\tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \right) \right) \left/ \right.$$

$$\left(\left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] + i \right. \right.$$

$$\left. \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] + \frac{1}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} (5 + 5i) \right.$$

$$\left. \left. \left. \left. \left. \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \left(\frac{1+i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right), \left(\frac{1-i}{2}\right) \left(1 + \cot\left[\frac{1}{2}(c+dx)\right]\right)\right] \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \right) \right) \right)$$

■ **Problem 108: Unable to integrate problem.**

$$\int \frac{\text{Csc}[c+dx]}{(a+a\text{Sin}[c+dx])^{1/3}} dx$$

Optimal (type 6, 77 leaves, 4 steps):

$$\frac{2^{1/6} \text{AppellF1}\left[\frac{1}{2}, 1, \frac{5}{6}, \frac{3}{2}, 1 - \text{Sin}[c+dx], \frac{1}{2} (1 - \text{Sin}[c+dx])\right] \text{Cos}[c+dx]}{d (1 + \text{Sin}[c+dx])^{1/6} (a + a \text{Sin}[c+dx])^{1/3}}$$

Result (type 8, 23 leaves):

$$\int \frac{\text{Csc}[c+dx]}{(a+a\text{Sin}[c+dx])^{1/3}} dx$$

■ **Problem 109: Mathematica result simpler than optimal antiderivative, IF it can be verified!**

$$\int \frac{\text{Csc}[c+dx]^2}{(a+a\text{Sin}[c+dx])^{1/3}} dx$$

Optimal (type 6, 77 leaves, 4 steps):

$$\frac{2^{1/6} \text{AppellF1}\left[\frac{1}{2}, 2, \frac{5}{6}, \frac{3}{2}, 1 - \text{Sin}[c+dx], \frac{1}{2} (1 - \text{Sin}[c+dx])\right] \text{Cos}[c+dx]}{d (1 + \text{Sin}[c+dx])^{1/6} (a + a \text{Sin}[c+dx])^{1/3}}$$

Result (type 5, 182 leaves):

$$\left(2 \times 2^{2/3} \operatorname{Cos}\left[\frac{1}{4} (2c - \pi + 2dx)\right]^{2/3} (\operatorname{Cos}[c + dx] + i \operatorname{Sin}[c + dx]) \right. \\ \left. \left(1 + 4 \operatorname{Sin}[c + dx] + 4i \operatorname{Cos}[c + dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -i e^{-i(c+dx)}\right] (1 + i \operatorname{Cos}[c + dx] + \operatorname{Sin}[c + dx])^{2/3} \right) \right) / \\ \left(5d \left(e^{-\frac{1}{4}i(2c+\pi+2dx)} (i + e^{i(c+dx)}) \right)^{2/3} (1 + e^{2i(c+dx)}) (a(1 + \operatorname{Sin}[c + dx]))^{1/3} \right)$$

■ **Problem 114: Unable to integrate problem.**

$$\int \frac{\operatorname{Csc}[c + dx]}{(a + a \operatorname{Sin}[c + dx])^{4/3}} dx$$

Optimal (type 6, 80 leaves, 4 steps):

$$\frac{\operatorname{AppellF1}\left[\frac{1}{2}, 1, \frac{11}{6}, \frac{3}{2}, 1 - \operatorname{Sin}[c + dx], \frac{1}{2}(1 - \operatorname{Sin}[c + dx])\right] \operatorname{Cos}[c + dx]}{2^{5/6} a d (1 + \operatorname{Sin}[c + dx])^{1/6} (a + a \operatorname{Sin}[c + dx])^{1/3}}$$

Result (type 8, 23 leaves):

$$\int \frac{\operatorname{Csc}[c + dx]}{(a + a \operatorname{Sin}[c + dx])^{4/3}} dx$$

■ **Problem 115: Mathematica result simpler than optimal antiderivative, IF it can be verified!**

$$\int \frac{\operatorname{Csc}[c + dx]^2}{(a + a \operatorname{Sin}[c + dx])^{4/3}} dx$$

Optimal (type 6, 80 leaves, 4 steps):

$$\frac{\operatorname{AppellF1}\left[\frac{1}{2}, 2, \frac{11}{6}, \frac{3}{2}, 1 - \operatorname{Sin}[c + dx], \frac{1}{2}(1 - \operatorname{Sin}[c + dx])\right] \operatorname{Cos}[c + dx]}{2^{5/6} a d (1 + \operatorname{Sin}[c + dx])^{1/6} (a + a \operatorname{Sin}[c + dx])^{1/3}}$$

Result (type 5, 308 leaves):

$$\left(4i 2^{2/3} \operatorname{Cos}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right)\right]^{8/3} \left(14 + 35 e^{i(-c+\frac{\pi}{2}-dx)} + 12 e^{2i(-c+\frac{\pi}{2}-dx)} + 35 e^{3i(-c+\frac{\pi}{2}-dx)} + \right. \right. \\ \left. 14 e^{4i(-c+\frac{\pi}{2}-dx)} + 14 \left(-1 + e^{i(-c+\frac{\pi}{2}-dx)}\right) \left(1 + e^{i(-c+\frac{\pi}{2}-dx)}\right)^{11/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{i(-c+\frac{\pi}{2}-dx)}\right] \right) / \\ \left(55d \left(-1 + e^{i(-c+\frac{\pi}{2}-dx)}\right) \left(1 + e^{i(-c+\frac{\pi}{2}-dx)}\right)^3 \left(e^{-\frac{1}{2}i(-c+\frac{\pi}{2}-dx)} \left(1 + e^{i(-c+\frac{\pi}{2}-dx)}\right)\right)^{2/3} (a + a \operatorname{Sin}[c + dx])^{4/3} \right)$$

■ **Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sin}[e + fx]^n (1 + \operatorname{Sin}[e + fx])^{3/2} dx$$

Optimal (type 5, 96 leaves, 4 steps):

$$\frac{2(5+4n)\cos[ex+fx]\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, 1-\sin[ex+fx]\right]}{f(3+2n)\sqrt{1+\sin[ex+fx]}} - \frac{2\cos[ex+fx]\sin[ex+fx]^{1+n}}{f(3+2n)\sqrt{1+\sin[ex+fx]}}$$

Result (type 6, 20237 leaves) : Display of huge result suppressed!

- **Problem 117: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin[ex+fx]^n \sqrt{1+\sin[ex+fx]} \, dx$$

Optimal (type 5, 43 leaves, 2 steps) :

$$\frac{2\cos[ex+fx]\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, 1-\sin[ex+fx]\right]}{f\sqrt{1+\sin[ex+fx]}}$$

Result (type 5, 196 leaves) :

$$\left(2^{1-n} (1 - e^{2i(ex+fx)})^{-n} (-i e^{-i(ex+fx)} (-1 + e^{2i(ex+fx)}))^n \left((1-2n) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}(-1-2n), -n, \frac{1}{4}(3-2n), e^{2i(ex+fx)}\right] + \right. \right. \\ \left. \left. i e^{i(ex+fx)} (1+2n) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}(1-2n), -n, \frac{1}{4}(5-2n), e^{2i(ex+fx)}\right] \right) \sqrt{1+\sin[ex+fx]} \right) / \left((i + e^{i(ex+fx)}) f (-1+2n) (1+2n) \right)$$

- **Problem 118: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin[ex+fx]^n}{\sqrt{1+\sin[ex+fx]}} \, dx$$

Optimal (type 6, 58 leaves, 3 steps) :

$$\frac{\operatorname{AppellF1}\left[\frac{1}{2}, -n, 1, \frac{3}{2}, 1-\sin[ex+fx], \frac{1}{2}(1-\sin[ex+fx])\right] \cos[ex+fx]}{f\sqrt{1+\sin[ex+fx]}}$$

Result (type 6, 387 leaves) :

$$\frac{1}{f} \operatorname{Sec}[e + f x] \operatorname{Sin}[e + f x]^n (1 + \operatorname{Sin}[e + f x])^{3/2}$$

$$\left(\left(4 \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x] \right] \right) \right) / \left(8 \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x] \right] + \right.$$

$$\left. \left(-4 n \operatorname{AppellF1} \left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x] \right] + \operatorname{AppellF1} \left[2, \frac{3}{2}, -n, 3, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x] \right] \right) \right.$$

$$\left. (1 + \operatorname{Sin}[e + f x]) \right) - \left((-1 + 2 n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{1}{1 + \operatorname{Sin}[e + f x]} \right] (-1 + \operatorname{Sin}[e + f x]) \right) /$$

$$\left((1 + 2 n) \left(2 \left(n \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{1}{1 + \operatorname{Sin}[e + f x]} \right] + \operatorname{AppellF1} \left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{2}{1 + \operatorname{Sin}[e + f x]} \right], \right. \right.$$

$$\left. \left. \frac{1}{1 + \operatorname{Sin}[e + f x]} \right) \right) + (-1 + 2 n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{1}{1 + \operatorname{Sin}[e + f x]} \right] (1 + \operatorname{Sin}[e + f x]) \right) \right)$$

■ **Problem 119: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^n}{(1 + \operatorname{Sin}[e + f x])^{3/2}} dx$$

Optimal (type 6, 60 leaves, 3 steps):

$$\frac{\operatorname{AppellF1} \left[\frac{1}{2}, -n, 2, \frac{3}{2}, 1 - \operatorname{Sin}[e + f x], \frac{1}{2} (1 - \operatorname{Sin}[e + f x]) \right] \operatorname{Cos}[e + f x]}{2 f \sqrt{1 + \operatorname{Sin}[e + f x]}}$$

Result (type 6, 624 leaves):

$$\frac{1}{2f} \operatorname{Sec}[e+fx] \operatorname{Sin}[e+fx]^n \sqrt{1+\operatorname{Sin}[e+fx]} \left(\left(4 \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\operatorname{Sin}[e+fx]), 1+\operatorname{Sin}[e+fx]\right] (1+\operatorname{Sin}[e+fx]) \right) / \right. \\ \left. \left(8 \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\operatorname{Sin}[e+fx]), 1+\operatorname{Sin}[e+fx]\right] + \left(-4n \operatorname{AppellF1}\left[2, \frac{1}{2}, 1-n, 3, \frac{1}{2}(1+\operatorname{Sin}[e+fx]), 1+\operatorname{Sin}[e+fx]\right] + \right. \right. \right. \\ \left. \left. \left. \operatorname{AppellF1}\left[2, \frac{3}{2}, -n, 3, \frac{1}{2}(1+\operatorname{Sin}[e+fx]), 1+\operatorname{Sin}[e+fx]\right] \right) (1+\operatorname{Sin}[e+fx]) \right) \right) - \\ \left((-1+2n) \operatorname{AppellF1}\left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\operatorname{Sin}[e+fx]}, \frac{1}{1+\operatorname{Sin}[e+fx]}\right] (-1+\operatorname{Sin}[e+fx]) (1+\operatorname{Sin}[e+fx]) \right) / \\ \left((1+2n) \left(2 \left(n \operatorname{AppellF1}\left[\frac{1}{2}-n, -\frac{1}{2}, 1-n, \frac{3}{2}-n, \frac{2}{1+\operatorname{Sin}[e+fx]}, \frac{1}{1+\operatorname{Sin}[e+fx]}\right] + \operatorname{AppellF1}\left[\frac{1}{2}-n, \frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\operatorname{Sin}[e+fx]}, \right. \right. \right. \\ \left. \left. \left. \frac{1}{1+\operatorname{Sin}[e+fx]}\right] \right) + (-1+2n) \operatorname{AppellF1}\left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\operatorname{Sin}[e+fx]}, \frac{1}{1+\operatorname{Sin}[e+fx]}\right] (1+\operatorname{Sin}[e+fx]) \right) \right) - \\ \left(2(-3+2n) \operatorname{AppellF1}\left[\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\operatorname{Sin}[e+fx]}, \frac{1}{1+\operatorname{Sin}[e+fx]}\right] (-1+\operatorname{Sin}[e+fx]) \right) / \\ \left((-1+2n) \left(2 \left(n \operatorname{AppellF1}\left[\frac{3}{2}-n, -\frac{1}{2}, 1-n, \frac{5}{2}-n, \frac{2}{1+\operatorname{Sin}[e+fx]}, \frac{1}{1+\operatorname{Sin}[e+fx]}\right] + \operatorname{AppellF1}\left[\frac{3}{2}-n, \frac{1}{2}, -n, \frac{5}{2}-n, \frac{2}{1+\operatorname{Sin}[e+fx]}, \right. \right. \right. \\ \left. \left. \left. \frac{1}{1+\operatorname{Sin}[e+fx]}\right] \right) + (-3+2n) \operatorname{AppellF1}\left[\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\operatorname{Sin}[e+fx]}, \frac{1}{1+\operatorname{Sin}[e+fx]}\right] (1+\operatorname{Sin}[e+fx]) \right) \right) \right) \right)$$

- **Problem 120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sin}[e+fx]^n (a+a \operatorname{Sin}[e+fx])^{3/2} dx$$

Optimal (type 5, 106 leaves, 4 steps):

$$\frac{2a^2(5+4n) \operatorname{Cos}[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, 1-\operatorname{Sin}[e+fx]\right]}{f(3+2n) \sqrt{a+a \operatorname{Sin}[e+fx]}} - \frac{2a^2 \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^{1+n}}{f(3+2n) \sqrt{a+a \operatorname{Sin}[e+fx]}}$$

Result (type 6, 20239 leaves): Display of huge result suppressed!

- **Problem 121: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sin}[e+fx]^n \sqrt{a+a \operatorname{Sin}[e+fx]} dx$$

Optimal (type 5, 46 leaves, 2 steps):

$$\frac{2a \operatorname{Cos}[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, 1-\operatorname{Sin}[e+fx]\right]}{f \sqrt{a+a \operatorname{Sin}[e+fx]}}$$

Result (type 5, 264 leaves):

$$\frac{1}{f(-1+2n)(1+2n)\left(\cos\left[\frac{1}{2}(e+fx)\right]+\sin\left[\frac{1}{2}(e+fx)\right]\right)}$$

$$(1+i)e^{-\frac{1}{2}ifx}\left(e^{ifx}(1+2n)\operatorname{Hypergeometric2F1}\left[\frac{1}{4}(1-2n),-n,\frac{1}{4}(5-2n),e^{2ifx}(\cos[e]+i\sin[e])^2\right]\left(\cos\left[\frac{e}{2}\right]+i\sin\left[\frac{e}{2}\right]\right)+\right.$$

$$\left.(-1+2n)\operatorname{Hypergeometric2F1}\left[\frac{1}{4}(-1-2n),-n,\frac{1}{4}(3-2n),e^{2ifx}(\cos[e]+i\sin[e])^2\right]\left(i\cos\left[\frac{e}{2}\right]+\sin\left[\frac{e}{2}\right]\right)\right)$$

$$(1-e^{2ifx}\cos[e]^2+e^{2ifx}\sin[e]^2-i e^{2ifx}\sin[2e])^{-n}\sin[e+fx]^n\sqrt{a(1+\sin[e+fx])}$$

■ **Problem 122: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]^n}{\sqrt{a+a\sin[e+fx]}} dx$$

Optimal (type 6, 60 leaves, 4 steps):

$$\frac{\operatorname{AppellF1}\left[\frac{1}{2},-n,1,\frac{3}{2},1-\sin[e+fx],\frac{1}{2}(1-\sin[e+fx])\right]\cos[e+fx]}{f\sqrt{a+a\sin[e+fx]}}$$

Result (type 6, 426 leaves):

$$\frac{1}{f\sqrt{a(1+\sin[e+fx])}}$$

$$\operatorname{Sec}[e+fx](1+\sin[e+fx])^2\left(\left(4\operatorname{AppellF1}\left[1,\frac{1}{2},-n,2,\frac{1}{2}(1+\sin[e+fx]),1+\sin[e+fx]\right](-\sin[e+fx])^{-n}(-\sin[e+fx]^2)^n\right)/\right.$$

$$\left(8\operatorname{AppellF1}\left[1,\frac{1}{2},-n,2,\frac{1}{2}(1+\sin[e+fx]),1+\sin[e+fx]\right]-\left(4n\operatorname{AppellF1}\left[2,\frac{1}{2},1-n,3,\frac{1}{2}(1+\sin[e+fx]),1+\sin[e+fx]\right]-\right.\right.$$

$$\left.\left.\operatorname{AppellF1}\left[2,\frac{3}{2},-n,3,\frac{1}{2}(1+\sin[e+fx]),1+\sin[e+fx]\right]\right)(1+\sin[e+fx])\right)-$$

$$\left(\left(-1+2n\right)\operatorname{AppellF1}\left[-\frac{1}{2}-n,-\frac{1}{2},-n,\frac{1}{2}-n,\frac{2}{1+\sin[e+fx]},\frac{1}{1+\sin[e+fx]}\right](-1+\sin[e+fx])\sin[e+fx]^n\right)/$$

$$\left(\left(1+2n\right)\left(2\left(n\operatorname{AppellF1}\left[\frac{1}{2}-n,-\frac{1}{2},1-n,\frac{3}{2}-n,\frac{2}{1+\sin[e+fx]},\frac{1}{1+\sin[e+fx]}\right]+\operatorname{AppellF1}\left[\frac{1}{2}-n,\frac{1}{2},-n,\frac{3}{2}-n,\frac{2}{1+\sin[e+fx]},\right.\right.\right.$$

$$\left.\left.\frac{1}{1+\sin[e+fx]}\right]\right)+(-1+2n)\operatorname{AppellF1}\left[-\frac{1}{2}-n,-\frac{1}{2},-n,\frac{1}{2}-n,\frac{2}{1+\sin[e+fx]},\frac{1}{1+\sin[e+fx]}\right](1+\sin[e+fx])\right)\right)$$

■ **Problem 123: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]^n}{(a+a\sin[e+fx])^{3/2}} dx$$

Optimal (type 6, 65 leaves, 4 steps):

$$\frac{\text{AppellF1}\left[\frac{1}{2}, -n, 2, \frac{3}{2}, 1 - \text{Sin}[e + f x], \frac{1}{2} (1 - \text{Sin}[e + f x])\right] \text{Cos}[e + f x]}{2 a f \sqrt{a + a \text{Sin}[e + f x]}}$$

Result (type 6, 796 leaves):

$$\begin{aligned} & \left(\text{Cos}[e + f x] \text{Sin}[e + f x]^n (1 + \text{Sin}[e + f x]) \left(\frac{-a + a (1 + \text{Sin}[e + f x])}{a} \right)^{-n} \right. \\ & \left(\left(4 a^2 \text{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \text{Sin}[e + f x]), 1 + \text{Sin}[e + f x]\right] (-\text{Sin}[e + f x])^{-n} (1 + \text{Sin}[e + f x]) \left(-\frac{(a - a (1 + \text{Sin}[e + f x]))^2}{a^2} \right)^n \right) / \right. \\ & \left(8 a \text{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \text{Sin}[e + f x]), 1 + \text{Sin}[e + f x]\right] + \right. \\ & a \left(-4 n \text{AppellF1}\left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2} (1 + \text{Sin}[e + f x]), 1 + \text{Sin}[e + f x]\right] + \text{AppellF1}\left[2, \frac{3}{2}, -n, 3, \frac{1}{2} (1 + \text{Sin}[e + f x]), 1 + \text{Sin}[e + f x]\right] \right) \\ & \left. (1 + \text{Sin}[e + f x]) \right) - \left(a (-1 + 2 n) \text{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]} \right] \right. \\ & \left. \text{Sin}[e + f x]^n (1 + \text{Sin}[e + f x]) (-2 a + a (1 + \text{Sin}[e + f x])) \right) \left. \right) / \\ & \left((1 + 2 n) \left(2 a \left(n \text{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]} \right] + \text{AppellF1}\left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]} \right] \right. \right. \right. \\ & \left. \left. \frac{1}{1 + \text{Sin}[e + f x]} \right) \right) + a (-1 + 2 n) \text{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]} \right] (1 + \text{Sin}[e + f x]) \right) \left. \right) - \\ & \left(2 a (-3 + 2 n) \text{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]} \right] \text{Sin}[e + f x]^n (-2 a + a (1 + \text{Sin}[e + f x])) \right) \left. \right) / \\ & \left((-1 + 2 n) \left(2 a \left(n \text{AppellF1}\left[\frac{3}{2} - n, -\frac{1}{2}, 1 - n, \frac{5}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]} \right] + \text{AppellF1}\left[\frac{3}{2} - n, \frac{1}{2}, -n, \frac{5}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]} \right] \right. \right. \right. \\ & \left. \left. \frac{1}{1 + \text{Sin}[e + f x]} \right) \right) + a (-3 + 2 n) \text{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]} \right] (1 + \text{Sin}[e + f x]) \right) \left. \right) \left. \right) / \\ & \left(2 a^2 f \sqrt{a (1 + \text{Sin}[e + f x])} \sqrt{\frac{2 a^2 (1 + \text{Sin}[e + f x]) - a^2 (1 + \text{Sin}[e + f x])^2}{a^2}} \sqrt{1 - \frac{(a + a (1 + \text{Sin}[e + f x]))^2}{a^2}} \right) \end{aligned}$$

■ **Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (\text{d Sin}[e + f x])^n (1 + \text{Sin}[e + f x])^{3/2} dx$$

Optimal (type 5, 130 leaves, 4 steps):

$$\frac{2 \text{Cos}[e + f x] (\text{d Sin}[e + f x])^{1+n}}{d f (3 + 2 n) \sqrt{1 + \text{Sin}[e + f x]}} + \frac{(5 + 4 n) \text{Cos}[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, 1 + n, 2 + n, \text{Sin}[e + f x]\right] (\text{d Sin}[e + f x])^{1+n}}{d f (1 + n) (3 + 2 n) \sqrt{1 - \text{Sin}[e + f x]} \sqrt{1 + \text{Sin}[e + f x]}}$$

Result (type 6, 20257 leaves) : Display of huge result suppressed!

■ **Problem 125: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (\text{d Sin}[e + f x])^n \sqrt{1 + \text{Sin}[e + f x]} \, dx$$

Optimal (type 5, 72 leaves, 2 steps) :

$$\frac{\text{Cos}[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, 1 + n, 2 + n, \text{Sin}[e + f x]\right] (\text{d Sin}[e + f x])^{1+n}}{df (1+n) \sqrt{1 - \text{Sin}[e + f x]} \sqrt{1 + \text{Sin}[e + f x]}}$$

Result (type 5, 231 leaves) :

$$\left((1 + i) e^{-\frac{1}{2}i(e+fx)} (2 - 2e^{2i(e+fx)})^{-n} (-i e^{-i(e+fx)} (-1 + e^{2i(e+fx)}))^n \left(i(-1 + 2n) \text{Hypergeometric2F1}\left[\frac{1}{4}(-1 - 2n), -n, \frac{1}{4}(3 - 2n), e^{2i(e+fx)}\right] + e^{i(e+fx)} (1 + 2n) \text{Hypergeometric2F1}\left[\frac{1}{4}(1 - 2n), -n, \frac{1}{4}(5 - 2n), e^{2i(e+fx)}\right] \right) \text{Sin}[e + f x]^{-n} (\text{d Sin}[e + f x])^n \sqrt{1 + \text{Sin}[e + f x]} \right) / \left(f(-1 + 2n)(1 + 2n) \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \right)$$

■ **Problem 126: Result more than twice size of optimal antiderivative.**

$$\int \frac{(\text{d Sin}[e + f x])^n}{\sqrt{1 + \text{Sin}[e + f x]}} \, dx$$

Optimal (type 6, 78 leaves, 4 steps) :

$$-\frac{1}{f \sqrt{1 + \text{Sin}[e + f x]}} \text{AppellF1}\left[\frac{1}{2}, -n, 1, \frac{3}{2}, 1 - \text{Sin}[e + f x], \frac{1}{2}(1 - \text{Sin}[e + f x])\right] \text{Cos}[e + f x] \text{Sin}[e + f x]^{-n} (\text{d Sin}[e + f x])^n$$

Result (type 6, 389 leaves) :

$$\frac{1}{f} \text{Sec}[e + f x] (\text{d Sin}[e + f x])^n (1 + \text{Sin}[e + f x])^{3/2} \left(\left(4 \text{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1 + \text{Sin}[e + f x]), 1 + \text{Sin}[e + f x]\right] \right) / \left(8 \text{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1 + \text{Sin}[e + f x]), 1 + \text{Sin}[e + f x]\right] + \left(-4n \text{AppellF1}\left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2}(1 + \text{Sin}[e + f x]), 1 + \text{Sin}[e + f x]\right] + \text{AppellF1}\left[2, \frac{3}{2}, -n, 3, \frac{1}{2}(1 + \text{Sin}[e + f x]), 1 + \text{Sin}[e + f x]\right] \right) (1 + \text{Sin}[e + f x]) \right) - \left((-1 + 2n) \text{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]}\right] (-1 + \text{Sin}[e + f x]) \right) / \left((1 + 2n) \left(2 \left(n \text{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]}\right] + \text{AppellF1}\left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]}\right] \right) + (-1 + 2n) \text{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \text{Sin}[e + f x]}, \frac{1}{1 + \text{Sin}[e + f x]}\right] (1 + \text{Sin}[e + f x]) \right) \right)$$

■ **Problem 127: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Sin}[e + f x])^n}{(1 + \operatorname{Sin}[e + f x])^{3/2}} dx$$

Optimal (type 6, 80 leaves, 4 steps):

$$-\frac{1}{2 f \sqrt{1 + \operatorname{Sin}[e + f x]}} \operatorname{AppellF1}\left[\frac{1}{2}, -n, 2, \frac{3}{2}, 1 - \operatorname{Sin}[e + f x], \frac{1}{2} (1 - \operatorname{Sin}[e + f x])\right] \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]^{-n} (d \operatorname{Sin}[e + f x])^n$$

Result (type 6, 626 leaves):

$$\begin{aligned} & \frac{1}{2 f} \operatorname{Sec}[e + f x] (d \operatorname{Sin}[e + f x])^n \sqrt{1 + \operatorname{Sin}[e + f x]} \left(\left(4 \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x]\right] (1 + \operatorname{Sin}[e + f x]) \right) / \right. \\ & \left(8 \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x]\right] + \left(-4 n \operatorname{AppellF1}\left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x]\right] + \right. \right. \\ & \left. \left. \operatorname{AppellF1}\left[2, \frac{3}{2}, -n, 3, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), 1 + \operatorname{Sin}[e + f x]\right] \right) (1 + \operatorname{Sin}[e + f x]) \right) - \\ & \left((-1 + 2 n) \operatorname{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{1}{1 + \operatorname{Sin}[e + f x]}\right] (-1 + \operatorname{Sin}[e + f x]) (1 + \operatorname{Sin}[e + f x]) \right) / \\ & \left((1 + 2 n) \left(2 \left(n \operatorname{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{1}{1 + \operatorname{Sin}[e + f x]}\right] + \operatorname{AppellF1}\left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \right. \right. \right. \\ & \left. \left. \frac{1}{1 + \operatorname{Sin}[e + f x]}\right] \right) + (-1 + 2 n) \operatorname{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{1}{1 + \operatorname{Sin}[e + f x]}\right] (1 + \operatorname{Sin}[e + f x]) \right) \right) - \\ & \left(2 (-3 + 2 n) \operatorname{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{1}{1 + \operatorname{Sin}[e + f x]}\right] (-1 + \operatorname{Sin}[e + f x]) \right) / \\ & \left((-1 + 2 n) \left(2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} - n, -\frac{1}{2}, 1 - n, \frac{5}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{1}{1 + \operatorname{Sin}[e + f x]}\right] + \operatorname{AppellF1}\left[\frac{3}{2} - n, \frac{1}{2}, -n, \frac{5}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \right. \right. \right. \\ & \left. \left. \frac{1}{1 + \operatorname{Sin}[e + f x]}\right] \right) + (-3 + 2 n) \operatorname{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \operatorname{Sin}[e + f x]}, \frac{1}{1 + \operatorname{Sin}[e + f x]}\right] (1 + \operatorname{Sin}[e + f x]) \right) \right) \end{aligned}$$

■ **Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d \operatorname{Sin}[e + f x])^n (a + a \operatorname{Sin}[e + f x])^{3/2} dx$$

Optimal (type 5, 131 leaves, 5 steps):

$$-\frac{2 a^2 (5 + 4 n) \operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, 1 - \operatorname{Sin}[e + f x]\right] \operatorname{Sin}[e + f x]^{-n} (d \operatorname{Sin}[e + f x])^n}{d f (3 + 2 n) \sqrt{a + a \operatorname{Sin}[e + f x]}} - \frac{2 a^2 \operatorname{Cos}[e + f x] (d \operatorname{Sin}[e + f x])^{1+n}}{d f (3 + 2 n) \sqrt{a + a \operatorname{Sin}[e + f x]}}$$

Result (type 6, 20259 leaves): Display of huge result suppressed!

- **Problem 129: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (\text{d Sin}[e + f x])^n \sqrt{a + a \text{Sin}[e + f x]} \, dx$$

Optimal (type 5, 66 leaves, 3 steps):

$$-\frac{2 a \text{Cos}[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, 1 - \text{Sin}[e + f x]\right] \text{Sin}[e + f x]^{-n} (\text{d Sin}[e + f x])^n}{f \sqrt{a + a \text{Sin}[e + f x]}}$$

Result (type 5, 266 leaves):

$$\frac{1}{f (-1 + 2n) (1 + 2n) \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)} \\ (1 + i) e^{-\frac{1}{2} i f x} \left(e^{i f x} (1 + 2n) \text{Hypergeometric2F1}\left[\frac{1}{4}(1 - 2n), -n, \frac{1}{4}(5 - 2n), e^{2 i f x} (\text{Cos}[e] + i \text{Sin}[e])^2\right] \left(\text{Cos}\left[\frac{e}{2}\right] + i \text{Sin}\left[\frac{e}{2}\right]\right) + \right. \\ \left. (-1 + 2n) \text{Hypergeometric2F1}\left[\frac{1}{4}(-1 - 2n), -n, \frac{1}{4}(3 - 2n), e^{2 i f x} (\text{Cos}[e] + i \text{Sin}[e])^2\right] \left(i \text{Cos}\left[\frac{e}{2}\right] + \text{Sin}\left[\frac{e}{2}\right]\right) \right) \\ (1 - e^{2 i f x} \text{Cos}[e]^2 + e^{2 i f x} \text{Sin}[e]^2 - i e^{2 i f x} \text{Sin}[2e])^{-n} (\text{d Sin}[e + f x])^n \sqrt{a (1 + \text{Sin}[e + f x])}$$

- **Problem 130: Result more than twice size of optimal antiderivative.**

$$\int \frac{(\text{d Sin}[e + f x])^n}{\sqrt{a + a \text{Sin}[e + f x]}} \, dx$$

Optimal (type 6, 80 leaves, 5 steps):

$$-\frac{1}{f \sqrt{a + a \text{Sin}[e + f x]}} \text{AppellF1}\left[\frac{1}{2}, -n, 1, \frac{3}{2}, 1 - \text{Sin}[e + f x], \frac{1}{2}(1 - \text{Sin}[e + f x])\right] \text{Cos}[e + f x] \text{Sin}[e + f x]^{-n} (\text{d Sin}[e + f x])^n$$

Result (type 6, 446 leaves):

$$\frac{1}{f \sqrt{a (1 + \sin[e + f x])}} \sec[e + f x] \sin[e + f x]^{-n} (d \sin[e + f x])^n (1 + \sin[e + f x])^2$$

$$\left(\left(4 \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), 1 + \sin[e + f x] \right] (-\sin[e + f x])^{-n} (-\sin[e + f x]^2)^n \right) / \right.$$

$$\left(8 \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), 1 + \sin[e + f x] \right] - \left(4 n \operatorname{AppellF1} \left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2} (1 + \sin[e + f x]), 1 + \sin[e + f x] \right] - \right.$$

$$\left. \left. \operatorname{AppellF1} \left[2, \frac{3}{2}, -n, 3, \frac{1}{2} (1 + \sin[e + f x]), 1 + \sin[e + f x] \right] \right) (1 + \sin[e + f x]) \right) -$$

$$\left((-1 + 2n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{1}{1 + \sin[e + f x]} \right] (-1 + \sin[e + f x]) \sin[e + f x]^n \right) /$$

$$\left((1 + 2n) \left(2 \left(n \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{1}{1 + \sin[e + f x]} \right] + \operatorname{AppellF1} \left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \right. \right. \right.$$

$$\left. \left. \frac{1}{1 + \sin[e + f x]} \right] \right) + (-1 + 2n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{1}{1 + \sin[e + f x]} \right] (1 + \sin[e + f x]) \right) \right)$$

■ **Problem 131: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \sin[e + f x])^n}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 6, 85 leaves, 5 steps):

$$- \frac{1}{2 a f \sqrt{a + a \sin[e + f x]}} \operatorname{AppellF1} \left[\frac{1}{2}, -n, 2, \frac{3}{2}, 1 - \sin[e + f x], \frac{1}{2} (1 - \sin[e + f x]) \right] \cos[e + f x] \sin[e + f x]^{-n} (d \sin[e + f x])^n$$

Result (type 6, 798 leaves):

$$\begin{aligned}
& \left(\cos[e+fx] (d \sin[e+fx])^n (1+\sin[e+fx]) \left(\frac{-a+a(1+\sin[e+fx])}{a} \right)^{-n} \right. \\
& \left(\left(4a^2 \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\sin[e+fx]), 1+\sin[e+fx] \right] (-\sin[e+fx])^{-n} (1+\sin[e+fx]) \left(-\frac{(a-a(1+\sin[e+fx]))^2}{a^2} \right)^n \right) / \right. \\
& \left(8a \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\sin[e+fx]), 1+\sin[e+fx] \right] + \right. \\
& \left. a \left(-4n \operatorname{AppellF1} \left[2, \frac{1}{2}, 1-n, 3, \frac{1}{2}(1+\sin[e+fx]), 1+\sin[e+fx] \right] + \operatorname{AppellF1} \left[2, \frac{3}{2}, -n, 3, \frac{1}{2}(1+\sin[e+fx]), 1+\sin[e+fx] \right] \right) \right. \\
& \left. (1+\sin[e+fx]) \right) - \left(a(-1+2n) \operatorname{AppellF1} \left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]} \right] \right. \\
& \left. \sin[e+fx]^n (1+\sin[e+fx]) (-2a+a(1+\sin[e+fx])) \right) \left. \right) / \\
& \left((1+2n) \left(2a \left(n \operatorname{AppellF1} \left[\frac{1}{2}-n, -\frac{1}{2}, 1-n, \frac{3}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]} \right] + \operatorname{AppellF1} \left[\frac{1}{2}-n, \frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]} \right], \right. \right. \\
& \left. \left. \frac{1}{1+\sin[e+fx]} \right) \right) + a(-1+2n) \operatorname{AppellF1} \left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]} \right] (1+\sin[e+fx]) \right) \left. \right) - \\
& \left(2a(-3+2n) \operatorname{AppellF1} \left[\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]} \right] \sin[e+fx]^n (-2a+a(1+\sin[e+fx])) \right) / \\
& \left((-1+2n) \left(2a \left(n \operatorname{AppellF1} \left[\frac{3}{2}-n, -\frac{1}{2}, 1-n, \frac{5}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]} \right] + \operatorname{AppellF1} \left[\frac{3}{2}-n, \frac{1}{2}, -n, \frac{5}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]} \right], \right. \right. \\
& \left. \left. \frac{1}{1+\sin[e+fx]} \right) \right) + a(-3+2n) \operatorname{AppellF1} \left[\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]} \right] (1+\sin[e+fx]) \right) \left. \right) \left. \right) / \\
& \left(2a^2 f \sqrt{a(1+\sin[e+fx])} \sqrt{\frac{2a^2(1+\sin[e+fx]) - a^2(1+\sin[e+fx])^2}{a^2}} \sqrt{1 - \frac{(a-a(1+\sin[e+fx]))^2}{a^2}} \right)
\end{aligned}$$

■ **Problem 132: Result more than twice size of optimal antiderivative.**

$$\int \sin[e+fx]^n (1+\sin[e+fx])^m dx$$

Optimal (type 6, 71 leaves, 2 steps):

$$\frac{{}_2F_1^{\frac{1}{2}+m} \operatorname{AppellF1} \left[\frac{1}{2}, -n, \frac{1}{2}-m, \frac{3}{2}, 1-\sin[e+fx], \frac{1}{2}(1-\sin[e+fx]) \right] \cos[e+fx]}{f \sqrt{1+\sin[e+fx]}}$$

Result (type 6, 2805 leaves):

$$-\left(\left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -n, 1+m+n, \frac{3}{2}, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \cos[e+fx] \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)^{-m} \right. \right.$$

$$\int (1 - \sin[e + f x])^m (-\sin[e + f x])^n dx$$

Optimal (type 6, 68 leaves, 2 steps):

$$\frac{2^{\frac{1}{2}+m} \text{AppellF1}\left[\frac{1}{2}, -n, \frac{1}{2} - m, \frac{3}{2}, 1 + \sin[e + f x], \frac{1}{2} (1 + \sin[e + f x])\right] \cos[e + f x]}{f \sqrt{1 - \sin[e + f x]}}$$

Result (type 6, 4138 leaves):

$$\begin{aligned} & - \left(\left(2 (3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{1+n} \right. \right. \\ & \quad \left. \left. (1 - \sin[e + f x])^m (-\sin[e + f x])^n \left(\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]\right)^2 \sin[e + f x]\right)^n \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}}\right)^{2m} \right) \right) / \\ & \left(f (1 + 2m) \left((3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\ & \quad 2 \left(n \text{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\ & \quad \left. \left. (1 + m + n) \text{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \\ & \left(\left((3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]\right)^2 \right)^n \right. \\ & \quad \left. \left(\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]\right)^2 \sin[e + f x]\right)^n \left(\frac{\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}}\right)^{2m} \right) / \end{aligned}$$

$$\begin{aligned}
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, -n, 1+m+n, \frac{3}{2}+m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \right. \\
& \quad 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2}+m, 1-n, 1+m+n, \frac{5}{2}+m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}+m, -n, 2+m+n, \frac{5}{2}+m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) - \\
& \left(2 (3+2m) (1+n) \operatorname{AppellF1} \left[\frac{1}{2}+m, -n, 1+m+n, \frac{3}{2}+m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \\
& \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^n \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e+fx] \right)^n \left(\frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right) / \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, -n, 1+m+n, \frac{3}{2}+m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \right. \\
& \quad 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2}+m, 1-n, 1+m+n, \frac{5}{2}+m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}+m, -n, 2+m+n, \frac{5}{2}+m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) + \\
& \left(2 (3+2m) \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e+fx] \right)^n \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(\frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right) \\
& \left(-\frac{1}{\frac{3}{2}+m} \left(\frac{1}{2}+m \right) n \operatorname{AppellF1} \left[\frac{3}{2}+m, 1-n, 1+m+n, \frac{5}{2}+m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \left. \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{\frac{3}{2}+m} \left(\frac{1}{2}+m \right) (1+m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, -n, 2+m+n, \right. \right.
\end{aligned}$$

$$\left. \begin{aligned}
& \left. \left. \left. \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) / \\
& \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) + \\
& \left(2 (3 + 2m) n \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{1+n} \right. \\
& \quad \left. \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \right)^{-1+n} \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \right) \right) \\
& \left. \left(-\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 + \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) / \\
& \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) + \\
& \left(4m (3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right)
\end{aligned} \right)$$

$$\begin{aligned}
& \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e + f x] \right)^n \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \\
& \left(\frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{-1+2m} \left(\frac{1}{2} \sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} - \frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{2 \sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right) \Bigg/ \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1+m+n, \frac{3}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \right. \\
& \quad 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1-n, 1+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, 2+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \Bigg) - \\
& \left(2 (3+2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1+m+n, \frac{3}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \right. \\
& \quad \left. \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e + f x] \right)^n \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(\frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right) \right. \\
& \left(-2 \left(n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1-n, 1+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, \right. \right. \right. \\
& \quad \left. \left. 2+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] + \right. \\
& \quad (3+2m) \left(-\frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m \right) n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1-n, 1+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \left. \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m \right) (1+m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, 2+m+n, \right. \right. \\
& \quad \left. \left. \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(n \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) (1 + m + n) \operatorname{AppellF1}\left[\frac{5}{2} + m, 1 - n, 2 + m + n, \frac{7}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right], \right. \right. \\
& \quad \left. \left. - \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) (1 - n) \operatorname{AppellF1}\left[\frac{5}{2} + m, 2 - n, \right. \right. \\
& \quad \left. \left. 1 + m + n, \frac{7}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + \\
& \quad (1 + m + n) \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) n \operatorname{AppellF1}\left[\frac{5}{2} + m, 1 - n, 2 + m + n, \frac{7}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) (2 + m + n) \operatorname{AppellF1}\left[\frac{5}{2} + m, -n, 3 + m + n, \frac{7}{2} + m, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \Bigg) / \\
& \quad \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] + (1 + m + n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 134: Result more than twice size of optimal antiderivative.**

$$\int (d \operatorname{Sin}[e + f x])^n (1 + \operatorname{Sin}[e + f x])^m dx$$

Optimal (type 6, 91 leaves, 3 steps):

$$-\frac{1}{f \sqrt{1 + \operatorname{Sin}[e + f x]}} 2^{\frac{1}{2} + m} \operatorname{AppellF1}\left[\frac{1}{2}, -n, \frac{1}{2} - m, \frac{3}{2}, 1 - \operatorname{Sin}[e + f x], \frac{1}{2} (1 - \operatorname{Sin}[e + f x])\right] \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]^{-n} (d \operatorname{Sin}[e + f x])^n$$

Result (type 6, 2813 leaves):

$$-\left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -n, 1 + m + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Cos}[e + f x] \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-m} \operatorname{Sin}[e + f x]^n \right. \right.$$

$$\int (1 - \sin[e + f x])^m (d \sin[e + f x])^n dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{1}{f \sqrt{1 - \sin[e + f x]}} 2^{\frac{1}{2} + m} \text{AppellF1}\left[\frac{1}{2}, -n, \frac{1}{2} - m, \frac{3}{2}, 1 + \sin[e + f x], \frac{1}{2} (1 + \sin[e + f x])\right] \cos[e + f x] (-\sin[e + f x])^{-n} (d \sin[e + f x])^n$$

Result (type 6, 4138 leaves):

$$- \left(\left(2 (3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{1+n} \right. \right. \\ \left. \left. (1 - \sin[e + f x])^m (d \sin[e + f x])^n \left(\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sin[e + f x]\right)^n \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}}\right)^{2m} \right) \right) / \\ \left(f (1 + 2m) \left((3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\ \left. \left. 2 \left(n \text{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \right. \\ \left. \left. \left. (1 + m + n) \text{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \\ \left(\left((3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^n \right. \right. \\ \left. \left. \left(\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sin[e + f x]\right)^n \left(\frac{\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}}\right)^{2m} \right) \right) /$$

$$\begin{aligned}
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, -n, 1+m+n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \\
& \quad 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2}+m, 1-n, 1+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}+m, -n, 2+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \left(2 (3+2m) (1+n) \operatorname{AppellF1} \left[\frac{1}{2}+m, -n, 1+m+n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \\
& \left(\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^n \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Sin} [e + f x] \right)^n \left(\frac{\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right) / \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, -n, 1+m+n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \\
& \quad 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2}+m, 1-n, 1+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}+m, -n, 2+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) + \\
& \left(2 (3+2m) \left(\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Sin} [e + f x] \right)^n \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right) \right) \\
& \left(-\frac{1}{\frac{3}{2}+m} \left(\frac{1}{2}+m \right) n \operatorname{AppellF1} \left[\frac{3}{2}+m, 1-n, 1+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{\frac{3}{2}+m} \left(\frac{1}{2}+m \right) (1+m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, -n, 2+m+n, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) / \\
& \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) + \\
& \left(2 (3 + 2m) n \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{1+n} \right. \\
& \quad \left. \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \right)^{-1+n} \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \right) \right) \\
& \left. \left(-\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 + \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) / \\
& \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) + \\
& \left(4m (3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e + f x] \right)^n \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \\
& \left(\frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{-1+2m} \left(\frac{1}{2} \sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} - \frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{2 \sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right) \Bigg/ \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1+m+n, \frac{3}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1-n, 1+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, 2+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \left(2 (3+2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1+m+n, \frac{3}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \right. \\
& \quad \left. \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e + f x] \right)^n \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(\frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right) \right. \\
& \left(-2 \left(n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1-n, 1+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, \right. \right. \right. \\
& \quad \left. \left. \left. 2+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] + \right. \\
& \quad \left. (3+2m) \left(-\frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m \right) n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1-n, 1+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m \right) (1+m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, 2+m+n, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) -
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(n \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) (1 + m + n) \operatorname{AppellF1}\left[\frac{5}{2} + m, 1 - n, 2 + m + n, \frac{7}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, \right. \right. \\
& \quad \left. \left. - \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) (1 - n) \operatorname{AppellF1}\left[\frac{5}{2} + m, 2 - n, \right. \right. \\
& \quad \left. \left. 1 + m + n, \frac{7}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + \\
& \quad (1 + m + n) \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) n \operatorname{AppellF1}\left[\frac{5}{2} + m, 1 - n, 2 + m + n, \frac{7}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) (2 + m + n) \operatorname{AppellF1}\left[\frac{5}{2} + m, -n, 3 + m + n, \frac{7}{2} + m, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \Bigg) / \\
& \quad \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] + (1 + m + n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 136: Result more than twice size of optimal antiderivative.**

$$\int \sin[e + f x]^n (a + a \sin[e + f x])^m dx$$

Optimal (type 6, 87 leaves, 3 steps):

$$-\frac{1}{f} 2^{\frac{1}{2} + m} \operatorname{AppellF1}\left[\frac{1}{2}, -n, \frac{1}{2} - m, \frac{3}{2}, 1 - \sin[e + f x], \frac{1}{2} (1 - \sin[e + f x])\right] \cos[e + f x] (1 + \sin[e + f x])^{-\frac{1}{2} - m} (a + a \sin[e + f x])^m$$

Result (type 6, 2807 leaves):

$$-\left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -n, 1 + m + n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \cos[e + f x] \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-m} \right. \right.$$

$$\int (-\sin[e + f x])^n (a - a \sin[e + f x])^m dx$$

Optimal (type 6, 85 leaves, 3 steps):

$$\frac{1}{f} 2^{\frac{1}{2}+m} \text{AppellF1}\left[\frac{1}{2}, -n, \frac{1}{2}-m, \frac{3}{2}, 1 + \sin[e + f x], \frac{1}{2} (1 + \sin[e + f x])\right] \cos[e + f x] (1 - \sin[e + f x])^{-\frac{1}{2}-m} (a - a \sin[e + f x])^m$$

Result (type 6, 4139 leaves):

$$- \left(\left(2 (3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{1+n} (-\sin[e + f x])^n \right. \right.$$

$$\left. \left(\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sin[e + f x] \right)^n (a - a \sin[e + f x])^m \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \right) /$$

$$\left(f (1 + 2m) \left((3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right.$$

$$2 \left(n \text{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.$$

$$\left. \left. (1 + m + n) \text{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)$$

$$\left(\left((3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^n \right. \right.$$

$$\left. \left(\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sin[e + f x] \right)^n \left(\frac{\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \right) /$$

$$\begin{aligned}
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, -n, 1+m+n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \\
& \quad 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2}+m, 1-n, 1+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}+m, -n, 2+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \left(2 (3+2m) (1+n) \operatorname{AppellF1} \left[\frac{1}{2}+m, -n, 1+m+n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \\
& \left(\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^n \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Sin} [e + f x] \right)^n \left(\frac{\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right) / \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, -n, 1+m+n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \\
& \quad 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2}+m, 1-n, 1+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}+m, -n, 2+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) + \\
& \left(2 (3+2m) \left(\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Sin} [e + f x] \right)^n \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right) \right) \\
& \left(-\frac{1}{\frac{3}{2}+m} \left(\frac{1}{2}+m \right) n \operatorname{AppellF1} \left[\frac{3}{2}+m, 1-n, 1+m+n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{\frac{3}{2}+m} \left(\frac{1}{2}+m \right) (1+m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, -n, 2+m+n, \right. \right.
\end{aligned}$$

$$\left. \begin{aligned}
& \left. \left. \left. \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) / \\
& \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) + \\
& \left(2 (3 + 2m) n \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{1+n} \right. \\
& \quad \left. \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \right)^{-1+n} \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \right) \right) \\
& \left. \left. \left. \left(-\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 + \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \right) / \\
& \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) + \\
& \left(4m (3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right)
\end{aligned} \right)$$

$$\begin{aligned}
& \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e + f x] \right)^n \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \\
& \left(\frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{-1+2m} \left(\frac{1}{2} \sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} - \frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{2 \sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right) \Bigg/ \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1+m+n, \frac{3}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \right. \\
& \quad 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1-n, 1+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, 2+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \Bigg) - \\
& \left(2 (3+2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1+m+n, \frac{3}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \right. \\
& \quad \left. \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e + f x] \right)^n \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(\frac{\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right) \right. \\
& \left(-2 \left(n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1-n, 1+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1+m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, \right. \right. \right. \\
& \quad \left. \left. 2+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] + \right. \\
& \quad (3+2m) \left(-\frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m \right) n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1-n, 1+m+n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \left. \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m \right) (1+m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, 2+m+n, \right. \right. \\
& \quad \left. \left. \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(n \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) (1 + m + n) \operatorname{AppellF1}\left[\frac{5}{2} + m, 1 - n, 2 + m + n, \frac{7}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, \right. \right. \\
& \quad \left. \left. - \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) (1 - n) \operatorname{AppellF1}\left[\frac{5}{2} + m, 2 - n, \right. \right. \\
& \quad \left. \left. 1 + m + n, \frac{7}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + \\
& \quad (1 + m + n) \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) n \operatorname{AppellF1}\left[\frac{5}{2} + m, 1 - n, 2 + m + n, \frac{7}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m\right) (2 + m + n) \operatorname{AppellF1}\left[\frac{5}{2} + m, -n, 3 + m + n, \frac{7}{2} + m, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \Bigg) / \\
& \quad \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] + (1 + m + n) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 138: Result more than twice size of optimal antiderivative.**

$$\int (d \operatorname{Sin}[e + f x])^n (a + a \operatorname{Sin}[e + f x])^m dx$$

Optimal (type 6, 107 leaves, 4 steps):

$$-\frac{1}{f} 2^{\frac{1}{2} + m} \operatorname{AppellF1}\left[\frac{1}{2}, -n, \frac{1}{2} - m, \frac{3}{2}, 1 - \operatorname{Sin}[e + f x], \frac{1}{2} (1 - \operatorname{Sin}[e + f x])\right]$$

$$\operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]^{-n} (d \operatorname{Sin}[e + f x])^n (1 + \operatorname{Sin}[e + f x])^{-\frac{1}{2} - m} (a + a \operatorname{Sin}[e + f x])^m$$

Result (type 6, 2815 leaves):

Problem 139: Result more than twice size of optimal antiderivative.

$$\int (d \operatorname{Sin}[e + f x])^n (a - a \operatorname{Sin}[e + f x])^m dx$$

Optimal (type 6, 107 leaves, 4 steps):

$$\frac{1}{f} 2^{\frac{1}{2}+m} \operatorname{AppellF1}\left[\frac{1}{2}, -n, \frac{1}{2}-m, \frac{3}{2}, 1 + \operatorname{Sin}[e + f x], \frac{1}{2} (1 + \operatorname{Sin}[e + f x])\right]$$

$$\operatorname{Cos}[e + f x] (1 - \operatorname{Sin}[e + f x])^{-\frac{1}{2}-m} (-\operatorname{Sin}[e + f x])^{-n} (d \operatorname{Sin}[e + f x])^n (a - a \operatorname{Sin}[e + f x])^m$$

Result (type 6, 4139 leaves):

$$- \left(\left(2 (3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{1+n} (d \operatorname{Sin}[e + f x])^n \right.$$

$$\left. \left(\operatorname{Sec}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \right)^n (a - a \operatorname{Sin}[e + f x])^m \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \right) /$$

$$\left(f (1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right.$$

$$2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.$$

$$\left. (1 + m + n) \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)$$

$$\left(\left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^n \right.$$

$$\begin{aligned}
& \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \right)^n \left(\frac{\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \Big/ \\
& \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \quad 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \Big) - \\
& \left(2 (3 + 2m) (1 + n) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \\
& \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^n \operatorname{Sin}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \right)^n \left(\frac{\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \Big/ \\
& \left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \quad 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \Big) + \\
& \left(2 (3 + 2m) \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{1+n} \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x] \right)^n \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \right) \\
& \left(-\frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m \right) n \operatorname{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
& \text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m\right) (1 + m + n) \text{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \right. \\
& \left. \frac{5}{2} + m, \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right] \Bigg/ \\
& \left((1 + 2m) \left((3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left(n \text{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) + \\
& \left(2 (3 + 2m) n \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\text{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{1+n} \right. \\
& \left. \left(\text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Sin}[e + f x] \right)^{-1+n} \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right)^{2m} \right) \right) \\
& \left. \left(-\text{Cos}[e + f x] \text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 + \text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Sin}[e + f x] \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right] \Bigg/ \\
& \left((1 + 2m) \left((3 + 2m) \text{AppellF1}\left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left(n \text{AppellF1}\left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + m + n) \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) +
\end{aligned}
\right.
\end{aligned}$$

$$\left(4 m (3 + 2 m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right.$$

$$\left. \left(\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Sin} [e + f x] \right)^n \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right.$$

$$\left. \left. \left(\frac{\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{-1+2m} \left(\frac{1}{2} \sqrt{\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} - \frac{\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{2 \sqrt{\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right) \right) \right) /$$

$$\left((1 + 2 m) \left((3 + 2 m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \right.$$

$$2 \left(n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1 + m + n) \right.$$

$$\left. \left. \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) -$$

$$\left(2 (3 + 2 m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \left(\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{1+n} \right.$$

$$\left. \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Sin} [e + f x] \right)^n \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right)^{2m} \right)$$

$$\left(-2 \left(n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1 + m + n) \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, \right. \right.$$

$$2 + m + n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] + \right.$$

$$\left. (3 + 2 m) \left(-\frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m \right) n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \right)$$

$$\begin{aligned}
& \left. \begin{aligned}
& \left. \left. \left. \left. \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{\frac{3}{2} + m} \left(\frac{1}{2} + m \right) (1 + m + n) \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, 2 + m + n, \right. \right. \right. \\
& \left. \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) - \\
& 2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(n \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m \right) (1 + m + n) \operatorname{AppellF1} \left[\frac{5}{2} + m, 1 - n, 2 + m + n, \frac{7}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] + \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m \right) (1 - n) \operatorname{AppellF1} \left[\frac{5}{2} + m, 2 - n, \right. \right. \right. \\
& \left. 1 + m + n, \frac{7}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) + \\
& (1 + m + n) \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m \right) n \operatorname{AppellF1} \left[\frac{5}{2} + m, 1 - n, 2 + m + n, \frac{7}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \\
& \left. \left. \left. \left. \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m \right) (2 + m + n) \operatorname{AppellF1} \left[\frac{5}{2} + m, -n, 3 + m + n, \frac{7}{2} + m, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) \right) \right) \right) \right) / \\
& \left((1 + 2 m) \left((3 + 2 m) \operatorname{AppellF1} \left[\frac{1}{2} + m, -n, 1 + m + n, \frac{3}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \right. \right. \\
& \left. 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2} + m, 1 - n, 1 + m + n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1 + m + n) \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. \left. \left. \operatorname{AppellF1} \left[\frac{3}{2} + m, -n, 2 + m + n, \frac{5}{2} + m, \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) \right) \right) \right) \right) \right) \right)
\end{aligned}
\right.
\end{aligned}$$

■ **Problem 141: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sin[c + dx]^3 (a + a \sin[c + dx])^n dx$$

Optimal (type 5, 215 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(4+n) \cos[c+dx] (a+a \sin[c+dx])^n}{d(1+n)(2+n)(3+n)} - \frac{\cos[c+dx] \sin[c+dx]^2 (a+a \sin[c+dx])^n}{d(3+n)} - \frac{1}{d(1+n)(2+n)(3+n)} \\
& 2^{\frac{1}{2}+n} n (5+3n+n^2) \cos[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}, \frac{1}{2}(1-\sin[c+dx])\right] (1+\sin[c+dx])^{-\frac{1}{2}-n} (a+a \sin[c+dx])^n - \\
& \frac{n \cos[c+dx] (a+a \sin[c+dx])^{1+n}}{ad(6+5n+n^2)}
\end{aligned}$$

Result (type 5, 316 leaves):

$$\begin{aligned}
& \frac{1}{d(9-10n^2+n^4)} \\
& 2^{-3-2n} e^{3i(c+dx)} (1+i e^{-i(c+dx)})^{-2n} \left(e^{-\frac{1}{4}i(2c+\pi+2dx)} (i+e^{i(c+dx)}) \right)^{2n} \left((3-n-3n^2+n^3) \operatorname{Hypergeometric2F1}[-3-n, -2n, -2-n, -i e^{-i(c+dx)}] - \right. \\
& e^{-2i(c+dx)} (3+n) (3(3-4n+n^2) \operatorname{Hypergeometric2F1}[-1-n, -2n, -n, -i e^{-i(c+dx)}] - \\
& e^{-4i(c+dx)} (1+n) (3e^{2i(c+dx)} (-3+n) \operatorname{Hypergeometric2F1}[1-n, -2n, 2-n, -i e^{-i(c+dx)}] - \\
& \left. (-1+n) \operatorname{Hypergeometric2F1}[3-n, -2n, 4-n, -i e^{-i(c+dx)}]) \right) (a(1+\sin[c+dx]))^n \sin\left[\frac{1}{4}(2c+\pi+2dx)\right]^{-2n}
\end{aligned}$$

■ **Problem 142: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sin[c+dx]^2 (a+a \sin[c+dx])^n dx$$

Optimal (type 5, 156 leaves, 4 steps):

$$\begin{aligned}
& \frac{\cos[c+dx] (a+a \sin[c+dx])^n}{d(2+3n+n^2)} - \frac{1}{d(1+n)(2+n)} \\
& 2^{\frac{1}{2}+n} (1+n+n^2) \cos[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}, \frac{1}{2}(1-\sin[c+dx])\right] (1+\sin[c+dx])^{-\frac{1}{2}-n} (a+a \sin[c+dx])^n - \\
& \frac{\cos[c+dx] (a+a \sin[c+dx])^{1+n}}{ad(2+n)}
\end{aligned}$$

Result (type 6, 29340 leaves): Display of huge result suppressed!

■ **Problem 143: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sin[c+dx] (a+a \sin[c+dx])^n dx$$

Optimal (type 5, 109 leaves, 3 steps):

$$\begin{aligned}
& - \frac{\cos[c+dx] (a+a \sin[c+dx])^n}{d(1+n)} - \frac{1}{d(1+n)} \\
& 2^{\frac{1}{2}+n} n \cos[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-n, \frac{3}{2}, \frac{1}{2}(1-\sin[c+dx])\right] (1+\sin[c+dx])^{-\frac{1}{2}-n} (a+a \sin[c+dx])^n
\end{aligned}$$

Result (type 5, 197 leaves) :

$$\frac{1}{d(-1+n)(1+n)} 2^{-1-2n} e^{-i(c+dx)} \left(1 + i e^{-i(c+dx)}\right)^{-2n} \left(e^{-\frac{1}{4}i(2c+\pi+2dx)} (i + e^{i(c+dx)})\right)^{2n} \\ (-e^{2i(c+dx)} (-1+n) \text{Hypergeometric2F1}[-1-n, -2n, -n, -i e^{-i(c+dx)}] + (1+n) \text{Hypergeometric2F1}[1-n, -2n, 2-n, -i e^{-i(c+dx)}]) \\ (a(1+\text{Sin}[c+dx]))^n \text{Sin}\left[\frac{1}{4}(2c+\pi+2dx)\right]^{-2n}$$

■ **Problem 145: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c+dx] (a + a \text{Sin}[c+dx])^n dx$$

Optimal (type 6, 85 leaves, 4 steps) :

$$-\frac{1}{d} 2^{\frac{1}{2}+n} \text{AppellF1}\left[\frac{1}{2}, 1, \frac{1}{2}-n, \frac{3}{2}, 1-\text{Sin}[c+dx], \frac{1}{2}(1-\text{Sin}[c+dx])\right] \text{Cos}[c+dx] (1+\text{Sin}[c+dx])^{-\frac{1}{2}-n} (a + a \text{Sin}[c+dx])^n$$

Result (type 6, 2560 leaves) :

$$-\left(\left(\text{Csc}[c+dx] \left(\text{Sec}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^{-n} (a + a \text{Sin}[c+dx])^n\right.\right. \\ \left.\left.\left(\text{AppellF1}\left[2n, n, n, 1+2n, -\frac{1+i}{-1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}, -\frac{1-i}{-1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right] \left(\frac{-i+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{-1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right)^n\right.\right. \\ \left.\left.\left(\frac{i+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{-1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right)^n - \text{AppellF1}\left[2n, n, n, 1+2n, \frac{1-i}{1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}, \frac{1+i}{1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right] \right.\right. \\ \left.\left.\left(\frac{-i+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right)^n \left(\frac{i+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right)^n\right)\right) / \\ \left(2dn \left(-\frac{1}{2} \left(\text{Sec}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^{-n} \text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] \left(\text{AppellF1}\left[2n, n, n, 1+2n, -\frac{1+i}{-1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]},\right.\right.\right.\right. \\ \left.\left.\left.\frac{1-i}{-1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right] \left(\frac{-i+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{-1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right)^n \left(\frac{i+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{-1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right)^n - \text{AppellF1}\left[2n, n, n, 1+2n,\right.\right.\right. \\ \left.\left.\left.\frac{1-i}{1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}, \frac{1+i}{1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right] \left(\frac{-i+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right)^n \left(\frac{i+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}\right)^n\right)\right) + \\ \frac{1}{2n} \left(\text{Sec}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right)^{-n} \left(\left(\left(1-i\right) n^2 \text{AppellF1}\left[1+2n, n, 1+n, 2+2n, -\frac{1+i}{-1+\text{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]},\right.\right.\right.\right.$$

$$n \operatorname{AppellF1}\left[2n, n, n, 1+2n, \frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^n$$

$$\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^{-1+n} \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\left(i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}\right)\right)$$

■ **Problem 146: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^2 (a+a\operatorname{Sin}[c+dx])^n dx$$

Optimal (type 6, 85 leaves, 4 steps):

$$-\frac{1}{d} 2^{\frac{1}{2}+n} \operatorname{AppellF1}\left[\frac{1}{2}, 2, \frac{1}{2}-n, \frac{3}{2}, 1-\operatorname{Sin}[c+dx], \frac{1}{2}(1-\operatorname{Sin}[c+dx])\right] \operatorname{Cos}[c+dx] (1+\operatorname{Sin}[c+dx])^{-\frac{1}{2}-n} (a+a\operatorname{Sin}[c+dx])^n$$

Result (type 6, 4206 leaves):

$$-\left(\operatorname{Csc}[c+dx]^2 \left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^{-n}\right.$$

$$(a+a\operatorname{Sin}[c+dx])^n \left(-\operatorname{AppellF1}\left[1+2n, n, n, 2+2n, \frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right]\right.$$

$$\left(-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right) \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^n \left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^n -$$

$$\operatorname{AppellF1}\left[1+2n, n, n, 2(1+n), -\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^n$$

$$\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^n \left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)\right) \left/d (1+2n) \left(-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)\right.$$

$$\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right) \left(-\frac{1}{2(1+2n)\left(-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2 \left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^{1-n}}\right.$$

$$\left(-\operatorname{AppellF1}\left[1+2n, n, n, 2+2n, \frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)\right)$$

$$\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^n \left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^n - \operatorname{AppellF1}\left[1+2n, n, n, 2(1+n), -\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]},\right.$$

$$\begin{aligned}
& n \operatorname{AppellF1}\left[1+2n, n, n, 2+2n, \frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right) \\
& \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^{-1+n} \left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^n \\
& \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\left(-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}\right) - n \operatorname{AppellF1}\left[1+2n, n, n, 2+2n, \right. \\
& \left.\frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right) \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^n \\
& \left.\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^{-1+n} \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\left(i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}\right)\right)\right)
\end{aligned}$$

■ **Problem 153: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx] (a+b \operatorname{Sin}[e+fx]) dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$bx - \frac{a \operatorname{ArcTanh}[\operatorname{Cos}[e+fx]]}{f}$$

Result (type 3, 43 leaves):

$$bx - \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{a \operatorname{Log}\left[\operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f}$$

■ **Problem 161: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx] (a+b \operatorname{Sin}[e+fx])^2 dx$$

Optimal (type 3, 35 leaves, 3 steps):

$$2abx - \frac{a^2 \operatorname{ArcTanh}[\operatorname{Cos}[e+fx]]}{f} - \frac{b^2 \operatorname{Cos}[e+fx]}{f}$$

Result (type 3, 76 leaves):

$$2abx - \frac{b^2 \operatorname{Cos}[e] \operatorname{Cos}[fx]}{f} - \frac{a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{a^2 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{b^2 \operatorname{Sin}[e] \operatorname{Sin}[fx]}{f}$$

■ **Problem 162: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e + f x]^2 (a + b \text{Sin}[e + f x])^2 dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$b^2 x - \frac{2 a b \text{ArcTanh}[\text{Cos}[e + f x]]}{f} - \frac{a^2 \text{Cot}[e + f x]}{f}$$

Result (type 3, 76 leaves):

$$\frac{1}{2 f} \left(-a^2 \text{Cot}\left[\frac{1}{2}(e + f x)\right] + 2 b \left(b e + b f x - 2 a \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right] + 2 a \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) + a^2 \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right)$$

■ **Problem 163: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e + f x]^3 (a + b \text{Sin}[e + f x])^2 dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\frac{(a^2 + 2 b^2) \text{ArcTanh}[\text{Cos}[e + f x]]}{2 f} - \frac{2 a b \text{Cot}[e + f x]}{f} - \frac{a^2 \text{Cot}[e + f x] \text{Csc}[e + f x]}{2 f}$$

Result (type 3, 133 leaves):

$$\frac{1}{8 f} \left(-8 a b \text{Cot}\left[\frac{1}{2}(e + f x)\right] - a^2 \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2 - 4 a^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right] - 8 b^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right] + 4 a^2 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 8 b^2 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + a^2 \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 + 8 a b \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right)$$

■ **Problem 165: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e + f x]^5 (a + b \text{Sin}[e + f x])^2 dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$-\frac{(3 a^2 + 4 b^2) \text{ArcTanh}[\text{Cos}[e + f x]]}{8 f} - \frac{2 a b \text{Cot}[e + f x]}{f} - \frac{2 a b \text{Cot}[e + f x]^3}{3 f} - \frac{(3 a^2 + 4 b^2) \text{Cot}[e + f x] \text{Csc}[e + f x]}{8 f} - \frac{a^2 \text{Cot}[e + f x] \text{Csc}[e + f x]^3}{4 f}$$

Result (type 3, 255 leaves):

$$-\frac{4 a b \text{Cot}[e + f x]}{3 f} - \frac{3 a^2 \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{32 f} - \frac{b^2 \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{8 f} - \frac{a^2 \text{Csc}\left[\frac{1}{2}(e + f x)\right]^4}{64 f} - \frac{2 a b \text{Cot}[e + f x] \text{Csc}[e + f x]^2}{3 f} - \frac{3 a^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right]}{8 f} - \frac{b^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right]}{2 f} + \frac{3 a^2 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{8 f} + \frac{b^2 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{2 f} + \frac{3 a^2 \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{32 f} + \frac{b^2 \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{8 f} + \frac{a^2 \text{Sec}\left[\frac{1}{2}(e + f x)\right]^4}{64 f}$$

■ **Problem 173: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e + f x]^4 (a + b \text{Sin}[e + f x])^3 dx$$

Optimal (type 3, 109 leaves, 6 steps):

$$\frac{b (3 a^2 + 2 b^2) \text{ArcTanh}[\text{Cos}[e + f x]]}{2 f} - \frac{a (2 a^2 + 9 b^2) \text{Cot}[e + f x]}{3 f} - \frac{7 a^2 b \text{Cot}[e + f x] \text{Csc}[e + f x]}{6 f} - \frac{a^2 \text{Cot}[e + f x] \text{Csc}[e + f x]^2 (a + b \text{Sin}[e + f x])}{3 f}$$

Result (type 3, 525 leaves):

$$\frac{(-2 a^3 \text{Cos}[\frac{1}{2} (e + f x)] - 9 a b^2 \text{Cos}[\frac{1}{2} (e + f x)]) \text{Csc}[\frac{1}{2} (e + f x)] (b + a \text{Csc}[e + f x])^3 \text{Sin}[e + f x]^3}{6 f (a + b \text{Sin}[e + f x])^3} - \frac{3 a^2 b \text{Csc}[\frac{1}{2} (e + f x)]^2 (b + a \text{Csc}[e + f x])^3 \text{Sin}[e + f x]^3 - a^3 \text{Cot}[\frac{1}{2} (e + f x)] \text{Csc}[\frac{1}{2} (e + f x)]^2 (b + a \text{Csc}[e + f x])^3 \text{Sin}[e + f x]^3}{8 f (a + b \text{Sin}[e + f x])^3} + \frac{(-3 a^2 b - 2 b^3) (b + a \text{Csc}[e + f x])^3 \text{Log}[\text{Cos}[\frac{1}{2} (e + f x)]] \text{Sin}[e + f x]^3}{2 f (a + b \text{Sin}[e + f x])^3} + \frac{(3 a^2 b + 2 b^3) (b + a \text{Csc}[e + f x])^3 \text{Log}[\text{Sin}[\frac{1}{2} (e + f x)]] \text{Sin}[e + f x]^3}{2 f (a + b \text{Sin}[e + f x])^3} + \frac{3 a^2 b (b + a \text{Csc}[e + f x])^3 \text{Sec}[\frac{1}{2} (e + f x)]^2 \text{Sin}[e + f x]^3}{8 f (a + b \text{Sin}[e + f x])^3} + \frac{(b + a \text{Csc}[e + f x])^3 \text{Sec}[\frac{1}{2} (e + f x)] (2 a^3 \text{Sin}[\frac{1}{2} (e + f x)] + 9 a b^2 \text{Sin}[\frac{1}{2} (e + f x)]) \text{Sin}[e + f x]^3}{6 f (a + b \text{Sin}[e + f x])^3} + \frac{a^3 (b + a \text{Csc}[e + f x])^3 \text{Sec}[\frac{1}{2} (e + f x)]^2 \text{Sin}[e + f x]^3 \text{Tan}[\frac{1}{2} (e + f x)]}{24 f (a + b \text{Sin}[e + f x])^3}$$

■ **Problem 174: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e + f x]^5 (a + b \text{Sin}[e + f x])^3 dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$\frac{3 a (a^2 + 4 b^2) \text{ArcTanh}[\text{Cos}[e + f x]]}{8 f} - \frac{b (2 a^2 + b^2) \text{Cot}[e + f x]}{f} - \frac{3 a (a^2 + 4 b^2) \text{Cot}[e + f x] \text{Csc}[e + f x]}{8 f} - \frac{3 a^2 b \text{Cot}[e + f x] \text{Csc}[e + f x]^2}{4 f} - \frac{a^2 \text{Cot}[e + f x] \text{Csc}[e + f x]^3 (a + b \text{Sin}[e + f x])}{4 f}$$

Result (type 3, 322 leaves):

$$\frac{(-2 a^2 b \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - b^3 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right] - 3(a^3 + 4 a b^2) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2 - a^2 b \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right] \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{2 f} - \frac{3(a^3 + 4 a b^2) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{32 f} - \frac{a^2 b \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right] \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{8 f} -$$

$$\frac{a^3 \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^4}{64 f} - \frac{3(a^3 + 4 a b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f} + \frac{3(a^3 + 4 a b^2) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f} + \frac{3(a^3 + 4 a b^2) \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{32 f} +$$

$$\frac{a^3 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^4}{64 f} + \frac{\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] (2 a^2 b \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + b^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right])}{2 f} + \frac{a^2 b \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{8 f}$$

■ **Problem 206: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Csc}[e+f x]^2 \sqrt{a+b \operatorname{Sin}[e+f x]} dx$$

Optimal (type 4, 213 leaves, 9 steps):

$$-\frac{\operatorname{Cot}[e+f x] \sqrt{a+b \operatorname{Sin}[e+f x]}}{f} - \frac{\operatorname{EllipticE}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right), \frac{2 b}{a+b}\right] \sqrt{a+b \operatorname{Sin}[e+f x]}}{f \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}} +$$

$$\frac{a \operatorname{EllipticF}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}{f \sqrt{a+b \operatorname{Sin}[e+f x]}} + \frac{b \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(e-\frac{\pi}{2}+f x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}{f \sqrt{a+b \operatorname{Sin}[e+f x]}}$$

Result (type 4, 425 leaves):

$$\begin{aligned}
& -\frac{\operatorname{Cot}[e+f x] \sqrt{a+b \operatorname{Sin}[e+f x]}}{f} + \frac{1}{4 f} \\
& b \left(-\frac{2 \operatorname{EllipticPi}\left[2, \frac{1}{2}(-e+\frac{\pi}{2}-f x), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}{\sqrt{a+b \operatorname{Sin}[e+f x]}} - \left(2 i \operatorname{Cos}[e+f x] \operatorname{Cos}[2(e+f x)] \left(2 a(a-b) \operatorname{EllipticE}\left[\right.\right.\right. \right. \\
& \quad \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[e+f x]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[e+f x]}\right], \frac{a+b}{a-b}\right] - \right.\right. \right. \\
& \quad \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[e+f x]}\right], \frac{a+b}{a-b}\right]\right)\right)\right) \sqrt{\frac{b-b \operatorname{Sin}[e+f x]}{a+b}} \sqrt{-\frac{b+b \operatorname{Sin}[e+f x]}{a-b}} \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\operatorname{Sin}[e+f x]}^2 (-2 a^2+b^2+4 a(a+b \operatorname{Sin}[e+f x]) - 2(a+b \operatorname{Sin}[e+f x])^2)} \right. \\
& \quad \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \operatorname{Sin}[e+f x])+(a+b \operatorname{Sin}[e+f x])^2}{b^2}} \right)
\end{aligned}$$

■ **Problem 210: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Csc}[e+f x]^2}{\sqrt{a+b \operatorname{Sin}[e+f x]}} dx$$

Optimal (type 4, 222 leaves, 9 steps):

$$\begin{aligned}
& -\frac{\operatorname{Cot}[e+f x] \sqrt{a+b \operatorname{Sin}[e+f x]}}{a f} - \frac{\operatorname{EllipticE}\left[\frac{1}{2}(e-\frac{\pi}{2}+f x), \frac{2 b}{a+b}\right] \sqrt{a+b \operatorname{Sin}[e+f x]}}{a f \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}} + \\
& \frac{\operatorname{EllipticF}\left[\frac{1}{2}(e-\frac{\pi}{2}+f x), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}{f \sqrt{a+b \operatorname{Sin}[e+f x]}} - \frac{b \operatorname{EllipticPi}\left[2, \frac{1}{2}(e-\frac{\pi}{2}+f x), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}{a f \sqrt{a+b \operatorname{Sin}[e+f x]}}
\end{aligned}$$

Result (type 4, 431 leaves):

$$\begin{aligned}
& -\frac{\text{Cot}[e+fx] \sqrt{a+b \text{Sin}[e+fx]}}{af} + \frac{1}{4af} \\
& b \left(\frac{6 \text{EllipticPi}\left[2, \frac{1}{2}(-e + \frac{\pi}{2} - fx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \text{Sin}[e+fx]}{a+b}}}{\sqrt{a+b \text{Sin}[e+fx]}} - \left(2i \text{Cos}[e+fx] \text{Cos}[2(e+fx)] \left(2a(a-b) \text{EllipticE}\left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. i \text{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \text{Sin}[e+fx]}\right], \frac{a+b}{a-b}\right] + b \left(2a \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \text{Sin}[e+fx]}\right], \frac{a+b}{a-b}\right] - \right. \right. \right. \\
& \quad \left. \left. \left. b \text{EllipticPi}\left[\frac{a+b}{a}, i \text{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \text{Sin}[e+fx]}\right], \frac{a+b}{a-b}\right]\right)\right) \sqrt{\frac{b-b \text{Sin}[e+fx]}{a+b}} \sqrt{-\frac{b+b \text{Sin}[e+fx]}{a-b}} \right) \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\text{Sin}[e+fx]}^2 (-2a^2+b^2+4a(a+b \text{Sin}[e+fx]) - 2(a+b \text{Sin}[e+fx])^2) \right. \\
& \quad \left. \sqrt{-\frac{a^2-b^2-2a(a+b \text{Sin}[e+fx])+(a+b \text{Sin}[e+fx])^2}{b^2}} \right)
\end{aligned}$$

- **Problem 211: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\text{Sin}[c+dx]} \sqrt{a+b \text{Sin}[c+dx]} dx$$

Optimal (type 4, 371 leaves, 7 steps):

$$\begin{aligned}
& -\frac{\text{Cos}[c+dx] \sqrt{a+b \text{Sin}[c+dx]}}{d \sqrt{\text{Sin}[c+dx]}} + \frac{(a-b) \sqrt{a+b} \sqrt{\frac{a(1-\text{Csc}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Csc}[c+dx])}{a-b}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sin}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Sin}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \text{Tan}[c+dx]}{ad} \\
& \frac{\sqrt{a+b} \sqrt{\frac{a(1-\text{Csc}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Csc}[c+dx])}{a-b}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sin}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Sin}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \text{Tan}[c+dx]}{d} + \frac{1}{bd} \\
& a \sqrt{a+b} \sqrt{\frac{a(1-\text{Csc}[c+dx])}{a+b}} \sqrt{\frac{a(1+\text{Csc}[c+dx])}{a-b}} \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sin}[c+dx]}}{\sqrt{a+b} \sqrt{\text{Sin}[c+dx]}}\right], -\frac{a+b}{a-b}\right] \text{Tan}[c+dx]
\end{aligned}$$

Result (type 4, 1001 leaves) :

$$\begin{aligned}
 & \frac{\sqrt{2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+2b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}}{d} - \\
 & \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} d \sqrt{\frac{a+2b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}} \sqrt{2} \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \\
 & \left(\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a+2b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) + i \left(b-\sqrt{-a^2+b^2} \right) \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{b+\sqrt{-a^2+b^2}}} \right. \\
 & \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) \right) \\
 & \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^{3/2} \sqrt{1+\frac{a}{b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}} - \\
 & a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[-\frac{i \left(b+\sqrt{-a^2+b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \\
 & \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^{3/2} \sqrt{1+\frac{a}{b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}} + \\
 & a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[\frac{i \left(b+\sqrt{-a^2+b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right]
 \end{aligned}$$

$$\left. \tan\left[\frac{1}{2}(c+dx)\right]^{3/2} \sqrt{1 + \frac{a}{b \tan\left[\frac{1}{2}(c+dx)\right] - \sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(c+dx)\right]}} \right)$$

■ **Problem 216: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \sin[e+fx])^m}{a+b \sin[e+fx]} dx$$

Optimal (type 6, 195 leaves, 5 steps):

$$-\frac{1}{(a^2-b^2)f} a d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2-b^2}\right] \cos[e+fx] (d \sin[e+fx])^{-1+m} (\sin[e+fx]^2)^{\frac{1-m}{2}} +$$

$$\frac{b \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2-b^2}\right] \cos[e+fx] (d \sin[e+fx])^m (\sin[e+fx]^2)^{-m/2}}{(a^2-b^2)f}$$

Result (type 6, 6755 leaves):

$$\left((d \sin[e+fx])^m \tan[e+fx] \left(\frac{\tan[e+fx]}{\sqrt{1+\tan[e+fx]^2}} \right)^m \right.$$

$$\left. - \left(\left(a^3 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[e+fx]^2\right] \right) / \left((1+m) \left(-a^2(3+m) \right. \right. \right.$$

$$\left. \left. \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[e+fx]^2\right] + \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\tan[e+fx]^2, \right. \right. \right.$$

$$\left. \left. \left(-1+\frac{b^2}{a^2}\right) \tan[e+fx]^2 \right) + a^2 m \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \frac{5+m}{2}, -\tan[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right)$$

$$\left. \left. \left. (-b^2 \tan[e+fx]^2 + a^2 (1+\tan[e+fx]^2)) \right) \right) \right) + \frac{1}{b(2+m)(b^2 \tan[e+fx]^2 - a^2(1+\tan[e+fx]^2))}$$

$$\tan[e+fx] \left(\left(a^2 (a^2-b^2) (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, \frac{1}{2}(-1+m), 1, \frac{4+m}{2}, -\tan[e+fx]^2, \frac{(-a^2+b^2) \tan[e+fx]^2}{a^2}\right] \sqrt{1+\tan[e+fx]^2} \right) / \right.$$

$$\left(-a^2 (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, \frac{1}{2}(-1+m), 1, \frac{4+m}{2}, -\tan[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[e+fx]^2\right] + \right.$$

$$\left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{4+m}{2}, \frac{1}{2}(-1+m), 2, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[e+fx]^2\right] + a^2 \right. \right.$$

$$\left. \left. (-1+m) \operatorname{AppellF1}\left[\frac{4+m}{2}, \frac{1+m}{2}, 1, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right) \tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) +$$

$$\begin{aligned}
& \left. \left. \left. \left. \text{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\tan[e+f x]^2 \right] (1+\tan[e+f x]^2)^{m/2} (-b^2 \tan[e+f x]^2 + a^2 (1+\tan[e+f x]^2)) \right] \right) \right) \right) / \\
& \left(f (a+b \sin[e+f x]) \left(\text{Sec}[e+f x]^2 \left(\frac{\tan[e+f x]}{\sqrt{1+\tan[e+f x]^2}} \right)^m \right. \right. \\
& \left. \left. - \left(\left(a^3 (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right) / \left((1+m) \left(-a^2 (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right) + \left(2 (a^2 - b^2) \text{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left. a^2 m \text{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \frac{5+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right) \tan[e+f x]^2 \right) \right) \right) \right) \right) + \frac{1}{b (2+m) (b^2 \tan[e+f x]^2 - a^2 (1+\tan[e+f x]^2))} \\
& \tan[e+f x] \left(\left(a^2 (a^2 - b^2) (4+m) \text{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, -\tan[e+f x]^2, \frac{(-a^2 + b^2) \tan[e+f x]^2}{a^2} \right] \sqrt{1+\tan[e+f x]^2} \right) / \right. \\
& \left(-a^2 (4+m) \text{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + \right. \\
& \left(2 (a^2 - b^2) \text{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 2, \frac{6+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + \right. \\
& \left. \left. a^2 (-1+m) \text{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 1, \frac{6+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right) \tan[e+f x]^2 \right) + \\
& \left. \left. \left. \left. \text{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\tan[e+f x]^2 \right] (1+\tan[e+f x]^2)^{m/2} (-b^2 \tan[e+f x]^2 + a^2 (1+\tan[e+f x]^2)) \right] \right) \right) \right) \right) + \\
& m \tan[e+f x] \left(\frac{\tan[e+f x]}{\sqrt{1+\tan[e+f x]^2}} \right)^{-1+m} \left(-\frac{\text{Sec}[e+f x]^2 \tan[e+f x]^2}{(1+\tan[e+f x]^2)^{3/2}} + \frac{\text{Sec}[e+f x]^2}{\sqrt{1+\tan[e+f x]^2}} \right) \\
& \left(- \left(\left(a^3 (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right) / \right. \right. \\
& \left((1+m) \left(-a^2 (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + \right. \right. \\
& \left. \left. \left(2 (a^2 - b^2) \text{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + a^2 m \text{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \right. \right. \right. \right. \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{5+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right) \tan[ex]^2 \left(-b^2 \tan[ex]^2 + a^2 (1 + \tan[ex]^2) \right) \Bigg) + \\
& \frac{1}{b(2+m) \left(b^2 \tan[ex]^2 - a^2 (1 + \tan[ex]^2) \right)} \tan[ex] \left(\left(a^2 (a^2 - b^2) (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \right. \right. \right. \\
& \left. \left. \left. \frac{4+m}{2}, -\tan[ex]^2, \frac{(-a^2 + b^2) \tan[ex]^2}{a^2} \right] \sqrt{1 + \tan[ex]^2} \right) / \left(-a^2 (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, \right. \right. \right. \\
& \left. \left. \left. -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 2, \frac{6+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \right. \right. \right. \right. \\
& \left. \left. \left. \tan[ex]^2 \right] + a^2 (-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 1, \frac{6+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] \right) \tan[ex]^2 \right) + \\
& \left. \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\tan[ex]^2 \right] (1 + \tan[ex]^2)^{m/2} (-b^2 \tan[ex]^2 + a^2 (1 + \tan[ex]^2)) \right) \Bigg) + \\
& \tan[ex] \left(\frac{\tan[ex]}{\sqrt{1 + \tan[ex]^2}} \right)^m \left(\left(a^3 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] \right. \right. \\
& \left. \left. (2 a^2 \operatorname{Sec}[ex]^2 \tan[ex] - 2 b^2 \operatorname{Sec}[ex]^2 \tan[ex]) \right) / \right. \\
& \left((1+m) \left(-a^2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] + a^2 m \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \right. \right. \right. \\
& \left. \left. \left. \frac{5+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right) \tan[ex]^2 \left(-b^2 \tan[ex]^2 + a^2 (1 + \tan[ex]^2) \right) \right)^2 - \right. \\
& \left. \left(a^3 (3+m) \left(-\frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1 + \frac{m}{2}, 1, 1 + \frac{3+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] \operatorname{Sec}[ex]^2 \tan[ex] + \right. \right. \\
& \left. \left. \frac{1}{3+m} 2 \left(-1 + \frac{b^2}{a^2}\right) (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, \frac{m}{2}, 2, 1 + \frac{3+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] \operatorname{Sec}[ex]^2 \tan[ex] \right) \right) / \\
& \left((1+m) \left(-a^2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] + \right. \right. \\
& \left. \left. a^2 m \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \frac{5+m}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[ex]^2 \right] \right) \tan[ex]^2 \right) \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) + \left(a^3 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right. \\
& \left(2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left. a^2 m \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \sec[e + f x]^2 \tan[e + f x] - a^2 (3+m) \right. \\
& \quad \left(-\frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1 + \frac{m}{2}, 1, 1 + \frac{3+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \frac{1}{3+m} \right. \\
& \quad \left. 2 \left(-1 + \frac{b^2}{a^2} \right) (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, \frac{m}{2}, 2, 1 + \frac{3+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) + \\
& \quad \tan[e + f x]^2 \left(2 (a^2 - b^2) \left(-\frac{1}{5+m} m (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1 + \frac{m}{2}, 2, 1 + \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right. \right. \\
& \quad \left. \left. \sec[e + f x]^2 \tan[e + f x] + \frac{1}{5+m} 4 \left(-1 + \frac{b^2}{a^2} \right) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, \frac{m}{2}, 3, 1 + \frac{5+m}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) + a^2 m \left(\frac{1}{5+m} 2 \left(-1 + \frac{b^2}{a^2} \right) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \frac{2+m}{2}, 2, 1 + \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] - \frac{1}{5+m} (2+m) (3+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1 + \frac{2+m}{2}, 1, 1 + \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \right) \Big/ \\
& \left((1+m) \left(-a^2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \left(2 (a^2 - b^2) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 m \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) - \\
& \frac{1}{b (2+m) (b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2))^2} \tan[e + f x] (-2 a^2 \sec[e + f x]^2 \tan[e + f x] + 2 b^2 \sec[e + f x]^2 \tan[e + f x]) \\
& \left(\left(a^2 (a^2 - b^2) (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \sqrt{1 + \tan[e + f x]^2} \right) \Big/ \right. \\
& \quad \left. \left(-a^2 (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 2, \frac{6+m}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \\
& \quad \left. a^2 (-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 1, \frac{6+m}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \operatorname{Tan}[e+fx]^2 \right) + \\
& \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\operatorname{Tan}[e+fx]^2 \right] (1 + \operatorname{Tan}[e+fx]^2)^{m/2} (-b^2 \operatorname{Tan}[e+fx]^2 + a^2 (1 + \operatorname{Tan}[e+fx]^2)) \Bigg) + \\
& \frac{1}{b (2+m) (b^2 \operatorname{Tan}[e+fx]^2 - a^2 (1 + \operatorname{Tan}[e+fx]^2))} \operatorname{Sec}[e+fx]^2 \\
& \left(\left(a^2 (a^2 - b^2) (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+fx]^2}{a^2} \right] \sqrt{1 + \operatorname{Tan}[e+fx]^2} \right) / \right. \\
& \quad \left(-a^2 (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 2, \frac{6+m}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \right. \\
& \quad \left. \left. a^2 (-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 1, \frac{6+m}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \operatorname{Tan}[e+fx]^2 \right) + \right. \\
& \quad \left. \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\operatorname{Tan}[e+fx]^2 \right] (1 + \operatorname{Tan}[e+fx]^2)^{m/2} (-b^2 \operatorname{Tan}[e+fx]^2 + a^2 (1 + \operatorname{Tan}[e+fx]^2)) \right) \Bigg) + \\
& \frac{1}{b (2+m) (b^2 \operatorname{Tan}[e+fx]^2 - a^2 (1 + \operatorname{Tan}[e+fx]^2))} \operatorname{Tan}[e+fx] \left(\operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\operatorname{Tan}[e+fx]^2 \right] \right. \\
& \quad \left. (2 a^2 \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - 2 b^2 \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]) (1 + \operatorname{Tan}[e+fx]^2)^{m/2} + \right. \\
& \quad \left. \left(a^2 (a^2 - b^2) (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+fx]^2}{a^2} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) / \right. \\
& \quad \left(\sqrt{1 + \operatorname{Tan}[e+fx]^2} \left(-a^2 (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \right. \\
& \quad \left. \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 2, \frac{6+m}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. a^2 (-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 1, \frac{6+m}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \operatorname{Tan}[e+fx]^2 \right) \right) \right) \Bigg) + \\
& \left(a^2 (a^2 - b^2) (4+m) \left(-\frac{1}{4+m} (-1+m) (2+m) \operatorname{AppellF1} \left[1 + \frac{2+m}{2}, 1 + \frac{1}{2} (-1+m), 1, 1 + \frac{4+m}{2}, -\operatorname{Tan}[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{(-a^2 + b^2) \operatorname{Tan}[e+fx]^2}{a^2} \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + \frac{1}{a^2 (4+m)} 2 (-a^2 + b^2) (2+m) \operatorname{AppellF1} \left[1 + \frac{2+m}{2}, \frac{1}{2} (-1+m), \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. 2, 1 + \frac{4+m}{2}, -\tan[e+fx]^2, \frac{(-a^2+b^2)\tan[e+fx]^2}{a^2} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \sqrt{1+\tan[e+fx]^2} \Bigg) / \\
& \left(-a^2(4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2}(-1+m), 1, \frac{4+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \\
& \left(2(a^2-b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2}(-1+m), 2, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \\
& \left. a^2(-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 1, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \Bigg) + \\
& m \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] (1+\tan[e+fx]^2)^{-1+\frac{m}{2}} \\
& (-b^2 \tan[e+fx]^2 + a^2(1+\tan[e+fx]^2)) + (2+m) \operatorname{Csc}[e+fx] \operatorname{Sec}[e+fx] (1+\tan[e+fx]^2)^{m/2} \\
& (-b^2 \tan[e+fx]^2 + a^2(1+\tan[e+fx]^2)) \left(-\operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\tan[e+fx]^2 \right] + (1+\tan[e+fx]^2)^{\frac{1}{2}(-1-m)} \right) - \\
& \left(a^2(a^2-b^2)(4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2}(-1+m), 1, \frac{4+m}{2}, -\tan[e+fx]^2, \frac{(-a^2+b^2)\tan[e+fx]^2}{a^2} \right] \right. \\
& \left. \sqrt{1+\tan[e+fx]^2} \left(2 \left(2(a^2-b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2}(-1+m), 2, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \right. \right. \\
& \left. \left. a^2(-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 1, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \right. \\
& a^2(4+m) \left(-\frac{1}{4+m}(-1+m)(2+m) \operatorname{AppellF1} \left[1 + \frac{2+m}{2}, 1 + \frac{1}{2}(-1+m), 1, 1 + \frac{4+m}{2}, -\tan[e+fx]^2, \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{1}{4+m} 2 \left(-1 + \frac{b^2}{a^2} \right) (2+m) \operatorname{AppellF1} \left[1 + \frac{2+m}{2}, \frac{1}{2}(-1+m), \right. \right. \\
& \left. \left. 2, 1 + \frac{4+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \tan[e+fx]^2 \\
& \left(2(a^2-b^2) \left(-\frac{1}{6+m}(-1+m)(4+m) \operatorname{AppellF1} \left[1 + \frac{4+m}{2}, 1 + \frac{1}{2}(-1+m), 2, 1 + \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right. \right. \\
& \left. \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{1}{6+m} 4 \left(-1 + \frac{b^2}{a^2} \right) (4+m) \operatorname{AppellF1} \left[1 + \frac{4+m}{2}, \frac{1}{2}(-1+m), 3, 1 + \frac{6+m}{2}, -\tan[e+fx]^2, \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + a^2(-1+m) \left(\frac{1}{6+m} 2 \left(-1 + \frac{b^2}{a^2} \right) (4+m) \operatorname{AppellF1} \left[1 + \frac{4+m}{2}, \right. \right. \\
& \left. \left. \frac{1+m}{2}, 2, 1 + \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{1}{6+m} (1+m)(4+m) \right.
\end{aligned}$$

$$\begin{aligned} & \left. \left. \left. \left. \left. \left. \text{AppellF1} \left[1 + \frac{4+m}{2}, 1 + \frac{1+m}{2}, 1, 1 + \frac{6+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] \right] \right) \right) \right) \right) \right) \right) / \\ & \left(-a^2 (4+m) \text{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 1, \frac{4+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \\ & \left. \left(2 (a^2 - b^2) \text{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 2, \frac{6+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \right. \\ & \left. \left. a^2 (-1+m) \text{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 1, \frac{6+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \right) \right) \right) \right) \right) \end{aligned}$$

■ **Problem 217: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \sin[e+fx])^m}{(a+b \sin[e+fx])^2} dx$$

Optimal (type 6, 306 leaves, 10 steps):

$$\begin{aligned} & - \frac{1}{(a^2 - b^2)^2 d f} b^2 \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-1-m), 2, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \sin[e+fx])^{1+m} (\sin[e+fx]^2)^{\frac{1}{2}(-1-m)} - \\ & \frac{1}{(a^2 - b^2)^2 f} a^2 d \text{AppellF1} \left[\frac{1}{2}, \frac{1-m}{2}, 2, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \sin[e+fx])^{-1+m} (\sin[e+fx]^2)^{\frac{1-m}{2}} + \\ & \frac{1}{(a^2 - b^2)^2 f} 2 a b \text{AppellF1} \left[\frac{1}{2}, -\frac{m}{2}, 2, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \sin[e+fx])^m (\sin[e+fx]^2)^{-m/2} \end{aligned}$$

Result (type 6, 8428 leaves):

$$\begin{aligned} & \left(a^2 (d \sin[e+fx])^m \text{Tan}[e+fx] \left(\frac{\text{Tan}[e+fx]}{\sqrt{1 + \text{Tan}[e+fx]^2}} \right)^m \right. \\ & \left. \left(\left((a^2 + b^2) (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \text{Tan}[e+fx]^2}{a^2} \right] \right) \right) / \right. \\ & \left. \left((1+m) \left(-a^2 (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \right. \right. \\ & \left. \left. \left(2 (a^2 - b^2) \text{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \right. \right. \\ & \left. \left. \left. a^2 m \text{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \frac{5+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left. \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) + \frac{1}{(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2))^2} 2 a b \\
& \left(\left(a b (3 + m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 2, \frac{3+m}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right) / \left((1 + m) \left(a^2 (3 + m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 2, \frac{3+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 3, \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. a^2 m \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 2, \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \right) + \\
& \left((a^2 - b^2) (4 + m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1 + m), 2, \frac{4+m}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x] \sqrt{1 + \tan[e + f x]^2} \right) / \\
& \left((2 + m) \left(a^2 (4 + m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1 + m), 2, \frac{4+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1 + m), 3, \frac{6+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \\
& \quad \left. \left. a^2 (-1 + m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 2, \frac{6+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \right) \right) / \\
& \left((-a^2 + b^2) f (a + b \sin[e + f x])^2 \left(\frac{1}{-a^2 + b^2} a^2 \sec[e + f x]^2 \left(\frac{\tan[e + f x]}{\sqrt{1 + \tan[e + f x]^2}} \right)^m \right. \right. \\
& \quad \left. \left. \left(\left((a^2 + b^2) (3 + m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right) / \right. \right. \\
& \quad \left((1 + m) \left(-a^2 (3 + m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 m \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \right) \\
& \quad \left. \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) + \frac{1}{(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2))^2} \\
& 2 a b \left(\left(a b (3 + m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 2, \frac{3+m}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right) / \left((1 + m) \left(a^2 (3 + m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. 2, \frac{3+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 3, \frac{5+m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan[e + f x]^2 \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{1 + \tan[e + f x]^2} \right) / \left((2 + m) \left(a^2 (4 + m) \operatorname{AppellF1} \left[\frac{2 + m}{2}, \frac{1}{2} (-1 + m), 2, \frac{4 + m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4 + m}{2}, \frac{1}{2} (-1 + m), 3, \frac{6 + m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \\
& \left. \left. a^2 (-1 + m) \operatorname{AppellF1} \left[\frac{4 + m}{2}, \frac{1 + m}{2}, 2, \frac{6 + m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \right) + \\
& \frac{1}{-a^2 + b^2} a^2 \tan[e + f x] \left(\frac{\tan[e + f x]}{\sqrt{1 + \tan[e + f x]^2}} \right)^m \left(- \left(\left((a^2 + b^2) (3 + m) \operatorname{AppellF1} \left[\frac{1 + m}{2}, \frac{m}{2}, 1, \frac{3 + m}{2}, -\tan[e + f x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] (2 a^2 \operatorname{Sec}[e + f x]^2 \tan[e + f x] - 2 b^2 \operatorname{Sec}[e + f x]^2 \tan[e + f x]) \right) \right) / \\
& \left((1 + m) \left(-a^2 (3 + m) \operatorname{AppellF1} \left[\frac{1 + m}{2}, \frac{m}{2}, 1, \frac{3 + m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3 + m}{2}, \frac{m}{2}, 2, \frac{5 + m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 m \operatorname{AppellF1} \left[\frac{3 + m}{2}, \frac{2 + m}{2}, 1, \right. \right. \right. \\
& \left. \left. \left. \frac{5 + m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2))^2 \right) \right) + \\
& \left((a^2 + b^2) (3 + m) \left(-\frac{1}{3 + m} m (1 + m) \operatorname{AppellF1} \left[1 + \frac{1 + m}{2}, 1 + \frac{m}{2}, 1, 1 + \frac{3 + m}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right. \right. \\
& \left. \left. \operatorname{Sec}[e + f x]^2 \tan[e + f x] + \frac{1}{a^2 (3 + m)} 2 (-a^2 + b^2) (1 + m) \operatorname{AppellF1} \left[1 + \frac{1 + m}{2}, \frac{m}{2}, 2, \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{3 + m}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \right) / \\
& \left((1 + m) \left(-a^2 (3 + m) \operatorname{AppellF1} \left[\frac{1 + m}{2}, \frac{m}{2}, 1, \frac{3 + m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3 + m}{2}, \frac{m}{2}, 2, \frac{5 + m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 m \operatorname{AppellF1} \left[\frac{3 + m}{2}, \frac{2 + m}{2}, 1, \right. \right. \right. \\
& \left. \left. \left. \frac{5 + m}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \right) - \\
& \frac{1}{(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2))^3} 4 a b (-2 a^2 \operatorname{Sec}[e + f x]^2 \tan[e + f x] + 2 b^2 \operatorname{Sec}[e + f x]^2 \tan[e + f x])
\end{aligned}$$

$$\begin{aligned}
& \left(\left(a b (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 2, \frac{3+m}{2}, -\tan[e+f x]^2, \frac{(-a^2+b^2) \tan[e+f x]^2}{a^2} \right] \right) / \left((1+m) \left(a^2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] - \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 3, \frac{5+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan[e+f x]^2 \right] + a^2 m \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 2, \frac{5+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right) \tan[e+f x]^2 \right) \right) + \\
& \left((a^2 - b^2) (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 2, \frac{4+m}{2}, -\tan[e+f x]^2, \frac{(-a^2+b^2) \tan[e+f x]^2}{a^2} \right] \tan[e+f x] \right. \\
& \quad \left. \sqrt{1 + \tan[e+f x]^2} \right) / \left((2+m) \left(a^2 (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 2, \frac{4+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 3, \frac{6+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left. a^2 (-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 2, \frac{6+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right) \tan[e+f x]^2 \right) \right) \right) - \\
& \left((a^2 + b^2) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\tan[e+f x]^2, \frac{(-a^2+b^2) \tan[e+f x]^2}{a^2} \right] \right. \\
& \quad \left(2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left. a^2 m \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \frac{5+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right) \sec[e+f x]^2 \tan[e+f x] - a^2 (3+m) \left(-\frac{1}{3+m} \right. \right. \\
& \quad \left. \left. m (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1 + \frac{m}{2}, 1, 1 + \frac{3+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \sec[e+f x]^2 \tan[e+f x] + \frac{1}{3+m} \right. \right. \\
& \quad \left. \left. 2 \left(-1 + \frac{b^2}{a^2} \right) (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, \frac{m}{2}, 2, 1 + \frac{3+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \sec[e+f x]^2 \tan[e+f x] \right) \right) + \\
& \quad \tan[e+f x]^2 \left(2 (a^2 - b^2) \left(-\frac{1}{5+m} m (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1 + \frac{m}{2}, 2, 1 + \frac{5+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right. \right. \\
& \quad \left. \left. \sec[e+f x]^2 \tan[e+f x] + \frac{1}{5+m} 4 \left(-1 + \frac{b^2}{a^2} \right) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, \frac{m}{2}, 3, 1 + \frac{5+m}{2}, -\tan[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \sec[e+f x]^2 \tan[e+f x] \right) + a^2 m \left(\frac{1}{5+m} 2 \left(-1 + \frac{b^2}{a^2} \right) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \frac{2+m}{2}, 2, 1 + \frac{5+m}{2}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \sec[e+f x]^2 \tan[e+f x] - \frac{1}{5+m} (2+m) (3+m) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \text{AppellF1} \left[1 + \frac{3+m}{2}, 1 + \frac{2+m}{2}, 1, 1 + \frac{5+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] \right] \right) \right) \right) \Big/ \\
& \left((1+m) \left(-a^2 (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 1, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \text{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 2, \frac{5+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + a^2 m \text{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1, \right. \right. \right. \\
& \left. \left. \left. \frac{5+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \left(-b^2 \text{Tan}[e+fx]^2 + a^2 (1 + \text{Tan}[e+fx]^2) \right) \right) \right) + \\
& \frac{1}{(b^2 \text{Tan}[e+fx]^2 - a^2 (1 + \text{Tan}[e+fx]^2))^2} 2 a b \left(\left(a b (3+m) \left(-1 / (3+m) m (1+m) \text{AppellF1} \left[1 + \frac{1+m}{2}, 1 + \frac{m}{2}, 2, \right. \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{3+m}{2}, -\text{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \text{Tan}[e+fx]^2}{a^2} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] + 1 / (a^2 (3+m)) 4 (-a^2 + b^2) (1+m) \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1} \left[1 + \frac{1+m}{2}, \frac{m}{2}, 3, 1 + \frac{3+m}{2}, -\text{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \text{Tan}[e+fx]^2}{a^2} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] \right) \right) \right) \Big/ \\
& \left((1+m) \left(a^2 (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 2, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] - \right. \right. \\
& \left. \left(4 (a^2 - b^2) \text{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 3, \frac{5+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \right. \\
& \left. \left. a^2 m \text{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 2, \frac{5+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \right) \right) + \\
& \left((a^2 - b^2) (4+m) \text{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 2, \frac{4+m}{2}, -\text{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \text{Tan}[e+fx]^2}{a^2} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx]^2 \right) \Big/ \\
& \left((2+m) \sqrt{1 + \text{Tan}[e+fx]^2} \left(a^2 (4+m) \text{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 2, \frac{4+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \right. \\
& \left. \left(-4 (a^2 - b^2) \text{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 3, \frac{6+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] - \right. \right. \\
& \left. \left. a^2 (-1+m) \text{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 2, \frac{6+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \right) \right) + \\
& \left((a^2 - b^2) (4+m) \text{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 2, \frac{4+m}{2}, -\text{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \text{Tan}[e+fx]^2}{a^2} \right] \text{Sec}[e+fx]^2 \right. \\
& \left. \sqrt{1 + \text{Tan}[e+fx]^2} \right) \Big/ \left((2+m) \left(a^2 (4+m) \text{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 2, \frac{4+m}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 3, \frac{6+m}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \\
& \quad \left. a^2 (-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 2, \frac{6+m}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^2 \right) + \\
& \left((a^2 - b^2) (4+m) \operatorname{Tan}[e+f x] \left[-\frac{1}{4+m} (-1+m) (2+m) \operatorname{AppellF1} \left[1 + \frac{2+m}{2}, 1 + \frac{1}{2} (-1+m), 2, 1 + \frac{4+m}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(-a^2 + b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \frac{1}{a^2 (4+m)} 4 (-a^2 + b^2) (2+m) \operatorname{AppellF1} \left[1 + \frac{2+m}{2}, \frac{1}{2} (-1+m), \right. \right. \\
& \quad \left. \left. 3, 1 + \frac{4+m}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \sqrt{1 + \operatorname{Tan}[e+f x]^2} \Big/ \\
& \left((2+m) \left(a^2 (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 2, \frac{4+m}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 3, \frac{6+m}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left. a^2 (-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 2, \frac{6+m}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^2 \right) \right) - \\
& \left(a b (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{m}{2}, 2, \frac{3+m}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \right. \\
& \quad \left. - 2 \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{m}{2}, 3, \frac{5+m}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left. a^2 m \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 2, \frac{5+m}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + a^2 (3+m) \right. \\
& \quad \left. \left(-\frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1 + \frac{m}{2}, 2, 1 + \frac{3+m}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \right. \right. \\
& \quad \left. \frac{1}{3+m} 4 \left(-1 + \frac{b^2}{a^2} \right) (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, \frac{m}{2}, 3, 1 + \frac{3+m}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \right. \\
& \quad \left. \operatorname{Tan}[e+f x] \right) - \operatorname{Tan}[e+f x]^2 \left(4 (a^2 - b^2) \left(-\frac{1}{5+m} m (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1 + \frac{m}{2}, 3, 1 + \frac{5+m}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \frac{1}{5+m} 6 \left(-1 + \frac{b^2}{a^2} \right) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, \frac{m}{2}, 4, 1 + \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + a^2 m \left(\frac{1}{5+m} 4 \left(-1 + \frac{b^2}{a^2} \right) (3+m) \operatorname{AppellF1} \left[1 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{3+m}{2}, \frac{2+m}{2}, 3, 1 + \frac{5+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \left] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{1}{5+m} (2+m) (3+m) \right. \\
& \left. \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 1 + \frac{2+m}{2}, 2, 1 + \frac{5+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Big) \Big) \Big) \Big) / \\
& \left((1+m) \left(a^2 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{m}{2}, 2, \frac{3+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] - \right. \right. \\
& \left. \left(4 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{m}{2}, 3, \frac{5+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \left. \left. a^2 m \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{2+m}{2}, 2, \frac{5+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right)^2 \Big) - \\
& \left((a^2 - b^2) (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 2, \frac{4+m}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \tan[e+fx] \right. \\
& \left. \sqrt{1 + \tan[e+fx]^2} \left(2 \left(-4 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 3, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] - \right. \right. \right. \\
& \left. \left. a^2 (-1+m) \operatorname{AppellF1}\left[\frac{4+m}{2}, \frac{1+m}{2}, 2, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right) \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \right. \\
& a^2 (4+m) \left(-\frac{1}{4+m} (-1+m) (2+m) \operatorname{AppellF1}\left[1 + \frac{2+m}{2}, 1 + \frac{1}{2} (-1+m), 2, 1 + \frac{4+m}{2}, -\tan[e+fx]^2, \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{1}{4+m} 4 \left(-1 + \frac{b^2}{a^2}\right) (2+m) \operatorname{AppellF1}\left[1 + \frac{2+m}{2}, \frac{1}{2} (-1+m), 3, \right. \right. \\
& \left. \left. 1 + \frac{4+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \tan[e+fx]^2 \left(-4 (a^2 - b^2) \right. \\
& \left. \left(-\frac{1}{6+m} (-1+m) (4+m) \operatorname{AppellF1}\left[1 + \frac{4+m}{2}, 1 + \frac{1}{2} (-1+m), 3, 1 + \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right. \right. \\
& \left. \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \frac{1}{6+m} 6 \left(-1 + \frac{b^2}{a^2}\right) (4+m) \operatorname{AppellF1}\left[1 + \frac{4+m}{2}, \frac{1}{2} (-1+m), 4, 1 + \frac{6+m}{2}, -\tan[e+fx]^2, \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) - a^2 (-1+m) \left(\frac{1}{6+m} 4 \left(-1 + \frac{b^2}{a^2}\right) (4+m) \operatorname{AppellF1}\left[1 + \frac{4+m}{2}, \right. \right. \\
& \left. \left. \frac{1+m}{2}, 3, 1 + \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{1}{6+m} (1+m) (4+m) \right. \\
& \left. \left. \operatorname{AppellF1}\left[1 + \frac{4+m}{2}, 1 + \frac{1+m}{2}, 2, 1 + \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \Big) \Big) \Big) \Big) /
\end{aligned}$$

$$\left((2+m) \left(a^2 (4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, \frac{1}{2} (-1+m), 2, \frac{4+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \right. \\ \left. \left. -4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1}{2} (-1+m), 3, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] - \right. \right. \\ \left. \left. a^2 (-1+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, \frac{1+m}{2}, 2, \frac{6+m}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right)^2 \right) \right) \right) \right) \right) \right) \right) \right)$$

■ **Problem 218: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \sin[e+fx])^m}{(a+b \sin[e+fx])^3} dx$$

Optimal (type 6, 406 leaves, 13 steps):

$$- \frac{1}{(a^2 - b^2)^3 d f} 3 a b^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-1-m), 3, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \sin[e+fx])^{1+m} (\sin[e+fx]^2)^{\frac{1}{2}(-1-m)} - \\ \frac{1}{(a^2 - b^2)^3 f} a^3 d \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1-m}{2}, 3, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \sin[e+fx])^{-1+m} (\sin[e+fx]^2)^{\frac{1-m}{2}} + \\ \frac{1}{(a^2 - b^2)^3 f} b^3 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-2-m), 3, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \sin[e+fx])^m (\sin[e+fx]^2)^{-m/2} + \\ \frac{1}{(a^2 - b^2)^3 f} 3 a^2 b \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{m}{2}, 3, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \sin[e+fx])^m (\sin[e+fx]^2)^{-m/2}$$

Result (type 6, 20820 leaves): Display of huge result suppressed!

■ **Problem 222: Unable to integrate problem.**

$$\int \sin[c+dx]^3 (a+b \sin[c+dx])^n dx$$

Optimal (type 6, 351 leaves, 9 steps):

$$\frac{2 a \cos [c+d x] (a+b \sin [c+d x])^{1+n}}{b^2 d (2+n)(3+n)} - \frac{\cos [c+d x] \sin [c+d x] (a+b \sin [c+d x])^{1+n}}{b d (3+n)} -$$

$$\left(\sqrt{2} (a+b) (2 a^2+b^2 (2+n)^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1-n, \frac{3}{2}, \frac{1}{2} (1-\sin [c+d x]), \frac{b (1-\sin [c+d x])}{a+b}\right] \right.$$

$$\left. \cos [c+d x] (a+b \sin [c+d x])^n \left(\frac{a+b \sin [c+d x]}{a+b} \right)^{-n} \right) / \left(b^3 d (2+n)(3+n) \sqrt{1+\sin [c+d x]} \right) +$$

$$\left(\sqrt{2} a (2 a^2+b^2 (4+5 n+n^2)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -n, \frac{3}{2}, \frac{1}{2} (1-\sin [c+d x]), \frac{b (1-\sin [c+d x])}{a+b}\right] \cos [c+d x] \right.$$

$$\left. (a+b \sin [c+d x])^n \left(\frac{a+b \sin [c+d x]}{a+b} \right)^{-n} \right) / \left(b^3 d (2+n)(3+n) \sqrt{1+\sin [c+d x]} \right)$$

Result (type 8, 23 leaves):

$$\int \sin [c+d x]^3 (a+b \sin [c+d x])^n dx$$

■ **Problem 223: Unable to integrate problem.**

$$\int \sin [c+d x]^2 (a+b \sin [c+d x])^n dx$$

Optimal (type 6, 274 leaves, 8 steps):

$$-\frac{\cos [c+d x] (a+b \sin [c+d x])^{1+n}}{b d (2+n)} +$$

$$\left(\sqrt{2} a (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1-n, \frac{3}{2}, \frac{1}{2} (1-\sin [c+d x]), \frac{b (1-\sin [c+d x])}{a+b}\right] \cos [c+d x] (a+b \sin [c+d x])^n \left(\frac{a+b \sin [c+d x]}{a+b} \right)^{-n} \right) /$$

$$\left(b^2 d (2+n) \sqrt{1+\sin [c+d x]} \right) - \left(\sqrt{2} (a^2+b^2 (1+n)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -n, \frac{3}{2}, \frac{1}{2} (1-\sin [c+d x]), \frac{b (1-\sin [c+d x])}{a+b}\right] \right.$$

$$\left. \cos [c+d x] (a+b \sin [c+d x])^n \left(\frac{a+b \sin [c+d x]}{a+b} \right)^{-n} \right) / \left(b^2 d (2+n) \sqrt{1+\sin [c+d x]} \right)$$

Result (type 8, 23 leaves):

$$\int \sin [c+d x]^2 (a+b \sin [c+d x])^n dx$$

■ **Problem 231: Result more than twice size of optimal antiderivative.**

$$\int \frac{a+a \sin [e+f x]}{c-c \sin [e+f x]} dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$-\frac{a x}{c} + \frac{2 a \cos [e+f x]}{f (c-c \sin [e+f x])}$$

Result (type 3, 83 leaves) :

$$\frac{a \left(-f x \cos \left[\frac{f x}{2} \right] + 4 \sin \left[\frac{f x}{2} \right] + f x \sin \left[e + \frac{f x}{2} \right] \right)}{c f \left(\cos \left[\frac{e}{2} \right] - \sin \left[\frac{e}{2} \right] \right) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)}$$

■ **Problem 232: Result more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e + f x]}{(c - c \sin[e + f x])^2} dx$$

Optimal (type 3, 30 leaves, 2 steps) :

$$\frac{a c \cos[e + f x]^3}{3 f (c - c \sin[e + f x])^3}$$

Result (type 3, 74 leaves) :

$$-\frac{a \left(-3 \cos \left[e + \frac{f x}{2} \right] + \cos \left[e + \frac{3 f x}{2} \right] \right)}{3 c^2 f \left(\cos \left[\frac{e}{2} \right] - \sin \left[\frac{e}{2} \right] \right) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^3}$$

■ **Problem 241: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^2}{c - c \sin[e + f x]} dx$$

Optimal (type 3, 57 leaves, 4 steps) :

$$-\frac{3 a^2 x}{c} + \frac{3 a^2 \cos[e + f x]}{c f} + \frac{2 a^2 c \cos[e + f x]^3}{f (c - c \sin[e + f x])^2}$$

Result (type 3, 130 leaves) :

$$\left(a^2 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(\cos \left[\frac{1}{2} (e + f x) \right] (3 (e + f x) - \cos[e + f x]) + (-8 - 3 e - 3 f x + \cos[e + f x]) \sin \left[\frac{1}{2} (e + f x) \right] \right) \right. \\ \left. (1 + \sin[e + f x])^2 \right) / \left(c f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \right)^4 (-1 + \sin[e + f x])$$

■ **Problem 243: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^2}{(c - c \sin[e + f x])^3} dx$$

Optimal (type 3, 34 leaves, 2 steps) :

$$\frac{a^2 c^2 \cos[e + f x]^5}{5 f (c - c \sin[e + f x])^5}$$

Result (type 3, 81 leaves) :

$$\frac{a^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(-10 \sin\left[\frac{1}{2}(e+fx)\right] - 5 \sin\left[\frac{3}{2}(e+fx)\right] + \sin\left[\frac{5}{2}(e+fx)\right] \right)}{10 c^3 f (-1 + \sin[e+fx])^3}$$

■ **Problem 255: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e+fx])^3}{(c - c \sin[e+fx])^3} dx$$

Optimal (type 3, 106 leaves, 5 steps):

$$-\frac{a^3 x}{c^3} + \frac{2 a^3 c^2 \cos[e+fx]^5}{5 f (c - c \sin[e+fx])^5} - \frac{2 a^3 \cos[e+fx]^3}{3 f (c - c \sin[e+fx])^3} + \frac{2 a^3 \cos[e+fx]}{f (c^3 - c^3 \sin[e+fx])}$$

Result (type 3, 249 leaves):

$$\frac{1}{15 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c - c \sin[e+fx])^3} \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(24 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) - 44 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 - 15 (e+fx) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 + 48 \sin\left[\frac{1}{2}(e+fx)\right] - 88 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] + 92 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 \sin\left[\frac{1}{2}(e+fx)\right] \right) (a + a \sin[e+fx])^3$$

■ **Problem 256: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e+fx])^3}{(c - c \sin[e+fx])^4} dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{a^3 c^3 \cos[e+fx]^7}{7 f (c - c \sin[e+fx])^7}$$

Result (type 3, 93 leaves):

$$\frac{a^3 \left(35 \cos\left[\frac{1}{2}(e+fx)\right] - 21 \cos\left[\frac{3}{2}(e+fx)\right] - 7 \cos\left[\frac{5}{2}(e+fx)\right] + \cos\left[\frac{7}{2}(e+fx)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)}{(28 c^4 f (-1 + \sin[e+fx])^4)}$$

■ **Problem 263: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \sin[e+fx])^2}{a + a \sin[e+fx]} dx$$

Optimal (type 3, 56 leaves, 4 steps):

$$-\frac{3c^2x}{a} - \frac{3c^2 \cos[e+fx]}{af} - \frac{2ac^2 \cos[e+fx]^3}{f(a+a\sin[e+fx])^2}$$

Result (type 3, 129 leaves):

$$-\left(c^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)\right) \left(\cos\left[\frac{1}{2}(e+fx)\right] (3(e+fx) + \cos[e+fx]) + (-8 + 3e + 3fx + \cos[e+fx]) \sin\left[\frac{1}{2}(e+fx)\right]\right) \\ (-1 + \sin[e+fx])^2 \Big/ \left(af \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)\right)^4 (1 + \sin[e+fx])$$

■ **Problem 264: Result more than twice size of optimal antiderivative.**

$$\int \frac{c - c \sin[e+fx]}{a + a \sin[e+fx]} dx$$

Optimal (type 3, 32 leaves, 2 steps):

$$-\frac{cx}{a} - \frac{2c \cos[e+fx]}{f(a+a\sin[e+fx])}$$

Result (type 3, 79 leaves):

$$-\frac{c \left(fx \cos\left[\frac{fx}{2}\right] - 4 \sin\left[\frac{fx}{2}\right] + fx \sin\left[e + \frac{fx}{2}\right] \right)}{af \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)}$$

■ **Problem 271: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \sin[e+fx])^3}{(a + a \sin[e+fx])^2} dx$$

Optimal (type 3, 90 leaves, 5 steps):

$$\frac{5c^3x}{a^2} + \frac{5c^3 \cos[e+fx]}{a^2 f} - \frac{2a^2 c^3 \cos[e+fx]^5}{3f(a+a\sin[e+fx])^4} + \frac{10c^3 \cos[e+fx]^3}{3f(a+a\sin[e+fx])^2}$$

Result (type 3, 210 leaves):

$$\frac{1}{3f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (a+a\sin[e+fx])^2} \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \\ \left(16 \sin\left[\frac{1}{2}(e+fx)\right] - 8 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) - 56 \sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 + \right. \\ \left. 15(e+fx) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 + 3 \cos[e+fx] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \right) (c - c \sin[e+fx])^3$$

■ **Problem 273: Result more than twice size of optimal antiderivative.**

$$\int \frac{c - c \sin[e + f x]}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 3, 29 leaves, 2 steps):

$$-\frac{a c \cos[e + f x]^3}{3 f (a + a \sin[e + f x])^3}$$

Result (type 3, 70 leaves):

$$\frac{c \left(-3 \cos\left[e + \frac{f x}{2}\right] + \cos\left[e + \frac{3 f x}{2}\right] \right)}{3 a^2 f \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3}$$

■ **Problem 280: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \sin[e + f x])^4}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 3, 124 leaves, 6 steps):

$$-\frac{7 c^4 x}{a^3} - \frac{7 c^4 \cos[e + f x]}{a^3 f} - \frac{2 a^3 c^4 \cos[e + f x]^7}{5 f (a + a \sin[e + f x])^6} + \frac{14 a c^4 \cos[e + f x]^5}{15 f (a + a \sin[e + f x])^4} - \frac{14 c^4 \cos[e + f x]^3}{3 a f (a + a \sin[e + f x])^2}$$

Result (type 3, 270 leaves):

$$\frac{1}{15 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^8 (a + a \sin[e + f x])^3} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(96 \sin\left[\frac{1}{2}(e + f x)\right] - 48 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) - 256 \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + 128 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 + 464 \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 - 105 (e + f x) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 - 15 \cos[e + f x] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \right) (c - c \sin[e + f x])^4$$

■ **Problem 281: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \sin[e + f x])^3}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 3, 103 leaves, 5 steps):

$$-\frac{c^3 x}{a^3} - \frac{2 a^2 c^3 \cos[e + f x]^5}{5 f (a + a \sin[e + f x])^5} + \frac{2 c^3 \cos[e + f x]^3}{3 f (a + a \sin[e + f x])^3} - \frac{2 c^3 \cos[e + f x]}{f (a^3 + a^3 \sin[e + f x])}$$

Result (type 3, 239 leaves):

$$\frac{1}{15 f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (a + a \sin[e+fx])^3}$$

$$\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(48 \sin\left[\frac{1}{2}(e+fx)\right] - 24 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) - \right.$$

$$88 \sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 + 44 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 +$$

$$\left. 92 \sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 - 15 (e+fx) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 \right) (c - c \sin[e+fx])^3$$

- **Problem 282: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \sin[e+fx])^2}{(a + a \sin[e+fx])^3} dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$-\frac{a^2 c^2 \cos[e+fx]^5}{5 f (a + a \sin[e+fx])^5}$$

Result (type 3, 81 leaves):

$$\frac{c^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(10 \sin\left[\frac{1}{2}(e+fx)\right] + 5 \sin\left[\frac{3}{2}(e+fx)\right] - \sin\left[\frac{5}{2}(e+fx)\right] \right)}{10 a^3 f (1 + \sin[e+fx])^3}$$

- **Problem 293: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e+fx]) \sqrt{c - c \sin[e+fx]} dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{2 a c^2 \cos[e+fx]^3}{3 f (c - c \sin[e+fx])^{3/2}}$$

Result (type 3, 71 leaves):

$$\frac{2 a \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \sqrt{c - c \sin[e+fx]}}{3 f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)}$$

- **Problem 294: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e+fx]}{\sqrt{c - c \sin[e+fx]}} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{2\sqrt{2} a \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{\sqrt{c} f} - \frac{2 a \cos[e+fx]}{f \sqrt{c-c \sin[e+fx]}}$$

Result (type 3, 162 leaves):

$$\begin{aligned} & - \left(2 a \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \right. \\ & \left. \left(\sqrt{c} + \sqrt{c} \sin[e+fx] - i \sqrt{2} \operatorname{Log}\left[\frac{2(-i\sqrt{2}\sqrt{c} + \sqrt{-c(1+\sin[e+fx])})}{\sqrt{c-c \sin[e+fx]}}\right] \sqrt{-c(1+\sin[e+fx])} \right) \right) / \\ & \left(\sqrt{c} f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c-c \sin[e+fx]} \right) \end{aligned}$$

■ **Problem 295: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e+fx]}{(c - c \sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 76 leaves, 4 steps):

$$-\frac{a \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{\sqrt{2} c^{3/2} f} + \frac{a \cos[e+fx]}{f (c - c \sin[e+fx])^{3/2}}$$

Result (type 3, 201 leaves):

$$\begin{aligned} & \left(a \sec[e+fx] \left(2\sqrt{c} - i\sqrt{2} \operatorname{Log}\left[\frac{2(-i\sqrt{2}\sqrt{c} + \sqrt{-c(1+\sin[e+fx])})}{\sqrt{c-c \sin[e+fx]}}\right] \sqrt{-c(1+\sin[e+fx])} + \right. \right. \\ & \left. \left. \sin[e+fx] \left(2\sqrt{c} + i\sqrt{2} \operatorname{Log}\left[\frac{2(-i\sqrt{2}\sqrt{c} + \sqrt{-c(1+\sin[e+fx])})}{\sqrt{c-c \sin[e+fx]}}\right] \sqrt{-c(1+\sin[e+fx])} \right) \right) \right) / (2c^{3/2} f \sqrt{c-c \sin[e+fx]}) \end{aligned}$$

■ **Problem 296: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e+fx]}{(c - c \sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 113 leaves, 5 steps):

$$-\frac{a \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{8\sqrt{2} c^{5/2} f} + \frac{a \cos[e+fx]}{2f (c - c \sin[e+fx])^{5/2}} - \frac{a \cos[e+fx]}{8cf (c - c \sin[e+fx])^{3/2}}$$

Result (type 3, 334 leaves):

$$\left(a \left(14 \sqrt{c} - 3 i \sqrt{2} \operatorname{Log} \left[\frac{2 \left(-i \sqrt{2} \sqrt{c} + \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right)}{\sqrt{c - c \operatorname{Sin}[e + f x]}} \right] \sqrt{-c (1 + \operatorname{Sin}[e + f x])} - \right. \right. \\ \left. \left. \operatorname{Cos}[2 (e + f x)] \left(2 \sqrt{c} - i \sqrt{2} \operatorname{Log} \left[\frac{2 \left(-i \sqrt{2} \sqrt{c} + \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right)}{\sqrt{c - c \operatorname{Sin}[e + f x]}} \right] \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right) + \right. \right. \\ \left. \left. 4 \operatorname{Sin}[e + f x] \left(4 \sqrt{c} + i \sqrt{2} \operatorname{Log} \left[\frac{2 \left(-i \sqrt{2} \sqrt{c} + \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right)}{\sqrt{c - c \operatorname{Sin}[e + f x]}} \right] \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right) \right) \right) \Bigg) / \\ \left(32 c^{5/2} f \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] - \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \right)^3 \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{c - c \operatorname{Sin}[e + f x]} \right)$$

■ **Problem 297: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \operatorname{Sin}[e + f x]}{(c - c \operatorname{Sin}[e + f x])^{7/2}} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$-\frac{a \operatorname{ArcTanh} \left[\frac{\sqrt{c} \operatorname{Cos}[e + f x]}{\sqrt{2} \sqrt{c - c \operatorname{Sin}[e + f x]}} \right]}{32 \sqrt{2} c^{7/2} f} + \frac{a \operatorname{Cos}[e + f x]}{3 f (c - c \operatorname{Sin}[e + f x])^{7/2}} - \frac{a \operatorname{Cos}[e + f x]}{24 c f (c - c \operatorname{Sin}[e + f x])^{5/2}} - \frac{a \operatorname{Cos}[e + f x]}{32 c^2 f (c - c \operatorname{Sin}[e + f x])^{3/2}}$$

Result (type 3, 426 leaves):

$$\left(a \left(6 \left(38 \sqrt{c} - 5 i \sqrt{2} \operatorname{Log} \left[\frac{2 \left(-i \sqrt{2} \sqrt{c} + \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right)}{\sqrt{c - c \operatorname{Sin}[e + f x]}} \right] \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right) - \right. \\ \left. 2 \operatorname{Cos}[2 (e + f x)] \left(14 \sqrt{c} - 9 i \sqrt{2} \operatorname{Log} \left[\frac{2 \left(-i \sqrt{2} \sqrt{c} + \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right)}{\sqrt{c - c \operatorname{Sin}[e + f x]}} \right] \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right) + \right. \\ \left. \operatorname{Sin}[e + f x] \left(262 \sqrt{c} + 45 i \sqrt{2} \operatorname{Log} \left[\frac{2 \left(-i \sqrt{2} \sqrt{c} + \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right)}{\sqrt{c - c \operatorname{Sin}[e + f x]}} \right] \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right) + \right. \\ \left. 3 \left(2 \sqrt{c} - i \sqrt{2} \operatorname{Log} \left[\frac{2 \left(-i \sqrt{2} \sqrt{c} + \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right)}{\sqrt{c - c \operatorname{Sin}[e + f x]}} \right] \sqrt{-c (1 + \operatorname{Sin}[e + f x])} \right) \operatorname{Sin}[3 (e + f x)] \right) \right) \Bigg) / \\ \left(768 c^{7/2} f \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] - \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \right)^5 \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{c - c \operatorname{Sin}[e + f x]} \right)$$

■ **Problem 298: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{7/2} dx$$

Optimal (type 3, 145 leaves, 5 steps):

$$\frac{256 a^2 c^6 \cos[e + f x]^5}{1155 f (c - c \sin[e + f x])^{5/2}} + \frac{64 a^2 c^5 \cos[e + f x]^5}{231 f (c - c \sin[e + f x])^{3/2}} + \frac{8 a^2 c^4 \cos[e + f x]^5}{33 f \sqrt{c - c \sin[e + f x]}} + \frac{2 a^2 c^3 \cos[e + f x]^5 \sqrt{c - c \sin[e + f x]}}{11 f}$$

Result (type 3, 1105 leaves):

■ **Problem 299: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2} dx$$

Optimal (type 3, 109 leaves, 4 steps):

$$\frac{64 a^2 c^5 \cos[e + f x]^5}{315 f (c - c \sin[e + f x])^{5/2}} + \frac{16 a^2 c^4 \cos[e + f x]^5}{63 f (c - c \sin[e + f x])^{3/2}} + \frac{2 a^2 c^3 \cos[e + f x]^5}{9 f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 921 leaves):

$$\begin{aligned} & \frac{3 \cos\left[\frac{1}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \\ & \frac{\cos\left[\frac{3}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2}}{6 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{\cos\left[\frac{5}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2}}{10 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \\ & \frac{\cos\left[\frac{7}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2}}{56 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{\cos\left[\frac{9}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2}}{72 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{3 \sin\left[\frac{1}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{(a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2} \sin\left[\frac{3}{2}(e + f x)\right]}{6 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{(a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2} \sin\left[\frac{5}{2}(e + f x)\right]}{10 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{(a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2} \sin\left[\frac{7}{2}(e + f x)\right]}{56 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{(a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2} \sin\left[\frac{9}{2}(e + f x)\right]}{72 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} \end{aligned}$$

■ **Problem 301: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^2 \sqrt{c - c \sin[e + f x]} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{2 a^2 c^3 \cos[e + f x]^5}{5 f (c - c \sin[e + f x])^{5/2}}$$

Result (type 3, 73 leaves):

$$\frac{2 a^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \sqrt{c - c \sin[e + f x]}}{5 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)}$$

■ **Problem 302: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^2}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 115 leaves, 5 steps):

$$\frac{4 \sqrt{2} a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{\sqrt{c} f} - \frac{2 a^2 c \cos[e + f x]^3}{3 f (c - c \sin[e + f x])^{3/2}} - \frac{4 a^2 \cos[e + f x]}{f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 130 leaves):

$$-\frac{1}{3 f \sqrt{c - c \sin[e + f x]}} + a^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left((24 + 24 i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] + 15 \cos\left[\frac{1}{2}(e + f x)\right] - \cos\left[\frac{3}{2}(e + f x)\right] + 15 \sin\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{3}{2}(e + f x)\right] \right)$$

■ **Problem 303: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^2}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 115 leaves, 5 steps):

$$-\frac{3 \sqrt{2} a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{c^{3/2} f} + \frac{a^2 c \cos[e + f x]^3}{f (c - c \sin[e + f x])^{5/2}} + \frac{3 a^2 \cos[e + f x]}{c f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 149 leaves):

$$-\left(a^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(3 \cos\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{3}{2}(e+fx)\right] + 3 \sin\left[\frac{1}{2}(e+fx)\right] - (6+6i)(-1)^{1/4} \right. \right. \\ \left. \left. \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] (-1 + \sin[e+fx]) - \sin\left[\frac{3}{2}(e+fx)\right] \right) \right) / \left(cf(-1 + \sin[e+fx]) \sqrt{c - c \sin[e+fx]} \right)$$

■ **Problem 304: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e+fx])^2}{(c - c \sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 122 leaves, 5 steps):

$$\frac{3 a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{4 \sqrt{2} c^{5/2} f} + \frac{a^2 c \cos[e+fx]^3}{2 f (c - c \sin[e+fx])^{7/2}} - \frac{3 a^2 \cos[e+fx]}{4 c f (c - c \sin[e+fx])^{3/2}}$$

Result (type 3, 163 leaves):

$$\left(a^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \right. \\ \left(3 \cos\left[\frac{1}{2}(e+fx)\right] - 5 \cos\left[\frac{3}{2}(e+fx)\right] + 3 \sin\left[\frac{1}{2}(e+fx)\right] + (3+3i)(-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \right) \\ \left. (-3 + \cos[2(e+fx)] + 4 \sin[e+fx]) + 5 \sin\left[\frac{3}{2}(e+fx)\right] \right) / \left(8 c^2 f (-1 + \sin[e+fx])^2 \sqrt{c - c \sin[e+fx]} \right)$$

■ **Problem 305: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e+fx])^2}{(c - c \sin[e+fx])^{7/2}} dx$$

Optimal (type 3, 156 leaves, 6 steps):

$$\frac{a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{16 \sqrt{2} c^{7/2} f} + \frac{a^2 c \cos[e+fx]^3}{3 f (c - c \sin[e+fx])^{9/2}} - \frac{a^2 \cos[e+fx]}{4 c f (c - c \sin[e+fx])^{5/2}} + \frac{a^2 \cos[e+fx]}{16 c^2 f (c - c \sin[e+fx])^{3/2}}$$

Result (type 3, 307 leaves):

$$\frac{1}{48 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 (c - c \sin[e+fx])^{7/2}} a^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)$$

$$\left(32 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) - 28 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 + 3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 - \right.$$

$$\left. (3 + 3i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 + 64 \sin\left[\frac{1}{2}(e+fx)\right] - \right.$$

$$\left. 56 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] + 6 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 \sin\left[\frac{1}{2}(e+fx)\right] \right) (1 + \sin[e+fx])^2$$

■ **Problem 306: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e+fx])^2}{(c - c \sin[e+fx])^{9/2}} dx$$

Optimal (type 3, 190 leaves, 7 steps):

$$\frac{3 a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{256 \sqrt{2} c^{9/2} f} + \frac{a^2 c \cos[e+fx]^3}{4 f (c - c \sin[e+fx])^{11/2}} -$$

$$\frac{a^2 \cos[e+fx]}{8 c f (c - c \sin[e+fx])^{7/2}} + \frac{a^2 \cos[e+fx]}{64 c^2 f (c - c \sin[e+fx])^{5/2}} + \frac{3 a^2 \cos[e+fx]}{256 c^3 f (c - c \sin[e+fx])^{3/2}}$$

Result (type 3, 371 leaves):

$$\frac{1}{256 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 (c - c \sin[e+fx])^{9/2}}$$

$$a^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(128 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) - \right.$$

$$\left. 96 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 + 4 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 + 3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 - \right.$$

$$\left. (3 + 3i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^8 + \right.$$

$$\left. 256 \sin\left[\frac{1}{2}(e+fx)\right] - 192 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] + \right.$$

$$\left. 8 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 \sin\left[\frac{1}{2}(e+fx)\right] + 6 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 \sin\left[\frac{1}{2}(e+fx)\right] \right) (1 + \sin[e+fx])^2$$

■ **Problem 307: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e+fx])^3 (c - c \sin[e+fx])^{7/2} dx$$

$$\frac{(a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{7/2} \sin\left[\frac{9}{2}(e + f x)\right]}{48 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} +$$

$$\frac{(a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{7/2} \sin\left[\frac{11}{2}(e + f x)\right]}{352 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} +$$

$$\frac{(a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{7/2} \sin\left[\frac{13}{2}(e + f x)\right]}{416 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6}$$

■ **Problem 308: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2} dx$$

Optimal (type 3, 109 leaves, 4 steps):

$$\frac{64 a^3 c^6 \cos[e + f x]^7}{693 f (c - c \sin[e + f x])^{7/2}} + \frac{16 a^3 c^5 \cos[e + f x]^7}{99 f (c - c \sin[e + f x])^{5/2}} + \frac{2 a^3 c^4 \cos[e + f x]^7}{11 f (c - c \sin[e + f x])^{3/2}}$$

Result (type 3, 1105 leaves):

■ **Problem 310: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^3 \sqrt{c - c \sin[e + f x]} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{2 a^3 c^4 \cos[e + f x]^7}{7 f (c - c \sin[e + f x])^{7/2}}$$

Result (type 3, 73 leaves):

$$\frac{2 a^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 \sqrt{c - c \sin[e + f x]}}{7 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)}$$

■ **Problem 311: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^3}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 151 leaves, 6 steps):

$$\frac{8 \sqrt{2} a^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{\sqrt{c} f} - \frac{2 a^3 c^2 \cos[e + f x]^5}{5 f (c - c \sin[e + f x])^{5/2}} - \frac{4 a^3 c \cos[e + f x]^3}{3 f (c - c \sin[e + f x])^{3/2}} - \frac{8 a^3 \cos[e + f x]}{f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 156 leaves):

$$-\frac{1}{30 f \sqrt{c - c \sin[e + f x]}} + a^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left((480 + 480 i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] + 330 \cos\left[\frac{1}{2}(e + f x)\right] - 35 \cos\left[\frac{3}{2}(e + f x)\right] - 3 \cos\left[\frac{5}{2}(e + f x)\right] + 330 \sin\left[\frac{1}{2}(e + f x)\right] + 35 \sin\left[\frac{3}{2}(e + f x)\right] - 3 \sin\left[\frac{5}{2}(e + f x)\right] \right)$$

■ **Problem 312: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^3}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 150 leaves, 6 steps):

$$-\frac{10 \sqrt{2} a^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{c^{3/2} f} + \frac{a^3 c^2 \cos[e + f x]^5}{f (c - c \sin[e + f x])^{7/2}} + \frac{5 a^3 \cos[e + f x]^3}{3 f (c - c \sin[e + f x])^{3/2}} + \frac{10 a^3 \cos[e + f x]}{c f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 173 leaves):

$$\begin{aligned}
& - \left(a^3 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(50 \cos \left[\frac{1}{2} (e + f x) \right] + 25 \cos \left[\frac{3}{2} (e + f x) \right] + \cos \left[\frac{5}{2} (e + f x) \right] + \right. \\
& \quad \left. 50 \sin \left[\frac{1}{2} (e + f x) \right] - (120 + 120 i) (-1)^{1/4} \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] (-1 + \sin[e + f x]) - \right. \\
& \quad \left. \left. 25 \sin \left[\frac{3}{2} (e + f x) \right] + \sin \left[\frac{5}{2} (e + f x) \right] \right) \right) / \left(6 c f (-1 + \sin[e + f x]) \sqrt{c - c \sin[e + f x]} \right)
\end{aligned}$$

■ **Problem 313: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^3}{(c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$\frac{15 a^3 \operatorname{ArcTanh} \left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}} \right]}{2 \sqrt{2} c^{5/2} f} + \frac{a^3 c^2 \cos[e + f x]^5}{2 f (c - c \sin[e + f x])^{9/2}} - \frac{5 a^3 \cos[e + f x]^3}{4 f (c - c \sin[e + f x])^{5/2}} - \frac{15 a^3 \cos[e + f x]}{4 c^2 f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 187 leaves):

$$\begin{aligned}
& \left(a^3 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(-5 \cos \left[\frac{1}{2} (e + f x) \right] - 15 \cos \left[\frac{3}{2} (e + f x) \right] + 2 \cos \left[\frac{5}{2} (e + f x) \right] - \right. \\
& \quad \left. 5 \sin \left[\frac{1}{2} (e + f x) \right] + (15 + 15 i) (-1)^{1/4} \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] (-3 + \cos[2(e + f x)] + 4 \sin[e + f x]) + \right. \\
& \quad \left. \left. 15 \sin \left[\frac{3}{2} (e + f x) \right] + 2 \sin \left[\frac{5}{2} (e + f x) \right] \right) \right) / \left(4 c^2 f (-1 + \sin[e + f x])^2 \sqrt{c - c \sin[e + f x]} \right)
\end{aligned}$$

■ **Problem 314: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^3}{(c - c \sin[e + f x])^{7/2}} dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$-\frac{5 a^3 \operatorname{ArcTanh} \left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}} \right]}{8 \sqrt{2} c^{7/2} f} + \frac{a^3 c^2 \cos[e + f x]^5}{3 f (c - c \sin[e + f x])^{11/2}} - \frac{5 a^3 \cos[e + f x]^3}{12 f (c - c \sin[e + f x])^{7/2}} + \frac{5 a^3 \cos[e + f x]}{8 c^2 f (c - c \sin[e + f x])^{3/2}}$$

Result (type 3, 307 leaves):

$$\frac{1}{24 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c - c \sin[e+fx])^{7/2}} a^3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)$$

$$\left(32 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) - 52 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 + 33 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 + \right.$$

$$\left. (15 + 15i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 + 64 \sin\left[\frac{1}{2}(e+fx)\right] - \right.$$

$$\left. 104 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] + 66 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 \sin\left[\frac{1}{2}(e+fx)\right] \right) (1 + \sin[e+fx])^3$$

■ **Problem 315: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e+fx])^3}{(c - c \sin[e+fx])^{9/2}} dx$$

Optimal (type 3, 191 leaves, 7 steps):

$$-\frac{5 a^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{128 \sqrt{2} c^{9/2} f} + \frac{a^3 c^2 \cos[e+fx]^5}{4 f (c - c \sin[e+fx])^{13/2}} -$$

$$\frac{5 a^3 \cos[e+fx]^3}{24 f (c - c \sin[e+fx])^{9/2}} + \frac{5 a^3 \cos[e+fx]}{32 c^2 f (c - c \sin[e+fx])^{5/2}} - \frac{5 a^3 \cos[e+fx]}{128 c^3 f (c - c \sin[e+fx])^{3/2}}$$

Result (type 3, 371 leaves):

$$\frac{1}{384 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c - c \sin[e+fx])^{9/2}}$$

$$a^3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(384 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) - \right.$$

$$\left. 544 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 + 236 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 - 15 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 + \right.$$

$$\left. (15 + 15i) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^8 + \right.$$

$$\left. 768 \sin\left[\frac{1}{2}(e+fx)\right] - 1088 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] + \right.$$

$$\left. 472 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 \sin\left[\frac{1}{2}(e+fx)\right] - 30 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 \sin\left[\frac{1}{2}(e+fx)\right] \right) (1 + \sin[e+fx])^3$$

■ **Problem 316: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e+fx])^3}{(c - c \sin[e+fx])^{11/2}} dx$$

Optimal (type 3, 225 leaves, 8 steps) :

$$\begin{aligned}
 & - \frac{3 a^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \operatorname{Cos}[e+f x]}{\sqrt{2} \sqrt{c-c \operatorname{Sin}[e+f x]}}\right]}{512 \sqrt{2} c^{11/2} f} + \frac{a^3 c^2 \operatorname{Cos}[e+f x]^5}{5 f (c-c \operatorname{Sin}[e+f x])^{15/2}} - \frac{a^3 \operatorname{Cos}[e+f x]^3}{8 f (c-c \operatorname{Sin}[e+f x])^{11/2}} + \\
 & \frac{a^3 \operatorname{Cos}[e+f x]}{16 c^2 f (c-c \operatorname{Sin}[e+f x])^{7/2}} - \frac{a^3 \operatorname{Cos}[e+f x]}{128 c^3 f (c-c \operatorname{Sin}[e+f x])^{5/2}} - \frac{3 a^3 \operatorname{Cos}[e+f x]}{512 c^4 f (c-c \operatorname{Sin}[e+f x])^{3/2}}
 \end{aligned}$$

Result (type 3, 1005 leaves) :

$$\begin{aligned}
 & \frac{4 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2 (a+a \operatorname{Sin}[e+f x])^3}{5 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}} - \\
 & \frac{21 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^4 (a+a \operatorname{Sin}[e+f x])^3}{20 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}} + \frac{31 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (a+a \operatorname{Sin}[e+f x])^3}{80 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}} - \\
 & \frac{\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^8 (a+a \operatorname{Sin}[e+f x])^3}{128 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}} - \frac{3 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^{10} (a+a \operatorname{Sin}[e+f x])^3}{512 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}} + \\
 & \left(\left(\frac{3}{512} + \frac{3 i}{512}\right) (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \operatorname{Sec}\left[\frac{1}{4}(e+f x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{4}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{4}(e+f x)\right]\right)\right) \\
 & \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^{11} (a+a \operatorname{Sin}[e+f x])^3 \Big/ \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}\right) + \\
 & \frac{8 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right) \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] (a+a \operatorname{Sin}[e+f x])^3}{5 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}} - \\
 & \frac{21 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] (a+a \operatorname{Sin}[e+f x])^3}{10 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}} + \\
 & \frac{31 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^5 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] (a+a \operatorname{Sin}[e+f x])^3}{40 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}} - \\
 & \frac{\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^7 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] (a+a \operatorname{Sin}[e+f x])^3}{64 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}} - \\
 & \frac{3 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^9 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] (a+a \operatorname{Sin}[e+f x])^3}{256 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^6 (c-c \operatorname{Sin}[e+f x])^{11/2}}
 \end{aligned}$$

■ **Problem 321: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \sin[e + f x]) \sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 83 leaves, 4 steps) :

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{\sqrt{2} a \sqrt{c} f} - \frac{\text{Sec}[e + f x] \sqrt{c - c \sin[e + f x]}}{a c f}$$

Result (type 3, 97 leaves) :

$$-\left(\cos[e + f x] \left(1 + (1 + i) (-1)^{1/4} \text{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)\right)\right) / \left(a f (1 + \sin[e + f x]) \sqrt{c - c \sin[e + f x]}\right)$$

■ **Problem 322: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \sin[e + f x]) (c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 117 leaves, 5 steps) :

$$\frac{3 \text{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{4 \sqrt{2} a c^{3/2} f} + \frac{3 \cos[e + f x]}{4 a f (c - c \sin[e + f x])^{3/2}} - \frac{\text{Sec}[e + f x]}{a c f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 125 leaves) :

$$-\left(\text{Sec}[e + f x] \left(1 + (3 + 3i) (-1)^{1/4} \text{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] - 3 \sin[e + f x]\right)\right)\right) / \left(4 a c f \sqrt{c - c \sin[e + f x]}\right)$$

■ **Problem 323: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \sin[e + f x]) (c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 156 leaves, 6 steps) :

$$\frac{15 \text{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{32 \sqrt{2} a c^{5/2} f} + \frac{15 \cos[e + f x]}{32 a c f (c - c \sin[e + f x])^{3/2}} + \frac{\text{Sec}[e + f x]}{4 a c f (c - c \sin[e + f x])^{3/2}} - \frac{5 \text{Sec}[e + f x]}{8 a c^2 f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 162 leaves) :

$$\left(\left(\frac{1}{128} + \frac{i}{128} \right) \cos[e + f x] \right. \\ \left. \left(-60 (-1)^{1/4} \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \right)^4 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) + \right. \\ \left. (1 - i) (-9 + 15 \cos[2(e + f x)] + 40 \sin[e + f x]) \right) / \left(a c^2 f (-1 + \sin[e + f x])^2 (1 + \sin[e + f x]) \sqrt{c - c \sin[e + f x]} \right)$$

- **Problem 328: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c - c \sin[e + f x]}}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$-\frac{2 \operatorname{Sec}[e + f x]^3 (c - c \sin[e + f x])^{3/2}}{3 a^2 c f}$$

Result (type 3, 73 leaves):

$$-\frac{2 \sqrt{c - c \sin[e + f x]}}{3 a^2 f \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^3}$$

- **Problem 329: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \sin[e + f x])^2 \sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 124 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}} \right]}{2 \sqrt{2} a^2 \sqrt{c} f} - \frac{\operatorname{Sec}[e + f x] \sqrt{c - c \sin[e + f x]}}{2 a^2 c f} - \frac{\operatorname{Sec}[e + f x]^3 (c - c \sin[e + f x])^{3/2}}{3 a^2 c^2 f}$$

Result (type 3, 109 leaves):

$$\left(\cos[e + f x] \left(-5 - (3 + 3i) (-1)^{1/4} \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^3 - 3 \sin[e + f x] \right) \right) / \left(6 a^2 f (1 + \sin[e + f x])^2 \sqrt{c - c \sin[e + f x]} \right)$$

- **Problem 330: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 155 leaves, 6 steps):

$$\frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{8 \sqrt{2} a^2 c^{3/2} f} + \frac{5 \cos[e+fx]}{8 a^2 f (c-c \sin[e+fx])^{3/2}} - \frac{5 \sec[e+fx]}{6 a^2 c f \sqrt{c-c \sin[e+fx]}} - \frac{\sec[e+fx]^3 \sqrt{c-c \sin[e+fx]}}{3 a^2 c^2 f}$$

Result (type 3, 164 leaves):

$$\left(\left(\frac{1}{96} + \frac{i}{96} \right) \cos[e+fx] \left(60 (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4} (e+fx) \right] \right) \right] \left(\cos\left[\frac{1}{2} (e+fx) \right] - \sin\left[\frac{1}{2} (e+fx) \right] \right)^2 \left(\cos\left[\frac{1}{2} (e+fx) \right] + \sin\left[\frac{1}{2} (e+fx) \right] \right)^3 + (1-i) (11 + 15 \cos[2(e+fx)] - 20 \sin[e+fx]) \right) \right) / \left(a^2 c f (-1 + \sin[e+fx]) (1 + \sin[e+fx])^2 \sqrt{c-c \sin[e+fx]} \right)$$

■ **Problem 331: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a \sin[e+fx])^2 (c-c \sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 192 leaves, 7 steps):

$$\frac{35 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{64 \sqrt{2} a^2 c^{5/2} f} + \frac{35 \cos[e+fx]}{64 a^2 c f (c-c \sin[e+fx])^{3/2}} + \frac{7 \sec[e+fx]}{24 a^2 c f (c-c \sin[e+fx])^{3/2}} - \frac{35 \sec[e+fx]}{48 a^2 c^2 f \sqrt{c-c \sin[e+fx]}} - \frac{\sec[e+fx]^3}{3 a^2 c^2 f \sqrt{c-c \sin[e+fx]}}$$

Result (type 3, 156 leaves):

$$-\left(\left(\frac{1}{1536} + \frac{i}{1536} \right) \sec[e+fx]^3 \left(840 (-1)^{1/4} \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4} (e+fx) \right] \right) \right] \left(\cos\left[\frac{1}{2} (e+fx) \right] - \sin\left[\frac{1}{2} (e+fx) \right] \right)^4 \left(\cos\left[\frac{1}{2} (e+fx) \right] + \sin\left[\frac{1}{2} (e+fx) \right] \right)^3 + (1-i) (102 + 70 \cos[2(e+fx)] - 329 \sin[e+fx] - 105 \sin[3(e+fx)]) \right) \right) / \left(a^2 c^2 f \sqrt{c-c \sin[e+fx]} \right)$$

■ **Problem 336: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c-c \sin[e+fx]}}{(a+a \sin[e+fx])^3} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$-\frac{2 \sec[e+fx]^5 (c-c \sin[e+fx])^{5/2}}{5 a^3 c^2 f}$$

Result (type 3, 73 leaves):

$$\frac{2\sqrt{c-c\sin[e+fx]}}{5a^3f\left(\cos\left[\frac{1}{2}(e+fx)\right]-\sin\left[\frac{1}{2}(e+fx)\right]\right)\left(\cos\left[\frac{1}{2}(e+fx)\right]+\sin\left[\frac{1}{2}(e+fx)\right]\right)^5}$$

■ **Problem 337: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a\sin[e+fx])^3\sqrt{c-c\sin[e+fx]}} dx$$

Optimal (type 3, 160 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c}\cos[e+fx]}{\sqrt{2}\sqrt{c-c\sin[e+fx]}}\right]}{4\sqrt{2}a^3\sqrt{c}f} - \frac{\text{Sec}[e+fx]\sqrt{c-c\sin[e+fx]}}{4a^3cf} - \frac{\text{Sec}[e+fx]^3(c-c\sin[e+fx])^{3/2}}{6a^3c^2f} - \frac{\text{Sec}[e+fx]^5(c-c\sin[e+fx])^{5/2}}{5a^3c^3f}$$

Result (type 3, 189 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right]-\sin\left[\frac{1}{2}(e+fx)\right]\right)\left(\cos\left[\frac{1}{2}(e+fx)\right]+\sin\left[\frac{1}{2}(e+fx)\right]\right)\right) \left(-12-10\left(\cos\left[\frac{1}{2}(e+fx)\right]+\sin\left[\frac{1}{2}(e+fx)\right]\right)^2-15\left(\cos\left[\frac{1}{2}(e+fx)\right]+\sin\left[\frac{1}{2}(e+fx)\right]\right)^4-(15+15i)(-1)^{1/4}\right) \text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{1/4}\left(1+\tan\left[\frac{1}{4}(e+fx)\right]\right)\left(\cos\left[\frac{1}{2}(e+fx)\right]+\sin\left[\frac{1}{2}(e+fx)\right]\right)^5\right]\right) / \left(60a^3f(1+\sin[e+fx])^3\sqrt{c-c\sin[e+fx]}\right)$$

■ **Problem 338: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a\sin[e+fx])^3(c-c\sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 191 leaves, 7 steps):

$$\frac{7\text{ArcTanh}\left[\frac{\sqrt{c}\cos[e+fx]}{\sqrt{2}\sqrt{c-c\sin[e+fx]}}\right]}{16\sqrt{2}a^3c^{3/2}f} + \frac{7\cos[e+fx]}{16a^3f(c-c\sin[e+fx])^{3/2}} - \frac{7\text{Sec}[e+fx]}{12a^3cf\sqrt{c-c\sin[e+fx]}} - \frac{7\text{Sec}[e+fx]^3\sqrt{c-c\sin[e+fx]}}{30a^3c^2f} - \frac{\text{Sec}[e+fx]^5(c-c\sin[e+fx])^{3/2}}{5a^3c^3f}$$

Result (type 3, 174 leaves):

$$\left(\left(\frac{1}{1920} + \frac{i}{1920} \right) \cos[e + f x] \right. \\ \left. \left(840 (-1)^{1/4} \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \right)^2 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^5 + \right. \\ \left. (1 - i) (206 + 350 \cos[2(e + f x)] - 231 \sin[e + f x] + 105 \sin[3(e + f x)]) \right) \Bigg) / \\ \left(a^3 c f (-1 + \sin[e + f x]) (1 + \sin[e + f x])^3 \sqrt{c - c \sin[e + f x]} \right)$$

■ **Problem 339: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 228 leaves, 8 steps):

$$\frac{63 \operatorname{ArcTanh} \left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}} \right]}{128 \sqrt{2} a^3 c^{5/2} f} + \frac{63 \cos[e + f x]}{128 a^3 c f (c - c \sin[e + f x])^{3/2}} + \frac{21 \operatorname{Sec}[e + f x]}{80 a^3 c f (c - c \sin[e + f x])^{3/2}} - \\ \frac{21 \operatorname{Sec}[e + f x]}{32 a^3 c^2 f \sqrt{c - c \sin[e + f x]}} - \frac{3 \operatorname{Sec}[e + f x]^3}{10 a^3 c^2 f \sqrt{c - c \sin[e + f x]}} - \frac{\operatorname{Sec}[e + f x]^5 \sqrt{c - c \sin[e + f x]}}{5 a^3 c^3 f}$$

Result (type 3, 443 leaves):

$$\frac{1}{640 a^3 f (1 + \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}} \\ \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(-240 \cos[e + f x]^4 - 32 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \right)^4 - \\ 80 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 + 20 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \\ \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^5 + 75 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^3 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^5 - \\ (315 + 315 i) (-1)^{1/4} \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 \\ \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^5 + 40 \sin \left[\frac{1}{2} (e + f x) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^5 + \\ 150 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 \sin \left[\frac{1}{2} (e + f x) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^5$$

- **Problem 344: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \sin[e + f x]}}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$-\frac{a \cos[e + f x] \operatorname{Log}[1 - \sin[e + f x]]}{f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 129 leaves):

$$\frac{\sqrt{2} (-i + e^{i(e+fx)}) (fx - 2 \operatorname{ArcTan}[e^{i(e+fx)}] + i \operatorname{Log}[1 + e^{2i(e+fx)}]) \sqrt{a(1 + \sin[e + f x])}}{\sqrt{i c e^{-i(e+fx)} (-i + e^{i(e+fx)})^2 (i + e^{i(e+fx)}) f}}$$

- **Problem 345: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \sin[e + f x]}}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 40 leaves, 1 step):

$$\frac{a \cos[e + f x]}{f \sqrt{a + a \sin[e + f x]} (c - c \sin[e + f x])^{3/2}}$$

Result (type 3, 84 leaves):

$$\frac{\sqrt{a(1 + \sin[e + f x])} \sqrt{c - c \sin[e + f x]}}{c^2 f (\cos[\frac{1}{2}(e + f x)] - \sin[\frac{1}{2}(e + f x)])^3 (\cos[\frac{1}{2}(e + f x)] + \sin[\frac{1}{2}(e + f x)])}$$

- **Problem 346: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \sin[e + f x]}}{(c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 43 leaves, 1 step):

$$\frac{a \cos[e + f x]}{2 f \sqrt{a + a \sin[e + f x]} (c - c \sin[e + f x])^{5/2}}$$

Result (type 3, 87 leaves):

$$\frac{\sqrt{a(1 + \sin[e + f x])} \sqrt{c - c \sin[e + f x]}}{2 c^3 f (\cos[\frac{1}{2}(e + f x)] - \sin[\frac{1}{2}(e + f x)])^5 (\cos[\frac{1}{2}(e + f x)] + \sin[\frac{1}{2}(e + f x)])}$$

■ **Problem 347: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \sin[e + f x]}}{(c - c \sin[e + f x])^{7/2}} dx$$

Optimal (type 3, 43 leaves, 1 step):

$$\frac{a \cos[e + f x]}{3 f \sqrt{a + a \sin[e + f x]} (c - c \sin[e + f x])^{7/2}}$$

Result (type 3, 87 leaves):

$$\frac{\sqrt{a(1 + \sin[e + f x])} \sqrt{c - c \sin[e + f x]}}{3 c^4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)}$$

■ **Problem 354: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^{3/2}}{(c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{\cos[e + f x] (a + a \sin[e + f x])^{3/2}}{4 f (c - c \sin[e + f x])^{5/2}}$$

Result (type 3, 99 leaves):

$$\frac{a \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \sin[e + f x] \sqrt{a(1 + \sin[e + f x])}}{c^2 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) (-1 + \sin[e + f x])^2 \sqrt{c - c \sin[e + f x]}}$$

■ **Problem 365: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^{5/2}}{(c - c \sin[e + f x])^{7/2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{\cos[e + f x] (a + a \sin[e + f x])^{5/2}}{6 f (c - c \sin[e + f x])^{7/2}}$$

Result (type 3, 110 leaves):

$$\frac{a^2 (-5 + 3 \cos[2(e + f x)]) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \sqrt{a(1 + \sin[e + f x])}}{6 c^3 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) (-1 + \sin[e + f x])^3 \sqrt{c - c \sin[e + f x]}}$$

■ **Problem 369: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^{7/2} (c - c \sin[e + f x])^{9/2} dx$$

Optimal (type 3, 179 leaves, 4 steps) :

$$\frac{a^4 \operatorname{Cos}[e + f x] (c - c \operatorname{Sin}[e + f x])^{9/2}}{35 f \sqrt{a + a \operatorname{Sin}[e + f x]}} - \frac{a^3 \operatorname{Cos}[e + f x] \sqrt{a + a \operatorname{Sin}[e + f x]} (c - c \operatorname{Sin}[e + f x])^{9/2}}{14 f} - \frac{3 a^2 \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^{3/2} (c - c \operatorname{Sin}[e + f x])^{9/2}}{28 f} - \frac{a \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^{5/2} (c - c \operatorname{Sin}[e + f x])^{9/2}}{8 f}$$

Result (type 3, 735 leaves) :

$$\frac{7 \operatorname{Cos}[2 (e + f x)] (a (1 + \operatorname{Sin}[e + f x]))^{7/2} (c - c \operatorname{Sin}[e + f x])^{9/2}}{128 f (\operatorname{Cos}[\frac{1}{2} (e + f x)] - \operatorname{Sin}[\frac{1}{2} (e + f x)])^9 (\operatorname{Cos}[\frac{1}{2} (e + f x)] + \operatorname{Sin}[\frac{1}{2} (e + f x)])^7} + \frac{7 \operatorname{Cos}[4 (e + f x)] (a (1 + \operatorname{Sin}[e + f x]))^{7/2} (c - c \operatorname{Sin}[e + f x])^{9/2}}{256 f (\operatorname{Cos}[\frac{1}{2} (e + f x)] - \operatorname{Sin}[\frac{1}{2} (e + f x)])^9 (\operatorname{Cos}[\frac{1}{2} (e + f x)] + \operatorname{Sin}[\frac{1}{2} (e + f x)])^7} + \frac{\operatorname{Cos}[6 (e + f x)] (a (1 + \operatorname{Sin}[e + f x]))^{7/2} (c - c \operatorname{Sin}[e + f x])^{9/2}}{128 f (\operatorname{Cos}[\frac{1}{2} (e + f x)] - \operatorname{Sin}[\frac{1}{2} (e + f x)])^9 (\operatorname{Cos}[\frac{1}{2} (e + f x)] + \operatorname{Sin}[\frac{1}{2} (e + f x)])^7} + \frac{\operatorname{Cos}[8 (e + f x)] (a (1 + \operatorname{Sin}[e + f x]))^{7/2} (c - c \operatorname{Sin}[e + f x])^{9/2}}{1024 f (\operatorname{Cos}[\frac{1}{2} (e + f x)] - \operatorname{Sin}[\frac{1}{2} (e + f x)])^9 (\operatorname{Cos}[\frac{1}{2} (e + f x)] + \operatorname{Sin}[\frac{1}{2} (e + f x)])^7} + \frac{35 \operatorname{Sin}[e + f x] (a (1 + \operatorname{Sin}[e + f x]))^{7/2} (c - c \operatorname{Sin}[e + f x])^{9/2}}{64 f (\operatorname{Cos}[\frac{1}{2} (e + f x)] - \operatorname{Sin}[\frac{1}{2} (e + f x)])^9 (\operatorname{Cos}[\frac{1}{2} (e + f x)] + \operatorname{Sin}[\frac{1}{2} (e + f x)])^7} + \frac{7 (a (1 + \operatorname{Sin}[e + f x]))^{7/2} (c - c \operatorname{Sin}[e + f x])^{9/2} \operatorname{Sin}[3 (e + f x)]}{64 f (\operatorname{Cos}[\frac{1}{2} (e + f x)] - \operatorname{Sin}[\frac{1}{2} (e + f x)])^9 (\operatorname{Cos}[\frac{1}{2} (e + f x)] + \operatorname{Sin}[\frac{1}{2} (e + f x)])^7} + \frac{7 (a (1 + \operatorname{Sin}[e + f x]))^{7/2} (c - c \operatorname{Sin}[e + f x])^{9/2} \operatorname{Sin}[5 (e + f x)]}{320 f (\operatorname{Cos}[\frac{1}{2} (e + f x)] - \operatorname{Sin}[\frac{1}{2} (e + f x)])^9 (\operatorname{Cos}[\frac{1}{2} (e + f x)] + \operatorname{Sin}[\frac{1}{2} (e + f x)])^7} + \frac{(a (1 + \operatorname{Sin}[e + f x]))^{7/2} (c - c \operatorname{Sin}[e + f x])^{9/2} \operatorname{Sin}[7 (e + f x)]}{448 f (\operatorname{Cos}[\frac{1}{2} (e + f x)] - \operatorname{Sin}[\frac{1}{2} (e + f x)])^9 (\operatorname{Cos}[\frac{1}{2} (e + f x)] + \operatorname{Sin}[\frac{1}{2} (e + f x)])^7}$$

■ **Problem 378: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x])^{7/2}}{(c - c \operatorname{Sin}[e + f x])^{9/2}} dx$$

Optimal (type 3, 42 leaves, 1 step) :

$$\frac{\operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^{7/2}}{8 f (c - c \operatorname{Sin}[e + f x])^{9/2}}$$

Result (type 3, 327 leaves) :

$$\frac{2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{9/2}} - \frac{4 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{9/2}} +$$

$$\frac{3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{9/2}} - \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{9/2}}$$

■ **Problem 379: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^{7/2}}{(c - c \sin[e + fx])^{11/2}} dx$$

Optimal (type 3, 88 leaves, 2 steps):

$$\frac{\cos[e + fx] (a + a \sin[e + fx])^{7/2}}{10 f (c - c \sin[e + fx])^{11/2}} + \frac{\cos[e + fx] (a + a \sin[e + fx])^{7/2}}{80 c f (c - c \sin[e + fx])^{9/2}}$$

Result (type 3, 331 leaves):

$$\frac{8 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (a(1+\sin[e+fx]))^{7/2}}{5 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{11/2}} - \frac{3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{11/2}} +$$

$$\frac{2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{11/2}} - \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (a(1+\sin[e+fx]))^{7/2}}{2 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{11/2}}$$

■ **Problem 380: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^{7/2}}{(c - c \sin[e + fx])^{13/2}} dx$$

Optimal (type 3, 133 leaves, 3 steps):

$$\frac{\cos[e + fx] (a + a \sin[e + fx])^{7/2}}{12 f (c - c \sin[e + fx])^{13/2}} + \frac{\cos[e + fx] (a + a \sin[e + fx])^{7/2}}{60 c f (c - c \sin[e + fx])^{11/2}} + \frac{\cos[e + fx] (a + a \sin[e + fx])^{7/2}}{480 c^2 f (c - c \sin[e + fx])^{9/2}}$$

Result (type 3, 335 leaves):

$$\frac{4 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (a(1+\sin[e+fx]))^{7/2}}{3 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{13/2}} - \frac{12 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (a(1+\sin[e+fx]))^{7/2}}{5 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{13/2}} +$$

$$\frac{3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 (a(1+\sin[e+fx]))^{7/2}}{2 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{13/2}} - \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (a(1+\sin[e+fx]))^{7/2}}{3 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{13/2}}$$

- **Problem 385: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c - c \sin[e + f x]}}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 49 leaves, 3 steps):

$$\frac{c \cos[e + f x] \operatorname{Log}[1 + \sin[e + f x]]}{f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 130 leaves):

$$\frac{\sqrt{2} \left(i + e^{i(e+fx)} \right) \left(f x + 2 \operatorname{ArcTan}\left[e^{i(e+fx)} \right] + i \operatorname{Log}\left[1 + e^{2i(e+fx)} \right] \right) \sqrt{c - c \sin[e + f x]}}{\left(-i + e^{i(e+fx)} \right) \sqrt{-i a e^{-i(e+fx)} \left(i + e^{i(e+fx)} \right)^2} f}$$

- **Problem 392: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c - c \sin[e + f x]}}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 41 leaves, 1 step):

$$\frac{c \cos[e + f x]}{f (a + a \sin[e + f x])^{3/2} \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 85 leaves):

$$\frac{\sqrt{a(1 + \sin[e + f x])} \sqrt{c - c \sin[e + f x]}}{a^2 f \left(\cos\left[\frac{1}{2}(e + f x) \right] - \sin\left[\frac{1}{2}(e + f x) \right] \right) \left(\cos\left[\frac{1}{2}(e + f x) \right] + \sin\left[\frac{1}{2}(e + f x) \right] \right)^3}$$

- **Problem 399: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \sin[e + f x])^{3/2}}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{\cos[e + f x] (c - c \sin[e + f x])^{3/2}}{4 f (a + a \sin[e + f x])^{5/2}}$$

Result (type 3, 86 leaves):

$$\frac{c \left(\cos\left[\frac{1}{2}(e + f x) \right] + \sin\left[\frac{1}{2}(e + f x) \right] \right) \sin[e + f x] \sqrt{c - c \sin[e + f x]}}{f \left(\cos\left[\frac{1}{2}(e + f x) \right] - \sin\left[\frac{1}{2}(e + f x) \right] \right) (a(1 + \sin[e + f x]))^{5/2}}$$

■ **Problem 400: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c - c \sin[e + f x]}}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 43 leaves, 1 step):

$$-\frac{c \cos[e + f x]}{2 f (a + a \sin[e + f x])^{5/2} \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 87 leaves):

$$-\frac{\sqrt{a(1 + \sin[e + f x])} \sqrt{c - c \sin[e + f x]}}{2 a^3 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5}$$

■ **Problem 404: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n dx$$

Optimal (type 5, 110 leaves, 4 steps):

$$\frac{1}{f(1+2m)} 2^{\frac{1}{2}+m} c \cos[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}(1+2m), \frac{1}{2}(1-2n), \frac{1}{2}(3+2m), \frac{1}{2}(1+\sin[e + f x])\right] \\ (1 - \sin[e + f x])^{\frac{1}{2}-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1+n}$$

Result (type 6, 4008 leaves):

$$-\left(\left(4(3+2n) \text{AppellF1}\left[\frac{1}{2}+n, -2m, 1+2(m+n), \frac{3}{2}+n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(\cos\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{1+2m} \right. \right. \\ \left. \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{2m} \right) / \right. \\ \left(f(1+2n) \left((3+2n) \text{AppellF1}\left[\frac{1}{2}+n, -2m, 1+2(m+n), \frac{3}{2}+n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \right. \\ \left. \left. 2 \left(2m \text{AppellF1}\left[\frac{3}{2}+n, 1-2m, 1+2(m+n), \frac{5}{2}+n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\ \left. \left. (1+2m+2n) \text{AppellF1}\left[\frac{3}{2}+n, -2m, 2(1+m+n), \frac{5}{2}+n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \\ \left(\left((3+2n) \text{AppellF1}\left[\frac{1}{2}+n, -2m, 1+2(m+n), \frac{3}{2}+n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\ \left. \left. \left(\cos\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{2m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2n} \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{2m} \right) / \right)$$

$$\begin{aligned}
& \left(-e + \frac{\pi}{2} - f x\right)^2 \Big] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{2\left(\frac{5}{2} + n\right)}(1 - 2m)\left(\frac{3}{2} + n\right) \operatorname{AppellF1}\left[\frac{5}{2} + n, 2 - 2m, \right. \\
& \left. 1 + 2(m + n), \frac{7}{2} + n, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \Big] + \\
& (1 + 2m + 2n) \left(-\frac{1}{\frac{5}{2} + n} m \left(\frac{3}{2} + n\right) \operatorname{AppellF1}\left[\frac{5}{2} + n, 1 - 2m, 2(1 + m + n), \frac{7}{2} + n, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \right. \\
& \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{5}{2} + n} \left(\frac{3}{2} + n\right) (1 + m + n) \operatorname{AppellF1}\left[\frac{5}{2} + n, -2m, 1 + 2(1 + m + n), \right. \\
& \left. \frac{7}{2} + n, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \Big) \Big) \Big) \Big) / \\
& \left((1 + 2n) \left((3 + 2n) \operatorname{AppellF1}\left[\frac{1}{2} + n, -2m, 1 + 2(m + n), \frac{3}{2} + n, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left(2m \operatorname{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 1 + 2(m + n), \frac{5}{2} + n, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] + (1 + 2m + 2n) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + n, -2m, 2(1 + m + n), \frac{5}{2} + n, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \Big) \Big) \Big) \Big)
\end{aligned}$$

■ **Problem 405: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c - c \sin[e + f x])^3 dx$$

Optimal (type 5, 86 leaves, 4 steps):

$$-\frac{1}{7f} 2^{\frac{1}{2}+m} a^4 c^3 \operatorname{Cos}[e + f x]^7 \operatorname{Hypergeometric2F1}\left[\frac{7}{2}, \frac{1}{2} - m, \frac{9}{2}, \frac{1}{2}(1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{\frac{1}{2}-m} (a + a \sin[e + f x])^{-4+m}$$

Result (type 6, 12670 leaves):

$$\begin{aligned}
& - \left(\left(\left(2048 \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^3 \right. \right. \right. \\
& \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^6 - 6 \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^5 \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] + \right. \\
& \left. 15 \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^4 \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^2 - 20 \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^3 \right. \\
& \left. \left. \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^3 + 15 \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^2 \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^4 - \right. \right.
\end{aligned}$$

$$\left(\text{AppellF1} \left[\frac{1}{2}, -2m, 7+2m, \frac{3}{2}, \text{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\text{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] - \frac{2}{3} \left(2m \text{AppellF1} \left[\frac{3}{2}, 1-2m, 7+2m, \frac{5}{2}, \text{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\text{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + (7+2m) \text{AppellF1} \left[\frac{3}{2}, -2m, 8+2m, \frac{5}{2}, \text{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\text{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)^2 \right)^2 \right)^2 \right)^2 \right)$$

- **Problem 406: Attempted integration timed out after 120 seconds.**

$$\int (a + a \sin[e + fx])^m (c - c \sin[e + fx])^2 dx$$

Optimal (type 5, 86 leaves, 4 steps):

$$-\frac{1}{5f} 2^{\frac{1}{2}+m} a^3 c^2 \cos[e + fx]^5 \text{Hypergeometric2F1} \left[\frac{5}{2}, \frac{1}{2} - m, \frac{7}{2}, \frac{1}{2} (1 - \sin[e + fx]) \right] (1 + \sin[e + fx])^{\frac{1}{2}-m} (a + a \sin[e + fx])^{-3+m}$$

Result (type 1, 1 leaves):

???

- **Problem 407: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m (c - c \sin[e + fx]) dx$$

Optimal (type 5, 84 leaves, 4 steps):

$$-\frac{1}{3f} 2^{\frac{1}{2}+m} a^2 c \cos[e + fx]^3 \text{Hypergeometric2F1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{5}{2}, \frac{1}{2} (1 - \sin[e + fx]) \right] (1 + \sin[e + fx])^{\frac{1}{2}-m} (a + a \sin[e + fx])^{-2+m}$$

Result (type 5, 283 leaves):

$$-\frac{1}{f(-1+m)m(1+m) \left(\cos \left[\frac{1}{2} (e + fx) \right] - \sin \left[\frac{1}{2} (e + fx) \right] \right)^2} \left(i 2^{-1-2m} c e^{-i(e+fx)} \left(1 + i e^{-i(e+fx)} \right)^{-2m} \left(e^{-\frac{1}{4}i(2e+\pi+2fx)} (i + e^{i(e+fx)}) \right)^{2m} \left(e^{2i(e+fx)} (-1+m)m \text{Hypergeometric2F1}[-1-m, -2m, -m, -i e^{-i(e+fx)}] + (1+m) \left(m \text{Hypergeometric2F1}[1-m, -2m, 2-m, -i e^{-i(e+fx)}] - 2 e^{i(e+fx)} (-1+m) \text{Hypergeometric2F1}[-2m, -m, 1-m, -i e^{-i(e+fx)}] \right) \right) (-1 + \sin[e + fx]) (a(1 + \sin[e + fx]))^m \sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^{-2m} \right)$$

- **Problem 408: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^m}{c - c \sin[e + fx]} dx$$

Optimal (type 5, 76 leaves, 4 steps):

$$\frac{1}{c f} 2^{\frac{1}{2}+m} \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{1}{2} (1 - \text{Sin}[e + f x])\right] \text{Sec}[e + f x] (1 + \text{Sin}[e + f x])^{\frac{1}{2}-m} (a + a \text{Sin}[e + f x])^m$$

Result (type 6, 4905 leaves):

$$\begin{aligned} & - \left(\left(\text{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{2m} \text{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right] \right)^2 (a + a \text{Sin}[e + f x])^m \right. \\ & \quad \left(-\text{AppellF1}\left[-\frac{1}{2}, -2m, 2m, \frac{1}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] / \left(\text{AppellF1}\left[-\frac{1}{2}, -2m, 2m, \frac{1}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right. \right. \right. \\ & \quad \left. \left. -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - 4m \left(\text{AppellF1}\left[\frac{1}{2}, 1-2m, 2m, \frac{3}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\ & \quad \left. \left. \text{AppellF1}\left[\frac{1}{2}, -2m, 1+2m, \frac{3}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) / \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \\ & \quad \left. 4m \left(\text{AppellF1}\left[\frac{3}{2}, 1-2m, 2m, \frac{5}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\ & \quad \left. \left. \text{AppellF1}\left[\frac{3}{2}, -2m, 1+2m, \frac{5}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) / \\ & \quad \left(2 f (c - c \text{Sin}[e + f x]) \left(\text{Cos}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \text{Sin}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^2 \left(-\frac{1}{8} \text{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{2m} \right. \\ & \quad \left. \text{Csc}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(-\left(\text{AppellF1}\left[-\frac{1}{2}, -2m, 2m, \frac{1}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] / \right. \right. \right. \\ & \quad \left(\text{AppellF1}\left[-\frac{1}{2}, -2m, 2m, \frac{1}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - 4m \right. \\ & \quad \left. \left(\text{AppellF1}\left[\frac{1}{2}, 1-2m, 2m, \frac{3}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\ & \quad \left. \left. \text{AppellF1}\left[\frac{1}{2}, -2m, 1+2m, \frac{3}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) + \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) / \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \\ & \quad \left. 4m \left(\text{AppellF1}\left[\frac{3}{2}, 1-2m, 2m, \frac{5}{2}, \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right. \\
& \quad \left(-2m \left(\operatorname{AppellF1}\left[\frac{3}{2}, 1-2m, 2m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -2m, 1+2m, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + 3 \left(-\frac{1}{3}m \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. 1-2m, 2m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \right. \right. \\
& \quad \left. \left. \frac{1}{3}m \operatorname{AppellF1}\left[\frac{3}{2}, -2m, 1+2m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right) - 4m \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \left(-\frac{6}{5}m \operatorname{AppellF1}\left[\frac{5}{2}, 1-2m, 1+2m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{3}{10}(1-2m) \operatorname{AppellF1}\left[\frac{5}{2}, 2-2m, 2m, \frac{7}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{3}{10}(1+2m) \operatorname{AppellF1}\left[\frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. -2m, 2+2m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right) \right) \Bigg) / \\
& \quad \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - 4m \left(\operatorname{AppellF1}\left[\frac{3}{2}, 1-2m, \right. \right. \right. \\
& \quad \left. \left. \left. 2m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, -2m, 1+2m, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 409: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x])^m}{(c - c \operatorname{Sin}[e + f x])^2} dx$$

Optimal (type 5, 86 leaves, 4 steps):

$$\frac{1}{3ac^2f} 2^{\frac{1}{2}+m} \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{1}{2}(1-\operatorname{Sin}[e+fx])\right] \operatorname{Sec}[e+fx]^3 (1+\operatorname{Sin}[e+fx])^{\frac{1}{2}-m} (a+a\operatorname{Sin}[e+fx])^{1-m}$$

Result (type 6, 10191 leaves):

$$-\left(\left(\operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^3 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^4 (a+a\operatorname{Sin}[e+fx])^m\right)$$

$$\begin{aligned}
& \left(5 \operatorname{AppellF1} \left[\frac{3}{2}, -2m, 2m, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^6 \right. \\
& \quad \left(-2m \left(\operatorname{AppellF1} \left[\frac{5}{2}, 1-2m, 2m, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) + \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{5}{2}, -2m, 1+2m, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \\
& \quad \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] + 5 \left(-\frac{3}{5} m \operatorname{AppellF1} \left[\frac{5}{2}, 1-2m, 2m, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \\
& \quad \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] - \frac{3}{5} m \operatorname{AppellF1} \left[\frac{5}{2}, -2m, 1+2m, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, \right. \\
& \quad \left. -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \right) - 4m \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \\
& \quad \left(-\frac{10}{7} m \operatorname{AppellF1} \left[\frac{7}{2}, 1-2m, 1+2m, \frac{9}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right. \\
& \quad \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] + \frac{5}{14} (1-2m) \operatorname{AppellF1} \left[\frac{7}{2}, 2-2m, 2m, \frac{9}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \\
& \quad \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] - \frac{5}{14} (1+2m) \operatorname{AppellF1} \left[\frac{7}{2}, -2m, 2+2m, \frac{9}{2}, \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \right) \right) / \\
& \quad \left(5 \operatorname{AppellF1} \left[\frac{3}{2}, -2m, 2m, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] - 4m \left(\operatorname{AppellF1} \left[\frac{5}{2}, 1-2m, \right. \right. \right. \\
& \quad \left. \left. 2m, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \operatorname{AppellF1} \left[\frac{5}{2}, -2m, 1+2m, \right. \right. \right. \\
& \quad \left. \left. \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 410: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^m}{(c - c \sin[e + fx])^3} dx$$

Optimal (type 5, 86 leaves, 4 steps):

$$\frac{1}{5 a^2 c^3 f} 2^{\frac{1}{2}+m} \operatorname{Hypergeometric2F1} \left[-\frac{5}{2}, \frac{1}{2} - m, -\frac{3}{2}, \frac{1}{2} (1 - \sin[e + fx]) \right] \operatorname{Sec}[e + fx]^5 (1 + \sin[e + fx])^{\frac{1}{2}-m} (a + a \sin[e + fx])^{2+m}$$

Result (type 6, 15208 leaves):

$$\begin{aligned}
& 2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \text{AppellF1}\left[\frac{5}{2}, -2m, 1+2m, \right. \\
& \left. \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 - \\
& \left(21 \text{AppellF1}\left[\frac{5}{2}, -2m, 2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^{10} \right. \\
& \left. \left(-2m \left(\text{AppellF1}\left[\frac{7}{2}, 1-2m, 2m, \frac{9}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \text{AppellF1}\left[\frac{7}{2}, -2m, 1+2m, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{9}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \right. \\
& \left. 7 \left(-\frac{5}{7}m \text{AppellF1}\left[\frac{7}{2}, 1-2m, 2m, \frac{9}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{5}{7}m \text{AppellF1}\left[\frac{7}{2}, -2m, 1+2m, \frac{9}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - 4m \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right. \\
& \left. \left(-\frac{14}{9}m \text{AppellF1}\left[\frac{9}{2}, 1-2m, 1+2m, \frac{11}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{7}{18}(1-2m) \text{AppellF1}\left[\frac{9}{2}, 2-2m, 2m, \frac{11}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{7}{18}(1+2m) \text{AppellF1}\left[\frac{9}{2}, -2m, 2+2m, \frac{11}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right) \Bigg) / \\
& \left(7 \text{AppellF1}\left[\frac{5}{2}, -2m, 2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - 4m \left(\text{AppellF1}\left[\frac{7}{2}, 1-2m, \right. \right. \right. \\
& \left. \left. 2m, \frac{9}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \text{AppellF1}\left[\frac{7}{2}, -2m, 1+2m, \right. \right. \\
& \left. \left. \frac{9}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 414:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sin[e + fx])^m}{\sqrt{c - c \sin[e + fx]}} dx$$

Optimal (type 5, 68 leaves, 3 steps):

$$\frac{\text{Cos}[e + f x] \text{Hypergeometric2F1}\left[1, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \text{Sin}[e + f x])\right] (a + a \text{Sin}[e + f x])^m}{f (1 + 2 m) \sqrt{c - c \text{Sin}[e + f x]}}$$

Result (type 6, 3268 leaves):

$$\begin{aligned} & \left(\sqrt{2} (1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \\ & \quad \left. \text{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{1+2m} \text{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right) (a + a \text{Sin}[e + f x])^m \right) / \\ & \left(f (1+2m) \left(2 (1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Cos}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 + \right. \\ & \quad \left(\text{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\ & \quad \left. m \text{AppellF1}\left[2+2m, 1+2m, 1, 3+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right) \\ & \sqrt{c - c \text{Sin}[e + f x]} \left(\text{Cos}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \text{Sin}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right) \left(\left((1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \right. \right. \right. \\ & \quad \left. \left. \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{1+2m} \text{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \text{Csc}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) / \\ & \left(\sqrt{2} (1+2m) \left(2 (1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \\ & \quad \left. \text{Cos}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 + \left(\text{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\ & \quad \left. \left. m \text{AppellF1}\left[2+2m, 1+2m, 1, 3+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) + \\ & \left((1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \\ & \quad \left. \text{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \text{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right) / \\ & \left(\sqrt{2} \left(2 (1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Cos}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 + \right. \right. \\ & \quad \left(\text{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\ & \quad \left. m \text{AppellF1}\left[2+2m, 1+2m, 1, 3+2m, \frac{1}{2} \left(1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \text{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) - \\ & \left(\sqrt{2} (1+m) \text{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{1+2m} \text{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(-\frac{1}{2(2+2m)} (1+2m) \text{AppellF1}\left[2+2m, 2m, 2, 3+2m, \right. \right. \right. \end{aligned}$$

$$\begin{aligned} & \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{4(3+2m)}(1+2m)(2+2m) \operatorname{AppellF1}\left[3+2m, 2+2m, 1, 4+2m, \right. \\ & \left. \frac{1}{2}\left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right) \bigg) \bigg) \bigg) / \\ & \left((1+2m) \left(2(1+m) \operatorname{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2}\left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \right. \\ & \left. \operatorname{Cos}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 + \left(\operatorname{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2}\left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) + m \right. \right. \\ & \left. \left. \operatorname{AppellF1}\left[2+2m, 1+2m, 1, 3+2m, \frac{1}{2}\left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right), 1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Cos}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \bigg) \bigg) \right) \end{aligned}$$

■ **Problem 415: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x])^m}{(c - c \operatorname{Sin}[e + f x])^{3/2}} dx$$

Optimal (type 5, 74 leaves, 3 steps):

$$\frac{\operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1}\left[2, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2}(1 + \operatorname{Sin}[e + f x])\right] (a + a \operatorname{Sin}[e + f x])^m}{2 c f (1 + 2m) \sqrt{c - c \operatorname{Sin}[e + f x]}}$$

Result (type 6, 7559 leaves):

$$\begin{aligned} & - \left(\left(\left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right)^3 (a + a \operatorname{Sin}[e + f x])^m \left(\frac{1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2m} \right. \right. \\ & \left. \left(-\operatorname{AppellF1}\left[1, -2m, 2m, 2, \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] / \left(-m \left(\operatorname{AppellF1}\left[2, 1 - 2m, 2m, 3, \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^2, \right. \right. \right. \right. \\ & \left. \left. - \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \operatorname{AppellF1}\left[2, -2m, 1 + 2m, 3, \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \right) + \right. \\ & \left. \operatorname{AppellF1}\left[1, -2m, 2m, 2, \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \right. \\ & \left(\operatorname{AppellF1}\left[1, -2m, 2m, 2, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) / \\ & \left(\operatorname{AppellF1}\left[1, -2m, 2m, 2, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \\ & \left. m \left(\operatorname{AppellF1}\left[2, 1 - 2m, 2m, 3, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) + \right. \\ & \left. \operatorname{AppellF1}\left[2, -2m, 1 + 2m, 3, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) - \end{aligned}$$

$$1, 3 + 2m, \frac{1}{2} - \frac{1}{2} \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \left(-1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^2\right)\right)\right)\right)$$

■ **Problem 416: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + fx])^m}{(c - c \operatorname{Sin}[e + fx])^{5/2}} dx$$

Optimal (type 5, 74 leaves, 3 steps):

$$\frac{\operatorname{Cos}[e + fx] \operatorname{Hypergeometric2F1}\left[3, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2}(1 + \operatorname{Sin}[e + fx])\right] (a + a \operatorname{Sin}[e + fx])^m}{4 c^2 f (1 + 2m) \sqrt{c - c \operatorname{Sin}[e + fx]}}$$

Result (type 6, 11641 leaves):

$$\begin{aligned} & - \left(\left(\operatorname{Cos}\left[\frac{1}{2}(e + fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + fx)\right] \right)^5 (a + a \operatorname{Sin}[e + fx])^m \right. \\ & \left. \left(\frac{1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right)^{2m} \left(- \left(8 \operatorname{AppellF1}\left[1, -2m, 2m, 2, \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) / \right. \right. \\ & \quad \left(-m \left(\operatorname{AppellF1}\left[2, 1 - 2m, 2m, 3, \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\ & \quad \quad \left. \operatorname{AppellF1}\left[2, -2m, 1 + 2m, 3, \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) + \\ & \quad \left. \operatorname{AppellF1}\left[1, -2m, 2m, 2, \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) + \\ & \left. \left(8 \operatorname{AppellF1}\left[1, -2m, 2m, 2, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) / \right. \\ & \quad \left(\operatorname{AppellF1}\left[1, -2m, 2m, 2, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \right. \\ & \quad \left. m \left(\operatorname{AppellF1}\left[2, 1 - 2m, 2m, 3, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\ & \quad \quad \left. \operatorname{AppellF1}\left[2, -2m, 1 + 2m, 3, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) + \\ & \left. \left(3 \operatorname{AppellF1}\left[2, -2m, 2m, 3, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \right) / \right. \\ & \quad \left. \left(3 \operatorname{AppellF1}\left[2, -2m, 2m, 3, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \right. \right. \end{aligned}$$

$$\begin{aligned}
& 2m \left(\text{AppellF1} \left[3, 1-2m, 2m, 4, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right. \\
& \quad \left. \text{AppellF1} \left[3, -2m, 1+2m, 4, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 - \\
& \left(3 \text{AppellF1} \left[2, -2m, 2m, 3, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) / \\
& \left(-2m \left(\text{AppellF1} \left[3, 1-2m, 2m, 4, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[3, -2m, 1+2m, 4, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 + \right. \\
& \quad \left. 3 \text{AppellF1} \left[2, -2m, 2m, 3, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^4 \right) - \\
& \left(24(1+m) \text{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \\
& \quad \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \left(-1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) / \\
& \left((1+2m) \left(-2(1+m) \text{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. \left(\text{AppellF1} \left[2+2m, 2m, 2, 3+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + m \text{AppellF1} \left[2+2m, \right. \right. \right. \\
& \quad \left. \left. \left. 1+2m, 1, 3+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \left(-1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) \right) \right) / \\
& \left(128 \sqrt{2} f (c - c \sin[e + fx])^{5/2} \left(\cos \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] - \sin \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] \right) \right)^5 \left(\frac{1}{64 \sqrt{2}} m \left(\frac{1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2}{1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2} \right)^{-1+2m} \right. \\
& \quad \left. \left(\frac{\sec \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)}{2 \left(1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)^2} - \frac{\sec \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]}{2 \left(1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)} \right) \right) \\
& \left(- \left(8 \text{AppellF1} \left[1, -2m, 2m, 2, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) / \left(-m \left(\text{AppellF1} \left[2, 1-2m, 2m, 3, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \text{AppellF1} \left[2, -2m, 1+2m, 3, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. fx \right) \right]^2 \right] \right) + \text{AppellF1} \left[1, -2m, 2m, 2, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) + \right.
\end{aligned}$$

$$\begin{aligned} & \left(\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right)^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{4(3+2m)} (1+2m)(2+2m) \operatorname{AppellF1} \left[3+2m, 2+2m, 1, 4+2m, \right. \\ & \left. \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \sec \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) / \\ & \left((1+2m) \left(-2(1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \right. \right. \\ & \left. \left(\operatorname{AppellF1} \left[2+2m, 2m, 2, 3+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + m \operatorname{AppellF1} \left[2+2m, 1+2m, \right. \right. \right. \\ & \left. \left. \left. 1, 3+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \left(-1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) \right) \end{aligned}$$

■ **Problem 417: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 5, 68 leaves, 3 steps):

$$\frac{\cos[e + f x] \operatorname{Hypergeometric2F1} \left[1, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]) \right] (a + a \sin[e + f x])^m}{f(1+2m) \sqrt{c - c \sin[e + f x]}}$$

Result (type 6, 3268 leaves):

$$\begin{aligned} & \left(\sqrt{2} (1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\ & \left. \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{1+2m} \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) (a + a \sin[e + f x])^m \right) / \\ & \left(f(1+2m) \left(2(1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \cos \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 + \right. \\ & \left(\operatorname{AppellF1} \left[2+2m, 2m, 2, 3+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \right. \\ & \left. m \operatorname{AppellF1} \left[2+2m, 1+2m, 1, 3+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) \\ & \sqrt{c - c \sin[e + f x]} \left(\cos \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x \right) \right] - \sin \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x \right) \right] \right) \left(\left((1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \right. \right. \right. \\ & \left. \left. \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{1+2m} \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \operatorname{Csc} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \\ & \left(\sqrt{2} (1+2m) \left(2(1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \end{aligned}$$

$$\begin{aligned}
& m \operatorname{AppellF1}\left[2+2m, 1+2m, 1, 3+2m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] + \\
& 2(1+m) \operatorname{Cos}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(-\frac{1}{2(2+2m)}(1+2m) \operatorname{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), \right. \right. \\
& \quad \left. \left. 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{2(2+2m)}m(1+2m) \operatorname{AppellF1}\left[2+2m, 1+ \right. \right. \\
& \quad \left. \left. 2m, 1, 3+2m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) + \\
& \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\frac{1}{3+2m}(2+2m) \operatorname{AppellF1}\left[3+2m, 2m, 3, 4+2m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), \right. \right. \\
& \quad \left. \left. 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{2(3+2m)}m(2+2m) \operatorname{AppellF1}\left[3+2m, 1+ \right. \right. \\
& \quad \left. \left. 2m, 2, 4+2m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) + \\
& m\left(-\frac{1}{2(3+2m)}(2+2m) \operatorname{AppellF1}\left[3+2m, 1+2m, 2, 4+2m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{4(3+2m)}(1+2m)(2+2m) \operatorname{AppellF1}\left[3+2m, 2+2m, 1, 4+2m, \right. \right. \\
& \quad \left. \left. \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) \right) / \\
& \left((1+2m)\left(2(1+m) \operatorname{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Cos}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 + \left(\operatorname{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + m \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[2+2m, 1+2m, 1, 3+2m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right) \right) \right)
\end{aligned}$$

■ **Problem 418: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(c + c \operatorname{Sin}[e + f x])^m}{\sqrt{a - a \operatorname{Sin}[e + f x]}} dx$$

Optimal (type 5, 68 leaves, 3 steps):

$$\frac{\operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2}(1 + \operatorname{Sin}[e + f x])\right] (c + c \operatorname{Sin}[e + f x])^m}{f(1 + 2m) \sqrt{a - a \operatorname{Sin}[e + f x]}}$$

Result (type 6, 3268 leaves):

$$\left((1+2m) \left(2(1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right. \right. \\ \left. \left. \cos \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 + \left(\operatorname{AppellF1} \left[2+2m, 2m, 2, 3+2m, \frac{1}{2} \left(1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + m \right. \right. \right. \\ \left. \left. \left. \operatorname{AppellF1} \left[2+2m, 1+2m, 1, 3+2m, \frac{1}{2} \left(1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) \right)$$

■ **Problem 421: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{-1-m} dx$$

Optimal (type 3, 46 leaves, 1 step):

$$\frac{\cos[e + fx] (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{-1-m}}{f(1+2m)}$$

Result (type 3, 107 leaves):

$$\frac{1}{cf(1+2m)} 2^{-m} \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^{-1-2m} \left(\cos \left[\frac{1}{2} (e + fx) \right] - \sin \left[\frac{1}{2} (e + fx) \right] \right)^{2m} \\ (a(1 + \sin[e + fx]))^m (c - c \sin[e + fx])^{-m} \sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]$$

■ **Problem 422: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{-m} dx$$

Optimal (type 5, 112 leaves, 4 steps):

$$\frac{1}{f(1+2m)} 2^{\frac{1}{2}-m} c \cos[e + fx] \operatorname{Hypergeometric2F1} \left[\frac{1}{2} (1+2m), \frac{1}{2} (1+2m), \frac{1}{2} (3+2m), \frac{1}{2} (1 + \sin[e + fx]) \right] \\ (1 - \sin[e + fx])^{\frac{1}{2}+m} (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{-1-m}$$

Result (type 6, 3987 leaves):

$$\left(2^{2-m} (-3+2m) \operatorname{AppellF1} \left[\frac{1}{2} - m, -2m, 1, \frac{3}{2} - m, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \left(\cos \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)^{1+2m} \right. \\ \left. \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^{-2m} \left(\cos \left[\frac{1}{2} (e + fx) \right] - \sin \left[\frac{1}{2} (e + fx) \right] \right)^{2m} (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{-m} \right. \\ \left. \left(\cos \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] - \sin \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] \right)^{-2m} \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \left(1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)^{2m} \right) / \\ \left(f(-1+2m) \left((-3+2m) \operatorname{AppellF1} \left[\frac{1}{2} - m, -2m, 1, \frac{3}{2} - m, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right.$$

$$\begin{aligned}
& (-3 + 2m) \left(-\frac{1}{\frac{3}{2} - m} \left(\frac{1}{2} - m \right) m \operatorname{AppellF1} \left[\frac{3}{2} - m, 1 - 2m, 1, \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{2 \left(\frac{3}{2} - m \right)} \left(\frac{1}{2} - m \right) \operatorname{AppellF1} \left[\frac{3}{2} - m, -2m, 2, \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \right] \right) + 2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \\
& \left(-\frac{1}{\frac{5}{2} - m} \left(\frac{3}{2} - m \right) m \operatorname{AppellF1} \left[\frac{5}{2} - m, 1 - 2m, 2, \frac{7}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right. \\
& \quad \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{\frac{5}{2} - m} \left(\frac{3}{2} - m \right) \operatorname{AppellF1} \left[\frac{5}{2} - m, -2m, 3, \frac{7}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \\
& \quad \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] + 2m \left(-\frac{1}{2 \left(\frac{5}{2} - m \right)} \left(\frac{3}{2} - m \right) \operatorname{AppellF1} \left[\frac{5}{2} - m, 1 - 2m, 2, \frac{7}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] + \frac{1}{2 \left(\frac{5}{2} - m \right)} (1 - 2m) \left(\frac{3}{2} - m \right) \operatorname{AppellF1} \left[\frac{5}{2} - m, 2 - 2m, \right. \right. \\
& \quad \left. \left. 1, \frac{7}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \right] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left((-1 + 2m) \left((-3 + 2m) \operatorname{AppellF1} \left[\frac{1}{2} - m, -2m, 1, \frac{3}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \left(2m \operatorname{AppellF1} \left[\frac{3}{2} - m, 1 - 2m, 1, \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \operatorname{AppellF1} \left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3}{2} - m, -2m, 2, \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 423: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{1-m} dx$$

Optimal (type 5, 114 leaves, 4 steps):

$$\frac{1}{f(1+2m)} 2^{\frac{3}{2}-m} c^2 \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2} (-1 + 2m), \frac{1}{2} (1 + 2m), \frac{1}{2} (3 + 2m), \frac{1}{2} (1 + \sin[e + f x]) \right] \\
(1 - \sin[e + f x])^{\frac{1}{2}+m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1-m}$$

Result (type 6, 7365 leaves):

$$\begin{aligned}
& - \left(\left(2^{5-m} (-3+2m) \cos \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^5 \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m} \right. \right. \\
& \quad \sin \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-2m} \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^{-2(1-m)} \\
& \quad (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{1-m} \left(\cos \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x \right) \right] - \sin \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x \right) \right] \right)^{2-2m} \\
& \quad \left(- \left(\left(\text{AppellF1} \left[\frac{1}{2} - m, -2m, 2, \frac{3}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \sec \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \right. \\
& \quad \left((-3+2m) \text{AppellF1} \left[\frac{1}{2} - m, -2m, 2, \frac{3}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) + \\
& \quad 4 \left(m \text{AppellF1} \left[\frac{3}{2} - m, 1-2m, 2, \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \text{AppellF1} \left[\right. \right. \\
& \quad \left. \left. \frac{3}{2} - m, -2m, 3, \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \Bigg) + \\
& \quad \text{AppellF1} \left[\frac{1}{2} - m, -2m, 3, \frac{3}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] / \\
& \quad \left((-3+2m) \text{AppellF1} \left[\frac{1}{2} - m, -2m, 3, \frac{3}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) + \\
& \quad 2 \left(2m \text{AppellF1} \left[\frac{3}{2} - m, 1-2m, 3, \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) + \\
& \quad \left. 3 \text{AppellF1} \left[\frac{3}{2} - m, -2m, 4, \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \Bigg) \Bigg) / \\
& \quad \left(f (-1+2m) \left(-\frac{1}{-1+2m} 2^{5-m} m (-3+2m) \cos \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^5 \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{1+2m} \sin \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1-2m} \right. \right. \\
& \quad \left(- \left(\left(\text{AppellF1} \left[\frac{1}{2} - m, -2m, 2, \frac{3}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \sec \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \right. \\
& \quad \left((-3+2m) \text{AppellF1} \left[\frac{1}{2} - m, -2m, 2, \frac{3}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) + \\
& \quad 4 \left(m \text{AppellF1} \left[\frac{3}{2} - m, 1-2m, 2, \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \text{AppellF1} \left[\frac{3}{2} - m, -2m, 3, \right. \right. \\
& \quad \left. \left. \frac{5}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) + \text{AppellF1} \left[\frac{1}{2} - m, -2m, 3, \frac{3}{2} - m, \right. \\
& \quad \left. \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] / \left((-3+2m) \text{AppellF1} \left[\frac{1}{2} - m, -2m, 3, \frac{3}{2} - m, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& 3, \frac{7}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{2\left(\frac{5}{2} - m\right)} \\
& 3\left(\frac{3}{2} - m\right) \operatorname{AppellF1}\left[\frac{5}{2} - m, -2m, 4, \frac{7}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right. \\
& \left. \left(-e + \frac{\pi}{2} - f x\right)\right] + m\left(-\frac{1}{\frac{5}{2} - m}\left(\frac{3}{2} - m\right) \operatorname{AppellF1}\left[\frac{5}{2} - m, 1 - 2m, 3, \frac{7}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \\
& \left. \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{2\left(\frac{5}{2} - m\right)}(1 - 2m)\left(\frac{3}{2} - m\right) \operatorname{AppellF1}\left[\frac{5}{2} - m, 2 - 2m, 2, \frac{7}{2} - m, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]\right)\right) / \\
& \left((-3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} - m, -2m, 2, \frac{3}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \left. 4\left(m \operatorname{AppellF1}\left[\frac{3}{2} - m, 1 - 2m, 2, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} - m, -2m, 3, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^2 - \\
& \left(\operatorname{AppellF1}\left[\frac{1}{2} - m, -2m, 3, \frac{3}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \\
& \left(\left(2m \operatorname{AppellF1}\left[\frac{3}{2} - m, 1 - 2m, 3, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + 3 \operatorname{AppellF1}\left[\frac{3}{2} - m, -2m, \right. \right. \right. \\
& \left. \left. 4, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \right. \\
& \left. (-3 + 2m) \left(-\frac{1}{\frac{3}{2} - m}\left(\frac{1}{2} - m\right) m \operatorname{AppellF1}\left[\frac{3}{2} - m, 1 - 2m, 3, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\
& \left. \left. \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{2\left(\frac{3}{2} - m\right)} 3\left(\frac{1}{2} - m\right) \operatorname{AppellF1}\left[\frac{3}{2} - m, -2m, 4, \frac{5}{2} - m, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)\right) + \\
& 2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(2m \left(-\frac{1}{2\left(\frac{5}{2} - m\right)} 3\left(\frac{3}{2} - m\right) \operatorname{AppellF1}\left[\frac{5}{2} - m, 1 - 2m, 4, \frac{7}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\left]\operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]+\frac{1}{2\left(\frac{5}{2}-m\right)}(1-2 m)\left(\frac{3}{2}-m\right)\operatorname{AppellF1}\left[\frac{5}{2}-m,\right. \\
& \left.2-2 m, 3, \frac{7}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)+ \\
& 3\left(-\frac{1}{\frac{5}{2}-m}\left(\frac{3}{2}-m\right) m \operatorname{AppellF1}\left[\frac{5}{2}-m, 1-2 m, 4, \frac{7}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{\frac{5}{2}-m} 2\left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m, -2 m, 5, \frac{7}{2}-m,\right. \right. \\
& \left. \left.\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\right)\right)\right) / \\
& \left((-3+2 m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2 m, 3, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+\right. \\
& \left.2\left(2 m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2 m, 3, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+\right. \right. \\
& \left. \left.3 \operatorname{AppellF1}\left[\frac{3}{2}-m, -2 m, 4, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right)\right)\right)\right)\right)\right)\right)\right)
\end{aligned}$$

■ **Problem 424: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a \sin [e+f x])^m (c-c \sin [e+f x])^{2-m} dx$$

Optimal (type 5, 114 leaves, 4 steps):

$$\begin{aligned}
& \frac{1}{f(1+2 m)} 2^{\frac{5}{2}-m} c^3 \cos [e+f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-3+2 m), \frac{1}{2}(1+2 m), \frac{1}{2}(3+2 m), \frac{1}{2}(1+\sin [e+f x])\right] \\
& (1-\sin [e+f x])^{\frac{1}{2}+m}(a+a \sin [e+f x])^m (c-c \sin [e+f x])^{-1-m}
\end{aligned}$$

Result (type 6, 11688 leaves):

$$\begin{aligned}
& -\left(\left(2^{8-3 m}(-3+2 m)\left(\cos \left[\frac{1}{2}(e+f x)\right]-\sin \left[\frac{1}{2}(e+f x)\right]\right)^{-2(2-m)}\right.\right. \\
& \left.\left.(a+a \sin [e+f x])^m (c-c \sin [e+f x])^{2-m}\left(\cos \left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right)\right]-\sin \left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right)\right]\right)^{4-2 m}\right.\right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\left(\frac{\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}\right)^{-2 m}\left(\frac{1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}\right)^{2 m}\right.\right)
\end{aligned}$$

$$2 \left(2 m \operatorname{AppellF1} \left[\frac{3}{2} - m, 1 - 2 m, 5, \frac{5}{2} - m, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \right. \\ \left. 5 \operatorname{AppellF1} \left[\frac{3}{2} - m, -2 m, 6, \frac{5}{2} - m, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^2 \right)$$

- **Problem 430: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e + f x]}{c + d \sin[e + f x]} dx$$

Optimal (type 3, 63 leaves, 4 steps):

$$\frac{a x}{d} - \frac{2 a (c - d) \operatorname{ArcTan} \left[\frac{d + c \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{c^2 - d^2}} \right]}{d \sqrt{c^2 - d^2} f}$$

Result (type 3, 182 leaves):

$$\left(a \left(-2 (c - d) \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{f x}{2} \right] (\cos[e] - i \sin[e]) (d \cos[e + \frac{f x}{2}] + c \sin[\frac{f x}{2}])}{\sqrt{c^2 - d^2} \sqrt{(\cos[e] - i \sin[e])^2}} \right] (\cos[e] - i \sin[e]) + \sqrt{c^2 - d^2} f x \sqrt{(\cos[e] - i \sin[e])^2} \right) \right. \\ \left. (1 + \sin[e + f x]) \right) / \left(d \sqrt{c^2 - d^2} f \sqrt{(\cos[e] - i \sin[e])^2} \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 \right)$$

- **Problem 431: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e + f x]}{(c + d \sin[e + f x])^2} dx$$

Optimal (type 3, 83 leaves, 5 steps):

$$\frac{2 a \operatorname{ArcTan} \left[\frac{d + c \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{c^2 - d^2}} \right]}{(c + d) \sqrt{c^2 - d^2} f} - \frac{a \cos[e + f x]}{(c + d) f (c + d \sin[e + f x])}$$

Result (type 3, 220 leaves):

$$\left(a (1 + \sin[e + f x]) \left(2 \sqrt{c^2 - d^2} \operatorname{Csc}[e] \sqrt{(\cos[e] - i \sin[e])^2} (c \cos[e] + d \sin[f x]) + \right. \right. \\ \left. \left. 4 d \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{f x}{2}\right] (\cos[e] - i \sin[e]) (d \cos\left[e + \frac{f x}{2}\right] + c \sin\left[\frac{f x}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\cos[e] - i \sin[e])^2}} \right] (\cos[e] - i \sin[e]) (c + d \sin[e + f x]) \right) \right) \Bigg) / \\ \left(2 d (c + d) \sqrt{c^2 - d^2} f \sqrt{(\cos[e] - i \sin[e])^2} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 (c + d \sin[e + f x]) \right)$$

■ **Problem 432: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + a \sin[e + f x]}{(c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 134 leaves, 6 steps):

$$\frac{a (2 c - d) \operatorname{ArcTan}\left[\frac{d + c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{c^2 - d^2}}\right]}{(c + d) (c^2 - d^2)^{3/2} f} - \frac{a \cos[e + f x]}{2 (c + d) f (c + d \sin[e + f x])^2} - \frac{a (c - 2 d) \cos[e + f x]}{2 (c - d) (c + d)^2 f (c + d \sin[e + f x])}$$

Result (type 3, 242 leaves):

$$\left(a (1 + \sin[e + f x]) \left(\frac{4 (2 c - d) \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{f x}{2}\right] (\cos[e] - i \sin[e]) (d \cos\left[e + \frac{f x}{2}\right] + c \sin\left[\frac{f x}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\cos[e] - i \sin[e])^2}}\right] (\cos[e] - i \sin[e])}{(c - d) \sqrt{c^2 - d^2} \sqrt{(\cos[e] - i \sin[e])^2}} + \frac{2 (c + d) \operatorname{Csc}[e] (c \cos[e] + d \sin[f x])}{d (c + d \sin[e + f x])^2} \right. \right. \\ \left. \left. \frac{(-4 c + 2 d) \cot[e] + 2 (c - 2 d) \operatorname{Csc}[e] \sin[f x]}{(c - d) (c + d \sin[e + f x])} \right) \right) / \left(4 (c + d)^2 f \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 \right)$$

■ **Problem 433: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e + f x]}{(c + d \sin[e + f x])^4} dx$$

Optimal (type 3, 192 leaves, 7 steps):

$$\frac{a (2 c^2 - 2 c d + d^2) \operatorname{ArcTan}\left[\frac{d+c \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c^2-d^2}}\right]}{(c+d) (c^2-d^2)^{5/2} f} - \frac{a \operatorname{Cos}[e+f x]}{3 (c+d) f (c+d \operatorname{Sin}[e+f x])^3} - \frac{a (2 c-3 d) \operatorname{Cos}[e+f x]}{6 (c-d) (c+d)^2 f (c+d \operatorname{Sin}[e+f x])^2} - \frac{a (c-4 d) (2 c-d) \operatorname{Cos}[e+f x]}{6 (c-d)^2 (c+d)^3 f (c+d \operatorname{Sin}[e+f x])}$$

Result (type 3, 428 leaves):

$$\frac{1}{24 (c-d)^2 (c+d)^3 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^2} a (1 + \operatorname{Sin}[e+f x]) \left(\frac{24 (2 c^2 - 2 c d + d^2) \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{f x}{2}\right] (\operatorname{Cos}[e] - i \operatorname{Sin}[e]) (d \operatorname{Cos}\left[e + \frac{f x}{2}\right] + c \operatorname{Sin}\left[\frac{f x}{2}\right])}{\sqrt{c^2-d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}}\right]}{\sqrt{c^2-d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}} \right) (\operatorname{Cos}[e] - i \operatorname{Sin}[e]) + \frac{1}{d (c+d \operatorname{Sin}[e+f x])^3} (2 c (4 c^4 - 18 c^3 d + 14 c^2 d^2 - 27 c d^3 + 12 d^4) \operatorname{Cot}[e] - d \operatorname{Csc}[e] (3 d (4 c^3 - 16 c^2 d + 6 c d^2 + d^3) \operatorname{Cos}[e+2 f x] - 3 d^2 (2 c^2 - 2 c d + d^2) \operatorname{Cos}[3 e+2 f x] - 24 c^4 \operatorname{Sin}[f x] + 78 c^3 d \operatorname{Sin}[f x] - 24 c^2 d^2 \operatorname{Sin}[f x] + 12 c d^3 \operatorname{Sin}[f x] - 12 d^4 \operatorname{Sin}[f x] + 30 c^3 d \operatorname{Sin}[2 e+f x] - 30 c^2 d^2 \operatorname{Sin}[2 e+f x] + 15 c d^3 \operatorname{Sin}[2 e+f x] + 2 c^2 d^2 \operatorname{Sin}[2 e+3 f x] - 9 c d^3 \operatorname{Sin}[2 e+3 f x] + 4 d^4 \operatorname{Sin}[2 e+3 f x])) \right)$$

■ **Problem 456: Result more than twice size of optimal antiderivative.**

$$\int \frac{c+d \operatorname{Sin}[e+f x]}{a+a \operatorname{Sin}[e+f x]} dx$$

Optimal (type 3, 35 leaves, 2 steps):

$$\frac{dx}{a} - \frac{(c-d) \operatorname{Cos}[e+f x]}{f (a+a \operatorname{Sin}[e+f x])}$$

Result (type 3, 79 leaves):

$$\frac{1}{a f (1 + \operatorname{Sin}[e+f x])} \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right) \left(d (e+f x) \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + (2 c+d (-2+e+f x)) \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)$$

■ **Problem 457: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{a+a \operatorname{Sin}[e+f x]} dx$$

Optimal (type 3, 23 leaves, 1 step):

$$-\frac{\text{Cos}[e + f x]}{f (a + a \text{Sin}[e + f x])}$$

Result (type 3, 48 leaves):

$$\frac{2 \text{Sin}\left[\frac{1}{2}(e + f x)\right] \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)}{f (a + a \text{Sin}[e + f x])}$$

■ **Problem 461: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \text{Sin}[e + f x])^5}{(a + a \text{Sin}[e + f x])^2} dx$$

Optimal (type 3, 260 leaves, 4 steps):

$$\begin{aligned} & \frac{5 (2 c - d) d^2 (2 c^2 - 3 c d + 2 d^2) x}{2 a^2} + \frac{2 d (c^4 + 10 c^3 d - 44 c^2 d^2 + 40 c d^3 - 12 d^4) \text{Cos}[e + f x]}{3 a^2 f} + \\ & \frac{d^2 (2 c^3 + 20 c^2 d - 57 c d^2 + 30 d^3) \text{Cos}[e + f x] \text{Sin}[e + f x]}{6 a^2 f} + \frac{d (c^2 + 10 c d - 12 d^2) \text{Cos}[e + f x] (c + d \text{Sin}[e + f x])^2}{3 a^2 f} - \\ & \frac{(c - d) (c + 10 d) \text{Cos}[e + f x] (c + d \text{Sin}[e + f x])^3}{3 a^2 f (1 + \text{Sin}[e + f x])} - \frac{(c - d) \text{Cos}[e + f x] (c + d \text{Sin}[e + f x])^4}{3 f (a + a \text{Sin}[e + f x])^2} \end{aligned}$$

Result (type 3, 837 leaves):

$$\frac{1}{48 a^2 f (1 + \sin[e + f x])^2} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(3 d (80 c^4 + 80 c^3 d (-4 + 3 e + 3 f x) - 80 c^2 d^2 (-5 + 6 e + 6 f x) + 35 c d^3 (-7 + 12 e + 12 f x) - 4 d^4 (-13 + 30 e + 30 f x)) \cos\left[\frac{1}{2}(e + f x)\right] - (16 c^5 + 160 c^4 d + 80 c^3 d^2 (-10 + 3 e + 3 f x) - 40 c^2 d^3 (-41 + 12 e + 12 f x) - 6 d^5 (-57 + 20 e + 20 f x) + 5 c d^4 (-239 + 84 e + 84 f x)) \cos\left[\frac{3}{2}(e + f x)\right] + 120 c^2 d^3 \cos\left[\frac{5}{2}(e + f x)\right] - 75 c d^4 \cos\left[\frac{5}{2}(e + f x)\right] + 30 d^5 \cos\left[\frac{5}{2}(e + f x)\right] + 15 c d^4 \cos\left[\frac{7}{2}(e + f x)\right] - 3 d^5 \cos\left[\frac{7}{2}(e + f x)\right] - d^5 \cos\left[\frac{9}{2}(e + f x)\right] + 48 c^5 \sin\left[\frac{1}{2}(e + f x)\right] + 240 c^4 d \sin\left[\frac{1}{2}(e + f x)\right] - 1440 c^3 d^2 \sin\left[\frac{1}{2}(e + f x)\right] + 2640 c^2 d^3 \sin\left[\frac{1}{2}(e + f x)\right] - 1905 c d^4 \sin\left[\frac{1}{2}(e + f x)\right] + 516 d^5 \sin\left[\frac{1}{2}(e + f x)\right] + 720 c^3 d^2 e \sin\left[\frac{1}{2}(e + f x)\right] - 1440 c^2 d^3 e \sin\left[\frac{1}{2}(e + f x)\right] + 1260 c d^4 e \sin\left[\frac{1}{2}(e + f x)\right] - 360 d^5 e \sin\left[\frac{1}{2}(e + f x)\right] + 720 c^3 d^2 f x \sin\left[\frac{1}{2}(e + f x)\right] - 1440 c^2 d^3 f x \sin\left[\frac{1}{2}(e + f x)\right] + 1260 c d^4 f x \sin\left[\frac{1}{2}(e + f x)\right] - 360 d^5 f x \sin\left[\frac{1}{2}(e + f x)\right] - 360 c^2 d^3 \sin\left[\frac{3}{2}(e + f x)\right] + 315 c d^4 \sin\left[\frac{3}{2}(e + f x)\right] - 118 d^5 \sin\left[\frac{3}{2}(e + f x)\right] + 240 c^3 d^2 e \sin\left[\frac{3}{2}(e + f x)\right] - 480 c^2 d^3 e \sin\left[\frac{3}{2}(e + f x)\right] + 420 c d^4 e \sin\left[\frac{3}{2}(e + f x)\right] - 120 d^5 e \sin\left[\frac{3}{2}(e + f x)\right] + 240 c^3 d^2 f x \sin\left[\frac{3}{2}(e + f x)\right] - 480 c^2 d^3 f x \sin\left[\frac{3}{2}(e + f x)\right] + 420 c d^4 f x \sin\left[\frac{3}{2}(e + f x)\right] - 120 d^5 f x \sin\left[\frac{3}{2}(e + f x)\right] - 120 c^2 d^3 \sin\left[\frac{5}{2}(e + f x)\right] + 75 c d^4 \sin\left[\frac{5}{2}(e + f x)\right] - 30 d^5 \sin\left[\frac{5}{2}(e + f x)\right] + 15 c d^4 \sin\left[\frac{7}{2}(e + f x)\right] - 3 d^5 \sin\left[\frac{7}{2}(e + f x)\right] + d^5 \sin\left[\frac{9}{2}(e + f x)\right] \right)$$

■ **Problem 464: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^2}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 3, 85 leaves, 3 steps):

$$\frac{d^2 x}{a^2} - \frac{(c - d)(c + 4d) \cos[e + f x]}{3 a^2 f (1 + \sin[e + f x])} - \frac{(c - d) \cos[e + f x] (c + d \sin[e + f x])}{3 f (a + a \sin[e + f x])^2}$$

Result (type 3, 172 leaves):

$$\frac{1}{3 a^2 f (1 + \sin[e + f x])^2} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(2 (c - d)^2 \sin\left[\frac{1}{2}(e + f x)\right] - (c - d)^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) + 2 (c^2 + 4 c d - 5 d^2) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + 3 d^2 (e + f x) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3$$

■ **Problem 470: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c + d \sin[e + f x])^6}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 3, 354 leaves, 5 steps):

$$\begin{aligned} & \frac{d^3 (40 c^3 - 90 c^2 d + 78 c d^2 - 23 d^3) x}{2 a^3} + \frac{2 d (2 c^5 + 18 c^4 d + 107 c^3 d^2 - 472 c^2 d^3 + 456 c d^4 - 136 d^5) \cos[e + f x]}{15 a^3 f} + \\ & \frac{d^2 (4 c^4 + 36 c^3 d + 216 c^2 d^2 - 626 c d^3 + 345 d^4) \cos[e + f x] \sin[e + f x]}{30 a^3 f} + \\ & \frac{d (2 c^3 + 18 c^2 d + 111 c d^2 - 136 d^3) \cos[e + f x] (c + d \sin[e + f x])^2}{15 a^3 f} - \frac{(c - d) (2 c^2 + 18 c d + 115 d^2) \cos[e + f x] (c + d \sin[e + f x])^3}{15 f (a^3 + a^3 \sin[e + f x])} - \\ & \frac{(c - d) (2 c + 13 d) \cos[e + f x] (c + d \sin[e + f x])^4}{15 a f (a + a \sin[e + f x])^2} - \frac{(c - d) \cos[e + f x] (c + d \sin[e + f x])^5}{5 f (a + a \sin[e + f x])^3} \end{aligned}$$

Result (type 3, 560 leaves):

$$\begin{aligned} & \frac{1}{120 a^3 f (1 + \sin[e + f x])^3} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(48 (c - d)^6 \sin\left[\frac{1}{2}(e + f x)\right] - 24 (c - d)^6 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) + \\ & 32 (c - d)^5 (c + 14 d) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 - 16 (c - d)^5 (c + 14 d) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 + \\ & 16 (c - d)^4 (2 c^2 + 26 c d + 197 d^2) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 - 60 d^3 (-40 c^3 + 90 c^2 d - 78 c d^2 + 23 d^3) \\ & (e + f x) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 + 10 d^6 \cos[3(e + f x)] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 - \\ & 45 d^4 (20 c^2 - 24 c d + 9 d^2) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (\cos[e + f x] - i \sin[e + f x]) - \\ & 45 d^4 (20 c^2 - 24 c d + 9 d^2) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (\cos[e + f x] + i \sin[e + f x]) - \\ & 45 i (2 c - d) d^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (\cos[2(e + f x)] - i \sin[2(e + f x)]) + \\ & 45 i (2 c - d) d^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (\cos[2(e + f x)] + i \sin[2(e + f x)]) \end{aligned}$$

■ **Problem 471: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^5}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 3, 278 leaves, 4 steps):

$$\frac{d^3 (20 c^2 - 30 c d + 13 d^2) x}{2 a^3} + \frac{2 d (2 c^4 + 15 c^3 d + 72 c^2 d^2 - 180 c d^3 + 76 d^4) \operatorname{Cos}[e + f x]}{15 a^3 f} +$$

$$\frac{d^2 (4 c^3 + 30 c^2 d + 146 c d^2 - 195 d^3) \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{30 a^3 f} - \frac{(c - d) (2 c^2 + 15 c d + 76 d^2) \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^2}{15 f (a^3 + a^3 \operatorname{Sin}[e + f x])} -$$

$$\frac{(c - d) (2 c + 11 d) \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^3}{15 a f (a + a \operatorname{Sin}[e + f x])^2} - \frac{(c - d) \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^4}{5 f (a + a \operatorname{Sin}[e + f x])^3}$$

Result (type 3, 992 leaves):

$$\frac{1}{480 f (a + a \operatorname{Sin}[e + f x])^3} \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] \right) \left(1200 c^4 d \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + 4800 c^3 d^2 \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - 21600 c^2 d^3 \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \right.$$

$$22500 c d^4 \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - 7560 d^5 \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + 12000 c^2 d^3 (e + f x) \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - 18000 c d^4 (e + f x) \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] +$$

$$7800 d^5 (e + f x) \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - 160 c^5 \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] - 1200 c^4 d \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] - 3200 c^3 d^2 \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] +$$

$$18400 c^2 d^3 \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] - 24300 c d^4 \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] + 9230 d^5 \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] - 6000 c^2 d^3 (e + f x) \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] +$$

$$9000 c d^4 (e + f x) \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] - 3900 d^5 (e + f x) \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] + 1500 c d^4 \operatorname{Cos}\left[\frac{5}{2} (e + f x)\right] - 750 d^5 \operatorname{Cos}\left[\frac{5}{2} (e + f x)\right] -$$

$$1200 c^2 d^3 (e + f x) \operatorname{Cos}\left[\frac{5}{2} (e + f x)\right] + 1800 c d^4 (e + f x) \operatorname{Cos}\left[\frac{5}{2} (e + f x)\right] - 780 d^5 (e + f x) \operatorname{Cos}\left[\frac{5}{2} (e + f x)\right] + 300 c d^4 \operatorname{Cos}\left[\frac{7}{2} (e + f x)\right] -$$

$$105 d^5 \operatorname{Cos}\left[\frac{7}{2} (e + f x)\right] - 15 d^5 \operatorname{Cos}\left[\frac{9}{2} (e + f x)\right] + 320 c^5 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 1200 c^4 d \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 6400 c^3 d^2 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] -$$

$$29600 c^2 d^3 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 35100 c d^4 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 12760 d^5 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 12000 c^2 d^3 (e + f x) \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] -$$

$$18000 c d^4 (e + f x) \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 7800 d^5 (e + f x) \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 2400 c^3 d^2 \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] - 7200 c^2 d^3 \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] +$$

$$4500 c d^4 \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] - 930 d^5 \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] + 6000 c^2 d^3 (e + f x) \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] - 9000 c d^4 (e + f x) \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] +$$

$$3900 d^5 (e + f x) \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] - 32 c^5 \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] - 240 c^4 d \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] - 1120 c^3 d^2 \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] + 5120 c^2 d^3 \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] -$$

$$7260 c d^4 \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] + 2782 d^5 \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] - 1200 c^2 d^3 (e + f x) \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] + 1800 c d^4 (e + f x) \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] -$$

$$780 d^5 (e + f x) \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] + 300 c d^4 \operatorname{Sin}\left[\frac{7}{2} (e + f x)\right] - 105 d^5 \operatorname{Sin}\left[\frac{7}{2} (e + f x)\right] + 15 d^5 \operatorname{Sin}\left[\frac{9}{2} (e + f x)\right] \left. \right)$$

■ **Problem 472: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^4}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 3, 195 leaves, 6 steps):

$$\frac{(4c - 3d) d^3 x}{a^3} + \frac{d^2 (2c^2 + 10cd - 27d^2) \cos[e + fx]}{15a^3 f} - \frac{(c - d)^2 (2c^2 + 12cd + 45d^2) \cos[e + fx]}{15f (a^3 + a^3 \sin[e + fx])} - \frac{(c - d) (2c + 9d) \cos[e + fx] (c + d \sin[e + fx])^2}{15af (a + a \sin[e + fx])^2} - \frac{(c - d) \cos[e + fx] (c + d \sin[e + fx])^3}{5f (a + a \sin[e + fx])^3}$$

Result (type 3, 683 leaves):

$$\frac{1}{120 a^3 f (1 + \sin[e + f x])^3} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(15d (16c^3 + 48c^2 d - 15d^3 (-5 + 4e + 4fx) + 16cd^2 (-9 + 5e + 5fx)) \cos\left[\frac{1}{2}(e + f x)\right] - 5(8c^4 + 48c^3 d + 96c^2 d^2 - 9d^4 (-27 + 10e + 10fx) + 8cd^3 (-46 + 15e + 15fx)) \cos\left[\frac{3}{2}(e + f x)\right] + 75d^4 \cos\left[\frac{5}{2}(e + f x)\right] - 120cd^3 e \cos\left[\frac{5}{2}(e + f x)\right] + 90d^4 e \cos\left[\frac{5}{2}(e + f x)\right] - 120cd^3 f x \cos\left[\frac{5}{2}(e + f x)\right] + 90d^4 f x \cos\left[\frac{5}{2}(e + f x)\right] + 15d^4 \cos\left[\frac{7}{2}(e + f x)\right] + 80c^4 \sin\left[\frac{1}{2}(e + f x)\right] + 240c^3 d \sin\left[\frac{1}{2}(e + f x)\right] + 960c^2 d^2 \sin\left[\frac{1}{2}(e + f x)\right] - 2960cd^3 \sin\left[\frac{1}{2}(e + f x)\right] + 1755d^4 \sin\left[\frac{1}{2}(e + f x)\right] + 1200cd^3 e \sin\left[\frac{1}{2}(e + f x)\right] - 900d^4 e \sin\left[\frac{1}{2}(e + f x)\right] + 1200cd^3 f x \sin\left[\frac{1}{2}(e + f x)\right] - 900d^4 f x \sin\left[\frac{1}{2}(e + f x)\right] + 360c^2 d^2 \sin\left[\frac{3}{2}(e + f x)\right] - 720cd^3 \sin\left[\frac{3}{2}(e + f x)\right] + 225d^4 \sin\left[\frac{3}{2}(e + f x)\right] + 600cd^3 e \sin\left[\frac{3}{2}(e + f x)\right] - 450d^4 e \sin\left[\frac{3}{2}(e + f x)\right] + 600cd^3 f x \sin\left[\frac{3}{2}(e + f x)\right] - 450d^4 f x \sin\left[\frac{3}{2}(e + f x)\right] - 8c^4 \sin\left[\frac{5}{2}(e + f x)\right] - 48c^3 d \sin\left[\frac{5}{2}(e + f x)\right] - 168c^2 d^2 \sin\left[\frac{5}{2}(e + f x)\right] + 512cd^3 \sin\left[\frac{5}{2}(e + f x)\right] - 363d^4 \sin\left[\frac{5}{2}(e + f x)\right] - 120cd^3 e \sin\left[\frac{5}{2}(e + f x)\right] + 90d^4 e \sin\left[\frac{5}{2}(e + f x)\right] - 120cd^3 f x \sin\left[\frac{5}{2}(e + f x)\right] + 90d^4 f x \sin\left[\frac{5}{2}(e + f x)\right] + 15d^4 \sin\left[\frac{7}{2}(e + f x)\right] \right)$$

■ **Problem 473: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^3}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$\frac{d^3 x}{a^3} - \frac{(c-d)^2 (2c+7d) \cos[e+fx]}{15af(a+a\sin[e+fx])^2} - \frac{(c-d)(2c^2+11cd+29d^2) \cos[e+fx]}{15f(a^3+a^3\sin[e+fx])} - \frac{(c-d) \cos[e+fx] (c+d\sin[e+fx])^2}{5f(a+a\sin[e+fx])^3}$$

Result (type 3, 408 leaves):

$$\frac{1}{60a^3f(1+\sin[e+fx])^3} \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(30d(3c^2+6cd+d^2(-9+5e+5fx)) \cos\left[\frac{1}{2}(e+fx)\right] - 5(4c^3+18c^2d+24cd^2+d^3(-46+15e+15fx)) \cos\left[\frac{3}{2}(e+fx)\right] - 15d^3e \cos\left[\frac{5}{2}(e+fx)\right] - 15d^3fx \cos\left[\frac{5}{2}(e+fx)\right] + 40c^3 \sin\left[\frac{1}{2}(e+fx)\right] + 90c^2d \sin\left[\frac{1}{2}(e+fx)\right] + 240cd^2 \sin\left[\frac{1}{2}(e+fx)\right] - 370d^3 \sin\left[\frac{1}{2}(e+fx)\right] + 150d^3e \sin\left[\frac{1}{2}(e+fx)\right] + 150d^3fx \sin\left[\frac{1}{2}(e+fx)\right] + 90cd^2 \sin\left[\frac{3}{2}(e+fx)\right] - 90d^3 \sin\left[\frac{3}{2}(e+fx)\right] + 75d^3e \sin\left[\frac{3}{2}(e+fx)\right] + 75d^3fx \sin\left[\frac{3}{2}(e+fx)\right] - 4c^3 \sin\left[\frac{5}{2}(e+fx)\right] - 18c^2d \sin\left[\frac{5}{2}(e+fx)\right] - 42cd^2 \sin\left[\frac{5}{2}(e+fx)\right] + 64d^3 \sin\left[\frac{5}{2}(e+fx)\right] - 15d^3e \sin\left[\frac{5}{2}(e+fx)\right] - 15d^3fx \sin\left[\frac{5}{2}(e+fx)\right] \right)$$

■ **Problem 479: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a\sin[e+fx])^3 (c+d\sin[e+fx])^3} dx$$

Optimal (type 3, 378 leaves, 9 steps):

$$\frac{d^3 (20c^2+30cd+13d^2) \operatorname{ArcTan}\left[\frac{d+c \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c^2-d^2}}\right]}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}f} - \frac{d(4c^3-30c^2d+146cd^2+195d^3) \cos[e+fx]}{30a^3(c-d)^4(c+d)f(c+d\sin[e+fx])^2} - \frac{\cos[e+fx]}{5(c-d)f(a+a\sin[e+fx])^3(c+d\sin[e+fx])^2} - \frac{(2c-11d) \cos[e+fx]}{15a(c-d)^2f(a+a\sin[e+fx])^2(c+d\sin[e+fx])^2} - \frac{(2c^2-15cd+76d^2) \cos[e+fx]}{15(c-d)^3f(a^3+a^3\sin[e+fx])(c+d\sin[e+fx])^2} - \frac{d(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4) \cos[e+fx]}{30a^3(c-d)^5(c+d)^2f(c+d\sin[e+fx])}$$

Result (type 3, 1066 leaves):

$$\begin{aligned}
& - \left(d^3 (20 c^2 + 30 c d + 13 d^2) \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right] \left(d \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + c \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right)}{\sqrt{c^2 - d^2}} \right] \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right)^6 \right) / \\
& \left((c - d)^5 (c + d)^2 \sqrt{c^2 - d^2} f (a + a \operatorname{Sin}[e + f x])^3 \right) + \\
& \frac{1}{480 (c - d)^5 (c + d)^2 f (a + a \operatorname{Sin}[e + f x])^3 (c + d \operatorname{Sin}[e + f x])^2} \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \\
& \left(-400 c^5 d \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + 3400 c^4 d^2 \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + 19340 c^3 d^3 \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + 30400 c^2 d^4 \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \right. \\
& 19940 c d^5 \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + 4810 d^6 \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] - 160 c^6 \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] + 848 c^5 d \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] - 2400 c^4 d^2 \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] - \\
& 19396 c^3 d^3 \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] - 35280 c^2 d^4 \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] - 24742 c d^5 \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] - 5810 d^6 \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] - \\
& 1260 c^3 d^3 \operatorname{Cos} \left[\frac{5}{2} (e + f x) \right] - 2640 c^2 d^4 \operatorname{Cos} \left[\frac{5}{2} (e + f x) \right] - 2250 c d^5 \operatorname{Cos} \left[\frac{5}{2} (e + f x) \right] - 870 d^6 \operatorname{Cos} \left[\frac{5}{2} (e + f x) \right] + 32 c^5 d \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] - \\
& 200 c^4 d^2 \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] + 836 c^3 d^3 \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] + 4480 c^2 d^4 \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] + 5747 c d^5 \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] + 2200 d^6 \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] - \\
& 135 c d^5 \operatorname{Cos} \left[\frac{9}{2} (e + f x) \right] - 90 d^6 \operatorname{Cos} \left[\frac{9}{2} (e + f x) \right] + 320 c^6 \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] - 1520 c^5 d \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + 4568 c^4 d^2 \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + \\
& 27340 c^3 d^3 \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + 40904 c^2 d^4 \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + 26020 c d^5 \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + 6318 d^6 \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + \\
& 800 c^4 d^2 \operatorname{Sin} \left[\frac{3}{2} (e + f x) \right] + 7500 c^3 d^3 \operatorname{Sin} \left[\frac{3}{2} (e + f x) \right] + 13280 c^2 d^4 \operatorname{Sin} \left[\frac{3}{2} (e + f x) \right] + 9690 c d^5 \operatorname{Sin} \left[\frac{3}{2} (e + f x) \right] + 2750 d^6 \operatorname{Sin} \left[\frac{3}{2} (e + f x) \right] - \\
& 32 c^6 \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] + 80 c^5 d \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] - 32 c^4 d^2 \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] - 6820 c^3 d^3 \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] - 18080 c^2 d^4 \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] - \\
& 15670 c d^5 \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] - 4266 d^6 \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] - 60 c^2 d^4 \operatorname{Sin} \left[\frac{7}{2} (e + f x) \right] + 135 c d^5 \operatorname{Sin} \left[\frac{7}{2} (e + f x) \right] + 60 d^6 \operatorname{Sin} \left[\frac{7}{2} (e + f x) \right] + \\
& \left. 8 c^4 d^2 \operatorname{Sin} \left[\frac{9}{2} (e + f x) \right] - 60 c^3 d^3 \operatorname{Sin} \left[\frac{9}{2} (e + f x) \right] + 284 c^2 d^4 \operatorname{Sin} \left[\frac{9}{2} (e + f x) \right] + 915 c d^5 \operatorname{Sin} \left[\frac{9}{2} (e + f x) \right] + 518 d^6 \operatorname{Sin} \left[\frac{9}{2} (e + f x) \right] \right)
\end{aligned}$$

■ **Problem 482: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sin}[e + f x]) (c + d \operatorname{Sin}[e + f x])^{5/2} dx$$

Optimal (type 4, 290 leaves, 8 steps):

$$\begin{aligned}
& - \frac{2 a (15 c^2 + 56 c d + 25 d^2) \operatorname{Cos}[e + f x] \sqrt{c + d \operatorname{Sin}[e + f x]}}{105 f} - \frac{2 a (5 c + 7 d) \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^{3/2}}{35 f} \\
& \frac{2 a \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^{5/2}}{7 f} + \frac{2 a (15 c^3 + 161 c^2 d + 145 c d^2 + 63 d^3) \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \operatorname{Sin}[e + f x]}}{105 d f \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}}} \\
& \frac{2 a (c^2 - d^2) (15 c^2 + 56 c d + 25 d^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}}}{105 d f \sqrt{c + d \operatorname{Sin}[e + f x]}}
\end{aligned}$$

Result (type 6, 3531 leaves):

$$\begin{aligned}
& a \left(\left(c^3 \operatorname{Sec}[e] (1 + \operatorname{Sin}[e + f x]) \right) \left(- \left(\operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Csc}[e] \left(c + d \operatorname{Cos}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \operatorname{Sin}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)}\right]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right) \operatorname{Cot}[e] \operatorname{Sin}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \right. \\
& \left. \left(\sqrt{1 + \operatorname{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \operatorname{Cot}[e]^2} + d \operatorname{Cos}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2}}{d \sqrt{1 + \operatorname{Cot}[e]^2} - c \operatorname{Csc}[e]}} \right) \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \operatorname{Cot}[e]^2} - d \operatorname{Cos}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2}}{d \sqrt{1 + \operatorname{Cot}[e]^2} + c \operatorname{Csc}[e]}} \sqrt{c + d \operatorname{Cos}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2} \operatorname{Sin}[e]} \right) - \\
& \left. \frac{2 d \operatorname{Sin}[e] \left(c + d \operatorname{Cos}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2} \operatorname{Sin}[e] \right)}{d^2 \operatorname{Cos}[e]^2 + d^2 \operatorname{Sin}[e]^2} - \frac{\operatorname{Cot}[e] \operatorname{Sin}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]]}{\sqrt{1 + \operatorname{Cot}[e]^2}} \right) / \left(7 f \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \\
& \sqrt{c + d \operatorname{Cos}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2} \operatorname{Sin}[e]}
\end{aligned}$$

$$\begin{aligned}
& \left(23 c^2 d \operatorname{Sec}[e] (1 + \sin[e + f x]) \left(\left(\operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)}\right]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \right. \\
& \left. - \frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \\
& \left(\sqrt{1 + \operatorname{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \operatorname{Cot}[e]^2} + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2}}{d \sqrt{1 + \operatorname{Cot}[e]^2} - c \operatorname{Csc}[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \operatorname{Cot}[e]^2} - d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2}}{d \sqrt{1 + \operatorname{Cot}[e]^2} + c \operatorname{Csc}[e]}} \sqrt{c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]} \right) - \\
& \left. \frac{\frac{2 d \sin[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d^2 \cos[e]^2 + d^2 \sin[e]^2} - \frac{\operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]]}{\sqrt{1 + \operatorname{Cot}[e]^2}}}{\sqrt{c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}} \right) / \left(15 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \\
& \left(29 c d^2 \operatorname{Sec}[e] (1 + \sin[e + f x]) \left(\left(\operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)}\right]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \right. \\
& \left. - \frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{1 + \cot[e]^2} \sqrt{\frac{d \sqrt{1 + \cot[e]^2} + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} - c \csc[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \cot[e]^2} - d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} + c \csc[e]}} \sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]} \right) - \\
& \frac{2 d \sin[e] \left(\frac{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}{d^2 \cos[e]^2 + d^2 \sin[e]^2} \right) - \frac{\cot[e] \sin[f x - \text{ArcTan}[\cot[e]]]}{\sqrt{1 + \cot[e]^2}}}{\sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}} \Bigg) / \left(21 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \\
& \left(3 d^3 \sec[e] (1 + \sin[e + f x]) \left(- \left(\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\csc[e] \left(c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e] \right)}{d \sqrt{1 + \cot[e]^2} \left(1 - \frac{c \csc[e]}{d \sqrt{1 + \cot[e]^2}} \right)} \right) \right. \right. \right. \\
& \left. \left. - \frac{\csc[e] \left(c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e] \right)}{d \sqrt{1 + \cot[e]^2} \left(-1 - \frac{c \csc[e]}{d \sqrt{1 + \cot[e]^2}} \right)} \right) \cot[e] \sin[f x - \text{ArcTan}[\cot[e]]] \right) / \\
& \left(\sqrt{1 + \cot[e]^2} \sqrt{\frac{d \sqrt{1 + \cot[e]^2} + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} - c \csc[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \cot[e]^2} - d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} + c \csc[e]}} \sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]} \right) - \\
& \frac{2 d \sin[e] \left(\frac{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}{d^2 \cos[e]^2 + d^2 \sin[e]^2} \right) - \frac{\cot[e] \sin[f x - \text{ArcTan}[\cot[e]]]}{\sqrt{1 + \cot[e]^2}}}{\sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}} \Bigg) / \left(5 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) +
\end{aligned}$$

$$\left((1 + \sin[e + f x]) \sqrt{c + d \sin[e + f x]} \left(- \frac{(180 c^2 + 308 c d + 115 d^2) \cos[e] \cos[f x]}{210 f} + \frac{d^2 \cos[3 e] \cos[3 f x]}{14 f} - \frac{d (15 c + 7 d) \cos[2 f x] \sin[2 e]}{35 f} + \frac{(180 c^2 + 308 c d + 115 d^2) \sin[e] \sin[f x]}{210 f} - \frac{d (15 c + 7 d) \cos[2 e] \sin[2 f x]}{35 f} - \frac{d^2 \sin[3 e] \sin[3 f x]}{14 f} + \frac{2 (15 c^3 + 161 c^2 d + 145 c d^2 + 63 d^3) \tan[e]}{105 d f} \right) \right) /$$

$$\left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 + \frac{1}{7 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}}$$

18

 c^2

$$\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]] \right) \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)}\right],$$

$$-\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]] \right) \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right]$$

$$\frac{\sec[e] \sec[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x])}{\sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \sec[e] + d \sqrt{1 + \tan[e]^2}}}}$$

$$\sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}}}$$

$$\frac{\sqrt{c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}} + 1}{d f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}}$$

$$\begin{aligned}
& 2 c^3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\operatorname{Sec}[e] \left(c + d \operatorname{Cos}[e] \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right] \sqrt{1 + \operatorname{Tan}[e]^2}\right)}{d \sqrt{1 + \operatorname{Tan}[e]^2} \left(1 - \frac{c \operatorname{Sec}[e]}{d \sqrt{1 + \operatorname{Tan}[e]^2}}\right)},\right. \\
& \left. -\frac{\operatorname{Sec}[e] \left(c + d \operatorname{Cos}[e] \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right] \sqrt{1 + \operatorname{Tan}[e]^2}\right)}{d \sqrt{1 + \operatorname{Tan}[e]^2} \left(-1 - \frac{c \operatorname{Sec}[e]}{d \sqrt{1 + \operatorname{Tan}[e]^2}}\right)}\right] \operatorname{Sec}[e] \\
& \operatorname{Sec}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] (1 + \operatorname{Sin}[e + f x]) \sqrt{\frac{d \sqrt{1 + \operatorname{Tan}[e]^2} - d \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \sqrt{1 + \operatorname{Tan}[e]^2}}{c \operatorname{Sec}[e] + d \sqrt{1 + \operatorname{Tan}[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1 + \operatorname{Tan}[e]^2} + d \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \sqrt{1 + \operatorname{Tan}[e]^2}}{-c \operatorname{Sec}[e] + d \sqrt{1 + \operatorname{Tan}[e]^2}}} \\
& \sqrt{c + d \operatorname{Cos}[e] \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \sqrt{1 + \operatorname{Tan}[e]^2}} + \\
& \frac{1}{15 f \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)^2 \sqrt{1 + \operatorname{Tan}[e]^2}} \\
& 34 c d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\operatorname{Sec}[e] \left(c + d \operatorname{Cos}[e] \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right] \sqrt{1 + \operatorname{Tan}[e]^2}\right)}{d \sqrt{1 + \operatorname{Tan}[e]^2} \left(1 - \frac{c \operatorname{Sec}[e]}{d \sqrt{1 + \operatorname{Tan}[e]^2}}\right)},\right. \\
& \left. -\frac{\operatorname{Sec}[e] \left(c + d \operatorname{Cos}[e] \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right] \sqrt{1 + \operatorname{Tan}[e]^2}\right)}{d \sqrt{1 + \operatorname{Tan}[e]^2} \left(-1 - \frac{c \operatorname{Sec}[e]}{d \sqrt{1 + \operatorname{Tan}[e]^2}}\right)}\right] \operatorname{Sec}[e] \\
& \operatorname{Sec}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] (1 + \operatorname{Sin}[e + f x]) \sqrt{\frac{d \sqrt{1 + \operatorname{Tan}[e]^2} - d \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \sqrt{1 + \operatorname{Tan}[e]^2}}{c \operatorname{Sec}[e] + d \sqrt{1 + \operatorname{Tan}[e]^2}}}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}} + \\
& \frac{1}{21 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}} \\
& 10 d^2 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)}, \right. \\
& \left. -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right] \sec[e] \\
& \sec[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x]) \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \sec[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}
\end{aligned}$$

- **Problem 483: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x]) (c + d \sin[e + f x])^{3/2} dx$$

Optimal (type 4, 231 leaves, 7 steps):

$$\frac{2 a (3 c+5 d) \cos [e+f x] \sqrt{c+d \sin [e+f x]}}{15 f}-\frac{2 a \cos [e+f x](c+d \sin [e+f x])^{3 / 2}}{5 f}+$$

$$\frac{2 a\left(3 c^2+20 c d+9 d^2\right) \operatorname{EllipticE}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right), \frac{2 d}{c+d}\right] \sqrt{c+d \sin [e+f x]}}{15 d f \sqrt{\frac{c+d \sin [e+f x]}{c+d}}}$$

$$\frac{2 a(3 c+5 d)\left(c^2-d^2\right) \operatorname{EllipticF}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right), \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \sin [e+f x]}{c+d}}}{15 d f \sqrt{c+d \sin [e+f x]}}$$

Result (type 6, 2625 leaves):

$$a \left(\left(c^2 \sec [e] (1+\sin [e+f x]) \right) \left(\operatorname{AppellF1}\left[-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}, \frac{1}{2},-\frac{\csc [e]\left(c+d \cos [f x-\operatorname{ArcTan}[\cot [e]]\right) \sqrt{1+\cot [e]^2} \sin [e]}\right]}{d \sqrt{1+\cot [e]^2}\left(1-\frac{c \csc [e]}{d \sqrt{1+\cot [e]^2}}\right)}\right), \right. \\ \left. -\frac{\csc [e]\left(c+d \cos [f x-\operatorname{ArcTan}[\cot [e]]\right) \sqrt{1+\cot [e]^2} \sin [e]}\right]}{d \sqrt{1+\cot [e]^2}\left(-1-\frac{c \csc [e]}{d \sqrt{1+\cot [e]^2}}\right)}\right] \cot [e] \sin [f x-\operatorname{ArcTan}[\cot [e]]] \Big/ \\ \left(\sqrt{1+\cot [e]^2} \sqrt{\frac{d \sqrt{1+\cot [e]^2}+d \cos [f x-\operatorname{ArcTan}[\cot [e]]] \sqrt{1+\cot [e]^2}}{d \sqrt{1+\cot [e]^2}-c \csc [e]}} \right. \\ \left. \sqrt{\frac{d \sqrt{1+\cot [e]^2}-d \cos [f x-\operatorname{ArcTan}[\cot [e]]] \sqrt{1+\cot [e]^2}}{d \sqrt{1+\cot [e]^2}+c \csc [e]}} \sqrt{c+d \cos [f x-\operatorname{ArcTan}[\cot [e]]] \sqrt{1+\cot [e]^2} \sin [e]} \right) - \\ \left. \frac{2 d \sin [e]\left(c+d \cos [f x-\operatorname{ArcTan}[\cot [e]]] \sqrt{1+\cot [e]^2} \sin [e]\right)-\frac{\cot [e] \sin [f x-\operatorname{ArcTan}[\cot [e]]]}{\sqrt{1+\cot [e]^2}}}{d^2 \cos [e]^2+d^2 \sin [e]^2}}{\sqrt{c+d \cos [f x-\operatorname{ArcTan}[\cot [e]]] \sqrt{1+\cot [e]^2} \sin [e]}} \right) \Big/ \left(5 f\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^2 \right) +$$

$$\begin{aligned}
& \left(4 c d \operatorname{Sec}[e] (1 + \sin[e + f x]) \left(- \left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \right. \\
& \left. - \frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \\
& \left(\sqrt{1 + \operatorname{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \operatorname{Cot}[e]^2} + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2}}{d \sqrt{1 + \operatorname{Cot}[e]^2} - c \operatorname{Csc}[e]}} \right. \\
& \left. - \sqrt{\frac{d \sqrt{1 + \operatorname{Cot}[e]^2} - d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2}}{d \sqrt{1 + \operatorname{Cot}[e]^2} + c \operatorname{Csc}[e]}} \sqrt{c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]} \right) - \\
& \left. \frac{\frac{2 d \sin[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d^2 \cos[e]^2 + d^2 \sin[e]^2} - \frac{\operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]]}{\sqrt{1 + \operatorname{Cot}[e]^2}}}{\sqrt{c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}} \right) / \left(3 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \\
& \left(3 d^2 \operatorname{Sec}[e] (1 + \sin[e + f x]) \left(- \left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \right. \\
& \left. - \frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{1 + \cot[e]^2} \sqrt{\frac{d \sqrt{1 + \cot[e]^2} + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} - c \csc[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \cot[e]^2} - d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} + c \csc[e]}} \sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]} \right) - \\
& \left. \frac{\frac{2 d \sin[e] \left(c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e] \right)}{d^2 \cos[e]^2 + d^2 \sin[e]^2} - \frac{\cot[e] \sin[f x - \text{ArcTan}[\cot[e]]]}{\sqrt{1 + \cot[e]^2}}}{\sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}} \right) \Bigg/ \left(5 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \\
& \left((1 + \sin[e + f x]) \sqrt{c + d \sin[e + f x]} \left(-\frac{2(6c + 5d) \cos[e] \cos[f x]}{15 f} - \frac{d \cos[2 f x] \sin[2 e]}{5 f} + \right. \right. \\
& \left. \left. \frac{2(6c + 5d) \sin[e] \sin[f x]}{15 f} - \frac{d \cos[2 e] \sin[2 f x]}{5 f} + \frac{2(3c^2 + 20cd + 9d^2) \tan[e]}{15 d f} \right) \right) \Bigg/ \\
& \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 + \frac{1}{5 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}} \\
& 8 \\
& c \\
& \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)}, \right. \\
& \left. -\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right] \\
& \text{Sec}[e] \text{Sec}[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x]) \\
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}}
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}}}{\sqrt{c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}} + 1} \\
& \frac{d f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}}{2 c^2 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)}\right], \\
& \quad - \frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right] \sec[e] \\
& \frac{\sec[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x]) \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \sec[e] + d \sqrt{1 + \tan[e]^2}}}}{\sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}}} \\
& \frac{\sqrt{c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}} + 1} \\
& \frac{3 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}}{2 d \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)}\right],
\end{aligned}$$

$$\begin{aligned}
& - \frac{\text{Sec}[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \text{Sec}[e] \\
& \text{Sec}[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x]) \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}
\end{aligned}$$

- **Problem 484: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x]) \sqrt{c + d \sin[e + f x]} dx$$

Optimal (type 4, 179 leaves, 6 steps):

$$\begin{aligned}
& - \frac{2 a \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{3 f} + \frac{2 a (c + 3 d) \text{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \sin[e + f x]}}{3 d f \sqrt{\frac{c + d \sin[e + f x]}{c + d}}} \\
& \frac{2 a (c^2 - d^2) \text{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{3 d f \sqrt{c + d \sin[e + f x]}}
\end{aligned}$$

Result (type 6, 1736 leaves):

$$a \left(\left(c \text{Sec}[e] (1 + \sin[e + f x]) \right) - \left(\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{\text{Csc}[e] \left(c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e] \right)}{d \sqrt{1 + \cot[e]^2} \left(1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \cot[e]^2}} \right)}\right] \right) \right)$$

$$\begin{aligned}
& \left. - \frac{\text{Csc}[e] \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)}{d \sqrt{1 + \text{Cot}[e]^2} \left(-1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \text{Cot}[e]^2}} \right)} \right] \text{Cot}[e] \sin[f x - \text{ArcTan}[\text{Cot}[e]]] \Bigg/ \\
& \left(\sqrt{1 + \text{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} - c \text{Csc}[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} - d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} + c \text{Csc}[e]}} \sqrt{c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e]} \right) - \\
& \left. \frac{2 d \sin[e] \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right) - \frac{\text{Cot}[e] \sin[f x - \text{ArcTan}[\text{Cot}[e]]]}{\sqrt{1 + \text{Cot}[e]^2}}}{d^2 \cos^2[e] + d^2 \sin^2[e]} \right) \Bigg/ \left(3 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \\
& \left. \sqrt{c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e]} \right) \Bigg/ \\
& \left(d \text{Sec}[e] (1 + \sin[e + f x]) \left(\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Csc}[e] \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)}{d \sqrt{1 + \text{Cot}[e]^2} \left(1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \text{Cot}[e]^2}} \right)} \right] \right. \right. \\
& \left. \left. - \frac{\text{Csc}[e] \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)}{d \sqrt{1 + \text{Cot}[e]^2} \left(-1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \text{Cot}[e]^2}} \right)} \right] \text{Cot}[e] \sin[f x - \text{ArcTan}[\text{Cot}[e]]] \right) \Bigg/ \\
& \left(\sqrt{1 + \text{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} - c \text{Csc}[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} - d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} + c \text{Csc}[e]}} \sqrt{c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e]} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\frac{2 d \sin[e] \left(c+d \cos[f x-\operatorname{ArcTan}[\cot[e]] \right) \sqrt{1+\cot[e]^2} \sin[e]}{d^2 \cos[e]^2+d^2 \sin[e]^2} - \frac{\cot[e] \sin[f x-\operatorname{ArcTan}[\cot[e]]]}{\sqrt{1+\cot[e]^2}}}{\sqrt{c+d \cos[f x-\operatorname{ArcTan}[\cot[e]]]} \sqrt{1+\cot[e]^2} \sin[e]}}{\right)}{f \left(\cos\left[\frac{e}{2}+\frac{f x}{2}\right] + \sin\left[\frac{e}{2}+\frac{f x}{2}\right] \right)^2} + \\
& \frac{(1+\sin[e+f x]) \sqrt{c+d \sin[e+f x]} \left(-\frac{2 \cos[e] \cos[f x]}{3 f} + \frac{2 \sin[e] \sin[f x]}{3 f} + \frac{2(c+3 d) \tan[e]}{3 d f} \right)}{\left(\cos\left[\frac{e}{2}+\frac{f x}{2}\right] + \sin\left[\frac{e}{2}+\frac{f x}{2}\right] \right)^2} + \\
& \frac{1}{3 f \left(\cos\left[\frac{e}{2}+\frac{f x}{2}\right] + \sin\left[\frac{e}{2}+\frac{f x}{2}\right] \right)^2 \sqrt{1+\tan[e]^2}} \\
& 2 \\
& \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\sec[e] \left(c+d \cos[e] \sin[f x+\operatorname{ArcTan}[\tan[e]] \right) \sqrt{1+\tan[e]^2}}{d \sqrt{1+\tan[e]^2} \left(1-\frac{c \sec[e]}{d \sqrt{1+\tan[e]^2}} \right)}\right], \\
& -\frac{\sec[e] \left(c+d \cos[e] \sin[f x+\operatorname{ArcTan}[\tan[e]] \right) \sqrt{1+\tan[e]^2}}{d \sqrt{1+\tan[e]^2} \left(-1-\frac{c \sec[e]}{d \sqrt{1+\tan[e]^2}} \right)} \right] \sec[e] \\
& \sec[f x+\operatorname{ArcTan}[\tan[e]]] (1+\sin[e+f x]) \sqrt{\frac{d \sqrt{1+\tan[e]^2} - d \sin[f x+\operatorname{ArcTan}[\tan[e]]] \sqrt{1+\tan[e]^2}}{c \sec[e] + d \sqrt{1+\tan[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1+\tan[e]^2} + d \sin[f x+\operatorname{ArcTan}[\tan[e]]] \sqrt{1+\tan[e]^2}}{-c \sec[e] + d \sqrt{1+\tan[e]^2}}} \\
& \sqrt{c+d \cos[e] \sin[f x+\operatorname{ArcTan}[\tan[e]]] \sqrt{1+\tan[e]^2}} + \\
& \frac{1}{d f \left(\cos\left[\frac{e}{2}+\frac{f x}{2}\right] + \sin\left[\frac{e}{2}+\frac{f x}{2}\right] \right)^2 \sqrt{1+\tan[e]^2}}
\end{aligned}$$

$$\begin{aligned}
& 2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\operatorname{Sec}[e] \left(c + d \operatorname{Cos}[e] \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right] \sqrt{1 + \operatorname{Tan}[e]^2}\right)}{d \sqrt{1 + \operatorname{Tan}[e]^2} \left(1 - \frac{c \operatorname{Sec}[e]}{d \sqrt{1 + \operatorname{Tan}[e]^2}}\right)}, \right. \\
& \left. -\frac{\operatorname{Sec}[e] \left(c + d \operatorname{Cos}[e] \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]\right] \sqrt{1 + \operatorname{Tan}[e]^2}\right)}{d \sqrt{1 + \operatorname{Tan}[e]^2} \left(-1 - \frac{c \operatorname{Sec}[e]}{d \sqrt{1 + \operatorname{Tan}[e]^2}}\right)}\right] \operatorname{Sec}[e] \\
& \operatorname{Sec}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] (1 + \operatorname{Sin}[e + f x]) \sqrt{\frac{d \sqrt{1 + \operatorname{Tan}[e]^2} - d \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \sqrt{1 + \operatorname{Tan}[e]^2}}{c \operatorname{Sec}[e] + d \sqrt{1 + \operatorname{Tan}[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1 + \operatorname{Tan}[e]^2} + d \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \sqrt{1 + \operatorname{Tan}[e]^2}}{-c \operatorname{Sec}[e] + d \sqrt{1 + \operatorname{Tan}[e]^2}}} \\
& \left. \sqrt{c + d \operatorname{Cos}[e] \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \sqrt{1 + \operatorname{Tan}[e]^2}}\right)
\end{aligned}$$

■ **Problem 485: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \operatorname{Sin}[e + f x]}{\sqrt{c + d \operatorname{Sin}[e + f x]}} dx$$

Optimal (type 4, 138 leaves, 5 steps):

$$\frac{2 a \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c+d}\right] \sqrt{c + d \operatorname{Sin}[e + f x]} - 2 a (c - d) \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c+d}\right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c+d}}}{d f \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c+d}}}$$

Result (type 6, 880 leaves):

$$a \left(\operatorname{Sec}[e] (1 + \operatorname{Sin}[e + f x]) - \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Csc}[e] \left(c + d \operatorname{Cos}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]\right] \sqrt{1 + \operatorname{Cot}[e]^2} \operatorname{Sin}[e]\right)}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}}\right)}, \right. \right.$$

$$\begin{aligned}
& \left. - \frac{\text{Csc}[e] \left(c + d \cos\left[fx - \text{ArcTan}[\text{Cot}[e]]\right] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)}{d \sqrt{1 + \text{Cot}[e]^2} \left(-1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \text{Cot}[e]^2}} \right)} \right] \text{Cot}[e] \sin\left[fx - \text{ArcTan}[\text{Cot}[e]]\right] \Bigg/ \\
& \left(\sqrt{1 + \text{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} + d \cos\left[fx - \text{ArcTan}[\text{Cot}[e]]\right] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} - c \text{Csc}[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} - d \cos\left[fx - \text{ArcTan}[\text{Cot}[e]]\right] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} + c \text{Csc}[e]}} \sqrt{c + d \cos\left[fx - \text{ArcTan}[\text{Cot}[e]]\right] \sqrt{1 + \text{Cot}[e]^2} \sin[e]} \right) - \\
& \left. \frac{2 d \sin[e] \left(c + d \cos\left[fx - \text{ArcTan}[\text{Cot}[e]]\right] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right) - \frac{\text{Cot}[e] \sin\left[fx - \text{ArcTan}[\text{Cot}[e]]\right]}{\sqrt{1 + \text{Cot}[e]^2}}}{d^2 \cos^2[e] + d^2 \sin^2[e]} \right) \Bigg/ \\
& \left. \sqrt{c + d \cos\left[fx - \text{ArcTan}[\text{Cot}[e]]\right] \sqrt{1 + \text{Cot}[e]^2} \sin[e]} \right) \Bigg/ \\
& \left(f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 + \frac{2 (1 + \sin[e + fx]) \sqrt{c + d \sin[e + fx]} \tan[e]}{d f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2} + \right. \\
& \left. \frac{1}{d f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 \sqrt{1 + \tan^2[e]}} \right) \\
& 2 \text{ AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, - \frac{\text{Sec}[e] \left(c + d \cos[e] \sin\left[fx + \text{ArcTan}[\tan[e]]\right] \sqrt{1 + \tan^2[e]} \right)}{d \sqrt{1 + \tan^2[e]} \left(1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan^2[e]}} \right)} \right], \\
& - \frac{\text{Sec}[e] \left(c + d \cos[e] \sin\left[fx + \text{ArcTan}[\tan[e]]\right] \sqrt{1 + \tan^2[e]} \right)}{d \sqrt{1 + \tan^2[e]} \left(-1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan^2[e]}} \right)} \Bigg] \text{Sec}[e] \\
& \text{Sec}\left[fx + \text{ArcTan}[\tan[e]]\right] (1 + \sin[e + fx]) \sqrt{\frac{d \sqrt{1 + \tan^2[e]} - d \sin\left[fx + \text{ArcTan}[\tan[e]]\right] \sqrt{1 + \tan^2[e]}}{c \text{Sec}[e] + d \sqrt{1 + \tan^2[e]}}}
\end{aligned}$$

$$\sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}}$$

$$\sqrt{c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}$$

- **Problem 486: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e + f x]}{(c + d \sin[e + f x])^{3/2}} dx$$

Optimal (type 4, 169 leaves, 6 steps):

$$-\frac{2 a \cos[e + f x]}{(c + d) f \sqrt{c + d \sin[e + f x]}} - \frac{2 a \text{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2d}{c+d}\right] \sqrt{c + d \sin[e + f x]}}{d (c + d) f \sqrt{\frac{c + d \sin[e + f x]}{c + d}}} + \frac{2 a \text{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2d}{c+d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{d f \sqrt{c + d \sin[e + f x]}}$$

Result (type 6, 938 leaves):

$$a \left(\frac{(1 + \sin[e + f x]) \sqrt{c + d \sin[e + f x]} \left(-\frac{2 \csc[e] \sec[e]}{d (c + d) f} + \frac{2 \csc[e] (c \cos[e] + d \sin[f x])}{d (c + d) f (c + d \sin[e + f x])} \right)}{\left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2} - \right.$$

$$\left. \left(\sec[e] (1 + \sin[e + f x]) \left(-\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\csc[e] \left(c + d \cos[f x - \text{ArcTan}[\cot[e]] \right) \sqrt{1 + \cot[e]^2} \sin[e]}{d \sqrt{1 + \cot[e]^2} \left(1 - \frac{c \csc[e]}{d \sqrt{1 + \cot[e]^2}} \right)} \right]}{d \sqrt{1 + \cot[e]^2} \left(-1 - \frac{c \csc[e]}{d \sqrt{1 + \cot[e]^2}} \right)} \right) \cot[e] \sin[f x - \text{ArcTan}[\cot[e]]] \right) \right) /$$

$$\begin{aligned}
& \left(\sqrt{1 + \cot[e]^2} \sqrt{\frac{d \sqrt{1 + \cot[e]^2} + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} - c \csc[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \cot[e]^2} - d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} + c \csc[e]}} \sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]} \right) - \\
& \left. \frac{2 d \sin[e] \left(\frac{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}{d^2 \cos[e]^2 + d^2 \sin[e]^2} \right) - \frac{\cot[e] \sin[f x - \text{ArcTan}[\cot[e]]]}{\sqrt{1 + \cot[e]^2}}}{\sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}} \right) \Bigg/ \\
& \left((c + d) f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \frac{1}{d (c + d) f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}} \\
& 2 \text{ AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)}, \right. \\
& \left. -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right] \sec[e] \\
& \sec[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x]) \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \sec[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}}
\end{aligned}$$

$$\left. \sqrt{c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]]]} \sqrt{1 + \tan[e]^2} \right)$$

■ **Problem 487: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e + fx]}{(c + d \sin[e + fx])^{5/2}} dx$$

Optimal (type 4, 237 leaves, 7 steps):

$$-\frac{2 a \cos[e + f x]}{3 (c + d) f (c + d \sin[e + f x])^{3/2}} - \frac{2 a (c - 3 d) \cos[e + f x]}{3 (c - d) (c + d)^2 f \sqrt{c + d \sin[e + f x]}}$$

$$+ \frac{2 a (c - 3 d) \text{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \sin[e + f x]}}{3 (c - d) d (c + d)^2 f \sqrt{\frac{c + d \sin[e + f x]}{c + d}}} + \frac{2 a \text{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{3 d (c + d) f \sqrt{c + d \sin[e + f x]}}$$

Result (type 6, 1870 leaves):

$$a \left(\left((1 + \sin[e + fx]) \sqrt{c + d \sin[e + fx]} \left(-\frac{2 (c - 3 d) \csc[e] \sec[e]}{3 (c - d) d (c + d)^2 f} + \frac{2 \csc[e] (c \cos[e] + d \sin[fx])}{3 d (c + d) f (c + d \sin[e + fx])^2} - \frac{2 \csc[e] (3 c \cos[e] - d \cos[e] - c \sin[fx] + 3 d \sin[fx])}{3 (c - d) (c + d)^2 f (c + d \sin[e + fx])} \right) \right) / \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 - \left(c \sec[e] (1 + \sin[e + fx]) \left(-\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\csc[e] (c + d \cos[fx - \text{ArcTan}[\cot[e]]) \sqrt{1 + \cot[e]^2} \sin[e]}{d \sqrt{1 + \cot[e]^2} \left(1 - \frac{c \csc[e]}{d \sqrt{1 + \cot[e]^2}}\right)}\right] \right) \right) / \left(\frac{\csc[e] (c + d \cos[fx - \text{ArcTan}[\cot[e]]) \sqrt{1 + \cot[e]^2} \sin[e]}{d \sqrt{1 + \cot[e]^2} \left(-1 - \frac{c \csc[e]}{d \sqrt{1 + \cot[e]^2}}\right)} \right) \cot[e] \sin[fx - \text{ArcTan}[\cot[e]]] \right) \right)$$

$$\begin{aligned}
& \left(\sqrt{1 + \cot[e]^2} \sqrt{\frac{d \sqrt{1 + \cot[e]^2} + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} - c \csc[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \cot[e]^2} - d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} + c \csc[e]}} \sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]} \right) - \\
& \left. \frac{2 d \sin[e] \left(\frac{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}{d^2 \cos[e]^2 + d^2 \sin[e]^2} \right) - \frac{\cot[e] \sin[f x - \text{ArcTan}[\cot[e]]]}{\sqrt{1 + \cot[e]^2}}}{\sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}} \right) \Bigg/ \left(3 (c - d) (c + d)^2 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \\
& \left(d \sec[e] (1 + \sin[e + f x]) \left(- \left(\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\csc[e] (c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e])}{d \sqrt{1 + \cot[e]^2} \left(1 - \frac{c \csc[e]}{d \sqrt{1 + \cot[e]^2}}\right)}\right] \right. \right. \right. \\
& \left. \left. - \frac{\csc[e] (c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e])}{d \sqrt{1 + \cot[e]^2} \left(-1 - \frac{c \csc[e]}{d \sqrt{1 + \cot[e]^2}}\right)} \right] \cot[e] \sin[f x - \text{ArcTan}[\cot[e]]] \right) \Bigg/ \\
& \left(\sqrt{1 + \cot[e]^2} \sqrt{\frac{d \sqrt{1 + \cot[e]^2} + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} - c \csc[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \cot[e]^2} - d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} + c \csc[e]}} \sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]} \right) - \\
& \left. \frac{2 d \sin[e] \left(\frac{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}{d^2 \cos[e]^2 + d^2 \sin[e]^2} \right) - \frac{\cot[e] \sin[f x - \text{ArcTan}[\cot[e]]]}{\sqrt{1 + \cot[e]^2}}}{\sqrt{c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]}} \right) \Bigg/
\end{aligned}$$

$$\left((c-d)(c+d)^2 f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 \right) - \frac{1}{3(c-d)(c+d)^2 f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}}$$

2

$$\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]] \right] \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)}\right],$$

$$-\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]] \right] \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)}$$

$$\text{Sec}[e] \text{Sec}[fx + \text{ArcTan}[\tan[e]]] (1 + \sin[e + fx])$$

$$\sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}}$$

$$\sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}}$$

$$\sqrt{c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}} +$$

1

$$(c-d) d (c+d)^2 f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}$$

$$2 c \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]] \right] \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)}\right],$$

$$-\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]] \right] \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \text{Sec}[e]$$

$$\left. \begin{aligned} & \text{Sec}[f x + \text{ArcTan}[\text{Tan}[e]]] (1 + \text{Sin}[e + f x]) \sqrt{\frac{d \sqrt{1 + \text{Tan}[e]^2} - d \text{Sin}[f x + \text{ArcTan}[\text{Tan}[e]]] \sqrt{1 + \text{Tan}[e]^2}}{c \text{Sec}[e] + d \sqrt{1 + \text{Tan}[e]^2}}} \\ & \sqrt{\frac{d \sqrt{1 + \text{Tan}[e]^2} + d \text{Sin}[f x + \text{ArcTan}[\text{Tan}[e]]] \sqrt{1 + \text{Tan}[e]^2}}{-c \text{Sec}[e] + d \sqrt{1 + \text{Tan}[e]^2}}} \\ & \sqrt{c + d \text{Cos}[e] \text{Sin}[f x + \text{ArcTan}[\text{Tan}[e]]] \sqrt{1 + \text{Tan}[e]^2}} \end{aligned} \right)$$

- **Problem 488: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \text{Sin}[e + f x]}{(c + d \text{Sin}[e + f x])^{7/2}} dx$$

Optimal (type 4, 318 leaves, 8 steps):

$$\begin{aligned} & -\frac{2 a \text{Cos}[e + f x]}{5 (c + d) f (c + d \text{Sin}[e + f x])^{5/2}} - \frac{2 a (3 c - 5 d) \text{Cos}[e + f x]}{15 (c - d) (c + d)^2 f (c + d \text{Sin}[e + f x])^{3/2}} - \frac{2 a (3 c^2 - 20 c d + 9 d^2) \text{Cos}[e + f x]}{15 (c - d)^2 (c + d)^3 f \sqrt{c + d \text{Sin}[e + f x]}} \\ & + \frac{2 a (3 c^2 - 20 c d + 9 d^2) \text{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \text{Sin}[e + f x]}}{15 (c - d)^2 d (c + d)^3 f \sqrt{\frac{c + d \text{Sin}[e + f x]}{c + d}}} + \frac{2 a (3 c - 5 d) \text{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \text{Sin}[e + f x]}{c + d}}}{15 (c - d) d (c + d)^2 f \sqrt{c + d \text{Sin}[e + f x]}} \end{aligned}$$

Result (type 6, 2815 leaves):

$$\begin{aligned} & a \left((1 + \text{Sin}[e + f x]) \sqrt{c + d \text{Sin}[e + f x]} \left(-\frac{2 (3 c^2 - 20 c d + 9 d^2) \text{Csc}[e] \text{Sec}[e]}{15 (c - d)^2 d (c + d)^3 f} + \right. \right. \\ & \left. \left. \frac{2 \text{Csc}[e] (c \text{Cos}[e] + d \text{Sin}[f x])}{5 d (c + d) f (c + d \text{Sin}[e + f x])^3} - \frac{2 \text{Csc}[e] (5 c \text{Cos}[e] - 3 d \text{Cos}[e] - 3 c \text{Sin}[f x] + 5 d \text{Sin}[f x])}{15 (c - d) (c + d)^2 f (c + d \text{Sin}[e + f x])^2} \right. \right. \\ & \left. \left. \left(2 \text{Csc}[e] (15 c^2 \text{Cos}[e] - 12 c d \text{Cos}[e] + 5 d^2 \text{Cos}[e] - 3 c^2 \text{Sin}[f x] + 20 c d \text{Sin}[f x] - 9 d^2 \text{Sin}[f x]) \right) / \right. \right. \\ & \left. \left. (15 (c - d)^2 (c + d)^3 f (c + d \text{Sin}[e + f x])) \right) \right) / \left(\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 - \end{aligned}$$

$$\begin{aligned}
& \left(c^2 \operatorname{Sec}[e] (1 + \sin[e + f x]) \left(- \left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \right. \\
& \left. - \frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \\
& \left(\sqrt{1 + \operatorname{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \operatorname{Cot}[e]^2} + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2}}{d \sqrt{1 + \operatorname{Cot}[e]^2} - c \operatorname{Csc}[e]}} \right. \\
& \left. - \sqrt{\frac{d \sqrt{1 + \operatorname{Cot}[e]^2} - d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2}}{d \sqrt{1 + \operatorname{Cot}[e]^2} + c \operatorname{Csc}[e]}} \sqrt{c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]} \right) - \\
& \left. \frac{\frac{2 d \sin[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d^2 \cos[e]^2 + d^2 \sin[e]^2} - \frac{\operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]]}{\sqrt{1 + \operatorname{Cot}[e]^2}}}{\sqrt{c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}} \right) / \left(5 (c - d)^2 (c + d)^3 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \\
& \left(4 c d \operatorname{Sec}[e] (1 + \sin[e + f x]) \left(- \left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \right. \\
& \left. - \frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(-1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \operatorname{Cot}[e] \sin[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right) / \\
& \left(\sqrt{1 + \operatorname{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \operatorname{Cot}[e]^2} + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \sqrt{1 + \operatorname{Cot}[e]^2}}{d \sqrt{1 + \operatorname{Cot}[e]^2} - c \operatorname{Csc}[e]}} \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{\frac{d\sqrt{1+\cot[e]^2} - d\cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1+\cot[e]^2}}{d\sqrt{1+\cot[e]^2} + c\csc[e]}} \sqrt{c+d\cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1+\cot[e]^2} \sin[e]}}}{\frac{2d\sin[e] \left(\frac{c+d\cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1+\cot[e]^2} \sin[e]}{d^2\cos[e]^2 + d^2\sin[e]^2} \right) - \frac{\cot[e] \sin[fx - \text{ArcTan}[\cot[e]]]}{\sqrt{1+\cot[e]^2}}}{\sqrt{c+d\cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1+\cot[e]^2} \sin[e]}}} \right) \Bigg/ \left(3(c-d)^2 (c+d)^3 f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 \right) - \\
& \left(3d^2 \sec[e] (1 + \sin[e + fx]) \left(- \left(\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\csc[e] \left(c + d \cos[fx - \text{ArcTan}[\cot[e]] \right) \sqrt{1 + \cot[e]^2} \sin[e]}{d\sqrt{1 + \cot[e]^2} \left(1 - \frac{c\csc[e]}{d\sqrt{1 + \cot[e]^2}} \right)} \right. \right. \right. \right. \\
& \left. \left. \left. - \frac{\csc[e] \left(c + d \cos[fx - \text{ArcTan}[\cot[e]] \right) \sqrt{1 + \cot[e]^2} \sin[e]}{d\sqrt{1 + \cot[e]^2} \left(-1 - \frac{c\csc[e]}{d\sqrt{1 + \cot[e]^2}} \right)} \right] \cot[e] \sin[fx - \text{ArcTan}[\cot[e]]] \right) \right) \Bigg/ \\
& \left(\sqrt{1 + \cot[e]^2} \sqrt{\frac{d\sqrt{1 + \cot[e]^2} + d\cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d\sqrt{1 + \cot[e]^2} - c\csc[e]}} \right. \\
& \left. \sqrt{\frac{d\sqrt{1 + \cot[e]^2} - d\cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d\sqrt{1 + \cot[e]^2} + c\csc[e]}} \sqrt{c + d\cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]} \right) - \\
& \left. \frac{2d\sin[e] \left(\frac{c+d\cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1+\cot[e]^2} \sin[e]}{d^2\cos[e]^2 + d^2\sin[e]^2} \right) - \frac{\cot[e] \sin[fx - \text{ArcTan}[\cot[e]]]}{\sqrt{1+\cot[e]^2}}}{\sqrt{c+d\cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1+\cot[e]^2} \sin[e]}}} \right) \Bigg/ \\
& \left(5(c-d)^2 (c+d)^3 f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 \right) - \frac{1}{5(c-d)^2 (c+d)^3 f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}}
\end{aligned}$$

c

$$\begin{aligned}
& \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]] \right] \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)}\right], \\
& -\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]] \right] \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \Big] \\
& \text{Sec}[e] \text{Sec}[fx + \text{ArcTan}[\tan[e]]] (1 + \sin[e + fx]) \\
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}} + \\
& \frac{1}{(c - d)^2 d (c + d)^3 f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}} \\
& 2 c^2 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]] \right] \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)}\right], \\
& -\frac{\text{Sec}[e] \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]] \right] \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \Big] \text{Sec}[e] \\
& \text{Sec}[fx + \text{ArcTan}[\tan[e]]] (1 + \sin[e + fx]) \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}} + \\
& \frac{1}{3 (c - d)^2 (c + d)^3 f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}} \\
& 2 d \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)}, \right. \\
& \left. -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right] \sec[e] \\
& \sec[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x]) \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \sec[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}
\end{aligned}$$

■ **Problem 516: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^{5/2}}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 4, 322 leaves, 8 steps):

$$\begin{aligned}
& - \frac{2(c-d)(c+3d)\cos[e+fx]\sqrt{c+d\sin[e+fx]}}{15af(a+a\sin[e+fx])^2} - \frac{(4c^2+15cd+27d^2)\cos[e+fx]\sqrt{c+d\sin[e+fx]}}{30f(a^3+a^3\sin[e+fx])} \\
& + \frac{(c-d)\cos[e+fx](c+d\sin[e+fx])^{3/2}}{5f(a+a\sin[e+fx])^3} - \frac{(4c^2+15cd+27d^2)\operatorname{EllipticE}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right]\sqrt{c+d\sin[e+fx]}}{30a^3f\sqrt{\frac{c+d\sin[e+fx]}{c+d}}} \\
& + \frac{(c+d)(4c^2+11cd+15d^2)\operatorname{EllipticF}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right]\sqrt{\frac{c+d\sin[e+fx]}{c+d}}}{30a^3f\sqrt{c+d\sin[e+fx]}}
\end{aligned}$$

Result (type 4, 662 leaves):

$$\begin{aligned}
& \frac{1}{f(a+a\sin[e+fx])^3} \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 \left(-\frac{(c-d)^2}{5\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} - \frac{(c-d)(2c+9d)}{15\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2} + \right. \\
& \quad \frac{2(2c^2\sin\left[\frac{1}{2}(e+fx)\right] + 7cd\sin\left[\frac{1}{2}(e+fx)\right] - 9d^2\sin\left[\frac{1}{2}(e+fx)\right])}{15\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^3} + \\
& \quad \left. \frac{2(c^2\sin\left[\frac{1}{2}(e+fx)\right] - 2cd\sin\left[\frac{1}{2}(e+fx)\right] + d^2\sin\left[\frac{1}{2}(e+fx)\right])}{5\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5} + \frac{4c^2\sin\left[\frac{1}{2}(e+fx)\right] + 15cd\sin\left[\frac{1}{2}(e+fx)\right] + 27d^2\sin\left[\frac{1}{2}(e+fx)\right]}{15\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)} \right) \\
& \sqrt{c+d\sin[e+fx]} - \frac{1}{60f(a+a\sin[e+fx])^3} d \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 \\
& \left(-\frac{2(cd-15d^2)\operatorname{EllipticF}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2d}{c+d}\right]\sqrt{\frac{c+d\sin[e+fx]}{c+d}}}{\sqrt{c+d\sin[e+fx]}} + \frac{2(4c^2+15cd+27d^2)\cos[e+fx]^2\sqrt{c+d\sin[e+fx]}}{d(1-\sin[e+fx])^2} - \right. \\
& \quad \left. \frac{1}{d(4c^2+15cd+27d^2)} \left(\frac{2(c+d)\operatorname{EllipticE}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2d}{c+d}\right]\sqrt{\frac{c+d\sin[e+fx]}{c+d}}}{\sqrt{c+d\sin[e+fx]}} - \frac{2c\operatorname{EllipticF}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2d}{c+d}\right]\sqrt{\frac{c+d\sin[e+fx]}{c+d}}}{\sqrt{c+d\sin[e+fx]}} \right) \right)
\end{aligned}$$

- **Problem 525: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \sin[e + f x]} \, dx$$

Optimal (type 3, 26 leaves, 1 step):

$$-\frac{2 a \cos[e + f x]}{f \sqrt{a + a \sin[e + f x]}}$$

Result (type 3, 65 leaves):

$$\frac{2 \left(-\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \sqrt{a(1 + \sin[e + f x])}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)}$$

- **Problem 526: Result is not expressed in closed-form.**

$$\int \frac{\sqrt{a + a \sin[e + f x]}}{c + d \sin[e + f x]} \, dx$$

Optimal (type 3, 61 leaves, 2 steps):

$$-\frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{d} \sqrt{c + d} f}$$

Result (type 7, 657 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{d} \sqrt{c+d} f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)} \left(\frac{1}{8} + \frac{i}{8} \right) \left(\text{RootSum}\left[-d+2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, \right. \right. \\
& \frac{1}{d-ic e^{ie} \#1^2} \left((1+i) d \sqrt{e^{-ie}} f x - (2-2i) d \sqrt{e^{-ie}} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] - i \sqrt{d} \sqrt{c+d} f x \#1 + 2 \sqrt{d} \sqrt{c+d} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1 + \right. \\
& \left. \frac{(1-i) c f x \#1^2}{\sqrt{e^{-ie}}} + \frac{(2+2i) c \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^2}{\sqrt{e^{-ie}}} - \sqrt{d} \sqrt{c+d} e^{ie} f x \#1^3 - 2i \sqrt{d} \sqrt{c+d} e^{ie} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^3 \right) \& \left. - \right. \\
& \left. i \text{RootSum}\left[-d+2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, \frac{1}{d-ic e^{ie} \#1^2} \left((1-i) d \sqrt{e^{-ie}} f x + (2+2i) d \sqrt{e^{-ie}} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] + \right. \right. \\
& \left. \left. \sqrt{d} \sqrt{c+d} f x \#1 + 2i \sqrt{d} \sqrt{c+d} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1 - \frac{(1+i) c f x \#1^2}{\sqrt{e^{-ie}}} + \frac{(2-2i) c \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^2}{\sqrt{e^{-ie}}} - \right. \right. \\
& \left. \left. i \sqrt{d} \sqrt{c+d} e^{ie} f x \#1^3 + 2 \sqrt{d} \sqrt{c+d} e^{ie} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^3 \right) \& \right) \left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right] \right) \sqrt{a(1+\sin[e+fx])}
\end{aligned}$$

■ **Problem 527: Result is not expressed in closed-form.**

$$\int \frac{\sqrt{a+a \sin[e+fx]}}{(c+d \sin[e+fx])^2} dx$$

Optimal (type 3, 105 leaves, 3 steps):

$$-\frac{\sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{c+d} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{d} (c+d)^{3/2} f} - \frac{a \cos[e+fx]}{(c+d) f \sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])}$$

Result (type 7, 871 leaves):

$$\begin{aligned}
& \frac{1}{\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]} \\
& \left(\frac{1}{4} + \frac{i}{4}\right) \sqrt{a(1 + \sin[e+fx])} \left(\left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right] \right) \left((-1+i)x \cos[e] + 1 / (4f) \operatorname{RootSum}\left[-d + 2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, \right. \right. \right. \\
& \left. \left. \left. 1 / (d - ic e^{ie} \#1^2) \left((1+i)d \sqrt{e^{-ie}} f x - (2-2i)d \sqrt{e^{-ie}} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] - i \sqrt{d} \sqrt{c+d} f x \#1 + 2 \sqrt{d} \sqrt{c+d} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1 + \right. \right. \right. \\
& \left. \left. \left. \frac{(1-i) c f x \#1^2}{\sqrt{e^{-ie}}} + \frac{(2+2i) c \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^2}{\sqrt{e^{-ie}}} - \sqrt{d} \sqrt{c+d} e^{ie} f x \#1^3 - 2i \sqrt{d} \sqrt{c+d} e^{ie} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^3 \right) \& \right) \\
& \left. \left. \left. \left(\cos[e] + i(-1 + \sin[e]) \right) \sqrt{\cos[e] - i \sin[e]} + (1+i)x \sin[e] \right) \right) \right) / \\
& \left(\sqrt{d} (c+d)^{3/2} (\cos[e] + i(-1 + \sin[e])) \sqrt{\cos[e] - i \sin[e]} \right) + \left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right] \right) \\
& \left((1-i)x \cos[e] - (1+i)x \sin[e] + 1 / (4f) \operatorname{RootSum}\left[-d + 2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, 1 / (d - ic e^{ie} \#1^2) \left((1-i)d \sqrt{e^{-ie}} f x + (2+2i)d \right. \right. \right. \\
& \left. \left. \left. \sqrt{e^{-ie}} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] + \sqrt{d} \sqrt{c+d} f x \#1 + 2i \sqrt{d} \sqrt{c+d} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1 - \frac{(1+i) c f x \#1^2}{\sqrt{e^{-ie}}} + \frac{(2-2i) c \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^2}{\sqrt{e^{-ie}}} - \right. \right. \\
& \left. \left. \left. i \sqrt{d} \sqrt{c+d} e^{ie} f x \#1^3 + 2 \sqrt{d} \sqrt{c+d} e^{ie} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^3 \right) \& \right) \sqrt{\cos[e] - i \sin[e]} (-1 - i \cos[e] + \sin[e]) \right) \right) / \\
& \left(\sqrt{d} (c+d)^{3/2} (\cos[e] + i(-1 + \sin[e])) \sqrt{\cos[e] - i \sin[e]} \right) - \frac{(2-2i) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)}{(c+d) f (c+d \sin[e+fx])} \right)
\end{aligned}$$

■ **Problem 528: Result is not expressed in closed-form.**

$$\int \frac{\sqrt{a + a \sin[e+fx]}}{(c+d \sin[e+fx])^3} dx$$

Optimal (type 3, 154 leaves, 4 steps):

$$\frac{3 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{c+d} \sqrt{a+a \sin[e+fx]}}\right]}{4 \sqrt{d} (c+d)^{5/2} f} - \frac{a \cos[e+fx]}{2 (c+d) f \sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])^2} - \frac{3 a \cos[e+fx]}{4 (c+d)^2 f \sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])}$$

Result (type 7, 920 leaves):

$$\frac{1}{\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]}$$

$$\left(\frac{1}{16} + \frac{i}{16}\right) \sqrt{a(1+\sin[e+fx])} \left(\left(3 \left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right]\right) \left((-1+i) \times \cos[e] + 1 / (4f) \operatorname{RootSum}\left[-d+2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, \right. \right. \right. \right.$$

$$\left. \left. \left. 1 / (d-ic e^{ie} \#1^2) \left((1+i) d \sqrt{e^{-ie}} f x - (2-2i) d \sqrt{e^{-ie}} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] - i \sqrt{d} \sqrt{c+d} f x \#1 + 2 \sqrt{d} \sqrt{c+d} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1 + \right. \right. \right.$$

$$\left. \left. \frac{(1-i) c f x \#1^2}{\sqrt{e^{-ie}}} + \frac{(2+2i) c \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^2}{\sqrt{e^{-ie}}} - \sqrt{d} \sqrt{c+d} e^{ie} f x \#1^3 - 2i \sqrt{d} \sqrt{c+d} e^{ie} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^3 \right) \& \right) (\cos[e] +$$

$$i(-1+\sin[e])) \sqrt{\cos[e] - i \sin[e]} + (1+i) \times \sin[e] \left. \right) \left. \right) / \left(\sqrt{d} (c+d)^{5/2} (\cos[e] + i(-1+\sin[e])) \sqrt{\cos[e] - i \sin[e]} \right) +$$

$$\left(3 \left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right]\right) \left((1-i) \times \cos[e] - (1+i) \times \sin[e] + 1 / (4f) \operatorname{RootSum}\left[-d+2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, \right. \right. \right.$$

$$\left. \left. \left. 1 / (d-ic e^{ie} \#1^2) \left((1-i) d \sqrt{e^{-ie}} f x + (2+2i) d \sqrt{e^{-ie}} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] + \sqrt{d} \sqrt{c+d} f x \#1 + 2i \sqrt{d} \sqrt{c+d} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1 - \right. \right. \right.$$

$$\left. \left. \frac{(1+i) c f x \#1^2}{\sqrt{e^{-ie}}} + \frac{(2-2i) c \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^2}{\sqrt{e^{-ie}}} - i \sqrt{d} \sqrt{c+d} e^{ie} f x \#1^3 + 2 \sqrt{d} \sqrt{c+d} e^{ie} \operatorname{Log}\left[e^{\frac{ifx}{2}} - \#1\right] \#1^3 \right) \& \right) \left. \right) \left. \right) / \left(\sqrt{d} (c+d)^{5/2} (\cos[e] + i(-1+\sin[e])) \sqrt{\cos[e] - i \sin[e]} \right) -$$

$$\frac{(4-4i) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)}{(c+d) f (c+d \sin[e+fx])^2} - \frac{(6-6i) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)}{(c+d)^2 f (c+d \sin[e+fx])}$$

■ **Problem 533: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^{3/2}}{c + d \sin[e + f x]} dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{2 a^{3/2} (c - d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c+d} \sqrt{a+a \sin[e + f x]}}\right]}{d^{3/2} \sqrt{c+d} f} - \frac{2 a^2 \cos[e + f x]}{d f \sqrt{a+a \sin[e + f x]}}$$

Result (type 3, 233 leaves):

$$\frac{1}{d^{3/2} \sqrt{c+d} f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3} \left(-2 \sqrt{d} \sqrt{c+d} \cos\left[\frac{1}{2}(e + f x)\right] + (c - d) \left(\log\left[-\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 \left(c + d + \sqrt{d} \sqrt{c+d} \cos\left[\frac{1}{2}(e + f x)\right] - \sqrt{d} \sqrt{c+d} \sin\left[\frac{1}{2}(e + f x)\right]\right)\right] - \log\left[(c + d) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 + \sqrt{d} \sqrt{c+d} \left(-1 + 2 \tan\left[\frac{1}{4}(e + f x)\right] + \tan\left[\frac{1}{4}(e + f x)\right]^2\right)\right] \right) + 2 \sqrt{d} \sqrt{c+d} \sin\left[\frac{1}{2}(e + f x)\right] \left(a (1 + \sin[e + f x])\right)^{3/2}$$

■ **Problem 534: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^{3/2}}{(c + d \sin[e + f x])^2} dx$$

Optimal (type 3, 119 leaves, 4 steps):

$$-\frac{a^{3/2} (c + 3 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c+d} \sqrt{a+a \sin[e + f x]}}\right]}{d^{3/2} (c + d)^{3/2} f} + \frac{a^2 (c - d) \cos[e + f x]}{d (c + d) f \sqrt{a+a \sin[e + f x]} (c + d \sin[e + f x])}$$

Result (type 3, 268 leaves):

$$-\left(\left((a (1 + \sin[e + f x]))^{3/2} \left(-2 (c - d) \sqrt{d} \sqrt{c+d} \cos\left[\frac{1}{2}(e + f x)\right] + 2 (c - d) \sqrt{d} \sqrt{c+d} \sin\left[\frac{1}{2}(e + f x)\right] + (c + 3 d) \left(\log\left[-\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 \left(c + d + \sqrt{d} \sqrt{c+d} \cos\left[\frac{1}{2}(e + f x)\right] - \sqrt{d} \sqrt{c+d} \sin\left[\frac{1}{2}(e + f x)\right]\right)\right] - \log\left[(c + d) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 + \sqrt{d} \sqrt{c+d} \left(-1 + 2 \tan\left[\frac{1}{4}(e + f x)\right] + \tan\left[\frac{1}{4}(e + f x)\right]^2\right)\right] \right) (c + d \sin[e + f x]) \right) \right) / \left(2 d^{3/2} (c + d)^{3/2} f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 (c + d \sin[e + f x]) \right)$$

■ **Problem 536: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])^3 dx$$

Optimal (type 3, 328 leaves, 6 steps):

$$\begin{aligned} & - \frac{4 a^3 (c + d) (15 c^2 + 10 c d + 7 d^2) (3 c^2 - 38 c d + 355 d^2) \cos[e + f x]}{3465 d^2 f \sqrt{a + a \sin[e + f x]}} - \frac{8 a^2 (5 c - d) (c + d) (3 c^2 - 38 c d + 355 d^2) \cos[e + f x] \sqrt{a + a \sin[e + f x]}}{3465 d f} \\ & + \frac{4 a (c + d) (3 c^2 - 38 c d + 355 d^2) \cos[e + f x] (a + a \sin[e + f x])^{3/2}}{1155 f} - \frac{2 a^3 (3 c^2 - 38 c d + 355 d^2) \cos[e + f x] (c + d \sin[e + f x])^3}{693 d^2 f \sqrt{a + a \sin[e + f x]}} \\ & + \frac{2 a^3 (3 c - 23 d) \cos[e + f x] (c + d \sin[e + f x])^4}{99 d^2 f \sqrt{a + a \sin[e + f x]}} - \frac{2 a^2 \cos[e + f x] \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^4}{11 d f} \end{aligned}$$

Result (type 3, 845 leaves):

$$\begin{aligned} & - \frac{(40 c^3 + 90 c^2 d + 78 c d^2 + 23 d^3) \cos\left[\frac{1}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{8 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} - \\ & + \frac{(20 c^3 + 66 c^2 d + 60 c d^2 + 19 d^3) \cos\left[\frac{3}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{24 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} + \\ & + \frac{(8 c^3 + 60 c^2 d + 72 c d^2 + 25 d^3) \cos\left[\frac{5}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{80 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} + \\ & - \frac{d (12 c^2 + 30 c d + 13 d^2) \cos\left[\frac{7}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{112 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} - \frac{d^2 (6 c + 5 d) \cos\left[\frac{9}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{144 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} - \\ & + \frac{d^3 \cos\left[\frac{11}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{176 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} + \frac{(40 c^3 + 90 c^2 d + 78 c d^2 + 23 d^3) \sin\left[\frac{1}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{8 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} - \\ & - \frac{(20 c^3 + 66 c^2 d + 60 c d^2 + 19 d^3) (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{3}{2}(e + f x)\right]}{24 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} - \\ & + \frac{(8 c^3 + 60 c^2 d + 72 c d^2 + 25 d^3) (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{5}{2}(e + f x)\right]}{80 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} + \frac{d (12 c^2 + 30 c d + 13 d^2) (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{7}{2}(e + f x)\right]}{112 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} + \\ & - \frac{d^2 (6 c + 5 d) (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{9}{2}(e + f x)\right]}{144 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} - \frac{d^3 (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{11}{2}(e + f x)\right]}{176 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} \end{aligned}$$

■ **Problem 540: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^{5/2}}{c + d \sin[e + f x]} dx$$

Optimal (type 3, 142 leaves, 4 steps):

$$-\frac{2 a^{5/2} (c-d)^2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{c+d} \sqrt{a+a \sin[e+fx]}}\right]}{d^{5/2} \sqrt{c+d} f} + \frac{2 a^3 (3c-7d) \cos[e+fx]}{3 d^2 f \sqrt{a+a \sin[e+fx]}} - \frac{2 a^2 \cos[e+fx] \sqrt{a+a \sin[e+fx]}}{3 d f}$$

Result (type 3, 330 leaves):

$$\frac{1}{6 d^{5/2} f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5} \\ (a(1 + \sin[e+fx]))^{5/2} \left(6(2c-5d)\sqrt{d} \cos\left[\frac{1}{2}(e+fx)\right] - 2d^{3/2} \cos\left[\frac{3}{2}(e+fx)\right] - \frac{1}{\sqrt{c+d}} 3(c-d)^2 \right. \\ \left. \left(e+fx - 2 \log\left[\sec\left[\frac{1}{4}(e+fx)\right]^2\right] + 2 \log\left[-\sec\left[\frac{1}{4}(e+fx)\right]^2\right] \left(c+d + \sqrt{d} \sqrt{c+d} \cos\left[\frac{1}{2}(e+fx)\right] - \sqrt{d} \sqrt{c+d} \sin\left[\frac{1}{2}(e+fx)\right]\right)\right)\right) + \frac{1}{\sqrt{c+d}} \\ 3(c-d)^2 \left(e+fx - 2 \log\left[\sec\left[\frac{1}{4}(e+fx)\right]^2\right] + 2 \log\left[(c+d) \sec\left[\frac{1}{4}(e+fx)\right]^2 + \sqrt{d} \sqrt{c+d} \left(-1 + 2 \tan\left[\frac{1}{4}(e+fx)\right] + \tan\left[\frac{1}{4}(e+fx)\right]^2\right)\right]\right) + \\ 6\sqrt{d}(-2c+5d) \sin\left[\frac{1}{2}(e+fx)\right] - 2d^{3/2} \sin\left[\frac{3}{2}(e+fx)\right]$$

■ **Problem 541: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^{5/2}}{(c + d \sin[e + f x])^2} dx$$

Optimal (type 3, 166 leaves, 4 steps):

$$\frac{a^{5/2} (c-d) (3c+5d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{c+d} \sqrt{a+a \sin[e+fx]}}\right]}{d^{5/2} (c+d)^{3/2} f} - \frac{a^3 (3c+d) \cos[e+fx]}{d^2 (c+d) f \sqrt{a+a \sin[e+fx]}} + \frac{a^2 (c-d) \cos[e+fx] \sqrt{a+a \sin[e+fx]}}{d (c+d) f (c+d \sin[e+fx])}$$

Result (type 3, 350 leaves):

$$\frac{1}{4 d^{5/2} f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^5}$$

$$\left(a (1 + \sin[e + f x]) \right)^{5/2} \left(-8 \sqrt{d} \cos \left[\frac{1}{2} (e + f x) \right] + \frac{1}{(c+d)^{3/2}} (3 c^2 + 2 c d - 5 d^2) \left(e + f x - 2 \log \left[\sec \left[\frac{1}{4} (e + f x) \right]^2 \right] + \right. \right.$$

$$\left. \left. 2 \log \left[-\sec \left[\frac{1}{4} (e + f x) \right]^2 \right] \left(c + d + \sqrt{d} \sqrt{c+d} \cos \left[\frac{1}{2} (e + f x) \right] - \sqrt{d} \sqrt{c+d} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) + \frac{1}{(c+d)^{3/2}} (-3 c^2 - 2 c d + 5 d^2) \right.$$

$$\left. \left(e + f x - 2 \log \left[\sec \left[\frac{1}{4} (e + f x) \right]^2 \right] + 2 \log \left[(c+d) \sec \left[\frac{1}{4} (e + f x) \right]^2 + \sqrt{d} \sqrt{c+d} \left(-1 + 2 \tan \left[\frac{1}{4} (e + f x) \right] + \tan \left[\frac{1}{4} (e + f x) \right]^2 \right) \right] \right) + \right.$$

$$\left. 8 \sqrt{d} \sin \left[\frac{1}{2} (e + f x) \right] - \frac{4 (c-d)^2 \sqrt{d} \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)}{(c+d) (c+d \sin[e + f x])} \right)$$

■ **Problem 543: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c + d \sin[e + f x])^3}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 178 leaves, 6 steps):

$$\frac{\sqrt{2} (c-d)^3 \operatorname{ArcTanh} \left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}} \right]}{\sqrt{a} f} - \frac{4 d (21 c^2 - 12 c d + 7 d^2) \cos[e + f x]}{15 f \sqrt{a + a \sin[e + f x]}}$$

$$\frac{2 (9 c - d) d^2 \cos[e + f x] \sqrt{a + a \sin[e + f x]}}{15 a f} - \frac{2 d \cos[e + f x] (c + d \sin[e + f x])^2}{5 f \sqrt{a + a \sin[e + f x]}}$$

Result (type 3, 155 leaves):

$$- \left(\left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \right.$$

$$\left. \left((-60 - 60 i) (-1)^{3/4} (c-d)^3 \operatorname{ArcTanh} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \left(-1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] - 2 d \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) \right.$$

$$\left. \left. \left(-90 c^2 + 30 c d - 29 d^2 + 3 d^2 \cos[2 (e + f x)] - 2 (15 c - d) d \sin[e + f x] \right) \right) \right) / \left(30 f \sqrt{a (1 + \sin[e + f x])} \right)$$

■ **Problem 544: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c + d \sin[e + f x])^2}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 123 leaves, 4 steps):

$$-\frac{\sqrt{2} (c-d)^2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{a} f} - \frac{4 (3c-d) d \cos[e+fx]}{3 f \sqrt{a+a \sin[e+fx]}} - \frac{2 d^2 \cos[e+fx] \sqrt{a+a \sin[e+fx]}}{3 a f}$$

Result (type 3, 125 leaves):

$$-\frac{1}{3 f \sqrt{a (1 + \sin[e+fx])}} 2 \left(\cos\left[\frac{1}{2} (e+fx)\right] + \sin\left[\frac{1}{2} (e+fx)\right] \right) \\ \left((-3-3i) (-1)^{3/4} (c-d)^2 \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e+fx)\right]\right)\right] + d \left(\cos\left[\frac{1}{2} (e+fx)\right] - \sin\left[\frac{1}{2} (e+fx)\right] \right) (6c-d+d \sin[e+fx]) \right)$$

■ **Problem 545: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{c+d \sin[e+fx]}{\sqrt{a+a \sin[e+fx]}} dx$$

Optimal (type 3, 79 leaves, 3 steps):

$$-\frac{\sqrt{2} (c-d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{a} f} - \frac{2 d \cos[e+fx]}{f \sqrt{a+a \sin[e+fx]}}$$

Result (type 3, 106 leaves):

$$\frac{1}{f \sqrt{a (1 + \sin[e+fx])}} 2 \left(\cos\left[\frac{1}{2} (e+fx)\right] + \sin\left[\frac{1}{2} (e+fx)\right] \right) \\ \left((1+i) (-1)^{3/4} (c-d) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e+fx)\right]\right)\right] + d \left(-\cos\left[\frac{1}{2} (e+fx)\right] + \sin\left[\frac{1}{2} (e+fx)\right] \right) \right)$$

■ **Problem 546: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+a \sin[e+fx]}} dx$$

Optimal (type 3, 47 leaves, 2 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{a} f}$$

Result (type 3, 73 leaves):

$$\frac{(2+2i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e+fx)\right] + \sin\left[\frac{1}{2} (e+fx)\right] \right)}{f \sqrt{a (1 + \sin[e+fx])}}$$

■ **Problem 547: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])} dx$$

Optimal (type 3, 123 leaves, 5 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} (c - d) f} + \frac{2 \sqrt{d} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} (c - d) \sqrt{c + d} f}$$

Result (type 3, 215 leaves):

$$\frac{1}{(c - d) \sqrt{c + d} f \sqrt{a} (1 + \sin[e + f x])} \left((2 + 2i) (-1)^{3/4} \sqrt{c + d} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \right) + \sqrt{d} \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c + d} + \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right) \right) - \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c + d} - \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right) \right) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right)$$

■ **Problem 548: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^2} dx$$

Optimal (type 3, 175 leaves, 6 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} (c - d)^2 f} + \frac{\sqrt{d} (3c + d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} (c - d)^2 (c + d)^{3/2} f} + \frac{d \cos[e + f x]}{(c^2 - d^2) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])}$$

Result (type 3, 324 leaves):

$$\frac{1}{4 (c - d)^2 f \sqrt{a} (1 + \sin[e + f x])} \left(\left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right) \left((8 + 8i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \right) + \frac{1}{(c + d)^{3/2}} \sqrt{d} (3c + d) \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c + d} + \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right) \right) \right) - \frac{1}{(c + d)^{3/2}} \sqrt{d} (3c + d) \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c + d} - \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right) \right) \right) + \frac{4 (c - d) d \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right]\right)}{(c + d) (c + d \sin[e + f x])}$$

■ **Problem 549: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 247 leaves, 7 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{a} (c-d)^3 f} + \frac{\sqrt{d} (15 c^2 + 10 c d + 7 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{c+d} \sqrt{a+a \sin[e+fx]}}\right]}{4 \sqrt{a} (c-d)^3 (c+d)^{5/2} f} +$$

$$\frac{d \cos[e + f x]}{2 (c^2 - d^2) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^2} + \frac{d (7 c + d) \cos[e + f x]}{4 (c^2 - d^2)^2 f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])}$$

Result (type 3, 414 leaves):

$$\frac{1}{16 f \sqrt{a} (1 + \sin[e + f x])}$$

$$\left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(\frac{(32 + 32 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} (-1 + \tan\left[\frac{1}{4} (e + f x)\right])\right]}{(c-d)^3} + \frac{1}{(c-d)^3 (c+d)^{5/2}} \sqrt{d} \right.$$

$$\left. (15 c^2 + 10 c d + 7 d^2) \left(e + f x - 2 \log\left[\sec\left[\frac{1}{4} (e + f x)\right]\right]^2 \right) + 2 \log\left[\sec\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) +$$

$$\frac{1}{(-c+d)^3 (c+d)^{5/2}} \sqrt{d} (15 c^2 + 10 c d + 7 d^2)$$

$$\left(e + f x - 2 \log\left[\sec\left[\frac{1}{4} (e + f x)\right]\right]^2 \right) + 2 \log\left[\sec\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) +$$

$$\frac{8 d (\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right])}{(c-d) (c+d) (c+d \sin[e + f x])^2} + \frac{4 d (7 c + d) (\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right])}{(c-d)^2 (c+d)^2 (c+d \sin[e + f x])}$$

■ **Problem 550: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c + d \sin[e + f x])^3}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 192 leaves, 6 steps):

$$-\frac{(c-d)^2 (c+11d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{2 \sqrt{2} a^{3/2} f} + \frac{d (3 c^2 - 24 c d + 13 d^2) \cos[e + f x]}{3 a f \sqrt{a + a \sin[e + f x]}}$$

$$\frac{(3 c - 7 d) d^2 \cos[e + f x] \sqrt{a + a \sin[e + f x]}}{6 a^2 f} - \frac{(c-d) \cos[e + f x] (c + d \sin[e + f x])^2}{2 f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 328 leaves) :

$$\frac{1}{6 f (a (1 + \sin[e + f x]))^{3/2}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(6 (c - d)^3 \sin\left[\frac{1}{2} (e + f x)\right] - 3 (c - d)^3 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) +$$

$$(3 + 3 i) (-1)^{3/4} (c - d)^2 (c + 11 d) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 -$$

$$18 (2 c - d) d^2 \cos\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - 2 d^3 \cos\left[\frac{3}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 +$$

$$18 (2 c - d) d^2 \sin\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - 2 d^3 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 \sin\left[\frac{3}{2} (e + f x)\right]$$

■ **Problem 551: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c + d \sin[e + f x])^2}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 138 leaves, 4 steps) :

$$-\frac{(c - d) (c + 7 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{2 \sqrt{2} a^{3/2} f} + \frac{(c - 5 d) d \cos[e + f x]}{2 a f \sqrt{a + a \sin[e + f x]}} - \frac{(c - d) \cos[e + f x] (c + d \sin[e + f x])}{2 f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 239 leaves) :

$$\frac{1}{2 f (a (1 + \sin[e + f x]))^{3/2}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(2 (c - d)^2 \sin\left[\frac{1}{2} (e + f x)\right] - (c - d)^2 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) +$$

$$(1 + i) (-1)^{3/4} (c^2 + 6 c d - 7 d^2) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 -$$

$$4 d^2 \cos\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 + 4 d^2 \sin\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2$$

■ **Problem 552: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{c + d \sin[e + f x]}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 87 leaves, 3 steps) :

$$-\frac{(c + 3 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{2 \sqrt{2} a^{3/2} f} - \frac{(c - d) \cos[e + f x]}{2 f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 150 leaves) :

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(2(c-d)\sin\left[\frac{1}{2}(e+fx)\right] + (-c+d) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) + (1+i)(-1)^{3/4}(c+3d) \right. \right. \\ \left. \left. \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4}\left(-1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \right) \right) / \left(2f(a(1+\sin[e+fx]))^{3/2} \right)$$

■ **Problem 553: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a\sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[e+fx]}{\sqrt{2}\sqrt{a+a\sin[e+fx]}}\right]}{2\sqrt{2}a^{3/2}f} - \frac{\cos[e+fx]}{2f(a+a\sin[e+fx])^{3/2}}$$

Result (type 3, 108 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(-\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] + (1+i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4}\left(-1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] (1+\sin[e+fx]) \right) \right) / \left(2f(a(1+\sin[e+fx]))^{3/2} \right)$$

■ **Problem 554: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a\sin[e+fx])^{3/2}(c+d\sin[e+fx])} dx$$

Optimal (type 3, 164 leaves, 6 steps):

$$-\frac{(c-5d)\operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[e+fx]}{\sqrt{2}\sqrt{a+a\sin[e+fx]}}\right]}{2\sqrt{2}a^{3/2}(c-d)^2f} - \frac{2d^{3/2}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{d}\cos[e+fx]}{\sqrt{c+d}\sqrt{a+a\sin[e+fx]}}\right]}{a^{3/2}(c-d)^2\sqrt{c+d}f} - \frac{\cos[e+fx]}{2(c-d)f(a+a\sin[e+fx])^{3/2}}$$

Result (type 3, 385 leaves):

$$\frac{1}{2(c-d)^2 f (a(1+\sin[ex+fx]))^{3/2}} \left(\cos\left[\frac{1}{2}(ex+fx)\right] + \sin\left[\frac{1}{2}(ex+fx)\right] \right) \left(2(c-d) \sin\left[\frac{1}{2}(ex+fx)\right] - (c-d) \left(\cos\left[\frac{1}{2}(ex+fx)\right] + \sin\left[\frac{1}{2}(ex+fx)\right] \right) \right) +$$

$$(1+i)(-1)^{3/4}(c-5d) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(ex+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(ex+fx)\right] + \sin\left[\frac{1}{2}(ex+fx)\right] \right)^2 - \frac{1}{\sqrt{c+d}}$$

$$d^{3/2} \left(ex+fx - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(ex+fx)\right]\right]^2 \right) + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(ex+fx)\right]\right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos\left[\frac{1}{2}(ex+fx)\right] - \sqrt{d} \sin\left[\frac{1}{2}(ex+fx)\right] \right) \left(\cos\left[\frac{1}{2}(ex+fx)\right] + \sin\left[\frac{1}{2}(ex+fx)\right] \right) \right)$$

$$\left(\cos\left[\frac{1}{2}(ex+fx)\right] + \sin\left[\frac{1}{2}(ex+fx)\right] \right)^2 + \frac{1}{\sqrt{c+d}} d^{3/2} \left(ex+fx - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(ex+fx)\right]\right]^2 \right) +$$

$$2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(ex+fx)\right]\right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos\left[\frac{1}{2}(ex+fx)\right] + \sqrt{d} \sin\left[\frac{1}{2}(ex+fx)\right] \right) \left(\cos\left[\frac{1}{2}(ex+fx)\right] + \sin\left[\frac{1}{2}(ex+fx)\right] \right)^2$$

■ **Problem 555: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a\sin[ex+fx])^{3/2} (c+d\sin[ex+fx])^2} dx$$

Optimal (type 3, 243 leaves, 7 steps):

$$-\frac{(c-9d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[ex+fx]}{\sqrt{2} \sqrt{a+a\sin[ex+fx]}}\right]}{2\sqrt{2} a^{3/2} (c-d)^3 f} - \frac{d^{3/2} (5c+3d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[ex+fx]}{\sqrt{c+d} \sqrt{a+a\sin[ex+fx]}}\right]}{a^{3/2} (c-d)^3 (c+d)^{3/2} f} -$$

$$\frac{\cos[ex+fx]}{2(c-d) f (a+a\sin[ex+fx])^{3/2} (c+d\sin[ex+fx])} - \frac{d(c+3d) \cos[ex+fx]}{2a(c-d)^2 (c+d) f \sqrt{a+a\sin[ex+fx]} (c+d\sin[ex+fx])}$$

Result (type 3, 491 leaves):

$$\frac{1}{4 f (a (1 + \sin[e + f x]))^{3/2}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(\frac{4 \sin\left[\frac{1}{2} (e + f x)\right]}{(c - d)^2} - \frac{2 (\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right])}{(c - d)^2} + \frac{1}{(c - d)^3} \right. \\ \left. (2 + 2 i) (-1)^{3/4} (c - 9 d) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 + \frac{1}{(-c + d)^3 (c + d)^{3/2}} \right. \\ \left. d^{3/2} (5 c + 3 d) \left(e + f x - 2 \operatorname{Log}\left[\sec\left[\frac{1}{4} (e + f x)\right]^2\right] + 2 \operatorname{Log}\left[\sec\left[\frac{1}{4} (e + f x)\right]^2 \left(\sqrt{c + d} + \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right)\right] \right) \right. \\ \left. \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 + \frac{1}{(c - d)^3 (c + d)^{3/2}} \right. \\ \left. d^{3/2} (5 c + 3 d) \left(e + f x - 2 \operatorname{Log}\left[\sec\left[\frac{1}{4} (e + f x)\right]^2\right] + 2 \operatorname{Log}\left[\sec\left[\frac{1}{4} (e + f x)\right]^2 \left(\sqrt{c + d} - \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right)\right] \right) \right. \\ \left. \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - \frac{4 d^2 (\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right]) (\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right])^2}{(c - d)^2 (c + d) (c + d \sin[e + f x])} \right)$$

■ **Problem 556: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 318 leaves, 8 steps):

$$- \frac{(c - 13 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{2 \sqrt{2} a^{3/2} (c - d)^4 f} - \frac{d^{3/2} (35 c^2 + 42 c d + 19 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{4 a^{3/2} (c - d)^4 (c + d)^{5/2} f} \\ - \frac{\cos[e + f x]}{2 (c - d) f (a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])^2} - \frac{d (c + 2 d) \cos[e + f x]}{2 a (c - d)^2 (c + d) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^2} \\ - \frac{d (2 c + d) (c + 7 d) \cos[e + f x]}{4 a (c - d)^3 (c + d)^2 f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])}$$

Result (type 3, 801 leaves):

$$\begin{aligned}
& \frac{\sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)}{(c-d)^3 f (a(1+\sin[e+fx]))^{3/2}} - \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{2(c-d)^3 f (a(1+\sin[e+fx]))^{3/2}} + \\
& \left((1+i)(c-13d) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4}(e+fx)\right] \left(\cos\left[\frac{1}{4}(e+fx)\right] - \sin\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^3 \right) / \\
& \left((2(-1)^{1/4}c^4 - 8(-1)^{1/4}c^3d + 12(-1)^{1/4}c^2d^2 - 8(-1)^{1/4}cd^3 + 2(-1)^{1/4}d^4) f (a(1+\sin[e+fx]))^{3/2} - \right. \\
& \left. \left(d^{3/2} (35c^2 + 42cd + 19d^2) \left(e+fx - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2\right] + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos\left[\frac{1}{2}(e+fx)\right] - \sqrt{d} \sin\left[\frac{1}{2}(e+fx)\right]\right)\right] \right) \right) \right) / \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \right) / (16(c-d)^4 (c+d)^{5/2} f (a(1+\sin[e+fx]))^{3/2}) + \\
& \left(d^{3/2} (35c^2 + 42cd + 19d^2) \left(e+fx - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2\right] + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos\left[\frac{1}{2}(e+fx)\right] + \sqrt{d} \sin\left[\frac{1}{2}(e+fx)\right]\right)\right] \right) \right) / \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \right) / (16(c-d)^4 (c+d)^{5/2} f (a(1+\sin[e+fx]))^{3/2}) + \\
& \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^3 \left(-d^2 \cos\left[\frac{1}{2}(e+fx)\right] + d^2 \sin\left[\frac{1}{2}(e+fx)\right]\right)}{2(c-d)^2 (c+d) f (a(1+\sin[e+fx]))^{3/2} (c+d \sin[e+fx])^2} + \\
& \left(\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \left(-11cd^2 \cos\left[\frac{1}{2}(e+fx)\right] - 5d^3 \cos\left[\frac{1}{2}(e+fx)\right] + 11cd^2 \sin\left[\frac{1}{2}(e+fx)\right] + 5d^3 \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\
& (4(c-d)^3 (c+d)^2 f (a(1+\sin[e+fx]))^{3/2} (c+d \sin[e+fx]))
\end{aligned}$$

■ **Problem 557: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \sin[e+fx])^3}{(a+a \sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 194 leaves, 6 steps):

$$\begin{aligned}
& \frac{3(c-d)(c^2+6cd+25d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{16\sqrt{2} a^{5/2} f} - \\
& \frac{(c-d)^2(3c+13d) \cos[e+fx]}{16af(a+a \sin[e+fx])^{3/2}} + \frac{(c-9d)d^2 \cos[e+fx]}{4a^2 f \sqrt{a+a \sin[e+fx]}} - \frac{(c-d) \cos[e+fx] (c+d \sin[e+fx])^2}{4f(a+a \sin[e+fx])^{5/2}}
\end{aligned}$$

Result (type 3, 400 leaves):

$$\frac{1}{32 f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(-11 c^3 \cos\left[\frac{1}{2}(e + f x)\right] + 9 c^2 d \cos\left[\frac{1}{2}(e + f x)\right] + 15 c d^2 \cos\left[\frac{1}{2}(e + f x)\right] - 45 d^3 \cos\left[\frac{1}{2}(e + f x)\right] - 3 c^3 \cos\left[\frac{3}{2}(e + f x)\right] - 15 c^2 d \cos\left[\frac{3}{2}(e + f x)\right] + 39 c d^2 \cos\left[\frac{3}{2}(e + f x)\right] - 69 d^3 \cos\left[\frac{3}{2}(e + f x)\right] + 16 d^3 \cos\left[\frac{5}{2}(e + f x)\right] + 11 c^3 \sin\left[\frac{1}{2}(e + f x)\right] - 9 c^2 d \sin\left[\frac{1}{2}(e + f x)\right] - 15 c d^2 \sin\left[\frac{1}{2}(e + f x)\right] + 45 d^3 \sin\left[\frac{1}{2}(e + f x)\right] + (6 + 6 i) (-1)^{3/4} (c^3 + 5 c^2 d + 19 c d^2 - 25 d^3) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4 - 3 c^3 \sin\left[\frac{3}{2}(e + f x)\right] - 15 c^2 d \sin\left[\frac{3}{2}(e + f x)\right] + 39 c d^2 \sin\left[\frac{3}{2}(e + f x)\right] - 69 d^3 \sin\left[\frac{3}{2}(e + f x)\right] - 16 d^3 \sin\left[\frac{5}{2}(e + f x)\right] \right)$$

■ **Problem 558: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c + d \sin[e + f x])^2}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 147 leaves, 4 steps):

$$-\frac{(3 c^2 + 10 c d + 19 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} f} - \frac{3 (c - d) (c + 3 d) \cos[e + f x]}{16 a f (a + a \sin[e + f x])^{3/2}} - \frac{(c - d) \cos[e + f x] (c + d \sin[e + f x])}{4 f (a + a \sin[e + f x])^{5/2}}$$

Result (type 3, 252 leaves):

$$\frac{1}{16 f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(8 (c - d)^2 \sin\left[\frac{1}{2}(e + f x)\right] - 4 (c - d)^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) + 2 (3 c^2 + 10 c d - 13 d^2) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 - (c - d) (3 c + 13 d) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 + (1 + i) (-1)^{3/4} (3 c^2 + 10 c d + 19 d^2) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4$$

■ **Problem 559: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{c + d \sin[e + f x]}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 126 leaves, 4 steps):

$$-\frac{(3 c + 5 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} f} - \frac{(c - d) \cos[e + f x]}{4 f (a + a \sin[e + f x])^{5/2}} - \frac{(3 c + 5 d) \cos[e + f x]}{16 a f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 227 leaves):

$$\frac{1}{16 f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(8 (c - d) \sin\left[\frac{1}{2} (e + f x)\right] + 4 (-c + d) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) +$$

$$2 (3 c + 5 d) \sin\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - (3 c + 5 d) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^3 +$$

$$(1 + i) (-1)^{3/4} (3 c + 5 d) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4$$

■ **Problem 560: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$-\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} f} - \frac{\cos[e + f x]}{4 f (a + a \sin[e + f x])^{5/2}} - \frac{3 \cos[e + f x]}{16 a f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 196 leaves):

$$\frac{1}{16 f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(8 \sin\left[\frac{1}{2} (e + f x)\right] - 4 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) +$$

$$6 \sin\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - 3 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^3 +$$

$$(3 + 3 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4$$

■ **Problem 561: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])} dx$$

Optimal (type 3, 218 leaves, 7 steps):

$$-\frac{(3 c^2 - 14 c d + 43 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} (c - d)^3 f} +$$

$$\frac{2 d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{a^{5/2} (c - d)^3 \sqrt{c + d} f} - \frac{\cos[e + f x]}{4 (c - d) f (a + a \sin[e + f x])^{5/2}} - \frac{(3 c - 11 d) \cos[e + f x]}{16 a (c - d)^2 f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 501 leaves):

$$\begin{aligned}
& \frac{1}{16 f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(\frac{8 \sin\left[\frac{1}{2} (e + f x)\right]}{c - d} - \frac{4 (\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right])}{c - d} \right) + \\
& \frac{2 (3 c - 11 d) \sin\left[\frac{1}{2} (e + f x)\right] (\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right])^2}{(c - d)^2} + \frac{(-3 c + 11 d) (\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right])^3}{(c - d)^2} + \frac{1}{(c - d)^3} \\
& (1 + i) (-1)^{3/4} (3 c^2 - 14 c d + 43 d^2) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right)^4 + \\
& \frac{1}{(c - d)^3 \sqrt{c + d}} 8 d^{5/2} \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c + d} + \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right) \right) \\
& \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right)^4 + \frac{1}{(-c + d)^3 \sqrt{c + d}} \\
& 8 d^{5/2} \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c + d} - \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right) \right) \\
& \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right)^4 \Big)
\end{aligned}$$

- **Problem 562: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])^2} dx$$

Optimal (type 3, 313 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(3 c^2 - 22 c d + 115 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} (c - d)^4 f} + \\
& \frac{d^{5/2} (7 c + 5 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{a^{5/2} (c - d)^4 (c + d)^{3/2} f} - \frac{\cos[e + f x]}{4 (c - d) f (a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])} - \\
& \frac{3 (c - 5 d) \cos[e + f x]}{16 a (c - d)^2 f (a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])} - \frac{(c - 7 d) d (3 c + 5 d) \cos[e + f x]}{16 a^2 (c - d)^3 (c + d) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])}
\end{aligned}$$

Result (type 3, 800 leaves):

$$\begin{aligned}
& \frac{\sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)}{2(c-d)^2 f (a(1+\sin[e+fx]))^{5/2}} - \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{4(c-d)^2 f (a(1+\sin[e+fx]))^{5/2}} + \\
& \frac{(-3c+19d) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{16(c-d)^3 f (a(1+\sin[e+fx]))^{5/2}} + \left((1+i) (3c^2 - 22cd + 115d^2) \right. \\
& \left. \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4}(e+fx)\right] \left(\cos\left[\frac{1}{4}(e+fx)\right] - \sin\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5\right) / \\
& \left((16(-1)^{1/4}c^4 - 64(-1)^{1/4}c^3d + 96(-1)^{1/4}c^2d^2 - 64(-1)^{1/4}cd^3 + 16(-1)^{1/4}d^4) f (a(1+\sin[e+fx]))^{5/2} \right) + \\
& \left(d^{5/2} (7c+5d) \left(e+fx - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2\right] + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos\left[\frac{1}{2}(e+fx)\right] - \sqrt{d} \sin\left[\frac{1}{2}(e+fx)\right]\right)\right] \right) \right) \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 / \left(4(c-d)^4 (c+d)^{3/2} f (a(1+\sin[e+fx]))^{5/2} \right) - \\
& \left(d^{5/2} (7c+5d) \left(e+fx - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2\right] + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos\left[\frac{1}{2}(e+fx)\right] + \sqrt{d} \sin\left[\frac{1}{2}(e+fx)\right]\right)\right] \right) \right) \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 / \left(4(c-d)^4 (c+d)^{3/2} f (a(1+\sin[e+fx]))^{5/2} \right) + \\
& \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^3 \left(3c \sin\left[\frac{1}{2}(e+fx)\right] - 19d \sin\left[\frac{1}{2}(e+fx)\right]\right)}{8(c-d)^3 f (a(1+\sin[e+fx]))^{5/2}} + \\
& \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5 \left(d^3 \cos\left[\frac{1}{2}(e+fx)\right] - d^3 \sin\left[\frac{1}{2}(e+fx)\right]\right)}{(c-d)^3 (c+d) f (a(1+\sin[e+fx]))^{5/2} (c+d \sin[e+fx])}
\end{aligned}$$

■ **Problem 563: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a \sin[e+fx])^{5/2} (c+d \sin[e+fx])^3} dx$$

Optimal (type 3, 400 leaves, 9 steps):

$$\begin{aligned}
& - \frac{3(c^2 - 10cd + 73d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{16\sqrt{2} a^{5/2} (c-d)^5 f \cos[e+fx]} + \frac{3d^{5/2} (21c^2 + 30cd + 13d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{c+d} \sqrt{a+a \sin[e+fx]}}\right]}{4a^{5/2} (c-d)^5 (c+d)^{5/2} f (3c-19d) \cos[e+fx]} - \\
& \frac{4(c-d) f (a+a \sin[e+fx])^{5/2} (c+d \sin[e+fx])^2}{d(3c^2 - 20cd - 31d^2) \cos[e+fx]} - \frac{16a(c-d)^2 f (a+a \sin[e+fx])^{3/2} (c+d \sin[e+fx])^2}{3d(c+3d) (c^2 - 10cd - 7d^2) \cos[e+fx]} - \\
& \frac{16a^2(c-d)^3 (c+d) f \sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])^2}{16a^2(c-d)^4 (c+d)^2 f \sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])}
\end{aligned}$$

Result (type 3, 958 leaves):

$$\begin{aligned}
& \frac{\sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)}{2(c-d)^3 f (a(1+\sin[e+fx]))^{5/2}} - \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{4(c-d)^3 f (a(1+\sin[e+fx]))^{5/2}} \\
& \frac{3(c-9d) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{16(c-d)^4 f (a(1+\sin[e+fx]))^{5/2}} + \left((3+3i) (c^2-10cd+73d^2) \right. \\
& \left. \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4}(e+fx)\right] \left(\cos\left[\frac{1}{4}(e+fx)\right] - \sin\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5 \right) / \\
& \left((16(-1)^{1/4}c^5 - 80(-1)^{1/4}c^4d + 160(-1)^{1/4}c^3d^2 - 160(-1)^{1/4}c^2d^3 + 80(-1)^{1/4}cd^4 - 16(-1)^{1/4}d^5) f (a(1+\sin[e+fx]))^{5/2} \right) + \left(3d^{5/2} \right. \\
& \left. (21c^2 + 30cd + 13d^2) \left(e+fx - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2\right] + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos\left[\frac{1}{2}(e+fx)\right] - \sqrt{d} \sin\left[\frac{1}{2}(e+fx)\right]\right)\right] \right) \right) \\
& \left. \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5 \right) / (16(c-d)^5 (c+d)^{5/2} f (a(1+\sin[e+fx]))^{5/2}) + \left(3d^{5/2} (21c^2 + 30cd + 13d^2) \right. \\
& \left. \left(e+fx - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2\right] + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e+fx)\right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos\left[\frac{1}{2}(e+fx)\right] + \sqrt{d} \sin\left[\frac{1}{2}(e+fx)\right]\right)\right] \right) \right) \\
& \left. \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5 \right) / (16(-c+d)^5 (c+d)^{5/2} f (a(1+\sin[e+fx]))^{5/2}) + \\
& \frac{3 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^3 (c \sin\left[\frac{1}{2}(e+fx)\right] - 9d \sin\left[\frac{1}{2}(e+fx)\right])}{8(c-d)^4 f (a(1+\sin[e+fx]))^{5/2}} + \\
& \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5 (d^3 \cos\left[\frac{1}{2}(e+fx)\right] - d^3 \sin\left[\frac{1}{2}(e+fx)\right])}{2(c-d)^3 (c+d) f (a(1+\sin[e+fx]))^{5/2} (c+d \sin[e+fx])^2} + \\
& \left(3 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5 \left(5cd^3 \cos\left[\frac{1}{2}(e+fx)\right] + 3d^4 \cos\left[\frac{1}{2}(e+fx)\right] - 5cd^3 \sin\left[\frac{1}{2}(e+fx)\right] - 3d^4 \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\
& (4(c-d)^4 (c+d)^2 f (a(1+\sin[e+fx]))^{5/2} (c+d \sin[e+fx]))
\end{aligned}$$

■ **Problem 564: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])^{5/2} dx$$

Optimal (type 3, 203 leaves, 5 steps):

$$\begin{aligned}
& \frac{5\sqrt{a} (c+d)^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{8\sqrt{d} f} - \frac{5a (c+d)^2 \cos[e+fx] \sqrt{c+d \sin[e+fx]}}{8f \sqrt{a+a \sin[e+fx]}} \\
& \frac{5a (c+d) \cos[e+fx] (c+d \sin[e+fx])^{3/2}}{12f \sqrt{a+a \sin[e+fx]}} - \frac{a \cos[e+fx] (c+d \sin[e+fx])^{5/2}}{3f \sqrt{a+a \sin[e+fx]}}
\end{aligned}$$

Result (type 3, 391 leaves) :

$$\begin{aligned}
 & - \frac{1}{48 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c+d \sin[e+fx]}} \\
 & \sqrt{a(1+\sin[e+fx])} \left(\frac{1}{\sqrt{d}} 15 (c+d)^3 \left[\operatorname{Log}\left[\frac{e^{-i} e \left(2(-1)^{1/4} c - 2(-1)^{3/4} d e^{i(e+fx)} + 2\sqrt{d} \sqrt{2c e^{i(e+fx)} - i d (-1 + e^{2i(e+fx)})} \right)}{\sqrt{d}} \right] - \right. \right. \\
 & \left. \left. \operatorname{Log}\left[\frac{2 e^{\frac{1}{2} i (e-2fx)} \left((-1)^{3/4} d + (-1)^{1/4} c e^{i(e+fx)} + i \sqrt{d} \sqrt{2c e^{i(e+fx)} - i d (-1 + e^{2i(e+fx)})} \right) f}{\sqrt{d}} \right] \right) \right) \\
 & \left(i \cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{(\cos[e+fx] + i \sin[e+fx]) (c+d \sin[e+fx])} + \\
 & \left. 2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (c+d \sin[e+fx]) (33c^2 + 40cd + 19d^2 - 4d^2 \cos[2(e+fx)] + 2d(13c+5d) \sin[e+fx]) \right)
 \end{aligned}$$

■ **Problem 565: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])^{3/2} dx$$

Optimal (type 3, 156 leaves, 4 steps) :

$$\frac{3 \sqrt{a} (c+d)^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{4 \sqrt{d} f} - \frac{3 a (c+d) \cos[e+fx] \sqrt{c+d \sin[e+fx]}}{4 f \sqrt{a+a \sin[e+fx]}} - \frac{a \cos[e+fx] (c+d \sin[e+fx])^{3/2}}{2 f \sqrt{a+a \sin[e+fx]}}$$

Result (type 3, 365 leaves) :

$$\begin{aligned}
& \frac{1}{8 f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{c + d \sin[e + f x]}} \\
& \sqrt{a (1 + \sin[e + f x])} \left(-\frac{1}{\sqrt{d}} 3 i (c + d)^2 \left(\operatorname{Log} \left[\frac{e^{-i e} \left(2 (-1)^{1/4} c - 2 (-1)^{3/4} d e^{i (e + f x)} + 2 \sqrt{d} \sqrt{2 c e^{i (e + f x)} - i d (-1 + e^{2 i (e + f x)})} \right)}{\sqrt{d}} \right) \right) - \right. \\
& \left. \operatorname{Log} \left[\frac{2 e^{\frac{1}{2} i (e - 2 f x)} \left((-1)^{3/4} d + (-1)^{1/4} c e^{i (e + f x)} + i \sqrt{d} \sqrt{2 c e^{i (e + f x)} - i d (-1 + e^{2 i (e + f x)})} \right) f}{\sqrt{d}} \right] \right) \\
& \left(\cos \left[\frac{1}{2} (e + f x) \right] - i \sin \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{(\cos[e + f x] + i \sin[e + f x]) (c + d \sin[e + f x])} - \\
& \left. 2 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) (c + d \sin[e + f x]) (5 c + 3 d + 2 d \sin[e + f x]) \right)
\end{aligned}$$

- **Problem 566: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]} dx$$

Optimal (type 3, 105 leaves, 3 steps):

$$\frac{\sqrt{a} (c + d) \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}} \right]}{\sqrt{d} f} - \frac{a \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{f \sqrt{a + a \sin[e + f x]}}$$

Result (type 3, 350 leaves):

$$\left(\sqrt{a(1 + \sin[e + fx])} \left(-\frac{2 \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right) (c + d \sin[e + fx])}{f} - \right. \right.$$

$$\left. \left. \frac{1}{(\sqrt{d} f) i (c + d)} \left(\operatorname{Log}\left[\frac{e^{-i} e \left(2(-1)^{1/4} c - 2(-1)^{3/4} d e^{i(e+fx)} + 2\sqrt{d} \sqrt{2c e^{i(e+fx)} - i d (-1 + e^{2i(e+fx)})} \right)}{\sqrt{d}} \right] - \right. \right.$$

$$\left. \left. \operatorname{Log}\left[\frac{2 e^{\frac{1}{2} i (e - 2fx)} \left((-1)^{3/4} d + (-1)^{1/4} c e^{i(e+fx)} + i \sqrt{d} \sqrt{2c e^{i(e+fx)} - i d (-1 + e^{2i(e+fx)})} \right) f}{\sqrt{d}} \right] \right) \right) /$$

$$\left(\cos\left[\frac{1}{2}(e + fx)\right] - i \sin\left[\frac{1}{2}(e + fx)\right] \right) \sqrt{(\cos[e + fx] + i \sin[e + fx]) (c + d \sin[e + fx])} \Bigg) /$$

$$\left(2 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right) \sqrt{c + d \sin[e + fx]} \right)$$

- **Problem 567: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \sin[e + fx]}}{\sqrt{c + d \sin[e + fx]}} dx$$

Optimal (type 3, 61 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + fx]}{\sqrt{a + a \sin[e + fx]} \sqrt{c + d \sin[e + fx]}} \right]}{\sqrt{d} f}$$

Result (type 3, 305 leaves):

$$\begin{aligned}
& - \left(i \left[\text{Log} \left[\frac{e^{-i} e \left(2 (-1)^{1/4} c - 2 (-1)^{3/4} d e^{i(e+fx)} + 2 \sqrt{d} \sqrt{2 c e^{i(e+fx)} - i d (-1 + e^{2i(e+fx)})} \right)}{\sqrt{d}} \right] - \right. \\
& \quad \left. \text{Log} \left[- \frac{(1+i) e^{\frac{1}{2} i(e-2fx)} \left(-(-1)^{1/4} d + (-1)^{3/4} c e^{i(e+fx)} - \sqrt{d} \sqrt{2 c e^{i(e+fx)} - i d (-1 + e^{2i(e+fx)})} \right) f}{\sqrt{d}} \right] \right) \\
& \quad \left(\text{Cos} \left[\frac{1}{2} (e+fx) \right] - i \text{Sin} \left[\frac{1}{2} (e+fx) \right] \right) \sqrt{a(1+\text{Sin}[e+fx])} \sqrt{(\text{Cos}[e+fx] + i \text{Sin}[e+fx]) (c+d \text{Sin}[e+fx])} \Big/ \\
& \quad \left(\sqrt{d} f \left(\text{Cos} \left[\frac{1}{2} (e+fx) \right] + \text{Sin} \left[\frac{1}{2} (e+fx) \right] \right) \sqrt{c+d \text{Sin}[e+fx]} \right)
\end{aligned}$$

■ **Problem 574: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sin}[e + f x])^{3/2}}{\sqrt{c + d \text{Sin}[e + f x]}} dx$$

Optimal (type 3, 111 leaves, 4 steps):

$$\frac{a^{3/2} (c - 3 d) \text{ArcTan} \left[\frac{\sqrt{a} \sqrt{d} \text{Cos}[e+fx]}{\sqrt{a+a \text{Sin}[e+fx]} \sqrt{c+d \text{Sin}[e+fx]}} \right]}{d^{3/2} f} - \frac{a^2 \text{Cos}[e+fx] \sqrt{c+d \text{Sin}[e+fx]}}{d f \sqrt{a+a \text{Sin}[e+fx]}}$$

Result (type 3, 301 leaves):

$$\begin{aligned}
& \frac{1}{2 d^{3/2} f \left(\text{Cos} \left[\frac{1}{2} (e+fx) \right] + \text{Sin} \left[\frac{1}{2} (e+fx) \right] \right)^3} \\
& (a(1+\text{Sin}[e+fx]))^{3/2} \left(-2(c-3d) \text{ArcTan} \left[\frac{\sqrt{2} \sqrt{d} \text{Sin} \left[\frac{1}{4} (2e-\pi+2fx) \right]}{\sqrt{c+d \text{Sin}[e+fx]}} \right] - (c-3d) \text{ArcTanh} \left[\frac{\sqrt{2} \sqrt{d} \text{Cos} \left[\frac{1}{4} (2e-\pi+2fx) \right]}{\sqrt{c+d \text{Sin}[e+fx]}} \right] + \right. \\
& \quad c \text{Log} \left[\sqrt{2} \sqrt{d} \text{Cos} \left[\frac{1}{4} (2e-\pi+2fx) \right] + \sqrt{c+d \text{Sin}[e+fx]} \right] - 3d \text{Log} \left[\sqrt{2} \sqrt{d} \text{Cos} \left[\frac{1}{4} (2e-\pi+2fx) \right] + \sqrt{c+d \text{Sin}[e+fx]} \right] - \\
& \quad \left. 2 \sqrt{d} \text{Cos} \left[\frac{1}{2} (e+fx) \right] \sqrt{c+d \text{Sin}[e+fx]} + 2 \sqrt{d} \text{Sin} \left[\frac{1}{2} (e+fx) \right] \sqrt{c+d \text{Sin}[e+fx]} \right)
\end{aligned}$$

■ **Problem 575: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sin}[e + f x])^{3/2}}{(c + d \text{Sin}[e + f x])^{3/2}} dx$$

Optimal (type 3, 117 leaves, 4 steps):

$$-\frac{2 a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+f x]}{\sqrt{a+a \sin[e+f x]} \sqrt{c+d \sin[e+f x]}}\right]}{d^{3/2} f} + \frac{2 a^2 (c-d) \cos[e+f x]}{d (c+d) f \sqrt{a+a \sin[e+f x]} \sqrt{c+d \sin[e+f x]}}$$

Result (type 3, 377 leaves):

$$\left((a (1 + \sin[e+f x]))^{3/2} \left(2 c \sqrt{d} \cos\left[\frac{1}{2} (e+f x)\right] - 2 d^{3/2} \cos\left[\frac{1}{2} (e+f x)\right] - \right. \right. \\ \left. \left. 2 c \sqrt{d} \sin\left[\frac{1}{2} (e+f x)\right] + 2 d^{3/2} \sin\left[\frac{1}{2} (e+f x)\right] + 2 (c+d) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{d} \sin\left[\frac{1}{4} (2 e-\pi+2 f x)\right]}{\sqrt{c+d \sin[e+f x]}}\right] \sqrt{c+d \sin[e+f x]} + \right. \right. \\ \left. \left. (c+d) \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{d} \cos\left[\frac{1}{4} (2 e-\pi+2 f x)\right]}{\sqrt{c+d \sin[e+f x]}}\right] \sqrt{c+d \sin[e+f x]} - c \operatorname{Log}\left[\sqrt{2} \sqrt{d} \cos\left[\frac{1}{4} (2 e-\pi+2 f x)\right] + \sqrt{c+d \sin[e+f x]}\right] \right. \right. \\ \left. \left. \sqrt{c+d \sin[e+f x]} - d \operatorname{Log}\left[\sqrt{2} \sqrt{d} \cos\left[\frac{1}{4} (2 e-\pi+2 f x)\right] + \sqrt{c+d \sin[e+f x]}\right] \sqrt{c+d \sin[e+f x]}\right] \right) / \\ \left(d^{3/2} (c+d) f \left(\cos\left[\frac{1}{2} (e+f x)\right] + \sin\left[\frac{1}{2} (e+f x)\right] \right)^3 \sqrt{c+d \sin[e+f x]} \right)$$

■ **Problem 588: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \sin[e+f x])^{5/2}}{\sqrt{a+a \sin[e+f x]}} dx$$

Optimal (type 3, 249 leaves, 7 steps):

$$-\frac{\sqrt{d} (15 c^2 - 10 c d + 7 d^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+f x]}{\sqrt{a+a \sin[e+f x]} \sqrt{c+d \sin[e+f x]}}\right]}{4 \sqrt{a} f} - \frac{\sqrt{2} (c-d)^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[e+f x]}{\sqrt{2} \sqrt{a+a \sin[e+f x]} \sqrt{c+d \sin[e+f x]}}\right]}{\sqrt{a} f} \\ - \frac{(7 c-d) d \cos[e+f x] \sqrt{c+d \sin[e+f x]}}{4 f \sqrt{a+a \sin[e+f x]}} - \frac{d \cos[e+f x] (c+d \sin[e+f x])^{3/2}}{2 f \sqrt{a+a \sin[e+f x]}}$$

Result (type 3, 1893 leaves):

$$\frac{1}{f \sqrt{a (1 + \sin[e+f x])}} \left(\cos\left[\frac{1}{2} (e+f x)\right] + \sin\left[\frac{1}{2} (e+f x)\right] \right) \sqrt{c+d \sin[e+f x]} \\ + \left(\frac{1}{4} d (-9 c + 2 d) \cos\left[\frac{1}{2} (e+f x)\right] - \frac{1}{4} d^2 \cos\left[\frac{3}{2} (e+f x)\right] - \frac{1}{4} d (-9 c + 2 d) \sin\left[\frac{1}{2} (e+f x)\right] - \frac{1}{4} d^2 \sin\left[\frac{3}{2} (e+f x)\right] \right) +$$

$$\left(\left(\sqrt{2} (c-d)^{5/2} \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \sqrt{2} (c-d)^{5/2} \operatorname{Log}\left[c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\operatorname{Cos}[e+fx]}} \sqrt{c+d \operatorname{Sin}[e+fx]} + (-c+d) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \right.$$

$$\left. \frac{1}{8} i \sqrt{d} (15c^2 - 10cd + 7d^2) \operatorname{Log}\left[\frac{2 \left(c - i \left(d + (1+i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\operatorname{Cos}[e+fx]}} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) + (-ic+d) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)}{d^{3/2} (15c^2 - 10cd + 7d^2) (i + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])} \right] + \right.$$

$$\left. \frac{1}{8} i \sqrt{d} (15c^2 - 10cd + 7d^2) \operatorname{Log}\left[\frac{2 \left(c + id + (1+i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\operatorname{Cos}[e+fx]}} \sqrt{c+d \operatorname{Sin}[e+fx]} + (ic+d) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)}{d^{3/2} (15c^2 - 10cd + 7d^2) (-i + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])} \right] \right)$$

$$\left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right) \left(\frac{c^3}{(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]) \sqrt{c+d \operatorname{Sin}[e+fx]}} - \right.$$

$$\frac{9c^2d}{8 (\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]) \sqrt{c+d \operatorname{Sin}[e+fx]}} + \frac{7cd^2}{4 (\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]) \sqrt{c+d \operatorname{Sin}[e+fx]}} -$$

$$\frac{d^3}{8 (\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]) \sqrt{c+d \operatorname{Sin}[e+fx]}} + \frac{15c^2d \operatorname{Sin}[e+fx]}{8 (\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]) \sqrt{c+d \operatorname{Sin}[e+fx]}} -$$

$$\left. \frac{5cd^2 \operatorname{Sin}[e+fx]}{4 (\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]) \sqrt{c+d \operatorname{Sin}[e+fx]}} + \frac{7d^3 \operatorname{Sin}[e+fx]}{8 (\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]) \sqrt{c+d \operatorname{Sin}[e+fx]}} \right) \Big/$$

$$\left(f \sqrt{a(1+\operatorname{Sin}[e+fx])} \left(\frac{(c-d)^{5/2} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{\sqrt{2} (1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right])} - \sqrt{2} (c-d)^{5/2} \right)$$

$$\begin{aligned}
& \left(\frac{1}{2} (-c+d) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{\sqrt{c-d} d \operatorname{Cos}[e+fx] \sqrt{\frac{1}{1+\operatorname{Cos}[e+fx]}}}{\sqrt{c+d \operatorname{Sin}[e+fx]}} + \sqrt{c-d} \left(\frac{1}{1+\operatorname{Cos}[e+fx]}\right)^{3/2} \operatorname{Sin}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \Bigg) / \\
& \left(c-d+2\sqrt{c-d} \sqrt{\frac{1}{1+\operatorname{Cos}[e+fx]}} \sqrt{c+d \operatorname{Sin}[e+fx]} + (-c+d) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - \\
& \left(i d^2 (15c^2 - 10cd + 7d^2)^2 \left(i + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \left(\left(\left(\frac{1}{2} (-ic+d) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 - i \left(\frac{(1+i) d^{3/2} \operatorname{Cos}[e+fx] \sqrt{\frac{1}{1+\operatorname{Cos}[e+fx]}}}{\sqrt{2} \sqrt{c+d \operatorname{Sin}[e+fx]}} + \right. \right. \right. \right. \\
& \left. \left. \left. \frac{(1+i) \sqrt{d} \left(\frac{1}{1+\operatorname{Cos}[e+fx]}\right)^{3/2} \operatorname{Sin}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{\sqrt{2}} \right) \right) \right) / \left(d^{3/2} (15c^2 - 10cd + 7d^2) \left(i + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) - \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left(c-i \left(d+(1+i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\operatorname{Cos}[e+fx]}} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) + (-ic+d) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) / \\
& \left(d^{3/2} (15c^2 - 10cd + 7d^2) \left(i + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)^2 \right) \Bigg) / \\
& \left(16 \left(c-i \left(d+(1+i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\operatorname{Cos}[e+fx]}} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) + (-ic+d) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) + \\
& \left(i d^2 (15c^2 - 10cd + 7d^2)^2 \left(-i + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \left(\left(\left(\frac{1}{2} (ic+d) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{(1+i) d^{3/2} \operatorname{Cos}[e+fx] \sqrt{\frac{1}{1+\operatorname{Cos}[e+fx]}}}{\sqrt{2} \sqrt{c+d \operatorname{Sin}[e+fx]}} + \right. \right. \right. \right.
\end{aligned}$$

$$\left. \left. \left. \frac{(1+i)\sqrt{d} \left(\frac{1}{1+\cos[e+fx]} \right)^{3/2} \sin[e+fx] \sqrt{c+d \sin[e+fx]}}{\sqrt{2}} \right) \right) / \left(d^{3/2} (15c^2 - 10cd + 7d^2) \left(-i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) - \right. \\ \left. \left(\sec\left[\frac{1}{2}(e+fx)\right] \right)^2 \left(c + id + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (ic+d) \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\ \left(d^{3/2} (15c^2 - 10cd + 7d^2) \left(-i + \tan\left[\frac{1}{2}(e+fx)\right] \right)^2 \right) \right) / \\ \left(16 \left(c + id + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (ic+d) \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right)$$

- **Problem 589: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \sin[e+fx])^{3/2}}{\sqrt{a+a \sin[e+fx]}} dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$-\frac{(3c-d)\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{d} \cos[e+fx]}{\sqrt{a+a \sin[e+fx]}\sqrt{c+d \sin[e+fx]}}\right]}{\sqrt{a}f} - \frac{\sqrt{2}(c-d)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c-d} \cos[e+fx]}{\sqrt{2}\sqrt{a+a \sin[e+fx]}\sqrt{c+d \sin[e+fx]}}\right]}{\sqrt{a}f} - \frac{d \cos[e+fx] \sqrt{c+d \sin[e+fx]}}{f \sqrt{a+a \sin[e+fx]}}$$

Result (type 3, 1639 leaves):

$$\frac{(\cos[\frac{1}{2}(e+fx)] + \sin[\frac{1}{2}(e+fx)]) (-d \cos[\frac{1}{2}(e+fx)] + d \sin[\frac{1}{2}(e+fx)]) \sqrt{c+d \sin[e+fx]}}{f \sqrt{a(1+\sin[e+fx])}} +$$

$$\left(\left(\sqrt{2}(c-d)^{3/2} \operatorname{Log}\left[1 + \tan\left[\frac{1}{2}(e+fx)\right]\right] - \sqrt{2}(c-d)^{3/2} \operatorname{Log}\left[c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (-c+d) \tan\left[\frac{1}{2}(e+fx)\right]\right] \right) + \right.$$

$$\frac{1}{2} i \sqrt{d} (-3c+d) \left(\text{Log} \left[\frac{2 i \left(i c+d+(1+i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\cos[e+f x]}} \sqrt{c+d \sin[e+f x]}+(c+i d) \tan \left[\frac{1}{2}(e+f x) \right] \right)}{d^{3/2}(-3c+d)\left(i+\tan \left[\frac{1}{2}(e+f x) \right]\right)} \right] \right) -$$

$$\text{Log} \left[-\frac{2 \left(c+i d+(1+i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\cos[e+f x]}} \sqrt{c+d \sin[e+f x]}+(i c+d) \tan \left[\frac{1}{2}(e+f x) \right] \right)}{d^{3/2}(-3c+d)\left(-i+\tan \left[\frac{1}{2}(e+f x) \right]\right)} \right] \right)$$

$$\left(\cos \left[\frac{1}{2}(e+f x) \right] + \sin \left[\frac{1}{2}(e+f x) \right] \right) \left(\frac{c^2}{\left(\cos \left[\frac{1}{2}(e+f x) \right] + \sin \left[\frac{1}{2}(e+f x) \right] \right) \sqrt{c+d \sin[e+f x]}} - \frac{c d}{2 \left(\cos \left[\frac{1}{2}(e+f x) \right] + \sin \left[\frac{1}{2}(e+f x) \right] \right) \sqrt{c+d \sin[e+f x]}} + \frac{d^2}{2 \left(\cos \left[\frac{1}{2}(e+f x) \right] + \sin \left[\frac{1}{2}(e+f x) \right] \right) \sqrt{c+d \sin[e+f x]}} + \frac{3 c d \sin[e+f x]}{2 \left(\cos \left[\frac{1}{2}(e+f x) \right] + \sin \left[\frac{1}{2}(e+f x) \right] \right) \sqrt{c+d \sin[e+f x]}} - \frac{d^2 \sin[e+f x]}{2 \left(\cos \left[\frac{1}{2}(e+f x) \right] + \sin \left[\frac{1}{2}(e+f x) \right] \right) \sqrt{c+d \sin[e+f x]}} \right) \right) /$$

$$\left(f \sqrt{a(1+\sin[e+f x])} \left(\frac{(c-d)^{3/2} \sec \left[\frac{1}{2}(e+f x) \right]^2}{\sqrt{2}\left(1+\tan \left[\frac{1}{2}(e+f x) \right]\right)} - \sqrt{2}(c-d)^{3/2} \right) \right)$$

$$\left(\frac{1}{2}(-c+d) \sec \left[\frac{1}{2}(e+f x) \right]^2 + \frac{\sqrt{c-d} d \cos[e+f x] \sqrt{\frac{1}{1+\cos[e+f x]}}}{\sqrt{c+d \sin[e+f x]}} + \sqrt{c-d} \left(\frac{1}{1+\cos[e+f x]} \right)^{3/2} \sin[e+f x] \sqrt{c+d \sin[e+f x]} \right) /$$

$$\left(c-d+2 \sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+f x]}} \sqrt{c+d \sin[e+f x]}+(-c+d) \tan \left[\frac{1}{2}(e+f x) \right] \right) +$$

$$\begin{aligned}
& \frac{1}{2} i \sqrt{d} (-3c+d) \left(- \left(i d^{3/2} (-3c+d) \left(i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(\left(2i \left[\frac{1}{2}(c+id) \sec\left[\frac{1}{2}(e+fx)\right]^2 + \frac{(1+i)d^{3/2} \cos[e+fx] \sqrt{\frac{1}{1+\cos[e+fx]}}}{\sqrt{2} \sqrt{c+d \sin[e+fx]}} + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{(1+i)\sqrt{d} \left(\frac{1}{1+\cos[e+fx]\right)^{3/2} \sin[e+fx] \sqrt{c+d \sin[e+fx]}}{\sqrt{2}} \right) \right) \right) / \left(d^{3/2} (-3c+d) \left(i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) - \right. \\
& \left. \left(i \sec\left[\frac{1}{2}(e+fx)\right]^2 \left(ic+d + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (c+id) \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) / \right. \\
& \left. \left(d^{3/2} (-3c+d) \left(i + \tan\left[\frac{1}{2}(e+fx)\right] \right)^2 \right) \right) / \\
& \left(2 \left(ic+d + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (c+id) \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& \left(d^{3/2} (-3c+d) \left(-i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \left(- \left(2 \left[\frac{1}{2}(ic+d) \sec\left[\frac{1}{2}(e+fx)\right]^2 + \frac{(1+i)d^{3/2} \cos[e+fx] \sqrt{\frac{1}{1+\cos[e+fx]}}}{\sqrt{2} \sqrt{c+d \sin[e+fx]}} + \right. \right. \right. \right. \\
& \left. \left. \left. \frac{(1+i)\sqrt{d} \left(\frac{1}{1+\cos[e+fx]\right)^{3/2} \sin[e+fx] \sqrt{c+d \sin[e+fx]}}{\sqrt{2}} \right) \right) \right) / \left(d^{3/2} (-3c+d) \left(-i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \left(c+id + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (ic+d) \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) /
\end{aligned}$$

$$\left(d^{3/2} (-3c+d) \left(-i + \tan\left[\frac{1}{2}(e+fx)\right] \right)^2 \right) \Bigg/$$

$$\left(2 \left(c + id + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d\sin[e+fx]} + (ic+d) \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg)$$

■ **Problem 590: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d\sin[e+fx]}}{\sqrt{a+a\sin[e+fx]}} dx$$

Optimal (type 3, 141 leaves, 5 steps):

$$\frac{2\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{d}\cos[e+fx]}{\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right] - \sqrt{2}\sqrt{c-d} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c-d}\cos[e+fx]}{\sqrt{2}\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{\sqrt{a}f}$$

Result (type 3, 1251 leaves):

$$\left(\left(\sqrt{2}\sqrt{c-d} \operatorname{Log}\left[1 + \tan\left[\frac{1}{2}(e+fx)\right]\right] \right) - \sqrt{2}\sqrt{c-d} \operatorname{Log}\left[c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d\sin[e+fx]} + (-c+d) \tan\left[\frac{1}{2}(e+fx)\right] \right] \right) -$$

$$i\sqrt{d} \operatorname{Log}\left[\frac{2 \left(c - id + (1-i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d\sin[e+fx]} + (-ic+d) \tan\left[\frac{1}{2}(e+fx)\right] \right)}{d^{3/2} \left(i + \tan\left[\frac{1}{2}(e+fx)\right] \right)} \right] -$$

$$\operatorname{Log}\left[\frac{2 \left(c + id + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d\sin[e+fx]} + (ic+d) \tan\left[\frac{1}{2}(e+fx)\right] \right)}{d^{3/2} \left(-i + \tan\left[\frac{1}{2}(e+fx)\right] \right)} \right] \sqrt{c+d\sin[e+fx]} \Bigg/$$

$$\begin{aligned}
& \left(\frac{f \sqrt{a (1 + \sin[e + f x])}}{\sqrt{2} (1 + \tan[\frac{1}{2} (e + f x)])} - \left(\sqrt{2} \sqrt{c-d} \left[\frac{1}{2} (-c+d) \sec\left[\frac{1}{2} (e + f x)\right]^2 + \right. \right. \right. \\
& \left. \left. \frac{\sqrt{c-d} d \cos[e + f x] \sqrt{\frac{1}{1+\cos[e+fx]}}}{\sqrt{c+d \sin[e+fx]}} + \sqrt{c-d} \left(\frac{1}{1+\cos[e+fx]} \right)^{3/2} \sin[e+fx] \sqrt{c+d \sin[e+fx]} \right] \right) / \\
& \left(c-d+2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (-c+d) \tan\left[\frac{1}{2} (e + f x)\right] \right) - \\
& i \sqrt{d} \left(\left(d^{3/2} \left(i + \tan\left[\frac{1}{2} (e + f x)\right] \right) \right) \left(\left(2 \left[\frac{1}{2} (-ic+d) \sec\left[\frac{1}{2} (e + f x)\right]^2 + \frac{(1-i) d^{3/2} \cos[e+fx] \sqrt{\frac{1}{1+\cos[e+fx]}}}{\sqrt{2} \sqrt{c+d \sin[e+fx]}} + \right. \right. \right. \right. \\
& \left. \left. \left. \frac{(1-i) \sqrt{d} \left(\frac{1}{1+\cos[e+fx]} \right)^{3/2} \sin[e+fx] \sqrt{c+d \sin[e+fx]}}{\sqrt{2}} \right] \right) / \left(d^{3/2} \left(i + \tan\left[\frac{1}{2} (e + f x)\right] \right) \right) - \left(\sec\left[\frac{1}{2} (e + f x)\right]^2 \left(c - id + \right. \right. \right. \\
& \left. \left. \left. (1-i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (-ic+d) \tan\left[\frac{1}{2} (e + f x)\right] \right) \right) / \left(d^{3/2} \left(i + \tan\left[\frac{1}{2} (e + f x)\right] \right)^2 \right) \right) / \\
& \left(2 \left(c - id + (1-i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (-ic+d) \tan\left[\frac{1}{2} (e + f x)\right] \right) \right) - \\
& \left(d^{3/2} \left(-i + \tan\left[\frac{1}{2} (e + f x)\right] \right) \right) \left(\left(2 \left[\frac{1}{2} (ic+d) \sec\left[\frac{1}{2} (e + f x)\right]^2 + \frac{(1+i) d^{3/2} \cos[e+fx] \sqrt{\frac{1}{1+\cos[e+fx]}}}{\sqrt{2} \sqrt{c+d \sin[e+fx]}} + \right. \right. \right. \\
& \left. \left. \left. \frac{(1+i) d^{3/2} \cos[e+fx] \sqrt{\frac{1}{1+\cos[e+fx]}}}{\sqrt{2} \sqrt{c+d \sin[e+fx]}} \right] \right) + \right. \\
& \left. \left(d^{3/2} \left(-i + \tan\left[\frac{1}{2} (e + f x)\right] \right) \right) \right)
\end{aligned}$$

$$\left. \left. \left. \frac{(1+i)\sqrt{d} \left(\frac{1}{1+\cos[ex+f x]}\right)^{3/2} \sin[ex+f x] \sqrt{c+d \sin[ex+f x]}}{\sqrt{2}} \right) \right) / \left(d^{3/2} \left(-i + \tan\left[\frac{1}{2}(ex+f x)\right] \right) \right) - \left(\sec\left[\frac{1}{2}(ex+f x)\right] \right)^2 \left(c + i d + \right. \right.$$

$$\left. \left. \left. (1+i)\sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\cos[ex+f x]} \sqrt{c+d \sin[ex+f x]} + (ic+d) \tan\left[\frac{1}{2}(ex+f x)\right]} \right) \right) / \left(d^{3/2} \left(-i + \tan\left[\frac{1}{2}(ex+f x)\right] \right)^2 \right) \right) /$$

$$\left(2 \left(c + i d + (1+i)\sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\cos[ex+f x]} \sqrt{c+d \sin[ex+f x]} + (ic+d) \tan\left[\frac{1}{2}(ex+f x)\right]} \right) \right) \right) \right)$$

■ **Problem 591: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a+a \sin[ex+f x]} \sqrt{c+d \sin[ex+f x]}} dx$$

Optimal (type 3, 79 leaves, 2 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[ex+f x]}{\sqrt{2} \sqrt{a+a \sin[ex+f x]} \sqrt{c+d \sin[ex+f x]}}\right]}{\sqrt{a} \sqrt{c-d} f}$$

Result (type 3, 283 leaves):

$$\left(\operatorname{Log}\left[1 + \tan\left[\frac{1}{2}(ex+f x)\right]\right] - \operatorname{Log}\left[c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[ex+f x]} \sqrt{c+d \sin[ex+f x]} + (-c+d) \tan\left[\frac{1}{2}(ex+f x)\right]}\right] \right) /$$

$$\left(f \sqrt{a(1+\sin[ex+f x])} \sqrt{c+d \sin[ex+f x]} \right)$$

$$\left(\frac{\sec\left[\frac{1}{2}(ex+f x)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(ex+f x)\right]} - \frac{-\frac{1}{2}(c-d) \sec\left[\frac{1}{2}(ex+f x)\right]^2 + \frac{\sqrt{c-d} \left(\frac{1}{1+\cos[ex+f x]}\right)^{3/2} (d+d \cos[ex+f x] + c \sin[ex+f x])}{\sqrt{c+d \sin[ex+f x]}}}{c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[ex+f x]} \sqrt{c+d \sin[ex+f x]} + (-c+d) \tan\left[\frac{1}{2}(ex+f x)\right]}} \right)$$

■ **Problem 592: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 131 leaves, 4 steps) :

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[e+f x]}{\sqrt{2} \sqrt{a+a \sin[e+f x]} \sqrt{c+d \sin[e+f x]}}\right]}{\sqrt{a} (c-d)^{3/2} f} + \frac{2 d \cos[e+f x]}{(c^2 - d^2) f \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}$$

Result (type 3, 306 leaves) :

$$\frac{\frac{2 d \cos[e+f x]}{c+d} + \frac{\operatorname{Log}\left[1 + \tan\left[\frac{1}{2}(e+f x)\right]\right] - \operatorname{Log}\left[c-d+2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+f x]}} \sqrt{c+d \sin[e+f x]} + (-c+d) \tan\left[\frac{1}{2}(e+f x)\right]\right]}{\frac{\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{2+2 \tan\left[\frac{1}{2}(e+f x)\right]} - \frac{\frac{1}{2}(c-d) \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 + \frac{\sqrt{c-d} \left(\frac{1}{1+\cos[e+f x]}\right)^{3/2} (d \cos[e+f x] - c \sin[e+f x])}{\sqrt{c+d \sin[e+f x]}}}{c-d+2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+f x]}} \sqrt{c+d \sin[e+f x]} + (-c+d) \tan\left[\frac{1}{2}(e+f x)\right]}}}{(c-d) f \sqrt{a(1+\sin[e+f x])} \sqrt{c+d \sin[e+f x]}}$$

■ **Problem 593: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 191 leaves, 5 steps) :

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[e+f x]}{\sqrt{2} \sqrt{a+a \sin[e+f x]} \sqrt{c+d \sin[e+f x]}}\right]}{\sqrt{a} (c-d)^{5/2} f} + \frac{2 d \cos[e+f x]}{3 (c^2 - d^2) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^{3/2}} + \frac{2 d (5 c + d) \cos[e+f x]}{3 (c^2 - d^2)^2 f \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}$$

Result (type 3, 387 leaves) :

$$\left(\left(2d \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) (6c^2 + cd - d^2 + d(5c+d)\sin[e+fx]) \right) / \right. \\ \left. ((c+d)^2(c+d\sin[e+fx])) + \right. \\ \left. \left(3 \left(\log\left[1 + \tan\left[\frac{1}{2}(e+fx)\right]\right] - \log\left[c-d + 2\sqrt{c-d}\sqrt{\frac{1}{1+\cos[e+fx]}}\sqrt{c+d\sin[e+fx]} + (-c+d)\tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \right) / \right. \\ \left. \left(\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2}{2 + 2\tan\left[\frac{1}{2}(e+fx)\right]} - \frac{-\frac{1}{2}(c-d)\sec\left[\frac{1}{2}(e+fx)\right]^2 + \frac{\sqrt{c-d}\left(\frac{1}{1+\cos[e+fx]}\right)^{3/2}(d+d\cos[e+fx]+c\sin[e+fx])}{\sqrt{c+d\sin[e+fx]}}}{c-d + 2\sqrt{c-d}\sqrt{\frac{1}{1+\cos[e+fx]}}\sqrt{c+d\sin[e+fx]} + (-c+d)\tan\left[\frac{1}{2}(e+fx)\right]} \right) \right) / \right. \\ \left. (3(c-d)^2 f \sqrt{a(1+\sin[e+fx])}\sqrt{c+d\sin[e+fx]}) \right)$$

■ **Problem 594: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d\sin[e+fx])^{5/2}}{(a+a\sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 251 leaves, 7 steps):

$$-\frac{(5c-3d)d^{3/2}\operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{d}\cos[e+fx]}{\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{a^{3/2}f} - \frac{(c-d)^{3/2}(c+9d)\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c-d}\cos[e+fx]}{\sqrt{2}\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{2\sqrt{2}a^{3/2}f} + \\ \frac{(c-3d)d\cos[e+fx]\sqrt{c+d\sin[e+fx]}}{2af\sqrt{a+a\sin[e+fx]}} - \frac{(c-d)\cos[e+fx](c+d\sin[e+fx])^{3/2}}{2f(a+a\sin[e+fx])^{3/2}}$$

Result (type 3, 1844 leaves):

$$\frac{1}{f(a(1+\sin[e+fx]))^{3/2}} \\ \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \left(-d^2\cos\left[\frac{1}{2}(e+fx)\right] + d^2\sin\left[\frac{1}{2}(e+fx)\right] - \frac{(c-d)^2}{2(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right])} + \right. \\ \left. \frac{c^2\sin\left[\frac{1}{2}(e+fx)\right] - 2cd\sin\left[\frac{1}{2}(e+fx)\right] + d^2\sin\left[\frac{1}{2}(e+fx)\right]}{(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right])^2} \right) \sqrt{c+d\sin[e+fx]} + \left(\frac{(c-d)^{3/2}(c+9d)\log\left[1 + \tan\left[\frac{1}{2}(e+fx)\right]\right]}{\sqrt{2}} + \right.$$

$$i (5c - 3d) d^{3/2} \operatorname{Log} \left[- \frac{i \left(-i c + d + (1 - i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (c - i d) \tan \left[\frac{1}{2} (e + f x) \right] \right)}{d^{5/2} (-5c + 3d) (-i + \tan \left[\frac{1}{2} (e + f x) \right])} \right] +$$

$$i d^{3/2} (-5c + 3d) \operatorname{Log} \left[\frac{i \left(i c + d + (1 + i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (c + i d) \tan \left[\frac{1}{2} (e + f x) \right] \right)}{d^{5/2} (-5c + 3d) (i + \tan \left[\frac{1}{2} (e + f x) \right])} \right] - \frac{1}{\sqrt{2}} (c - d)^{3/2} (c + 9d)$$

$$\operatorname{Log} \left[c - d + 2 \sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan \left[\frac{1}{2} (e + f x) \right] \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^3$$

$$\left(\frac{c^3}{4 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{c + d \sin[e + f x]}} + \frac{7 c^2 d}{4 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{c + d \sin[e + f x]}} - \frac{7 c d^2}{4 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{c + d \sin[e + f x]}} + \frac{3 d^3}{4 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{c + d \sin[e + f x]}} + \frac{5 c d^2 \sin[e + f x]}{2 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{c + d \sin[e + f x]}} - \frac{3 d^3 \sin[e + f x]}{2 \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{c + d \sin[e + f x]}} \right) \Big/$$

$$\left(f (a (1 + \sin[e + f x]))^{3/2} \left(\frac{(c - d)^{3/2} (c + 9d) \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}{2 \sqrt{2} (1 + \tan \left[\frac{1}{2} (e + f x) \right])} - (c - d)^{3/2} (c + 9d) \right) \right)$$

$$\left(\frac{1}{2} (-c + d) \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 + \frac{\sqrt{c - d} d \cos[e + f x] \sqrt{\frac{1}{1 + \cos[e + f x]}}}{\sqrt{c + d \sin[e + f x]}} + \sqrt{c - d} \left(\frac{1}{1 + \cos[e + f x]} \right)^{3/2} \sin[e + f x] \sqrt{c + d \sin[e + f x]} \right) \Big/$$

$$\begin{aligned}
& \left(\sqrt{2} \left(c - d + 2 \sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right] \right) \right) - \\
& \left((5c - 3d) d^4 (-5c + 3d) \left(-i + \tan\left[\frac{1}{2}(e + f x)\right] \right) \left(-i \left(\frac{1}{2}(c - id) \sec\left[\frac{1}{2}(e + f x)\right]^2 + \frac{(1 - i) d^{3/2} \cos[e + f x] \sqrt{\frac{1}{1 + \cos[e + f x]}}}{\sqrt{2} \sqrt{c + d \sin[e + f x]}} \right. \right. \right. \\
& \left. \left. \left. \frac{(1 - i) \sqrt{d} \left(\frac{1}{1 + \cos[e + f x]}\right)^{3/2} \sin[e + f x] \sqrt{c + d \sin[e + f x]}}{\sqrt{2}} \right) \right) \right) / \left(d^{5/2} (-5c + 3d) \left(-i + \tan\left[\frac{1}{2}(e + f x)\right] \right) \right) + \\
& \left(i \sec\left[\frac{1}{2}(e + f x)\right]^2 \left(-ic + d + (1 - i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (c - id) \tan\left[\frac{1}{2}(e + f x)\right] \right) \right) / \\
& \left(2d^{5/2} (-5c + 3d) \left(-i + \tan\left[\frac{1}{2}(e + f x)\right] \right)^2 \right) \right) / \\
& \left(-ic + d + (1 - i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (c - id) \tan\left[\frac{1}{2}(e + f x)\right] \right) + \\
& \left(d^4 (-5c + 3d)^2 \left(i + \tan\left[\frac{1}{2}(e + f x)\right] \right) \left(i \left(\frac{1}{2}(c + id) \sec\left[\frac{1}{2}(e + f x)\right]^2 + \frac{(1 + i) d^{3/2} \cos[e + f x] \sqrt{\frac{1}{1 + \cos[e + f x]}}}{\sqrt{2} \sqrt{c + d \sin[e + f x]}} \right. \right. \right. \\
& \left. \left. \left. \frac{(1 + i) \sqrt{d} \left(\frac{1}{1 + \cos[e + f x]}\right)^{3/2} \sin[e + f x] \sqrt{c + d \sin[e + f x]}}{\sqrt{2}} \right) \right) \right) / \left(d^{5/2} (-5c + 3d) \left(i + \tan\left[\frac{1}{2}(e + f x)\right] \right) \right) -
\end{aligned}$$

$$\left(\left(i \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \right)^2 \left(i c+d+(1+i)\sqrt{2}\sqrt{d}\sqrt{\frac{1}{1+\cos[e+fx]}}\sqrt{c+d\sin[e+fx]}+(c+id)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) /$$

$$\left(2d^{5/2}(-5c+3d)\left(i+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)^2 \right) /$$

$$\left(i c+d+(1+i)\sqrt{2}\sqrt{d}\sqrt{\frac{1}{1+\cos[e+fx]}}\sqrt{c+d\sin[e+fx]}+(c+id)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) /$$

- **Problem 595: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d\sin[e+fx])^{3/2}}{(a+a\sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 194 leaves, 6 steps):

$$\frac{2d^{3/2}\operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{d}\cos[e+fx]}{\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{a^{3/2}f} - \frac{\sqrt{c-d}(c+5d)\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c-d}\cos[e+fx]}{\sqrt{2}\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{2\sqrt{2}a^{3/2}f} - \frac{(c-d)\cos[e+fx]\sqrt{c+d\sin[e+fx]}}{2f(a+a\sin[e+fx])^{3/2}}$$

Result (type 3, 1625 leaves):

$$\frac{1}{f(a(1+\sin[e+fx]))^{3/2}}$$

$$\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \left(\frac{-c+d}{2\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)} + \frac{c\sin\left[\frac{1}{2}(e+fx)\right] - d\sin\left[\frac{1}{2}(e+fx)\right]}{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2} \right) \sqrt{c+d\sin[e+fx]} +$$

$$\left(\left(\sqrt{2}(c^2+4cd-5d^2)\operatorname{Log}\left[1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \right.$$

$$\left. \left. \sqrt{2}(c^2+4cd-5d^2)\operatorname{Log}\left[c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos[e+fx]}}\sqrt{c+d\sin[e+fx]}+(-c+d)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - \right. \right.$$

$$\begin{aligned}
& 4 i \sqrt{c-d} d^{3/2} \left(\operatorname{Log} \left[\frac{c-i \left(d+(1+i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\cos [e+f x]}} \sqrt{c+d \sin [e+f x]} \right) + (-i c+d) \tan \left[\frac{1}{2} (e+f x) \right]}{2 d^{5/2} \left(i+\tan \left[\frac{1}{2} (e+f x) \right] \right)} \right] - \right. \\
& \left. \operatorname{Log} \left[\frac{c+i d+(1+i) \sqrt{2} \sqrt{d} \sqrt{\frac{1}{1+\cos [e+f x]}} \sqrt{c+d \sin [e+f x]} + (i c+d) \tan \left[\frac{1}{2} (e+f x) \right]}{2 d^{5/2} \left(-i+\tan \left[\frac{1}{2} (e+f x) \right] \right)} \right] \right) \left(\cos \left[\frac{1}{2} (e+f x) \right] + \sin \left[\frac{1}{2} (e+f x) \right] \right)^3 \\
& \left(\frac{c^2}{4 \left(\cos \left[\frac{1}{2} (e+f x) \right] + \sin \left[\frac{1}{2} (e+f x) \right] \right) \sqrt{c+d \sin [e+f x]}} + \frac{c d}{\left(\cos \left[\frac{1}{2} (e+f x) \right] + \sin \left[\frac{1}{2} (e+f x) \right] \right) \sqrt{c+d \sin [e+f x]}} - \right. \\
& \left. \frac{d^2}{4 \left(\cos \left[\frac{1}{2} (e+f x) \right] + \sin \left[\frac{1}{2} (e+f x) \right] \right) \sqrt{c+d \sin [e+f x]}} + \frac{d^2 \sin [e+f x]}{\left(\cos \left[\frac{1}{2} (e+f x) \right] + \sin \left[\frac{1}{2} (e+f x) \right] \right) \sqrt{c+d \sin [e+f x]}} \right) \Big/ \\
& \left(f (a (1+\sin [e+f x]))^{3/2} \left(\frac{\left(c^2+4 c d-5 d^2 \right) \sec \left[\frac{1}{2} (e+f x) \right]^2}{\sqrt{2} \left(1+\tan \left[\frac{1}{2} (e+f x) \right] \right)} - \sqrt{2} \left(c^2+4 c d-5 d^2 \right) \right. \right. \\
& \left. \left. \left(\frac{1}{2} (-c+d) \sec \left[\frac{1}{2} (e+f x) \right]^2 + \frac{\sqrt{c-d} d \cos [e+f x] \sqrt{\frac{1}{1+\cos [e+f x]}}}{\sqrt{c+d \sin [e+f x]}} + \sqrt{c-d} \left(\frac{1}{1+\cos [e+f x]} \right)^{3/2} \sin [e+f x] \sqrt{c+d \sin [e+f x]} \right) \right) \Big/ \\
& \left(c-d+2 \sqrt{c-d} \sqrt{\frac{1}{1+\cos [e+f x]}} \sqrt{c+d \sin [e+f x]} + (-c+d) \tan \left[\frac{1}{2} (e+f x) \right] \right) - \\
& 4 i \sqrt{c-d} d^{3/2} \left(\left(2 d^{5/2} \left(i+\tan \left[\frac{1}{2} (e+f x) \right] \right) \right) \left(\left(\frac{1}{2} (-i c+d) \sec \left[\frac{1}{2} (e+f x) \right]^2 - i \left(\frac{(1+i) d^{3/2} \cos [e+f x] \sqrt{\frac{1}{1+\cos [e+f x]}}}{\sqrt{2} \sqrt{c+d \sin [e+f x]}} + \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{(1+i)\sqrt{d}\left(\frac{1}{1+\cos[e+fx]}\right)^{3/2}\sin[e+fx]\sqrt{c+d\sin[e+fx]}}{\sqrt{2}} \right) \right) \right) / \left(2d^{5/2} \left(i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) - \\
& \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \left(c - i \left(d + (1+i)\sqrt{2}\sqrt{d}\sqrt{\frac{1}{1+\cos[e+fx]}\sqrt{c+d\sin[e+fx]}} \right) + (-ic+d)\tan\left[\frac{1}{2}(e+fx)\right] \right) \right) / \left(4d^{5/2} \right. \\
& \left. \left. \left. \left(i + \tan\left[\frac{1}{2}(e+fx)\right] \right)^2 \right) \right) \right) / \left(c - i \left(d + (1+i)\sqrt{2}\sqrt{d}\sqrt{\frac{1}{1+\cos[e+fx]}\sqrt{c+d\sin[e+fx]}} \right) + (-ic+d)\tan\left[\frac{1}{2}(e+fx)\right] \right) - \\
& \left(2d^{5/2} \left(-i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \left(\left(\frac{1}{2}(ic+d)\sec\left[\frac{1}{2}(e+fx)\right]^2 + \frac{(1+i)d^{3/2}\cos[e+fx]\sqrt{\frac{1}{1+\cos[e+fx]}}}{\sqrt{2}\sqrt{c+d\sin[e+fx]}} + \right. \right. \\
& \left. \left. \frac{(1+i)\sqrt{d}\left(\frac{1}{1+\cos[e+fx]}\right)^{3/2}\sin[e+fx]\sqrt{c+d\sin[e+fx]}}{\sqrt{2}} \right) / \left(2d^{5/2} \left(-i + \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) - \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \left(c + id + \right. \right. \right. \\
& \left. \left. \left. (1+i)\sqrt{2}\sqrt{d}\sqrt{\frac{1}{1+\cos[e+fx]}\sqrt{c+d\sin[e+fx]}} + (ic+d)\tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \left(4d^{5/2} \left(-i + \tan\left[\frac{1}{2}(e+fx)\right] \right)^2 \right) \right) \right) / \\
& \left(c + id + (1+i)\sqrt{2}\sqrt{d}\sqrt{\frac{1}{1+\cos[e+fx]}\sqrt{c+d\sin[e+fx]}} + (ic+d)\tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 596: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d\sin[e+fx]}}{(a+a\sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 126 leaves, 4 steps) :

$$-\frac{(c+d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{2 \sqrt{2} a^{3/2} \sqrt{c-d} f} - \frac{\cos[e+fx] \sqrt{c+d \sin[e+fx]}}{2 f (a+a \sin[e+fx])^{3/2}}$$

Result (type 3, 372 leaves) :

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \left(-\frac{2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (c+d \sin[e+fx])}{\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]} + \right. \right. \\ \left. \left. (c+d) \left(\log\left[1 + \tan\left[\frac{1}{2}(e+fx)\right]\right] - \log\left[c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (-c+d) \tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \right) \right) / \\ \left(\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(e+fx)\right]} - \frac{-\frac{1}{2}(c-d) \sec\left[\frac{1}{2}(e+fx)\right]^2 + \frac{\sqrt{c-d} \left(\frac{1}{1+\cos[e+fx]}\right)^{3/2} (d+d \cos[e+fx] + c \sin[e+fx])}{\sqrt{c+d \sin[e+fx]}}}{c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (-c+d) \tan\left[\frac{1}{2}(e+fx)\right]} \right) \right) / \\ (4 f (a (1 + \sin[e+fx]))^{3/2} \sqrt{c+d \sin[e+fx]})$$

■ **Problem 597: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a \sin[e+fx])^{3/2} \sqrt{c+d \sin[e+fx]}} dx$$

Optimal (type 3, 135 leaves, 4 steps) :

$$-\frac{(c-3d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{2 \sqrt{2} a^{3/2} (c-d)^{3/2} f} - \frac{\cos[e+fx] \sqrt{c+d \sin[e+fx]}}{2 (c-d) f (a+a \sin[e+fx])^{3/2}}$$

Result (type 3, 381 leaves) :

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \left(-\frac{2\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)(c+d\sin[e+fx])}{\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]} + \right. \right. \\ \left. \left. (c-3d) \left(\log\left[1 + \tan\left[\frac{1}{2}(e+fx)\right]\right] - \log\left[c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos[e+fx]}}\sqrt{c+d\sin[e+fx]} + (-c+d)\tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \right) \right) / \\ \left(\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2}{2+2\tan\left[\frac{1}{2}(e+fx)\right]} - \frac{-\frac{1}{2}(c-d)\sec\left[\frac{1}{2}(e+fx)\right]^2 + \frac{\sqrt{c-d}\left(\frac{1}{1+\cos[e+fx]}\right)^{3/2}(d+d\cos[e+fx]+c\sin[e+fx])}{\sqrt{c+d\sin[e+fx]}}}{c-d+2\sqrt{c-d}\sqrt{\frac{1}{1+\cos[e+fx]}}\sqrt{c+d\sin[e+fx]} + (-c+d)\tan\left[\frac{1}{2}(e+fx)\right]} \right) \right) / \\ \left(4(c-d)f(a(1+\sin[e+fx]))^{3/2}\sqrt{c+d\sin[e+fx]} \right)$$

■ **Problem 598: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a\sin[e+fx])^{3/2}(c+d\sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 197 leaves, 5 steps):

$$\frac{(c-7d)\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c-d}\cos[e+fx]}{\sqrt{2}\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{2\sqrt{2}a^{3/2}(c-d)^{5/2}f} - \frac{\cos[e+fx]}{2(c-d)f(a+a\sin[e+fx])^{3/2}\sqrt{c+d\sin[e+fx]}} - \frac{d(c+5d)\cos[e+fx]}{2a(c-d)^2(c+d)f\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}$$

Result (type 3, 401 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \left(-\frac{2\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)\left(c^2 + cd + 4d^2 + d(c+5d)\sin[e+fx]\right)}{(c+d)\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)} + \right. \right. \\ \left. \left. (c-7d) \left(\log\left[1 + \tan\left[\frac{1}{2}(e+fx)\right]\right] - \log\left[c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d\sin[e+fx]} + (-c+d)\tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \right) \right) / \\ \left(\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2}{2 + 2\tan\left[\frac{1}{2}(e+fx)\right]} - \frac{-\frac{1}{2}(c-d)\sec\left[\frac{1}{2}(e+fx)\right]^2 + \frac{\sqrt{c-d}\left(\frac{1}{1+\cos[e+fx]}\right)^{3/2}(d+d\cos[e+fx]+c\sin[e+fx])}{\sqrt{c+d\sin[e+fx]}}}{c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d\sin[e+fx]} + (-c+d)\tan\left[\frac{1}{2}(e+fx)\right]} \right) \right) / \\ \left(4(c-d)^2 f (a(1+\sin[e+fx]))^{3/2} \sqrt{c+d\sin[e+fx]} \right)$$

■ **Problem 599: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a\sin[e+fx])^{3/2} (c+d\sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 271 leaves, 6 steps):

$$\frac{(c-11d) \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c-d}\cos[e+fx]}{\sqrt{2}\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{2\sqrt{2}a^{3/2}(c-d)^{7/2}f} - \frac{\cos[e+fx]}{2(c-d)f(a+a\sin[e+fx])^{3/2}(c+d\sin[e+fx])^{3/2}} - \\ \frac{d(3c+7d)\cos[e+fx]}{6a(c-d)^2(c+d)f\sqrt{a+a\sin[e+fx]}(c+d\sin[e+fx])^{3/2}} - \frac{d(3c^2+38cd+19d^2)\cos[e+fx]}{6a(c-d)^3(c+d)^2f\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}$$

Result (type 3, 601 leaves):

$$\frac{1}{f (a (1 + \sin[e + f x]))^{3/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \sqrt{c + d \sin[e + f x]}$$

$$\left(\frac{\sin\left[\frac{1}{2}(e + f x)\right]}{(c - d)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^2} - \frac{1}{2(c - d)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)} - \frac{2(d^2 \cos\left[\frac{1}{2}(e + f x)\right] - d^2 \sin\left[\frac{1}{2}(e + f x)\right])}{3(c - d)^2 (c + d) (c + d \sin[e + f x])^2} - \frac{8(2cd^2 \cos\left[\frac{1}{2}(e + f x)\right] + d^3 \cos\left[\frac{1}{2}(e + f x)\right] - 2cd^2 \sin\left[\frac{1}{2}(e + f x)\right] - d^3 \sin\left[\frac{1}{2}(e + f x)\right])}{3(c - d)^3 (c + d)^2 (c + d \sin[e + f x])} \right) +$$

$$\left((c - 11d) \left(\log\left[1 + \tan\left[\frac{1}{2}(e + f x)\right]\right] - \log\left[c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]\right] \right) \right)$$

$$\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \Big/ \left(4(c - d)^3 f (a (1 + \sin[e + f x]))^{3/2} \sqrt{c + d \sin[e + f x]} \right)$$

$$\left(\frac{\sec\left[\frac{1}{2}(e + f x)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]} - \frac{-\frac{1}{2}(c - d) \sec\left[\frac{1}{2}(e + f x)\right]^2 + \frac{\sqrt{c - d} \left(\frac{1}{1 + \cos[e + f x]}\right)^{3/2} (d + d \cos[e + f x] + c \sin[e + f x])}{\sqrt{c + d \sin[e + f x]}}}{c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]} \right)$$

■ **Problem 600: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^{5/2}}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 260 leaves, 7 steps):

$$\frac{2 d^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}\right]}{a^{5/2} f} - \frac{\sqrt{c - d} (3 c^2 + 14 c d + 43 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c - d} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} f} - \frac{(c - d) (3 c + 11 d) \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{16 a f (a + a \sin[e + f x])^{3/2}} - \frac{(c - d) \cos[e + f x] (c + d \sin[e + f x])^{3/2}}{4 f (a + a \sin[e + f x])^{5/2}}$$

Result (type 3, 1845 leaves):

$$\frac{1}{f (a (1 + \sin[e + f x]))^{5/2}}$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 \left[-\frac{(c-d)^2}{4 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3} - \frac{3(c-d)(c+5d)}{16 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)} + \right. \\
& \left. \frac{3(c^2 \sin\left[\frac{1}{2}(e+fx)\right] + 4cd \sin\left[\frac{1}{2}(e+fx)\right] - 5d^2 \sin\left[\frac{1}{2}(e+fx)\right])}{8 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2} + \frac{c^2 \sin\left[\frac{1}{2}(e+fx)\right] - 2cd \sin\left[\frac{1}{2}(e+fx)\right] + d^2 \sin\left[\frac{1}{2}(e+fx)\right]}{2 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^4} \right] \\
& \sqrt{c+d \sin[e+fx]} + \left(\left(\sqrt{2} (3c^3 + 11c^2d + 29cd^2 - 43d^3) \log\left[1 + \tan\left[\frac{1}{2}(e+fx)\right]\right] - \right. \right. \\
& \left. \left. \sqrt{2} (3c^3 + 11c^2d + 29cd^2 - 43d^3) \log\left[c-d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (-c+d) \tan\left[\frac{1}{2}(e+fx)\right]\right] - \right. \right. \\
& \left. \left. 32i\sqrt{c-d}d^{5/2} \left[\log\left[\frac{c-i \left(d + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} \right) + (-ic+d) \tan\left[\frac{1}{2}(e+fx)\right]}{16d^{7/2} (i + \tan\left[\frac{1}{2}(e+fx)\right])} \right] - \right. \right. \\
& \left. \left. \left. \log\left[\frac{c+id + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[e+fx]}} \sqrt{c+d \sin[e+fx]} + (ic+d) \tan\left[\frac{1}{2}(e+fx)\right]}{16d^{7/2} (-i + \tan\left[\frac{1}{2}(e+fx)\right])} \right] \right] \right) \right) \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 \left(\frac{3c^3}{32 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c+d \sin[e+fx]}} + \right. \\
& \frac{11c^2d}{32 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c+d \sin[e+fx]}} + \frac{29cd^2}{32 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c+d \sin[e+fx]}} - \\
& \left. \frac{11d^3}{32 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c+d \sin[e+fx]}} + \frac{d^3 \sin[e+fx]}{\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c+d \sin[e+fx]}} \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(f (a (1 + \sin[efx]))^{5/2} \left(\frac{(3c^3 + 11c^2d + 29cd^2 - 43d^3) \operatorname{Sec}\left[\frac{1}{2}(efx)\right]^2}{\sqrt{2} (1 + \tan\left[\frac{1}{2}(efx)\right])} - \sqrt{2} (3c^3 + 11c^2d + 29cd^2 - 43d^3) \right. \right. \\
& \left. \left. \left(\frac{1}{2} (-c + d) \operatorname{Sec}\left[\frac{1}{2}(efx)\right]^2 + \frac{\sqrt{c-d} d \cos[efx] \sqrt{\frac{1}{1+\cos[efx]}}}{\sqrt{c+d \sin[efx]}} + \sqrt{c-d} \left(\frac{1}{1+\cos[efx]}\right)^{3/2} \sin[efx] \sqrt{c+d \sin[efx]} \right) \right) \right) / \\
& \left(c - d + 2\sqrt{c-d} \sqrt{\frac{1}{1+\cos[efx]}} \sqrt{c+d \sin[efx]} + (-c+d) \tan\left[\frac{1}{2}(efx)\right] \right) - \\
& 32i\sqrt{c-d} d^{5/2} \left(\left(16d^{7/2} \left(i + \tan\left[\frac{1}{2}(efx)\right] \right) \right) \left(\frac{1}{2} (-ic+d) \operatorname{Sec}\left[\frac{1}{2}(efx)\right]^2 - i \left(\frac{(1+i)d^{3/2} \cos[efx] \sqrt{\frac{1}{1+\cos[efx]}}}{\sqrt{2} \sqrt{c+d \sin[efx]}} + \right. \right. \right. \\
& \left. \left. \left. \frac{(1+i)\sqrt{d} \left(\frac{1}{1+\cos[efx]}\right)^{3/2} \sin[efx] \sqrt{c+d \sin[efx]}}{\sqrt{2}} \right) \right) \right) / \left(16d^{7/2} \left(i + \tan\left[\frac{1}{2}(efx)\right] \right) \right) - \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(efx)\right]^2 \left(c - i \left(d + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[efx]}} \sqrt{c+d \sin[efx]} \right) + (-ic+d) \tan\left[\frac{1}{2}(efx)\right] \right) \right) / \left(32d^{7/2} \right. \\
& \left. \left(i + \tan\left[\frac{1}{2}(efx)\right] \right)^2 \right) \right) / \left(c - i \left(d + (1+i)\sqrt{2}\sqrt{d} \sqrt{\frac{1}{1+\cos[efx]}} \sqrt{c+d \sin[efx]} \right) + (-ic+d) \tan\left[\frac{1}{2}(efx)\right] \right) - \\
& \left(16d^{7/2} \left(-i + \tan\left[\frac{1}{2}(efx)\right] \right) \right) \left(\frac{1}{2} (ic+d) \operatorname{Sec}\left[\frac{1}{2}(efx)\right]^2 + \frac{(1+i)d^{3/2} \cos[efx] \sqrt{\frac{1}{1+\cos[efx]}}}{\sqrt{2} \sqrt{c+d \sin[efx]}} + \right.
\end{aligned}$$

$$\frac{1}{f (a (1 + \sin[e + f x]))^{5/2}}$$

$$\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \left(\frac{-c + d}{4 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3} + \frac{-3c - 7d}{16 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)} + \frac{c \sin\left[\frac{1}{2}(e + f x)\right] - d \sin\left[\frac{1}{2}(e + f x)\right]}{2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4} + \frac{3c \sin\left[\frac{1}{2}(e + f x)\right] + 7d \sin\left[\frac{1}{2}(e + f x)\right]}{8 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2} \right) \sqrt{c + d \sin[e + f x]} +$$

$$\left(3(c + d)^2 \left(\log\left[1 + \tan\left[\frac{1}{2}(e + f x)\right]\right] - \log\left[c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]\right] \right) \right)$$

$$\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 \left/ \left(32 f (a (1 + \sin[e + f x]))^{5/2} \sqrt{c + d \sin[e + f x]} \right) \right.$$

$$\left. \left(\frac{\sec\left[\frac{1}{2}(e + f x)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]} - \frac{-\frac{1}{2}(c - d) \sec\left[\frac{1}{2}(e + f x)\right]^2 + \frac{\sqrt{c - d} \left(\frac{1}{1 + \cos[e + f x]}\right)^{3/2} (d + d \cos[e + f x] + c \sin[e + f x])}{\sqrt{c + d \sin[e + f x]}}}{c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]} \right) \right)$$

■ **Problem 602: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c + d \sin[e + f x]}}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 191 leaves, 5 steps):

$$\frac{(3c - 5d)(c + d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c - d} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} (c - d)^{3/2} f} - \frac{\cos[e + f x] \sqrt{c + d \sin[e + f x]}}{4 f (a + a \sin[e + f x])^{5/2}} - \frac{(3c - d) \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{16 a (c - d) f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 552 leaves):

$$\frac{1}{f (a (1 + \sin[e + f x]))^{5/2}}$$

$$\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \left(\frac{\sin\left[\frac{1}{2}(e + f x)\right]}{2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \frac{1}{4 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3} + \frac{-3c + d}{16(c - d) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)} + \frac{3c \sin\left[\frac{1}{2}(e + f x)\right] - d \sin\left[\frac{1}{2}(e + f x)\right]}{8(c - d) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^2} \right) \sqrt{c + d \sin[e + f x]} + \left((3c^2 - 2cd - 5d^2) \left(\log\left[1 + \tan\left[\frac{1}{2}(e + f x)\right]\right] - \log\left[c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]\right] \right) \right)$$

$$\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 \left/ \left(32(c - d) f (a (1 + \sin[e + f x]))^{5/2} \sqrt{c + d \sin[e + f x]} \right) \right.$$

$$\left. \left(\frac{\sec\left[\frac{1}{2}(e + f x)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]} - \frac{-\frac{1}{2}(c - d) \sec\left[\frac{1}{2}(e + f x)\right]^2 + \frac{\sqrt{c - d} \left(\frac{1}{1 + \cos[e + f x]}\right)^{3/2} (d + d \cos[e + f x] + c \sin[e + f x])}{\sqrt{c + d \sin[e + f x]}}}{c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]} \right) \right)$$

■ **Problem 603: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \sin[e + f x])^{5/2} \sqrt{c + d \sin[e + f x]}} dx$$

Optimal (type 3, 201 leaves, 5 steps):

$$-\frac{(3c^2 - 10cd + 19d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c - d} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}\right]}{16\sqrt{2} a^{5/2} (c - d)^{5/2} f} - \frac{\cos[e + f x] \sqrt{c + d \sin[e + f x]}}{4(c - d) f (a + a \sin[e + f x])^{5/2}} - \frac{3(c - 3d) \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{16a(c - d)^2 f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 565 leaves):

$$\frac{1}{f (a (1 + \sin[e + f x]))^{5/2}}$$

$$\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \left(\frac{\sin\left[\frac{1}{2}(e + f x)\right]}{2(c-d) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \frac{1}{4(c-d) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3} - \frac{3(c-3d)}{16(c-d)^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)} + \frac{3(c \sin\left[\frac{1}{2}(e + f x)\right] - 3d \sin\left[\frac{1}{2}(e + f x)\right])}{8(c-d)^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^2} \sqrt{c+d \sin[e + f x]} + \left((3c^2 - 10cd + 19d^2) \left(\log\left[1 + \tan\left[\frac{1}{2}(e + f x)\right]\right] - \log\left[c - d + 2\sqrt{c-d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c+d \sin[e + f x]} + (-c+d) \tan\left[\frac{1}{2}(e + f x)\right]\right] \right) \right) \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 \Bigg/ \left(32(c-d)^2 f (a (1 + \sin[e + f x]))^{5/2} \sqrt{c+d \sin[e + f x]} \right)$$

$$\left(\frac{\sec\left[\frac{1}{2}(e + f x)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]} - \frac{-\frac{1}{2}(c-d) \sec\left[\frac{1}{2}(e + f x)\right]^2 + \frac{\sqrt{c-d} \left(\frac{1}{1 + \cos[e + f x]}\right)^{3/2} (d+d \cos[e + f x] + c \sin[e + f x])}{\sqrt{c+d \sin[e + f x]}}}{c - d + 2\sqrt{c-d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c+d \sin[e + f x]} + (-c+d) \tan\left[\frac{1}{2}(e + f x)\right]} \right)$$

■ **Problem 604: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 270 leaves, 6 steps):

$$\frac{3(c^2 - 6cd + 25d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}\right]}{16\sqrt{2} a^{5/2} (c-d)^{7/2} f} - \frac{\cos[e + f x]}{4(c-d) f (a + a \sin[e + f x])^{5/2} \sqrt{c + d \sin[e + f x]}}$$

$$\frac{(3c - 13d) \cos[e + f x]}{16a(c-d)^2 f (a + a \sin[e + f x])^{3/2} \sqrt{c + d \sin[e + f x]}} - \frac{(c-7d)d(3c+7d) \cos[e + f x]}{16a^2(c-d)^3(c+d) f \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}$$

Result (type 3, 622 leaves):

$$\frac{1}{f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \sqrt{c + d \sin[e + f x]} \left(\frac{\sin\left[\frac{1}{2}(e + f x)\right]}{2 (c - d)^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \frac{1}{4 (c - d)^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3} + \frac{-3c + 17d}{16 (c - d)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)} + \frac{3c \sin\left[\frac{1}{2}(e + f x)\right] - 17d \sin\left[\frac{1}{2}(e + f x)\right]}{8 (c - d)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^2} + \frac{2 (d^3 \cos\left[\frac{1}{2}(e + f x)\right] - d^3 \sin\left[\frac{1}{2}(e + f x)\right])}{(c - d)^3 (c + d) (c + d \sin[e + f x])} \right) + \left(3 (c^2 - 6cd + 25d^2) \left(\log\left[1 + \tan\left[\frac{1}{2}(e + f x)\right]\right] - \log\left[c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]\right] \right) \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 \Bigg/ \left(32 (c - d)^3 f (a (1 + \sin[e + f x]))^{5/2} \sqrt{c + d \sin[e + f x]} \right) \left(\frac{\sec\left[\frac{1}{2}(e + f x)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]} - \frac{-\frac{1}{2}(c - d) \sec\left[\frac{1}{2}(e + f x)\right]^2 + \frac{\sqrt{c - d} \left(\frac{1}{1 + \cos[e + f x]}\right)^{3/2} (d + d \cos[e + f x] + c \sin[e + f x])}{\sqrt{c + d \sin[e + f x]}}}{c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]} \right)$$

■ **Problem 605: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 355 leaves, 7 steps):

$$\frac{(3c^2 - 26cd + 163d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c - d} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}\right]}{16\sqrt{2} a^{5/2} (c - d)^{9/2} f} - \frac{\cos[e + f x]}{4(c - d) f (a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])^{3/2}} - \frac{(3c - 17d) \cos[e + f x]}{16a(c - d)^2 f (a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])^{3/2}} - \frac{d(9c^2 - 54cd - 95d^2) \cos[e + f x]}{48a^2(c - d)^3 (c + d) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^{3/2}} - \frac{d(9c^3 - 57c^2d - 493cd^2 - 299d^3) \cos[e + f x]}{48a^2(c - d)^4 (c + d)^2 f \sqrt{a + a \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}$$

Result (type 3, 717 leaves):

$$\frac{1}{f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \sqrt{c + d \sin[e + f x]} \left(\frac{\sin\left[\frac{1}{2}(e + f x)\right]}{2 (c - d)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \frac{1}{4 (c - d)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^3} + \frac{-3c + 25d}{16 (c - d)^4 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)} + \frac{3c \sin\left[\frac{1}{2}(e + f x)\right] - 25d \sin\left[\frac{1}{2}(e + f x)\right]}{8 (c - d)^4 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^2} + \frac{2 (d^3 \cos\left[\frac{1}{2}(e + f x)\right] - d^3 \sin\left[\frac{1}{2}(e + f x)\right])}{3 (c - d)^3 (c + d) (c + d \sin[e + f x])^2} + \frac{2 (11c d^3 \cos\left[\frac{1}{2}(e + f x)\right] + 7d^4 \cos\left[\frac{1}{2}(e + f x)\right] - 11c d^3 \sin\left[\frac{1}{2}(e + f x)\right] - 7d^4 \sin\left[\frac{1}{2}(e + f x)\right])}{3 (c - d)^4 (c + d)^2 (c + d \sin[e + f x])} \right) + \left((3c^2 - 26cd + 163d^2) \left(\log\left[1 + \tan\left[\frac{1}{2}(e + f x)\right]\right] - \log\left[c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]\right] \right) \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 \Bigg/ \left(32 (c - d)^4 f (a (1 + \sin[e + f x]))^{5/2} \sqrt{c + d \sin[e + f x]} \right) \left(\frac{\sec\left[\frac{1}{2}(e + f x)\right]^2}{2 + 2 \tan\left[\frac{1}{2}(e + f x)\right]} - \frac{-\frac{1}{2}(c - d) \sec\left[\frac{1}{2}(e + f x)\right]^2 + \frac{\sqrt{c - d} \left(\frac{1}{1 + \cos[e + f x]}\right)^{3/2} (d + d \cos[e + f x] + c \sin[e + f x])}{\sqrt{c + d \sin[e + f x]}}}{c - d + 2\sqrt{c - d} \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{c + d \sin[e + f x]} + (-c + d) \tan\left[\frac{1}{2}(e + f x)\right]} \right)$$

■ **Problem 606: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 129 leaves, 4 steps):

$$\left(\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, -n, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d}\right] \cos[e + f x] (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c - d}\right)^{-n} \right) \Bigg/ \left(f (1 + 2m) \sqrt{1 - \sin[e + f x]} \right)$$

Result (type 6, 373 leaves):

$$\begin{aligned}
& \left(6 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -n, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right. \\
& \quad \left. \left(\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right)^{-\frac{1}{2}+m} \cot \left[\frac{1}{4} (2e + \pi + 2fx) \right] (a(1 + \sin[e + fx]))^m (c+d \sin[e + fx])^n \left(\sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2 \right)^{\frac{1}{2}-m} \right) / \\
& \left(f \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -n, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] + \right. \right. \\
& \quad \left. \left(4dn \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 1-n, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] + \right. \right. \\
& \quad \left. \left. (c+d)(-1+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, -n, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right) \Bigg)
\end{aligned}$$

■ **Problem 607: Attempted integration timed out after 120 seconds.**

$$\int (a + a \sin[e + fx])^m (c + d \sin[e + fx])^3 dx$$

Optimal (type 5, 320 leaves, 6 steps):

$$\begin{aligned}
& \frac{d (d^2 (4+m) - cd (5-3m-2m^2) + 2c^2 (8+6m+m^2)) \cos[e + fx] (a + a \sin[e + fx])^m}{f (1+m) (2+m) (3+m)} - \\
& \frac{1}{f (1+m) (2+m) (3+m)} 2^{\frac{1}{2}+m} (d^3 m (5+3m+m^2) + 3c^2 d m (6+5m+m^2) + 3cd^2 (3+4m+4m^2+m^3) + c^3 (6+11m+6m^2+m^3)) \\
& \cos[e + fx] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + fx]) \right] (1 + \sin[e + fx])^{-\frac{1}{2}-m} (a + a \sin[e + fx])^m - \\
& \frac{d^2 (dm + c(5+m)) \cos[e + fx] (a + a \sin[e + fx])^{1+m}}{af (2+m) (3+m)} - \frac{d \cos[e + fx] (a + a \sin[e + fx])^m (c + d \sin[e + fx])^2}{f (3+m)}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 608: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m (c + d \sin[e + fx])^2 dx$$

Optimal (type 5, 193 leaves, 4 steps):

$$\frac{d(d-2c(2+m))\cos[e+fx](a+a\sin[e+fx])^m}{f(1+m)(2+m)} - \frac{1}{f(1+m)(2+m)} 2^{\frac{1}{2}+m} (2cdm(2+m) + d^2(1+m+m^2) + c^2(2+3m+m^2)) \cos[e+fx]$$

$$\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-m, \frac{3}{2}, \frac{1}{2}(1-\sin[e+fx])\right] (1+\sin[e+fx])^{-\frac{1}{2}-m} (a+a\sin[e+fx])^m - \frac{d^2 \cos[e+fx](a+a\sin[e+fx])^{1+m}}{af(2+m)}$$

Result (type 5, 1774 leaves):

$$-\left(2\left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^{\frac{1}{2}-m}\left(1-\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^{-\frac{1}{2}+m}\left(c+d-2d\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right)^2$$

$$\left(4\Gamma\left[\frac{3}{2}-m\right]\text{HypergeometricPFQ}\left[\left\{\frac{3}{2}, 2, \frac{3}{2}-m\right\}, \left\{1, \frac{9}{2}\right\}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right.$$

$$\left.\left(c+d-2d\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2+16\Gamma\left[\frac{3}{2}-m\right]\text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{3}{2}-m, \frac{9}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right.$$

$$\left.\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\left(c^2+cd\left(2-3\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)+d^2\left(1-3\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2+2\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4\right)\right)\right)+$$

$$7\Gamma\left[\frac{1}{2}-m\right]\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-m, \frac{7}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\left(15c^2+10cd\left(3-2\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)+\right.$$

$$\left.\left.d^2\left(15-20\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2+12\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4\right)\right)\right)\left(a+a\sin[e+fx]\right)^m \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)/$$

$$\left(f\left(4\Gamma\left[\frac{3}{2}-m\right]\text{HypergeometricPFQ}\left[\left\{\frac{3}{2}, 2, \frac{3}{2}-m\right\}, \left\{1, \frac{9}{2}\right\}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right.\right.$$

$$\left.\left(c+d-2d\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2+16\Gamma\left[\frac{3}{2}-m\right]\text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{3}{2}-m, \frac{9}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right.\right.$$

$$\left.\left.\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\left(c^2+cd\left(2-3\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)+d^2\left(1-3\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2+2\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4\right)\right)\right)\right)+$$

$$7\Gamma\left[\frac{1}{2}-m\right]\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-m, \frac{7}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]$$

$$\left(15c^2+10cd\left(3-2\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)+d^2\left(15-20\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2+12\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4\right)\right)+$$

$$\frac{2}{3}\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\left(-48d\Gamma\left[\frac{3}{2}-m\right]\text{HypergeometricPFQ}\left[\left\{\frac{3}{2}, 2, \frac{3}{2}-m\right\}, \left\{1, \frac{9}{2}\right\}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right.$$

$$\left.\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\left(c+d-2d\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right)+$$

$$12\Gamma\left[\frac{3}{2}-m\right]\text{HypergeometricPFQ}\left[\left\{\frac{3}{2}, 2, \frac{3}{2}-m\right\}, \left\{1, \frac{9}{2}\right\}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\left(c+d-2d\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right)^2-$$

$$\begin{aligned}
& 4 (-3 + 2 m) \text{Gamma}\left[\frac{3}{2} - m\right] \text{HypergeometricPFQ}\left[\left\{\frac{5}{2}, 3, \frac{5}{2} - m\right\}, \left\{2, \frac{11}{2}\right\}, \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \\
& \left(c + d - 2 d \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^2 + 48 d \text{Gamma}\left[\frac{3}{2} - m\right] \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{3}{2} - m, \frac{9}{2}, \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \\
& \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(-3 c \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] + d \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(-3 + 4 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)\right) + \\
& 84 d \text{Gamma}\left[\frac{1}{2} - m\right] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{7}{2}, \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(-5 c + d \left(-5 + 6 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)\right) + \\
& 48 \text{Gamma}\left[\frac{3}{2} - m\right] \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{3}{2} - m, \frac{9}{2}, \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \\
& \left(c^2 + c d \left(2 - 3 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) + d^2 \left(1 - 3 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 + 2 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4\right)\right) - \\
& 8 (-3 + 2 m) \text{Gamma}\left[\frac{3}{2} - m\right] \text{Hypergeometric2F1}\left[\frac{5}{2}, \frac{5}{2} - m, \frac{11}{2}, \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \\
& \left(c^2 + c d \left(2 - 3 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) + d^2 \left(1 - 3 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 + 2 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4\right)\right) + \\
& 3 \left(\frac{1}{2} - m\right) \text{Gamma}\left[\frac{1}{2} - m\right] \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{3}{2} - m, \frac{9}{2}, \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \\
& \left(15 c^2 + 10 c d \left(3 - 2 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) + d^2 \left(15 - 20 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 + 12 \text{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4\right)\right) \right) \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 609: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \text{Sin}[e + f x])^m (c + d \text{Sin}[e + f x]) dx$$

Optimal (type 5, 117 leaves, 3 steps):

$$-\frac{d \text{Cos}[e + f x] (a + a \text{Sin}[e + f x])^m}{f (1 + m)} - \frac{1}{f (1 + m)}$$

$$2^{\frac{1}{2}+m} (c + c m + d m) \text{Cos}[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2} (1 - \text{Sin}[e + f x])\right] (1 + \text{Sin}[e + f x])^{-\frac{1}{2}-m} (a + a \text{Sin}[e + f x])^m$$

Result (type 5, 295 leaves):

$$\begin{aligned}
& -\frac{1}{f} (a (1 + \sin[e + f x]))^m \left(\frac{1}{-1 + m^2} 2^{-1-2m} d e^{-i(e+fx)} (1 + i e^{-i(e+fx)})^{-2m} \left(e^{-\frac{i}{4}(2e+\pi+2fx)} (i + e^{i(e+fx)}) \right)^{2m} \right. \\
& \quad \left. (e^{2i(e+fx)} (-1 + m) \operatorname{Hypergeometric2F1}[-1 - m, -2m, -m, -i e^{-i(e+fx)}] - (1 + m) \operatorname{Hypergeometric2F1}[1 - m, -2m, 2 - m, -i e^{-i(e+fx)}]) + \right. \\
& \quad \left. \left(2\sqrt{2} c \cos\left[\frac{1}{4}(2e - \pi + 2fx)\right] \right)^{1+2m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{3}{2} + m, \sin\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2\right] \sin\left[\frac{1}{4}(2e - \pi + 2fx)\right] \right) / \\
& \quad \left. \left((1 + 2m) \sqrt{1 - \sin[e + f x]} \right) \right) \sin\left[\frac{1}{4}(2e + \pi + 2fx)\right]^{-2m}
\end{aligned}$$

■ **Problem 611: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m}{c + d \sin[e + f x]} dx$$

Optimal (type 6, 100 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, 1, \frac{3}{2} + m, \frac{1}{2}(1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c-d}\right] \cos[e + f x] (a + a \sin[e + f x])^m}{(c - d) f (1 + 2m) \sqrt{1 - \sin[e + f x]}}$$

Result (type 6, 363 leaves):

$$\begin{aligned}
& -\left(\left(6(c + d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} - m, 1, \frac{3}{2}, \cos\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4}(2e - \pi + 2fx)\right]^2}{c + d}\right] \right)^2 \right. \\
& \quad \left. \left(\cos\left[\frac{1}{4}(2e - \pi + 2fx)\right]^2 \right)^{-\frac{1}{2}+m} \cot\left[\frac{1}{4}(2e + \pi + 2fx)\right] (a(1 + \sin[e + f x]))^m \left(\sin\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2 \right)^{\frac{1}{2}-m} \right) / \right. \\
& \quad \left(f(c + d \sin[e + f x]) \left(3(c + d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} - m, 1, \frac{3}{2}, \cos\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4}(2e - \pi + 2fx)\right]^2}{c + d}\right] \right)^2 + \right. \\
& \quad \left. \left(4d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - m, 2, \frac{5}{2}, \cos\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4}(2e - \pi + 2fx)\right]^2}{c + d}\right] \right)^2 - \right. \\
& \quad \left. \left. (c + d) (-1 + 2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} - m, 1, \frac{5}{2}, \cos\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4}(2e - \pi + 2fx)\right]^2}{c + d}\right] \right) \sin\left[\frac{1}{4}(2e - \pi + 2fx)\right]^2 \right) \right)
\end{aligned}$$

■ **Problem 612: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m}{(c + d \sin[e + f x])^2} dx$$

Optimal (type 6, 100 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, 2, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c - d}\right] \cos[e + f x] (a + a \sin[e + f x])^m}{(c - d)^2 f (1 + 2m) \sqrt{1 - \sin[e + f x]}}$$

Result (type 6, 363 leaves):

$$\begin{aligned} & - \left(\left(6 (c + d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} - m, 2, \frac{3}{2}, \cos\left[\frac{1}{4} (2e + \pi + 2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2}{c + d}\right] \right. \right. \\ & \quad \left. \left(\cos\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2 \right)^{-\frac{1}{2} + m} \cot\left[\frac{1}{4} (2e + \pi + 2fx)\right] (a (1 + \sin[e + f x]))^m \left(\sin\left[\frac{1}{4} (2e + \pi + 2fx)\right]^2 \right)^{\frac{1}{2} - m} \right) / \right. \\ & \quad \left. \left(f (c + d \sin[e + f x])^2 \left(3 (c + d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} - m, 2, \frac{3}{2}, \cos\left[\frac{1}{4} (2e + \pi + 2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2}{c + d}\right] \right. \right. \right. \\ & \quad \left. \left. \left(8d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - m, 3, \frac{5}{2}, \cos\left[\frac{1}{4} (2e + \pi + 2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2}{c + d}\right] \right) - \right. \right. \\ & \quad \left. \left. \left. (c + d) (-1 + 2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} - m, 2, \frac{5}{2}, \cos\left[\frac{1}{4} (2e + \pi + 2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2}{c + d}\right] \right) \sin\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2 \right) \right) \right) \end{aligned}$$

■ **Problem 613: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m}{(c + d \sin[e + f x])^3} dx$$

Optimal (type 6, 100 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, 3, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c - d}\right] \cos[e + f x] (a + a \sin[e + f x])^m}{(c - d)^3 f (1 + 2m) \sqrt{1 - \sin[e + f x]}}$$

Result (type 6, 363 leaves):

$$\begin{aligned}
& - \left(\left(6 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 3, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right. \right. \\
& \quad \left. \left(\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right)^{-\frac{1}{2}+m} \cot \left[\frac{1}{4} (2e + \pi + 2fx) \right] (a (1 + \sin[e + fx]))^m \left(\sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2 \right)^{\frac{1}{2}-m} \right) / \\
& \quad \left(f (c+d \sin[e + fx])^3 \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 3, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right. \right. \\
& \quad \left. \left(12d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 4, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right. \right. \\
& \quad \left. \left. (c+d) (-1+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, 3, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 614: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m (c + d \sin[e + fx])^{5/2} dx$$

Optimal (type 6, 138 leaves, 4 steps):

$$\begin{aligned}
& \left(\sqrt{2} (c-d)^2 \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, -\frac{5}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + fx]), -\frac{d(1 + \sin[e + fx])}{c-d} \right] \cos[e + fx] (a + a \sin[e + fx])^m \sqrt{c + d \sin[e + fx]} \right) / \\
& \left(f (1 + 2m) \sqrt{1 - \sin[e + fx]} \sqrt{\frac{c + d \sin[e + fx]}{c - d}} \right)
\end{aligned}$$

Result (type 6, 365 leaves):

$$\begin{aligned}
& - \left(\left(3 \sqrt{2} (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -\frac{5}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \right. \\
& \quad \left. \sqrt{1 + \sin[e + fx]} (a(1 + \sin[e + fx]))^m (c + d \sin[e + fx])^{5/2} \tan \left[\frac{1}{4} (2e - \pi + 2fx) \right] \right) / \\
& \left(f \sqrt{\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2} \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -\frac{5}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) + \right. \\
& \quad \left(10d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, -\frac{3}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) + \\
& \quad \left. (c+d) (-1 + 2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, -\frac{5}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right) \right)
\end{aligned}$$

■ **Problem 615: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m (c + d \sin[e + fx])^{3/2} dx$$

Optimal (type 6, 136 leaves, 4 steps):

$$\begin{aligned}
& \left(\sqrt{2} (c-d) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, -\frac{3}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + fx]), -\frac{d(1 + \sin[e + fx])}{c-d} \right] \cos[e + fx] (a + a \sin[e + fx])^m \sqrt{c + d \sin[e + fx]} \right) / \\
& \left(f (1 + 2m) \sqrt{1 - \sin[e + fx]} \sqrt{\frac{c + d \sin[e + fx]}{c - d}} \right)
\end{aligned}$$

Result (type 6, 365 leaves):

$$\begin{aligned}
& - \left(\left(3 \sqrt{2} (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -\frac{3}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right. \right. \\
& \quad \left. \left. \sqrt{1 + \sin[e + fx]} (a(1 + \sin[e + fx]))^m (c+d \sin[e + fx])^{3/2} \tan \left[\frac{1}{4} (2e - \pi + 2fx) \right] \right) \right) / \\
& \left(f \sqrt{\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2} \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -\frac{3}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] + \right. \right. \\
& \quad \left(6d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, -\frac{1}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] + \right. \\
& \quad \left. \left. (c+d) (-1 + 2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, -\frac{3}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 616: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m \sqrt{c + d \sin[e + fx]} \, dx$$

Optimal (type 6, 131 leaves, 4 steps):

$$\begin{aligned}
& \left(\sqrt{2} \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + fx]), -\frac{d(1 + \sin[e + fx])}{c-d} \right] \cos[e + fx] (a + a \sin[e + fx])^m \sqrt{c + d \sin[e + fx]} \right) / \\
& \left(f (1 + 2m) \sqrt{1 - \sin[e + fx]} \sqrt{\frac{c + d \sin[e + fx]}{c - d}} \right)
\end{aligned}$$

Result (type 6, 365 leaves):

$$\begin{aligned}
& - \left(\left(3 \sqrt{2} (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -\frac{1}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \right. \\
& \quad \left. \sqrt{1 + \sin[e + fx]} (a(1 + \sin[e + fx]))^m \sqrt{c + d \sin[e + fx]} \tan \left[\frac{1}{4} (2e - \pi + 2fx) \right] \right) / \\
& \left(f \sqrt{\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2} \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -\frac{1}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) + \right. \\
& \quad \left(2d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{1}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) + \\
& \quad \left. (c+d) (-1 + 2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, -\frac{1}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right) \right)
\end{aligned}$$

■ **Problem 617: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^m}{\sqrt{c + d \sin[e + fx]}} dx$$

Optimal (type 6, 131 leaves, 4 steps):

$$\begin{aligned}
& \left(\sqrt{2} \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + fx]), -\frac{d(1 + \sin[e + fx])}{c-d} \right] \cos[e + fx] (a + a \sin[e + fx])^m \sqrt{\frac{c + d \sin[e + fx]}{c-d}} \right) / \\
& \left(f (1 + 2m) \sqrt{1 - \sin[e + fx]} \sqrt{c + d \sin[e + fx]} \right)
\end{aligned}$$

Result (type 6, 373 leaves):

$$\begin{aligned}
& - \left(\left(6 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{1}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \right. \\
& \quad \left. \left(\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right] \right)^{-\frac{1}{2}+m} \cot \left[\frac{1}{4} (2e + \pi + 2fx) \right] (a (1 + \sin[e + fx]))^m \left(\sin \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right)^{\frac{1}{2}-m} \right) / \\
& \quad \left(f \sqrt{c+d \sin[e + fx]} \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{1}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \right. \\
& \quad \left. \left(2d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) - \right. \\
& \quad \left. \left. (c+d) (-1+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, \frac{1}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \right) \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right) \right)
\end{aligned}$$

■ **Problem 618: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^m}{(c + d \sin[e + fx])^{3/2}} dx$$

Optimal (type 6, 138 leaves, 4 steps):

$$\begin{aligned}
& \left(\sqrt{2} \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + fx]), -\frac{d (1 + \sin[e + fx])}{c-d} \right] \cos[e + fx] (a + a \sin[e + fx])^m \sqrt{\frac{c+d \sin[e + fx]}{c-d}} \right) / \\
& \quad \left((c-d) f (1+2m) \sqrt{1 - \sin[e + fx]} \sqrt{c+d \sin[e + fx]} \right)
\end{aligned}$$

Result (type 6, 373 leaves):

$$\begin{aligned}
& - \left(\left(6 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right. \right. \\
& \quad \left. \left(\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right)^{-\frac{1}{2}+m} \cot \left[\frac{1}{4} (2e + \pi + 2fx) \right] (a (1 + \sin[e + fx]))^m \left(\sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2 \right)^{\frac{1}{2}-m} \right) / \\
& \quad \left(f (c+d \sin[e + fx])^{3/2} \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right. \right. \\
& \quad \left. \left(6d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{5}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right. \right. \\
& \quad \left. \left. (c+d) (-1+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, \frac{3}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 619: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^m}{(c + d \sin[e + fx])^{5/2}} dx$$

Optimal (type 6, 138 leaves, 4 steps):

$$\begin{aligned}
& \left(\sqrt{2} \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, \frac{5}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + fx]), -\frac{d (1 + \sin[e + fx])}{c-d} \right] \cos[e + fx] (a + a \sin[e + fx])^m \sqrt{\frac{c + d \sin[e + fx]}{c-d}} \right) / \\
& \left((c-d)^2 f (1+2m) \sqrt{1 - \sin[e + fx]} \sqrt{c + d \sin[e + fx]} \right)
\end{aligned}$$

Result (type 6, 373 leaves):

$$\begin{aligned}
& - \left(\left(6 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{5}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \right. \\
& \quad \left. \left(\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right)^{-\frac{1}{2}+m} \cot \left[\frac{1}{4} (2e + \pi + 2fx) \right] (a (1 + \sin[e + fx]))^m \left(\sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2 \right)^{\frac{1}{2}-m} \right) / \right. \\
& \quad \left(f (c+d \sin[e + fx])^{5/2} \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{5}{2}, \frac{3}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \right. \\
& \quad \left. \left(10d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{7}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) - \right. \\
& \quad \left. \left. (c+d) (-1+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, \frac{5}{2}, \frac{5}{2}, \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2, \frac{2d \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{c+d} \right) \right) \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right) \Big) \Big)
\end{aligned}$$

■ **Problem 628: Result more than twice size of optimal antiderivative.**

$$\int (3 - 3 \sin[e + fx])^{-1-m} (1 + \sin[e + fx])^m dx$$

Optimal (type 3, 43 leaves, 1 step):

$$\frac{\cos[e + fx] (3 - 3 \sin[e + fx])^{-1-m} (1 + \sin[e + fx])^m}{f (1 + 2m)}$$

Result (type 3, 97 leaves):

$$\frac{1}{3 (f + 2fm)} \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right]^{-1-2m} \left(\cos \left[\frac{1}{2} (e + fx) \right] - \sin \left[\frac{1}{2} (e + fx) \right] \right)^{2m} (6 - 6 \sin[e + fx])^{-m} (1 + \sin[e + fx])^m \sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]$$

■ **Problem 629: Result more than twice size of optimal antiderivative.**

$$\int (3 - 4 \sin[e + fx])^{-1-m} (1 + \sin[e + fx])^m dx$$

Optimal (type 5, 83 leaves, 2 steps):

$$\frac{1}{f (1 + \sin[e + fx])} 2^{1+m} \cos[e + fx] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 + m, \frac{3}{2}, \frac{7 (1 - \sin[e + fx])}{1 + \sin[e + fx]} \right] (3 - 4 \sin[e + fx])^{-m} (-3 + 4 \sin[e + fx])^m$$

Result (type 5, 176 leaves):

$$\frac{1}{f} 2 \left(\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right)^{-\frac{1}{2}+m} \cot \left[\frac{1}{4} (2e + \pi + 2fx) \right] \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{7 \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{3 - 4 \sin[e + fx]} \right]$$

$$(3 - 4 \sin[e + fx])^{-m} (1 + \sin[e + fx])^m \left(\frac{\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{-3 + 4 \sin[e + fx]} \right)^{\frac{1}{2}-m} \left(\sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2 \right)^{\frac{1}{2}-m}$$

■ **Problem 630: Result more than twice size of optimal antiderivative.**

$$\int (3 - 5 \sin[e + fx])^{-1-m} (1 + \sin[e + fx])^m dx$$

Optimal (type 5, 78 leaves, 2 steps):

$$\frac{\cos[e + fx] \text{Hypergeometric2F1} \left[\frac{1}{2}, 1 + m, \frac{3}{2}, \frac{4(1 - \sin[e + fx])}{1 + \sin[e + fx]} \right] (3 - 5 \sin[e + fx])^{-m} (-3 + 5 \sin[e + fx])^m}{f (1 + \sin[e + fx])}$$

Result (type 5, 184 leaves):

$$\frac{1}{f} 2^{\frac{1}{2}-m} \left(\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2 \right)^{-\frac{1}{2}+m} \cot \left[\frac{1}{4} (2e + \pi + 2fx) \right] \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{8 \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{3 - 5 \sin[e + fx]} \right]$$

$$(3 - 5 \sin[e + fx])^{-m} (1 + \sin[e + fx])^m \left(\frac{\cos \left[\frac{1}{4} (2e - \pi + 2fx) \right]^2}{-3 + 5 \sin[e + fx]} \right)^{\frac{1}{2}-m} \left(\sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^2 \right)^{\frac{1}{2}-m}$$

■ **Problem 633: Result more than twice size of optimal antiderivative.**

$$\int (3 + 3 \sin[e + fx])^{-1-m} (a + a \sin[e + fx])^m dx$$

Optimal (type 3, 39 leaves, 2 steps):

$$\frac{\cos[e + fx] (3 + 3 \sin[e + fx])^{-1-m} (a + a \sin[e + fx])^m}{f}$$

Result (type 3, 104 leaves):

$$-\frac{1}{f} 2^{-m} 3^{-1-m} \cos \left[\frac{1}{4} (2e + \pi + 2fx) \right] \left(\cos \left[\frac{1}{2} (e + fx) \right] + \sin \left[\frac{1}{2} (e + fx) \right] \right)^{2(1+m)}$$

$$(1 + \sin[e + fx])^{-1-m} (a (1 + \sin[e + fx]))^m \sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^{-1-2m}$$

■ **Problem 634: Result more than twice size of optimal antiderivative.**

$$\int (3 + 2 \sin[e + fx])^{-1-m} (a + a \sin[e + fx])^m dx$$

Optimal (type 5, 83 leaves, 2 steps):

$$-\frac{1}{f} \left(\frac{5}{2}\right)^{-1-m} \cos[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{a-a\sin[e+fx]}{5(a+a\sin[e+fx])}\right] (1+\sin[e+fx])^{-1-m} (a+a\sin[e+fx])^m$$

Result (type 5, 179 leaves):

$$-\frac{1}{f} 2 \times 5^{-1-m} \left(\cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^2\right)^{-\frac{1}{2}+m} \cot\left[\frac{1}{4}(2e+\pi+2fx)\right] \\ \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{1}{5}\cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2 \sec\left[\frac{1}{4}(2e-\pi+2fx)\right]^2\right] (a(1+\sin[e+fx]))^m \\ (3+2\sin[e+fx])^{-m} \left(\sec\left[\frac{1}{4}(2e-\pi+2fx)\right]^2 (3+2\sin[e+fx])\right)^m \left(\sin\left[\frac{1}{4}(2e+\pi+2fx)\right]^2\right)^{\frac{1}{2}-m}$$

■ **Problem 635: Result more than twice size of optimal antiderivative.**

$$\int (3+\sin[e+fx])^{-1-m} (a+a\sin[e+fx])^m dx$$

Optimal (type 5, 81 leaves, 2 steps):

$$-\frac{1}{f} 2^{-1-m} \cos[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{a-a\sin[e+fx]}{2(a+a\sin[e+fx])}\right] (1+\sin[e+fx])^{-1-m} (a+a\sin[e+fx])^m$$

Result (type 5, 166 leaves):

$$-\frac{1}{f} 2^{-1-2m} \cot\left[\frac{1}{4}(2e+\pi+2fx)\right] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{1}{2}\cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2 \sec\left[\frac{1}{4}(2e-\pi+2fx)\right]^2\right] \\ (a(1+\sin[e+fx]))^m \left(\frac{\cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{3+\sin[e+fx]}\right)^m \left(\sec\left[\frac{1}{4}(2e-\pi+2fx)\right]^2 (3+\sin[e+fx])\right)^m \left(\sin\left[\frac{1}{4}(2e+\pi+2fx)\right]^2\right)^{-m}$$

■ **Problem 637: Result more than twice size of optimal antiderivative.**

$$\int (3-\sin[e+fx])^{-1-m} (a+a\sin[e+fx])^m dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$-\frac{1}{f} \cos[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{2(a-a\sin[e+fx])}{a+a\sin[e+fx]}\right] (1+\sin[e+fx])^{-1-m} (a+a\sin[e+fx])^m$$

Result (type 5, 184 leaves):

$$-\frac{1}{f} 2^{\frac{1}{2}-m} \left(\cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^2\right)^{-\frac{1}{2}+m} \cot\left[\frac{1}{4}(2e+\pi+2fx)\right] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-m, \frac{3}{2}, -\frac{4\sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{-3+\sin[e+fx]}\right] \\ (3-\sin[e+fx])^{-m} \left(-\frac{\cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{-3+\sin[e+fx]}\right)^{\frac{1}{2}-m} (a(1+\sin[e+fx]))^m \left(\sin\left[\frac{1}{4}(2e+\pi+2fx)\right]^2\right)^{\frac{1}{2}-m}$$

■ **Problem 638: Result more than twice size of optimal antiderivative.**

$$\int (3 - 2 \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m dx$$

Optimal (type 5, 77 leaves, 2 steps):

$$-\frac{1}{f} 2^{1+m} \cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{5(a - a \sin[e + f x])}{a + a \sin[e + f x]}\right] (1 + \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m$$

Result (type 5, 179 leaves):

$$-\frac{1}{f} 2 \left(\cos\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2 \right)^{-\frac{1}{2}+m} \cot\left[\frac{1}{4} (2e + \pi + 2fx)\right] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{5 \sin\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2}{3 - 2 \sin[e + f x]}\right]^2$$

$$(3 - 2 \sin[e + f x])^{-m} (a (1 + \sin[e + f x]))^m \left(-\frac{\cos\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2}{-3 + 2 \sin[e + f x]} \right)^{\frac{1}{2}-m} \left(\sin\left[\frac{1}{4} (2e + \pi + 2fx)\right]^2 \right)^{\frac{1}{2}-m}$$

■ **Problem 639: Result more than twice size of optimal antiderivative.**

$$\int (3 - 3 \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m dx$$

Optimal (type 3, 45 leaves, 1 step):

$$\frac{\cos[e + f x] (3 - 3 \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m}{f (1 + 2m)}$$

Result (type 3, 99 leaves):

$$\frac{1}{3(f + 2fm)} \cos\left[\frac{1}{4} (2e + \pi + 2fx)\right]^{-1-2m} \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^{2m} (6 - 6 \sin[e + f x])^{-m} (a (1 + \sin[e + f x]))^m \sin\left[\frac{1}{4} (2e + \pi + 2fx)\right]$$

■ **Problem 642: Result more than twice size of optimal antiderivative.**

$$\int (-3 + 5 \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{\cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, \frac{4(a - a \sin[e + f x])}{a + a \sin[e + f x]}\right] (1 + \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m}{f}$$

Result (type 5, 154 leaves):

$$-\frac{1}{f} \left(\cos\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2 \right)^{-\frac{1}{2}+m} \cot\left[\frac{1}{4} (2e + \pi + 2fx)\right] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, 4 \tan\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2\right]^2$$

$$(a (1 + \sin[e + f x]))^m \left(\sec\left[\frac{1}{4} (2e - \pi + 2fx)\right]^2 (-3 + 5 \sin[e + f x]) \right)^m (-6 + 10 \sin[e + f x])^{-m} \left(\sin\left[\frac{1}{4} (2e + \pi + 2fx)\right]^2 \right)^{\frac{1}{2}-m}$$

■ **Problem 644: Result more than twice size of optimal antiderivative.**

$$\int (-3 + 3 \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m dx$$

Optimal (type 3, 45 leaves, 1 step):

$$\frac{\cos[e + f x] (-3 + 3 \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m}{f (1 + 2 m)}$$

Result (type 3, 110 leaves):

$$\frac{1}{f + 2 f m} 2^{-m} 3^{-1-m} \cos\left[\frac{1}{4} (2 e + \pi + 2 f x)\right]^{-1-2m} \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right]\right)^{2(1+m)} \\ (-1 + \sin[e + f x])^{-1-m} (a (1 + \sin[e + f x]))^m \sin\left[\frac{1}{4} (2 e + \pi + 2 f x)\right]$$

■ **Problem 650: Result more than twice size of optimal antiderivative.**

$$\int (-3 - 3 \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m dx$$

Optimal (type 3, 39 leaves, 2 steps):

$$\frac{\cos[e + f x] (-3 - 3 \sin[e + f x])^{-1-m} (a + a \sin[e + f x])^m}{f}$$

Result (type 3, 106 leaves):

$$-\frac{1}{f} 2^{-m} 3^{-1-m} \cos\left[\frac{1}{4} (2 e + \pi + 2 f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right)^{2(1+m)} \\ (-1 - \sin[e + f x])^{-1-m} (a (1 + \sin[e + f x]))^m \sin\left[\frac{1}{4} (2 e + \pi + 2 f x)\right]^{-1-2m}$$

■ **Problem 655: Unable to integrate problem.**

$$\int (a + a \sin[e + f x])^3 (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 107 leaves, 3 steps):

$$-\frac{1}{f \sqrt{1 + \sin[e + f x]}} \\ 8 \sqrt{2} a^3 \text{AppellF1}\left[\frac{1}{2}, -\frac{5}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 8, 27 leaves):

$$\int (a + a \sin[e + f x])^3 (c + d \sin[e + f x])^n dx$$

■ **Problem 656: Unable to integrate problem.**

$$\int (a + a \sin[e + f x])^2 (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 107 leaves, 3 steps):

$$-\frac{1}{f \sqrt{1 + \sin[e + f x]}} + 4 \sqrt{2} a^2 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 8, 27 leaves):

$$\int (a + a \sin[e + f x])^2 (c + d \sin[e + f x])^n dx$$

■ **Problem 657: Unable to integrate problem.**

$$\int (a + a \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 105 leaves, 3 steps):

$$-\frac{1}{f \sqrt{1 + \sin[e + f x]}} + 2 \sqrt{2} a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 8, 25 leaves):

$$\int (a + a \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

■ **Problem 659: Unable to integrate problem.**

$$\int \frac{(c + d \sin[e + f x])^n}{a + a \sin[e + f x]} dx$$

Optimal (type 6, 107 leaves, 3 steps):

$$-\frac{1}{\sqrt{2} a f \sqrt{1 + \sin[e + f x]}} + \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 8, 27 leaves):

$$\int \frac{(c + d \sin[e + f x])^n}{a + a \sin[e + f x]} dx$$

■ **Problem 660: Unable to integrate problem.**

$$\int \frac{(c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 6, 109 leaves, 3 steps):

$$-\left(\text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -n, \frac{3}{2}, \frac{1}{2}(1 - \sin[e + f x]), \frac{d(1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n} \right) / \left(2\sqrt{2} a^2 f \sqrt{1 + \sin[e + f x]}\right)$$

Result (type 8, 27 leaves):

$$\int \frac{(c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^2} dx$$

■ **Problem 661: Unable to integrate problem.**

$$\int \frac{(c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 6, 109 leaves, 3 steps):

$$-\left(\text{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -n, \frac{3}{2}, \frac{1}{2}(1 - \sin[e + f x]), \frac{d(1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n} \right) / \left(4\sqrt{2} a^3 f \sqrt{1 + \sin[e + f x]}\right)$$

Result (type 8, 27 leaves):

$$\int \frac{(c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^3} dx$$

■ **Problem 662: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])^n dx$$

Optimal (type 5, 257 leaves, 5 steps):

$$\frac{2a^3(3c - d(11 + 4n)) \cos[e + f x] (c + d \sin[e + f x])^{1+n}}{d^2 f (3 + 2n)(5 + 2n) \sqrt{a + a \sin[e + f x]}} - \frac{2a^2 \cos[e + f x] \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^{1+n}}{d f (5 + 2n)} - \left(2a^3(3c^2 - 2cd(7 + 4n) + d^2(43 + 56n + 16n^2)) \cos[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, \frac{d(1 - \sin[e + f x])}{c + d}\right] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}\right) / \left(d^2 f (3 + 2n)(5 + 2n) \sqrt{a + a \sin[e + f x]}\right)$$

Result (type 6, 577 leaves):

$$\begin{aligned}
& \left(2 a \operatorname{Cos}[e+f x] (1+\operatorname{Sin}[e+f x]) (c+d \operatorname{Sin}[e+f x])^{2 n} \left(c+\frac{d(-a+a(1+\operatorname{Sin}[e+f x]))}{a} \right)^{-n} \right. \\
& \left. \left(-\frac{1}{d^2(1+n)} a(c+d) \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, 1+n, 2+n, \frac{a c-a d+a d(1+\operatorname{Sin}[e+f x])}{a c+a d}\right] \right. \right. \\
& \quad \left. \sqrt{\frac{d(2 a-a(1+\operatorname{Sin}[e+f x]))}{a(c+d)}} (a(c-d)+a d(1+\operatorname{Sin}[e+f x]))+\frac{1}{d(1+n)} \right. \\
& \quad \left. 2 a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+n, 2+n, \frac{a c-a d+a d(1+\operatorname{Sin}[e+f x])}{a c+a d}\right] \sqrt{\frac{d(2 a-a(1+\operatorname{Sin}[e+f x]))}{a(c+d)}} (a(c-d)+a d(1+\operatorname{Sin}[e+f x]))+\right. \\
& \quad \left. \left(3 a^2(c-d) \operatorname{AppellF1}\left[2, -\frac{1}{2}, -n, 3, \frac{1}{2}(1+\operatorname{Sin}[e+f x]), -\frac{a d(1+\operatorname{Sin}[e+f x])}{a c-a d}\right] (1+\operatorname{Sin}[e+f x])^2(-2 a+a(1+\operatorname{Sin}[e+f x])) \right) \right. \\
& \quad \left. \left(12 a(c-d) \operatorname{AppellF1}\left[2, -\frac{1}{2}, -n, 3, \frac{1}{2}(1+\operatorname{Sin}[e+f x]), -\frac{a d(1+\operatorname{Sin}[e+f x])}{a c-a d}\right] + \right. \right. \\
& \quad \left. \left. a\left(4 d n \operatorname{AppellF1}\left[3, -\frac{1}{2}, 1-n, 4, \frac{1}{2}(1+\operatorname{Sin}[e+f x]), -\frac{a d(1+\operatorname{Sin}[e+f x])}{a c-a d}\right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-c+d) \operatorname{AppellF1}\left[3, \frac{1}{2}, -n, 4, \frac{1}{2}(1+\operatorname{Sin}[e+f x]), -\frac{a d(1+\operatorname{Sin}[e+f x])}{a c-a d}\right] (1+\operatorname{Sin}[e+f x]) \right) \right) \right) \right) \Bigg/ \\
& \left(f \sqrt{a(1+\operatorname{Sin}[e+f x])} \sqrt{\frac{2 a^2(1+\operatorname{Sin}[e+f x]) - a^2(1+\operatorname{Sin}[e+f x])^2}{a^2}} \sqrt{1-\frac{(-a+a(1+\operatorname{Sin}[e+f x]))^2}{a^2}} \right)
\end{aligned}$$

■ **Problem 664: Unable to integrate problem.**

$$\int \sqrt{a+a \operatorname{Sin}[e+f x]} (c+d \operatorname{Sin}[e+f x])^n dx$$

Optimal (type 5, 85 leaves, 3 steps):

$$\frac{2 a \operatorname{Cos}[e+f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, \frac{d(1-\operatorname{Sin}[e+f x])}{c+d}\right] (c+d \operatorname{Sin}[e+f x])^n \left(\frac{c+d \operatorname{Sin}[e+f x]}{c+d}\right)^{-n}}{f \sqrt{a+a \operatorname{Sin}[e+f x]}}$$

Result (type 8, 29 leaves):

$$\int \sqrt{a+a \operatorname{Sin}[e+f x]} (c+d \operatorname{Sin}[e+f x])^n dx$$

■ **Problem 665: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^n}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$-\frac{1}{f \sqrt{a + a \sin[e + f x]}} \operatorname{AppellF1}\left[\frac{1}{2}, -n, 1, \frac{3}{2}, \frac{d(1 - \sin[e + f x])}{c + d}, \frac{1}{2}(1 - \sin[e + f x])\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 6, 494 leaves):

$$\begin{aligned} & \frac{1}{a f} \operatorname{Sec}[e + f x] (a (1 + \sin[e + f x]))^{3/2} (c + d \sin[e + f x])^n \\ & \left(\left(4 (c - d) \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1 + \sin[e + f x]), \frac{d(1 + \sin[e + f x])}{-c + d}\right] \right) / \left(8 a (c - d) \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \right. \right. \right. \\ & \quad \left. \left. \frac{1}{2}(1 + \sin[e + f x]), \frac{d(1 + \sin[e + f x])}{-c + d}\right] + a \left(4 d n \operatorname{AppellF1}\left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2}(1 + \sin[e + f x]), \frac{d(1 + \sin[e + f x])}{-c + d}\right] + \right. \right. \\ & \quad \left. \left. (c - d) \operatorname{AppellF1}\left[2, \frac{3}{2}, -n, 3, \frac{1}{2}(1 + \sin[e + f x]), \frac{d(1 + \sin[e + f x])}{-c + d}\right] (1 + \sin[e + f x]) \right) - \right. \\ & \quad \left. \left(d (-1 + 2 n) \operatorname{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d + d \sin[e + f x]}\right] (-1 + \sin[e + f x]) \right) / \right. \\ & \quad \left. \left(a (1 + 2 n) \left(-2 (c - d) n \operatorname{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d + d \sin[e + f x]}\right] + \right. \right. \\ & \quad \left. \left. 2 d \operatorname{AppellF1}\left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d + d \sin[e + f x]}\right] + \right. \right. \\ & \quad \left. \left. d (-1 + 2 n) \operatorname{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d + d \sin[e + f x]}\right] (1 + \sin[e + f x]) \right) \right) \right) \end{aligned}$$

■ **Problem 666: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 6, 104 leaves, 3 steps):

$$-\frac{1}{2 a f \sqrt{a + a \sin[e + f x]}} \operatorname{AppellF1}\left[\frac{1}{2}, -n, 2, \frac{3}{2}, \frac{d(1 - \sin[e + f x])}{c + d}, \frac{1}{2}(1 - \sin[e + f x])\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 6, 918 leaves):

$$\begin{aligned}
& \left(\cos[e + f x] (1 + \sin[e + f x]) (c + d \sin[e + f x])^{2n} \left(c + \frac{d(-a + a(1 + \sin[e + f x]))}{a} \right)^{-n} \right. \\
& \left(\left(4 a^2 (c - d) \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), -\frac{ad(1 + \sin[e + f x])}{ac - ad} \right] (1 + \sin[e + f x]) \right) \right) / \\
& \left(8 a (c - d) \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), -\frac{ad(1 + \sin[e + f x])}{ac - ad} \right] + a \left(4 d n \operatorname{AppellF1} \left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2} (1 + \sin[e + f x]), \right. \right. \right. \\
& \left. \left. \left. -\frac{ad(1 + \sin[e + f x])}{ac - ad} \right] + (c - d) \operatorname{AppellF1} \left[2, \frac{3}{2}, -n, 3, \frac{1}{2} (1 + \sin[e + f x]), -\frac{ad(1 + \sin[e + f x])}{ac - ad} \right] \right) (1 + \sin[e + f x]) \right) - \\
& \left(a d (-1 + 2 n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] (1 + \sin[e + f x]) (-2 a + a(1 + \sin[e + f x])) \right) / \\
& \left((1 + 2 n) \left(2 a \left((-c + d) n \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] + \right. \right. \right. \\
& \left. \left. \left. d \operatorname{AppellF1} \left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] \right) \right) + \\
& \left. a d (-1 + 2 n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] (1 + \sin[e + f x]) \right) \right) - \\
& \left(2 a d (-3 + 2 n) \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] (-2 a + a(1 + \sin[e + f x])) \right) / \\
& \left((-1 + 2 n) \left(2 a \left((-c + d) n \operatorname{AppellF1} \left[\frac{3}{2} - n, -\frac{1}{2}, 1 - n, \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] + \right. \right. \right. \\
& \left. \left. \left. d \operatorname{AppellF1} \left[\frac{3}{2} - n, \frac{1}{2}, -n, \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] \right) \right) + \\
& \left. a d (-3 + 2 n) \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] (1 + \sin[e + f x]) \right) \right) \right) / \\
& \left(2 a^2 f \sqrt{a(1 + \sin[e + f x])} \sqrt{\frac{2 a^2 (1 + \sin[e + f x]) - a^2 (1 + \sin[e + f x])^2}{a^2}} \right. \\
& \left. \sqrt{1 - \frac{(-a + a(1 + \sin[e + f x]))^2}{a^2}} \right)
\end{aligned}$$

■ **Problem 667: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 6, 104 leaves, 3 steps):

$$-\frac{1}{4 a^2 f \sqrt{a + a \sin[e + f x]}}$$

$$\text{AppellF1}\left[\frac{1}{2}, -n, 3, \frac{3}{2}, \frac{d(1 - \sin[e + f x])}{c + d}, \frac{1}{2}(1 - \sin[e + f x])\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 6, 1377 leaves):

$$\frac{1}{a f \sqrt{1 - \frac{(-a + a(1 + \sin[e + f x]))^2}{a^2}}} 2 \cos[e + f x] (c + d \sin[e + f x])^n \left(c + \frac{d(-a + a(1 + \sin[e + f x]))}{a}\right)^{-n}$$

$$\left((c - d) \text{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1 + \sin[e + f x]), -\frac{ad(1 + \sin[e + f x])}{ac - ad}\right] (a(1 + \sin[e + f x]))^{3/2} (c + d \sin[e + f x])^n \right) /$$

$$\left(2 a^2 \left(8 a (c - d) \text{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1 + \sin[e + f x]), -\frac{ad(1 + \sin[e + f x])}{ac - ad}\right] + \right. \right.$$

$$\left. a \left(4 d n \text{AppellF1}\left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2}(1 + \sin[e + f x]), -\frac{ad(1 + \sin[e + f x])}{ac - ad}\right] + (c - d) \text{AppellF1}\left[2, \frac{3}{2}, -n, 3, \right. \right.$$

$$\left. \left. \frac{1}{2}(1 + \sin[e + f x]), -\frac{ad(1 + \sin[e + f x])}{ac - ad}\right] \right) (1 + \sin[e + f x]) \right) \sqrt{\frac{2 a^2 (1 + \sin[e + f x]) - a^2 (1 + \sin[e + f x])^2}{a^2}} -$$

$$\left(d(-1 + 2n) \text{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])}\right] (a(1 + \sin[e + f x]))^{3/2} \right.$$

$$\left. (c + d \sin[e + f x])^n (-2a + a(1 + \sin[e + f x])) \right) /$$

$$\left(8 a^3 (1 + 2n) \left(2 a \left((-c + d) n \text{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])}\right] + \right. \right.$$

$$\left. d \text{AppellF1}\left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])}\right] \right) + a d (-1 + 2n) \text{AppellF1}\left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \right.$$

$$\left. \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])}\right] (1 + \sin[e + f x]) \right) \sqrt{\frac{2 a^2 (1 + \sin[e + f x]) - a^2 (1 + \sin[e + f x])^2}{a^2}} -$$

$$\left(d(-3 + 2n) \text{AppellF1}\left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])}\right] \sqrt{a(1 + \sin[e + f x])} \right)$$

$$\begin{aligned}
& \left. (c + d \sin[e + f x])^n (-2a + a(1 + \sin[e + f x])) \right) / \\
& \left(4a^2 (-1 + 2n) \left(2a \left((-c + d)n \operatorname{AppellF1} \left[\frac{3}{2} - n, -\frac{1}{2}, 1 - n, \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] + \right. \right. \right. \\
& \quad \left. \left. d \operatorname{AppellF1} \left[\frac{3}{2} - n, \frac{1}{2}, -n, \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] + a d (-3 + 2n) \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, -n, \right. \right. \\
& \quad \left. \left. \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] (1 + \sin[e + f x]) \right) \sqrt{\frac{2a^2(1 + \sin[e + f x]) - a^2(1 + \sin[e + f x])^2}{a^2}} \right) - \\
& \left(d (-5 + 2n) \operatorname{AppellF1} \left[\frac{3}{2} - n, -\frac{1}{2}, -n, \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] (c + d \sin[e + f x])^n (-2a + a(1 + \sin[e + f x])) \right) / \\
& \left(2a (-3 + 2n) \sqrt{a(1 + \sin[e + f x])} \left(2a \left((-c + d)n \operatorname{AppellF1} \left[\frac{5}{2} - n, -\frac{1}{2}, 1 - n, \frac{7}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] + \right. \right. \right. \\
& \quad \left. \left. d \operatorname{AppellF1} \left[\frac{5}{2} - n, \frac{1}{2}, -n, \frac{7}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] + a d (-5 + 2n) \operatorname{AppellF1} \left[\frac{3}{2} - n, -\frac{1}{2}, -n, \right. \right. \\
& \quad \left. \left. \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d(1 + \sin[e + f x])} \right] (1 + \sin[e + f x]) \right) \sqrt{\frac{2a^2(1 + \sin[e + f x]) - a^2(1 + \sin[e + f x])^2}{a^2}} \right) \Bigg)
\end{aligned}$$

■ **Problem 668: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x]) (c + d \sin[e + f x])^{1/3} dx$$

Optimal (type 6, 107 leaves, 3 steps):

$$\frac{2\sqrt{2} a \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d(1 - \sin[e + f x])}{c + d} \right] \cos[e + f x] (c + d \sin[e + f x])^{1/3}}{f \sqrt{1 + \sin[e + f x]} \left(\frac{c + d \sin[e + f x]}{c + d} \right)^{1/3}}$$

Result (type 6, 1736 leaves):

$$a \left(\left(c \operatorname{Sec}[e] (1 + \sin[e + f x]) \right) - \left(\operatorname{AppellF1} \left[-\frac{2}{3}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, \frac{\operatorname{Csc}[e] \left(c + d \cos[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]] \right) \sqrt{1 + \operatorname{Cot}[e]^2} \sin[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2} \left(1 - \frac{c \operatorname{Csc}[e]}{d \sqrt{1 + \operatorname{Cot}[e]^2}} \right)} \right] \right) \right)$$

$$\begin{aligned}
& \left. - \frac{\text{Csc}[e] \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)}{d \sqrt{1 + \text{Cot}[e]^2} \left(-1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \text{Cot}[e]^2}} \right)} \right] \text{Cot}[e] \sin[f x - \text{ArcTan}[\text{Cot}[e]]] \Bigg/ \\
& \left(\sqrt{1 + \text{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} - c \text{Csc}[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} - d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} + c \text{Csc}[e]}} \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)^{2/3}} \right) - \\
& \left. \frac{\frac{3 d \sin[e] \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)}{d^2 \cos[e]^2 + d^2 \sin[e]^2} - \frac{\text{Cot}[e] \sin[f x - \text{ArcTan}[\text{Cot}[e]]]}{\sqrt{1 + \text{Cot}[e]^2}}}{\left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)^{2/3}} \right) \Bigg/ \left(4 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2} \right] + \sin\left[\frac{e}{2} + \frac{f x}{2} \right] \right)^2 \right) + \\
& \left(d \text{Sec}[e] (1 + \sin[e + f x]) \left(- \left(\text{AppellF1}\left[-\frac{2}{3}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{\text{Csc}[e] \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)}{d \sqrt{1 + \text{Cot}[e]^2} \left(1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \text{Cot}[e]^2}} \right)} \right] \right. \right. \right. \\
& \left. \left. - \frac{\text{Csc}[e] \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)}{d \sqrt{1 + \text{Cot}[e]^2} \left(-1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \text{Cot}[e]^2}} \right)} \right] \text{Cot}[e] \sin[f x - \text{ArcTan}[\text{Cot}[e]]] \right) \Bigg/ \\
& \left(\sqrt{1 + \text{Cot}[e]^2} \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} - c \text{Csc}[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \text{Cot}[e]^2} - d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2}}{d \sqrt{1 + \text{Cot}[e]^2} + c \text{Csc}[e]}} \left(c + d \cos[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{1 + \text{Cot}[e]^2} \sin[e] \right)^{2/3}} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\frac{3 d \sin[e] \left(c + d \cos[f x - \text{ArcTan}[\cot[e]] \right) \sqrt{1 + \cot[e]^2} \sin[e]}{d^2 \cos[e]^2 + d^2 \sin[e]^2} - \frac{\cot[e] \sin[f x - \text{ArcTan}[\cot[e]]]}{\sqrt{1 + \cot[e]^2}}}{\left(c + d \cos[f x - \text{ArcTan}[\cot[e]] \right) \sqrt{1 + \cot[e]^2} \sin[e]} \right)^{2/3}}{\left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2} + \\
& \frac{(1 + \sin[e + f x]) (c + d \sin[e + f x])^{1/3} \left(-\frac{3 \cos[e] \cos[f x]}{4 f} + \frac{3 \sin[e] \sin[f x]}{4 f} + \frac{3 (c + 4 d) \tan[e]}{4 d f} \right)}{\left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2} + \\
& \frac{1}{4 f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}} \\
& 3 \\
& \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]] \right) \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right], \\
& -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]] \right) \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right] \sec[e] \\
& \frac{\sec[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x]) \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \sec[e] + d \sqrt{1 + \tan[e]^2}}}}{\sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}}} \\
& \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)^{1/3} + \\
& \frac{1}{d f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}} \\
& 3 c \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]] \right) \sqrt{1 + \tan[e]^2}}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right],
\end{aligned}$$

$$\begin{aligned}
& - \frac{\text{Sec}[e] \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \text{Sec}[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \text{Sec}[e] \\
& \text{Sec}[fx + \text{ArcTan}[\tan[e]]] (1 + \sin[e + fx]) \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \text{Sec}[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \left(c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)^{1/3}
\end{aligned}$$

■ **Problem 669: Result more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e + fx]}{(c + d \sin[e + fx])^{1/3}} dx$$

Optimal (type 6, 107 leaves, 3 steps):

$$- \frac{2 \sqrt{2} a \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + fx]), \frac{d (1 - \sin[e + fx])}{c + d}\right] \cos[e + fx] \left(\frac{c + d \sin[e + fx]}{c + d}\right)^{1/3}}{f \sqrt{1 + \sin[e + fx]} (c + d \sin[e + fx])^{1/3}}$$

Result (type 6, 886 leaves):

$$\begin{aligned}
& a \left(\left(\text{Sec}[e] (1 + \sin[e + fx]) \left(- \left(\text{AppellF1}\left[-\frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, \frac{2}{3}, -\frac{\text{Csc}[e] \left(c + d \cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]\right)}{d \sqrt{1 + \cot[e]^2} \left(1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \cot[e]^2}} \right)} \right)} \right) \right) \right. \\
& \left. - \frac{\text{Csc}[e] \left(c + d \cos[fx - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e]\right)}{d \sqrt{1 + \cot[e]^2} \left(-1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \cot[e]^2}} \right)} \right) \cot[e] \sin[fx - \text{ArcTan}[\cot[e]]] \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{1 + \cot[e]^2} \sqrt{\frac{d \sqrt{1 + \cot[e]^2} + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} - c \csc[e]}} \right. \\
& \left. \sqrt{\frac{d \sqrt{1 + \cot[e]^2} - d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} + c \csc[e]}} \left(c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e] \right)^{1/3}} \right) - \\
& \left. \frac{\frac{3 d \sin[e] \left(c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e] \right) - \frac{\cot[e] \sin[f x - \text{ArcTan}[\cot[e]]]}{\sqrt{1 + \cot[e]^2}}}{2 (d^2 \cos[e]^2 + d^2 \sin[e]^2)}}{\left(c + d \cos[f x - \text{ArcTan}[\cot[e]]] \sqrt{1 + \cot[e]^2} \sin[e] \right)^{1/3}}}{\left. \right)} \\
& \left(f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \frac{3 (1 + \sin[e + f x]) (c + d \sin[e + f x])^{2/3} \tan[e]}{2 d f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2} + \\
& \frac{1}{2 d f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1 + \tan[e]^2}} \\
& 3 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \right], \\
& - \frac{\sec[e] \left(c + d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2} \right)}{d \sqrt{1 + \tan[e]^2} \left(-1 - \frac{c \sec[e]}{d \sqrt{1 + \tan[e]^2}} \right)} \sec[e] \\
& \sec[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x]) \sqrt{\frac{d \sqrt{1 + \tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{c \sec[e] + d \sqrt{1 + \tan[e]^2}}} \\
& \sqrt{\frac{d \sqrt{1 + \tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-c \sec[e] + d \sqrt{1 + \tan[e]^2}}}
\end{aligned}$$

$$\left((c + d \cos[e] \sin[fx + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2})^{2/3} \right)$$

■ **Problem 670: Result more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sin[e + fx]}{(c + d \sin[e + fx])^{4/3}} dx$$

Optimal (type 6, 112 leaves, 3 steps):

$$\frac{2\sqrt{2} a \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, \frac{4}{3}, \frac{3}{2}, \frac{1}{2}(1 - \sin[e + fx]), \frac{d(1 - \sin[e + fx])}{c+d}\right] \cos[e + fx] \left(\frac{c+d \sin[e + fx]}{c+d}\right)^{1/3}}{(c+d) f \sqrt{1 + \sin[e + fx]} (c+d \sin[e + fx])^{1/3}}$$

Result (type 6, 942 leaves):

$$a \left(\frac{(1 + \sin[e + fx]) (c + d \sin[e + fx])^{2/3} \left(-\frac{3 \text{Csc}[e] \text{Sec}[e]}{d(c+d)f} + \frac{3 \text{Csc}[e] (c \cos[e] + d \sin[fx])}{d(c+d)f(c+d \sin[e + fx])} \right)}{\left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right)^2} - \right. \\ \left. \left(2 \text{Sec}[e] (1 + \sin[e + fx]) \left(-\left(\text{AppellF1}\left[-\frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, \frac{2}{3}, -\frac{\text{Csc}[e] (c + d \cos[fx - \text{ArcTan}[\cot[e]]) \sqrt{1 + \cot[e]^2} \sin[e]}{d \sqrt{1 + \cot[e]^2} \left(1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \cot[e]^2}\right)}\right]} \right) \right. \right. \right. \\ \left. \left. - \frac{\text{Csc}[e] (c + d \cos[fx - \text{ArcTan}[\cot[e]]) \sqrt{1 + \cot[e]^2} \sin[e]}{d \sqrt{1 + \cot[e]^2} \left(-1 - \frac{c \text{Csc}[e]}{d \sqrt{1 + \cot[e]^2}\right)} \right) \cot[e] \sin[fx - \text{ArcTan}[\cot[e]]] \right) / \right. \\ \left. \left(\sqrt{1 + \cot[e]^2} \sqrt{\frac{d \sqrt{1 + \cot[e]^2} + d \cos[fx - \text{ArcTan}[\cot[e]]) \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} - c \text{Csc}[e]}} \right) \right. \\ \left. \sqrt{\frac{d \sqrt{1 + \cot[e]^2} - d \cos[fx - \text{ArcTan}[\cot[e]]) \sqrt{1 + \cot[e]^2}}{d \sqrt{1 + \cot[e]^2} + c \text{Csc}[e]}} (c + d \cos[fx - \text{ArcTan}[\cot[e]]) \sqrt{1 + \cot[e]^2} \sin[e] \right)^{1/3} - \right)$$

$$\left. \left. \frac{\frac{3 d \sin[e] \left(c+d \cos[f x - \text{ArcTan}[\cot[e]] \right) \sqrt{1+\cot[e]^2} \sin[e]}{2 \left(d^2 \cos[e]^2 + d^2 \sin[e]^2 \right)} - \frac{\cot[e] \sin[f x - \text{ArcTan}[\cot[e]]]}{\sqrt{1+\cot[e]^2}}}{\left(c+d \cos[f x - \text{ArcTan}[\cot[e]] \right) \sqrt{1+\cot[e]^2} \sin[e]} \right)^{1/3}}{\right.} \right.$$

$$\left((c+d) f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \right) + \frac{1}{2 d (c+d) f \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right)^2 \sqrt{1+\tan[e]^2}}$$

$$3 \text{ AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, -\frac{\sec[e] \left(c+d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]] \right) \sqrt{1+\tan[e]^2}}{d \sqrt{1+\tan[e]^2} \left(1 - \frac{c \sec[e]}{d \sqrt{1+\tan[e]^2}} \right)} \right],$$

$$-\frac{\sec[e] \left(c+d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]] \right) \sqrt{1+\tan[e]^2}}{d \sqrt{1+\tan[e]^2} \left(-1 - \frac{c \sec[e]}{d \sqrt{1+\tan[e]^2}} \right)} \right] \sec[e]$$

$$\sec[f x + \text{ArcTan}[\tan[e]]] (1 + \sin[e + f x]) \sqrt{\frac{d \sqrt{1+\tan[e]^2} - d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1+\tan[e]^2}}{c \sec[e] + d \sqrt{1+\tan[e]^2}}}$$

$$\sqrt{\frac{d \sqrt{1+\tan[e]^2} + d \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1+\tan[e]^2}}{-c \sec[e] + d \sqrt{1+\tan[e]^2}}}$$

$$\left(c+d \cos[e] \sin[f x + \text{ArcTan}[\tan[e]]] \sqrt{1+\tan[e]^2} \right)^{2/3} \right)$$

■ **Problem 692: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \sin[e + f x])^3}{(c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 255 leaves, 6 steps):

$$\frac{b^3 x}{d^3} - \frac{(9 a^2 b c d^4 - a^3 d^3 (2 c^2 + d^2) - 3 a b^2 d^3 (c^2 + 2 d^2) + b^3 (2 c^5 - 5 c^3 d^2 + 6 c d^4)) \operatorname{ArcTan}\left[\frac{d+c \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c^2-d^2}}\right]}{d^3 (c^2-d^2)^{5/2} f} +$$

$$\frac{(b c - a d)^2 \operatorname{Cos}[e+f x] (a+b \operatorname{Sin}[e+f x])}{2 d (c^2-d^2) f (c+d \operatorname{Sin}[e+f x])^2} + \frac{(b c - a d)^2 (2 b c^2 + 3 a c d - 5 b d^2) \operatorname{Cos}[e+f x]}{2 d^2 (c^2-d^2)^2 f (c+d \operatorname{Sin}[e+f x])}$$

Result (type 3, 521 leaves):

$$\frac{1}{4 d^3 f} \left(-\frac{1}{(c^2-d^2)^{5/2}} 4 (9 a^2 b c d^4 - a^3 d^3 (2 c^2 + d^2) - 3 a b^2 d^3 (c^2 + 2 d^2) + b^3 (2 c^5 - 5 c^3 d^2 + 6 c d^4)) \operatorname{ArcTan}\left[\frac{d+c \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c^2-d^2}}\right] + \right.$$

$$\frac{1}{(c^2-d^2)^2 (c+d \operatorname{Sin}[e+f x])^2}$$

$$\left. \begin{aligned} & (4 b^3 c^6 e - 6 b^3 c^4 d^2 e + 2 b^3 d^6 e + 4 b^3 c^6 f x - 6 b^3 c^4 d^2 f x + 2 b^3 d^6 f x - 2 d (b c - a d)^2 (-2 b c^3 - 4 a c^2 d + 5 b c d^2 + a d^3) \operatorname{Cos}[e+f x] - \\ & 2 b^3 (-c^2 d + d^3)^2 (e+f x) \operatorname{Cos}[2(e+f x)] + 8 b^3 c^5 d e \operatorname{Sin}[e+f x] - 16 b^3 c^3 d^3 e \operatorname{Sin}[e+f x] + 8 b^3 c d^5 e \operatorname{Sin}[e+f x] + \\ & 8 b^3 c^5 d f x \operatorname{Sin}[e+f x] - 16 b^3 c^3 d^3 f x \operatorname{Sin}[e+f x] + 8 b^3 c d^5 f x \operatorname{Sin}[e+f x] + 3 b^3 c^4 d^2 \operatorname{Sin}[2(e+f x)] - 3 a b^2 c^3 d^3 \operatorname{Sin}[2(e+f x)] - \\ & 3 a^2 b c^2 d^4 \operatorname{Sin}[2(e+f x)] - 6 b^3 c^2 d^4 \operatorname{Sin}[2(e+f x)] + 3 a^3 c d^5 \operatorname{Sin}[2(e+f x)] + 12 a b^2 c d^5 \operatorname{Sin}[2(e+f x)] - 6 a^2 b d^6 \operatorname{Sin}[2(e+f x)] \end{aligned} \right)$$

■ **Problem 714: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c+d \operatorname{Sin}[e+f x])^5}{(a+b \operatorname{Sin}[e+f x])^3} dx$$

Optimal (type 3, 534 leaves, 8 steps):

$$-\frac{d^3 (30 a b c d - 12 a^2 d^2 - b^2 (20 c^2 + d^2)) x}{2 b^5} + \frac{1}{b^5 (a^2 - b^2)^{5/2} f}$$

$$(b c - a d)^3 (6 a^3 b c d - 12 a b^3 c d + 12 a^4 d^2 + a^2 b^2 (2 c^2 - 29 d^2) + b^4 (c^2 + 20 d^2)) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{a^2-b^2}}\right] - \frac{1}{2 b^4 (a^2-b^2)^2 f}$$

$$d (30 a^4 b c d^3 - 12 a^5 d^4 - a^3 b^2 d^2 (16 c^2 - 21 d^2) - b^5 c d (17 c^2 - 10 d^2) - a^2 b^3 c d (4 c^2 + 55 d^2) + a b^4 (6 c^4 + 43 c^2 d^2 - 6 d^4)) \operatorname{Cos}[e+f x] +$$

$$\frac{1}{2 b^3 (a^2 - b^2)^2 f} d^2 (7 a^3 b c d^2 - 6 a^4 d^3 + b^4 d (8 c^2 - d^2) + a^2 b^2 d (c^2 + 10 d^2) - a b^3 c (3 c^2 + 16 d^2)) \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x] +$$

$$\frac{(b c - a d)^2 (3 a b c + 4 a^2 d - 7 b^2 d) \operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^2}{2 b^2 (a^2 - b^2)^2 f (a+b \operatorname{Sin}[e+f x])} + \frac{(b c - a d)^2 \operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^3}{2 b (a^2 - b^2) f (a+b \operatorname{Sin}[e+f x])^2}$$

Result (type 3, 341 leaves):

$$\frac{1}{4 b^5 f} \left(2 d^3 (-30 a b c d + 12 a^2 d^2 + b^2 (20 c^2 + d^2)) (e + f x) + \frac{1}{(a^2 - b^2)^{5/2}} \right. \\ \left. 4 (b c - a d)^3 (6 a^3 b c d - 12 a b^3 c d + 12 a^4 d^2 + a^2 b^2 (2 c^2 - 29 d^2)) + b^4 (c^2 + 20 d^2) \right) \text{ArcTan} \left[\frac{b + a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right] + \\ 2 b d^4 (-5 b c + 3 a d) (\text{Cos}[e + f x] - i \text{Sin}[e + f x]) + 2 b d^4 (-5 b c + 3 a d) (\text{Cos}[e + f x] + i \text{Sin}[e + f x]) - \\ \frac{2 b (b c - a d)^5 \text{Cos}[e + f x]}{(-a^2 + b^2) (a + b \text{Sin}[e + f x])^2} + \frac{2 b (b c - a d)^4 (3 a b c + 7 a^2 d - 10 b^2 d) \text{Cos}[e + f x]}{(a^2 - b^2)^2 (a + b \text{Sin}[e + f x])} - b^2 d^5 \text{Sin}[2 (e + f x)] \left. \right)$$

■ **Problem 715: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \text{Sin}[e + f x])^4}{(a + b \text{Sin}[e + f x])^3} dx$$

Optimal (type 3, 318 leaves, 7 steps):

$$\frac{d^3 (4 b c - 3 a d) x}{b^4} + \frac{(b c - a d)^2 (4 a^3 b c d - 10 a b^3 c d + 6 a^4 d^2 + a^2 b^2 (2 c^2 - 15 d^2)) \text{ArcTan} \left[\frac{b + a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right]}{b^4 (a^2 - b^2)^{5/2} f} + \\ \frac{d^2 (2 a b c d - 3 a^2 d^2 - b^2 (c^2 - 2 d^2)) \text{Cos}[e + f x]}{2 b^3 (a^2 - b^2) f} + \frac{3 (b c - a d)^3 (a b c + a^2 d - 2 b^2 d) \text{Cos}[e + f x]}{2 b^3 (a^2 - b^2)^2 f (a + b \text{Sin}[e + f x])} + \frac{(b c - a d)^2 \text{Cos}[e + f x] (c + d \text{Sin}[e + f x])^2}{2 b (a^2 - b^2) f (a + b \text{Sin}[e + f x])^2}$$

Result (type 3, 894 leaves):

$$\frac{1}{4 b^4 f} \left(\frac{1}{(a^2 - b^2)^{5/2}} 4 (b c - a d)^2 (4 a^3 b c d - 10 a b^3 c d + 6 a^4 d^2 + a^2 b^2 (2 c^2 - 15 d^2)) \text{ArcTan} \left[\frac{b + a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right] + \right. \\ \frac{1}{(a^2 - b^2)^2 (a + b \text{Sin}[e + f x])^2} (16 a^6 b c d^3 e - 24 a^4 b^3 c d^3 e + 8 b^7 c d^3 e - 12 a^7 d^4 e + 18 a^5 b^2 d^4 e - \\ 6 a b^5 d^4 e + 16 a^6 b c d^3 f x - 24 a^4 b^3 c d^3 f x + 8 b^7 c d^3 f x - 12 a^7 d^4 f x + 18 a^5 b^2 d^4 f x - 6 a b^5 d^4 f x - \\ b (8 a b^5 c^3 d - 16 a^5 b c d^3 + 12 a^6 d^4 - 21 a^4 b^2 d^4 + 8 a^3 b^3 c d (2 c^2 + 5 d^2)) + b^6 (2 c^4 + d^4) + 2 a^2 b^4 (-4 c^4 - 18 c^2 d^2 + d^4)) \text{Cos}[e + f x] + \\ 2 b^2 (a^2 - b^2)^2 d^3 (-4 b c + 3 a d) (e + f x) \text{Cos}[2 (e + f x)] + a^4 b^3 d^4 \text{Cos}[3 (e + f x)] - 2 a^2 b^5 d^4 \text{Cos}[3 (e + f x)] + b^7 d^4 \text{Cos}[3 (e + f x)] + \\ 32 a^5 b^2 c d^3 e \text{Sin}[e + f x] - 64 a^3 b^4 c d^3 e \text{Sin}[e + f x] + 32 a b^6 c d^3 e \text{Sin}[e + f x] - 24 a^6 b d^4 e \text{Sin}[e + f x] + 48 a^4 b^3 d^4 e \text{Sin}[e + f x] - \\ 24 a^2 b^5 d^4 e \text{Sin}[e + f x] + 32 a^5 b^2 c d^3 f x \text{Sin}[e + f x] - 64 a^3 b^4 c d^3 f x \text{Sin}[e + f x] + 32 a b^6 c d^3 f x \text{Sin}[e + f x] - \\ 24 a^6 b d^4 f x \text{Sin}[e + f x] + 48 a^4 b^3 d^4 f x \text{Sin}[e + f x] - 24 a^2 b^5 d^4 f x \text{Sin}[e + f x] + 3 a b^6 c^4 \text{Sin}[2 (e + f x)] - 4 a^2 b^5 c^3 d \text{Sin}[2 (e + f x)] - \\ 8 b^7 c^3 d \text{Sin}[2 (e + f x)] - 6 a^3 b^4 c^2 d^2 \text{Sin}[2 (e + f x)] + 24 a b^6 c^2 d^2 \text{Sin}[2 (e + f x)] + 12 a^4 b^3 c d^3 \text{Sin}[2 (e + f x)] - \\ \left. 24 a^2 b^5 c d^3 \text{Sin}[2 (e + f x)] - 9 a^5 b^2 d^4 \text{Sin}[2 (e + f x)] + 16 a^3 b^4 d^4 \text{Sin}[2 (e + f x)] - 4 a b^6 d^4 \text{Sin}[2 (e + f x)] \right)$$

■ **Problem 716: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^3}{(a + b \sin[e + f x])^3} dx$$

Optimal (type 3, 248 leaves, 6 steps):

$$\frac{d^3 x}{b^3} + \frac{(bc - ad) (2a^3 bcd - 8ab^3cd + 2a^4d^2 + a^2b^2(2c^2 - 5d^2) + b^4(c^2 + 6d^2)) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right]}{b^3(a^2-b^2)^{5/2}f} +$$

$$\frac{(bc - ad)^2(3abc + 2a^2d - 5b^2d) \cos[e + fx]}{2b^2(a^2 - b^2)^2 f (a + b \sin[e + fx])} + \frac{(bc - ad)^2 \cos[e + fx] (c + d \sin[e + fx])}{2b(a^2 - b^2) f (a + b \sin[e + fx])^2}$$

Result (type 3, 524 leaves):

$$\frac{1}{4b^3f} \left(-\frac{1}{(a^2 - b^2)^{5/2}} 4(2a^5d^3 - 5a^3b^2d^3 + 3ab^4d(3c^2 + 2d^2) - a^2b^3c(2c^2 + 3d^2) - b^5c(c^2 + 6d^2)) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] + \right.$$

$$\frac{1}{(a^2 - b^2)^2 (a + b \sin[e + fx])^2} (4a^6d^3e - 6a^4b^2d^3e + 2b^6d^3e + 4a^6d^3fx - 6a^4b^2d^3fx + 2b^6d^3fx - 2b(bc - ad)^2(-4a^2bc + b^3c - 2a^3d + 5ab^2d) \cos[e + fx] -$$

$$2(-a^2b + b^3)^2d^3(e + fx) \cos[2(e + fx)] + 8a^5bd^3e \sin[e + fx] - 16a^3b^3d^3e \sin[e + fx] + 8ab^5d^3e \sin[e + fx] +$$

$$8a^5bd^3fx \sin[e + fx] - 16a^3b^3d^3fx \sin[e + fx] + 8ab^5d^3fx \sin[e + fx] + 3ab^5c^3 \sin[2(e + fx)] - 3a^2b^4c^2d \sin[2(e + fx)] -$$

$$\left. 6b^6c^2d \sin[2(e + fx)] - 3a^3b^3cd^2 \sin[2(e + fx)] + 12ab^5cd^2 \sin[2(e + fx)] + 3a^4b^2d^3 \sin[2(e + fx)] - 6a^2b^4d^3 \sin[2(e + fx)] \right)$$

■ **Problem 722: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sin[e + f x])^3 (c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 669 leaves, 11 steps):

$$\frac{b^3 (10 a^3 b c d - 4 a b^3 c d - 20 a^4 d^2 - a^2 b^2 (2 c^2 - 29 d^2) - b^4 (c^2 + 12 d^2)) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2} (b c-a d)^5 f}$$

$$\frac{d^3 (a^2 d^2 (2 c^2 + d^2) - a b (10 c^3 d - 4 c d^3) + b^2 (20 c^4 - 29 c^2 d^2 + 12 d^4)) \operatorname{ArcTan}\left[\frac{d+c \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c^2-d^2}}\right]}{(b c-a d)^5 (c^2-d^2)^{5/2} f}$$

$$\frac{d (a^4 d^3 - b^4 d (5 c^2 - 6 d^2) + 2 a^2 b^2 d (4 c^2 - 5 d^2) - 3 a b^3 c (c^2 - d^2)) \operatorname{Cos}[e+f x]}{2 (a^2-b^2)^2 (b c-a d)^3 (c^2-d^2) f (c+d \operatorname{Sin}[e+f x])^2} +$$

$$\frac{b^2 \operatorname{Cos}[e+f x]}{2 (a^2-b^2) (b c-a d) f (a+b \operatorname{Sin}[e+f x])^2 (c+d \operatorname{Sin}[e+f x])^2} + \frac{b^2 (3 a b c - 7 a^2 d + 4 b^2 d) \operatorname{Cos}[e+f x]}{2 (a^2-b^2)^2 (b c-a d)^2 f (a+b \operatorname{Sin}[e+f x]) (c+d \operatorname{Sin}[e+f x])^2} +$$

$$\frac{(3 d (a^5 c d^4 - 2 a^3 b^2 c d^4 + a b^4 c (c^4 - 2 c^2 d^2 + 2 d^4) + b^5 d (2 c^4 - 7 c^2 d^2 + 4 d^4) - a^2 b^3 d (3 c^4 - 12 c^2 d^2 + 7 d^4) - a^4 b (3 c^2 d^3 - 2 d^5)) \operatorname{Cos}[e+f x]}{(2 (a^2-b^2)^2 (b c-a d)^4 (c^2-d^2)^2 f (c+d \operatorname{Sin}[e+f x]))}$$

Result (type 3, 1815 leaves):

$$\begin{aligned}
& - \left(b^3 (2 a^2 b^2 c^2 + b^4 c^2 - 10 a^3 b c d + 4 a b^3 c d + 20 a^4 d^2 - 29 a^2 b^2 d^2 + 12 b^4 d^2) \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right] (b \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right])}{\sqrt{a^2 - b^2}} \right] \right) / \\
& \left((a^2 - b^2)^{5/2} (-bc + ad)^5 f \right) - \frac{1}{(bc - ad)^5 (c^2 - d^2)^{5/2} f} \\
& d^3 (20 b^2 c^4 - 10 a b c^3 d + 2 a^2 c^2 d^2 - 29 b^2 c^2 d^2 + 4 a b c d^3 + a^2 d^4 + 12 b^2 d^4) \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right] (d \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + c \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right])}{\sqrt{c^2 - d^2}} \right] + \\
& (32 a^2 b^5 c^7 \operatorname{Cos}[e + f x] - 8 b^7 c^7 \operatorname{Cos}[e + f x] - 80 a^3 b^4 c^6 d \operatorname{Cos}[e + f x] + 68 a b^6 c^6 d \operatorname{Cos}[e + f x] - 92 a^2 b^5 c^5 d^2 \operatorname{Cos}[e + f x] + \\
& 38 b^7 c^5 d^2 \operatorname{Cos}[e + f x] + 140 a^3 b^4 c^4 d^3 \operatorname{Cos}[e + f x] - 122 a b^6 c^4 d^3 \operatorname{Cos}[e + f x] - 80 a^6 b c^3 d^4 \operatorname{Cos}[e + f x] + 140 a^4 b^3 c^3 d^4 \operatorname{Cos}[e + f x] + \\
& 48 a^2 b^5 c^3 d^4 \operatorname{Cos}[e + f x] - 72 b^7 c^3 d^4 \operatorname{Cos}[e + f x] + 32 a^7 c^2 d^5 \operatorname{Cos}[e + f x] - 92 a^5 b^2 c^2 d^5 \operatorname{Cos}[e + f x] + 48 a^3 b^4 c^2 d^5 \operatorname{Cos}[e + f x] + \\
& 12 a b^6 c^2 d^5 \operatorname{Cos}[e + f x] + 68 a^6 b c d^6 \operatorname{Cos}[e + f x] - 122 a^4 b^3 c d^6 \operatorname{Cos}[e + f x] + 12 a^2 b^5 c d^6 \operatorname{Cos}[e + f x] + 36 b^7 c d^6 \operatorname{Cos}[e + f x] - \\
& 8 a^7 d^7 \operatorname{Cos}[e + f x] + 38 a^5 b^2 d^7 \operatorname{Cos}[e + f x] - 72 a^3 b^4 d^7 \operatorname{Cos}[e + f x] + 36 a b^6 d^7 \operatorname{Cos}[e + f x] - 12 a b^6 c^6 d \operatorname{Cos}[3(e + f x)] + \\
& 28 a^2 b^5 c^5 d^2 \operatorname{Cos}[3(e + f x)] - 22 b^7 c^5 d^2 \operatorname{Cos}[3(e + f x)] + 20 a^3 b^4 c^4 d^3 \operatorname{Cos}[3(e + f x)] + 10 a b^6 c^4 d^3 \operatorname{Cos}[3(e + f x)] + \\
& 20 a^4 b^3 c^3 d^4 \operatorname{Cos}[3(e + f x)] - 96 a^2 b^5 c^3 d^4 \operatorname{Cos}[3(e + f x)] + 64 b^7 c^3 d^4 \operatorname{Cos}[3(e + f x)] + 28 a^5 b^2 c^2 d^5 \operatorname{Cos}[3(e + f x)] - \\
& 96 a^3 b^4 c^2 d^5 \operatorname{Cos}[3(e + f x)] + 44 a b^6 c^2 d^5 \operatorname{Cos}[3(e + f x)] - 12 a^6 b c d^6 \operatorname{Cos}[3(e + f x)] + 10 a^4 b^3 c d^6 \operatorname{Cos}[3(e + f x)] + \\
& 44 a^2 b^5 c d^6 \operatorname{Cos}[3(e + f x)] - 36 b^7 c d^6 \operatorname{Cos}[3(e + f x)] - 22 a^5 b^2 d^7 \operatorname{Cos}[3(e + f x)] + 64 a^3 b^4 d^7 \operatorname{Cos}[3(e + f x)] - \\
& 36 a b^6 d^7 \operatorname{Cos}[3(e + f x)] + 12 a b^6 c^7 \operatorname{Sin}[2(e + f x)] - 4 a^2 b^5 c^6 d \operatorname{Sin}[2(e + f x)] + 16 b^7 c^6 d \operatorname{Sin}[2(e + f x)] - \\
& 80 a^3 b^4 c^5 d^2 \operatorname{Sin}[2(e + f x)] + 38 a b^6 c^5 d^2 \operatorname{Sin}[2(e + f x)] - 10 a^2 b^5 c^4 d^3 \operatorname{Sin}[2(e + f x)] - 20 b^7 c^4 d^3 \operatorname{Sin}[2(e + f x)] - \\
& 80 a^5 b^2 c^3 d^4 \operatorname{Sin}[2(e + f x)] + 320 a^3 b^4 c^3 d^4 \operatorname{Sin}[2(e + f x)] - 192 a b^6 c^3 d^4 \operatorname{Sin}[2(e + f x)] - 4 a^6 b c^2 d^5 \operatorname{Sin}[2(e + f x)] - \\
& 10 a^4 b^3 c^2 d^5 \operatorname{Sin}[2(e + f x)] + 64 a^2 b^5 c^2 d^5 \operatorname{Sin}[2(e + f x)] - 26 b^7 c^2 d^5 \operatorname{Sin}[2(e + f x)] + 12 a^7 c d^6 \operatorname{Sin}[2(e + f x)] + \\
& 38 a^5 b^2 c d^6 \operatorname{Sin}[2(e + f x)] - 192 a^3 b^4 c d^6 \operatorname{Sin}[2(e + f x)] + 124 a b^6 c d^6 \operatorname{Sin}[2(e + f x)] + 16 a^6 b d^7 \operatorname{Sin}[2(e + f x)] - \\
& 20 a^4 b^3 d^7 \operatorname{Sin}[2(e + f x)] - 26 a^2 b^5 d^7 \operatorname{Sin}[2(e + f x)] + 24 b^7 d^7 \operatorname{Sin}[2(e + f x)] - 3 a b^6 c^5 d^2 \operatorname{Sin}[4(e + f x)] + \\
& 9 a^2 b^5 c^4 d^3 \operatorname{Sin}[4(e + f x)] - 6 b^7 c^4 d^3 \operatorname{Sin}[4(e + f x)] + 6 a b^6 c^3 d^4 \operatorname{Sin}[4(e + f x)] + 9 a^4 b^3 c^2 d^5 \operatorname{Sin}[4(e + f x)] - \\
& 36 a^2 b^5 c^2 d^5 \operatorname{Sin}[4(e + f x)] + 21 b^7 c^2 d^5 \operatorname{Sin}[4(e + f x)] - 3 a^5 b^2 c d^6 \operatorname{Sin}[4(e + f x)] + 6 a^3 b^4 c d^6 \operatorname{Sin}[4(e + f x)] - \\
& 6 a b^6 c d^6 \operatorname{Sin}[4(e + f x)] - 6 a^4 b^3 d^7 \operatorname{Sin}[4(e + f x)] + 21 a^2 b^5 d^7 \operatorname{Sin}[4(e + f x)] - 12 b^7 d^7 \operatorname{Sin}[4(e + f x)]) / \\
& (16 (a^2 - b^2)^2 (-bc + ad)^4 (c^2 - d^2)^2 f (a + b \operatorname{Sin}[e + f x])^2 (c + d \operatorname{Sin}[e + f x])^2)
\end{aligned}$$

- **Problem 745: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \operatorname{Sin}[e + f x])^{5/2}}{a + b \operatorname{Sin}[e + f x]} dx$$

Optimal (type 4, 296 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 d^2 \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{3 b f} + \frac{2 d (7 b c - 3 a d) \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \sin[e + f x]}}{3 b^2 f \sqrt{\frac{c + d \sin[e + f x]}{c + d}}} \\
& \frac{2 d (6 a b c d - 3 a^2 d^2 - b^2 (2 c^2 + d^2)) \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{3 b^3 f \sqrt{c + d \sin[e + f x]}} + \\
& \frac{2 (b c - a d)^3 \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{b^3 (a + b) f \sqrt{c + d \sin[e + f x]}}
\end{aligned}$$

Result (type 4, 868 leaves):

$$\begin{aligned}
& \frac{2d^2 \cos[e+fx] \sqrt{c+d \sin[e+fx]}}{3bf} - \\
& \frac{1}{6bf} \left(\frac{2(-6bc^3 - 7bcd^2 + ad^3) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(-e + \frac{\pi}{2} - fx), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{(a+b) \sqrt{c+d \sin[e+fx]}} - \left(2i(-18bc^2d + 4acd^2 - 2bd^3) \right. \right. \\
& \left. \left. \cos[e+fx] \left((bc-ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + ad \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, \right. \right. \right. \\
& \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) \right) / \\
& \left(bd^2 \sqrt{-\frac{1}{c+d}} (bc-ad) (a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) - \\
& \left(2i(7bcd^2 - 3ad^3) \cos[e+fx] \cos[2(e+fx)] \left(2b(c-d) (bc-ad) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + \right. \right. \\
& \left. \left. d \left(-2(a+b) (-bc+ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + \right. \right. \\
& \left. \left. \left(2a^2 - b^2 \right) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \\
& \left. \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) / \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc-ad) (a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \right. \\
& \left. \left. (-2c^2+d^2+4c(c+d \sin[e+fx]) - 2(c+d \sin[e+fx])^2) \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) \right)
\end{aligned}$$

- **Problem 746: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c + d \sin[e + f x])^{3/2}}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 229 leaves, 8 steps):

$$\frac{2 d \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c+d}\right] \sqrt{c + d \sin[e + f x]}}{b f \sqrt{\frac{c+d \sin[e+f x]}{c+d}}} +$$

$$\frac{2 d (b c - a d) \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \sin[e+f x]}{c+d}}}{b^2 f \sqrt{c + d \sin[e + f x]}} + \frac{2 (b c - a d)^2 \operatorname{EllipticPi}\left[\frac{2 b}{a+b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \sin[e+f x]}{c+d}}}{b^2 (a + b) f \sqrt{c + d \sin[e + f x]}}$$

Result (type 4, 242 leaves):

$$\frac{1}{b^2 \sqrt{-\frac{1}{c+d}} f} 2 i \left(b (c - d) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[e + f x]}\right], \frac{c + d}{c - d}\right] + \right.$$

$$(a d + b (-2 c + d)) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[e + f x]}\right], \frac{c + d}{c - d}\right] +$$

$$\left. (b c - a d) \operatorname{EllipticPi}\left[\frac{b (c + d)}{b c - a d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[e + f x]}\right], \frac{c + d}{c - d}\right] \right)$$

$$\operatorname{Sec}[e + f x] \sqrt{-\frac{d (-1 + \sin[e + f x])}{c + d}} \sqrt{-\frac{d (1 + \sin[e + f x])}{c - d}}$$

- **Problem 749: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sin[e + f x]) (c + d \sin[e + f x])^{3/2}} dx$$

Optimal (type 4, 220 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 d^2 \operatorname{Cos}[e + f x]}{(b c - a d) (c^2 - d^2) f \sqrt{c + d \operatorname{Sin}[e + f x]}} - \\
& \frac{2 d \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \operatorname{Sin}[e + f x]}}{(b c - a d) (c^2 - d^2) f \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}}} + \frac{2 b \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}}}{(a + b) (b c - a d) f \sqrt{c + d \operatorname{Sin}[e + f x]}}
\end{aligned}$$

Result (type 4, 877 leaves):

$$\begin{aligned}
& - \frac{2 d^2 \operatorname{Cos}[e+f x]}{(b c-a d)\left(c^2-d^2\right) f \sqrt{c+d \operatorname{Sin}[e+f x]}} - \\
& \frac{1}{2(c-d)(c+d)(b c-a d) f} \left(- \frac{2\left(-2 b c^2+2 a c d+3 b d^2\right) \operatorname{EllipticPi}\left[\frac{2 b}{a+b}, \frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right), \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \operatorname{Sin}[e+f x]}{c+d}}}{(a+b) \sqrt{c+d \operatorname{Sin}[e+f x]}} - \right. \\
& \left(2 i\left(2 b c d+2 a d^2\right) \operatorname{Cos}[e+f x] \left((b c-a d) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right]+a d \operatorname{EllipticPi}\left[\frac{b(c+d)}{b c-a d}, \right. \right. \\
& \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right]\right) \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} \sqrt{-\frac{d+d \operatorname{Sin}[e+f x]}{c-d}}(-b c+a d+b(c+d \operatorname{Sin}[e+f x])) \right) / \\
& \left(b d^2 \sqrt{-\frac{1}{c+d}}(b c-a d)(a+b \operatorname{Sin}[e+f x]) \sqrt{1-\operatorname{Sin}[e+f x]^2} \sqrt{-\frac{c^2-d^2-2 c(c+d \operatorname{Sin}[e+f x])+(c+d \operatorname{Sin}[e+f x])^2}{d^2}} \right) + \\
& \left(2 i d \operatorname{Cos}[e+f x] \operatorname{Cos}[2(e+f x)] \left(2 b(c-d)(b c-a d) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right]+ \right. \right. \\
& \left. \left. d \left(-2(a+b)(-b c+a d) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right]+ \right. \right. \\
& \left. \left. \left(2 a^2-b^2\right) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{b c-a d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right]\right) \right) \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} \\
& \left. \sqrt{-\frac{d+d \operatorname{Sin}[e+f x]}{c-d}}(-b c+a d+b(c+d \operatorname{Sin}[e+f x])) \right) / \left(b \sqrt{-\frac{1}{c+d}}(b c-a d)(a+b \operatorname{Sin}[e+f x]) \sqrt{1-\operatorname{Sin}[e+f x]^2} \right. \\
& \left. \left. \left. (-2 c^2+d^2+4 c(c+d \operatorname{Sin}[e+f x])-2(c+d \operatorname{Sin}[e+f x])^2\right) \sqrt{-\frac{c^2-d^2-2 c(c+d \operatorname{Sin}[e+f x])+(c+d \operatorname{Sin}[e+f x])^2}{d^2}} \right) \right)
\end{aligned}$$

- **Problem 750: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sin[e + f x]) (c + d \sin[e + f x])^{5/2}} dx$$

Optimal (type 4, 399 leaves, 10 steps):

$$\begin{aligned} & - \frac{2 d^2 \cos[e + f x]}{3 (b c - a d) (c^2 - d^2) f (c + d \sin[e + f x])^{3/2}} - \frac{2 d^2 (7 b c^2 - 4 a c d - 3 b d^2) \cos[e + f x]}{3 (b c - a d)^2 (c^2 - d^2)^2 f \sqrt{c + d \sin[e + f x]}} - \\ & \frac{2 d (7 b c^2 - 4 a c d - 3 b d^2) \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \sin[e + f x]}}{3 (b c - a d)^2 (c^2 - d^2)^2 f \sqrt{\frac{c + d \sin[e + f x]}{c + d}}} + \\ & \frac{2 d \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{3 (b c - a d) (c^2 - d^2) f \sqrt{c + d \sin[e + f x]}} + \frac{2 b^2 \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{(a + b) (b c - a d)^2 f \sqrt{c + d \sin[e + f x]}} \end{aligned}$$

Result (type 4, 1079 leaves):

$$\begin{aligned}
& \frac{\sqrt{c+d \sin[ex+f]}}{f} \left(-\frac{2d^2 \cos[ex+f]}{3(bc-ad)(c^2-d^2)(c+d \sin[ex+f])^2} + \frac{2(-7bc^2d^2 \cos[ex+f] + 4acd^3 \cos[ex+f] + 3bd^4 \cos[ex+f])}{3(bc-ad)^2(c^2-d^2)^2(c+d \sin[ex+f])} \right) + \frac{1}{6(c-d)^2(c+d)^2(bc-ad)^2f} \\
& \left(-\left(2(6b^2c^4 - 12abc^3d + 6a^2c^2d^2 - 19b^2c^2d^2 + 8abcd^3 + 2a^2d^4 + 9b^2d^4) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[ex+f]}{c+d}} \right) / \right. \\
& \left. \left((a+b) \sqrt{c+d \sin[ex+f]} \right) - \left(2i(-12b^2c^3d - 8abc^2d^2 + 8a^2cd^3 + 4b^2cd^3 + 8abd^4) \cos[ex+f] \right. \right. \\
& \left. \left((bc-ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[ex+f]}\right], \frac{c+d}{c-d}\right] + ad \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, \right. \right. \right. \\
& \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[ex+f]}\right], \frac{c+d}{c-d}\right] \right) \sqrt{\frac{d-d \sin[ex+f]}{c+d}} \sqrt{-\frac{d+d \sin[ex+f]}{c-d}} (-bc+ad+b(c+d \sin[ex+f])) \right) / \right. \\
& \left. \left(bd^2 \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[ex+f]) \sqrt{1-\sin[ex+f]^2} \sqrt{-\frac{c^2-d^2-2c(c+d \sin[ex+f])+(c+d \sin[ex+f])^2}{d^2}} \right) - \right. \\
& \left. \left(2i(7b^2c^2d^2 - 4abcd^3 - 3b^2d^4) \cos[ex+f] \cos[2(ex+f)] \left(2b(c-d)(bc-ad) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[ex+f]}\right], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{c+d}{c-d} \right] + d \left(-2(a+b)(-bc+ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[ex+f]}\right], \frac{c+d}{c-d}\right] + \right. \right. \right. \\
& \left. \left. \left. (2a^2-b^2)d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[ex+f]}\right], \frac{c+d}{c-d}\right] \right) \right) \sqrt{\frac{d-d \sin[ex+f]}{c+d}} \right. \\
& \left. \left. \sqrt{-\frac{d+d \sin[ex+f]}{c-d}} (-bc+ad+b(c+d \sin[ex+f])) \right) / \left(bd^2 \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[ex+f]) \sqrt{1-\sin[ex+f]^2} \right. \right. \\
& \left. \left. (-2c^2+d^2+4c(c+d \sin[ex+f]) - 2(c+d \sin[ex+f])^2) \sqrt{-\frac{c^2-d^2-2c(c+d \sin[ex+f])+(c+d \sin[ex+f])^2}{d^2}} \right) \right)
\end{aligned}$$

■ **Problem 751: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + f x])^{7/2}}{(a + b \sin[e + f x])^2} dx$$

Optimal (type 4, 534 leaves, 10 steps):

$$\frac{d (6 a b c d - 5 a^2 d^2 - b^2 (3 c^2 - 2 d^2)) \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{3 b^2 (a^2 - b^2) f} + \frac{(b c - a d)^2 \cos[e + f x] (c + d \sin[e + f x])^{3/2}}{b (a^2 - b^2) f (a + b \sin[e + f x])} +$$

$$\left((29 a^2 b c d^2 - 15 a^3 d^3 + b^3 (3 c^3 - 20 c d^2) - a b^2 (9 c^2 d - 12 d^3)) \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \sin[e + f x]}\right) /$$

$$\left(3 b^3 (a^2 - b^2) f \sqrt{\frac{c + d \sin[e + f x]}{c + d}} \right) - \left((24 a^3 b c d^3 - 15 a^4 d^4 - 12 a b^3 c d (c^2 + 3 d^2) + 2 a^2 b^2 d^2 (c^2 + 8 d^2) + b^4 (3 c^4 + 16 c^2 d^2 + 2 d^4)) \right.$$

$$\left. \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}} \right) / \left(3 b^4 (a^2 - b^2) f \sqrt{c + d \sin[e + f x]} \right) +$$

$$\frac{(b c - a d)^3 (2 a b c + 5 a^2 d - 7 b^2 d) \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{(a - b) b^4 (a + b)^2 f \sqrt{c + d \sin[e + f x]}}$$

Result (type 4, 1109 leaves):

$$\frac{\sqrt{c + d \sin[e + f x]} \left(-\frac{2 d^3 \cos[e + f x]}{3 b^2} + \frac{-b^3 c^3 \cos[e + f x] + 3 a b^2 c^2 d \cos[e + f x] - 3 a^2 b c d^2 \cos[e + f x] + a^3 d^3 \cos[e + f x]}{b^2 (-a^2 + b^2) (a + b \sin[e + f x])} \right)}{f} -$$

$$\frac{1}{12 (a - b) b^2 (a + b) f} \left(- \left(2 (-12 a b^2 c^4 + 39 b^3 c^3 d - 45 a b^2 c^2 d^2 + a^2 b c d^3 + 20 b^3 c d^3 + 5 a^3 d^4 - 8 a b^2 d^4) \right. \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}} \right) / \left((a + b) \sqrt{c + d \sin[e + f x]} \right) -$$

$$\left(2 i (-12 a b^2 c^3 d - 36 a^2 b c^2 d^2 + 72 b^3 c^2 d^2 + 20 a^3 c d^3 - 56 a b^2 c d^3 + 8 a^2 b d^4 + 4 b^3 d^4) \cos[e + f x] \right.$$

$$\left. \left((b c - a d) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c + d}} \sqrt{c + d \sin[e + f x]}\right], \frac{c + d}{c - d}\right] + a d \operatorname{EllipticPi}\left[\frac{b (c + d)}{b c - a d}, \right. \right. \right.$$

$$\begin{aligned}
& \left. i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \Bigg/ \\
& \left(b d^2 \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) - \\
& \left(2i(3b^3c^3d-9ab^2c^2d^2+29a^2bcd^3-20b^3cd^3-15a^3d^4+12ab^2d^4) \cos[e+fx] \cos[2(e+fx)] \right. \\
& \left(2b(c-d)(bc-ad) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right] + \right. \\
& \left. d \left(-2(a+b)(-bc+ad) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right] + \right. \right. \\
& \left. \left. (2a^2-b^2)d \operatorname{EllipticPi} \left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right] \right) \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \\
& \left. \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) \Bigg/ \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \right. \\
& \left. \left. (-2c^2+d^2+4c(c+d \sin[e+fx])-2(c+d \sin[e+fx])^2) \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) \right) \Bigg)
\end{aligned}$$

■ **Problem 752: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \sin[e+fx])^{5/2}}{(a+b \sin[e+fx])^2} dx$$

Optimal (type 4, 390 leaves, 9 steps):

$$\frac{(bc-ad)^2 \cos[e+fx] \sqrt{c+d \sin[e+fx]} - (2abcd-3a^2d^2-b^2(c^2-2d^2)) \operatorname{EllipticE}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right] \sqrt{c+d \sin[e+fx]}}{b(a^2-b^2) f (a+b \sin[e+fx])} + \frac{b^2(a^2-b^2) f \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{b^2(a^2-b^2) f \sqrt{c+d \sin[e+fx]}}$$

$$\frac{(bc-ad)(2abcd+3a^2d^2-b^2(c^2+4d^2)) \operatorname{EllipticF}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{b^3(a^2-b^2) f \sqrt{c+d \sin[e+fx]}} +$$

$$\frac{(bc-ad)^2(2abc+3a^2d-5b^2d) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{(a-b) b^3 (a+b)^2 f \sqrt{c+d \sin[e+fx]}}$$

Result (type 4, 986 leaves):

$$\frac{(-b^2c^2 \cos[e+fx] + 2abcd \cos[e+fx] - a^2d^2 \cos[e+fx]) \sqrt{c+d \sin[e+fx]}}{b(-a^2+b^2) f (a+b \sin[e+fx])} +$$

$$\frac{1}{4(a-b) b (a+b) f} \left(- \left(2(4abc^3 - 9b^2c^2d + 6abcd^2 + a^2d^3 - 2b^2d^3) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}} \right) / \right.$$

$$\left. \left((a+b) \sqrt{c+d \sin[e+fx]} \right) - \right.$$

$$\left. \left(2i(4abc^2d + 4a^2cd^2 - 12b^2cd^2 + 4abd^3) \cos[e+fx] \left((bc-ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + \right. \right. \right.$$

$$\left. \left. \left. ad \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \right) \right.$$

$$\left. \left. \left. \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) / \right) \right.$$

$$\left. \left(bd^2 \sqrt{-\frac{1}{c+d}} (bc-ad) (a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) - \right.$$

$$\left. \left(2i(-b^2c^2d + 2abcd^2 - 3a^2d^3 + 2b^2d^3) \cos[e+fx] \cos[2(e+fx)] \right) \right)$$

$$\begin{aligned}
& \left(2 b (c-d) (b c-a d) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin [e+f x]}\right], \frac{c+d}{c-d}\right] + \right. \\
& d \left(-2 (a+b) (-b c+a d) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin [e+f x]}\right], \frac{c+d}{c-d}\right] + \right. \\
& \left. \left. (2 a^2-b^2) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{b c-a d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin [e+f x]}\right], \frac{c+d}{c-d}\right]\right) \right) \sqrt{\frac{d-d \sin [e+f x]}{c+d}} \\
& \left. \sqrt{-\frac{d+d \sin [e+f x]}{c-d}} (-b c+a d+b(c+d \sin [e+f x]))\right) / \left(b^2 d \sqrt{-\frac{1}{c+d}} (b c-a d)(a+b \sin [e+f x]) \sqrt{1-\sin [e+f x]^2} \right. \\
& \left. (-2 c^2+d^2+4 c(c+d \sin [e+f x])-2(c+d \sin [e+f x])^2) \sqrt{-\frac{c^2-d^2-2 c(c+d \sin [e+f x])+(c+d \sin [e+f x])^2}{d^2}} \right)
\end{aligned}$$

■ **Problem 753: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \sin [e+f x])^{3/2}}{(a+b \sin [e+f x])^2} dx$$

Optimal (type 4, 351 leaves, 9 steps):

$$\begin{aligned}
& \frac{(b c-a d) \cos [e+f x] \sqrt{c+d \sin [e+f x]}}{(a^2-b^2) f (a+b \sin [e+f x])} + \frac{(b c-a d) \operatorname{EllipticE}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right), \frac{2 d}{c+d}\right] \sqrt{c+d \sin [e+f x]}}{b\left(a^2-b^2\right) f \sqrt{\frac{c+d \sin [e+f x]}{c+d}}} + \\
& \frac{\left(2 a b c d+a^2 d^2-b^2\left(c^2+2 d^2\right)\right) \operatorname{EllipticF}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right), \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \sin [e+f x]}{c+d}}}{b^2\left(a^2-b^2\right) f \sqrt{c+d \sin [e+f x]}} + \\
& \frac{(b c-a d)\left(2 a b c+a^2 d-3 b^2 d\right) \operatorname{EllipticPi}\left[\frac{2 b}{a+b}, \frac{1}{2}\left(e-\frac{\pi}{2}+f x\right), \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \sin [e+f x]}{c+d}}}{(a-b) b^2(a+b)^2 f \sqrt{c+d \sin [e+f x]}}
\end{aligned}$$

Result (type 4, 891 leaves):

$$\begin{aligned}
& \frac{(bc \cos[ex + f] - ad \cos[ex + f]) \sqrt{c + d \sin[ex + f]}}{(a^2 - b^2) f (a + b \sin[ex + f])} + \\
& \frac{1}{4(a-b)(a+b)f} \left(- \frac{2(4ac^2 - 5bcd + ad^2) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(-e + \frac{\pi}{2} - fx), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[ex + f]}{c+d}}}{(a+b) \sqrt{c + d \sin[ex + f]}} - \right. \\
& \left(2i(4acd - 4bd^2) \cos[ex + f] \left((bc - ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + f]}\right], \frac{c+d}{c-d}\right] + ad \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc - ad}, \right. \right. \\
& \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + f]}\right], \frac{c+d}{c-d}\right] \right) \sqrt{\frac{d - d \sin[ex + f]}{c+d}} \sqrt{-\frac{d + d \sin[ex + f]}{c-d}} (-bc + ad + b(c + d \sin[ex + f])) \right) / \\
& \left(bd^2 \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[ex + f]) \sqrt{1 - \sin[ex + f]^2} \sqrt{-\frac{c^2 - d^2 - 2c(c + d \sin[ex + f]) + (c + d \sin[ex + f])^2}{d^2}} - \right. \\
& \left. 2i(-bcd + ad^2) \cos[ex + f] \cos[2(ex + f)] \left(2b(c - d) (bc - ad) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + f]}\right], \frac{c+d}{c-d}\right] + \right. \right. \\
& \left. \left. d \left(-2(a+b)(-bc + ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + f]}\right], \frac{c+d}{c-d}\right] + \right. \right. \right. \\
& \left. \left. \left. (2a^2 - b^2) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc - ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + f]}\right], \frac{c+d}{c-d}\right] \right) \right) \sqrt{\frac{d - d \sin[ex + f]}{c+d}} \right. \\
& \left. \sqrt{-\frac{d + d \sin[ex + f]}{c-d}} (-bc + ad + b(c + d \sin[ex + f])) \right) / \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[ex + f]) \sqrt{1 - \sin[ex + f]^2} \right. \\
& \left. \left. (-2c^2 + d^2 + 4c(c + d \sin[ex + f]) - 2(c + d \sin[ex + f])^2) \sqrt{-\frac{c^2 - d^2 - 2c(c + d \sin[ex + f]) + (c + d \sin[ex + f])^2}{d^2}} \right) \right)
\end{aligned}$$

- **Problem 754: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d \sin[e+f x]}}{(a+b \sin[e+f x])^2} dx$$

Optimal (type 4, 307 leaves, 9 steps):

$$\frac{b \cos[e+f x] \sqrt{c+d \sin[e+f x]}}{(a^2-b^2) f (a+b \sin[e+f x])} + \frac{\text{EllipticE}\left[\frac{1}{2} \left(e-\frac{\pi}{2}+f x\right), \frac{2d}{c+d}\right] \sqrt{c+d \sin[e+f x]}}{(a^2-b^2) f \sqrt{\frac{c+d \sin[e+f x]}{c+d}}} -$$

$$\frac{(bc-ad) \text{EllipticF}\left[\frac{1}{2} \left(e-\frac{\pi}{2}+f x\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+f x]}{c+d}}}{b (a^2-b^2) f \sqrt{c+d \sin[e+f x]}} + \frac{(2abc-a^2d-b^2d) \text{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2} \left(e-\frac{\pi}{2}+f x\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+f x]}{c+d}}}{(a-b) b (a+b)^2 f \sqrt{c+d \sin[e+f x]}}$$

Result (type 4, 846 leaves):

$$\begin{aligned}
& -\frac{b \cos[e+fx] \sqrt{c+d \sin[e+fx]}}{(-a^2+b^2) f (a+b \sin[e+fx])} + \frac{1}{4(a-b)(a+b) f} \left(\frac{2(4ac-bd) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(-e+\frac{\pi}{2}-fx), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{(a+b) \sqrt{c+d \sin[e+fx]}} - \right. \\
& \left. \left(8 i a \cos[e+fx] \left((bc-ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + a d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, \right. \right. \right. \\
& \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) / \right. \\
& \left. \left(b d \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) + \right. \\
& \left. \left(2 i \cos[e+fx] \cos[2(e+fx)] \left(2 b(c-d)(bc-ad) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + \right. \right. \right. \\
& \left. \left. \left. d \left(-2(a+b)(-bc+ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + \right. \right. \right. \\
& \left. \left. \left. (2a^2-b^2) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \right. \\
& \left. \left. \left. \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) / \left(b \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \right. \right. \\
& \left. \left. \left. (-2c^2+d^2+4c(c+d \sin[e+fx]) - 2(c+d \sin[e+fx])^2) \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) \right) \right)
\end{aligned}$$

■ **Problem 755: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \sin[e+fx])^2 \sqrt{c+d \sin[e+fx]}} dx$$

Optimal (type 4, 325 leaves, 9 steps) :

$$\frac{b^2 \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{(a^2 - b^2) (b c - a d) f (a + b \sin[e + f x])} + \frac{b \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2d}{c+d}\right] \sqrt{c + d \sin[e + f x]}}{(a^2 - b^2) (b c - a d) f \sqrt{\frac{c+d \sin[e+f x]}{c+d}}} -$$

$$\frac{\operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+f x]}{c+d}}}{(a^2 - b^2) f \sqrt{c + d \sin[e + f x]}} + \frac{(2 a b c - 3 a^2 d + b^2 d) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+f x]}{c+d}}}{(a - b) (a + b)^2 (b c - a d) f \sqrt{c + d \sin[e + f x]}}$$

Result (type 4, 871 leaves) :

$$\begin{aligned}
& - \frac{b^2 \operatorname{Cos}[e + f x] \sqrt{c + d \operatorname{Sin}[e + f x]}}{(a^2 - b^2) (-bc + ad) f (a + b \operatorname{Sin}[e + f x])} + \\
& \frac{1}{4(a-b)(a+b)(-bc+ad)f} \left(- \frac{2(-4abc + 4a^2d - 3b^2d) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(-e + \frac{\pi}{2} - fx), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \operatorname{Sin}[e+fx]}{c+d}}}{(a+b) \sqrt{c+d \operatorname{Sin}[e+fx]}} + \right. \\
& \left(8i a \operatorname{Cos}[e + f x] \left((bc - ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+fx]}\right], \frac{c+d}{c-d}\right] + a d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, \right. \right. \\
& \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \sqrt{\frac{d-d \operatorname{Sin}[e+fx]}{c+d}} \sqrt{-\frac{d+d \operatorname{Sin}[e+fx]}{c-d}} (-bc+ad+b(c+d \operatorname{Sin}[e+fx])) \right) / \\
& \left(d \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \operatorname{Sin}[e + f x]) \sqrt{1 - \operatorname{Sin}[e + f x]^2} \sqrt{-\frac{c^2 - d^2 - 2c(c+d \operatorname{Sin}[e+fx]) + (c+d \operatorname{Sin}[e+fx])^2}{d^2}} \right) - \\
& \left(2i \operatorname{Cos}[e + f x] \operatorname{Cos}[2(e + f x)] \left(2b(c-d)(bc - ad) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+fx]}\right], \frac{c+d}{c-d}\right] + \right. \right. \\
& \left. \left(-2(a+b)(-bc+ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+fx]}\right], \frac{c+d}{c-d}\right] + \right. \right. \\
& \left. \left. (2a^2 - b^2) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \right) \sqrt{\frac{d-d \operatorname{Sin}[e+fx]}{c+d}} \\
& \left. \sqrt{-\frac{d+d \operatorname{Sin}[e+fx]}{c-d}} (-bc+ad+b(c+d \operatorname{Sin}[e+fx])) \right) / \left(\sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \operatorname{Sin}[e + f x]) \sqrt{1 - \operatorname{Sin}[e + f x]^2} \right. \\
& \left. \left. (-2c^2 + d^2 + 4c(c+d \operatorname{Sin}[e+fx]) - 2(c+d \operatorname{Sin}[e+fx])^2) \sqrt{-\frac{c^2 - d^2 - 2c(c+d \operatorname{Sin}[e+fx]) + (c+d \operatorname{Sin}[e+fx])^2}{d^2}} \right) \right)
\end{aligned}$$

■ **Problem 756: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sin[e + f x])^2 (c + d \sin[e + f x])^{3/2}} dx$$

Optimal (type 4, 449 leaves, 10 steps):

$$\frac{d (2 a^2 d^2 + b^2 (c^2 - 3 d^2)) \cos[e + f x]}{(a^2 - b^2) (b c - a d)^2 (c^2 - d^2) f \sqrt{c + d \sin[e + f x]}} + \frac{b^2 \cos[e + f x]}{(a^2 - b^2) (b c - a d) f (a + b \sin[e + f x]) \sqrt{c + d \sin[e + f x]}} +$$

$$\frac{(2 a^2 d^2 + b^2 (c^2 - 3 d^2)) \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \sin[e + f x]} - b \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{(a^2 - b^2) (b c - a d)^2 (c^2 - d^2) f \sqrt{\frac{c + d \sin[e + f x]}{c + d}}} +$$

$$\frac{b (2 a b c - 5 a^2 d + 3 b^2 d) \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}}}{(a - b) (a + b)^2 (b c - a d)^2 f \sqrt{c + d \sin[e + f x]}}$$

Result (type 4, 1057 leaves):

$$\frac{\sqrt{c + d \sin[e + f x]} \left(\frac{b^3 \cos[e + f x]}{(a^2 - b^2) (-b c + a d)^2 (a + b \sin[e + f x])} + \frac{2 d^3 \cos[e + f x]}{(b c - a d)^2 (c^2 - d^2) (c + d \sin[e + f x])} \right)}{f} +$$

$$\frac{1}{4 (a - b) (a + b) (c - d) (c + d) (-b c + a d)^2 f} \left(- \left(2 (4 a b^2 c^3 - 8 a^2 b c^2 d + 7 b^3 c^2 d + 4 a^3 c d^2 - 8 a b^2 c d^2 + 10 a^2 b d^3 - 9 b^3 d^3) \right. \right.$$

$$\left. \left. \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}} \right) / \left((a + b) \sqrt{c + d \sin[e + f x]} \right) - \right.$$

$$\left. \left(2 i (4 a b^2 c^2 d + 4 a^2 b c d^2 - 4 b^3 c d^2 + 4 a^3 d^3 - 8 a b^2 d^3) \cos[e + f x] \left((b c - a d) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c + d}} \sqrt{c + d \sin[e + f x]}\right], \frac{c + d}{c - d}\right] \right. \right. \right.$$

$$\left. \left. \left. \frac{c + d}{c - d} \right) + a d \operatorname{EllipticPi}\left[\frac{b (c + d)}{b c - a d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c + d}} \sqrt{c + d \sin[e + f x]}\right], \frac{c + d}{c - d}\right] \right) \right)$$

$$\left. \left. \left. \sqrt{\frac{d - d \sin[e + f x]}{c + d}} \sqrt{-\frac{d + d \sin[e + f x]}{c - d}} (-b c + a d + b (c + d \sin[e + f x])) \right) \right) / \right)$$

$$\begin{aligned}
& \left(b d^2 \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[ex + fx]) \sqrt{1 - \sin[ex + fx]^2} \sqrt{-\frac{c^2 - d^2 - 2c(c + d \sin[ex + fx]) + (c + d \sin[ex + fx])^2}{d^2}} \right) - \\
& \left(2i(-b^3 c^2 d - 2a^2 b d^3 + 3b^3 d^3) \cos[ex + fx] \cos[2(ex + fx)] \left(2b(c - d)(bc - ad) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + fx]} \right], \right. \right. \right. \\
& \left. \left. \left. \frac{c+d}{c-d} \right] + d \left(-2(a+b)(-bc + ad) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + fx]} \right], \frac{c+d}{c-d} \right] + \right. \right. \\
& \left. \left. (2a^2 - b^2) d \operatorname{EllipticPi} \left[\frac{b(c+d)}{bc - ad}, i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + fx]} \right], \frac{c+d}{c-d} \right] \right) \right) \sqrt{\frac{d - d \sin[ex + fx]}{c+d}} \\
& \left. \sqrt{-\frac{d + d \sin[ex + fx]}{c-d}} (-bc + ad + b(c + d \sin[ex + fx])) \right) / \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[ex + fx]) \sqrt{1 - \sin[ex + fx]^2} \right. \\
& \left. \left. (-2c^2 + d^2 + 4c(c + d \sin[ex + fx]) - 2(c + d \sin[ex + fx])^2) \sqrt{-\frac{c^2 - d^2 - 2c(c + d \sin[ex + fx]) + (c + d \sin[ex + fx])^2}{d^2}} \right) \right)
\end{aligned}$$

■ **Problem 757: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + b \sin[ex + fx])^2 (c + d \sin[ex + fx])^{5/2}} dx$$

Optimal (type 4, 661 leaves, 11 steps):

$$\frac{d (2 a^2 d^2 + b^2 (3 c^2 - 5 d^2)) \operatorname{Cos}[e + f x]}{3 (a^2 - b^2) (b c - a d)^2 (c^2 - d^2) f (c + d \operatorname{Sin}[e + f x])^{3/2}} + \frac{b^2 \operatorname{Cos}[e + f x]}{(a^2 - b^2) (b c - a d) f (a + b \operatorname{Sin}[e + f x]) (c + d \operatorname{Sin}[e + f x])^{3/2}} -$$

$$\frac{(8 a^3 c d^4 - 8 a b^2 c d^4 - 4 a^2 b d^3 (5 c^2 - 3 d^2) - b^3 (3 c^4 d - 26 c^2 d^3 + 15 d^5)) \operatorname{Cos}[e + f x]}{3 (a^2 - b^2) (b c - a d)^3 (c^2 - d^2)^2 f \sqrt{c + d \operatorname{Sin}[e + f x]}} -$$

$$\left((8 a^3 c d^3 - 8 a b^2 c d^3 - 4 a^2 b d^2 (5 c^2 - 3 d^2) - b^3 (3 c^4 - 26 c^2 d^2 + 15 d^4)) \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \operatorname{Sin}[e + f x]} \right) /$$

$$\left(3 (a^2 - b^2) (b c - a d)^3 (c^2 - d^2)^2 f \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}} \right) - \frac{(2 a^2 d^2 + b^2 (3 c^2 - 5 d^2)) \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}}}{3 (a^2 - b^2) (b c - a d)^2 (c^2 - d^2) f \sqrt{c + d \operatorname{Sin}[e + f x]}} +$$

$$\frac{b^2 (2 a b c - 7 a^2 d + 5 b^2 d) \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}}}{(a - b) (a + b)^2 (b c - a d)^3 f \sqrt{c + d \operatorname{Sin}[e + f x]}}$$

Result (type 4, 1319 leaves):

$$\frac{1}{f} \sqrt{c + d \operatorname{Sin}[e + f x]} \left(- \frac{b^4 \operatorname{Cos}[e + f x]}{(a^2 - b^2) (-b c + a d)^3 (a + b \operatorname{Sin}[e + f x])} + \frac{2 d^3 \operatorname{Cos}[e + f x]}{3 (b c - a d)^2 (c^2 - d^2) (c + d \operatorname{Sin}[e + f x])^2} - \right.$$

$$\left. \frac{4 (-5 b c^2 d^3 \operatorname{Cos}[e + f x] + 2 a c d^4 \operatorname{Cos}[e + f x] + 3 b d^5 \operatorname{Cos}[e + f x])}{3 (b c - a d)^3 (c^2 - d^2)^2 (c + d \operatorname{Sin}[e + f x])} \right) + \frac{1}{12 (a - b) (a + b) (c - d)^2 (c + d)^2 (-b c + a d)^3 f}$$

$$\left(- \left(2 (-12 a b^3 c^5 + 36 a^2 b^2 c^4 d - 33 b^4 c^4 d - 36 a^3 b c^3 d^2 + 60 a b^3 c^3 d^2 + 12 a^4 c^2 d^3 - 104 a^2 b^2 c^2 d^3 + 86 b^4 c^2 d^3 + 28 a^3 b c d^4 - 40 a b^3 c d^4 + \right. \right.$$

$$\left. 4 a^4 d^5 + 44 a^2 b^2 d^5 - 45 b^4 d^5) \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}} \right) / \left((a + b) \sqrt{c + d \operatorname{Sin}[e + f x]} \right) -$$

$$\left(2 i (-12 a b^3 c^4 d - 36 a^2 b^2 c^3 d^2 + 36 b^4 c^3 d^2 - 28 a^3 b c^2 d^3 + 52 a b^3 c^2 d^3 + 16 a^4 c d^4 + 4 a^2 b^2 c d^4 - 20 b^4 c d^4 + 28 a^3 b d^5 - 40 a b^3 d^5) \right.$$

$$\left. \operatorname{Cos}[e + f x] \left((b c - a d) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c + d}} \sqrt{c + d \operatorname{Sin}[e + f x]}\right], \frac{c + d}{c - d}\right] + a d \operatorname{EllipticPi}\left[\frac{b (c + d)}{b c - a d}, \right. \right.$$

$$\begin{aligned}
& \left. i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \Big/ \\
& \left(b d^2 \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) - \\
& \left(2i(3b^4c^4d+20a^2b^2c^2d^3-26b^4c^2d^3-8a^3bcd^4+8ab^3cd^4-12a^2b^2d^5+15b^4d^5) \cos[e+fx] \right. \\
& \cos[2(e+fx)] \left(2b(c-d)(bc-ad) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right] + \right. \\
& d \left(-2(a+b)(-bc+ad) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right] + \right. \\
& \left. \left. (2a^2-b^2)d \operatorname{EllipticPi} \left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right] \right) \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \\
& \left. \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) \Big/ \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \right. \\
& \left. \left. (-2c^2+d^2+4c(c+d \sin[e+fx])-2(c+d \sin[e+fx])^2) \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) \right)
\end{aligned}$$

■ **Problem 758: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c+d \sin[e+fx])^{9/2}}{(a+b \sin[e+fx])^3} dx$$

Optimal (type 4, 816 leaves, 11 steps):

$$\begin{aligned}
& \frac{1}{12 b^3 (a^2 - b^2)^2 f} d (36 a^3 b c d^2 - 35 a^4 d^3 + b^4 d (45 c^2 - 8 d^2) - 18 a b^3 c (c^2 + 5 d^2) + a^2 b^2 d (9 c^2 + 61 d^2)) \operatorname{Cos}[e + f x] \sqrt{c + d \operatorname{Sin}[e + f x]} + \\
& \frac{(b c - a d)^2 (6 a b c + 7 a^2 d - 13 b^2 d) \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^{3/2}}{4 b^2 (a^2 - b^2)^2 f (a + b \operatorname{Sin}[e + f x])} + \frac{(b c - a d)^2 \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^{5/2}}{2 b (a^2 - b^2) f (a + b \operatorname{Sin}[e + f x])^2} + \\
& \left((185 a^4 b c d^3 - 105 a^5 d^4 - b^5 c d (51 c^2 - 104 d^2) - 15 a^3 b^2 d^2 (3 c^2 - 13 d^2) - a^2 b^3 c d (21 c^2 + 361 d^2) + 9 a b^4 (2 c^4 + 17 c^2 d^2 - 8 d^4)) \right. \\
& \left. \operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{c + d \operatorname{Sin}[e + f x]}\right) / \left(12 b^4 (a^2 - b^2)^2 f \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}} \right) - \\
& \left((150 a^5 b c d^4 - 105 a^6 d^5 - 12 a^3 b^3 c d^2 (4 c^2 + 29 d^2) + a^4 b^2 d^3 (26 c^2 + 223 d^2) - b^6 d (57 c^4 + 136 c^2 d^2 + 8 d^4) + 6 a b^5 c (3 c^4 + 38 c^2 d^2 + 48 d^4) - \right. \\
& \left. a^2 b^4 d (33 c^4 + 70 c^2 d^2 + 128 d^4)) \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}} \right) / \left(12 b^5 (a^2 - b^2)^2 f \sqrt{c + d \operatorname{Sin}[e + f x]} \right) + \\
& \left((b c - a d)^3 (20 a^3 b c d - 44 a b^3 c d + 35 a^4 d^2 + 2 a^2 b^2 (4 c^2 - 43 d^2) + b^4 (4 c^2 + 63 d^2)) \operatorname{EllipticPi}\left[\frac{2 b}{a + b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2 d}{c + d}\right] \right. \\
& \left. \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}} \right) / \left(4 (a - b)^2 b^5 (a + b)^3 f \sqrt{c + d \operatorname{Sin}[e + f x]} \right)
\end{aligned}$$

Result (type 4, 1526 leaves):

$$\begin{aligned}
& \frac{1}{f} \sqrt{c + d \operatorname{Sin}[e + f x]} \\
& \left(-\frac{2 d^4 \operatorname{Cos}[e + f x]}{3 b^3} + (-b^4 c^4 \operatorname{Cos}[e + f x] + 4 a b^3 c^3 d \operatorname{Cos}[e + f x] - 6 a^2 b^2 c^2 d^2 \operatorname{Cos}[e + f x] + 4 a^3 b c d^3 \operatorname{Cos}[e + f x] - a^4 d^4 \operatorname{Cos}[e + f x]) / \right. \\
& \left(2 b^3 (-a^2 + b^2) (a + b \operatorname{Sin}[e + f x])^2 \right) + (6 a b^4 c^4 \operatorname{Cos}[e + f x] - 7 a^2 b^3 c^3 d \operatorname{Cos}[e + f x] - 17 b^5 c^3 d \operatorname{Cos}[e + f x] - 15 a^3 b^2 c^2 d^2 \operatorname{Cos}[e + f x] + \\
& 51 a b^4 c^2 d^2 \operatorname{Cos}[e + f x] + 27 a^4 b c d^3 \operatorname{Cos}[e + f x] - 51 a^2 b^3 c d^3 \operatorname{Cos}[e + f x] - 11 a^5 d^4 \operatorname{Cos}[e + f x] + 17 a^3 b^2 d^4 \operatorname{Cos}[e + f x]) / \\
& \left. (4 b^3 (-a^2 + b^2)^2 (a + b \operatorname{Sin}[e + f x])) \right) - \frac{1}{48 (a - b)^2 b^3 (a + b)^2 f} \\
& \left(-\left(2 (-48 a^2 b^3 c^5 - 24 b^5 c^5 + 306 a b^4 c^4 d - 177 a^2 b^3 c^3 d^2 - 327 b^5 c^3 d^2 - 105 a^3 b^2 c^2 d^3 + 501 a b^4 c^2 d^3 + 13 a^4 b c d^4 - 53 a^2 b^3 c d^4 - 104 b^5 c d^4 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (35 a^5 d^5 - 73 a^3 b^2 d^5 + 56 a b^4 d^5) \operatorname{EllipticPi}\left[\frac{2 b}{a+b}, \frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right), \frac{2 d}{c+d}\right] \sqrt{\frac{c+d \operatorname{Sin}[e+f x]}{c+d}} \Big/ \left((a+b) \sqrt{c+d \operatorname{Sin}[e+f x]}\right) - \\
& \left(2 i \left(-60 a^2 b^3 c^4 d - 12 b^5 c^4 d + 36 a^3 b^2 c^3 d^2 + 252 a b^4 c^3 d^2 - 228 a^4 b c^2 d^3 + 276 a^2 b^3 c^2 d^3 - 480 b^5 c^2 d^3 + 140 a^5 c d^4 - 364 a^3 b^2 c d^4 + \right. \right. \\
& \left. 512 a b^4 c d^4 + 56 a^4 b d^5 - 112 a^2 b^3 d^5 - 16 b^5 d^5\right) \operatorname{Cos}[e+f x] \left((b c - a d) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right] + \right. \\
& \left. a d \operatorname{EllipticPi}\left[\frac{b(c+d)}{b c - a d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right] \right) \\
& \left. \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} \sqrt{-\frac{d+d \operatorname{Sin}[e+f x]}{c-d}} (-b c + a d + b(c+d \operatorname{Sin}[e+f x])) \right) \Big/ \\
& \left(b d^2 \sqrt{-\frac{1}{c+d}} (b c - a d) (a + b \operatorname{Sin}[e+f x]) \sqrt{1 - \operatorname{Sin}[e+f x]^2} \sqrt{-\frac{c^2 - d^2 - 2 c(c+d \operatorname{Sin}[e+f x]) + (c+d \operatorname{Sin}[e+f x])^2}{d^2}} \right) - \\
& \left(2 i \left(18 a b^4 c^4 d - 21 a^2 b^3 c^3 d^2 - 51 b^5 c^3 d^2 - 45 a^3 b^2 c^2 d^3 + 153 a b^4 c^2 d^3 + 185 a^4 b c d^4 - 361 a^2 b^3 c d^4 + 104 b^5 c d^4 - 105 a^5 d^5 + 195 a^3 b^2 d^5 - \right. \right. \\
& \left. 72 a b^4 d^5\right) \operatorname{Cos}[e+f x] \operatorname{Cos}[2(e+f x)] \left(2 b(c-d)(b c - a d) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right] + \right. \\
& \left. d \left(-2(a+b)(-b c + a d) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right] + \right. \right. \\
& \left. \left. (2 a^2 - b^2) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{b c - a d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \operatorname{Sin}[e+f x]}\right], \frac{c+d}{c-d}\right] \right) \right) \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} \\
& \left. \sqrt{-\frac{d+d \operatorname{Sin}[e+f x]}{c-d}} (-b c + a d + b(c+d \operatorname{Sin}[e+f x])) \right) \Big/ \left(b^2 d \sqrt{-\frac{1}{c+d}} (b c - a d) (a + b \operatorname{Sin}[e+f x]) \sqrt{1 - \operatorname{Sin}[e+f x]^2} \right)
\end{aligned}$$

$$\left(-2c^2 + d^2 + 4c(c + d \sin[e + fx]) - 2(c + d \sin[e + fx])^2 \right) \sqrt{-\frac{c^2 - d^2 - 2c(c + d \sin[e + fx]) + (c + d \sin[e + fx])^2}{d^2}}$$

■ **Problem 759: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c + d \sin[e + fx])^{7/2}}{(a + b \sin[e + fx])^3} dx$$

Optimal (type 4, 605 leaves, 10 steps):

$$\frac{(bc - ad)^2 (6abc + 5a^2d - 11b^2d) \cos[e + fx] \sqrt{c + d \sin[e + fx]} + \frac{(bc - ad)^2 \cos[e + fx] (c + d \sin[e + fx])^{3/2}}{2b(a^2 - b^2)f(a + b \sin[e + fx])^2} - \left((8a^3bcd^2 - 15a^4d^3 + b^4d(13c^2 - 8d^2) - 2ab^3c(3c^2 + 13d^2) + a^2b^2d(5c^2 + 29d^2)) \operatorname{EllipticE}\left[\frac{1}{2}\left(e - \frac{\pi}{2} + fx\right), \frac{2d}{c+d}\right] \sqrt{c + d \sin[e + fx]} \right)}{4b^2(a^2 - b^2)^2 f(a + b \sin[e + fx])} + \left(4b^3(a^2 - b^2)^2 f \sqrt{\frac{c + d \sin[e + fx]}{c + d}} \right) + \left(3(bc - ad)(4a^3bcd^2 + 5a^4d^3 + a^2b^2d(c^2 - 11d^2) - 2ab^3c(c^2 + 5d^2) + b^4d(5c^2 + 8d^2)) \operatorname{EllipticF}\left[\frac{1}{2}\left(e - \frac{\pi}{2} + fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c + d \sin[e + fx]}{c + d}} \right) / \left(4b^4(a^2 - b^2)^2 f \sqrt{c + d \sin[e + fx]} \right) + \left((bc - ad)^2 (12a^3bcd - 36ab^3cd + 15a^4d^2 + 2a^2b^2(4c^2 - 19d^2) + b^4(4c^2 + 35d^2)) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c + d \sin[e + fx]}{c + d}} \right) / \left(4(a - b)^2 b^4 (a + b)^3 f \sqrt{c + d \sin[e + fx]} \right)$$

Result (type 4, 1323 leaves):

$$\frac{1}{f} \sqrt{c + d \sin[e + fx]} \left(\frac{-b^3c^3 \cos[e + fx] + 3a^2b^2c^2d \cos[e + fx] - 3a^2bcd^2 \cos[e + fx] + a^3d^3 \cos[e + fx]}{2b^2(-a^2 + b^2)(a + b \sin[e + fx])^2} + \frac{(6ab^3c^3 \cos[e + fx] - 5a^2b^2c^2d \cos[e + fx] - 13b^4c^2d \cos[e + fx] - 8a^3bcd^2 \cos[e + fx] + 26ab^3cd^2 \cos[e + fx] + 7a^4d^3 \cos[e + fx] - 13a^2b^2d^3 \cos[e + fx])}{(4b^2(-a^2 + b^2)^2(a + b \sin[e + fx]))} \right) + \frac{1}{16(a - b)^2 b^2 (a + b)^2 f} \left(- \left(2(16a^2b^2c^4 + 8b^4c^4 - 78ab^3c^3d + 33a^2b^2c^2d^2 + 57b^4c^2d^2 + 8a^3bcd^3 - 50ab^3cd^3 + 5a^4d^4 - 7a^2b^2d^4 + 8b^4d^4) \right) \right)$$

$$\begin{aligned}
& \text{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}} \Big/ \left((a+b) \sqrt{c+d \sin[e+fx]}\right) - \\
& \left(2i \left(20a^2b^2c^3d + 4b^4c^3d - 8a^3bc^2d^2 - 64ab^3c^2d^2 + 20a^4cd^3 - 12a^2b^2cd^3 + 64b^4cd^3 + 8a^3bd^4 - 32ab^3d^4 \right) \right. \\
& \cos[e+fx] \left((bc-ad) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + ad \text{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, \right. \right. \\
& \left. \left. i \text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \sqrt{\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \Big/ \\
& \left(bd^2 \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) - \\
& \left(2i \left(-6ab^3c^3d + 5a^2b^2c^2d^2 + 13b^4c^2d^2 + 8a^3bcd^3 - 26ab^3cd^3 - 15a^4d^4 + 29a^2b^2d^4 - 8b^4d^4 \right) \cos[e+fx] \right. \\
& \cos[2(e+fx)] \left(2b(c-d)(bc-ad) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + \right. \\
& d \left(-2(a+b)(-bc+ad) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + \right. \\
& \left. \left. (2a^2-b^2)d \text{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \\
& \left. \sqrt{\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) \Big/ \left(b^2d \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \right. \\
& \left. \left. (-2c^2+d^2+4c(c+d \sin[e+fx]) - 2(c+d \sin[e+fx])^2) \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) \right)
\end{aligned}$$

Problem 760: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(c + d \sin[e + f x])^{5/2}}{(a + b \sin[e + f x])^3} dx$$

Optimal (type 4, 549 leaves, 10 steps):

$$\frac{(bc - ad)^2 \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{2b(a^2 - b^2)f(a + b \sin[e + f x])^2} + \frac{3(bc - ad)(2abc + a^2d - 3b^2d) \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{4b(a^2 - b^2)^2 f(a + b \sin[e + f x])} +$$

$$\frac{3(bc - ad)(2abc + a^2d - 3b^2d) \operatorname{EllipticE}\left[\frac{1}{2}\left(e - \frac{\pi}{2} + f x\right), \frac{2d}{c+d}\right] \sqrt{c + d \sin[e + f x]}}{4b^2(a^2 - b^2)^2 f \sqrt{\frac{c + d \sin[e + f x]}{c + d}}} +$$

$$\left((4a^3bcd^2 + 3a^4d^3 + a^2b^2d(7c^2 - 5d^2) + b^4d(11c^2 + 8d^2) - 2ab^3c(3c^2 + 11d^2)) \operatorname{EllipticF}\left[\frac{1}{2}\left(e - \frac{\pi}{2} + f x\right), \frac{2d}{c+d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}} \right) /$$

$$(4b^3(a^2 - b^2)^2 f \sqrt{c + d \sin[e + f x]}) + \left((bc - ad)(4a^3bcd - 28ab^3cd + 3a^4d^2 + 2a^2b^2(4c^2 - 3d^2) + b^4(4c^2 + 15d^2)) \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right), \frac{2d}{c+d}\right] \sqrt{\frac{c + d \sin[e + f x]}{c + d}} \right) / (4(a - b)^2 b^3 (a + b)^3 f \sqrt{c + d \sin[e + f x]})$$

Result (type 4, 1149 leaves):

$$\frac{1}{f} \sqrt{c + d \sin[e + f x]} \left(\frac{-b^2 c^2 \cos[e + f x] + 2abcd \cos[e + f x] - a^2 d^2 \cos[e + f x]}{2b(-a^2 + b^2)(a + b \sin[e + f x])^2} - \right.$$

$$\left. \frac{(3(-2ab^2c^2 \cos[e + f x] + a^2bcd \cos[e + f x] + 3b^3cd \cos[e + f x] + a^3d^2 \cos[e + f x] - 3ab^2d^2 \cos[e + f x]))}{(4b(-a^2 + b^2)^2(a + b \sin[e + f x]))} - \frac{1}{16(a - b)^2 b(a + b)^2 f} \right)$$

$$\left(- \left(2(-16a^2bc^3 - 8b^3c^3 + 54ab^2c^2d - 15a^2bcd^2 - 21b^3cd^2 + a^3d^3 + 5ab^2d^3) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right), \frac{2d}{c+d}\right] \right. \right.$$

$$\left. \left. \sqrt{\frac{c + d \sin[e + f x]}{c + d}} \right) / \left((a + b) \sqrt{c + d \sin[e + f x]} \right) - \left(2i(-20a^2bc^2d - 4b^3c^2d + 4a^3cd^2 + 44ab^2cd^2 - 8a^2bd^3 - 16b^3d^3) \right. \right.$$

$$\begin{aligned}
& \cos[e + f x] \left((bc - ad) \operatorname{EllipticF}\left[\operatorname{i ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e + f x]}\right], \frac{c+d}{c-d}\right] + a d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, \right. \right. \\
& \quad \left. \left. \operatorname{i ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e + f x]}\right], \frac{c+d}{c-d}\right] \sqrt{\frac{d-d \sin[e + f x]}{c+d}} \sqrt{\frac{d+d \sin[e + f x]}{c-d}} (-bc+ad+b(c+d \sin[e + f x])) \right) / \\
& \left(b d^2 \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[e + f x]) \sqrt{1 - \sin[e + f x]^2} \sqrt{-\frac{c^2 - d^2 - 2c(c+d \sin[e + f x]) + (c+d \sin[e + f x])^2}{d^2}} \right) - \\
& \left(2 i (6 a b^2 c^2 d - 3 a^2 b c d^2 - 9 b^3 c d^2 - 3 a^3 d^3 + 9 a b^2 d^3) \cos[e + f x] \cos[2(e + f x)] \right. \\
& \quad \left(2 b(c-d)(bc - ad) \operatorname{EllipticE}\left[\operatorname{i ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e + f x]}\right], \frac{c+d}{c-d}\right] + \right. \\
& \quad \left. d \left(-2(a+b)(-bc+ad) \operatorname{EllipticF}\left[\operatorname{i ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e + f x]}\right], \frac{c+d}{c-d}\right] + \right. \right. \\
& \quad \left. \left. (2 a^2 - b^2) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, \operatorname{i ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e + f x]}\right], \frac{c+d}{c-d}\right] \right) \right) \sqrt{\frac{d-d \sin[e + f x]}{c+d}} \\
& \quad \left. \sqrt{-\frac{d+d \sin[e + f x]}{c-d}} (-bc+ad+b(c+d \sin[e + f x])) \right) / \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[e + f x]) \sqrt{1 - \sin[e + f x]^2} \right. \\
& \quad \left. \left. (-2 c^2 + d^2 + 4 c(c+d \sin[e + f x]) - 2(c+d \sin[e + f x])^2) \sqrt{-\frac{c^2 - d^2 - 2c(c+d \sin[e + f x]) + (c+d \sin[e + f x])^2}{d^2}} \right) \right)
\end{aligned}$$

■ **Problem 761: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \sin[e + f x])^{3/2}}{(a+b \sin[e + f x])^3} dx$$

Optimal (type 4, 472 leaves, 10 steps):

$$\frac{(bc - ad) \cos[e + fx] \sqrt{c + d \sin[e + fx]}}{2(a^2 - b^2) f (a + b \sin[e + fx])^2} + \frac{(6abc - a^2d - 5b^2d) \cos[e + fx] \sqrt{c + d \sin[e + fx]}}{4(a^2 - b^2)^2 f (a + b \sin[e + fx])} +$$

$$\frac{(6abc - a^2d - 5b^2d) \operatorname{EllipticE}\left[\frac{1}{2}\left(e - \frac{\pi}{2} + fx\right), \frac{2d}{c+d}\right] \sqrt{c + d \sin[e + fx]}}{4b(a^2 - b^2)^2 f \sqrt{\frac{c + d \sin[e + fx]}{c+d}}} -$$

$$\frac{(bc - ad)(6abc + a^2d - 7b^2d) \operatorname{EllipticF}\left[\frac{1}{2}\left(e - \frac{\pi}{2} + fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c + d \sin[e + fx]}{c+d}}}{4b^2(a^2 - b^2)^2 f \sqrt{c + d \sin[e + fx]}} -$$

$$\left(\frac{(4a^3bcd + 20ab^3cd + a^4d^2 - b^4(4c^2 + 3d^2) - 2a^2b^2(4c^2 + 5d^2)) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c + d \sin[e + fx]}{c+d}}}{4(a-b)^2 b^2 (a+b)^3 f \sqrt{c + d \sin[e + fx]}} \right) /$$

Result (type 4, 1001 leaves):

$$\frac{\sqrt{c + d \sin[e + fx]} \left(\frac{bc \cos[e + fx] - ad \cos[e + fx]}{2(a^2 - b^2)(a + b \sin[e + fx])^2} + \frac{6abc \cos[e + fx] - a^2d \cos[e + fx] - 5b^2d \cos[e + fx]}{4(a^2 - b^2)^2(a + b \sin[e + fx])} \right)}{f} +$$

$$\frac{1}{16(a-b)^2(a+b)^2 f} \left(- \left(2(16a^2c^2 + 8b^2c^2 - 30abcd + 5a^2d^2 + b^2d^2) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c + d \sin[e + fx]}{c+d}} \right) / \right.$$

$$\left. \left((a+b) \sqrt{c + d \sin[e + fx]} \right) - \right.$$

$$\left. \left(2i(20a^2cd + 4b^2cd - 24abd^2) \cos[e + fx] \left((bc - ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[e + fx]}\right], \frac{c+d}{c-d}\right] + \right. \right.$$

$$\left. \left. ad \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc - ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[e + fx]}\right], \frac{c+d}{c-d}\right] \right) \right.$$

$$\left. \left. \sqrt{\frac{d - d \sin[e + fx]}{c+d}} \sqrt{-\frac{d + d \sin[e + fx]}{c-d}} (-bc + ad + b(c + d \sin[e + fx])) \right) / \right.$$

$$\begin{aligned}
& \left(b d^2 \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[e + fx]) \sqrt{1 - \sin[e + fx]^2} \sqrt{-\frac{c^2 - d^2 - 2c(c + d \sin[e + fx]) + (c + d \sin[e + fx])^2}{d^2}} \right) - \\
& \left(2i(-6abcd + a^2d^2 + 5b^2d^2) \cos[e + fx] \cos[2(e + fx)] \left(2b(c - d)(bc - ad) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[e + fx]}\right]\right], \right. \right. \\
& \quad \left. \left. \frac{c+d}{c-d}\right] + d \left(-2(a+b)(-bc + ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[e + fx]}\right]\right], \frac{c+d}{c-d}\right] + \right. \\
& \quad \left. \left. (2a^2 - b^2)d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc - ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[e + fx]}\right], \frac{c+d}{c-d}\right]\right) \right) \sqrt{\frac{d - d \sin[e + fx]}{c+d}} \\
& \left. \sqrt{-\frac{d + d \sin[e + fx]}{c-d}} (-bc + ad + b(c + d \sin[e + fx])) \right) / \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[e + fx]) \sqrt{1 - \sin[e + fx]^2} \right. \\
& \quad \left. \left. (-2c^2 + d^2 + 4c(c + d \sin[e + fx]) - 2(c + d \sin[e + fx])^2) \sqrt{-\frac{c^2 - d^2 - 2c(c + d \sin[e + fx]) + (c + d \sin[e + fx])^2}{d^2}} \right) \right)
\end{aligned}$$

- **Problem 762: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c + d \sin[e + fx]}}{(a + b \sin[e + fx])^3} dx$$

Optimal (type 4, 487 leaves, 10 steps):

$$\frac{b \cos[e+fx] \sqrt{c+d \sin[e+fx]}}{2(a^2-b^2) f (a+b \sin[e+fx])^2} + \frac{b(6abc-5a^2d-b^2d) \cos[e+fx] \sqrt{c+d \sin[e+fx]}}{4(a^2-b^2)^2 (bc-ad) f (a+b \sin[e+fx])} +$$

$$\frac{(6abc-5a^2d-b^2d) \operatorname{EllipticE}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right] \sqrt{c+d \sin[e+fx]}}{4(a^2-b^2)^2 (bc-ad) f \sqrt{\frac{c+d \sin[e+fx]}{c+d}}} -$$

$$\frac{3(2abc-a^2d-b^2d) \operatorname{EllipticF}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{4b(a^2-b^2)^2 f \sqrt{c+d \sin[e+fx]}} -$$

$$\left((12a^3bcd+12ab^3cd-3a^4d^2-b^4(4c^2-d^2)-2a^2b^2(4c^2+5d^2)) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}} \right) /$$

$$(4(a-b)^2 b (a+b)^3 (bc-ad) f \sqrt{c+d \sin[e+fx]})$$

Result (type 4, 1038 leaves):

$$\frac{\sqrt{c+d \sin[e+fx]} \left(\frac{b \cos[e+fx]}{2(a^2-b^2)(a+b \sin[e+fx])^2} - \frac{6abc \cos[e+fx]-5a^2bd \cos[e+fx]-b^3d \cos[e+fx]}{4(a^2-b^2)^2(-bc+ad)(a+b \sin[e+fx])} \right)}{f} +$$

$$\frac{1}{16(a-b)^2(a+b)^2(-bc+ad) f} \left(- \left(2(-16a^2bc^2-8b^3c^2+16a^3cd+14ab^2cd-9a^2bd^2+3b^3d^2) \right. \right.$$

$$\left. \left. \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}} \right) / \left((a+b) \sqrt{c+d \sin[e+fx]} \right) - \right.$$

$$\left. \left(2i(-20a^2bcd-4b^3cd+16a^3d^2+8ab^2d^2) \cos[e+fx] \left((bc-ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]} \right], \frac{c+d}{c-d}\right] + \right. \right. \right.$$

$$\left. \left. ad \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]} \right], \frac{c+d}{c-d}\right] \right) \right.$$

$$\left. \left. \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) / \right)$$

$$\begin{aligned}
& \left(b d^2 \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[ex + fx]) \sqrt{1 - \sin[ex + fx]^2} \sqrt{-\frac{c^2 - d^2 - 2c(c + d \sin[ex + fx]) + (c + d \sin[ex + fx])^2}{d^2}} \right) - \\
& \left(2i (6ab^2cd - 5a^2bd^2 - b^3d^2) \cos[ex + fx] \cos[2(ex + fx)] \left(2b(c - d)(bc - ad) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + fx]} \right], \right. \right. \right. \\
& \left. \left. \left. \frac{c+d}{c-d} \right] + d \left(-2(a + b)(-bc + ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + fx]} \right], \frac{c+d}{c-d} \right] + \right. \right. \\
& \left. \left. (2a^2 - b^2) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc - ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c + d \sin[ex + fx]} \right], \frac{c+d}{c-d} \right] \right) \right) \sqrt{\frac{d - d \sin[ex + fx]}{c+d}} \\
& \left. \sqrt{-\frac{d + d \sin[ex + fx]}{c-d}} (-bc + ad + b(c + d \sin[ex + fx])) \right) / \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[ex + fx]) \sqrt{1 - \sin[ex + fx]^2} \right. \\
& \left. (-2c^2 + d^2 + 4c(c + d \sin[ex + fx]) - 2(c + d \sin[ex + fx])^2) \sqrt{-\frac{c^2 - d^2 - 2c(c + d \sin[ex + fx]) + (c + d \sin[ex + fx])^2}{d^2}} \right)
\end{aligned}$$

- **Problem 763: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sin[ex + fx])^3 \sqrt{c + d \sin[ex + fx]}} dx$$

Optimal (type 4, 503 leaves, 10 steps):

$$\frac{b^2 \cos[e+fx] \sqrt{c+d \sin[e+fx]}}{2(a^2-b^2)(bc-ad)f(a+b \sin[e+fx])^2} + \frac{3b^2(2abc-3a^2d+b^2d) \cos[e+fx] \sqrt{c+d \sin[e+fx]}}{4(a^2-b^2)^2(bc-ad)^2f(a+b \sin[e+fx])} +$$

$$\frac{3b(2abc-3a^2d+b^2d) \operatorname{EllipticE}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right] \sqrt{c+d \sin[e+fx]}}{4(a^2-b^2)^2(bc-ad)^2f \sqrt{\frac{c+d \sin[e+fx]}{c+d}}} -$$

$$\frac{(6abc-7a^2d+b^2d) \operatorname{EllipticF}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{4(a^2-b^2)^2(bc-ad)f \sqrt{c+d \sin[e+fx]}} -$$

$$\left(\frac{(20a^3bcd+4ab^3cd-15a^4d^2-2a^2b^2(4c^2-3d^2)-b^4(4c^2+3d^2)) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(e-\frac{\pi}{2}+fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{4(a-b)^2(a+b)^3(bc-ad)^2f \sqrt{c+d \sin[e+fx]}} \right) /$$

Result (type 4, 1069 leaves):

$$\frac{\sqrt{c+d \sin[e+fx]} \left(-\frac{b^2 \cos[e+fx]}{2(a^2-b^2)(-bc+ad)(a+b \sin[e+fx])^2} + \frac{3(2ab^3c \cos[e+fx]-3a^2b^2d \cos[e+fx]+b^4d \cos[e+fx])}{4(a^2-b^2)^2(-bc+ad)^2(a+b \sin[e+fx])} \right)}{f} +$$

$$\frac{1}{16(a-b)^2(a+b)^2(-bc+ad)^2f} \left(- \left(2(16a^2b^2c^2+8b^4c^2-32a^3bcd+2ab^3cd+16a^4d^2-19a^2b^2d^2+9b^4d^2) \right. \right.$$

$$\left. \left. \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}} \right) / \left((a+b) \sqrt{c+d \sin[e+fx]} \right) - \right.$$

$$\left. \left(2i(20a^2b^2cd+4b^4cd-32a^3bd^2+8ab^3d^2) \cos[e+fx] \left((bc-ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]} \right], \frac{c+d}{c-d} \right] + \right. \right. \right.$$

$$\left. \left. ad \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]} \right], \frac{c+d}{c-d} \right] \right) \right.$$

$$\left. \left. \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) / \right)$$

$$\begin{aligned}
& \left(b d^2 \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[ex + fx]) \sqrt{1 - \sin[ex + fx]^2} \sqrt{-\frac{c^2 - d^2 - 2c(c+d \sin[ex + fx]) + (c+d \sin[ex + fx])^2}{d^2}} \right) - \\
& \left(2i(-6ab^3cd + 9a^2b^2d^2 - 3b^4d^2) \cos[ex + fx] \cos[2(ex + fx)] \left(2b(c-d)(bc - ad) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[ex + fx]} \right], \right. \right. \right. \\
& \left. \left. \left. \frac{c+d}{c-d} \right] + d \left(-2(a+b)(-bc + ad) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[ex + fx]} \right], \frac{c+d}{c-d} \right] + \right. \right. \\
& \left. \left. (2a^2 - b^2) d \operatorname{EllipticPi}\left[\frac{b(c+d)}{bc - ad}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[ex + fx]} \right], \frac{c+d}{c-d} \right] \right) \right) \sqrt{\frac{d - d \sin[ex + fx]}{c+d}} \\
& \left. \sqrt{-\frac{d + d \sin[ex + fx]}{c-d}} (-bc + ad + b(c + d \sin[ex + fx])) \right) / \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc - ad) (a + b \sin[ex + fx]) \sqrt{1 - \sin[ex + fx]^2} \right. \\
& \left. \left. (-2c^2 + d^2 + 4c(c + d \sin[ex + fx]) - 2(c + d \sin[ex + fx])^2) \sqrt{-\frac{c^2 - d^2 - 2c(c+d \sin[ex + fx]) + (c+d \sin[ex + fx])^2}{d^2}} \right) \right)
\end{aligned}$$

■ **Problem 764: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + b \sin[ex + fx])^3 (c + d \sin[ex + fx])^{3/2}} dx$$

Optimal (type 4, 682 leaves, 11 steps):

$$\begin{aligned}
& - \frac{d \left(8 a^4 d^3 + a^2 b^2 d \left(13 c^2 - 29 d^2 \right) - b^4 d \left(7 c^2 - 15 d^2 \right) - 6 a b^3 c \left(c^2 - d^2 \right) \right) \operatorname{Cos}[e + f x]}{4 \left(a^2 - b^2 \right)^2 (b c - a d)^3 \left(c^2 - d^2 \right) f \sqrt{c + d} \operatorname{Sin}[e + f x]} + \\
& \frac{b^2 \operatorname{Cos}[e + f x]}{2 \left(a^2 - b^2 \right) (b c - a d) f \left(a + b \operatorname{Sin}[e + f x] \right)^2 \sqrt{c + d} \operatorname{Sin}[e + f x]} + \frac{b^2 \left(6 a b c - 11 a^2 d + 5 b^2 d \right) \operatorname{Cos}[e + f x]}{4 \left(a^2 - b^2 \right)^2 (b c - a d)^2 f \left(a + b \operatorname{Sin}[e + f x] \right) \sqrt{c + d} \operatorname{Sin}[e + f x]} - \\
& \left(\left(8 a^4 d^3 + a^2 b^2 d \left(13 c^2 - 29 d^2 \right) - b^4 d \left(7 c^2 - 15 d^2 \right) - 6 a b^3 c \left(c^2 - d^2 \right) \right) \operatorname{EllipticE} \left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x \right), \frac{2 d}{c + d} \right] \sqrt{c + d} \operatorname{Sin}[e + f x] \right) / \\
& \left(4 \left(a^2 - b^2 \right)^2 (b c - a d)^3 \left(c^2 - d^2 \right) f \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}} \right) - \frac{b \left(6 a b c - 11 a^2 d + 5 b^2 d \right) \operatorname{EllipticF} \left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x \right), \frac{2 d}{c + d} \right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}}}{4 \left(a^2 - b^2 \right)^2 (b c - a d)^2 f \sqrt{c + d} \operatorname{Sin}[e + f x]} - \\
& \left(b \left(28 a^3 b c d - 4 a b^3 c d - 35 a^4 d^2 - 2 a^2 b^2 \left(4 c^2 - 19 d^2 \right) - b^4 \left(4 c^2 + 15 d^2 \right) \right) \operatorname{EllipticPi} \left[\frac{2 b}{a + b}, \frac{1}{2} \left(e - \frac{\pi}{2} + f x \right), \frac{2 d}{c + d} \right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}} \right) / \\
& \left(4 (a - b)^2 (a + b)^3 (b c - a d)^3 f \sqrt{c + d} \operatorname{Sin}[e + f x] \right)
\end{aligned}$$

Result (type 4, 1318 leaves):

$$\begin{aligned}
& \frac{1}{f} \sqrt{c + d} \operatorname{Sin}[e + f x] \left(\frac{b^3 \operatorname{Cos}[e + f x]}{2 \left(a^2 - b^2 \right) (-b c + a d)^2 \left(a + b \operatorname{Sin}[e + f x] \right)^2} - \frac{6 a b^4 c \operatorname{Cos}[e + f x] - 13 a^2 b^3 d \operatorname{Cos}[e + f x] + 7 b^5 d \operatorname{Cos}[e + f x]}{4 \left(a^2 - b^2 \right)^2 (-b c + a d)^3 \left(a + b \operatorname{Sin}[e + f x] \right)} - \right. \\
& \left. \frac{2 d^4 \operatorname{Cos}[e + f x]}{(b c - a d)^3 \left(c^2 - d^2 \right) (c + d) \operatorname{Sin}[e + f x]} \right) + \frac{1}{16 (a - b)^2 (a + b)^2 (c - d) (c + d) (-b c + a d)^3 f} \\
& \left(- \left(2 \left(-16 a^2 b^3 c^4 - 8 b^5 c^4 + 48 a^3 b^2 c^3 d - 18 a b^4 c^3 d - 48 a^4 b c^2 d^2 + 95 a^2 b^3 c^2 d^2 - 29 b^5 c^2 d^2 + 16 a^5 c d^3 - 80 a^3 b^2 c d^3 + 34 a b^4 c d^3 + \right. \right. \right. \\
& \left. \left. \left. 56 a^4 b d^4 - 95 a^2 b^3 d^4 + 45 b^5 d^4 \right) \operatorname{EllipticPi} \left[\frac{2 b}{a + b}, \frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right), \frac{2 d}{c + d} \right] \sqrt{\frac{c + d \operatorname{Sin}[e + f x]}{c + d}} \right) / \left((a + b) \sqrt{c + d} \operatorname{Sin}[e + f x] \right) - \\
& \left(2 i \left(-20 a^2 b^3 c^3 d - 4 b^5 c^3 d + 48 a^3 b^2 c^2 d^2 - 24 a b^4 c^2 d^2 + 16 a^4 b c d^3 - 12 a^2 b^3 c d^3 + 20 b^5 c d^3 + 16 a^5 d^4 - 80 a^3 b^2 d^4 + 40 a b^4 d^4 \right) \right. \\
& \left. \operatorname{Cos}[e + f x] \left((b c - a d) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c + d}} \sqrt{c + d} \operatorname{Sin}[e + f x]} \right], \frac{c + d}{c - d} \right] + a d \operatorname{EllipticPi} \left[\frac{b (c + d)}{b c - a d}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \Big/ \\
& \left(b d^2 \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) - \\
& \left(2i(6ab^4c^3d-13a^2b^3c^2d^2+7b^5c^2d^2-6ab^4cd^3-8a^4bd^4+29a^2b^3d^4-15b^5d^4) \cos[e+fx] \right. \\
& \cos[2(e+fx)] \left(2b(c-d)(bc-ad) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right] + \right. \\
& d \left(-2(a+b)(-bc+ad) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right] + \right. \\
& \left. \left. (2a^2-b^2)d \operatorname{EllipticPi} \left[\frac{b(c+d)}{bc-ad}, i \operatorname{ArcSinh} \left[\sqrt{-\frac{1}{c+d} \sqrt{c+d \sin[e+fx]}} \right], \frac{c+d}{c-d} \right] \right) \right) \sqrt{\frac{d-d \sin[e+fx]}{c+d}} \\
& \left. \sqrt{-\frac{d+d \sin[e+fx]}{c-d}} (-bc+ad+b(c+d \sin[e+fx])) \right) \Big/ \left(b^2 d \sqrt{-\frac{1}{c+d}} (bc-ad)(a+b \sin[e+fx]) \sqrt{1-\sin[e+fx]^2} \right. \\
& \left. \left. (-2c^2+d^2+4c(c+d \sin[e+fx])-2(c+d \sin[e+fx])^2) \sqrt{-\frac{c^2-d^2-2c(c+d \sin[e+fx])+(c+d \sin[e+fx])^2}{d^2}} \right) \right)
\end{aligned}$$

■ **Problem 765: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \sin[e+fx]} (c+d \sin[e+fx])^{5/2} dx$$

Optimal (type 4, 888 leaves, 8 steps):

$$\frac{1}{24 b^2 (b c - a d) f} \sqrt{a+b} (c-d) \sqrt{c+d} (14 a b c d - 3 a^2 d^2 + b^2 (33 c^2 + 16 d^2)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+f x]}{\sqrt{c+d} \sqrt{a+b} \sin[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\operatorname{Sec}[e+f x] \sqrt{-\frac{(b c - a d)(1 - \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e+f x])}{(c-d)(a+b \sin[e+f x])}} (a+b \sin[e+f x]) - \frac{1}{8 b^3 \sqrt{a+b} d f}$$

$$\sqrt{c+d} (5 a^2 b c d^2 - a^3 d^3 - a b^2 d (15 c^2 + 4 d^2) - 5 b^3 (c^3 + 4 c d^2)) \operatorname{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+f x]}{\sqrt{c+d} \sqrt{a+b} \sin[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\operatorname{Sec}[e+f x] \sqrt{-\frac{(b c - a d)(1 - \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e+f x])}{(c-d)(a+b \sin[e+f x])}} (a+b \sin[e+f x]) -$$

$$\frac{(14 a b c d - 3 a^2 d^2 + b^2 (33 c^2 + 16 d^2)) \cos[e+f x] \sqrt{c+d} \sin[e+f x]}{24 b f \sqrt{a+b} \sin[e+f x]} - \frac{d (13 b c - 3 a d) \cos[e+f x] \sqrt{a+b} \sin[e+f x] \sqrt{c+d} \sin[e+f x]}{12 b f}$$

$$\frac{d^2 \cos[e+f x] (a+b \sin[e+f x])^{3/2} \sqrt{c+d} \sin[e+f x]}{3 b f} + \frac{1}{24 b^3 \sqrt{c+d} f}$$

$$(a+b)^{3/2} (3 a^2 d^2 - 6 a b d (2 c+d) + b^2 (33 c^2 + 26 c d + 16 d^2)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sin[e+f x]}{\sqrt{a+b} \sqrt{c+d} \sin[e+f x]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]$$

$$\operatorname{Sec}[e+f x] \sqrt{\frac{(b c - a d)(1 - \sin[e+f x])}{(a+b)(c+d \sin[e+f x])}} \sqrt{-\frac{(b c - a d)(1 + \sin[e+f x])}{(a-b)(c+d \sin[e+f x])}} (c+d \sin[e+f x])$$

Result (type 4, 1948 leaves):

$$-\frac{1}{48 b f} \left(- \left(4 (-b c + a d) (-48 a b c^3 - 59 b^2 c^2 d - 58 a b c d^2 + a^2 d^3 - 16 b^2 d^3) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{-c+d}} \right. \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (c+d \sin[e+f x])}{-b c + a d}}}{\sqrt{2}}}\right], \frac{2(-b c + a d)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^4 \right.$$

$$\left. \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (a+b \sin[e+f x])}{-b c + a d}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (c+d \sin[e+f x])}{-b c + a d}} \right) /$$

$$\begin{aligned}
& \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) - 4(-bc+ad) \left(-48b^2c^3 - 92abc^2d + 4a^2cd^2 - 76b^2cd^2 - 28abd^3 \right) \\
& \left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right. \\
& \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) - \\
& \left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right) / \\
& \left((a+b)d \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) + 2(33b^2c^2d + 14abcd^2 - 3a^2d^3 + 16b^2d^3) \left(\frac{\cos[e+fx] \sqrt{c+d \sin[e+fx]}}{d \sqrt{a+b \sin[e+fx]}} + \right. \\
& \left. \sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{\frac{a+b \sin[e+fx]}{a+b}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \right] \sqrt{c+d \sin[e+fx]} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(b d \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \sin[e+f x]}} \sqrt{a+b \sin[e+f x]} \sqrt{\frac{a+b \sin[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \right) - \\
& \frac{1}{b d} 2(-b c+a d) \left(\left((a+b) c+a d \right) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}}{-b c+a d}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \sec[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) - \\
& \left((b c+a d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\frac{\sqrt{-(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}}{-b c+a d}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \sec[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b) d \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) \right) \right) + \\
& \frac{\sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \left(-\frac{d(13 b c+a d) \cos[e+f x]}{12 b} - \frac{1}{6} d^2 \sin[2(e+f x)] \right)}{f}
\end{aligned}$$

■ **Problem 766: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \sin[e+f x]} (c+d \sin[e+f x])^{3/2} dx$$

Optimal (type 4, 784 leaves, 8 steps):

$$\frac{1}{4 b (b c - a d) f} \sqrt{a+b} (c-d) \sqrt{c+d} (5 b c + a d) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+f x]}{\sqrt{c+d} \sqrt{a+b \sin[e+f x]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+f x] \sqrt{-\frac{(b c - a d)(1 - \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e+f x])}{(c-d)(a+b \sin[e+f x])}} (a+b \sin[e+f x]) + \frac{1}{4 b^2 \sqrt{a+b} d f}$$

$$\sqrt{c+d} (6 a b c d - a^2 d^2 + b^2 (3 c^2 + 4 d^2)) \text{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+f x]}{\sqrt{c+d} \sqrt{a+b \sin[e+f x]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+f x] \sqrt{-\frac{(b c - a d)(1 - \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e+f x])}{(c-d)(a+b \sin[e+f x])}} (a+b \sin[e+f x]) +$$

$$\frac{(b c - a d) \cos[e+f x] \sqrt{c+d} \sin[e+f x]}{2 f \sqrt{a+b \sin[e+f x]}} - \frac{(5 b c + a d) \cos[e+f x] \sqrt{c+d} \sin[e+f x]}{4 f \sqrt{a+b \sin[e+f x]}} + \frac{1}{4 b^2 \sqrt{c+d} f}$$

$$(a+b)^{3/2} (5 b c - a d + 2 b d) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \sin[e+f x]}}{\sqrt{a+b} \sqrt{c+d \sin[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \text{Sec}[e+f x]$$

$$\sqrt{\frac{(b c - a d)(1 - \sin[e+f x])}{(a+b)(c+d \sin[e+f x])}} \sqrt{-\frac{(b c - a d)(1 + \sin[e+f x])}{(a-b)(c+d \sin[e+f x])}} (c+d \sin[e+f x]) - \frac{b \cos[e+f x] (c+d \sin[e+f x])^{3/2}}{2 f \sqrt{a+b \sin[e+f x]}}$$

Result (type 4, 1849 leaves):

$$-\frac{d \cos[e+f x] \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]}}{2 f} +$$

$$\frac{1}{8 f} \left(- \left(4 (-b c + a d) (8 a c^2 + 7 b c d + 3 a d^2) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-c+d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c+d \sin[e+f x])}{-b c + a d}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2 (-b c + a d)}{(a+b)(-c+d)} \right] \text{Sec}[e+f x] \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a+b \sin[e+f x])}{-b c + a d}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c+d \sin[e+f x])}{-b c + a d}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) -$$

$$\begin{aligned}
& 4(-bc+ad)(8bc^2+12acd+4bd^2) \left(\left(\sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b)\operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}{-bc+ad}} \right] \right) / \left((a+b)(c+d)\sqrt{a+b\operatorname{Sin}[e+fx]}\sqrt{c+d\operatorname{Sin}[e+fx]} \right) - \\
& \left(\sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b)\operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \right. \\
& \left. \left((a+b)d\sqrt{a+b\operatorname{Sin}[e+fx]}\sqrt{c+d\operatorname{Sin}[e+fx]} \right) \right) + 2(-5bcd-ad^2) \left(\frac{\operatorname{Cos}[e+fx]\sqrt{c+d\operatorname{Sin}[e+fx]}}{d\sqrt{a+b\operatorname{Sin}[e+fx]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}} (a+b)\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}}\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}}{\sqrt{\frac{a+b\operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \right] \sqrt{c+d\operatorname{Sin}[e+fx]} \right) / \right. \\
& \left. \left(bd\sqrt{\frac{(a+b)\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b\operatorname{Sin}[e+fx]}} \sqrt{a+b\operatorname{Sin}[e+fx]} \sqrt{\frac{a+b\operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d\operatorname{Sin}[e+fx])}{(c+d)(a+b\operatorname{Sin}[e+fx])}} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{bd} 2 (-bc+ad) \left(\left((a+b) c+ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}\right]}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& \left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}\right]}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 767: Humongous result has more than 200000 leaves.**

$$\int \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} dx$$

Optimal (type 4, 628 leaves, 7 steps):

$$\frac{1}{(bc-ad)f} \sqrt{a+b} (c-d) \sqrt{c+d} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\sin[e+fx])}{(c+d)(a+b\sin[e+fx])}} \sqrt{\frac{(bc-ad)(1+\sin[e+fx])}{(c-d)(a+b\sin[e+fx])}} (a+b\sin[e+fx]) +$$

$$\frac{1}{b\sqrt{a+b}df} \sqrt{c+d} (bc+ad) \text{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\sin[e+fx])}{(c+d)(a+b\sin[e+fx])}} \sqrt{\frac{(bc-ad)(1+\sin[e+fx])}{(c-d)(a+b\sin[e+fx])}} (a+b\sin[e+fx]) -$$

$$\frac{b\cos[e+fx] \sqrt{c+d} \sin[e+fx]}{f\sqrt{a+b} \sin[e+fx]} + \frac{1}{b\sqrt{c+d}f} (a+b)^{3/2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]}{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]$$

$$\text{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d\sin[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d\sin[e+fx])}} (c+d\sin[e+fx])$$

Result (type ?, 228392 leaves): Display of huge result suppressed!

■ **Problem 768: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b\sin[e+fx]}}{\sqrt{c+d\sin[e+fx]}} dx$$

Optimal (type 4, 198 leaves, 1 step):

$$\frac{1}{\sqrt{a+b}df} 2\sqrt{c+d} \text{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\sin[e+fx])}{(c+d)(a+b\sin[e+fx])}} \sqrt{\frac{(bc-ad)(1+\sin[e+fx])}{(c-d)(a+b\sin[e+fx])}} (a+b\sin[e+fx])$$

Result (type 4, 578 leaves):

$$\begin{aligned}
& -\frac{1}{f} 4(-bc+ad) \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right. \right. \\
& \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \right. \\
& \left. \left(b \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right. \right. \\
& \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right)
\end{aligned}$$

■ **Problem 769: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Sin}[e+fx]}}{(c+d \operatorname{Sin}[e+fx])^{3/2}} dx$$

Optimal (type 4, 409 leaves, 3 steps):

$$\begin{aligned}
& - \left(2 (a-b) \sqrt{a+b} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b \sin[e+fx]}}{\sqrt{a+b} \sqrt{c+d \sin[e+fx]}} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \quad \left. \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d \sin[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d \sin[e+fx])}} (c+d \sin[e+fx]) \right) / \left((c-d) \sqrt{c+d} (bc-ad) f \right) + \\
& \left(2 (a-b) \sqrt{a+b} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b \sin[e+fx]}}{\sqrt{a+b} \sqrt{c+d \sin[e+fx]}} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d \sin[e+fx])}} \right. \\
& \quad \left. \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d \sin[e+fx])}} (c+d \sin[e+fx]) \right) / \left((c-d) \sqrt{c+d} (bc-ad) f \right)
\end{aligned}$$

Result (type 4, 1830 leaves):

$$\begin{aligned}
& - \frac{2d \cos[e+fx] \sqrt{a+b \sin[e+fx]}}{(-c^2+d^2) f \sqrt{c+d \sin[e+fx]}} + \frac{1}{(c-d)(c+d) f} \\
& \left(- \left(4ac(-bc+ad) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}} \right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \right. \right. \\
& \quad \left. \left. \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right) / \right. \\
& \quad \left. \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) - 4(-bc+ad)(bc+ad) \right. \\
& \quad \left. \left(\left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}} \right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right. \right. \right. \\
& \quad \left. \left. \operatorname{Sec}[e+fx] \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) / \\
& \left. \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) - 2bd \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{\sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \operatorname{Sin}[e+fx]} \right) / \right. \\
& \left. \left(bd \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b \operatorname{Sin}[e+fx]}} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \right) - \right. \\
& \left. \frac{1}{bd} 2(-bc+ad) \left((a+b)(c+d) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right)
\end{aligned}$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right/ \left((a+b) (c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}}$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right/ \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right) \right)$$

■ **Problem 770: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \operatorname{Sin}[e+fx]}}{(c+d \operatorname{Sin}[e+fx])^{5/2}} dx$$

Optimal (type 4, 489 leaves, 4 steps):

$$\frac{2 d \operatorname{Cos}[e+f x] \sqrt{a+b \operatorname{Sin}[e+f x]}}{3\left(c^2-d^2\right) f\left(c+d \operatorname{Sin}[e+f x]\right)^{3 / 2}} +$$

$$\left(2(a-b) \sqrt{a+b}\left(4 a c d-b\left(3 c^2+d^2\right)\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+f x]\right.$$

$$\left.\sqrt{\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \sqrt{-\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}}(c+d \operatorname{Sin}[e+f x])}\right) / \left(3(c-d)^2(c+d)^{3 / 2}(b c-a d)^2 f\right) +$$

$$\left(2(a-b) \sqrt{a+b}(3 c+d) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+f x]\right.$$

$$\left.\sqrt{\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \sqrt{-\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}}(c+d \operatorname{Sin}[e+f x])}\right) / \left(3(c-d)^2(c+d)^{3 / 2}(b c-a d) f\right)$$

Result (type 4, 2037 leaves):

$$\frac{1}{f} \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]} \left(\frac{2 d \operatorname{Cos}[e+f x]}{3\left(c^2-d^2\right)\left(c+d \operatorname{Sin}[e+f x]\right)^2} + \frac{2\left(3 b c^2 d \operatorname{Cos}[e+f x]-4 a c d^2 \operatorname{Cos}[e+f x]+b d^3 \operatorname{Cos}[e+f x]\right)}{3(b c-a d)\left(c^2-d^2\right)^2\left(c+d \operatorname{Sin}[e+f x]\right)} \right) +$$

$$\frac{1}{3(c-d)^2(c+d)^2(-b c+a d) f} \left(-4(-b c+a d)\left(-3 a b c^3+3 a^2 c^2 d+b^2 c^2 d-a b c d^2+a^2 d^3-b^2 d^3\right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \right.$$

$$\left. \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}\right) /$$

$$\left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}\right)-4(-b c+a d)\left(-3 b^2 c^3+a b c^2 d+4 a^2 c d^2-b^2 c d^2-a b d^3\right)$$

$$\left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right.$$

$$\left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right.$$

$$\left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) /$$

$$\left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) + 2(3b^2c^2d - 4abcd^2 + b^2d^3) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} +$$

$$\left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}}{\sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \operatorname{Sin}[e+fx]} \right) /$$

$$\left(bd \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b \operatorname{Sin}[e+fx]}} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \right) -$$

$$\begin{aligned}
& \frac{1}{bd} 2 (-bc+ad) \left(\left((a+b) c+ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}\right]}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& \left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}\right]}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 772: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sin}[e+fx])^{3/2} (c+d \operatorname{Sin}[e+fx])^{3/2} dx$$

Optimal (type 4, 870 leaves, 8 steps):

$$\frac{1}{24 b d (b c - a d) f} \sqrt{a+b} (c-d) \sqrt{c+d} (38 a b c d + 3 a^2 d^2 + b^2 (3 c^2 + 16 d^2)) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+f x]}{\sqrt{c+d} \sqrt{a+b} \sin[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+f x] \sqrt{-\frac{(b c - a d)(1 - \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e+f x])}{(c-d)(a+b \sin[e+f x])}} (a+b \sin[e+f x]) + \frac{1}{8 b^2 \sqrt{a+b} d^2 f}$$

$$\sqrt{c+d} (b c + a d) (10 a b c d - a^2 d^2 - b^2 (c^2 - 12 d^2)) \text{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+f x]}{\sqrt{c+d} \sqrt{a+b} \sin[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+f x] \sqrt{-\frac{(b c - a d)(1 - \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e+f x])}{(c-d)(a+b \sin[e+f x])}} (a+b \sin[e+f x]) -$$

$$\frac{(38 a b c d + 3 a^2 d^2 + b^2 (3 c^2 + 16 d^2)) \cos[e+f x] \sqrt{c+d} \sin[e+f x]}{24 d f \sqrt{a+b} \sin[e+f x]} -$$

$$\frac{(3 b c + 7 a d) \cos[e+f x] \sqrt{a+b} \sin[e+f x] \sqrt{c+d} \sin[e+f x]}{12 f} - \frac{1}{24 b^2 d \sqrt{c+d} f}$$

$$(a+b)^{3/2} (3 a^2 d^2 - 6 a b d (4 c+d) - b^2 (3 c^2 + 14 c d + 16 d^2)) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sin[e+f x]}{\sqrt{a+b} \sqrt{c+d} \sin[e+f x]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \text{Sec}[e+f x]$$

$$\sqrt{\frac{(b c - a d)(1 - \sin[e+f x])}{(a+b)(c+d \sin[e+f x])}} \sqrt{-\frac{(b c - a d)(1 + \sin[e+f x])}{(a-b)(c+d \sin[e+f x])}} (c+d \sin[e+f x]) - \frac{b \cos[e+f x] \sqrt{a+b} \sin[e+f x] (c+d \sin[e+f x])^{3/2}}{3 f}$$

Result (type 4, 1922 leaves):

$$\frac{1}{48 f} \left(- \left(4 (-b c + a d) (48 a^2 c^2 + 17 b^2 c^2 + 82 a b c d + 17 a^2 d^2 + 16 b^2 d^2) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-c+d}} \right. \right.$$

$$\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c+d \sin[e+f x])}{-b c + a d}}}{\sqrt{2}}}\right], \frac{2(-b c + a d)}{(a+b)(-c+d)}\right] \text{Sec}[e+f x] \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \right. \right.$$

$$\left. \left. \sqrt{\frac{(c+d) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a+b \sin[e+f x])}{-b c + a d}} \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c+d \sin[e+f x])}{-b c + a d}} \right) \right)$$

$$\begin{aligned}
& \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) - 4(-bc+ad) (68abc^2 + 68a^2cd + 52b^2cd + 52abd^2) \\
& \left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \\
& \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) - \\
& \left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right. \\
& \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right) / \\
& \left((a+b)d \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) + 2(-3b^2c^2 - 38abcd - 3a^2d^2 - 16b^2d^2) \left(\frac{\cos[e+fx] \sqrt{c+d \sin[e+fx]}}{d \sqrt{a+b \sin[e+fx]}} + \right. \\
& \left. \sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{\frac{a+b \sin[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \sin[e+fx]} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(b d \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \sin[e+f x]}} \sqrt{a+b \sin[e+f x]} \sqrt{\frac{a+b \sin[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \right) - \\
& \frac{1}{b d} 2(-b c+a d) \left(\left((a+b) c+a d \right) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) - \\
& \left((b c+a d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b) d \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) \left. \right) \left. \right) + \\
& \frac{\sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \left(-\frac{7}{12}(b c+a d) \cos[e+f x]-\frac{1}{6} b d \sin[2(e+f x)]\right)}{f}
\end{aligned}$$

■ **Problem 773: Result more than twice size of optimal antiderivative.**

$$\int (a+b \sin[e+f x])^{3/2} \sqrt{c+d \sin[e+f x]} dx$$

Optimal (type 4, 740 leaves, 7 steps):

$$\frac{1}{4d(bc-ad)f} \sqrt{a+b} (c-d) \sqrt{c+d} (bc+5ad) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}{\sqrt{c+d} \sqrt{a+b \sin[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\operatorname{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\sin[e+fx])}{(c+d)(a+b \sin[e+fx])}} \sqrt{\frac{(bc-ad)(1+\sin[e+fx])}{(c-d)(a+b \sin[e+fx])}} (a+b \sin[e+fx]) + \frac{1}{4b \sqrt{a+b} d^2 f}$$

$$\sqrt{c+d} (6abcd+3a^2d^2-b^2(c^2-4d^2)) \operatorname{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}{\sqrt{c+d} \sqrt{a+b \sin[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\operatorname{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\sin[e+fx])}{(c+d)(a+b \sin[e+fx])}} \sqrt{\frac{(bc-ad)(1+\sin[e+fx])}{(c-d)(a+b \sin[e+fx])}} (a+b \sin[e+fx]) -$$

$$\frac{b(bc+5ad) \cos[e+fx] \sqrt{c+d} \sin[e+fx]}{4df \sqrt{a+b \sin[e+fx]}} - \frac{b \cos[e+fx] \sqrt{a+b \sin[e+fx]} \sqrt{c+d} \sin[e+fx]}{2f} + \frac{1}{4bd \sqrt{c+d} f}$$

$$(a+b)^{3/2} (3ad+b(c+2d)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \sin[e+fx]}}{\sqrt{a+b} \sqrt{c+d \sin[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]$$

$$\operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d \sin[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d \sin[e+fx])}} (c+d \sin[e+fx])$$

Result (type 4, 1849 leaves):

$$-\frac{b \cos[e+fx] \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}{2f} +$$

$$\frac{1}{8f} \left(- \left(4(-bc+ad)(8a^2c+3b^2c+7abd) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) -$$

$$\begin{aligned}
& 4(-bc+ad)(12abc+8a^2d+4b^2d) \left(\left(\sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b)\operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}{-bc+ad}} \right] \right) / \left((a+b)(c+d)\sqrt{a+b\operatorname{Sin}[e+fx]}\sqrt{c+d\operatorname{Sin}[e+fx]} \right) - \\
& \left(\sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b)\operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \right. \\
& \left. \left((a+b)d\sqrt{a+b\operatorname{Sin}[e+fx]}\sqrt{c+d\operatorname{Sin}[e+fx]} \right) \right) + 2(-b^2c-5abd) \left(\frac{\operatorname{Cos}[e+fx]\sqrt{c+d\operatorname{Sin}[e+fx]}}{d\sqrt{a+b\operatorname{Sin}[e+fx]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}} (a+b)\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}}\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}}{\sqrt{\frac{a+b\operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \right] \sqrt{c+d\operatorname{Sin}[e+fx]} \right) / \right. \\
& \left. \left(bd\sqrt{\frac{(a+b)\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b\operatorname{Sin}[e+fx]}} \sqrt{a+b\operatorname{Sin}[e+fx]} \sqrt{\frac{a+b\operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d\operatorname{Sin}[e+fx])}{(c+d)(a+b\operatorname{Sin}[e+fx])}} \right) - \right.
\end{aligned}$$

$$\frac{1}{bd} 2 (-bc + ad) \left(\left((a+b) c + ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right.$$

$$\left. \frac{2 (-bc + ad)}{(a+b) (-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b) (c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left((bc + ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc + ad}{(a+b) d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2 (-bc + ad)}{(a+b) (-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right)$$

■ **Problem 774: Humongous result has more than 200000 leaves.**

$$\int \frac{(a+b \operatorname{Sin}[e+fx])^{3/2}}{\sqrt{c+d \operatorname{Sin}[e+fx]}} dx$$

Optimal (type 4, 644 leaves, 6 steps):

$$\begin{aligned}
& - \frac{b \cos[e + f x] \sqrt{a + b \sin[e + f x]}}{f \sqrt{c + d \sin[e + f x]}} - \frac{1}{d (bc - ad) f} (a - b) b \sqrt{a + b} \sqrt{c + d} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right] \\
& \operatorname{Sec}[e + f x] \sqrt{\frac{(bc - ad)(1 - \sin[e + f x])}{(a + b)(c + d \sin[e + f x])}} \sqrt{-\frac{(bc - ad)(1 + \sin[e + f x])}{(a - b)(c + d \sin[e + f x])}} (c + d \sin[e + f x]) + \\
& \frac{1}{d^2 \sqrt{c + d} f} \sqrt{a + b} (b(c - d) - 2ad) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right] \\
& \operatorname{Sec}[e + f x] \sqrt{\frac{(bc - ad)(1 - \sin[e + f x])}{(a + b)(c + d \sin[e + f x])}} \sqrt{-\frac{(bc - ad)(1 + \sin[e + f x])}{(a - b)(c + d \sin[e + f x])}} (c + d \sin[e + f x]) - \\
& \frac{1}{d^2 \sqrt{c + d} f} \sqrt{a + b} (bc - 3ad) \operatorname{EllipticPi}\left[\frac{(a + b)d}{b(c + d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right] \\
& \operatorname{Sec}[e + f x] \sqrt{\frac{(bc - ad)(1 - \sin[e + f x])}{(a + b)(c + d \sin[e + f x])}} \sqrt{-\frac{(bc - ad)(1 + \sin[e + f x])}{(a - b)(c + d \sin[e + f x])}} (c + d \sin[e + f x])
\end{aligned}$$

Result (type ?, 222963 leaves) : Display of huge result suppressed!

■ **Problem 775: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \sin[e + f x])^{3/2}}{(c + d \sin[e + f x])^{3/2}} dx$$

Optimal (type 4, 600 leaves, 5 steps) :

$$\frac{1}{(c-d)d\sqrt{c+d}f} 2(a-b)\sqrt{a+b} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{a+b}\sin[e+fx]}{\sqrt{a+b}\sqrt{c+d}\sin[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]$$

$$\operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d)\sin[e+fx]}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d)\sin[e+fx]}} (c+d)\sin[e+fx] -$$

$$\frac{1}{(c-d)d^2\sqrt{c+d}f} 2\sqrt{a+b}(b(c-2d)+ad) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{a+b}\sin[e+fx]}{\sqrt{a+b}\sqrt{c+d}\sin[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]$$

$$\operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d)\sin[e+fx]}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d)\sin[e+fx]}} (c+d)\sin[e+fx] +$$

$$\frac{1}{d^2\sqrt{c+d}f} 2b\sqrt{a+b} \operatorname{EllipticPi}\left[\frac{(a+b)d}{b(c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{a+b}\sin[e+fx]}{\sqrt{a+b}\sqrt{c+d}\sin[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]$$

$$\operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d)\sin[e+fx]}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d)\sin[e+fx]}} (c+d)\sin[e+fx]$$

Result (type 4, 1866 leaves):

$$-\frac{2(bc\cos[e+fx] - ad\cos[e+fx])\sqrt{a+b}\sin[e+fx]}{(c^2-d^2)f\sqrt{c+d}\sin[e+fx]} + \frac{1}{(c-d)(c+d)f}$$

$$\left(- \left(4(-bc+ad)(a^2c-abd) \sqrt{\frac{(c+d)\cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\sin[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right.$$

$$\operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b)\sin[e+fx]}{-bc+ad}}$$

$$\left. \sqrt{-\frac{(a+b)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\sin[e+fx]}{-bc+ad}} \right) / \left((a+b)(c+d)\sqrt{a+b}\sin[e+fx]\sqrt{c+d}\sin[e+fx] \right) -$$

$$\begin{aligned}
& 4(-bc+ad)(a^2d-b^2d) \left(\left(\sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right. \\
& \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b)\operatorname{Sin}[e+fx]}{-bc+ad}} \\
& \left. \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)(c+d)\sqrt{a+b\operatorname{Sin}[e+fx]}\sqrt{c+d\operatorname{Sin}[e+fx]} \right) - \\
& \left(\sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b)\operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \\
& \left. (a+b)d\sqrt{a+b\operatorname{Sin}[e+fx]}\sqrt{c+d\operatorname{Sin}[e+fx]} \right) + 2(b^2c-abd) \left(\frac{\operatorname{Cos}[e+fx]\sqrt{c+d\operatorname{Sin}[e+fx]}}{d\sqrt{a+b\operatorname{Sin}[e+fx]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}}(a+b)\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}}\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}}{\sqrt{\frac{a+b\operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \right] \sqrt{c+d\operatorname{Sin}[e+fx]} \right) / \\
& \left. bd \sqrt{\frac{(a+b)\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b\operatorname{Sin}[e+fx]}} \sqrt{a+b\operatorname{Sin}[e+fx]} \sqrt{\frac{a+b\operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d\operatorname{Sin}[e+fx])}{(c+d)(a+b\operatorname{Sin}[e+fx])}} \right) -
\end{aligned}$$

$$\frac{1}{bd} 2 (-bc + ad) \left(\left((a+b) c + ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right.$$

$$\left. \frac{2 (-bc + ad)}{(a+b) (-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b) (c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left((bc + ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc + ad}{(a+b) d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2 (-bc + ad)}{(a+b) (-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right)$$

■ **Problem 776: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Sin}[e+fx])^{3/2}}{(c+d \operatorname{Sin}[e+fx])^{5/2}} dx$$

Optimal (type 4, 497 leaves, 4 steps):

$$\frac{2 (b c - a d) \operatorname{Cos}[e + f x] \sqrt{a + b \operatorname{Sin}[e + f x]}}{3 (c^2 - d^2) f (c + d \operatorname{Sin}[e + f x])^{3/2}} -$$

$$\left(8 (a - b) \sqrt{a + b} (a c - b d) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{c + d \operatorname{Sin}[e + f x]}}\right], \frac{(a + b) (c - d)}{(a - b) (c + d)}\right] \operatorname{Sec}[e + f x] \right.$$

$$\left. \sqrt{\frac{(b c - a d) (1 - \operatorname{Sin}[e + f x])}{(a + b) (c + d \operatorname{Sin}[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Sin}[e + f x])}{(a - b) (c + d \operatorname{Sin}[e + f x])}} (c + d \operatorname{Sin}[e + f x])} \right) / (3 (c - d)^2 (c + d)^{3/2} (b c - a d) f) +$$

$$\left(2 (a - b) \sqrt{a + b} (a (3 c + d) - b (c + 3 d)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{c + d \operatorname{Sin}[e + f x]}}\right], \frac{(a + b) (c - d)}{(a - b) (c + d)}\right] \operatorname{Sec}[e + f x] \right.$$

$$\left. \sqrt{\frac{(b c - a d) (1 - \operatorname{Sin}[e + f x])}{(a + b) (c + d \operatorname{Sin}[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Sin}[e + f x])}{(a - b) (c + d \operatorname{Sin}[e + f x])}} (c + d \operatorname{Sin}[e + f x])} \right) / (3 (c - d)^2 (c + d)^{3/2} (b c - a d) f)$$

Result (type 4, 1982 leaves):

$$\frac{\sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]} \left(-\frac{2 (b c \operatorname{Cos}[e + f x] - a d \operatorname{Cos}[e + f x])}{3 (c^2 - d^2) (c + d \operatorname{Sin}[e + f x])^2} - \frac{8 (-a c d \operatorname{Cos}[e + f x] + b d^2 \operatorname{Cos}[e + f x])}{3 (c^2 - d^2)^2 (c + d \operatorname{Sin}[e + f x])} \right)}{f} +$$

$$\frac{1}{3 (c - d)^2 (c + d)^2 f} \left(- \left(4 (-b c + a d) (3 a^2 c^2 + b^2 c^2 - 4 a b c d + a^2 d^2 - b^2 d^2) \sqrt{\frac{(c + d) \operatorname{Cot}\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^2}{-c + d}} \right. \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^2 (c + d \operatorname{Sin}[e + f x])}{-b c + a d}}}{\sqrt{2}}}\right], \frac{2 (-b c + a d)}{(a + b) (-c + d)}\right] \operatorname{Sec}[e + f x] \operatorname{Sin}\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^4 \right.$$

$$\left. \sqrt{\frac{(c + d) \operatorname{Csc}\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^2 (a + b \operatorname{Sin}[e + f x])}{-b c + a d}} \sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^2 (c + d \operatorname{Sin}[e + f x])}{-b c + a d}} \right) /$$

$$\left((a + b) (c + d) \sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]} \right) - 4 (-b c + a d) (4 a b c^2 + 4 a^2 c d - 4 b^2 c d - 4 a b d^2)$$

$$\left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right.$$

$$\left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right.$$

$$\left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) /$$

$$\left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) + 2(-4abcd+4b^2d^2) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} +$$

$$\left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}}{\sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) /$$

$$\left(bd \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b \operatorname{Sin}[e+fx]}} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \right) -$$

$$\begin{aligned}
& \frac{1}{bd} 2 (-bc+ad) \left(\left((a+b) c+ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& \left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 779: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sin}[e+fx])^{5/2} \sqrt{c+d \operatorname{Sin}[e+fx]} dx$$

Optimal (type 4, 894 leaves, 8 steps):

$$\frac{1}{24 d^2 (bc - ad) f} \sqrt{a+b} (c-d) \sqrt{c+d} (14 a b c d + 33 a^2 d^2 - b^2 (3 c^2 - 16 d^2)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \operatorname{Sin}[e+f x]}{\sqrt{c+d} \sqrt{a+b} \operatorname{Sin}[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\operatorname{Sec}[e+f x] \sqrt{-\frac{(bc-ad)(1-\operatorname{Sin}[e+f x])}{(c+d)(a+b \operatorname{Sin}[e+f x])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sin}[e+f x])}{(c-d)(a+b \operatorname{Sin}[e+f x])}} (a+b \operatorname{Sin}[e+f x]) + \frac{1}{8 b \sqrt{a+b} d^3 f}}$$

$$\sqrt{c+d} (15 a^2 b c d^2 + 5 a^3 d^3 - 5 a b^2 d (c^2 - 4 d^2) + b^3 (c^3 + 4 c d^2)) \operatorname{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \operatorname{Sin}[e+f x]}{\sqrt{c+d} \sqrt{a+b} \operatorname{Sin}[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\operatorname{Sec}[e+f x] \sqrt{-\frac{(bc-ad)(1-\operatorname{Sin}[e+f x])}{(c+d)(a+b \operatorname{Sin}[e+f x])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sin}[e+f x])}{(c-d)(a+b \operatorname{Sin}[e+f x])}} (a+b \operatorname{Sin}[e+f x]) -$$

$$\frac{b(14 a b c d + 33 a^2 d^2 - b^2 (3 c^2 - 16 d^2)) \operatorname{Cos}[e+f x] \sqrt{c+d} \operatorname{Sin}[e+f x]}{24 d^2 f \sqrt{a+b} \operatorname{Sin}[e+f x]} +$$

$$\frac{b(3 b c - 13 a d) \operatorname{Cos}[e+f x] \sqrt{a+b} \operatorname{Sin}[e+f x] \sqrt{c+d} \operatorname{Sin}[e+f x]}{12 d f} + \frac{1}{24 b d^2 \sqrt{c+d} f}$$

$$(a+b)^{3/2} (15 a^2 d^2 + 6 a b d (2 c + 3 d) - b^2 (3 c^2 - 2 c d - 16 d^2)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \operatorname{Sin}[e+f x]}{\sqrt{a+b} \sqrt{c+d} \operatorname{Sin}[e+f x]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+f x]$$

$$\sqrt{\frac{(bc-ad)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}} (c+d \operatorname{Sin}[e+f x]) - \frac{b^2 \operatorname{Cos}[e+f x] \sqrt{a+b} \operatorname{Sin}[e+f x] (c+d \operatorname{Sin}[e+f x])^{3/2}}{3 d f}}$$

Result (type 4, 1949 leaves):

$$\frac{1}{48 d f} \left(- \left(4 (-bc + ad) (-b^3 c^2 + 48 a^3 c d + 58 a b^2 c d + 59 a^2 b d^2 + 16 b^3 d^2) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{-c+d}} \right. \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^4 \right.$$

$$\left. \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (a+b \operatorname{Sin}[e+f x])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-bc+ad}} \right) /$$

$$\begin{aligned}
& \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) - 4(-bc+ad) \left(-4ab^2c^2 + 92a^2bcd + 28b^3cd + 48a^3d^2 + 76ab^2d^2 \right) \\
& \left(\left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right. \\
& \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) - \right. \\
& \left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right. \\
& \left. \left. \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right) / \right. \\
& \left. \left((a+b)d \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) \right) + 2(3b^3c^2 - 14ab^2cd - 33a^2bd^2 - 16b^3d^2) \left(\frac{\cos[e+fx] \sqrt{c+d \sin[e+fx]}}{d \sqrt{a+b \sin[e+fx]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{\sqrt{\frac{a+b \sin[e+fx]}{a+b}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \sin[e+fx]} \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(b d \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \sin[e+f x]}} \sqrt{a+b \sin[e+f x]} \sqrt{\frac{a+b \sin[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \right) - \\
& \frac{1}{b d} 2(-b c+a d) \left(\left((a+b) c+a d \right) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) - \\
& \left((b c+a d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b) d \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) \left. \right) \left. \right) + \\
& \frac{\sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \left(-\frac{b(b c+13 a d) \cos[e+f x]}{12 d} - \frac{1}{6} b^2 \sin[2(e+f x)] \right)}{f}
\end{aligned}$$

■ **Problem 780: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \sin[e+f x])^{5/2}}{\sqrt{c+d \sin[e+f x]}} dx$$

Optimal (type 4, 745 leaves, 7 steps) :

$$\begin{aligned}
 & -\frac{1}{4d^2(bc-ad)f} 3b\sqrt{a+b}(c-d)\sqrt{c+d}(bc-3ad)\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sqrt{c+d}\sin[efx]}{\sqrt{c+d}\sqrt{a+b}\sin[efx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
 & \operatorname{Sec}[efx] \sqrt{\frac{(bc-ad)(1-\sin[efx])}{(c+d)(a+b\sin[efx])}} \sqrt{\frac{(bc-ad)(1+\sin[efx])}{(c-d)(a+b\sin[efx])}} (a+b\sin[efx]) - \frac{1}{4\sqrt{a+b}d^3f} \\
 & \sqrt{c+d}(10abcd-15a^2d^2-b^2(3c^2+4d^2))\operatorname{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sqrt{c+d}\sin[efx]}{\sqrt{c+d}\sqrt{a+b}\sin[efx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
 & \operatorname{Sec}[efx] \sqrt{\frac{(bc-ad)(1-\sin[efx])}{(c+d)(a+b\sin[efx])}} \sqrt{\frac{(bc-ad)(1+\sin[efx])}{(c-d)(a+b\sin[efx])}} (a+b\sin[efx]) + \\
 & \frac{3b^2(bc-3ad)\cos[efx]\sqrt{c+d}\sin[efx]}{4d^2f\sqrt{a+b}\sin[efx]} - \frac{b^2\cos[efx]\sqrt{a+b}\sin[efx]\sqrt{c+d}\sin[efx]}{2df} - \frac{1}{4d^2\sqrt{c+d}f} \\
 & (a+b)^{3/2}(3bc-7ad-2bd)\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{a+b}\sin[efx]}{\sqrt{a+b}\sqrt{c+d}\sin[efx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \\
 & \operatorname{Sec}[efx] \sqrt{\frac{(bc-ad)(1-\sin[efx])}{(a+b)(c+d)\sin[efx]}} \sqrt{-\frac{(bc-ad)(1+\sin[efx])}{(a-b)(c+d)\sin[efx]}} (c+d\sin[efx])
 \end{aligned}$$

Result (type 4, 1864 leaves) :

$$\begin{aligned}
 & -\frac{b^2\cos[efx]\sqrt{a+b}\sin[efx]\sqrt{c+d}\sin[efx]}{2df} + \\
 & \frac{1}{8df} \left(-\left(4(-bc+ad)(-b^3c+8a^3d+11ab^2d) \sqrt{\frac{(c+d)\cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\sin[efx]}}{-bc+ad}}}{\sqrt{2}}\right], \right. \right. \\
 & \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b)\sin[efx]}{-bc+ad}} \right. \right. \\
 & \left. \left. \sqrt{-\frac{(a+b)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d)\sin[efx]}{-bc+ad}} \right) \right] / \left((a+b)(c+d)\sqrt{a+b}\sin[efx]\sqrt{c+d}\sin[efx] \right) -
 \end{aligned}$$

$$\begin{aligned}
& 4(-bc+ad)(-4ab^2c+24a^2bd+4b^3d) \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \right. \\
& \left. \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \right. \right. \\
& \left. \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \right. \\
& \left. \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) + 2(3b^3c-9ab^2d) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}}{\sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \right] \sqrt{c+d \operatorname{Sin}[e+fx]} \right) / \right. \\
& \left. \left(bd \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b \operatorname{Sin}[e+fx]}} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{bd} 2 (-bc + ad) \left(\left((a+b) c + ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2 (-bc + ad)}{(a+b) (-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) \right] / \left((a+b) (c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \right. \\
& \left((bc + ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc + ad}{(a+b) d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2 (-bc + ad)}{(a+b) (-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) \right] / \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 781: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Sin}[e+fx])^{5/2}}{(c+d \operatorname{Sin}[e+fx])^{3/2}} dx$$

Optimal (type 4, 780 leaves, 7 steps):

$$\begin{aligned}
& - \frac{1}{d^2 \sqrt{c+d} (bc-ad) f} \sqrt{a+b} (4abcd - 2a^2d^2 - b^2(3c^2 - d^2)) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \text{Sin}[e+fx]}{\sqrt{c+d} \sqrt{a+b} \text{Sin}[e+fx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \text{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\text{Sin}[e+fx])}{(c+d)(a+b\text{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\text{Sin}[e+fx])}{(c-d)(a+b\text{Sin}[e+fx])}} (a+b\text{Sin}[e+fx]) - \\
& \frac{1}{\sqrt{a+b} d^3 f} b \sqrt{c+d} (3bc - 5ad) \text{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \text{Sin}[e+fx]}{\sqrt{c+d} \sqrt{a+b} \text{Sin}[e+fx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \text{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\text{Sin}[e+fx])}{(c+d)(a+b\text{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\text{Sin}[e+fx])}{(c-d)(a+b\text{Sin}[e+fx])}} (a+b\text{Sin}[e+fx]) + \\
& \frac{2(bc-ad)^2 \text{Cos}[e+fx] \sqrt{a+b\text{Sin}[e+fx]}}{d(c^2-d^2) f \sqrt{c+d\text{Sin}[e+fx]}} + \frac{b(4abcd - 2a^2d^2 - b^2(3c^2 - d^2)) \text{Cos}[e+fx] \sqrt{c+d\text{Sin}[e+fx]}}{d^2(c^2-d^2) f \sqrt{a+b\text{Sin}[e+fx]}} - \\
& \frac{1}{d^2(c+d)^{3/2} f} (a+b)^{3/2} (2ad - b(3c+d)) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b\text{Sin}[e+fx]}}{\sqrt{a+b} \sqrt{c+d\text{Sin}[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \\
& \text{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\text{Sin}[e+fx])}{(a+b)(c+d\text{Sin}[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\text{Sin}[e+fx])}{(a-b)(c+d\text{Sin}[e+fx])}} (c+d\text{Sin}[e+fx])
\end{aligned}$$

Result (type 4, 1976 leaves):

$$\begin{aligned}
& - \frac{2(b^2c^2 \text{Cos}[e+fx] - 2abcd \text{Cos}[e+fx] + a^2d^2 \text{Cos}[e+fx]) \sqrt{a+b\text{Sin}[e+fx]}}{d(-c^2+d^2) f \sqrt{c+d\text{Sin}[e+fx]}} - \\
& \frac{1}{2(c-d) d (c+d) f} \left(- \left(4(-bc+ad) (-b^3c^2 - 2a^3cd - 2ab^2cd + 4a^2bd^2 + b^3d^2) \sqrt{\frac{(c+d) \text{Cot}\left[\frac{1}{2}(-e + \frac{\pi}{2} - fx)\right]^2}{-c+d}} \right. \right. \\
& \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \text{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - fx)\right]^2 (c+d\text{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \text{Sec}[e+fx] \text{Sin}\left[\frac{1}{2}(-e + \frac{\pi}{2} - fx)\right]^4 \right. \right. \\
& \left. \left. \sqrt{\frac{(c+d) \text{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - fx)\right]^2 (a+b\text{Sin}[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \text{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - fx)\right]^2 (c+d\text{Sin}[e+fx])}{-bc+ad}} \right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left((a+b)(c+d) \sqrt{a+b \sin[ex+fx]} \sqrt{c+d \sin[ex+fx]} \right) - 4(-bc+ad) \left(-4ab^2c^2 + 2a^2bcd - 2b^3cd - 2a^3d^2 + 6ab^2d^2 \right) \\
& \left(\left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[ex+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right. \right. \\
& \operatorname{Sec}[ex+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[ex+fx])}{-bc+ad}} \\
& \left. \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[ex+fx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[ex+fx]} \sqrt{c+d \sin[ex+fx]} \right) - \right. \\
& \left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[ex+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[ex+fx] \right. \\
& \left. \left. \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[ex+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[ex+fx])}{-bc+ad}} \right) / \right. \\
& \left. \left((a+b)d \sqrt{a+b \sin[ex+fx]} \sqrt{c+d \sin[ex+fx]} \right) \right) + 2(3b^3c^2 - 4ab^2cd + 2a^2bd^2 - b^3d^2) \left(\frac{\cos[ex+fx] \sqrt{c+d \sin[ex+fx]}}{d \sqrt{a+b \sin[ex+fx]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{\sqrt{\frac{a+b \sin[ex+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \right] \sqrt{c+d \sin[ex+fx]} \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(b d \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \sin[e+f x]}} \sqrt{a+b \sin[e+f x]} \sqrt{\frac{a+b \sin[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \right) - \\
& \frac{1}{b d} 2(-b c+a d) \left(\left((a+b) c+a d \right) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) - \\
& \left((b c+a d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b) d \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) \right) \right)
\end{aligned}$$

■ **Problem 782: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \sin[e+f x])^{5/2}}{(c+d \sin[e+f x])^{5/2}} dx$$

Optimal (type 4, 737 leaves, 6 steps):

$$\frac{2 (b c - a d)^2 \operatorname{Cos}[e + f x] \sqrt{a + b \operatorname{Sin}[e + f x]}}{3 d (c^2 - d^2) f (c + d \operatorname{Sin}[e + f x])^{3/2}} + \frac{1}{3 (c - d)^2 d^2 (c + d)^{3/2} f}$$

$$2 (a - b) \sqrt{a + b} (3 b c^2 + 4 a c d - 7 b d^2) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{c + d \operatorname{Sin}[e + f x]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right] \operatorname{Sec}[e + f x]$$

$$\sqrt{\frac{(b c - a d)(1 - \operatorname{Sin}[e + f x])}{(a + b)(c + d \operatorname{Sin}[e + f x])}} \sqrt{-\frac{(b c - a d)(1 + \operatorname{Sin}[e + f x])}{(a - b)(c + d \operatorname{Sin}[e + f x])}} (c + d \operatorname{Sin}[e + f x]) - \frac{1}{3 (c - d)^2 d^3 (c + d)^{3/2} f} 2 \sqrt{a + b}}$$

$$(a^2 d^2 (3 c + d) + a b d (3 c^2 - 4 c d - 7 d^2) + b^2 (3 c^3 - 6 c^2 d - 2 c d^2 + 9 d^3)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{c + d \operatorname{Sin}[e + f x]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{\frac{(b c - a d)(1 - \operatorname{Sin}[e + f x])}{(a + b)(c + d \operatorname{Sin}[e + f x])}} \sqrt{-\frac{(b c - a d)(1 + \operatorname{Sin}[e + f x])}{(a - b)(c + d \operatorname{Sin}[e + f x])}} (c + d \operatorname{Sin}[e + f x]) +$$

$$\frac{1}{d^3 \sqrt{c + d} f} 2 b^2 \sqrt{a + b} \operatorname{EllipticPi}\left[\frac{(a + b) d}{b (c + d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{c + d \operatorname{Sin}[e + f x]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{\frac{(b c - a d)(1 - \operatorname{Sin}[e + f x])}{(a + b)(c + d \operatorname{Sin}[e + f x])}} \sqrt{-\frac{(b c - a d)(1 + \operatorname{Sin}[e + f x])}{(a - b)(c + d \operatorname{Sin}[e + f x])}} (c + d \operatorname{Sin}[e + f x])$$

Result (type 4, 2139 leaves):

$$\frac{1}{f} \frac{1}{\sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]}} \left(-\frac{2 (b^2 c^2 \operatorname{Cos}[e + f x] - 2 a b c d \operatorname{Cos}[e + f x] + a^2 d^2 \operatorname{Cos}[e + f x])}{3 d (-c^2 + d^2) (c + d \operatorname{Sin}[e + f x])^2} - \right.$$

$$\left. \frac{(2 (3 b^2 c^3 \operatorname{Cos}[e + f x] + a b c^2 d \operatorname{Cos}[e + f x] - 4 a^2 c d^2 \operatorname{Cos}[e + f x] - 7 b^2 c d^2 \operatorname{Cos}[e + f x] + 7 a b d^3 \operatorname{Cos}[e + f x]))}{(3 d (-c^2 + d^2)^2 (c + d \operatorname{Sin}[e + f x]))} + \frac{1}{3 (c - d)^2 d (c + d)^2 f} \right)$$

$$\left(-\frac{1}{(a + b)(c + d) \sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]}} 4 (-b c + a d) (-b^3 c^3 + 3 a^3 c^2 d + 2 a b^2 c^2 d - 8 a^2 b c d^2 + b^3 c d^2 + a^3 d^3 + 2 a b^2 d^3) \right.$$

$$\left. \sqrt{\frac{(c + d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-c + d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c + d \operatorname{Sin}[e + f x])}{-b c + a d}}}{\sqrt{2}}}\right], \frac{2 (-b c + a d)}{(a + b)(-c + d)}\right] \operatorname{Sec}[e + f x] \right)$$

$$\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \sqrt{\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} -$$

$$4(-bc+ad)(-4ab^2c^3 + 3a^2bc^2d + b^3c^2d + 4a^3cd^2 - 7a^2bd^3 + 3b^3d^3)$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right)$$

$$\operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}}$$

$$\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \left/ \left((a+b)(c+d) \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) - \right.$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right)$$

$$\operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}}$$

$$\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \left/ \left((a+b)d \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) \right) +$$

$$2(3b^3c^3 + ab^2c^2d - 4a^2bcd^2 - 7b^3cd^2 + 7ab^2d^3) \left(\frac{\cos[efx] \sqrt{c+d \sin[efx]}}{d \sqrt{a+b \sin[efx]}} + \right.$$

$$\begin{aligned}
& \left(\sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{\frac{a+b \sin[efx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \sin[efx]} \right) / \\
& \left(bd \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{a+b \sin[efx]}} \sqrt{a+b \sin[efx]} \sqrt{\frac{a+b \sin[efx]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[efx])}{(c+d)(a+b \sin[efx])}} \right) - \\
& \frac{1}{bd} 2(-bc+ad) \left((a+b)c+ad \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) - \\
& \left((bc+ad) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) \right) \right) \right)
\end{aligned}$$

■ Problem 783: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d \sin[e + f x])^{5/2}}{\sqrt{a + b \sin[e + f x]}} dx$$

Optimal (type 4, 772 leaves, 7 steps):

$$\frac{1}{4 b^2 (b c - a d) f} \sqrt{a + b} (c - d) d \sqrt{c + d} (3 b c - a d) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}\right], \frac{(a - b)(c + d)}{(a + b)(c - d)}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{-\frac{(b c - a d)(1 - \sin[e + f x])}{(c + d)(a + b \sin[e + f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e + f x])}{(c - d)(a + b \sin[e + f x])}} (a + b \sin[e + f x]) - \frac{1}{4 b^3 \sqrt{a + b} f}$$

$$\sqrt{c + d} (10 a b c d - 3 a^2 d^2 - b^2 (15 c^2 + 4 d^2)) \operatorname{EllipticPi}\left[\frac{b(c + d)}{(a + b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}\right], \frac{(a - b)(c + d)}{(a + b)(c - d)}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{-\frac{(b c - a d)(1 - \sin[e + f x])}{(c + d)(a + b \sin[e + f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e + f x])}{(c - d)(a + b \sin[e + f x])}} (a + b \sin[e + f x]) -$$

$$\frac{3 d (3 b c - a d) \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{4 b f \sqrt{a + b \sin[e + f x]}} - \frac{d^2 \cos[e + f x] \sqrt{a + b \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}{2 b f} + \frac{1}{4 b^3 \sqrt{c + d} f}$$

$$\sqrt{a + b} (3 a^2 d^2 - a b d (7 c + 3 d) + b^2 (8 c^2 + 9 c d + 2 d^2)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{\frac{(b c - a d)(1 - \sin[e + f x])}{(a + b)(c + d \sin[e + f x])}} \sqrt{-\frac{(b c - a d)(1 + \sin[e + f x])}{(a - b)(c + d \sin[e + f x])}} (c + d \sin[e + f x])$$

Result (type 4, 1864 leaves):

$$-\frac{d^2 \cos[e + f x] \sqrt{a + b \sin[e + f x]} \sqrt{c + d \sin[e + f x]}}{2 b f} +$$

$$\frac{1}{8 b f} \left(- \left(4 (-b c + a d) (8 b c^3 + 11 b c d^2 - a d^3) \sqrt{\frac{(c + d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-c + d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c + d \sin[e + f x])}{-b c + a d}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2 (-b c + a d)}{(a + b)(-c + d)} \right] \operatorname{Sec}[e + f x] \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \sqrt{\frac{(c + d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a + b \sin[e + f x])}{-b c + a d}}$$

$$\begin{aligned}
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& 4(-bc+ad)(24bc^2d-4acd^2+4bd^3) \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}}}\right. \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \right. \\
& \left. \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) / \\
& \left. \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) + 2(-9bcd^2+3ad^3) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} + \right.
\end{aligned}$$

$$\left(\sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{\frac{a+b \sin[efx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \sin[efx]} \right) /$$

$$\left(bd \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{a+b \sin[efx]}} \sqrt{a+b \sin[efx]} \sqrt{\frac{a+b \sin[efx]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[efx])}{(c+d)(a+b \sin[efx])}} \right) -$$

$$\frac{1}{bd} 2(-bc+ad) \left((a+b)c+ad \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}\right], \frac{1}{\sqrt{2}}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) -$$

$$\left((bc+ad) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}\right], \frac{1}{\sqrt{2}}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right.$$

$$\left. \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) \right) \right) \right)$$

■ Problem 784: Humongous result has more than 200000 leaves.

$$\int \frac{(c + d \sin[e + f x])^{3/2}}{\sqrt{a + b \sin[e + f x]}} dx$$

Optimal (type 4, 644 leaves, 6 steps):

$$\frac{1}{b(b c - a d) f} \sqrt{a + b} (c - d) d \sqrt{c + d} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}\right], \frac{(a - b)(c + d)}{(a + b)(c - d)}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{-\frac{(b c - a d)(1 - \sin[e + f x])}{(c + d)(a + b \sin[e + f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e + f x])}{(c - d)(a + b \sin[e + f x])}} (a + b \sin[e + f x]) + \frac{1}{b^2 \sqrt{a + b} f}$$

$$\sqrt{c + d} (3 b c - a d) \operatorname{EllipticPi}\left[\frac{b(c + d)}{(a + b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}\right], \frac{(a - b)(c + d)}{(a + b)(c - d)}\right] \operatorname{Sec}[e + f x]$$

$$\sqrt{-\frac{(b c - a d)(1 - \sin[e + f x])}{(c + d)(a + b \sin[e + f x])}} \sqrt{\frac{(b c - a d)(1 + \sin[e + f x])}{(c - d)(a + b \sin[e + f x])}} (a + b \sin[e + f x]) - \frac{d \cos[e + f x] \sqrt{c + d \sin[e + f x]}}{f \sqrt{a + b \sin[e + f x]}}$$

$$\frac{1}{b^2 \sqrt{c + d} f} \sqrt{a + b} (a d - b(2 c + d)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{\frac{(b c - a d)(1 - \sin[e + f x])}{(a + b)(c + d \sin[e + f x])}} \sqrt{-\frac{(b c - a d)(1 + \sin[e + f x])}{(a - b)(c + d \sin[e + f x])}} (c + d \sin[e + f x])$$

Result (type ?, 222963 leaves): Display of huge result suppressed!

■ **Problem 785: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c + d \sin[e + f x]}}{\sqrt{a + b \sin[e + f x]}} dx$$

Optimal (type 4, 198 leaves, 1 step):

$$\frac{1}{b \sqrt{c + d} f} 2 \sqrt{a + b} \operatorname{EllipticPi}\left[\frac{(a + b)d}{b(c + d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{\frac{(b c - a d)(1 - \sin[e + f x])}{(a + b)(c + d \sin[e + f x])}} \sqrt{-\frac{(b c - a d)(1 + \sin[e + f x])}{(a - b)(c + d \sin[e + f x])}} (c + d \sin[e + f x])$$

Result (type 4, 578 leaves):

$$\begin{aligned}
& -\frac{1}{f} 4 (bc - ad) \left(\left(\sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{bc-ad}}}{\sqrt{2}}}\right], \frac{2 (bc - ad)}{(-a+b) (c+d)}\right] \right. \right. \\
& \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{-\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{bc-ad}} \\
& \left. \left. \sqrt{\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{bc-ad}} \right) / \left((a+b) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \right. \\
& \left. \left(d \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-a+b}} \operatorname{EllipticPi}\left[\frac{bc-ad}{b(c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{bc-ad}}}{\sqrt{2}}}\right], \frac{2 (bc - ad)}{(-a+b) (c+d)}\right] \right) \right. \\
& \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{-\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{bc-ad}} \\
& \left. \left. \sqrt{\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{bc-ad}} \right) / \left(b(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right)
\end{aligned}$$

■ **Problem 787: Humongous result has more than 200000 leaves.**

$$\int \frac{1}{\sqrt{a+b \operatorname{Sin}[e+fx]} (c+d \operatorname{Sin}[e+fx])^{3/2}} dx$$

Optimal (type 4, 405 leaves, 3 steps):

$$\left(2 (a-b) \sqrt{a+b} d \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]}{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+fx] \right. \\ \left. \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d)\sin[e+fx]}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d)\sin[e+fx]}} (c+d)\sin[e+fx] \right) / \left((c-d) \sqrt{c+d} (bc-ad)^2 f \right) + \\ \left(2 \sqrt{a+b} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]}{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d)\sin[e+fx]}} \right. \\ \left. \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d)\sin[e+fx]}} (c+d)\sin[e+fx] \right) / \left((c-d) \sqrt{c+d} (bc-ad) f \right)$$

Result (type ?, 204942 leaves): Display of huge result suppressed!

■ **Problem 788: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a+b \sin[e+fx]} (c+d \sin[e+fx])^{5/2}} dx$$

Optimal (type 4, 521 leaves, 4 steps):

$$-\frac{2 d^2 \cos[e+fx] \sqrt{a+b \sin[e+fx]}}{3 (bc-ad) (c^2-d^2) f (c+d \sin[e+fx])^{3/2}} - \\ \left(4 (a-b) \sqrt{a+b} d (2acd-b(3c^2-d^2)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]}{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+fx] \right. \\ \left. \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d)\sin[e+fx]}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d)\sin[e+fx]}} (c+d)\sin[e+fx] \right) / \left(3 (c-d)^2 (c+d)^{3/2} (bc-ad)^3 f \right) - \\ \left(2 \sqrt{a+b} (ad(3c+d) - b(3c^2+3cd-2d^2)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]}{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+fx] \right. \\ \left. \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d)\sin[e+fx]}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d)\sin[e+fx]}} (c+d)\sin[e+fx] \right) / \left(3 (c-d)^2 (c+d)^{3/2} (bc-ad)^2 f \right)$$

Result (type 4, 2072 leaves):

$$\frac{1}{f} \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]}$$

$$\begin{aligned}
& \left(-\frac{2d^2 \cos[e+fx]}{3(bc-ad)(c^2-d^2)(c+d \sin[e+fx])^2} + \frac{4(-3bc^2d^2 \cos[e+fx] + 2acd^3 \cos[e+fx] + bd^4 \cos[e+fx])}{3(bc-ad)^2(c^2-d^2)^2(c+d \sin[e+fx])} \right) + \\
& \frac{1}{3(c-d)^2(c+d)^2(bc-ad)^2f} \\
& \left(-\frac{1}{(a+b)(c+d)\sqrt{a+b \sin[e+fx]}\sqrt{c+d \sin[e+fx]}} 4(-bc+ad)(3b^2c^4 - 6abc^3d + 3a^2c^2d^2 - 5b^2c^2d^2 + 2abc d^3 + a^2d^4 + 2b^2d^4) \right. \\
& \left. \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right. \\
& \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(a+b \sin[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d \sin[e+fx])}{-bc+ad}} - \right. \\
& \left. 4(-bc+ad)(-6b^2c^3d - 2abc^2d^2 + 4a^2cd^3 + 2b^2cd^3 + 2abd^4) \right. \\
& \left. \left(\left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right) \right. \\
& \left. \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(a+b \sin[e+fx])}{-bc+ad}} \right. \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d \sin[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d)\sqrt{a+b \sin[e+fx]}\sqrt{c+d \sin[e+fx]} \right) - \right. \\
& \left. \left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \\
& \left. \left((a+b) d \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) + 2 (6b^2 c^2 d^2 - 4abcd^3 - 2b^2 d^4) \left(\frac{\cos[efx] \sqrt{c+d \sin[efx]}}{d \sqrt{a+b \sin[efx]}} + \right. \right. \\
& \left. \left. \sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{\frac{a+b \sin[efx]}{a+b}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \sin[efx]} \right) / \right. \\
& \left. \left(bd \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{a+b \sin[efx]}} \sqrt{a+b \sin[efx]} \sqrt{\frac{a+b \sin[efx]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[efx])}{(c+d)(a+b \sin[efx])}} \right) - \right. \\
& \frac{1}{bd} 2(-bc+ad) \left(\left((a+b) c+ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \sec[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right. \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) - \right. \\
& \left. \left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right.
\end{aligned}$$

$$\left. \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}} \right) / \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right)$$

■ **Problem 789: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \operatorname{Sin}[e+fx])^{5/2}}{(a+b \operatorname{Sin}[e+fx])^{3/2}} dx$$

Optimal (type 4, 822 leaves, 7 steps):

$$\left((c-d) \sqrt{c+d} (2b^2c^2 - 4abcd + 3a^2d^2 - b^2d^2) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+fx]}}{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \operatorname{Sec}[e+fx] \right. \\ \left. \sqrt{-\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(c-d)(a+b \operatorname{Sin}[e+fx])}} (a+b \operatorname{Sin}[e+fx])} \right) / \left((a-b) b^2 \sqrt{a+b} (bc-ad) f \right) + \\ \frac{1}{b^3 \sqrt{a+b} f} d \sqrt{c+d} (5bc-3ad) \operatorname{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+fx]}}{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\ \operatorname{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(c-d)(a+b \operatorname{Sin}[e+fx])}} (a+b \operatorname{Sin}[e+fx])} + \\ \frac{2(bc-ad)^2 \operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{b(a^2-b^2) f \sqrt{a+b \operatorname{Sin}[e+fx]}} + \frac{(4abcd-3a^2d^2-b^2(2c^2-d^2)) \operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{b(a^2-b^2) f \sqrt{a+b \operatorname{Sin}[e+fx]}} - \frac{1}{(a-b) b^3 \sqrt{c+d} f} \\ \sqrt{a+b} (3a^2d^2 - 2abd(c+3d) - b^2(2c^2 - 6cd - d^2)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+fx]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \\ \operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(a+b)(c+d \operatorname{Sin}[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(a-b)(c+d \operatorname{Sin}[e+fx])}} (c+d \operatorname{Sin}[e+fx])} \right)$$

Result (type 4, 1975 leaves):

$$\frac{2(b^2c^2 \operatorname{Cos}[e+fx] - 2abcd \operatorname{Cos}[e+fx] + a^2d^2 \operatorname{Cos}[e+fx]) \sqrt{c+d \operatorname{Sin}[e+fx]}}{b(-a^2+b^2) f \sqrt{a+b \operatorname{Sin}[e+fx]}} +$$

$$\begin{aligned}
& \frac{1}{2(a-b)b(a+b)f} \left(- \left(4(-bc+ad)(2abc^3 - 4b^2c^2d + 2abcd^2 + a^2d^3 - b^2d^3) \sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \right. \right. \\
& \left. \left. \sqrt{\frac{(c+d)\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(a+b)\operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \right. \\
& \left. \left((a+b)(c+d)\sqrt{a+b\operatorname{Sin}[e+fx]}\sqrt{c+d\operatorname{Sin}[e+fx]} \right) - 4(-bc+ad)(2b^2c^3 - 2abc^2d + 4a^2cd^2 - 6b^2cd^2 + 2abd^3) \right. \\
& \left. \left(\left(\sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right. \right. \\
& \left. \left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d)\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(a+b)\operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)(c+d)\sqrt{a+b\operatorname{Sin}[e+fx]}\sqrt{c+d\operatorname{Sin}[e+fx]} \right) - \right. \\
& \left. \left(\sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d)\operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \\
& \left. \left((a+b) d \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) + 2(-2b^2c^2d + 4abcd^2 - 3a^2d^3 + b^2d^3) \left(\frac{\cos[efx] \sqrt{c+d \sin[efx]}}{d \sqrt{a+b \sin[efx]}} + \right. \right. \\
& \left. \left. \left(\sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{\frac{a+b \sin[efx]}{a+b}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \sin[efx]} \right) / \right. \right. \\
& \left. \left. \left(bd \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{a+b \sin[efx]}} \sqrt{a+b \sin[efx]} \sqrt{\frac{a+b \sin[efx]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[efx])}{(c+d)(a+b \sin[efx])}} \right) - \right. \right. \\
& \left. \left. \frac{1}{bd} 2(-bc+ad) \left((a+b)c+ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \sec[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) - \right. \right. \\
& \left. \left. \left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}}{\sqrt{2}}}\right], \right. \right.
\end{aligned}$$

$$\left. \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right. \right. \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right] \right/ \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right)$$

■ **Problem 790: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \operatorname{Sin}[e+fx])^{3/2}}{(a+b \operatorname{Sin}[e+fx])^{3/2}} dx$$

Optimal (type 4, 600 leaves, 5 steps):

$$\begin{aligned} & \frac{1}{(a-b)b\sqrt{a+bf}} 2(c-d)\sqrt{c+d} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sqrt{c+d \operatorname{Sin}[e+fx]}}{\sqrt{c+d}\sqrt{a+b \operatorname{Sin}[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\ & \operatorname{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(c-d)(a+b \operatorname{Sin}[e+fx])}} (a+b \operatorname{Sin}[e+fx]) + \\ & \frac{1}{b^2\sqrt{a+bf}} 2d\sqrt{c+d} \operatorname{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sqrt{c+d \operatorname{Sin}[e+fx]}}{\sqrt{c+d}\sqrt{a+b \operatorname{Sin}[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\ & \operatorname{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(c-d)(a+b \operatorname{Sin}[e+fx])}} (a+b \operatorname{Sin}[e+fx]) + \\ & \frac{1}{(a-b)b^2\sqrt{c+d}f} 2\sqrt{a+b}(b(c-2d)+ad) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{a+b \operatorname{Sin}[e+fx]}}{\sqrt{a+b}\sqrt{c+d \operatorname{Sin}[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \\ & \operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(a+b)(c+d \operatorname{Sin}[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(a-b)(c+d \operatorname{Sin}[e+fx])}} (c+d \operatorname{Sin}[e+fx]) \end{aligned}$$

Result (type 4, 1866 leaves):

$$-\frac{2(-bc \operatorname{Cos}[e+fx] + ad \operatorname{Cos}[e+fx]) \sqrt{c+d \operatorname{Sin}[e+fx]}}{(a^2-b^2)f\sqrt{a+b \operatorname{Sin}[e+fx]}} + \frac{1}{(a-b)(a+b)f}$$

$$\begin{aligned}
& \left(- \left[4 (-bc+ad) (ac^2-bcd) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right. \\
& \quad \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \\
& \quad \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right] / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& \quad 4(-bc+ad)(bc^2-bd^2) \left(\left[\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right. \\
& \quad \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \\
& \quad \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right] / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& \quad \left[\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \\
& \quad \left. \operatorname{Sin}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right] /
\end{aligned}$$

$$\begin{aligned}
& \left. \left((a+b) d \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) + 2(-bcd+ad^2) \left(\frac{\cos[e+fx] \sqrt{c+d \sin[e+fx]}}{d \sqrt{a+b \sin[e+fx]}} + \right. \right. \\
& \left. \left. \left(\sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{\sqrt{\frac{a+b \sin[e+fx]}{a+b}}}\right]}, \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \sin[e+fx]} \right) / \right. \\
& \left. \left(bd \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b \sin[e+fx]}} \sqrt{a+b \sin[e+fx]} \sqrt{\frac{a+b \sin[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[e+fx])}{(c+d)(a+b \sin[e+fx])}} \right) - \right. \\
& \left. \frac{1}{bd} 2(-bc+ad) \left((a+b) c+ad \right) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}\right], \sqrt{2} \right), \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) - \\
& \left((bc+ad) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}\right], \sqrt{2} \right), \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \right), \right.
\end{aligned}$$

$$\left. \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) \right/ \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right)$$

- **Problem 792: Humongous result has more than 200000 leaves.**

$$\int \frac{1}{(a+b \operatorname{Sin}[e+fx])^{3/2} \sqrt{c+d \operatorname{Sin}[e+fx]}} dx$$

Optimal (type 4, 405 leaves, 3 steps):

$$\left(2b(c-d) \sqrt{c+d} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+fx]}}{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \operatorname{Sec}[e+fx] \right. \\ \left. \sqrt{-\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(c-d)(a+b \operatorname{Sin}[e+fx])}} (a+b \operatorname{Sin}[e+fx])} \right/ \left((a-b) \sqrt{a+b} (bc-ad)^2 f \right) + \\ \left(2\sqrt{a+b} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+fx]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(a+b)(c+d \operatorname{Sin}[e+fx])}} \right. \right. \\ \left. \left. \sqrt{-\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(a-b)(c+d \operatorname{Sin}[e+fx])}} (c+d \operatorname{Sin}[e+fx])} \right) \right/ \left((a-b) \sqrt{c+d} (bc-ad) f \right)$$

Result (type ?, 204906 leaves): Display of huge result suppressed!

- **Problem 793: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \operatorname{Sin}[e+fx])^{3/2} (c+d \operatorname{Sin}[e+fx])^{3/2}} dx$$

Optimal (type 4, 495 leaves, 4 steps):

$$\frac{2 b^2 \operatorname{Cos}[e+f x]}{(a^2-b^2)(b c-a d) f \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}} -$$

$$\left(2 \left(a^2 d^2 + b^2 (c^2 - 2 d^2) \right) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \operatorname{Sec}[e+f x] \right.$$

$$\left. \sqrt{\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \sqrt{-\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}} (c+d \operatorname{Sin}[e+f x])} \right) / \left(\sqrt{a+b} (c-d) \sqrt{c+d} (b c-a d)^3 f \right) +$$

$$\left(2 (b(c-2 d)-a d) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \operatorname{Sec}[e+f x] \sqrt{\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \right.$$

$$\left. \sqrt{-\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}} (c+d \operatorname{Sin}[e+f x])} \right) / \left(\sqrt{a+b} (c-d) \sqrt{c+d} (b c-a d)^2 f \right)$$

Result (type 4, 2052 leaves):

$$\frac{\sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]} \left(\frac{2 b^3 \operatorname{Cos}[e+f x]}{(a^2-b^2)(-b c+a d)^2 (a+b \operatorname{Sin}[e+f x])} + \frac{2 d^3 \operatorname{Cos}[e+f x]}{(b c-a d)^2 (c^2-d^2)(c+d \operatorname{Sin}[e+f x])} \right)}{f} + \frac{1}{(a-b)(a+b)(c-d)(c+d)(-b c+a d)^2 f}$$

$$\left(-\frac{1}{(a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}} 4(-b c+a d) (a b^2 c^3 - 2 a^2 b c^2 d + 2 b^3 c^2 d + a^3 c d^2 - 2 a b^2 c d^2 + 2 a^2 b d^3 - 2 b^3 d^3) \right.$$

$$\left. \sqrt{\frac{(c+d) \operatorname{Cot} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{-c+d}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{-\frac{(a+b) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}} \right], \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \right.$$

$$\left. \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^4 \sqrt{\frac{(c+d) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \sqrt{-\frac{(a+b) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}} - \right.$$

$$\left. 4(-b c+a d) (b^3 c^3 + a b^2 c^2 d + a^2 b c d^2 - 2 b^3 c d^2 + a^3 d^3 - 2 a b^2 d^3) \right.$$

$$\left. \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{-c+d}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{-\frac{(a+b) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}} \right], \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \right) \right)$$

$$\begin{aligned}
& \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}\right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}\right) - \\
& \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}\right) / \right. \\
& \left. \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}\right) + 2\left(-b^3 c^2 d-a^2 b d^3+2 b^3 d^3\right) \left(\frac{\operatorname{Cos}[e+f x] \sqrt{c+d \operatorname{Sin}[e+f x]}}{d \sqrt{a+b \operatorname{Sin}[e+f x]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}}(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}}{\sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}\right], \frac{2(-b c+a d)}{(a-b)(c+d)}\right] \sqrt{c+d \operatorname{Sin}[e+f x]}\right) / \right. \\
& \left. \left(b d \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \operatorname{Sin}[e+f x]}} \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+f x])}{(c+d)(a+b \operatorname{Sin}[e+f x])}}\right) - \right. \\
& \left. \frac{1}{b d} 2(-b c+a d) \left((a+b) c+a d \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2(-bc+ad)}{(a+b)(-c+d)} \left[\sec[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right] / \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) - \\
& \left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right] \right), \\
& \frac{2(-bc+ad)}{(a+b)(-c+d)} \left[\sec[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \right] / \left((a+b)d \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 794: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \sin[e+fx])^{3/2} (c+d \sin[e+fx])^{5/2}} dx$$

Optimal (type 4, 681 leaves, 5 steps):

$$\frac{2 b^2 \operatorname{Cos}[e+f x]}{(a^2-b^2)(b c-a d) f \sqrt{a+b \operatorname{Sin}[e+f x]}(c+d \operatorname{Sin}[e+f x])^{3/2}} + \frac{2 d\left(a^2 d^2+b^2\left(3 c^2-4 d^2\right)\right) \operatorname{Cos}[e+f x] \sqrt{a+b \operatorname{Sin}[e+f x]}}{3\left(a^2-b^2\right)(b c-a d)^2\left(c^2-d^2\right) f(c+d \operatorname{Sin}[e+f x])^{3/2}} +$$

$$\left(2\left(4 a^3 c d^3-4 a b^2 c d^3-a^2 b d^2\left(9 c^2-5 d^2\right)-b^3\left(3 c^4-15 c^2 d^2+8 d^4\right)\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+f x] \sqrt{\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \sqrt{-\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}}(c+d \operatorname{Sin}[e+f x])}\right) / \left(3 \sqrt{a+b}(c-d)^2(c+d)^{3/2}(b c-a d)^4 f\right) +$$

$$\left(2\left(a^2 d^2(3 c+d)-6 a b d\left(c^2-d^2\right)+b^2\left(3 c^3-9 c^2 d-6 c d^2+8 d^3\right)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+f x] \sqrt{\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \sqrt{-\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}}(c+d \operatorname{Sin}[e+f x])}\right) / \left(3 \sqrt{a+b}(c-d)^2(c+d)^{3/2}(b c-a d)^3 f\right)$$

Result (type 4, 2320 leaves):

$$\frac{1}{f} \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]} \left(-\frac{2 b^4 \operatorname{Cos}[e+f x]}{\left(a^2-b^2\right)(-b c+a d)^3(a+b \operatorname{Sin}[e+f x])} + \frac{2 d^3 \operatorname{Cos}[e+f x]}{3(b c-a d)^2\left(c^2-d^2\right)(c+d \operatorname{Sin}[e+f x])^2} - \frac{2\left(-9 b c^2 d^3 \operatorname{Cos}[e+f x]+4 a c d^4 \operatorname{Cos}[e+f x]+5 b d^5 \operatorname{Cos}[e+f x]\right)}{3(b c-a d)^3\left(c^2-d^2\right)^2(c+d \operatorname{Sin}[e+f x])} \right) +$$

$$\frac{1}{3(a-b)(a+b)(c-d)^2(c+d)^2(-b c+a d)^3 f} \left(-\frac{1}{(a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}} 4(-b c+a d)\left(-3 a b^3 c^5+9 a^2 b^2 c^4 d-9 b^4 c^4 d-9 a^3 b c^3 d^2+15 a b^3 c^3 d^2+3 a^4 c^2 d^3-20 a^2 b^2 c^2 d^3+17 b^4 c^2 d^3+5 a^3 b c d^4-8 a b^3 c d^4+a^4 d^5+7 a^2 b^2 d^5-8 b^4 d^5\right) \right.$$

$$\left. \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x] \right.$$

$$\left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \operatorname{Sin}[e+f x])}{-b c+a d}} - 4(-b c+a d)\left(-3 b^4 c^5-3 a b^3 c^4 d-9 a^2 b^2 c^3 d^2+15 b^4 c^3 d^2-5 a^3 b c^2 d^3+11 a b^3 c^2 d^3+4 a^4 c d^4+a^2 b^2 c d^4-8 b^4 c d^4+5 a^3 b d^5-8 a b^3 d^5\right) \right.$$

$$\left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right.$$

$$\left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right.$$

$$\left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) +$$

$$2(3b^4c^4d+9a^2b^2c^2d^3-15b^4c^2d^3-4a^3bcd^4+4ab^3cd^4-5a^2b^2d^5+8b^4d^5) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} +$$

$$\left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{\sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \operatorname{Sin}[e+fx]} \right) /$$

$$\left(b d \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \sin[e+f x]}} \sqrt{a+b \sin[e+f x]} \sqrt{\frac{a+b \sin[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \right) -$$

$$\frac{1}{b d} 2(-b c+a d) \left(\left((a+b) c+a d \right) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}}$$

$$\left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) -$$

$$\left((b c+a d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}}$$

$$\left. \left. \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b) d \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) \right) \right) \right)$$

■ **Problem 795: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \sin[e+f x])^{5/2}}{(a+b \sin[e+f x])^{5/2}} dx$$

Optimal (type 4, 736 leaves, 6 steps):

$$\frac{1}{3(a-b)^2 b^2 (a+b)^{3/2} f} 2(c-d) \sqrt{c+d} (4abc + 3a^2 d - 7b^2 d) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \text{Sin}[e+fx]}{\sqrt{c+d} \sqrt{a+b} \text{Sin}[e+fx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\text{Sin}[e+fx])}{(c+d)(a+b\text{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\text{Sin}[e+fx])}{(c-d)(a+b\text{Sin}[e+fx])}} (a+b\text{Sin}[e+fx]) +$$

$$\frac{1}{b^3 \sqrt{a+b} f} 2d^2 \sqrt{c+d} \text{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \text{Sin}[e+fx]}{\sqrt{c+d} \sqrt{a+b} \text{Sin}[e+fx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \text{Sec}[e+fx]$$

$$\sqrt{-\frac{(bc-ad)(1-\text{Sin}[e+fx])}{(c+d)(a+b\text{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\text{Sin}[e+fx])}{(c-d)(a+b\text{Sin}[e+fx])}} (a+b\text{Sin}[e+fx]) + \frac{2(bc-ad)^2 \text{Cos}[e+fx] \sqrt{c+d} \text{Sin}[e+fx]}{3b(a^2-b^2)f(a+b\text{Sin}[e+fx])^{3/2}} +$$

$$\left(2(3a^2 b(c-2d)d + 3a^3 d^2 + ab^2(3c^2 - 4cd - 2d^2) + b^3(c^2 - 7cd + 9d^2)) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \text{Sin}[e+fx]}{\sqrt{a+b} \sqrt{c+d} \text{Sin}[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]\right)$$

$$\text{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\text{Sin}[e+fx])}{(a+b)(c+d\text{Sin}[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\text{Sin}[e+fx])}{(a-b)(c+d\text{Sin}[e+fx])}} (c+d\text{Sin}[e+fx])} \Big/ (3(a-b)^2 b^3 \sqrt{a+b} \sqrt{c+d} f)$$

Result (type 4, 2142 leaves):

$$\frac{1}{f} \sqrt{a+b\text{Sin}[e+fx]} \sqrt{c+d\text{Sin}[e+fx]} \left(-\frac{2(b^2 c^2 \text{Cos}[e+fx] - 2abcd \text{Cos}[e+fx] + a^2 d^2 \text{Cos}[e+fx])}{3b(-a^2+b^2)(a+b\text{Sin}[e+fx])^2} - \right.$$

$$\left. (2(-4ab^2 c^2 \text{Cos}[e+fx] + a^2 bcd \text{Cos}[e+fx] + 7b^3 cd \text{Cos}[e+fx] + 3a^3 d^2 \text{Cos}[e+fx] - 7ab^2 d^2 \text{Cos}[e+fx])) \Big/ \right.$$

$$\left. (3b(-a^2+b^2)^2 (a+b\text{Sin}[e+fx])) \right) - \frac{1}{3(a-b)^2 b(a+b)^2 f}$$

$$\left(-\frac{1}{(a+b)(c+d) \sqrt{a+b\text{Sin}[e+fx]} \sqrt{c+d\text{Sin}[e+fx]}} 4(-bc+ad) (-3a^2 bc^3 - b^3 c^3 + 8ab^2 c^2 d - 2a^2 bcd^2 - 2b^3 cd^2 + a^3 d^3 - ab^2 d^3) \right.$$

$$\left. \sqrt{\frac{(c+d) \text{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2}{-c+d}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(a+b) \text{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (c+d\text{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \text{Sec}[e+fx] \right.$$

$$\left. \text{Sin}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^4 \sqrt{\frac{(c+d) \text{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (a+b\text{Sin}[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \text{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (c+d\text{Sin}[e+fx])}{-bc+ad}} - \right.$$

$$4(-bc+ad) (-4ab^2 c^3 - 3a^2 bc^2 d + 7b^3 c^2 d + 4a^3 cd^2 - a^2 bd^3 - 3b^3 d^3)$$

$$\left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right.$$

$$\left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right.$$

$$\left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) +$$

$$2(4ab^2c^2d - a^2bcd^2 - 7b^3cd^2 - 3a^3d^3 + 7ab^2d^3) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} +$$

$$\left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{\sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \operatorname{Sin}[e+fx]} \right) /$$

$$\begin{aligned}
& \left(b d \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \sin[e+f x]}} \sqrt{a+b \sin[e+f x]} \sqrt{\frac{a+b \sin[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \right) - \\
& \frac{1}{b d} 2(-b c+a d) \left(\left((a+b) c+a d \right) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) - \\
& \left((b c+a d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b) d \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) \right) \right)
\end{aligned}$$

■ **Problem 796: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+d \sin[e+f x])^{3/2}}{(a+b \sin[e+f x])^{5/2}} dx$$

Optimal (type 4, 497 leaves, 4 steps):

$$\left(8 (c-d) \sqrt{c+d} (a c-b d) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \operatorname{Sin}[e+f x]}{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \operatorname{Sec}[e+f x] \sqrt{-\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(c+d)(a+b \operatorname{Sin}[e+f x])}}\right. \right. \\ \left. \left. \sqrt{\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(c-d)(a+b \operatorname{Sin}[e+f x])}} (a+b \operatorname{Sin}[e+f x])\right) \right] / \left(3 (a-b)^2 (a+b)^{3/2} (b c-a d) f \right) + \frac{2 (b c-a d) \operatorname{Cos}[e+f x] \sqrt{c+d} \operatorname{Sin}[e+f x]}{3 (a^2-b^2) f (a+b \operatorname{Sin}[e+f x])^{3/2}} + \\ \left(2 (c-d) (3 a c+b c-a d-3 b d) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+f x] \right. \\ \left. \sqrt{\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \sqrt{-\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}} (c+d \operatorname{Sin}[e+f x])\right) \right] / \left(3 (a-b)^2 \sqrt{a+b} \sqrt{c+d} (b c-a d) f \right)$$

Result (type 4, 1982 leaves):

$$\frac{\sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]} \left(-\frac{2(-b c \operatorname{Cos}[e+f x]+a d \operatorname{Cos}[e+f x])}{3(a^2-b^2)(a+b \operatorname{Sin}[e+f x])^2} - \frac{8(-a b c \operatorname{Cos}[e+f x]+b^2 d \operatorname{Cos}[e+f x])}{3(a^2-b^2)^2(a+b \operatorname{Sin}[e+f x])} \right)}{f} + \\ \frac{1}{3(a-b)^2(a+b)^2 f} \left(- \left(4(-b c+a d) (3 a^2 c^2+b^2 c^2-4 a b c d+a^2 d^2-b^2 d^2) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{-c+d}} \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2(c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^4 \right. \right. \\ \left. \left. \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2(a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2(c+d \operatorname{Sin}[e+f x])}{-b c+a d}} \right) \right] / \\ \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]} \right) - 4(-b c+a d) (4 a b c^2+4 a^2 c d-4 b^2 c d-4 a b d^2) \\ \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2(c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \right. \right.$$

$$\begin{aligned}
& \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}\right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}\right) - \\
& \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}\right) / \right. \\
& \left. \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}\right) + 2(-4 a b c d+4 b^2 d^2) \left(\frac{\operatorname{Cos}[e+f x] \sqrt{c+d \operatorname{Sin}[e+f x]}}{d \sqrt{a+b \operatorname{Sin}[e+f x]}} + \right. \right. \\
& \left. \left. \sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{\sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}\right], \frac{2(-b c+a d)}{(a-b)(c+d)}\right] \sqrt{c+d \operatorname{Sin}[e+f x]}\right) / \right. \\
& \left. \left(b d \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \operatorname{Sin}[e+f x]}} \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+f x])}{(c+d)(a+b \operatorname{Sin}[e+f x])}}\right) - \right. \\
& \left. \frac{1}{b d} 2(-b c+a d) \left((a+b) c+a d \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2(-bc+ad)}{(a+b)(-c+d)} \left[\sec[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \\
& \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \left/ \left((a+b)(c+d) \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) \right. - \\
& \left. \left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right] \right), \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \left[\sec[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \sin[e+fx])}{-bc+ad}} \left/ \left((a+b)d \sqrt{a+b \sin[e+fx]} \sqrt{c+d \sin[e+fx]} \right) \right) \right] \right) \right) \right)
\end{aligned}$$

■ **Problem 797: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d \sin[e+fx]}}{(a+b \sin[e+fx])^{5/2}} dx$$

Optimal (type 4, 489 leaves, 4 steps):

$$\begin{aligned}
& \left(2 (c-d) \sqrt{c+d} (4abc - 3a^2d - b^2d) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a+b} \sqrt{c+d} \operatorname{Sin}[e+fx]}{\sqrt{c+d} \sqrt{a+b} \operatorname{Sin}[e+fx]} \right], \frac{(a-b)(c+d)}{(a+b)(c-d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \left. \sqrt{-\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(c+d)(a+b\operatorname{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(c-d)(a+b\operatorname{Sin}[e+fx])}} (a+b\operatorname{Sin}[e+fx])} \right] / (3(a-b)^2(a+b)^{3/2}(bc-ad)^2f) + \\
& \frac{2b \operatorname{Cos}[e+fx] \sqrt{c+d} \operatorname{Sin}[e+fx]}{3(a^2-b^2)f(a+b\operatorname{Sin}[e+fx])^{3/2}} + \left(2(3a+b)(c-d) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b} \operatorname{Sin}[e+fx]}{\sqrt{a+b} \sqrt{c+d} \operatorname{Sin}[e+fx]} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \left. \sqrt{\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(a+b)(c+d\operatorname{Sin}[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(a-b)(c+d\operatorname{Sin}[e+fx])}} (c+d\operatorname{Sin}[e+fx])} \right] / (3(a-b)^2\sqrt{a+b}\sqrt{c+d}(bc-ad)f)
\end{aligned}$$

Result (type 4, 2037 leaves):

$$\begin{aligned}
& \frac{1}{f} \sqrt{a+b\operatorname{Sin}[e+fx]} \sqrt{c+d\operatorname{Sin}[e+fx]} \left(\frac{2b \operatorname{Cos}[e+fx]}{3(a^2-b^2)(a+b\operatorname{Sin}[e+fx])^2} + \frac{2(-4ab^2c \operatorname{Cos}[e+fx] + 3a^2bd \operatorname{Cos}[e+fx] + b^3d \operatorname{Cos}[e+fx])}{3(a^2-b^2)^2(-bc+ad)(a+b\operatorname{Sin}[e+fx])} \right) + \\
& \frac{1}{3(a-b)^2(a+b)^2(-bc+ad)f} \left(- \left(4(-bc+ad)(-3a^2bc^2 - b^3c^2 + 3a^3cd + ab^2cd - a^2bd^2 + b^3d^2) \sqrt{\frac{(c+d) \operatorname{Cot} \left[\frac{1}{2}(-e + \frac{\pi}{2} - fx) \right]^2}{-c+d}} \right. \right. \\
& \left. \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc} \left[\frac{1}{2}(-e + \frac{\pi}{2} - fx) \right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}} \right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin} \left[\frac{1}{2}(-e + \frac{\pi}{2} - fx) \right]^4 \right. \right. \\
& \left. \left. \sqrt{\frac{(c+d) \operatorname{Csc} \left[\frac{1}{2}(-e + \frac{\pi}{2} - fx) \right]^2 (a+b\operatorname{Sin}[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc} \left[\frac{1}{2}(-e + \frac{\pi}{2} - fx) \right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right] / \right. \\
& \left. \left((a+b)(c+d) \sqrt{a+b\operatorname{Sin}[e+fx]} \sqrt{c+d\operatorname{Sin}[e+fx]} \right) - 4(-bc+ad)(-4ab^2c^2 - a^2bcd + b^3cd + 3a^3d^2 + ab^2d^2) \right. \\
& \left. \left(\sqrt{\frac{(c+d) \operatorname{Cot} \left[\frac{1}{2}(-e + \frac{\pi}{2} - fx) \right]^2}{-c+d}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc} \left[\frac{1}{2}(-e + \frac{\pi}{2} - fx) \right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}} \right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}\right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}\right) - \\
& \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x] \right. \\
& \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b \operatorname{Sin}[e+f x])}{-b c+a d}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}\right) / \right. \\
& \left. \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}\right) + 2\left(4 a b^2 c d-3 a^2 b d^2-b^3 d^2\right) \left(\frac{\operatorname{Cos}[e+f x] \sqrt{c+d \operatorname{Sin}[e+f x]}}{d \sqrt{a+b \operatorname{Sin}[e+f x]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}}(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}}{\sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}\right], \frac{2(-b c+a d)}{(a-b)(c+d)}\right] \sqrt{c+d \operatorname{Sin}[e+f x]}\right) / \right. \\
& \left. \left(b d \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \operatorname{Sin}[e+f x]}} \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+f x])}{(c+d)(a+b \operatorname{Sin}[e+f x])}}\right) - \right. \\
& \left. \frac{1}{b d} 2(-b c+a d) \left(\left((a+b) c+a d\right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \\
& \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \left/ \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) - \\
& \left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right] \right), \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \left/ \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 798: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \operatorname{Sin}[e+fx])^{5/2} \sqrt{c+d \operatorname{Sin}[e+fx]}} dx$$

Optimal (type 4, 516 leaves, 4 steps):

$$\left(4 b (c-d) \sqrt{c+d} (2 a b c - 3 a^2 d + b^2 d) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \operatorname{Sin}[e+f x]}{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \right. \\ \left. \operatorname{Sec}[e+f x] \sqrt{-\frac{(bc-ad)(1-\operatorname{Sin}[e+f x])}{(c+d)(a+b \operatorname{Sin}[e+f x])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sin}[e+f x])}{(c-d)(a+b \operatorname{Sin}[e+f x])}} (a+b \operatorname{Sin}[e+f x])} \right) / \\ (3(a-b)^2 (a+b)^{3/2} (bc-ad)^3 f) + \frac{2 b^2 \operatorname{Cos}[e+f x] \sqrt{c+d} \operatorname{Sin}[e+f x]}{3(a^2-b^2)(bc-ad) f (a+b \operatorname{Sin}[e+f x])^{3/2}} + \\ \left(2 (3 a b (c-d) - 3 a^2 d + b^2 (c+2 d)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+f x] \right. \\ \left. \sqrt{\frac{(bc-ad)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}} (c+d \operatorname{Sin}[e+f x])} \right) / (3(a-b)^2 \sqrt{a+b} \sqrt{c+d} (bc-ad)^2 f) \right)$$

Result (type 4, 2072 leaves):

$$\frac{1}{f} \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]} \\ \left(-\frac{2 b^2 \operatorname{Cos}[e+f x]}{3(a^2-b^2)(-bc+ad)(a+b \operatorname{Sin}[e+f x])^2} + \frac{4(2 a b^3 c \operatorname{Cos}[e+f x] - 3 a^2 b^2 d \operatorname{Cos}[e+f x] + b^4 d \operatorname{Cos}[e+f x])}{3(a^2-b^2)^2(-bc+ad)^2(a+b \operatorname{Sin}[e+f x])} \right) + \\ \frac{1}{3(a-b)^2(a+b)^2(-bc+ad)^2 f} \\ \left(-\frac{1}{(a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}} 4(-bc+ad)(3 a^2 b^2 c^2 + b^4 c^2 - 6 a^3 b c d + 2 a b^3 c d + 3 a^4 d^2 - 5 a^2 b^2 d^2 + 2 b^4 d^2) \right. \\ \left. \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-bc+ad}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x] \right. \\ \left. \operatorname{Sin}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 (a+b \operatorname{Sin}[e+f x])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 (c+d \operatorname{Sin}[e+f x])}{-bc+ad}} \right) - \\ 4(-bc+ad)(4 a b^3 c^2 - 2 a^2 b^2 c d + 2 b^4 c d - 6 a^3 b d^2 + 2 a b^3 d^2)$$

$$\left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right.$$

$$\left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right.$$

$$\left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) /$$

$$\left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) + 2(-4ab^3cd + 6a^2b^2d^2 - 2b^4d^2) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} +$$

$$\left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}}{\sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \operatorname{Sin}[e+fx]} \right) /$$

$$\left(bd \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b \operatorname{Sin}[e+fx]}} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \right) -$$

$$\begin{aligned}
& \frac{1}{bd} 2 (-bc+ad) \left(\left((a+b) c+ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}\right]}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& \left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}}}\right]}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 799: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \operatorname{Sin}[e+fx])^{5/2} (c+d \operatorname{Sin}[e+fx])^{3/2}} dx$$

Optimal (type 4, 688 leaves, 5 steps):

$$\frac{2 b^2 \operatorname{Cos}[e+f x]}{3\left(a^2-b^2\right)(b c-a d) f\left(a+b \operatorname{Sin}[e+f x]\right)^{3 / 2} \sqrt{c+d \operatorname{Sin}[e+f x]}}+\frac{8 b^2\left(a b c-2 a^2 d+b^2 d\right) \operatorname{Cos}[e+f x]}{3\left(a^2-b^2\right)^2(b c-a d)^2 f \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}}+ \\ \left(2\left(3 a^4 d^3-b^4 d\left(5 c^2-8 d^2\right)+3 a^2 b^2 d\left(3 c^2-5 d^2\right)-4 a b^3 c\left(c^2-d^2\right)\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]\right) \\ \operatorname{Sec}[e+f x] \sqrt{\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \sqrt{-\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}}(c+d \operatorname{Sin}[e+f x])}} \\ \left(3 \sqrt{a+b}\left(a^2-b^2\right)(c-d) \sqrt{c+d}(b c-a d)^4 f\right)-\left(2\left(3 a^2 b(2 c-3 d) d-3 a^3 d^2-3 a b^2\left(c^2-2 d^2\right)+b^3\left(c^2-6 c d+8 d^2\right)\right)\right) \\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+f x] \sqrt{\frac{(b c-a d)(1-\operatorname{Sin}[e+f x])}{(a+b)(c+d \operatorname{Sin}[e+f x])}} \\ \sqrt{-\frac{(b c-a d)(1+\operatorname{Sin}[e+f x])}{(a-b)(c+d \operatorname{Sin}[e+f x])}}(c+d \operatorname{Sin}[e+f x])}} \left(3 \sqrt{a+b}\left(a^2-b^2\right)(c-d) \sqrt{c+d}(b c-a d)^3 f\right)$$

Result (type 4, 2322 leaves):

$$\frac{1}{f} \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]} \\ \left(\frac{2 b^3 \operatorname{Cos}[e+f x]}{3\left(a^2-b^2\right)(-b c+a d)^2\left(a+b \operatorname{Sin}[e+f x]\right)^2}-\frac{2\left(4 a b^4 c \operatorname{Cos}[e+f x]-9 a^2 b^3 d \operatorname{Cos}[e+f x]+5 b^5 d \operatorname{Cos}[e+f x]\right)}{3\left(a^2-b^2\right)^2(-b c+a d)^3\left(a+b \operatorname{Sin}[e+f x]\right)}-\right. \\ \left.\frac{2 d^4 \operatorname{Cos}[e+f x]}{(b c-a d)^3\left(c^2-d^2\right)(c+d \operatorname{Sin}[e+f x])}\right)+\frac{1}{3(a-b)^2(a+b)^2(c-d)(c+d)(-b c+a d)^3 f} \\ \left(-\frac{1}{(a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}} 4(-b c+a d)\left(-3 a^2 b^3 c^4-b^5 c^4+9 a^3 b^2 c^3 d-5 a b^4 c^3 d-9 a^4 b c^2 d^2+\right.\right. \\ \left.\left.20 a^2 b^3 c^2 d^2-7 b^5 c^2 d^2+3 a^5 c d^3-15 a^3 b^2 c d^3+8 a b^4 c d^3+9 a^4 b d^4-17 a^2 b^3 d^4+8 b^5 d^4\right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}}\right)$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d\text{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \text{Sec}[e+fx] \text{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \\
& \sqrt{\frac{(c+d)\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b\text{Sin}[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b)\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d\text{Sin}[e+fx])}{-bc+ad}} - \\
& 4(-bc+ad)(-4ab^4c^4+5a^2b^3c^3d-5b^5c^3d+9a^3b^2c^2d^2-ab^4c^2d^2+3a^4bcd^3-11a^2b^3cd^3+8b^5cd^3+3a^5d^4-15a^3b^2d^4+8ab^4d^4) \\
& \left(\left(\sqrt{\frac{(c+d)\text{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d\text{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right. \\
& \left. \left. \text{Sec}[e+fx] \text{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b\text{Sin}[e+fx])}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d\text{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d)\sqrt{a+b\text{Sin}[e+fx]}\sqrt{c+d\text{Sin}[e+fx]} \right) - \right. \\
& \left. \left(\sqrt{\frac{(c+d)\text{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \text{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d\text{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right. \\
& \left. \left. \text{Sec}[e+fx] \text{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d)\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b\text{Sin}[e+fx])}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d\text{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b)d\sqrt{a+b\text{Sin}[e+fx]}\sqrt{c+d\text{Sin}[e+fx]} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 (4 a b^4 c^3 d - 9 a^2 b^3 c^2 d^2 + 5 b^5 c^2 d^2 - 4 a b^4 c d^3 - 3 a^4 b d^4 + 15 a^2 b^3 d^4 - 8 b^5 d^4) \left(\frac{\cos[e + f x] \sqrt{c + d \sin[e + f x]}}{d \sqrt{a + b \sin[e + f x]}} + \right. \\
& \left. \left(\sqrt{\frac{a - b}{a + b}} (a + b) \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a - b}{a + b}} \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\frac{a + b \sin[e + f x]}{a + b}}}\right], \frac{2(-bc + ad)}{(a - b)(c + d)}\right] \sqrt{c + d \sin[e + f x]}\right) / \right. \\
& \left. \left(b d \sqrt{\frac{(a + b) \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{a + b \sin[e + f x]}} \sqrt{a + b \sin[e + f x]} \sqrt{\frac{a + b \sin[e + f x]}{a + b}} \sqrt{\frac{(a + b)(c + d \sin[e + f x])}{(c + d)(a + b \sin[e + f x])}} \right) - \right. \\
& \left. \frac{1}{b d} 2(-bc + ad) \left((a + b) c + ad \right) \sqrt{\frac{(c + d) \cot\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-c + d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c + d \sin[e + f x])}{-bc + ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc + ad)}{(a + b)(-c + d)} \right] \sec[e + f x] \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \sqrt{\frac{(c + d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a + b \sin[e + f x])}{-bc + ad}} \right. \\
& \left. \left. \sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c + d \sin[e + f x])}{-bc + ad}} \right) / \left((a + b)(c + d) \sqrt{a + b \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) - \right. \\
& \left. \left((bc + ad) \sqrt{\frac{(c + d) \cot\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-c + d}} \operatorname{EllipticPi}\left[\frac{-bc + ad}{(a + b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c + d \sin[e + f x])}{-bc + ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc + ad)}{(a + b)(-c + d)} \right] \sec[e + f x] \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \sqrt{\frac{(c + d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a + b \sin[e + f x])}{-bc + ad}} \right), \right.
\end{aligned}$$

$$\left. \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) \right/ \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right)$$

■ **Problem 800: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \operatorname{Sin}[e+fx])^{5/2} (c+d \operatorname{Sin}[e+fx])^{5/2}} dx$$

Optimal (type 4, 941 leaves, 6 steps):

$$\frac{2 b^2 \operatorname{Cos}[e+fx]}{3 (a^2-b^2) (bc-ad) f (a+b \operatorname{Sin}[e+fx])^{3/2} (c+d \operatorname{Sin}[e+fx])^{3/2}} + \frac{4 b^2 (2 a b c-5 a^2 d+3 b^2 d) \operatorname{Cos}[e+fx]}{3 (a^2-b^2)^2 (bc-ad)^2 f \sqrt{a+b \operatorname{Sin}[e+fx]} (c+d \operatorname{Sin}[e+fx])^{3/2}} -$$

$$\frac{2 d (a^4 d^3+a^2 b^2 d (11 c^2-13 d^2)-b^4 d (7 c^2-8 d^2)-4 a b^3 c (c^2-d^2)) \operatorname{Cos}[e+fx] \sqrt{a+b \operatorname{Sin}[e+fx]}}{3 (a^2-b^2)^2 (bc-ad)^3 (c^2-d^2) f (c+d \operatorname{Sin}[e+fx])^{3/2}} -$$

$$\left(8 (a^5 c d^4-2 a^3 b^2 c d^4+a b^4 c (c^4-2 c^2 d^2+2 d^4)+b^5 d (2 c^4-7 c^2 d^2+4 d^4)-a^2 b^3 d (3 c^4-12 c^2 d^2+7 d^4)-a^4 b (3 c^2 d^3-2 d^5)) \right.$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+fx]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(a+b)(c+d \operatorname{Sin}[e+fx])}}$$

$$\left. \sqrt{-\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(a-b)(c+d \operatorname{Sin}[e+fx])}} (c+d \operatorname{Sin}[e+fx])} \right/ \left(3 \sqrt{a+b} (a^2-b^2) (c-d)^2 (c+d)^{3/2} (bc-ad)^5 f \right) -$$

$$\left(2 (a^4 d^3 (3 c+d)-9 a^3 b d^2 (c^2-d^2)+a^2 b^2 d (9 c^3-18 c^2 d-15 c d^2+16 d^3)+b^4 (c^4-9 c^3 d+16 c^2 d^2+12 c d^3-16 d^4)-3 a b^3 (c^4-5 c^2 d^2+4 d^4)) \right.$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sin}[e+fx]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sin}[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\operatorname{Sin}[e+fx])}{(a+b)(c+d \operatorname{Sin}[e+fx])}}$$

$$\left. \sqrt{-\frac{(bc-ad)(1+\operatorname{Sin}[e+fx])}{(a-b)(c+d \operatorname{Sin}[e+fx])}} (c+d \operatorname{Sin}[e+fx])} \right/ \left(3 \sqrt{a+b} (a^2-b^2) (c-d)^2 (c+d)^{3/2} (bc-ad)^4 f \right)$$

Result (type 4, 2639 leaves):

$$\frac{1}{f} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]}$$

$$\begin{aligned}
& \left(-\frac{2b^4 \cos[e+fx]}{3(a^2-b^2)(-bc+ad)^3(a+b\sin[e+fx])^2} + \frac{8(a^5c \cos[e+fx] - 3a^2b^4d \cos[e+fx] + 2b^6d \cos[e+fx])}{3(a^2-b^2)^2(-bc+ad)^4(a+b\sin[e+fx])} - \right. \\
& \left. \frac{2d^4 \cos[e+fx]}{3(bc-ad)^3(c^2-d^2)(c+d\sin[e+fx])^2} + \frac{8(-3bc^2d^4 \cos[e+fx] + acd^5 \cos[e+fx] + 2bd^6 \cos[e+fx])}{3(bc-ad)^4(c^2-d^2)^2(c+d\sin[e+fx])} \right) + \\
& \frac{1}{3(a-b)^2(a+b)^2(c-d)^2(c+d)^2(-bc+ad)^4f} \left(-\frac{1}{(a+b)(c+d)\sqrt{a+b\sin[e+fx]}\sqrt{c+d\sin[e+fx]}} 4(-bc+ad) \right. \\
& (3a^2b^4c^6 + b^6c^6 - 12a^3b^3c^5d + 8ab^5c^5d + 18a^4b^2c^4d^2 - 41a^2b^4c^4d^2 + 15b^6c^4d^2 - 12a^5bc^3d^3 + 48a^3b^3c^3d^3 - 28ab^5c^3d^3 + \\
& 3a^6c^2d^4 - 41a^4b^2c^2d^4 + 74a^2b^4c^2d^4 - 32b^6c^2d^4 + 8a^5bcd^5 - 28a^3b^3cd^5 + 16ab^5cd^5 + a^6d^6 + 15a^4b^2d^6 - 32a^2b^4d^6 + 16b^6d^6) \\
& \left. \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d\sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right. \\
& \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(a+b\sin[e+fx])}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d\sin[e+fx])}{-bc+ad}} - \right. \\
& 4(-bc+ad)(4ab^5c^6 - 8a^2b^4c^5d + 8b^6c^5d - 12a^3b^3c^4d^2 - 12a^4b^2c^3d^3 + 40a^2b^4c^3d^3 - 28b^6c^3d^3 - 8a^5bc^2d^4 + \\
& 40a^3b^3c^2d^4 - 20ab^5c^2d^4 + 4a^6cd^5 - 20a^2b^4cd^5 + 16b^6cd^5 + 8a^5bd^6 - 28a^3b^3d^6 + 16ab^5d^6) \\
& \left. \left(\left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d\sin[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(a+b\sin[e+fx])}{-bc+ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2(c+d\sin[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d)\sqrt{a+b\sin[e+fx]}\sqrt{c+d\sin[e+fx]} \right) - \right.
\end{aligned}$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right.$$

$$\left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) +$$

$$2(-4ab^5c^5d + 12a^2b^4c^4d^2 - 8b^6c^4d^2 + 8ab^5c^3d^3 + 12a^4b^2c^2d^4 - 48a^2b^4c^2d^4 + 28b^6c^2d^4 - 4a^5bcd^5 + 8a^3b^3cd^5 -$$

$$8ab^5cd^5 - 8a^4b^2d^6 + 28a^2b^4d^6 - 16b^6d^6) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} +$$

$$\left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}}{\sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \right] \sqrt{c+d \operatorname{Sin}[e+fx]} \right) /$$

$$\left(bd \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b \operatorname{Sin}[e+fx]}} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \right) -$$

$$\frac{1}{bd} 2(-bc+ad) \left((a+b)c+ad \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}{\sqrt{2}}}\right],$$

$$\frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}}$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right/ \left((a+b) (c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}}}$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right/ \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right)$$

■ **Problem 802: Unable to integrate problem.**

$$\int (a+b \operatorname{Sin}[e+fx])^m (c+d \operatorname{Sin}[e+fx])^2 dx$$

Optimal (type 6, 311 leaves, 8 steps):

$$-\frac{d^2 \operatorname{Cos}[e+fx] (a+b \operatorname{Sin}[e+fx])^{1+m}}{bf(2+m)} +$$

$$\left(\sqrt{2} (a+b) d (ad-2bc(2+m)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1-m, \frac{3}{2}, \frac{1}{2} (1-\operatorname{Sin}[e+fx]), \frac{b(1-\operatorname{Sin}[e+fx])}{a+b}\right] \operatorname{Cos}[e+fx] \right.$$

$$\left. (a+b \operatorname{Sin}[e+fx])^m \left(\frac{a+b \operatorname{Sin}[e+fx]}{a+b}\right)^{-m} \right) / \left(b^2 f (2+m) \sqrt{1+\operatorname{Sin}[e+fx]} \right) -$$

$$\left(\sqrt{2} (ad(ad-2bc(2+m)) + b^2 (d^2(1+m) + c^2(2+m))) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1-\operatorname{Sin}[e+fx]), \frac{b(1-\operatorname{Sin}[e+fx])}{a+b}\right] \right.$$

$$\left. \operatorname{Cos}[e+fx] (a+b \operatorname{Sin}[e+fx])^m \left(\frac{a+b \operatorname{Sin}[e+fx]}{a+b}\right)^{-m} \right) / \left(b^2 f (2+m) \sqrt{1+\operatorname{Sin}[e+fx]} \right)$$

Result (type 8, 27 leaves):

$$\int (a+b \operatorname{Sin}[e+fx])^m (c+d \operatorname{Sin}[e+fx])^2 dx$$

■ **Problem 814: Unable to integrate problem.**

$$\int (d \operatorname{Csc}[e + f x])^n (a + a \operatorname{Sin}[e + f x])^3 dx$$

Optimal (type 5, 272 leaves, 8 steps):

$$\frac{a^3 d^3 (1 - 2n) \operatorname{Cot}[e + f x] (d \operatorname{Csc}[e + f x])^{-3+n}}{f (1 - n) (2 - n)} + \frac{d^3 \operatorname{Cot}[e + f x] (d \operatorname{Csc}[e + f x])^{-3+n} (a^3 + a^3 \operatorname{Csc}[e + f x])}{f (1 - n)} +$$

$$\frac{a^3 d^3 (5 - 4n) \operatorname{Cos}[e + f x] (d \operatorname{Csc}[e + f x])^{-3+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3-n}{2}, \frac{5-n}{2}, \operatorname{Sin}[e + f x]^2\right]}{f (1 - n) (3 - n) \sqrt{\operatorname{Cos}[e + f x]^2}} +$$

$$\frac{a^3 d^4 (11 - 4n) \operatorname{Cos}[e + f x] (d \operatorname{Csc}[e + f x])^{-4+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4-n}{2}, \frac{6-n}{2}, \operatorname{Sin}[e + f x]^2\right]}{f (2 - n) (4 - n) \sqrt{\operatorname{Cos}[e + f x]^2}}$$

Result (type 9, 28213 leaves): Display of huge result suppressed!

■ **Problem 816: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d \operatorname{Csc}[e + f x])^n (a + a \operatorname{Sin}[e + f x]) dx$$

Optimal (type 5, 149 leaves, 6 steps):

$$\frac{a d \operatorname{Cos}[e + f x] (d \operatorname{Csc}[e + f x])^{-1+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e + f x]^2\right]}{f (1 - n) \sqrt{\operatorname{Cos}[e + f x]^2}} +$$

$$\frac{a d^2 \operatorname{Cos}[e + f x] (d \operatorname{Csc}[e + f x])^{-2+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2-n}{2}, \frac{4-n}{2}, \operatorname{Sin}[e + f x]^2\right]}{f (2 - n) \sqrt{\operatorname{Cos}[e + f x]^2}}$$

Result (type 5, 278 leaves):

$$\frac{1}{f (-1 + n) n (1 + n) \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right)^2} 2^{-1+n} a e^{-i (e+f n x)} \left(1 - e^{2 i (e+f x)}\right)^n \left(\frac{i e^i (e+f x)}{-1 + e^{2 i (e+f x)}}\right)^n$$

$$\operatorname{Csc}[e + f x]^{-1-n} (d \operatorname{Csc}[e + f x])^n (1 + \operatorname{Csc}[e + f x]) \left(e^{i f (-1+n) x} (1 + n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2} (-1 + n), n, \frac{1 + n}{2}, e^{2 i (e+f x)}\right] - \right.$$

$$\left. e^{i e} (-1 + n) \left(2 i e^{i f n x} (1 + n) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, n, \frac{2 + n}{2}, e^{2 i (e+f x)}\right] + e^{i (e+f (1+n) x)} n \operatorname{Hypergeometric2F1}\left[n, \frac{1 + n}{2}, \frac{3 + n}{2}, e^{2 i (e+f x)}\right] \right) \right)$$

■ **Problem 817: Unable to integrate problem.**

$$\int \frac{(d \operatorname{Csc}[e + f x])^n}{a + a \operatorname{Sin}[e + f x]} dx$$

Optimal (type 5, 171 leaves, 7 steps):

$$-\frac{\text{Cot}[e + f x] (d \text{Csc}[e + f x])^n}{f (a + a \text{Csc}[e + f x])} + \frac{d n \text{Cos}[e + f x] (d \text{Csc}[e + f x])^{-1+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \text{Sin}[e + f x]^2\right]}{a f (1-n) \sqrt{\text{Cos}[e + f x]^2}} +$$

$$\frac{\text{Cos}[e + f x] (d \text{Csc}[e + f x])^n \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \text{Sin}[e + f x]^2\right]}{a f \sqrt{\text{Cos}[e + f x]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{(d \text{Csc}[e + f x])^n}{a + a \text{Sin}[e + f x]} dx$$

■ **Problem 818: Unable to integrate problem.**

$$\int \frac{(d \text{Csc}[e + f x])^n}{(a + a \text{Sin}[e + f x])^2} dx$$

Optimal (type 5, 231 leaves, 8 steps):

$$-\frac{2 n \text{Cot}[e + f x] (d \text{Csc}[e + f x])^{2+n}}{3 a^2 d^2 f (1 + \text{Csc}[e + f x])} + \frac{\text{Cot}[e + f x] (d \text{Csc}[e + f x])^{2+n}}{3 d^2 f (a + a \text{Csc}[e + f x])^2} +$$

$$\frac{2 n \text{Cos}[e + f x] (d \text{Csc}[e + f x])^{2+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (-2 - n), -\frac{n}{2}, \text{Sin}[e + f x]^2\right]}{3 a^2 d^2 f \sqrt{\text{Cos}[e + f x]^2}} -$$

$$\frac{(1 + 2 n) \text{Cos}[e + f x] (d \text{Csc}[e + f x])^{1+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (-1 - n), \frac{1-n}{2}, \text{Sin}[e + f x]^2\right]}{3 a^2 d f \sqrt{\text{Cos}[e + f x]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{(d \text{Csc}[e + f x])^n}{(a + a \text{Sin}[e + f x])^2} dx$$

■ **Problem 819: Result more than twice size of optimal antiderivative.**

$$\int (c (d \text{Sin}[e + f x])^p)^n (a + a \text{Sin}[e + f x])^m dx$$

Optimal (type 6, 113 leaves, 5 steps):

$$-\frac{1}{f} 2^{\frac{1}{2}+m} \text{AppellF1}\left[\frac{1}{2}, -n p, \frac{1}{2} - m, \frac{3}{2}, 1 - \text{Sin}[e + f x], \frac{1}{2} (1 - \text{Sin}[e + f x])\right]$$

$$\text{Cos}[e + f x] \text{Sin}[e + f x]^{-n p} (c (d \text{Sin}[e + f x])^p)^n (1 + \text{Sin}[e + f x])^{-\frac{1}{2}-m} (a + a \text{Sin}[e + f x])^m$$

Result (type 6, 2967 leaves):

$$-\left(\left(3 \text{AppellF1}\left[\frac{1}{2}, -n p, 1 + m + n p, \frac{3}{2}, \text{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \text{Cos}[e + f x] \left(\text{Sec}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-m} \text{Sin}[e + f x]^{n p} \right.$$

$$\int (c (d \sin[e + f x])^p)^n (a + a \sin[e + f x])^3 dx$$

Optimal (type 5, 299 leaves, 7 steps):

$$\begin{aligned} & - \frac{a^3 (7 + 2 n p) \cos[e + f x] \sin[e + f x] (c (d \sin[e + f x])^p)^n}{f (2 + n p) (3 + n p)} + \\ & \left(a^3 (5 + 4 n p) \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (1 + n p), \frac{1}{2} (3 + n p), \sin[e + f x]^2 \right] \sin[e + f x] (c (d \sin[e + f x])^p)^n \right) / \\ & \left(f (1 + n p) (2 + n p) \sqrt{\cos[e + f x]^2} \right) + \\ & \left(a^3 (11 + 4 n p) \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (2 + n p), \frac{1}{2} (4 + n p), \sin[e + f x]^2 \right] \sin[e + f x]^2 (c (d \sin[e + f x])^p)^n \right) / \\ & \left(f (2 + n p) (3 + n p) \sqrt{\cos[e + f x]^2} \right) - \frac{\cos[e + f x] \sin[e + f x] (c (d \sin[e + f x])^p)^n (a^3 + a^3 \sin[e + f x])}{f (3 + n p)} \end{aligned}$$

Result (type 9, 26224 leaves): Display of huge result suppressed!

■ **Problem 822: Result unnecessarily involves imaginary or complex numbers.**

$$\int (c (d \sin[e + f x])^p)^n (a + a \sin[e + f x]) dx$$

Optimal (type 5, 163 leaves, 4 steps):

$$\begin{aligned} & \left(a \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (1 + n p), \frac{1}{2} (3 + n p), \sin[e + f x]^2 \right] \sin[e + f x] (c (d \sin[e + f x])^p)^n \right) / \left(f (1 + n p) \sqrt{\cos[e + f x]^2} \right) + \\ & \left(a \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (2 + n p), \frac{1}{2} (4 + n p), \sin[e + f x]^2 \right] \sin[e + f x]^2 (c (d \sin[e + f x])^p)^n \right) / \left(f (2 + n p) \sqrt{\cos[e + f x]^2} \right) \end{aligned}$$

Result (type 5, 307 leaves):

$$\begin{aligned} & \frac{1}{f n p (-1 + n p) (1 + n p) \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^2} \\ & 2^{-1 - n p} a e^{-i (e + f x)} \left(1 - e^{2 i (e + f x)} \right)^{-n p} \left(-i e^{-i (e + f x)} (-1 + e^{2 i (e + f x)}) \right)^{n p} \left(2 i e^{i (e + f x)} (-1 + n^2 p^2) \operatorname{Hypergeometric2F1} \left[-n p, -\frac{n p}{2}, 1 - \frac{n p}{2}, e^{2 i (e + f x)} \right] + \right. \\ & n p (1 - n p) \operatorname{Hypergeometric2F1} \left[-n p, \frac{1}{2} (-1 - n p), \frac{1}{2} (1 - n p), e^{2 i (e + f x)} \right] + \\ & \left. e^{2 i (e + f x)} n p (1 + n p) \operatorname{Hypergeometric2F1} \left[-n p, \frac{1}{2} (1 - n p), \frac{1}{2} (3 - n p), e^{2 i (e + f x)} \right] \right) \sin[e + f x]^{-n p} (c (d \sin[e + f x])^p)^n (1 + \sin[e + f x]) \end{aligned}$$

■ **Problem 823: Unable to integrate problem.**

$$\int \frac{(c (d \sin[e + f x])^p)^n}{a + a \sin[e + f x]} dx$$

Optimal (type 5, 189 leaves, 5 steps) :

$$\frac{\cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{np}{2}, \frac{1}{2}(2 + np), \sin[e + f x]^2\right] (c (d \sin[e + f x])^p)^n}{a f \sqrt{\cos[e + f x]^2}} - \left(np \cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}(1 + np), \frac{1}{2}(3 + np), \sin[e + f x]^2\right] \sin[e + f x] (c (d \sin[e + f x])^p)^n \right) / \left(a f (1 + np) \sqrt{\cos[e + f x]^2} \right) - \frac{\cos[e + f x] (c (d \sin[e + f x])^p)^n}{f (a + a \sin[e + f x])}$$

Result (type 8, 29 leaves) :

$$\int \frac{(c (d \sin[e + f x])^p)^n}{a + a \sin[e + f x]} dx$$

■ **Problem 824: Unable to integrate problem.**

$$\int \frac{(c (d \sin[e + f x])^p)^n}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 5, 288 leaves, 6 steps) :

$$- \left(np (1 - 2np) \cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}(1 + np), \frac{1}{2}(3 + np), \sin[e + f x]^2\right] \sin[e + f x] (c (d \sin[e + f x])^p)^n \right) / \left(3 a^2 f (1 + np) \sqrt{\cos[e + f x]^2} \right) + \left(2 (1 - n^2 p^2) \cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}(2 + np), \frac{1}{2}(4 + np), \sin[e + f x]^2\right] \sin[e + f x]^2 (c (d \sin[e + f x])^p)^n \right) / \left(3 a^2 f (2 + np) \sqrt{\cos[e + f x]^2} \right) + \frac{2 (1 - np) \cos[e + f x] \sin[e + f x] (c (d \sin[e + f x])^p)^n}{3 a^2 f (1 + \sin[e + f x])} + \frac{\cos[e + f x] \sin[e + f x] (c (d \sin[e + f x])^p)^n}{3 f (a + a \sin[e + f x])^2}$$

Result (type 8, 29 leaves) :

$$\int \frac{(c (d \sin[e + f x])^p)^n}{(a + a \sin[e + f x])^2} dx$$

■ **Problem 828: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Csc}[e + f x])^n}{a + b \sin[e + f x]} dx$$

Optimal (type 6, 204 leaves, 7 steps) :

$$\frac{1}{(a^2 - b^2) df} b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{n}{2}, 1, \frac{3}{2}, \cos[e + fx]^2, -\frac{b^2 \cos[e + fx]^2}{a^2 - b^2}\right] \cos[e + fx] (d \operatorname{Csc}[e + fx])^{1+n} \sin[e + fx] (\sin[e + fx]^2)^{n/2} -$$

$$\frac{1}{(a^2 - b^2) df} a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1+n}{2}, 1, \frac{3}{2}, \cos[e + fx]^2, -\frac{b^2 \cos[e + fx]^2}{a^2 - b^2}\right] \cos[e + fx] (d \operatorname{Csc}[e + fx])^{1+n} (\sin[e + fx]^2)^{\frac{1+n}{2}}$$

Result (type 6, 7063 leaves):

$$\left((d \operatorname{Csc}[e + fx])^n \tan[e + fx] \left(\cot[e + fx] \sqrt{1 + \tan[e + fx]^2} \right)^n \right.$$

$$\left(\left(a^3 (-3 + n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \right) / \right.$$

$$\left((-1 + n) \left(-a^2 (-3 + n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \right) + \right.$$

$$\left(a^2 n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1 - \frac{n}{2}, 1, \frac{5-n}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + \right.$$

$$\left. \left. 2 (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3-n}{2}, -\frac{n}{2}, 2, \frac{5-n}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right)$$

$$\left(-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2) \right) \left. + \frac{1}{b (-2 + n) (b^2 \tan[e + fx]^2 - a^2 (1 + \tan[e + fx]^2))} \right.$$

$$\tan[e + fx] \left(- \left(\left(a^2 (a^2 - b^2) (-4 + n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2} (-1 - n), 1, 2 - \frac{n}{2}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \sqrt{1 + \tan[e + fx]^2} \right) / \right.$$

$$\left(-a^2 (-4 + n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2} (-1 - n), 1, 2 - \frac{n}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + \right.$$

$$\left(-2 (a^2 - b^2) \operatorname{AppellF1}\left[2 - \frac{n}{2}, \frac{1}{2} (-1 - n), 2, 3 - \frac{n}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + \right.$$

$$\left. \left. a^2 (1 + n) \operatorname{AppellF1}\left[2 - \frac{n}{2}, \frac{1-n}{2}, 1, 3 - \frac{n}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right) -$$

$$\operatorname{Hypergeometric2F1}\left[\frac{1}{2} - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, -\tan[e + fx]^2\right] (1 + \tan[e + fx]^2)^{-n/2} (-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2)) \left. \right) \left. \right) \left. \right) /$$

$$\left(f (a + b \sin[e + fx]) \left(\sec[e + fx]^2 \left(\cot[e + fx] \sqrt{1 + \tan[e + fx]^2} \right)^n \right.$$

$$\left(\left(a^3 (-3 + n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \right) / \right.$$

$$\begin{aligned}
& \left((-1+n) \left(-a^2 (-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \right. \\
& \quad \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1 - \frac{n}{2}, 1, \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \\
& \quad \left. \left. 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 2, \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \\
& \quad \left. (-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2)) \right) + \frac{1}{b (-2+n) (b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2))} \\
& \tan[e+fx] \left(- \left(\left(a^2 (a^2 - b^2) (-4+n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 1, 2 - \frac{n}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \right. \right. \right. \\
& \quad \left. \left. \sqrt{1 + \tan[e+fx]^2} \right) / \left(-a^2 (-4+n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 1, 2 - \frac{n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \right. \\
& \quad \left. \left. (-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 3 - \frac{n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \right. \\
& \quad \left. \left. a^2 (1+n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1-n}{2}, 1, 3 - \frac{n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) - \\
& \quad \left. \operatorname{Hypergeometric2F1} \left[\frac{1}{2} - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, -\tan[e+fx]^2 \right] (1 + \tan[e+fx]^2)^{-n/2} (-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2)) \right) \right) + \\
& n \tan[e+fx] \left(\cot[e+fx] \sqrt{1 + \tan[e+fx]^2} \right)^{-1+n} \left(\frac{\sec[e+fx]^2}{\sqrt{1 + \tan[e+fx]^2}} - \csc[e+fx]^2 \sqrt{1 + \tan[e+fx]^2} \right) \\
& \left(\left(a^3 (-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) / \right. \\
& \quad \left((-1+n) \left(-a^2 (-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \right. \\
& \quad \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1 - \frac{n}{2}, 1, \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + \right. \\
& \quad \left. \left. 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 2, \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \\
& \quad \left. (-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2)) \right) + \frac{1}{b (-2+n) (b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2))} \\
& \tan[e+fx] \left(- \left(\left(a^2 (a^2 - b^2) (-4+n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 1, 2 - \frac{n}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{1 + \tan[e + f x]^2} \right) / \left(-a^2 (-4 + n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1 - n), 1, 2 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \\
& \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1 - n), 2, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \\
& \left. a^2 (1 + n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1 - n}{2}, 1, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^2 \right) - \\
& \left. \operatorname{Hypergeometric2F1} \left[\frac{1}{2} - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, -\tan[e + f x]^2 \right] (1 + \tan[e + f x]^2)^{-n/2} (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \Bigg) + \\
& \tan[e + f x] \left(\cot[e + f x] \sqrt{1 + \tan[e + f x]^2} \right)^n \left(- \left(\left(a^3 (-3 + n) \operatorname{AppellF1} \left[\frac{1 - n}{2}, -\frac{n}{2}, 1, \frac{3 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right. \right. \right. \\
& \left. \left. \left(2 a^2 \sec[e + f x]^2 \tan[e + f x] - 2 b^2 \sec[e + f x]^2 \tan[e + f x] \right) \right) \right) / \\
& \left((-1 + n) \left(-a^2 (-3 + n) \operatorname{AppellF1} \left[\frac{1 - n}{2}, -\frac{n}{2}, 1, \frac{3 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3 - n}{2}, 1 - \frac{n}{2}, 1, \frac{5 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3 - n}{2}, -\frac{n}{2}, 2, \frac{5 - n}{2}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2))^2 \right) \right) + \left(a^3 (-3 + n) \right. \\
& \left. \left(\frac{1}{3 - n} (1 - n) n \operatorname{AppellF1} \left[1 + \frac{1 - n}{2}, 1 - \frac{n}{2}, 1, 1 + \frac{3 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \frac{1}{3 - n} \right. \right. \\
& \left. \left. 2 \left(-1 + \frac{b^2}{a^2} \right) (1 - n) \operatorname{AppellF1} \left[1 + \frac{1 - n}{2}, -\frac{n}{2}, 2, 1 + \frac{3 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \Bigg) / \\
& \left((-1 + n) \left(-a^2 (-3 + n) \operatorname{AppellF1} \left[\frac{1 - n}{2}, -\frac{n}{2}, 1, \frac{3 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3 - n}{2}, 1 - \frac{n}{2}, 1, \frac{5 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3 - n}{2}, -\frac{n}{2}, \right. \right. \\
& \left. \left. 2, \frac{5 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) - \\
& \left(a^3 (-3 + n) \operatorname{AppellF1} \left[\frac{1 - n}{2}, -\frac{n}{2}, 1, \frac{3 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right. \\
& \left. \left(2 \left(a^2 n \operatorname{AppellF1} \left[\frac{3 - n}{2}, 1 - \frac{n}{2}, 1, \frac{5 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2(-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3-n}{2}, -\frac{n}{2}, 2, \frac{5-n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - a^2(-3+n) \\
& \left(\frac{1}{3-n}(1-n)n \operatorname{AppellF1}\left[1 + \frac{1-n}{2}, 1 - \frac{n}{2}, 1, 1 + \frac{3-n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + \frac{1}{3-n}\right. \\
& \left.2\left(-1 + \frac{b^2}{a^2}\right)(1-n) \operatorname{AppellF1}\left[1 + \frac{1-n}{2}, -\frac{n}{2}, 2, 1 + \frac{3-n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]\right) + \\
& \operatorname{Tan}[e+fx]^2 \left(a^2 n \left(\frac{1}{5-n} 2\left(-1 + \frac{b^2}{a^2}\right)(3-n) \operatorname{AppellF1}\left[1 + \frac{3-n}{2}, 1 - \frac{n}{2}, 2, 1 + \frac{5-n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right]\right.\right. \\
& \left.\left.\operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \frac{1}{5-n} 2(3-n)\left(1 - \frac{n}{2}\right) \operatorname{AppellF1}\left[1 + \frac{3-n}{2}, 2 - \frac{n}{2}, 1, 1 + \frac{5-n}{2}, -\operatorname{Tan}[e+fx]^2, \right.\right.\right. \\
& \left.\left.\left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]\right) + 2(-a^2 + b^2) \left(\frac{1}{5-n}(3-n)n \operatorname{AppellF1}\left[1 + \frac{3-n}{2}, 1 - \frac{n}{2}, \right.\right. \\
& \left.\left.2, 1 + \frac{5-n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + \frac{1}{5-n} 4\left(-1 + \frac{b^2}{a^2}\right)(3-n)\right. \\
& \left.\left.\operatorname{AppellF1}\left[1 + \frac{3-n}{2}, -\frac{n}{2}, 3, 1 + \frac{5-n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]\right)\right)\right) \Big/ \\
& \left((-1+n) \left(-a^2(-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] + \right.\right. \\
& \left.\left(a^2 n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1 - \frac{n}{2}, 1, \frac{5-n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] + 2(-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3-n}{2}, -\frac{n}{2}, 2, \right.\right.\right. \\
& \left.\left.\frac{5-n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right]\right) \operatorname{Tan}[e+fx]^2 \left(-b^2 \operatorname{Tan}[e+fx]^2 + a^2(1 + \operatorname{Tan}[e+fx]^2)\right)\right) - \\
& \frac{1}{b(-2+n)(b^2 \operatorname{Tan}[e+fx]^2 - a^2(1 + \operatorname{Tan}[e+fx]^2))^2} \operatorname{Tan}[e+fx] \left(-2a^2 \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + 2b^2 \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]\right) \\
& \left(-\left(\left(a^2(a^2 - b^2)(-4+n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2}(-1-n), 1, 2 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+fx]^2}{a^2}\right] \sqrt{1 + \operatorname{Tan}[e+fx]^2}\right)\right.\right. \\
& \left.\left(-a^2(-4+n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2}(-1-n), 1, 2 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] + \right.\right. \\
& \left.\left(-2(a^2 - b^2) \operatorname{AppellF1}\left[2 - \frac{n}{2}, \frac{1}{2}(-1-n), 2, 3 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right] + \right.\right. \\
& \left.\left.a^2(1+n) \operatorname{AppellF1}\left[2 - \frac{n}{2}, \frac{1-n}{2}, 1, 3 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+fx]^2\right]\right) \operatorname{Tan}[e+fx]^2\right) \Big) -
\end{aligned}$$

$$\begin{aligned}
& \left. \text{Hypergeometric2F1}\left[\frac{1}{2} - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, -\text{Tan}[e + f x]^2\right] (1 + \text{Tan}[e + f x]^2)^{-n/2} (-b^2 \text{Tan}[e + f x]^2 + a^2 (1 + \text{Tan}[e + f x]^2)) \right) + \\
& \frac{1}{b(-2+n)(b^2 \text{Tan}[e + f x]^2 - a^2 (1 + \text{Tan}[e + f x]^2))} \text{Sec}[e + f x]^2 \\
& \left(- \left(\left(a^2 (a^2 - b^2) (-4+n) \text{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2}(-1-n), 1, 2 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \text{Tan}[e + f x]^2}{a^2}\right] \sqrt{1 + \text{Tan}[e + f x]^2} \right) / \right. \right. \\
& \quad \left(-a^2 (-4+n) \text{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2}(-1-n), 1, 2 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \text{Tan}[e + f x]^2\right] + \right. \\
& \quad \left. \left(-2(a^2 - b^2) \text{AppellF1}\left[2 - \frac{n}{2}, \frac{1}{2}(-1-n), 2, 3 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \text{Tan}[e + f x]^2\right] + \right. \right. \\
& \quad \left. \left. a^2(1+n) \text{AppellF1}\left[2 - \frac{n}{2}, \frac{1-n}{2}, 1, 3 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \text{Tan}[e + f x]^2\right] \right) \text{Tan}[e + f x]^2 \right) \right) - \\
& \left. \text{Hypergeometric2F1}\left[\frac{1}{2} - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, -\text{Tan}[e + f x]^2\right] (1 + \text{Tan}[e + f x]^2)^{-n/2} (-b^2 \text{Tan}[e + f x]^2 + a^2 (1 + \text{Tan}[e + f x]^2)) \right) + \\
& \frac{1}{b(-2+n)(b^2 \text{Tan}[e + f x]^2 - a^2 (1 + \text{Tan}[e + f x]^2))} \text{Tan}[e + f x] \left(-\text{Hypergeometric2F1}\left[\frac{1}{2} - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, -\text{Tan}[e + f x]^2\right] \right. \\
& \quad \left(2a^2 \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - 2b^2 \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) (1 + \text{Tan}[e + f x]^2)^{-n/2} - \\
& \quad \left(a^2 (a^2 - b^2) (-4+n) \text{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2}(-1-n), 1, 2 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \text{Tan}[e + f x]^2}{a^2}\right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) / \\
& \quad \left(\sqrt{1 + \text{Tan}[e + f x]^2} \left(-a^2 (-4+n) \text{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2}(-1-n), 1, 2 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \text{Tan}[e + f x]^2\right] + \right. \right. \\
& \quad \left. \left(-2(a^2 - b^2) \text{AppellF1}\left[2 - \frac{n}{2}, \frac{1}{2}(-1-n), 2, 3 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \text{Tan}[e + f x]^2\right] + \right. \right. \\
& \quad \left. \left. a^2(1+n) \text{AppellF1}\left[2 - \frac{n}{2}, \frac{1-n}{2}, 1, 3 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \text{Tan}[e + f x]^2\right] \right) \text{Tan}[e + f x]^2 \right) \right) - \\
& \quad \left(a^2 (a^2 - b^2) (-4+n) \left(-\frac{1}{2 - \frac{n}{2}}(-1-n) \left(1 - \frac{n}{2}\right) \text{AppellF1}\left[2 - \frac{n}{2}, 1 + \frac{1}{2}(-1-n), 1, 3 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{(-a^2 + b^2) \text{Tan}[e + f x]^2}{a^2}\right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] + \frac{1}{a^2(2 - \frac{n}{2})} 2(-a^2 + b^2) \left(1 - \frac{n}{2}\right) \text{AppellF1}\left[2 - \frac{n}{2}, \frac{1}{2}(-1-n), \right. \right. \right. \\
& \quad \left. \left. \left. 2, 3 - \frac{n}{2}, -\text{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \text{Tan}[e + f x]^2}{a^2}\right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) \sqrt{1 + \text{Tan}[e + f x]^2} \right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-a^2 (-4+n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 1, 2 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \\
& \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 3 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \\
& \left. a^2 (1+n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1-n}{2}, 1, 3 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 + \\
& n \operatorname{Hypergeometric2F1} \left[\frac{1}{2} - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] (1 + \operatorname{Tan}[e+fx]^2)^{-\frac{n}{2}} \\
& (-b^2 \operatorname{Tan}[e+fx]^2 + a^2 (1 + \operatorname{Tan}[e+fx]^2)) - 2 \left(1 - \frac{n}{2} \right) \operatorname{Csc}[e+fx] \operatorname{Sec}[e+fx] (1 + \operatorname{Tan}[e+fx]^2)^{-n/2} \\
& (-b^2 \operatorname{Tan}[e+fx]^2 + a^2 (1 + \operatorname{Tan}[e+fx]^2)) \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2} - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2 \right] + (1 + \operatorname{Tan}[e+fx]^2)^{-\frac{1}{2} + \frac{n}{2}} \right) + \\
& \left(a^2 (a^2 - b^2) (-4+n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 1, 2 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+fx]^2}{a^2} \right] \right. \\
& \left. \sqrt{1 + \operatorname{Tan}[e+fx]^2} \left(2 \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 3 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \right. \right. \\
& \left. \left. a^2 (1+n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1-n}{2}, 1, 3 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \right. \\
& \left. a^2 (-4+n) \left(-\frac{1}{2 - \frac{n}{2}} (-1-n) \left(1 - \frac{n}{2} \right) \operatorname{AppellF1} \left[2 - \frac{n}{2}, 1 + \frac{1}{2} (-1-n), 1, 3 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \right. \right. \\
& \left. \left. \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + \frac{1}{2 - \frac{n}{2}} 2 \left(-1 + \frac{b^2}{a^2} \right) \left(1 - \frac{n}{2} \right) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, \right. \right. \right. \\
& \left. \left. 3 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) + \operatorname{Tan}[e+fx]^2 \\
& \left(-2 (a^2 - b^2) \left(-\frac{1}{3 - \frac{n}{2}} (-1-n) \left(2 - \frac{n}{2} \right) \operatorname{AppellF1} \left[3 - \frac{n}{2}, 1 + \frac{1}{2} (-1-n), 2, 4 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \right. \right. \\
& \left. \left. \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + \frac{1}{3 - \frac{n}{2}} 4 \left(-1 + \frac{b^2}{a^2} \right) \left(2 - \frac{n}{2} \right) \operatorname{AppellF1} \left[3 - \frac{n}{2}, \frac{1}{2} (-1-n), 3, 4 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) + a^2 (1+n) \left(-\frac{1}{3 - \frac{n}{2}} (1-n) \left(2 - \frac{n}{2} \right) \operatorname{AppellF1} \left[3 - \frac{n}{2}, 1 + \frac{1-n}{2}, \right. \right. \\
& \left. \left. 1, 4 - \frac{n}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + \frac{1}{3 - \frac{n}{2}} 2 \left(-1 + \frac{b^2}{a^2} \right) \left(2 - \frac{n}{2} \right) \right.
\end{aligned}$$

$$\begin{aligned} & \left. \left. \left. \left. \left. \text{AppellF1} \left[3 - \frac{n}{2}, \frac{1-n}{2}, 2, 4 - \frac{n}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] \right] \right) \right) \right) \right) \right) / \\ & \left(-a^2 (-4+n) \text{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 1, 2 - \frac{n}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \\ & \left. \left(-2 (a^2 - b^2) \text{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 3 - \frac{n}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \right. \\ & \left. \left. a^2 (1+n) \text{AppellF1} \left[2 - \frac{n}{2}, \frac{1-n}{2}, 1, 3 - \frac{n}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \right) \right) \right) \right) \end{aligned}$$

■ **Problem 829: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Csc}[e+fx])^n}{(a+b \operatorname{Sin}[e+fx])^2} dx$$

Optimal (type 6, 321 leaves, 10 steps):

$$\begin{aligned} & - \frac{1}{(a^2 - b^2)^2 d^2 f} \\ & b^2 \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-1+n), 2, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \operatorname{Csc}[e+fx])^{2+n} \sin[e+fx]^3 (\sin[e+fx]^2)^{\frac{1}{2}(-1+n)} - \\ & \frac{1}{(a^2 - b^2)^2 d^2 f} a^2 \text{AppellF1} \left[\frac{1}{2}, \frac{1+n}{2}, 2, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \operatorname{Csc}[e+fx])^{2+n} \sin[e+fx] (\sin[e+fx]^2)^{\frac{1+n}{2}} + \\ & \frac{1}{(a^2 - b^2)^2 d^2 f} 2 a b \text{AppellF1} \left[\frac{1}{2}, \frac{n}{2}, 2, \frac{3}{2}, \cos[e+fx]^2, -\frac{b^2 \cos[e+fx]^2}{a^2 - b^2} \right] \cos[e+fx] (d \operatorname{Csc}[e+fx])^{2+n} (\sin[e+fx]^2)^{\frac{2+n}{2}} \end{aligned}$$

Result (type 6, 8880 leaves):

$$\begin{aligned} & \left(a^2 (d \operatorname{Csc}[e+fx])^n \text{Tan}[e+fx] \left(\cot[e+fx] \sqrt{1 + \text{Tan}[e+fx]^2} \right)^n \right. \\ & \left. - \left(\left((a^2 + b^2) (-3+n) \text{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\text{Tan}[e+fx]^2, \frac{(-a^2 + b^2) \text{Tan}[e+fx]^2}{a^2} \right] \right) \right) / \right. \\ & \left. \left((-1+n) \left(-a^2 (-3+n) \text{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + \right. \right. \right. \\ & \left. \left. \left(a^2 n \text{AppellF1} \left[\frac{3-n}{2}, 1 - \frac{n}{2}, 1, \frac{5-n}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] + 2 (-a^2 + b^2) \text{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 2, \right. \right. \right. \right. \\ & \left. \left. \left. \frac{5-n}{2}, -\text{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \right) (-b^2 \text{Tan}[e+fx]^2 + a^2 (1 + \text{Tan}[e+fx]^2)) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \frac{1}{(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2))^2} 2 a b \left(- \left(a b (-3 + n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right) / \right. \\
& \left((-1 + n) \left(a^2 (-3 + n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1 - \frac{n}{2}, 2, \frac{5-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left. 4 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 3, \frac{5-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \right) - \\
& \left((a^2 - b^2) (-4 + n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1 - n), 2, 2 - \frac{n}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x] \sqrt{1 + \tan[e + f x]^2} \right) / \\
& \left((-2 + n) \left(a^2 (-4 + n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1 - n), 2, 2 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1 - n), 3, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \\
& \left. \left. a^2 (1 + n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1-n}{2}, 2, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \right) \right) / \\
& \left((-a^2 + b^2) f (a + b \sin[e + f x])^2 \left(\frac{1}{-a^2 + b^2} a^2 \operatorname{Sec}[e + f x]^2 \left(\cot[e + f x] \sqrt{1 + \tan[e + f x]^2} \right)^n \right. \right. \\
& \left. \left. - \left(\left((a^2 + b^2) (-3 + n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right) / \right. \right. \\
& \left. \left((-1 + n) \left(-a^2 (-3 + n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1 - \frac{n}{2}, 1, \frac{5-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 2, \right. \right. \\
& \left. \left. \frac{5-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \right) \right) + \\
& \frac{1}{(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2))^2} 2 a b \left(- \left(a b (-3 + n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \left. \left. \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right) / \left((-1 + n) \left(a^2 (-3 + n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1-\frac{n}{2}, 2, \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \\
& \quad \left. 4(-a^2+b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 3, \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \Bigg) - \\
& \left((a^2-b^2)(-4+n) \operatorname{AppellF1} \left[1-\frac{n}{2}, \frac{1}{2}(-1-n), 2, 2-\frac{n}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Tan}[e+f x] \right. \\
& \quad \left. \sqrt{1+\operatorname{Tan}[e+f x]^2} \right) / \left((-2+n) \left(a^2(-4+n) \operatorname{AppellF1} \left[1-\frac{n}{2}, \frac{1}{2}(-1-n), 2, 2-\frac{n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left(4(a^2-b^2) \operatorname{AppellF1} \left[2-\frac{n}{2}, \frac{1}{2}(-1-n), 3, 3-\frac{n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left. a^2(1+n) \operatorname{AppellF1} \left[2-\frac{n}{2}, \frac{1-n}{2}, 2, 3-\frac{n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) \Bigg) + \\
& \frac{1}{-a^2+b^2} a^2 n \operatorname{Tan}[e+f x] \left(\operatorname{Cot}[e+f x] \sqrt{1+\operatorname{Tan}[e+f x]^2} \right)^{-1+n} \left(\frac{\operatorname{Sec}[e+f x]^2}{\sqrt{1+\operatorname{Tan}[e+f x]^2}} - \operatorname{Csc}[e+f x]^2 \sqrt{1+\operatorname{Tan}[e+f x]^2} \right) \\
& \left(- \left(\left((a^2+b^2)(-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \right) / \right. \right. \\
& \quad \left((-1+n) \left(-a^2(-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1-\frac{n}{2}, 1, \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + 2(-a^2+b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 2, \right. \right. \\
& \quad \left. \left. \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) (-b^2 \operatorname{Tan}[e+f x]^2 + a^2(1+\operatorname{Tan}[e+f x]^2)) \Bigg) \Bigg) + \\
& \frac{1}{(b^2 \operatorname{Tan}[e+f x]^2 - a^2(1+\operatorname{Tan}[e+f x]^2))^2} 2 a b \left(- \left(a b(-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \right) / \left((-1+n) \left(a^2(-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1-\frac{n}{2}, 2, \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left. 4(-a^2+b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 3, \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left((a^2 - b^2) (-4 + n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1 - n), 2, 2 - \frac{n}{2}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Tan}[e + f x] \right. \\
& \left. \sqrt{1 + \operatorname{Tan}[e + f x]^2} \right) / \left((-2 + n) \left(a^2 (-4 + n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1 - n), 2, 2 - \frac{n}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1 - n), 3, 3 - \frac{n}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] - \right. \right. \\
& \left. \left. a^2 (1 + n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1 - n}{2}, 2, 3 - \frac{n}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) \Bigg) + \\
& \frac{1}{-a^2 + b^2} a^2 \operatorname{Tan}[e + f x] \left(\operatorname{Cot}[e + f x] \sqrt{1 + \operatorname{Tan}[e + f x]^2} \right)^n \left(\left((a^2 + b^2) (-3 + n) \operatorname{AppellF1} \left[\frac{1 - n}{2}, -\frac{n}{2}, 1, \frac{3 - n}{2}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] (2 a^2 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - 2 b^2 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]) \right) \right) / \\
& \left((-1 + n) \left(-a^2 (-3 + n) \operatorname{AppellF1} \left[\frac{1 - n}{2}, -\frac{n}{2}, 1, \frac{3 - n}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3 - n}{2}, 1 - \frac{n}{2}, 1, \frac{5 - n}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3 - n}{2}, -\frac{n}{2}, 2, \right. \right. \right. \\
& \left. \left. \left. \frac{5 - n}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) (-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2))^2 \right) - \\
& \left((a^2 + b^2) (-3 + n) \left(\frac{1}{3 - n} (1 - n) n \operatorname{AppellF1} \left[1 + \frac{1 - n}{2}, 1 - \frac{n}{2}, 1, 1 + \frac{3 - n}{2}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \right. \right. \\
& \left. \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \frac{1}{a^2 (3 - n)} 2 (-a^2 + b^2) (1 - n) \operatorname{AppellF1} \left[1 + \frac{1 - n}{2}, -\frac{n}{2}, 2, \right. \right. \\
& \left. \left. \left. 1 + \frac{3 - n}{2}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \right) / \\
& \left((-1 + n) \left(-a^2 (-3 + n) \operatorname{AppellF1} \left[\frac{1 - n}{2}, -\frac{n}{2}, 1, \frac{3 - n}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3 - n}{2}, 1 - \frac{n}{2}, 1, \frac{5 - n}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3 - n}{2}, -\frac{n}{2}, \right. \right. \right. \\
& \left. \left. \left. 2, \frac{5 - n}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) (-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)) \right) - \\
& \frac{1}{(b^2 \operatorname{Tan}[e + f x]^2 - a^2 (1 + \operatorname{Tan}[e + f x]^2))^3} 4 a b (-2 a^2 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + 2 b^2 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x])
\end{aligned}$$

$$\begin{aligned}
& \left(- \left(a b (-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \right) / \right. \\
& \left((-1+n) \left(a^2 (-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1 - \frac{n}{2}, 2, \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left. 4 (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 3, \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) - \\
& \left((a^2-b^2) (-4+n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 2 - \frac{n}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Tan}[e+f x] \right. \\
& \left. \sqrt{1 + \operatorname{Tan}[e+f x]^2} \right) / \left((-2+n) \left(a^2 (-4+n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 2 - \frac{n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left(4 (a^2-b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1-n), 3, 3 - \frac{n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \left. \left. a^2 (1+n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1-n}{2}, 2, 3 - \frac{n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) \right) + \\
& \left((a^2+b^2) (-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \right. \\
& \left(2 \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1 - \frac{n}{2}, 1, \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left. 2 (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 2, \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - a^2 (-3+n) \right. \\
& \left. \left(\frac{1}{3-n} (1-n) n \operatorname{AppellF1} \left[1 + \frac{1-n}{2}, 1 - \frac{n}{2}, 1, 1 + \frac{3-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \frac{1}{3-n} \right. \right. \\
& \left. \left. 2 \left(-1 + \frac{b^2}{a^2} \right) (1-n) \operatorname{AppellF1} \left[1 + \frac{1-n}{2}, -\frac{n}{2}, 2, 1 + \frac{3-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) + \\
& \operatorname{Tan}[e+f x]^2 \left(a^2 n \left(\frac{1}{5-n} 2 \left(-1 + \frac{b^2}{a^2} \right) (3-n) \operatorname{AppellF1} \left[1 + \frac{3-n}{2}, 1 - \frac{n}{2}, 2, 1 + \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right. \right. \\
& \left. \left. \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{5-n} 2 (3-n) \left(1 - \frac{n}{2} \right) \operatorname{AppellF1} \left[1 + \frac{3-n}{2}, 2 - \frac{n}{2}, 1, 1 + \frac{5-n}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + 2 (-a^2+b^2) \left(\frac{1}{5-n} (3-n) n \operatorname{AppellF1} \left[1 + \frac{3-n}{2}, 1 - \frac{n}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2, 1 + \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \left] \sec[e+fx]^2 \tan[e+fx] + \frac{1}{5-n} 4 \left(-1 + \frac{b^2}{a^2}\right) (3-n) \right. \\
& \left. \operatorname{AppellF1}\left[1 + \frac{3-n}{2}, -\frac{n}{2}, 3, 1 + \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right] \right) \Big/ \\
& \left((-1+n) \left(-a^2 (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -\frac{n}{2}, 1, \frac{3-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1 - \frac{n}{2}, 1, \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + 2 (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3-n}{2}, -\frac{n}{2}, 2, \right. \right. \right. \\
& \left. \left. \left. \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \left(-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2) \right) \right) \right) + \\
& \frac{1}{(b^2 \tan[e+fx]^2 - a^2 (1 + \tan[e+fx]^2))^2} 2 a b \left(- \left(a b (-3+n) \left(1 / (3-n) (1-n) n \operatorname{AppellF1}\left[1 + \frac{1-n}{2}, 1 - \frac{n}{2}, 2, \right. \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{3-n}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \sec[e+fx]^2 \tan[e+fx] + 1 / (a^2 (3-n)) 4 (-a^2 + b^2) (1-n) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[1 + \frac{1-n}{2}, -\frac{n}{2}, 3, 1 + \frac{3-n}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) \Big/ \\
& \left((-1+n) \left(a^2 (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] - \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1 - \frac{n}{2}, 2, \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \left. \left. 4 (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3-n}{2}, -\frac{n}{2}, 3, \frac{5-n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \right) - \\
& \left((a^2 - b^2) (-4+n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 2 - \frac{n}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \sec[e+fx]^2 \tan[e+fx]^2 \right) \Big/ \\
& \left((-2+n) \sqrt{1 + \tan[e+fx]^2} \left(a^2 (-4+n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 2 - \frac{n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \left. \left(4 (a^2 - b^2) \operatorname{AppellF1}\left[2 - \frac{n}{2}, \frac{1}{2} (-1-n), 3, 3 - \frac{n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] - \right. \right. \\
& \left. \left. a^2 (1+n) \operatorname{AppellF1}\left[2 - \frac{n}{2}, \frac{1-n}{2}, 2, 3 - \frac{n}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \right) - \\
& \left((a^2 - b^2) (-4+n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 2 - \frac{n}{2}, -\tan[e+fx]^2, \frac{(-a^2 + b^2) \tan[e+fx]^2}{a^2} \right] \sec[e+fx]^2 \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{1 + \tan[e + f x]^2} \right) / \left((-2 + n) \left(a^2 (-4 + n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1 - n), 2, 2 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1 - n), 3, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \\
& \left. \left. a^2 (1 + n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1 - n}{2}, 2, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) - \\
& \left((a^2 - b^2) (-4 + n) \tan[e + f x] \left(-\frac{1}{2 - \frac{n}{2}} (-1 - n) \left(1 - \frac{n}{2} \right) \operatorname{AppellF1} \left[2 - \frac{n}{2}, 1 + \frac{1}{2} (-1 - n), 2, 3 - \frac{n}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \left. \left. \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] + \frac{1}{a^2 (2 - \frac{n}{2})} 4 (-a^2 + b^2) \left(1 - \frac{n}{2} \right) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1 - n), \right. \right. \\
& \left. \left. 3, 3 - \frac{n}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \sqrt{1 + \tan[e + f x]^2} \Big/ \\
& \left((-2 + n) \left(a^2 (-4 + n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1 - n), 2, 2 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1 - n), 3, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \\
& \left. \left. a^2 (1 + n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1 - n}{2}, 2, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) + \\
& \left(a b (-3 + n) \operatorname{AppellF1} \left[\frac{1 - n}{2}, -\frac{n}{2}, 2, \frac{3 - n}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right. \\
& \left. -2 \left(a^2 n \operatorname{AppellF1} \left[\frac{3 - n}{2}, 1 - \frac{n}{2}, 2, \frac{5 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 4 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3 - n}{2}, -\frac{n}{2}, 3, \right. \right. \right. \\
& \left. \left. \frac{5 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] + a^2 (-3 + n) \left(\frac{1}{3 - n} (1 - n) n \operatorname{AppellF1} \left[\right. \right. \right. \\
& \left. \left. \left. 1 + \frac{1 - n}{2}, 1 - \frac{n}{2}, 2, 1 + \frac{3 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] + \frac{1}{3 - n} 4 \left(-1 + \frac{b^2}{a^2} \right) \right. \right. \right. \\
& \left. \left. (1 - n) \operatorname{AppellF1} \left[1 + \frac{1 - n}{2}, -\frac{n}{2}, 3, 1 + \frac{3 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) - \right. \\
& \left. \tan[e + f x]^2 \left(a^2 n \left(\frac{1}{5 - n} 4 \left(-1 + \frac{b^2}{a^2} \right) (3 - n) \operatorname{AppellF1} \left[1 + \frac{3 - n}{2}, 1 - \frac{n}{2}, 3, 1 + \frac{5 - n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right. \right. \right. \\
& \left. \left. \left. \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \frac{1}{5 - n} 2 (3 - n) \left(1 - \frac{n}{2} \right) \operatorname{AppellF1} \left[1 + \frac{3 - n}{2}, 2 - \frac{n}{2}, 2, 1 + \frac{5 - n}{2}, -\tan[e + f x]^2, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \left[\sec[e + f x]^2 \tan[e + f x] \right] + 4 (-a^2 + b^2) \left(\frac{1}{5-n} (3-n) n \operatorname{AppellF1} \left[1 + \frac{3-n}{2}, 1 - \frac{n}{2}, \right. \right. \\
& \left. \left. 3, 1 + \frac{5-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \frac{1}{5-n} 6 \left(-1 + \frac{b^2}{a^2} \right) (3-n) \right. \\
& \left. \left. \operatorname{AppellF1} \left[1 + \frac{3-n}{2}, -\frac{n}{2}, 4, 1 + \frac{5-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \Bigg) \Bigg) / \\
& \left((-1+n) \left(a^2 (-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, -\frac{n}{2}, 2, \frac{3-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \\
& \left. \left(a^2 n \operatorname{AppellF1} \left[\frac{3-n}{2}, 1 - \frac{n}{2}, 2, \frac{5-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left. 4 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3-n}{2}, -\frac{n}{2}, 3, \frac{5-n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \Bigg) + \\
& \left((a^2 - b^2) (-4+n) \operatorname{AppellF1} \left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 2 - \frac{n}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x] \right. \\
& \left. \sqrt{1 + \tan[e + f x]^2} \left(2 \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1}{2} (-1-n), 3, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \right. \\
& \left. \left. a^2 (1+n) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \frac{1-n}{2}, 2, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \sec[e + f x]^2 \tan[e + f x] + \right. \\
& \left. a^2 (-4+n) \left(-\frac{1}{2 - \frac{n}{2}} (-1-n) \left(1 - \frac{n}{2} \right) \operatorname{AppellF1} \left[2 - \frac{n}{2}, 1 + \frac{1}{2} (-1-n), 2, 3 - \frac{n}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \frac{1}{2 - \frac{n}{2}} 4 \left(-1 + \frac{b^2}{a^2} \right) \left(1 - \frac{n}{2} \right) \operatorname{AppellF1} \left[2 - \frac{n}{2}, \right. \right. \\
& \left. \left. \frac{1}{2} (-1-n), 3, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) + \tan[e + f x]^2 \right. \\
& \left. \left(4 (a^2 - b^2) \left(-\frac{1}{3 - \frac{n}{2}} (-1-n) \left(2 - \frac{n}{2} \right) \operatorname{AppellF1} \left[3 - \frac{n}{2}, 1 + \frac{1}{2} (-1-n), 3, 4 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right. \right. \right. \\
& \left. \left. \sec[e + f x]^2 \tan[e + f x] + \frac{1}{3 - \frac{n}{2}} 6 \left(-1 + \frac{b^2}{a^2} \right) \left(2 - \frac{n}{2} \right) \operatorname{AppellF1} \left[3 - \frac{n}{2}, \frac{1}{2} (-1-n), 4, 4 - \frac{n}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) - a^2 (1+n) \left(-\frac{1}{3 - \frac{n}{2}} (1-n) \left(2 - \frac{n}{2} \right) \operatorname{AppellF1} \left[3 - \frac{n}{2}, 1 + \frac{1-n}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2, 4 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2 \left[\sec[e + f x]^2 \tan[e + f x] + \frac{1}{3 - \frac{n}{2}} 4 \left(-1 + \frac{b^2}{a^2}\right) \left(2 - \frac{n}{2}\right) \right. \\
& \left. \text{AppellF1}\left[3 - \frac{n}{2}, \frac{1-n}{2}, 3, 4 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right] \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left((-2+n) \left(a^2 (-4+n) \text{AppellF1}\left[1 - \frac{n}{2}, \frac{1}{2} (-1-n), 2, 2 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(4 (a^2 - b^2) \text{AppellF1}\left[2 - \frac{n}{2}, \frac{1}{2} (-1-n), 3, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2 \right] - \right. \right. \\
& \left. \left. a^2 (1+n) \text{AppellF1}\left[2 - \frac{n}{2}, \frac{1-n}{2}, 2, 3 - \frac{n}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2 \right] \tan[e + f x]^2 \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 830: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Csc}[e + f x])^n}{(a + b \operatorname{Sin}[e + f x])^3} dx$$

Optimal (type 6, 432 leaves, 12 steps):

$$\begin{aligned}
& - \frac{1}{(a^2 - b^2)^3 d^3 f} 3 a b^2 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1+n), 3, \frac{3}{2}, \cos[e + f x]^2, -\frac{b^2 \cos[e + f x]^2}{a^2 - b^2}\right] \\
& \cos[e + f x] (d \operatorname{Csc}[e + f x])^{3+n} \operatorname{Sin}[e + f x]^4 (\operatorname{Sin}[e + f x]^2)^{\frac{1}{2}(-1+n)} + \frac{1}{(a^2 - b^2)^3 d^3 f} \\
& b^3 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-2+n), 3, \frac{3}{2}, \cos[e + f x]^2, -\frac{b^2 \cos[e + f x]^2}{a^2 - b^2}\right] \cos[e + f x] (d \operatorname{Csc}[e + f x])^{3+n} \operatorname{Sin}[e + f x]^3 (\operatorname{Sin}[e + f x]^2)^{n/2} + \\
& \frac{1}{(a^2 - b^2)^3 d^3 f} 3 a^2 b \text{AppellF1}\left[\frac{1}{2}, \frac{n}{2}, 3, \frac{3}{2}, \cos[e + f x]^2, -\frac{b^2 \cos[e + f x]^2}{a^2 - b^2}\right] \cos[e + f x] (d \operatorname{Csc}[e + f x])^{3+n} \operatorname{Sin}[e + f x]^3 (\operatorname{Sin}[e + f x]^2)^{n/2} - \\
& \frac{1}{(a^2 - b^2)^3 d^3 f} a^3 \text{AppellF1}\left[\frac{1}{2}, \frac{1+n}{2}, 3, \frac{3}{2}, \cos[e + f x]^2, -\frac{b^2 \cos[e + f x]^2}{a^2 - b^2}\right] \cos[e + f x] (d \operatorname{Csc}[e + f x])^{3+n} (\operatorname{Sin}[e + f x]^2)^{\frac{3+n}{2}}
\end{aligned}$$

Result (type 6, 21714 leaves): Display of huge result suppressed!

■ **Problem 835: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c (d \operatorname{Sin}[e + f x])^p)^n}{a + b \operatorname{Sin}[e + f x]} dx$$

Optimal (type 6, 204 leaves, 6 steps):

$$\frac{1}{(a^2 - b^2) f} b \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, \cos[e + fx]^2, -\frac{b^2 \cos[e + fx]^2}{a^2 - b^2}\right] \cos[e + fx] (\sin[e + fx]^2)^{-\frac{np}{2}} (c (d \sin[e + fx])^p)^n - \frac{1}{(a^2 - b^2) f}$$

$$a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (1 - np), 1, \frac{3}{2}, \cos[e + fx]^2, -\frac{b^2 \cos[e + fx]^2}{a^2 - b^2}\right] \cot[e + fx] (\sin[e + fx]^2)^{\frac{1}{2} (1 - np)} (c (d \sin[e + fx])^p)^n$$

Result (type 6, 7184 leaves):

$$\left((c (d \sin[e + fx])^p)^n \tan[e + fx] \left(\frac{\tan[e + fx]}{\sqrt{1 + \tan[e + fx]^2}} \right)^{np} \right.$$

$$\left. - \frac{\operatorname{Hypergeometric2F1}\left[\frac{1}{2} + \frac{np}{2}, 1 + \frac{np}{2}, 2 + \frac{np}{2}, -\tan[e + fx]^2\right] \tan[e + fx] (1 + \tan[e + fx]^2)^{\frac{np}{2}}}{2b + bnp} - \right.$$

$$\left(a^2 (a^2 - b^2) (4 + np) \operatorname{AppellF1}\left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx] \right.$$

$$\left. \sqrt{1 + \tan[e + fx]^2} \right) / \left(b (2 + np) \left(a^2 (4 + np) \operatorname{AppellF1}\left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + \right.$$

$$\left(-2 (a^2 - b^2) \operatorname{AppellF1}\left[2 + \frac{np}{2}, \frac{1}{2} (-1 + np), 2, 3 + \frac{np}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 (1 - np) \operatorname{AppellF1}\left[2 + \frac{np}{2}, \frac{1}{2} \right.$$

$$\left. (1 + np), 1, 3 + \frac{np}{2}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \tan[e + fx]^2 (b^2 \tan[e + fx]^2 - a^2 (1 + \tan[e + fx]^2)) \right) -$$

$$\left(a^3 (3 + np) \operatorname{AppellF1}\left[\frac{1}{2} (1 + np), \frac{np}{2}, 1, \frac{1}{2} (3 + np), -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \right) /$$

$$\left((1 + np) \left(-a^2 (3 + np) \operatorname{AppellF1}\left[\frac{1}{2} (1 + np), \frac{np}{2}, 1, \frac{1}{2} (3 + np), -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + \right.$$

$$\left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2} (3 + np), \frac{np}{2}, 2, \frac{1}{2} (5 + np), -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + \right.$$

$$\left. a^2 np \operatorname{AppellF1}\left[\frac{1}{2} (3 + np), 1 + \frac{np}{2}, 1, \frac{1}{2} (5 + np), -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \right) \tan[e + fx]^2 \right)$$

$$\left. (-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2)) \right) / \left(f (a + b \sin[e + fx]) \left(\sec[e + fx]^2 \left(\frac{\tan[e + fx]}{\sqrt{1 + \tan[e + fx]^2}} \right)^{np} \right. \right.$$

$$\left. - \frac{\operatorname{Hypergeometric2F1}\left[\frac{1}{2} + \frac{np}{2}, 1 + \frac{np}{2}, 2 + \frac{np}{2}, -\tan[e + fx]^2\right] \tan[e + fx] (1 + \tan[e + fx]^2)^{\frac{np}{2}}}{2b + bnp} - \right)$$

$$\begin{aligned}
& \left(a^2 (a^2 - b^2) (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Tan}[e + f x] \right. \\
& \quad \left. \sqrt{1 + \operatorname{Tan}[e + f x]^2} \right) / \left(b (2 + np) \left(a^2 (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (-1 + np), 2, 3 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + a^2 (1 - np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{1}{2} (1 + np), 1, 3 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \left(b^2 \operatorname{Tan}[e + f x]^2 - a^2 (1 + \operatorname{Tan}[e + f x]^2) \right) \left. \right) - \\
& \left(a^3 (3 + np) \operatorname{AppellF1} \left[\frac{1}{2} (1 + np), \frac{np}{2}, 1, \frac{1}{2} (3 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) / \\
& \left((1 + np) \left(-a^2 (3 + np) \operatorname{AppellF1} \left[\frac{1}{2} (1 + np), \frac{np}{2}, 1, \frac{1}{2} (3 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3 + np), \frac{np}{2}, 2, \frac{1}{2} (5 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + a^2 np \operatorname{AppellF1} \left[\frac{1}{2} (3 + np), 1 + \right. \right. \right. \\
& \quad \left. \left. \frac{np}{2}, 1, \frac{1}{2} (5 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \left(-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2) \right) \left. \right) \right) + \\
& np \operatorname{Tan}[e + f x] \left(\frac{\operatorname{Tan}[e + f x]}{\sqrt{1 + \operatorname{Tan}[e + f x]^2}} \right)^{-1+np} \left(-\frac{\operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]^2}{(1 + \operatorname{Tan}[e + f x]^2)^{3/2}} + \frac{\operatorname{Sec}[e + f x]^2}{\sqrt{1 + \operatorname{Tan}[e + f x]^2}} \right) \\
& \left(-\frac{\operatorname{Hypergeometric2F1} \left[\frac{1}{2} + \frac{np}{2}, 1 + \frac{np}{2}, 2 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2 \right] \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{\frac{np}{2}}}{2b + bnp} - \right. \\
& \left(a^2 (a^2 - b^2) (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Tan}[e + f x] \right. \\
& \quad \left. \sqrt{1 + \operatorname{Tan}[e + f x]^2} \right) / \left(b (2 + np) \left(a^2 (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (-1 + np), 2, 3 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + a^2 (1 - np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{1}{2} (1 + np), 1, 3 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \left(b^2 \operatorname{Tan}[e + f x]^2 - a^2 (1 + \operatorname{Tan}[e + f x]^2) \right) \left. \right) - \\
& \left(a^3 (3 + np) \operatorname{AppellF1} \left[\frac{1}{2} (1 + np), \frac{np}{2}, 1, \frac{1}{2} (3 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left((1 + np) \left(-a^2 (3 + np) \operatorname{AppellF1} \left[\frac{1}{2} (1 + np), \frac{np}{2}, 1, \frac{1}{2} (3 + np), -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3 + np), \frac{np}{2}, 2, \frac{1}{2} (5 + np), -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] + a^2 np \operatorname{AppellF1} \left[\frac{1}{2} (3 + np), 1 + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{np}{2}, 1, \frac{1}{2} (5 + np), -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \operatorname{Tan}[e + fx]^2 \right) \left(-b^2 \operatorname{Tan}[e + fx]^2 + a^2 (1 + \operatorname{Tan}[e + fx]^2) \right) \right) \right) + \\
& \operatorname{Tan}[e + fx] \left(\frac{\operatorname{Tan}[e + fx]}{\sqrt{1 + \operatorname{Tan}[e + fx]^2}} \right)^{np} \left(-\frac{\operatorname{Hypergeometric2F1} \left[\frac{1}{2} + \frac{np}{2}, 1 + \frac{np}{2}, 2 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2 \right] \operatorname{Sec}[e + fx]^2 (1 + \operatorname{Tan}[e + fx]^2)^{\frac{np}{2}}}{2b + bnp} - \right. \\
& \quad \left. \frac{1}{2b + bnp} np \operatorname{Hypergeometric2F1} \left[\frac{1}{2} + \frac{np}{2}, 1 + \frac{np}{2}, 2 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2 \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx]^2 (1 + \operatorname{Tan}[e + fx]^2)^{-1 + \frac{np}{2}} + \right. \\
& \quad \left. \left(a^2 (a^2 - b^2) (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + fx]^2}{a^2} \right] \right) \right. \\
& \quad \left. \operatorname{Tan}[e + fx] \left(-2 a^2 \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] + 2 b^2 \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] \right) \sqrt{1 + \operatorname{Tan}[e + fx]^2} \right) / \\
& \left(b (2 + np) \left(a^2 (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (-1 + np), 2, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] + a^2 (1 - np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{2} (1 + np), 1, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \right) \operatorname{Tan}[e + fx]^2 \right) \left(b^2 \operatorname{Tan}[e + fx]^2 - a^2 (1 + \operatorname{Tan}[e + fx]^2) \right)^2 \right) - \\
& \left(a^2 (a^2 - b^2) (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + fx]^2}{a^2} \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx]^2 \right) / \\
& \left(b (2 + np) \sqrt{1 + \operatorname{Tan}[e + fx]^2} \left(a^2 (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] + \right. \right. \\
& \quad \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (-1 + np), 2, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] + a^2 (1 - np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{2} (1 + np), 1, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \right) \operatorname{Tan}[e + fx]^2 \right) \left(b^2 \operatorname{Tan}[e + fx]^2 - a^2 (1 + \operatorname{Tan}[e + fx]^2) \right) \right) - \\
& \left(a^2 (a^2 - b^2) (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 1, 2 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + fx]^2}{a^2} \right] \operatorname{Sec}[e + fx]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 + \tan[e + f x]^2} \Bigg/ \left(b (2 + n p) \left(a^2 (4 + n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 1, 2 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 2, 3 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 (1 - n p) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} (1 + n p), 1, 3 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2) \right) \right) - \\
& \left(a^2 (a^2 - b^2) (4 + n p) \tan[e + f x] \left(\frac{1}{a^2 (2 + \frac{n p}{2})} 2 (-a^2 + b^2) \left(1 + \frac{n p}{2} \right) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 2, 3 + \frac{n p}{2}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \frac{1}{2 + \frac{n p}{2}} \left(1 + \frac{n p}{2} \right) (-1 + n p) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, 1 + \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} (-1 + n p), 1, 3 + \frac{n p}{2}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \sqrt{1 + \tan[e + f x]^2} \right) \Bigg/ \\
& \left(b (2 + n p) \left(a^2 (4 + n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 1, 2 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 2, 3 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 (1 - n p) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} (1 + n p), 1, 3 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2) \right) \right) + \\
& \left(a^3 (3 + n p) \operatorname{AppellF1} \left[\frac{1}{2} (1 + n p), \frac{n p}{2}, 1, \frac{1}{2} (3 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right. \\
& \left. \left(2 a^2 \operatorname{Sec}[e + f x]^2 \tan[e + f x] - 2 b^2 \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \right) \Bigg/ \\
& \left((1 + n p) \left(-a^2 (3 + n p) \operatorname{AppellF1} \left[\frac{1}{2} (1 + n p), \frac{n p}{2}, 1, \frac{1}{2} (3 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3 + n p), \frac{n p}{2}, 2, \frac{1}{2} (5 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3 + n p), 1 + \right. \right. \right. \\
& \left. \left. \left. \frac{n p}{2}, 1, \frac{1}{2} (5 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right)^2 \right) - \\
& \left(a^3 (3 + n p) \left(\frac{1}{3 + n p} 2 \left(-1 + \frac{b^2}{a^2} \right) (1 + n p) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1 + n p), \frac{n p}{2}, 2, 1 + \frac{1}{2} (3 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \right. \right. \right. \\
& \left. \left. \left. \tan[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \frac{1}{3 + n p} n p (1 + n p) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1 + n p), 1 + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{np}{2}, 1, 1 + \frac{1}{2}(3+np), -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \Big) \Big) / \\
& \left((1+np) \left(-a^2(3+np) \operatorname{AppellF1} \left[\frac{1}{2}(1+np), \frac{np}{2}, 1, \frac{1}{2}(3+np), -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \quad \left. \left(2(a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}(3+np), \frac{np}{2}, 2, \frac{1}{2}(5+np), -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \\
& \quad \left. \left. a^2 np \operatorname{AppellF1} \left[\frac{1}{2}(3+np), 1 + \frac{np}{2}, 1, \frac{1}{2}(5+np), -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \\
& \quad \left. (-b^2 \tan[e+fx]^2 + a^2(1 + \tan[e+fx]^2)) \right) - \frac{1}{2b+bnp} 2 \left(1 + \frac{np}{2} \right) \operatorname{Sec}[e+fx]^2 (1 + \tan[e+fx]^2)^{\frac{np}{2}} \\
& \quad \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2} + \frac{np}{2}, 1 + \frac{np}{2}, 2 + \frac{np}{2}, -\tan[e+fx]^2 \right] + (1 + \tan[e+fx]^2)^{-\frac{1}{2} - \frac{np}{2}} \right) + \\
& \quad \left(a^2(a^2-b^2)(4+np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2}(-1+np), 1, 2 + \frac{np}{2}, -\tan[e+fx]^2, \frac{(-a^2+b^2)\tan[e+fx]^2}{a^2} \right] \tan[e+fx] \right. \\
& \quad \left. \sqrt{1 + \tan[e+fx]^2} \left(2 \left(-2(a^2-b^2) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2}(-1+np), 2, 3 + \frac{np}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. a^2(1-np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2}(1+np), 1, 3 + \frac{np}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right) \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \right. \\
& \quad \left. a^2(4+np) \left(\frac{1}{2 + \frac{np}{2}} 2 \left(-1 + \frac{b^2}{a^2}\right) \left(1 + \frac{np}{2} \right) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2}(-1+np), 2, 3 + \frac{np}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{1}{2 + \frac{np}{2}} \left(1 + \frac{np}{2} \right) (-1+np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, 1 + \frac{1}{2}(-1+np), 1, \right. \right. \right. \\
& \quad \left. \left. \left. 3 + \frac{np}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \tan[e+fx]^2 \right. \\
& \quad \left. \left(-2(a^2-b^2) \left(\frac{1}{3 + \frac{np}{2}} 4 \left(-1 + \frac{b^2}{a^2}\right) \left(2 + \frac{np}{2} \right) \operatorname{AppellF1} \left[3 + \frac{np}{2}, \frac{1}{2}(-1+np), 3, 4 + \frac{np}{2}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \right. \right. \right. \\
& \quad \left. \left. \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \frac{1}{3 + \frac{np}{2}} \left(2 + \frac{np}{2} \right) (-1+np) \operatorname{AppellF1} \left[3 + \frac{np}{2}, 1 + \frac{1}{2}(-1+np), 2, 4 + \frac{np}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + a^2(1-np) \left(\frac{1}{3 + \frac{np}{2}} 2 \left(-1 + \frac{b^2}{a^2}\right) \left(2 + \frac{np}{2} \right) \operatorname{AppellF1} \left[3 + \frac{np}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} (1 + n p), 2, 4 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2 \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \frac{1}{3 + \frac{n p}{2}} \left(2 + \frac{n p}{2}\right) (1 + n p) \\
& \operatorname{AppellF1}\left[3 + \frac{n p}{2}, 1 + \frac{1}{2} (1 + n p), 1, 4 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2 \operatorname{Sec}[e + f x]^2 \tan[e + f x]\right] \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left(b (2 + n p) \left(a^2 (4 + n p) \operatorname{AppellF1}\left[1 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 1, 2 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + \right. \right. \\
& \left. \left(-2 (a^2 - b^2) \operatorname{AppellF1}\left[2 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 2, 3 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + \right. \right. \\
& \left. \left. a^2 (1 - n p) \operatorname{AppellF1}\left[2 + \frac{n p}{2}, \frac{1}{2} (1 + n p), 1, 3 + \frac{n p}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \right) \tan[e + f x]^2 \right)^2 \\
& \left. (b^2 \tan[e + f x]^2 - a^2 (1 + \tan[e + f x]^2)) \right) + \left(a^3 (3 + n p) \operatorname{AppellF1}\left[\frac{1}{2} (1 + n p), \frac{n p}{2}, 1, \frac{1}{2} (3 + n p), -\tan[e + f x]^2, \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \left(2 \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2} (3 + n p), \frac{n p}{2}, 2, \frac{1}{2} (5 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + \right. \right. \right. \\
& \left. \left. a^2 n p \operatorname{AppellF1}\left[\frac{1}{2} (3 + n p), 1 + \frac{n p}{2}, 1, \frac{1}{2} (5 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \right) \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \\
& \left. a^2 (3 + n p) \left(\frac{1}{3 + n p} 2 \left(-1 + \frac{b^2}{a^2}\right) (1 + n p) \operatorname{AppellF1}\left[1 + \frac{1}{2} (1 + n p), \frac{n p}{2}, 2, 1 + \frac{1}{2} (3 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \right. \right. \\
& \left. \left. \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \frac{1}{3 + n p} n p (1 + n p) \operatorname{AppellF1}\left[1 + \frac{1}{2} (1 + n p), 1 + \frac{n p}{2}, 1, 1 + \frac{1}{2} (3 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \right. \right. \\
& \left. \left. \tan[e + f x]^2 \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) + \tan[e + f x]^2 \left(2 (a^2 - b^2) \left(\frac{1}{5 + n p} 4 \left(-1 + \frac{b^2}{a^2}\right) (3 + n p) \operatorname{AppellF1}\left[1 + \frac{1}{2} (3 + n p), \right. \right. \right. \\
& \left. \left. \frac{n p}{2}, 3, 1 + \frac{1}{2} (5 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \frac{1}{5 + n p} n p (3 + n p) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[1 + \frac{1}{2} (3 + n p), 1 + \frac{n p}{2}, 2, 1 + \frac{1}{2} (5 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) + \right. \\
& \left. a^2 n p \left(\frac{1}{5 + n p} 2 \left(-1 + \frac{b^2}{a^2}\right) (3 + n p) \operatorname{AppellF1}\left[1 + \frac{1}{2} (3 + n p), 1 + \frac{n p}{2}, 2, 1 + \frac{1}{2} (5 + n p), -\tan[e + f x]^2, \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \frac{1}{5 + n p} 2 \left(1 + \frac{n p}{2}\right) (3 + n p) \operatorname{AppellF1}\left[1 + \frac{1}{2} (3 + n p), \right. \right. \\
& \left. \left. 2 + \frac{n p}{2}, 1, 1 + \frac{1}{2} (5 + n p), -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \Bigg) \Bigg) \Bigg) \Bigg) /
\end{aligned}$$

$$\left((1+n p) \left(-a^2 (3+n p) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 1, \frac{1}{2} (3+n p), -\tan[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + \left(2(a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), \frac{n p}{2}, 2, \frac{1}{2} (5+n p), -\tan[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), 1+\frac{n p}{2}, 1, \frac{1}{2} (5+n p), -\tan[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right) \tan[e+f x]^2 \right) \left(-b^2 \tan[e+f x]^2 + a^2 (1+\tan[e+f x]^2) \right) \right) \right)$$

■ **Problem 836: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c(d \sin[e+f x])^p)^n}{(a+b \sin[e+f x])^2} dx$$

Optimal (type 6, 322 leaves, 11 steps):

$$\frac{1}{(a^2-b^2)^2 f} 2 a b \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, \cos[e+f x]^2, -\frac{b^2 \cos[e+f x]^2}{a^2-b^2} \right] \cos[e+f x] (\sin[e+f x]^2)^{-\frac{n p}{2}} (c(d \sin[e+f x])^p)^n - \frac{1}{(a^2-b^2)^2 f} b^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-1-n p), 2, \frac{3}{2}, \cos[e+f x]^2, -\frac{b^2 \cos[e+f x]^2}{a^2-b^2} \right] \cos[e+f x] \sin[e+f x] (\sin[e+f x]^2)^{\frac{1}{2}(-1-n p)} (c(d \sin[e+f x])^p)^n - \frac{1}{(a^2-b^2)^2 f} a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (1-n p), 2, \frac{3}{2}, \cos[e+f x]^2, -\frac{b^2 \cos[e+f x]^2}{a^2-b^2} \right] \cot[e+f x] (\sin[e+f x]^2)^{\frac{1}{2}(1-n p)} (c(d \sin[e+f x])^p)^n$$

Result (type 6, 9486 leaves):

$$\left(a^2 (c(d \sin[e+f x])^p)^n \tan[e+f x] \left(\frac{\tan[e+f x]}{\sqrt{1+\tan[e+f x]^2}} \right)^{n p} \right. \\ \left. \left(\left(2 a b (a^2-b^2) (4+n p) \operatorname{AppellF1} \left[1+\frac{n p}{2}, \frac{1}{2} (-1+n p), 2, 2+\frac{n p}{2}, -\tan[e+f x]^2, \frac{(-a^2+b^2) \tan[e+f x]^2}{a^2} \right] \tan[e+f x] \sqrt{1+\tan[e+f x]^2} \right) / \left((2+n p) \left(a^2 (4+n p) \operatorname{AppellF1} \left[1+\frac{n p}{2}, \frac{1}{2} (-1+n p), 2, 2+\frac{n p}{2}, -\tan[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + \left(-4(a^2-b^2) \operatorname{AppellF1} \left[2+\frac{n p}{2}, \frac{1}{2} (-1+n p), 3, 3+\frac{n p}{2}, -\tan[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] + a^2 (1-n p) \operatorname{AppellF1} \left[2+\frac{n p}{2}, \frac{1}{2} (1+n p), 2, 3+\frac{n p}{2}, -\tan[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] \right) \tan[e+f x]^2 \right) \right) + \frac{1}{1+n p} (3+n p) \left(\left(2 a^2 b^2 \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 2, \frac{1}{2} (3+n p), -\tan[e+f x]^2, \frac{(-a^2+b^2) \tan[e+f x]^2}{a^2} \right] \right) / \left(a^2 (3+n p) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 2, \frac{1}{2} (3+n p), -\tan[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \tan[e+f x]^2 \right] - \right. \right.$$

$$\begin{aligned}
& \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3 + np), \frac{np}{2}, 3, \frac{1}{2} (5 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + a^2 np \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{1}{2} (3 + np), 1 + \frac{np}{2}, 2, \frac{1}{2} (5 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 + \\
& \left((a^2 + b^2) \operatorname{AppellF1} \left[\frac{1}{2} (1 + np), \frac{np}{2}, 1, \frac{1}{2} (3 + np), -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] (-b^2 \operatorname{Tan}[e + f x]^2 + \right. \\
& \quad \left. a^2 (1 + \operatorname{Tan}[e + f x]^2)) \right) / \left(-a^2 (3 + np) \operatorname{AppellF1} \left[\frac{1}{2} (1 + np), \frac{np}{2}, 1, \frac{1}{2} (3 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \\
& \quad \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3 + np), \frac{np}{2}, 2, \frac{1}{2} (5 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + a^2 np \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{1}{2} (3 + np), 1 + \frac{np}{2}, 1, \frac{1}{2} (5 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) / \\
& \left((-a^2 + b^2) f (a + b \operatorname{Sin}[e + f x])^2 (b^2 \operatorname{Tan}[e + f x]^2 - a^2 (1 + \operatorname{Tan}[e + f x]^2))^2 \left(-\frac{1}{(-a^2 + b^2) (b^2 \operatorname{Tan}[e + f x]^2 - a^2 (1 + \operatorname{Tan}[e + f x]^2))^3} \right. \right. \\
& \quad \left. \left. 2 a^2 \operatorname{Tan}[e + f x] (-2 a^2 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + 2 b^2 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]) \left(\frac{\operatorname{Tan}[e + f x]}{\sqrt{1 + \operatorname{Tan}[e + f x]^2}} \right)^{np} \right) \right. \\
& \quad \left(\left(2 a b (a^2 - b^2) (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 2, 2 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Tan}[e + f x] \right. \right. \\
& \quad \left. \left. \sqrt{1 + \operatorname{Tan}[e + f x]^2} \right) / \left((2 + np) \left(a^2 (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 2, 2 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \quad \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (-1 + np), 3, 3 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \\
& \quad \left. \left. a^2 (1 - np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (1 + np), 2, 3 + \frac{np}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) + \\
& \frac{1}{1 + np} (3 + np) \left(\left(2 a^2 b^2 \operatorname{AppellF1} \left[\frac{1}{2} (1 + np), \frac{np}{2}, 2, \frac{1}{2} (3 + np), -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \right) / \right. \\
& \quad \left(a^2 (3 + np) \operatorname{AppellF1} \left[\frac{1}{2} (1 + np), \frac{np}{2}, 2, \frac{1}{2} (3 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] - \right. \\
& \quad \left. \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3 + np), \frac{np}{2}, 3, \frac{1}{2} (5 + np), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), 1 + \frac{n p}{2}, 2, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^2 \Bigg) + \\
& \left((a^2 + b^2) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 1, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \right. \\
& \left. (-b^2 \operatorname{Tan}[e+f x]^2 + a^2 (1 + \operatorname{Tan}[e+f x]^2)) \right) \Bigg/ \left(-a^2 (3+n p) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 1, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), \frac{n p}{2}, 2, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left. a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), 1 + \frac{n p}{2}, 1, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) \Bigg) + \\
& \frac{1}{(-a^2 + b^2) (b^2 \operatorname{Tan}[e+f x]^2 - a^2 (1 + \operatorname{Tan}[e+f x]^2))^2} a^2 \operatorname{Sec}[e+f x]^2 \left(\frac{\operatorname{Tan}[e+f x]}{\sqrt{1 + \operatorname{Tan}[e+f x]^2}} \right)^{n p} \\
& \left(\left(2 a b (a^2 - b^2) (4+n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 2, 2 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Tan}[e+f x] \right. \right. \\
& \left. \left. \sqrt{1 + \operatorname{Tan}[e+f x]^2} \right) \Bigg/ \left((2+n p) \left(a^2 (4+n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 2, 2 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left. (-4 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 3, 3 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left. a^2 (1-n p) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (1+n p), 2, 3 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) + \\
& \frac{1}{1+n p} (3+n p) \left(\left(2 a^2 b^2 \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 2, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \right) \Bigg/ \right. \\
& \left(a^2 (3+n p) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 2, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] - \right. \\
& \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), \frac{n p}{2}, 3, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] + \right. \\
& \left. a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), 1 + \frac{n p}{2}, 2, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \Bigg) + \\
& \left((a^2 + b^2) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 1, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] (-b^2 \operatorname{Tan}[e+f x]^2 + \right. \\
& \left. a^2 (1 + \operatorname{Tan}[e+f x]^2)) \right) \Bigg/ \left(-a^2 (3+n p) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 1, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3 + n p), \frac{n p}{2}, 2, \frac{1}{2} (5 + n p), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \\
& \quad \left. a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3 + n p), 1 + \frac{n p}{2}, 1, \frac{1}{2} (5 + n p), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Tan}[e + f x]^2 \right) \Bigg) \\
& \frac{1}{(-a^2 + b^2) (b^2 \operatorname{Tan}[e + f x]^2 - a^2 (1 + \operatorname{Tan}[e + f x]^2))^2} a^2 n p \operatorname{Tan}[e + f x] \left(\frac{\operatorname{Tan}[e + f x]}{\sqrt{1 + \operatorname{Tan}[e + f x]^2}} \right)^{-1 + n p} \\
& \left(-\frac{\operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]^2}{(1 + \operatorname{Tan}[e + f x]^2)^{3/2}} + \frac{\operatorname{Sec}[e + f x]^2}{\sqrt{1 + \operatorname{Tan}[e + f x]^2}} \right) \\
& \left(\left(2 a b (a^2 - b^2) (4 + n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 2, 2 + \frac{n p}{2}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Tan}[e + f x] \right. \right. \\
& \quad \left. \left. \sqrt{1 + \operatorname{Tan}[e + f x]^2} \right) / \left((2 + n p) \left(a^2 (4 + n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 2, 2 + \frac{n p}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (-1 + n p), 3, 3 + \frac{n p}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. a^2 (1 - n p) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (1 + n p), 2, 3 + \frac{n p}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) + \\
& \frac{1}{1 + n p} (3 + n p) \left(\left(2 a^2 b^2 \operatorname{AppellF1} \left[\frac{1}{2} (1 + n p), \frac{n p}{2}, 2, \frac{1}{2} (3 + n p), -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \right) / \right. \\
& \quad \left(a^2 (3 + n p) \operatorname{AppellF1} \left[\frac{1}{2} (1 + n p), \frac{n p}{2}, 2, \frac{1}{2} (3 + n p), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] - \right. \\
& \quad \left(4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3 + n p), \frac{n p}{2}, 3, \frac{1}{2} (5 + n p), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \\
& \quad \left. \left. a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3 + n p), 1 + \frac{n p}{2}, 2, \frac{1}{2} (5 + n p), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) + \\
& \left((a^2 + b^2) \operatorname{AppellF1} \left[\frac{1}{2} (1 + n p), \frac{n p}{2}, 1, \frac{1}{2} (3 + n p), -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] (-b^2 \operatorname{Tan}[e + f x]^2 + \right. \\
& \quad \left. a^2 (1 + \operatorname{Tan}[e + f x]^2)) \right) / \left(-a^2 (3 + n p) \operatorname{AppellF1} \left[\frac{1}{2} (1 + n p), \frac{n p}{2}, 1, \frac{1}{2} (3 + n p), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3 + n p), \frac{n p}{2}, 2, \frac{1}{2} (5 + n p), -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), 1 + \frac{n p}{2}, 1, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^2 \Bigg) \Bigg) + \\
& \frac{1}{(-a^2+b^2) (b^2 \operatorname{Tan}[e+f x]^2 - a^2 (1 + \operatorname{Tan}[e+f x]^2))^2} a^2 \operatorname{Tan}[e+f x] \left(\frac{\operatorname{Tan}[e+f x]}{\sqrt{1 + \operatorname{Tan}[e+f x]^2}} \right)^{n p} \\
& \left(\left(2 a b (a^2 - b^2) (4+n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 2, 2 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \right. \right. \\
& \left. \left. \operatorname{Tan}[e+f x]^2 \right) / \left((2+n p) \sqrt{1 + \operatorname{Tan}[e+f x]^2} \left(a^2 (4+n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 2, 2 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right) + \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 3, 3 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left. a^2 (1-n p) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (1+n p), 2, 3 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) + \\
& \left(2 a b (a^2 - b^2) (4+n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 2, 2 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \right. \\
& \left. \sqrt{1 + \operatorname{Tan}[e+f x]^2} \right) / \left((2+n p) \left(a^2 (4+n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 2, 2 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 3, 3 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left. a^2 (1-n p) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (1+n p), 2, 3 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) + \\
& \left(2 a b (a^2 - b^2) (4+n p) \operatorname{Tan}[e+f x] \left(\frac{1}{a^2 (2 + \frac{n p}{2})} 4 (-a^2+b^2) \left(1 + \frac{n p}{2} \right) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 3, 3 + \frac{n p}{2}, \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{2 + \frac{n p}{2}} \left(1 + \frac{n p}{2} \right) (-1+n p) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \right. \right. \\
& \left. \left. 1 + \frac{1}{2} (-1+n p), 2, 3 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \sqrt{1 + \operatorname{Tan}[e+f x]^2} \Bigg) / \\
& \left((2+n p) \left(a^2 (4+n p) \operatorname{AppellF1} \left[1 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 2, 2 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \left. \left(-4 (a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{n p}{2}, \frac{1}{2} (-1+n p), 3, 3 + \frac{n p}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& a^2 (1 - np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (1 + np), 2, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \operatorname{Tan}[e + fx]^2 \Bigg) - \\
& \left(2ab(a^2 - b^2) (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 2, 2 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + fx]^2}{a^2} \right] \operatorname{Tan}[e + fx] \right. \\
& \sqrt{1 + \operatorname{Tan}[e + fx]^2} \left(2 \left(-4(a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (-1 + np), 3, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] + \right. \right. \\
& a^2 (1 - np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (1 + np), 2, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] + \\
& a^2 (4 + np) \left(\frac{1}{2 + \frac{np}{2}} 4 \left(-1 + \frac{b^2}{a^2} \right) \left(1 + \frac{np}{2} \right) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (-1 + np), 3, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \right. \\
& \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] - \frac{1}{2 + \frac{np}{2}} \left(1 + \frac{np}{2} \right) (-1 + np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, 1 + \frac{1}{2} (-1 + np), 2, \right. \\
& \left. \left. 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] \right) + \operatorname{Tan}[e + fx]^2 \\
& \left(-4(a^2 - b^2) \left(\frac{1}{3 + \frac{np}{2}} 6 \left(-1 + \frac{b^2}{a^2} \right) \left(2 + \frac{np}{2} \right) \operatorname{AppellF1} \left[3 + \frac{np}{2}, \frac{1}{2} (-1 + np), 4, 4 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \right. \right. \\
& \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] - \frac{1}{3 + \frac{np}{2}} \left(2 + \frac{np}{2} \right) (-1 + np) \operatorname{AppellF1} \left[3 + \frac{np}{2}, 1 + \frac{1}{2} (-1 + np), 3, 4 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] \right) + a^2 (1 - np) \left(\frac{1}{3 + \frac{np}{2}} 4 \left(-1 + \frac{b^2}{a^2} \right) \left(2 + \frac{np}{2} \right) \operatorname{AppellF1} \left[3 + \frac{np}{2}, \right. \right. \\
& \left. \left. \frac{1}{2} (1 + np), 3, 4 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] - \frac{1}{3 + \frac{np}{2}} \left(2 + \frac{np}{2} \right) (1 + np) \right. \\
& \left. \left. \operatorname{AppellF1} \left[3 + \frac{np}{2}, 1 + \frac{1}{2} (1 + np), 2, 4 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] \right) \right) \Bigg) \Bigg) / \\
& \left((2 + np) \left(a^2 (4 + np) \operatorname{AppellF1} \left[1 + \frac{np}{2}, \frac{1}{2} (-1 + np), 2, 2 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] + \right. \right. \\
& \left(-4(a^2 - b^2) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (-1 + np), 3, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] + \right. \\
& \left. \left. a^2 (1 - np) \operatorname{AppellF1} \left[2 + \frac{np}{2}, \frac{1}{2} (1 + np), 2, 3 + \frac{np}{2}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + fx]^2 \right] \operatorname{Tan}[e + fx]^2 \right) \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{1+n p} (3+n p) \left(\left((a^2+b^2) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 1, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \right. \right. \\
& \quad \left. \left. (2 a^2 \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - 2 b^2 \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]) \right) \right) / \\
& \left(-a^2 (3+n p) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 1, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \\
& \quad \left(2 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), \frac{n p}{2}, 2, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \\
& \quad \left. a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), 1 + \frac{n p}{2}, 1, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \Big) + \\
& \left(2 a^2 b^2 \left(\frac{1}{a^2 (3+n p)} 4 (-a^2+b^2) (1+n p) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1+n p), \frac{n p}{2}, 3, 1 + \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{3+n p} n p (1+n p) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1+n p), \right. \right. \\
& \quad \left. \left. 1 + \frac{n p}{2}, 2, 1 + \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \Big) / \\
& \left(a^2 (3+n p) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 2, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \\
& \quad \left(4 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), \frac{n p}{2}, 3, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \\
& \quad \left. a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), 1 + \frac{n p}{2}, 2, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \Big) + \\
& \left((a^2+b^2) \left(\frac{1}{a^2 (3+n p)} 2 (-a^2+b^2) (1+n p) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1+n p), \frac{n p}{2}, 2, 1 + \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{3+n p} n p (1+n p) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1+n p), \right. \right. \\
& \quad \left. \left. 1 + \frac{n p}{2}, 1, 1 + \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \Big) \\
& \quad \left. (-b^2 \operatorname{Tan}[e+f x]^2 + a^2 (1 + \operatorname{Tan}[e+f x]^2)) \right) / \left(-a^2 (3+n p) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 1, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(2 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), \frac{n p}{2}, 2, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& a^2 n p \operatorname{AppellF1}\left[\frac{1}{2}(3+n p), 1+\frac{n p}{2}, 1, \frac{1}{2}(5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Tan}[e+f x]^2 - \\
& \left((a^2+b^2) \operatorname{AppellF1}\left[\frac{1}{2}(1+n p), \frac{n p}{2}, 1, \frac{1}{2}(3+n p), -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2}\right] (-b^2 \operatorname{Tan}[e+f x]^2 + \right. \\
& a^2(1+\operatorname{Tan}[e+f x]^2) \left. \right) \left(2 \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}(3+n p), \frac{n p}{2}, 2, \frac{1}{2}(5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] + \right. \right. \\
& a^2 n p \operatorname{AppellF1}\left[\frac{1}{2}(3+n p), 1+\frac{n p}{2}, 1, \frac{1}{2}(5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \left. \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \\
& a^2(3+n p) \left(\frac{1}{3+n p} 2 \left(-1+\frac{b^2}{a^2}\right) (1+n p) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+n p), \frac{n p}{2}, 2, 1+\frac{1}{2}(3+n p), -\operatorname{Tan}[e+f x]^2, \right. \right. \\
& \left. \left. \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{3+n p} n p (1+n p) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+n p), \right. \right. \\
& \left. \left. 1+\frac{n p}{2}, 1, 1+\frac{1}{2}(3+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \\
& \operatorname{Tan}[e+f x]^2 \left(2(a^2-b^2) \left(\frac{1}{5+n p} 4 \left(-1+\frac{b^2}{a^2}\right) (3+n p) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+n p), \frac{n p}{2}, 3, 1+\frac{1}{2}(5+n p), \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{5+n p} n p (3+n p) \operatorname{AppellF1}\left[\right. \right. \\
& \left. \left. 1+\frac{1}{2}(3+n p), 1+\frac{n p}{2}, 2, 1+\frac{1}{2}(5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \\
& a^2 n p \left(\frac{1}{5+n p} 2 \left(-1+\frac{b^2}{a^2}\right) (3+n p) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+n p), 1+\frac{n p}{2}, 2, 1+\frac{1}{2}(5+n p), -\operatorname{Tan}[e+f x]^2, \right. \right. \\
& \left. \left. \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{5+n p} 2 \left(1+\frac{n p}{2}\right) (3+n p) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+n p), \right. \right. \\
& \left. \left. 2+\frac{n p}{2}, 1, 1+\frac{1}{2}(5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \Big) \Big) \Big) \Big) / \\
& \left(-a^2(3+n p) \operatorname{AppellF1}\left[\frac{1}{2}(1+n p), \frac{n p}{2}, 1, \frac{1}{2}(3+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] + \right. \\
& \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{2}(3+n p), \frac{n p}{2}, 2, \frac{1}{2}(5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] + \right. \right. \\
& \left. \left. a^2 n p \operatorname{AppellF1}\left[\frac{1}{2}(3+n p), 1+\frac{n p}{2}, 1, \frac{1}{2}(5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right)^2 -
\end{aligned}$$

$$\begin{aligned}
& \left(2 a^2 b^2 \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 2, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \right. \\
& \left(-2 \left(4 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), \frac{n p}{2}, 3, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left. a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), 1+\frac{n p}{2}, 2, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \right. \\
& a^2 (3+n p) \left(\frac{1}{3+n p} 4 \left(-1+\frac{b^2}{a^2} \right) (1+n p) \operatorname{AppellF1} \left[1+\frac{1}{2} (1+n p), \frac{n p}{2}, 3, 1+\frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \right. \right. \\
& \quad \left. \left. \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{3+n p} n p (1+n p) \operatorname{AppellF1} \left[1+\frac{1}{2} (1+n p), \right. \right. \\
& \quad \left. \left. 1+\frac{n p}{2}, 2, 1+\frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) - \\
& \operatorname{Tan}[e+f x]^2 \left(4 (a^2-b^2) \left(\frac{1}{5+n p} 6 \left(-1+\frac{b^2}{a^2} \right) (3+n p) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+n p), \frac{n p}{2}, 4, 1+\frac{1}{2} (5+n p), \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{5+n p} n p (3+n p) \operatorname{AppellF1} \left[\right. \right. \\
& \quad \left. \left. 1+\frac{1}{2} (3+n p), 1+\frac{n p}{2}, 3, 1+\frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \\
& a^2 n p \left(\frac{1}{5+n p} 4 \left(-1+\frac{b^2}{a^2} \right) (3+n p) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+n p), 1+\frac{n p}{2}, 3, 1+\frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \right. \right. \\
& \quad \left. \left. \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{1}{5+n p} 2 \left(1+\frac{n p}{2} \right) (3+n p) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+n p), \right. \right. \\
& \quad \left. \left. 2+\frac{n p}{2}, 2, 1+\frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \right) / \\
& \left(a^2 (3+n p) \operatorname{AppellF1} \left[\frac{1}{2} (1+n p), \frac{n p}{2}, 2, \frac{1}{2} (3+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \\
& \left(4 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), \frac{n p}{2}, 3, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \\
& \quad \left. \left. a^2 n p \operatorname{AppellF1} \left[\frac{1}{2} (3+n p), 1+\frac{n p}{2}, 2, \frac{1}{2} (5+n p), -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 837: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c (d \sin[e + f x])^p)^n}{(a + b \sin[e + f x])^3} dx$$

Optimal (type 6, 428 leaves, 14 steps):

$$\begin{aligned} & \frac{1}{(a^2 - b^2)^3 f} {}_3F_2 \left[\frac{1}{2}, -\frac{np}{2}, 3, \frac{3}{2}, \cos[e + f x]^2, -\frac{b^2 \cos[e + f x]^2}{a^2 - b^2} \right] \cos[e + f x] (\sin[e + f x]^2)^{-\frac{np}{2}} (c (d \sin[e + f x])^p)^n + \frac{1}{(a^2 - b^2)^3 f} \\ & b^3 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-2 - np), 3, \frac{3}{2}, \cos[e + f x]^2, -\frac{b^2 \cos[e + f x]^2}{a^2 - b^2} \right] \cos[e + f x] (\sin[e + f x]^2)^{-\frac{np}{2}} (c (d \sin[e + f x])^p)^n - \frac{1}{(a^2 - b^2)^3 f} \\ & 3 a b^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-1 - np), 3, \frac{3}{2}, \cos[e + f x]^2, -\frac{b^2 \cos[e + f x]^2}{a^2 - b^2} \right] \cos[e + f x] \sin[e + f x] (\sin[e + f x]^2)^{\frac{1}{2}(-1 - np)} (c (d \sin[e + f x])^p)^n - \\ & \frac{1}{(a^2 - b^2)^3 f} a^3 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (1 - np), 3, \frac{3}{2}, \cos[e + f x]^2, -\frac{b^2 \cos[e + f x]^2}{a^2 - b^2} \right] \cot[e + f x] (\sin[e + f x]^2)^{\frac{1}{2}(1 - np)} (c (d \sin[e + f x])^p)^n \end{aligned}$$

Result (type 6, 22711 leaves): Display of huge result suppressed!

Test results for the 1563 problems in "4.1.2.2 (g cos)^p (a+b sin)^m (c+d sin)^n.m"

- Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos[e + f x]^2 \sqrt{a + a \sin[e + f x]}}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 99 leaves, 5 steps):

$$-\frac{2 a \cos[e + f x] \operatorname{Log}[1 - \sin[e + f x]]}{c f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} - \frac{\cos[e + f x] \sqrt{a + a \sin[e + f x]}}{c f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 127 leaves):

$$\begin{aligned} & - \left(\left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \sqrt{a(1 + \sin[e + f x])} (-2 i f x + 4 i \operatorname{ArcTan}[e^{i(e + f x)}] + 2 \operatorname{Log}[1 + e^{2 i(e + f x)}] + \sin[e + f x]) \right) / \\ & \left(f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) (c - c \sin[e + f x])^{3/2} \right) \end{aligned}$$

- Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos[e + f x]^2 \sqrt{a + a \sin[e + f x]}}{(c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 97 leaves, 5 steps):

$$\frac{\cos[e + f x] \sqrt{a + a \sin[e + f x]}}{c f (c - c \sin[e + f x])^{3/2}} + \frac{a \cos[e + f x] \log[1 - \sin[e + f x]]}{c^2 f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 118 leaves):

$$\frac{1}{c^2 f \sqrt{c - c \sin[e + f x]}} \operatorname{Sec}[e + f x] \sqrt{a (1 + \sin[e + f x])} \left(2 - i f x + \log[1 + e^{2i(e+fx)}] - 2i \operatorname{ArcTan}[e^{i(e+fx)}] (-1 + \sin[e + f x]) + (i f x - \log[1 + e^{2i(e+fx)}]) \sin[e + f x] \right)$$

■ **Problem 13: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^2 (a + a \sin[e + f x])^{3/2}}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 45 leaves, 2 steps):

$$\frac{\cos[e + f x] (a + a \sin[e + f x])^{5/2}}{3 a f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 111 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) (a (1 + \sin[e + f x]))^{3/2} (-6 \cos[2(e + f x)] + 15 \sin[e + f x] - \sin[3(e + f x)]) \right) / \left(12 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \sqrt{c - c \sin[e + f x]} \right)$$

■ **Problem 17: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^2 (a + a \sin[e + f x])^{3/2}}{(c - c \sin[e + f x])^{9/2}} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$\frac{\cos[e + f x] (a + a \sin[e + f x])^{5/2}}{6 a c f (c - c \sin[e + f x])^{7/2}}$$

Result (type 3, 110 leaves):

$$\frac{a (-5 + 3 \cos[2(e + f x)]) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \sqrt{a (1 + \sin[e + f x])}}{6 c^4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) (-1 + \sin[e + f x])^4 \sqrt{c - c \sin[e + f x]}}$$

■ **Problem 23: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^2 (a + a \sin[e + f x])^{5/2}}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 45 leaves, 2 steps):

$$\frac{\cos[e + f x] (a + a \sin[e + f x])^{7/2}}{4 a f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 119 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) (a(1 + \sin[e + f x]))^{5/2} (-28 \cos[2(e + f x)] + \cos[4(e + f x)] + 56 \sin[e + f x] - 8 \sin[3(e + f x)]) \right) / \left(32 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \sqrt{c - c \sin[e + f x]} \right)$$

■ **Problem 28: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^2 (a + a \sin[e + f x])^{5/2}}{(c - c \sin[e + f x])^{11/2}} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$\frac{\cos[e + f x] (a + a \sin[e + f x])^{7/2}}{8 a c f (c - c \sin[e + f x])^{9/2}}$$

Result (type 3, 329 leaves):

$$\frac{2 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 (a(1 + \sin[e + f x]))^{5/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{11/2}} - \frac{4 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (a(1 + \sin[e + f x]))^{5/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{11/2}} + \frac{3 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a(1 + \sin[e + f x]))^{5/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{11/2}} - \frac{\left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a(1 + \sin[e + f x]))^{5/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{11/2}}$$

■ **Problem 29: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^2 (a + a \sin[e + f x])^{5/2}}{(c - c \sin[e + f x])^{13/2}} dx$$

Optimal (type 3, 97 leaves, 3 steps):

$$\frac{\cos[e + f x] (a + a \sin[e + f x])^{7/2}}{10 a c f (c - c \sin[e + f x])^{11/2}} + \frac{\cos[e + f x] (a + a \sin[e + f x])^{7/2}}{80 a c^2 f (c - c \sin[e + f x])^{9/2}}$$

Result (type 3, 333 leaves):

$$\frac{8 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 (a(1 + \sin[e + f x]))^{5/2}}{5 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{13/2}} - \frac{3 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (a(1 + \sin[e + f x]))^{5/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{13/2}} + \frac{2 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a(1 + \sin[e + f x]))^{5/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{13/2}} - \frac{\left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a(1 + \sin[e + f x]))^{5/2}}{2 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{13/2}}$$

■ **Problem 30: Result more than twice size of optimal antiderivative.**

$$\int \cos[e + f x]^2 (a + a \sin[e + f x])^{7/2} (c - c \sin[e + f x])^{9/2} dx$$

Optimal (type 3, 236 leaves, 6 steps):

$$\begin{aligned} & - \frac{4 a^4 \cos[e + f x] (c - c \sin[e + f x])^{11/2}}{315 c f \sqrt{a + a \sin[e + f x]}} - \frac{4 a^3 \cos[e + f x] \sqrt{a + a \sin[e + f x]} (c - c \sin[e + f x])^{11/2}}{105 c f} \\ & - \frac{a^2 \cos[e + f x] (a + a \sin[e + f x])^{3/2} (c - c \sin[e + f x])^{11/2}}{15 c f} \\ & - \frac{4 a \cos[e + f x] (a + a \sin[e + f x])^{5/2} (c - c \sin[e + f x])^{11/2}}{45 c f} - \frac{\cos[e + f x] (a + a \sin[e + f x])^{7/2} (c - c \sin[e + f x])^{11/2}}{10 c f} \end{aligned}$$

Result (type 3, 919 leaves):

$$\begin{aligned}
& \frac{21 \operatorname{Cos}[2(e+fx)] (a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2}}{512 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7} + \\
& \frac{3 \operatorname{Cos}[4(e+fx)] (a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2}}{128 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7} + \\
& \frac{9 \operatorname{Cos}[6(e+fx)] (a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2}}{1024 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7} + \\
& \frac{\operatorname{Cos}[8(e+fx)] (a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2}}{512 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7} + \\
& \frac{\operatorname{Cos}[10(e+fx)] (a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2}}{5120 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7} + \\
& \frac{63 \operatorname{Sin}[e+fx] (a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2}}{128 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7} + \\
& \frac{7 (a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2} \operatorname{Sin}[3(e+fx)]}{64 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7} + \\
& \frac{9 (a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2} \operatorname{Sin}[5(e+fx)]}{320 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7} + \\
& \frac{9 (a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2} \operatorname{Sin}[7(e+fx)]}{1792 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7} + \\
& \frac{(a(1+\operatorname{Sin}[e+fx]))^{7/2} (c-c\operatorname{Sin}[e+fx])^{9/2} \operatorname{Sin}[9(e+fx)]}{2304 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^9 \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^7}
\end{aligned}$$

■ **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e+fx]^2 (a+a\operatorname{Sin}[e+fx])^{7/2}}{\sqrt{c-c\operatorname{Sin}[e+fx]}} dx$$

Optimal (type 3, 45 leaves, 2 steps):

$$\frac{\operatorname{Cos}[e+fx] (a+a\operatorname{Sin}[e+fx])^{9/2}}{5 a f \sqrt{c-c\operatorname{Sin}[e+fx]}}$$

Result (type 3, 142 leaves):

$$\left(a^3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (1 + \sin[e+fx])^3 \sqrt{a(1 + \sin[e+fx])} \right. \\ \left. (-120 \cos[2(e+fx)] + 10 \cos[4(e+fx)] + 210 \sin[e+fx] - 45 \sin[3(e+fx)] + \sin[5(e+fx)]) \right) / \\ \left(80 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 \sqrt{c - c \sin[e+fx]} \right)$$

■ **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^2 (a + a \sin[e+fx])^{7/2}}{(c - c \sin[e+fx])^{13/2}} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$\frac{\cos[e+fx] (a + a \sin[e+fx])^{9/2}}{10 a c f (c - c \sin[e+fx])^{11/2}}$$

Result (type 3, 412 leaves):

$$\frac{16 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (a(1 + \sin[e+fx]))^{7/2}}{5 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{13/2}} - \\ \frac{8 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 (a(1 + \sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{13/2}} + \frac{8 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (a(1 + \sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{13/2}} - \\ \frac{4 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^9 (a(1 + \sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{13/2}} + \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^{11} (a(1 + \sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{13/2}}$$

■ **Problem 42: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^2 (a + a \sin[e+fx])^{7/2}}{(c - c \sin[e+fx])^{15/2}} dx$$

Optimal (type 3, 97 leaves, 3 steps):

$$\frac{\cos[e+fx] (a + a \sin[e+fx])^{9/2}}{12 a c f (c - c \sin[e+fx])^{13/2}} + \frac{\cos[e+fx] (a + a \sin[e+fx])^{9/2}}{120 a c^2 f (c - c \sin[e+fx])^{11/2}}$$

Result (type 3, 419 leaves):

$$\frac{8 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (a(1+\sin[e+fx]))^{7/2}}{3f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{15/2}} - \frac{32 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 (a(1+\sin[e+fx]))^{7/2}}{5f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{15/2}} + \frac{6 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{15/2}} - \frac{8 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^9 (a(1+\sin[e+fx]))^{7/2}}{3f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{15/2}} + \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^{11} (a(1+\sin[e+fx]))^{7/2}}{2f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{15/2}}$$

■ **Problem 43: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^2 (a+a \sin[e+fx])^{7/2}}{(c-c \sin[e+fx])^{17/2}} dx$$

Optimal (type 3, 145 leaves, 4 steps):

$$\frac{\cos[e+fx] (a+a \sin[e+fx])^{9/2}}{14acf (c-c \sin[e+fx])^{15/2}} + \frac{\cos[e+fx] (a+a \sin[e+fx])^{9/2}}{84ac^2f (c-c \sin[e+fx])^{13/2}} + \frac{\cos[e+fx] (a+a \sin[e+fx])^{9/2}}{840ac^3f (c-c \sin[e+fx])^{11/2}}$$

Result (type 3, 419 leaves):

$$\frac{16 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (a(1+\sin[e+fx]))^{7/2}}{7f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{17/2}} - \frac{16 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 (a(1+\sin[e+fx]))^{7/2}}{3f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{17/2}} + \frac{24 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (a(1+\sin[e+fx]))^{7/2}}{5f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{17/2}} - \frac{2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^9 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{17/2}} + \frac{\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^{11} (a(1+\sin[e+fx]))^{7/2}}{3f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c - c \sin[e+fx])^{17/2}}$$

■ **Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^2 (c-c \sin[e+fx])^{5/2}}{\sqrt{a+a \sin[e+fx]}} dx$$

Optimal (type 3, 45 leaves, 2 steps):

$$\frac{\cos[e+fx] (c-c \sin[e+fx])^{7/2}}{4cf \sqrt{a+a \sin[e+fx]}}$$

Result (type 3, 134 leaves):

$$\left(c^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) (-1 + \sin[e+fx])^2 \sqrt{c - c \sin[e+fx]} (28 \cos[2(e+fx)] - \cos[4(e+fx)] + 56 \sin[e+fx] - 8 \sin[3(e+fx)]) \right) / \left(32 f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 \sqrt{a(1 + \sin[e+fx])} \right)$$

■ **Problem 45: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^2 (c - c \sin[e+fx])^{3/2}}{\sqrt{a + a \sin[e+fx]}} dx$$

Optimal (type 3, 45 leaves, 2 steps):

$$-\frac{\cos[e+fx] (c - c \sin[e+fx])^{5/2}}{3 c f \sqrt{a + a \sin[e+fx]}}$$

Result (type 3, 120 leaves):

$$-\left(c \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) (-1 + \sin[e+fx]) \sqrt{c - c \sin[e+fx]} (6 \cos[2(e+fx)] + 15 \sin[e+fx] - \sin[3(e+fx)]) \right) / \left(12 f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \sqrt{a(1 + \sin[e+fx])} \right)$$

■ **Problem 53: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[e+fx]^2 \sqrt{c - c \sin[e+fx]}}{(a + a \sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 96 leaves, 5 steps):

$$\frac{2 c \cos[e+fx] \operatorname{Log}[1 + \sin[e+fx]]}{a f \sqrt{a + a \sin[e+fx]} \sqrt{c - c \sin[e+fx]}} + \frac{\cos[e+fx] \sqrt{c - c \sin[e+fx]}}{a f \sqrt{a + a \sin[e+fx]}}$$

Result (type 3, 127 leaves):

$$-\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (2 i f x + 4 i \operatorname{ArcTan}[e^{i(e+fx)}] - 2 \operatorname{Log}[1 + e^{2i(e+fx)}] + \sin[e+fx]) \sqrt{c - c \sin[e+fx]} \right) / \left(f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (a(1 + \sin[e+fx]))^{3/2} \right)$$

■ **Problem 61: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[e+fx]^2 \sqrt{c - c \sin[e+fx]}}{(a + a \sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 97 leaves, 5 steps):

$$-\frac{c \operatorname{Cos}[e + f x] \operatorname{Log}[1 + \operatorname{Sin}[e + f x]]}{a^2 f \sqrt{a + a \operatorname{Sin}[e + f x]} \sqrt{c - c \operatorname{Sin}[e + f x]}} - \frac{\operatorname{Cos}[e + f x] \sqrt{c - c \operatorname{Sin}[e + f x]}}{a f (a + a \operatorname{Sin}[e + f x])^{3/2}}$$

Result (type 3, 121 leaves):

$$\frac{1}{a^2 f \sqrt{a (1 + \operatorname{Sin}[e + f x])}} \operatorname{Sec}[e + f x] \sqrt{c - c \operatorname{Sin}[e + f x]} (2 i + f x + i \operatorname{Log}[1 + e^{2 i (e + f x)}]) + (f x + i \operatorname{Log}[1 + e^{2 i (e + f x)}]) \operatorname{Sin}[e + f x] + 2 \operatorname{ArcTan}[e^{i (e + f x)}] (1 + \operatorname{Sin}[e + f x])$$

■ **Problem 65: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e + f x]^2 (a + a \operatorname{Sin}[e + f x])^m (c - c \operatorname{Sin}[e + f x])^n dx$$

Optimal (type 5, 114 leaves, 5 steps):

$$\frac{1}{f (3 + 2 m)} 2^{3+n} c^2 \operatorname{Cos}[e + f x]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2} (3 + 2 m), \frac{1}{2} (-1 - 2 n), \frac{1}{2} (5 + 2 m), \frac{1}{2} (1 + \operatorname{Sin}[e + f x])\right] (1 - \operatorname{Sin}[e + f x])^{\frac{1}{2}-n} (a + a \operatorname{Sin}[e + f x])^m (c - c \operatorname{Sin}[e + f x])^{-2+n}$$

Result (type 6, 15688 leaves):

$$-\left(4^{3+n} (3 + 2 n) \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a + a \operatorname{Sin}[e + f x])^m (c - c \operatorname{Sin}[e + f x])^n \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]}{1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}\right)^{2n} \left(\frac{1 - \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}\right)^{2m} \left(\left(5 \operatorname{AppellF1}\left[\frac{1}{2} + n, -2 m, 3 + 2 (m + n), \frac{3}{2} + n, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^2\right) / \left(- (3 + 2 n) \operatorname{AppellF1}\left[\frac{1}{2} + n, -2 m, 3 + 2 (m + n), \frac{3}{2} + n, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + 2 \left(2 m \operatorname{AppellF1}\left[\frac{3}{2} + n, 1 - 2 m, 3 + 2 (m + n), \frac{5}{2} + n, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (3 + 2 m + 2 n) \operatorname{AppellF1}\left[\frac{3}{2} + n, -2 m, 2 (2 + m + n), \frac{5}{2} + n, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) + \left(4 \operatorname{AppellF1}\left[\frac{1}{2} + n, -2 m, 5 + 2 (m + n), \frac{3}{2} + n, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) / \left(- (3 + 2 n) \operatorname{AppellF1}\left[\frac{1}{2} + n, -2 m, 5 + 2 (m + n), \frac{3}{2} + n, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) +$$

$$\begin{aligned}
& \frac{1}{(1+2n) \left(1 + \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]\right)^5} 4^{2+n} (3+2n) \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]}{1 + \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}\right)^{2n} \\
& \left(\frac{1 - \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}\right)^{2m} \\
& \left(\left(5 \operatorname{AppellF1}\left[\frac{1}{2} + n, -2m, 3 + 2(m+n), \frac{3}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 + \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)\right) \right) / \\
& \left(-(3+2n) \operatorname{AppellF1}\left[\frac{1}{2} + n, -2m, 3 + 2(m+n), \frac{3}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \quad \left. 2 \left(2m \operatorname{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 3 + 2(m+n), \frac{5}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (3 + 2m + 2n) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + n, -2m, 2(2+m+n), \frac{5}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \\
& \left(5 \left(-\frac{1}{\frac{3}{2} + n} m \left(\frac{1}{2} + n\right) \operatorname{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 3 + 2(m+n), \frac{5}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{2 \left(\frac{3}{2} + n\right)} \left(\frac{1}{2} + n\right) (3 + 2(m+n)) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + n, -2m, 4 + 2(m+n), \frac{5}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \right) \left(1 + \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^2 \right) / \\
& \left(-(3+2n) \operatorname{AppellF1}\left[\frac{1}{2} + n, -2m, 3 + 2(m+n), \frac{3}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \quad \left. 2 \left(2m \operatorname{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 3 + 2(m+n), \frac{5}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (3 + 2m + 2n) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + n, -2m, 2(2+m+n), \frac{5}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \\
& \left(4 \left(-\frac{1}{\frac{3}{2} + n} m \left(\frac{1}{2} + n\right) \operatorname{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 5 + 2(m+n), \frac{5}{2} + n, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{2\left(\frac{3}{2} + n\right)}\left(\frac{1}{2} + n\right) (5 + 2(m+n)) \text{AppellF1}\left[\frac{3}{2} + n, -2m, 6 + 2(m+n), \right. \right. \right. \\
& \left. \left. \left. \frac{5}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) / \\
& \left(-(3+2n) \text{AppellF1}\left[\frac{1}{2} + n, -2m, 5 + 2(m+n), \frac{3}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \left. 2\left(2m \text{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 5 + 2(m+n), \frac{5}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (5 + 2m + 2n) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2} + n, -2m, 2(3+m+n), \frac{5}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) - \\
& \left(3 \text{AppellF1}\left[\frac{1}{2} + n, -2m, 2(1+m+n), \frac{3}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 + \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^2 \right) / \\
& \left(2\left(- (3 + 2n) \text{AppellF1}\left[\frac{1}{2} + n, -2m, 2(1+m+n), \frac{3}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\
& \left. 4\left(m \text{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 2(1+m+n), \frac{5}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1+m+n) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2} + n, -2m, 3 + 2(m+n), \frac{5}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) - \\
& \left(\left(-\frac{1}{\frac{3}{2} + n} m \left(\frac{1}{2} + n\right) \text{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 2(1+m+n), \frac{5}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{3}{2} + n} \left(\frac{1}{2} + n\right) (1+m+n) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2} + n, -2m, 1 + 2(1+m+n), \frac{5}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \left(1 + \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^3 \right) / \\
& \left(-(3+2n) \text{AppellF1}\left[\frac{1}{2} + n, -2m, 2(1+m+n), \frac{3}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \left. 4\left(m \text{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 2(1+m+n), \frac{5}{2} + n, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1+m+n) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{\frac{3}{2}+n}\left(\frac{1}{2}+n\right) \operatorname{AppellF1}\left[\frac{3}{2}+n,-2 m,\right. \\
& \left.1+2(1+m+n), \frac{5}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)+ \\
& 4 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\left(m\left(-\frac{1}{\frac{5}{2}+n}\left(\frac{3}{2}+n\right)(1+m+n) \operatorname{AppellF1}\left[\frac{5}{2}+n, 1-2 m, 1+2(1+m+n), \frac{7}{2}+n,\right.\right.\right. \\
& \left.\left.\left.\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]+ \right. \\
& \left.\frac{1}{2\left(\frac{5}{2}+n\right)}(1-2 m)\left(\frac{3}{2}+n\right) \operatorname{AppellF1}\left[\frac{5}{2}+n, 2-2 m, 2(1+m+n), \frac{7}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,\right.\right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\right)+(1+m+n) \\
& \left(-\frac{1}{\frac{5}{2}+n} m\left(\frac{3}{2}+n\right) \operatorname{AppellF1}\left[\frac{5}{2}+n, 1-2 m, 3+2(m+n), \frac{7}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{2\left(\frac{5}{2}+n\right)}\left(\frac{3}{2}+n\right)(3+2(m+n)) \operatorname{AppellF1}\left[\frac{5}{2}+n,-2 m, 4+2(m+n),\right.\right. \\
& \left.\left.\frac{7}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\right)\right) / \\
& \left(-\left(3+2 n\right) \operatorname{AppellF1}\left[\frac{1}{2}+n,-2 m, 2(1+m+n), \frac{3}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+ \right. \\
& \left.4\left(m \operatorname{AppellF1}\left[\frac{3}{2}+n, 1-2 m, 2(1+m+n), \frac{5}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+(1+m+n) \right. \right. \\
& \left. \left.\operatorname{AppellF1}\left[\frac{3}{2}+n,-2 m, 3+2(m+n), \frac{5}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^2+ \\
& \left(8 \operatorname{AppellF1}\left[\frac{1}{2}+n,-2 m, 2(2+m+n), \frac{3}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\left(1+\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\right. \\
& \left.2\left(m \operatorname{AppellF1}\left[\frac{3}{2}+n, 1-2 m, 2(2+m+n), \frac{5}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+(2+m+n) \right. \right. \\
& \left. \left.\operatorname{AppellF1}\left[\frac{3}{2}+n,-2 m, 5+2(m+n), \frac{5}{2}+n, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)
\end{aligned}$$

$$\begin{aligned}
& \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - (3 + 2n) \left(-\frac{1}{\frac{3}{2} + n} m \left(\frac{1}{2} + n\right) \text{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 2(2 + m + n), \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, \right. \\
& \quad \left. - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{3}{2} + n} \left(\frac{1}{2} + n\right) (2 + m + n) \text{AppellF1}\left[\frac{3}{2} + n, -2m, \right. \\
& \quad \left. 1 + 2(2 + m + n), \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \Bigg) + \\
& 4 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(m \left(-\frac{1}{\frac{5}{2} + n} \left(\frac{3}{2} + n\right) (2 + m + n) \text{AppellF1}\left[\frac{5}{2} + n, 1 - 2m, 1 + 2(2 + m + n), \frac{7}{2} + n, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \right. \\
& \quad \left. \frac{1}{2\left(\frac{5}{2} + n\right)} (1 - 2m) \left(\frac{3}{2} + n\right) \text{AppellF1}\left[\frac{5}{2} + n, 2 - 2m, 2(2 + m + n), \frac{7}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right. \right. \\
& \quad \left. \left. - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + (2 + m + n) \\
& \left(-\frac{1}{\frac{5}{2} + n} m \left(\frac{3}{2} + n\right) \text{AppellF1}\left[\frac{5}{2} + n, 1 - 2m, 5 + 2(m + n), \frac{7}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \right. \\
& \quad \left. \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{2\left(\frac{5}{2} + n\right)} \left(\frac{3}{2} + n\right) (5 + 2(m + n)) \text{AppellF1}\left[\frac{5}{2} + n, -2m, 6 + 2(m + n), \right. \right. \\
& \quad \left. \left. \frac{7}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \Bigg) \Bigg) / \\
& \left(-(3 + 2n) \text{AppellF1}\left[\frac{1}{2} + n, -2m, 2(2 + m + n), \frac{3}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] + \right. \\
& \quad \left. 4 \left(m \text{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 2(2 + m + n), \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] + (2 + m + n) \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{3}{2} + n, -2m, 5 + 2(m + n), \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^2 - \right. \\
& \left. \left(5 \text{AppellF1}\left[\frac{1}{2} + n, -2m, 3 + 2(m + n), \frac{3}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] \left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^2 \right. \right. \\
& \quad \left. \left. \left(\left(2m \text{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 3 + 2(m + n), \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (3 + 2m + 2n) \operatorname{AppellF1}\left[\frac{3}{2} + n, -2m, 2(2 + m + n), \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \\
& \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - (3 + 2n) \left(-\frac{1}{\frac{3}{2} + n} m \left(\frac{1}{2} + n\right) \operatorname{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 3 + 2(m + n), \right. \right. \\
& \quad \left. \left. \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{1}{2\left(\frac{3}{2} + n\right)} \right. \\
& \quad \left. \left(\frac{1}{2} + n\right) (3 + 2(m + n)) \operatorname{AppellF1}\left[\frac{3}{2} + n, -2m, 4 + 2(m + n), \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + 2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \left(2m \left(-\frac{1}{2\left(\frac{5}{2} + n\right)} \right. \right. \right. \\
& \quad \left. \left. \left(\frac{3}{2} + n\right) (3 + 2(m + n)) \operatorname{AppellF1}\left[\frac{5}{2} + n, 1 - 2m, 4 + 2(m + n), \frac{7}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{1}{2\left(\frac{5}{2} + n\right)} (1 - 2m) \left(\frac{3}{2} + n\right) \operatorname{AppellF1}\left[\frac{5}{2} + n, 2 - 2m, 3 + 2(m + n), \frac{7}{2} + \right. \right. \\
& \quad \left. \left. n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) + (3 + 2m + 2n) \\
& \quad \left(-\frac{1}{\frac{5}{2} + n} m \left(\frac{3}{2} + n\right) \operatorname{AppellF1}\left[\frac{5}{2} + n, 1 - 2m, 2(2 + m + n), \frac{7}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{1}{\frac{5}{2} + n} \left(\frac{3}{2} + n\right) (2 + m + n) \operatorname{AppellF1}\left[\frac{5}{2} + n, -2m, 1 + 2(2 + m + n), \right. \right. \\
& \quad \left. \left. \frac{7}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right) \Bigg/ \\
& \left(-(3 + 2n) \operatorname{AppellF1}\left[\frac{1}{2} + n, -2m, 3 + 2(m + n), \frac{3}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \quad \left. 2 \left(2m \operatorname{AppellF1}\left[\frac{3}{2} + n, 1 - 2m, 3 + 2(m + n), \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + (3 + 2m + 2n) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} + n, -2m, 2(2 + m + n), \frac{5}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 - \right. \\
& \quad \left. \left(4 \operatorname{AppellF1}\left[\frac{1}{2} + n, -2m, 5 + 2(m + n), \frac{3}{2} + n, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \right)
\end{aligned}$$

Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \cos[e + f x]^2 (a + a \sin[e + f x])^m (c - c \sin[e + f x])^3 dx$$

Optimal (type 5, 86 leaves, 4 steps):

$$-\frac{1}{9f} 2^{\frac{3}{2}+m} a^4 c^3 \cos[e + f x]^9 \operatorname{Hypergeometric2F1}\left[\frac{9}{2}, -\frac{1}{2}-m, \frac{11}{2}, \frac{1}{2}(1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{2}-m} (a + a \sin[e + f x])^{-4+m}$$

Result (type 6, 17864 leaves):

$$\begin{aligned} & - \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^3 \right. \\ & \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^{10} - 6 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^9 \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] + \right. \\ & 13 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^8 \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^2 - \\ & 8 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^7 \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^3 - 14 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^6 \\ & \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^4 + 28 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^5 \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^5 - \\ & 14 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^4 \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^6 - 8 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^3 \\ & \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^7 + 13 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^2 \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^8 - \\ & \left. 6 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^9 + \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^{10} \right) \\ & \left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 7+2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right. \right. \\ & \left. \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{2(1+m)} \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{7+2m} \right) / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 7+2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right. \right. \\ & \left. \left. -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - 2 \left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 7+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\ & \left. \left. (7+2m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 2(4+m), \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \\ & \left. \left(18 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 9+2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \end{aligned}$$

$$\begin{aligned}
& (5+m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 11+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 7+2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right. \\
& \left. \left(1 - \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right)^{2(1+m)} \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2} \right)^{7+2m} \right. \\
& \left. \left(- \left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 7+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + (7+2m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), \right. \right. \right. \\
& \left. \left. \left. 2(4+m), \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \right. \\
& \left. 3 \left(-\frac{1}{6} (7+2m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 8+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{3} (1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1-2(1+m), 7+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \right. \\
& \left. \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) - 2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(2(1+m) \left(-\frac{3}{10} (7+2m) \operatorname{AppellF1}\left[\frac{5}{2}, -1-2m, \right. \right. \right. \\
& \left. \left. \left. 8+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \right. \right. \\
& \left. \left. \frac{3}{10} (-1-2m) \operatorname{AppellF1}\left[\frac{5}{2}, -2m, 7+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) + (7+2m) \left(-\frac{3}{5} (4+m) \operatorname{AppellF1}\left[\frac{5}{2}, -2(1+m), 1+2(4+m), \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{3}{5} (1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1-2(1+m), \right. \right. \right. \\
& \left. \left. \left. 2(4+m), \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \right) \Bigg/ \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 7+2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] - \right. \\
& \left. 2 \left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 7+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + (7+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 2(4+m), \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{2(1+m)} \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}\right)^{11+2m} \\
& \left(-\left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 11+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (11+2m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), \right.\right.\right. \\
& \quad \left.\left.2(6+m), \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \\
& \quad 3\left(-\frac{1}{6}(11+2m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 12+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right. \\
& \quad \left.\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{3}(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1-2(1+m), 11+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right]\right) \\
& \quad \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - 2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(2(1+m) \left(-\frac{3}{10}(11+2m) \operatorname{AppellF1}\left[\frac{5}{2}, -1-2m, \right.\right.\right. \\
& \quad \left.\left.12+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \right. \\
& \quad \left.\frac{3}{10}(-1-2m) \operatorname{AppellF1}\left[\frac{5}{2}, -2m, 11+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right. \\
& \quad \left.\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right) + (11+2m) \left(-\frac{3}{5}(6+m) \operatorname{AppellF1}\left[\frac{5}{2}, -2(1+m), 1+2(6+m), \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right.\right. \\
& \quad \left.\left.-\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{3}{5}(1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1-2(1+m), \right.\right. \\
& \quad \left.\left.2(6+m), \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)\right) \Bigg) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 11+2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \\
& \quad \left.2\left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 11+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (11+2m) \right. \right. \\
& \quad \left. \left.\operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 2(6+m), \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 + \right. \\
& \quad \left. \left(12 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 2(4+m), \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right. \right. \\
& \quad \left. \left. \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{2(1+m)} \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}\right)^{2m} \left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{2m-2(4+m)}\right) \right)
\end{aligned}$$

$$\begin{aligned}
& 3 \left(-\frac{1}{3} (5+m) \operatorname{AppellF1} \left[\frac{3}{2}, -2(1+m), 1+2(5+m), \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{3} (1+m) \operatorname{AppellF1} \left[\frac{3}{2}, 1-2(1+m), 2(5+m), \frac{5}{2}, \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) - \\
& 4 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left((1+m) \left(-\frac{3}{5} (5+m) \operatorname{AppellF1} \left[\frac{5}{2}, -1-2m, 1+2(5+m), \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] + \frac{3}{10} (-1-2m) \operatorname{AppellF1} \left[\frac{5}{2}, -2m, 2(5+m), \right. \right. \\
& \quad \left. \left. \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) + (5+m) \\
& \quad \left(-\frac{3}{10} (11+2m) \operatorname{AppellF1} \left[\frac{5}{2}, -2(1+m), 12+2m, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{3}{5} (1+m) \operatorname{AppellF1} \left[\frac{5}{2}, 1-2(1+m), 11+2m, \frac{7}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) \right) \Bigg/ \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -2(1+m), 2(5+m), \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \\
& 4 \left((1+m) \operatorname{AppellF1} \left[\frac{3}{2}, -1-2m, 2(5+m), \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (5+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, -2(1+m), 11+2m, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 67: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[e + f x]^2 (a + a \sin[e + f x])^m (c - c \sin[e + f x])^2 dx$$

Optimal (type 5, 86 leaves, 4 steps):

$$-\frac{1}{7f} 2^{\frac{3}{2}+m} a^3 c^2 \cos[e + f x]^7 \operatorname{Hypergeometric2F1} \left[\frac{7}{2}, -\frac{1}{2}-m, \frac{9}{2}, \frac{1}{2} (1 - \sin[e + f x]) \right] (1 + \sin[e + f x])^{-\frac{1}{2}-m} (a + a \sin[e + f x])^{-3+m}$$

Result (type 6, 13077 leaves):

$$-\left(\left(12288 c^2 \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-2m} (a + a \sin[e + f x])^m \right. \right.$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{3}(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1-2(1+m),\right. \\
& \left.2(4+m), \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-4 \\
& \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\left((1+m)\left(-\frac{3}{10}(8+2 m) \operatorname{AppellF1}\left[\frac{5}{2},-1-2 m, 9+2 m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,\right.\right.\right. \\
& \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]+\frac{3}{10}(-1-2 m) \operatorname{AppellF1}\left[\frac{5}{2},-2 m,\right.\right. \\
& \left.\left.8+2 m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)+ \\
& (4+m)\left(-\frac{3}{10}(9+2 m) \operatorname{AppellF1}\left[\frac{5}{2},-2(1+m), 10+2 m, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{3}{5}(1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1-2(1+m), 9+2 m, \frac{7}{2},\right.\right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2},-2(1+m), 2(4+m), \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]-4\right. \\
& \left.\left((1+m) \operatorname{AppellF1}\left[\frac{3}{2},-1-2 m, 8+2 m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+ \right. \\
& \left.\left.(4+m) \operatorname{AppellF1}\left[\frac{3}{2},-2(1+m), 9+2 m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^2\right)
\end{aligned}$$

■ **Problem 68: Attempted integration timed out after 120 seconds.**

$$\int \cos [e+f x]^2(a+a \sin [e+f x])^m(c-c \sin [e+f x]) d x$$

Optimal (type 5, 84 leaves, 4 steps):

$$-\frac{1}{5 f} 2^{\frac{3}{2}+m} a^2 c \cos [e+f x]^5 \operatorname{Hypergeometric2F1}\left[\frac{5}{2},-\frac{1}{2}-m, \frac{7}{2}, \frac{1}{2}(1-\sin [e+f x])\right](1+\sin [e+f x])^{-\frac{1}{2}-m}(a+a \sin [e+f x])^{-2+m}$$

Result (type 1, 1 leaves):

???

■ **Problem 69: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [e+f x]^2(a+a \sin [e+f x])^m d x$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{1}{3f} 2^{\frac{3}{2}+m} a \operatorname{Cos}[e+fx]^3 \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -\frac{1}{2}-m, \frac{5}{2}, \frac{1}{2}(1-\operatorname{Sin}[e+fx])\right] (1+\operatorname{Sin}[e+fx])^{-\frac{1}{2}-m} (a+a\operatorname{Sin}[e+fx])^{-1+m}$$

Result (type 6, 6167 leaves):

$$\begin{aligned} & -\left(\left(192 \left(\operatorname{Cos}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2\right)^{5+2m} \operatorname{Cos}[e+fx]^2 (a+a\operatorname{Sin}[e+fx])^m \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right. \\ & \left.\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^{2(1+m)} \left(-\left(\operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 5+2m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right)/\right. \\ & \left.\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 5+2m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]-\right. \right. \\ & \left.\left.2\left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 5+2m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right)+(5+2m)\right. \right. \\ & \left.\left.\operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 2(3+m), \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)\right) + \\ & \left(\operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 2(2+m), \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)/ \\ & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 2(2+m), \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]-\right. \\ & \left.4\left((1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 4+2m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]+ \right. \right. \\ & \left.\left.(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 5+2m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)\right)/ \\ & \left(f\left(48 \operatorname{Cos}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^8 \left(\operatorname{Cos}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^{2m} \left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^{2(1+m)}\right. \\ & \left.-\left(\operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 5+2m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right)/\right. \\ & \left.\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 5+2m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]-2\right. \right. \\ & \left.\left.2\left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 5+2m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right)+(5+2m)\right. \right. \\ & \left.\left.\operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 2(3+m), \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)\right) + \\ & \left(\operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 2(2+m), \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)/ \\ & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 2(2+m), \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]-\right. \end{aligned}$$

$$\begin{aligned}
& (2+m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 5+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \\
& \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + 3\left(-\frac{1}{3}(2+m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 1+2(2+m), \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{3}(1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1-2(1+m), \right. \right. \\
& \quad \left. \left. 2(2+m), \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) - 4 \\
& \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left((1+m) \left(-\frac{3}{10}(4+2m) \operatorname{AppellF1}\left[\frac{5}{2}, -1-2m, 5+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \frac{3}{10}(-1-2m) \operatorname{AppellF1}\left[\frac{5}{2}, -2m, \right. \right. \\
& \quad \left. \left. 4+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) + \\
& (2+m) \left(-\frac{3}{10}(5+2m) \operatorname{AppellF1}\left[\frac{5}{2}, -2(1+m), 6+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{3}{5}(1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 1-2(1+m), 5+2m, \frac{7}{2}, \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) \right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 2(2+m), \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] - 4 \right. \\
& \quad \left. \left((1+m) \operatorname{AppellF1}\left[\frac{3}{2}, -1-2m, 4+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (2+m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 5+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 70: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^2 (a+a \sin[e+fx])^m}{c-c \sin[e+fx]} dx$$

Optimal (type 5, 77 leaves, 3 steps):

$$-\frac{1}{cf} 2^{\frac{3}{2}+m} \cos[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1}{2}-m, \frac{3}{2}, \frac{1}{2}(1-\sin[e+fx])\right] (1+\sin[e+fx])^{-\frac{1}{2}-m} (a+a \sin[e+fx])^m$$

Result (type 6, 6494 leaves):

$$-\frac{1}{f(c-c \sin[e+fx])} \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-2m} \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^2 (a+a \sin[e+fx])^m$$

$$\begin{aligned}
& \left(-\frac{1}{2} i \left(-\frac{1}{1+m} 4^{-m} e^{-i(-e+\frac{\pi}{2}-fx)} \left(1 + e^{i(-e+\frac{\pi}{2}-fx)} \right)^{-2m} \left(e^{-\frac{1}{2}i(-e+\frac{\pi}{2}-fx)} \left(1 + e^{i(-e+\frac{\pi}{2}-fx)} \right) \right)^{2m} \text{Hypergeometric2F1}[-1-m, -2m, -m, -e^{i(-e+\frac{\pi}{2}-fx)}] - \frac{1}{-1+m} \right. \right. \\
& \quad \left. \left. 4^{-m} e^{i(-e+\frac{\pi}{2}-fx)} \left(1 + e^{i(-e+\frac{\pi}{2}-fx)} \right)^{-2m} \left(e^{-\frac{1}{2}i(-e+\frac{\pi}{2}-fx)} \left(1 + e^{i(-e+\frac{\pi}{2}-fx)} \right) \right)^{2m} \text{Hypergeometric2F1}[1-m, -2m, 2-m, -e^{i(-e+\frac{\pi}{2}-fx)}] \right) - \right. \\
& \quad \left(2 \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right] \right)^{3+2m} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (3+2m), \frac{1}{2} (5+2m), \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right] \right) / \\
& \quad \left((3+2m) \sqrt{\sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2} \right) + \\
& \quad \left(8 \left(\cos \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \right)^{3+2m} \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^{2m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \right)^{2m} \right. \\
& \quad \left(\left(3 \text{AppellF1} \left[\frac{1}{2}, -2m, 2+2m, \frac{3}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \sec \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) / \right. \\
& \quad \left(3 \text{AppellF1} \left[\frac{1}{2}, -2m, 2+2m, \frac{3}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] - \right. \\
& \quad \left. 4 \left(m \text{AppellF1} \left[\frac{3}{2}, 1-2m, 2+2m, \frac{5}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right. \\
& \quad \left. (1+m) \text{AppellF1} \left[\frac{3}{2}, -2m, 3+2m, \frac{5}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 - \text{AppellF1} \left[\right. \\
& \quad \left. \frac{1}{2}, -2m, 3+2m, \frac{3}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] / \left(\text{AppellF1} \left[\frac{1}{2}, -2m, 3+2m, \frac{3}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] - \frac{2}{3} \left(2m \text{AppellF1} \left[\frac{3}{2}, 1-2m, 3+2m, \frac{5}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (3+2m) \text{AppellF1} \left[\frac{3}{2}, -2m, 4+2m, \frac{5}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) / \\
& \quad \left(2 \cos \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \right)^4 \left(\cos \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \right)^{2m} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \right)^{2m} \\
& \quad \left(\left(3 \text{AppellF1} \left[\frac{1}{2}, -2m, 2+2m, \frac{3}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \sec \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) / \right. \\
& \quad \left(3 \text{AppellF1} \left[\frac{1}{2}, -2m, 2+2m, \frac{3}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] - \right. \\
& \quad \left. 4 \left(m \text{AppellF1} \left[\frac{3}{2}, 1-2m, 2+2m, \frac{5}{2}, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + (1+m) \text{AppellF1} \left[\frac{3}{2}, -2m, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{1}{3} \left(2m \operatorname{AppellF1}\left[\frac{3}{2}, 1-2m, 3+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \quad \left. (3+2m) \operatorname{AppellF1}\left[\frac{3}{2}, -2m, 4+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \\
& \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{2}{3} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \left(2m \left(-\frac{3}{10} (3+2m) \operatorname{AppellF1}\left[\frac{5}{2}, 1-2m, 4+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{3}{10} (1-2m) \operatorname{AppellF1}\left[\frac{5}{2}, 2-2m, 3+2m, \right. \right. \\
& \quad \left. \left. \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) + (3+2m) \\
& \quad \left(-\frac{3}{5} m \operatorname{AppellF1}\left[\frac{5}{2}, 1-2m, 4+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{3}{10} (4+2m) \operatorname{AppellF1}\left[\frac{5}{2}, -2m, 5+2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \\
& \quad \left. \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \Big/ \left(\operatorname{AppellF1}\left[\frac{1}{2}, -2m, 3+2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \frac{2}{3} \left(2m \operatorname{AppellF1}\left[\frac{3}{2}, 1-2m, 3+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (3+2m) \operatorname{AppellF1}\left[\frac{3}{2}, -2m, 4+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \Big)
\end{aligned}$$

■ **Problem 71: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^2 (a+a \sin[e+fx])^m}{(c-c \sin[e+fx])^2} dx$$

Optimal (type 5, 81 leaves, 4 steps):

$$\frac{1}{a c^2 f} 2^{\frac{3}{2}+m} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{1}{2}-m, \frac{1}{2}, \frac{1}{2} (1-\sin[e+fx])\right] \sec[e+fx] (1+\sin[e+fx])^{-\frac{1}{2}-m} (a+a \sin[e+fx])^{1+m}$$

Result (type 6, 6360 leaves):

$$\begin{aligned}
& - \left(\left(\cos\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right)^{2m} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 (a+a \sin[e+fx])^m \frac{\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right]^2}{\left(\cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right]\right)^2} + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{3}{10}(-1 - 2m) \operatorname{AppellF1}\left[\frac{5}{2}, -2m, 1 + 2m, \frac{7}{2}, \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + \\
& (1 + 2m) \left(-\frac{3}{5}(1 + m) \operatorname{AppellF1}\left[\frac{5}{2}, -2(1 + m), 1 + 2(1 + m), \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \\
& \left(\operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{3}{5}(1 + m) \operatorname{AppellF1}\left[\frac{5}{2}, 1 - 2(1 + m), 2(1 + m), \frac{7}{2}, \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) / \\
& \left(-3 \operatorname{AppellF1}\left[\frac{1}{2}, -2(1 + m), 1 + 2m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + 2 \right. \\
& \left. \left(2(1 + m) \operatorname{AppellF1}\left[\frac{3}{2}, -1 - 2m, 1 + 2m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (1 + 2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -2(1 + m), 2(1 + m), \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right)
\end{aligned}$$

■ **Problem 72: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^2 (a + a \sin[e + f x])^m}{(c - c \sin[e + f x])^3} dx$$

Optimal (type 5, 86 leaves, 4 steps):

$$\frac{1}{3 a^2 c^3 f} 2^{\frac{3}{2}+m} \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{1}{2}(1 - \sin[e + f x])\right] \operatorname{Sec}[e + f x]^3 (1 + \sin[e + f x])^{-\frac{1}{2}-m} (a + a \sin[e + f x])^{2+m}$$

Result (type 6, 6298 leaves):

$$\begin{aligned}
& - \left(\left(\operatorname{Cos}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{2m} \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^3 \right. \\
& \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right)^6 (a + a \sin[e + f x])^m \left(\frac{\operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^2}{\left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] - \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]\right)^4} + \right. \\
& \left. \frac{2 \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]}{\left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] - \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]\right)^4} + \frac{\operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^2}{\left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] - \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]\right)^4} \right) \\
& \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{2(1+m)} \left(- \left(\operatorname{AppellF1}\left[-\frac{3}{2}, -2(1 + m), 2m, -\frac{1}{2}, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] / \right. \right.
\end{aligned}$$

$$\frac{1}{6} (-1 - 2m) \operatorname{AppellF1}\left[\frac{3}{2}, -2m, 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \\ \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + m \left(-\frac{1}{6} (1 + 2m) \operatorname{AppellF1}\left[\frac{3}{2}, -2(1+m), 2+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \\ \left. \left. -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{1}{3} (1+m) \operatorname{AppellF1}\left[\frac{3}{2}, 1-2(1+m), \right. \right. \\ \left. \left. 1+2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)\right) / \\ \left(\operatorname{AppellF1}\left[-\frac{1}{2}, -2(1+m), 2m, \frac{1}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - 4 \left((1+m) \operatorname{AppellF1}\left[\frac{1}{2}, -1-2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \right. \right. \\ \left. \left. -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\ \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}, -2(1+m), 1+2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\right)$$

- **Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[e + fx]^2 (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{5/2} dx$$

Optimal (type 3, 244 leaves, 5 steps):

$$\frac{768 c^3 \cos[e + fx] (a + a \sin[e + fx])^{1+m}}{a f (7 + 2m) (9 + 2m) (15 + 16m + 4m^2) \sqrt{c - c \sin[e + fx]}} + \frac{192 c^2 \cos[e + fx] (a + a \sin[e + fx])^{1+m} \sqrt{c - c \sin[e + fx]}}{a f (9 + 2m) (35 + 24m + 4m^2)} + \\ \frac{24 c \cos[e + fx] (a + a \sin[e + fx])^{1+m} (c - c \sin[e + fx])^{3/2}}{a f (63 + 32m + 4m^2)} + \frac{2 \cos[e + fx] (a + a \sin[e + fx])^{1+m} (c - c \sin[e + fx])^{5/2}}{a f (9 + 2m)}$$

Result (type 3, 695 leaves):

$$\begin{aligned}
& \frac{1}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5} \\
& (a(1+\sin[e+fx]))^m (c-c\sin[e+fx])^{5/2} \left(\frac{\left(2205+590m+108m^2+8m^3\right) \left(\left(\frac{3}{8}+\frac{3i}{8}\right)\cos\left[\frac{1}{2}(e+fx)\right] + \left(\frac{3}{8}-\frac{3i}{8}\right)\sin\left[\frac{1}{2}(e+fx)\right]\right)}{(3+2m)(5+2m)(7+2m)(9+2m)} + \right. \\
& \frac{\left(2205+590m+108m^2+8m^3\right) \left(\left(\frac{3}{8}-\frac{3i}{8}\right)\cos\left[\frac{1}{2}(e+fx)\right] + \left(\frac{3}{8}+\frac{3i}{8}\right)\sin\left[\frac{1}{2}(e+fx)\right]\right)}{(3+2m)(5+2m)(7+2m)(9+2m)} + \\
& \frac{\left(191m+48m^2+4m^3\right) \left((1-i)\cos\left[\frac{3}{2}(e+fx)\right] - (1+i)\sin\left[\frac{3}{2}(e+fx)\right]\right)}{(3+2m)(5+2m)(7+2m)(9+2m)} + \\
& \frac{\left(191m+48m^2+4m^3\right) \left((1+i)\cos\left[\frac{3}{2}(e+fx)\right] - (1-i)\sin\left[\frac{3}{2}(e+fx)\right]\right)}{(3+2m)(5+2m)(7+2m)(9+2m)} + \\
& \frac{(21+2m) \left(\left(\frac{3}{2}+\frac{3i}{2}\right)\cos\left[\frac{5}{2}(e+fx)\right] + \left(\frac{3}{2}-\frac{3i}{2}\right)\sin\left[\frac{5}{2}(e+fx)\right]\right)}{(5+2m)(7+2m)(9+2m)} + \frac{(21+2m) \left(\left(\frac{3}{2}-\frac{3i}{2}\right)\cos\left[\frac{5}{2}(e+fx)\right] + \left(\frac{3}{2}+\frac{3i}{2}\right)\sin\left[\frac{5}{2}(e+fx)\right]\right)}{(5+2m)(7+2m)(9+2m)} + \\
& \frac{(15+2m) \left(\left(\frac{3}{16}-\frac{3i}{16}\right)\cos\left[\frac{7}{2}(e+fx)\right] - \left(\frac{3}{16}+\frac{3i}{16}\right)\sin\left[\frac{7}{2}(e+fx)\right]\right)}{(7+2m)(9+2m)} + \frac{(15+2m) \left(\left(\frac{3}{16}+\frac{3i}{16}\right)\cos\left[\frac{7}{2}(e+fx)\right] - \left(\frac{3}{16}-\frac{3i}{16}\right)\sin\left[\frac{7}{2}(e+fx)\right]\right)}{(7+2m)(9+2m)} + \\
& \left. \frac{\left(-\frac{1}{16}+\frac{i}{16}\right)\cos\left[\frac{9}{2}(e+fx)\right] - \left(\frac{1}{16}+\frac{i}{16}\right)\sin\left[\frac{9}{2}(e+fx)\right]}{9+2m} + \frac{\left(-\frac{1}{16}-\frac{i}{16}\right)\cos\left[\frac{9}{2}(e+fx)\right] - \left(\frac{1}{16}-\frac{i}{16}\right)\sin\left[\frac{9}{2}(e+fx)\right]}{9+2m} \right)
\end{aligned}$$

■ **Problem 77: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^2 (a+a\sin[e+fx])^m}{(c-c\sin[e+fx])^{3/2}} dx$$

Optimal (type 5, 76 leaves, 4 steps):

$$\frac{\cos[e+fx] \operatorname{Hypergeometric2F1}\left[1, \frac{3}{2}+m, \frac{5}{2}+m, \frac{1}{2}(1+\sin[e+fx])\right] (a+a\sin[e+fx])^{1+m}}{a c f (3+2m) \sqrt{c-c\sin[e+fx]}}$$

Result (type 6, 3587 leaves):

$$\begin{aligned}
& \left(2\sqrt{2} (2+m) \operatorname{AppellF1}\left[3+2m, 2+2m, 1, 4+2m, \frac{1}{2}\left(1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^2, 1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^3 \right. \\
& \left. \cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^3 (a+a\sin[e+fx])^m \frac{\left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^{2m} \cos\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right]^2}{\cos\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right] - \sin\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right]} + \right.
\end{aligned}$$

$$\begin{aligned} & \left((3 + 2m) \left(2(2 + m) \operatorname{AppellF1} \left[3 + 2m, 2 + 2m, 1, 4 + 2m, \frac{1}{2} \left(1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right. \right. \\ & \quad \left. \left. \operatorname{Cos} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 + \left(\operatorname{AppellF1} \left[4 + 2m, 2 + 2m, 2, 5 + 2m, \frac{1}{2} \left(1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + (1 + m) \right. \right. \right. \\ & \quad \left. \left. \left. \operatorname{AppellF1} \left[4 + 2m, 3 + 2m, 1, 5 + 2m, \frac{1}{2} \left(1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) \right) \end{aligned}$$

■ **Problem 78: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e + fx]^2 (a + a \operatorname{Sin}[e + fx])^m}{(c - c \operatorname{Sin}[e + fx])^{5/2}} dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{\operatorname{Cos}[e + fx] \operatorname{Hypergeometric2F1} \left[2, \frac{3}{2} + m, \frac{5}{2} + m, \frac{1}{2} (1 + \operatorname{Sin}[e + fx]) \right] (a + a \operatorname{Sin}[e + fx])^{1+m}}{2 a c^2 f (3 + 2m) \sqrt{c - c \operatorname{Sin}[e + fx]}}$$

Result (type 6, 7834 leaves):

$$\begin{aligned} & - \left(\left(\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^{-2m} \left(\operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] - \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] \right) \right)^5 (a + a \operatorname{Sin}[e + fx])^m \right. \\ & \quad \left(\frac{\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^{2m} \operatorname{Cos} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right]^2}{\left(\operatorname{Cos} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] - \operatorname{Sin} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] \right)^3} + \frac{2 \operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^{2m} \operatorname{Cos} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] \operatorname{Sin} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right]}{\left(\operatorname{Cos} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] - \operatorname{Sin} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] \right)^3} + \right. \\ & \quad \left. \frac{\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^{2m} \operatorname{Sin} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right]^2}{\left(\operatorname{Cos} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] - \operatorname{Sin} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] \right)^3} \right) \left(\frac{1 - \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2} \right)^{2m} \\ & \quad \left(-\operatorname{AppellF1} \left[1, -2m, 2m, 2, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] / \left(-m \left(\operatorname{AppellF1} \left[2, 1 - 2m, 2m, 3, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \operatorname{AppellF1} \left[2, -2m, 1 + 2m, 3, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \right) + \\ & \quad \operatorname{AppellF1} \left[1, -2m, 2m, 2, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) + \\ & \quad \left(\operatorname{AppellF1} \left[1, -2m, 2m, 2, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) / \\ & \quad \left(\operatorname{AppellF1} \left[1, -2m, 2m, 2, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] - \right. \\ & \quad \left. m \left(\operatorname{AppellF1} \left[2, 1 - 2m, 2m, 3, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[2, -2m, 1+2m, 3, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 + \\
& \left(4(1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2}-\frac{1}{2}\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, 1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \\
& \left. \cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(-1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)\right) / \\
& \left((1+2m) \left(-2(1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2}-\frac{1}{2}\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, 1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right. \\
& \left. \left(\text{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2}-\frac{1}{2}\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, 1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + m \text{AppellF1}\left[2+2m, 1+2m, \right. \right. \right. \\
& \left. \left. \left. 1, 3+2m, \frac{1}{2}-\frac{1}{2}\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, 1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right)\right) \left(-1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)\right) / \left(4\sqrt{2}f\right) \\
& (c - c \sin[e + fx])^{5/2} \left(\frac{1}{2\sqrt{2}} m \left(\frac{1 - \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{1 + \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}\right)^{-1+2m} \left(-\frac{\sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(1 - \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)}{2 \left(1 + \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^2} \right. \right. \\
& \left. \left. \frac{\sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]}{2 \left(1 + \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)}\right) \left(-\text{AppellF1}\left[1, -2m, 2m, 2, \cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] / \right. \right. \\
& \left. \left(-m \left(\text{AppellF1}\left[2, 1-2m, 2m, 3, \cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right. \\
& \left. \left. \text{AppellF1}\left[2, -2m, 1+2m, 3, \cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right) + \right. \\
& \left. \text{AppellF1}\left[1, -2m, 2m, 2, \cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) + \\
& \left(\text{AppellF1}\left[1, -2m, 2m, 2, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) / \\
& \left(\text{AppellF1}\left[1, -2m, 2m, 2, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] - \right. \\
& \left. m \left(\text{AppellF1}\left[2, 1-2m, 2m, 3, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \text{AppellF1}\left[2, -2m, 1+2m, 3, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 + \left(4(1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2}-\right. \right. \right. \\
& \left. \left. \left. \frac{1}{2}\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, 1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(-1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)\right) / \right)
\end{aligned}$$

$$1, 3+2m, \frac{1}{2} - \frac{1}{2} \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\left(-1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^2\right)^2\right)^2\right)^2\right)\right)\right)\right)$$

■ **Problem 83: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^2 (a+a\sin[e+fx])^m (c-c\sin[e+fx])^{-3-m} dx$$

Optimal (type 3, 54 leaves, 2 steps):

$$\frac{\cos[e+fx] (a+a\sin[e+fx])^{1+m} (c-c\sin[e+fx])^{-2-m}}{acf(3+2m)}$$

Result (type 3, 109 leaves):

$$\frac{1}{c^3 f (3+2m)} 2^{-m} \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^{-3-2m} \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^{2m} \\ (a(1+\sin[e+fx]))^m (c-c\sin[e+fx])^{-m} \sin\left[\frac{1}{4}(2e+\pi+2fx)\right]^3$$

■ **Problem 84: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^2 (a+a\sin[e+fx])^m (c-c\sin[e+fx])^{-2-m} dx$$

Optimal (type 5, 113 leaves, 5 steps):

$$\frac{1}{f(3+2m)} 2^{-\frac{1}{2}-m} \cos[e+fx]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(3+2m), \frac{1}{2}(3+2m), \frac{1}{2}(5+2m), \frac{1}{2}(1+\sin[e+fx])\right] \\ (1-\sin[e+fx])^{\frac{1}{2}+m} (a+a\sin[e+fx])^m (c-c\sin[e+fx])^{-2-m}$$

Result (type 6, 5056 leaves):

$$-\left(\left(2^{-m} \left(\cos\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{2m} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right. \right. \\ \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^{-2(-2-m)} (a+a\sin[e+fx])^m (c-c\sin[e+fx])^{-2-m} \right. \\ \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right]^2 \left(\cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right]\right)^{-2-2m} + \right. \\ \left. 2 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2m} \cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] \left(\cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right]\right)^{-2-2m} \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] \right. \\ \left. \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2m} \left(\cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right]\right)^{-2-2m} \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right]^2 \right)$$

$$\begin{aligned}
& \left. \text{AppellF1}\left[\frac{3}{2}-m, -2(1+m), 2, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) \right) - \\
& 2^{-2-m} \left(\cos\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{2m} \csc\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \sin\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-2m} \\
& \left(-\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2}-m, -2(1+m), \frac{1}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]}{1+2m} + \left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2(1+m), 1, \frac{3}{2}-m, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] \sin\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^{2(1+m)}\right) \right) / \\
& \left((-1+2m) \left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2(1+m), 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] + 2 \right. \right. \\
& \left. \left. \left(2(1+m) \text{AppellF1}\left[\frac{3}{2}-m, -1-2m, 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] + \text{AppellF1}\left[\frac{3}{2}-m, -2(1+m), \right. \right. \right. \right. \\
& \left. \left. \left. 2, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] \right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) \right) + 2^{-m} \left(\cos\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{2m} \\
& \cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \sin\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-2m} \left(\left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2(1+m), 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] \cos\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \sin\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)^{2(1+m)}\right) \right) / \\
& \left(2(-1+2m) \left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2(1+m), 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] + 2 \right. \right. \\
& \left. \left. \left(2(1+m) \text{AppellF1}\left[\frac{3}{2}-m, -1-2m, 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] + \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{3}{2}-m, -2(1+m), 2, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] \right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) \right) + \\
& \left((-3+2m) \sin\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(-\frac{1}{2\left(\frac{3}{2}-m\right)} \left(\frac{1}{2}-m\right) \text{AppellF1}\left[\frac{3}{2}-m, -2(1+m), 2, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{\frac{3}{2}-m} \left(\frac{1}{2}-m\right) (1+m) \text{AppellF1}\left[\frac{3}{2}-m, 1-2(1+m), \right. \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{2(1+m)} \Big/ \left((-1+2m) \left((-3+2m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2(1+m), 1, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + 2 \left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}-m, -1-2m, 1, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \quad \left. \operatorname{AppellF1}\left[\frac{3}{2}-m, -2(1+m), 2, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) - \\
& \left((1+m) (-3+2m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2(1+m), 1, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \\
& \quad \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^3 \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{-1+2(1+m)} \Big/ \\
& \left((-1+2m) \left((-3+2m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2(1+m), 1, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + 2 \right. \\
& \quad \left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}-m, -1-2m, 1, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) + \\
& \quad \left. \operatorname{AppellF1}\left[\frac{3}{2}-m, -2(1+m), 2, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) - \\
& \frac{1}{2(1+2m)} \left(-\frac{1}{2}-m\right) \operatorname{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}-m, -2(1+m), \frac{1}{2}- \right. \right. \\
& \quad \left. \left. m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{2(1+m)}\right) - \\
& \left((-3+2m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2(1+m), 1, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \operatorname{Sin}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \\
& \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{2(1+m)} \left(\left(2(1+m) \operatorname{AppellF1}\left[\frac{3}{2}-m, -1-2m, 1, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}-m, -2(1+m), 2, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \right) \\
& \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + (-3+2m) \left(-\frac{1}{2\left(\frac{3}{2}-m\right)}\left(\frac{1}{2}-m\right) \operatorname{AppellF1}\left[\frac{3}{2}-m, -2(1+m), 2, \right. \right. \\
& \quad \left. \left. \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{3}{2}-m} \\
& \left(\frac{1}{2}-m\right) (1+m) \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2(1+m), 1, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)
\end{aligned}$$

$$\left. \left(\text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + 2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(-\frac{1}{\frac{5}{2} - m}\left(\frac{3}{2} - m\right) \text{AppellF1}\left[\frac{5}{2} - m, -2(1+m), 3, \frac{7}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{5}{2} - m}\left(\frac{3}{2} - m\right) (1+m) \text{AppellF1}\left[\frac{5}{2} - m, 1 - 2(1+m), 2, \frac{7}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + 2(1+m) \left(-\frac{1}{2\left(\frac{5}{2} - m\right)}\left(\frac{3}{2} - m\right) \text{AppellF1}\left[\frac{5}{2} - m, -1 - 2m, 2, \frac{7}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{2\left(\frac{5}{2} - m\right)}(-1 - 2m) \left(\frac{3}{2} - m\right) \text{AppellF1}\left[\frac{5}{2} - m, -2m, 1, \frac{7}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \Bigg/$$

$$\left((-1 + 2m) \left((-3 + 2m) \text{AppellF1}\left[\frac{1}{2} - m, -2(1+m), 1, \frac{3}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + 2 \left(2(1+m) \text{AppellF1}\left[\frac{3}{2} - m, -1 - 2m, 1, \frac{5}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2} - m, -2(1+m), 2, \frac{5}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right)$$

Problem 85: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Cos}[e + f x]^2 (a + a \text{Sin}[e + f x])^m (c - c \text{Sin}[e + f x])^{-1-m} dx$$

Optimal (type 5, 114 leaves, 5 steps):

$$\frac{1}{f(3+2m)} 2^{\frac{1}{2}-m} c \text{Cos}[e + f x]^3 \text{Hypergeometric2F1}\left[\frac{1}{2}(1+2m), \frac{1}{2}(3+2m), \frac{1}{2}(5+2m), \frac{1}{2}(1+\text{Sin}[e + f x])\right] (1 - \text{Sin}[e + f x])^{\frac{1}{2}+m} (a + a \text{Sin}[e + f x])^m (c - c \text{Sin}[e + f x])^{-2-m}$$

Result (type 6, 11925 leaves):

$$-\left(8^{1-m} (-3 + 2m) \text{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right] \right)^{-2(-1-m)} (a + a \text{Sin}[e + f x])^m (c - c \text{Sin}[e + f x])^{-1-m} \left(\text{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \text{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]^2 \left(\text{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] - \text{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] \right)^{-2m} + \right)$$

$$\begin{aligned}
& \left((-3+2m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2m, 2, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \\
& 4 \left(m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2m, 2, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-m, -2m, 3, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) - \\
& \left(4 \operatorname{AppellF1}\left[\frac{1}{2}-m, -2m, 3, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) / \\
& \left((-3+2m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2m, 3, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \\
& 2 \left(2m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2m, 3, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \\
& \quad \left. \left. 3 \operatorname{AppellF1}\left[\frac{3}{2}-m, -2m, 4, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \Bigg) + \\
& \frac{1}{(-1+2m) \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^3} 8^{1-m} (-3+2m) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]}{1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2} \right)^{-2m} \\
& \left(\frac{1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2} \right)^{2m} \\
& - \left(\left(\operatorname{AppellF1}\left[\frac{1}{2}-m, -2m, 1, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right. \right. \\
& \quad \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \Bigg) / \left((-3+2m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2m, 1, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + 2 \left(2m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2m, 1, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-m, -2m, 2, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \Bigg) - \\
& \left(\left(-\frac{1}{\frac{3}{2}-m} \left(\frac{1}{2}-m \right) m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2m, 1, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{2\left(\frac{3}{2}-m\right)} \left(\frac{1}{2}-m \right) \operatorname{AppellF1}\left[\frac{3}{2}-m, -2m, 2, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}-m, -2m, 3, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) + \\
& \left(5 \text{AppellF1}\left[\frac{1}{2}-m, -2m, 3, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \left(1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^2\right) / \right. \\
& \left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2m, 3, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + 2 \right. \\
& \left. \left(2m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 3, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right. \\
& \left. \left. 3 \text{AppellF1}\left[\frac{3}{2}-m, -2m, 4, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) - \\
& \left(8 \text{AppellF1}\left[\frac{1}{2}-m, -2m, 4, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \left(1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)\right) / \right. \\
& \left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2m, 4, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + 4 \right. \\
& \left. \left(m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 4, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right. \\
& \left. \left. 2 \text{AppellF1}\left[\frac{3}{2}-m, -2m, 5, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) + \\
& \left(4 \text{AppellF1}\left[\frac{1}{2}-m, -2m, 5, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right) / \right. \\
& \left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2m, 5, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + 2 \right. \\
& \left. \left(2m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 5, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right. \\
& \left. \left. 5 \text{AppellF1}\left[\frac{3}{2}-m, -2m, 6, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) \right) + \\
& \frac{1}{(-1+2m) \left(1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^5} 2^{4-3m} (-3+2m) \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(\frac{\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]}{1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}\right)^{-2m} \\
& \left(\frac{1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}\right)^{2m} \\
& \left(-\left(\left(\text{AppellF1}\left[\frac{1}{2}-m, -2m, 2, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \left(1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^3\right)\right) / \right.
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)+4 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \\
& \left(m\left(-\frac{1}{\frac{5}{2}-m} 2\left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m, 1-2 m, 5, \frac{7}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right.\right. \\
& \left.\left.\operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]+\frac{1}{2\left(\frac{5}{2}-m\right)}(1-2 m)\left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m, 2-2 m, 4, \right.\right.\right. \\
& \left.\left.\frac{7}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\right)+ \\
& 2\left(-\frac{1}{\frac{5}{2}-m}\left(\frac{3}{2}-m\right) m \operatorname{AppellF1}\left[\frac{5}{2}-m, 1-2 m, 5, \frac{7}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{2\left(\frac{5}{2}-m\right)} 5\left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m, -2 m, 6, \frac{7}{2}-m, \right.\right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\right)\right)\right)\right) / \\
& \left((-3+2 m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2 m, 4, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+ \right. \\
& \left.4\left(m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2 m, 4, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+ \right. \right. \\
& \left. \left.2 \operatorname{AppellF1}\left[\frac{3}{2}-m, -2 m, 5, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^2 - \\
& \left(4 \operatorname{AppellF1}\left[\frac{1}{2}-m, -2 m, 5, \frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right) \\
& \left(\left(2 m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2 m, 5, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+5 \operatorname{AppellF1}\left[\frac{3}{2}-m, -2 m, \right.\right. \right. \\
& \left.\left.\frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]+ \right. \\
& \left.(-3+2 m)\left(-\frac{1}{\frac{3}{2}-m}\left(\frac{1}{2}-m\right) m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2 m, 5, \frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{2\left(\frac{3}{2} - m\right)} 5 \left(\frac{1}{2} - m\right) \text{AppellF1}\left[\frac{3}{2} - m, -2m, 6, \frac{5}{2} - m, \right. \\
& \left. \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right] + \\
& 2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(2m \left(-\frac{1}{2\left(\frac{5}{2} - m\right)} 5 \left(\frac{3}{2} - m\right) \text{AppellF1}\left[\frac{5}{2} - m, 1 - 2m, 6, \frac{7}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right. \right. \right. \\
& \left. \left. -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{2\left(\frac{5}{2} - m\right)} (1 - 2m) \left(\frac{3}{2} - m\right) \text{AppellF1}\left[\frac{5}{2} - m, \right. \right. \right. \\
& \left. \left. 2 - 2m, 5, \frac{7}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + \\
& 5 \left(-\frac{1}{\frac{5}{2} - m} \left(\frac{3}{2} - m\right) m \text{AppellF1}\left[\frac{5}{2} - m, 1 - 2m, 6, \frac{7}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{\frac{5}{2} - m} 3 \left(\frac{3}{2} - m\right) \text{AppellF1}\left[\frac{5}{2} - m, -2m, 7, \frac{7}{2} - m, \right. \right. \\
& \left. \left. \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \Bigg) / \\
& \left((-3 + 2m) \text{AppellF1}\left[\frac{1}{2} - m, -2m, 5, \frac{3}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \left. 2 \left(2m \text{AppellF1}\left[\frac{3}{2} - m, 1 - 2m, 5, \frac{5}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\
& \left. \left. 5 \text{AppellF1}\left[\frac{3}{2} - m, -2m, 6, \frac{5}{2} - m, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 87: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[e + f x]^2 (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{1-m} dx$$

Optimal (type 5, 116 leaves, 5 steps):

$$\frac{1}{f(3+2m)} 2^{\frac{5}{2}-m} c^3 \cos[e + f x]^3 \text{Hypergeometric2F1}\left[\frac{1}{2}(-3+2m), \frac{1}{2}(3+2m), \frac{1}{2}(5+2m), \frac{1}{2}(1+\sin[e + f x])\right]$$

$$(1 - \sin[e + f x])^{\frac{1}{2}+m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-2-m}$$

$$\begin{aligned}
& \left. \left(\sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{1}{\frac{5}{2}-m} 3 \left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m, -2m, 7, \frac{7}{2}-m, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right) \Big/ \\
& \left((-3+2m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2m, 5, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \left. 2 \left(2m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2m, 5, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\
& \left. \left. 5 \operatorname{AppellF1}\left[\frac{3}{2}-m, -2m, 6, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^2 - \\
& \left(12 \operatorname{AppellF1}\left[\frac{1}{2}-m, -2m, 6, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right. \\
& \left. \left(2 \left(m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2m, 6, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + 3 \operatorname{AppellF1}\left[\frac{3}{2}-m, -2m, \right. \right. \right. \right. \\
& \left. \left. \left. 7, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \right. \\
& \left. (-3+2m) \left(-\frac{1}{\frac{3}{2}-m} \left(\frac{1}{2}-m\right) m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2m, 6, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \right. \\
& \left. \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{1}{\frac{3}{2}-m} 3 \left(\frac{1}{2}-m\right) \operatorname{AppellF1}\left[\frac{3}{2}-m, -2m, 7, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) + 4 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \\
& \left(m \left(-\frac{1}{\frac{5}{2}-m} 3 \left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m, 1-2m, 7, \frac{7}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \right. \\
& \left. \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{1}{2\left(\frac{5}{2}-m\right)} (1-2m) \left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m, 2-2m, 6, \right. \right. \\
& \left. \left. \frac{7}{2}-m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& - \left((2 - 2i) c^4 e^{-\frac{3}{2}i(e+fx)} (-i + e^{i(e+fx)}) (g \cos[e+fx])^{3/2} \right. \\
& \quad \left. \left(1 + e^{2i(e+fx)} + (-1 + e^{2ie}) \sqrt{1 + e^{2i(e+fx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(e+fx)} \right] \right) \sqrt{a(1 + \sin[e+fx])} \right) / \\
& \quad \left((-1 + e^{2ie}) \sqrt{i c e^{-i(e+fx)} (-i + e^{i(e+fx)})^2 \sqrt{e^{-i(e+fx)} (1 + e^{2i(e+fx)})} f \cos[e+fx]^{3/2} \left(\cos \left[\frac{e}{2} + \frac{fx}{2} \right] + \sin \left[\frac{e}{2} + \frac{fx}{2} \right] \right) \right) + \\
& \quad \frac{1}{\cos \left[\frac{e}{2} + \frac{fx}{2} \right] + \sin \left[\frac{e}{2} + \frac{fx}{2} \right]} (g \cos[e+fx])^{3/2} \operatorname{Sec}[e+fx] \\
& \quad \left(\frac{58 c^3 \cos \left[\frac{fx}{2} \right] (\cos \left[\frac{e}{2} \right] + \sin \left[\frac{e}{2} \right])}{77 f} - \frac{3 c^3 \cos \left[\frac{3fx}{2} \right] (\cos \left[\frac{3e}{2} \right] - \sin \left[\frac{3e}{2} \right])}{308 f} + \frac{145 c^3 \cos \left[\frac{5fx}{2} \right] (\cos \left[\frac{5e}{2} \right] + \sin \left[\frac{5e}{2} \right])}{924 f} + \right. \\
& \quad \frac{19 c^3 \cos \left[\frac{7fx}{2} \right] (\cos \left[\frac{7e}{2} \right] - \sin \left[\frac{7e}{2} \right])}{264 f} - \frac{c^3 \cos \left[\frac{9fx}{2} \right] (\cos \left[\frac{9e}{2} \right] + \sin \left[\frac{9e}{2} \right])}{88 f} + \frac{58 c^3 (\cos \left[\frac{e}{2} \right] - \sin \left[\frac{e}{2} \right]) \sin \left[\frac{fx}{2} \right]}{77 f} + \\
& \quad \frac{3 c^3 (\cos \left[\frac{3e}{2} \right] + \sin \left[\frac{3e}{2} \right]) \sin \left[\frac{3fx}{2} \right]}{308 f} + \frac{145 c^3 (\cos \left[\frac{5e}{2} \right] - \sin \left[\frac{5e}{2} \right]) \sin \left[\frac{5fx}{2} \right]}{924 f} - \frac{19 c^3 (\cos \left[\frac{7e}{2} \right] + \sin \left[\frac{7e}{2} \right]) \sin \left[\frac{7fx}{2} \right]}{264 f} - \\
& \quad \left. \frac{c^3 (\cos \left[\frac{9e}{2} \right] - \sin \left[\frac{9e}{2} \right]) \sin \left[\frac{9fx}{2} \right]}{88 f} - \frac{2 c^3 \cot[e]}{f (\cos \left[\frac{e}{2} + \frac{fx}{2} \right] - \sin \left[\frac{e}{2} + \frac{fx}{2} \right])} \right) \sqrt{a(1 + \sin[e+fx])} \sqrt{c - c \sin[e+fx]}
\end{aligned}$$

■ **Problem 89: Result unnecessarily involves higher level functions.**

$$\int (g \cos[e+fx])^{3/2} \sqrt{a+a \sin[e+fx]} (c - c \sin[e+fx])^{5/2} dx$$

Optimal (type 4, 290 leaves, 7 steps):

$$\begin{aligned}
& \frac{22 a c^3 (g \cos[e+fx])^{5/2}}{45 f g \sqrt{a+a \sin[e+fx]} \sqrt{c - c \sin[e+fx]}} + \frac{22 a c^3 g \sqrt{\cos[e+fx]} \sqrt{g \cos[e+fx]} \operatorname{EllipticE} \left[\frac{1}{2} (e+fx), 2 \right]}{15 f \sqrt{a+a \sin[e+fx]} \sqrt{c - c \sin[e+fx]}} + \\
& \frac{22 a c^2 (g \cos[e+fx])^{5/2} \sqrt{c - c \sin[e+fx]}}{105 f g \sqrt{a+a \sin[e+fx]}} + \frac{2 a c (g \cos[e+fx])^{5/2} (c - c \sin[e+fx])^{3/2}}{21 f g \sqrt{a+a \sin[e+fx]}} - \frac{2 a (g \cos[e+fx])^{5/2} (c - c \sin[e+fx])^{5/2}}{9 f g \sqrt{a+a \sin[e+fx]}}
\end{aligned}$$

Result (type 5, 205 leaves):

$$\begin{aligned}
& \left(c^3 g^2 \left(\cos \left[\frac{1}{2} (e+fx) \right] - \sin \left[\frac{1}{2} (e+fx) \right] \right) \sqrt{a(1 + \sin[e+fx])} \left(3696 i e^{-i(e+fx)} \sqrt{1 + e^{2i(e+fx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(e+fx)} \right] + \right. \right. \\
& \quad \left. \left. 4 \cos[e+fx] (-924 i + \cos[e+fx] (180 + 180 \cos[2(e+fx)] + 273 \sin[e+fx] - 35 \sin[3(e+fx)])) \right) \right) / \\
& \quad \left(2520 f \sqrt{g \cos[e+fx]} \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right) \sqrt{c - c \sin[e+fx]} \right)
\end{aligned}$$

■ **Problem 90: Result unnecessarily involves higher level functions.**

$$\int (g \cos[e + f x])^{3/2} \sqrt{a + a \sin[e + f x]} (c - c \sin[e + f x])^{3/2} dx$$

Optimal (type 4, 235 leaves, 6 steps):

$$\frac{2 a c^2 (g \cos[e + f x])^{5/2}}{5 f g \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} + \frac{6 a c^2 g \sqrt{\cos[e + f x]} \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e + f x), 2\right]}{5 f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} +$$

$$\frac{6 a c (g \cos[e + f x])^{5/2} \sqrt{c - c \sin[e + f x]}}{35 f g \sqrt{a + a \sin[e + f x]}} - \frac{2 a (g \cos[e + f x])^{5/2} (c - c \sin[e + f x])^{3/2}}{7 f g \sqrt{a + a \sin[e + f x]}}$$

Result (type 5, 147 leaves):

$$\frac{1}{140 f (g \cos[e + f x])^{3/2}} c g^3 \sqrt{a (1 + \sin[e + f x])} \sqrt{c - c \sin[e + f x]} \left(168 i e^{-i(e+f x)} \sqrt{1 + e^{2i(e+f x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(e+f x)}\right] + \right.$$

$$\left. 4 \cos[e + f x] (-42 i + \cos[e + f x] (5 + 5 \cos[2(e + f x)] + 14 \sin[e + f x])) \right)$$

■ **Problem 91: Result unnecessarily involves higher level functions.**

$$\int (g \cos[e + f x])^{3/2} \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]} dx$$

Optimal (type 4, 178 leaves, 5 steps):

$$\frac{2 a c (g \cos[e + f x])^{5/2}}{5 f g \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} +$$

$$\frac{6 a c g \sqrt{\cos[e + f x]} \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e + f x), 2\right]}{5 f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} - \frac{2 a (g \cos[e + f x])^{5/2} \sqrt{c - c \sin[e + f x]}}{5 f g \sqrt{a + a \sin[e + f x]}}$$

Result (type 5, 249 leaves):

$$\frac{1}{40 f} (g \cos[e + f x])^{3/2} \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x]^3 \sqrt{a (1 + \sin[e + f x])}$$

$$\sqrt{c - c \sin[e + f x]} \left(-11 \cos[f x] - 13 \cos[2 e + f x] + \cos[2 e + 3 f x] - \cos[4 e + 3 f x] + \right.$$

$$12 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i f x} (\cos[e] + i \sin[e])^2\right] (\cos[f x] - i \sin[f x]) \sqrt{1 + \cos[2(e + f x)] + i \sin[2(e + f x)]} +$$

$$\left. 4 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i f x} (\cos[e] + i \sin[e])^2\right] (\cos[f x] + i \sin[f x]) \sqrt{1 + \cos[2(e + f x)] + i \sin[2(e + f x)]} \right)$$

■ **Problem 92: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \cos[e + f x])^{3/2} \sqrt{a + a \sin[e + f x]}}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 4, 122 leaves, 4 steps):

$$-\frac{2 a (g \cos[e + f x])^{5/2}}{3 f g \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} + \frac{2 a g \sqrt{\cos[e + f x]} \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e + f x), 2\right]}{f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}}$$

Result (type 5, 147 leaves):

$$-\left(g \sqrt{g \cos[e + f x]} \left(1 + 6 i \cos[e + f x] + \cos[2(e + f x)] - 6 i e^{-i(e + f x)} \sqrt{1 + e^{2i(e + f x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(e + f x)}\right] \right) \sqrt{a(1 + \sin[e + f x])} \right) / \left(3 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \sqrt{c - c \sin[e + f x]} \right)$$

■ **Problem 93: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \cos[e + f x])^{3/2} \sqrt{a + a \sin[e + f x]}}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 4, 123 leaves, 4 steps):

$$\frac{4 a (g \cos[e + f x])^{5/2}}{f g \sqrt{a + a \sin[e + f x]} (c - c \sin[e + f x])^{3/2}} - \frac{6 a g \sqrt{\cos[e + f x]} \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e + f x), 2\right]}{c f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}}$$

Result (type 5, 184 leaves):

$$\left(2 i g \sqrt{g \cos[e + f x]} \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \sqrt{a(1 + \sin[e + f x])} \right. \\ \left. \left(-3 \cos[e + f x] + 3 e^{-i(e + f x)} \sqrt{1 + e^{2i(e + f x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(e + f x)}\right] + 2 i (1 + \sin[e + f x]) \right) \right) / \\ \left(c f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 (-1 + \sin[e + f x]) \sqrt{c - c \sin[e + f x]} \right)$$

■ **Problem 94: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \cos[e + f x])^{3/2} \sqrt{a + a \sin[e + f x]}}{(c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 4, 182 leaves, 5 steps):

$$\frac{4 a (g \cos [e+f x])^{5/2}}{5 f g \sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])^{5/2}} -$$

$$\frac{6 a (g \cos [e+f x])^{5/2}}{5 c f g \sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])^{3/2}} + \frac{6 a g \sqrt{\cos [e+f x]} \sqrt{g \cos [e+f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e+f x), 2\right]}{5 c^2 f \sqrt{a+a \sin [e+f x]} \sqrt{c-c \sin [e+f x]}}$$

Result (type 5, 194 leaves):

$$\left(e^{-\frac{3}{2} i (e+f x)} g^3 \left(-3 - i e^{i (e+f x)} + e^{2 i (e+f x)} - 5 i e^{3 i (e+f x)} - 3 (-i + e^{i (e+f x)})^2 \sqrt{1 + e^{2 i (e+f x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (e+f x)}\right] \right) \right. \\ \left. \left(\cos \left[\frac{1}{2} (e+f x) \right] - i \sin \left[\frac{1}{2} (e+f x) \right] \right) \sqrt{a (1 + \sin [e+f x])} \right) / \left(5 c^2 f (g \cos [e+f x])^{3/2} \sqrt{c-c \sin [e+f x]} \right)$$

■ **Problem 95: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \cos [e+f x])^{3/2} \sqrt{a+a \sin [e+f x]}}{(c-c \sin [e+f x])^{7/2}} dx$$

Optimal (type 4, 237 leaves, 6 steps):

$$\frac{4 a (g \cos [e+f x])^{5/2}}{9 f g \sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])^{7/2}} - \frac{2 a (g \cos [e+f x])^{5/2}}{15 c f g \sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])^{5/2}} -$$

$$\frac{2 a (g \cos [e+f x])^{5/2}}{15 c^2 f g \sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])^{3/2}} + \frac{2 a g \sqrt{\cos [e+f x]} \sqrt{g \cos [e+f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e+f x), 2\right]}{15 c^3 f \sqrt{a+a \sin [e+f x]} \sqrt{c-c \sin [e+f x]}}$$

Result (type 5, 237 leaves):

$$\left(g^2 (\cos [e+f x] - i \sin [e+f x]) \sqrt{a (1 + \sin [e+f x])} \left(7 i + 8 \cos [e+f x] - 13 i \cos [2 (e+f x)] - \right. \right. \\ \left. \left. \frac{3}{2} i e^{-2 i (e+f x)} (-i + e^{i (e+f x)})^4 \sqrt{1 + e^{2 i (e+f x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (e+f x)}\right] + 20 i \sin [e+f x] + 16 \sin [2 (e+f x)] \right) \right) / \\ \left(45 c^3 f \sqrt{g \cos [e+f x]} \left(\cos \left[\frac{1}{2} (e+f x) \right] - \sin \left[\frac{1}{2} (e+f x) \right] \right)^3 \left(\cos \left[\frac{1}{2} (e+f x) \right] + \sin \left[\frac{1}{2} (e+f x) \right] \right) \sqrt{c-c \sin [e+f x]} \right)$$

■ **Problem 96: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \cos [e+f x])^{3/2} \sqrt{a+a \sin [e+f x]}}{(c-c \sin [e+f x])^{9/2}} dx$$

Optimal (type 4, 292 leaves, 7 steps):

$$\frac{4 a (g \cos [e+f x])^{5/2}}{13 f g \sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])^{9/2}} -$$

$$\frac{2 a (g \cos [e+f x])^{5/2}}{39 c f g \sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])^{7/2}} - \frac{2 a (g \cos [e+f x])^{5/2}}{65 c^2 f g \sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])^{5/2}} -$$

$$\frac{2 a (g \cos [e+f x])^{5/2}}{65 c^3 f g \sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])^{3/2}} + \frac{2 a g \sqrt{\cos [e+f x]} \sqrt{g \cos [e+f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e+f x), 2\right]}{65 c^4 f \sqrt{a+a \sin [e+f x]} \sqrt{c-c \sin [e+f x]}}$$

Result (type 5, 255 leaves):

$$\left(g^2 (\cos [e+f x] - i \sin [e+f x]) \sqrt{a(1+\sin [e+f x])} \right.$$

$$\left. \left(54 i + 75 \cos [e+f x] - 66 i \cos [2(e+f x)] + \cos [3(e+f x)] + \frac{3}{2} e^{-3 i(e+f x)} (-i + e^{i(e+f x)})^6 \sqrt{1 + e^{2 i(e+f x)}} \right. \right.$$

$$\left. \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(e+f x)}\right] + 118 i \sin [e+f x] + 84 \sin [2(e+f x)] - 2 i \sin [3(e+f x)] \right) \right) /$$

$$\left(390 c^4 f \sqrt{g \cos [e+f x]} \left(\cos \left[\frac{1}{2}(e+f x) \right] - \sin \left[\frac{1}{2}(e+f x) \right] \right)^5 \left(\cos \left[\frac{1}{2}(e+f x) \right] + \sin \left[\frac{1}{2}(e+f x) \right] \right) \sqrt{c-c \sin [e+f x]} \right)$$

■ **Problem 99: Result unnecessarily involves higher level functions.**

$$\int (g \cos [e+f x])^{3/2} (a+a \sin [e+f x])^{3/2} \sqrt{c-c \sin [e+f x]} dx$$

Optimal (type 4, 235 leaves, 6 steps):

$$-\frac{2 a^2 c (g \cos [e+f x])^{5/2}}{5 f g \sqrt{a+a \sin [e+f x]} \sqrt{c-c \sin [e+f x]}} + \frac{6 a^2 c g \sqrt{\cos [e+f x]} \sqrt{g \cos [e+f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e+f x), 2\right]}{5 f \sqrt{a+a \sin [e+f x]} \sqrt{c-c \sin [e+f x]}}$$

$$-\frac{6 a c (g \cos [e+f x])^{5/2} \sqrt{a+a \sin [e+f x]}}{35 f g \sqrt{c-c \sin [e+f x]}} + \frac{2 c (g \cos [e+f x])^{5/2} (a+a \sin [e+f x])^{3/2}}{7 f g \sqrt{c-c \sin [e+f x]}}$$

Result (type 5, 195 leaves):

$$\left(a^2 g^2 \left(\cos \left[\frac{1}{2}(e+f x) \right] + \sin \left[\frac{1}{2}(e+f x) \right] \right) \sqrt{c-c \sin [e+f x]} \left(168 i e^{-i(e+f x)} \sqrt{1 + e^{2 i(e+f x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(e+f x)}\right] + \right. \right.$$

$$\left. \left. 4 \cos [e+f x] (-42 i + \cos [e+f x] (-5 - 5 \cos [2(e+f x)] + 14 \sin [e+f x])) \right) \right) /$$

$$\left(140 f \sqrt{g \cos [e+f x]} \left(\cos \left[\frac{1}{2}(e+f x) \right] - \sin \left[\frac{1}{2}(e+f x) \right] \right) \sqrt{a(1+\sin [e+f x])} \right)$$

■ **Problem 108: Result unnecessarily involves higher level functions.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^{5/2} \sqrt{c - c \sin[e + f x]} dx$$

Optimal (type 4, 290 leaves, 7 steps):

$$\begin{aligned} & - \frac{22 a^3 c (g \cos[e + f x])^{5/2}}{45 f g \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} + \frac{22 a^3 c g \sqrt{\cos[e + f x]} \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e + f x), 2\right]}{15 f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} - \\ & \frac{22 a^2 c (g \cos[e + f x])^{5/2} \sqrt{a + a \sin[e + f x]}}{105 f g \sqrt{c - c \sin[e + f x]}} - \frac{2 a c (g \cos[e + f x])^{5/2} (a + a \sin[e + f x])^{3/2}}{21 f g \sqrt{c - c \sin[e + f x]}} + \frac{2 c (g \cos[e + f x])^{5/2} (a + a \sin[e + f x])^{5/2}}{9 f g \sqrt{c - c \sin[e + f x]}} \end{aligned}$$

Result (type 5, 182 leaves):

$$\begin{aligned} & - \left(a^3 g \sqrt{g \cos[e + f x]} \sqrt{c - c \sin[e + f x]} \left(-3696 i e^{-i(e+fx)} \sqrt{1 + e^{2i(e+fx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(e+fx)}\right] + \right. \right. \\ & \quad \left. \left. 4 \cos[e + f x] (924 i + \cos[e + f x] (180 + 180 \cos[2(e + f x)] - 273 \sin[e + f x] + 35 \sin[3(e + f x)])) \right) \right) / \\ & \left(2520 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \sqrt{a(1 + \sin[e + f x])} \right) \end{aligned}$$

■ **Problem 118: Result unnecessarily involves higher level functions.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^{7/2} \sqrt{c - c \sin[e + f x]} dx$$

Optimal (type 4, 343 leaves, 8 steps):

$$\begin{aligned} & - \frac{2 a^4 c (g \cos[e + f x])^{5/2}}{3 f g \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} + \frac{2 a^4 c g \sqrt{\cos[e + f x]} \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e + f x), 2\right]}{f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} - \\ & \frac{2 a^3 c (g \cos[e + f x])^{5/2} \sqrt{a + a \sin[e + f x]}}{7 f g \sqrt{c - c \sin[e + f x]}} - \frac{10 a^2 c (g \cos[e + f x])^{5/2} (a + a \sin[e + f x])^{3/2}}{77 f g \sqrt{c - c \sin[e + f x]}} - \\ & \frac{2 a c (g \cos[e + f x])^{5/2} (a + a \sin[e + f x])^{5/2}}{33 f g \sqrt{c - c \sin[e + f x]}} + \frac{2 c (g \cos[e + f x])^{5/2} (a + a \sin[e + f x])^{7/2}}{11 f g \sqrt{c - c \sin[e + f x]}} \end{aligned}$$

Result (type 5, 676 leaves):

$$\begin{aligned}
& \left((2+2i) a^4 e^{-\frac{3}{2}i(e+fx)} (i+e^{i(e+fx)}) (g \cos[e+fx])^{3/2} \right. \\
& \quad \left. \left(1+e^{2i(e+fx)} + (-1+e^{2ie}) \sqrt{1+e^{2i(e+fx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(e+fx)}\right] \right) \sqrt{c-c \sin[e+fx]} \right) / \\
& \left((-1+e^{2ie}) \sqrt{-i a e^{-i(e+fx)} (i+e^{i(e+fx)})^2} \sqrt{e^{-i(e+fx)} (1+e^{2i(e+fx)})} f \cos[e+fx]^{3/2} \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right] \right) \right) + \\
& \frac{1}{\cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right]} (g \cos[e+fx])^{3/2} \operatorname{Sec}[e+fx] \\
& \left(-\frac{58 a^3 \cos\left[\frac{fx}{2}\right] \left(\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]\right)}{77 f} + \frac{3 a^3 \cos\left[\frac{3fx}{2}\right] \left(\cos\left[\frac{3e}{2}\right] + \sin\left[\frac{3e}{2}\right]\right)}{308 f} - \frac{145 a^3 \cos\left[\frac{5fx}{2}\right] \left(\cos\left[\frac{5e}{2}\right] - \sin\left[\frac{5e}{2}\right]\right)}{924 f} - \right. \\
& \frac{19 a^3 \cos\left[\frac{7fx}{2}\right] \left(\cos\left[\frac{7e}{2}\right] + \sin\left[\frac{7e}{2}\right]\right)}{264 f} + \frac{a^3 \cos\left[\frac{9fx}{2}\right] \left(\cos\left[\frac{9e}{2}\right] - \sin\left[\frac{9e}{2}\right]\right)}{88 f} + \frac{58 a^3 \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]\right) \sin\left[\frac{fx}{2}\right]}{77 f} + \\
& \frac{3 a^3 \left(\cos\left[\frac{3e}{2}\right] - \sin\left[\frac{3e}{2}\right]\right) \sin\left[\frac{3fx}{2}\right]}{308 f} + \frac{145 a^3 \left(\cos\left[\frac{5e}{2}\right] + \sin\left[\frac{5e}{2}\right]\right) \sin\left[\frac{5fx}{2}\right]}{924 f} - \frac{19 a^3 \left(\cos\left[\frac{7e}{2}\right] - \sin\left[\frac{7e}{2}\right]\right) \sin\left[\frac{7fx}{2}\right]}{264 f} - \\
& \left. \frac{a^3 \left(\cos\left[\frac{9e}{2}\right] + \sin\left[\frac{9e}{2}\right]\right) \sin\left[\frac{9fx}{2}\right]}{88 f} - \frac{2 a^3 \operatorname{Cot}[e]}{f \left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right)} \right) \sqrt{a+a \sin[e+fx]} \sqrt{c-c \sin[e+fx]}
\end{aligned}$$

■ **Problem 129: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \cos[e+fx])^{3/2} \sqrt{c-c \sin[e+fx]}}{\sqrt{a+a \sin[e+fx]}} dx$$

Optimal (type 4, 122 leaves, 4 steps):

$$\frac{2 c (g \cos[e+fx])^{5/2}}{3 f g \sqrt{a+a \sin[e+fx]} \sqrt{c-c \sin[e+fx]}} + \frac{2 c g \sqrt{\cos[e+fx]} \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[\frac{1}{2}(e+fx), 2\right]}{f \sqrt{a+a \sin[e+fx]} \sqrt{c-c \sin[e+fx]}}$$

Result (type 5, 149 leaves):

$$\left(g \sqrt{g \cos[e+fx]} \left(1 - 6 i \cos[e+fx] + \cos[2(e+fx)] + 6 i e^{-i(e+fx)} \sqrt{1+e^{2i(e+fx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(e+fx)}\right] \right) \right. \\
\left. \sqrt{c-c \sin[e+fx]} \right) / \left(3 f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sqrt{a(1+\sin[e+fx])} \right)$$

■ **Problem 137: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \cos[e+fx])^{3/2} \sqrt{c-c \sin[e+fx]}}{(a+a \sin[e+fx])^{3/2}} dx$$

Optimal (type 4, 123 leaves, 4 steps) :

$$-\frac{4 c (g \operatorname{Cos}[e+f x])^{5/2}}{f g (a+a \operatorname{Sin}[e+f x])^{3/2} \sqrt{c-c \operatorname{Sin}[e+f x]}} - \frac{6 c g \sqrt{\operatorname{Cos}[e+f x]} \sqrt{g \operatorname{Cos}[e+f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e+f x), 2\right]}{a f \sqrt{a+a \operatorname{Sin}[e+f x]} \sqrt{c-c \operatorname{Sin}[e+f x]}}$$

Result (type 5, 170 leaves) :

$$\left(2 g \sqrt{g \operatorname{Cos}[e+f x]} \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right)^2 \\ \left(-2+3 i \operatorname{Cos}[e+f x]-3 i e^{-i(e+f x)} \sqrt{1+e^{2 i(e+f x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(e+f x)}\right]+2 \operatorname{Sin}[e+f x]\right) \sqrt{c-c \operatorname{Sin}[e+f x]} \Bigg/ \\ \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2 (a(1+\operatorname{Sin}[e+f x]))^{3/2}\right)$$

■ **Problem 146: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \operatorname{Cos}[e+f x])^{3/2} \sqrt{c-c \operatorname{Sin}[e+f x]}}{(a+a \operatorname{Sin}[e+f x])^{5/2}} dx$$

Optimal (type 4, 182 leaves, 5 steps) :

$$-\frac{4 c (g \operatorname{Cos}[e+f x])^{5/2}}{5 f g (a+a \operatorname{Sin}[e+f x])^{5/2} \sqrt{c-c \operatorname{Sin}[e+f x]}} + \\ \frac{6 c (g \operatorname{Cos}[e+f x])^{5/2}}{5 a f g (a+a \operatorname{Sin}[e+f x])^{3/2} \sqrt{c-c \operatorname{Sin}[e+f x]}} + \frac{6 c g \sqrt{\operatorname{Cos}[e+f x]} \sqrt{g \operatorname{Cos}[e+f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e+f x), 2\right]}{5 a^2 f \sqrt{a+a \operatorname{Sin}[e+f x]} \sqrt{c-c \operatorname{Sin}[e+f x]}}$$

Result (type 5, 194 leaves) :

$$-\left(e^{-\frac{3}{2} i(e+f x)} g^3 \left(-3+i e^{i(e+f x)}+e^{2 i(e+f x)}+5 i e^{3 i(e+f x)}-3(i+e^{i(e+f x)})^2 \sqrt{1+e^{2 i(e+f x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(e+f x)}\right]\right)\right) \\ \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]-i \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right) \sqrt{c-c \operatorname{Sin}[e+f x]} \Bigg/ \left(5 a^2 f (g \operatorname{Cos}[e+f x])^{3/2} \sqrt{a(1+\operatorname{Sin}[e+f x])}\right)$$

■ **Problem 152: Attempted integration timed out after 120 seconds.**

$$\int (g \operatorname{Cos}[e+f x])^{3/2} (a+a \operatorname{Sin}[e+f x])^m (c-c \operatorname{Sin}[e+f x])^3 dx$$

Optimal (type 5, 93 leaves, 4 steps) :

$$-\frac{1}{17 f g^7} 2^{\frac{9}{4}+m} a^4 c^3 (g \operatorname{Cos}[e+f x])^{17/2} \operatorname{Hypergeometric2F1}\left[\frac{17}{4}, -\frac{1}{4}-m, \frac{21}{4}, \frac{1}{2}(1-\operatorname{Sin}[e+f x])\right] (1+\operatorname{Sin}[e+f x])^{-\frac{1}{4}-m} (a+a \operatorname{Sin}[e+f x])^{-4+m}$$

Result (type 1, 1 leaves) :

???

■ **Problem 153: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^2 dx$$

Optimal (type 5, 93 leaves, 4 steps):

$$-\frac{1}{13 f g^5} 2^{\frac{9}{4}+m} a^3 c^2 (g \cos[e + f x])^{13/2} \text{Hypergeometric2F1}\left[\frac{13}{4}, -\frac{1}{4}-m, \frac{17}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^{-3+m}$$

Result (type 1, 1 leaves):

???

■ **Problem 154: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x]) dx$$

Optimal (type 5, 91 leaves, 4 steps):

$$-\frac{1}{9 f g^3} 2^{\frac{9}{4}+m} a^2 c (g \cos[e + f x])^{9/2} \text{Hypergeometric2F1}\left[\frac{9}{4}, -\frac{1}{4}-m, \frac{13}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^{-2+m}$$

Result (type 1, 1 leaves):

???

■ **Problem 155: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m dx$$

Optimal (type 5, 88 leaves, 3 steps):

$$-\frac{1}{5 f g} 2^{\frac{9}{4}+m} a (g \cos[e + f x])^{5/2} \text{Hypergeometric2F1}\left[\frac{5}{4}, -\frac{1}{4}-m, \frac{9}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^{-1+m}$$

Result (type 6, 13703 leaves):

$$\left(64 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m \right. \\ \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos[e + f x]^{7/2} + \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos[e + f x]^{3/2} \sin[e + f x]^2 \right) \right. \\ \left. \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{2m} \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{3+2m} \sqrt{\frac{\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^3}{\left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^2}} \right. \\ \left. \left(\left(25 \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}-2m, 3+2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \right) \right) / \right.$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2m, 4 + 2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] / \\
& \left(-9 \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2m, 4 + 2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \quad 2 \left(4(2+m) \text{AppellF1}\left[\frac{9}{4}, -\frac{1}{2} - 2m, 5 + 2m, \frac{13}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \quad \left. \left.(1+4m) \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2} - 2m, 4 + 2m, \frac{13}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \Bigg) \Bigg) - \\
& \frac{1}{5 \sqrt{\frac{\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^3}{\left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^2}}} 32 \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{2m} \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}\right)^{3+2m} \\
& \left(\frac{\frac{1}{4} \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 - \frac{3}{4} \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{\left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^2} - \right. \\
& \quad \left. \frac{\text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^3\right)}{\left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^3}\right) \\
& \left(\left(25 \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2} - 2m, 3 + 2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\right) / \right. \\
& \quad \left(-5 \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2} - 2m, 3 + 2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \quad 2 \left((6+4m) \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2m, 4 + 2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + (1+4m) \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2} - 2m, 3 + 2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) + \\
& \left(25 \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2} - 2m, 4 + 2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) / \\
& \left(5 \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2} - 2m, 4 + 2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \right. \\
& \quad 2 \left(4(2+m) \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2m, 5 + 2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + (1+4m) \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2} - 2m, 4 + 2m, \frac{9}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) +
\end{aligned}$$

$$\left. \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2} - 2m, 4 + 2m, \frac{13}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^2 \right)$$

■ **Problem 156: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + fx])^{3/2} (a + a \sin[e + fx])^m}{c - c \sin[e + fx]} dx$$

Optimal (type 5, 84 leaves, 4 steps):

$$-\frac{1}{cf} 2^{\frac{9}{4}+m} g \sqrt{g \cos[e + fx]} \text{Hypergeometric2F1}\left[\frac{1}{4}, -\frac{1}{4} - m, \frac{5}{4}, \frac{1}{2} (1 - \sin[e + fx])\right] (1 + \sin[e + fx])^{-\frac{1}{4}-m} (a + a \sin[e + fx])^m$$

Result (type 6, 9339 leaves):

$$\begin{aligned} & -\frac{1}{f \cos[e + fx]^{3/2} (c - c \sin[e + fx])} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} (g \cos[e + fx])^{3/2} \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^2 \\ & (a + a \sin[e + fx])^m \left(-\left(\left(10 \sqrt{2} \text{AppellF1}\left[\frac{1}{4}, -\frac{3}{2} - 2m, 2 + 2m, \frac{5}{4}, \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \right. \right. \\ & \left. \left. \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2+4m} \cos[e + fx] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) / \left(\left(\cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] \right) \right) \right. \\ & \left(-8(1+m) \text{AppellF1}\left[\frac{5}{4}, -\frac{3}{2} - 2m, 3 + 2m, \frac{9}{4}, \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \right. \\ & \left. 2(3+4m) \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{9}{4}, \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\ & \left. 5 \text{AppellF1}\left[\frac{1}{4}, -\frac{3}{2} - 2m, 2 + 2m, \frac{5}{4}, \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \\ & \left. - \left(\left(5 \text{AppellF1}\left[\frac{1}{4}, -\frac{3}{2} - 2m, 2 + 2m, \frac{5}{4}, \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{1+2m} \sqrt{\cos[e + fx]} \right. \right. \right. \\ & \left. \left. \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) / \left(-8(1+m) \text{AppellF1}\left[\frac{5}{4}, -\frac{3}{2} - 2m, 3 + 2m, \frac{9}{4}, \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \right. \\ & \left. \left. \frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 - 2(3+4m) \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{9}{4}, \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\ & \left. 5 \text{AppellF1}\left[\frac{1}{4}, -\frac{3}{2} - 2m, 2 + 2m, \frac{5}{4}, \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) - \\ & \left(10 \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{1+2m} \sqrt{\cos[e + fx]} \left(\frac{1}{10} (2+2m) \text{AppellF1}\left[\frac{5}{4}, -\frac{3}{2} - 2m, 3 + 2m, \frac{9}{4}, \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& 2(3+4m) \left(\frac{5}{18} (2+2m) \operatorname{AppellF1} \left[\frac{9}{4}, -\frac{1}{2} - 2m, 3+2m, \frac{13}{4}, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \operatorname{Csc} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - \frac{5}{18} \left(-\frac{1}{2} - 2m \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2} - 2m, 2+2m, \frac{13}{4}, \right. \\
& \quad \left. \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \operatorname{Csc} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) + \\
& \quad \frac{5}{2} \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{3}{2} - 2m, 2+2m, \frac{5}{4}, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \\
& \quad \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] + 5 \left(\frac{1}{10} (2+2m) \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{3}{2} - 2m, 3+2m, \frac{9}{4}, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \left. \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \operatorname{Csc} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - \frac{1}{10} \left(-\frac{3}{2} - 2m \right) \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2} - 2m, 2+2m, \frac{9}{4}, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \operatorname{Csc} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \\
& \quad \left(-8(1+m) \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{3}{2} - 2m, 3+2m, \frac{9}{4}, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \\
& \quad \left. 2(3+4m) \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2} - 2m, 2+2m, \frac{9}{4}, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \right. \\
& \quad \left. 5 \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{3}{2} - 2m, 2+2m, \frac{5}{4}, \operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Cot} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) + \\
& \quad \left(7\sqrt{2} \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2} - 2m, 2+2m, \frac{7}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \left. \operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{4m} \right. \\
& \quad \left. \operatorname{Cos}[e + f x] \right. \\
& \quad \left. \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \\
& \quad \left(\left(\operatorname{Cos} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x \right) \right] - \operatorname{Sin} \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x \right) \right] \right) \right. \\
& \quad \left(21 \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2} - 2m, 2+2m, \frac{7}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - \right. \\
& \quad \left. 6 \left(4(1+m) \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2} - 2m, 3+2m, \frac{11}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \right. \\
& \quad \left. \left. (1+4m) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2} - 2m, 2+2m, \frac{11}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)
\end{aligned}$$

$$\left(\text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{7}{22}\left(\frac{1}{2} - 2m\right) \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2} - 2m, 2 + 2m, \frac{15}{4}, \right.\right. \\ \left.\left. \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) / \\ \left(-21 \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{7}{4}, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + 6 \right. \\ \left. \left(4 (1+m) \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2} - 2m, 3 + 2m, \frac{11}{4}, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\ \left. \left. (1+4m) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2} - 2m, 2 + 2m, \frac{11}{4}, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right)$$

■ Problem 157: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(g \text{Cos}[e + fx])^{3/2} (a + a \text{Sin}[e + fx])^m}{(c - c \text{Sin}[e + fx])^2} dx$$

Optimal (type 5, 93 leaves, 4 steps):

$$\frac{1}{3 a c^2 f (g \text{Cos}[e + fx])^{3/2}} 2^{9+m} g^3 \text{Hypergeometric2F1}\left[-\frac{3}{4}, -\frac{1}{4} - m, \frac{1}{4}, \frac{1}{2} (1 - \text{Sin}[e + fx])\right] (1 + \text{Sin}[e + fx])^{-\frac{1}{4}-m} (a + a \text{Sin}[e + fx])^{1+m}$$

Result (type 6, 13626 leaves):

$$-\frac{1}{f \text{Cos}[e + fx]^{3/2} (c - c \text{Sin}[e + fx])^2} \text{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} (g \text{Cos}[e + fx])^{3/2} \left(\text{Cos}\left[\frac{1}{2}(e + fx)\right] - \text{Sin}\left[\frac{1}{2}(e + fx)\right] \right)^4 \\ (a + a \text{Sin}[e + fx])^m \left(\left(3 \text{AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \right. \\ \left. \left. \text{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{4m} \text{Cos}[e + fx] \text{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \text{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \text{Sin}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) / \right. \\ \left(2 \sqrt{2} \left(8 m \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 1 + 2m, \frac{7}{4}, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\ \left. \left. (2 + 8 m) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2} - 2m, 2m, \frac{7}{4}, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - 3 \text{AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \right. \right. \\ \left. \left. \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \text{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \left(\text{Cos}\left[\frac{\pi}{4} + \frac{1}{2}(e - \frac{\pi}{2} + fx)\right] - \text{Sin}\left[\frac{\pi}{4} + \frac{1}{2}(e - \frac{\pi}{2} + fx)\right] \right)^3 \\ \left. - \left(\left(3 \text{AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \text{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2m} \sqrt{\text{Cos}[e + fx]} \right. \right. \\ \left. \left. \text{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \text{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) / \left(4 \left(8 m \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 1 + 2m, \frac{7}{4}, \text{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \right. \right.$$

$$\begin{aligned}
& \cos[e + f x] \\
& \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \\
& \csc\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \\
& \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \Big/ \\
& \left(2\sqrt{2}\left(8\text{m AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 1 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\
& \quad (2 + 8m)\text{ AppellF1}\left[\frac{3}{4}, \frac{1}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \\
& \quad \left. 3\text{ AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \\
& \left(\cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right)\right]\right)^3 \\
& \left(\left(3\text{ AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\
& \quad \left. \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \sqrt{\cos[e + f x]} \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \csc\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \Big/ \\
& \left(4\left(8\text{m AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 1 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + (2 + 8m) \right. \right. \\
& \quad \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - 3 \right. \\
& \quad \left. \left. \text{AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)\right) + \\
& \left(3\text{ AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \sqrt{\cos[e + f x]} \right. \\
& \quad \left. \csc\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^4\right) \Big/ \left(8\left(8\text{m AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 1 + 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\
& \quad (2 + 8m)\text{ AppellF1}\left[\frac{3}{4}, \frac{1}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - 3 \\
& \quad \left. \left. \text{AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)\right) + \\
& \left(3\text{m AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-1+2m} \right. \\
& \quad \left. \sqrt{\cos[e + f x]} \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \csc\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \frac{3}{4}, \frac{1}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + 8m \\
& \left(-\frac{3}{14}(1+2m) \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2} - 2m, 2+2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right.\right. \\
& \left.\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{3}{14}\left(-\frac{1}{2} - 2m\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2} - 2m, 1+2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + (2+8m)\left(-\frac{3}{7}m \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2} - 2m, 1+2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2,\right.\right.\right. \\
& \left.\left.-\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{3}{14}\left(\frac{1}{2} - 2m\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2} - 2m,\right.\right. \\
& \left.\left.2m, \frac{11}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)\right)\right) / \\
& \left(2\left(8m \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2} - 2m, 1+2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + (2+8m)\right.\right. \\
& \left.\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2} - 2m, 2m, \frac{7}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \right. \\
& \left.3 \operatorname{AppellF1}\left[-\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{3}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\right) - \\
& \left(\left(\cos\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{-1+2m} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{1+2m} \sqrt{\cos[e+fx]}\right. \\
& \left.\sqrt{\cos\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)}\right) \\
& \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{2m} \\
& \left(\left(\operatorname{AppellF1}\left[-\frac{3}{4}, -\frac{1}{2} - 2m, 2m, \frac{1}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right) / \right. \\
& \left(\operatorname{AppellF1}\left[-\frac{3}{4}, -\frac{1}{2} - 2m, 2m, \frac{1}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \right. \\
& \left.2\left(4m \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2} - 2m, 1+2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \left.\left.(1+4m) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2} - 2m, 2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right) + \\
& \left(15 \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{5}{4}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) /
\end{aligned}$$

$$\begin{aligned}
& 5 \left(-\frac{1}{5} m \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2} - 2m, 1 + 2m, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] + \frac{1}{10} \left(-\frac{1}{2} - 2m \right) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2} - 2m, 2m, \frac{9}{4}, \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) - \\
& 2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(4m \left(-\frac{5}{18} (1 + 2m) \operatorname{AppellF1} \left[\frac{9}{4}, -\frac{1}{2} - 2m, 2 + 2m, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] + \frac{5}{18} \left(-\frac{1}{2} - 2m \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2} - 2m, \right. \right. \\
& \quad \left. \left. 1 + 2m, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) \right) + \\
& (1 + 4m) \left(-\frac{5}{9} m \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2} - 2m, 1 + 2m, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \quad \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] + \frac{5}{18} \left(\frac{1}{2} - 2m \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2} - 2m, 2m, \frac{13}{4}, \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) \right) \Bigg) / \\
& \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2} - 2m, 2m, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] - 2 \left(4m \operatorname{AppellF1} \left[\frac{5}{4}, \right. \right. \right. \\
& \quad \left. \left. -\frac{1}{2} - 2m, 1 + 2m, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (1 + 4m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2} - 2m, 2m, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 158: Attempted integration timed out after 120 seconds.**

$$\int \frac{(g \operatorname{Cos}[e + f x])^{3/2} (a + a \operatorname{Sin}[e + f x])^m}{(c - c \operatorname{Sin}[e + f x])^3} dx$$

Optimal (type 5, 93 leaves, 4 steps):

$$\left(2^{\frac{9}{4}+m} g^5 \operatorname{Hypergeometric2F1} \left[-\frac{7}{4}, -\frac{1}{4} - m, -\frac{3}{4}, \frac{1}{2} (1 - \operatorname{Sin}[e + f x]) \right] (1 + \operatorname{Sin}[e + f x])^{-\frac{1}{4}-m} (a + a \operatorname{Sin}[e + f x])^{2+m} \right) / (7 a^2 c^3 f (g \operatorname{Cos}[e + f x])^{7/2})$$

Result (type 1, 1 leaves):

???

■ **Problem 159: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{5/2} dx$$

Optimal (type 5, 114 leaves, 4 steps):

$$-\frac{1}{15 f g^6} 2^{\frac{9}{4}+m} a^3 c^2 (g \cos[e + f x])^{15/2} \text{Hypergeometric2F1}\left[\frac{15}{4}, -\frac{1}{4}-m, \frac{19}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] \\ \sec[e + f x] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^{-3+m} \sqrt{c - c \sin[e + f x]}$$

Result (type 1, 1 leaves):

???

■ **Problem 160: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{3/2} dx$$

Optimal (type 5, 112 leaves, 4 steps):

$$-\frac{1}{11 f g^4} 2^{\frac{9}{4}+m} a^2 c (g \cos[e + f x])^{11/2} \text{Hypergeometric2F1}\left[\frac{11}{4}, -\frac{1}{4}-m, \frac{15}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] \\ \sec[e + f x] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^{-2+m} \sqrt{c - c \sin[e + f x]}$$

Result (type 1, 1 leaves):

???

■ **Problem 161: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m \sqrt{c - c \sin[e + f x]} dx$$

Optimal (type 5, 109 leaves, 4 steps):

$$-\frac{1}{7 f g^2} 2^{\frac{9}{4}+m} a (g \cos[e + f x])^{7/2} \text{Hypergeometric2F1}\left[\frac{7}{4}, -\frac{1}{4}-m, \frac{11}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] \\ \sec[e + f x] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^{-1+m} \sqrt{c - c \sin[e + f x]}$$

Result (type 1, 1 leaves):

???

■ **Problem 162: Attempted integration timed out after 120 seconds.**

$$\int \frac{(g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 5, 106 leaves, 4 steps):

$$-\frac{1}{3 f \sqrt{c - c \sin[e + f x]}}$$

$$2^{\frac{9}{4}+m} a \cos[e + f x] (g \cos[e + f x])^{3/2} \text{Hypergeometric2F1}\left[\frac{3}{4}, -\frac{1}{4} - m, \frac{7}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^{-1+m}$$

Result (type 1, 1 leaves):

???

■ **Problem 163: Attempted integration timed out after 120 seconds.**

$$\int \frac{(g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 5, 106 leaves, 4 steps):

$$\left(2^{\frac{9}{4}+m} g^2 \cos[e + f x] \text{Hypergeometric2F1}\left[-\frac{1}{4}, -\frac{1}{4} - m, \frac{3}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^m \right) / \left(c f \sqrt{g \cos[e + f x]} \sqrt{c - c \sin[e + f x]} \right)$$

Result (type 1, 1 leaves):

???

■ **Problem 164: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m}{(c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 5, 114 leaves, 4 steps):

$$\left(2^{\frac{9}{4}+m} g^4 \cos[e + f x] \text{Hypergeometric2F1}\left[-\frac{5}{4}, -\frac{1}{4} - m, -\frac{1}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^{1+m} \right) / \left(5 a c^2 f (g \cos[e + f x])^{5/2} \sqrt{c - c \sin[e + f x]} \right)$$

Result (type 6, 20476 leaves): Display of huge result suppressed!

■ **Problem 165: Attempted integration timed out after 120 seconds.**

$$\int \frac{(g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 5, 106 leaves, 4 steps):

$$-\frac{1}{3 f \sqrt{c - c \sin[e + f x]}}$$

$$2^{\frac{9}{4}+m} a \cos[e + f x] (g \cos[e + f x])^{3/2} \text{Hypergeometric2F1}\left[\frac{3}{4}, -\frac{1}{4} - m, \frac{7}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (a + a \sin[e + f x])^{-1+m}$$

Result (type 1, 1 leaves):

???

■ **Problem 166: Attempted integration timed out after 120 seconds.**

$$\int \frac{(g \cos[e + f x])^{3/2} (c + c \sin[e + f x])^m}{\sqrt{a - a \sin[e + f x]}} dx$$

Optimal (type 5, 106 leaves, 4 steps):

$$-\frac{1}{3 f \sqrt{a - a \sin[e + f x]}} \\ 2^{\frac{9}{4}+m} c \cos[e + f x] (g \cos[e + f x])^{3/2} \text{Hypergeometric2F1}\left[\frac{3}{4}, -\frac{1}{4}-m, \frac{7}{4}, \frac{1}{2} (1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{4}-m} (c + c \sin[e + f x])^{-1+m}$$

Result (type 1, 1 leaves):

???

■ **Problem 167: Result more than twice size of optimal antiderivative.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-3-m} dx$$

Optimal (type 5, 123 leaves, 4 steps):

$$\frac{1}{c^2 f g (5 + 4 m)} 2^{-\frac{3}{4}-m} (g \cos[e + f x])^{5/2} \text{Hypergeometric2F1}\left[\frac{1}{4} (5 + 4 m), \frac{1}{4} (11 + 4 m), \frac{1}{4} (9 + 4 m), \frac{1}{2} (1 + \sin[e + f x])\right] \\ (1 - \sin[e + f x])^{-\frac{1}{4}+m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1-m}$$

Result (type 5, 2502 leaves):

$$\left(2^{-4-m} \left(\cos\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-1+2m} \sqrt{\cos[e + f x]} (g \cos[e + f x])^{3/2} \right. \\ \left((-3 + 8 m + 16 m^2) \cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \text{Hypergeometric2F1}\left[-\frac{3}{2} - 2 m, -\frac{7}{4} - m, -\frac{3}{4} - m, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\ (7 + 4 m) \left(2 (-1 + 4 m) \cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Hypergeometric2F1}\left[-\frac{3}{2} - 2 m, -\frac{3}{4} - m, \frac{1}{4} - m, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right)^2 + \\ \left. \left. (3 + 4 m) \text{Hypergeometric2F1}\left[-\frac{3}{2} - 2 m, \frac{1}{4} - m, \frac{5}{4} - m, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \right) \\ \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{-2(-3-m)} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-3-m} \\ \left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^{-6-2m} /$$

$$\left(f (-1+4m) (3+4m) (7+4m) \sqrt{1 - \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right.$$

$$\left. - \frac{1}{(-1+4m) (3+4m) (7+4m) \left(1 - \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{3/2}} 2^{-6-m} \left(\operatorname{Cos}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-1+2m} \sqrt{\operatorname{Cos}[e + f x]} \right.$$

$$\left((-3+8m+16m^2) \operatorname{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-2m, -\frac{7}{4}-m, -\frac{3}{4}-m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.$$

$$(7+4m) \left(2(-1+4m) \operatorname{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-2m, -\frac{3}{4}-m, \frac{1}{4}-m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.$$

$$\left. \left. (3+4m) \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-2m, \frac{1}{4}-m, \frac{5}{4}-m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right.$$

$$\left. \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{(-1+4m) (3+4m) (7+4m) \sqrt{1 - \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right.$$

$$2^{-4-m} m \left(\operatorname{Cos}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-1+2m} \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \sqrt{\operatorname{Cos}[e + f x]}$$

$$\left((-3+8m+16m^2) \operatorname{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-2m, -\frac{7}{4}-m, -\frac{3}{4}-m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.$$

$$(7+4m) \left(2(-1+4m) \operatorname{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-2m, -\frac{3}{4}-m, \frac{1}{4}-m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.$$

$$\left. \left. (3+4m) \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-2m, \frac{1}{4}-m, \frac{5}{4}-m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right]\right) \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-1-2m} + \right.$$

$$\left. \frac{1}{(-1+4m) (3+4m) (7+4m) \sqrt{1 - \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}} 2^{-5-m} (-1+2m) \operatorname{Cos}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(\operatorname{Cos}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-2+2m} \right.$$

$$\left. \sqrt{\operatorname{Cos}[e + f x]} \left((-3+8m+16m^2) \operatorname{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-2m, -\frac{7}{4}-m, -\frac{3}{4}-m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right.$$

$$\left. \left. (7+4m) \left(2(-1+4m) \operatorname{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-2m, -\frac{3}{4}-m, \frac{1}{4}-m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \right.$$

$$\begin{aligned}
& (3 + 4m) \operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - 2m, \frac{1}{4} - m, \frac{5}{4} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \sin\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} - \\
& \left(2^{-5-m} \left(\cos\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{-1+2m} \left((-3 + 8m + 16m^2) \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - 2m, -\frac{7}{4} - m, -\frac{3}{4} - m, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + (7 + 4m) \left(2(-1 + 4m) \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - 2m, -\frac{3}{4} - m, \frac{1}{4} - \right. \right. \right. \\
& \left. \left. \left. m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + (3 + 4m) \operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - 2m, \frac{1}{4} - m, \frac{5}{4} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right)\right) \\
& \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} \sin[efx]\right) / \left((-1 + 4m)(3 + 4m)(7 + 4m) \sqrt{\cos[efx]} \sqrt{1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right) - \\
& \frac{1}{(-1 + 4m)(3 + 4m)(7 + 4m) \sqrt{1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}} 2^{-4-m} \left(\cos\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{-1+2m} \sqrt{\cos[efx]} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} \\
& \left(-(-3 + 8m + 16m^2) \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^3 \operatorname{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - 2m, -\frac{7}{4} - m, -\frac{3}{4} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \left. \frac{1}{2}\left(-\frac{7}{4} - m\right) (-3 + 8m + 16m^2) \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^3 \operatorname{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right. \\
& \left. \left(-\operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - 2m, -\frac{7}{4} - m, -\frac{3}{4} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{\frac{3}{2}+2m} \right) + (7 + 4m) \right. \\
& \left. \left(-(-1 + 4m) \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - 2m, -\frac{3}{4} - m, \frac{1}{4} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\
& \left. \left. \left(-\frac{3}{4} - m\right) (-1 + 4m) \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \left(-\operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - 2m, -\frac{3}{4} - m, \frac{1}{4} - m, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{\frac{3}{2}+2m} \right) + \frac{1}{2}\left(\frac{1}{4} - m\right) (3 + 4m) \operatorname{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \\
& \left. \left. \left(-\operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - 2m, \frac{1}{4} - m, \frac{5}{4} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{\frac{3}{2}+2m} \right) \right) \right) \right)
\end{aligned}$$

■ Problem 169: Attempted integration timed out after 120 seconds.

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1-m} dx$$

Optimal (type 5, 120 leaves, 4 steps):

$$\frac{1}{f g (5 + 4 m)} 2^{\frac{5}{4}-m} (g \cos[e + f x])^{5/2} \text{Hypergeometric2F1}\left[\frac{1}{4} (3 + 4 m), \frac{1}{4} (5 + 4 m), \frac{1}{4} (9 + 4 m), \frac{1}{2} (1 + \sin[e + f x])\right] \\ (1 - \sin[e + f x])^{-\frac{1}{4}+m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1-m}$$

Result (type 1, 1 leaves):

???

■ **Problem 170: Unable to integrate problem.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-m} dx$$

Optimal (type 5, 121 leaves, 4 steps):

$$\frac{1}{f g (5 + 4 m)} 2^{\frac{9}{4}-m} c (g \cos[e + f x])^{5/2} \text{Hypergeometric2F1}\left[\frac{1}{4} (-1 + 4 m), \frac{1}{4} (5 + 4 m), \frac{1}{4} (9 + 4 m), \frac{1}{2} (1 + \sin[e + f x])\right] \\ (1 - \sin[e + f x])^{-\frac{1}{4}+m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1-m}$$

Result (type 8, 42 leaves):

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-m} dx$$

■ **Problem 171: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{1-m} dx$$

Optimal (type 5, 123 leaves, 4 steps):

$$\frac{1}{f g (5 + 4 m)} 2^{\frac{13}{4}-m} c^2 (g \cos[e + f x])^{5/2} \text{Hypergeometric2F1}\left[\frac{1}{4} (-5 + 4 m), \frac{1}{4} (5 + 4 m), \frac{1}{4} (9 + 4 m), \frac{1}{2} (1 + \sin[e + f x])\right] \\ (1 - \sin[e + f x])^{-\frac{1}{4}+m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1-m}$$

Result (type 1, 1 leaves):

???

■ **Problem 172: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{3/2} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{2-m} dx$$

Optimal (type 5, 123 leaves, 4 steps):

$$\frac{1}{f g (5 + 4 m)} 2^{\frac{17}{4} - m} c^3 (g \cos[e + f x])^{5/2} \text{Hypergeometric2F1}\left[\frac{1}{4}(-9 + 4 m), \frac{1}{4}(5 + 4 m), \frac{1}{4}(9 + 4 m), \frac{1}{2}(1 + \sin[e + f x])\right] \\ (1 - \sin[e + f x])^{-\frac{1}{4} + m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1 - m}$$

Result (type 1, 1 leaves):

???

■ **Problem 174: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{1 - 2m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1 + m} dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$\frac{g (g \cos[e + f x])^{-2m} \log[1 - \sin[e + f x]] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^m}{c f}$$

Result (type 1, 1 leaves):

???

■ **Problem 175: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (g \cos[e + f x])^{5 - 2m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n dx$$

Optimal (type 3, 203 leaves, 3 steps):

$$\frac{8 a^3 (g \cos[e + f x])^{6 - 2m} (a + a \sin[e + f x])^{-3 + m} (c - c \sin[e + f x])^n}{f g (3 - m + n) (4 - m + n) (5 - m + n)} - \frac{4 a^2 (g \cos[e + f x])^{6 - 2m} (a + a \sin[e + f x])^{-2 + m} (c - c \sin[e + f x])^n}{f g (4 - m + n) (5 - m + n)} - \frac{a (g \cos[e + f x])^{6 - 2m} (a + a \sin[e + f x])^{-1 + m} (c - c \sin[e + f x])^n}{f g (5 - m + n)}$$

Result (type 3, 1513 leaves):

$$\frac{1}{f} \cos[e + f x]^{-5 + 2n} (g \cos[e + f x])^{5 - 2m} (a (1 + \sin[e + f x]))^m (c - c \sin[e + f x])^{n - \frac{n (\log[a (1 + \sin[e + f x])] + \log[c - c \sin[e + f x]])}{\log[c - c \sin[e + f x]]}}$$

$$\left(\frac{e^{n (-2 \log[\cos[e + f x]] + \log[a (1 + \sin[e + f x]]) + \log[c - c \sin[e + f x]])} (256 - 41 m + 3 m^2 + 41 n - 6 m n + 3 n^2)}{8 (-5 + m - n) (-4 + m - n) (-3 + m - n)} + \right.$$

$$\left. \left((300 - 23 m + m^2 + 23 n - 2 m n + n^2) \left(-\frac{1}{16} i e^{n (-2 \log[\cos[e + f x]] + \log[a (1 + \sin[e + f x]]) + \log[c - c \sin[e + f x]])} \cos[e + f x] - \right. \right. \right.$$

$$\left. \left. \frac{1}{16} e^{n (-2 \log[\cos[e + f x]] + \log[a (1 + \sin[e + f x]]) + \log[c - c \sin[e + f x]])} \sin[e + f x] \right) \right) / ((-5 + m - n) (-4 + m - n) (-3 + m - n)) +$$

$$\left((300 - 23 m + m^2 + 23 n - 2 m n + n^2) \left(\frac{1}{16} i e^{n (-2 \log[\cos[e + f x]] + \log[a (1 + \sin[e + f x]]) + \log[c - c \sin[e + f x]])} \cos[e + f x] - \right. \right.$$

$$\begin{aligned}
& \left. \left. \left. \frac{1}{16} e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Sin}[e+fx] \right) \right) \right) / ((-5+m-n)(-4+m-n)(-3+m-n)) + \\
& \left((-11m+m^2+11n-2mn+n^2) \left(\frac{1}{4} e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Cos}[2(e+fx)] - \right. \right. \\
& \left. \left. \frac{1}{4} i e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Sin}[2(e+fx)] \right) \right) / ((-5+m-n)(-4+m-n)(-3+m-n)) + \\
& \left((-11m+m^2+11n-2mn+n^2) \left(\frac{1}{4} e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Cos}[2(e+fx)] + \right. \right. \\
& \left. \left. \frac{1}{4} i e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Sin}[2(e+fx)] \right) \right) / ((-5+m-n)(-4+m-n)(-3+m-n)) + \\
& \left((100-53m+3m^2+53n-6mn+3n^2) \left(-\frac{1}{32} i e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Cos}[3(e+fx)] - \right. \right. \\
& \left. \left. \frac{1}{32} e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Sin}[3(e+fx)] \right) \right) / ((-5+m-n)(-4+m-n)(-3+m-n)) + \\
& \left((100-53m+3m^2+53n-6mn+3n^2) \left(\frac{1}{32} i e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Cos}[3(e+fx)] - \right. \right. \\
& \left. \left. \frac{1}{32} e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Sin}[3(e+fx)] \right) \right) / ((-5+m-n)(-4+m-n)(-3+m-n)) + \\
& \frac{1}{(-5+m-n)(-4+m-n)} (m-n) \left(\frac{1}{16} e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Cos}[4(e+fx)] - \right. \\
& \left. \frac{1}{16} i e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Sin}[4(e+fx)] \right) + \\
& \frac{1}{(-5+m-n)(-4+m-n)} (m-n) \left(\frac{1}{16} e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Cos}[4(e+fx)] + \right. \\
& \left. \frac{1}{16} i e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Sin}[4(e+fx)] \right) + \\
& \frac{1}{-5+m-n} \left(-\frac{1}{32} i e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Cos}[5(e+fx)] - \right. \\
& \left. \frac{1}{32} e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Sin}[5(e+fx)] \right) + \\
& \frac{1}{-5+m-n} \left(\frac{1}{32} i e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Cos}[5(e+fx)] - \right. \\
& \left. \frac{1}{32} e^{n(-2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + \operatorname{Log}[a(1+\operatorname{Sin}[e+fx])] + \operatorname{Log}[c-c \operatorname{Sin}[e+fx]])} \operatorname{Sin}[5(e+fx)] \right) \Big)
\end{aligned}$$

■ Problem 178: Attempted integration timed out after 120 seconds.

$$\int (g \cos[e + f x])^{-1-2m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n dx$$

Optimal (type 5, 81 leaves, 4 steps):

$$\frac{1}{2 f g (m-n)} (g \cos[e + f x])^{-2m} \text{Hypergeometric2F1}\left[1, -m+n, 1-m+n, \frac{1}{2} (1 - \sin[e + f x])\right] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n$$

Result (type 1, 1 leaves):

???

■ **Problem 179: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{-3-2m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n dx$$

Optimal (type 5, 85 leaves, 4 steps):

$$\frac{1}{4 f g^3 (1+m-n)} c (g \cos[e + f x])^{-2m} \text{Hypergeometric2F1}\left[2, -1-m+n, -m+n, \frac{1}{2} (1 - \sin[e + f x])\right] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1+n}$$

Result (type 1, 1 leaves):

???

■ **Problem 180: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{-5-2m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n dx$$

Optimal (type 5, 88 leaves, 4 steps):

$$\frac{1}{8 f g^5 (2+m-n)} c^2 (g \cos[e + f x])^{-2m} \text{Hypergeometric2F1}\left[3, -2-m+n, -1-m+n, \frac{1}{2} (1 - \sin[e + f x])\right] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-2+n}$$

Result (type 1, 1 leaves):

???

■ **Problem 182: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{-1-m-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{3+n} dx$$

Optimal (type 5, 134 leaves, 4 steps):

$$\frac{1}{f g (m-n)} 2^{3-\frac{m}{2}-\frac{n}{2}} c^3 (g \cos[e + f x])^{-m-n} \text{Hypergeometric2F1}\left[\frac{1}{2} (-4+m-n), \frac{m-n}{2}, \frac{1}{2} (2+m-n), \frac{1}{2} (1 + \sin[e + f x])\right] (1 - \sin[e + f x])^{\frac{m-n}{2}} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n$$

Result (type 1, 1 leaves):

???

■ **Problem 183: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{-1-m-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{2+n} dx$$

Optimal (type 5, 134 leaves, 4 steps):

$$\frac{1}{f g (m-n)} 2^{2-\frac{m}{2}+\frac{n}{2}} c^2 (g \cos[e + f x])^{-m-n} \text{Hypergeometric2F1}\left[\frac{1}{2}(-2+m-n), \frac{m-n}{2}, \frac{1}{2}(2+m-n), \frac{1}{2}(1+\sin[e + f x])\right] \\ (1 - \sin[e + f x])^{\frac{m-n}{2}} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n$$

Result (type 1, 1 leaves):

???

■ **Problem 184: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^{-1-m-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{1+n} dx$$

Optimal (type 5, 131 leaves, 4 steps):

$$\frac{1}{f g (m-n)} 2^{1-\frac{m}{2}+\frac{n}{2}} c (g \cos[e + f x])^{-m-n} \text{Hypergeometric2F1}\left[\frac{m-n}{2}, \frac{m-n}{2}, \frac{1}{2}(2+m-n), \frac{1}{2}(1+\sin[e + f x])\right] \\ (1 - \sin[e + f x])^{\frac{m-n}{2}} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n$$

Result (type 1, 1 leaves):

???

■ **Problem 187: Result more than twice size of optimal antiderivative.**

$$\int (g \cos[e + f x])^{-1-m-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-2+n} dx$$

Optimal (type 3, 204 leaves, 3 steps):

$$\frac{(g \cos[e + f x])^{-m-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-2+n}}{f g (4 + m - n)} + \\ \frac{2 (g \cos[e + f x])^{-m-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1+n}}{c f g (2 + m - n) (4 + m - n)} + \frac{2 (g \cos[e + f x])^{-m-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n}{c^2 f g (m - n) (2 + m - n) (4 + m - n)}$$

Result (type 3, 2259 leaves):

$$\left(2^{-2-m+2n} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{2m} \cos[e + f x] (g \cos[e + f x])^{-1-m-n} \\ \text{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^4 \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - f x\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right)^{2n} \\ \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - f x\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - f x\right)\right] - \sin\left[\frac{5}{8}\left(-e + \frac{\pi}{2} - f x\right)\right] + \sin\left[\frac{7}{8}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right)^{-m-n}$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^{-2(-2+n)} (a+a\sin[e+fx])^m (c-c\sin[e+fx])^{-2+n} \\
& \left(3+4m+m^2-4n-2mn+n^2 + \cos\left[2\left(-e+\frac{\pi}{2}-fx\right)\right] - 2(2+m-n)\sin[e+fx] \right) \left(\cos\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right] - \sin\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right] \right)^{-5+2n} / \\
& \left(f(m-n)(2+m-n)(4+m-n) \left(-\frac{1}{(m-n)(2+m-n)(4+m-n)} \right. \right. \\
& \quad \left. \left. 2^{-2-m+2n} \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{2m} \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right)^{2n} \right. \\
& \quad \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] - \sin\left[\frac{5}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{7}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right)^{-m-n} \\
& \quad \left. \left(2(2+m-n)\cos[e+fx] - 2\sin\left[2\left(-e+\frac{\pi}{2}-fx\right)\right] \right) + \frac{1}{(m-n)(2+m-n)(4+m-n)} \right) \\
& 2^{-2-m+2n} m \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-1+2m} \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^3 \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right)^{2n} \\
& \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] - \sin\left[\frac{5}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{7}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right)^{-m-n} \\
& \left(3+4m+m^2-4n-2mn+n^2 + \cos\left[2\left(-e+\frac{\pi}{2}-fx\right)\right] - 2(2+m-n)\sin[e+fx] \right) + \frac{1}{(m-n)(2+m-n)(4+m-n)} \\
& 2^{-1-m+2n} \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{1+2m} \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^5 \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right)^{2n} \\
& \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] - \sin\left[\frac{5}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{7}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right)^{-m-n} \\
& \left(3+4m+m^2-4n-2mn+n^2 + \cos\left[2\left(-e+\frac{\pi}{2}-fx\right)\right] - 2(2+m-n)\sin[e+fx] \right) - \frac{1}{(m-n)(2+m-n)(4+m-n)} \\
& 2^{-1-m+2n} n \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{2m} \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right)^{-1+2n} \\
& \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\frac{1}{8}\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \frac{3}{8}\cos\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) - \right. \\
& \quad \left. \frac{1}{8}\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \\
& \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] - \sin\left[\frac{5}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{7}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right)^{-m-n} \\
& \left(3+4m+m^2-4n-2mn+n^2 + \cos\left[2\left(-e+\frac{\pi}{2}-fx\right)\right] - 2(2+m-n)\sin[e+fx] \right) - \frac{1}{(m-n)(2+m-n)(4+m-n)}
\end{aligned}$$

$$\begin{aligned}
& 2^{-2-m+2n} (-m-n) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{2m} \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)\right)^{2n} \\
& \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] - \sin\left[\frac{5}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{7}{8}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)\right)^{-1-m-n} \\
& \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\frac{1}{8}\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \frac{3}{8}\cos\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{5}{8}\cos\left[\frac{5}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \frac{7}{8}\cos\left[\frac{7}{8}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) - \right. \\
& \left. \frac{1}{8}\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] - \sin\left[\frac{5}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{7}{8}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)\right) \\
& \left(3+4m+m^2-4n-2mn+n^2+\cos\left[2\left(-e+\frac{\pi}{2}-fx\right)\right]-2(2+m-n)\sin[e+fx]\right) \left(\cos\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right] + \sin\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right]\right)
\end{aligned}$$

■ **Problem 188: Result more than twice size of optimal antiderivative.**

$$\int (g \cos[e+fx])^{-1-m-n} (a+a \sin[e+fx])^m (c-c \sin[e+fx])^{-3+n} dx$$

Optimal (type 3, 290 leaves, 4 steps):

$$\begin{aligned}
& \frac{(g \cos[e+fx])^{-m-n} (a+a \sin[e+fx])^m (c-c \sin[e+fx])^{-3+n}}{f g (6+m-n)} + \frac{3 (g \cos[e+fx])^{-m-n} (a+a \sin[e+fx])^m (c-c \sin[e+fx])^{-2+n}}{c f g (4+m-n) (6+m-n)} + \\
& \frac{6 (g \cos[e+fx])^{-m-n} (a+a \sin[e+fx])^m (c-c \sin[e+fx])^{-1+n}}{c^2 f g (2+m-n) (4+m-n) (6+m-n)} + \frac{6 (g \cos[e+fx])^{-m-n} (a+a \sin[e+fx])^m (c-c \sin[e+fx])^n}{c^3 f g (m-n) (2+m-n) (4+m-n) (6+m-n)}
\end{aligned}$$

Result (type 3, 2681 leaves):

$$\begin{aligned}
& - \left(2^{-4-m+2n} \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{2m} \cos[e+fx] (g \cos[e+fx])^{-1-m-n} \right. \\
& \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^6 \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)\right)^{2n} \\
& \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] - \sin\left[\frac{5}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{7}{8}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)\right)^{-m-n} \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^{-2(-3+n)} (a+a \sin[e+fx])^m (c-c \sin[e+fx])^{-3+n} \\
& \left(-30-46m-18m^2-2m^3+46n+36mn+6m^2n-18n^2-6m^2n+2n^3-6(3+m-n)\cos\left[2\left(-e+\frac{\pi}{2}-fx\right)\right]+3\cos\left[3\left(-e+\frac{\pi}{2}-fx\right)\right]+ \right. \\
& \left. 3(15+2m^2-4m(-3+n)-12n+2n^2)\sin[e+fx]\right) \left(\cos\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right] - \sin\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right]\right)^{-7+2n} \Big/ \\
& \left(f(m-n)(2+m-n)(4+m-n)(6+m-n) \left(\frac{1}{(m-n)(2+m-n)(4+m-n)(6+m-n)} 2^{-4-m+2n} \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{2m} \right. \right. \\
& \left. \left. \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^6 \left(\cos\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right] + \sin\left[\frac{3}{8}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)\right)\right)^{2n} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] - \sin\left[\frac{5}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{7}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right)^{-m-n} \\
& \left(-3(15 + 2m^2 - 4m(-3 + n) - 12n + 2n^2) \cos[efx] + 12(3 + m - n) \sin\left[2\left(-e + \frac{\pi}{2} - fx\right)\right] - 9 \sin\left[3\left(-e + \frac{\pi}{2} - fx\right)\right] \right) - \\
& \frac{1}{(m-n)(2+m-n)(4+m-n)(6+m-n)} 2^{-4-m+2n} m \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-1+2m} \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^5 \\
& \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right)^{2n} \\
& \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] - \sin\left[\frac{5}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{7}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right)^{-m-n} \\
& \left(-30 - 46m - 18m^2 - 2m^3 + 46n + 36mn + 6m^2n - 18n^2 - 6mn^2 + 2n^3 - 6(3 + m - n) \cos\left[2\left(-e + \frac{\pi}{2} - fx\right)\right] + \right. \\
& \quad \left. 3 \cos\left[3\left(-e + \frac{\pi}{2} - fx\right)\right] + 3(15 + 2m^2 - 4m(-3 + n) - 12n + 2n^2) \sin[efx] \right) - \frac{1}{(m-n)(2+m-n)(4+m-n)(6+m-n)} \\
& 3 \times 2^{-4-m+2n} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{1+2m} \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^7 \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right)^{2n} \\
& \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] - \sin\left[\frac{5}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{7}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right)^{-m-n} \\
& \left(-30 - 46m - 18m^2 - 2m^3 + 46n + 36mn + 6m^2n - 18n^2 - 6mn^2 + 2n^3 - 6(3 + m - n) \cos\left[2\left(-e + \frac{\pi}{2} - fx\right)\right] + 3 \cos\left[3\left(-e + \frac{\pi}{2} - fx\right)\right] + \right. \\
& \quad \left. 3(15 + 2m^2 - 4m(-3 + n) - 12n + 2n^2) \sin[efx] \right) + \frac{1}{(m-n)(2+m-n)(4+m-n)(6+m-n)} 2^{-3-m+2n} n \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2m} \\
& \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^6 \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right)^{-1+2n} \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right. \\
& \quad \left. \left(-\frac{1}{8} \cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{3}{8} \cos\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) - \frac{1}{8} \sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right) \right) \\
& \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] - \sin\left[\frac{5}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{7}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right)^{-m-n} \\
& \left(-30 - 46m - 18m^2 - 2m^3 + 46n + 36mn + 6m^2n - 18n^2 - 6mn^2 + 2n^3 - 6(3 + m - n) \cos\left[2\left(-e + \frac{\pi}{2} - fx\right)\right] + 3 \cos\left[3\left(-e + \frac{\pi}{2} - fx\right)\right] + \right. \\
& \quad \left. 3(15 + 2m^2 - 4m(-3 + n) - 12n + 2n^2) \sin[efx] \right) + \frac{1}{(m-n)(2+m-n)(4+m-n)(6+m-n)} 2^{-4-m+2n} (-m-n) \\
& \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2m} \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^6 \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right)^{2n} \\
& \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] - \sin\left[\frac{5}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{7}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right)^{-1-m-n}
\end{aligned}$$

$$\begin{aligned} & \left(\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\frac{1}{8}\cos\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{3}{8}\cos\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{5}{8}\cos\left[\frac{5}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \frac{7}{8}\cos\left[\frac{7}{8}\left(-e + \frac{\pi}{2} - fx\right)\right]\right) - \\ & \frac{1}{8}\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(-\sin\left[\frac{1}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{3}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] - \sin\left[\frac{5}{8}\left(-e + \frac{\pi}{2} - fx\right)\right] + \sin\left[\frac{7}{8}\left(-e + \frac{\pi}{2} - fx\right)\right]\right) \\ & \left(-30 - 46m - 18m^2 - 2m^3 + 46n + 36mn + 6m^2n - 18n^2 - 6mn^2 + 2n^3 - 6(3+m-n)\cos\left[2\left(-e + \frac{\pi}{2} - fx\right)\right] + 3\cos\left[3\left(-e + \frac{\pi}{2} - fx\right)\right] + \right. \\ & \left. 3(15 + 2m^2 - 4m(-3+n) - 12n + 2n^2)\sin[e + fx]\right) \left(\cos\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] + \sin\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right]\right) \end{aligned}$$

■ **Problem 212: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx] \csc[c + dx]^3 (a + a \sin[c + dx])^3 dx$$

Optimal (type 3, 65 leaves, 4 steps):

$$-\frac{3a^3 \csc[c + dx]}{d} - \frac{3a^3 \csc[c + dx]^2}{2d} - \frac{a^3 \csc[c + dx]^3}{3d} + \frac{a^3 \log[\sin[c + dx]]}{d}$$

Result (type 3, 146 leaves):

$$\begin{aligned} a^3 \left(-\frac{19 \cot\left[\frac{1}{2}(c + dx)\right]}{12d} - \frac{3 \csc\left[\frac{1}{2}(c + dx)\right]^2}{8d} - \frac{\cot\left[\frac{1}{2}(c + dx)\right] \csc\left[\frac{1}{2}(c + dx)\right]^2}{24d} + \right. \\ \left. \frac{\log[\sin[c + dx]]}{d} - \frac{3 \sec\left[\frac{1}{2}(c + dx)\right]^2}{8d} - \frac{19 \tan\left[\frac{1}{2}(c + dx)\right]}{12d} - \frac{\sec\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right]}{24d} \right) \end{aligned}$$

■ **Problem 230: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx] \csc[c + dx]^2}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 63 leaves, 4 steps):

$$\frac{\csc[c + dx]}{ad} - \frac{\csc[c + dx]^2}{2ad} + \frac{\log[\sin[c + dx]]}{ad} - \frac{\log[1 + \sin[c + dx]]}{ad}$$

Result (type 3, 127 leaves):

$$\begin{aligned} \frac{1}{8ad(1 + \sin[c + dx])} \left(\csc\left[\frac{1}{2}(c + dx)\right] + \sec\left[\frac{1}{2}(c + dx)\right] \right)^2 \left(-1 - 2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \\ \cos[2(c + dx)] \left(2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log[\sin[c + dx]] \right) + \log[\sin[c + dx]] + 2 \sin[c + dx] \end{aligned}$$

■ **Problem 235: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] \sin[c + dx]}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 3, 37 leaves, 4 steps) :

$$\frac{\text{Log}[1 + \sin[c + dx]]}{a^2 d} + \frac{1}{d (a^2 + a^2 \sin[c + dx])}$$

Result (type 3, 88 leaves) :

$$\frac{1}{d (a + a \sin[c + dx])^2} \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right] \right)^2 \left(1 + 2 \text{Log}\left[\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right]\right] \right) \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right] \right)^2$$

■ **Problem 236: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx]}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 3, 52 leaves, 3 steps) :

$$\frac{\text{Log}[\sin[c + dx]]}{a^2 d} - \frac{\text{Log}[1 + \sin[c + dx]]}{a^2 d} + \frac{1}{d (a^2 + a^2 \sin[c + dx])}$$

Result (type 3, 112 leaves) :

$$\frac{1}{a^2 d (1 + \sin[c + dx])^2} \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right] \right)^2 \left(1 - 2 \text{Log}\left[\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right]\right] \right) + \text{Log}[\sin[c + dx]] + \left(-2 \text{Log}\left[\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right]\right] + \text{Log}[\sin[c + dx]] \right) \sin[c + dx]$$

■ **Problem 238: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx] \csc[c + dx]^2}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 3, 85 leaves, 4 steps) :

$$\frac{2 \csc[c + dx]}{a^2 d} - \frac{\csc[c + dx]^2}{2 a^2 d} + \frac{3 \text{Log}[\sin[c + dx]]}{a^2 d} - \frac{3 \text{Log}[1 + \sin[c + dx]]}{a^2 d} + \frac{1}{d (a^2 + a^2 \sin[c + dx])}$$

Result (type 3, 214 leaves) :

$$\frac{1}{16 a^2 d (1 + \sin[c + dx])^2} \left(\csc\left[\frac{1}{2}(c + dx)\right] + \sec\left[\frac{1}{2}(c + dx)\right] \right)^2$$

$$\left(4 - 12 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + 6 \cos[2(c + dx)] \left(-1 + 2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}[\sin[c + dx]] \right) \right. +$$

$$6 \operatorname{Log}[\sin[c + dx]] + \left. \left(6 - 18 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + 9 \operatorname{Log}[\sin[c + dx]] \right) \sin[c + dx] + \right.$$

$$\left. 6 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[3(c + dx)] - 3 \operatorname{Log}[\sin[c + dx]] \sin[3(c + dx)] \right)$$

■ **Problem 239: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx] \csc[c + dx]^3}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 3, 101 leaves, 4 steps):

$$-\frac{3 \csc[c + dx]}{a^2 d} + \frac{\csc[c + dx]^2}{a^2 d} - \frac{\csc[c + dx]^3}{3 a^2 d} - \frac{4 \operatorname{Log}[\sin[c + dx]]}{a^2 d} + \frac{4 \operatorname{Log}[1 + \sin[c + dx]]}{a^2 d} - \frac{1}{d (a^2 + a^2 \sin[c + dx])}$$

Result (type 3, 298 leaves):

$$-\frac{1}{48 a^2 d (1 + \sin[c + dx])^2} \csc\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{1}{2}(c + dx)\right] \left(\csc\left[\frac{1}{2}(c + dx)\right] + \sec\left[\frac{1}{2}(c + dx)\right] \right)^2$$

$$\left(8 - 18 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + 6 \cos[2(c + dx)] \left(-1 + 4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 2 \operatorname{Log}[\sin[c + dx]] \right) \right. +$$

$$9 \operatorname{Log}[\sin[c + dx]] + \cos[4(c + dx)] \left(-6 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + 3 \operatorname{Log}[\sin[c + dx]] \right) +$$

$$14 \sin[c + dx] - 36 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[c + dx] + 18 \operatorname{Log}[\sin[c + dx]] \sin[c + dx] -$$

$$\left. 6 \sin[3(c + dx)] + 12 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[3(c + dx)] - 6 \operatorname{Log}[\sin[c + dx]] \sin[3(c + dx)] \right)$$

■ **Problem 240: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] \sin[c + dx]^5}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 111 leaves, 4 steps):

$$-\frac{10 \operatorname{Log}[1 + \sin[c + dx]]}{a^3 d} + \frac{6 \sin[c + dx]}{a^3 d} - \frac{3 \sin[c + dx]^2}{2 a^3 d} + \frac{\sin[c + dx]^3}{3 a^3 d} + \frac{1}{2 a d (a + a \sin[c + dx])^2} - \frac{5}{d (a^3 + a^3 \sin[c + dx])}$$

Result (type 3, 450 leaves):

$$\frac{1}{32} \left(\frac{24 \cos[2c] \cos[2dx]}{a^3 d} - \frac{448 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{a^3 d} + x \left(\frac{224 \cos\left[\frac{c}{2}\right]}{a^3 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)} - \frac{224 \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right)}{a^3 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)} - \frac{224 \sin\left[\frac{c}{2}\right]}{a^3 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)} \right) + \frac{168 \cos[dx] \sin[c]}{a^3 d} - \frac{8 \cos[3dx] \sin[3c]}{3 a^3 d} + \frac{168 \cos[c] \sin[dx]}{a^3 d} - \frac{24 \sin[2c] \sin[2dx]}{a^3 d} - \frac{8 \cos[3c] \sin[3dx]}{3 a^3 d} + \frac{3}{a^3 d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} - \frac{70}{a^3 d \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} \right) - \frac{1}{4 a^3 d} \left(24 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] - 4 \cos[dx] \sin[c] - 4 \cos[c] \sin[dx] - \frac{1}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{10}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right) + \frac{5(-1 - 2 \sin[c+dx])}{32 a^3 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4}$$

■ **Problem 246: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx] \csc[c+dx]}{(a+a \sin[c+dx])^3} dx$$

Optimal (type 3, 90 leaves, 4 steps):

$$-\frac{\csc[c+dx]}{a^3 d} - \frac{3 \log[\sin[c+dx]]}{a^3 d} + \frac{3 \log[1 + \sin[c+dx]]}{a^3 d} - \frac{1}{2 a d (a + a \sin[c+dx])^2} - \frac{2}{d (a^3 + a^3 \sin[c+dx])}$$

Result (type 3, 218 leaves):

$$\frac{1}{2 d (a + a \sin[c+dx])^3} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \left(-1 - 4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)^2 - \cot\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 + 12 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 - 6 \log[\sin[c+dx]] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 - \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 \tan\left[\frac{1}{2}(c+dx)\right]$$

■ **Problem 247: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx] \csc[c+dx]^2}{(a+a \sin[c+dx])^3} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{3 \operatorname{Csc}[c + dx]}{a^3 d} - \frac{\operatorname{Csc}[c + dx]^2}{2 a^3 d} + \frac{6 \operatorname{Log}[\operatorname{Sin}[c + dx]]}{a^3 d} - \frac{6 \operatorname{Log}[1 + \operatorname{Sin}[c + dx]]}{a^3 d} + \frac{1}{2 a d (a + a \operatorname{Sin}[c + dx])^2} + \frac{3}{d (a^3 + a^3 \operatorname{Sin}[c + dx])}$$

Result (type 3, 275 leaves):

$$\frac{1}{8 a^3 d (1 + \operatorname{Sin}[c + dx])^3} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 \left(4 - \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right] \right)^4 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]^2 + 24 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 + 12 \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^4 - 96 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^4 + 48 \operatorname{Log}[\operatorname{Sin}[c + dx]] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^4 + 12 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^4 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right)^4 \right)$$

■ **Problem 248: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c + dx] \operatorname{Csc}[c + dx]^3}{(a + a \operatorname{Sin}[c + dx])^3} dx$$

Optimal (type 3, 126 leaves, 4 steps):

$$-\frac{6 \operatorname{Csc}[c + dx]}{a^3 d} + \frac{3 \operatorname{Csc}[c + dx]^2}{2 a^3 d} - \frac{\operatorname{Csc}[c + dx]^3}{3 a^3 d} - \frac{10 \operatorname{Log}[\operatorname{Sin}[c + dx]]}{a^3 d} + \frac{10 \operatorname{Log}[1 + \operatorname{Sin}[c + dx]]}{a^3 d} - \frac{1}{2 a d (a + a \operatorname{Sin}[c + dx])^2} - \frac{4}{d (a^3 + a^3 \operatorname{Sin}[c + dx])}$$

Result (type 3, 335 leaves):

$$\begin{aligned} & \frac{1}{24 a^3 d (1 + \sin[c + dx])^3} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \\ & \left(-12 - \cos\left[\frac{1}{2}(c + dx)\right] \right) \left(1 + \cot\left[\frac{1}{2}(c + dx)\right] \right)^4 \sin\left[\frac{1}{2}(c + dx)\right] + 9 \left(1 + \cot\left[\frac{1}{2}(c + dx)\right] \right)^4 \sin\left[\frac{1}{2}(c + dx)\right]^2 - \\ & 96 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 - 74 \cot\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 + \\ & 480 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 - \\ & 240 \operatorname{Log}[\sin[c + dx]] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 - 74 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 \tan\left[\frac{1}{2}(c + dx)\right] + \\ & 9 \cos\left[\frac{1}{2}(c + dx)\right]^2 \left(1 + \tan\left[\frac{1}{2}(c + dx)\right] \right)^4 - \frac{1}{2} \sin[c + dx] \left(1 + \tan\left[\frac{1}{2}(c + dx)\right] \right)^4 \end{aligned}$$

■ **Problem 255: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx] \operatorname{Csc}[c + dx]}{(a + a \sin[c + dx])^4} dx$$

Optimal (type 3, 111 leaves, 4 steps):

$$-\frac{\operatorname{Csc}[c + dx]}{a^4 d} - \frac{4 \operatorname{Log}[\sin[c + dx]]}{a^4 d} + \frac{4 \operatorname{Log}[1 + \sin[c + dx]]}{a^4 d} - \frac{1}{3 a d (a + a \sin[c + dx])^3} - \frac{1}{d (a^2 + a^2 \sin[c + dx])^2} - \frac{3}{d (a^4 + a^4 \sin[c + dx])}$$

Result (type 3, 243 leaves):

$$\begin{aligned} & \frac{1}{6 d (a + a \sin[c + dx])^4} \\ & \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \left(-2 - 6 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 - 18 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 - \right. \\ & 3 \cot\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 + 48 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 - \\ & \left. 24 \operatorname{Log}[\sin[c + dx]] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 - 3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 \tan\left[\frac{1}{2}(c + dx)\right] \right) \end{aligned}$$

■ **Problem 256: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx] \operatorname{Csc}[c + dx]^2}{(a + a \sin[c + dx])^4} dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$\frac{4 \operatorname{Csc}[c + dx]}{a^4 d} - \frac{\operatorname{Csc}[c + dx]^2}{2 a^4 d} + \frac{10 \operatorname{Log}[\operatorname{Sin}[c + dx]]}{a^4 d} - \frac{10 \operatorname{Log}[1 + \operatorname{Sin}[c + dx]]}{a^4 d} +$$

$$\frac{1}{3 a d (a + a \operatorname{Sin}[c + dx])^3} + \frac{3}{2 d (a^2 + a^2 \operatorname{Sin}[c + dx])^2} + \frac{6}{d (a^4 + a^4 \operatorname{Sin}[c + dx])}$$

Result (type 3, 300 leaves):

$$\frac{1}{24 a^4 d (1 + \operatorname{Sin}[c + dx])^4} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2$$

$$\left(8 - 3 \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right] \right) \right)^6 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]^4 + 36 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 + 144 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^4 +$$

$$48 \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6 - 480 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right]$$

$$\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6 + 240 \operatorname{Log}[\operatorname{Sin}[c + dx]] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6 +$$

$$48 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] - 3 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^4 \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right)^6$$

■ **Problem 257: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + dx] \sqrt{a + a \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 51 leaves, 4 steps):

$$-\frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a + a \operatorname{Sin}[c + dx]}}{\sqrt{a}}\right]}{d} + \frac{2 \sqrt{a + a \operatorname{Sin}[c + dx]}}{d}$$

Result (type 3, 118 leaves):

$$\left(\left(2 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + 2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right.$$

$$\left. \sqrt{a (1 + \operatorname{Sin}[c + dx])} \right) / \left(d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right)$$

■ **Problem 258: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^n (a + a \operatorname{Sin}[c + dx])^4 dx$$

Optimal (type 3, 114 leaves, 3 steps):

$$\frac{a^4 \operatorname{Sin}[c + dx]^{1+n}}{d (1+n)} + \frac{4 a^4 \operatorname{Sin}[c + dx]^{2+n}}{d (2+n)} + \frac{6 a^4 \operatorname{Sin}[c + dx]^{3+n}}{d (3+n)} + \frac{4 a^4 \operatorname{Sin}[c + dx]^{4+n}}{d (4+n)} + \frac{a^4 \operatorname{Sin}[c + dx]^{5+n}}{d (5+n)}$$

Result (type 3, 457 leaves):

$$\frac{1}{d \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^8} \sin[c + dx]^n (a + a \sin[c + dx])^4 \left(\frac{22 + 7n}{2(2+n)(4+n)} + \frac{(315 + 300n + 49n^2) \left(-\frac{1}{16} i \cos[c + dx] + \frac{1}{16} \sin[c + dx] \right)}{(1+n)(3+n)(5+n)} + \frac{(315 + 300n + 49n^2) \left(\frac{1}{16} i \cos[c + dx] + \frac{1}{16} \sin[c + dx] \right)}{(1+n)(3+n)(5+n)} + \frac{(3+n)(-2 \cos[2(c + dx)] - 2i \sin[2(c + dx)])}{(2+n)(4+n)} + \frac{(3+n)(-2 \cos[2(c + dx)] + 2i \sin[2(c + dx)])}{(2+n)(4+n)} + \frac{(135 + 29n) \left(-\frac{1}{32} i \cos[3(c + dx)] - \frac{1}{32} \sin[3(c + dx)] \right)}{(3+n)(5+n)} + \frac{(135 + 29n) \left(\frac{1}{32} i \cos[3(c + dx)] - \frac{1}{32} \sin[3(c + dx)] \right)}{(3+n)(5+n)} + \frac{\frac{1}{4} \cos[4(c + dx)] - \frac{1}{4} i \sin[4(c + dx)]}{4+n} + \frac{\frac{1}{4} \cos[4(c + dx)] + \frac{1}{4} i \sin[4(c + dx)]}{4+n} + \frac{-\frac{1}{32} i \cos[5(c + dx)] + \frac{1}{32} \sin[5(c + dx)]}{5+n} + \frac{\frac{1}{32} i \cos[5(c + dx)] + \frac{1}{32} \sin[5(c + dx)]}{5+n} \right)$$

■ **Problem 259: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] \sin[c + dx]^n (a + a \sin[c + dx])^3 dx$$

Optimal (type 3, 91 leaves, 3 steps):

$$\frac{a^3 \sin[c + dx]^{1+n}}{d(1+n)} + \frac{3a^3 \sin[c + dx]^{2+n}}{d(2+n)} + \frac{3a^3 \sin[c + dx]^{3+n}}{d(3+n)} + \frac{a^3 \sin[c + dx]^{4+n}}{d(4+n)}$$

Result (type 3, 363 leaves):

$$\frac{1}{d \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^6} \sin[c + dx]^n (a + a \sin[c + dx])^3 \left(\frac{3(18 + 5n)}{8(2+n)(4+n)} + \frac{(21 + 13n) \left(-\frac{1}{8} i \cos[c + dx] + \frac{1}{8} \sin[c + dx] \right)}{(1+n)(3+n)} + \frac{(21 + 13n) \left(\frac{1}{8} i \cos[c + dx] + \frac{1}{8} \sin[c + dx] \right)}{(1+n)(3+n)} + \frac{(-7 - 2n) \left(\frac{1}{2} \cos[2(c + dx)] - \frac{1}{2} i \sin[2(c + dx)] \right)}{(2+n)(4+n)} + \frac{(-7 - 2n) \left(\frac{1}{2} \cos[2(c + dx)] + \frac{1}{2} i \sin[2(c + dx)] \right)}{(2+n)(4+n)} + \frac{-\frac{3}{8} i \cos[3(c + dx)] - \frac{3}{8} \sin[3(c + dx)]}{3+n} + \frac{\frac{3}{8} i \cos[3(c + dx)] - \frac{3}{8} \sin[3(c + dx)]}{3+n} + \frac{\frac{1}{16} \cos[4(c + dx)] - \frac{1}{16} i \sin[4(c + dx)]}{4+n} + \frac{\frac{1}{16} \cos[4(c + dx)] + \frac{1}{16} i \sin[4(c + dx)]}{4+n} \right)$$

■ **Problem 262: Unable to integrate problem.**

$$\int \frac{\cos[c + dx] \sin[c + dx]^n}{a + a \sin[c + dx]} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}[1, 1 + n, 2 + n, -\sin[c + dx]] \sin[c + dx]^{1+n}}{a d (1 + n)}$$

Result (type 8, 29 leaves):

$$\int \frac{\cos[c + dx] \sin[c + dx]^n}{a + a \sin[c + dx]} dx$$

■ **Problem 263: Unable to integrate problem.**

$$\int \frac{\cos[c + dx] \sin[c + dx]^n}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}[2, 1 + n, 2 + n, -\sin[c + dx]] \sin[c + dx]^{1+n}}{a^2 d (1 + n)}$$

Result (type 8, 29 leaves):

$$\int \frac{\cos[c + dx] \sin[c + dx]^n}{(a + a \sin[c + dx])^2} dx$$

■ **Problem 264: Unable to integrate problem.**

$$\int \frac{\cos[c + dx] \sin[c + dx]^n}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}[3, 1 + n, 2 + n, -\sin[c + dx]] \sin[c + dx]^{1+n}}{a^3 d (1 + n)}$$

Result (type 8, 29 leaves):

$$\int \frac{\cos[c + dx] \sin[c + dx]^n}{(a + a \sin[c + dx])^3} dx$$

■ **Problem 265: Unable to integrate problem.**

$$\int \frac{\cos[c + dx] \sin[c + dx]^n}{(a + a \sin[c + dx])^4} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}[4, 1 + n, 2 + n, -\text{Sin}[c + d x]] \text{Sin}[c + d x]^{1+n}}{a^4 d (1 + n)}$$

Result (type 8, 29 leaves):

$$\int \frac{\text{Cos}[c + d x] \text{Sin}[c + d x]^n}{(a + a \text{Sin}[c + d x])^4} dx$$

■ **Problem 282: Result more than twice size of optimal antiderivative.**

$$\int \text{Cot}[c + d x]^2 \text{Csc}[c + d x]^3 (a + a \text{Sin}[c + d x])^2 dx$$

Optimal (type 3, 82 leaves, 9 steps):

$$\frac{5 a^2 \text{ArcTanh}[\text{Cos}[c + d x]]}{8 d} - \frac{2 a^2 \text{Cot}[c + d x]^3}{3 d} - \frac{3 a^2 \text{Cot}[c + d x] \text{Csc}[c + d x]}{8 d} - \frac{a^2 \text{Cot}[c + d x] \text{Csc}[c + d x]^3}{4 d}$$

Result (type 3, 209 leaves):

$$a^2 \left(\frac{\text{Cot}\left[\frac{1}{2}(c + d x)\right]}{3 d} - \frac{3 \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{32 d} - \frac{\text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{12 d} - \frac{\text{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{64 d} + \frac{5 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} - \frac{5 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{3 \text{Sec}\left[\frac{1}{2}(c + d x)\right]^2}{32 d} + \frac{\text{Sec}\left[\frac{1}{2}(c + d x)\right]^4}{64 d} - \frac{\text{Tan}\left[\frac{1}{2}(c + d x)\right]}{3 d} + \frac{\text{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{12 d} \right)$$

■ **Problem 292: Result more than twice size of optimal antiderivative.**

$$\int \text{Cot}[c + d x]^2 \text{Csc}[c + d x]^4 (a + a \text{Sin}[c + d x])^3 dx$$

Optimal (type 3, 100 leaves, 12 steps):

$$\frac{7 a^3 \text{ArcTanh}[\text{Cos}[c + d x]]}{8 d} - \frac{4 a^3 \text{Cot}[c + d x]^3}{3 d} - \frac{a^3 \text{Cot}[c + d x]^5}{5 d} - \frac{a^3 \text{Cot}[c + d x] \text{Csc}[c + d x]}{8 d} - \frac{3 a^3 \text{Cot}[c + d x] \text{Csc}[c + d x]^3}{4 d}$$

Result (type 3, 267 leaves):

$$a^3 \left(\frac{17 \text{Cot}\left[\frac{1}{2}(c + d x)\right]}{30 d} - \frac{\text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{32 d} - \frac{59 \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{480 d} - \frac{3 \text{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{64 d} - \frac{\text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{160 d} + \frac{7 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} - \frac{7 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{\text{Sec}\left[\frac{1}{2}(c + d x)\right]^2}{32 d} + \frac{3 \text{Sec}\left[\frac{1}{2}(c + d x)\right]^4}{64 d} - \frac{17 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{30 d} + \frac{59 \text{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{480 d} + \frac{\text{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{160 d} \right)$$

■ **Problem 293: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^2 \operatorname{Csc}[c + d x]^5 (a + a \operatorname{Sin}[c + d x])^3 dx$$

Optimal (type 3, 124 leaves, 14 steps):

$$\frac{7 a^3 \operatorname{ArcTanh}[\operatorname{Cos}[c + d x]]}{16 d} - \frac{4 a^3 \operatorname{Cot}[c + d x]^3}{3 d} - \frac{3 a^3 \operatorname{Cot}[c + d x]^5}{5 d} +$$

$$\frac{7 a^3 \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]}{16 d} - \frac{17 a^3 \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^3}{24 d} - \frac{a^3 \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^5}{6 d}$$

Result (type 3, 252 leaves):

$$-\frac{1}{1920 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^6}$$

$$a^3 \left(\operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^6 (18 + 5 \operatorname{Csc}[c + d x]) + \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^4 (34 + 90 \operatorname{Csc}[c + d x]) - 2 \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2 (176 + 105 \operatorname{Csc}[c + d x]) - \right.$$

$$840 \operatorname{Csc}[c + d x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + (97 + 159 \operatorname{Cos}[c + d x] + 44 \operatorname{Cos}[2(c + d x)]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^6 +$$

$$\left. 840 \operatorname{Csc}[c + d x]^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^2 - 1440 \operatorname{Csc}[c + d x]^5 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^4 - 320 \operatorname{Csc}[c + d x]^7 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^6 \right) \operatorname{Sin}[c + d x] (1 + \operatorname{Sin}[c + d x])^3$$

■ **Problem 297: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^2 \operatorname{Sin}[c + d x]^4}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 104 leaves, 6 steps):

$$\frac{3 x}{8 a} + \frac{\operatorname{Cos}[c + d x]}{a d} - \frac{2 \operatorname{Cos}[c + d x]^3}{3 a d} + \frac{\operatorname{Cos}[c + d x]^5}{5 a d} - \frac{3 \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{8 a d} - \frac{\operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]^3}{4 a d}$$

Result (type 3, 281 leaves):

$$\frac{1}{480} \left(\frac{180 x}{a} + \frac{300 \operatorname{Cos}[c] \operatorname{Cos}[d x]}{a d} - \frac{50 \operatorname{Cos}[3 c] \operatorname{Cos}[3 d x]}{a d} + \frac{6 \operatorname{Cos}[5 c] \operatorname{Cos}[5 d x]}{a d} - \frac{120 \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{a d} + \right.$$

$$\frac{15 \operatorname{Cos}[4 d x] \operatorname{Sin}[4 c]}{a d} - \frac{300 \operatorname{Sin}[c] \operatorname{Sin}[d x]}{a d} - \frac{120 \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{a d} + \frac{50 \operatorname{Sin}[3 c] \operatorname{Sin}[3 d x]}{a d} + \frac{15 \operatorname{Cos}[4 c] \operatorname{Sin}[4 d x]}{a d} -$$

$$\left. \frac{6 \operatorname{Sin}[5 c] \operatorname{Sin}[5 d x]}{a d} - \frac{60 \operatorname{Sin}\left[\frac{d x}{2}\right]}{a d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)} + \frac{30 \operatorname{Sin}[c + d x]}{a d (1 + \operatorname{Sin}[c + d x])} + \frac{60 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^2}{d (a + a \operatorname{Sin}[c + d x])} \right)$$

■ **Problem 298: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 \sin[c + dx]^3}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 87 leaves, 6 steps):

$$-\frac{3x}{8a} - \frac{\cos[c + dx]}{ad} + \frac{\cos[c + dx]^3}{3ad} + \frac{3 \cos[c + dx] \sin[c + dx]}{8ad} + \frac{\cos[c + dx] \sin[c + dx]^3}{4ad}$$

Result (type 3, 271 leaves):

$$\frac{1}{192ad \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(24(c - 3dx) \cos\left[\frac{c}{2}\right] - 72 \cos\left[\frac{c}{2} + dx\right] - 72 \cos\left[\frac{3c}{2} + dx\right] + 24 \cos\left[\frac{3c}{2} + 2dx\right] - 24 \cos\left[\frac{5c}{2} + 2dx\right] + 8 \cos\left[\frac{5c}{2} + 3dx\right] + 8 \cos\left[\frac{7c}{2} + 3dx\right] - \right. \\ \left. 3 \cos\left[\frac{7c}{2} + 4dx\right] + 3 \cos\left[\frac{9c}{2} + 4dx\right] - 48 \sin\left[\frac{c}{2}\right] + 24c \sin\left[\frac{c}{2}\right] - 72dx \sin\left[\frac{c}{2}\right] + 72 \sin\left[\frac{c}{2} + dx\right] - 72 \sin\left[\frac{3c}{2} + dx\right] + \right. \\ \left. 24 \sin\left[\frac{3c}{2} + 2dx\right] + 24 \sin\left[\frac{5c}{2} + 2dx\right] - 8 \sin\left[\frac{5c}{2} + 3dx\right] + 8 \sin\left[\frac{7c}{2} + 3dx\right] - 3 \sin\left[\frac{7c}{2} + 4dx\right] - 3 \sin\left[\frac{9c}{2} + 4dx\right] \right)$$

■ **Problem 300: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 \sin[c + dx]}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 45 leaves, 4 steps):

$$-\frac{x}{2a} - \frac{\cos[c + dx]}{ad} + \frac{\cos[c + dx] \sin[c + dx]}{2ad}$$

Result (type 3, 161 leaves):

$$\frac{1}{8ad \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(2(c - 2dx) \cos\left[\frac{c}{2}\right] - 4 \cos\left[\frac{c}{2} + dx\right] - 4 \cos\left[\frac{3c}{2} + dx\right] + \cos\left[\frac{3c}{2} + 2dx\right] - \right. \\ \left. \cos\left[\frac{5c}{2} + 2dx\right] - 4 \sin\left[\frac{c}{2}\right] + 2c \sin\left[\frac{c}{2}\right] - 4dx \sin\left[\frac{c}{2}\right] + 4 \sin\left[\frac{c}{2} + dx\right] - 4 \sin\left[\frac{3c}{2} + dx\right] + \sin\left[\frac{3c}{2} + 2dx\right] + \sin\left[\frac{5c}{2} + 2dx\right] \right)$$

■ **Problem 302: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx]^2}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 29 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\cos[c + dx]]}{ad} - \frac{\cot[c + dx]}{ad}$$

Result (type 3, 69 leaves):

$$-\frac{1}{2ad} \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}[c+dx] + \left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) \operatorname{Sin}[c+dx] \right)$$

- **Problem 308: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]^3}{(a+a\operatorname{Sin}[c+dx])^2} dx$$

Optimal (type 3, 83 leaves, 9 steps):

$$\frac{3x}{a^2} + \frac{3\operatorname{Cos}[c+dx]}{a^2d} - \frac{\operatorname{Cos}[c+dx]^3}{3a^2d} - \frac{\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{a^2d} + \frac{2\operatorname{Cos}[c+dx]}{a^2d(1+\operatorname{Sin}[c+dx])}$$

Result (type 3, 183 leaves):

$$\left((2+72dx)\operatorname{Cos}\left[\frac{dx}{2}\right] + 31\operatorname{Cos}\left[c+\frac{dx}{2}\right] + 27\operatorname{Cos}\left[c+\frac{3dx}{2}\right] + 5\operatorname{Cos}\left[3c+\frac{5dx}{2}\right] - \operatorname{Cos}\left[3c+\frac{7dx}{2}\right] - 131\operatorname{Sin}\left[\frac{dx}{2}\right] + 2\operatorname{Sin}\left[c+\frac{dx}{2}\right] + 72dx\operatorname{Sin}\left[c+\frac{dx}{2}\right] + 27\operatorname{Sin}\left[2c+\frac{3dx}{2}\right] - 5\operatorname{Sin}\left[2c+\frac{5dx}{2}\right] - \operatorname{Sin}\left[4c+\frac{7dx}{2}\right] \right) / \left(24a^2d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

- **Problem 310: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{(a+a\operatorname{Sin}[c+dx])^2} dx$$

Optimal (type 3, 47 leaves, 3 steps):

$$\frac{2x}{a^2} + \frac{\operatorname{Cos}[c+dx]}{a^2d} + \frac{2\operatorname{Cos}[c+dx]}{d(a^2+a^2\operatorname{Sin}[c+dx])}$$

Result (type 3, 129 leaves):

$$\left((1+12dx)\operatorname{Cos}\left[\frac{dx}{2}\right] + 2\operatorname{Cos}\left[c+\frac{dx}{2}\right] + 3\operatorname{Cos}\left[c+\frac{3dx}{2}\right] - 28\operatorname{Sin}\left[\frac{dx}{2}\right] + \operatorname{Sin}\left[c+\frac{dx}{2}\right] + 12dx\operatorname{Sin}\left[c+\frac{dx}{2}\right] + 3\operatorname{Sin}\left[2c+\frac{3dx}{2}\right] \right) / \left(6a^2d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

- **Problem 311: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx] \operatorname{Cot}[c+dx]}{(a+a\operatorname{Sin}[c+dx])^2} dx$$

Optimal (type 3, 40 leaves, 5 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{a^2d} + \frac{2\operatorname{Cos}[c+dx]}{a^2d(1+\operatorname{Sin}[c+dx])}$$

Result (type 3, 115 leaves) :

$$-\frac{1}{a^2 d (1 + \sin[c + dx])^2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3 \\ \left(\cos\left[\frac{1}{2}(c + dx)\right] \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \left(4 + \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \sin\left[\frac{1}{2}(c + dx)\right] \right)$$

■ **Problem 312: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx]^2}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 3, 54 leaves, 7 steps) :

$$\frac{2 \operatorname{ArcTanh}[\cos[c + dx]]}{a^2 d} - \frac{\cot[c + dx]}{a^2 d} - \frac{2 \cot[c + dx]}{a^2 d (1 + \csc[c + dx])}$$

Result (type 3, 216 leaves) :

$$-\frac{1}{4 a^2 d (1 + \sin[c + dx])^2} \csc\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{1}{2}(c + dx)\right] \\ \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3 \left(\cos\left[\frac{3}{2}(c + dx)\right] \left(5 + 2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - 2 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right. \\ \left. \cos\left[\frac{1}{2}(c + dx)\right] \left(-3 - 2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + 2 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + 2 \left(-2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + \right. \right. \\ \left. \left. 2 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] + \cos[c + dx] \left(1 - 2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + 2 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \right) \right) \sin\left[\frac{1}{2}(c + dx)\right]$$

■ **Problem 313: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx]^2 \csc[c + dx]}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 3, 78 leaves, 9 steps) :

$$-\frac{5 \operatorname{ArcTanh}[\cos[c + dx]]}{2 a^2 d} + \frac{2 \cot[c + dx]}{a^2 d} - \frac{\cot[c + dx] \csc[c + dx]}{2 a^2 d} + \frac{2 \cos[c + dx]}{a^2 d (1 + \sin[c + dx])}$$

Result (type 3, 364 leaves) :

$$\begin{aligned}
& - \frac{4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3}{d(a+a\operatorname{Sin}[c+dx])^2} + \\
& \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4}{d(a+a\operatorname{Sin}[c+dx])^2} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4}{8d(a+a\operatorname{Sin}[c+dx])^2} - \\
& \frac{5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4}{2d(a+a\operatorname{Sin}[c+dx])^2} + \frac{5 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4}{2d(a+a\operatorname{Sin}[c+dx])^2} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4}{8d(a+a\operatorname{Sin}[c+dx])^2} - \frac{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{d(a+a\operatorname{Sin}[c+dx])^2}
\end{aligned}$$

■ **Problem 314: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^2 \operatorname{Csc}[c+dx]^2}{(a+a\operatorname{Sin}[c+dx])^2} dx$$

Optimal (type 3, 91 leaves, 11 steps):

$$\frac{3 \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{a^2 d} - \frac{3 \operatorname{Cot}[c+dx]}{a^2 d} - \frac{\operatorname{Cot}[c+dx]^3}{3 a^2 d} + \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{a^2 d} - \frac{2 \operatorname{Cos}[c+dx]}{a^2 d (1 + \operatorname{Sin}[c+dx])}$$

Result (type 3, 472 leaves):

$$\begin{aligned}
& \frac{1}{192 a^2 d (1 + \operatorname{Sin}[c+dx])^2} \\
& \left(\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \right)^3 \left(-10 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] + 20 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] - 9 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \right) + \\
& 9 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] + 3 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \left(8 + 9 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] - 9 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) - \\
& 3 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] \left(14 + 9 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] - 9 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 9 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \\
& 9 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 12 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 27 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - \\
& 27 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 6 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 27 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] - \\
& 27 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] - 2 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] - 9 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + 9 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \\
& \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + 8 \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] - 9 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] + 9 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]
\end{aligned}$$

■ **Problem 315: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 \sin[c + dx]^3}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 97 leaves, 9 steps) :

$$-\frac{11x}{2a^3} - \frac{3 \cos[c + dx]}{a^3 d} + \frac{\cos[c + dx] \sin[c + dx]}{2a^3 d} + \frac{2 \cos[c + dx]}{3a^3 d (1 + \sin[c + dx])^2} - \frac{19 \cos[c + dx]}{3a^3 d (1 + \sin[c + dx])}$$

Result (type 3, 235 leaves) :

$$\frac{1}{240 a^3 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} \left((9 - 1980 dx) \cos\left[\frac{dx}{2}\right] + 1326 \cos\left[c + \frac{dx}{2}\right] - 2012 \cos\left[c + \frac{3dx}{2}\right] - 3 \cos\left[2c + \frac{3dx}{2}\right] + 660 dx \cos\left[2c + \frac{3dx}{2}\right] + 135 \cos\left[3c + \frac{5dx}{2}\right] - 15 \cos\left[3c + \frac{7dx}{2}\right] + 3216 \sin\left[\frac{dx}{2}\right] + 9 \sin\left[c + \frac{dx}{2}\right] - 1980 dx \sin\left[c + \frac{dx}{2}\right] + 3 \sin\left[c + \frac{3dx}{2}\right] - 660 dx \sin\left[c + \frac{3dx}{2}\right] - 498 \sin\left[2c + \frac{3dx}{2}\right] - 135 \sin\left[2c + \frac{5dx}{2}\right] - 15 \sin\left[4c + \frac{7dx}{2}\right] \right)$$

■ **Problem 317: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 \sin[c + dx]}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 61 leaves, 3 steps) :

$$-\frac{x}{a^3} - \frac{7 \cos[c + dx]}{3a^3 d (1 + \sin[c + dx])} + \frac{2 \cos[c + dx]}{3a d (a + a \sin[c + dx])^2}$$

Result (type 3, 183 leaves) :

$$\left((9 - 180 dx) \cos\left[\frac{dx}{2}\right] + 351 \cos\left[c + \frac{dx}{2}\right] - 277 \cos\left[c + \frac{3dx}{2}\right] - 3 \cos\left[2c + \frac{3dx}{2}\right] + 60 dx \cos\left[2c + \frac{3dx}{2}\right] + 471 \sin\left[\frac{dx}{2}\right] + 9 \sin\left[c + \frac{dx}{2}\right] - 180 dx \sin\left[c + \frac{dx}{2}\right] + 3 \sin\left[c + \frac{3dx}{2}\right] - 60 dx \sin\left[c + \frac{3dx}{2}\right] - 3 \sin\left[2c + \frac{3dx}{2}\right] \right) / \left(120 a^3 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3 \right)$$

■ **Problem 318: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] \cot[c + dx]}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 68 leaves, 7 steps) :

$$-\frac{\text{ArcTanh}[\text{Cos}[c+dx]]}{a^3 d} + \frac{2 \text{Cos}[c+dx]}{3 a^3 d (1+\text{Sin}[c+dx])^2} + \frac{5 \text{Cos}[c+dx]}{3 a^3 d (1+\text{Sin}[c+dx])}$$

Result (type 3, 185 leaves):

$$\frac{1}{3 d (a+a \text{Sin}[c+dx])^3} \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 \\ - \left(-4 \text{Sin}\left[\frac{1}{2}(c+dx)\right] + 2 \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right) - 10 \text{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^2 - \right. \\ \left. 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 + 3 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 \right)$$

■ **Problem 319: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c+dx]^2}{(a+a \text{Sin}[c+dx])^3} dx$$

Optimal (type 3, 82 leaves, 10 steps):

$$\frac{3 \text{ArcTanh}[\text{Cos}[c+dx]]}{a^3 d} - \frac{\text{Cot}[c+dx]}{a^3 d} + \frac{2 \text{Cot}[c+dx]}{3 a^3 d (1+\text{Csc}[c+dx])^2} - \frac{13 \text{Cot}[c+dx]}{3 a^3 d (1+\text{Csc}[c+dx])}$$

Result (type 3, 255 leaves):

$$\frac{1}{6 d (a+a \text{Sin}[c+dx])^3} \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 \\ - \left(8 \text{Sin}\left[\frac{1}{2}(c+dx)\right] - 4 \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right) + 44 \text{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^2 - \right. \\ \left. 3 \text{Cot}\left[\frac{1}{2}(c+dx)\right] \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 + 18 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 - \right. \\ \left. 18 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 + 3 \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 \text{Tan}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 320: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c+dx]^2 \text{Csc}[c+dx]}{(a+a \text{Sin}[c+dx])^3} dx$$

Optimal (type 3, 106 leaves, 11 steps):

$$-\frac{11 \text{ArcTanh}[\text{Cos}[c+dx]]}{2 a^3 d} + \frac{3 \text{Cot}[c+dx]}{a^3 d} - \frac{\text{Cot}[c+dx] \text{Csc}[c+dx]}{2 a^3 d} + \frac{2 \text{Cos}[c+dx]}{3 a^3 d (1+\text{Sin}[c+dx])^2} + \frac{17 \text{Cos}[c+dx]}{3 a^3 d (1+\text{Sin}[c+dx])}$$

Result (type 3, 465 leaves):

$$\begin{aligned}
& - \frac{4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3}{3d(a+a\operatorname{Sin}[c+dx])^3} + \\
& \frac{2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4}{3d(a+a\operatorname{Sin}[c+dx])^3} - \frac{34 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5}{3d(a+a\operatorname{Sin}[c+dx])^3} + \\
& \frac{3 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{2d(a+a\operatorname{Sin}[c+dx])^3} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a\operatorname{Sin}[c+dx])^3} - \\
& \frac{11 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{2d(a+a\operatorname{Sin}[c+dx])^3} + \frac{11 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{2d(a+a\operatorname{Sin}[c+dx])^3} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a\operatorname{Sin}[c+dx])^3} - \frac{3 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2d(a+a\operatorname{Sin}[c+dx])^3}
\end{aligned}$$

■ **Problem 326: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^2 \sqrt{a+a\operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 89 leaves, 4 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c+dx]}{\sqrt{a+a\operatorname{Sin}[c+dx]}}\right]}{d} + \frac{3a \operatorname{Cos}[c+dx]}{d\sqrt{a+a\operatorname{Sin}[c+dx]}} - \frac{\operatorname{Cot}[c+dx] \sqrt{a+a\operatorname{Sin}[c+dx]}}{d}$$

Result (type 3, 206 leaves):

$$\begin{aligned}
& \left(\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \sqrt{a(1+\operatorname{Sin}[c+dx])} \left(-4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 2 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] + 4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] + \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] + 2 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] \right) \right) / \\
& \left(d \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \right) \left(\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right] \right) \left(\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right] \right) \right)
\end{aligned}$$

■ **Problem 327: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^2 \operatorname{Csc}[c+dx] \sqrt{a+a\operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 101 leaves, 5 steps):

$$\frac{5\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c+dx]}{\sqrt{a+a\operatorname{Sin}[c+dx]}}\right]}{4d} - \frac{a \operatorname{Cot}[c+dx]}{4d\sqrt{a+a\operatorname{Sin}[c+dx]}} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx] \sqrt{a+a\operatorname{Sin}[c+dx]}}{2d}$$

Result (type 3, 249 leaves):

$$\begin{aligned}
& - \frac{1}{4 d \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]\right) \left(\operatorname{Csc}\left[\frac{1}{4}(c + d x)\right]^2 - \operatorname{Sec}\left[\frac{1}{4}(c + d x)\right]^2\right)^2} \\
& \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^7 \sqrt{a(1 + \operatorname{Sin}[c + d x])} \left(2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + 6 \operatorname{Cos}\left[\frac{3}{2}(c + d x)\right] - 5 \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\
& \quad \left. 5 \operatorname{Cos}[2(c + d x)] \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 5 \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \right. \\
& \quad \left. 5 \operatorname{Cos}[2(c + d x)] \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 6 \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right]\right)
\end{aligned}$$

■ **Problem 328: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + d x]^2 \operatorname{Csc}[c + d x]^2 \sqrt{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 137 leaves, 6 steps):

$$\frac{3 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c + d x]}{\sqrt{a + a \operatorname{Sin}[c + d x]}}\right]}{8 d} + \frac{3 a \operatorname{Cot}[c + d x]}{8 d \sqrt{a + a \operatorname{Sin}[c + d x]}} - \frac{a \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]}{12 d \sqrt{a + a \operatorname{Sin}[c + d x]}} - \frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2 \sqrt{a + a \operatorname{Sin}[c + d x]}}{3 d}$$

Result (type 3, 285 leaves):

$$\begin{aligned}
& - \frac{1}{24 d \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]\right) \left(\operatorname{Csc}\left[\frac{1}{4}(c + d x)\right]^2 - \operatorname{Sec}\left[\frac{1}{4}(c + d x)\right]^2\right)^3} \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^{10} \sqrt{a(1 + \operatorname{Sin}[c + d x])} \\
& \left(-12 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + 58 \operatorname{Cos}\left[\frac{3}{2}(c + d x)\right] + 18 \operatorname{Cos}\left[\frac{5}{2}(c + d x)\right] + 12 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - 27 \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) \\
& \quad \operatorname{Sin}[c + d x] + 27 \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[c + d x] + 58 \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] - 18 \operatorname{Sin}\left[\frac{5}{2}(c + d x)\right] + \\
& \quad 9 \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[3(c + d x)] - 9 \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[3(c + d x)]
\end{aligned}$$

■ **Problem 334: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + d x]^2 \operatorname{Csc}[c + d x] (a + a \operatorname{Sin}[c + d x])^{3/2} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c + d x]}{\sqrt{a + a \operatorname{Sin}[c + d x]}}\right]}{4 d} + \frac{13 a^2 \operatorname{Cos}[c + d x]}{4 d \sqrt{a + a \operatorname{Sin}[c + d x]}} - \frac{3 a \operatorname{Cot}[c + d x] \sqrt{a + a \operatorname{Sin}[c + d x]}}{4 d} - \frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x] (a + a \operatorname{Sin}[c + d x])^{3/2}}{2 d}$$

Result (type 3, 271 leaves):

$$\begin{aligned}
& - \frac{1}{4 d \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right) \left(\csc\left[\frac{1}{4}(c + dx)\right]^2 - \sec\left[\frac{1}{4}(c + dx)\right]^2\right)^2} a \csc\left[\frac{1}{2}(c + dx)\right]^7 \sqrt{a(1 + \sin[c + dx])} \\
& \left(-22 \cos\left[\frac{1}{2}(c + dx)\right] + 22 \cos\left[\frac{3}{2}(c + dx)\right] + 8 \cos\left[\frac{5}{2}(c + dx)\right] - \log\left[1 + \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + \right. \\
& \left. \cos[2(c + dx)] \log\left[1 + \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + \log\left[1 - \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - \right. \\
& \left. \cos[2(c + dx)] \log\left[1 - \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + 22 \sin\left[\frac{1}{2}(c + dx)\right] + 22 \sin\left[\frac{3}{2}(c + dx)\right] - 8 \sin\left[\frac{5}{2}(c + dx)\right]\right)
\end{aligned}$$

■ **Problem 335: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx]^2 \csc[c + dx]^2 (a + a \sin[c + dx])^{3/2} dx$$

Optimal (type 3, 139 leaves, 6 steps):

$$\begin{aligned}
& \frac{13 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{8 d} + \frac{5 a^2 \cot[c + dx]}{24 d \sqrt{a + a \sin[c + dx]}} - \\
& \frac{a \cot[c + dx] \csc[c + dx] \sqrt{a + a \sin[c + dx]}}{4 d} - \frac{\cot[c + dx] \csc[c + dx]^2 (a + a \sin[c + dx])^{3/2}}{3 d}
\end{aligned}$$

Result (type 3, 286 leaves):

$$\begin{aligned}
& - \frac{1}{24 d \left(1 + \cot\left[\frac{1}{2}(c + dx)\right]\right) \left(\csc\left[\frac{1}{4}(c + dx)\right]^2 - \sec\left[\frac{1}{4}(c + dx)\right]^2\right)^3} a \csc\left[\frac{1}{2}(c + dx)\right]^{10} \sqrt{a(1 + \sin[c + dx])} \\
& \left(12 \cos\left[\frac{1}{2}(c + dx)\right] + 70 \cos\left[\frac{3}{2}(c + dx)\right] - 18 \cos\left[\frac{5}{2}(c + dx)\right] - 12 \sin\left[\frac{1}{2}(c + dx)\right] - 117 \log\left[1 + \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \\
& \sin[c + dx] + 117 \log\left[1 - \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[c + dx] + 70 \sin\left[\frac{3}{2}(c + dx)\right] + 18 \sin\left[\frac{5}{2}(c + dx)\right] + \\
& 39 \log\left[1 + \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[3(c + dx)] - 39 \log\left[1 - \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[3(c + dx)]
\end{aligned}$$

■ **Problem 340: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx]^2}{\sqrt{a + a \sin[c + dx]}} dx$$

Optimal (type 3, 62 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{\sqrt{a} d} - \frac{\cot[c + dx]}{d \sqrt{a + a \sin[c + dx]}}$$

Result (type 3, 138 leaves) :

$$\frac{1}{8 d \sqrt{a (1 + \sin [c + d x])}} \operatorname{Csc} \left[\frac{1}{4} (c + d x) \right] \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right] \left(-2 \operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + 2 \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] + \right. \\ \left. \left(\operatorname{Log} \left[1 + \operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] - \operatorname{Log} \left[1 - \operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \right) \operatorname{Sin} [c + d x] \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right)$$

■ **Problem 341: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot} [c + d x]^2 \operatorname{Csc} [c + d x]}{\sqrt{a + a \operatorname{Sin} [c + d x]}} dx$$

Optimal (type 3, 100 leaves, 5 steps) :

$$\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a} \operatorname{Cos} [c + d x]}{\sqrt{a + a \operatorname{Sin} [c + d x]}} \right]}{4 \sqrt{a} d} + \frac{\operatorname{Cot} [c + d x]}{4 d \sqrt{a + a \operatorname{Sin} [c + d x]}} - \frac{\operatorname{Cot} [c + d x] \operatorname{Csc} [c + d x]}{2 d \sqrt{a + a \operatorname{Sin} [c + d x]}}$$

Result (type 3, 272 leaves) :

$$\frac{1}{32 d \sqrt{a (1 + \operatorname{Sin} [c + d x])}} \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right) \left(-8 + 4 \operatorname{Cot} \left[\frac{1}{4} (c + d x) \right] - \operatorname{Csc} \left[\frac{1}{4} (c + d x) \right]^2 + 4 \operatorname{Log} \left[1 + \operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] - \right. \\ \left. 4 \operatorname{Log} \left[1 - \operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] + \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 + \frac{2}{\left(\operatorname{Cos} \left[\frac{1}{4} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{4} (c + d x) \right] \right)^2} - \right. \\ \left. \frac{8 \operatorname{Sin} \left[\frac{1}{4} (c + d x) \right]}{\operatorname{Cos} \left[\frac{1}{4} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{4} (c + d x) \right]} - \frac{2}{\left(\operatorname{Cos} \left[\frac{1}{4} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{4} (c + d x) \right] \right)^2} + \frac{8 \operatorname{Sin} \left[\frac{1}{4} (c + d x) \right]}{\operatorname{Cos} \left[\frac{1}{4} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{4} (c + d x) \right]} + 4 \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right] \right)$$

■ **Problem 342: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot} [c + d x]^2 \operatorname{Csc} [c + d x]^2}{\sqrt{a + a \operatorname{Sin} [c + d x]}} dx$$

Optimal (type 3, 135 leaves, 6 steps) :

$$\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a} \operatorname{Cos} [c + d x]}{\sqrt{a + a \operatorname{Sin} [c + d x]}} \right]}{8 \sqrt{a} d} + \frac{\operatorname{Cot} [c + d x]}{8 d \sqrt{a + a \operatorname{Sin} [c + d x]}} + \frac{\operatorname{Cot} [c + d x] \operatorname{Csc} [c + d x]}{12 d \sqrt{a + a \operatorname{Sin} [c + d x]}} - \frac{\operatorname{Cot} [c + d x] \operatorname{Csc} [c + d x]^2}{3 d \sqrt{a + a \operatorname{Sin} [c + d x]}}$$

Result (type 3, 292 leaves) :

$$\frac{1}{24 d \left(\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 - \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \right)^3 \sqrt{a(1+\operatorname{Sin}[c+dx])}} \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^9 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \\ \left(-60 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 2 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] - 6 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] + 60 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 9 \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] - \right. \\ \left. 9 \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] + 2 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 6 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] - \right. \\ \left. 3 \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] + 3 \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] \right)$$

■ **Problem 343: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]^3}{(a+a \operatorname{Sin}[c+dx])^{3/2}} dx$$

Optimal (type 3, 184 leaves, 8 steps):

$$\frac{2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sin}[c+dx]}}\right]}{a^{3/2} d} - \frac{344 \operatorname{Cos}[c+dx]}{105 a d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \\ \frac{16 \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]^2}{35 a d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \frac{2 \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]^3}{7 a d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \frac{76 \operatorname{Cos}[c+dx] \sqrt{a+a \operatorname{Sin}[c+dx]}}{105 a^2 d}$$

Result (type 3, 201 leaves):

$$\frac{1}{420 a^2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)} \\ \sqrt{a(1+\operatorname{Sin}[c+dx])} \left((1680+1680i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{dx}{4}\right] \left(\operatorname{Cos}\left[\frac{1}{4}(2c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(2c+dx)\right] \right) \right] - \right. \\ \left. 1365 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 245 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] + 63 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] - 15 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] + \right. \\ \left. 1365 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 245 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] - 63 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] - 15 \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] \right)$$

■ **Problem 344: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]^2}{(a+a \operatorname{Sin}[c+dx])^{3/2}} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$-\frac{2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sin}[c+dx]}}\right]}{a^{3/2} d} + \frac{18 \operatorname{Cos}[c+dx]}{5 a d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \frac{2 \operatorname{Cos}[c+dx]^3}{5 a d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \frac{4 \operatorname{Cos}[c+dx] \sqrt{a+a \operatorname{Sin}[c+dx]}}{5 a^2 d}$$

Result (type 3, 150 leaves) :

$$\left(\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \left((40+40i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] + 30 \cos\left[\frac{1}{2}(c+dx)\right] - 5 \cos\left[\frac{3}{2}(c+dx)\right] - \cos\left[\frac{5}{2}(c+dx)\right] - 30 \sin\left[\frac{1}{2}(c+dx)\right] - 5 \sin\left[\frac{3}{2}(c+dx)\right] + \sin\left[\frac{5}{2}(c+dx)\right] \right) \right) / (10d(a(1+\sin[c+dx]))^{3/2})$$

■ **Problem 345: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^2 \sin[c+dx]}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 108 leaves, 4 steps) :

$$\frac{2\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{2}\sqrt{a+\sin[c+dx]}}\right]}{a^{3/2}d} - \frac{10\cos[c+dx]}{3ad\sqrt{a+\sin[c+dx]}} + \frac{2\cos[c+dx]\sqrt{a+a\sin[c+dx]}}{3a^2d}$$

Result (type 3, 149 leaves) :

$$\left(\sqrt{a(1+\sin[c+dx])} \left((12+12i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \sec\left[\frac{dx}{4}\right] \left(\cos\left[\frac{1}{4}(2c+dx)\right] - \sin\left[\frac{1}{4}(2c+dx)\right] \right) \right] - 9 \cos\left[\frac{1}{2}(c+dx)\right] + \cos\left[\frac{3}{2}(c+dx)\right] + 9 \sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{3}{2}(c+dx)\right] \right) \right) / \left(3a^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

■ **Problem 346: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx] \cot[c+dx]}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 85 leaves, 6 steps) :

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{a+\sin[c+dx]}}\right]}{a^{3/2}d} + \frac{2\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{2}\sqrt{a+\sin[c+dx]}}\right]}{a^{3/2}d}$$

Result (type 3, 130 leaves) :

$$-\frac{1}{d(a(1+\sin[c+dx]))^{3/2}} \left((4+4i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] \right) + \log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3$$

■ **Problem 347: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cot[c+dx]^2}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 113 leaves, 6 steps) :

$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{a^{3/2} d} - \frac{2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{a^{3/2} d} - \frac{\cot[c+dx]}{a d \sqrt{a+a \sin[c+dx]}}$$

Result (type 3, 206 leaves) :

$$\frac{1}{4 d (a (1 + \sin[c + dx]))^{3/2}} \left(\cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right] \right)^3 \left((16 + 16 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (c + dx)\right]\right)\right] - \cot\left[\frac{1}{4} (c + dx)\right] + 2 \left(3 \operatorname{Log}\left[1 + \cos\left[\frac{1}{2} (c + dx)\right] - \sin\left[\frac{1}{2} (c + dx)\right]\right] - 3 \operatorname{Log}\left[1 - \cos\left[\frac{1}{2} (c + dx)\right] + \sin\left[\frac{1}{2} (c + dx)\right]\right] + \sec\left[\frac{1}{2} (c + dx)\right] + \csc[c + dx] \sin\left[\frac{1}{4} (c + dx)\right]^2 - \csc[c + dx] \sin\left[\frac{1}{4} (c + dx)\right] \sin\left[\frac{3}{4} (c + dx)\right] \right)$$

■ **Problem 348: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx]^2 \csc[c+dx]}{(a+a \sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 153 leaves, 8 steps) :

$$-\frac{11 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{4 a^{3/2} d} + \frac{2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{a^{3/2} d} + \frac{5 \cot[c+dx]}{4 a d \sqrt{a+a \sin[c+dx]}} - \frac{\cot[c+dx] \csc[c+dx]}{2 a d \sqrt{a+a \sin[c+dx]}}$$

Result (type 3, 791 leaves) :

$$\begin{aligned}
& - \frac{3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3}{4d(a(1+\sin[c+dx]))^{3/2}} + \frac{1}{d(a(1+\sin[c+dx]))^{3/2}} \\
& (4+4i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3 + \\
& \frac{3 \operatorname{Cot}\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{8d(a(1+\sin[c+dx]))^{3/2}} - \frac{\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{32d(a(1+\sin[c+dx]))^{3/2}} - \\
& \frac{11 \operatorname{Log}\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{8d(a(1+\sin[c+dx]))^{3/2}} + \\
& \frac{11 \operatorname{Log}\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{8d(a(1+\sin[c+dx]))^{3/2}} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{32d(a(1+\sin[c+dx]))^{3/2}} + \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{16d\left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)^2(a(1+\sin[c+dx]))^{3/2}} - \\
& \frac{3 \sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{4d\left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)(a(1+\sin[c+dx]))^{3/2}} - \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{16d\left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)^2(a(1+\sin[c+dx]))^{3/2}} + \\
& \frac{3 \sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3}{4d\left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)(a(1+\sin[c+dx]))^{3/2}} + \frac{3\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{8d(a(1+\sin[c+dx]))^{3/2}}
\end{aligned}$$

■ **Problem 349: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cot}[c+dx]^2 \operatorname{Csc}[c+dx]^2}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 191 leaves, 9 steps):

$$\frac{23 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a\sin[c+dx]}}\right]}{8a^{3/2}d} - \frac{2\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2}\sqrt{a+a\sin[c+dx]}}\right]}{a^{3/2}d} - \frac{9 \operatorname{Cot}[c+dx]}{8ad\sqrt{a+a\sin[c+dx]}} + \frac{7 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{12ad\sqrt{a+a\sin[c+dx]}} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{3ad\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 332 leaves):

$$\frac{1}{192 d (a (1 + \sin[c + dx]))^{3/2}} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3$$

$$\left((768 + 768 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c + dx)\right]\right)\right] - \frac{1}{\left(\operatorname{Csc}\left[\frac{1}{4}(c + dx)\right]^2 - \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2\right)^3} 8 \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^9 \right.$$

$$\left. \left(228 \cos\left[\frac{1}{2}(c + dx)\right] - 110 \cos\left[\frac{3}{2}(c + dx)\right] - 54 \cos\left[\frac{5}{2}(c + dx)\right] - 228 \sin\left[\frac{1}{2}(c + dx)\right] - 207 \log\left[1 + \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \right. \right.$$

$$\left. \sin[c + dx] + 207 \log\left[1 - \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[c + dx] - 110 \sin\left[\frac{3}{2}(c + dx)\right] + 54 \sin\left[\frac{5}{2}(c + dx)\right] + \right.$$

$$\left. 69 \log\left[1 + \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[3(c + dx)] - 69 \log\left[1 - \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[3(c + dx)] \right)$$

■ **Problem 376: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx]^4 \operatorname{Csc}[c + dx]^4 (a + a \sin[c + dx]) dx$$

Optimal (type 3, 114 leaves, 8 steps):

$$\frac{a \operatorname{ArcTanh}[\cos[c + dx]]}{16 d} - \frac{a \cot[c + dx]^5}{5 d} - \frac{a \cot[c + dx]^7}{7 d} -$$

$$\frac{a \cot[c + dx] \operatorname{Csc}[c + dx]}{16 d} + \frac{a \cot[c + dx] \operatorname{Csc}[c + dx]^3}{8 d} - \frac{a \cot[c + dx]^3 \operatorname{Csc}[c + dx]^3}{6 d}$$

Result (type 3, 239 leaves):

$$-\frac{2 a \cot[c + dx]}{35 d} - \frac{a \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{64 d} + \frac{a \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^4}{64 d} - \frac{a \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^6}{384 d} - \frac{a \cot[c + dx] \operatorname{Csc}[c + dx]^2}{35 d} + \frac{8 a \cot[c + dx] \operatorname{Csc}[c + dx]^4}{35 d} -$$

$$\frac{a \cot[c + dx] \operatorname{Csc}[c + dx]^6}{7 d} - \frac{a \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right]}{16 d} + \frac{a \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right]}{16 d} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2}{64 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4}{64 d} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^6}{384 d}$$

■ **Problem 377: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx]^4 \operatorname{Csc}[c + dx]^5 (a + a \sin[c + dx]) dx$$

Optimal (type 3, 136 leaves, 9 steps):

$$\frac{3 a \operatorname{ArcTanh}[\cos[c + dx]]}{128 d} - \frac{a \cot[c + dx]^5}{5 d} - \frac{a \cot[c + dx]^7}{7 d} - \frac{3 a \cot[c + dx] \operatorname{Csc}[c + dx]}{128 d} -$$

$$\frac{a \cot[c + dx] \operatorname{Csc}[c + dx]^3}{64 d} + \frac{a \cot[c + dx] \operatorname{Csc}[c + dx]^5}{16 d} - \frac{a \cot[c + dx]^3 \operatorname{Csc}[c + dx]^5}{8 d}$$

Result (type 3, 279 leaves):

$$\begin{aligned}
& - \frac{2 a \operatorname{Cot}[c+d x]}{35 d} - \frac{3 a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{512 d} + \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4}{1024 d} + \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^6}{512 d} - \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^8}{2048 d} \\
& \frac{a \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^2}{35 d} + \frac{8 a \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^4}{35 d} - \frac{a \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^6}{7 d} - \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right]}{128 d} + \\
& \frac{3 a \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{128 d} + \frac{3 a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{512 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4}{1024 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^6}{512 d} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^8}{2048 d}
\end{aligned}$$

■ **Problem 388: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+d x]^4 \operatorname{Csc}[c+d x]^3 (a+a \operatorname{Sin}[c+d x])^2 dx$$

Optimal (type 3, 132 leaves, 11 steps):

$$\begin{aligned}
& - \frac{7 a^2 \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{16 d} - \frac{2 a^2 \operatorname{Cot}[c+d x]^5}{5 d} + \frac{5 a^2 \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]}{16 d} - \\
& \frac{a^2 \operatorname{Cot}[c+d x]^3 \operatorname{Csc}[c+d x]}{4 d} + \frac{a^2 \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^3}{8 d} - \frac{a^2 \operatorname{Cot}[c+d x]^3 \operatorname{Csc}[c+d x]^3}{6 d}
\end{aligned}$$

Result (type 3, 267 leaves):

$$\begin{aligned}
& a^2 \left(- \frac{\operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]}{5 d} + \frac{9 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{64 d} + \frac{7 \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{80 d} - \right. \\
& \frac{\operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4}{80 d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^6}{384 d} - \frac{7 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right]}{16 d} + \frac{7 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{16 d} - \frac{9 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{64 d} + \\
& \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^6}{384 d} + \frac{\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{5 d} - \frac{7 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{80 d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{80 d} \right)
\end{aligned}$$

■ **Problem 398: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+d x] \operatorname{Cot}[c+d x]^3 (a+a \operatorname{Sin}[c+d x])^3 dx$$

Optimal (type 3, 137 leaves, 15 steps):

$$\begin{aligned}
& - \frac{33 a^3 x}{8} - \frac{3 a^3 \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{2 d} + \frac{2 a^3 \operatorname{Cos}[c+d x]}{d} + \frac{a^3 \operatorname{Cos}[c+d x]^3}{d} - \\
& \frac{3 a^3 \operatorname{Cot}[c+d x]}{d} - \frac{a^3 \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]}{2 d} - \frac{7 a^3 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{8 d} - \frac{a^3 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^3}{4 d}
\end{aligned}$$

Result (type 3, 564 leaves):

$$\begin{aligned}
& - \frac{33 (c + d x) (a + a \sin[c + d x])^3}{8 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} + \frac{11 \cos[c + d x] (a + a \sin[c + d x])^3}{4 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} + \\
& \frac{\cos[3 (c + d x)] (a + a \sin[c + d x])^3}{4 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} - \frac{3 \cot\left[\frac{1}{2} (c + d x)\right] (a + a \sin[c + d x])^3}{2 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} - \frac{\csc\left[\frac{1}{2} (c + d x)\right]^2 (a + a \sin[c + d x])^3}{8 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} - \\
& \frac{3 \log\left[\cos\left[\frac{1}{2} (c + d x)\right]\right] (a + a \sin[c + d x])^3}{2 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} + \frac{3 \log\left[\sin\left[\frac{1}{2} (c + d x)\right]\right] (a + a \sin[c + d x])^3}{2 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} + \frac{\sec\left[\frac{1}{2} (c + d x)\right]^2 (a + a \sin[c + d x])^3}{8 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} - \\
& \frac{(a + a \sin[c + d x])^3 \sin[2 (c + d x)]}{2 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} + \frac{(a + a \sin[c + d x])^3 \sin[4 (c + d x)]}{32 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6} + \frac{3 (a + a \sin[c + d x])^3 \tan\left[\frac{1}{2} (c + d x)\right]}{2 d \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right)^6}
\end{aligned}$$

■ **Problem 403: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + d x]^4 \csc[c + d x]^4 (a + a \sin[c + d x])^3 dx$$

Optimal (type 3, 150 leaves, 14 steps):

$$\begin{aligned}
& - \frac{9 a^3 \operatorname{ArcTanh}\left[\cos[c + d x]\right]}{16 d} - \frac{4 a^3 \cot[c + d x]^5}{5 d} - \frac{a^3 \cot[c + d x]^7}{7 d} + \frac{3 a^3 \cot[c + d x] \csc[c + d x]}{16 d} - \\
& \frac{a^3 \cot[c + d x]^3 \csc[c + d x]}{4 d} + \frac{3 a^3 \cot[c + d x] \csc[c + d x]^3}{8 d} - \frac{a^3 \cot[c + d x]^3 \csc[c + d x]^3}{2 d}
\end{aligned}$$

Result (type 3, 363 leaves):

$$\begin{aligned}
& a^3 \left(- \frac{23 \cot\left[\frac{1}{2} (c + d x)\right]}{70 d} + \frac{7 \csc\left[\frac{1}{2} (c + d x)\right]^2}{64 d} + \frac{297 \cot\left[\frac{1}{2} (c + d x)\right] \csc\left[\frac{1}{2} (c + d x)\right]^2}{2240 d} + \frac{\csc\left[\frac{1}{2} (c + d x)\right]^4}{32 d} - \right. \\
& \frac{31 \cot\left[\frac{1}{2} (c + d x)\right] \csc\left[\frac{1}{2} (c + d x)\right]^4}{2240 d} - \frac{\csc\left[\frac{1}{2} (c + d x)\right]^6}{128 d} - \frac{\cot\left[\frac{1}{2} (c + d x)\right] \csc\left[\frac{1}{2} (c + d x)\right]^6}{896 d} - \frac{9 \log\left[\cos\left[\frac{1}{2} (c + d x)\right]\right]}{16 d} + \\
& \frac{9 \log\left[\sin\left[\frac{1}{2} (c + d x)\right]\right]}{16 d} - \frac{7 \sec\left[\frac{1}{2} (c + d x)\right]^2}{64 d} - \frac{\sec\left[\frac{1}{2} (c + d x)\right]^4}{32 d} + \frac{\sec\left[\frac{1}{2} (c + d x)\right]^6}{128 d} + \frac{23 \tan\left[\frac{1}{2} (c + d x)\right]}{70 d} - \\
& \left. \frac{297 \sec\left[\frac{1}{2} (c + d x)\right]^2 \tan\left[\frac{1}{2} (c + d x)\right]}{2240 d} + \frac{31 \sec\left[\frac{1}{2} (c + d x)\right]^4 \tan\left[\frac{1}{2} (c + d x)\right]}{2240 d} + \frac{\sec\left[\frac{1}{2} (c + d x)\right]^6 \tan\left[\frac{1}{2} (c + d x)\right]}{896 d} \right)
\end{aligned}$$

■ **Problem 404: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + d x]^4 \csc[c + d x]^5 (a + a \sin[c + d x])^3 dx$$

Optimal (type 3, 176 leaves, 16 steps):

$$\begin{aligned}
& - \frac{27 a^3 \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{128 d} - \frac{4 a^3 \operatorname{Cot}[c+d x]^5}{5 d} - \frac{3 a^3 \operatorname{Cot}[c+d x]^7}{7 d} - \frac{27 a^3 \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]}{128 d} + \\
& \frac{23 a^3 \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^3}{64 d} - \frac{a^3 \operatorname{Cot}[c+d x]^3 \operatorname{Csc}[c+d x]^3}{2 d} + \frac{a^3 \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^5}{16 d} - \frac{a^3 \operatorname{Cot}[c+d x]^3 \operatorname{Csc}[c+d x]^5}{8 d}
\end{aligned}$$

Result (type 3, 1027 leaves) :

$$\begin{aligned}
& - \frac{13 \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] (a+a \operatorname{Sin}[c+d x])^3}{70 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} - \frac{27 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 (a+a \operatorname{Sin}[c+d x])^3}{512 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \\
& \frac{107 \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 (a+a \operatorname{Sin}[c+d x])^3}{2240 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \frac{49 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4 (a+a \operatorname{Sin}[c+d x])^3}{1024 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \\
& \frac{19 \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4 (a+a \operatorname{Sin}[c+d x])^3}{2240 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} - \frac{3 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^6 (a+a \operatorname{Sin}[c+d x])^3}{512 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} - \\
& \frac{3 \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^6 (a+a \operatorname{Sin}[c+d x])^3}{896 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^8 (a+a \operatorname{Sin}[c+d x])^3}{2048 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} - \\
& \frac{27 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] (a+a \operatorname{Sin}[c+d x])^3}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \frac{27 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+a \operatorname{Sin}[c+d x])^3}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \frac{27 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 (a+a \operatorname{Sin}[c+d x])^3}{512 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} - \\
& \frac{49 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 (a+a \operatorname{Sin}[c+d x])^3}{1024 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \frac{3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^6 (a+a \operatorname{Sin}[c+d x])^3}{512 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^8 (a+a \operatorname{Sin}[c+d x])^3}{2048 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \\
& \frac{13 (a+a \operatorname{Sin}[c+d x])^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{70 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} - \frac{107 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 (a+a \operatorname{Sin}[c+d x])^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2240 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} - \\
& \frac{19 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 (a+a \operatorname{Sin}[c+d x])^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2240 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6} + \frac{3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^6 (a+a \operatorname{Sin}[c+d x])^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{896 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6}
\end{aligned}$$

■ **Problem 410: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x]^4 \operatorname{Sin}[c+d x]^3}{a+a \operatorname{Sin}[c+d x]} dx$$

Optimal (type 3, 117 leaves, 8 steps) :

$$\begin{aligned}
& - \frac{x}{16 a} - \frac{\operatorname{Cos}[c+d x]^3}{3 a d} + \frac{\operatorname{Cos}[c+d x]^5}{5 a d} - \frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{16 a d} + \frac{\operatorname{Cos}[c+d x]^3 \operatorname{Sin}[c+d x]}{8 a d} + \frac{\operatorname{Cos}[c+d x]^3 \operatorname{Sin}[c+d x]^3}{6 a d}
\end{aligned}$$

Result (type 3, 377 leaves) :

$$\frac{1}{1920 a d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(30 (3 c - 4 d x) \cos\left[\frac{c}{2}\right] - 120 \cos\left[\frac{c}{2} + d x\right] - 120 \cos\left[\frac{3 c}{2} + d x\right] + 15 \cos\left[\frac{3 c}{2} + 2 d x\right] - 15 \cos\left[\frac{5 c}{2} + 2 d x\right] - 20 \cos\left[\frac{5 c}{2} + 3 d x\right] - 20 \cos\left[\frac{7 c}{2} + 3 d x\right] + 15 \cos\left[\frac{7 c}{2} + 4 d x\right] - 15 \cos\left[\frac{9 c}{2} + 4 d x\right] + 12 \cos\left[\frac{9 c}{2} + 5 d x\right] + 12 \cos\left[\frac{11 c}{2} + 5 d x\right] - 5 \cos\left[\frac{11 c}{2} + 6 d x\right] + 5 \cos\left[\frac{13 c}{2} + 6 d x\right] - 180 \sin\left[\frac{c}{2}\right] + 90 c \sin\left[\frac{c}{2}\right] - 120 d x \sin\left[\frac{c}{2}\right] + 120 \sin\left[\frac{c}{2} + d x\right] - 120 \sin\left[\frac{3 c}{2} + d x\right] + 15 \sin\left[\frac{3 c}{2} + 2 d x\right] + 15 \sin\left[\frac{5 c}{2} + 2 d x\right] + 20 \sin\left[\frac{5 c}{2} + 3 d x\right] - 20 \sin\left[\frac{7 c}{2} + 3 d x\right] + 15 \sin\left[\frac{7 c}{2} + 4 d x\right] + 15 \sin\left[\frac{9 c}{2} + 4 d x\right] - 12 \sin\left[\frac{9 c}{2} + 5 d x\right] + 12 \sin\left[\frac{11 c}{2} + 5 d x\right] - 5 \sin\left[\frac{11 c}{2} + 6 d x\right] - 5 \sin\left[\frac{13 c}{2} + 6 d x\right] \right)$$

■ **Problem 411: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^4 \sin[c + d x]^2}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 91 leaves, 7 steps) :

$$\frac{x}{8 a} + \frac{\cos[c + d x]^3}{3 a d} - \frac{\cos[c + d x]^5}{5 a d} + \frac{\cos[c + d x] \sin[c + d x]}{8 a d} - \frac{\cos[c + d x]^3 \sin[c + d x]}{4 a d}$$

Result (type 3, 258 leaves) :

$$\frac{1}{960 a d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(120 d x \cos\left[\frac{c}{2}\right] + 60 \cos\left[\frac{c}{2} + d x\right] + 60 \cos\left[\frac{3 c}{2} + d x\right] + 10 \cos\left[\frac{5 c}{2} + 3 d x\right] + 10 \cos\left[\frac{7 c}{2} + 3 d x\right] - 15 \cos\left[\frac{7 c}{2} + 4 d x\right] + 15 \cos\left[\frac{9 c}{2} + 4 d x\right] - 6 \cos\left[\frac{9 c}{2} + 5 d x\right] - 6 \cos\left[\frac{11 c}{2} + 5 d x\right] + 120 \sin\left[\frac{c}{2}\right] + 120 d x \sin\left[\frac{c}{2}\right] - 60 \sin\left[\frac{c}{2} + d x\right] + 60 \sin\left[\frac{3 c}{2} + d x\right] - 10 \sin\left[\frac{5 c}{2} + 3 d x\right] + 10 \sin\left[\frac{7 c}{2} + 3 d x\right] - 15 \sin\left[\frac{7 c}{2} + 4 d x\right] - 15 \sin\left[\frac{9 c}{2} + 4 d x\right] + 6 \sin\left[\frac{9 c}{2} + 5 d x\right] - 6 \sin\left[\frac{11 c}{2} + 5 d x\right] \right)$$

■ **Problem 412: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^4 \sin[c + d x]}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 73 leaves, 6 steps) :

$$\frac{x}{8 a} - \frac{\cos[c + d x]^3}{3 a d} - \frac{\cos[c + d x] \sin[c + d x]}{8 a d} + \frac{\cos[c + d x]^3 \sin[c + d x]}{4 a d}$$

Result (type 3, 219 leaves) :

$$\begin{aligned}
 & - \left(-24 (c - dx) \cos\left[\frac{c}{2}\right] + 24 \cos\left[\frac{c}{2} + dx\right] + 24 \cos\left[\frac{3c}{2} + dx\right] + 8 \cos\left[\frac{5c}{2} + 3dx\right] + 8 \cos\left[\frac{7c}{2} + 3dx\right] - \right. \\
 & \quad \left. 3 \cos\left[\frac{7c}{2} + 4dx\right] + 3 \cos\left[\frac{9c}{2} + 4dx\right] + 48 \sin\left[\frac{c}{2}\right] - 24c \sin\left[\frac{c}{2}\right] + 24dx \sin\left[\frac{c}{2}\right] - 24 \sin\left[\frac{c}{2} + dx\right] + 24 \sin\left[\frac{3c}{2} + dx\right] - \right. \\
 & \quad \left. 8 \sin\left[\frac{5c}{2} + 3dx\right] + 8 \sin\left[\frac{7c}{2} + 3dx\right] - 3 \sin\left[\frac{7c}{2} + 4dx\right] - 3 \sin\left[\frac{9c}{2} + 4dx\right] \right) / \left(192ad \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \right)
 \end{aligned}$$

■ **Problem 416: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx]^4}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 58 leaves, 5 steps) :

$$- \frac{\text{ArcTanh}[\cos[c + dx]]}{2ad} - \frac{\cot[c + dx]^3}{3ad} + \frac{\cot[c + dx] \csc[c + dx]}{2ad}$$

Result (type 3, 124 leaves) :

$$\begin{aligned}
 & - \frac{1}{96ad(1 + \sin[c + dx])} \csc\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{1}{2}(c + dx)\right] \left(\csc\left[\frac{1}{2}(c + dx)\right] + \sec\left[\frac{1}{2}(c + dx)\right] \right)^2 \\
 & \quad \left(\cos[3(c + dx)] + \cos[c + dx] (3 - 6 \sin[c + dx]) + 6 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \right) \sin[c + dx]^3
 \end{aligned}$$

■ **Problem 420: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^4 \sin[c + dx]^5}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 3, 147 leaves, 11 steps) :

$$\begin{aligned}
 & - \frac{5x}{8a^2} - \frac{2 \cos[c + dx]}{a^2 d} + \frac{5 \cos[c + dx]^3}{3a^2 d} - \frac{4 \cos[c + dx]^5}{5a^2 d} + \frac{\cos[c + dx]^7}{7a^2 d} + \\
 & \quad \frac{5 \cos[c + dx] \sin[c + dx]}{8a^2 d} + \frac{5 \cos[c + dx] \sin[c + dx]^3}{12a^2 d} + \frac{\cos[c + dx] \sin[c + dx]^5}{3a^2 d}
 \end{aligned}$$

Result (type 3, 414 leaves) :

$$\frac{1}{13440 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(-8400 d x \cos\left[\frac{c}{2}\right] - 7875 \cos\left[\frac{c}{2} + d x\right] - 7875 \cos\left[\frac{3c}{2} + d x\right] + 3150 \cos\left[\frac{3c}{2} + 2 d x\right] - 3150 \cos\left[\frac{5c}{2} + 2 d x\right] + 1435 \cos\left[\frac{5c}{2} + 3 d x\right] + 1435 \cos\left[\frac{7c}{2} + 3 d x\right] - 630 \cos\left[\frac{7c}{2} + 4 d x\right] + 630 \cos\left[\frac{9c}{2} + 4 d x\right] - 231 \cos\left[\frac{9c}{2} + 5 d x\right] - 231 \cos\left[\frac{11c}{2} + 5 d x\right] + 70 \cos\left[\frac{11c}{2} + 6 d x\right] - 70 \cos\left[\frac{13c}{2} + 6 d x\right] + 15 \cos\left[\frac{13c}{2} + 7 d x\right] + 15 \cos\left[\frac{15c}{2} + 7 d x\right] + 420 \sin\left[\frac{c}{2}\right] - 8400 d x \sin\left[\frac{c}{2}\right] + 7875 \sin\left[\frac{c}{2} + d x\right] - 7875 \sin\left[\frac{3c}{2} + d x\right] + 3150 \sin\left[\frac{3c}{2} + 2 d x\right] + 3150 \sin\left[\frac{5c}{2} + 2 d x\right] - 1435 \sin\left[\frac{5c}{2} + 3 d x\right] + 1435 \sin\left[\frac{7c}{2} + 3 d x\right] - 630 \sin\left[\frac{7c}{2} + 4 d x\right] - 630 \sin\left[\frac{9c}{2} + 4 d x\right] + 231 \sin\left[\frac{9c}{2} + 5 d x\right] - 231 \sin\left[\frac{11c}{2} + 5 d x\right] + 70 \sin\left[\frac{11c}{2} + 6 d x\right] + 70 \sin\left[\frac{13c}{2} + 6 d x\right] - 15 \sin\left[\frac{13c}{2} + 7 d x\right] + 15 \sin\left[\frac{15c}{2} + 7 d x\right] \right)$$

■ **Problem 422: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^4 \sin[c + d x]^3}{(a + a \sin[c + d x])^2} dx$$

Optimal (type 3, 102 leaves, 10 steps):

$$-\frac{3x}{4a^2} - \frac{2\cos[c + dx]}{a^2 d} + \frac{\cos[c + dx]^3}{a^2 d} - \frac{\cos[c + dx]^5}{5a^2 d} + \frac{3\cos[c + dx]\sin[c + dx]}{4a^2 d} + \frac{\cos[c + dx]\sin[c + dx]^3}{2a^2 d}$$

Result (type 3, 304 leaves):

$$-\frac{1}{160 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(120 d x \cos\left[\frac{c}{2}\right] + 110 \cos\left[\frac{c}{2} + d x\right] + 110 \cos\left[\frac{3c}{2} + d x\right] - 40 \cos\left[\frac{3c}{2} + 2 d x\right] + 40 \cos\left[\frac{5c}{2} + 2 d x\right] - 15 \cos\left[\frac{5c}{2} + 3 d x\right] - 15 \cos\left[\frac{7c}{2} + 3 d x\right] + 5 \cos\left[\frac{7c}{2} + 4 d x\right] - 5 \cos\left[\frac{9c}{2} + 4 d x\right] + \cos\left[\frac{9c}{2} + 5 d x\right] + \cos\left[\frac{11c}{2} + 5 d x\right] - 10 \sin\left[\frac{c}{2}\right] + 120 d x \sin\left[\frac{c}{2}\right] - 110 \sin\left[\frac{c}{2} + d x\right] + 110 \sin\left[\frac{3c}{2} + d x\right] - 40 \sin\left[\frac{3c}{2} + 2 d x\right] - 40 \sin\left[\frac{5c}{2} + 2 d x\right] + 15 \sin\left[\frac{5c}{2} + 3 d x\right] - 15 \sin\left[\frac{7c}{2} + 3 d x\right] + 5 \sin\left[\frac{7c}{2} + 4 d x\right] + 5 \sin\left[\frac{9c}{2} + 4 d x\right] - \sin\left[\frac{9c}{2} + 5 d x\right] + \sin\left[\frac{11c}{2} + 5 d x\right] \right)$$

■ **Problem 423: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^4 \sin[c + d x]^2}{(a + a \sin[c + d x])^2} dx$$

Optimal (type 3, 87 leaves, 10 steps):

$$\frac{7x}{8a^2} + \frac{2\cos[c+dx]}{a^2d} - \frac{2\cos[c+dx]^3}{3a^2d} - \frac{7\cos[c+dx]\sin[c+dx]}{8a^2d} - \frac{\cos[c+dx]\sin[c+dx]^3}{4a^2d}$$

Result (type 3, 258 leaves):

$$\frac{1}{192a^2d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(168dx \cos\left[\frac{c}{2}\right] + 144 \cos\left[\frac{c}{2} + dx\right] + 144 \cos\left[\frac{3c}{2} + dx\right] - 48 \cos\left[\frac{3c}{2} + 2dx\right] + 48 \cos\left[\frac{5c}{2} + 2dx\right] - 16 \cos\left[\frac{5c}{2} + 3dx\right] - 16 \cos\left[\frac{7c}{2} + 3dx\right] + 3 \cos\left[\frac{7c}{2} + 4dx\right] - 3 \cos\left[\frac{9c}{2} + 4dx\right] + 8 \sin\left[\frac{c}{2}\right] + 168dx \sin\left[\frac{c}{2}\right] - 144 \sin\left[\frac{c}{2} + dx\right] + 144 \sin\left[\frac{3c}{2} + dx\right] - 48 \sin\left[\frac{3c}{2} + 2dx\right] - 48 \sin\left[\frac{5c}{2} + 2dx\right] + 16 \sin\left[\frac{5c}{2} + 3dx\right] - 16 \sin\left[\frac{7c}{2} + 3dx\right] + 3 \sin\left[\frac{7c}{2} + 4dx\right] + 3 \sin\left[\frac{9c}{2} + 4dx\right] \right)$$

■ **Problem 424: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^4 \sin[c+dx]}{(a+a\sin[c+dx])^2} dx$$

Optimal (type 3, 70 leaves, 4 steps):

$$-\frac{x}{a^2} - \frac{2\cos[c+dx]^3}{3a^2d} - \frac{\cos[c+dx]\sin[c+dx]}{a^2d} - \frac{\cos[c+dx]^5}{d(a+a\sin[c+dx])^2}$$

Result (type 3, 200 leaves):

$$\frac{1}{24a^2d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(-24dx \cos\left[\frac{c}{2}\right] - 21 \cos\left[\frac{c}{2} + dx\right] - 21 \cos\left[\frac{3c}{2} + dx\right] + 6 \cos\left[\frac{3c}{2} + 2dx\right] - 6 \cos\left[\frac{5c}{2} + 2dx\right] + \cos\left[\frac{5c}{2} + 3dx\right] + \cos\left[\frac{7c}{2} + 3dx\right] + 4 \sin\left[\frac{c}{2}\right] - 24dx \sin\left[\frac{c}{2}\right] + 21 \sin\left[\frac{c}{2} + dx\right] - 21 \sin\left[\frac{3c}{2} + dx\right] + 6 \sin\left[\frac{3c}{2} + 2dx\right] + 6 \sin\left[\frac{5c}{2} + 2dx\right] - \sin\left[\frac{5c}{2} + 3dx\right] + \sin\left[\frac{7c}{2} + 3dx\right] \right)$$

■ **Problem 426: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2 \cot[c+dx]^2}{(a+a\sin[c+dx])^2} dx$$

Optimal (type 3, 35 leaves, 6 steps):

$$\frac{x}{a^2} + \frac{2\operatorname{ArcTanh}[\cos[c+dx]]}{a^2d} - \frac{\cot[c+dx]}{a^2d}$$

Result (type 3, 98 leaves):

$$\frac{1}{2d(a + a \sin[c + dx])^2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 \left(2(c + dx) - \cot\left[\frac{1}{2}(c + dx)\right] + 4 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - 4 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] + \tan\left[\frac{1}{2}(c + dx)\right] \right)$$

■ **Problem 433: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^4 \sin[c + dx]^2}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 87 leaves, 9 steps):

$$-\frac{11x}{2a^3} - \frac{5 \cos[c + dx]}{a^3 d} + \frac{\cos[c + dx]^3}{3a^3 d} + \frac{3 \cos[c + dx] \sin[c + dx]}{2a^3 d} - \frac{4 \cos[c + dx]}{a^3 d (1 + \sin[c + dx])}$$

Result (type 3, 181 leaves):

$$\left((1 - 660dx) \cos\left[\frac{dx}{2}\right] - 286 \cos\left[c + \frac{dx}{2}\right] - 240 \cos\left[c + \frac{3dx}{2}\right] - 40 \cos\left[3c + \frac{5dx}{2}\right] + 5 \cos\left[3c + \frac{7dx}{2}\right] + 1244 \sin\left[\frac{dx}{2}\right] + \sin\left[c + \frac{dx}{2}\right] - 660dx \sin\left[c + \frac{dx}{2}\right] - 240 \sin\left[2c + \frac{3dx}{2}\right] + 40 \sin\left[2c + \frac{5dx}{2}\right] + 5 \sin\left[4c + \frac{7dx}{2}\right] \right) / \left(120a^3 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)$$

■ **Problem 435: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3 \cot[c + dx]}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 45 leaves, 5 steps):

$$\frac{x}{a^3} - \frac{\text{ArcTanh}[\cos[c + dx]]}{a^3 d} + \frac{4 \cos[c + dx]}{a^3 d (1 + \sin[c + dx])}$$

Result (type 3, 122 leaves):

$$\frac{1}{a^3 d (1 + \sin[c + dx])^3} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 \left(\cos\left[\frac{1}{2}(c + dx)\right] \left(c + dx - \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \right) + \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \left(-8 + c + dx - \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \sin\left[\frac{1}{2}(c + dx)\right]$$

■ **Problem 436: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 \cot[c + dx]^2}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 54 leaves, 7 steps):

$$\frac{3 \operatorname{ArcTanh}[\cos[c + dx]]}{a^3 d} - \frac{\cot[c + dx]}{a^3 d} - \frac{4 \cos[c + dx]}{a^3 d (1 + \sin[c + dx])}$$

Result (type 3, 156 leaves):

$$-\frac{1}{2 a^3 d (1 + \sin[c + dx])^3} \left(\cos\left[\frac{1}{2}(c + dx)\right] \left(-17 + \cot\left[\frac{1}{2}(c + dx)\right]^2 - 6 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + 6 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] + \cot\left[\frac{1}{2}(c + dx)\right] \left(1 - 6 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + 6 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \right) - \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 \tan\left[\frac{1}{2}(c + dx)\right] \right)$$

■ **Problem 437: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] \cot[c + dx]^3}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 78 leaves, 9 steps):

$$-\frac{9 \operatorname{ArcTanh}[\cos[c + dx]]}{2 a^3 d} + \frac{3 \cot[c + dx]}{a^3 d} - \frac{\cot[c + dx] \operatorname{Csc}[c + dx]}{2 a^3 d} + \frac{4 \cos[c + dx]}{a^3 d (1 + \sin[c + dx])}$$

Result (type 3, 369 leaves):

$$-\frac{8 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5}{d (a + a \sin[c + dx])^3} + \frac{3 \cot\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6}{2 d (a + a \sin[c + dx])^3} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6}{8 d (a + a \sin[c + dx])^3} - \frac{9 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6}{2 d (a + a \sin[c + dx])^3} + \frac{9 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6}{2 d (a + a \sin[c + dx])^3} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6}{8 d (a + a \sin[c + dx])^3} - \frac{3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 \tan\left[\frac{1}{2}(c + dx)\right]}{2 d (a + a \sin[c + dx])^3}$$

■ **Problem 438: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx]^4}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 96 leaves, 11 steps):

$$\frac{11 \operatorname{ArcTanh}[\cos[c + dx]]}{2 a^3 d} - \frac{5 \cot[c + dx]}{a^3 d} - \frac{\cot[c + dx]^3}{3 a^3 d} + \frac{3 \cot[c + dx] \operatorname{Csc}[c + dx]}{2 a^3 d} - \frac{4 \cot[c + dx]}{a^3 d (1 + \operatorname{Csc}[c + dx])}$$

Result (type 3, 497 leaves):

$$\begin{aligned}
& \frac{8 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5}{d(a+a \operatorname{Sin}[c+dx])^3} - \\
& \frac{7 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{3d(a+a \operatorname{Sin}[c+dx])^3} + \frac{3 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a \operatorname{Sin}[c+dx])^3} - \\
& \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{24d(a+a \operatorname{Sin}[c+dx])^3} + \frac{11 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{2d(a+a \operatorname{Sin}[c+dx])^3} - \\
& \frac{11 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{2d(a+a \operatorname{Sin}[c+dx])^3} - \frac{3 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a \operatorname{Sin}[c+dx])^3} + \\
& \frac{7 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{3d(a+a \operatorname{Sin}[c+dx])^3} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24d(a+a \operatorname{Sin}[c+dx])^3}
\end{aligned}$$

■ **Problem 439: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^4 \operatorname{Csc}[c+dx]}{(a+a \operatorname{Sin}[c+dx])^3} dx$$

Optimal (type 3, 117 leaves, 14 steps):

$$-\frac{51 \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{8a^3d} + \frac{7 \operatorname{Cot}[c+dx]}{a^3d} + \frac{\operatorname{Cot}[c+dx]^3}{a^3d} - \frac{19 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{8a^3d} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{4a^3d} + \frac{4 \operatorname{Cos}[c+dx]}{a^3d(1+\operatorname{Sin}[c+dx])}$$

Result (type 3, 601 leaves):

$$\begin{aligned}
& -\frac{8 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5}{d(a+a \operatorname{Sin}[c+dx])^3} + \\
& \frac{3 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{d(a+a \operatorname{Sin}[c+dx])^3} - \frac{19 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{32d(a+a \operatorname{Sin}[c+dx])^3} + \\
& \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a \operatorname{Sin}[c+dx])^3} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{64d(a+a \operatorname{Sin}[c+dx])^3} - \\
& \frac{51 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a \operatorname{Sin}[c+dx])^3} + \frac{51 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a \operatorname{Sin}[c+dx])^3} + \\
& \frac{19 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{32d(a+a \operatorname{Sin}[c+dx])^3} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{64d(a+a \operatorname{Sin}[c+dx])^3} - \\
& \frac{3 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{d(a+a \operatorname{Sin}[c+dx])^3} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{8d(a+a \operatorname{Sin}[c+dx])^3}
\end{aligned}$$

■ **Problem 440: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[e + f x]^4 \text{Sin}[e + f x]}{(a + a \text{Sin}[e + f x])^6} dx$$

Optimal (type 3, 58 leaves, 2 steps):

$$\frac{\text{Cos}[e + f x]^5}{7 f (a + a \text{Sin}[e + f x])^6} - \frac{6 \text{Cos}[e + f x]^5}{35 a f (a + a \text{Sin}[e + f x])^5}$$

Result (type 3, 237 leaves):

$$\frac{1}{4620 a^6 f \left(\text{Cos}\left[\frac{e}{2}\right] + \text{Sin}\left[\frac{e}{2}\right] \right) \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right] \right)^7} \left(35 \text{Cos}\left[\frac{f x}{2}\right] + 4585 \text{Cos}\left[e + \frac{f x}{2}\right] - 2982 \text{Cos}\left[e + \frac{3 f x}{2}\right] - 21 \text{Cos}\left[2 e + \frac{3 f x}{2}\right] - 7 \text{Cos}\left[2 e + \frac{5 f x}{2}\right] - 1148 \text{Cos}\left[3 e + \frac{5 f x}{2}\right] + 197 \text{Cos}\left[3 e + \frac{7 f x}{2}\right] + \text{Cos}\left[4 e + \frac{7 f x}{2}\right] + 2275 \text{Sin}\left[\frac{f x}{2}\right] + 35 \text{Sin}\left[e + \frac{f x}{2}\right] + 21 \text{Sin}\left[e + \frac{3 f x}{2}\right] + 1134 \text{Sin}\left[2 e + \frac{3 f x}{2}\right] - 224 \text{Sin}\left[2 e + \frac{5 f x}{2}\right] - 7 \text{Sin}\left[3 e + \frac{5 f x}{2}\right] - \text{Sin}\left[3 e + \frac{7 f x}{2}\right] + \text{Sin}\left[4 e + \frac{7 f x}{2}\right] \right)$$

■ **Problem 441: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[e + f x]^4 \text{Sin}[e + f x]^2}{(a + a \text{Sin}[e + f x])^7} dx$$

Optimal (type 3, 89 leaves, 18 steps):

$$-\frac{a \text{Cos}[e + f x]^7}{18 f (a + a \text{Sin}[e + f x])^8} + \frac{25 \text{Cos}[e + f x]^5}{126 a f (a + a \text{Sin}[e + f x])^6} - \frac{47 \text{Cos}[e + f x]^5}{315 a^2 f (a + a \text{Sin}[e + f x])^5}$$

Result (type 3, 293 leaves):

$$\frac{1}{720 720 a^7 f \left(\text{Cos}\left[\frac{e}{2}\right] + \text{Sin}\left[\frac{e}{2}\right] \right) \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right] \right)^9} \left(1890 \text{Cos}\left[\frac{f x}{2}\right] + 718 830 \text{Cos}\left[e + \frac{f x}{2}\right] - 467 208 \text{Cos}\left[e + \frac{3 f x}{2}\right] - 1260 \text{Cos}\left[2 e + \frac{3 f x}{2}\right] - 540 \text{Cos}\left[2 e + \frac{5 f x}{2}\right] - 179 640 \text{Cos}\left[3 e + \frac{5 f x}{2}\right] + 30 753 \text{Cos}\left[3 e + \frac{7 f x}{2}\right] + 135 \text{Cos}\left[4 e + \frac{7 f x}{2}\right] + 15 \text{Cos}\left[4 e + \frac{9 f x}{2}\right] - 15 \text{Cos}\left[5 e + \frac{9 f x}{2}\right] + 971 082 \text{Sin}\left[\frac{f x}{2}\right] + 1890 \text{Sin}\left[e + \frac{f x}{2}\right] + 1260 \text{Sin}\left[e + \frac{3 f x}{2}\right] + 659 400 \text{Sin}\left[2 e + \frac{3 f x}{2}\right] - 303 192 \text{Sin}\left[2 e + \frac{5 f x}{2}\right] - 540 \text{Sin}\left[3 e + \frac{5 f x}{2}\right] - 135 \text{Sin}\left[3 e + \frac{7 f x}{2}\right] - 89 955 \text{Sin}\left[4 e + \frac{7 f x}{2}\right] + 13 427 \text{Sin}\left[4 e + \frac{9 f x}{2}\right] + 15 \text{Sin}\left[5 e + \frac{9 f x}{2}\right] \right)$$

■ **Problem 442: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^4 \sin[e + f x]^3}{(a + a \sin[e + f x])^8} dx$$

Optimal (type 3, 157 leaves, 24 steps):

$$\frac{4 \cos[e + f x]}{11 a^8 f (1 + \sin[e + f x])^6} - \frac{52 \cos[e + f x]}{33 a^8 f (1 + \sin[e + f x])^5} + \frac{617 \cos[e + f x]}{231 a^8 f (1 + \sin[e + f x])^4} - \frac{846 \cos[e + f x]}{385 a^8 f (1 + \sin[e + f x])^3} + \frac{1003 \cos[e + f x]}{1155 a^8 f (1 + \sin[e + f x])^2} - \frac{152 \cos[e + f x]}{1155 a^8 f (1 + \sin[e + f x])}$$

Result (type 3, 343 leaves):

$$\frac{1}{240240 a^8 f \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^{11}} \left(462 \cos\left[\frac{f x}{2}\right] + 486024 \cos\left[e + \frac{f x}{2}\right] - 351450 \cos\left[e + \frac{3 f x}{2}\right] - 330 \cos\left[2e + \frac{3 f x}{2}\right] - 165 \cos\left[2e + \frac{5 f x}{2}\right] - 180015 \cos\left[3e + \frac{5 f x}{2}\right] + 63580 \cos\left[3e + \frac{7 f x}{2}\right] + 55 \cos\left[4e + \frac{7 f x}{2}\right] + 11 \cos\left[4e + \frac{9 f x}{2}\right] + 15004 \cos\left[5e + \frac{9 f x}{2}\right] - 1975 \cos\left[5e + \frac{11 f x}{2}\right] - \cos\left[6e + \frac{11 f x}{2}\right] + 425964 \sin\left[\frac{f x}{2}\right] + 462 \sin\left[e + \frac{f x}{2}\right] + 330 \sin\left[e + \frac{3 f x}{2}\right] + 299970 \sin\left[2e + \frac{3 f x}{2}\right] - 145695 \sin\left[2e + \frac{5 f x}{2}\right] - 165 \sin\left[3e + \frac{5 f x}{2}\right] - 55 \sin\left[3e + \frac{7 f x}{2}\right] - 44990 \sin\left[4e + \frac{7 f x}{2}\right] + 6710 \sin\left[4e + \frac{9 f x}{2}\right] + 11 \sin\left[5e + \frac{9 f x}{2}\right] + \sin\left[5e + \frac{11 f x}{2}\right] - \sin\left[6e + \frac{11 f x}{2}\right] \right)$$

■ **Problem 443: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^4 \sin[c + d x]^2 \sqrt{a + a \sin[c + d x]} dx$$

Optimal (type 3, 156 leaves, 5 steps):

$$-\frac{1472 a^3 \cos[c + d x]^5}{45045 d (a + a \sin[c + d x])^{5/2}} - \frac{368 a^2 \cos[c + d x]^5}{9009 d (a + a \sin[c + d x])^{3/2}} - \frac{46 a \cos[c + d x]^5}{1287 d \sqrt{a + a \sin[c + d x]}} + \frac{20 \cos[c + d x]^5 \sqrt{a + a \sin[c + d x]}}{143 d} - \frac{2 \cos[c + d x]^5 (a + a \sin[c + d x])^{3/2}}{13 a d}$$

Result (type 3, 619 leaves):

$$\begin{aligned}
& \frac{(5 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] - 3 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right]) \sqrt{a(1+\operatorname{Sin}[c+dx])}}{480 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \frac{(9 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] - 7 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right]) \sqrt{a(1+\operatorname{Sin}[c+dx])}}{1008 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{(13 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] - 11 \operatorname{Cos}\left[\frac{13}{2}(c+dx)\right]) \sqrt{a(1+\operatorname{Sin}[c+dx])}}{4576 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{\left(\frac{2 \operatorname{Cos}\left[\frac{dx}{2}\right] \operatorname{Sin}\left[\frac{c}{2}\right]}{d} + \frac{2 \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d}\right) \sqrt{a(1+\operatorname{Sin}[c+dx])}}{16 \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{\left(-\frac{2 \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Cos}\left[\frac{dx}{2}\right]}{d} + \frac{2 \operatorname{Sin}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d}\right) \sqrt{a(1+\operatorname{Sin}[c+dx])}}{16 \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{\sqrt{a(1+\operatorname{Sin}[c+dx])} (5 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 3 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right])}{480 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{\sqrt{a(1+\operatorname{Sin}[c+dx])} (9 \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] + 7 \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right])}{1008 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \frac{\sqrt{a(1+\operatorname{Sin}[c+dx])} (13 \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right] + 11 \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right])}{4576 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 449: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^4 \operatorname{Csc}[c+dx] \sqrt{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 173 leaves, 9 steps):

$$\begin{aligned}
& -\frac{67 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c+dx]}{\sqrt{a+a \operatorname{Sin}[c+dx]}}\right]}{64 d} + \frac{61 a \operatorname{Cot}[c+dx]}{64 d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \\
& \frac{61 a \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{96 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \frac{a \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{24 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3 \sqrt{a+a \operatorname{Sin}[c+dx]}}{4 d}
\end{aligned}$$

Result (type 3, 367 leaves):

$$\begin{aligned}
& -\frac{1}{192 d \left(1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right) \left(\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 - \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\right)^4} \\
& \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^{13} \sqrt{a(1+\operatorname{Sin}[c+dx])} \left(442 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - 162 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] + 122 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] + 366 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] + \right. \\
& 603 \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 804 \operatorname{Cos}[2(c+dx)] \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \\
& 201 \operatorname{Cos}[4(c+dx)] \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 603 \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \\
& 804 \operatorname{Cos}[2(c+dx)] \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 201 \operatorname{Cos}[4(c+dx)] \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left. \right) - \\
& 442 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 162 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] - 122 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + 366 \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]
\end{aligned}$$

■ **Problem 453: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^4 \sin[c + dx]^2 (a + a \sin[c + dx])^{3/2} dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$\begin{aligned} & - \frac{256 a^4 \cos[c + dx]^5}{6435 d (a + a \sin[c + dx])^{5/2}} - \frac{64 a^3 \cos[c + dx]^5}{1287 d (a + a \sin[c + dx])^{3/2}} - \frac{56 a^2 \cos[c + dx]^5}{1287 d \sqrt{a + a \sin[c + dx]}} - \\ & \frac{14 a \cos[c + dx]^5 \sqrt{a + a \sin[c + dx]}}{429 d} + \frac{4 \cos[c + dx]^5 (a + a \sin[c + dx])^{3/2}}{39 d} - \frac{2 \cos[c + dx]^5 (a + a \sin[c + dx])^{5/2}}{15 a d} \end{aligned}$$

Result (type 3, 859 leaves):

$$\begin{aligned}
& \frac{a \left(-\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{a(1+\sin[c+dx])}}{8d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)} - \\
& \frac{a \left(3 \cos\left[\frac{1}{2}(c+dx)\right] + \cos\left[\frac{3}{2}(c+dx)\right] - 4 \sin\left[\frac{1}{2}(c+dx)\right]^3 \right) \sqrt{a(1+\sin[c+dx])}}{48d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)} + \\
& \frac{a \sqrt{a(1+\sin[c+dx])} \left(5 \cos\left[\frac{3}{2}(c+dx)\right] - 3 \cos\left[\frac{5}{2}(c+dx)\right] + 5 \sin\left[\frac{3}{2}(c+dx)\right] + 3 \sin\left[\frac{5}{2}(c+dx)\right] \right)}{480d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)} + \\
& \left(a \sqrt{a(1+\sin[c+dx])} \left(105 \cos\left[\frac{1}{2}(c+dx)\right] + 35 \cos\left[\frac{3}{2}(c+dx)\right] - 21 \cos\left[\frac{5}{2}(c+dx)\right] - 15 \cos\left[\frac{7}{2}(c+dx)\right] - 105 \sin\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. 35 \sin\left[\frac{3}{2}(c+dx)\right] + 21 \sin\left[\frac{5}{2}(c+dx)\right] - 15 \sin\left[\frac{7}{2}(c+dx)\right] \right) \right) / \left(6720d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) - \\
& \frac{a \sqrt{a(1+\sin[c+dx])} \left(9 \cos\left[\frac{7}{2}(c+dx)\right] - 7 \cos\left[\frac{9}{2}(c+dx)\right] + 9 \sin\left[\frac{7}{2}(c+dx)\right] + 7 \sin\left[\frac{9}{2}(c+dx)\right] \right)}{1008d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)} - \\
& \left(a \sqrt{a(1+\sin[c+dx])} \left(693 \cos\left[\frac{5}{2}(c+dx)\right] + 495 \cos\left[\frac{7}{2}(c+dx)\right] - 385 \cos\left[\frac{9}{2}(c+dx)\right] - 315 \cos\left[\frac{11}{2}(c+dx)\right] - 693 \sin\left[\frac{5}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. 495 \sin\left[\frac{7}{2}(c+dx)\right] + 385 \sin\left[\frac{9}{2}(c+dx)\right] - 315 \sin\left[\frac{11}{2}(c+dx)\right] \right) \right) / \left(110880d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) - \\
& \left(a \sqrt{a(1+\sin[c+dx])} \left(13 \cos\left[\frac{11}{2}(c+dx)\right] - 11 \cos\left[\frac{13}{2}(c+dx)\right] + 13 \sin\left[\frac{11}{2}(c+dx)\right] + 11 \sin\left[\frac{13}{2}(c+dx)\right] \right) \right) / \\
& \quad \left(4576d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) - \\
& \left(a \sqrt{a(1+\sin[c+dx])} \left(715 \cos\left[\frac{9}{2}(c+dx)\right] + 585 \cos\left[\frac{11}{2}(c+dx)\right] - 495 \cos\left[\frac{13}{2}(c+dx)\right] - 429 \cos\left[\frac{15}{2}(c+dx)\right] - 715 \sin\left[\frac{9}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. 585 \sin\left[\frac{11}{2}(c+dx)\right] + 495 \sin\left[\frac{13}{2}(c+dx)\right] - 429 \sin\left[\frac{15}{2}(c+dx)\right] \right) \right) / \left(411840d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)
\end{aligned}$$

■ **Problem 454: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 \sin[c+dx] (a+a \sin[c+dx])^{3/2} dx$$

Optimal (type 3, 156 leaves, 5 steps):

$$\begin{aligned}
& - \frac{256 a^4 \cos[c+dx]^5}{5005 d (a+a \sin[c+dx])^{5/2}} - \frac{64 a^3 \cos[c+dx]^5}{1001 d (a+a \sin[c+dx])^{3/2}} - \\
& \frac{8 a^2 \cos[c+dx]^5}{143 d \sqrt{a+a \sin[c+dx]}} - \frac{6 a \cos[c+dx]^5 \sqrt{a+a \sin[c+dx]}}{143 d} - \frac{2 \cos[c+dx]^5 (a+a \sin[c+dx])^{3/2}}{13 d}
\end{aligned}$$

Result (type 3, 673 leaves) :

$$\begin{aligned}
 & \frac{a \left(3 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] - 4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^3 \right) \sqrt{a(1+\operatorname{Sin}[c+dx])}}{24 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)} - \\
 & \left(a \sqrt{a(1+\operatorname{Sin}[c+dx])} \left(30 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 5 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] - 3 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] - \right. \right. \\
 & \quad \left. \left. 30 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 5 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 3 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] \right) \right) / \left(240 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) - \\
 & \left(3 a \sqrt{a(1+\operatorname{Sin}[c+dx])} \left(7 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] + 5 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] - 7 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + 5 \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] \right) \right) / \\
 & \quad \left(560 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) + \\
 & \left(a \sqrt{a(1+\operatorname{Sin}[c+dx])} \left(105 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] - 63 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] - 45 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] + 35 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] + 105 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \right. \right. \\
 & \quad \left. \left. 63 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] - 45 \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] - 35 \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right] \right) \right) / \left(3360 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) - \\
 & \left(a \sqrt{a(1+\operatorname{Sin}[c+dx])} \left(11 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] + 9 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] - 11 \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right] + 9 \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right] \right) \right) / \\
 & \quad \left(1584 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) + \\
 & \left(a \sqrt{a(1+\operatorname{Sin}[c+dx])} \left(1287 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] - 1001 \operatorname{Cos}\left[\frac{9}{2}(c+dx)\right] - 819 \operatorname{Cos}\left[\frac{11}{2}(c+dx)\right] + 693 \operatorname{Cos}\left[\frac{13}{2}(c+dx)\right] + 1287 \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] + \right. \right. \\
 & \quad \left. \left. 1001 \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right] - 819 \operatorname{Sin}\left[\frac{11}{2}(c+dx)\right] - 693 \operatorname{Sin}\left[\frac{13}{2}(c+dx)\right] \right) \right) / \left(288288 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right)
 \end{aligned}$$

■ **Problem 462: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^4 \operatorname{Csc}[c+dx]^4 (a+a \operatorname{Sin}[c+dx])^{3/2} dx$$

Optimal (type 3, 291 leaves, 16 steps) :

$$\begin{aligned}
 & \frac{171 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c+dx]}{\sqrt{a+a \operatorname{Sin}[c+dx]}}\right]}{1024 d} - \frac{171 a^2 \operatorname{Cot}[c+dx]}{1024 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \frac{57 a^2 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{512 d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \\
 & \frac{199 a^2 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{640 d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \frac{1237 a^2 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{2240 d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \frac{9 a^2 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^4}{40 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \\
 & \frac{a \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^5 \sqrt{a+a \operatorname{Sin}[c+dx]}}{28 d} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^6 (a+a \operatorname{Sin}[c+dx])^{3/2}}{7 d}
 \end{aligned}$$

Result (type 3, 2055 leaves) :

$$\begin{aligned}
& \frac{3861 (a (1 + \sin[c + dx]))^{3/2}}{71680 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \frac{3861 \cot\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}}{143360 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \frac{43 \csc\left[\frac{1}{4}(c + dx)\right]^2 (a (1 + \sin[c + dx]))^{3/2}}{8192 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{619 \cot\left[\frac{1}{4}(c + dx)\right] \csc\left[\frac{1}{4}(c + dx)\right]^2 (a (1 + \sin[c + dx]))^{3/2}}{286720 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \frac{11 \csc\left[\frac{1}{4}(c + dx)\right]^4 (a (1 + \sin[c + dx]))^{3/2}}{8192 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{9 \cot\left[\frac{1}{4}(c + dx)\right] \csc\left[\frac{1}{4}(c + dx)\right]^4 (a (1 + \sin[c + dx]))^{3/2}}{143360 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \frac{\csc\left[\frac{1}{4}(c + dx)\right]^6 (a (1 + \sin[c + dx]))^{3/2}}{16384 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \\
& \frac{\cot\left[\frac{1}{4}(c + dx)\right] \csc\left[\frac{1}{4}(c + dx)\right]^6 (a (1 + \sin[c + dx]))^{3/2}}{114688 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \frac{171 \log\left[1 + \cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] (a (1 + \sin[c + dx]))^{3/2}}{2048 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{171 \log\left[1 - \cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] (a (1 + \sin[c + dx]))^{3/2}}{2048 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \frac{43 \sec\left[\frac{1}{4}(c + dx)\right]^2 (a (1 + \sin[c + dx]))^{3/2}}{8192 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \\
& \frac{11 \sec\left[\frac{1}{4}(c + dx)\right]^4 (a (1 + \sin[c + dx]))^{3/2}}{8192 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \frac{\sec\left[\frac{1}{4}(c + dx)\right]^6 (a (1 + \sin[c + dx]))^{3/2}}{16384 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{1792 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^6 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3}{403 (a (1 + \sin[c + dx]))^{3/2}} - \\
& \frac{71680 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3}{443 (a (1 + \sin[c + dx]))^{3/2}} + \\
& \frac{71680 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3}{\sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}} + \\
& \frac{7168 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^7 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3}{9 \sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}} - \\
& \frac{17920 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^5 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3}{619 \sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}} + \\
& \frac{71680 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3}{(a (1 + \sin[c + dx]))^{3/2}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{3861 \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{71680 d \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{\operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{7168 d \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right)^7 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{3(a(1+\operatorname{Sin}[c+dx]))^{3/2}}{7168 d \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right)^6 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{9 \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{17920 d \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right)^5 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{367(a(1+\operatorname{Sin}[c+dx]))^{3/2}}{71680 d \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right)^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{619 \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{71680 d \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right)^3 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{531(a(1+\operatorname{Sin}[c+dx]))^{3/2}}{35840 d \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right)^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{3861 \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{71680 d \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \\
& \frac{3861(a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{143360 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{619 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{286720 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{9 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{143360 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^6 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{114688 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3}
\end{aligned}$$

■ **Problem 463: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^4 \operatorname{Csc}[c+dx]^5 (a+a \operatorname{Sin}[c+dx])^{3/2} dx$$

Optimal (type 3, 329 leaves, 18 steps):

$$\begin{aligned}
& \frac{1587 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{a+a \sin [c+d x]}}\right]}{16384 d} - \frac{1587 a^2 \cot [c+d x]}{16384 d \sqrt{a+a \sin [c+d x]}} - \frac{529 a^2 \cot [c+d x] \csc [c+d x]}{8192 d \sqrt{a+a \sin [c+d x]}} - \\
& \frac{529 a^2 \cot [c+d x] \csc [c+d x]^2}{10240 d \sqrt{a+a \sin [c+d x]}} + \frac{8653 a^2 \cot [c+d x] \csc [c+d x]^3}{35840 d \sqrt{a+a \sin [c+d x]}} + \frac{1957 a^2 \cot [c+d x] \csc [c+d x]^4}{4480 d \sqrt{a+a \sin [c+d x]}} + \\
& \frac{83 a^2 \cot [c+d x] \csc [c+d x]^5}{448 d \sqrt{a+a \sin [c+d x]}} - \frac{3 a \cot [c+d x] \csc [c+d x]^6 \sqrt{a+a \sin [c+d x]}}{112 d} - \frac{\cot [c+d x] \csc [c+d x]^7 (a+a \sin [c+d x])^{3/2}}{8 d}
\end{aligned}$$

Result (type 3, 2303 leaves):

$$\begin{aligned}
& \frac{6053 (a (1 + \sin [c+d x]))^{3/2}}{143360 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} - \frac{6053 \cot [\frac{1}{4} (c+d x)] (a (1 + \sin [c+d x]))^{3/2}}{286720 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} - \\
& \frac{179 \csc [\frac{1}{4} (c+d x)]^2 (a (1 + \sin [c+d x]))^{3/2}}{131072 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} + \frac{107 \cot [\frac{1}{4} (c+d x)] \csc [\frac{1}{4} (c+d x)]^2 (a (1 + \sin [c+d x]))^{3/2}}{573440 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} + \\
& \frac{113 \csc [\frac{1}{4} (c+d x)]^4 (a (1 + \sin [c+d x]))^{3/2}}{262144 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} + \frac{31 \cot [\frac{1}{4} (c+d x)] \csc [\frac{1}{4} (c+d x)]^4 (a (1 + \sin [c+d x]))^{3/2}}{143360 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} + \\
& \frac{\csc [\frac{1}{4} (c+d x)]^6 (a (1 + \sin [c+d x]))^{3/2}}{131072 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} - \frac{3 \cot [\frac{1}{4} (c+d x)] \csc [\frac{1}{4} (c+d x)]^6 (a (1 + \sin [c+d x]))^{3/2}}{229376 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} - \\
& \frac{\csc [\frac{1}{4} (c+d x)]^8 (a (1 + \sin [c+d x]))^{3/2}}{524288 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} - \frac{1587 \log [1 + \cos [\frac{1}{2} (c+d x)] - \sin [\frac{1}{2} (c+d x)]] (a (1 + \sin [c+d x]))^{3/2}}{32768 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} + \\
& \frac{1587 \log [1 - \cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)]] (a (1 + \sin [c+d x]))^{3/2}}{32768 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} + \frac{179 \sec [\frac{1}{4} (c+d x)]^2 (a (1 + \sin [c+d x]))^{3/2}}{131072 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} - \\
& \frac{113 \sec [\frac{1}{4} (c+d x)]^4 (a (1 + \sin [c+d x]))^{3/2}}{262144 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} - \frac{\sec [\frac{1}{4} (c+d x)]^6 (a (1 + \sin [c+d x]))^{3/2}}{131072 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} + \\
& \frac{\sec [\frac{1}{4} (c+d x)]^8 (a (1 + \sin [c+d x]))^{3/2}}{524288 d (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} + \frac{(a (1 + \sin [c+d x]))^{3/2}}{32768 d (\cos [\frac{1}{4} (c+d x)] - \sin [\frac{1}{4} (c+d x)])^8 (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} + \\
& \frac{5 (a (1 + \sin [c+d x]))^{3/2}}{114688 d (\cos [\frac{1}{4} (c+d x)] - \sin [\frac{1}{4} (c+d x)])^6 (\cos [\frac{1}{2} (c+d x)] + \sin [\frac{1}{2} (c+d x)])^3} -
\end{aligned}$$

$$\begin{aligned}
& \frac{5939 (a (1 + \sin[c + dx]))^{3/2}}{2293760 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{5409 (a (1 + \sin[c + dx]))^{3/2}}{2293760 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{3 \sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}}{14336 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^7 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \\
& \frac{31 \sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}}{17920 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^5 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \\
& \frac{107 \sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}}{143360 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{6053 \sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}}{143360 d \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \\
& \frac{(a (1 + \sin[c + dx]))^{3/2}}{32768 d \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^8 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \\
& \frac{3 \sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}}{14336 d \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^7 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{19 (a (1 + \sin[c + dx]))^{3/2}}{114688 d \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^6 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{31 \sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}}{17920 d \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^5 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{1971 (a (1 + \sin[c + dx]))^{3/2}}{2293760 d \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{107 \sin\left[\frac{1}{4}(c + dx)\right] (a (1 + \sin[c + dx]))^{3/2}}{143360 d \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} - \\
& \frac{7121 (a (1 + \sin[c + dx]))^{3/2}}{2293760 d \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3} -
\end{aligned}$$

$$\frac{6053 \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] (a(1+\operatorname{Sin}[c+dx]))^{3/2}}{143360 d \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} -$$

$$\frac{6053 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{286720 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{107 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{573440 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} +$$

$$\frac{31 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{143360 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{3 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^6 (a(1+\operatorname{Sin}[c+dx]))^{3/2} \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{229376 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3}$$

■ **Problem 468: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx] \operatorname{Cot}[c+dx]^3}{\sqrt{a+a \operatorname{Sin}[c+dx]}} dx$$

Optimal (type 3, 125 leaves, 11 steps):

$$\frac{9 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c+dx]}{\sqrt{a+a \operatorname{Sin}[c+dx]}}\right]}{4 \sqrt{a} d} - \frac{2 \operatorname{Cos}[c+dx]}{d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \frac{\operatorname{Cot}[c+dx]}{4 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{2 d \sqrt{a+a \operatorname{Sin}[c+dx]}}$$

Result (type 3, 773 leaves):

$$\begin{aligned}
& - \frac{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]}{4d\sqrt{a(1+\sin[c+dx])}} - \frac{2\cos\left[\frac{1}{2}(c+dx)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{d\sqrt{a(1+\sin[c+dx])}} + \\
& \frac{\cot\left[\frac{1}{4}(c+dx)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{8d\sqrt{a(1+\sin[c+dx])}} - \frac{\csc\left[\frac{1}{4}(c+dx)\right]^2\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{32d\sqrt{a(1+\sin[c+dx])}} + \\
& \frac{9\log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{8d\sqrt{a(1+\sin[c+dx])}} - \\
& \frac{9\log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{8d\sqrt{a(1+\sin[c+dx])}} + \frac{\sec\left[\frac{1}{4}(c+dx)\right]^2\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{32d\sqrt{a(1+\sin[c+dx])}} + \\
& \frac{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]}{16d\left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)^2\sqrt{a(1+\sin[c+dx])}} - \frac{\sin\left[\frac{1}{4}(c+dx)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{4d\left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)\sqrt{a(1+\sin[c+dx])}} - \\
& \frac{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]}{16d\left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)^2\sqrt{a(1+\sin[c+dx])}} + \frac{\sin\left[\frac{1}{4}(c+dx)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{4d\left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)\sqrt{a(1+\sin[c+dx])}} + \\
& \frac{2\sin\left[\frac{1}{2}(c+dx)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{d\sqrt{a(1+\sin[c+dx])}} + \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\tan\left[\frac{1}{4}(c+dx)\right]}{8d\sqrt{a(1+\sin[c+dx])}}
\end{aligned}$$

■ **Problem 469: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx]^4}{\sqrt{a+a\sin[c+dx]}} dx$$

Optimal (type 3, 135 leaves, 11 steps):

$$- \frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{a+a\sin[c+dx]}}\right]}{8\sqrt{a}d} + \frac{9\cot[c+dx]}{8d\sqrt{a+a\sin[c+dx]}} + \frac{\cot[c+dx]\csc[c+dx]}{12d\sqrt{a+a\sin[c+dx]}} - \frac{\cot[c+dx]\csc[c+dx]^2}{3d\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 292 leaves):

$$\frac{1}{24 d \left(\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 - \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \right)^3 \sqrt{a(1+\operatorname{Sin}[c+dx])}}$$

$$\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^9 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \left(36 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - 46 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] - \right.$$

$$54 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] - 36 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 63 \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] +$$

$$63 \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] - 46 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 54 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] +$$

$$\left. 21 \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] - 21 \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] \right)$$

■ **Problem 470: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^4 \operatorname{Csc}[c+dx]}{\sqrt{a+a \operatorname{Sin}[c+dx]}} dx$$

Optimal (type 3, 170 leaves, 15 steps):

$$-\frac{11 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cos}[c+dx]}{\sqrt{a+a \operatorname{Sin}[c+dx]}}\right]}{64 \sqrt{a} d} - \frac{11 \operatorname{Cot}[c+dx]}{64 d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \frac{53 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{96 d \sqrt{a+a \operatorname{Sin}[c+dx]}} + \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{24 d \sqrt{a+a \operatorname{Sin}[c+dx]}} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{4 d \sqrt{a+a \operatorname{Sin}[c+dx]}}$$

Result (type 3, 374 leaves):

$$\frac{1}{192 d \left(\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 - \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \right)^4 \sqrt{a(1+\operatorname{Sin}[c+dx])}}$$

$$\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^{12} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \left(214 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - 558 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] - 490 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] + \right.$$

$$66 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] - 99 \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 132 \operatorname{Cos}[2(c+dx)] \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] -$$

$$33 \operatorname{Cos}[4(c+dx)] \operatorname{Log}\left[1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 99 \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] -$$

$$132 \operatorname{Cos}[2(c+dx)] \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 33 \operatorname{Cos}[4(c+dx)] \operatorname{Log}\left[1 - \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] -$$

$$\left. 214 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 558 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 490 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + 66 \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] \right)$$

■ **Problem 476: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 \operatorname{Cot}[c+dx]^2}{(a+a \operatorname{Sin}[c+dx])^{3/2}} dx$$

Optimal (type 3, 94 leaves, 9 steps) :

$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{a^{3/2} d} - \frac{\cos[c+dx]}{a d \sqrt{a+a \sin[c+dx]}} - \frac{\cot[c+dx] \sqrt{a+a \sin[c+dx]}}{a^2 d}$$

Result (type 3, 220 leaves) :

$$\left(\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \left(2 - 8 \cos\left[\frac{1}{2}(c+dx)\right] - \cot\left[\frac{1}{4}(c+dx)\right] \right) + \right. \\ \left. 6 \log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - 6 \log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + \frac{2 \sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} - \right. \\ \left. \frac{2 \sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]} + 8 \sin\left[\frac{1}{2}(c+dx)\right] - \tan\left[\frac{1}{4}(c+dx)\right] \right) \Big/ (4 d (a (1 + \sin[c+dx]))^{3/2})$$

■ **Problem 477: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx] \cot[c+dx]^3}{(a+a \sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 106 leaves, 8 steps) :

$$-\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{4 a^{3/2} d} + \frac{7 \cot[c+dx]}{4 a d \sqrt{a+a \sin[c+dx]}} - \frac{\cot[c+dx] \operatorname{Csc}[c+dx] \sqrt{a+a \sin[c+dx]}}{2 a^2 d}$$

Result (type 3, 274 leaves) :

$$\frac{1}{32 d (a (1 + \sin[c+dx]))^{3/2}} \\ \left(\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \left(-24 + 12 \cot\left[\frac{1}{4}(c+dx)\right] - \operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 - 12 \log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right. \\ \left. 12 \log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + \sec\left[\frac{1}{4}(c+dx)\right]^2 + \frac{2}{\left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)^2} - \right. \\ \left. \frac{24 \sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} - \frac{2}{\left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)^2} + \frac{24 \sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]} + 12 \tan\left[\frac{1}{4}(c+dx)\right] \right)$$

■ **Problem 478: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx]^4}{(a+a \sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 144 leaves, 10 steps) :

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{a+a\sin[c+dx]}}\right]}{8a^{3/2}d} - \frac{\text{Cot}[c+dx]}{8ad\sqrt{a+a\sin[c+dx]}} + \frac{11\text{Cot}[c+dx]\text{Csc}[c+dx]}{12ad\sqrt{a+a\sin[c+dx]}} - \frac{\text{Cot}[c+dx]\text{Csc}[c+dx]^2\sqrt{a+a\sin[c+dx]}}{3a^2d}$$

Result (type 3, 294 leaves) :

$$\frac{1}{24d\left(\text{Csc}\left[\frac{1}{4}(c+dx)\right]^2 - \text{Sec}\left[\frac{1}{4}(c+dx)\right]^2\right)^3\left(a(1+\sin[c+dx])\right)^{3/2}}$$

$$\text{Csc}\left[\frac{1}{2}(c+dx)\right]^9\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3\left(-132\cos\left[\frac{1}{2}(c+dx)\right] + 62\cos\left[\frac{3}{2}(c+dx)\right] +\right.$$

$$6\cos\left[\frac{5}{2}(c+dx)\right] + 132\sin\left[\frac{1}{2}(c+dx)\right] - 9\log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]\sin[c+dx] +$$

$$9\log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]\sin[c+dx] + 62\sin\left[\frac{3}{2}(c+dx)\right] - 6\sin\left[\frac{5}{2}(c+dx)\right] +$$

$$\left.3\log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]\sin[3(c+dx)] - 3\log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]\sin[3(c+dx)]\right)$$

■ **Problem 479: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c+dx]^4 \text{Csc}[c+dx]}{(a+a\sin[c+dx])^{3/2}} dx$$

Optimal (type 3, 182 leaves, 12 steps) :

$$-\frac{3\text{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{a+a\sin[c+dx]}}\right]}{64a^{3/2}d} - \frac{3\text{Cot}[c+dx]}{64ad\sqrt{a+a\sin[c+dx]}} -$$

$$\frac{\text{Cot}[c+dx]\text{Csc}[c+dx]}{32ad\sqrt{a+a\sin[c+dx]}} + \frac{5\text{Cot}[c+dx]\text{Csc}[c+dx]^2}{8ad\sqrt{a+a\sin[c+dx]}} - \frac{\text{Cot}[c+dx]\text{Csc}[c+dx]^3\sqrt{a+a\sin[c+dx]}}{4a^2d}$$

Result (type 3, 376 leaves) :

$$\begin{aligned}
& - \frac{1}{64 d \left(\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 - \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \right)^4 (a(1+\sin[c+dx]))^{3/2}} \\
& \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^{12} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \left(446 \cos\left[\frac{1}{2}(c+dx)\right] - 182 \cos\left[\frac{3}{2}(c+dx)\right] - 2 \cos\left[\frac{5}{2}(c+dx)\right] - \right. \\
& 6 \cos\left[\frac{7}{2}(c+dx)\right] + 9 \log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - 12 \cos[2(c+dx)] \log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \\
& 3 \cos[4(c+dx)] \log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - 9 \log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + \\
& 12 \cos[2(c+dx)] \log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] - 3 \cos[4(c+dx)] \log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] - \\
& \left. 446 \sin\left[\frac{1}{2}(c+dx)\right] - 182 \sin\left[\frac{3}{2}(c+dx)\right] + 2 \sin\left[\frac{5}{2}(c+dx)\right] - 6 \sin\left[\frac{7}{2}(c+dx)\right] \right)
\end{aligned}$$

■ **Problem 481: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^4 \sin[c+dx]^4}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 260 leaves, 18 steps):

$$\begin{aligned}
& - \frac{4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{a^{5/2} d} + \frac{4496 \cos[c+dx]}{693 a^2 d \sqrt{a+a \sin[c+dx]}} + \frac{200 \cos[c+dx] \sin[c+dx]^2}{231 a^2 d \sqrt{a+a \sin[c+dx]}} - \\
& \frac{424 \cos[c+dx] \sin[c+dx]^3}{693 a^2 d \sqrt{a+a \sin[c+dx]}} + \frac{46 \cos[c+dx] \sin[c+dx]^4}{99 a^2 d \sqrt{a+a \sin[c+dx]}} - \frac{2 \cos[c+dx] \sin[c+dx]^5}{11 a^2 d \sqrt{a+a \sin[c+dx]}} - \frac{1048 \cos[c+dx] \sqrt{a+a \sin[c+dx]}}{693 a^3 d}
\end{aligned}$$

Result (type 3, 224 leaves):

$$\begin{aligned}
& \frac{1}{11088 d (a(1+\sin[c+dx]))^{5/2}} \\
& \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 \left((88704 + 88704 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] + 73458 \cos\left[\frac{1}{2}(c+dx)\right] - \right. \\
& 15246 \cos\left[\frac{3}{2}(c+dx)\right] - 4851 \cos\left[\frac{5}{2}(c+dx)\right] + 1485 \cos\left[\frac{7}{2}(c+dx)\right] + 385 \cos\left[\frac{9}{2}(c+dx)\right] - 63 \cos\left[\frac{11}{2}(c+dx)\right] - \\
& \left. 73458 \sin\left[\frac{1}{2}(c+dx)\right] - 15246 \sin\left[\frac{3}{2}(c+dx)\right] + 4851 \sin\left[\frac{5}{2}(c+dx)\right] + 1485 \sin\left[\frac{7}{2}(c+dx)\right] - 385 \sin\left[\frac{9}{2}(c+dx)\right] - 63 \sin\left[\frac{11}{2}(c+dx)\right] \right)
\end{aligned}$$

■ **Problem 482: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^4 \sin[c+dx]^3}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 222 leaves, 16 steps):

$$\frac{4\sqrt{2}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{2}\sqrt{a+a\sin[c+dx]}}\right]}{a^{5/2}d} - \frac{2048\cos[c+dx]}{315a^2d\sqrt{a+a\sin[c+dx]}} - \frac{92\cos[c+dx]\sin[c+dx]^2}{105a^2d\sqrt{a+a\sin[c+dx]}} +$$

$$\frac{38\cos[c+dx]\sin[c+dx]^3}{63a^2d\sqrt{a+a\sin[c+dx]}} - \frac{2\cos[c+dx]\sin[c+dx]^4}{9a^2d\sqrt{a+a\sin[c+dx]}} + \frac{472\cos[c+dx]\sqrt{a+a\sin[c+dx]}}{315a^3d}$$

Result (type 3, 225 leaves):

$$\frac{1}{2520a^3d\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}$$

$$\sqrt{a(1+\sin[c+dx])}\left((20160+20160i)(-1)^{3/4}\operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\operatorname{Sec}\left[\frac{dx}{4}\right]\left(\cos\left[\frac{1}{4}(2c+dx)\right]-\sin\left[\frac{1}{4}(2c+dx)\right]\right)\right]\right)-$$

$$16380\cos\left[\frac{1}{2}(c+dx)\right]+3150\cos\left[\frac{3}{2}(c+dx)\right]+882\cos\left[\frac{5}{2}(c+dx)\right]-225\cos\left[\frac{7}{2}(c+dx)\right]-35\cos\left[\frac{9}{2}(c+dx)\right]+$$

$$16380\sin\left[\frac{1}{2}(c+dx)\right]+3150\sin\left[\frac{3}{2}(c+dx)\right]-882\sin\left[\frac{5}{2}(c+dx)\right]-225\sin\left[\frac{7}{2}(c+dx)\right]+35\sin\left[\frac{9}{2}(c+dx)\right]\right]$$

■ **Problem 483: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^4 \sin[c+dx]^2}{(a+a\sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 169 leaves, 6 steps):

$$-\frac{4\sqrt{2}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\cos[c+dx]}{\sqrt{2}\sqrt{a+a\sin[c+dx]}}\right]}{a^{5/2}d} + \frac{4\cos[c+dx]^5}{7d(a+a\sin[c+dx])^{5/2}} + \frac{2\cos[c+dx]^3}{3ad(a+a\sin[c+dx])^{3/2}} - \frac{2\cos[c+dx]^5}{7ad(a+a\sin[c+dx])^{3/2}} + \frac{4\cos[c+dx]}{a^2d\sqrt{a+a\sin[c+dx]}}$$

Result (type 3, 201 leaves):

$$\frac{1}{84a^3d\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}$$

$$\sqrt{a(1+\sin[c+dx])}\left((672+672i)(-1)^{3/4}\operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\operatorname{Sec}\left[\frac{dx}{4}\right]\left(\cos\left[\frac{1}{4}(2c+dx)\right]-\sin\left[\frac{1}{4}(2c+dx)\right]\right)\right]\right)-$$

$$525\cos\left[\frac{1}{2}(c+dx)\right]+91\cos\left[\frac{3}{2}(c+dx)\right]+21\cos\left[\frac{5}{2}(c+dx)\right]-3\cos\left[\frac{7}{2}(c+dx)\right]+$$

$$525\sin\left[\frac{1}{2}(c+dx)\right]+91\sin\left[\frac{3}{2}(c+dx)\right]-21\sin\left[\frac{5}{2}(c+dx)\right]-3\sin\left[\frac{7}{2}(c+dx)\right]\right]$$

■ **Problem 484: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^4 \sin [c+d x]}{(a+a \sin [c+d x])^{5 / 2}} d x$$

Optimal (type 3, 137 leaves, 5 steps) :

$$\frac{4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{2} \sqrt{a+a \sin [c+d x]}}\right]}{a^{5 / 2} d}-\frac{2 \cos [c+d x]^5}{5 d(a+a \sin [c+d x])^{5 / 2}}-\frac{2 \cos [c+d x]^3}{3 a d(a+a \sin [c+d x])^{3 / 2}}-\frac{4 \cos [c+d x]}{a^2 d \sqrt{a+a \sin [c+d x]}}$$

Result (type 3, 177 leaves) :

$$\left(\sqrt{a(1+\sin [c+d x])}\left((240+240 i)(-1)^{3 / 4} \operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3 / 4} \operatorname{Sec}\left[\frac{d x}{4}\right]\left(\cos \left[\frac{1}{4}(2 c+d x)\right]-\sin \left[\frac{1}{4}(2 c+d x)\right]\right)\right]-180 \cos \left[\frac{1}{2}(c+d x)\right]+25 \cos \left[\frac{3}{2}(c+d x)\right]+3 \cos \left[\frac{5}{2}(c+d x)\right]+180 \sin \left[\frac{1}{2}(c+d x)\right]+25 \sin \left[\frac{3}{2}(c+d x)\right]-3 \sin \left[\frac{5}{2}(c+d x)\right]\right)\right) / \left(30 a^3 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)\right)$$

■ **Problem 485: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^3 \cot [c+d x]}{(a+a \sin [c+d x])^{5 / 2}} d x$$

Optimal (type 3, 113 leaves, 9 steps) :

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{a+a \sin [c+d x]}}\right]}{a^{5 / 2} d}+\frac{4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [c+d x]}{\sqrt{2} \sqrt{a+a \sin [c+d x]}}\right]}{a^{5 / 2} d}-\frac{2 \cos [c+d x]}{a^2 d \sqrt{a+a \sin [c+d x]}}$$

Result (type 3, 154 leaves) :

$$-\frac{1}{d(a(1+\sin [c+d x]))^{5 / 2}}\left((8+8 i)(-1)^{3 / 4} \operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3 / 4}\left(-1+\tan \left[\frac{1}{4}(c+d x)\right]\right)\right]+2 \cos \left[\frac{1}{2}(c+d x)\right]+\log \left[1+\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]-\log \left[1-\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]-2 \sin \left[\frac{1}{2}(c+d x)\right]\right)\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^5$$

■ **Problem 486: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^2 \cot [c+d x]^2}{(a+a \sin [c+d x])^{5 / 2}} d x$$

Optimal (type 3, 113 leaves, 12 steps) :

$$\frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{a^{5/2} d} - \frac{4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{a^{5/2} d} - \frac{\cot[c+dx]}{a^2 d \sqrt{a+a \sin[c+dx]}}$$

Result (type 3, 170 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 \right. \\ \left. \left((32 + 32i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c+dx)\right]\right)\right] - \cot\left[\frac{1}{4}(c+dx)\right] + 10 \log\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \right. \right. \\ \left. \left. 10 \log\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + 2 \sec\left[\frac{1}{2}(c+dx)\right] - \tan\left[\frac{1}{4}(c+dx)\right] \right) \right) / \left(4 d (a (1 + \sin[c+dx]))^{5/2} \right)$$

■ **Problem 487: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx] \cot[c+dx]^3}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 153 leaves, 14 steps):

$$-\frac{23 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{4 a^{5/2} d} + \frac{4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{a^{5/2} d} + \frac{9 \cot[c+dx]}{4 a^2 d \sqrt{a+a \sin[c+dx]}} - \frac{\cot[c+dx] \operatorname{Csc}[c+dx]}{2 a^2 d \sqrt{a+a \sin[c+dx]}}$$

Result (type 3, 791 leaves):

$$\begin{aligned}
& - \frac{5 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{4d \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} + \frac{1}{d \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} \\
& (8+8i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 + \\
& \frac{5 \operatorname{Cot}\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{8d \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} - \frac{\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{32d \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} - \\
& \frac{23 \operatorname{Log}\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{8d \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} + \\
& \frac{23 \operatorname{Log}\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{8d \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{32d \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} + \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{16d \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)^2 \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} - \\
& \frac{5 \sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{4d \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right) \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} - \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{16d \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^2 \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} + \\
& \frac{5 \sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{4d \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right) \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}} + \frac{5 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{8d \left(a \left(1 + \sin[c+dx] \right) \right)^{5/2}}
\end{aligned}$$

■ **Problem 488: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cot}[c+dx]^4}{(a+a \sin[c+dx])^{5/2}} dx$$

Optimal (type 3, 191 leaves, 16 steps):

$$\begin{aligned}
& \frac{45 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{a+a \sin[c+dx]}}\right]}{8 a^{5/2} d} - \frac{4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c+dx]}{\sqrt{2} \sqrt{a+a \sin[c+dx]}}\right]}{a^{5/2} d} - \\
& \frac{19 \operatorname{Cot}[c+dx]}{8 a^2 d \sqrt{a+a \sin[c+dx]}} + \frac{13 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{12 a^2 d \sqrt{a+a \sin[c+dx]}} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{3 a^2 d \sqrt{a+a \sin[c+dx]}}
\end{aligned}$$

Result (type 3, 332 leaves):

$$\frac{1}{192 d (a (1 + \sin[c + d x]))^{5/2}} \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^5$$

$$\left((1536 + 1536 i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(c + d x)\right]\right)\right] - \frac{1}{\left(\operatorname{Csc}\left[\frac{1}{4}(c + d x)\right]^2 - \operatorname{Sec}\left[\frac{1}{4}(c + d x)\right]^2\right)^3} 8 \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^9 \right.$$

$$\left. \left(396 \cos\left[\frac{1}{2}(c + d x)\right] - 218 \cos\left[\frac{3}{2}(c + d x)\right] - 114 \cos\left[\frac{5}{2}(c + d x)\right] - 396 \sin\left[\frac{1}{2}(c + d x)\right] - 405 \log\left[1 + \cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] \right.$$

$$\left. \sin[c + d x] + 405 \log\left[1 - \cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \sin[c + d x] - 218 \sin\left[\frac{3}{2}(c + d x)\right] + 114 \sin\left[\frac{5}{2}(c + d x)\right] + \right.$$

$$\left. 135 \log\left[1 + \cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] \sin[3(c + d x)] - 135 \log\left[1 - \cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \sin[3(c + d x)] \right)$$

- **Problem 489: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + d x]^4 \operatorname{Csc}[c + d x]}{(a + a \sin[c + d x])^{5/2}} dx$$

Optimal (type 3, 229 leaves, 18 steps):

$$-\frac{363 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{a + a \sin[c + d x]}}\right]}{64 a^{5/2} d} + \frac{4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[c + d x]}{\sqrt{2} \sqrt{a + a \sin[c + d x]}}\right]}{a^{5/2} d} + \frac{149 \cot[c + d x]}{64 a^2 d \sqrt{a + a \sin[c + d x]}}$$

$$-\frac{107 \cot[c + d x] \operatorname{Csc}[c + d x]}{96 a^2 d \sqrt{a + a \sin[c + d x]}} + \frac{17 \cot[c + d x] \operatorname{Csc}[c + d x]^2}{24 a^2 d \sqrt{a + a \sin[c + d x]}} - \frac{\cot[c + d x] \operatorname{Csc}[c + d x]^3}{4 a^2 d \sqrt{a + a \sin[c + d x]}}$$

Result (type 3, 1327 leaves):

$$\begin{aligned}
& - \frac{155 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^5}{96 d (a (1 + \sin[c+dx]))^{5/2}} + \frac{1}{d (a (1 + \sin[c+dx]))^{5/2}} \\
& (8 + 8i) (-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 + \\
& \frac{155 \operatorname{Cot}\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{192 d (a (1 + \sin[c+dx]))^{5/2}} - \frac{51 \operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{512 d (a (1 + \sin[c+dx]))^{5/2}} + \\
& \frac{5 \operatorname{Cot}\left[\frac{1}{4}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{384 d (a (1 + \sin[c+dx]))^{5/2}} - \frac{\operatorname{Csc}\left[\frac{1}{4}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{1024 d (a (1 + \sin[c+dx]))^{5/2}} - \\
& \frac{363 \operatorname{Log}\left[1 + \cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{128 d (a (1 + \sin[c+dx]))^{5/2}} + \\
& \frac{363 \operatorname{Log}\left[1 - \cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{128 d (a (1 + \sin[c+dx]))^{5/2}} + \\
& \frac{51 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{512 d (a (1 + \sin[c+dx]))^{5/2}} + \frac{\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{1024 d (a (1 + \sin[c+dx]))^{5/2}} + \\
& \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{133 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5} + \\
& \frac{256 d \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)^4 (a (1 + \sin[c+dx]))^{5/2}}{768 d \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)^2 (a (1 + \sin[c+dx]))^{5/2}} - \\
& \frac{5 \sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{155 \sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5} - \\
& \frac{96 d \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)^3 (a (1 + \sin[c+dx]))^{5/2}}{96 d \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right) (a (1 + \sin[c+dx]))^{5/2}} - \\
& \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{5 \sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5} + \\
& \frac{256 d \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)^4 (a (1 + \sin[c+dx]))^{5/2}}{96 d \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)^3 (a (1 + \sin[c+dx]))^{5/2}} - \\
& \frac{173 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5}{155 \sin\left[\frac{1}{4}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5} + \\
& \frac{768 d \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)^2 (a (1 + \sin[c+dx]))^{5/2}}{96 d \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right) (a (1 + \sin[c+dx]))^{5/2}} + \\
& \frac{155 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{192 d (a (1 + \sin[c+dx]))^{5/2}} + \frac{5 \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^5 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{384 d (a (1 + \sin[c+dx]))^{5/2}}
\end{aligned}$$

■ **Problem 490: Unable to integrate problem.**

$$\int \cos[c+dx]^4 \sin[c+dx]^n (a + a \sin[c+dx])^2 dx$$

Optimal (type 5, 200 leaves, 5 steps):

$$\frac{a^2 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{1+n}}{d(1+n) \sqrt{\cos^2[c+dx]}} +$$

$$\frac{2a^2 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{2+n}}{d(2+n) \sqrt{\cos^2[c+dx]}} +$$

$$\frac{a^2 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{3+n}}{d(3+n) \sqrt{\cos^2[c+dx]}}$$

Result (type 8, 31 leaves):

$$\int \cos[c+dx]^4 \sin[c+dx]^n (a + a \sin[c+dx])^2 dx$$

■ **Problem 491: Unable to integrate problem.**

$$\int \cos[c+dx]^4 \sin[c+dx]^n (a + a \sin[c+dx]) dx$$

Optimal (type 5, 129 leaves, 3 steps):

$$\frac{a \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{1+n}}{d(1+n) \sqrt{\cos^2[c+dx]}} +$$

$$\frac{a \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{2+n}}{d(2+n) \sqrt{\cos^2[c+dx]}}$$

Result (type 8, 29 leaves):

$$\int \cos[c+dx]^4 \sin[c+dx]^n (a + a \sin[c+dx]) dx$$

■ **Problem 492: Unable to integrate problem.**

$$\int \frac{\cos[c+dx]^4 \sin[c+dx]^n}{a + a \sin[c+dx]} dx$$

Optimal (type 5, 134 leaves, 3 steps):

$$\frac{\cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{1+n}}{ad(1+n) \sqrt{\cos^2[c+dx]}} -$$

$$\frac{\cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{2+n}}{ad(2+n) \sqrt{\cos^2[c+dx]}}$$

Result (type 9, 23962 leaves) : Display of huge result suppressed!

■ **Problem 504: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^5 \operatorname{Csc}[c + d x] (a + a \operatorname{Sin}[c + d x]) dx$$

Optimal (type 3, 86 leaves, 4 steps) :

$$-\frac{a \operatorname{Csc}[c + d x]}{d} + \frac{a \operatorname{Csc}[c + d x]^2}{d} + \frac{2 a \operatorname{Csc}[c + d x]^3}{3 d} - \frac{a \operatorname{Csc}[c + d x]^4}{4 d} - \frac{a \operatorname{Csc}[c + d x]^5}{5 d} + \frac{a \operatorname{Log}[\operatorname{Sin}[c + d x]]}{d}$$

Result (type 3, 198 leaves) :

$$-\frac{89 a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]}{240 d} + \frac{31 a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{480 d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{160 d} + \frac{a \operatorname{Csc}[c + d x]^2}{d} - \frac{a \operatorname{Csc}[c + d x]^4}{4 d} + \frac{a \operatorname{Log}[\operatorname{Sin}[c + d x]]}{d} - \frac{89 a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{240 d} + \frac{31 a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{480 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{160 d}$$

■ **Problem 505: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^5 \operatorname{Csc}[c + d x]^2 (a + a \operatorname{Sin}[c + d x]) dx$$

Optimal (type 3, 61 leaves, 6 steps) :

$$-\frac{a \operatorname{Cot}[c + d x]^6}{6 d} - \frac{a \operatorname{Csc}[c + d x]}{d} + \frac{2 a \operatorname{Csc}[c + d x]^3}{3 d} - \frac{a \operatorname{Csc}[c + d x]^5}{5 d}$$

Result (type 3, 173 leaves) :

$$-\frac{89 a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]}{240 d} - \frac{a \operatorname{Cot}[c + d x]^6}{6 d} + \frac{31 a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{480 d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{160 d} - \frac{89 a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{240 d} + \frac{31 a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{480 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{160 d}$$

■ **Problem 506: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^5 \operatorname{Csc}[c + d x]^3 (a + a \operatorname{Sin}[c + d x]) dx$$

Optimal (type 3, 65 leaves, 6 steps) :

$$-\frac{a \operatorname{Cot}[c + d x]^6}{6 d} - \frac{a \operatorname{Csc}[c + d x]^3}{3 d} + \frac{2 a \operatorname{Csc}[c + d x]^5}{5 d} - \frac{a \operatorname{Csc}[c + d x]^7}{7 d}$$

Result (type 3, 233 leaves) :

$$\begin{aligned}
& - \frac{103 a \cot\left[\frac{1}{2}(c+dx)\right]}{3360 d} - \frac{a \cot[c+dx]^6}{6 d} - \frac{103 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{6720 d} + \\
& \frac{9 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{1120 d} - \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{896 d} - \frac{103 a \tan\left[\frac{1}{2}(c+dx)\right]}{3360 d} - \\
& \frac{103 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{6720 d} + \frac{9 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{1120 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{896 d}
\end{aligned}$$

■ **Problem 507: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^5 \csc[c+dx]^4 (a+a \sin[c+dx]) dx$$

Optimal (type 3, 81 leaves, 7 steps):

$$-\frac{a \cot[c+dx]^6}{6 d} - \frac{a \cot[c+dx]^8}{8 d} - \frac{a \csc[c+dx]^3}{3 d} + \frac{2 a \csc[c+dx]^5}{5 d} - \frac{a \csc[c+dx]^7}{7 d}$$

Result (type 3, 265 leaves):

$$\begin{aligned}
& - \frac{103 a \cot\left[\frac{1}{2}(c+dx)\right]}{3360 d} - \frac{103 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{6720 d} + \frac{9 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{1120 d} - \\
& \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{896 d} - \frac{a \csc[c+dx]^4}{4 d} + \frac{a \csc[c+dx]^6}{3 d} - \frac{a \csc[c+dx]^8}{8 d} - \frac{103 a \tan\left[\frac{1}{2}(c+dx)\right]}{3360 d} - \\
& \frac{103 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{6720 d} + \frac{9 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{1120 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{896 d}
\end{aligned}$$

■ **Problem 508: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^5 \csc[c+dx]^5 (a+a \sin[c+dx]) dx$$

Optimal (type 3, 81 leaves, 7 steps):

$$-\frac{a \cot[c+dx]^6}{6 d} - \frac{a \cot[c+dx]^8}{8 d} - \frac{a \csc[c+dx]^5}{5 d} + \frac{2 a \csc[c+dx]^7}{7 d} - \frac{a \csc[c+dx]^9}{9 d}$$

Result (type 3, 325 leaves):

$$\begin{aligned}
& - \frac{649 a \cot\left[\frac{1}{2}(c+dx)\right]}{80640d} - \frac{649 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{161280d} - \frac{31 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{53760d} + \\
& \frac{37 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{32256d} - \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^8}{4608d} - \frac{a \csc[c+dx]^4}{4d} + \\
& \frac{a \csc[c+dx]^6}{3d} - \frac{a \csc[c+dx]^8}{8d} - \frac{649 a \tan\left[\frac{1}{2}(c+dx)\right]}{80640d} - \frac{649 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{161280d} - \\
& \frac{31 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{53760d} + \frac{37 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{32256d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{4608d}
\end{aligned}$$

■ **Problem 509: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^5 \csc[c+dx]^6 (a+a \sin[c+dx]) dx$$

Optimal (type 3, 97 leaves, 4 steps):

$$-\frac{a \csc[c+dx]^5}{5d} - \frac{a \csc[c+dx]^6}{6d} + \frac{2 a \csc[c+dx]^7}{7d} + \frac{a \csc[c+dx]^8}{4d} - \frac{a \csc[c+dx]^9}{9d} - \frac{a \csc[c+dx]^{10}}{10d}$$

Result (type 3, 325 leaves):

$$\begin{aligned}
& - \frac{649 a \cot\left[\frac{1}{2}(c+dx)\right]}{80640d} - \frac{649 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{161280d} - \frac{31 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{53760d} + \\
& \frac{37 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{32256d} - \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^8}{4608d} - \frac{a \csc[c+dx]^6}{6d} + \\
& \frac{a \csc[c+dx]^8}{4d} - \frac{a \csc[c+dx]^{10}}{10d} - \frac{649 a \tan\left[\frac{1}{2}(c+dx)\right]}{80640d} - \frac{649 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{161280d} - \\
& \frac{31 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{53760d} + \frac{37 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{32256d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{4608d}
\end{aligned}$$

■ **Problem 510: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^5 \csc[c+dx]^7 (a+a \sin[c+dx]) dx$$

Optimal (type 3, 97 leaves, 4 steps):

$$-\frac{a \csc[c+dx]^6}{6d} - \frac{a \csc[c+dx]^7}{7d} + \frac{a \csc[c+dx]^8}{4d} + \frac{2 a \csc[c+dx]^9}{9d} - \frac{a \csc[c+dx]^{10}}{10d} - \frac{a \csc[c+dx]^{11}}{11d}$$

Result (type 3, 385 leaves):

$$\begin{aligned}
& - \frac{1109 a \cot\left[\frac{1}{2}(c+dx)\right]}{354816 d} - \frac{1109 a \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{709632 d} - \frac{13 a \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{29568 d} + \\
& \frac{173 a \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6}{1419264 d} + \frac{17 a \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^8}{101376 d} - \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^{10}}{22528 d} - \frac{a \operatorname{Csc}[c+dx]^6}{6 d} + \\
& \frac{a \operatorname{Csc}[c+dx]^8}{4 d} - \frac{a \operatorname{Csc}[c+dx]^{10}}{10 d} - \frac{1109 a \tan\left[\frac{1}{2}(c+dx)\right]}{354816 d} - \frac{1109 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{709632 d} - \frac{13 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{29568 d} + \\
& \frac{173 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{1419264 d} + \frac{17 a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{101376 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^{10} \tan\left[\frac{1}{2}(c+dx)\right]}{22528 d}
\end{aligned}$$

■ **Problem 520: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^5 \operatorname{Csc}[c+dx]^2 (a+a \sin[c+dx])^2 dx$$

Optimal (type 3, 119 leaves, 4 steps):

$$-\frac{2 a^2 \operatorname{Csc}[c+dx]}{d} + \frac{a^2 \operatorname{Csc}[c+dx]^2}{2 d} + \frac{4 a^2 \operatorname{Csc}[c+dx]^3}{3 d} + \frac{a^2 \operatorname{Csc}[c+dx]^4}{4 d} - \frac{2 a^2 \operatorname{Csc}[c+dx]^5}{5 d} - \frac{a^2 \operatorname{Csc}[c+dx]^6}{6 d} + \frac{a^2 \operatorname{Log}[\sin[c+dx]]}{d}$$

Result (type 3, 280 leaves):

$$\begin{aligned}
& a^2 \left(-\frac{89 \cot\left[\frac{1}{2}(c+dx)\right]}{120 d} + \frac{9 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{64 d} + \frac{31 \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{240 d} + \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{128 d} - \right. \\
& \frac{\cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{80 d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6}{384 d} + \frac{\operatorname{Log}[\sin[c+dx]]}{d} + \frac{9 \sec\left[\frac{1}{2}(c+dx)\right]^2}{64 d} + \frac{\sec\left[\frac{1}{2}(c+dx)\right]^4}{128 d} - \\
& \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^6}{384 d} - \frac{89 \tan\left[\frac{1}{2}(c+dx)\right]}{120 d} + \frac{31 \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{240 d} - \frac{\sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{80 d} \right)
\end{aligned}$$

■ **Problem 529: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^5 \operatorname{Csc}[c+dx]^2 (a+a \sin[c+dx])^3 dx$$

Optimal (type 3, 133 leaves, 4 steps):

$$-\frac{a^3 \operatorname{Csc}[c+dx]}{d} + \frac{5 a^3 \operatorname{Csc}[c+dx]^2}{2 d} + \frac{5 a^3 \operatorname{Csc}[c+dx]^3}{3 d} - \frac{a^3 \operatorname{Csc}[c+dx]^4}{4 d} - \frac{3 a^3 \operatorname{Csc}[c+dx]^5}{5 d} - \frac{a^3 \operatorname{Csc}[c+dx]^6}{6 d} + \frac{3 a^3 \operatorname{Log}[\sin[c+dx]]}{d} + \frac{a^3 \sin[c+dx]}{d}$$

Result (type 3, 291 leaves):

$$a^3 \left(-\frac{47 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{240d} + \frac{37 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{64d} + \frac{73 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{480d} - \frac{3 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{128d} - \frac{3 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{160d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6}{384d} + \frac{3 \operatorname{Log}[\operatorname{Sin}[c+dx]]}{d} + \frac{37 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{64d} - \frac{3 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}{128d} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6}{384d} + \frac{\operatorname{Sin}[c+dx]}{d} - \frac{47 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{240d} + \frac{73 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{480d} - \frac{3 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{160d} \right)$$

■ **Problem 530: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx] \operatorname{Cot}[c+dx]^4 (a+a \operatorname{Sin}[c+dx])^4 dx$$

Optimal (type 3, 145 leaves, 4 steps):

$$\frac{4a^4 \operatorname{Csc}[c+dx]}{d} - \frac{2a^4 \operatorname{Csc}[c+dx]^2}{d} - \frac{a^4 \operatorname{Csc}[c+dx]^3}{3d} - \frac{4a^4 \operatorname{Log}[\operatorname{Sin}[c+dx]]}{d} - \frac{10a^4 \operatorname{Sin}[c+dx]}{d} - \frac{2a^4 \operatorname{Sin}[c+dx]^2}{d} + \frac{4a^4 \operatorname{Sin}[c+dx]^3}{3d} + \frac{a^4 \operatorname{Sin}[c+dx]^4}{d} + \frac{a^4 \operatorname{Sin}[c+dx]^5}{5d}$$

Result (type 3, 636 leaves):

$$\frac{\operatorname{Cos}[2(c+dx)](a+a \operatorname{Sin}[c+dx])^4}{2d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} + \frac{\operatorname{Cos}[4(c+dx)](a+a \operatorname{Sin}[c+dx])^4}{8d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} - \frac{25 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right](a+a \operatorname{Sin}[c+dx])^4}{12d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2(a+a \operatorname{Sin}[c+dx])^4}{2d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2(a+a \operatorname{Sin}[c+dx])^4}{24d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} - \frac{4 \operatorname{Log}[\operatorname{Sin}[c+dx]](a+a \operatorname{Sin}[c+dx])^4}{d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2(a+a \operatorname{Sin}[c+dx])^4}{2d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} - \frac{71 \operatorname{Sin}[c+dx](a+a \operatorname{Sin}[c+dx])^4}{8d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} - \frac{19(a+a \operatorname{Sin}[c+dx])^4 \operatorname{Sin}[3(c+dx)]}{48d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} + \frac{(a+a \operatorname{Sin}[c+dx])^4 \operatorname{Sin}[5(c+dx)]}{80d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} - \frac{25(a+a \operatorname{Sin}[c+dx])^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{12d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2(a+a \operatorname{Sin}[c+dx])^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24d(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])^8}$$

■ **Problem 561: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^5}{(a+a \operatorname{Sin}[c+dx])^3} dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$\frac{4 \operatorname{Csc}[c+dx]}{a^3 d} - \frac{2 \operatorname{Csc}[c+dx]^2}{a^3 d} + \frac{\operatorname{Csc}[c+dx]^3}{a^3 d} - \frac{\operatorname{Csc}[c+dx]^4}{4a^3 d} + \frac{4 \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^3 d} - \frac{4 \operatorname{Log}[1 + \operatorname{Sin}[c+dx]]}{a^3 d}$$

Result (type 3, 558 leaves) :

$$\begin{aligned}
 & \frac{9 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{4 d (a+a \operatorname{Sin}[c+dx])^3} - \frac{17 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{32 d (a+a \operatorname{Sin}[c+dx])^3} + \\
 & \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8 d (a+a \operatorname{Sin}[c+dx])^3} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{64 d (a+a \operatorname{Sin}[c+dx])^3} - \\
 & \frac{8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{d (a+a \operatorname{Sin}[c+dx])^3} + \frac{4 \operatorname{Log}[\operatorname{Sin}[c+dx]] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{d (a+a \operatorname{Sin}[c+dx])^3} - \\
 & \frac{17 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{32 d (a+a \operatorname{Sin}[c+dx])^3} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{64 d (a+a \operatorname{Sin}[c+dx])^3} + \\
 & \frac{9 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{4 d (a+a \operatorname{Sin}[c+dx])^3} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{8 d (a+a \operatorname{Sin}[c+dx])^3}
 \end{aligned}$$

■ **Problem 563: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^5}{(a+a \operatorname{Sin}[c+dx])^4} dx$$

Optimal (type 3, 120 leaves, 3 steps) :

$$\frac{12 \operatorname{Csc}[c+dx]}{a^4 d} - \frac{4 \operatorname{Csc}[c+dx]^2}{a^4 d} + \frac{4 \operatorname{Csc}[c+dx]^3}{3 a^4 d} - \frac{\operatorname{Csc}[c+dx]^4}{4 a^4 d} + \frac{16 \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^4 d} - \frac{16 \operatorname{Log}[1+\operatorname{Sin}[c+dx]]}{a^4 d} + \frac{4}{d (a^4 + a^4 \operatorname{Sin}[c+dx])}$$

Result (type 3, 598 leaves) :

$$\begin{aligned}
& \frac{4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{d(a+a\sin[c+dx])^4} + \frac{19 \cot\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{3d(a+a\sin[c+dx])^4} - \\
& \frac{33 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{32d(a+a\sin[c+dx])^4} + \frac{\cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{6d(a+a\sin[c+dx])^4} - \\
& \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{64d(a+a\sin[c+dx])^4} - \frac{32 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{d(a+a\sin[c+dx])^4} + \\
& \frac{16 \operatorname{Log}[\sin[c+dx]] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{d(a+a\sin[c+dx])^4} - \frac{33 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{32d(a+a\sin[c+dx])^4} - \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{64d(a+a\sin[c+dx])^4} + \frac{19 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{3d(a+a\sin[c+dx])^4} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{6d(a+a\sin[c+dx])^4}
\end{aligned}$$

■ **Problem 564: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx]^5 \operatorname{Csc}[c+dx]}{(a+a\sin[c+dx])^4} dx$$

Optimal (type 3, 135 leaves, 4 steps):

$$\begin{aligned}
& -\frac{16 \operatorname{Csc}[c+dx]}{a^4 d} + \frac{6 \operatorname{Csc}[c+dx]^2}{a^4 d} - \frac{8 \operatorname{Csc}[c+dx]^3}{3a^4 d} + \frac{\operatorname{Csc}[c+dx]^4}{a^4 d} - \\
& \frac{\operatorname{Csc}[c+dx]^5}{5a^4 d} - \frac{20 \operatorname{Log}[\sin[c+dx]]}{a^4 d} + \frac{20 \operatorname{Log}[1+\sin[c+dx]]}{a^4 d} - \frac{4}{d(a^4+a^4\sin[c+dx])}
\end{aligned}$$

Result (type 3, 726 leaves):

$$\begin{aligned}
& - \frac{4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{d(a+a\sin[c+dx])^4} - \frac{2089 \cot\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{240 d(a+a\sin[c+dx])^4} + \\
& \frac{13 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{8 d(a+a\sin[c+dx])^4} - \frac{169 \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{480 d(a+a\sin[c+dx])^4} + \\
& \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{16 d(a+a\sin[c+dx])^4} - \frac{\cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{160 d(a+a\sin[c+dx])^4} + \\
& \frac{40 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{d(a+a\sin[c+dx])^4} - \frac{20 \operatorname{Log}[\sin[c+dx]] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{d(a+a\sin[c+dx])^4} + \\
& \frac{13 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{8 d(a+a\sin[c+dx])^4} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8}{16 d(a+a\sin[c+dx])^4} - \\
& \frac{2089 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{240 d(a+a\sin[c+dx])^4} - \frac{169 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{480 d(a+a\sin[c+dx])^4} - \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{160 d(a+a\sin[c+dx])^4}
\end{aligned}$$

- **Problem 565: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^5 \sin[c+dx]^n (a+a\sin[c+dx])^3 dx$$

Optimal (type 3, 181 leaves, 3 steps):

$$\begin{aligned}
& \frac{a^3 \sin[c+dx]^{1+n}}{d(1+n)} + \frac{3 a^3 \sin[c+dx]^{2+n}}{d(2+n)} + \frac{a^3 \sin[c+dx]^{3+n}}{d(3+n)} - \\
& \frac{5 a^3 \sin[c+dx]^{4+n}}{d(4+n)} - \frac{5 a^3 \sin[c+dx]^{5+n}}{d(5+n)} + \frac{a^3 \sin[c+dx]^{6+n}}{d(6+n)} + \frac{3 a^3 \sin[c+dx]^{7+n}}{d(7+n)} + \frac{a^3 \sin[c+dx]^{8+n}}{d(8+n)}
\end{aligned}$$

Result (type 3, 843 leaves):

$$\frac{1}{d \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^6}$$

$$\sin[c + dx]^n (a + a \sin[c + dx])^3 \left(\frac{18064 + 5508n + 596n^2 + 27n^3}{128(2+n)(4+n)(6+n)(8+n)} + \frac{(5775 + 3015n + 329n^2 + 17n^3) \left(-\frac{1}{128} i \cos[c + dx] + \frac{1}{128} \sin[c + dx] \right)}{(1+n)(3+n)(5+n)(7+n)} + \right.$$

$$\frac{(5775 + 3015n + 329n^2 + 17n^3) \left(\frac{1}{128} i \cos[c + dx] + \frac{1}{128} \sin[c + dx] \right)}{(1+n)(3+n)(5+n)(7+n)} +$$

$$\frac{(-3168 - 368n + 38n^2 + 3n^3) \left(\frac{1}{64} \cos[2(c + dx)] - \frac{1}{64} i \sin[2(c + dx)] \right)}{(2+n)(4+n)(6+n)(8+n)} +$$

$$\frac{(-3168 - 368n + 38n^2 + 3n^3) \left(\frac{1}{64} \cos[2(c + dx)] + \frac{1}{64} i \sin[2(c + dx)] \right)}{(2+n)(4+n)(6+n)(8+n)} +$$

$$\frac{(595 + 304n + 21n^2) \left(-\frac{1}{128} i \cos[3(c + dx)] + \frac{1}{128} \sin[3(c + dx)] \right)}{(3+n)(5+n)(7+n)} + \frac{(595 + 304n + 21n^2) \left(\frac{1}{128} i \cos[3(c + dx)] + \frac{1}{128} \sin[3(c + dx)] \right)}{(3+n)(5+n)(7+n)} +$$

$$\frac{(-600 - 138n - 7n^2) \left(\frac{1}{64} \cos[4(c + dx)] - \frac{1}{64} i \sin[4(c + dx)] \right)}{(4+n)(6+n)(8+n)} + \frac{(-600 - 138n - 7n^2) \left(\frac{1}{64} \cos[4(c + dx)] + \frac{1}{64} i \sin[4(c + dx)] \right)}{(4+n)(6+n)(8+n)} +$$

$$\frac{(-35 + n) \left(-\frac{1}{128} i \cos[5(c + dx)] + \frac{1}{128} \sin[5(c + dx)] \right)}{(5+n)(7+n)} + \frac{(-35 + n) \left(\frac{1}{128} i \cos[5(c + dx)] + \frac{1}{128} \sin[5(c + dx)] \right)}{(5+n)(7+n)} +$$

$$\frac{(-20 - 3n) \left(\frac{1}{64} \cos[6(c + dx)] - \frac{1}{64} i \sin[6(c + dx)] \right)}{(6+n)(8+n)} + \frac{(-20 - 3n) \left(\frac{1}{64} \cos[6(c + dx)] + \frac{1}{64} i \sin[6(c + dx)] \right)}{(6+n)(8+n)} +$$

$$\frac{-\frac{3}{128} i \cos[7(c + dx)] - \frac{3}{128} \sin[7(c + dx)]}{7+n} + \frac{\frac{3}{128} i \cos[7(c + dx)] - \frac{3}{128} \sin[7(c + dx)]}{7+n} +$$

$$\left. \frac{\frac{1}{256} \cos[8(c + dx)] - \frac{1}{256} i \sin[8(c + dx)]}{8+n} + \frac{\frac{1}{256} \cos[8(c + dx)] + \frac{1}{256} i \sin[8(c + dx)]}{8+n} \right)$$

■ **Problem 566: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^5 \sin[c + dx]^n (a + a \sin[c + dx])^2 dx$$

Optimal (type 3, 160 leaves, 3 steps):

$$\frac{a^2 \sin[c + dx]^{1+n}}{d(1+n)} + \frac{2a^2 \sin[c + dx]^{2+n}}{d(2+n)} - \frac{a^2 \sin[c + dx]^{3+n}}{d(3+n)} - \frac{4a^2 \sin[c + dx]^{4+n}}{d(4+n)} - \frac{a^2 \sin[c + dx]^{5+n}}{d(5+n)} + \frac{2a^2 \sin[c + dx]^{6+n}}{d(6+n)} + \frac{a^2 \sin[c + dx]^{7+n}}{d(7+n)}$$

Result (type 3, 705 leaves):

$$\frac{1}{d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4}$$

$$\sin[c+dx]^n (a + a \sin[c+dx])^2 \left(\frac{88 + 14n + n^2}{8(2+n)(4+n)(6+n)} + \frac{(4725 + 1853n + 211n^2 + 11n^3) \left(-\frac{1}{128}i \cos[c+dx] + \frac{1}{128} \sin[c+dx]\right)}{(1+n)(3+n)(5+n)(7+n)} + \right.$$

$$\frac{(4725 + 1853n + 211n^2 + 11n^3) \left(\frac{1}{128}i \cos[c+dx] + \frac{1}{128} \sin[c+dx]\right)}{(1+n)(3+n)(5+n)(7+n)} + \frac{(-120 + 6n + n^2) \left(\frac{1}{32} \cos[2(c+dx)] - \frac{1}{32}i \sin[2(c+dx)]\right)}{(2+n)(4+n)(6+n)} +$$

$$\frac{(-120 + 6n + n^2) \left(\frac{1}{32} \cos[2(c+dx)] + \frac{1}{32}i \sin[2(c+dx)]\right)}{(2+n)(4+n)(6+n)} + \frac{(665 + 224n + 15n^2) \left(-\frac{1}{128}i \cos[3(c+dx)] + \frac{1}{128} \sin[3(c+dx)]\right)}{(3+n)(5+n)(7+n)} +$$

$$\frac{(665 + 224n + 15n^2) \left(\frac{1}{128}i \cos[3(c+dx)] + \frac{1}{128} \sin[3(c+dx)]\right)}{(3+n)(5+n)(7+n)} + \frac{(-12 - n) \left(\frac{1}{16} \cos[4(c+dx)] - \frac{1}{16}i \sin[4(c+dx)]\right)}{(4+n)(6+n)} +$$

$$\frac{(-12 - n) \left(\frac{1}{16} \cos[4(c+dx)] + \frac{1}{16}i \sin[4(c+dx)]\right)}{(4+n)(6+n)} + \frac{(7 + 3n) \left(-\frac{1}{128}i \cos[5(c+dx)] + \frac{1}{128} \sin[5(c+dx)]\right)}{(5+n)(7+n)} +$$

$$\frac{(7 + 3n) \left(\frac{1}{128}i \cos[5(c+dx)] + \frac{1}{128} \sin[5(c+dx)]\right)}{(5+n)(7+n)} + \frac{-\frac{1}{32} \cos[6(c+dx)] - \frac{1}{32}i \sin[6(c+dx)]}{6+n} +$$

$$\frac{-\frac{1}{32} \cos[6(c+dx)] + \frac{1}{32}i \sin[6(c+dx)]}{6+n} + \frac{-\frac{1}{128}i \cos[7(c+dx)] - \frac{1}{128} \sin[7(c+dx)]}{7+n} + \frac{\frac{1}{128}i \cos[7(c+dx)] - \frac{1}{128} \sin[7(c+dx)]}{7+n} \Bigg)$$

■ **Problem 567: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^5 \sin[c+dx]^n (a + a \sin[c+dx]) dx$$

Optimal (type 3, 123 leaves, 3 steps):

$$\frac{a \sin[c+dx]^{1+n}}{d(1+n)} + \frac{a \sin[c+dx]^{2+n}}{d(2+n)} - \frac{2a \sin[c+dx]^{3+n}}{d(3+n)} - \frac{2a \sin[c+dx]^{4+n}}{d(4+n)} + \frac{a \sin[c+dx]^{5+n}}{d(5+n)} + \frac{a \sin[c+dx]^{6+n}}{d(6+n)}$$

Result (type 3, 345 leaves):

$$\frac{1}{16d(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)}$$

$$a \sin[c+dx]^{1+n} \left(8544 + 10520n + 4888n^2 + 1114n^3 + 128n^4 + 6n^5 + 8(336 + 692n + 484n^2 + 147n^3 + 20n^4 + n^5) \cos[2(c+dx)] + \right.$$

$$2(144 + 324n + 260n^2 + 95n^3 + 16n^4 + n^5) \cos[4(c+dx)] + 2640 \sin[c+dx] + 4468n \sin[c+dx] + 2258n^2 \sin[c+dx] +$$

$$474n^3 \sin[c+dx] + 46n^4 \sin[c+dx] + 2n^5 \sin[c+dx] + 840 \sin[3(c+dx)] + 1798n \sin[3(c+dx)] +$$

$$1331n^2 \sin[3(c+dx)] + 431n^3 \sin[3(c+dx)] + 61n^4 \sin[3(c+dx)] + 3n^5 \sin[3(c+dx)] + 120 \sin[5(c+dx)] +$$

$$274n \sin[5(c+dx)] + 225n^2 \sin[5(c+dx)] + 85n^3 \sin[5(c+dx)] + 15n^4 \sin[5(c+dx)] + n^5 \sin[5(c+dx)] \Bigg)$$

■ **Problem 570: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^5 \sin [c+d x]^n}{(a+a \sin [c+d x])^3} d x$$

Optimal (type 5, 85 leaves, 4 steps) :

$$-\frac{3 \sin [c+d x]^{1+n}}{a^3 d (1+n)} + \frac{4 \operatorname{Hypergeometric2F1}[1, 1+n, 2+n, -\sin [c+d x]] \sin [c+d x]^{1+n}}{a^3 d (1+n)} + \frac{\sin [c+d x]^{2+n}}{a^3 d (2+n)}$$

Result (type 8, 31 leaves) :

$$\int \frac{\cos [c+d x]^5 \sin [c+d x]^n}{(a+a \sin [c+d x])^3} d x$$

■ **Problem 571: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^5 \sin [c+d x]^n}{(a+a \sin [c+d x])^4} d x$$

Optimal (type 5, 88 leaves, 4 steps) :

$$\frac{\sin [c+d x]^{1+n}}{a^4 d (1+n)} - \frac{4 \operatorname{Hypergeometric2F1}[1, 1+n, 2+n, -\sin [c+d x]] \sin [c+d x]^{1+n}}{a^4 d} + \frac{4 \sin [c+d x]^{1+n}}{d (a^4 + a^4 \sin [c+d x])}$$

Result (type 8, 31 leaves) :

$$\int \frac{\cos [c+d x]^5 \sin [c+d x]^n}{(a+a \sin [c+d x])^4} d x$$

■ **Problem 585: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^6 \csc [c+d x]^4 (a+a \sin [c+d x]) d x$$

Optimal (type 3, 138 leaves, 9 steps) :

$$\frac{5 a \operatorname{ArcTanh}[\cos [c+d x]]}{128 d} - \frac{a \cot [c+d x]^7}{7 d} - \frac{a \cot [c+d x]^9}{9 d} + \frac{5 a \cot [c+d x] \csc [c+d x]}{128 d} - \frac{5 a \cot [c+d x] \csc [c+d x]^3}{64 d} + \frac{5 a \cot [c+d x]^3 \csc [c+d x]^3}{48 d} - \frac{a \cot [c+d x]^5 \csc [c+d x]^3}{8 d}$$

Result (type 3, 301 leaves) :

$$\frac{2 a \cot [c+d x]}{63 d} + \frac{5 a \csc \left[\frac{1}{2}(c+d x)\right]^2}{512 d} - \frac{15 a \csc \left[\frac{1}{2}(c+d x)\right]^4}{1024 d} + \frac{7 a \csc \left[\frac{1}{2}(c+d x)\right]^6}{1536 d} - \frac{a \csc \left[\frac{1}{2}(c+d x)\right]^8}{2048 d} + \frac{a \cot [c+d x] \csc [c+d x]^2}{63 d} - \frac{5 a \cot [c+d x] \csc [c+d x]^4}{21 d} + \frac{19 a \cot [c+d x] \csc [c+d x]^6}{63 d} - \frac{a \cot [c+d x] \csc [c+d x]^8}{9 d} + \frac{5 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right]}{128 d} - \frac{5 a \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right]}{128 d} - \frac{5 a \sec \left[\frac{1}{2}(c+d x)\right]^2}{512 d} + \frac{15 a \sec \left[\frac{1}{2}(c+d x)\right]^4}{1024 d} - \frac{7 a \sec \left[\frac{1}{2}(c+d x)\right]^6}{1536 d} + \frac{a \sec \left[\frac{1}{2}(c+d x)\right]^8}{2048 d}$$

■ **Problem 586: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^6 \csc [c+d x]^5 (a+a \sin [c+d x]) dx$$

Optimal (type 3, 160 leaves, 10 steps):

$$\frac{3 a \operatorname{ArcTanh}[\cos [c+d x]]}{256 d} - \frac{a \cot [c+d x]^7}{7 d} - \frac{a \cot [c+d x]^9}{9 d} + \frac{3 a \cot [c+d x] \csc [c+d x]}{256 d} + \frac{a \cot [c+d x] \csc [c+d x]^3}{128 d} - \frac{a \cot [c+d x] \csc [c+d x]^5}{32 d} + \frac{a \cot [c+d x]^3 \csc [c+d x]^5}{16 d} - \frac{a \cot [c+d x]^5 \csc [c+d x]^5}{10 d}$$

Result (type 3, 341 leaves):

$$\frac{2 a \cot [c+d x]}{63 d} + \frac{3 a \csc \left[\frac{1}{2}(c+d x)\right]^2}{1024 d} - \frac{a \csc \left[\frac{1}{2}(c+d x)\right]^4}{1024 d} - \frac{3 a \csc \left[\frac{1}{2}(c+d x)\right]^6}{2048 d} + \frac{3 a \csc \left[\frac{1}{2}(c+d x)\right]^8}{4096 d} - \frac{a \csc \left[\frac{1}{2}(c+d x)\right]^{10}}{10240 d} + \frac{a \cot [c+d x] \csc [c+d x]^2}{63 d} - \frac{5 a \cot [c+d x] \csc [c+d x]^4}{21 d} + \frac{19 a \cot [c+d x] \csc [c+d x]^6}{63 d} - \frac{a \cot [c+d x] \csc [c+d x]^8}{9 d} + \frac{3 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right]}{256 d} - \frac{3 a \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right]}{256 d} - \frac{3 a \sec \left[\frac{1}{2}(c+d x)\right]^2}{1024 d} + \frac{a \sec \left[\frac{1}{2}(c+d x)\right]^4}{1024 d} + \frac{3 a \sec \left[\frac{1}{2}(c+d x)\right]^6}{2048 d} - \frac{3 a \sec \left[\frac{1}{2}(c+d x)\right]^8}{4096 d} + \frac{a \sec \left[\frac{1}{2}(c+d x)\right]^{10}}{10240 d}$$

■ **Problem 587: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^6 \csc [c+d x]^6 (a+a \sin [c+d x]) dx$$

Optimal (type 3, 176 leaves, 10 steps):

$$\frac{3 a \operatorname{ArcTanh}[\cos [c+d x]]}{256 d} - \frac{a \cot [c+d x]^7}{7 d} - \frac{2 a \cot [c+d x]^9}{9 d} - \frac{a \cot [c+d x]^{11}}{11 d} + \frac{3 a \cot [c+d x] \csc [c+d x]}{256 d} + \frac{a \cot [c+d x] \csc [c+d x]^3}{128 d} - \frac{a \cot [c+d x] \csc [c+d x]^5}{32 d} + \frac{a \cot [c+d x]^3 \csc [c+d x]^5}{16 d} - \frac{a \cot [c+d x]^5 \csc [c+d x]^5}{10 d}$$

Result (type 3, 363 leaves):

$$\begin{aligned} & \frac{8 a \cot [c+d x]}{693 d} + \frac{3 a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{1024 d} - \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4}{1024 d} - \frac{3 a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^6}{2048 d} + \frac{3 a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^8}{4096 d} - \\ & \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^{10}}{10240 d} + \frac{4 a \cot [c+d x] \operatorname{Csc}[c+d x]^2}{693 d} + \frac{a \cot [c+d x] \operatorname{Csc}[c+d x]^4}{231 d} - \frac{113 a \cot [c+d x] \operatorname{Csc}[c+d x]^6}{693 d} + \\ & \frac{23 a \cot [c+d x] \operatorname{Csc}[c+d x]^8}{99 d} - \frac{a \cot [c+d x] \operatorname{Csc}[c+d x]^{10}}{11 d} + \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right]}{256 d} - \frac{3 a \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{256 d} - \\ & \frac{3 a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{1024 d} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4}{1024 d} + \frac{3 a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^6}{2048 d} - \frac{3 a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^8}{4096 d} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^{10}}{10240 d} \end{aligned}$$

■ **Problem 594: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^3 \cot [c+d x]^3 (a+a \sin [c+d x])^2 dx$$

Optimal (type 3, 140 leaves, 16 steps):

$$\begin{aligned} & -\frac{15 a^2 x}{4} + \frac{3 a^2 \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{2 d} - \frac{a^2 \operatorname{Cos}[c+d x]}{d} + \frac{a^2 \operatorname{Cos}[c+d x]^5}{5 d} - \\ & \frac{2 a^2 \cot [c+d x]}{d} - \frac{a^2 \cot [c+d x] \operatorname{Csc}[c+d x]}{2 d} - \frac{9 a^2 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{4 d} + \frac{a^2 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^3}{2 d} \end{aligned}$$

Result (type 3, 607 leaves):

$$\begin{aligned} & -\frac{15 (c+d x) (a+a \sin [c+d x])^2}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{7 \operatorname{Cos}[c+d x] (a+a \sin [c+d x])^2}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{\operatorname{Cos}[3(c+d x)] (a+a \sin [c+d x])^2}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \\ & \frac{\operatorname{Cos}[5(c+d x)] (a+a \sin [c+d x])^2}{80 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{\cot \left[\frac{1}{2}(c+d x)\right] (a+a \sin [c+d x])^2}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 (a+a \sin [c+d x])^2}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \\ & \frac{3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] (a+a \sin [c+d x])^2}{2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+a \sin [c+d x])^2}{2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 (a+a \sin [c+d x])^2}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \\ & \frac{(a+a \sin [c+d x])^2 \operatorname{Sin}[2(c+d x)]}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{(a+a \sin [c+d x])^2 \operatorname{Sin}[4(c+d x)]}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{(a+a \sin [c+d x])^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} \end{aligned}$$

■ **Problem 600: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^6 \operatorname{Csc}[c+d x]^3 (a+a \sin [c+d x])^2 dx$$

Optimal (type 3, 182 leaves, 13 steps):

$$\frac{45 a^2 \operatorname{ArcTanh}[\cos [c+d x]]}{128 d}-\frac{2 a^2 \cot [c+d x]^7}{7 d}-\frac{35 a^2 \cot [c+d x] \operatorname{Csc}[c+d x]}{128 d}+\frac{5 a^2 \cot [c+d x]^3 \operatorname{Csc}[c+d x]}{24 d}-\frac{a^2 \cot [c+d x]^5 \operatorname{Csc}[c+d x]}{6 d}-\frac{5 a^2 \cot [c+d x] \operatorname{Csc}[c+d x]^3}{64 d}+\frac{5 a^2 \cot [c+d x]^3 \operatorname{Csc}[c+d x]^3}{48 d}-\frac{a^2 \cot [c+d x]^5 \operatorname{Csc}[c+d x]^3}{8 d}$$

Result (type 3, 401 leaves):

$$a^2 \left(\frac{\cot \left[\frac{1}{2} (c+d x) \right]}{7 d}-\frac{83 \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^2}{512 d}-\frac{19 \cot \left[\frac{1}{2} (c+d x) \right] \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^2}{224 d}+\frac{17 \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^4}{1024 d}+\frac{5 \cot \left[\frac{1}{2} (c+d x) \right] \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^4}{224 d}+\frac{\operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^6}{512 d}-\frac{\cot \left[\frac{1}{2} (c+d x) \right] \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^6}{448 d}-\frac{\operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^8}{2048 d}+\frac{45 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+d x) \right] \right]}{128 d}-\frac{45 \operatorname{Log} \left[\sin \left[\frac{1}{2} (c+d x) \right] \right]}{128 d}+\frac{83 \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^2}{512 d}-\frac{17 \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^4}{1024 d}-\frac{\operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^6}{512 d}+\frac{\operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^8}{2048 d}-\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{7 d}+\frac{19 \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right]}{224 d}-\frac{5 \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^4 \tan \left[\frac{1}{2} (c+d x) \right]}{224 d}+\frac{\operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^6 \tan \left[\frac{1}{2} (c+d x) \right]}{448 d} \right)$$

■ **Problem 601: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^6 \operatorname{Csc}[c+d x]^4 (a+a \sin [c+d x])^2 dx$$

Optimal (type 3, 152 leaves, 12 steps):

$$\frac{5 a^2 \operatorname{ArcTanh}[\cos [c+d x]]}{64 d}-\frac{2 a^2 \cot [c+d x]^7}{7 d}-\frac{a^2 \cot [c+d x]^9}{9 d}+\frac{5 a^2 \cot [c+d x] \operatorname{Csc}[c+d x]}{64 d}-\frac{5 a^2 \cot [c+d x] \operatorname{Csc}[c+d x]^3}{32 d}+\frac{5 a^2 \cot [c+d x]^3 \operatorname{Csc}[c+d x]^3}{24 d}-\frac{a^2 \cot [c+d x]^5 \operatorname{Csc}[c+d x]^3}{4 d}$$

Result (type 3, 313 leaves):

$$\begin{aligned}
& - \frac{1}{1\,032\,192\,d} a^2 \operatorname{Csc}[c + dx]^9 \left(72\,576 \operatorname{Cos}[c + dx] + 37\,632 \operatorname{Cos}[3(c + dx)] + 6912 \operatorname{Cos}[5(c + dx)] - 1728 \operatorname{Cos}[7(c + dx)] - \right. \\
& \quad 704 \operatorname{Cos}[9(c + dx)] - 39\,690 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[c + dx] + 39\,690 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[c + dx] + \\
& \quad 36\,540 \operatorname{Sin}[2(c + dx)] + 26\,460 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[3(c + dx)] - 26\,460 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[3(c + dx)] + \\
& \quad 20\,916 \operatorname{Sin}[4(c + dx)] - 11\,340 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[5(c + dx)] + 11\,340 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[5(c + dx)] + \\
& \quad 16\,044 \operatorname{Sin}[6(c + dx)] + 2835 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[7(c + dx)] - 2835 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[7(c + dx)] + \\
& \quad \left. 630 \operatorname{Sin}[8(c + dx)] - 315 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[9(c + dx)] + 315 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[9(c + dx)] \right)
\end{aligned}$$

■ **Problem 611: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx]^3 \operatorname{Cot}[c + dx]^3 (a + a \operatorname{Sin}[c + dx])^3 dx$$

Optimal (type 3, 181 leaves, 17 steps):

$$\begin{aligned}
& - \frac{85 a^3 x}{16} - \frac{a^3 \operatorname{ArcTanh}[\operatorname{Cos}[c + dx]]}{2d} + \frac{a^3 \operatorname{Cos}[c + dx]}{d} + \frac{2 a^3 \operatorname{Cos}[c + dx]^3}{3d} + \frac{3 a^3 \operatorname{Cos}[c + dx]^5}{5d} - \frac{3 a^3 \operatorname{Cot}[c + dx]}{d} - \\
& \frac{a^3 \operatorname{Cot}[c + dx] \operatorname{Csc}[c + dx]}{2d} - \frac{43 a^3 \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{16d} + \frac{5 a^3 \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^3}{24d} + \frac{a^3 \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^5}{6d}
\end{aligned}$$

Result (type 3, 664 leaves):

$$\begin{aligned}
& - \frac{85 (c + dx) (a + a \operatorname{Sin}[c + dx])^3}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} + \frac{15 \operatorname{Cos}[c + dx] (a + a \operatorname{Sin}[c + dx])^3}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} + \\
& \frac{17 \operatorname{Cos}[3(c + dx)] (a + a \operatorname{Sin}[c + dx])^3}{48 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} + \frac{3 \operatorname{Cos}[5(c + dx)] (a + a \operatorname{Sin}[c + dx])^3}{80 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} - \\
& \frac{3 \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right] (a + a \operatorname{Sin}[c + dx])^3}{2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2 (a + a \operatorname{Sin}[c + dx])^3}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} - \frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right] (a + a \operatorname{Sin}[c + dx])^3}{2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} + \\
& \frac{\operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] (a + a \operatorname{Sin}[c + dx])^3}{2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a + a \operatorname{Sin}[c + dx])^3}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} - \frac{81 (a + a \operatorname{Sin}[c + dx])^3 \operatorname{Sin}[2(c + dx)]}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} - \\
& \frac{3 (a + a \operatorname{Sin}[c + dx])^3 \operatorname{Sin}[4(c + dx)]}{64 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} + \frac{(a + a \operatorname{Sin}[c + dx])^3 \operatorname{Sin}[6(c + dx)]}{192 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} + \frac{3 (a + a \operatorname{Sin}[c + dx])^3 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6}
\end{aligned}$$

■ **Problem 618: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^6 \operatorname{Csc}[c + d x]^4 (a + a \operatorname{Sin}[c + d x])^3 dx$$

Optimal (type 3, 200 leaves, 16 steps):

$$\frac{55 a^3 \operatorname{ArcTanh}[\operatorname{Cos}[c + d x]]}{128 d} - \frac{4 a^3 \cot [c + d x]^7}{7 d} - \frac{a^3 \cot [c + d x]^9}{9 d} - \frac{25 a^3 \cot [c + d x] \operatorname{Csc}[c + d x]}{128 d} + \frac{5 a^3 \cot [c + d x]^3 \operatorname{Csc}[c + d x]}{24 d} - \frac{a^3 \cot [c + d x]^5 \operatorname{Csc}[c + d x]}{6 d} - \frac{15 a^3 \cot [c + d x] \operatorname{Csc}[c + d x]^3}{64 d} + \frac{5 a^3 \cot [c + d x]^3 \operatorname{Csc}[c + d x]^3}{16 d} - \frac{3 a^3 \cot [c + d x]^5 \operatorname{Csc}[c + d x]^3}{8 d}$$

Result (type 3, 459 leaves):

$$a^3 \left(\frac{29 \cot \left[\frac{1}{2} (c + d x) \right]}{126 d} - \frac{73 \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^2}{512 d} - \frac{4163 \cot \left[\frac{1}{2} (c + d x) \right] \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^2}{32 256 d} - \frac{13 \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^4}{1024 d} + \frac{319 \cot \left[\frac{1}{2} (c + d x) \right] \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^4}{10 752 d} + \frac{17 \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^6}{1536 d} - \frac{53 \cot \left[\frac{1}{2} (c + d x) \right] \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^6}{32 256 d} - \frac{3 \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^8}{2048 d} - \frac{\cot \left[\frac{1}{2} (c + d x) \right] \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^8}{4608 d} + \frac{55 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] \right]}{128 d} - \frac{55 \operatorname{Log} \left[\operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right]}{128 d} + \frac{73 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{512 d} + \frac{13 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^4}{1024 d} - \frac{17 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^6}{1536 d} + \frac{3 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^8}{2048 d} - \frac{29 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{126 d} + \frac{4163 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{32 256 d} + \frac{319 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^4 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{10 752 d} + \frac{53 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^6 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{32 256 d} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^8 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{4608 d} \right)$$

■ **Problem 624: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^6 \operatorname{Sin}[c + d x]^4}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 159 leaves, 9 steps):

$$\frac{3 x}{128 a} + \frac{\operatorname{Cos}[c + d x]^5}{5 a d} - \frac{2 \operatorname{Cos}[c + d x]^7}{7 a d} + \frac{\operatorname{Cos}[c + d x]^9}{9 a d} + \frac{3 \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{128 a d} + \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sin}[c + d x]}{64 a d} - \frac{\operatorname{Cos}[c + d x]^5 \operatorname{Sin}[c + d x]}{16 a d} - \frac{\operatorname{Cos}[c + d x]^5 \operatorname{Sin}[c + d x]^3}{8 a d}$$

Result (type 3, 429 leaves):

$$\frac{1}{645120 a d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)}$$

$$\left(2520 (5 c + 6 d x) \cos\left[\frac{c}{2}\right] + 7560 \cos\left[\frac{c}{2} + d x\right] + 7560 \cos\left[\frac{3 c}{2} + d x\right] + 1680 \cos\left[\frac{5 c}{2} + 3 d x\right] + 1680 \cos\left[\frac{7 c}{2} + 3 d x\right] - 2520 \cos\left[\frac{7 c}{2} + 4 d x\right] + \right.$$

$$2520 \cos\left[\frac{9 c}{2} + 4 d x\right] - 1008 \cos\left[\frac{9 c}{2} + 5 d x\right] - 1008 \cos\left[\frac{11 c}{2} + 5 d x\right] - 180 \cos\left[\frac{13 c}{2} + 7 d x\right] - 180 \cos\left[\frac{15 c}{2} + 7 d x\right] +$$

$$315 \cos\left[\frac{15 c}{2} + 8 d x\right] - 315 \cos\left[\frac{17 c}{2} + 8 d x\right] + 140 \cos\left[\frac{17 c}{2} + 9 d x\right] + 140 \cos\left[\frac{19 c}{2} + 9 d x\right] + 12600 \sin\left[\frac{c}{2}\right] +$$

$$12600 c \sin\left[\frac{c}{2}\right] + 15120 d x \sin\left[\frac{c}{2}\right] - 7560 \sin\left[\frac{c}{2} + d x\right] + 7560 \sin\left[\frac{3 c}{2} + d x\right] - 1680 \sin\left[\frac{5 c}{2} + 3 d x\right] + 1680 \sin\left[\frac{7 c}{2} + 3 d x\right] -$$

$$2520 \sin\left[\frac{7 c}{2} + 4 d x\right] - 2520 \sin\left[\frac{9 c}{2} + 4 d x\right] + 1008 \sin\left[\frac{9 c}{2} + 5 d x\right] - 1008 \sin\left[\frac{11 c}{2} + 5 d x\right] + 180 \sin\left[\frac{13 c}{2} + 7 d x\right] -$$

$$\left. 180 \sin\left[\frac{15 c}{2} + 7 d x\right] + 315 \sin\left[\frac{15 c}{2} + 8 d x\right] + 315 \sin\left[\frac{17 c}{2} + 8 d x\right] - 140 \sin\left[\frac{17 c}{2} + 9 d x\right] + 140 \sin\left[\frac{19 c}{2} + 9 d x\right] \right)$$

■ **Problem 625: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^6 \sin[c + d x]^3}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 141 leaves, 9 steps):

$$-\frac{3 x}{128 a} - \frac{\cos[c + d x]^5}{5 a d} + \frac{\cos[c + d x]^7}{7 a d} - \frac{3 \cos[c + d x] \sin[c + d x]}{128 a d} -$$

$$\frac{\cos[c + d x]^3 \sin[c + d x]}{64 a d} + \frac{\cos[c + d x]^5 \sin[c + d x]}{16 a d} + \frac{\cos[c + d x]^5 \sin[c + d x]^3}{8 a d}$$

Result (type 3, 375 leaves):

$$\frac{1}{71680 a d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)}$$

$$\left(1680 (c - d x) \cos\left[\frac{c}{2}\right] - 1680 \cos\left[\frac{c}{2} + d x\right] - 1680 \cos\left[\frac{3 c}{2} + d x\right] - 560 \cos\left[\frac{5 c}{2} + 3 d x\right] - 560 \cos\left[\frac{7 c}{2} + 3 d x\right] + 280 \cos\left[\frac{7 c}{2} + 4 d x\right] - \right.$$

$$280 \cos\left[\frac{9 c}{2} + 4 d x\right] + 112 \cos\left[\frac{9 c}{2} + 5 d x\right] + 112 \cos\left[\frac{11 c}{2} + 5 d x\right] + 80 \cos\left[\frac{13 c}{2} + 7 d x\right] + 80 \cos\left[\frac{15 c}{2} + 7 d x\right] -$$

$$35 \cos\left[\frac{15 c}{2} + 8 d x\right] + 35 \cos\left[\frac{17 c}{2} + 8 d x\right] - 3360 \sin\left[\frac{c}{2}\right] + 1680 c \sin\left[\frac{c}{2}\right] - 1680 d x \sin\left[\frac{c}{2}\right] + 1680 \sin\left[\frac{c}{2} + d x\right] -$$

$$1680 \sin\left[\frac{3 c}{2} + d x\right] + 560 \sin\left[\frac{5 c}{2} + 3 d x\right] - 560 \sin\left[\frac{7 c}{2} + 3 d x\right] + 280 \sin\left[\frac{7 c}{2} + 4 d x\right] + 280 \sin\left[\frac{9 c}{2} + 4 d x\right] - 112 \sin\left[\frac{9 c}{2} + 5 d x\right] +$$

$$\left. 112 \sin\left[\frac{11 c}{2} + 5 d x\right] - 80 \sin\left[\frac{13 c}{2} + 7 d x\right] + 80 \sin\left[\frac{15 c}{2} + 7 d x\right] - 35 \sin\left[\frac{15 c}{2} + 8 d x\right] - 35 \sin\left[\frac{17 c}{2} + 8 d x\right] \right)$$

■ **Problem 626: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^6 \sin[c+dx]^2}{a+a\sin[c+dx]} dx$$

Optimal (type 3, 115 leaves, 8 steps):

$$\frac{x}{16a} + \frac{\cos[c+dx]^5}{5ad} - \frac{\cos[c+dx]^7}{7ad} + \frac{\cos[c+dx]\sin[c+dx]}{16ad} + \frac{\cos[c+dx]^3\sin[c+dx]}{24ad} - \frac{\cos[c+dx]^5\sin[c+dx]}{6ad}$$

Result (type 3, 414 leaves):

$$\frac{1}{13440ad \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(840dx \cos\left[\frac{c}{2}\right] + 315 \cos\left[\frac{c}{2} + dx\right] + 315 \cos\left[\frac{3c}{2} + dx\right] + 105 \cos\left[\frac{3c}{2} + 2dx\right] - 105 \cos\left[\frac{5c}{2} + 2dx\right] + 105 \cos\left[\frac{5c}{2} + 3dx\right] + 105 \cos\left[\frac{7c}{2} + 3dx\right] - 105 \cos\left[\frac{7c}{2} + 4dx\right] + 105 \cos\left[\frac{9c}{2} + 4dx\right] - 21 \cos\left[\frac{9c}{2} + 5dx\right] - 21 \cos\left[\frac{11c}{2} + 5dx\right] - 35 \cos\left[\frac{11c}{2} + 6dx\right] + 35 \cos\left[\frac{13c}{2} + 6dx\right] - 15 \cos\left[\frac{13c}{2} + 7dx\right] - 15 \cos\left[\frac{15c}{2} + 7dx\right] + 1050 \sin\left[\frac{c}{2}\right] + 840dx \sin\left[\frac{c}{2}\right] - 315 \sin\left[\frac{c}{2} + dx\right] + 315 \sin\left[\frac{3c}{2} + dx\right] + 105 \sin\left[\frac{3c}{2} + 2dx\right] + 105 \sin\left[\frac{5c}{2} + 2dx\right] - 105 \sin\left[\frac{5c}{2} + 3dx\right] + 105 \sin\left[\frac{7c}{2} + 3dx\right] - 105 \sin\left[\frac{7c}{2} + 4dx\right] - 105 \sin\left[\frac{9c}{2} + 4dx\right] + 21 \sin\left[\frac{9c}{2} + 5dx\right] - 21 \sin\left[\frac{11c}{2} + 5dx\right] - 35 \sin\left[\frac{11c}{2} + 6dx\right] - 35 \sin\left[\frac{13c}{2} + 6dx\right] + 15 \sin\left[\frac{13c}{2} + 7dx\right] - 15 \sin\left[\frac{15c}{2} + 7dx\right] \right)$$

■ **Problem 627: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^6 \sin[c+dx]}{a+a\sin[c+dx]} dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$-\frac{x}{16a} - \frac{\cos[c+dx]^5}{5ad} - \frac{\cos[c+dx]\sin[c+dx]}{16ad} - \frac{\cos[c+dx]^3\sin[c+dx]}{24ad} + \frac{\cos[c+dx]^5\sin[c+dx]}{6ad}$$

Result (type 3, 377 leaves):

$$\begin{aligned}
& - \frac{1}{1920 a d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \\
& \left(-30 (5 c - 4 d x) \cos\left[\frac{c}{2}\right] + 120 \cos\left[\frac{c}{2} + d x\right] + 120 \cos\left[\frac{3 c}{2} + d x\right] + 15 \cos\left[\frac{3 c}{2} + 2 d x\right] - 15 \cos\left[\frac{5 c}{2} + 2 d x\right] + 60 \cos\left[\frac{5 c}{2} + 3 d x\right] + \right. \\
& 60 \cos\left[\frac{7 c}{2} + 3 d x\right] - 15 \cos\left[\frac{7 c}{2} + 4 d x\right] + 15 \cos\left[\frac{9 c}{2} + 4 d x\right] + 12 \cos\left[\frac{9 c}{2} + 5 d x\right] + 12 \cos\left[\frac{11 c}{2} + 5 d x\right] - 5 \cos\left[\frac{11 c}{2} + 6 d x\right] + \\
& 5 \cos\left[\frac{13 c}{2} + 6 d x\right] + 300 \sin\left[\frac{c}{2}\right] - 150 c \sin\left[\frac{c}{2}\right] + 120 d x \sin\left[\frac{c}{2}\right] - 120 \sin\left[\frac{c}{2} + d x\right] + 120 \sin\left[\frac{3 c}{2} + d x\right] + \\
& 15 \sin\left[\frac{3 c}{2} + 2 d x\right] + 15 \sin\left[\frac{5 c}{2} + 2 d x\right] - 60 \sin\left[\frac{5 c}{2} + 3 d x\right] + 60 \sin\left[\frac{7 c}{2} + 3 d x\right] - 15 \sin\left[\frac{7 c}{2} + 4 d x\right] - \\
& \left. 15 \sin\left[\frac{9 c}{2} + 4 d x\right] - 12 \sin\left[\frac{9 c}{2} + 5 d x\right] + 12 \sin\left[\frac{11 c}{2} + 5 d x\right] - 5 \sin\left[\frac{11 c}{2} + 6 d x\right] - 5 \sin\left[\frac{13 c}{2} + 6 d x\right] \right)
\end{aligned}$$

■ **Problem 632: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x] \cot[c + d x]^5}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 102 leaves, 7 steps):

$$-\frac{x}{a} - \frac{3 \operatorname{ArcTanh}[\cos[c + d x]]}{8 a d} - \frac{\cot[c + d x]}{a d} + \frac{\cot[c + d x]^3}{3 a d} + \frac{3 \cot[c + d x] \csc[c + d x]}{8 a d} - \frac{\cot[c + d x]^3 \csc[c + d x]}{4 a d}$$

Result (type 3, 232 leaves):

$$\begin{aligned}
& - \frac{1}{192 a d (1 + \sin[c + d x])} \operatorname{Csc}[c + d x]^4 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2 \\
& \left(72 c + 72 d x + 18 \cos[c + d x] + 30 \cos[3(c + d x)] + 24 c \cos[4(c + d x)] + 24 d x \cos[4(c + d x)] + 27 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\
& 9 \cos[4(c + d x)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - 12 \cos[2(c + d x)] \left(8 c + 8 d x + 3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - 3 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \right) - \\
& \left. 27 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] - 9 \cos[4(c + d x)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] + 32 \sin[2(c + d x)] - 32 \sin[4(c + d x)] \right)
\end{aligned}$$

■ **Problem 633: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + d x]^6}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$\frac{3 \operatorname{ArcTanh}[\cos[c + d x]]}{8 a d} - \frac{\cot[c + d x]^5}{5 a d} - \frac{3 \cot[c + d x] \csc[c + d x]}{8 a d} + \frac{\cot[c + d x]^3 \csc[c + d x]}{4 a d}$$

Result (type 3, 189 leaves):

$$\begin{aligned}
& - \frac{1}{640 a d} \operatorname{Csc}[c + d x]^5 \\
& \left(80 \operatorname{Cos}[c + d x] + 40 \operatorname{Cos}[3(c + d x)] + 8 \operatorname{Cos}[5(c + d x)] - 150 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[c + d x] + 150 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[c + d x] + \right. \\
& \quad 20 \operatorname{Sin}[2(c + d x)] + 75 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[3(c + d x)] - 75 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[3(c + d x)] - \\
& \quad \left. 50 \operatorname{Sin}[4(c + d x)] - 15 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[5(c + d x)] + 15 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[5(c + d x)] \right)
\end{aligned}$$

■ **Problem 634: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^6 \operatorname{Sin}[c + d x]^3}{(a + a \operatorname{Sin}[c + d x])^2} dx$$

Optimal (type 3, 135 leaves, 13 steps):

$$-\frac{x}{8 a^2} - \frac{2 \operatorname{Cos}[c + d x]^3}{3 a^2 d} + \frac{3 \operatorname{Cos}[c + d x]^5}{5 a^2 d} - \frac{\operatorname{Cos}[c + d x]^7}{7 a^2 d} - \frac{\operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{8 a^2 d} + \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sin}[c + d x]}{4 a^2 d} + \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sin}[c + d x]^3}{3 a^2 d}$$

Result (type 3, 414 leaves):

$$\begin{aligned}
& - \frac{1}{13440 a^2 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right)} \\
& \left(1680 d x \operatorname{Cos}\left[\frac{c}{2}\right] + 1365 \operatorname{Cos}\left[\frac{c}{2} + d x\right] + 1365 \operatorname{Cos}\left[\frac{3c}{2} + d x\right] - 210 \operatorname{Cos}\left[\frac{3c}{2} + 2 d x\right] + 210 \operatorname{Cos}\left[\frac{5c}{2} + 2 d x\right] + 175 \operatorname{Cos}\left[\frac{5c}{2} + 3 d x\right] + \right. \\
& \quad 175 \operatorname{Cos}\left[\frac{7c}{2} + 3 d x\right] - 210 \operatorname{Cos}\left[\frac{7c}{2} + 4 d x\right] + 210 \operatorname{Cos}\left[\frac{9c}{2} + 4 d x\right] - 147 \operatorname{Cos}\left[\frac{9c}{2} + 5 d x\right] - 147 \operatorname{Cos}\left[\frac{11c}{2} + 5 d x\right] + 70 \operatorname{Cos}\left[\frac{11c}{2} + 6 d x\right] - \\
& \quad 70 \operatorname{Cos}\left[\frac{13c}{2} + 6 d x\right] + 15 \operatorname{Cos}\left[\frac{13c}{2} + 7 d x\right] + 15 \operatorname{Cos}\left[\frac{15c}{2} + 7 d x\right] - 420 \operatorname{Sin}\left[\frac{c}{2}\right] + 1680 d x \operatorname{Sin}\left[\frac{c}{2}\right] - 1365 \operatorname{Sin}\left[\frac{c}{2} + d x\right] + 1365 \operatorname{Sin}\left[\frac{3c}{2} + d x\right] - \\
& \quad 210 \operatorname{Sin}\left[\frac{3c}{2} + 2 d x\right] - 210 \operatorname{Sin}\left[\frac{5c}{2} + 2 d x\right] - 175 \operatorname{Sin}\left[\frac{5c}{2} + 3 d x\right] + 175 \operatorname{Sin}\left[\frac{7c}{2} + 3 d x\right] - 210 \operatorname{Sin}\left[\frac{7c}{2} + 4 d x\right] - 210 \operatorname{Sin}\left[\frac{9c}{2} + 4 d x\right] + \\
& \quad \left. 147 \operatorname{Sin}\left[\frac{9c}{2} + 5 d x\right] - 147 \operatorname{Sin}\left[\frac{11c}{2} + 5 d x\right] + 70 \operatorname{Sin}\left[\frac{11c}{2} + 6 d x\right] + 70 \operatorname{Sin}\left[\frac{13c}{2} + 6 d x\right] - 15 \operatorname{Sin}\left[\frac{13c}{2} + 7 d x\right] + 15 \operatorname{Sin}\left[\frac{15c}{2} + 7 d x\right] \right)
\end{aligned}$$

■ **Problem 635: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^6 \operatorname{Sin}[c + d x]^2}{(a + a \operatorname{Sin}[c + d x])^2} dx$$

Optimal (type 3, 104 leaves, 6 steps):

$$\frac{3 x}{16 a^2} + \frac{\operatorname{Cos}[c + d x]^5}{10 a^2 d} + \frac{3 \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{16 a^2 d} + \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sin}[c + d x]}{8 a^2 d} + \frac{\operatorname{Cos}[c + d x]^3 (a - a \operatorname{Sin}[c + d x])^3}{6 a^5 d}$$

Result (type 3, 362 leaves):

$$\frac{1}{1920 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)}$$

$$\left(360 dx \cos\left[\frac{c}{2}\right] + 240 \cos\left[\frac{c}{2} + dx\right] + 240 \cos\left[\frac{3c}{2} + dx\right] - 15 \cos\left[\frac{3c}{2} + 2dx\right] + 15 \cos\left[\frac{5c}{2} + 2dx\right] + 40 \cos\left[\frac{5c}{2} + 3dx\right] + 40 \cos\left[\frac{7c}{2} + 3dx\right] - 45 \cos\left[\frac{7c}{2} + 4dx\right] + 45 \cos\left[\frac{9c}{2} + 4dx\right] - 24 \cos\left[\frac{9c}{2} + 5dx\right] - 24 \cos\left[\frac{11c}{2} + 5dx\right] + 5 \cos\left[\frac{11c}{2} + 6dx\right] - 5 \cos\left[\frac{13c}{2} + 6dx\right] + 50 \sin\left[\frac{c}{2}\right] + 360 dx \sin\left[\frac{c}{2}\right] - 240 \sin\left[\frac{c}{2} + dx\right] + 240 \sin\left[\frac{3c}{2} + dx\right] - 15 \sin\left[\frac{3c}{2} + 2dx\right] - 15 \sin\left[\frac{5c}{2} + 2dx\right] - 40 \sin\left[\frac{5c}{2} + 3dx\right] + 40 \sin\left[\frac{7c}{2} + 3dx\right] - 45 \sin\left[\frac{7c}{2} + 4dx\right] - 45 \sin\left[\frac{9c}{2} + 4dx\right] + 24 \sin\left[\frac{9c}{2} + 5dx\right] - 24 \sin\left[\frac{11c}{2} + 5dx\right] + 5 \sin\left[\frac{11c}{2} + 6dx\right] + 5 \sin\left[\frac{13c}{2} + 6dx\right] \right)$$

■ **Problem 636: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^6 \sin[c + dx]}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{x}{4a^2} - \frac{2 \cos[c + dx]^5}{15a^2 d} - \frac{\cos[c + dx] \sin[c + dx]}{4a^2 d} - \frac{\cos[c + dx]^3 \sin[c + dx]}{6a^2 d} - \frac{\cos[c + dx]^7}{3d(a + a \sin[c + dx])^2}$$

Result (type 3, 258 leaves):

$$\frac{1}{480 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)}$$

$$\left(-120 dx \cos\left[\frac{c}{2}\right] - 90 \cos\left[\frac{c}{2} + dx\right] - 90 \cos\left[\frac{3c}{2} + dx\right] - 25 \cos\left[\frac{5c}{2} + 3dx\right] - 25 \cos\left[\frac{7c}{2} + 3dx\right] + 15 \cos\left[\frac{7c}{2} + 4dx\right] - 15 \cos\left[\frac{9c}{2} + 4dx\right] + 3 \cos\left[\frac{9c}{2} + 5dx\right] + 3 \cos\left[\frac{11c}{2} + 5dx\right] + 50 \sin\left[\frac{c}{2}\right] - 120 dx \sin\left[\frac{c}{2}\right] + 90 \sin\left[\frac{c}{2} + dx\right] - 90 \sin\left[\frac{3c}{2} + dx\right] + 25 \sin\left[\frac{5c}{2} + 3dx\right] - 25 \sin\left[\frac{7c}{2} + 3dx\right] + 15 \sin\left[\frac{7c}{2} + 4dx\right] + 15 \sin\left[\frac{9c}{2} + 4dx\right] - 3 \sin\left[\frac{9c}{2} + 5dx\right] + 3 \sin\left[\frac{11c}{2} + 5dx\right] \right)$$

■ **Problem 644: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^6 \sin[c + dx]^3}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 129 leaves, 14 steps):

$$-\frac{23x}{16a^3} - \frac{4 \cos[c + dx]}{a^3 d} + \frac{7 \cos[c + dx]^3}{3a^3 d} - \frac{3 \cos[c + dx]^5}{5a^3 d} + \frac{23 \cos[c + dx] \sin[c + dx]}{16a^3 d} + \frac{23 \cos[c + dx] \sin[c + dx]^3}{24a^3 d} + \frac{\cos[c + dx] \sin[c + dx]^5}{6a^3 d}$$

Result (type 3, 362 leaves):

$$\begin{aligned}
& - \frac{1}{1920 a^3 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \\
& \left(2760 d x \cos\left[\frac{c}{2}\right] + 2520 \cos\left[\frac{c}{2} + d x\right] + 2520 \cos\left[\frac{3c}{2} + d x\right] - 945 \cos\left[\frac{3c}{2} + 2 d x\right] + 945 \cos\left[\frac{5c}{2} + 2 d x\right] - 380 \cos\left[\frac{5c}{2} + 3 d x\right] - \right. \\
& 380 \cos\left[\frac{7c}{2} + 3 d x\right] + 135 \cos\left[\frac{7c}{2} + 4 d x\right] - 135 \cos\left[\frac{9c}{2} + 4 d x\right] + 36 \cos\left[\frac{9c}{2} + 5 d x\right] + 36 \cos\left[\frac{11c}{2} + 5 d x\right] - 5 \cos\left[\frac{11c}{2} + 6 d x\right] + \\
& 5 \cos\left[\frac{13c}{2} + 6 d x\right] - 18 \sin\left[\frac{c}{2}\right] + 2760 d x \sin\left[\frac{c}{2}\right] - 2520 \sin\left[\frac{c}{2} + d x\right] + 2520 \sin\left[\frac{3c}{2} + d x\right] - \\
& 945 \sin\left[\frac{3c}{2} + 2 d x\right] - 945 \sin\left[\frac{5c}{2} + 2 d x\right] + 380 \sin\left[\frac{5c}{2} + 3 d x\right] - 380 \sin\left[\frac{7c}{2} + 3 d x\right] + 135 \sin\left[\frac{7c}{2} + 4 d x\right] + \\
& \left. 135 \sin\left[\frac{9c}{2} + 4 d x\right] - 36 \sin\left[\frac{9c}{2} + 5 d x\right] + 36 \sin\left[\frac{11c}{2} + 5 d x\right] - 5 \sin\left[\frac{11c}{2} + 6 d x\right] - 5 \sin\left[\frac{13c}{2} + 6 d x\right] \right)
\end{aligned}$$

■ **Problem 645: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^6 \sin[c + d x]^2}{(a + a \sin[c + d x])^3} dx$$

Optimal (type 3, 105 leaves, 12 steps):

$$\frac{13 x}{8 a^3} + \frac{4 \cos[c + d x]}{a^3 d} - \frac{5 \cos[c + d x]^3}{3 a^3 d} + \frac{\cos[c + d x]^5}{5 a^3 d} - \frac{13 \cos[c + d x] \sin[c + d x]}{8 a^3 d} - \frac{3 \cos[c + d x] \sin[c + d x]^3}{4 a^3 d}$$

Result (type 3, 310 leaves):

$$\begin{aligned}
& \frac{1}{960 a^3 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \\
& \left(1560 d x \cos\left[\frac{c}{2}\right] + 1380 \cos\left[\frac{c}{2} + d x\right] + 1380 \cos\left[\frac{3c}{2} + d x\right] - 480 \cos\left[\frac{3c}{2} + 2 d x\right] + 480 \cos\left[\frac{5c}{2} + 2 d x\right] - 170 \cos\left[\frac{5c}{2} + 3 d x\right] - \right. \\
& 170 \cos\left[\frac{7c}{2} + 3 d x\right] + 45 \cos\left[\frac{7c}{2} + 4 d x\right] - 45 \cos\left[\frac{9c}{2} + 4 d x\right] + 6 \cos\left[\frac{9c}{2} + 5 d x\right] + 6 \cos\left[\frac{11c}{2} + 5 d x\right] + \\
& 10 \sin\left[\frac{c}{2}\right] + 1560 d x \sin\left[\frac{c}{2}\right] - 1380 \sin\left[\frac{c}{2} + d x\right] + 1380 \sin\left[\frac{3c}{2} + d x\right] - 480 \sin\left[\frac{3c}{2} + 2 d x\right] - 480 \sin\left[\frac{5c}{2} + 2 d x\right] + \\
& \left. 170 \sin\left[\frac{5c}{2} + 3 d x\right] - 170 \sin\left[\frac{7c}{2} + 3 d x\right] + 45 \sin\left[\frac{7c}{2} + 4 d x\right] + 45 \sin\left[\frac{9c}{2} + 4 d x\right] - 6 \sin\left[\frac{9c}{2} + 5 d x\right] + 6 \sin\left[\frac{11c}{2} + 5 d x\right] \right)
\end{aligned}$$

■ **Problem 646: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^6 \sin[c + d x]}{(a + a \sin[c + d x])^3} dx$$

Optimal (type 3, 84 leaves, 5 steps):

$$-\frac{15x}{8a^3} - \frac{4\cos[c+dx]}{a^3d} + \frac{\cos[c+dx]^3}{a^3d} + \frac{15\cos[c+dx]\sin[c+dx]}{8a^3d} + \frac{\cos[c+dx]\sin[c+dx]^3}{4a^3d}$$

Result (type 3, 252 leaves):

$$-\frac{1}{64a^3d\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)} \left(120dx\cos\left[\frac{c}{2}\right] + 104\cos\left[\frac{c}{2} + dx\right] + 104\cos\left[\frac{3c}{2} + dx\right] - 32\cos\left[\frac{3c}{2} + 2dx\right] + 32\cos\left[\frac{5c}{2} + 2dx\right] - 8\cos\left[\frac{5c}{2} + 3dx\right] - \right. \\ \left. 8\cos\left[\frac{7c}{2} + 3dx\right] + \cos\left[\frac{7c}{2} + 4dx\right] - \cos\left[\frac{9c}{2} + 4dx\right] - 2\sin\left[\frac{c}{2}\right] + 120dx\sin\left[\frac{c}{2}\right] - 104\sin\left[\frac{c}{2} + dx\right] + 104\sin\left[\frac{3c}{2} + dx\right] - \right. \\ \left. 32\sin\left[\frac{3c}{2} + 2dx\right] - 32\sin\left[\frac{5c}{2} + 2dx\right] + 8\sin\left[\frac{5c}{2} + 3dx\right] - 8\sin\left[\frac{7c}{2} + 3dx\right] + \sin\left[\frac{7c}{2} + 4dx\right] + \sin\left[\frac{9c}{2} + 4dx\right] \right)$$

■ **Problem 648: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^4 \cot[c+dx]^2}{(a+a\sin[c+dx])^3} dx$$

Optimal (type 3, 49 leaves, 7 steps):

$$\frac{3x}{a^3} + \frac{3\operatorname{ArcTanh}[\cos[c+dx]]}{a^3d} + \frac{\cos[c+dx]}{a^3d} - \frac{\cot[c+dx]}{a^3d}$$

Result (type 3, 106 leaves):

$$\frac{1}{2d(a+a\sin[c+dx])^3} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 \\ \left(6(c+dx) + 2\cos[c+dx] - \cot\left[\frac{1}{2}(c+dx)\right] + 6\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] - 6\operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] + \tan\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 649: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^3 \cot[c+dx]^3}{(a+a\sin[c+dx])^3} dx$$

Optimal (type 3, 60 leaves, 8 steps):

$$-\frac{x}{a^3} - \frac{7\operatorname{ArcTanh}[\cos[c+dx]]}{2a^3d} + \frac{3\cot[c+dx]}{a^3d} - \frac{\cot[c+dx]\csc[c+dx]}{2a^3d}$$

Result (type 3, 126 leaves):

$$\frac{1}{8d(a+a\sin[c+dx])^3} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 \left(-8(c+dx) + 12\cot\left[\frac{1}{2}(c+dx)\right] - \right. \\ \left. \csc\left[\frac{1}{2}(c+dx)\right]^2 - 28\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] + 28\operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] + \sec\left[\frac{1}{2}(c+dx)\right]^2 - 12\tan\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 651: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx] \cot[c+dx]^5}{(a+a\sin[c+dx])^3} dx$$

Optimal (type 3, 93 leaves, 12 steps):

$$-\frac{15 \operatorname{ArcTanh}[\cos[c+dx]]}{8a^3d} + \frac{4 \cot[c+dx]}{a^3d} + \frac{\cot[c+dx]^3}{a^3d} - \frac{15 \cot[c+dx] \operatorname{Csc}[c+dx]}{8a^3d} - \frac{\cot[c+dx] \operatorname{Csc}[c+dx]^3}{4a^3d}$$

Result (type 3, 555 leaves):

$$\begin{aligned} & \frac{3 \cot\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{2d(a+a\sin[c+dx])^3} - \frac{15 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{32d(a+a\sin[c+dx])^3} + \\ & \frac{\cot\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a\sin[c+dx])^3} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{64d(a+a\sin[c+dx])^3} - \\ & \frac{15 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a\sin[c+dx])^3} + \frac{15 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a\sin[c+dx])^3} + \\ & \frac{15 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{32d(a+a\sin[c+dx])^3} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{64d(a+a\sin[c+dx])^3} - \\ & \frac{3 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2d(a+a\sin[c+dx])^3} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{8d(a+a\sin[c+dx])^3} \end{aligned}$$

■ **Problem 653: Unable to integrate problem.**

$$\int \cos[c+dx]^6 \sin[c+dx]^{11} (a+a\sin[c+dx])^3 dx$$

Optimal (type 5, 267 leaves, 6 steps):

$$\frac{a^3 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{1+n}}{d(1+n) \sqrt{\cos[c+dx]^2}} +$$

$$\frac{3a^3 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{2+n}}{d(2+n) \sqrt{\cos[c+dx]^2}} +$$

$$\frac{3a^3 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{3+n}}{d(3+n) \sqrt{\cos[c+dx]^2}} +$$

$$\frac{a^3 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{4+n}{2}, \frac{6+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{4+n}}{d(4+n) \sqrt{\cos[c+dx]^2}}$$

Result (type 8, 31 leaves):

$$\int \cos[c+dx]^6 \sin[c+dx]^n (a + a \sin[c+dx])^3 dx$$

■ **Problem 654: Unable to integrate problem.**

$$\int \cos[c+dx]^6 \sin[c+dx]^n (a + a \sin[c+dx])^2 dx$$

Optimal (type 5, 200 leaves, 5 steps):

$$\frac{a^2 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{1+n}}{d(1+n) \sqrt{\cos[c+dx]^2}} +$$

$$\frac{2a^2 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{2+n}}{d(2+n) \sqrt{\cos[c+dx]^2}} +$$

$$\frac{a^2 \cos[c+dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \sin^2[c+dx]\right] \sin[c+dx]^{3+n}}{d(3+n) \sqrt{\cos[c+dx]^2}}$$

Result (type 8, 31 leaves):

$$\int \cos[c+dx]^6 \sin[c+dx]^n (a + a \sin[c+dx])^2 dx$$

■ **Problem 655: Unable to integrate problem.**

$$\int \cos[c+dx]^6 \sin[c+dx]^n (a + a \sin[c+dx]) dx$$

Optimal (type 5, 129 leaves, 3 steps):

$$\frac{a \operatorname{Cos}[c + d x] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \operatorname{Sin}[c + d x]^2\right] \operatorname{Sin}[c + d x]^{1+n}}{d(1+n) \sqrt{\operatorname{Cos}[c + d x]^2}} +$$

$$\frac{a \operatorname{Cos}[c + d x] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \operatorname{Sin}[c + d x]^2\right] \operatorname{Sin}[c + d x]^{2+n}}{d(2+n) \sqrt{\operatorname{Cos}[c + d x]^2}}$$

Result (type 8, 29 leaves):

$$\int \operatorname{Cos}[c + d x]^6 \operatorname{Sin}[c + d x]^n (a + a \operatorname{Sin}[c + d x]) dx$$

■ **Problem 669: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + d x]^7 \operatorname{Csc}[c + d x] (a + a \operatorname{Sin}[c + d x]) dx$$

Optimal (type 3, 119 leaves, 4 steps):

$$\frac{a \operatorname{Csc}[c + d x]}{d} - \frac{3 a \operatorname{Csc}[c + d x]^2}{2 d} - \frac{a \operatorname{Csc}[c + d x]^3}{d} + \frac{3 a \operatorname{Csc}[c + d x]^4}{4 d} + \frac{3 a \operatorname{Csc}[c + d x]^5}{5 d} - \frac{a \operatorname{Csc}[c + d x]^6}{6 d} - \frac{a \operatorname{Csc}[c + d x]^7}{7 d} - \frac{a \operatorname{Log}[\operatorname{Sin}[c + d x]]}{d}$$

Result (type 3, 278 leaves):

$$\frac{381 a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]}{1120 d} - \frac{179 a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{2240 d} + \frac{a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{70 d} -$$

$$\frac{a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^6}{896 d} - \frac{3 a \operatorname{Csc}[c + d x]^2}{2 d} + \frac{3 a \operatorname{Csc}[c + d x]^4}{4 d} - \frac{a \operatorname{Csc}[c + d x]^6}{6 d} - \frac{a \operatorname{Log}[\operatorname{Sin}[c + d x]]}{d} + \frac{381 a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{1120 d} -$$

$$\frac{179 a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{2240 d} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{70 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^6 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{896 d}$$

■ **Problem 670: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + d x]^7 \operatorname{Csc}[c + d x]^2 (a + a \operatorname{Sin}[c + d x]) dx$$

Optimal (type 3, 74 leaves, 6 steps):

$$-\frac{a \operatorname{Cot}[c + d x]^8}{8 d} + \frac{a \operatorname{Csc}[c + d x]}{d} - \frac{a \operatorname{Csc}[c + d x]^3}{d} + \frac{3 a \operatorname{Csc}[c + d x]^5}{5 d} - \frac{a \operatorname{Csc}[c + d x]^7}{7 d}$$

Result (type 3, 233 leaves):

$$\frac{381 a \cot\left[\frac{1}{2}(c+dx)\right]}{1120 d} - \frac{a \cot[c+dx]^8}{8 d} - \frac{179 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{2240 d} +$$

$$\frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{70 d} - \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{896 d} + \frac{381 a \tan\left[\frac{1}{2}(c+dx)\right]}{1120 d} -$$

$$\frac{179 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{2240 d} + \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{70 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{896 d}$$

■ **Problem 671: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^7 \csc[c+dx]^3 (a+a \sin[c+dx]) dx$$

Optimal (type 3, 81 leaves, 6 steps):

$$-\frac{a \cot[c+dx]^8}{8 d} + \frac{a \csc[c+dx]^3}{3 d} - \frac{3 a \csc[c+dx]^5}{5 d} + \frac{3 a \csc[c+dx]^7}{7 d} - \frac{a \csc[c+dx]^9}{9 d}$$

Result (type 3, 293 leaves):

$$\frac{1823 a \cot\left[\frac{1}{2}(c+dx)\right]}{80640 d} - \frac{a \cot[c+dx]^8}{8 d} + \frac{1823 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{161280 d} -$$

$$\frac{463 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{53760 d} + \frac{73 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{32256 d} -$$

$$\frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^8}{4608 d} + \frac{1823 a \tan\left[\frac{1}{2}(c+dx)\right]}{80640 d} + \frac{1823 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{161280 d} -$$

$$\frac{463 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{53760 d} + \frac{73 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{32256 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{4608 d}$$

■ **Problem 672: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^7 \csc[c+dx]^4 (a+a \sin[c+dx]) dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$-\frac{a \cot[c+dx]^8}{8 d} - \frac{a \cot[c+dx]^{10}}{10 d} + \frac{a \csc[c+dx]^3}{3 d} - \frac{3 a \csc[c+dx]^5}{5 d} + \frac{3 a \csc[c+dx]^7}{7 d} - \frac{a \csc[c+dx]^9}{9 d}$$

Result (type 3, 341 leaves):

$$\frac{1823 a \cot\left[\frac{1}{2}(c+dx)\right]}{80640 d} + \frac{1823 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{161280 d} - \frac{463 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{53760 d} +$$

$$\frac{73 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{32256 d} - \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^8}{4608 d} + \frac{a \csc[c+dx]^4}{4 d} - \frac{a \csc[c+dx]^6}{2 d} +$$

$$\frac{3 a \csc[c+dx]^8}{8 d} - \frac{a \csc[c+dx]^{10}}{10 d} + \frac{1823 a \tan\left[\frac{1}{2}(c+dx)\right]}{80640 d} + \frac{1823 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{161280 d} -$$

$$\frac{463 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{53760 d} + \frac{73 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{32256 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{4608 d}$$

■ **Problem 673: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^7 \csc[c+dx]^5 (a + a \sin[c+dx]) dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$-\frac{a \cot[c+dx]^8}{8 d} - \frac{a \cot[c+dx]^{10}}{10 d} + \frac{a \csc[c+dx]^5}{5 d} - \frac{3 a \csc[c+dx]^7}{7 d} + \frac{a \csc[c+dx]^9}{3 d} - \frac{a \csc[c+dx]^{11}}{11 d}$$

Result (type 3, 401 leaves):

$$\frac{2911 a \cot\left[\frac{1}{2}(c+dx)\right]}{591360 d} + \frac{2911 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{1182720 d} +$$

$$\frac{27 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{197120 d} - \frac{485 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{473088 d} + \frac{13 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^8}{33792 d} -$$

$$\frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^{10}}{22528 d} + \frac{a \csc[c+dx]^4}{4 d} - \frac{a \csc[c+dx]^6}{2 d} + \frac{3 a \csc[c+dx]^8}{8 d} - \frac{a \csc[c+dx]^{10}}{10 d} +$$

$$\frac{2911 a \tan\left[\frac{1}{2}(c+dx)\right]}{591360 d} + \frac{2911 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1182720 d} + \frac{27 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{197120 d} -$$

$$\frac{485 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{473088 d} + \frac{13 a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{33792 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^{10} \tan\left[\frac{1}{2}(c+dx)\right]}{22528 d}$$

■ **Problem 674: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^7 \csc[c+dx]^6 (a + a \sin[c+dx]) dx$$

Optimal (type 3, 113 leaves, 8 steps):

$$-\frac{a \cot[c+dx]^8}{8 d} - \frac{a \cot[c+dx]^{10}}{5 d} - \frac{a \cot[c+dx]^{12}}{12 d} + \frac{a \csc[c+dx]^5}{5 d} - \frac{3 a \csc[c+dx]^7}{7 d} + \frac{a \csc[c+dx]^9}{3 d} - \frac{a \csc[c+dx]^{11}}{11 d}$$

Result (type 3, 401 leaves):

$$\begin{aligned}
& \frac{2911 a \cot\left[\frac{1}{2}(c+dx)\right]}{591360d} + \frac{2911 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{1182720d} + \\
& \frac{27 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{197120d} - \frac{485 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{473088d} + \frac{13 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^8}{33792d} - \\
& \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^{10}}{22528d} + \frac{a \csc[c+dx]^6}{6d} - \frac{3 a \csc[c+dx]^8}{8d} + \frac{3 a \csc[c+dx]^{10}}{10d} - \frac{a \csc[c+dx]^{12}}{12d} + \\
& \frac{2911 a \tan\left[\frac{1}{2}(c+dx)\right]}{591360d} + \frac{2911 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1182720d} + \frac{27 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{197120d} - \\
& \frac{485 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{473088d} + \frac{13 a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{33792d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^{10} \tan\left[\frac{1}{2}(c+dx)\right]}{22528d}
\end{aligned}$$

■ **Problem 675: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^7 \csc[c+dx]^7 (a+a \sin[c+dx]) dx$$

Optimal (type 3, 113 leaves, 8 steps):

$$-\frac{a \cot[c+dx]^8}{8d} - \frac{a \cot[c+dx]^{10}}{5d} - \frac{a \cot[c+dx]^{12}}{12d} + \frac{a \csc[c+dx]^7}{7d} - \frac{a \csc[c+dx]^9}{3d} + \frac{3 a \csc[c+dx]^{11}}{11d} - \frac{a \csc[c+dx]^{13}}{13d}$$

Result (type 3, 461 leaves):

$$\begin{aligned}
& \frac{10027 a \cot\left[\frac{1}{2}(c+dx)\right]}{6150144d} + \frac{10027 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{12300288d} + \frac{755 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{410096d} - \\
& \frac{101 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{768768d} - \frac{101 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^8}{878592d} + \frac{79 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^{10}}{1171456d} - \\
& \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^{12}}{106496d} + \frac{a \csc[c+dx]^6}{6d} - \frac{3 a \csc[c+dx]^8}{8d} + \frac{3 a \csc[c+dx]^{10}}{10d} - \frac{a \csc[c+dx]^{12}}{12d} + \frac{10027 a \tan\left[\frac{1}{2}(c+dx)\right]}{6150144d} + \\
& \frac{10027 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{12300288d} + \frac{755 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{410096d} - \frac{101 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{768768d} - \\
& \frac{101 a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{878592d} + \frac{79 a \sec\left[\frac{1}{2}(c+dx)\right]^{10} \tan\left[\frac{1}{2}(c+dx)\right]}{1171456d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^{12} \tan\left[\frac{1}{2}(c+dx)\right]}{106496d}
\end{aligned}$$

■ **Problem 676: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^7 \csc[c+dx]^8 (a+a \sin[c+dx]) dx$$

Optimal (type 3, 129 leaves, 4 steps):

$$\frac{a \operatorname{Csc}[c + dx]^7}{7d} + \frac{a \operatorname{Csc}[c + dx]^8}{8d} - \frac{a \operatorname{Csc}[c + dx]^9}{3d} - \frac{3a \operatorname{Csc}[c + dx]^{10}}{10d} + \frac{3a \operatorname{Csc}[c + dx]^{11}}{11d} + \frac{a \operatorname{Csc}[c + dx]^{12}}{4d} - \frac{a \operatorname{Csc}[c + dx]^{13}}{13d} - \frac{a \operatorname{Csc}[c + dx]^{14}}{14d}$$

Result (type 3, 461 leaves):

$$\begin{aligned} & \frac{10027 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{6150144 d} + \frac{10027 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{12300288 d} + \frac{755 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{4100096 d} - \\ & \frac{101 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6}{768768 d} - \frac{101 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^8}{878592 d} + \frac{79 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^{10}}{1171456 d} - \\ & \frac{a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^{12}}{106496 d} + \frac{a \operatorname{Csc}[c+dx]^8}{8d} - \frac{3a \operatorname{Csc}[c+dx]^{10}}{10d} + \frac{a \operatorname{Csc}[c+dx]^{12}}{4d} - \frac{a \operatorname{Csc}[c+dx]^{14}}{14d} + \frac{10027 a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{6150144 d} + \\ & \frac{10027 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{12300288 d} + \frac{755 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{4100096 d} - \frac{101 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{768768 d} - \\ & \frac{101 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{878592 d} + \frac{79 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^{10} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1171456 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^{12} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{106496 d} \end{aligned}$$

- **Problem 697: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^7 \sin[c+dx]^n (a+a \sin[c+dx])^3 dx$$

Optimal (type 3, 184 leaves, 3 steps):

$$\begin{aligned} & \frac{a^3 \sin[c+dx]^{1+n}}{d(1+n)} + \frac{3a^3 \sin[c+dx]^{2+n}}{d(2+n)} - \frac{8a^3 \sin[c+dx]^{4+n}}{d(4+n)} - \frac{6a^3 \sin[c+dx]^{5+n}}{d(5+n)} + \\ & \frac{6a^3 \sin[c+dx]^{6+n}}{d(6+n)} + \frac{8a^3 \sin[c+dx]^{7+n}}{d(7+n)} - \frac{3a^3 \sin[c+dx]^{9+n}}{d(9+n)} - \frac{a^3 \sin[c+dx]^{10+n}}{d(10+n)} \end{aligned}$$

Result (type 3, 1019 leaves):

$$\begin{aligned}
& \frac{1}{d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6} \\
& \sin[c+dx]^n (a + a \sin[c+dx])^3 \left(\frac{3 \left(11792 + 2276n + 260n^2 + 11n^3 \right) \left(28665 + 4541n + 471n^2 + 19n^3 \right) \left(-\frac{1}{256}i \cos[c+dx] + \frac{1}{256} \sin[c+dx] \right)}{256(2+n)(4+n)(6+n)(10+n)} + \frac{\left(28665 + 4541n + 471n^2 + 19n^3 \right) \left(-\frac{1}{256}i \cos[c+dx] + \frac{1}{256} \sin[c+dx] \right)}{(1+n)(5+n)(7+n)(9+n)} + \right. \\
& \frac{\left(28665 + 4541n + 471n^2 + 19n^3 \right) \left(\frac{1}{256}i \cos[c+dx] + \frac{1}{256} \sin[c+dx] \right)}{(1+n)(5+n)(7+n)(9+n)} + \\
& \frac{\left(-21840 + 2252n + 492n^2 + 25n^3 \right) \left(\frac{1}{512} \cos[2(c+dx)] - \frac{1}{512}i \sin[2(c+dx)] \right)}{(2+n)(4+n)(6+n)(10+n)} + \\
& \frac{\left(-21840 + 2252n + 492n^2 + 25n^3 \right) \left(\frac{1}{512} \cos[2(c+dx)] + \frac{1}{512}i \sin[2(c+dx)] \right)}{(2+n)(4+n)(6+n)(10+n)} + \\
& \frac{\left(735 + 108n + 5n^2 \right) \left(-\frac{3}{128}i \cos[3(c+dx)] + \frac{3}{128} \sin[3(c+dx)] \right)}{(5+n)(7+n)(9+n)} + \frac{\left(735 + 108n + 5n^2 \right) \left(\frac{3}{128}i \cos[3(c+dx)] + \frac{3}{128} \sin[3(c+dx)] \right)}{(5+n)(7+n)(9+n)} + \\
& \frac{\left(-1320 - 166n - 7n^2 \right) \left(\frac{1}{128} \cos[4(c+dx)] - \frac{1}{128}i \sin[4(c+dx)] \right)}{(4+n)(6+n)(10+n)} + \frac{\left(-1320 - 166n - 7n^2 \right) \left(\frac{1}{128} \cos[4(c+dx)] + \frac{1}{128}i \sin[4(c+dx)] \right)}{(4+n)(6+n)(10+n)} + \\
& \frac{\left(63 + 76n + 5n^2 \right) \left(-\frac{1}{128}i \cos[5(c+dx)] + \frac{1}{128} \sin[5(c+dx)] \right)}{(5+n)(7+n)(9+n)} + \frac{\left(63 + 76n + 5n^2 \right) \left(\frac{1}{128}i \cos[5(c+dx)] + \frac{1}{128} \sin[5(c+dx)] \right)}{(5+n)(7+n)(9+n)} + \\
& \frac{\left(230 + 17n \right) \left(-\frac{3 \cos[6(c+dx)]}{1024} - \frac{3i \sin[6(c+dx)]}{1024} \right)}{(6+n)(10+n)} + \frac{\left(230 + 17n \right) \left(-\frac{3 \cos[6(c+dx)]}{1024} + \frac{3i \sin[6(c+dx)]}{1024} \right)}{(6+n)(10+n)} + \\
& \frac{\left(99 + 5n \right) \left(-\frac{1}{512}i \cos[7(c+dx)] - \frac{1}{512} \sin[7(c+dx)] \right)}{(7+n)(9+n)} + \frac{\left(99 + 5n \right) \left(\frac{1}{512}i \cos[7(c+dx)] - \frac{1}{512} \sin[7(c+dx)] \right)}{(7+n)(9+n)} + \\
& \frac{-\frac{5}{512} \cos[8(c+dx)] - \frac{5}{512}i \sin[8(c+dx)]}{10+n} + \frac{-\frac{5}{512} \cos[8(c+dx)] + \frac{5}{512}i \sin[8(c+dx)]}{10+n} + \frac{-\frac{3}{512}i \cos[9(c+dx)] - \frac{3}{512} \sin[9(c+dx)]}{9+n} \\
& \left. \frac{\frac{3}{512}i \cos[9(c+dx)] - \frac{3}{512} \sin[9(c+dx)]}{9+n} + \frac{\frac{\cos[10(c+dx)]}{1024} - \frac{i \sin[10(c+dx)]}{1024}}{10+n} + \frac{\frac{\cos[10(c+dx)]}{1024} + \frac{i \sin[10(c+dx)]}{1024}}{10+n} \right)
\end{aligned}$$

■ **Problem 698: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^7 \sin[c+dx]^n (a + a \sin[c+dx])^2 dx$$

Optimal (type 3, 184 leaves, 3 steps):

$$\frac{a^2 \operatorname{Sin}[c+dx]^{1+n}}{d(1+n)} + \frac{2a^2 \operatorname{Sin}[c+dx]^{2+n}}{d(2+n)} - \frac{2a^2 \operatorname{Sin}[c+dx]^{3+n}}{d(3+n)} - \frac{6a^2 \operatorname{Sin}[c+dx]^{4+n}}{d(4+n)} +$$

$$\frac{6a^2 \operatorname{Sin}[c+dx]^{6+n}}{d(6+n)} + \frac{2a^2 \operatorname{Sin}[c+dx]^{7+n}}{d(7+n)} - \frac{2a^2 \operatorname{Sin}[c+dx]^{8+n}}{d(8+n)} - \frac{a^2 \operatorname{Sin}[c+dx]^{9+n}}{d(9+n)}$$

Result (type 3, 925 leaves):

$$\frac{1}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^4}$$

$$\operatorname{Sin}[c+dx]^n (a + a \operatorname{Sin}[c+dx])^2 \left(\frac{4464 + 892n + 108n^2 + 5n^3}{64(2+n)(4+n)(6+n)(8+n)} + \frac{(14553 + 2547n + 295n^2 + 13n^3) \left(-\frac{1}{256}i \operatorname{Cos}[c+dx] + \frac{1}{256} \operatorname{Sin}[c+dx] \right)}{(1+n)(3+n)(7+n)(9+n)} + \right.$$

$$\frac{(14553 + 2547n + 295n^2 + 13n^3) \left(\frac{1}{256}i \operatorname{Cos}[c+dx] + \frac{1}{256} \operatorname{Sin}[c+dx] \right)}{(1+n)(3+n)(7+n)(9+n)} + \frac{(-672 + 80n + 18n^2 + n^3) \left(\frac{1}{32} \operatorname{Cos}[2(c+dx)] - \frac{1}{32}i \operatorname{Sin}[2(c+dx)] \right)}{(2+n)(4+n)(6+n)(8+n)} +$$

$$\frac{(-672 + 80n + 18n^2 + n^3) \left(\frac{1}{32} \operatorname{Cos}[2(c+dx)] + \frac{1}{32}i \operatorname{Sin}[2(c+dx)] \right)}{(2+n)(4+n)(6+n)(8+n)} + \frac{(1323 + 218n + 11n^2) \left(-\frac{1}{128}i \operatorname{Cos}[3(c+dx)] + \frac{1}{128} \operatorname{Sin}[3(c+dx)] \right)}{(3+n)(7+n)(9+n)} +$$

$$\frac{(1323 + 218n + 11n^2) \left(\frac{1}{128}i \operatorname{Cos}[3(c+dx)] + \frac{1}{128} \operatorname{Sin}[3(c+dx)] \right)}{(3+n)(7+n)(9+n)} + \frac{(-168 - 22n - n^2) \left(\frac{1}{32} \operatorname{Cos}[4(c+dx)] - \frac{1}{32}i \operatorname{Sin}[4(c+dx)] \right)}{(4+n)(6+n)(8+n)} +$$

$$\frac{(-168 - 22n - n^2) \left(\frac{1}{32} \operatorname{Cos}[4(c+dx)] + \frac{1}{32}i \operatorname{Sin}[4(c+dx)] \right)}{(4+n)(6+n)(8+n)} + \frac{(63 + 5n) \left(-\frac{1}{128}i \operatorname{Cos}[5(c+dx)] + \frac{1}{128} \operatorname{Sin}[5(c+dx)] \right)}{(7+n)(9+n)} +$$

$$\frac{(63 + 5n) \left(\frac{1}{128}i \operatorname{Cos}[5(c+dx)] + \frac{1}{128} \operatorname{Sin}[5(c+dx)] \right)}{(7+n)(9+n)} + \frac{(-12 - n) \left(\frac{1}{32} \operatorname{Cos}[6(c+dx)] - \frac{1}{32}i \operatorname{Sin}[6(c+dx)] \right)}{(6+n)(8+n)} +$$

$$\frac{(-12 - n) \left(\frac{1}{32} \operatorname{Cos}[6(c+dx)] + \frac{1}{32}i \operatorname{Sin}[6(c+dx)] \right)}{(6+n)(8+n)} + \frac{(-9 + n) \left(-\frac{1}{512}i \operatorname{Cos}[7(c+dx)] + \frac{1}{512} \operatorname{Sin}[7(c+dx)] \right)}{(7+n)(9+n)} +$$

$$\frac{(-9 + n) \left(\frac{1}{512}i \operatorname{Cos}[7(c+dx)] + \frac{1}{512} \operatorname{Sin}[7(c+dx)] \right)}{(7+n)(9+n)} + \frac{-\frac{1}{128} \operatorname{Cos}[8(c+dx)] - \frac{1}{128}i \operatorname{Sin}[8(c+dx)]}{8+n} +$$

$$\left. \frac{-\frac{1}{128} \operatorname{Cos}[8(c+dx)] + \frac{1}{128}i \operatorname{Sin}[8(c+dx)]}{8+n} + \frac{-\frac{1}{512}i \operatorname{Cos}[9(c+dx)] - \frac{1}{512} \operatorname{Sin}[9(c+dx)]}{9+n} + \frac{\frac{1}{512}i \operatorname{Cos}[9(c+dx)] - \frac{1}{512} \operatorname{Sin}[9(c+dx)]}{9+n} \right)$$

■ **Problem 699: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^7 \operatorname{Sin}[c+dx]^n (a + a \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 167 leaves, 3 steps):

$$\frac{a \operatorname{Sin}[c+dx]^{1+n}}{d(1+n)} + \frac{a \operatorname{Sin}[c+dx]^{2+n}}{d(2+n)} - \frac{3a \operatorname{Sin}[c+dx]^{3+n}}{d(3+n)} -$$

$$\frac{3a \operatorname{Sin}[c+dx]^{4+n}}{d(4+n)} + \frac{3a \operatorname{Sin}[c+dx]^{5+n}}{d(5+n)} + \frac{3a \operatorname{Sin}[c+dx]^{6+n}}{d(6+n)} - \frac{a \operatorname{Sin}[c+dx]^{7+n}}{d(7+n)} - \frac{a \operatorname{Sin}[c+dx]^{8+n}}{d(8+n)}$$

Result (type 3, 966 leaves):

$$\frac{1}{\left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} a \operatorname{Sin}[c+dx]^n (1 + \operatorname{Sin}[c+dx])$$

$$\left(\frac{4464 + 892n + 108n^2 + 5n^3}{128d(2+n)(4+n)(6+n)(8+n)} + \frac{(3675 + 691n + 93n^2 + 5n^3) \left(-\frac{i \operatorname{Cos}[c+dx]}{128d} + \frac{\operatorname{Sin}[c+dx]}{128d}\right)}{(1+n)(3+n)(5+n)(7+n)} + \frac{(3675 + 691n + 93n^2 + 5n^3) \left(\frac{i \operatorname{Cos}[c+dx]}{128d} + \frac{\operatorname{Sin}[c+dx]}{128d}\right)}{(1+n)(3+n)(5+n)(7+n)} + \right.$$

$$\frac{(-672 + 80n + 18n^2 + n^3) \left(\frac{\operatorname{Cos}[2c+2dx]}{64d} - \frac{i \operatorname{Sin}[2c+2dx]}{64d}\right)}{(2+n)(4+n)(6+n)(8+n)} + \frac{(-672 + 80n + 18n^2 + n^3) \left(\frac{\operatorname{Cos}[2c+2dx]}{64d} + \frac{i \operatorname{Sin}[2c+2dx]}{64d}\right)}{(2+n)(4+n)(6+n)(8+n)} +$$

$$\frac{(245 + 48n + 3n^2) \left(-\frac{3i \operatorname{Cos}[3c+3dx]}{128d} + \frac{3 \operatorname{Sin}[3c+3dx]}{128d}\right)}{(3+n)(5+n)(7+n)} + \frac{(245 + 48n + 3n^2) \left(\frac{3i \operatorname{Cos}[3c+3dx]}{128d} + \frac{3 \operatorname{Sin}[3c+3dx]}{128d}\right)}{(3+n)(5+n)(7+n)} +$$

$$\frac{(168 + 22n + n^2) \left(-\frac{\operatorname{Cos}[4c+4dx]}{64d} - \frac{i \operatorname{Sin}[4c+4dx]}{64d}\right)}{(4+n)(6+n)(8+n)} + \frac{(-168 - 22n - n^2) \left(\frac{\operatorname{Cos}[4c+4dx]}{64d} - \frac{i \operatorname{Sin}[4c+4dx]}{64d}\right)}{(4+n)(6+n)(8+n)} + \frac{(49 + 5n) \left(-\frac{i \operatorname{Cos}[5c+5dx]}{128d} + \frac{\operatorname{Sin}[5c+5dx]}{128d}\right)}{(5+n)(7+n)} +$$

$$\frac{(49 + 5n) \left(\frac{i \operatorname{Cos}[5c+5dx]}{128d} + \frac{\operatorname{Sin}[5c+5dx]}{128d}\right)}{(5+n)(7+n)} + \frac{(12+n) \left(-\frac{\operatorname{Cos}[6c+6dx]}{64d} - \frac{i \operatorname{Sin}[6c+6dx]}{64d}\right)}{(6+n)(8+n)} + \frac{(-12-n) \left(\frac{\operatorname{Cos}[6c+6dx]}{64d} - \frac{i \operatorname{Sin}[6c+6dx]}{64d}\right)}{(6+n)(8+n)} +$$

$$\left. \frac{-\frac{i \operatorname{Cos}[7c+7dx]}{128d} + \frac{\operatorname{Sin}[7c+7dx]}{128d}}{7+n} + \frac{\frac{i \operatorname{Cos}[7c+7dx]}{128d} + \frac{\operatorname{Sin}[7c+7dx]}{128d}}{7+n} + \frac{-\frac{\operatorname{Cos}[8c+8dx]}{256d} - \frac{i \operatorname{Sin}[8c+8dx]}{256d}}{8+n} + \frac{-\frac{\operatorname{Cos}[8c+8dx]}{256d} + \frac{i \operatorname{Sin}[8c+8dx]}{256d}}{8+n} \right)$$

■ **Problem 700: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^7 \operatorname{Sin}[c+dx]^n}{a + a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 137 leaves, 3 steps):

$$\frac{\operatorname{Sin}[c+dx]^{1+n}}{ad(1+n)} - \frac{\operatorname{Sin}[c+dx]^{2+n}}{ad(2+n)} - \frac{2 \operatorname{Sin}[c+dx]^{3+n}}{ad(3+n)} + \frac{2 \operatorname{Sin}[c+dx]^{4+n}}{ad(4+n)} + \frac{\operatorname{Sin}[c+dx]^{5+n}}{ad(5+n)} - \frac{\operatorname{Sin}[c+dx]^{6+n}}{ad(6+n)}$$

Result (type 3, 380 leaves):

$$\begin{aligned}
& - \frac{1}{16 a d (1+n)(2+n)(3+n)(4+n)(5+n)(6+n)(1+\sin[c+dx])} \\
& \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \sin[c+dx]^{1+n} \left(-8544 - 10520n - 4888n^2 - 1114n^3 - 128n^4 - 6n^5 - \right. \\
& 8(336 + 692n + 484n^2 + 147n^3 + 20n^4 + n^5) \cos[2(c+dx)] - 2(144 + 324n + 260n^2 + 95n^3 + 16n^4 + n^5) \cos[4(c+dx)] + 2640 \sin[c+dx] + \\
& 4468n \sin[2(c+dx)] + 2258n^2 \sin[2(c+dx)] + 474n^3 \sin[2(c+dx)] + 46n^4 \sin[2(c+dx)] + 2n^5 \sin[2(c+dx)] + 840 \sin[3(c+dx)] + \\
& 1798n \sin[3(c+dx)] + 1331n^2 \sin[3(c+dx)] + 431n^3 \sin[3(c+dx)] + 61n^4 \sin[3(c+dx)] + 3n^5 \sin[3(c+dx)] + \\
& \left. 120 \sin[5(c+dx)] + 274n \sin[5(c+dx)] + 225n^2 \sin[5(c+dx)] + 85n^3 \sin[5(c+dx)] + 15n^4 \sin[5(c+dx)] + n^5 \sin[5(c+dx)] \right)
\end{aligned}$$

- **Problem 701: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^7 \sin[c+dx]^n}{(a+a\sin[c+dx])^2} dx$$

Optimal (type 3, 92 leaves, 3 steps):

$$\frac{\sin[c+dx]^{1+n}}{a^2 d (1+n)} - \frac{2 \sin[c+dx]^{2+n}}{a^2 d (2+n)} + \frac{2 \sin[c+dx]^{4+n}}{a^2 d (4+n)} - \frac{\sin[c+dx]^{5+n}}{a^2 d (5+n)}$$

Result (type 3, 405 leaves):

$$\begin{aligned}
& \frac{1}{d(a+a\sin[c+dx])^2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 \sin[c+dx]^n \\
& \left(-\frac{10+n}{4(2+n)(4+n)} + \frac{(35+3n)\left(-\frac{1}{16}i\cos[c+dx] + \frac{1}{16}\sin[c+dx]\right)}{(1+n)(5+n)} + \frac{(35+3n)\left(\frac{1}{16}i\cos[c+dx] + \frac{1}{16}\sin[c+dx]\right)}{(1+n)(5+n)} + \right. \\
& \frac{\cos[2(c+dx)] - i\sin[2(c+dx)]}{(2+n)(4+n)} + \frac{\cos[2(c+dx)] + i\sin[2(c+dx)]}{(2+n)(4+n)} + \frac{-\frac{5}{32}i\cos[3(c+dx)] + \frac{5}{32}\sin[3(c+dx)]}{5+n} + \\
& \frac{\frac{5}{32}i\cos[3(c+dx)] + \frac{5}{32}\sin[3(c+dx)]}{5+n} + \frac{\frac{1}{8}\cos[4(c+dx)] - \frac{1}{8}i\sin[4(c+dx)]}{4+n} + \frac{\frac{1}{8}\cos[4(c+dx)] + \frac{1}{8}i\sin[4(c+dx)]}{4+n} + \\
& \left. \frac{-\frac{1}{32}i\cos[5(c+dx)] - \frac{1}{32}\sin[5(c+dx)]}{5+n} + \frac{\frac{1}{32}i\cos[5(c+dx)] - \frac{1}{32}\sin[5(c+dx)]}{5+n} \right)
\end{aligned}$$

- **Problem 702: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^7 \sin[c+dx]^n}{(a+a\sin[c+dx])^3} dx$$

Optimal (type 3, 92 leaves, 3 steps):

$$\frac{\sin[c+dx]^{1+n}}{a^3 d (1+n)} - \frac{3 \sin[c+dx]^{2+n}}{a^3 d (2+n)} + \frac{3 \sin[c+dx]^{3+n}}{a^3 d (3+n)} - \frac{\sin[c+dx]^{4+n}}{a^3 d (4+n)}$$

Result (type 3, 363 leaves):

$$\frac{1}{d (a + a \sin[c + dx])^3} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 \sin[c + dx]^n$$

$$\left(-\frac{3(18 + 5n)}{8(2+n)(4+n)} + \frac{(21 + 13n)\left(-\frac{1}{8}i \cos[c + dx] + \frac{1}{8}\sin[c + dx]\right)}{(1+n)(3+n)} + \frac{(21 + 13n)\left(\frac{1}{8}i \cos[c + dx] + \frac{1}{8}\sin[c + dx]\right)}{(1+n)(3+n)} + \right.$$

$$\frac{(7 + 2n)\left(\frac{1}{2}\cos[2(c + dx)] - \frac{1}{2}i \sin[2(c + dx)]\right)}{(2+n)(4+n)} + \frac{(7 + 2n)\left(\frac{1}{2}\cos[2(c + dx)] + \frac{1}{2}i \sin[2(c + dx)]\right)}{(2+n)(4+n)} +$$

$$\frac{-\frac{3}{8}i \cos[3(c + dx)] - \frac{3}{8}\sin[3(c + dx)]}{3+n} + \frac{\frac{3}{8}i \cos[3(c + dx)] - \frac{3}{8}\sin[3(c + dx)]}{3+n} +$$

$$\left. \frac{-\frac{1}{16}\cos[4(c + dx)] - \frac{1}{16}i \sin[4(c + dx)]}{4+n} + \frac{-\frac{1}{16}\cos[4(c + dx)] + \frac{1}{16}i \sin[4(c + dx)]}{4+n} \right)$$

■ **Problem 703: Unable to integrate problem.**

$$\int \frac{\cos[c + dx]^7 \sin[c + dx]^n}{(a + a \sin[c + dx])^4} dx$$

Optimal (type 5, 109 leaves, 8 steps):

$$-\frac{7 \sin[c + dx]^{1+n}}{a^4 d (1+n)} + \frac{8 \text{Hypergeometric2F1}[1, 1+n, 2+n, -\sin[c + dx]] \sin[c + dx]^{1+n}}{a^4 d (1+n)} + \frac{4 \sin[c + dx]^{2+n}}{a^4 d (2+n)} - \frac{\sin[c + dx]^{3+n}}{a^4 d (3+n)}$$

Result (type 8, 31 leaves):

$$\int \frac{\cos[c + dx]^7 \sin[c + dx]^n}{(a + a \sin[c + dx])^4} dx$$

■ **Problem 704: Unable to integrate problem.**

$$\int \frac{\cos[c + dx]^7 \sin[c + dx]^n}{(a + a \sin[c + dx])^5} dx$$

Optimal (type 5, 160 leaves, 4 steps):

$$-\frac{4(3 + 2n) \text{Hypergeometric2F1}[1, 1+n, 2+n, -\sin[c + dx]] \sin[c + dx]^{1+n}}{a^5 d (1+n)}$$

$$\frac{\sin[c + dx]^{1+n} (a - a \sin[c + dx])^2}{d(2+n)(a^7 + a^7 \sin[c + dx])} + \frac{\sin[c + dx]^{1+n} (a(27 + 30n + 8n^2) + a(7 + 2n) \sin[c + dx])}{d(2 + 3n + n^2)(a^6 + a^6 \sin[c + dx])}$$

Result (type 8, 31 leaves):

$$\int \frac{\cos[c + dx]^7 \sin[c + dx]^n}{(a + a \sin[c + dx])^5} dx$$

■ **Problem 705: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^8 \sin[c + dx]^5}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 209 leaves, 11 steps):

$$\frac{5x}{1024a} - \frac{\cos[c + dx]^7}{7ad} + \frac{2\cos[c + dx]^9}{9ad} - \frac{\cos[c + dx]^{11}}{11ad} - \frac{5\cos[c + dx]\sin[c + dx]}{1024ad} - \frac{5\cos[c + dx]^3\sin[c + dx]}{1536ad} - \frac{\cos[c + dx]^5\sin[c + dx]}{384ad} + \frac{\cos[c + dx]^7\sin[c + dx]}{64ad} + \frac{\cos[c + dx]^7\sin[c + dx]^3}{24ad} + \frac{\cos[c + dx]^7\sin[c + dx]^5}{12ad}$$

Result (type 3, 1247 leaves):

$$\frac{1}{2048} \left(-\frac{9x}{a} - \frac{16\cos[c]\cos[dx]}{ad} + \frac{4\cos[3c]\cos[3dx]}{ad} - \frac{8\cos[5c]\cos[5dx]}{5ad} + \frac{4\cos[7c]\cos[7dx]}{7ad} + \frac{7\cos[2dx]\sin[2c]}{ad} - \frac{5\cos[4dx]\sin[4c]}{2ad} + \frac{\cos[6dx]\sin[6c]}{ad} - \frac{\cos[8dx]\sin[8c]}{4ad} + \frac{16\sin[c]\sin[dx]}{ad} + \frac{7\cos[2c]\sin[2dx]}{ad} - \frac{4\sin[3c]\sin[3dx]}{ad} - \frac{5\cos[4c]\sin[4dx]}{2ad} + \frac{8\sin[5c]\sin[5dx]}{5ad} + \frac{\cos[6c]\sin[6dx]}{ad} - \frac{4\sin[7c]\sin[7dx]}{7ad} - \frac{\cos[8c]\sin[8dx]}{4ad} + \frac{2\sin\left[\frac{dx}{2}\right]}{ad \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \frac{1}{24576a} \right) + 5 \left(30x + \frac{48\cos[c]\cos[dx]}{d} - \frac{8\cos[3c]\cos[3dx]}{d} - \frac{18\cos[2dx]\sin[2c]}{d} + \frac{3\cos[4dx]\sin[4c]}{d} - \frac{48\sin[c]\sin[dx]}{d} - \frac{18\cos[2c]\sin[2dx]}{d} + \frac{8\sin[3c]\sin[3dx]}{d} + \frac{3\cos[4c]\sin[4dx]}{d} - \frac{12\sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} \right) + \frac{1}{4096a} \left(-25 \left(-3x - \frac{4\cos[c]\cos[dx]}{d} + \frac{\cos[2dx]\sin[2c]}{d} + \frac{4\sin[c]\sin[dx]}{d} + \frac{\cos[2c]\sin[2dx]}{d} + \frac{2\sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} \right) - \frac{1}{61440a} \right) + 7 \left(-210x - \frac{360\cos[c]\cos[dx]}{d} + \frac{80\cos[3c]\cos[3dx]}{d} - \frac{24\cos[5c]\cos[5dx]}{d} + \frac{150\cos[2dx]\sin[2c]}{d} - \frac{45\cos[4dx]\sin[4c]}{d} + \frac{10\cos[6dx]\sin[6c]}{d} + \frac{360\sin[c]\sin[dx]}{d} + \frac{150\cos[2c]\sin[2dx]}{d} - \frac{80\sin[3c]\sin[3dx]}{d} - \frac{45\cos[4c]\sin[4dx]}{d} + \right)$$

$$\begin{aligned}
& \left. \frac{24 \operatorname{Sin}[5 c] \operatorname{Sin}[5 d x]}{d} + \frac{10 \operatorname{Cos}[6 c] \operatorname{Sin}[6 d x]}{d} + \frac{60 \operatorname{Sin}\left[\frac{d x}{2}\right]}{d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} \right\} + \\
& \frac{5 \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) \left((c+d x) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + (-2+c+d x) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}{1024 a d (1+\operatorname{Sin}[c+d x])} + \frac{1}{1720320 a d} \\
& \left(-13860(c+d x) - 25200 \operatorname{Cos}[c+d x] + 6720 \operatorname{Cos}[3(c+d x)] - 3024 \operatorname{Cos}[5(c+d x)] + \right. \\
& \left. 1440 \operatorname{Cos}[7(c+d x)] - 560 \operatorname{Cos}[9(c+d x)] + \frac{2520 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]} + 11340 \operatorname{Sin}[2(c+d x)] - \right. \\
& \left. 4410 \operatorname{Sin}[4(c+d x)] + 2100 \operatorname{Sin}[6(c+d x)] - 945 \operatorname{Sin}[8(c+d x)] + 252 \operatorname{Sin}[10(c+d x)] \right) + \\
& \frac{1}{56770560 a d} \left(180180(c+d x) + 332640 \operatorname{Cos}[c+d x] - 92400 \operatorname{Cos}[3(c+d x)] + 44352 \operatorname{Cos}[5(c+d x)] - 23760 \operatorname{Cos}[7(c+d x)] + \right. \\
& \left. 12320 \operatorname{Cos}[9(c+d x)] - 5040 \operatorname{Cos}[11(c+d x)] - \frac{27720 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]} - 152460 \operatorname{Sin}[2(c+d x)] + \right. \\
& \left. 62370 \operatorname{Sin}[4(c+d x)] - 32340 \operatorname{Sin}[6(c+d x)] + 17325 \operatorname{Sin}[8(c+d x)] - 8316 \operatorname{Sin}[10(c+d x)] + 2310 \operatorname{Sin}[12(c+d x)] \right)
\end{aligned}$$

■ **Problem 706: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x]^8 \operatorname{Sin}[c+d x]^4}{a+a \operatorname{Sin}[c+d x]} dx$$

Optimal (type 3, 183 leaves, 10 steps):

$$\begin{aligned}
& \frac{3 x}{256 a} + \frac{\operatorname{Cos}[c+d x]^7}{7 a d} - \frac{2 \operatorname{Cos}[c+d x]^9}{9 a d} + \frac{\operatorname{Cos}[c+d x]^{11}}{11 a d} + \frac{3 \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{256 a d} + \\
& \frac{\operatorname{Cos}[c+d x]^3 \operatorname{Sin}[c+d x]}{128 a d} + \frac{\operatorname{Cos}[c+d x]^5 \operatorname{Sin}[c+d x]}{160 a d} - \frac{3 \operatorname{Cos}[c+d x]^7 \operatorname{Sin}[c+d x]}{80 a d} - \frac{\operatorname{Cos}[c+d x]^7 \operatorname{Sin}[c+d x]^3}{10 a d}
\end{aligned}$$

Result (type 3, 1084 leaves):

$$\begin{aligned}
& \frac{1}{1024} \left(-\frac{8 x}{a} - \frac{14 \operatorname{Cos}[c] \operatorname{Cos}[d x]}{a d} + \frac{10 \operatorname{Cos}[3 c] \operatorname{Cos}[3 d x]}{3 a d} - \frac{6 \operatorname{Cos}[5 c] \operatorname{Cos}[5 d x]}{5 a d} + \frac{2 \operatorname{Cos}[7 c] \operatorname{Cos}[7 d x]}{7 a d} + \frac{6 \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{a d} - \right. \\
& \left. \frac{2 \operatorname{Cos}[4 d x] \operatorname{Sin}[4 c]}{a d} + \frac{2 \operatorname{Cos}[6 d x] \operatorname{Sin}[6 c]}{3 a d} + \frac{14 \operatorname{Sin}[c] \operatorname{Sin}[d x]}{a d} + \frac{6 \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{a d} - \frac{10 \operatorname{Sin}[3 c] \operatorname{Sin}[3 d x]}{3 a d} - \frac{2 \operatorname{Cos}[4 c] \operatorname{Sin}[4 d x]}{a d} + \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{6 \sin[5c] \sin[5dx]}{5ad} + \frac{2 \cos[6c] \sin[6dx]}{3ad} - \frac{2 \sin[7c] \sin[7dx]}{7ad} + \frac{2 \sin\left[\frac{dx}{2}\right]}{ad \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} \right) - \\
& \frac{1}{2560a} \left(30x + \frac{50 \cos[c] \cos[dx]}{d} - \frac{10 \cos[3c] \cos[3dx]}{d} + \frac{2 \cos[5c] \cos[5dx]}{d} - \frac{20 \cos[2dx] \sin[2c]}{d} + \right. \\
& \frac{5 \cos[4dx] \sin[4c]}{d} - \frac{50 \sin[c] \sin[dx]}{d} - \frac{20 \cos[2c] \sin[2dx]}{d} + \frac{10 \sin[3c] \sin[3dx]}{d} + \\
& \left. \frac{5 \cos[4c] \sin[4dx]}{d} - \frac{2 \sin[5c] \sin[5dx]}{d} - \frac{10 \sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} \right) + \\
& \frac{x + \frac{\cos[c] \cos[dx]}{d} - \frac{\sin[c] \sin[dx]}{d} - \frac{\sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}}{128a} - \frac{1}{3072a} \\
& 17 \left(-6x - \frac{9 \cos[c] \cos[dx]}{d} + \frac{\cos[3c] \cos[3dx]}{d} + \frac{3 \cos[2dx] \sin[2c]}{d} + \frac{9 \sin[c] \sin[dx]}{d} + \frac{3 \cos[2c] \sin[2dx]}{d} - \frac{\sin[3c] \sin[3dx]}{d} + \right. \\
& \left. \frac{3 \sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} \right) + \frac{7 \sin\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}{512d(a + a \sin[c+dx])} + \\
& \frac{1}{64512ad} \left(1260(c+dx) + 2268 \cos[c+dx] - 588 \cos[3(c+dx)] + 252 \cos[5(c+dx)] - 108 \cos[7(c+dx)] + 28 \cos[9(c+dx)] - \right. \\
& \left. \frac{252 \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} - 1008 \sin[2(c+dx)] + 378 \sin[4(c+dx)] - 168 \sin[6(c+dx)] + 63 \sin[8(c+dx)] \right) + \\
& \frac{1}{2365440ad} \left(-13860(c+dx) - 25410 \cos[c+dx] + 6930 \cos[3(c+dx)] - 3234 \cos[5(c+dx)] + 1650 \cos[7(c+dx)] - \right. \\
& \left. 770 \cos[9(c+dx)] + 210 \cos[11(c+dx)] + \frac{2310 \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} + 11550 \sin[2(c+dx)] - \right. \\
& \left. 4620 \sin[4(c+dx)] + 2310 \sin[6(c+dx)] - 1155 \sin[8(c+dx)] + 462 \sin[10(c+dx)] \right)
\end{aligned}$$

■ **Problem 707: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^8 \sin[c+dx]^3}{a + a \sin[c+dx]} dx$$

Optimal (type 3, 165 leaves, 10 steps):

$$-\frac{3x}{256a} - \frac{\cos[c+dx]^7}{7ad} + \frac{\cos[c+dx]^9}{9ad} - \frac{3\cos[c+dx]\sin[c+dx]}{256ad} - \frac{\cos[c+dx]^3\sin[c+dx]}{128ad} - \frac{\cos[c+dx]^5\sin[c+dx]}{160ad} + \frac{3\cos[c+dx]^7\sin[c+dx]}{80ad} + \frac{\cos[c+dx]^7\sin[c+dx]^3}{10ad}$$

Result (type 3, 1078 leaves):

$$\begin{aligned} &-\frac{1}{1024} 5 \left(\frac{9x}{a} + \frac{16\cos[c]\cos[dx]}{ad} - \frac{4\cos[3c]\cos[3dx]}{ad} + \frac{8\cos[5c]\cos[5dx]}{5ad} - \frac{4\cos[7c]\cos[7dx]}{7ad} - \right. \\ &\quad \frac{7\cos[2dx]\sin[2c]}{ad} + \frac{5\cos[4dx]\sin[4c]}{2ad} - \frac{\cos[6dx]\sin[6c]}{ad} + \frac{\cos[8dx]\sin[8c]}{4ad} - \frac{16\sin[c]\sin[dx]}{ad} - \\ &\quad \frac{7\cos[2c]\sin[2dx]}{ad} + \frac{4\sin[3c]\sin[3dx]}{ad} + \frac{5\cos[4c]\sin[4dx]}{2ad} - \frac{8\sin[5c]\sin[5dx]}{5ad} - \frac{\cos[6c]\sin[6dx]}{ad} + \\ &\quad \left. \frac{4\sin[7c]\sin[7dx]}{7ad} + \frac{\cos[8c]\sin[8dx]}{4ad} - \frac{2\sin\left[\frac{dx}{2}\right]}{ad\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} \right) + \frac{1}{6144a} \\ &5 \left(30x + \frac{48\cos[c]\cos[dx]}{d} - \frac{8\cos[3c]\cos[3dx]}{d} - \frac{18\cos[2dx]\sin[2c]}{d} + \frac{3\cos[4dx]\sin[4c]}{d} - \frac{48\sin[c]\sin[dx]}{d} - \right. \\ &\quad \left. \frac{18\cos[2c]\sin[2dx]}{d} + \frac{8\sin[3c]\sin[3dx]}{d} + \frac{3\cos[4c]\sin[4dx]}{d} - \frac{12\sin\left[\frac{dx}{2}\right]}{d\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} \right) + \\ &\frac{1}{512a} 11 \left(-3x - \frac{4\cos[c]\cos[dx]}{d} + \frac{\cos[2dx]\sin[2c]}{d} + \frac{4\sin[c]\sin[dx]}{d} + \frac{\cos[2c]\sin[2dx]}{d} + \right. \\ &\quad \left. \frac{2\sin\left[\frac{dx}{2}\right]}{d\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} \right) - \frac{1}{30720a} \\ &7 \left(-210x - \frac{360\cos[c]\cos[dx]}{d} + \frac{80\cos[3c]\cos[3dx]}{d} - \frac{24\cos[5c]\cos[5dx]}{d} + \frac{150\cos[2dx]\sin[2c]}{d} - \frac{45\cos[4dx]\sin[4c]}{d} + \right. \\ &\quad \frac{10\cos[6dx]\sin[6c]}{d} + \frac{360\sin[c]\sin[dx]}{d} + \frac{150\cos[2c]\sin[2dx]}{d} - \frac{80\sin[3c]\sin[3dx]}{d} - \frac{45\cos[4c]\sin[4dx]}{d} + \\ &\quad \left. \frac{24\sin[5c]\sin[5dx]}{d} + \frac{10\cos[6c]\sin[6dx]}{d} + \frac{60\sin\left[\frac{dx}{2}\right]}{d\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} \right) + \\ &\frac{7\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\left((c+dx)\cos\left[\frac{1}{2}(c+dx)\right] + (-2+c+dx)\sin\left[\frac{1}{2}(c+dx)\right]\right)}{512ad(1+\sin[c+dx])} - \end{aligned}$$

$$\frac{1}{1290240ad} \left(-13860(c+dx) - 25200 \cos[c+dx] + 6720 \cos[3(c+dx)] - 3024 \cos[5(c+dx)] + \right. \\ \left. 1440 \cos[7(c+dx)] - 560 \cos[9(c+dx)] + \frac{2520 \sin\left[\frac{1}{2}(c+dx)\right]}{\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]} + 11340 \sin[2(c+dx)] - \right. \\ \left. 4410 \sin[4(c+dx)] + 2100 \sin[6(c+dx)] - 945 \sin[8(c+dx)] + 252 \sin[10(c+dx)] \right)$$

■ **Problem 708: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^8 \sin[c+dx]^2}{a+a \sin[c+dx]} dx$$

Optimal (type 3, 139 leaves, 9 steps):

$$\frac{5x}{128a} + \frac{\cos[c+dx]^7}{7ad} - \frac{\cos[c+dx]^9}{9ad} + \frac{5 \cos[c+dx] \sin[c+dx]}{128ad} + \\ \frac{5 \cos[c+dx]^3 \sin[c+dx]}{192ad} + \frac{\cos[c+dx]^5 \sin[c+dx]}{48ad} - \frac{\cos[c+dx]^7 \sin[c+dx]}{8ad}$$

Result (type 3, 479 leaves):

$$-\frac{1}{1290240ad} \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \\ \left(2520(c-2dx) \cos\left[\frac{c}{2}\right] - 1512 \cos\left[\frac{c}{2}+dx\right] - 1512 \cos\left[\frac{3c}{2}+dx\right] - 1008 \cos\left[\frac{3c}{2}+2dx\right] + 1008 \cos\left[\frac{5c}{2}+2dx\right] - 672 \cos\left[\frac{5c}{2}+3dx\right] - \right. \\ \left. 672 \cos\left[\frac{7c}{2}+3dx\right] + 504 \cos\left[\frac{7c}{2}+4dx\right] - 504 \cos\left[\frac{9c}{2}+4dx\right] + 336 \cos\left[\frac{11c}{2}+6dx\right] - 336 \cos\left[\frac{13c}{2}+6dx\right] + 108 \cos\left[\frac{13c}{2}+7dx\right] + \right. \\ \left. 108 \cos\left[\frac{15c}{2}+7dx\right] + 63 \cos\left[\frac{15c}{2}+8dx\right] - 63 \cos\left[\frac{17c}{2}+8dx\right] + 28 \cos\left[\frac{17c}{2}+9dx\right] + 28 \cos\left[\frac{19c}{2}+9dx\right] - 7560 \sin\left[\frac{c}{2}\right] + \right. \\ \left. 2520c \sin\left[\frac{c}{2}\right] - 5040dx \sin\left[\frac{c}{2}\right] + 1512 \sin\left[\frac{c}{2}+dx\right] - 1512 \sin\left[\frac{3c}{2}+dx\right] - 1008 \sin\left[\frac{3c}{2}+2dx\right] - 1008 \sin\left[\frac{5c}{2}+2dx\right] + \right. \\ \left. 672 \sin\left[\frac{5c}{2}+3dx\right] - 672 \sin\left[\frac{7c}{2}+3dx\right] + 504 \sin\left[\frac{7c}{2}+4dx\right] + 504 \sin\left[\frac{9c}{2}+4dx\right] + 336 \sin\left[\frac{11c}{2}+6dx\right] + 336 \sin\left[\frac{13c}{2}+6dx\right] - \right. \\ \left. 108 \sin\left[\frac{13c}{2}+7dx\right] + 108 \sin\left[\frac{15c}{2}+7dx\right] + 63 \sin\left[\frac{15c}{2}+8dx\right] + 63 \sin\left[\frac{17c}{2}+8dx\right] - 28 \sin\left[\frac{17c}{2}+9dx\right] + 28 \sin\left[\frac{19c}{2}+9dx\right] \right)$$

■ **Problem 709: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^8 \sin[c + dx]}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 121 leaves, 8 steps):

$$\frac{5x}{128a} - \frac{\cos[c + dx]^7}{7ad} - \frac{5 \cos[c + dx] \sin[c + dx]}{128ad} - \frac{5 \cos[c + dx]^3 \sin[c + dx]}{192ad} - \frac{\cos[c + dx]^5 \sin[c + dx]}{48ad} + \frac{\cos[c + dx]^7 \sin[c + dx]}{8ad}$$

Result (type 3, 481 leaves):

$$\begin{aligned} & - \frac{1}{43008ad \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \\ & \left(-336(7c - 5dx) \cos\left[\frac{c}{2}\right] + 1680 \cos\left[\frac{c}{2} + dx\right] + 1680 \cos\left[\frac{3c}{2} + dx\right] + 336 \cos\left[\frac{3c}{2} + 2dx\right] - 336 \cos\left[\frac{5c}{2} + 2dx\right] + 1008 \cos\left[\frac{5c}{2} + 3dx\right] + \right. \\ & 1008 \cos\left[\frac{7c}{2} + 3dx\right] - 168 \cos\left[\frac{7c}{2} + 4dx\right] + 168 \cos\left[\frac{9c}{2} + 4dx\right] + 336 \cos\left[\frac{9c}{2} + 5dx\right] + 336 \cos\left[\frac{11c}{2} + 5dx\right] - 112 \cos\left[\frac{11c}{2} + 6dx\right] + \\ & 112 \cos\left[\frac{13c}{2} + 6dx\right] + 48 \cos\left[\frac{13c}{2} + 7dx\right] + 48 \cos\left[\frac{15c}{2} + 7dx\right] - 21 \cos\left[\frac{15c}{2} + 8dx\right] + 21 \cos\left[\frac{17c}{2} + 8dx\right] + 4704 \sin\left[\frac{c}{2}\right] - \\ & 2352c \sin\left[\frac{c}{2}\right] + 1680dx \sin\left[\frac{c}{2}\right] - 1680 \sin\left[\frac{c}{2} + dx\right] + 1680 \sin\left[\frac{3c}{2} + dx\right] + 336 \sin\left[\frac{3c}{2} + 2dx\right] + 336 \sin\left[\frac{5c}{2} + 2dx\right] - \\ & 1008 \sin\left[\frac{5c}{2} + 3dx\right] + 1008 \sin\left[\frac{7c}{2} + 3dx\right] - 168 \sin\left[\frac{7c}{2} + 4dx\right] - 168 \sin\left[\frac{9c}{2} + 4dx\right] - 336 \sin\left[\frac{9c}{2} + 5dx\right] + 336 \sin\left[\frac{11c}{2} + 5dx\right] - \\ & \left. 112 \sin\left[\frac{11c}{2} + 6dx\right] - 112 \sin\left[\frac{13c}{2} + 6dx\right] - 48 \sin\left[\frac{13c}{2} + 7dx\right] + 48 \sin\left[\frac{15c}{2} + 7dx\right] - 21 \sin\left[\frac{15c}{2} + 8dx\right] - 21 \sin\left[\frac{17c}{2} + 8dx\right] \right) \end{aligned}$$

■ **Problem 716: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] \cot[c + dx]^7}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 142 leaves, 9 steps):

$$\frac{x}{a} + \frac{5 \operatorname{ArcTanh}[\cos[c + dx]]}{16ad} + \frac{\cot[c + dx]}{ad} - \frac{\cot[c + dx]^3}{3ad} + \frac{\cot[c + dx]^5}{5ad} - \frac{5 \cot[c + dx] \operatorname{Csc}[c + dx]}{16ad} + \frac{5 \cot[c + dx]^3 \operatorname{Csc}[c + dx]}{24ad} - \frac{\cot[c + dx]^5 \operatorname{Csc}[c + dx]}{6ad}$$

Result (type 3, 317 leaves):

$$\begin{aligned}
& - \frac{1}{7680 a d (1 + \sin[c + d x])} \operatorname{Csc}[c + d x]^6 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 \\
& \left(-2400 c - 2400 d x + 900 \operatorname{Cos}[c + d x] + 50 \operatorname{Cos}[3(c + d x)] - 1440 c \operatorname{Cos}[4(c + d x)] - 1440 d x \operatorname{Cos}[4(c + d x)] + 330 \operatorname{Cos}[5(c + d x)] + \right. \\
& 240 c \operatorname{Cos}[6(c + d x)] + 240 d x \operatorname{Cos}[6(c + d x)] - 750 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] - 450 \operatorname{Cos}[4(c + d x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] + \\
& 75 \operatorname{Cos}[6(c + d x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] + 225 \operatorname{Cos}[2(c + d x)] \left(16(c + d x) + 5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] - 5 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \left. \right) + \\
& 750 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 450 \operatorname{Cos}[4(c + d x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \\
& \left. 75 \operatorname{Cos}[6(c + d x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 1200 \operatorname{Sin}[2(c + d x)] + 768 \operatorname{Sin}[4(c + d x)] - 368 \operatorname{Sin}[6(c + d x)] \right)
\end{aligned}$$

■ **Problem 717: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c + d x]^8}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 106 leaves, 7 steps):

$$- \frac{5 \operatorname{ArcTanh}[\operatorname{Cos}[c + d x]]}{16 a d} - \frac{\operatorname{Cot}[c + d x]^7}{7 a d} + \frac{5 \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]}{16 a d} - \frac{5 \operatorname{Cot}[c + d x]^3 \operatorname{Csc}[c + d x]}{24 a d} + \frac{\operatorname{Cot}[c + d x]^5 \operatorname{Csc}[c + d x]}{6 a d}$$

Result (type 3, 284 leaves):

$$\begin{aligned}
& - \frac{1}{86016 a d (1 + \operatorname{Sin}[c + d x])} \\
& \operatorname{Csc}[c + d x]^5 \left(\operatorname{Csc}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \right)^2 \left(1680 \operatorname{Cos}[c + d x] + 1008 \operatorname{Cos}[3(c + d x)] + 336 \operatorname{Cos}[5(c + d x)] + 48 \operatorname{Cos}[7(c + d x)] + \right. \\
& 3675 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[c + d x] - 3675 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[c + d x] - 1190 \operatorname{Sin}[2(c + d x)] - \\
& 2205 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[3(c + d x)] + 2205 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[3(c + d x)] + \\
& 392 \operatorname{Sin}[4(c + d x)] + 735 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[5(c + d x)] - 735 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[5(c + d x)] - \\
& \left. 462 \operatorname{Sin}[6(c + d x)] - 105 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[7(c + d x)] + 105 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sin}[7(c + d x)] \right)
\end{aligned}$$

■ **Problem 718: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c + d x]^8 \operatorname{Csc}[c + d x]}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 134 leaves, 8 steps):

$$\frac{5 \operatorname{ArcTanh}[\cos[c + dx]]}{128 ad} + \frac{\cot[c + dx]^7}{7 ad} + \frac{5 \cot[c + dx] \operatorname{Csc}[c + dx]}{128 ad} -$$

$$\frac{5 \cot[c + dx] \operatorname{Csc}[c + dx]^3}{64 ad} + \frac{5 \cot[c + dx]^3 \operatorname{Csc}[c + dx]^3}{48 ad} - \frac{\cot[c + dx]^5 \operatorname{Csc}[c + dx]^3}{8 ad}$$

Result (type 3, 291 leaves):

$$\frac{1}{344064 ad} \operatorname{Csc}[c + dx]^8 \left(-24710 \cos[c + dx] - 12530 \cos[3(c + dx)] - 5558 \cos[5(c + dx)] - 210 \cos[7(c + dx)] + \right.$$

$$3675 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - 5880 \cos[2(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + 2940 \cos[4(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] -$$

$$840 \cos[6(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + 105 \cos[8(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - 3675 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] +$$

$$5880 \cos[2(c + dx)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] - 2940 \cos[4(c + dx)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] + 840 \cos[6(c + dx)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] -$$

$$\left. 105 \cos[8(c + dx)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] + 5376 \sin[2(c + dx)] + 5376 \sin[4(c + dx)] + 2304 \sin[6(c + dx)] + 384 \sin[8(c + dx)] \right)$$

■ **Problem 719: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c + dx]^8 \operatorname{Csc}[c + dx]^2}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 152 leaves, 9 steps):

$$-\frac{5 \operatorname{ArcTanh}[\cos[c + dx]]}{128 ad} - \frac{\cot[c + dx]^7}{7 ad} - \frac{\cot[c + dx]^9}{9 ad} - \frac{5 \cot[c + dx] \operatorname{Csc}[c + dx]}{128 ad} +$$

$$\frac{5 \cot[c + dx] \operatorname{Csc}[c + dx]^3}{64 ad} - \frac{5 \cot[c + dx]^3 \operatorname{Csc}[c + dx]^3}{48 ad} + \frac{\cot[c + dx]^5 \operatorname{Csc}[c + dx]^3}{8 ad}$$

Result (type 3, 313 leaves):

$$\begin{aligned}
& - \frac{1}{2064384ad} \operatorname{Csc}[c+dx]^9 \left(129024 \operatorname{Cos}[c+dx] + 75264 \operatorname{Cos}[3(c+dx)] + 23040 \operatorname{Cos}[5(c+dx)] + 2304 \operatorname{Cos}[7(c+dx)] - \right. \\
& \quad 256 \operatorname{Cos}[9(c+dx)] + 39690 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] - 39690 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] - \\
& \quad 36540 \operatorname{Sin}[2(c+dx)] - 26460 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] + 26460 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] - \\
& \quad 20916 \operatorname{Sin}[4(c+dx)] + 11340 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[5(c+dx)] - 11340 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[5(c+dx)] - \\
& \quad 16044 \operatorname{Sin}[6(c+dx)] - 2835 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[7(c+dx)] + 2835 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[7(c+dx)] - \\
& \quad \left. 630 \operatorname{Sin}[8(c+dx)] + 315 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[9(c+dx)] - 315 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[9(c+dx)] \right)
\end{aligned}$$

■ **Problem 720: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^8 \operatorname{Csc}[c+dx]^3}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 176 leaves, 10 steps):

$$\begin{aligned}
& \frac{3 \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{256ad} + \frac{\operatorname{Cot}[c+dx]^7}{7ad} + \frac{\operatorname{Cot}[c+dx]^9}{9ad} + \frac{3 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{256ad} + \\
& \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{128ad} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^5}{32ad} + \frac{\operatorname{Cot}[c+dx]^3 \operatorname{Csc}[c+dx]^5}{16ad} - \frac{\operatorname{Cot}[c+dx]^5 \operatorname{Csc}[c+dx]^5}{10ad}
\end{aligned}$$

Result (type 3, 386 leaves):

$$\begin{aligned}
& - \frac{1}{165150720ad(1+\operatorname{Csc}[c+dx])} \operatorname{Csc}[c+dx]^9 \left(\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \right)^2 \\
& \left(2367540 \operatorname{Cos}[c+dx] + 1307880 \operatorname{Cos}[3(c+dx)] + 436968 \operatorname{Cos}[5(c+dx)] + 18270 \operatorname{Cos}[7(c+dx)] - 1890 \operatorname{Cos}[9(c+dx)] - \right. \\
& \quad 119070 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] + 198450 \operatorname{Cos}[2(c+dx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] - 113400 \operatorname{Cos}[4(c+dx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] + \\
& \quad 42525 \operatorname{Cos}[6(c+dx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] - 9450 \operatorname{Cos}[8(c+dx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] + 945 \operatorname{Cos}[10(c+dx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \left. \right) + \\
& \quad 119070 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 198450 \operatorname{Cos}[2(c+dx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 113400 \operatorname{Cos}[4(c+dx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \\
& \quad 42525 \operatorname{Cos}[6(c+dx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 9450 \operatorname{Cos}[8(c+dx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 945 \operatorname{Cos}[10(c+dx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left. \right) - \\
& \quad 537600 \operatorname{Sin}[2(c+dx)] - 522240 \operatorname{Sin}[4(c+dx)] - 207360 \operatorname{Sin}[6(c+dx)] - 25600 \operatorname{Sin}[8(c+dx)] + 2560 \operatorname{Sin}[10(c+dx)] \left. \right)
\end{aligned}$$

■ **Problem 722: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^8 \sin[c + dx]^5}{(a + a \sin[c + dx])^2} dx$$

Optimal (type 3, 203 leaves, 15 steps):

$$\frac{3x}{128a^2} - \frac{2\cos[c + dx]^5}{5a^2d} + \frac{5\cos[c + dx]^7}{7a^2d} - \frac{4\cos[c + dx]^9}{9a^2d} + \frac{\cos[c + dx]^{11}}{11a^2d} - \frac{3\cos[c + dx]\sin[c + dx]}{128a^2d} - \frac{\cos[c + dx]^3\sin[c + dx]}{64a^2d} + \frac{\cos[c + dx]^5\sin[c + dx]}{16a^2d} + \frac{\cos[c + dx]^5\sin[c + dx]^3}{8a^2d} + \frac{\cos[c + dx]^5\sin[c + dx]^5}{5a^2d}$$

Result (type 3, 1469 leaves):

$$\frac{5(-3 + \cos[c + dx] + \cos[2(c + dx)] - 4\sin[c + dx] + \sin[2(c + dx)])}{3072a^2d(1 + \sin[c + dx])^2} + \frac{1}{86016a^2d} \left(27720(c + dx) + 41580\cos[c + dx] - 7056\cos[3(c + dx)] + 1764\cos[5(c + dx)] - 360\cos[7(c + dx)] + 28\cos[9(c + dx)] + \frac{42\sin[\frac{1}{2}(c + dx)]}{(\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)])^3} - \frac{21}{(\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)])^2} - \frac{15204\sin[\frac{1}{2}(c + dx)]}{\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)]} - 15120\sin[2(c + dx)] + 3528\sin[4(c + dx)] - 840\sin[6(c + dx)] + 126\sin[8(c + dx)] \right) + \frac{1}{2027520a^2d} \left(-360360(c + dx) - 566280\cos[c + dx] + 108900\cos[3(c + dx)] - 33264\cos[5(c + dx)] + 9900\cos[7(c + dx)] - 2200\cos[9(c + dx)] + 180\cos[11(c + dx)] - \frac{330\sin[\frac{1}{2}(c + dx)]}{(\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)])^3} + \frac{165}{(\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)])^2} + \frac{166980\sin[\frac{1}{2}(c + dx)]}{\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)]} + 217800\sin[2(c + dx)] - 59400\sin[4(c + dx)] + 18480\sin[6(c + dx)] - 4950\sin[8(c + dx)] + 792\sin[10(c + dx)] \right) + \left(25 \left(36dx\cos\left[\frac{dx}{2}\right] - 21\cos\left[c + \frac{dx}{2}\right] + 35\cos\left[c + \frac{3dx}{2}\right] - 12dx\cos\left[2c + \frac{3dx}{2}\right] - 3\cos\left[3c + \frac{5dx}{2}\right] - 57\sin\left[\frac{dx}{2}\right] + 36dx\sin\left[c + \frac{dx}{2}\right] + 12dx\sin\left[c + \frac{3dx}{2}\right] + 9\sin\left[2c + \frac{3dx}{2}\right] + 3\sin\left[2c + \frac{5dx}{2}\right] \right) / \left(12288a^2d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3 \right) + \left(5 \left(180dx\cos\left[\frac{dx}{2}\right] - 21\cos\left[c + \frac{dx}{2}\right] + 147\cos\left[c + \frac{3dx}{2}\right] - 60dx\cos\left[2c + \frac{3dx}{2}\right] - 15\cos\left[3c + \frac{5dx}{2}\right] + 3\cos\left[3c + \frac{7dx}{2}\right] + \cos\left[5c + \frac{9dx}{2}\right] - 201 \right)$$

$$\begin{aligned} & \left(\sin\left[\frac{dx}{2}\right] + 180 dx \sin\left[c + \frac{dx}{2}\right] + 60 dx \sin\left[c + \frac{3dx}{2}\right] + 73 \sin\left[2c + \frac{3dx}{2}\right] + 15 \sin\left[2c + \frac{5dx}{2}\right] + 3 \sin\left[4c + \frac{7dx}{2}\right] - \sin\left[4c + \frac{9dx}{2}\right] \right) / \\ & \left(12288 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \right) - \frac{1}{30720 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3} \\ & 7 \left(2520 dx \cos\left[\frac{dx}{2}\right] + 165 \cos\left[c + \frac{dx}{2}\right] + 1905 \cos\left[c + \frac{3dx}{2}\right] - 840 dx \cos\left[2c + \frac{3dx}{2}\right] - 210 \cos\left[3c + \frac{5dx}{2}\right] + 42 \cos\left[3c + \frac{7dx}{2}\right] + \right. \\ & 14 \cos\left[5c + \frac{9dx}{2}\right] - 6 \cos\left[5c + \frac{11dx}{2}\right] - 3 \cos\left[7c + \frac{13dx}{2}\right] - 2355 \sin\left[\frac{dx}{2}\right] + 2520 dx \sin\left[c + \frac{dx}{2}\right] + 840 dx \sin\left[c + \frac{3dx}{2}\right] + \\ & \left. 1175 \sin\left[2c + \frac{3dx}{2}\right] + 210 \sin\left[2c + \frac{5dx}{2}\right] + 42 \sin\left[4c + \frac{7dx}{2}\right] - 14 \sin\left[4c + \frac{9dx}{2}\right] - 6 \sin\left[6c + \frac{11dx}{2}\right] + 3 \sin\left[6c + \frac{13dx}{2}\right] \right) + \\ & \frac{1}{43008 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3} \\ & \left(7560 dx \cos\left[\frac{dx}{2}\right] + 1239 \cos\left[c + \frac{dx}{2}\right] + 5467 \cos\left[c + \frac{3dx}{2}\right] - 2520 dx \cos\left[2c + \frac{3dx}{2}\right] - 630 \cos\left[3c + \frac{5dx}{2}\right] + \right. \\ & 126 \cos\left[3c + \frac{7dx}{2}\right] + 42 \cos\left[5c + \frac{9dx}{2}\right] - 18 \cos\left[5c + \frac{11dx}{2}\right] - 9 \cos\left[7c + \frac{13dx}{2}\right] + 5 \cos\left[7c + \frac{15dx}{2}\right] + 3 \cos\left[9c + \frac{17dx}{2}\right] - \\ & 6321 \sin\left[\frac{dx}{2}\right] + 7560 dx \sin\left[c + \frac{dx}{2}\right] + 2520 dx \sin\left[c + \frac{3dx}{2}\right] + 3773 \sin\left[2c + \frac{3dx}{2}\right] + 630 \sin\left[2c + \frac{5dx}{2}\right] + \\ & \left. 126 \sin\left[4c + \frac{7dx}{2}\right] - 42 \sin\left[4c + \frac{9dx}{2}\right] - 18 \sin\left[6c + \frac{11dx}{2}\right] + 9 \sin\left[6c + \frac{13dx}{2}\right] + 5 \sin\left[8c + \frac{15dx}{2}\right] - 3 \sin\left[8c + \frac{17dx}{2}\right] \right) \end{aligned}$$

■ **Problem 723: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^8 \sin[c+dx]^4}{(a+a \sin[c+dx])^2} dx$$

Optimal (type 3, 185 leaves, 17 steps):

$$\begin{aligned} & \frac{9x}{256a^2} + \frac{2 \cos[c+dx]^5}{5a^2d} - \frac{4 \cos[c+dx]^7}{7a^2d} + \frac{2 \cos[c+dx]^9}{9a^2d} + \frac{9 \cos[c+dx] \sin[c+dx]}{256a^2d} + \\ & \frac{3 \cos[c+dx]^3 \sin[c+dx]}{128a^2d} - \frac{3 \cos[c+dx]^5 \sin[c+dx]}{32a^2d} - \frac{3 \cos[c+dx]^5 \sin[c+dx]^3}{16a^2d} - \frac{\cos[c+dx]^5 \sin[c+dx]^5}{10a^2d} \end{aligned}$$

Result (type 3, 585 leaves):

$$\begin{aligned}
& \frac{1}{1290240 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \\
& \left(-2520 (187 c - 18 d x) \cos\left[\frac{c}{2}\right] + 30240 \cos\left[\frac{c}{2} + d x\right] + 30240 \cos\left[\frac{3c}{2} + d x\right] - 1260 \cos\left[\frac{3c}{2} + 2 d x\right] + 1260 \cos\left[\frac{5c}{2} + 2 d x\right] + 6720 \cos\left[\frac{5c}{2} + 3 d x\right] + \right. \\
& 6720 \cos\left[\frac{7c}{2} + 3 d x\right] - 7560 \cos\left[\frac{7c}{2} + 4 d x\right] + 7560 \cos\left[\frac{9c}{2} + 4 d x\right] - 4032 \cos\left[\frac{9c}{2} + 5 d x\right] - 4032 \cos\left[\frac{11c}{2} + 5 d x\right] + \\
& 630 \cos\left[\frac{11c}{2} + 6 d x\right] - 630 \cos\left[\frac{13c}{2} + 6 d x\right] - 720 \cos\left[\frac{13c}{2} + 7 d x\right] - 720 \cos\left[\frac{15c}{2} + 7 d x\right] + 945 \cos\left[\frac{15c}{2} + 8 d x\right] - 945 \cos\left[\frac{17c}{2} + 8 d x\right] + \\
& 560 \cos\left[\frac{17c}{2} + 9 d x\right] + 560 \cos\left[\frac{19c}{2} + 9 d x\right] - 126 \cos\left[\frac{19c}{2} + 10 d x\right] + 126 \cos\left[\frac{21c}{2} + 10 d x\right] + 327180 \sin\left[\frac{c}{2}\right] - 471240 c \sin\left[\frac{c}{2}\right] + \\
& 45360 d x \sin\left[\frac{c}{2}\right] - 30240 \sin\left[\frac{c}{2} + d x\right] + 30240 \sin\left[\frac{3c}{2} + d x\right] - 1260 \sin\left[\frac{3c}{2} + 2 d x\right] - 1260 \sin\left[\frac{5c}{2} + 2 d x\right] - 6720 \sin\left[\frac{5c}{2} + 3 d x\right] + \\
& 6720 \sin\left[\frac{7c}{2} + 3 d x\right] - 7560 \sin\left[\frac{7c}{2} + 4 d x\right] - 7560 \sin\left[\frac{9c}{2} + 4 d x\right] + 4032 \sin\left[\frac{9c}{2} + 5 d x\right] - 4032 \sin\left[\frac{11c}{2} + 5 d x\right] + \\
& 630 \sin\left[\frac{11c}{2} + 6 d x\right] + 630 \sin\left[\frac{13c}{2} + 6 d x\right] + 720 \sin\left[\frac{13c}{2} + 7 d x\right] - 720 \sin\left[\frac{15c}{2} + 7 d x\right] + 945 \sin\left[\frac{15c}{2} + 8 d x\right] + \\
& \left. 945 \sin\left[\frac{17c}{2} + 8 d x\right] - 560 \sin\left[\frac{17c}{2} + 9 d x\right] + 560 \sin\left[\frac{19c}{2} + 9 d x\right] - 126 \sin\left[\frac{19c}{2} + 10 d x\right] - 126 \sin\left[\frac{21c}{2} + 10 d x\right] \right)
\end{aligned}$$

■ **Problem 724: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^8 \sin[c + d x]^3}{(a + a \sin[c + d x])^2} dx$$

Optimal (type 3, 159 leaves, 14 steps):

$$\begin{aligned}
& -\frac{3x}{64a^2} - \frac{2\cos[c + dx]^5}{5a^2d} + \frac{3\cos[c + dx]^7}{7a^2d} - \frac{\cos[c + dx]^9}{9a^2d} - \frac{3\cos[c + dx]\sin[c + dx]}{64a^2d} - \\
& \frac{\cos[c + dx]^3\sin[c + dx]}{32a^2d} + \frac{\cos[c + dx]^5\sin[c + dx]}{8a^2d} + \frac{\cos[c + dx]^5\sin[c + dx]^3}{4a^2d}
\end{aligned}$$

Result (type 3, 429 leaves):

$$\begin{aligned}
& - \frac{1}{322\,560\,a^2\,d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \\
& \left(2520 (55\,c + 6\,d\,x) \cos\left[\frac{c}{2}\right] + 11\,340 \cos\left[\frac{c}{2} + d\,x\right] + 11\,340 \cos\left[\frac{3\,c}{2} + d\,x\right] + 3360 \cos\left[\frac{5\,c}{2} + 3\,d\,x\right] + 3360 \cos\left[\frac{7\,c}{2} + 3\,d\,x\right] - \right. \\
& 2520 \cos\left[\frac{7\,c}{2} + 4\,d\,x\right] + 2520 \cos\left[\frac{9\,c}{2} + 4\,d\,x\right] - 1008 \cos\left[\frac{9\,c}{2} + 5\,d\,x\right] - 1008 \cos\left[\frac{11\,c}{2} + 5\,d\,x\right] - 450 \cos\left[\frac{13\,c}{2} + 7\,d\,x\right] - \\
& 450 \cos\left[\frac{15\,c}{2} + 7\,d\,x\right] + 315 \cos\left[\frac{15\,c}{2} + 8\,d\,x\right] - 315 \cos\left[\frac{17\,c}{2} + 8\,d\,x\right] + 70 \cos\left[\frac{17\,c}{2} + 9\,d\,x\right] + 70 \cos\left[\frac{19\,c}{2} + 9\,d\,x\right] - 81\,900 \sin\left[\frac{c}{2}\right] + \\
& 138\,600\,c \sin\left[\frac{c}{2}\right] + 15\,120\,d\,x \sin\left[\frac{c}{2}\right] - 11\,340 \sin\left[\frac{c}{2} + d\,x\right] + 11\,340 \sin\left[\frac{3\,c}{2} + d\,x\right] - 3360 \sin\left[\frac{5\,c}{2} + 3\,d\,x\right] + 3360 \sin\left[\frac{7\,c}{2} + 3\,d\,x\right] - \\
& 2520 \sin\left[\frac{7\,c}{2} + 4\,d\,x\right] - 2520 \sin\left[\frac{9\,c}{2} + 4\,d\,x\right] + 1008 \sin\left[\frac{9\,c}{2} + 5\,d\,x\right] - 1008 \sin\left[\frac{11\,c}{2} + 5\,d\,x\right] + 450 \sin\left[\frac{13\,c}{2} + 7\,d\,x\right] - \\
& \left. 450 \sin\left[\frac{15\,c}{2} + 7\,d\,x\right] + 315 \sin\left[\frac{15\,c}{2} + 8\,d\,x\right] + 315 \sin\left[\frac{17\,c}{2} + 8\,d\,x\right] - 70 \sin\left[\frac{17\,c}{2} + 9\,d\,x\right] + 70 \sin\left[\frac{19\,c}{2} + 9\,d\,x\right] \right)
\end{aligned}$$

■ **Problem 725: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d\,x]^8 \sin[c + d\,x]^2}{(a + a \sin[c + d\,x])^2} dx$$

Optimal (type 3, 141 leaves, 15 steps):

$$\begin{aligned}
& \frac{11\,x}{128\,a^2} + \frac{2 \cos[c + d\,x]^5}{5\,a^2\,d} - \frac{2 \cos[c + d\,x]^7}{7\,a^2\,d} + \frac{11 \cos[c + d\,x] \sin[c + d\,x]}{128\,a^2\,d} + \\
& \frac{11 \cos[c + d\,x]^3 \sin[c + d\,x]}{192\,a^2\,d} - \frac{11 \cos[c + d\,x]^5 \sin[c + d\,x]}{48\,a^2\,d} - \frac{\cos[c + d\,x]^5 \sin[c + d\,x]^3}{8\,a^2\,d}
\end{aligned}$$

Result (type 3, 481 leaves):

$$\frac{1}{215\,040 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)}$$

$$\left(9240 (15 c + 2 d x) \cos\left[\frac{c}{2}\right] + 10\,080 \cos\left[\frac{c}{2} + d x\right] + 10\,080 \cos\left[\frac{3 c}{2} + d x\right] + 1680 \cos\left[\frac{3 c}{2} + 2 d x\right] - 1680 \cos\left[\frac{5 c}{2} + 2 d x\right] + 3360 \cos\left[\frac{5 c}{2} + 3 d x\right] + \right.$$

$$3360 \cos\left[\frac{7 c}{2} + 3 d x\right] - 2520 \cos\left[\frac{7 c}{2} + 4 d x\right] + 2520 \cos\left[\frac{9 c}{2} + 4 d x\right] - 672 \cos\left[\frac{9 c}{2} + 5 d x\right] - 672 \cos\left[\frac{11 c}{2} + 5 d x\right] - 560 \cos\left[\frac{11 c}{2} + 6 d x\right] +$$

$$560 \cos\left[\frac{13 c}{2} + 6 d x\right] - 480 \cos\left[\frac{13 c}{2} + 7 d x\right] - 480 \cos\left[\frac{15 c}{2} + 7 d x\right] + 105 \cos\left[\frac{15 c}{2} + 8 d x\right] - 105 \cos\left[\frac{17 c}{2} + 8 d x\right] - 79\,800 \sin\left[\frac{c}{2}\right] +$$

$$138\,600 c \sin\left[\frac{c}{2}\right] + 18\,480 d x \sin\left[\frac{c}{2}\right] - 10\,080 \sin\left[\frac{c}{2} + d x\right] + 10\,080 \sin\left[\frac{3 c}{2} + d x\right] + 1680 \sin\left[\frac{3 c}{2} + 2 d x\right] + 1680 \sin\left[\frac{5 c}{2} + 2 d x\right] -$$

$$3360 \sin\left[\frac{5 c}{2} + 3 d x\right] + 3360 \sin\left[\frac{7 c}{2} + 3 d x\right] - 2520 \sin\left[\frac{7 c}{2} + 4 d x\right] - 2520 \sin\left[\frac{9 c}{2} + 4 d x\right] + 672 \sin\left[\frac{9 c}{2} + 5 d x\right] - 672 \sin\left[\frac{11 c}{2} + 5 d x\right] -$$

$$\left. 560 \sin\left[\frac{11 c}{2} + 6 d x\right] - 560 \sin\left[\frac{13 c}{2} + 6 d x\right] + 480 \sin\left[\frac{13 c}{2} + 7 d x\right] - 480 \sin\left[\frac{15 c}{2} + 7 d x\right] + 105 \sin\left[\frac{15 c}{2} + 8 d x\right] + 105 \sin\left[\frac{17 c}{2} + 8 d x\right] \right)$$

■ **Problem 726: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^8 \sin[c + d x]}{(a + a \sin[c + d x])^2} dx$$

Optimal (type 3, 124 leaves, 6 steps):

$$-\frac{x}{8 a^2} - \frac{2 \cos[c + d x]^7}{35 a^2 d} - \frac{\cos[c + d x] \sin[c + d x]}{8 a^2 d} - \frac{\cos[c + d x]^3 \sin[c + d x]}{12 a^2 d} - \frac{\cos[c + d x]^5 \sin[c + d x]}{15 a^2 d} - \frac{\cos[c + d x]^9}{5 d (a + a \sin[c + d x])^2}$$

Result (type 3, 414 leaves):

$$-\frac{1}{13\,440 a^2 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)}$$

$$\left(1680 d x \cos\left[\frac{c}{2}\right] + 1155 \cos\left[\frac{c}{2} + d x\right] + 1155 \cos\left[\frac{3 c}{2} + d x\right] + 210 \cos\left[\frac{3 c}{2} + 2 d x\right] - 210 \cos\left[\frac{5 c}{2} + 2 d x\right] + 525 \cos\left[\frac{5 c}{2} + 3 d x\right] + \right.$$

$$525 \cos\left[\frac{7 c}{2} + 3 d x\right] - 210 \cos\left[\frac{7 c}{2} + 4 d x\right] + 210 \cos\left[\frac{9 c}{2} + 4 d x\right] + 63 \cos\left[\frac{9 c}{2} + 5 d x\right] + 63 \cos\left[\frac{11 c}{2} + 5 d x\right] - 70 \cos\left[\frac{11 c}{2} + 6 d x\right] +$$

$$70 \cos\left[\frac{13 c}{2} + 6 d x\right] - 15 \cos\left[\frac{13 c}{2} + 7 d x\right] - 15 \cos\left[\frac{15 c}{2} + 7 d x\right] - 980 \sin\left[\frac{c}{2}\right] + 1680 d x \sin\left[\frac{c}{2}\right] - 1155 \sin\left[\frac{c}{2} + d x\right] + 1155 \sin\left[\frac{3 c}{2} + d x\right] +$$

$$210 \sin\left[\frac{3 c}{2} + 2 d x\right] + 210 \sin\left[\frac{5 c}{2} + 2 d x\right] - 525 \sin\left[\frac{5 c}{2} + 3 d x\right] + 525 \sin\left[\frac{7 c}{2} + 3 d x\right] - 210 \sin\left[\frac{7 c}{2} + 4 d x\right] - 210 \sin\left[\frac{9 c}{2} + 4 d x\right] -$$

$$\left. 63 \sin\left[\frac{9 c}{2} + 5 d x\right] + 63 \sin\left[\frac{11 c}{2} + 5 d x\right] - 70 \sin\left[\frac{11 c}{2} + 6 d x\right] - 70 \sin\left[\frac{13 c}{2} + 6 d x\right] + 15 \sin\left[\frac{13 c}{2} + 7 d x\right] - 15 \sin\left[\frac{15 c}{2} + 7 d x\right] \right)$$

■ **Problem 732: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^2 \cot [c+d x]^6}{(a+a \sin [c+d x])^2} d x$$

Optimal (type 3, 118 leaves, 11 steps):

$$\frac{x}{a^2} + \frac{3 \operatorname{ArcTanh}[\cos [c+d x]]}{4 a^2 d} + \frac{\cot [c+d x]}{a^2 d} - \frac{\cot [c+d x]^3}{3 a^2 d} - \frac{\cot [c+d x]^5}{5 a^2 d} - \frac{3 \cot [c+d x] \operatorname{Csc}[c+d x]}{4 a^2 d} + \frac{\cot [c+d x]^3 \operatorname{Csc}[c+d x]}{2 a^2 d}$$

Result (type 3, 254 leaves):

$$\begin{aligned} & \frac{1}{960 a^2 d} \operatorname{Csc}[c+d x]^5 \\ & \left(-40 \cos [c+d x] - 220 \cos [3(c+d x)] + 68 \cos [5(c+d x)] + 600 c \sin [c+d x] + 600 d x \sin [c+d x] + 450 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right] \sin [c+d x] - \right. \\ & 450 \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right] \sin [c+d x] - 60 \sin [2(c+d x)] - 300 c \sin [3(c+d x)] - 300 d x \sin [3(c+d x)] - \\ & 225 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right] \sin [3(c+d x)] + 225 \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right] \sin [3(c+d x)] + 150 \sin [4(c+d x)] + \\ & \left. 60 c \sin [5(c+d x)] + 60 d x \sin [5(c+d x)] + 45 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right] \sin [5(c+d x)] - 45 \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right] \sin [5(c+d x)] \right) \end{aligned}$$

■ **Problem 734: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot [c+d x]^8}{(a+a \sin [c+d x])^2} d x$$

Optimal (type 3, 124 leaves, 19 steps):

$$\frac{\operatorname{ArcTanh}[\cos [c+d x]]}{8 a^2 d} - \frac{2 \cot [c+d x]^5}{5 a^2 d} - \frac{\cot [c+d x]^7}{7 a^2 d} + \frac{\cot [c+d x] \operatorname{Csc}[c+d x]}{8 a^2 d} - \frac{7 \cot [c+d x] \operatorname{Csc}[c+d x]^3}{12 a^2 d} + \frac{\cot [c+d x] \operatorname{Csc}[c+d x]^5}{3 a^2 d}$$

Result (type 3, 251 leaves):

$$\begin{aligned} & -\frac{1}{53760 a^2 d} \\ & \operatorname{Csc}[c+d x]^7 \left(5880 \cos [c+d x] + 2184 \cos [3(c+d x)] - 168 \cos [5(c+d x)] - 216 \cos [7(c+d x)] - 3675 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right] \sin [c+d x] + \right. \\ & 3675 \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right] \sin [c+d x] - 2170 \sin [2(c+d x)] + 2205 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right] \sin [3(c+d x)] - \\ & 2205 \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right] \sin [3(c+d x)] - 3080 \sin [4(c+d x)] - 735 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right] \sin [5(c+d x)] + 735 \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right] \\ & \left. \sin [5(c+d x)] - 210 \sin [6(c+d x)] + 105 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right] \sin [7(c+d x)] - 105 \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right] \sin [7(c+d x)] \right) \end{aligned}$$

■ **Problem 739: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^8 \sin[c + dx]^3}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 161 leaves, 18 steps):

$$\begin{aligned} & -\frac{29x}{128a^3} - \frac{4\cos[c + dx]^3}{3a^3d} + \frac{7\cos[c + dx]^5}{5a^3d} - \frac{3\cos[c + dx]^7}{7a^3d} - \frac{29\cos[c + dx]\sin[c + dx]}{128a^3d} + \\ & \frac{29\cos[c + dx]^3\sin[c + dx]}{64a^3d} + \frac{29\cos[c + dx]^3\sin[c + dx]^3}{48a^3d} + \frac{\cos[c + dx]^3\sin[c + dx]^5}{8a^3d} \end{aligned}$$

Result (type 3, 481 leaves):

$$\begin{aligned} & \frac{1}{215040a^3d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \\ & \left(840(1287c - 58dx)\cos\left[\frac{c}{2}\right] - 38640\cos\left[\frac{c}{2} + dx\right] - 38640\cos\left[\frac{3c}{2} + dx\right] + 6720\cos\left[\frac{3c}{2} + 2dx\right] - 6720\cos\left[\frac{5c}{2} + 2dx\right] - 3920\cos\left[\frac{5c}{2} + 3dx\right] - \right. \\ & 3920\cos\left[\frac{7c}{2} + 3dx\right] + 5880\cos\left[\frac{7c}{2} + 4dx\right] - 5880\cos\left[\frac{9c}{2} + 4dx\right] + 4368\cos\left[\frac{9c}{2} + 5dx\right] + 4368\cos\left[\frac{11c}{2} + 5dx\right] - 2240\cos\left[\frac{11c}{2} + 6dx\right] + \\ & 2240\cos\left[\frac{13c}{2} + 6dx\right] - 720\cos\left[\frac{13c}{2} + 7dx\right] - 720\cos\left[\frac{15c}{2} + 7dx\right] + 105\cos\left[\frac{15c}{2} + 8dx\right] - 105\cos\left[\frac{17c}{2} + 8dx\right] - 998340\sin\left[\frac{c}{2}\right] + \\ & 1081080c\sin\left[\frac{c}{2}\right] - 48720dx\sin\left[\frac{c}{2}\right] + 38640\sin\left[\frac{c}{2} + dx\right] - 38640\sin\left[\frac{3c}{2} + dx\right] + 6720\sin\left[\frac{3c}{2} + 2dx\right] + 6720\sin\left[\frac{5c}{2} + 2dx\right] + \\ & 3920\sin\left[\frac{5c}{2} + 3dx\right] - 3920\sin\left[\frac{7c}{2} + 3dx\right] + 5880\sin\left[\frac{7c}{2} + 4dx\right] + 5880\sin\left[\frac{9c}{2} + 4dx\right] - 4368\sin\left[\frac{9c}{2} + 5dx\right] + 4368\sin\left[\frac{11c}{2} + 5dx\right] - \\ & \left. 2240\sin\left[\frac{11c}{2} + 6dx\right] - 2240\sin\left[\frac{13c}{2} + 6dx\right] + 720\sin\left[\frac{13c}{2} + 7dx\right] - 720\sin\left[\frac{15c}{2} + 7dx\right] + 105\sin\left[\frac{15c}{2} + 8dx\right] + 105\sin\left[\frac{17c}{2} + 8dx\right] \right) \end{aligned}$$

■ **Problem 740: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^8 \sin[c + dx]^2}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 133 leaves, 16 steps):

$$\frac{5x}{16a^3} + \frac{4\cos[c + dx]^3}{3a^3d} - \frac{\cos[c + dx]^5}{a^3d} + \frac{\cos[c + dx]^7}{7a^3d} + \frac{5\cos[c + dx]\sin[c + dx]}{16a^3d} - \frac{5\cos[c + dx]^3\sin[c + dx]}{8a^3d} - \frac{\cos[c + dx]^3\sin[c + dx]^3}{2a^3d}$$

Result (type 3, 429 leaves):

$$\frac{1}{2688 a^3 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(-168 (99 c - 5 d x) \cos\left[\frac{c}{2}\right] + 609 \cos\left[\frac{c}{2} + d x\right] + 609 \cos\left[\frac{3 c}{2} + d x\right] - 63 \cos\left[\frac{3 c}{2} + 2 d x\right] + 63 \cos\left[\frac{5 c}{2} + 2 d x\right] + 91 \cos\left[\frac{5 c}{2} + 3 d x\right] + 91 \cos\left[\frac{7 c}{2} + 3 d x\right] - 105 \cos\left[\frac{7 c}{2} + 4 d x\right] + 105 \cos\left[\frac{9 c}{2} + 4 d x\right] - 63 \cos\left[\frac{9 c}{2} + 5 d x\right] - 63 \cos\left[\frac{11 c}{2} + 5 d x\right] + 21 \cos\left[\frac{11 c}{2} + 6 d x\right] - 21 \cos\left[\frac{13 c}{2} + 6 d x\right] + 3 \cos\left[\frac{13 c}{2} + 7 d x\right] + 3 \cos\left[\frac{15 c}{2} + 7 d x\right] + 16996 \sin\left[\frac{c}{2}\right] - 16632 c \sin\left[\frac{c}{2}\right] + 840 d x \sin\left[\frac{c}{2}\right] - 609 \sin\left[\frac{c}{2} + d x\right] + 609 \sin\left[\frac{3 c}{2} + d x\right] - 63 \sin\left[\frac{3 c}{2} + 2 d x\right] - 63 \sin\left[\frac{5 c}{2} + 2 d x\right] - 91 \sin\left[\frac{5 c}{2} + 3 d x\right] + 91 \sin\left[\frac{7 c}{2} + 3 d x\right] - 105 \sin\left[\frac{7 c}{2} + 4 d x\right] - 105 \sin\left[\frac{9 c}{2} + 4 d x\right] + 63 \sin\left[\frac{9 c}{2} + 5 d x\right] - 63 \sin\left[\frac{11 c}{2} + 5 d x\right] + 21 \sin\left[\frac{11 c}{2} + 6 d x\right] + 21 \sin\left[\frac{13 c}{2} + 6 d x\right] - 3 \sin\left[\frac{13 c}{2} + 7 d x\right] + 3 \sin\left[\frac{15 c}{2} + 7 d x\right] \right)$$

■ **Problem 741: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^8 \sin[c + d x]}{(a + a \sin[c + d x])^3} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$-\frac{7 x}{16 a^3} - \frac{7 \cos[c + d x]^5}{30 a^3 d} - \frac{7 \cos[c + d x] \sin[c + d x]}{16 a^3 d} - \frac{7 \cos[c + d x]^3 \sin[c + d x]}{24 a^3 d} - \frac{\cos[c + d x]^9}{3 d (a + a \sin[c + d x])^3} - \frac{\cos[c + d x]^7}{6 d (a^3 + a^3 \sin[c + d x])}$$

Result (type 3, 362 leaves):

$$\frac{1}{1920 a^3 d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right)} \left(-840 d x \cos\left[\frac{c}{2}\right] - 600 \cos\left[\frac{c}{2} + d x\right] - 600 \cos\left[\frac{3 c}{2} + d x\right] + 15 \cos\left[\frac{3 c}{2} + 2 d x\right] - 15 \cos\left[\frac{5 c}{2} + 2 d x\right] - 140 \cos\left[\frac{5 c}{2} + 3 d x\right] - 140 \cos\left[\frac{7 c}{2} + 3 d x\right] + 105 \cos\left[\frac{7 c}{2} + 4 d x\right] - 105 \cos\left[\frac{9 c}{2} + 4 d x\right] + 36 \cos\left[\frac{9 c}{2} + 5 d x\right] + 36 \cos\left[\frac{11 c}{2} + 5 d x\right] - 5 \cos\left[\frac{11 c}{2} + 6 d x\right] + 5 \cos\left[\frac{13 c}{2} + 6 d x\right] + 42 \sin\left[\frac{c}{2}\right] - 840 d x \sin\left[\frac{c}{2}\right] + 600 \sin\left[\frac{c}{2} + d x\right] - 600 \sin\left[\frac{3 c}{2} + d x\right] + 15 \sin\left[\frac{3 c}{2} + 2 d x\right] + 15 \sin\left[\frac{5 c}{2} + 2 d x\right] + 140 \sin\left[\frac{5 c}{2} + 3 d x\right] - 140 \sin\left[\frac{7 c}{2} + 3 d x\right] + 105 \sin\left[\frac{7 c}{2} + 4 d x\right] + 105 \sin\left[\frac{9 c}{2} + 4 d x\right] - 36 \sin\left[\frac{9 c}{2} + 5 d x\right] + 36 \sin\left[\frac{11 c}{2} + 5 d x\right] - 5 \sin\left[\frac{11 c}{2} + 6 d x\right] - 5 \sin\left[\frac{13 c}{2} + 6 d x\right] \right)$$

■ **Problem 745: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^4 \cot[c + d x]^4}{(a + a \sin[c + d x])^3} dx$$

Optimal (type 3, 92 leaves, 11 steps) :

$$-\frac{3x}{a^3} - \frac{\text{ArcTanh}[\text{Cos}[c+dx]]}{2a^3d} - \frac{\text{Cos}[c+dx]}{a^3d} - \frac{3\text{Cot}[c+dx]}{a^3d} - \frac{\text{Cot}[c+dx]^3}{3a^3d} + \frac{3\text{Cot}[c+dx]\text{Csc}[c+dx]}{2a^3d}$$

Result (type 3, 538 leaves) :

$$\begin{aligned} & -\frac{3(c+dx)\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{d(a+a\text{Sin}[c+dx])^3} - \frac{\text{Cos}[c+dx]\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{d(a+a\text{Sin}[c+dx])^3} - \\ & \frac{4\text{Cot}\left[\frac{1}{2}(c+dx)\right]\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{3d(a+a\text{Sin}[c+dx])^3} + \frac{3\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a\text{Sin}[c+dx])^3} - \\ & \frac{\text{Cot}\left[\frac{1}{2}(c+dx)\right]\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{24d(a+a\text{Sin}[c+dx])^3} - \frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right]\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{2d(a+a\text{Sin}[c+dx])^3} + \\ & \frac{\text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right]\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{2d(a+a\text{Sin}[c+dx])^3} - \frac{3\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6}{8d(a+a\text{Sin}[c+dx])^3} + \\ & \frac{4\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6\text{Tan}\left[\frac{1}{2}(c+dx)\right]}{3d(a+a\text{Sin}[c+dx])^3} + \frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2\left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6\text{Tan}\left[\frac{1}{2}(c+dx)\right]}{24d(a+a\text{Sin}[c+dx])^3} \end{aligned}$$

■ **Problem 746: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[c+dx]^3 \text{Cot}[c+dx]^5}{(a+a\text{Sin}[c+dx])^3} dx$$

Optimal (type 3, 97 leaves, 12 steps) :

$$\frac{x}{a^3} + \frac{13\text{ArcTanh}[\text{Cos}[c+dx]]}{8a^3d} + \frac{\text{Cot}[c+dx]}{a^3d} + \frac{\text{Cot}[c+dx]^3}{a^3d} - \frac{11\text{Cot}[c+dx]\text{Csc}[c+dx]}{8a^3d} - \frac{\text{Cot}[c+dx]\text{Csc}[c+dx]^3}{4a^3d}$$

Result (type 3, 495 leaves) :

$$\begin{aligned}
& \frac{(c+dx) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{d(a+a\sin[c+dx])^3} - \frac{11 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{32d(a+a\sin[c+dx])^3} + \\
& \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{8d(a+a\sin[c+dx])^3} - \\
& \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{64d(a+a\sin[c+dx])^3} + \frac{13 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{8d(a+a\sin[c+dx])^3} - \\
& \frac{13 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{8d(a+a\sin[c+dx])^3} + \frac{11 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{32d(a+a\sin[c+dx])^3} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}{64d(a+a\sin[c+dx])^3} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{8d(a+a\sin[c+dx])^3}
\end{aligned}$$

■ **Problem 750: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^8 \operatorname{Csc}[c+dx]}{(a+a\sin[c+dx])^3} dx$$

Optimal (type 3, 166 leaves, 18 steps):

$$\begin{aligned}
& \frac{29 \operatorname{ArcTanh}[\cos[c+dx]]}{128a^3d} + \frac{4 \operatorname{Cot}[c+dx]^3}{3a^3d} + \frac{7 \operatorname{Cot}[c+dx]^5}{5a^3d} + \frac{3 \operatorname{Cot}[c+dx]^7}{7a^3d} + \\
& \frac{29 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{128a^3d} + \frac{29 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{192a^3d} - \frac{23 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^5}{48a^3d} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^7}{8a^3d}
\end{aligned}$$

Result (type 3, 1027 leaves):

$$\begin{aligned}
& - \frac{19 \cot\left[\frac{1}{2}(c+dx)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{105 d (a + a \sin[c+dx])^3} + \frac{29 \operatorname{csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{512 d (a + a \sin[c+dx])^3} - \\
& \frac{83 \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{csc}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{6720 d (a + a \sin[c+dx])^3} + \frac{\operatorname{csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{1024 d (a + a \sin[c+dx])^3} + \\
& \frac{23 \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{csc}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{2240 d (a + a \sin[c+dx])^3} - \frac{13 \operatorname{csc}\left[\frac{1}{2}(c+dx)\right]^6 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{1536 d (a + a \sin[c+dx])^3} + \\
& \frac{3 \cot\left[\frac{1}{2}(c+dx)\right] \operatorname{csc}\left[\frac{1}{2}(c+dx)\right]^6 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{896 d (a + a \sin[c+dx])^3} - \frac{\operatorname{csc}\left[\frac{1}{2}(c+dx)\right]^8 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{2048 d (a + a \sin[c+dx])^3} + \\
& \frac{29 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{128 d (a + a \sin[c+dx])^3} - \frac{29 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{128 d (a + a \sin[c+dx])^3} - \\
& \frac{29 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{512 d (a + a \sin[c+dx])^3} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{1024 d (a + a \sin[c+dx])^3} + \\
& \frac{13 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{1536 d (a + a \sin[c+dx])^3} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^8 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6}{2048 d (a + a \sin[c+dx])^3} + \\
& \frac{19 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{105 d (a + a \sin[c+dx])^3} + \frac{83 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{6720 d (a + a \sin[c+dx])^3} - \\
& \frac{23 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2240 d (a + a \sin[c+dx])^3} - \\
& \frac{3 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{896 d (a + a \sin[c+dx])^3}
\end{aligned}$$

■ **Problem 757: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^3 \operatorname{Sec}[c+dx]^2 (a + a \sin[c+dx]) dx$$

Optimal (type 3, 75 leaves, 8 steps):

$$- \frac{3 a \operatorname{ArcTanh}[\cos[c+dx]]}{2 d} - \frac{a \cot[c+dx]}{d} + \frac{3 a \operatorname{Sec}[c+dx]}{2 d} - \frac{a \operatorname{Csc}[c+dx]^2 \operatorname{Sec}[c+dx]}{2 d} + \frac{a \operatorname{Tan}[c+dx]}{d}$$

Result (type 3, 172 leaves):

$$-\frac{2 a \cot [2 (c+d x)]}{d}-\frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{8 d}-\frac{3 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+\frac{3 a \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+$$

$$\frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{8 d}+\frac{a \sin \left[\frac{1}{2}(c+d x)\right]}{d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)}-\frac{a \sin \left[\frac{1}{2}(c+d x)\right]}{d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)}$$

■ **Problem 758: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+d x]^4 \operatorname{Sec}[c+d x]^2 (a+a \sin [c+d x]) dx$$

Optimal (type 3, 91 leaves, 8 steps):

$$-\frac{3 a \operatorname{ArcTanh}\left[\cos [c+d x]\right]}{2 d}-\frac{2 a \cot [c+d x]}{d}-\frac{a \cot [c+d x]^3}{3 d}+\frac{3 a \operatorname{Sec}[c+d x]}{2 d}-\frac{a \operatorname{Csc}[c+d x]^2 \operatorname{Sec}[c+d x]}{2 d}+\frac{a \tan [c+d x]}{d}$$

Result (type 3, 205 leaves):

$$-\frac{5 a \cot [c+d x]}{3 d}-\frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{8 d}-\frac{a \cot [c+d x] \operatorname{Csc}[c+d x]^2}{3 d}-\frac{3 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+\frac{3 a \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+$$

$$\frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{8 d}+\frac{a \sin \left[\frac{1}{2}(c+d x)\right]}{d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)}-\frac{a \sin \left[\frac{1}{2}(c+d x)\right]}{d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)}+\frac{a \tan [c+d x]}{d}$$

■ **Problem 760: Result more than twice size of optimal antiderivative.**

$$\int (a+a \sin [c+d x])^2 \tan [c+d x]^2 dx$$

Optimal (type 3, 71 leaves, 6 steps):

$$-\frac{5 a^2 x}{2}+\frac{2 a^2 \cos [c+d x]}{d}+\frac{2 a^2 \cos [c+d x]}{d(1-\sin [c+d x])}+\frac{a^2 \cos [c+d x] \sin [c+d x]}{2 d}$$

Result (type 3, 145 leaves):

$$-\left(a^2(1+\sin [c+d x])^2\right.$$

$$\left.\left(\cos \left[\frac{1}{2}(c+d x)\right](10(c+d x)-8 \cos [c+d x]-\sin [2(c+d x)])+\sin \left[\frac{1}{2}(c+d x)\right](-2(8+5 c+5 d x)+8 \cos [c+d x]+\sin [2(c+d x)])\right)\right) /$$

$$\left(4 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4\right)$$

■ **Problem 761: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x](a+a \sin [c+d x])^2 \tan [c+d x] dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$-2 a^2 x + \frac{2 a^2 \cos [c+d x]}{d} + \frac{\sec [c+d x] (a+a \sin [c+d x])^2}{d}$$

Result (type 3, 90 leaves):

$$\frac{\left(-2(c+d x) + \cos [c+d x] + \frac{4 \sin \left[\frac{1}{2}(c+d x)\right]}{\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]}\right) (a+a \sin [c+d x])^2}{d \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^4}$$

■ **Problem 767: Result more than twice size of optimal antiderivative.**

$$\int \sec [c+d x] (a+a \sin [c+d x])^3 \tan [c+d x] dx$$

Optimal (type 3, 67 leaves, 2 steps):

$$-\frac{9 a^3 x}{2} + \frac{6 a^3 \cos [c+d x]}{d} + \frac{3 a^3 \cos [c+d x] \sin [c+d x]}{2 d} + \frac{\sec [c+d x] (a+a \sin [c+d x])^3}{d}$$

Result (type 3, 145 leaves):

$$-\left(a^3 (1+\sin [c+d x])^3 \left(\cos \left[\frac{1}{2}(c+d x)\right] (18(c+d x) - 12 \cos [c+d x] - \sin [2(c+d x)]) + \sin \left[\frac{1}{2}(c+d x)\right] (-2(16+9 c+9 d x) + 12 \cos [c+d x] + \sin [2(c+d x)])\right)\right) / \left(4 d \left(\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right) \left(\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right)^6\right)$$

■ **Problem 771: Result more than twice size of optimal antiderivative.**

$$\int \csc [c+d x]^4 \sec [c+d x]^2 (a+a \sin [c+d x])^3 dx$$

Optimal (type 3, 98 leaves, 10 steps):

$$-\frac{11 a^3 \operatorname{ArcTanh}[\cos [c+d x]]}{2 d} - \frac{5 a^3 \cot [c+d x]}{d} - \frac{a^3 \cot [c+d x]^3}{3 d} - \frac{3 a^3 \cot [c+d x] \csc [c+d x]}{2 d} + \frac{4 a^3 \cos [c+d x]}{d (1-\sin [c+d x])}$$

Result (type 3, 211 leaves):

$$a^3 \left(-\frac{7 \cot \left[\frac{1}{2}(c+d x)\right]}{3 d} - \frac{3 \csc \left[\frac{1}{2}(c+d x)\right]^2}{8 d} - \frac{\cot \left[\frac{1}{2}(c+d x)\right] \csc \left[\frac{1}{2}(c+d x)\right]^2}{24 d} - \frac{11 \log [\cos \left[\frac{1}{2}(c+d x)\right]]}{2 d} + \frac{11 \log [\sin \left[\frac{1}{2}(c+d x)\right]]}{2 d} + \frac{3 \sec \left[\frac{1}{2}(c+d x)\right]^2}{8 d} + \frac{8 \sin \left[\frac{1}{2}(c+d x)\right]}{d \left(\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right)} + \frac{7 \tan \left[\frac{1}{2}(c+d x)\right]}{3 d} + \frac{\sec \left[\frac{1}{2}(c+d x)\right]^2 \tan \left[\frac{1}{2}(c+d x)\right]}{24 d} \right)$$

■ **Problem 774: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c + dx]^2}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 50 leaves, 5 steps):

$$\frac{\sec[c + dx]}{ad} - \frac{\sec[c + dx]^3}{3ad} + \frac{\tan[c + dx]^3}{3ad}$$

Result (type 3, 106 leaves):

$$\frac{6 - 10 \cos[c + dx] + 2 \cos[2(c + dx)] + 8 \sin[c + dx] - 5 \sin[2(c + dx)]}{12ad \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) (1 + \sin[c + dx])}$$

■ **Problem 775: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx] \tan[c + dx]}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 37 leaves, 5 steps):

$$\frac{\sec[c + dx]^3}{3ad} - \frac{\tan[c + dx]^3}{3ad}$$

Result (type 3, 104 leaves):

$$\frac{-3 + \cos[c + dx] + \cos[2(c + dx)] - 2 \sin[c + dx] + \frac{1}{2} \sin[2(c + dx)]}{6ad \left(-\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) (1 + \sin[c + dx])}$$

■ **Problem 777: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c + dx]^2 \sec[c + dx]^2}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 93 leaves, 8 steps):

$$\frac{\operatorname{ArcTanh}[\cos[c + dx]]}{ad} - \frac{\cot[c + dx]}{ad} - \frac{\sec[c + dx]}{ad} - \frac{\sec[c + dx]^3}{3ad} + \frac{2 \tan[c + dx]}{ad} + \frac{\tan[c + dx]^3}{3ad}$$

Result (type 3, 245 leaves):

$$\begin{aligned}
& - \left(\left(\text{Csc}[c + dx]^3 \left(2 + 10 \text{Cos}[2(c + dx)] + 8 \text{Cos}[3(c + dx)] + 3 \text{Cos}[3(c + dx)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] - 3 \text{Cos}[3(c + dx)] \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right. \right. \\
& \quad \left. \left. \text{Cos}[c + dx] \left(-8 - 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] + 3 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) \right) + 4 \text{Sin}[c + dx] - 16 \text{Sin}[2(c + dx)] - \right. \\
& \quad \left. 6 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[2(c + dx)] + 6 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[2(c + dx)] + 8 \text{Sin}[3(c + dx)] \right) \Big/ \\
& \quad \left(3 a d \left(\text{Csc}\left[\frac{1}{2}(c + dx)\right] - \text{Sec}\left[\frac{1}{2}(c + dx)\right] \right) \left(\text{Csc}\left[\frac{1}{2}(c + dx)\right] + \text{Sec}\left[\frac{1}{2}(c + dx)\right] \right) (1 + \text{Sin}[c + dx]) \right) \Big)
\end{aligned}$$

■ **Problem 785: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + dx]^2 \text{Sec}[c + dx]^2}{(a + a \text{Sin}[c + dx])^2} dx$$

Optimal (type 3, 130 leaves, 12 steps):

$$\frac{2 \text{ArcTanh}[\text{Cos}[c + dx]]}{a^2 d} - \frac{\text{Cot}[c + dx]}{a^2 d} - \frac{2 \text{Sec}[c + dx]}{a^2 d} - \frac{2 \text{Sec}[c + dx]^3}{3 a^2 d} - \frac{2 \text{Sec}[c + dx]^5}{5 a^2 d} + \frac{4 \text{Tan}[c + dx]}{a^2 d} + \frac{5 \text{Tan}[c + dx]^3}{3 a^2 d} + \frac{2 \text{Tan}[c + dx]^5}{5 a^2 d}$$

Result (type 3, 289 leaves):

$$\begin{aligned}
& - \frac{1}{15 a^2 d \left(\text{Csc}\left[\frac{1}{2}(c + dx)\right]^2 - \text{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \right) (1 + \text{Sin}[c + dx])^2} \\
& \quad \text{Csc}[c + dx]^3 \left(40 + 48 \text{Cos}[2(c + dx)] + 112 \text{Cos}[3(c + dx)] - 28 \text{Cos}[4(c + dx)] + 60 \text{Cos}[3(c + dx)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] - \right. \\
& \quad \quad 4 \text{Cos}[c + dx] \left(28 + 15 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] - 15 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) - 60 \text{Cos}[3(c + dx)] \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \\
& \quad \quad 58 \text{Sin}[c + dx] - 168 \text{Sin}[2(c + dx)] - 90 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[2(c + dx)] + 90 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[2(c + dx)] + \\
& \quad \quad \left. 82 \text{Sin}[3(c + dx)] + 28 \text{Sin}[4(c + dx)] + 15 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[4(c + dx)] - 15 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[4(c + dx)] \right)
\end{aligned}$$

■ **Problem 786: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + dx]^3 \text{Sec}[c + dx]^2}{(a + a \text{Sin}[c + dx])^2} dx$$

Optimal (type 3, 158 leaves, 15 steps):

$$\begin{aligned}
& - \frac{9 \text{ArcTanh}[\text{Cos}[c + dx]]}{2 a^2 d} + \frac{2 \text{Cot}[c + dx]}{a^2 d} + \frac{9 \text{Sec}[c + dx]}{2 a^2 d} + \frac{3 \text{Sec}[c + dx]^3}{2 a^2 d} + \\
& \quad \frac{9 \text{Sec}[c + dx]^5}{10 a^2 d} - \frac{\text{Csc}[c + dx]^2 \text{Sec}[c + dx]^5}{2 a^2 d} - \frac{6 \text{Tan}[c + dx]}{a^2 d} - \frac{2 \text{Tan}[c + dx]^3}{a^2 d} - \frac{2 \text{Tan}[c + dx]^5}{5 a^2 d}
\end{aligned}$$

Result (type 3, 328 leaves) :

$$\begin{aligned}
 & - \frac{1}{320 a^2 d (1 + \sin[c + dx])^2} \operatorname{Csc}[c + dx]^2 \operatorname{Sec}[c + dx] \\
 & \left(-348 + 176 \cos[2(c + dx)] - 651 \cos[3(c + dx)] + 332 \cos[4(c + dx)] + 93 \cos[5(c + dx)] - 630 \cos[3(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \right) + \\
 & 90 \cos[5(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + 18 \cos[c + dx] \left(31 + 30 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - 30 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \\
 & 630 \cos[3(c + dx)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] - 90 \cos[5(c + dx)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] - 432 \sin[c + dx] + 744 \sin[2(c + dx)] + \\
 & 720 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \sin[2(c + dx)] - 720 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[2(c + dx)] - 176 \sin[3(c + dx)] - \\
 & 372 \sin[4(c + dx)] - 360 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \sin[4(c + dx)] + 360 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[4(c + dx)] + 128 \sin[5(c + dx)] \right)
 \end{aligned}$$

■ **Problem 793: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c + dx] \operatorname{Sec}[c + dx]^2}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 151 leaves, 14 steps) :

$$\begin{aligned}
 & - \frac{\operatorname{ArcTanh}[\cos[c + dx]]}{a^3 d} + \frac{\operatorname{Sec}[c + dx]}{a^3 d} + \frac{\operatorname{Sec}[c + dx]^3}{3 a^3 d} + \frac{\operatorname{Sec}[c + dx]^5}{5 a^3 d} + \\
 & \frac{4 \operatorname{Sec}[c + dx]^7}{7 a^3 d} - \frac{3 \operatorname{Tan}[c + dx]}{a^3 d} - \frac{10 \operatorname{Tan}[c + dx]^3}{3 a^3 d} - \frac{11 \operatorname{Tan}[c + dx]^5}{5 a^3 d} - \frac{4 \operatorname{Tan}[c + dx]^7}{7 a^3 d}
 \end{aligned}$$

Result (type 3, 341 leaves) :

$$\begin{aligned}
 & \frac{1}{840 d (a + a \sin[c + dx])^3} \\
 & \left(60 - \frac{120 \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]} - 324 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) + 162 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 - \right. \\
 & 706 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^3 + 353 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 - \\
 & 2281 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^5 - 840 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 + \\
 & \left. 840 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 + \frac{105 \sin\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6}{\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]} \right)
 \end{aligned}$$

■ **Problem 794: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c + dx]^2 \sec[c + dx]^2}{(a + a \sin[c + dx])^3} dx$$

Optimal (type 3, 162 leaves, 14 steps):

$$\frac{3 \operatorname{ArcTanh}[\cos[c + dx]]}{a^3 d} - \frac{\cot[c + dx]}{a^3 d} - \frac{3 \sec[c + dx]}{a^3 d} - \frac{\sec[c + dx]^3}{a^3 d} - \frac{3 \sec[c + dx]^5}{5 a^3 d} - \frac{4 \sec[c + dx]^7}{7 a^3 d} + \frac{7 \tan[c + dx]}{a^3 d} + \frac{5 \tan[c + dx]^3}{a^3 d} + \frac{13 \tan[c + dx]^5}{5 a^3 d} + \frac{4 \tan[c + dx]^7}{7 a^3 d}$$

Result (type 3, 351 leaves):

$$\frac{1}{140 a^3 d \left(\csc\left[\frac{1}{2}(c + dx)\right]^2 - \sec\left[\frac{1}{2}(c + dx)\right]^2 \right) (1 + \sin[c + dx])^3} \csc[c + dx]^3$$

$$\left(-966 - 440 \cos[2(c + dx)] - 2640 \cos[3(c + dx)] + 846 \cos[4(c + dx)] + 176 \cos[5(c + dx)] - 1575 \cos[3(c + dx)] \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \right) +$$

$$105 \cos[5(c + dx)] \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] + 14 \cos[c + dx] \left(176 + 105 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - 105 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \right) +$$

$$1575 \cos[3(c + dx)] \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] - 105 \cos[5(c + dx)] \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] - 1316 \sin[c + dx] + 3520 \sin[2(c + dx)] +$$

$$2100 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \sin[2(c + dx)] - 2100 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[2(c + dx)] - 1380 \sin[3(c + dx)] -$$

$$1056 \sin[4(c + dx)] - 630 \log\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \sin[4(c + dx)] + 630 \log\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \sin[4(c + dx)] + 176 \sin[5(c + dx)] \right)$$

■ **Problem 803: Result more than twice size of optimal antiderivative.**

$$\int \csc[c + dx]^3 \sec[c + dx]^4 (a + a \sin[c + dx]) dx$$

Optimal (type 3, 110 leaves, 9 steps):

$$-\frac{5 a \operatorname{ArcTanh}[\cos[c + dx]]}{2 d} - \frac{a \cot[c + dx]}{d} + \frac{5 a \sec[c + dx]}{2 d} + \frac{5 a \sec[c + dx]^3}{6 d} - \frac{a \csc[c + dx]^2 \sec[c + dx]^3}{2 d} + \frac{2 a \tan[c + dx]}{d} + \frac{a \tan[c + dx]^3}{3 d}$$

Result (type 3, 359 leaves):

$$\begin{aligned}
& -\frac{a \operatorname{Cot}[c+dx]}{d} - \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{8d} - \frac{5a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{5a \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \\
& \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{8d} + \frac{a}{12d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{6d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{13a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{6d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} - \frac{a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{6d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{a}{12d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - \\
& \frac{13a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{6d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{5a \operatorname{Tan}[c+dx]}{3d} + \frac{a \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3d}
\end{aligned}$$

■ **Problem 818: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^4 \operatorname{Sec}[c+dx]^4 (a+a \operatorname{Sin}[c+dx])^3 dx$$

Optimal (type 3, 128 leaves, 12 steps):

$$-\frac{17a^3 \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{2d} - \frac{6a^3 \operatorname{Cot}[c+dx]}{d} - \frac{a^3 \operatorname{Cot}[c+dx]^3}{3d} - \frac{3a^3 \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{2d} + \frac{2a^3 \operatorname{Cos}[c+dx]}{3d(1-\operatorname{Sin}[c+dx])^2} + \frac{23a^3 \operatorname{Cos}[c+dx]}{3d(1-\operatorname{Sin}[c+dx])}$$

Result (type 3, 287 leaves):

$$\begin{aligned}
& a^3 \left(-\frac{17 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{6d} - \frac{3 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{8d} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{24d} - \frac{17 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{17 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \right. \\
& \frac{3 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{8d} + \frac{2}{3d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{3d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \left. \frac{46 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{3d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{17 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{6d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24d} \right)
\end{aligned}$$

■ **Problem 827: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx]^4}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 115 leaves, 7 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{ad} + \frac{\operatorname{Sec}[c+dx]}{ad} + \frac{\operatorname{Sec}[c+dx]^3}{3ad} + \frac{\operatorname{Sec}[c+dx]^5}{5ad} - \frac{\operatorname{Tan}[c+dx]}{ad} - \frac{2 \operatorname{Tan}[c+dx]^3}{3ad} - \frac{\operatorname{Tan}[c+dx]^5}{5ad}$$

Result (type 3, 267 leaves):

$$\begin{aligned}
& - \frac{1}{120 a d (1 + \sin[c + d x])} \\
& \sec[c + d x]^3 \left(-100 - 76 \cos[2(c + d x)] + \frac{149}{4} \cos[3(c + d x)] - 8 \cos[4(c + d x)] + 30 \cos[3(c + d x)] \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\
& \cos[c + d x] \left(\frac{447}{4} + 90 \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - 90 \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \right) - 30 \cos[3(c + d x)] \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] - \\
& 22 \sin[c + d x] + \frac{149}{4} \sin[2(c + d x)] + 30 \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] \sin[2(c + d x)] - 30 \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \sin[2(c + d x)] - \\
& 14 \sin[3(c + d x)] + \frac{149}{8} \sin[4(c + d x)] + 15 \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] \sin[4(c + d x)] - 15 \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \sin[4(c + d x)] \Big)
\end{aligned}$$

■ **Problem 828: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c + d x]^2 \sec[c + d x]^4}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 126 leaves, 8 steps):

$$\frac{\operatorname{ArcTanh}[\cos[c + d x]]}{a d} - \frac{\cot[c + d x]}{a d} - \frac{\sec[c + d x]}{a d} - \frac{\sec[c + d x]^3}{3 a d} - \frac{\sec[c + d x]^5}{5 a d} + \frac{3 \tan[c + d x]}{a d} + \frac{\tan[c + d x]^3}{a d} + \frac{\tan[c + d x]^5}{5 a d}$$

Result (type 3, 341 leaves):

$$\begin{aligned}
& - \frac{1}{3840 a d (1 + \sin[c + d x])} \\
& \csc\left[\frac{1}{2}(c + d x)\right] \sec\left[\frac{1}{2}(c + d x)\right] \sec[c + d x]^3 \left(176 + 1216 \cos[2(c + d x)] + 149 \cos[3(c + d x)] + 528 \cos[4(c + d x)] + 149 \cos[5(c + d x)] + \right. \\
& 120 \cos[3(c + d x)] \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] + 120 \cos[5(c + d x)] \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - 120 \cos[3(c + d x)] \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] - \\
& 120 \cos[5(c + d x)] \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] + \cos[c + d x] \left(-298 - 240 \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] + 240 \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \right) + 352 \sin[c + d x] - \\
& 596 \sin[2(c + d x)] - 480 \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] \sin[2(c + d x)] + 480 \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \sin[2(c + d x)] + 864 \sin[3(c + d x)] - \\
& \left. 298 \sin[4(c + d x)] - 240 \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] \sin[4(c + d x)] + 240 \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] \sin[4(c + d x)] + 384 \sin[5(c + d x)] \right)
\end{aligned}$$

■ **Problem 836: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c + d x] \sec[c + d x]^4}{(a + a \sin[c + d x])^2} dx$$

Optimal (type 3, 149 leaves, 11 steps):

$$-\frac{\text{ArcTanh}[\text{Cos}[c+dx]]}{a^2 d} + \frac{\text{Sec}[c+dx]}{a^2 d} + \frac{\text{Sec}[c+dx]^3}{3 a^2 d} + \frac{\text{Sec}[c+dx]^5}{5 a^2 d} +$$

$$\frac{2 \text{Sec}[c+dx]^7}{7 a^2 d} - \frac{2 \text{Tan}[c+dx]}{a^2 d} - \frac{2 \text{Tan}[c+dx]^3}{a^2 d} - \frac{6 \text{Tan}[c+dx]^5}{5 a^2 d} - \frac{2 \text{Tan}[c+dx]^7}{7 a^2 d}$$

Result (type 3, 352 leaves):

$$\left(6216 + 5312 \text{Cos}[2(c+dx)] - 1677 \text{Cos}[3(c+dx)] + 696 \text{Cos}[4(c+dx)] + 559 \text{Cos}[5(c+dx)] - 1260 \text{Cos}[3(c+dx)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right] + \right.$$

$$420 \text{Cos}[5(c+dx)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right] - 14 \text{Cos}[c+dx] \left(559 + 420 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right] - 420 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) +$$

$$1260 \text{Cos}[3(c+dx)] \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 420 \text{Cos}[5(c+dx)] \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 2464 \text{Sin}[c+dx] - 4472 \text{Sin}[2(c+dx)] -$$

$$3360 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \text{Sin}[2(c+dx)] + 3360 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \text{Sin}[2(c+dx)] + 2208 \text{Sin}[3(c+dx)] - 2236 \text{Sin}[4(c+dx)] -$$

$$1680 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \text{Sin}[4(c+dx)] + 1680 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \text{Sin}[4(c+dx)] + 384 \text{Sin}[5(c+dx)] \right) /$$

$$\left(6720 a^2 d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] - \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3 \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^7 \right)$$

■ **Problem 837: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c+dx]^2 \text{Sec}[c+dx]^4}{(a+a \text{Sin}[c+dx])^2} dx$$

Optimal (type 3, 164 leaves, 12 steps):

$$\frac{2 \text{ArcTanh}[\text{Cos}[c+dx]]}{a^2 d} - \frac{\text{Cot}[c+dx]}{a^2 d} - \frac{2 \text{Sec}[c+dx]}{a^2 d} - \frac{2 \text{Sec}[c+dx]^3}{3 a^2 d} -$$

$$\frac{2 \text{Sec}[c+dx]^5}{5 a^2 d} - \frac{2 \text{Sec}[c+dx]^7}{7 a^2 d} + \frac{5 \text{Tan}[c+dx]}{a^2 d} + \frac{3 \text{Tan}[c+dx]^3}{a^2 d} + \frac{7 \text{Tan}[c+dx]^5}{5 a^2 d} + \frac{2 \text{Tan}[c+dx]^7}{7 a^2 d}$$

Result (type 3, 442 leaves):

$$\frac{1}{a^2} 16 \left(-\frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{32d} + \frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} - \frac{\operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \right.$$

$$\frac{1}{768d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{384d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{13 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{384d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} +$$

$$\frac{\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{224d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^7} - \frac{1}{448d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6} + \frac{3 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{140d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} -$$

$$\frac{3}{280d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{997 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{13440d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} -$$

$$\left. \frac{997}{26880d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{4777 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{13440d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{32d} \right)$$

■ **Problem 852: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sin}[c + dx]) \operatorname{Tan}[c + dx]^5 dx$$

Optimal (type 3, 115 leaves, 3 steps):

$$-\frac{23 a \operatorname{Log}[1 - \operatorname{Sin}[c + dx]]}{16 d} + \frac{7 a \operatorname{Log}[1 + \operatorname{Sin}[c + dx]]}{16 d} - \frac{a \operatorname{Sin}[c + dx]}{d} + \frac{a^3}{8 d (a - a \operatorname{Sin}[c + dx])^2} - \frac{a^2}{d (a - a \operatorname{Sin}[c + dx])} + \frac{a^2}{8 d (a + a \operatorname{Sin}[c + dx])}$$

Result (type 3, 246 leaves):

$$-\frac{a \operatorname{Log}[\operatorname{Cos}[c + dx]]}{d} - \frac{15 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} + \frac{15 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} -$$

$$\frac{a \operatorname{Sec}[c + dx]^2}{d} + \frac{a \operatorname{Sec}[c + dx]^4}{4 d} + \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{9 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} -$$

$$\frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{9 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a \operatorname{Sin}[c + dx]}{d}$$

■ **Problem 853: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx] (a + a \operatorname{Sin}[c + dx]) \operatorname{Tan}[c + dx]^4 dx$$

Optimal (type 3, 105 leaves, 4 steps):

$$-\frac{11 a \operatorname{Log}[1 - \operatorname{Sin}[c + dx]]}{16 d} - \frac{5 a \operatorname{Log}[1 + \operatorname{Sin}[c + dx]]}{16 d} + \frac{a^3}{8 d (a - a \operatorname{Sin}[c + dx])^2} - \frac{3 a^2}{4 d (a - a \operatorname{Sin}[c + dx])} - \frac{a^2}{8 d (a + a \operatorname{Sin}[c + dx])}$$

Result (type 3, 234 leaves) :

$$\begin{aligned} & - \frac{a \operatorname{Log}[\operatorname{Cos}[c + d x]]}{d} - \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} - \\ & \frac{a \operatorname{Sec}[c + d x]^2}{d} + \frac{a \operatorname{Sec}[c + d x]^4}{4 d} + \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{5 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \\ & \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{5 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} \end{aligned}$$

■ **Problem 854: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^2 (a + a \operatorname{Sin}[c + d x]) \operatorname{Tan}[c + d x]^3 dx$$

Optimal (type 3, 84 leaves, 5 steps) :

$$\frac{3 a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a^3}{8 d (a - a \operatorname{Sin}[c + d x])^2} - \frac{a^2}{2 d (a - a \operatorname{Sin}[c + d x])} + \frac{a^2}{8 d (a + a \operatorname{Sin}[c + d x])}$$

Result (type 3, 207 leaves) :

$$\begin{aligned} & - \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \\ & \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{5 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \\ & \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{5 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{a \operatorname{Tan}[c + d x]^4}{4 d} \end{aligned}$$

■ **Problem 855: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^3 (a + a \operatorname{Sin}[c + d x]) \operatorname{Tan}[c + d x]^2 dx$$

Optimal (type 3, 84 leaves, 5 steps) :

$$- \frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a^3}{8 d (a - a \operatorname{Sin}[c + d x])^2} - \frac{a^2}{4 d (a - a \operatorname{Sin}[c + d x])} - \frac{a^2}{8 d (a + a \operatorname{Sin}[c + d x])}$$

Result (type 3, 207 leaves) :

$$\frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} - \frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} +$$

$$\frac{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4}{a} - \frac{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{a} -$$

$$\frac{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4}{a} + \frac{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{a} + \frac{a \operatorname{Tan}[c+dx]^4}{4d}$$

■ **Problem 856: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^4 (a + a \sin[c+dx]) \operatorname{Tan}[c+dx] dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$-\frac{a \operatorname{ArcTanh}[\sin[c+dx]]}{8d} + \frac{a^3}{8d(a-a\sin[c+dx])^2} + \frac{a^2}{8d(a+a\sin[c+dx])}$$

Result (type 3, 207 leaves):

$$\frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} - \frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} +$$

$$\frac{a \sec[c+dx]^4}{4d} + \frac{a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} -$$

$$\frac{a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}$$

■ **Problem 857: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx] \sec[c+dx]^5 (a + a \sin[c+dx]) dx$$

Optimal (type 3, 117 leaves, 4 steps):

$$-\frac{11a \operatorname{Log}[1 - \sin[c+dx]]}{16d} + \frac{a \operatorname{Log}[\sin[c+dx]]}{d} - \frac{5a \operatorname{Log}[1 + \sin[c+dx]]}{16d} +$$

$$\frac{d}{8d(a-a\sin[c+dx])^2} + \frac{d}{2d(a-a\sin[c+dx])} + \frac{d}{8d(a+a\sin[c+dx])}$$

Result (type 3, 248 leaves):

$$\begin{aligned}
& - \frac{a \operatorname{Log}[\operatorname{Cos}[c + dx]]}{d} - \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \\
& \frac{a \operatorname{Log}[\operatorname{Sin}[c + dx]]}{d} + \frac{a \operatorname{Sec}[c + dx]^2}{2 d} + \frac{a \operatorname{Sec}[c + dx]^4}{4 d} + \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4} + \\
& \frac{3 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4} - \frac{3 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2}
\end{aligned}$$

■ **Problem 858: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + dx]^2 \operatorname{Sec}[c + dx]^5 (a + a \operatorname{Sin}[c + dx]) dx$$

Optimal (type 3, 129 leaves, 4 steps):

$$\begin{aligned}
& - \frac{a \operatorname{Csc}[c + dx]}{d} - \frac{23 a \operatorname{Log}[1 - \operatorname{Sin}[c + dx]]}{16 d} + \frac{a \operatorname{Log}[\operatorname{Sin}[c + dx]]}{d} + \\
& \frac{7 a \operatorname{Log}[1 + \operatorname{Sin}[c + dx]]}{16 d} + \frac{a^3}{8 d (a - a \operatorname{Sin}[c + dx])^2} + \frac{3 a^2}{4 d (a - a \operatorname{Sin}[c + dx])} - \frac{a^2}{8 d (a + a \operatorname{Sin}[c + dx])}
\end{aligned}$$

Result (type 3, 284 leaves):

$$\begin{aligned}
& - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right]}{2 d} - \frac{a \operatorname{Log}[\operatorname{Cos}[c + dx]]}{d} - \frac{15 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \frac{15 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \\
& \frac{a \operatorname{Log}[\operatorname{Sin}[c + dx]]}{d} + \frac{a \operatorname{Sec}[c + dx]^2}{2 d} + \frac{a \operatorname{Sec}[c + dx]^4}{4 d} + \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{7 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} - \\
& \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4} - \frac{7 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{2 d}
\end{aligned}$$

■ **Problem 859: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + dx]^3 \operatorname{Sec}[c + dx]^5 (a + a \operatorname{Sin}[c + dx]) dx$$

Optimal (type 3, 143 leaves, 4 steps):

$$\begin{aligned}
& - \frac{a \operatorname{Csc}[c + dx]}{d} - \frac{a \operatorname{Csc}[c + dx]^2}{2 d} - \frac{39 a \operatorname{Log}[1 - \operatorname{Sin}[c + dx]]}{16 d} + \frac{3 a \operatorname{Log}[\operatorname{Sin}[c + dx]]}{d} - \\
& \frac{9 a \operatorname{Log}[1 + \operatorname{Sin}[c + dx]]}{16 d} + \frac{a^3}{8 d (a - a \operatorname{Sin}[c + dx])^2} + \frac{a^2}{d (a - a \operatorname{Sin}[c + dx])} + \frac{a^2}{8 d (a + a \operatorname{Sin}[c + dx])}
\end{aligned}$$

Result (type 3, 298 leaves):

$$\begin{aligned}
& - \frac{a \cot\left[\frac{1}{2}(c+dx)\right]}{2d} - \frac{a \csc[c+dx]^2}{2d} - \frac{3a \log[\cos[c+dx]]}{d} - \frac{15a \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \\
& \frac{15a \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{3a \log[\sin[c+dx]]}{d} + \frac{a \sec[c+dx]^2}{d} + \frac{a \sec[c+dx]^4}{4d} + \\
& \frac{a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{7a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \\
& \frac{a \tan\left[\frac{1}{2}(c+dx)\right]}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{7a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a \tan\left[\frac{1}{2}(c+dx)\right]}{2d}
\end{aligned}$$

■ **Problem 860: Result more than twice size of optimal antiderivative.**

$$\int \csc[c+dx]^4 \sec[c+dx]^5 (a + a \sin[c+dx]) dx$$

Optimal (type 3, 162 leaves, 4 steps):

$$\begin{aligned}
& - \frac{3a \csc[c+dx]}{d} - \frac{a \csc[c+dx]^2}{2d} - \frac{a \csc[c+dx]^3}{3d} - \frac{59a \log[1 - \sin[c+dx]]}{16d} + \frac{3a \log[\sin[c+dx]]}{d} + \\
& \frac{11a \log[1 + \sin[c+dx]]}{16d} + \frac{a^3}{8d(a - a \sin[c+dx])^2} + \frac{5a^2}{4d(a - a \sin[c+dx])} - \frac{d}{8d(a + a \sin[c+dx])}
\end{aligned}$$

Result (type 3, 358 leaves):

$$\begin{aligned}
& - \frac{19a \cot\left[\frac{1}{2}(c+dx)\right]}{12d} - \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{24d} - \frac{a \csc[c+dx]^2}{2d} - \\
& \frac{3a \log[\cos[c+dx]]}{d} - \frac{35a \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{35a \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \\
& \frac{3a \log[\sin[c+dx]]}{d} + \frac{a \sec[c+dx]^2}{d} + \frac{a \sec[c+dx]^4}{4d} + \frac{a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \\
& \frac{11a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \\
& \frac{11a}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{19a \tan\left[\frac{1}{2}(c+dx)\right]}{12d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{24d}
\end{aligned}$$

■ **Problem 863: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^2 (a + a \sin[c+dx])^2 \tan[c+dx]^3 dx$$

Optimal (type 3, 87 leaves, 4 steps):

$$-\frac{7 a^2 \operatorname{Log}[1 - \operatorname{Sin}[c + d x]]}{8 d} - \frac{a^2 \operatorname{Log}[1 + \operatorname{Sin}[c + d x]]}{8 d} + \frac{a^4}{4 d (a - a \operatorname{Sin}[c + d x])^2} - \frac{5 a^3}{4 d (a - a \operatorname{Sin}[c + d x])}$$

Result (type 3, 270 leaves):

$$-\frac{a^2 \operatorname{Log}[\operatorname{Cos}[c + d x]]}{d} - \frac{3 a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{4 d} + \frac{3 a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{4 d} -$$

$$\frac{a^2 \operatorname{Sec}[c + d x]^2}{d} + \frac{a^2 \operatorname{Sec}[c + d x]^4}{4 d} + \frac{a^2}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{5 a^2}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} -$$

$$\frac{a^2}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{5 a^2}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{a^2 \operatorname{Tan}[c + d x]^4}{4 d}$$

■ **Problem 869: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + d x]^4 \operatorname{Sec}[c + d x]^5 (a + a \operatorname{Sin}[c + d x])^2 dx$$

Optimal (type 3, 150 leaves, 4 steps):

$$-\frac{4 a^2 \operatorname{Csc}[c + d x]}{d} - \frac{a^2 \operatorname{Csc}[c + d x]^2}{d} - \frac{a^2 \operatorname{Csc}[c + d x]^3}{3 d} - \frac{49 a^2 \operatorname{Log}[1 - \operatorname{Sin}[c + d x]]}{8 d} +$$

$$\frac{6 a^2 \operatorname{Log}[\operatorname{Sin}[c + d x]]}{d} + \frac{a^2 \operatorname{Log}[1 + \operatorname{Sin}[c + d x]]}{8 d} + \frac{a^4}{4 d (a - a \operatorname{Sin}[c + d x])^2} + \frac{9 a^3}{4 d (a - a \operatorname{Sin}[c + d x])}$$

Result (type 3, 652 leaves):

$$\begin{aligned}
& - \frac{25 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] (a+a \operatorname{Sin}[c+dx])^2}{12 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 (a+a \operatorname{Sin}[c+dx])^2}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \\
& \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 (a+a \operatorname{Sin}[c+dx])^2}{24 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{49 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] (a+a \operatorname{Sin}[c+dx])^2}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \\
& \frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] (a+a \operatorname{Sin}[c+dx])^2}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{6 \operatorname{Log}\left[\operatorname{Sin}[c+dx]\right] (a+a \operatorname{Sin}[c+dx])^2}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (a+a \operatorname{Sin}[c+dx])^2}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{(a+a \operatorname{Sin}[c+dx])^2}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \\
& \frac{9 (a+a \operatorname{Sin}[c+dx])^2}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \\
& \frac{25 (a+a \operatorname{Sin}[c+dx])^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{12 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (a+a \operatorname{Sin}[c+dx])^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4}
\end{aligned}$$

■ **Problem 882: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+dx]^7}{a+a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 130 leaves, 8 steps):

$$\begin{aligned}
& - \frac{35 \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{128 a d} + \frac{35 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{128 a d} - \\
& \frac{35 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^3}{192 a d} + \frac{7 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^5}{48 a d} - \frac{\operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^7}{8 a d} + \frac{\operatorname{Tan}[c+dx]^8}{8 a d}
\end{aligned}$$

Result (type 3, 342 leaves):

$$\frac{1}{384 d (a + a \sin[c + dx])} \left(-192 + \frac{6}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^6} - \frac{40}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \right. \\ \left. \frac{114}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + 105 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 - \right. \\ \left. 105 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 + \frac{4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^6} - \right. \\ \left. \frac{27 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{87 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} \right)$$

■ **Problem 883: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx] \tan[c + dx]^6}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 134 leaves, 8 steps):

$$-\frac{5 \operatorname{ArcTanh}[\sin[c + dx]]}{128 a d} - \frac{5 \sec[c + dx] \tan[c + dx]}{128 a d} + \\ \frac{5 \sec[c + dx]^3 \tan[c + dx]}{64 a d} - \frac{5 \sec[c + dx]^3 \tan[c + dx]^3}{48 a d} + \frac{\sec[c + dx]^3 \tan[c + dx]^5}{8 a d} - \frac{\tan[c + dx]^8}{8 a d}$$

Result (type 3, 342 leaves):

$$\frac{1}{384 d (a + a \sin[c + dx])} \left(60 - \frac{6}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^6} + \frac{32}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} - \right. \\ \left. \frac{66}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + 15 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 - \right. \\ \left. 15 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 + \frac{4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^6} - \right. \\ \left. \frac{21 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{45 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} \right)$$

■ **Problem 884: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^2 \tan[c + dx]^5}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 152 leaves, 9 steps) :

$$\frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{128 a d} + \frac{5 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{128 a d} - \frac{5 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{64 a d} +$$

$$\frac{5 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]^3}{48 a d} - \frac{\operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]^5}{8 a d} + \frac{\operatorname{Tan}[c + d x]^6}{6 a d} + \frac{\operatorname{Tan}[c + d x]^8}{8 a d}$$

Result (type 3, 340 leaves) :

$$\frac{1}{384 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^6 (a + a \operatorname{Sin}[c + d x])}$$

$$\left(6 - 24 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 + 30 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^4 - 15 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right] \right)$$

$$\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^8 + 15 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^8 +$$

$$\frac{4 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^8}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^6} - \frac{15 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^8}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^4} + \frac{15 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^8}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2}$$

■ **Problem 885: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]^4}{a + a \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 150 leaves, 9 steps) :

$$\frac{3 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{128 a d} + \frac{3 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{128 a d} + \frac{\operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{64 a d} -$$

$$\frac{\operatorname{Sec}[c + d x]^5 \operatorname{Tan}[c + d x]}{16 a d} + \frac{\operatorname{Sec}[c + d x]^5 \operatorname{Tan}[c + d x]^3}{8 a d} - \frac{\operatorname{Tan}[c + d x]^6}{6 a d} - \frac{\operatorname{Tan}[c + d x]^8}{8 a d}$$

Result (type 3, 342 leaves) :

$$\begin{aligned}
& - \frac{1}{384 d (a + a \sin[c + dx])} \left(12 + \frac{6}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^6} - \frac{16}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \right. \\
& \quad \left. \frac{6}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + 9 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 - \right. \\
& \quad \left. 9 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 - \frac{4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^6} + \right. \\
& \quad \left. \frac{9 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} \right)
\end{aligned}$$

■ **Problem 886: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^4 \tan[c + dx]^3}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 150 leaves, 9 steps):

$$\begin{aligned}
& - \frac{3 \operatorname{ArcTanh}[\sin[c + dx]]}{128 a d} - \frac{\sec[c + dx]^6}{6 a d} + \frac{\sec[c + dx]^8}{8 a d} - \frac{3 \sec[c + dx] \tan[c + dx]}{128 a d} - \\
& \frac{\sec[c + dx]^3 \tan[c + dx]}{64 a d} + \frac{\sec[c + dx]^5 \tan[c + dx]}{16 a d} - \frac{\sec[c + dx]^5 \tan[c + dx]^3}{8 a d}
\end{aligned}$$

Result (type 3, 340 leaves):

$$\begin{aligned}
& - \frac{1}{384 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^6 (a + a \sin[c + dx])} \\
& \left(-6 + 8 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 + 6 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4 - 9 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \\
& \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^8 + 9 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^8 - \\
& \frac{4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^8}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^6} + \frac{3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^8}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{9 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^8}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}
\end{aligned}$$

■ **Problem 887: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^5 \tan[c + dx]^2}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 148 leaves, 9 steps):

$$-\frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{128 ad} + \frac{\operatorname{Sec}[c+dx]^6}{6 ad} - \frac{\operatorname{Sec}[c+dx]^8}{8 ad} - \frac{5 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{128 ad} -$$

$$\frac{5 \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{192 ad} - \frac{\operatorname{Sec}[c+dx]^5 \operatorname{Tan}[c+dx]}{48 ad} + \frac{\operatorname{Sec}[c+dx]^7 \operatorname{Tan}[c+dx]}{8 ad}$$

Result (type 3, 317 leaves):

$$\frac{1}{384 d (a + a \operatorname{Sin}[c+dx])}$$

$$\left(12 - \frac{6}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6} + \frac{6}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + 15 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right.$$

$$\left. \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 - 15 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 + \right.$$

$$\left. \frac{4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6} + \frac{3 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{3 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 888: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^6 \operatorname{Tan}[c+dx]}{a + a \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 130 leaves, 8 steps):

$$\frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{128 ad} + \frac{\operatorname{Sec}[c+dx]^8}{8 ad} + \frac{5 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{128 ad} +$$

$$\frac{5 \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{192 ad} + \frac{\operatorname{Sec}[c+dx]^5 \operatorname{Tan}[c+dx]}{48 ad} - \frac{\operatorname{Sec}[c+dx]^7 \operatorname{Tan}[c+dx]}{8 ad}$$

Result (type 3, 340 leaves):

$$\frac{1}{384 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6 (a + a \operatorname{Sin}[c+dx])}$$

$$\left(6 + 8 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 + 6 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4 - 15 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right.$$

$$\left. \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8 + 15 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8 + \right.$$

$$\left. \frac{4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^6} + \frac{9 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{15 \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^8}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 889: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^7}{a + a \text{Sin}[c + d x]} dx$$

Optimal (type 3, 165 leaves, 4 steps):

$$\frac{35 \text{ArcTanh}[\text{Sin}[c + d x]]}{128 a d} + \frac{a^2}{96 d (a - a \text{Sin}[c + d x])^3} + \frac{5 a}{128 d (a - a \text{Sin}[c + d x])^2} + \frac{15}{128 d (a - a \text{Sin}[c + d x])} - \frac{a^3}{64 d (a + a \text{Sin}[c + d x])^4} - \frac{a^2}{24 d (a + a \text{Sin}[c + d x])^3} - \frac{5 a}{64 d (a + a \text{Sin}[c + d x])^2} - \frac{5}{32 d (a + a \text{Sin}[c + d x])}$$

Result (type 3, 342 leaves):

$$\frac{1}{384 d (a + a \text{Sin}[c + d x])} \left(-60 - \frac{6}{(\text{Cos}[\frac{1}{2} (c + d x)] + \text{Sin}[\frac{1}{2} (c + d x)])^6} - \frac{16}{(\text{Cos}[\frac{1}{2} (c + d x)] + \text{Sin}[\frac{1}{2} (c + d x)])^4} - \frac{30}{(\text{Cos}[\frac{1}{2} (c + d x)] + \text{Sin}[\frac{1}{2} (c + d x)])^2} - 105 \text{Log} \left[\text{Cos}[\frac{1}{2} (c + d x)] - \text{Sin}[\frac{1}{2} (c + d x)] \right] \left(\text{Cos}[\frac{1}{2} (c + d x)] + \text{Sin}[\frac{1}{2} (c + d x)] \right)^2 + 105 \text{Log} \left[\text{Cos}[\frac{1}{2} (c + d x)] + \text{Sin}[\frac{1}{2} (c + d x)] \right] \left(\text{Cos}[\frac{1}{2} (c + d x)] + \text{Sin}[\frac{1}{2} (c + d x)] \right)^2 + \frac{4 (\text{Cos}[\frac{1}{2} (c + d x)] + \text{Sin}[\frac{1}{2} (c + d x)])^2}{(\text{Cos}[\frac{1}{2} (c + d x)] - \text{Sin}[\frac{1}{2} (c + d x)])^6} + \frac{15 (\text{Cos}[\frac{1}{2} (c + d x)] + \text{Sin}[\frac{1}{2} (c + d x)])^2}{(\text{Cos}[\frac{1}{2} (c + d x)] - \text{Sin}[\frac{1}{2} (c + d x)])^4} + \frac{45 (\text{Cos}[\frac{1}{2} (c + d x)] + \text{Sin}[\frac{1}{2} (c + d x)])^2}{(\text{Cos}[\frac{1}{2} (c + d x)] - \text{Sin}[\frac{1}{2} (c + d x)])^2} \right)$$

■ **Problem 891: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + d x]^2 \text{Sec}[c + d x]^7}{a + a \text{Sin}[c + d x]} dx$$

Optimal (type 3, 217 leaves, 4 steps):

$$-\frac{\text{Csc}[c + d x]}{a d} - \frac{187 \text{Log}[1 - \text{Sin}[c + d x]]}{256 a d} - \frac{\text{Log}[\text{Sin}[c + d x]]}{a d} + \frac{443 \text{Log}[1 + \text{Sin}[c + d x]]}{256 a d} + \frac{a^2}{96 d (a - a \text{Sin}[c + d x])^3} + \frac{9 a}{128 d (a - a \text{Sin}[c + d x])^2} + \frac{47}{128 d (a - a \text{Sin}[c + d x])} - \frac{a^3}{64 d (a + a \text{Sin}[c + d x])^4} - \frac{a^2}{12 d (a + a \text{Sin}[c + d x])^3} - \frac{19 a}{64 d (a + a \text{Sin}[c + d x])^2} - \frac{35}{32 d (a + a \text{Sin}[c + d x])}$$

Result (type 3, 628 leaves):

$$\begin{aligned}
& - \frac{35}{32 d (a + a \sin[c + d x])} - \frac{1}{64 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^6 (a + a \sin[c + d x])} - \\
& \frac{1}{12 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^4 (a + a \sin[c + d x])} - \frac{19}{64 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2 (a + a \sin[c + d x])} - \\
& \frac{\cot\left[\frac{1}{2}(c + d x)\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2}{2 d (a + a \sin[c + d x])} - \frac{187 \log\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2}{128 d (a + a \sin[c + d x])} + \\
& \frac{443 \log\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2}{128 d (a + a \sin[c + d x])} - \frac{\log[\sin[c + d x]] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2}{d (a + a \sin[c + d x])} + \\
& \frac{\left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2}{96 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right] \right)^6 (a + a \sin[c + d x])} + \frac{9 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2}{128 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right] \right)^4 (a + a \sin[c + d x])} + \\
& \frac{47 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2}{128 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right] \right)^2 (a + a \sin[c + d x])} - \frac{\left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right] \right)^2 \tan\left[\frac{1}{2}(c + d x)\right]}{2 d (a + a \sin[c + d x])}
\end{aligned}$$

■ **Problem 892: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c + d x]^3 \sec[c + d x]^7}{a + a \sin[c + d x]} dx$$

Optimal (type 3, 232 leaves, 4 steps):

$$\begin{aligned}
& \frac{\csc[c + d x]}{a d} - \frac{\csc[c + d x]^2}{2 a d} - \frac{325 \log[1 - \sin[c + d x]]}{256 a d} + \frac{5 \log[\sin[c + d x]]}{a d} - \\
& \frac{955 \log[1 + \sin[c + d x]]}{256 a d} + \frac{a^2}{96 d (a - a \sin[c + d x])^3} + \frac{a d}{128 d (a - a \sin[c + d x])^2} + \frac{69}{128 d (a - a \sin[c + d x])} + \\
& \frac{a^3}{64 d (a + a \sin[c + d x])^4} + \frac{5 a^2}{48 d (a + a \sin[c + d x])^3} + \frac{29 a}{64 d (a + a \sin[c + d x])^2} + \frac{2}{d (a + a \sin[c + d x])}
\end{aligned}$$

Result (type 3, 734 leaves):

$$\begin{aligned}
& \frac{2}{d(a+a\sin[c+dx])} + \frac{1}{64d\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6(a+a\sin[c+dx])} + \\
& \frac{5}{48d\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4(a+a\sin[c+dx])} + \frac{29}{64d\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2(a+a\sin[c+dx])} + \\
& \frac{\cot\left[\frac{1}{2}(c+dx)\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{2d(a+a\sin[c+dx])} - \frac{\csc\left[\frac{1}{2}(c+dx)\right]^2\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{8d(a+a\sin[c+dx])} - \\
& \frac{325\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{128d(a+a\sin[c+dx])} - \\
& \frac{955\log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{128d(a+a\sin[c+dx])} + \\
& \frac{5\log[\sin[c+dx]]\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{d(a+a\sin[c+dx])} - \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{8d(a+a\sin[c+dx])} + \\
& \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{11\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{96d\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^6(a+a\sin[c+dx])}{128d\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4(a+a\sin[c+dx])} + \\
& \frac{69\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{128d\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2(a+a\sin[c+dx])} + \frac{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 \tan\left[\frac{1}{2}(c+dx)\right]}{2d(a+a\sin[c+dx])}
\end{aligned}$$

■ **Problem 893: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c+dx]^4 \sec[c+dx]^7}{a+a\sin[c+dx]} dx$$

Optimal (type 3, 253 leaves, 4 steps):

$$\begin{aligned}
& -\frac{5\csc[c+dx]}{ad} + \frac{\csc[c+dx]^2}{2ad} - \frac{\csc[c+dx]^3}{3ad} - \frac{515\log[1-\sin[c+dx]]}{256ad} - \frac{5\log[\sin[c+dx]]}{ad} + \\
& \frac{1795\log[1+\sin[c+dx]]}{256ad} + \frac{1}{96d(a-a\sin[c+dx])^3} + \frac{13a}{128d(a-a\sin[c+dx])^2} + \frac{95}{128d(a-a\sin[c+dx])} - \\
& \frac{1}{64d(a+a\sin[c+dx])^4} - \frac{1}{8d(a+a\sin[c+dx])^3} - \frac{41a}{64d(a+a\sin[c+dx])^2} - \frac{105}{32d(a+a\sin[c+dx])}
\end{aligned}$$

Result (type 3, 864 leaves):

$$\begin{aligned}
& - \frac{105}{32 d (a + a \sin[c + dx])} - \frac{1}{64 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 (a + a \sin[c + dx])} - \\
& \frac{1}{8 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 (a + a \sin[c + dx])} - \frac{41}{64 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 (a + a \sin[c + dx])} - \\
& \frac{31 \cot\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{12 d (a + a \sin[c + dx])} + \frac{\csc\left[\frac{1}{2}(c + dx)\right]^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{8 d (a + a \sin[c + dx])} - \\
& \frac{\cot\left[\frac{1}{2}(c + dx)\right] \csc\left[\frac{1}{2}(c + dx)\right]^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{24 d (a + a \sin[c + dx])} - \\
& \frac{515 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{128 d (a + a \sin[c + dx])} + \\
& \frac{1795 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{128 d (a + a \sin[c + dx])} - \frac{5 \log[\sin[c + dx]] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{d (a + a \sin[c + dx])} + \\
& \frac{\sec\left[\frac{1}{2}(c + dx)\right]^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{8 d (a + a \sin[c + dx])} + \frac{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{96 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 (a + a \sin[c + dx])} + \\
& \frac{13 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{128 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 (a + a \sin[c + dx])} + \frac{95 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{128 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 (a + a \sin[c + dx])} - \\
& \frac{31 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \tan\left[\frac{1}{2}(c + dx)\right]}{12 d (a + a \sin[c + dx])} - \frac{\sec\left[\frac{1}{2}(c + dx)\right]^2 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \tan\left[\frac{1}{2}(c + dx)\right]}{24 d (a + a \sin[c + dx])}
\end{aligned}$$

■ **Problem 897: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin[c + dx] \tan[c + dx]^9}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 233 leaves, 4 steps):

$$\begin{aligned}
& - \frac{193 \log[1 - \sin[c + dx]]}{512 a d} - \frac{319 \log[1 + \sin[c + dx]]}{512 a d} + \frac{a^3}{256 d (a - a \sin[c + dx])^4} - \\
& \frac{7 a^2}{192 d (a - a \sin[c + dx])^3} + \frac{81 a}{512 d (a - a \sin[c + dx])^2} - \frac{61}{128 d (a - a \sin[c + dx])} - \frac{a^4}{160 d (a + a \sin[c + dx])^5} + \\
& \frac{15 a^3}{256 d (a + a \sin[c + dx])^4} - \frac{95 a^2}{384 d (a + a \sin[c + dx])^3} + \frac{325 a}{512 d (a + a \sin[c + dx])^2} - \frac{315}{256 d (a + a \sin[c + dx])}
\end{aligned}$$

Result (type 3, 586 leaves):

$$\begin{aligned}
& - \frac{315}{256 d (a + a \sin[c + dx])} - \frac{1}{160 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^8 (a + a \sin[c + dx])} + \\
& \frac{15}{256 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 (a + a \sin[c + dx])} - \\
& \frac{95}{384 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 (a + a \sin[c + dx])} + \frac{325}{512 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 (a + a \sin[c + dx])} - \\
& \frac{193 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{256 d (a + a \sin[c + dx])} - \\
& \frac{319 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{256 d (a + a \sin[c + dx])} + \\
& \frac{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{256 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^8 (a + a \sin[c + dx])} - \frac{7 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{192 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^6 (a + a \sin[c + dx])} + \\
& \frac{81 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{512 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4 (a + a \sin[c + dx])} - \frac{61 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}{128 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2 (a + a \sin[c + dx])}
\end{aligned}$$

■ **Problem 898: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c + dx]^9}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 154 leaves, 9 steps):

$$\begin{aligned}
& \frac{63 \operatorname{ArcTanh}[\sin[c + dx]]}{256 a d} - \frac{63 \operatorname{Sec}[c + dx] \tan[c + dx]}{256 a d} + \frac{21 \operatorname{Sec}[c + dx] \tan[c + dx]^3}{128 a d} - \\
& \frac{21 \operatorname{Sec}[c + dx] \tan[c + dx]^5}{160 a d} + \frac{9 \operatorname{Sec}[c + dx] \tan[c + dx]^7}{80 a d} - \frac{\operatorname{Sec}[c + dx] \tan[c + dx]^9}{10 a d} + \frac{\tan[c + dx]^{10}}{10 a d}
\end{aligned}$$

Result (type 3, 417 leaves):

$$\frac{1}{2560 d (a + a \sin[c + dx])} \left(\frac{1280 + \frac{16}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^8} - \frac{130}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6} + \frac{460}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{935}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - 630 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 + 630 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 + \frac{10 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^8} - \frac{80 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^6} + \frac{285 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{650 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 899: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx] \tan[c+dx]^8}{a + a \sin[c+dx]} dx$$

Optimal (type 3, 160 leaves, 9 steps):

$$\frac{7 \operatorname{ArcTanh}[\sin[c+dx]]}{256 a d} + \frac{7 \sec[c+dx] \tan[c+dx]}{256 a d} - \frac{7 \sec[c+dx]^3 \tan[c+dx]}{128 a d} + \frac{7 \sec[c+dx]^3 \tan[c+dx]^3}{96 a d} - \frac{7 \sec[c+dx]^3 \tan[c+dx]^5}{80 a d} + \frac{\sec[c+dx]^3 \tan[c+dx]^7}{10 a d} - \frac{\tan[c+dx]^{10}}{10 a d}$$

Result (type 3, 417 leaves):

$$\frac{1}{7680 d (a + a \sin[c + dx])} \left(-1050 - \frac{48}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^8} + \frac{330}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6} - \frac{940}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{1395}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - 210 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 + 210 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 + \frac{30 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^8} - \frac{200 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^6} + \frac{555 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{840 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 900: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^2 \tan[c+dx]^7}{a + a \sin[c+dx]} dx$$

Optimal (type 3, 178 leaves, 10 steps):

$$-\frac{7 \operatorname{ArcTanh}[\sin[c+dx]]}{256 ad} - \frac{7 \sec[c+dx] \tan[c+dx]}{256 ad} + \frac{7 \sec[c+dx]^3 \tan[c+dx]}{128 ad} - \frac{7 \sec[c+dx]^3 \tan[c+dx]^3}{96 ad} + \frac{7 \sec[c+dx]^3 \tan[c+dx]^5}{80 ad} - \frac{\sec[c+dx]^3 \tan[c+dx]^7}{10 ad} + \frac{\tan[c+dx]^8}{8 ad} + \frac{\tan[c+dx]^{10}}{10 ad}$$

Result (type 3, 415 leaves):

$$\frac{1}{7680 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^8 (a + a \sin[c+dx])} \left(48 - 270 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 + 580 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 - 525 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6 + 210 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^{10} - 210 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^{10} + \frac{30 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^8} - \frac{160 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^6} + \frac{315 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{210 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 901: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^3 \text{Tan}[c + d x]^6}{a + a \text{Sin}[c + d x]} dx$$

Optimal (type 3, 176 leaves, 10 steps):

$$\frac{3 \text{ArcTanh}[\text{Sin}[c + d x]]}{256 a d} - \frac{3 \text{Sec}[c + d x] \text{Tan}[c + d x]}{256 a d} - \frac{\text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{128 a d} +$$

$$\frac{\text{Sec}[c + d x]^5 \text{Tan}[c + d x]}{32 a d} - \frac{\text{Sec}[c + d x]^5 \text{Tan}[c + d x]^3}{16 a d} + \frac{\text{Sec}[c + d x]^5 \text{Tan}[c + d x]^5}{10 a d} - \frac{\text{Tan}[c + d x]^8}{8 a d} - \frac{\text{Tan}[c + d x]^{10}}{10 a d}$$

Result (type 3, 417 leaves):

$$\frac{1}{2560 d (a + a \text{Sin}[c + d x])}$$

$$\left(50 - \frac{16}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^8} + \frac{70}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^6} - \frac{100}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} +$$

$$\frac{25}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + 30 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 -$$

$$30 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 + \frac{10 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^8} -$$

$$\frac{40 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^6} + \frac{45 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{20 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} \right)$$

■ **Problem 902: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^4 \text{Tan}[c + d x]^5}{a + a \text{Sin}[c + d x]} dx$$

Optimal (type 3, 194 leaves, 11 steps):

$$\frac{3 \text{ArcTanh}[\text{Sin}[c + d x]]}{256 a d} + \frac{\text{Sec}[c + d x]^6}{6 a d} - \frac{\text{Sec}[c + d x]^8}{4 a d} + \frac{\text{Sec}[c + d x]^{10}}{10 a d} + \frac{3 \text{Sec}[c + d x] \text{Tan}[c + d x]}{256 a d} +$$

$$\frac{\text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{128 a d} - \frac{\text{Sec}[c + d x]^5 \text{Tan}[c + d x]}{32 a d} + \frac{\text{Sec}[c + d x]^5 \text{Tan}[c + d x]^3}{16 a d} - \frac{\text{Sec}[c + d x]^5 \text{Tan}[c + d x]^5}{10 a d}$$

Result (type 3, 415 leaves):

$$\frac{1}{7680 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 (a + a \sin[c+dx])}$$

$$\left(48 - 150 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + 100 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 + \right.$$

$$75 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 - 90 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10} +$$

$$90 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10} + \frac{30 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^8} -$$

$$\left. \frac{80 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^6} + \frac{15 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^4} + \frac{90 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} \right)$$

■ **Problem 904: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^6 \tan[c+dx]^3}{a + a \sin[c+dx]} dx$$

Optimal (type 3, 174 leaves, 10 steps):

$$-\frac{3 \operatorname{ArcTanh}[\sin[c+dx]]}{256 a d} - \frac{\sec[c+dx]^8}{8 a d} + \frac{\sec[c+dx]^{10}}{10 a d} - \frac{3 \sec[c+dx] \tan[c+dx]}{256 a d} -$$

$$\frac{\sec[c+dx]^3 \tan[c+dx]}{128 a d} - \frac{\sec[c+dx]^5 \tan[c+dx]}{160 a d} + \frac{3 \sec[c+dx]^7 \tan[c+dx]}{80 a d} - \frac{\sec[c+dx]^7 \tan[c+dx]^3}{10 a d}$$

Result (type 3, 365 leaves):

$$-\frac{1}{2560 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 (a + a \sin[c+dx])}$$

$$\left(-16 + 10 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + 20 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 + \right.$$

$$15 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 - 30 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10} +$$

$$30 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10} - \frac{10 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^8} +$$

$$\left. \frac{15 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^4} + \frac{30 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} \right)$$

■ **Problem 905: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^7 \text{Tan}[c + d x]^2}{a + a \text{Sin}[c + d x]} dx$$

Optimal (type 3, 172 leaves, 10 steps):

$$\begin{aligned} & - \frac{7 \text{ArcTanh}[\text{Sin}[c + d x]]}{256 a d} + \frac{\text{Sec}[c + d x]^8}{8 a d} - \frac{\text{Sec}[c + d x]^{10}}{10 a d} - \frac{7 \text{Sec}[c + d x] \text{Tan}[c + d x]}{256 a d} \\ & - \frac{7 \text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{384 a d} - \frac{7 \text{Sec}[c + d x]^5 \text{Tan}[c + d x]}{480 a d} - \frac{\text{Sec}[c + d x]^7 \text{Tan}[c + d x]}{80 a d} + \frac{\text{Sec}[c + d x]^9 \text{Tan}[c + d x]}{10 a d} \end{aligned}$$

Result (type 3, 417 leaves):

$$\begin{aligned} & \frac{1}{7680 d (a + a \text{Sin}[c + d x])} \\ & \left(150 - \frac{48}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^8} - \frac{30}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^6} + \frac{20}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \right. \\ & \frac{75}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + 210 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 - \\ & 210 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 + \frac{30 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^8} + \\ & \left. \frac{40 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^6} + \frac{15 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{60 \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} \right) \end{aligned}$$

■ **Problem 906: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^8 \text{Tan}[c + d x]}{a + a \text{Sin}[c + d x]} dx$$

Optimal (type 3, 154 leaves, 9 steps):

$$\begin{aligned} & \frac{7 \text{ArcTanh}[\text{Sin}[c + d x]]}{256 a d} + \frac{\text{Sec}[c + d x]^{10}}{10 a d} + \frac{7 \text{Sec}[c + d x] \text{Tan}[c + d x]}{256 a d} + \\ & \frac{7 \text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{384 a d} + \frac{7 \text{Sec}[c + d x]^5 \text{Tan}[c + d x]}{480 a d} + \frac{\text{Sec}[c + d x]^7 \text{Tan}[c + d x]}{80 a d} - \frac{\text{Sec}[c + d x]^9 \text{Tan}[c + d x]}{10 a d} \end{aligned}$$

Result (type 3, 415 leaves):

1

$$\begin{aligned}
& 7680 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^8 (a + a \sin[c+dx]) \\
& \left(48 + 90 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 + 100 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 + \right. \\
& 75 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 - 210 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10} + \\
& 210 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10} + \frac{30 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^8} + \\
& \left. \frac{80 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^6} + \frac{135 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^4} + \frac{210 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^{10}}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} \right)
\end{aligned}$$

■ **Problem 909: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c+dx]^2 \sec[c+dx]^9}{a + a \sin[c+dx]} dx$$

Optimal (type 3, 262 leaves, 4 steps):

$$\begin{aligned}
& -\frac{\csc[c+dx]}{ad} - \frac{437 \operatorname{Log}[1 - \sin[c+dx]]}{512 ad} - \frac{\operatorname{Log}[\sin[c+dx]]}{ad} + \frac{949 \operatorname{Log}[1 + \sin[c+dx]]}{512 ad} + \frac{a^3}{256 d (a - a \sin[c+dx])^4} + \\
& \frac{5 a^2}{192 d (a - a \sin[c+dx])^3} + \frac{57 a}{512 d (a - a \sin[c+dx])^2} + \frac{61}{128 d (a - a \sin[c+dx])} - \frac{a^4}{160 d (a + a \sin[c+dx])^5} - \\
& \frac{9 a^3}{256 d (a + a \sin[c+dx])^4} - \frac{47 a^2}{384 d (a + a \sin[c+dx])^3} - \frac{187 a}{512 d (a + a \sin[c+dx])^2} - \frac{315}{256 d (a + a \sin[c+dx])}
\end{aligned}$$

Result (type 3, 737 leaves):

$$\begin{aligned}
& - \frac{315}{256 d (a + a \sin[c + dx])} - \frac{1}{160 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^8 (a + a \sin[c + dx])} \\
& \frac{9}{256 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 (a + a \sin[c + dx])} - \frac{47}{384 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 (a + a \sin[c + dx])} \\
& \frac{187}{512 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 (a + a \sin[c + dx])} - \frac{\cot\left[\frac{1}{2}(c + dx)\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{2 d (a + a \sin[c + dx])} \\
& \frac{437 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{256 d (a + a \sin[c + dx])} + \\
& \frac{949 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{256 d (a + a \sin[c + dx])} - \\
& \frac{\log[\sin[c + dx]] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{d (a + a \sin[c + dx])} + \frac{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{256 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^8 (a + a \sin[c + dx])} + \\
& \frac{5 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{192 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^6 (a + a \sin[c + dx])} + \frac{57 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{512 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^4 (a + a \sin[c + dx])} \\
& \frac{61 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{128 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 (a + a \sin[c + dx])} - \frac{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \tan\left[\frac{1}{2}(c + dx)\right]}{2 d (a + a \sin[c + dx])}
\end{aligned}$$

■ **Problem 910: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c + dx]^3 \sec[c + dx]^9}{a + a \sin[c + dx]} dx$$

Optimal (type 3, 279 leaves, 4 steps):

$$\begin{aligned}
& \frac{\csc[c + dx]}{a d} - \frac{\csc[c + dx]^2}{2 a d} - \frac{843 \log[1 - \sin[c + dx]]}{512 a d} + \frac{6 \log[\sin[c + dx]]}{a d} - \frac{2229 \log[1 + \sin[c + dx]]}{512 a d} + \\
& \frac{256 d (a - a \sin[c + dx])^4}{a^4} + \frac{32 d (a - a \sin[c + dx])^3}{11 a^3} + \frac{512 d (a - a \sin[c + dx])^2}{23 a^2} + \frac{256 d (a - a \sin[c + dx])}{325 a} + \\
& \frac{160 d (a + a \sin[c + dx])^5}{256 d (a + a \sin[c + dx])^4} + \frac{128 d (a + a \sin[c + dx])^3}{512 d (a + a \sin[c + dx])^2} + \frac{5}{2 d (a + a \sin[c + dx])}
\end{aligned}$$

Result (type 3, 845 leaves):

$$\begin{aligned}
& \frac{5}{2 d (a + a \sin [c + d x])} + \frac{1}{160 d \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^8 (a + a \sin [c + d x])} + \\
& \frac{11}{256 d \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^6 (a + a \sin [c + d x])} + \frac{23}{128 d \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^4 (a + a \sin [c + d x])} + \\
& \frac{325}{512 d \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2 (a + a \sin [c + d x])} + \frac{\cot \left[\frac{1}{2} (c + d x) \right] \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{2 d (a + a \sin [c + d x])} - \\
& \frac{\operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^2 \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{8 d (a + a \sin [c + d x])} - \frac{843 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{256 d (a + a \sin [c + d x])} - \\
& \frac{2229 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{256 d (a + a \sin [c + d x])} + \frac{6 \operatorname{Log} [\sin [c + d x]] \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{d (a + a \sin [c + d x])} - \\
& \frac{\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{8 d (a + a \sin [c + d x])} + \frac{\left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{256 d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^8 (a + a \sin [c + d x])} + \\
& \frac{\left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{32 d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^6 (a + a \sin [c + d x])} + \frac{81 \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{512 d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^4 (a + a \sin [c + d x])} + \\
& \frac{203 \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2}{256 d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^2 (a + a \sin [c + d x])} + \frac{\left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{2 d (a + a \sin [c + d x])}
\end{aligned}$$

■ **Problem 911: Result more than twice size of optimal antiderivative.**

$$\int (g \operatorname{Sec}[e + f x])^p (d \sin[e + f x])^n (a + a \sin[e + f x])^m dx$$

Optimal (type 6, 127 leaves, 5 steps):

$$\begin{aligned}
& \frac{1}{d f (1+n)} \operatorname{AppellF1} \left[1+n, \frac{1+p}{2}, \frac{1}{2} (1-2m+p), 2+n, \sin[e + f x], -\sin[e + f x] \right] \operatorname{Sec}[e + f x] \\
& (g \operatorname{Sec}[e + f x])^p (1 - \sin[e + f x])^{\frac{1+p}{2}} (d \sin[e + f x])^{1+n} (1 + \sin[e + f x])^{\frac{1}{2} (1-2m+p)} (a + a \sin[e + f x])^m
\end{aligned}$$

Result (type 6, 3577 leaves):

$$\begin{aligned}
& \left((-3+p) \operatorname{AppellF1} \left[\frac{1}{2} - \frac{p}{2}, -n, 1+m+n-p, \frac{3}{2} - \frac{p}{2}, \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \left. \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-m} \operatorname{Sec}[e + f x]^{-1+p} (g \operatorname{Sec}[e + f x])^p \sin[e + f x]^n (d \sin[e + f x])^n (a + a \sin[e + f x])^m \right) /
\end{aligned}$$

$$\begin{aligned}
& 1+m+n-p, \frac{7}{2}-\frac{p}{2}, \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \Bigg) + \\
& (1+m+n-p) \left(-\frac{1}{\frac{5}{2}-\frac{p}{2}} n \left(\frac{3}{2}-\frac{p}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{p}{2}, 1-n, 2+m+n-p, \frac{7}{2}-\frac{p}{2}, \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \\
& \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{\frac{5}{2}-\frac{p}{2}} (2+m+n-p) \left(\frac{3}{2}-\frac{p}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{p}{2}, -n, 3+m+n-p, \right. \\
& \left. \left. \left. \frac{7}{2}-\frac{p}{2}, \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right] \right) \Bigg) \Bigg) / \\
& \left((-1+p) \left((-3+p) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{p}{2}, -n, 1+m+n-p, \frac{3}{2}-\frac{p}{2}, \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) + \right. \\
& \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2}-\frac{p}{2}, 1-n, 1+m+n-p, \frac{5}{2}-\frac{p}{2}, \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + (1+m+n-p) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-\frac{p}{2}, -n, 2+m+n-p, \frac{5}{2}-\frac{p}{2}, \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 913: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[e+fx] (a+a \sin[e+fx])^4 (c+d \sin[e+fx])^n dx$$

Optimal (type 3, 175 leaves, 3 steps):

$$\begin{aligned}
& \frac{a^4 (c-d)^4 (c+d \sin[e+fx])^{1+n}}{d^5 f (1+n)} - \frac{4 a^4 (c-d)^3 (c+d \sin[e+fx])^{2+n}}{d^5 f (2+n)} + \\
& \frac{6 a^4 (c-d)^2 (c+d \sin[e+fx])^{3+n}}{d^5 f (3+n)} - \frac{4 a^4 (c-d) (c+d \sin[e+fx])^{4+n}}{d^5 f (4+n)} + \frac{a^4 (c+d \sin[e+fx])^{5+n}}{d^5 f (5+n)}
\end{aligned}$$

Result (type 3, 1209 leaves):

$$\frac{1}{f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^8} (a + a \sin[e + f x])^4 (c + d \sin[e + f x])^n$$

$$\left(\left(192 c^5 - 960 c^4 d + 1920 c^3 d^2 - 1920 c^2 d^3 + 960 c d^4 + 1320 d^5 - 192 c^4 d n + 912 c^3 d^2 n - 1744 c^2 d^3 n + 1730 c d^4 n + 2444 d^5 n + 144 c^3 d^2 n^2 - \right. \right.$$

$$\left. 672 c^2 d^3 n^2 + 1297 c d^4 n^2 + 1436 d^5 n^2 - 80 c^2 d^3 n^3 + 370 c d^4 n^3 + 340 d^5 n^3 + 35 c d^4 n^4 + 28 d^5 n^4 \right) / \left(8 d^5 (1 + n) (2 + n) (3 + n) (4 + n) (5 + n) \right) +$$

$$\left(\left(2520 d^4 - 192 c^4 n + 960 c^3 d n - 1968 c^2 d^2 n + 2160 c d^3 n + 4290 d^4 n + 192 c^3 d n^2 - 936 c^2 d^2 n^2 + 1912 c d^3 n^2 + 2507 d^4 n^2 - \right. \right.$$

$$\left. 120 c^2 d^2 n^3 + 576 c d^3 n^3 + 594 d^4 n^3 + 56 c d^3 n^4 + 49 d^4 n^4 \right) \left(-\frac{i \cos[e + f x]}{16 d^4} + \frac{\sin[e + f x]}{16 d^4} \right) / \left((1 + n) (2 + n) (3 + n) (4 + n) (5 + n) \right) +$$

$$\left(\left(2520 d^4 - 192 c^4 n + 960 c^3 d n - 1968 c^2 d^2 n + 2160 c d^3 n + 4290 d^4 n + 192 c^3 d n^2 - 936 c^2 d^2 n^2 + 1912 c d^3 n^2 + 2507 d^4 n^2 - \right. \right.$$

$$\left. 120 c^2 d^2 n^3 + 576 c d^3 n^3 + 594 d^4 n^3 + 56 c d^3 n^4 + 49 d^4 n^4 \right) \left(\frac{i \cos[e + f x]}{16 d^4} + \frac{\sin[e + f x]}{16 d^4} \right) / \left((1 + n) (2 + n) (3 + n) (4 + n) (5 + n) \right) +$$

$$\left(\left(-360 d^3 - 12 c^3 n + 60 c^2 d n - 126 c d^2 n - 312 d^3 n + 12 c^2 d n^2 - 59 c d^2 n^2 - 88 d^3 n^2 - 7 c d^2 n^3 - 8 d^3 n^3 \right) \right.$$

$$\left. \left(\frac{\cos[2(e + f x)]}{4 d^3} - \frac{i \sin[2(e + f x)]}{4 d^3} \right) \right) / \left((2 + n) (3 + n) (4 + n) (5 + n) \right) +$$

$$\left(\left(-360 d^3 - 12 c^3 n + 60 c^2 d n - 126 c d^2 n - 312 d^3 n + 12 c^2 d n^2 - 59 c d^2 n^2 - 88 d^3 n^2 - 7 c d^2 n^3 - 8 d^3 n^3 \right) \right.$$

$$\left. \left(\frac{\cos[2(e + f x)]}{4 d^3} + \frac{i \sin[2(e + f x)]}{4 d^3} \right) \right) / \left((2 + n) (3 + n) (4 + n) (5 + n) \right) +$$

$$\frac{(540 d^2 - 16 c^2 n + 80 c d n + 251 d^2 n + 16 c d n^2 + 29 d^2 n^2) \left(-\frac{i \cos[3(e + f x)]}{32 d^2} - \frac{\sin[3(e + f x)]}{32 d^2} \right)}{(3 + n) (4 + n) (5 + n)} +$$

$$\frac{(540 d^2 - 16 c^2 n + 80 c d n + 251 d^2 n + 16 c d n^2 + 29 d^2 n^2) \left(\frac{i \cos[3(e + f x)]}{32 d^2} - \frac{\sin[3(e + f x)]}{32 d^2} \right)}{(3 + n) (4 + n) (5 + n)} +$$

$$\frac{(20 d + c n + 4 d n) \left(\frac{\cos[4(e + f x)]}{16 d} - \frac{i \sin[4(e + f x)]}{16 d} \right)}{(4 + n) (5 + n)} + \frac{(20 d + c n + 4 d n) \left(\frac{\cos[4(e + f x)]}{16 d} + \frac{i \sin[4(e + f x)]}{16 d} \right)}{(4 + n) (5 + n)} +$$

$$\left. \frac{-\frac{1}{32} i \cos[5(e + f x)] + \frac{1}{32} \sin[5(e + f x)]}{5 + n} + \frac{\frac{1}{32} i \cos[5(e + f x)] + \frac{1}{32} \sin[5(e + f x)]}{5 + n} \right)$$

■ **Problem 914: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[e + f x] (a + a \sin[e + f x])^3 (c + d \sin[e + f x])^n dx$$

Optimal (type 3, 139 leaves, 3 steps):

$$-\frac{a^3 (c-d)^3 (c+d \operatorname{Sin}[e+f x])^{1+n}}{d^4 f (1+n)} + \frac{3 a^3 (c-d)^2 (c+d \operatorname{Sin}[e+f x])^{2+n}}{d^4 f (2+n)} - \frac{3 a^3 (c-d) (c+d \operatorname{Sin}[e+f x])^{3+n}}{d^4 f (3+n)} + \frac{a^3 (c+d \operatorname{Sin}[e+f x])^{4+n}}{d^4 f (4+n)}$$

Result (type 3, 784 leaves):

$$\frac{1}{f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^6}$$

$$(a + a \operatorname{Sin}[e+f x])^3 (c+d \operatorname{Sin}[e+f x])^n \left(\begin{aligned} & \left(-48 c^4 + 192 c^3 d - 288 c^2 d^2 + 192 c d^3 + 162 d^4 + 48 c^3 d n - 180 c^2 d^2 n + \right. \\ & \quad \left. 256 c d^3 n + 261 d^4 n - 36 c^2 d^2 n^2 + 132 c d^3 n^2 + 114 d^4 n^2 + 20 c d^3 n^3 + 15 d^4 n^3 \right) / \left(8 d^4 (1+n) (2+n) (3+n) (4+n) \right) + \\ & \left(\left(168 d^3 + 24 c^3 n - 96 c^2 d n + 150 c d^2 n + 230 d^3 n - 24 c^2 d n^2 + 93 c d^2 n^2 + 99 d^3 n^2 + 15 c d^2 n^3 + 13 d^3 n^3 \right) \left(-\frac{i \operatorname{Cos}[e+f x]}{8 d^3} + \frac{\operatorname{Sin}[e+f x]}{8 d^3} \right) \right) / \\ & \left((1+n) (2+n) (3+n) (4+n) \right) + \\ & \left(\left(168 d^3 + 24 c^3 n - 96 c^2 d n + 150 c d^2 n + 230 d^3 n - 24 c^2 d n^2 + 93 c d^2 n^2 + 99 d^3 n^2 + 15 c d^2 n^3 + 13 d^3 n^3 \right) \left(\frac{i \operatorname{Cos}[e+f x]}{8 d^3} + \frac{\operatorname{Sin}[e+f x]}{8 d^3} \right) \right) / \\ & \left((1+n) (2+n) (3+n) (4+n) \right) + \frac{\left(-42 d^2 + 3 c^2 n - 12 c d n - 26 d^2 n - 3 c d n^2 - 4 d^2 n^2 \right) \left(\frac{\operatorname{Cos}[2(e+f x)]}{4 d^2} - \frac{i \operatorname{Sin}[2(e+f x)]}{4 d^2} \right)}{(2+n) (3+n) (4+n)} + \\ & \frac{\left(-42 d^2 + 3 c^2 n - 12 c d n - 26 d^2 n - 3 c d n^2 - 4 d^2 n^2 \right) \left(\frac{\operatorname{Cos}[2(e+f x)]}{4 d^2} + \frac{i \operatorname{Sin}[2(e+f x)]}{4 d^2} \right)}{(2+n) (3+n) (4+n)} + \frac{(12 d + c n + 3 d n) \left(-\frac{i \operatorname{Cos}[3(e+f x)]}{8 d} - \frac{\operatorname{Sin}[3(e+f x)]}{8 d} \right)}{(3+n) (4+n)} + \\ & \frac{(12 d + c n + 3 d n) \left(\frac{i \operatorname{Cos}[3(e+f x)]}{8 d} - \frac{\operatorname{Sin}[3(e+f x)]}{8 d} \right)}{(3+n) (4+n)} + \frac{\frac{1}{16} \operatorname{Cos}[4(e+f x)] - \frac{1}{16} i \operatorname{Sin}[4(e+f x)]}{4+n} + \frac{\frac{1}{16} \operatorname{Cos}[4(e+f x)] + \frac{1}{16} i \operatorname{Sin}[4(e+f x)]}{4+n} \end{aligned} \right)$$

■ **Problem 917: Unable to integrate problem.**

$$\int \frac{\operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^n}{a + a \operatorname{Sin}[e+f x]} dx$$

Optimal (type 5, 60 leaves, 2 steps):

$$-\frac{\operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c+d \operatorname{Sin}[e+f x]}{c-d}\right] (c+d \operatorname{Sin}[e+f x])^{1+n}}{a (c-d) f (1+n)}$$

Result (type 8, 33 leaves):

$$\int \frac{\operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^n}{a + a \operatorname{Sin}[e+f x]} dx$$

■ **Problem 918: Unable to integrate problem.**

$$\int \frac{\operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^n}{(a + a \operatorname{Sin}[e+f x])^2} dx$$

Optimal (type 5, 60 leaves, 2 steps) :

$$\frac{d \operatorname{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{c+d \operatorname{Sin}[e+f x]}{c-d}\right] (c+d \operatorname{Sin}[e+f x])^{1+n}}{a^2 (c-d)^2 f (1+n)}$$

Result (type 8, 33 leaves) :

$$\int \frac{\operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^n}{(a+a \operatorname{Sin}[e+f x])^2} dx$$

■ **Problem 919: Unable to integrate problem.**

$$\int \frac{\operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^n}{(a+a \operatorname{Sin}[e+f x])^3} dx$$

Optimal (type 5, 63 leaves, 2 steps) :

$$\frac{d^2 \operatorname{Hypergeometric2F1}\left[3, 1+n, 2+n, \frac{c+d \operatorname{Sin}[e+f x]}{c-d}\right] (c+d \operatorname{Sin}[e+f x])^{1+n}}{a^3 (c-d)^3 f (1+n)}$$

Result (type 8, 33 leaves) :

$$\int \frac{\operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^n}{(a+a \operatorname{Sin}[e+f x])^3} dx$$

■ **Problem 920: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e+f x] (a+a \operatorname{Sin}[e+f x])^m (c+d \operatorname{Sin}[e+f x])^4 dx$$

Optimal (type 3, 170 leaves, 3 steps) :

$$\frac{(c-d)^4 (a+a \operatorname{Sin}[e+f x])^{1+m}}{a f (1+m)} + \frac{4 (c-d)^3 d (a+a \operatorname{Sin}[e+f x])^{2+m}}{a^2 f (2+m)} + \frac{6 (c-d)^2 d^2 (a+a \operatorname{Sin}[e+f x])^{3+m}}{a^3 f (3+m)} + \frac{4 (c-d) d^3 (a+a \operatorname{Sin}[e+f x])^{4+m}}{a^4 f (4+m)} + \frac{d^4 (a+a \operatorname{Sin}[e+f x])^{5+m}}{a^5 f (5+m)}$$

Result (type 3, 1457 leaves) :

$$\begin{aligned}
& \frac{1}{f} (a (1 + \text{Sin}[e + f x]))^m \\
& \left(\left(960 c^4 - 960 c^3 d + 1920 c^2 d^2 - 600 c d^3 + 192 d^4 + 1232 c^4 m + 208 c^3 d m + 1344 c^2 d^2 m + 300 c d^3 m + 66 d^4 m + 568 c^4 m^2 + 560 c^3 d m^2 + 792 c^2 d^2 m^2 + \right. \right. \\
& \quad \left. \left. 204 c d^3 m^2 + 81 d^4 m^2 + 112 c^4 m^3 + 176 c^3 d m^3 + 240 c^2 d^2 m^3 + 84 c d^3 m^3 + 18 d^4 m^3 + 8 c^4 m^4 + 16 c^3 d m^4 + 24 c^2 d^2 m^4 + 12 c d^3 m^4 + 3 d^4 m^4 \right) / \right. \\
& \quad \left. (8 (1 + m) (2 + m) (3 + m) (4 + m) (5 + m)) + \frac{1}{(1 + m) (2 + m) (3 + m) (4 + m) (5 + m)} \right. \\
& \quad \left(960 c^4 + 1440 c^2 d^2 + 120 d^4 + 1232 c^4 m + 1920 c^3 d m + 888 c^2 d^2 m + 1200 c d^3 m + 10 d^4 m + 568 c^4 m^2 + 1504 c^3 d m^2 + 900 c^2 d^2 m^2 + 600 c d^3 m^2 + \right. \\
& \quad \left. 103 d^4 m^2 + 112 c^4 m^3 + 384 c^3 d m^3 + 336 c^2 d^2 m^3 + 192 c d^3 m^3 + 26 d^4 m^3 + 8 c^4 m^4 + 32 c^3 d m^4 + 36 c^2 d^2 m^4 + 24 c d^3 m^4 + 5 d^4 m^4 \right) \\
& \quad \left(-\frac{1}{16} i \text{Cos}[e + f x] + \frac{1}{16} \text{Sin}[e + f x] \right) + \frac{1}{(1 + m) (2 + m) (3 + m) (4 + m) (5 + m)} \left(960 c^4 + 1440 c^2 d^2 + 120 d^4 + 1232 c^4 m + \right. \\
& \quad \left. 1920 c^3 d m + 888 c^2 d^2 m + 1200 c d^3 m + 10 d^4 m + 568 c^4 m^2 + 1504 c^3 d m^2 + 900 c^2 d^2 m^2 + 600 c d^3 m^2 + 103 d^4 m^2 + 112 c^4 m^3 + 384 c^3 d m^3 + \right. \\
& \quad \left. 336 c^2 d^2 m^3 + 192 c d^3 m^3 + 26 d^4 m^3 + 8 c^4 m^4 + 32 c^3 d m^4 + 36 c^2 d^2 m^4 + 24 c d^3 m^4 + 5 d^4 m^4 \right) \left(\frac{1}{16} i \text{Cos}[e + f x] + \frac{1}{16} \text{Sin}[e + f x] \right) + \\
& \quad \left(\left(-240 c^3 d - 120 c d^3 - 188 c^3 d m - 120 c^2 d^2 m - 64 c d^3 m - 18 d^4 m - 48 c^3 d m^2 - 54 c^2 d^2 m^2 - 28 c d^3 m^2 - 5 d^4 m^2 - 4 c^3 d m^3 - \right. \right. \\
& \quad \left. \left. 6 c^2 d^2 m^3 - 4 c d^3 m^3 - d^4 m^3 \right) \left(\frac{1}{4} \text{Cos}[2 (e + f x)] - \frac{1}{4} i \text{Sin}[2 (e + f x)] \right) \right) / \left((2 + m) (3 + m) (4 + m) (5 + m) \right) + \\
& \quad \left(\left(-240 c^3 d - 120 c d^3 - 188 c^3 d m - 120 c^2 d^2 m - 64 c d^3 m - 18 d^4 m - 48 c^3 d m^2 - 54 c^2 d^2 m^2 - 28 c d^3 m^2 - 5 d^4 m^2 - 4 c^3 d m^3 - \right. \right. \\
& \quad \left. \left. 6 c^2 d^2 m^3 - 4 c d^3 m^3 - d^4 m^3 \right) \left(\frac{1}{4} \text{Cos}[2 (e + f x)] + \frac{1}{4} i \text{Sin}[2 (e + f x)] \right) \right) / \left((2 + m) (3 + m) (4 + m) (5 + m) \right) + \\
& \quad \frac{1}{(3 + m) (4 + m) (5 + m)} \left(480 c^2 d^2 + 60 d^4 + 216 c^2 d^2 m + 80 c d^3 m + 19 d^4 m + 24 c^2 d^2 m^2 + 16 c d^3 m^2 + 5 d^4 m^2 \right) \\
& \quad \left(-\frac{1}{32} i \text{Cos}[3 (e + f x)] - \frac{1}{32} \text{Sin}[3 (e + f x)] \right) + \frac{1}{(3 + m) (4 + m) (5 + m)} \\
& \quad \left(480 c^2 d^2 + 60 d^4 + 216 c^2 d^2 m + 80 c d^3 m + 19 d^4 m + 24 c^2 d^2 m^2 + 16 c d^3 m^2 + 5 d^4 m^2 \right) \left(\frac{1}{32} i \text{Cos}[3 (e + f x)] - \frac{1}{32} \text{Sin}[3 (e + f x)] \right) + \\
& \quad \frac{(20 c d^3 + 4 c d^3 m + d^4 m) \left(\frac{1}{16} \text{Cos}[4 (e + f x)] - \frac{1}{16} i \text{Sin}[4 (e + f x)] \right)}{(4 + m) (5 + m)} + \\
& \quad \frac{(20 c d^3 + 4 c d^3 m + d^4 m) \left(\frac{1}{16} \text{Cos}[4 (e + f x)] + \frac{1}{16} i \text{Sin}[4 (e + f x)] \right)}{(4 + m) (5 + m)} + \\
& \quad \left. \frac{-\frac{1}{32} i d^4 \text{Cos}[5 (e + f x)] + \frac{1}{32} d^4 \text{Sin}[5 (e + f x)]}{5 + m} + \frac{\frac{1}{32} i d^4 \text{Cos}[5 (e + f x)] + \frac{1}{32} d^4 \text{Sin}[5 (e + f x)]}{5 + m} \right)
\end{aligned}$$

■ **Problem 921: Result more than twice size of optimal antiderivative.**

$$\int \cos[e + f x] (a + a \sin[e + f x])^m (c + d \sin[e + f x])^3 dx$$

Optimal (type 3, 133 leaves, 3 steps):

$$\frac{(c-d)^3 (a+a \sin[e+fx])^{1+m}}{af(1+m)} + \frac{3(c-d)^2 d (a+a \sin[e+fx])^{2+m}}{a^2 f(2+m)} + \frac{3(c-d)d^2 (a+a \sin[e+fx])^{3+m}}{a^3 f(3+m)} + \frac{d^3 (a+a \sin[e+fx])^{4+m}}{a^4 f(4+m)}$$

Result (type 3, 295 leaves):

$$\frac{1}{4f(1+m)(2+m)(3+m)(4+m)} \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 (a(1+\sin[e+fx]))^m$$

$$(96c^3 - 144c^2d + 144cd^2 - 36d^3 + 104c^3m - 84c^2dm + 108cd^2m - 18d^3m + 36c^3m^2 - 12c^2dm^2 + 42cd^2m^2 - 6d^3m^2 + 4c^3m^3 + 6cd^2m^3 + 6d^2(2+3m+m^2)(d-c(4+m)) \cos[2(e+fx)] + 3d(1+m)(-8cd(4+m) + d^2(14+5m+m^2) + 4c^2(12+7m+m^2)) \sin[e+fx] - 6d^3 \sin[3(e+fx)] - 11d^3m \sin[3(e+fx)] - 6d^3m^2 \sin[3(e+fx)] - d^3m^3 \sin[3(e+fx)])$$

■ **Problem 926: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x] (a + a \sin[e + f x])^m}{(c + d \sin[e + f x])^3} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}\left[3, 1+m, 2+m, -\frac{d(1+\sin[e+fx])}{c-d}\right] (a+a \sin[e+fx])^{1+m}}{a(c-d)^3 f(1+m)}$$

Result (type 5, 203 leaves):

$$\left(2(a(1+\sin[e+fx]))^m \left(\frac{c-d}{c+d \sin[e+fx]} \right)^m \right.$$

$$\left. \left(-2d(1+m) \cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^2 \text{Hypergeometric2F1}\left[m, 2+m, 3+m, \frac{2d \cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{c+d \sin[e+fx]} \right] + \right.$$

$$\left. (2+m) \text{Hypergeometric2F1}\left[m, 1+m, 2+m, \frac{2d \cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{c+d \sin[e+fx]} \right] (c+d \sin[e+fx]) \right)^2$$

$$\left. \sin\left[\frac{1}{4}(2e+\pi+2fx)\right]^2 \right) / ((c-d)^2 f(1+m)(2+m)(c+d \sin[e+fx])^2)$$

■ **Problem 932: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cot[c + d x] (a + a \sin[c + d x])^m dx$$

Optimal (type 5, 43 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}\left[1, 1+m, 2+m, 1+\text{Sin}[c+dx]\right] (a+a\text{Sin}[c+dx])^{1+m}}{ad(1+m)}$$

Result (type 6, 12204 leaves):

$$\begin{aligned} & -\frac{1}{d} \text{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^{-2m} (a+a\text{Sin}[c+dx])^m \\ & \left(\frac{1}{2^m} \text{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^{2m} \left(-1+(-\text{Csc}[c+dx])^m \text{Hypergeometric2F1}\left[m, m, 1+m, 2\text{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \text{Csc}[c+dx]\right]\right) + \right. \\ & \left. \left(\text{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^{2+2m} \text{Csc}[c+dx] \left(\text{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-m} \right. \right. \\ & \left. \left(4m \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \left(\text{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^m \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] + \right. \right. \\ & \left. \left. \text{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(\frac{-i+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \right. \right. \\ & \left. \left. \left(\frac{i+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m - \text{AppellF1}\left[2m, m, m, 1+2m, \frac{1-i}{1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \right. \right. \\ & \left. \left. \left(\frac{-i+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \left(\frac{i+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m\right)\right) / \\ & \left(16m \left(-\frac{1}{8} \left(\text{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-m} \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] \left(4m \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right. \right. \right. \\ & \left. \left. \left(\text{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^m \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] + \text{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \right. \right. \right. \\ & \left. \left. -\frac{1-i}{-1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(\frac{-i+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \left(\frac{i+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m - \text{AppellF1}\left[2m, m, m, 1+2m, \right. \right. \\ & \left. \left. \frac{1-i}{1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(\frac{-i+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \left(\frac{i+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m\right) + \\ & \left.\frac{1}{8m} \left(\text{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-m} \left(2m \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \left(\text{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{1+m} + \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^{-1+m} \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \\
& \left(-\frac{\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2(-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right])}{2\left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2} + \frac{\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)} \right) -m \operatorname{AppellF1}\left[2m, m, m, \right. \\
& \left. 1+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^{-1+m} \\
& \left(-\frac{\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2(i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right])}{2\left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2} + \frac{\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)} \right) + 2m \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right)^{1+m} \\
& \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-1-m} \right) \right) \Bigg) - \\
& \left(\cos\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \operatorname{Csc}[c+dx] \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right)^{-m} \sin\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right. \\
& \left. \left(4m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right)^m \tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] - \right. \right. \\
& \left. \operatorname{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \right. \\
& \left. \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m + \operatorname{AppellF1}\left[2m, m, m, 1+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \right. \\
& \left. \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \right) \Bigg) / \\
& \left(16m \left(\frac{1}{8} \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right)^{-m} \tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] \left(4m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right. \right. \right. \\
& \left. \left. \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right)^m \tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] - \operatorname{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \right. \right. \right. \\
& \left. \left. \left. -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m + \operatorname{AppellF1}\left[2m, m, m, 1+2m, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \right) - \\
& \frac{1}{8m} \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right)^{-m} \left(2m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right)^{1+m} + \right. \\
& 4m^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right)^m \tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 - \left((1-i)m^2 \right. \\
& \left. \operatorname{AppellF1}\left[1+2m, m, 1+m, 2+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right) / \\
& \left((1+2m) \left(-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] \right)^2 \right) + \left((1+i)m^2 \operatorname{AppellF1}\left[1+2m, 1+m, m, 2+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right. \right. \\
& \left. \left. -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right) / \left((1+2m) \left(-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] \right)^2 \right) \right) \\
& \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m - m \operatorname{AppellF1}\left[2m, m, m, 1+2m, \right. \\
& \left. -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^{-1+m} \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \\
& \left(\frac{\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)} - \frac{\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\left(-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{2\left(-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2} \right) - m \operatorname{AppellF1}\left[2m, m, m, 1+2m, \right. \\
& \left. -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^{-1+m} \\
& \left(\frac{\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)} - \frac{\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\left(i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{2\left(-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2} \right) + \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \\
& \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right)^m \left(- \left((1+i)m^2 \operatorname{AppellF1}\left[1+2m, m, 1+m, 2+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right. \right. \right. \\
& \left. \left. \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]} \right] \sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \right) / \left((1+2m) \left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] \right)^2 \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left((1-i)^m \operatorname{AppellF1}\left[1+2m, 1+m, m, 2+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) / \left((1+2m) \left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2 \right) + m \operatorname{AppellF1}\left[2m, m, m, 1+2m, \right. \\
& \quad \left. \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^{-1+m} \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \\
& \quad \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2(-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right])}{2\left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}\right) + m \operatorname{AppellF1}\left[2m, m, m, \right. \\
& \quad \left. 1+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^{-1+m} \\
& \quad \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2(i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right])}{2\left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}\right) + 2m \left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{1+m} \\
& \quad \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] + \left(1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-1-m}\right)\right)\right) + \\
& \frac{1}{d} \operatorname{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^{-2m} (a+a\operatorname{Sin}[c+dx])^m \left(-\frac{1}{2m} \operatorname{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^{2m} (-1+(-\operatorname{Csc}[c+dx])^m) \right. \\
& \quad \left. \operatorname{Hypergeometric2F1}\left[m, m, 1+m, 2\operatorname{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 \operatorname{Csc}[c+dx]\right]\right) + \\
& \left(\operatorname{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^{2+2m} \left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-m} \left(4m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \right. \right. \\
& \quad \left. \left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^m \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] + \right. \\
& \quad \left. \operatorname{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \right. \\
& \quad \left. \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m - \right.
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[2m, m, m, 1+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \\
& \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \Big/ \\
& \left(16m \left(\cos\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] - \sin\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right) \left(\cos\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] + \sin\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)\right) \\
& \left(-\frac{1}{8} \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2\right)^{-m} \tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] \\
& \left(4m \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2\right)^m \tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right] + \\
& \text{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \\
& \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m - \text{AppellF1}\left[2m, m, m, 1+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \\
& \left(\frac{-i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m \left(\frac{i+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m + \\
& \frac{1}{8m} \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2\right)^{-m} \left(2m \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2\right)^{1+m} + \\
& 4m^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right] \left(\sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2\right)^m \tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2 + \\
& \left(\left((1-i)m^2 \text{AppellF1}\left[1+2m, m, 1+m, 2+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right]\right) \right. \\
& \left. \sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) \Big/ \left((1+2m) \left(-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2\right) + \left((1+i)m^2 \right. \\
& \left. \text{AppellF1}\left[1+2m, 1+m, m, 2+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right] \sec\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\left(i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^2}+\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)}+2m\left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{1+m} \right. \\
& \left. \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2},1+m,\frac{3}{2},-\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right]+\left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-1-m}\right)\right) \Bigg) - \\
& \left(\operatorname{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)^{2m}\left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-m}\operatorname{Sin}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\left(4m\operatorname{Hypergeometric2F1}\left[\frac{1}{2},1+m,\frac{3}{2},\right. \right. \\
& \left. \left.-\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right]\left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^m\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]-\right. \\
& \left.\operatorname{AppellF1}\left[2m,m,m,1+2m,-\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]},-\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right]\right) \\
& \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m+ \\
& \left.\operatorname{AppellF1}\left[2m,m,m,1+2m,\frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]},\frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right]\right) \\
& \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m\left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m\Bigg) / \\
& \left(16m\left(\operatorname{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]-\operatorname{Sin}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)\left(\operatorname{Cos}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]+\operatorname{Sin}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right)\right) \\
& \left(\frac{1}{8}\left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^{-m}\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]\right) \\
& \left(4m\operatorname{Hypergeometric2F1}\left[\frac{1}{2},1+m,\frac{3}{2},-\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right]\left(\operatorname{Sec}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]^2\right)^m\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]-\right. \\
& \left.\operatorname{AppellF1}\left[2m,m,m,1+2m,-\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]},-\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right]\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m\right) \\
& \left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right)^m+\operatorname{AppellF1}\left[2m,m,m,1+2m,\frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]},\frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)\right]}\right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^m \left(\frac{i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^m - \\
& \frac{1}{8m} \left(\sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2 \right)^{-m} \left({}_2F_1\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2\right] \left(\sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2 \right)^{1+m} + \right. \\
& \left. 4m^2 {}_2F_1\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2\right] \left(\sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2 \right)^m \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2 - \right. \\
& \left. \left(\left((1-i)m^2 \operatorname{AppellF1}\left[1+2m, m, 1+m, 2+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}\right] \right. \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2 \right) / \left((1+2m) \left(-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right] \right)^2 \right) + \left((1+i)m^2 \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[1+2m, 1+m, m, 2+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}\right] \sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2 \right) \right) / \\
& \left. \left((1+2m) \left(-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right] \right)^2 \right) \right) \left(\frac{-i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^m \left(\frac{i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^m - \\
& m \operatorname{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}\right] \left(\frac{-i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^{-1+m} \\
& \left(\frac{i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^m \left(\frac{\sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2}{2(-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right])} - \frac{\sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2 (-i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right])}{2(-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right])^2} \right) - \\
& m \operatorname{AppellF1}\left[2m, m, m, 1+2m, -\frac{1+i}{-1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}, -\frac{1-i}{-1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}\right] \\
& \left(\frac{-i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^m \left(\frac{i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^{-1+m} \left(\frac{\sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2}{2(-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right])} - \right. \\
& \left. \frac{\sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2 (i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right])}{2(-1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right])^2} \right) + \left(\frac{-i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^m \left(\frac{i + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}{1 + \tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]} \right)^m \\
& \left. - \left((1+i)m^2 \operatorname{AppellF1}\left[1+2m, m, 1+m, 2+2m, \frac{1-i}{1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}, \frac{1+i}{1+\tan\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]}\right] \sec\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx)\right]^2 \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left((1+2m) \left(1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] \right)^2 \right) - \left((1-i) m^2 \operatorname{AppellF1}\left[1+2m, 1+m, m, 2+2m, \frac{1-i}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}, \right. \right. \\
& \left. \left. \frac{1+i}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 \right) / \left((1+2m) \left(1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] \right)^2 \right) + m \operatorname{AppellF1}\left[2m, m, m, \right. \\
& \left. 1+2m, \frac{1-i}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}, \frac{1+i}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]} \right] \left(\frac{-i + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]} \right)^{-1+m} \left(\frac{i + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]} \right)^m \\
& \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 (-i + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}{2 \left(1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] \right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2}{2 \left(1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] \right)} \right) + m \operatorname{AppellF1}\left[2m, m, m, \right. \\
& \left. 1+2m, \frac{1-i}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}, \frac{1+i}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]} \right] \left(\frac{-i + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]} \right)^m \left(\frac{i + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]}{1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]} \right)^{-1+m} \\
& \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2 (i + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right])}{2 \left(1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] \right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2}{2 \left(1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] \right)} \right) + 2m \left(\operatorname{Sec}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] \right)^{1+m} \\
& \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right]^2\right] + \left(1 + \tan\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right)\right] \right)^2 \right)^{-1-m} \Big) \Big) \Big) \Big) \Big)
\end{aligned}$$

- **Problem 933: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cot[c+dx] \operatorname{Csc}[c+dx] (a+a\sin[c+dx])^m dx$$

Optimal (type 5, 42 leaves, 3 steps):

$$\frac{\operatorname{Hypergeometric2F1}\left[2, 1+m, 2+m, 1+\sin[c+dx]\right] (a+a\sin[c+dx])^{1+m}}{a d (1+m)}$$

Result (type 6, 20340 leaves): Display of huge result suppressed!

- **Problem 934: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cot[c+dx] \operatorname{Csc}[c+dx]^2 (a+a\sin[c+dx])^m dx$$

Optimal (type 5, 43 leaves, 3 steps):

$$\frac{\operatorname{Hypergeometric2F1}\left[3, 1+m, 2+m, 1+\sin[c+dx]\right] (a+a\sin[c+dx])^{1+m}}{a d (1+m)}$$

Result (type 6, 35073 leaves): Display of huge result suppressed!

■ **Problem 936: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[e + f x]^2}{(a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])} dx$$

Optimal (type 3, 123 leaves, 6 steps):

$$-\frac{2\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{a^{3/2} (c-d) f} + \frac{2\sqrt{c+d} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{c+d} \sqrt{a+a \sin[e+fx]}}\right]}{a^{3/2} (c-d) \sqrt{d} f}$$

Result (type 3, 220 leaves):

$$\frac{1}{\sqrt{d} (-c+d) f (a (1 + \sin[e + f x]))^{3/2}} (-1)^{3/4} \left((-4 - 4i) \sqrt{d} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right]\right) +$$

$$(-1)^{1/4} \sqrt{c+d} \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) -$$

$$\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]\right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^3$$

■ **Problem 937: Humongous result has more than 200000 leaves.**

$$\int \frac{\cos[e + f x]^2}{(a + a \sin[e + f x])^{3/2} \sqrt{c + d \sin[e + f x]}} dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{a^{3/2} \sqrt{d} f} - \frac{2\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{a^{3/2} \sqrt{c-d} f}$$

Result (type ?, 208404 leaves): Display of huge result suppressed!

■ **Problem 938: Result more than twice size of optimal antiderivative.**

$$\int \cos[e + f x]^2 (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 135 leaves, 4 steps):

$$\left(2\sqrt{2} \operatorname{AppellF1}\left[\frac{3}{2} + m, -\frac{1}{2}, -n, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c-d}\right] \right)$$

$$\cos[e + f x] (a + a \sin[e + f x])^{1+m} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c - d} \right)^{-n} / \left(a f (3 + 2m) \sqrt{1 - \sin[e + f x]} \right)$$

Result (type 6, 391 leaves):

$$\left(10 (c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -n, \frac{5}{2}, \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{c+d}\right] \left(\cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^2\right)^{\frac{1}{2}+m} \right. \\ \left. \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2 \operatorname{Csc}[e+fx] (a(1+\sin[e+fx]))^m (c+d \sin[e+fx])^n \sin[2(e+fx)] \left(\sin\left[\frac{1}{4}(2e+\pi+2fx)\right]^2\right)^{-\frac{1}{2}-m} \right) / \\ \left(3f \left(-5(c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -n, \frac{5}{2}, \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{c+d}\right] + \right. \right. \\ \left. \left(4dn \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}-m, 1-n, \frac{7}{2}, \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{c+d}\right] + \right. \right. \\ \left. \left. (c+d)(1+2m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}-m, -n, \frac{7}{2}, \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2, \frac{2d \sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2 \right) \right)$$

■ **Problem 939: Attempted integration timed out after 120 seconds.**

$$\int \cos[e+fx]^2 (a+a \sin[e+fx])^3 (c+d \sin[e+fx])^n dx$$

Optimal (type 6, 119 leaves, 3 steps):

$$-\frac{1}{3f\sqrt{1+\sin[e+fx]}} 16\sqrt{2} a^3 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{7}{2}, -n, \frac{5}{2}, \frac{1}{2}(1-\sin[e+fx]), \frac{d(1-\sin[e+fx])}{c+d}\right] \\ \cos[e+fx] (1-\sin[e+fx]) (c+d \sin[e+fx])^n \left(\frac{c+d \sin[e+fx]}{c+d}\right)^{-n}$$

Result (type 1, 1 leaves):

???

■ **Problem 940: Unable to integrate problem.**

$$\int \cos[e+fx]^2 (a+a \sin[e+fx])^2 (c+d \sin[e+fx])^n dx$$

Optimal (type 6, 119 leaves, 3 steps):

$$-\frac{1}{3f\sqrt{1+\sin[e+fx]}} 8\sqrt{2} a^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{5}{2}, -n, \frac{5}{2}, \frac{1}{2}(1-\sin[e+fx]), \frac{d(1-\sin[e+fx])}{c+d}\right] \\ \cos[e+fx] (1-\sin[e+fx]) (c+d \sin[e+fx])^n \left(\frac{c+d \sin[e+fx]}{c+d}\right)^{-n}$$

Result (type 8, 35 leaves):

$$\int \cos[e + f x]^2 (a + a \sin[e + f x])^2 (c + d \sin[e + f x])^n dx$$

■ **Problem 941: Unable to integrate problem.**

$$\int \cos[e + f x]^2 (a + a \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 117 leaves, 3 steps):

$$-\frac{1}{3 f \sqrt{1 + \sin[e + f x]}} 4 \sqrt{2} a \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}, -n, \frac{5}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (1 - \sin[e + f x]) (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 8, 33 leaves):

$$\int \cos[e + f x]^2 (a + a \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

■ **Problem 943: Unable to integrate problem.**

$$\int \frac{\cos[e + f x]^2 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 6, 119 leaves, 3 steps):

$$-\left(\operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -n, \frac{5}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (1 - \sin[e + f x]) (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}\right) / \left(3 \sqrt{2} a^2 f \sqrt{1 + \sin[e + f x]}\right)$$

Result (type 8, 35 leaves):

$$\int \frac{\cos[e + f x]^2 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^2} dx$$

■ **Problem 944: Unable to integrate problem.**

$$\int \frac{\cos[e + f x]^2 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 6, 119 leaves, 3 steps):

$$-\left(\operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, -n, \frac{5}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (1 - \sin[e + f x]) (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}\right) / \left(6 \sqrt{2} a^3 f \sqrt{1 + \sin[e + f x]}\right)$$

Result (type 8, 35 leaves):

$$\int \frac{\cos[e + f x]^2 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^3} dx$$

■ **Problem 945: Result more than twice size of optimal antiderivative.**

$$\int \cos[e + f x]^4 (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 135 leaves, 4 steps):

$$\left(4 \sqrt{2} \operatorname{AppellF1} \left[\frac{5}{2} + m, -\frac{3}{2}, -n, \frac{7}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \right. \\ \left. \cos[e + f x] (a + a \sin[e + f x])^{2+m} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c - d} \right)^{-n} \right] / \left(a^2 f (5 + 2m) \sqrt{1 - \sin[e + f x]} \right)$$

Result (type 6, 455 leaves):

$$\left(224 (c + d) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{3}{2} - m, -n, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right. \\ \left. \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^3 \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2} (-3 - 2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^5 \right. \\ \left. \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{3}{2} + m} \left(c + d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^n (a + a \sin[e + f x])^m \right] / \\ \left(5 f \left(-7 (c + d) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{3}{2} - m, -n, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right. \right. \\ \left. \left. \left(4 d n \operatorname{AppellF1} \left[\frac{7}{2}, -\frac{3}{2} - m, 1 - n, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \right. \right. \\ \left. \left. (c + d) (3 + 2m) \operatorname{AppellF1} \left[\frac{7}{2}, -\frac{1}{2} - m, -n, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right)$$

■ **Problem 946: Unable to integrate problem.**

$$\int \cos[e + f x]^4 (a + a \sin[e + f x])^2 (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$-\frac{1}{5 f \sqrt{1+\sin [e+f x]}} 16 \sqrt{2} a^2 \operatorname{AppellF1}\left[\frac{5}{2},-\frac{7}{2},-n,\frac{7}{2},\frac{1}{2}(1-\sin [e+f x]),\frac{d(1-\sin [e+f x])}{c+d}\right]$$

$$\cos [e+f x](1-\sin [e+f x])^2(c+d \sin [e+f x])^n\left(\frac{c+d \sin [e+f x]}{c+d}\right)^{-n}$$

Result (type 8, 35 leaves):

$$\int \cos [e+f x]^4(a+a \sin [e+f x])^2(c+d \sin [e+f x])^n dx$$

■ **Problem 947: Unable to integrate problem.**

$$\int \cos [e+f x]^4(a+a \sin [e+f x])(c+d \sin [e+f x])^n dx$$

Optimal (type 6, 119 leaves, 3 steps):

$$-\frac{1}{5 f(1+\sin [e+f x])^{3/2}} 8 \sqrt{2} a \operatorname{AppellF1}\left[\frac{5}{2},-\frac{5}{2},-n,\frac{7}{2},\frac{1}{2}(1-\sin [e+f x]),\frac{d(1-\sin [e+f x])}{c+d}\right]$$

$$\cos [e+f x]^3(1-\sin [e+f x])(c+d \sin [e+f x])^n\left(\frac{c+d \sin [e+f x]}{c+d}\right)^{-n}$$

Result (type 8, 33 leaves):

$$\int \cos [e+f x]^4(a+a \sin [e+f x])(c+d \sin [e+f x])^n dx$$

■ **Problem 948: Unable to integrate problem.**

$$\int \frac{\cos [e+f x]^4(c+d \sin [e+f x])^n}{a+a \sin [e+f x]} dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$-\frac{1}{5 a f \sqrt{1+\sin [e+f x]}} 2 \sqrt{2} \operatorname{AppellF1}\left[\frac{5}{2},-\frac{1}{2},-n,\frac{7}{2},\frac{1}{2}(1-\sin [e+f x]),\frac{d(1-\sin [e+f x])}{c+d}\right]$$

$$\cos [e+f x](1-\sin [e+f x])^2(c+d \sin [e+f x])^n\left(\frac{c+d \sin [e+f x]}{c+d}\right)^{-n}$$

Result (type 8, 35 leaves):

$$\int \frac{\cos [e+f x]^4(c+d \sin [e+f x])^n}{a+a \sin [e+f x]} dx$$

■ **Problem 949: Unable to integrate problem.**

$$\int \frac{\cos [e+f x]^4(c+d \sin [e+f x])^n}{(a+a \sin [e+f x])^2} dx$$

Optimal (type 6, 121 leaves, 4 steps):

$$-\frac{1}{5 a^2 f \sqrt{1 + \sin[e + f x]}}$$

$$\sqrt{2} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -n, \frac{7}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (1 - \sin[e + f x])^2 (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 8, 35 leaves):

$$\int \frac{\cos[e + f x]^4 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^2} dx$$

■ **Problem 950: Unable to integrate problem.**

$$\int \frac{\cos[e + f x]^4 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$-\left(\operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, -n, \frac{7}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (1 - \sin[e + f x])^2 (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}\right) /$$

$$\left(5 \sqrt{2} a^3 f \sqrt{1 + \sin[e + f x]}\right)$$

Result (type 8, 35 leaves):

$$\int \frac{\cos[e + f x]^4 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^3} dx$$

■ **Problem 951: Unable to integrate problem.**

$$\int \frac{\cos[e + f x]^4 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^4} dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$-\left(\operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}, -n, \frac{7}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (1 - \sin[e + f x])^2 (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}\right) /$$

$$\left(10 \sqrt{2} a^4 f \sqrt{1 + \sin[e + f x]}\right)$$

Result (type 8, 35 leaves):

$$\int \frac{\cos[e + f x]^4 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^4} dx$$

■ **Problem 952: Unable to integrate problem.**

$$\int \frac{\cos[e + f x]^4 (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^5} dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$-\left(\text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}, -n, \frac{7}{2}, \frac{1}{2}(1 - \sin[e + fx]), \frac{d(1 - \sin[e + fx])}{c + d}\right] \cos[e + fx] (1 - \sin[e + fx])^2 (c + d \sin[e + fx])^n \left(\frac{c + d \sin[e + fx]}{c + d}\right)^{-n}\right) / (20 \sqrt{2} a^5 f \sqrt{1 + \sin[e + fx]})$$

Result (type 8, 35 leaves):

$$\int \frac{\cos[e + fx]^4 (c + d \sin[e + fx])^n}{(a + a \sin[e + fx])^5} dx$$

■ **Problem 957: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx] (a + a \sin[c + dx]) (A + B \sin[c + dx]) dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{a(A + B) \log[1 - \sin[c + dx]]}{d} - \frac{aB \sin[c + dx]}{d}$$

Result (type 3, 172 leaves):

$$-\frac{aA \log[\cos[c + dx]]}{d} - \frac{aB \log[\cos[c + dx]]}{d} - \frac{aA \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{aA \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} - \frac{aB \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{aB \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} - \frac{aB \sin[c + dx]}{d}$$

■ **Problem 958: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^3 (a + a \sin[c + dx]) (A + B \sin[c + dx]) dx$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{a(A - B) \text{ArcTanh}[\sin[c + dx]]}{2d} + \frac{a^2(A + B)}{2d(a - a \sin[c + dx])}$$

Result (type 3, 260 leaves):

$$\frac{1}{4d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} a \left(2A + 2B + iAx - iBx - 2A \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 2B \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + A \log\left[\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2\right] - B \log\left[\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2\right] + 2i(A - B) \text{ArcTan}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right] \right) (-1 + \sin[c + dx]) + (A - B) \left(-ix + 2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2\right]\right) \sin[c + dx]$$

- **Problem 959: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^5 (a + a \sin[c + dx]) (A + B \sin[c + dx]) dx$$

Optimal (type 3, 100 leaves, 4 steps):

$$\frac{a (3A - B) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a^3 (A + B)}{8d (a - a \sin[c + dx])^2} + \frac{a^2 A}{4d (a - a \sin[c + dx])} - \frac{a^2 (A - B)}{8d (a + a \sin[c + dx])}$$

Result (type 3, 357 leaves):

$$\frac{1}{16 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^4} a \left(\frac{2(-A + B)}{d} + i(3A - B) \times \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 - \frac{2i(3A - B) \operatorname{ArcTan}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{d} - \frac{2(3A - B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{d} + \frac{(3A - B) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2\right] \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{d} + \frac{2(A + B) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^4} + \frac{4A \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2}{d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2} \right) (1 + \sin[c + dx])$$

- **Problem 960: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^7 (a + a \sin[c + dx]) (A + B \sin[c + dx]) dx$$

Optimal (type 3, 157 leaves, 4 steps):

$$\frac{a (5A - B) \operatorname{ArcTanh}[\sin[c + dx]]}{16d} + \frac{a^4 (A + B)}{24d (a - a \sin[c + dx])^3} + \frac{a^3 (3A + B)}{32d (a - a \sin[c + dx])^2} + \frac{3a^2 A}{16d (a - a \sin[c + dx])} - \frac{a^3 (A - B)}{32d (a + a \sin[c + dx])^2} - \frac{a^2 (2A - B)}{16d (a + a \sin[c + dx])}$$

Result (type 3, 451 leaves):

$$\frac{1}{96 \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6}$$

$$a \left(\frac{3(-A+B)}{d} - \frac{6(2A-B) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2}{d} + 3i(5A-B) \times \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4 - \frac{6i(5A-B) \operatorname{ArcTan}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4}{d} - \frac{6(5A-B) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4}{d} + \frac{3(5A-B) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4}{d} + \frac{4(A+B) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4}{d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^6} + \frac{3(3A+B) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4}{d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^4} + \frac{18A \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^4}{d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} \right) (1 + \sin[c+dx])$$

■ **Problem 964: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^2 (a + a \sin[c+dx]) (A + B \sin[c+dx]) dx$$

Optimal (type 3, 29 leaves, 2 steps):

$$-a B x + \frac{(A+B) \sec[c+dx] (a + a \sin[c+dx])}{d}$$

Result (type 3, 85 leaves):

$$\frac{a \left(-B dx \cos\left[\frac{dx}{2}\right] + 2(A+B) \sin\left[\frac{dx}{2}\right] + B dx \sin\left[c + \frac{dx}{2}\right] \right)}{d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)}$$

■ **Problem 966: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^6 (a + a \sin[c+dx]) (A + B \sin[c+dx]) dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$\frac{(A+B) \sec[c+dx]^5 (a + a \sin[c+dx])}{5d} + \frac{a(4A-B) \tan[c+dx]}{5d} + \frac{a(4A-B) \tan[c+dx]^3}{15d}$$

Result (type 3, 223 leaves):

$$\begin{aligned} & (a \operatorname{Sec}[c] (240 B \operatorname{Cos}[c] - 54 (A + B) \operatorname{Cos}[c + dx] - 18 A \operatorname{Cos}[3(c + dx)] - 18 B \operatorname{Cos}[3(c + dx)] + 128 A \operatorname{Cos}[c + 2dx] - \\ & 32 B \operatorname{Cos}[c + 2dx] + 64 A \operatorname{Cos}[3c + 4dx] - 16 B \operatorname{Cos}[3c + 4dx] + 384 A \operatorname{Sin}[dx] - 96 B \operatorname{Sin}[dx] + 18 A \operatorname{Sin}[2(c + dx)] + \\ & 18 B \operatorname{Sin}[2(c + dx)] + 9 A \operatorname{Sin}[4(c + dx)] + 9 B \operatorname{Sin}[4(c + dx)] + 128 A \operatorname{Sin}[2c + 3dx] - 32 B \operatorname{Sin}[2c + 3dx])) / \\ & \left(960 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^5 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^3 \right) \end{aligned}$$

■ **Problem 967: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^8 (a + a \operatorname{Sin}[c + dx]) (A + B \operatorname{Sin}[c + dx]) dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$\frac{(A + B) \operatorname{Sec}[c + dx]^7 (a + a \operatorname{Sin}[c + dx])}{7d} + \frac{a(6A - B) \operatorname{Tan}[c + dx]}{7d} + \frac{2a(6A - B) \operatorname{Tan}[c + dx]^3}{21d} + \frac{a(6A - B) \operatorname{Tan}[c + dx]^5}{35d}$$

Result (type 3, 315 leaves):

$$\begin{aligned} & (a \operatorname{Sec}[c] (8960 B \operatorname{Cos}[c] - 1500 (A + B) \operatorname{Cos}[c + dx] - 750 A \operatorname{Cos}[3(c + dx)] - 750 B \operatorname{Cos}[3(c + dx)] - \\ & 150 A \operatorname{Cos}[5(c + dx)] - 150 B \operatorname{Cos}[5(c + dx)] + 3840 A \operatorname{Cos}[c + 2dx] - 640 B \operatorname{Cos}[c + 2dx] + 3072 A \operatorname{Cos}[3c + 4dx] - \\ & 512 B \operatorname{Cos}[3c + 4dx] + 768 A \operatorname{Cos}[5c + 6dx] - 128 B \operatorname{Cos}[5c + 6dx] + 15360 A \operatorname{Sin}[dx] - 2560 B \operatorname{Sin}[dx] + \\ & 375 A \operatorname{Sin}[2(c + dx)] + 375 B \operatorname{Sin}[2(c + dx)] + 300 A \operatorname{Sin}[4(c + dx)] + 300 B \operatorname{Sin}[4(c + dx)] + 75 A \operatorname{Sin}[6(c + dx)] + \\ & 75 B \operatorname{Sin}[6(c + dx)] + 7680 A \operatorname{Sin}[2c + 3dx] - 1280 B \operatorname{Sin}[2c + 3dx] + 1536 A \operatorname{Sin}[4c + 5dx] - 256 B \operatorname{Sin}[4c + 5dx])) / \\ & \left(53760 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^7 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^5 \right) \end{aligned}$$

■ **Problem 968: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^{10} (a + a \operatorname{Sin}[c + dx]) (A + B \operatorname{Sin}[c + dx]) dx$$

Optimal (type 3, 119 leaves, 3 steps):

$$\frac{(A + B) \operatorname{Sec}[c + dx]^9 (a + a \operatorname{Sin}[c + dx])}{9d} + \frac{a(8A - B) \operatorname{Tan}[c + dx]}{9d} + \frac{a(8A - B) \operatorname{Tan}[c + dx]^3}{9d} + \frac{a(8A - B) \operatorname{Tan}[c + dx]^5}{15d} + \frac{a(8A - B) \operatorname{Tan}[c + dx]^7}{63d}$$

Result (type 3, 1683 leaves):

$$\begin{aligned} & a \left(\frac{(A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right]) (1 + \operatorname{Sin}[c + dx])}{112 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^9} + \right. \\ & \frac{(-A \operatorname{Cos}\left[\frac{c}{2}\right] + B \operatorname{Cos}\left[\frac{c}{2}\right] + A \operatorname{Sin}\left[\frac{c}{2}\right] - B \operatorname{Sin}\left[\frac{c}{2}\right]) (1 + \operatorname{Sin}[c + dx])}{224 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^8} + \frac{(41 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 27 B \operatorname{Sin}\left[\frac{dx}{2}\right]) (1 + \operatorname{Sin}[c + dx])}{1120 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^7} + \\ & \left. \frac{(-41 A \operatorname{Cos}\left[\frac{c}{2}\right] + 27 B \operatorname{Cos}\left[\frac{c}{2}\right] + 41 A \operatorname{Sin}\left[\frac{c}{2}\right] - 27 B \operatorname{Sin}\left[\frac{c}{2}\right]) (1 + \operatorname{Sin}[c + dx])}{2240 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^6} + \frac{(689 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 283 B \operatorname{Sin}\left[\frac{dx}{2}\right]) (1 + \operatorname{Sin}[c + dx])}{6720 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^5} \right) \end{aligned}$$

$$\begin{aligned}
& \frac{(-689 A \cos[\frac{c}{2}] + 283 B \cos[\frac{c}{2}] + 689 A \sin[\frac{c}{2}] - 283 B \sin[\frac{c}{2}]) (1 + \sin[c + dx])}{13440 d (\cos[\frac{c}{2}] + \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^4} + \frac{(5053 A \sin[\frac{dx}{2}] - 1091 B \sin[\frac{dx}{2}]) (1 + \sin[c + dx])}{13440 d (\cos[\frac{c}{2}] + \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^3} + \\
& \frac{(A \sin[\frac{dx}{2}] + B \sin[\frac{dx}{2}]) (1 + \sin[c + dx])}{72 d (\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] - \sin[\frac{c}{2} + \frac{dx}{2}])^9 (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^2} + \\
& \frac{(A \cos[\frac{c}{2}] + B \cos[\frac{c}{2}] + A \sin[\frac{c}{2}] + B \sin[\frac{c}{2}]) (1 + \sin[c + dx])}{144 d (\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] - \sin[\frac{c}{2} + \frac{dx}{2}])^8 (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^2} + \\
& \frac{(22 A \sin[\frac{dx}{2}] + 13 B \sin[\frac{dx}{2}]) (1 + \sin[c + dx])}{504 d (\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] - \sin[\frac{c}{2} + \frac{dx}{2}])^7 (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^2} + \\
& \frac{(22 A \cos[\frac{c}{2}] + 13 B \cos[\frac{c}{2}] + 22 A \sin[\frac{c}{2}] + 13 B \sin[\frac{c}{2}]) (1 + \sin[c + dx])}{1008 d (\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] - \sin[\frac{c}{2} + \frac{dx}{2}])^6 (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^2} + \\
& \frac{(149 A \sin[\frac{dx}{2}] + 47 B \sin[\frac{dx}{2}]) (1 + \sin[c + dx])}{1680 d (\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] - \sin[\frac{c}{2} + \frac{dx}{2}])^5 (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^2} + \\
& \frac{(149 A \cos[\frac{c}{2}] + 47 B \cos[\frac{c}{2}] + 149 A \sin[\frac{c}{2}] + 47 B \sin[\frac{c}{2}]) (1 + \sin[c + dx])}{3360 d (\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] - \sin[\frac{c}{2} + \frac{dx}{2}])^4 (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^2} + \\
& \frac{(823 A \sin[\frac{dx}{2}] + 94 B \sin[\frac{dx}{2}]) (1 + \sin[c + dx])}{5040 d (\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] - \sin[\frac{c}{2} + \frac{dx}{2}])^3 (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^2} + \\
& \frac{(823 A \cos[\frac{c}{2}] + 94 B \cos[\frac{c}{2}] + 823 A \sin[\frac{c}{2}] + 94 B \sin[\frac{c}{2}]) (1 + \sin[c + dx])}{10080 d (\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] - \sin[\frac{c}{2} + \frac{dx}{2}])^2 (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^2} + \\
& \frac{(17609 A \sin[\frac{dx}{2}] - 823 B \sin[\frac{dx}{2}]) (1 + \sin[c + dx])}{40320 d (\cos[\frac{c}{2}] - \sin[\frac{c}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] - \sin[\frac{c}{2} + \frac{dx}{2}]) (\cos[\frac{c}{2} + \frac{dx}{2}] + \sin[\frac{c}{2} + \frac{dx}{2}])^2}
\end{aligned}$$

■ **Problem 974: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^3 (a + a \sin[c + dx])^2 (A + B \sin[c + dx]) dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$\frac{a^2 B \operatorname{Log}[1 - \sin[c + dx]]}{d} + \frac{a^3 (A + B)}{d (a - a \sin[c + dx])}$$

Result (type 3, 95 leaves):

$$\left(a^2 \left(A + B + 2 B \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] - 2 B \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[c + d x] \right) \right) /$$

$$\left(d \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right) \right)^2$$

■ **Problem 976: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^7 (a + a \operatorname{Sin}[c + d x])^2 (A + B \operatorname{Sin}[c + d x]) dx$$

Optimal (type 3, 132 leaves, 4 steps):

$$\frac{a^2 (2 A - B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a^5 (A + B)}{12 d (a - a \operatorname{Sin}[c + d x])^3} + \frac{a^4 A}{8 d (a - a \operatorname{Sin}[c + d x])^2} + \frac{a^3 (3 A - B)}{16 d (a - a \operatorname{Sin}[c + d x])} - \frac{a^3 (A - B)}{16 d (a + a \operatorname{Sin}[c + d x])}$$

Result (type 3, 319 leaves):

$$\frac{1}{48 d \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^6}$$

$$\left(3 (-A + B) + 6 (-2 A + B) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^2 + \right.$$

$$6 (2 A - B) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^2 + \frac{4 (A + B) \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^2}{\left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^6} +$$

$$\left. \frac{6 A \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^2}{\left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^4} + \frac{3 (3 A - B) \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^2}{\left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^2} \right) (a + a \operatorname{Sin}[c + d x])^2$$

■ **Problem 984: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^{10} (a + a \operatorname{Sin}[c + d x])^2 (A + B \operatorname{Sin}[c + d x]) dx$$

Optimal (type 3, 154 leaves, 4 steps):

$$\frac{a^2 (7 A - 2 B) \operatorname{Sec}[c + d x]^7}{63 d} + \frac{(A + B) \operatorname{Sec}[c + d x]^9 (a + a \operatorname{Sin}[c + d x])^2}{9 d} +$$

$$\frac{a^2 (7 A - 2 B) \operatorname{Tan}[c + d x]}{9 d} + \frac{a^2 (7 A - 2 B) \operatorname{Tan}[c + d x]^3}{9 d} + \frac{a^2 (7 A - 2 B) \operatorname{Tan}[c + d x]^5}{15 d} + \frac{a^2 (7 A - 2 B) \operatorname{Tan}[c + d x]^7}{63 d}$$

Result (type 3, 324 leaves):

$$\begin{aligned}
& - \frac{1}{1290240d} \\
& a^2 \operatorname{Sec}[c+dx]^9 (1 + \operatorname{Sin}[c+dx])^2 (-184320B + 1125(49A + 13B) \operatorname{Cos}[c+dx] - 20480(7A - 2B) \operatorname{Cos}[2(c+dx)] + 23275A \operatorname{Cos}[3(c+dx)] + \\
& 6175B \operatorname{Cos}[3(c+dx)] - 114688A \operatorname{Cos}[4(c+dx)] + 32768B \operatorname{Cos}[4(c+dx)] + 1225A \operatorname{Cos}[5(c+dx)] + \\
& 325B \operatorname{Cos}[5(c+dx)] - 28672A \operatorname{Cos}[6(c+dx)] + 8192B \operatorname{Cos}[6(c+dx)] - 1225A \operatorname{Cos}[7(c+dx)] - 325B \operatorname{Cos}[7(c+dx)] - \\
& 322560A \operatorname{Sin}[c+dx] + 92160B \operatorname{Sin}[c+dx] - 24500A \operatorname{Sin}[2(c+dx)] - 6500B \operatorname{Sin}[2(c+dx)] - 136192A \operatorname{Sin}[3(c+dx)] + \\
& 38912B \operatorname{Sin}[3(c+dx)] - 19600A \operatorname{Sin}[4(c+dx)] - 5200B \operatorname{Sin}[4(c+dx)] - 7168A \operatorname{Sin}[5(c+dx)] + \\
& 2048B \operatorname{Sin}[5(c+dx)] - 4900A \operatorname{Sin}[6(c+dx)] - 1300B \operatorname{Sin}[6(c+dx)] + 7168A \operatorname{Sin}[7(c+dx)] - 2048B \operatorname{Sin}[7(c+dx)])
\end{aligned}$$

■ **Problem 985: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{12} (a + a \operatorname{Sin}[c+dx])^2 (A + B \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 179 leaves, 4 steps):

$$\begin{aligned}
& \frac{a^2 (9A - 2B) \operatorname{Sec}[c+dx]^9}{99d} + \frac{(A + B) \operatorname{Sec}[c+dx]^{11} (a + a \operatorname{Sin}[c+dx])^2}{11d} + \frac{a^2 (9A - 2B) \operatorname{Tan}[c+dx]}{11d} + \\
& \frac{4a^2 (9A - 2B) \operatorname{Tan}[c+dx]^3}{33d} + \frac{6a^2 (9A - 2B) \operatorname{Tan}[c+dx]^5}{55d} + \frac{4a^2 (9A - 2B) \operatorname{Tan}[c+dx]^7}{77d} + \frac{a^2 (9A - 2B) \operatorname{Tan}[c+dx]^9}{99d}
\end{aligned}$$

Result (type 3, 1349 leaves):

$$\begin{aligned}
& \frac{(-A + B) (a + a \operatorname{Sin}[c+dx])^2}{448d (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^{10}} + \frac{(-24A + 17B) (a + a \operatorname{Sin}[c+dx])^2}{2240d (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^8} + \\
& \frac{(-927A + 451B) (a + a \operatorname{Sin}[c+dx])^2}{26880d (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^6} + \frac{(A + B) (a + a \operatorname{Sin}[c+dx])^2}{176d (\operatorname{Cos}[\frac{1}{2}(c+dx)] - \operatorname{Sin}[\frac{1}{2}(c+dx)])^{10} (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^4} + \\
& \frac{(27A + 16B) (a + a \operatorname{Sin}[c+dx])^2}{1584d (\operatorname{Cos}[\frac{1}{2}(c+dx)] - \operatorname{Sin}[\frac{1}{2}(c+dx)])^8 (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^4} + \\
& \frac{(711A + 227B) (a + a \operatorname{Sin}[c+dx])^2}{22176d (\operatorname{Cos}[\frac{1}{2}(c+dx)] - \operatorname{Sin}[\frac{1}{2}(c+dx)])^6 (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^4} + \\
& \frac{(1866A + 227B) (a + a \operatorname{Sin}[c+dx])^2}{36960d (\operatorname{Cos}[\frac{1}{2}(c+dx)] - \operatorname{Sin}[\frac{1}{2}(c+dx)])^4 (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^4} + \\
& \frac{(70281A - 2143B) (a + a \operatorname{Sin}[c+dx])^2}{887040d (\operatorname{Cos}[\frac{1}{2}(c+dx)] - \operatorname{Sin}[\frac{1}{2}(c+dx)])^2 (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^4} + \\
& \frac{(167301A \operatorname{Sin}[\frac{1}{2}(c+dx)] - 26398B \operatorname{Sin}[\frac{1}{2}(c+dx)]) (a + a \operatorname{Sin}[c+dx])^2}{443520d (\operatorname{Cos}[\frac{1}{2}(c+dx)] - \operatorname{Sin}[\frac{1}{2}(c+dx)]) (\operatorname{Cos}[\frac{1}{2}(c+dx)] + \operatorname{Sin}[\frac{1}{2}(c+dx)])^4} +
\end{aligned}$$

$$\begin{aligned}
& \frac{(70281 A \sin[\frac{1}{2}(c+dx)] - 2143 B \sin[\frac{1}{2}(c+dx)]) (a + a \sin[c+dx])^2}{443520 d (\cos[\frac{1}{2}(c+dx)] - \sin[\frac{1}{2}(c+dx)])^3 (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^4} + \\
& \frac{(3867 A \sin[\frac{1}{2}(c+dx)] - 1186 B \sin[\frac{1}{2}(c+dx)]) (a + a \sin[c+dx])^2}{13440 d (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^5} + \frac{(927 A \sin[\frac{1}{2}(c+dx)] - 451 B \sin[\frac{1}{2}(c+dx)]) (a + a \sin[c+dx])^2}{13440 d (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^7} + \\
& \frac{(24 A \sin[\frac{1}{2}(c+dx)] - 17 B \sin[\frac{1}{2}(c+dx)]) (a + a \sin[c+dx])^2}{1120 d (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^9} + \frac{(A \sin[\frac{1}{2}(c+dx)] - B \sin[\frac{1}{2}(c+dx)]) (a + a \sin[c+dx])^2}{224 d (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^{11}} + \\
& \frac{(A \sin[\frac{1}{2}(c+dx)] + B \sin[\frac{1}{2}(c+dx)]) (a + a \sin[c+dx])^2}{88 d (\cos[\frac{1}{2}(c+dx)] - \sin[\frac{1}{2}(c+dx)])^{11} (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^4} + \\
& \frac{(27 A \sin[\frac{1}{2}(c+dx)] + 16 B \sin[\frac{1}{2}(c+dx)]) (a + a \sin[c+dx])^2}{792 d (\cos[\frac{1}{2}(c+dx)] - \sin[\frac{1}{2}(c+dx)])^9 (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^4} + \\
& \frac{(711 A \sin[\frac{1}{2}(c+dx)] + 227 B \sin[\frac{1}{2}(c+dx)]) (a + a \sin[c+dx])^2}{11088 d (\cos[\frac{1}{2}(c+dx)] - \sin[\frac{1}{2}(c+dx)])^7 (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^4} + \\
& \frac{(1866 A \sin[\frac{1}{2}(c+dx)] + 227 B \sin[\frac{1}{2}(c+dx)]) (a + a \sin[c+dx])^2}{18480 d (\cos[\frac{1}{2}(c+dx)] - \sin[\frac{1}{2}(c+dx)])^5 (\cos[\frac{1}{2}(c+dx)] + \sin[\frac{1}{2}(c+dx)])^4}
\end{aligned}$$

■ **Problem 989: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx] (a + a \sin[c+dx])^3 (A + B \sin[c+dx]) dx$$

Optimal (type 3, 51 leaves, 3 steps):

$$\frac{(A - B) (a + a \sin[c+dx])^4}{4 a d} + \frac{B (a + a \sin[c+dx])^5}{5 a^2 d}$$

Result (type 3, 116 leaves):

$$\begin{aligned}
& ((a + a \sin[c+dx])^3 \\
& (-20 (7 A + 5 B) \cos[2(c+dx)] + 5 (A + 3 B) \cos[4(c+dx)] + 140 (2 A + B) \sin[c+dx] - 10 (4 A + 5 B) \sin[3(c+dx)] + 2 B \sin[5(c+dx)]) \Big/ \\
& \left(160 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^6 \right)
\end{aligned}$$

■ **Problem 994: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^9 (a + a \sin[c+dx])^3 (A + B \sin[c+dx]) dx$$

Optimal (type 3, 162 leaves, 4 steps) :

$$\frac{a^3 (5A - 3B) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{32d} + \frac{a^7 (A + B)}{16d (a - a \operatorname{Sin}[c + dx])^4} + \frac{a^6 A}{12d (a - a \operatorname{Sin}[c + dx])^3} + \frac{a^5 (3A - B)}{32d (a - a \operatorname{Sin}[c + dx])^2} + \frac{a^4 (2A - B)}{16d (a - a \operatorname{Sin}[c + dx])} - \frac{a^4 (A - B)}{32d (a + a \operatorname{Sin}[c + dx])}$$

Result (type 3, 378 leaves) :

$$\frac{1}{96d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^8} \left(3(-A + B) + 3(-5A + 3B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 + 3(5A - 3B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 + \frac{6(A + B) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^8} + \frac{8A \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6} + \frac{3(3A - B) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^4} + \frac{6(2A - B) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2} \right) (a + a \operatorname{Sin}[c + dx])^3$$

■ **Problem 1007: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx] (A + B \operatorname{Sin}[c + dx])}{a + a \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 45 leaves, 4 steps) :

$$\frac{(A + B) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2ad} - \frac{A - B}{2d (a + a \operatorname{Sin}[c + dx])}$$

Result (type 3, 126 leaves) :

$$\frac{1}{2ad (1 + \operatorname{Sin}[c + dx])} \left(-A + B - (A + B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 + (A + B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 \right)$$

■ **Problem 1008: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^3 (A + B \operatorname{Sin}[c + dx])}{a + a \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 91 leaves, 4 steps):

$$\frac{(3A+B) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{8ad} + \frac{A+B}{8d(a-a\operatorname{Sin}[c+dx])} - \frac{a(A-B)}{8d(a+a\operatorname{Sin}[c+dx])^2} - \frac{A}{4d(a+a\operatorname{Sin}[c+dx])}$$

Result (type 3, 210 leaves):

$$\frac{1}{8ad(1+\operatorname{Sin}[c+dx])} \left(-2A + \frac{-A+B}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - (3A+B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 + \right. \\ \left. (3A+B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 + \frac{(A+B)\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 1009: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^5 (A+B\operatorname{Sin}[c+dx])}{a+a\operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 146 leaves, 4 steps):

$$\frac{(5A+B) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{16ad} + \frac{a(A+B)}{32d(a-a\operatorname{Sin}[c+dx])^2} + \frac{2A+B}{16d(a-a\operatorname{Sin}[c+dx])} - \frac{a^2(A-B)}{24d(a+a\operatorname{Sin}[c+dx])^3} - \frac{a(3A-B)}{32d(a+a\operatorname{Sin}[c+dx])^2} - \frac{3A}{16d(a+a\operatorname{Sin}[c+dx])}$$

Result (type 3, 298 leaves):

$$\frac{1}{96ad(1+\operatorname{Sin}[c+dx])} \left(-18A + \frac{4(-A+B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{3(-3A+B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - 6(5A+B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right. \\ \left. \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 + 6(5A+B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2 + \right. \\ \left. \frac{3(A+B)\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{6(2A+B)\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 1014: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx] (A+B\operatorname{Sin}[c+dx])}{(a+a\operatorname{Sin}[c+dx])^2} dx$$

Optimal (type 3, 44 leaves, 3 steps) :

$$\frac{B \operatorname{Log}[1 + \operatorname{Sin}[c + d x]]}{a^2 d} - \frac{A - B}{d (a^2 + a^2 \operatorname{Sin}[c + d x])}$$

Result (type 3, 92 leaves) :

$$\frac{1}{d (a + a \operatorname{Sin}[c + d x])^2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 \left(-A + B + 2 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2$$

■ **Problem 1015: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x] (A + B \operatorname{Sin}[c + d x])}{(a + a \operatorname{Sin}[c + d x])^2} dx$$

Optimal (type 3, 71 leaves, 4 steps) :

$$\frac{(A + B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{4 a^2 d} - \frac{A - B}{4 d (a + a \operatorname{Sin}[c + d x])^2} - \frac{A + B}{4 d (a^2 + a^2 \operatorname{Sin}[c + d x])}$$

Result (type 3, 153 leaves) :

$$-\frac{1}{4 a^2 d (1 + \operatorname{Sin}[c + d x])^2} \left(A - B + (A + B) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 + (A + B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^4 - (A + B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^4 \right)$$

■ **Problem 1020: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e + f x]^7 (a + a \operatorname{Sin}[e + f x])^m (A + B \operatorname{Sin}[e + f x]) dx$$

Optimal (type 3, 159 leaves, 3 steps) :

$$\frac{8 (A - B) (a + a \operatorname{Sin}[e + f x])^{4+m}}{a^4 f (4 + m)} - \frac{4 (3 A - 5 B) (a + a \operatorname{Sin}[e + f x])^{5+m}}{a^5 f (5 + m)} + \frac{6 (A - 3 B) (a + a \operatorname{Sin}[e + f x])^{6+m}}{a^6 f (6 + m)} - \frac{(A - 7 B) (a + a \operatorname{Sin}[e + f x])^{7+m}}{a^7 f (7 + m)} - \frac{B (a + a \operatorname{Sin}[e + f x])^{8+m}}{a^8 f (8 + m)}$$

Result (type 3, 1361 leaves) :

$$\frac{1}{f} (a (1 + \operatorname{Sin}[e + f x]))^m \left(393 216 A - 29 400 B + 118 528 A m + 26 822 B m + 16 160 A m^2 + 2407 B m^2 + 1256 A m^3 + 166 B m^3 + 40 A m^4 + 5 B m^4 \right) /$$

$$\begin{aligned}
& (128 (4+m) (5+m) (6+m) (7+m) (8+m)) + \\
& \left((235200A + 50024Am + 29400Bm + 3946A^2m^2 + 2578Bm^2 + 211Am^3 + 171Bm^3 + 5A^4m^4 + 5B^4m^4) \left(-\frac{1}{128} i \cos[e+fx] + \frac{1}{128} \sin[e+fx] \right) \right) / \\
& ((4+m) (5+m) (6+m) (7+m) (8+m)) + \\
& \left((235200A + 50024Am + 29400Bm + 3946A^2m^2 + 2578Bm^2 + 211Am^3 + 171Bm^3 + 5A^4m^4 + 5B^4m^4) \left(\frac{1}{128} i \cos[e+fx] + \frac{1}{128} \sin[e+fx] \right) \right) / \\
& ((4+m) (5+m) (6+m) (7+m) (8+m)) + \\
& \left((-11760B + 19296Am - 8932Bm + 5028A^2m^2 - 94Bm^2 + 447Am^3 + 19Bm^3 + 15A^4m^4 + B^4m^4) \left(\frac{1}{64} \cos[2(e+fx)] - \frac{1}{64} i \sin[2(e+fx)] \right) \right) / \\
& ((4+m) (5+m) (6+m) (7+m) (8+m)) + \\
& \left((-11760B + 19296Am - 8932Bm + 5028A^2m^2 - 94Bm^2 + 447Am^3 + 19Bm^3 + 15A^4m^4 + B^4m^4) \left(\frac{1}{64} \cos[2(e+fx)] + \frac{1}{64} i \sin[2(e+fx)] \right) \right) / \\
& ((4+m) (5+m) (6+m) (7+m) (8+m)) + \\
& \left((15680A + 10520Am + 1960Bm + 1814A^2m^2 + 1070Bm^2 + 117Am^3 + 93Bm^3 + 3A^4m^4 + 3B^4m^4) \left(-\frac{3}{128} i \cos[3(e+fx)] + \frac{3}{128} \sin[3(e+fx)] \right) \right) / \\
& ((4+m) (5+m) (6+m) (7+m) (8+m)) + \\
& \left((15680A + 10520Am + 1960Bm + 1814A^2m^2 + 1070Bm^2 + 117Am^3 + 93Bm^3 + 3A^4m^4 + 3B^4m^4) \left(\frac{3}{128} i \cos[3(e+fx)] + \frac{3}{128} \sin[3(e+fx)] \right) \right) / \\
& ((4+m) (5+m) (6+m) (7+m) (8+m)) + \\
& \left((-5880B + 2112Am - 4466Bm + 1080A^2m^2 - 989Bm^2 + 150Am^3 - 52Bm^3 + 6A^4m^4 - B^4m^4) \left(\frac{1}{64} \cos[4(e+fx)] - \frac{1}{64} i \sin[4(e+fx)] \right) \right) / \\
& ((4+m) (5+m) (6+m) (7+m) (8+m)) + \\
& \left((-5880B + 2112Am - 4466Bm + 1080A^2m^2 - 989Bm^2 + 150Am^3 - 52Bm^3 + 6A^4m^4 - B^4m^4) \left(\frac{1}{64} \cos[4(e+fx)] + \frac{1}{64} i \sin[4(e+fx)] \right) \right) / \\
& ((4+m) (5+m) (6+m) (7+m) (8+m)) + \\
& \left((2352A + 1118Am + 294Bm + 143A^2m^2 + 103Bm^2 + 5Am^3 + 5Bm^3) \left(-\frac{1}{128} i \cos[5(e+fx)] + \frac{1}{128} \sin[5(e+fx)] \right) \right) / \\
& ((5+m) (6+m) (7+m) (8+m)) + \\
& \left((2352A + 1118Am + 294Bm + 143A^2m^2 + 103Bm^2 + 5Am^3 + 5Bm^3) \left(\frac{1}{128} i \cos[5(e+fx)] + \frac{1}{128} \sin[5(e+fx)] \right) \right) / \\
& ((5+m) (6+m) (7+m) (8+m)) + \frac{(-84B + 8Am - 26Bm + Am^2 - Bm^2) \left(\frac{1}{64} \cos[6(e+fx)] - \frac{1}{64} i \sin[6(e+fx)] \right)}{(6+m) (7+m) (8+m)} + \\
& \frac{(-84B + 8Am - 26Bm + Am^2 - Bm^2) \left(\frac{1}{64} \cos[6(e+fx)] + \frac{1}{64} i \sin[6(e+fx)] \right)}{(6+m) (7+m) (8+m)} + \\
& \frac{(8A + Am + Bm) \left(-\frac{1}{128} i \cos[7(e+fx)] + \frac{1}{128} \sin[7(e+fx)] \right)}{(7+m) (8+m)} + \frac{(8A + Am + Bm) \left(\frac{1}{128} i \cos[7(e+fx)] + \frac{1}{128} \sin[7(e+fx)] \right)}{(7+m) (8+m)} +
\end{aligned}$$

$$\left. \begin{aligned} & \frac{-\frac{1}{256} B \cos[8(e+fx)] - \frac{1}{256} i B \sin[8(e+fx)]}{8+m} + \frac{-\frac{1}{256} B \cos[8(e+fx)] + \frac{1}{256} i B \sin[8(e+fx)]}{8+m} \end{aligned} \right)$$

- **Problem 1021: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^5 (a+a \sin[e+fx])^m (A+B \sin[e+fx]) dx$$

Optimal (type 3, 123 leaves, 3 steps):

$$\frac{4(A-B)(a+a \sin[e+fx])^{3+m}}{a^3 f(3+m)} - \frac{4(A-2B)(a+a \sin[e+fx])^{4+m}}{a^4 f(4+m)} + \frac{(A-5B)(a+a \sin[e+fx])^{5+m}}{a^5 f(5+m)} + \frac{B(a+a \sin[e+fx])^{6+m}}{a^6 f(6+m)}$$

Result (type 3, 869 leaves):

$$\begin{aligned} & \frac{1}{f} (a(1+\sin[e+fx]))^m \left(\frac{3072A-300B+1004Am+277Bm+118Am^2+22Bm^2+6Am^3+Bm^3}{16(3+m)(4+m)(5+m)(6+m)} + \right. \\ & \frac{(1800A+438Am+300Bm+29Am^2+23Bm^2+Am^3+Bm^3) \left(-\frac{1}{16} i \cos[e+fx] + \frac{1}{16} \sin[e+fx] \right)}{(3+m)(4+m)(5+m)(6+m)} + \\ & \frac{(1800A+438Am+300Bm+29Am^2+23Bm^2+Am^3+Bm^3) \left(\frac{1}{16} i \cos[e+fx] + \frac{1}{16} \sin[e+fx] \right)}{(3+m)(4+m)(5+m)(6+m)} + \\ & \left. \left((-900B+1056Am-705Bm+272Am^2-4Bm^2+16Am^3+Bm^3) \left(\frac{1}{64} \cos[2(e+fx)] - \frac{1}{64} i \sin[2(e+fx)] \right) \right) / ((3+m)(4+m)(5+m)(6+m)) + \right. \\ & \left. \left((-900B+1056Am-705Bm+272Am^2-4Bm^2+16Am^3+Bm^3) \left(\frac{1}{64} \cos[2(e+fx)] + \frac{1}{64} i \sin[2(e+fx)] \right) \right) / ((3+m)(4+m)(5+m)(6+m)) + \right. \\ & \left. \left((600A+418Am+100Bm+71Am^2+53Bm^2+3Am^3+3Bm^3) \left(-\frac{1}{32} i \cos[3(e+fx)] + \frac{1}{32} \sin[3(e+fx)] \right) \right) / ((3+m)(4+m)(5+m)(6+m)) + \right. \\ & \left. \left((600A+418Am+100Bm+71Am^2+53Bm^2+3Am^3+3Bm^3) \left(\frac{1}{32} i \cos[3(e+fx)] + \frac{1}{32} \sin[3(e+fx)] \right) \right) / ((3+m)(4+m)(5+m)(6+m)) + \right. \\ & \frac{(-60B+12Am-27Bm+2Am^2-Bm^2) \left(\frac{1}{32} \cos[4(e+fx)] - \frac{1}{32} i \sin[4(e+fx)] \right)}{(4+m)(5+m)(6+m)} + \\ & \frac{(-60B+12Am-27Bm+2Am^2-Bm^2) \left(\frac{1}{32} \cos[4(e+fx)] + \frac{1}{32} i \sin[4(e+fx)] \right)}{(4+m)(5+m)(6+m)} + \\ & \frac{(6A+Am+Bm) \left(-\frac{1}{32} i \cos[5(e+fx)] + \frac{1}{32} \sin[5(e+fx)] \right)}{(5+m)(6+m)} + \frac{(6A+Am+Bm) \left(\frac{1}{32} i \cos[5(e+fx)] + \frac{1}{32} \sin[5(e+fx)] \right)}{(5+m)(6+m)} + \\ & \left. \left. \frac{-\frac{1}{64} B \cos[6(e+fx)] - \frac{1}{64} i B \sin[6(e+fx)]}{6+m} + \frac{-\frac{1}{64} B \cos[6(e+fx)] + \frac{1}{64} i B \sin[6(e+fx)]}{6+m} \right) \right) \end{aligned}$$

■ **Problem 1024: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sin}[e + f x])^m (A + B \operatorname{Sin}[e + f x]) dx$$

Optimal (type 5, 80 leaves, 3 steps):

$$\frac{(A - B) (a + a \operatorname{Sin}[e + f x])^m}{2 f m} + \frac{(A + B) \operatorname{Hypergeometric2F1}\left[1, 1 + m, 2 + m, \frac{1}{2} (1 + \operatorname{Sin}[e + f x])\right] (a + a \operatorname{Sin}[e + f x])^{1+m}}{4 a f (1 + m)}$$

Result (type 6, 16323 leaves):

$$\begin{aligned} & - \left(\left(\operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{-2m} \operatorname{Cot}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \right)^2 \\ & (a + a \operatorname{Sin}[e + f x])^m \left(\left(A \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{2m} \right) / \left(\left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right) \right. \\ & \quad \left. \left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] + \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right) \right) + \left(B \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{2m} \operatorname{Sin}[e + f x] / \\ & \quad \left(\left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right) \left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] + \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right) \right) \right) \\ & \left(\frac{1 - \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2m} \left(\left(2 A \operatorname{AppellF1}\left[1, 1 - 2m, 2m, 2, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \right) / \right. \\ & \quad \left(\left(-1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left(-2 \operatorname{AppellF1}\left[1, 1 - 2m, 2m, 2, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) + \right. \\ & \quad \left. \left(2m \operatorname{AppellF1}\left[2, 1 - 2m, 1 + 2m, 3, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) + (-1 + 2m) \right. \\ & \quad \left. \operatorname{AppellF1}\left[2, 2 - 2m, 2m, 3, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) - \\ & \left(2 B \operatorname{AppellF1}\left[1, 1 - 2m, 2m, 2, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \right) / \\ & \quad \left(\left(-1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left(-2 \operatorname{AppellF1}\left[1, 1 - 2m, 2m, 2, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) + \right. \\ & \quad \left. \left(2m \operatorname{AppellF1}\left[2, 1 - 2m, 1 + 2m, 3, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) + (-1 + 2m) \right. \\ & \quad \left. \operatorname{AppellF1}\left[2, 2 - 2m, 2m, 3, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \\ & \left(4 B \operatorname{AppellF1}\left[1, -2m, 1 + 2m, 2, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \right) / \end{aligned}$$

Optimal (type 5, 100 leaves, 3 steps):

$$-\frac{a(A(2-m) - Bm) \operatorname{Hypergeometric2F1}\left[1, -1+m, m, \frac{1}{2}(1 + \sin[e+fx])\right] (a + a \sin[e+fx])^{-1+m}}{4f(1-m)} + \frac{a^2(A+B)(a + a \sin[e+fx])^{-1+m}}{2f(a - a \sin[e+fx])}$$

Result (type 6, 46238 leaves): Display of huge result suppressed!

■ **Problem 1026: Unable to integrate problem.**

$$\int \sec[e+fx]^5 (a + a \sin[e+fx])^m (A + B \sin[e+fx]) dx$$

Optimal (type 5, 104 leaves, 3 steps):

$$-\frac{a^2(A(4-m) - Bm) \operatorname{Hypergeometric2F1}\left[2, -2+m, -1+m, \frac{1}{2}(1 + \sin[e+fx])\right] (a + a \sin[e+fx])^{-2+m}}{16f(2-m)} + \frac{a^4(A+B)(a + a \sin[e+fx])^{-2+m}}{4f(a - a \sin[e+fx])^2}$$

Result (type 8, 33 leaves):

$$\int \sec[e+fx]^5 (a + a \sin[e+fx])^m (A + B \sin[e+fx]) dx$$

■ **Problem 1027: Unable to integrate problem.**

$$\int \cos[e+fx]^6 (a + a \sin[e+fx])^m (A + B \sin[e+fx]) dx$$

Optimal (type 5, 129 leaves, 4 steps):

$$-\frac{1}{7f(7+m)} 2^{\frac{7}{2}+m} a^3 (Bm + A(7+m)) \cos[e+fx]^7 \operatorname{Hypergeometric2F1}\left[\frac{7}{2}, -\frac{5}{2}-m, \frac{9}{2}, \frac{1}{2}(1 - \sin[e+fx])\right] \\ (1 + \sin[e+fx])^{-\frac{1}{2}-m} (a + a \sin[e+fx])^{-3+m} - \frac{B \cos[e+fx]^7 (a + a \sin[e+fx])^m}{f(7+m)}$$

Result (type 8, 33 leaves):

$$\int \cos[e+fx]^6 (a + a \sin[e+fx])^m (A + B \sin[e+fx]) dx$$

■ **Problem 1028: Unable to integrate problem.**

$$\int \cos[e+fx]^4 (a + a \sin[e+fx])^m (A + B \sin[e+fx]) dx$$

Optimal (type 5, 129 leaves, 4 steps):

$$-\frac{1}{5f(5+m)} 2^{\frac{5}{2}+m} a^2 (Bm + A(5+m)) \cos[e+fx]^5 \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, -\frac{3}{2}-m, \frac{7}{2}, \frac{1}{2}(1 - \sin[e+fx])\right] \\ (1 + \sin[e+fx])^{-\frac{1}{2}-m} (a + a \sin[e+fx])^{-2+m} - \frac{B \cos[e+fx]^5 (a + a \sin[e+fx])^m}{f(5+m)}$$

Result (type 8, 33 leaves):

$$\int \cos[e + f x]^4 (a + a \sin[e + f x])^m (A + B \sin[e + f x]) dx$$

■ **Problem 1029: Attempted integration timed out after 120 seconds.**

$$\int \cos[e + f x]^2 (a + a \sin[e + f x])^m (A + B \sin[e + f x]) dx$$

Optimal (type 5, 127 leaves, 4 steps):

$$-\frac{1}{3 f (3+m)} 2^{\frac{3}{2}+m} a (B m + A (3+m)) \cos[e + f x]^3 \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -\frac{1}{2}-m, \frac{5}{2}, \frac{1}{2} (1 - \sin[e + f x])\right] \\ (1 + \sin[e + f x])^{\frac{1}{2}-m} (a + a \sin[e + f x])^{-1+m} - \frac{B \cos[e + f x]^3 (a + a \sin[e + f x])^m}{f (3+m)}$$

Result (type 1, 1 leaves):

???

■ **Problem 1030: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sec[e + f x]^2 (a + a \sin[e + f x])^m (A + B \sin[e + f x]) dx$$

Optimal (type 5, 123 leaves, 4 steps):

$$\frac{B \sec[e + f x] (a + a \sin[e + f x])^m}{f (1-m)} + \frac{1}{f (1-m)} \\ 2^{-\frac{1}{2}+m} (A (1-m) - B m) \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{3}{2}-m, \frac{1}{2}, \frac{1}{2} (1 - \sin[e + f x])\right] \sec[e + f x] (1 + \sin[e + f x])^{\frac{1}{2}-m} (a + a \sin[e + f x])^m$$

Result (type 6, 12366 leaves):

$$-\left(\left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} \cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \right. \right. \\ \left. \left. (a + a \sin[e + f x])^m \left(\left(A \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \right) / \left(\left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^2 \right. \right. \right. \\ \left. \left. \left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] + \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^2 \right) + \left(B \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \sin[e + f x] \right) / \right. \right. \\ \left. \left. \left(\left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^2 \left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] + \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^2 \right) \right) \right) \\ \left(1 - \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{2(-1+m)} \left(\frac{1}{1 + \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2m} \\ \left(- \left(\left(15 (A + B) \operatorname{AppellF1}\left[-\frac{1}{2}, 2 - 2m, 2m, \frac{1}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) / \right. \right.$$

Problem 1031: Unable to integrate problem.

$$\int \text{Sec}[e + f x]^4 (a + a \text{Sin}[e + f x])^m (A + B \text{Sin}[e + f x]) dx$$

Optimal (type 5, 135 leaves, 4 steps):

$$\frac{B \text{Sec}[e + f x]^3 (a + a \text{Sin}[e + f x])^m}{f (3 - m)} + \frac{1}{3 a f (3 - m)}$$

$$2^{-\frac{3}{2}+m} (A (3 - m) - B m) \text{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{5}{2} - m, -\frac{1}{2}, \frac{1}{2} (1 - \text{Sin}[e + f x])\right] \text{Sec}[e + f x]^3 (1 + \text{Sin}[e + f x])^{\frac{1}{2}-m} (a + a \text{Sin}[e + f x])^{1+m}$$

Result (type 8, 33 leaves):

$$\int \text{Sec}[e + f x]^4 (a + a \text{Sin}[e + f x])^m (A + B \text{Sin}[e + f x]) dx$$

■ **Problem 1032: Unable to integrate problem.**

$$\int \text{Sec}[e + f x]^6 (a + a \text{Sin}[e + f x])^m (A + B \text{Sin}[e + f x]) dx$$

Optimal (type 5, 135 leaves, 4 steps):

$$\frac{B \text{Sec}[e + f x]^5 (a + a \text{Sin}[e + f x])^m}{f (5 - m)} + \frac{1}{5 a^2 f (5 - m)}$$

$$2^{-\frac{5}{2}+m} (A (5 - m) - B m) \text{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{7}{2} - m, -\frac{3}{2}, \frac{1}{2} (1 - \text{Sin}[e + f x])\right] \text{Sec}[e + f x]^5 (1 + \text{Sin}[e + f x])^{\frac{1}{2}-m} (a + a \text{Sin}[e + f x])^{2+m}$$

Result (type 8, 33 leaves):

$$\int \text{Sec}[e + f x]^6 (a + a \text{Sin}[e + f x])^m (A + B \text{Sin}[e + f x]) dx$$

■ **Problem 1033: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (g \text{Cos}[e + f x])^p (A + B \text{Sin}[e + f x]) (c - c \text{Sin}[e + f x])^{-4-p} dx$$

Optimal (type 3, 239 leaves, 4 steps):

$$\frac{(A + B) (g \text{Cos}[e + f x])^{1+p} (c - c \text{Sin}[e + f x])^{-4-p}}{f g (7 + p)} + \frac{(3 A - B (4 + p)) (g \text{Cos}[e + f x])^{1+p} (c - c \text{Sin}[e + f x])^{-3-p}}{c f g (5 + p) (7 + p)} +$$

$$\frac{2 (3 A - B (4 + p)) (g \text{Cos}[e + f x])^{1+p} (c - c \text{Sin}[e + f x])^{-2-p}}{c^2 f g (3 + p) (5 + p) (7 + p)} + \frac{2 (3 A - B (4 + p)) (g \text{Cos}[e + f x])^{1+p} (c - c \text{Sin}[e + f x])^{-1-p}}{c^3 f g (1 + p) (3 + p) (5 + p) (7 + p)}$$

Result (type 5, 1100 leaves):

$$\begin{aligned}
& \frac{1}{f \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^7} \cos[e+fx]^{-p} (g \cos[e+fx])^p \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^{-2(-4-p)-2p} (c - c \sin[e+fx])^{-4-p} \\
& \left(\frac{1 - \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\sec\left[\frac{1}{2}(e+fx)\right]^2}}\right)^{2p} \left(\frac{1 - \tan\left[\frac{1}{2}(e+fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}\right)^p \left(\frac{1 - \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}}\right)^{-2p} \left(-\frac{A \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^6 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)}{1+p}\right) \\
& \frac{2^{4+p} A \operatorname{Hypergeometric2F1}\left[-7-p, -p, -6-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p}}{7+p} - \\
& \frac{2^{4+p} B \operatorname{Hypergeometric2F1}\left[-7-p, -p, -6-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p}}{7+p} - \frac{1}{6+p} \\
& 3 \times 2^{4+p} A \operatorname{Hypergeometric2F1}\left[-6-p, -p, -5-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} - \\
& \frac{1}{6+p} 3 \times 2^{4+p} B \operatorname{Hypergeometric2F1}\left[-6-p, -p, -5-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} - \\
& \frac{1}{5+p} 9 \times 2^{3+p} A \operatorname{Hypergeometric2F1}\left[-5-p, -p, -4-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^2 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} - \\
& \frac{1}{5+p} 2^{6+p} B \operatorname{Hypergeometric2F1}\left[-5-p, -p, -4-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^2 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} - \\
& \frac{1}{4+p} 2^{6+p} A \operatorname{Hypergeometric2F1}\left[-4-p, -p, -3-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^3 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} - \\
& \frac{1}{4+p} 3 \times 2^{4+p} B \operatorname{Hypergeometric2F1}\left[-4-p, -p, -3-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^3 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} - \\
& \frac{1}{3+p} 9 \times 2^{2+p} A \operatorname{Hypergeometric2F1}\left[-3-p, -p, -2-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^4 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} - \\
& \frac{1}{3+p} 5 \times 2^{2+p} B \operatorname{Hypergeometric2F1}\left[-3-p, -p, -2-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^4 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} - \\
& \frac{1}{2+p} 3 \times 2^{2+p} A \operatorname{Hypergeometric2F1}\left[-2-p, -p, -1-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^5 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p} - \\
& \frac{1}{2+p} 2^{2+p} B \operatorname{Hypergeometric2F1}\left[-2-p, -p, -1-p, \frac{1}{2}\left(1 - \tan\left[\frac{1}{2}(e+fx)\right]\right)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^5 \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]\right)^{-p}
\end{aligned}$$

■ **Problem 1036: Unable to integrate problem.**

$$\int (g \cos[e + f x])^p (A + B \sin[e + f x]) (c - c \sin[e + f x])^{-1-p} dx$$

Optimal (type 5, 151 leaves, 4 steps):

$$\frac{(A + B) (g \cos[e + f x])^{1+p} (c - c \sin[e + f x])^{-1-p}}{f g (1 + p)} - \frac{1}{f g (1 + p)}$$

$$2^{\frac{1}{2}-\frac{p}{2}} B (g \cos[e + f x])^{1+p} \text{Hypergeometric2F1}\left[\frac{1+p}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2} (1 + \sin[e + f x])\right] (1 - \sin[e + f x])^{\frac{1+p}{2}} (c - c \sin[e + f x])^{-1-p}$$

Result (type 8, 40 leaves):

$$\int (g \cos[e + f x])^p (A + B \sin[e + f x]) (c - c \sin[e + f x])^{-1-p} dx$$

■ **Problem 1037: Unable to integrate problem.**

$$\int (g \cos[e + f x])^p (A + B \sin[e + f x]) (c - c \sin[e + f x])^{-p} dx$$

Optimal (type 5, 147 leaves, 4 steps):

$$\frac{1}{f g (1 + p)} 2^{\frac{1}{2}-\frac{p}{2}} c (A + B p) (g \cos[e + f x])^{1+p} \text{Hypergeometric2F1}\left[\frac{1+p}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2} (1 + \sin[e + f x])\right]$$

$$(1 - \sin[e + f x])^{\frac{1+p}{2}} (c - c \sin[e + f x])^{-1-p} - \frac{B (g \cos[e + f x])^{1+p} (c - c \sin[e + f x])^{-p}}{f g}$$

Result (type 8, 38 leaves):

$$\int (g \cos[e + f x])^p (A + B \sin[e + f x]) (c - c \sin[e + f x])^{-p} dx$$

■ **Problem 1038: Unable to integrate problem.**

$$\int (g \cos[e + f x])^p (A + B \sin[e + f x]) (c - c \sin[e + f x])^{1-p} dx$$

Optimal (type 5, 160 leaves, 4 steps):

$$\frac{1}{f g (1 + p)} 2^{\frac{1}{2}-\frac{p}{2}} c^2 (2A - B(1-p)) (g \cos[e + f x])^{1+p} \text{Hypergeometric2F1}\left[\frac{1}{2} (-1 + p), \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2} (1 + \sin[e + f x])\right]$$

$$(1 - \sin[e + f x])^{\frac{1+p}{2}} (c - c \sin[e + f x])^{-1-p} - \frac{B (g \cos[e + f x])^{1+p} (c - c \sin[e + f x])^{1-p}}{2 f g}$$

Result (type 8, 40 leaves):

$$\int (g \cos[e + f x])^p (A + B \sin[e + f x]) (c - c \sin[e + f x])^{1-p} dx$$

■ **Problem 1039: Attempted integration timed out after 120 seconds.**

$$\int (g \cos[e + f x])^p (A + B \sin[e + f x]) (c - c \sin[e + f x])^{2-p} dx$$

Optimal (type 5, 163 leaves, 4 steps):

$$\frac{1}{3 f g (1+p)} 2^{\frac{5-p}{2}} c^3 (3 A - B (2-p)) (g \cos[e + f x])^{1+p} \text{Hypergeometric2F1}\left[\frac{1}{2}(-3+p), \frac{1+p}{2}, \frac{3+p}{2}, \frac{1}{2}(1+\sin[e + f x])\right] \\ (1 - \sin[e + f x])^{\frac{1+p}{2}} (c - c \sin[e + f x])^{-1-p} - \frac{B (g \cos[e + f x])^{1+p} (c - c \sin[e + f x])^{2-p}}{3 f g}$$

Result (type 1, 1 leaves):

???

■ **Problem 1042: Result more than twice size of optimal antiderivative.**

$$\int (g \cos[e + f x])^p (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 168 leaves, 4 steps):

$$\frac{1}{a f (1+2m+p)} 2^{\frac{1-p}{2}} g \text{AppellF1}\left[\frac{1}{2}(1+2m+p), \frac{1-p}{2}, -n, \frac{1}{2}(3+2m+p), \frac{1}{2}(1+\sin[e + f x]), -\frac{d(1+\sin[e + f x])}{c-d}\right] \\ (g \cos[e + f x])^{-1+p} (1 - \sin[e + f x])^{\frac{1-p}{2}} (a + a \sin[e + f x])^{1+m} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c-d}\right)^{-n}$$

Result (type 6, 4377 leaves):

$$- \left(\left(2 (c+d) (3+p) \text{AppellF1}\left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \cos[e + f x]^p \right. \right. \\ \left. \left. (g \cos[e + f x])^p \left(\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-1-m} (a + a \sin[e + f x])^m (c + d \sin[e + f x])^{2n} \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) / \\ \left(f (1+p) \left((c+d) (3+p) \text{AppellF1}\left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - \right. \right. \\ \left. \left. 2 \left((-c+d) n \text{AppellF1}\left[\frac{3+p}{2}, 1+m+n+p, 1-n, \frac{5+p}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + (c+d) (1+m+n+p) \right. \right. \right. \\ \left. \left. \left. \text{AppellF1}\left[\frac{3+p}{2}, 2+m+n+p, -n, \frac{5+p}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right)$$

$$\begin{aligned}
& \left((c+d) (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right. \\
& \quad \left. \operatorname{Cos}[e+f x]^p \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-m} (c+d \operatorname{Sin}[e+f x])^n \right) / \\
& \left((1+p) \left((c+d) (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - 2 \left(-c+d \right) \right. \right. \\
& \quad \left. n \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+m+n+p, 1-n, \frac{5+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (1+m+n+p) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3+p}{2}, 2+m+n+p, -n, \frac{5+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) - \\
& \left(2 d (c+d) n (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right. \\
& \quad \left. \operatorname{Cos}[e+f x]^{1+p} \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-1-m} (c+d \operatorname{Sin}[e+f x])^{-1+n} \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) / \\
& \left((1+p) \left((c+d) (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - 2 \left(-c+d \right) \right. \right. \\
& \quad \left. n \operatorname{AppellF1} \left[\frac{3+p}{2}, 1+m+n+p, 1-n, \frac{5+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (1+m+n+p) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3+p}{2}, 2+m+n+p, -n, \frac{5+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) + \\
& \left(2 (c+d) p (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right. \\
& \quad \left. \operatorname{Cos}[e+f x]^{-1+p} \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-1-m} \operatorname{Sin}[e+f x] (c+d \operatorname{Sin}[e+f x])^n \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) / \\
& \left((1+p) \left((c+d) (3+p) \operatorname{AppellF1} \left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - 2 \left(-c+d \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& n \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+m+n+p, 1-n, \frac{5+p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + (c+d)(1+m+n+p) \\
& \operatorname{AppellF1}\left[\frac{3+p}{2}, 2+m+n+p, -n, \frac{5+p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) + \\
& \left(2(c+d)(-1-m)(3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right. \\
& \left.\operatorname{Cos}[e+f x]^p \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-1-m} (c+d) \operatorname{Sin}[e+f x]^n \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) / \\
& \left((1+p) \left((c+d)(3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] - 2\left(-c+d\right)\right.\right. \\
& \left.\left.n \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+m+n+p, 1-n, \frac{5+p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + (c+d)(1+m+n+p)\right.\right. \\
& \left.\left.\operatorname{AppellF1}\left[\frac{3+p}{2}, 2+m+n+p, -n, \frac{5+p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) + \right. \\
& \left.2(c+d)(3+p) \operatorname{Cos}[e+f x]^p \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-1-m} (c+d) \operatorname{Sin}[e+f x]^n \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(\frac{1}{(c+d)(3+p)}\right.\right. \\
& \left.\left.(c-d) n(1+p) \operatorname{AppellF1}\left[1+\frac{1+p}{2}, 1+m+n+p, 1-n, 1+\frac{3+p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) \\
& \left.\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] - \frac{1}{3+p}(1+p)(1+m+n+p) \operatorname{AppellF1}\left[1+\frac{1+p}{2}, 2+m+n+p, -n, 1+\frac{3+p}{2},\right.\right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right) / \\
& \left((1+p) \left((c+d)(3+p) \operatorname{AppellF1}\left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] - 2\left(-c+d\right)\right.\right. \\
& \left.\left.n \operatorname{AppellF1}\left[\frac{3+p}{2}, 1+m+n+p, 1-n, \frac{5+p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + (c+d)(1+m+n+p)\right)
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3+p}{2}, 2+m+n+p, -n, \frac{5+p}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) - \\
& \left(2(c+d)(3+p)\text{AppellF1}\left[\frac{1+p}{2}, 1+m+n+p, -n, \frac{3+p}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right. \\
& \left.\text{Cos}[e+fx]^p\left(\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{-1-m}(c+d\text{Sin}[e+fx])^n\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right. \\
& \left.-2\left((-c+d)n\text{AppellF1}\left[\frac{3+p}{2}, 1+m+n+p, 1-n, \frac{5+p}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]+(c+d)\right. \\
& \left.(1+m+n+p)\text{AppellF1}\left[\frac{3+p}{2}, 2+m+n+p, -n, \frac{5+p}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \\
& \left.\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]+(c+d)(3+p)\left(\frac{1}{(c+d)(3+p)}(c-d)n(1+p)\right.\right. \\
& \left.\left.\text{AppellF1}\left[1+\frac{1+p}{2}, 1+m+n+p, 1-n, 1+\frac{3+p}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right. \right. \\
& \left.\left.\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]-\frac{1}{3+p}(1+p)(1+m+n+p)\text{AppellF1}\left[1+\frac{1+p}{2}, 2+m+n+p, -n, \right.\right. \\
& \left.\left.1+\frac{3+p}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) - \\
& 2\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\left((-c+d)n\left(-\frac{1}{(c+d)(5+p)}(c-d)(1-n)(3+p)\text{AppellF1}\left[1+\frac{3+p}{2}, 1+m+n+p, 2-n, \right.\right.\right. \\
& \left.\left.\left.1+\frac{5+p}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]-\right. \\
& \left.\frac{1}{5+p}(3+p)(1+m+n+p)\text{AppellF1}\left[1+\frac{3+p}{2}, 2+m+n+p, 1-n, 1+\frac{5+p}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \\
& \left.\left.-\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right)+ (c+d)(1+m+n+p)\left(\frac{1}{(c+d)(5+p)}\right. \\
& \left.(c-d)n(3+p)\text{AppellF1}\left[1+\frac{3+p}{2}, 2+m+n+p, 1-n, 1+\frac{5+p}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right)
\end{aligned}$$

Result (type 8, 35 leaves) :

$$\int (g \cos[e + f x])^p (a + a \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

■ **Problem 1045: Unable to integrate problem.**

$$\int \frac{(g \cos[e + f x])^p (c + d \sin[e + f x])^n}{a + a \sin[e + f x]} dx$$

Optimal (type 6, 149 leaves, 3 steps) :

$$-\frac{1}{a f g (1+p)} 2^{-\frac{1}{2} + \frac{p}{2}} \text{AppellF1}\left[\frac{1+p}{2}, \frac{3-p}{2}, -n, \frac{3+p}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c+d}\right] \\ (g \cos[e + f x])^{1+p} (1 + \sin[e + f x])^{-1 + \frac{1-p}{2}} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c+d}\right)^{-n}$$

Result (type 8, 37 leaves) :

$$\int \frac{(g \cos[e + f x])^p (c + d \sin[e + f x])^n}{a + a \sin[e + f x]} dx$$

■ **Problem 1046: Unable to integrate problem.**

$$\int \frac{(g \cos[e + f x])^p (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 6, 149 leaves, 3 steps) :

$$-\frac{1}{a^2 f g (1+p)} 2^{-\frac{3}{2} + \frac{p}{2}} \text{AppellF1}\left[\frac{1+p}{2}, \frac{5-p}{2}, -n, \frac{3+p}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c+d}\right] \\ (g \cos[e + f x])^{1+p} (1 + \sin[e + f x])^{-2 + \frac{3-p}{2}} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c+d}\right)^{-n}$$

Result (type 8, 37 leaves) :

$$\int \frac{(g \cos[e + f x])^p (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^2} dx$$

■ **Problem 1047: Unable to integrate problem.**

$$\int \frac{(g \cos[e + f x])^p (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 6, 149 leaves, 3 steps) :

$$-\frac{1}{a^3 f g (1+p)} 2^{-\frac{5}{2} + \frac{p}{2}} \text{AppellF1}\left[\frac{1+p}{2}, \frac{7-p}{2}, -n, \frac{3+p}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c+d}\right] \\ (g \cos[e + f x])^{1+p} (1 + \sin[e + f x])^{-3 + \frac{5-p}{2}} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c+d}\right)^{-n}$$

Result (type 8, 37 leaves) :

$$\int \frac{(g \cos[e + f x])^p (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^3} dx$$

■ **Problem 1048: Unable to integrate problem.**

$$\int \frac{(g \cos[e + f x])^p (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^4} dx$$

Optimal (type 6, 149 leaves, 3 steps) :

$$-\frac{1}{a^4 f g (1+p)} 2^{-\frac{7}{2} + \frac{p}{2}} \text{AppellF1}\left[\frac{1+p}{2}, \frac{9-p}{2}, -n, \frac{3+p}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c+d}\right] \\ (g \cos[e + f x])^{1+p} (1 + \sin[e + f x])^{-4 + \frac{7-p}{2}} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c+d}\right)^{-n}$$

Result (type 8, 37 leaves) :

$$\int \frac{(g \cos[e + f x])^p (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^4} dx$$

■ **Problem 1049: Result more than twice size of optimal antiderivative.**

$$\int (g \sec[e + f x])^p (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 175 leaves, 5 steps) :

$$\frac{1}{a f (1+2m-p)} 2^{\frac{1-p}{2}} \text{AppellF1}\left[\frac{1}{2} (1+2m-p), \frac{1+p}{2}, -n, \frac{1}{2} (3+2m-p), \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c-d}\right] \\ \sec[e + f x] (g \sec[e + f x])^p (1 - \sin[e + f x])^{\frac{1+p}{2}} (a + a \sin[e + f x])^{1+m} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c-d}\right)^{-n}$$

Result (type 6, 4680 leaves) :

$$-\left(\left(2 (c+d) (-3+p) \text{AppellF1}\left[\frac{1-p}{2}, 1+m+n-p, -n, \frac{3-p}{2}, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right]\right.\right. \\ \left.\left.\left(\sec\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{-1-m} \sec[e + f x]^p (g \sec[e + f x])^p (a + a \sin[e + f x])^m (c + d \sin[e + f x])^{2n} \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]\right)\right) / \\ \left(f (1-p) \left((c+d) (-3+p) \text{AppellF1}\left[\frac{1-p}{2}, 1+m+n-p, -n, \frac{3-p}{2}, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right]\right) -\right.$$

$$\begin{aligned}
& \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-1-m} \operatorname{Sec}[e + f x]^{1+p} \operatorname{Sin}[e + f x] (c + d \operatorname{Sin}[e + f x])^n \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \Big/ \\
& \left((1-p) \left((c+d) (-3+p) \operatorname{AppellF1}\left[\frac{1-p}{2}, 1+m+n-p, -n, \frac{3-p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - 2 \left((c-d) \right. \right. \right. \\
& \quad n \operatorname{AppellF1}\left[\frac{3-p}{2}, 1+m+n-p, 1-n, \frac{5-p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - (c+d) (1+m+n-p) \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3-p}{2}, 2+m+n-p, -n, \frac{5-p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) + \\
& \left(2 (c+d) (-1-m) (-3+p) \operatorname{AppellF1}\left[\frac{1-p}{2}, 1+m+n-p, -n, \frac{3-p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \\
& \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-1-m} \operatorname{Sec}[e + f x]^p (c + d \operatorname{Sin}[e + f x])^n \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \Big/ \\
& \left((1-p) \left((c+d) (-3+p) \operatorname{AppellF1}\left[\frac{1-p}{2}, 1+m+n-p, -n, \frac{3-p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - 2 \left((c-d) \right. \right. \right. \\
& \quad n \operatorname{AppellF1}\left[\frac{3-p}{2}, 1+m+n-p, 1-n, \frac{5-p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - (c+d) (1+m+n-p) \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3-p}{2}, 2+m+n-p, -n, \frac{5-p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) + \\
& \left(2 (c+d) (-3+p) \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-1-m} \operatorname{Sec}[e + f x]^p (c + d \operatorname{Sin}[e + f x])^n \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{1}{(c+d) (3-p)} \right. \right. \\
& \quad (c-d) n (1-p) \operatorname{AppellF1}\left[1 + \frac{1-p}{2}, 1+m+n-p, 1-n, 1 + \frac{3-p}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{3-p} (1-p) (1+m+n-p) \operatorname{AppellF1}\left[1 + \frac{1-p}{2}, 2+m+n-p, -n, 1 + \frac{3-p}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left((1-p) \left((c+d) (-3+p) \operatorname{AppellF1} \left[\frac{1-p}{2}, 1+m+n-p, -n, \frac{3-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - 2 \left((c-d) \right. \right. \right. \\
& \quad \left. \left. \left. n \operatorname{AppellF1} \left[\frac{3-p}{2}, 1+m+n-p, 1-n, \frac{5-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - (c+d) (1+m+n-p) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[\frac{3-p}{2}, 2+m+n-p, -n, \frac{5-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \left(2 (c+d) (-3+p) \operatorname{AppellF1} \left[\frac{1-p}{2}, 1+m+n-p, -n, \frac{3-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \\
& \quad \left(\operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-1-m} \operatorname{Sec} [e+f x]^p (c+d \operatorname{Sin} [e+f x])^n \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \\
& \quad \left(-2 \left((c-d) n \operatorname{AppellF1} \left[\frac{3-p}{2}, 1+m+n-p, 1-n, \frac{5-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - (c+d) \right. \right. \\
& \quad \left. \left. (1+m+n-p) \operatorname{AppellF1} \left[\frac{3-p}{2}, 2+m+n-p, -n, \frac{5-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \right) \\
& \quad \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] + (c+d) (-3+p) \left(\frac{1}{(c+d) (3-p)} (c-d) n (1-p) \right. \\
& \quad \left. \operatorname{AppellF1} \left[1 + \frac{1-p}{2}, 1+m+n-p, 1-n, 1 + \frac{3-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \\
& \quad \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{3-p} (1-p) (1+m+n-p) \operatorname{AppellF1} \left[1 + \frac{1-p}{2}, 2+m+n-p, -n, \right. \\
& \quad \left. 1 + \frac{3-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) - \\
& \quad 2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left((c-d) n \left(-\frac{1}{(c+d) (5-p)} (c-d) (1-n) (3-p) \operatorname{AppellF1} \left[1 + \frac{3-p}{2}, 1+m+n-p, 2-n, \right. \right. \right. \\
& \quad \left. \left. \left. 1 + \frac{5-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) - \right. \\
& \quad \left. \frac{1}{5-p} (3-p) (1+m+n-p) \operatorname{AppellF1} \left[1 + \frac{3-p}{2}, 2+m+n-p, 1-n, 1 + \frac{5-p}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d} \operatorname{Sec}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right) - (c+d) (1+m+n-p) \left(\frac{1}{(c+d) (5-p)} \right. \\
& (c-d) n (3-p) \operatorname{AppellF1}\left[1 + \frac{3-p}{2}, 2+m+n-p, 1-n, 1 + \frac{5-p}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, - \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d} \right] \\
& \operatorname{Sec}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{5-p} (3-p) (2+m+n-p) \operatorname{AppellF1}\left[1 + \frac{3-p}{2}, 3+m+n-p, -n, 1 + \frac{5-p}{2}, \right. \\
& \left. -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, - \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d} \right] \operatorname{Sec}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \Bigg) \Bigg) / \\
& \left((1-p) \left((c+d) (-3+p) \operatorname{AppellF1}\left[\frac{1-p}{2}, 1+m+n-p, -n, \frac{3-p}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, - \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d} \right] - \right. \right. \\
& \left. \left. 2 \left((c-d) n \operatorname{AppellF1}\left[\frac{3-p}{2}, 1+m+n-p, 1-n, \frac{5-p}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, - \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d} \right] - \right. \right. \right. \\
& \left. \left. (c+d) (1+m+n-p) \operatorname{AppellF1}\left[\frac{3-p}{2}, 2+m+n-p, -n, \frac{5-p}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right. \right. \right. \\
& \left. \left. \left. - \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d} \right] \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 1065: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^2 \operatorname{Csc}[c+dx]^2 (a+b \operatorname{Sin}[c+dx])^2 dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$-b^2 x + \frac{a b \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{d} + \frac{(a^2 - 2b^2) \operatorname{Cot}[c+dx]}{3d} - \frac{a b \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{3d} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2 (a+b \operatorname{Sin}[c+dx])^2}{3d}$$

Result (type 3, 538 leaves):

$$\begin{aligned}
& - \frac{b^2 (c + dx) (b + a \operatorname{Csc}[c + dx])^2 \operatorname{Sin}[c + dx]^2}{d (a + b \operatorname{Sin}[c + dx])^2} + \frac{(a^2 \operatorname{Cos}[\frac{1}{2}(c + dx)] - 3b^2 \operatorname{Cos}[\frac{1}{2}(c + dx)]) \operatorname{Csc}[\frac{1}{2}(c + dx)] (b + a \operatorname{Csc}[c + dx])^2 \operatorname{Sin}[c + dx]^2}{6d (a + b \operatorname{Sin}[c + dx])^2} \\
& - \frac{ab \operatorname{Csc}[\frac{1}{2}(c + dx)]^2 (b + a \operatorname{Csc}[c + dx])^2 \operatorname{Sin}[c + dx]^2}{4d (a + b \operatorname{Sin}[c + dx])^2} - \frac{a^2 \operatorname{Cot}[\frac{1}{2}(c + dx)] \operatorname{Csc}[\frac{1}{2}(c + dx)]^2 (b + a \operatorname{Csc}[c + dx])^2 \operatorname{Sin}[c + dx]^2}{24d (a + b \operatorname{Sin}[c + dx])^2} + \\
& - \frac{ab (b + a \operatorname{Csc}[c + dx])^2 \operatorname{Log}[\operatorname{Cos}[\frac{1}{2}(c + dx)]] \operatorname{Sin}[c + dx]^2}{d (a + b \operatorname{Sin}[c + dx])^2} - \\
& + \frac{ab (b + a \operatorname{Csc}[c + dx])^2 \operatorname{Log}[\operatorname{Sin}[\frac{1}{2}(c + dx)]] \operatorname{Sin}[c + dx]^2}{d (a + b \operatorname{Sin}[c + dx])^2} + \frac{ab (b + a \operatorname{Csc}[c + dx])^2 \operatorname{Sec}[\frac{1}{2}(c + dx)]^2 \operatorname{Sin}[c + dx]^2}{4d (a + b \operatorname{Sin}[c + dx])^2} + \\
& + \frac{(b + a \operatorname{Csc}[c + dx])^2 \operatorname{Sec}[\frac{1}{2}(c + dx)] (-a^2 \operatorname{Sin}[\frac{1}{2}(c + dx)] + 3b^2 \operatorname{Sin}[\frac{1}{2}(c + dx)]) \operatorname{Sin}[c + dx]^2}{6d (a + b \operatorname{Sin}[c + dx])^2} + \\
& + \frac{a^2 (b + a \operatorname{Csc}[c + dx])^2 \operatorname{Sec}[\frac{1}{2}(c + dx)]^2 \operatorname{Sin}[c + dx]^2 \operatorname{Tan}[\frac{1}{2}(c + dx)]}{24d (a + b \operatorname{Sin}[c + dx])^2}
\end{aligned}$$

■ **Problem 1066: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + dx]^2 \operatorname{Csc}[c + dx]^3 (a + b \operatorname{Sin}[c + dx])^2 dx$$

Optimal (type 3, 123 leaves, 8 steps):

$$\begin{aligned}
& \frac{(a^2 + 4b^2) \operatorname{ArcTanh}[\operatorname{Cos}[c + dx]]}{8d} + \frac{2ab \operatorname{Cot}[c + dx]}{3d} + \frac{(a^2 - 2b^2) \operatorname{Cot}[c + dx] \operatorname{Csc}[c + dx]}{8d} - \\
& - \frac{ab \operatorname{Cot}[c + dx] \operatorname{Csc}[c + dx]^2}{6d} - \frac{\operatorname{Cot}[c + dx] \operatorname{Csc}[c + dx]^3 (a + b \operatorname{Sin}[c + dx])^2}{4d}
\end{aligned}$$

Result (type 3, 579 leaves):

$$\begin{aligned}
& \frac{a b \cot\left[\frac{1}{2}(c+dx)\right] (b+a \csc[c+dx])^2 \sin[c+dx]^2}{3 d (a+b \sin[c+dx])^2} + \frac{(a^2-4 b^2) \csc\left[\frac{1}{2}(c+dx)\right]^2 (b+a \csc[c+dx])^2 \sin[c+dx]^2}{32 d (a+b \sin[c+dx])^2} - \\
& \frac{a b \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2 (b+a \csc[c+dx])^2 \sin[c+dx]^2}{12 d (a+b \sin[c+dx])^2} - \frac{a^2 \csc\left[\frac{1}{2}(c+dx)\right]^4 (b+a \csc[c+dx])^2 \sin[c+dx]^2}{64 d (a+b \sin[c+dx])^2} + \\
& \frac{(a^2+4 b^2) (b+a \csc[c+dx])^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] \sin[c+dx]^2}{8 d (a+b \sin[c+dx])^2} + \frac{(-a^2-4 b^2) (b+a \csc[c+dx])^2 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] \sin[c+dx]^2}{8 d (a+b \sin[c+dx])^2} + \\
& \frac{(-a^2+4 b^2) (b+a \csc[c+dx])^2 \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx]^2}{32 d (a+b \sin[c+dx])^2} + \frac{a^2 (b+a \csc[c+dx])^2 \sec\left[\frac{1}{2}(c+dx)\right]^4 \sin[c+dx]^2}{64 d (a+b \sin[c+dx])^2} - \\
& \frac{a b (b+a \csc[c+dx])^2 \sin[c+dx]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{3 d (a+b \sin[c+dx])^2} + \frac{a b (b+a \csc[c+dx])^2 \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{12 d (a+b \sin[c+dx])^2}
\end{aligned}$$

■ **Problem 1074: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^2 \csc[c+dx]^2 (a+b \sin[c+dx])^3 dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$\begin{aligned}
& -3 a b^2 x + \frac{b (3 a^2 - 2 b^2) \operatorname{ArcTanh}[\cos[c+dx]]}{2 d} + \frac{11 b^3 \cos[c+dx]}{6 d} + \frac{a (a^2 - 3 b^2) \cot[c+dx]}{3 d} - \\
& \frac{b \cot[c+dx] \csc[c+dx] (a+b \sin[c+dx])^2}{2 d} - \frac{\cot[c+dx] \csc[c+dx]^2 (a+b \sin[c+dx])^3}{3 d}
\end{aligned}$$

Result (type 3, 615 leaves):

$$\begin{aligned}
& - \frac{3 a b^2 (c+d x) (b+a \operatorname{Csc}[c+d x])^3 \operatorname{Sin}[c+d x]^3}{d (a+b \operatorname{Sin}[c+d x])^3} + \frac{b^3 \operatorname{Cos}[c+d x] (b+a \operatorname{Csc}[c+d x])^3 \operatorname{Sin}[c+d x]^3}{d (a+b \operatorname{Sin}[c+d x])^3} + \\
& \frac{\left(a^3 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - 9 a b^2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right] (b+a \operatorname{Csc}[c+d x])^3 \operatorname{Sin}[c+d x]^3}{6 d (a+b \operatorname{Sin}[c+d x])^3} - \\
& \frac{3 a^2 b \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 (b+a \operatorname{Csc}[c+d x])^3 \operatorname{Sin}[c+d x]^3}{8 d (a+b \operatorname{Sin}[c+d x])^3} - \frac{a^3 \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 (b+a \operatorname{Csc}[c+d x])^3 \operatorname{Sin}[c+d x]^3}{24 d (a+b \operatorname{Sin}[c+d x])^3} + \\
& \frac{\left(3 a^2 b - 2 b^3\right) (b+a \operatorname{Csc}[c+d x])^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sin}[c+d x]^3}{2 d (a+b \operatorname{Sin}[c+d x])^3} + \\
& \frac{\left(-3 a^2 b + 2 b^3\right) (b+a \operatorname{Csc}[c+d x])^3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sin}[c+d x]^3}{2 d (a+b \operatorname{Sin}[c+d x])^3} + \frac{3 a^2 b (b+a \operatorname{Csc}[c+d x])^3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sin}[c+d x]^3}{8 d (a+b \operatorname{Sin}[c+d x])^3} + \\
& \frac{(b+a \operatorname{Csc}[c+d x])^3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \left(-a^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 9 a b^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sin}[c+d x]^3}{6 d (a+b \operatorname{Sin}[c+d x])^3} + \\
& \frac{a^3 (b+a \operatorname{Csc}[c+d x])^3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sin}[c+d x]^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{24 d (a+b \operatorname{Sin}[c+d x])^3}
\end{aligned}$$

■ **Problem 1075: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+d x]^2 \operatorname{Csc}[c+d x]^3 (a+b \operatorname{Sin}[c+d x])^3 dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\begin{aligned}
& -b^3 x + \frac{a \left(a^2 + 12 b^2\right) \operatorname{ArcTanh}\left[\operatorname{Cos}[c+d x]\right]}{8 d} + \frac{b \left(2 a^2 - b^2\right) \operatorname{Cot}[c+d x]}{2 d} + \frac{a \left(a^2 - 2 b^2\right) \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]}{8 d} - \\
& \frac{b \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^2 (a+b \operatorname{Sin}[c+d x])^2}{4 d} - \frac{\operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^3 (a+b \operatorname{Sin}[c+d x])^3}{4 d}
\end{aligned}$$

Result (type 3, 690 leaves):

$$\begin{aligned}
& - \frac{b^3 (c + dx) (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Sin}[c + dx]^3}{d (a + b \operatorname{Sin}[c + dx])^3} + \frac{(a^2 b \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - b^3 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right] (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Sin}[c + dx]^3}{2d (a + b \operatorname{Sin}[c + dx])^3} \\
& - \frac{(a^3 - 12ab^2) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2 (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Sin}[c + dx]^3}{32d (a + b \operatorname{Sin}[c + dx])^3} - \frac{a^2 b \operatorname{Cot}\left[\frac{1}{2}(c + dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2 (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Sin}[c + dx]^3}{8d (a + b \operatorname{Sin}[c + dx])^3} \\
& + \frac{a^3 \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^4 (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Sin}[c + dx]^3}{64d (a + b \operatorname{Sin}[c + dx])^3} + \frac{(a^3 + 12ab^2) (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[c + dx]^3}{8d (a + b \operatorname{Sin}[c + dx])^3} \\
& + \frac{(-a^3 - 12ab^2) (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sin}[c + dx]^3}{8d (a + b \operatorname{Sin}[c + dx])^3} + \\
& + \frac{(-a^3 + 12ab^2) (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sin}[c + dx]^3}{32d (a + b \operatorname{Sin}[c + dx])^3} + \frac{a^3 (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \operatorname{Sin}[c + dx]^3}{64d (a + b \operatorname{Sin}[c + dx])^3} \\
& + \frac{(b + a \operatorname{Csc}[c + dx])^3 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] (-a^2 b \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + b^3 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]) \operatorname{Sin}[c + dx]^3}{2d (a + b \operatorname{Sin}[c + dx])^3} + \\
& + \frac{a^2 b (b + a \operatorname{Csc}[c + dx])^3 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sin}[c + dx]^3 \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{8d (a + b \operatorname{Sin}[c + dx])^3}
\end{aligned}$$

- **Problem 1091: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e + fx]^2}{\sqrt{d \operatorname{Sin}[e + fx]} (a + b \operatorname{Sin}[e + fx])^{5/2}} dx$$

Optimal (type 4, 347 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 \operatorname{Cos}[e + fx] \sqrt{d \operatorname{Sin}[e + fx]}}{3 a d f (a + b \operatorname{Sin}[e + fx])^{3/2}} + \frac{4 b \operatorname{Cos}[e + fx]}{3 a (a^2 - b^2) f \sqrt{d \operatorname{Sin}[e + fx]} \sqrt{a + b \operatorname{Sin}[e + fx]}} - \\
& \frac{4 b \sqrt{\frac{a(1 - \operatorname{Csc}[e + fx])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Csc}[e + fx])}{a - b}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \operatorname{Sin}[e + fx]}}{\sqrt{a + b} \sqrt{d \operatorname{Sin}[e + fx]}}\right], -\frac{a + b}{a - b}\right] \operatorname{Tan}[e + fx]}{3 a^3 \sqrt{a + b} \sqrt{d} f} - \\
& \frac{4 \sqrt{\frac{a(1 - \operatorname{Csc}[e + fx])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Csc}[e + fx])}{a - b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \operatorname{Sin}[e + fx]}}{\sqrt{a + b} \sqrt{d \operatorname{Sin}[e + fx]}}\right], -\frac{a + b}{a - b}\right] \operatorname{Tan}[e + fx]}{3 a^2 \sqrt{a + b} \sqrt{d} f}
\end{aligned}$$

Result (type 4, 9146 leaves):

$$\frac{\operatorname{Sin}[e + fx] \sqrt{a + b \operatorname{Sin}[e + fx]} \left(\frac{2 \operatorname{Cos}[e + fx]}{3 a (a + b \operatorname{Sin}[e + fx])^2} - \frac{4 b^2 \operatorname{Cos}[e + fx]}{3 a^2 (a^2 - b^2) (a + b \operatorname{Sin}[e + fx])} \right)}{f \sqrt{d \operatorname{Sin}[e + fx]}} - \frac{1}{3 (a^2 - b^2) f \sqrt{d \operatorname{Sin}[e + fx]}}$$

$$\begin{aligned}
& 8 \sqrt{\sin[e+fx]} \left(\left(\sqrt{\cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(a+b)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \sin[e+fx]}}{a}}\right], \frac{2a}{a+b}\right] \sec[e+fx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \right. \\
& \left. \sqrt{-\frac{(a+b)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \sin[e+fx]}{a}} \sqrt{\frac{\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b\sin[e+fx])}{a}} \right) / \left(\sqrt{\sin[e+fx]} \sqrt{a+b\sin[e+fx]} \right) - \\
& \left(b \sqrt{\cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2} \operatorname{EllipticPi}\left[\frac{a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{-(a+b)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \sin[e+fx]}}{a}}\right], \frac{2a}{a+b}\right] \sec[e+fx] \right. \\
& \left. \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{-\frac{(a+b)\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \sin[e+fx]}{a}} \right. \\
& \left. \sqrt{\frac{\csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b\sin[e+fx])}{a}} \right) / \left((a+b) \sqrt{\sin[e+fx]} \sqrt{a+b\sin[e+fx]} \right) - \\
& \left(4b\sin[e+fx] \sqrt{a+b\sin[e+fx]} \left(\sqrt{2} \tan\left[\frac{1}{2}(e+fx)\right] \sqrt{\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1+\tan\left[\frac{1}{2}(e+fx)\right]^2}} \sqrt{\frac{a+2b\tan\left[\frac{1}{2}(e+fx)\right]+a\tan\left[\frac{1}{2}(e+fx)\right]^2}{1+\tan\left[\frac{1}{2}(e+fx)\right]^2}} \right. \right. \\
& \left. \left. \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} (a+2b\tan\left[\frac{1}{2}(e+fx)\right]+a\tan\left[\frac{1}{2}(e+fx)\right]^2)} \sqrt{2} \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a+2b\cot\left[\frac{1}{2}(e+fx)\right]+a\cot\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} i \left(b - \sqrt{-a^2+b^2} \right) \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \sqrt{\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{1 + \frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}}} \\
& \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] \right) - \\
& \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \sqrt{\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{1 + \frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}}} \text{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, \right. \\
& \left. i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] + \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \\
& \left. \sqrt{\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{1 + \frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}}} \text{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] \right) \\
& \left(\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^{3/2} \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e+fx)\right] + a \tan\left[\frac{1}{2}(e+fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}} \right) / \\
& \left(3 a^2 (a^2 - b^2) f \sqrt{d \sin[e+fx]} \left(\frac{\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sqrt{\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}} \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e+fx)\right] + a \tan\left[\frac{1}{2}(e+fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}}}{\sqrt{2}} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}} \left(a + 2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2}} \sqrt{2} \left(\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a + 2b \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \right. \\
& \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} i \left(b - \sqrt{-a^2+b^2} \right) \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \\
& \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) - \right. \\
& \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, \right. \\
& i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \\
& \left. \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) \\
& \left(b \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + a \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^{3/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{a + 2 b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}} - \frac{1}{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}} \left(a + 2 b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right)}} \\
& 2 \sqrt{2} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \left(\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}} \left(a + 2 b \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] + a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^2\right)} + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right) \\
& i \left(b - \sqrt{-a^2 + b^2}\right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}}} \\
& \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) - \\
& \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} a \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}}} \operatorname{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2 + b^2}\right)}{a} \right], \\
& i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} a \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \\
& \sqrt{\frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2 + b^2}\right)}{a} \right], i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right]} \\
& \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \right)^{3/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \sqrt{\frac{a + 2 b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}} \left(a + 2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)}} \\
& \sqrt{2} \left(\frac{i a \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{4 \left(1 - i \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}} - \frac{i a \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{4 \left(1 + i \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}} \right) + \\
& \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}} \left(-b \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 - a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right] \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \frac{1}{4 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}}} \\
& i \left(b - \sqrt{-a^2+b^2} \right) \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \\
& \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{4 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \\
& \operatorname{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 - \\
& \frac{1}{4 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, \right.
\end{aligned}$$

$$\begin{aligned}
& i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 - \left(i a \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2 \right. \\
& \left. \left(\operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] - \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) / \\
& \left(4 \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right) - \left(i a (b - \sqrt{-a^2 + b^2}) \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2 \right. \\
& \left. \left(\operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] - \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) / \\
& \left(4 (b + \sqrt{-a^2 + b^2}) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right) + \\
& \left(a^2 \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{EllipticPi} \left[-\frac{i (b + \sqrt{-a^2 + b^2})}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \right. \right. \\
& \left. \left. \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) / \left(4 (b - \sqrt{-a^2 + b^2}) \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(a^2 \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticPi}\left[-\frac{i\left(b + \sqrt{-a^2 + b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \right) / \\
& \left(4 \left(b + \sqrt{-a^2 + b^2}\right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]} \right) - \\
& \left(a^2 \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticPi}\left[\frac{i\left(b + \sqrt{-a^2 + b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \right. \right. \\
& \left. \left. \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \right) / \left(4 \left(b - \sqrt{-a^2 + b^2}\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]} \right) - \\
& \left(a^2 \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticPi}\left[\frac{i\left(b + \sqrt{-a^2 + b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \right) / \\
& \left(4 \left(b + \sqrt{-a^2 + b^2}\right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]} \right) + \\
& \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} i \left(b - \sqrt{-a^2 + b^2}\right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2 + b^2}}}
\end{aligned}$$

$$\left(\frac{i \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{4 \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}} - \frac{i \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{4 \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}} \right)$$

$$\left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^{3/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \sqrt{\frac{a + 2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} +$$

$$\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \sqrt{\frac{a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2 \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)} \right)}{\sqrt{2} \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}}}$$

$$\frac{1}{\sqrt{2} \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a + 2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)}$$

$$3 \left(\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a + 2b \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right)$$

$$i \left(b - \sqrt{-a^2+b^2} \right) \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}}$$

$$\left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] \right) -$$

$$\begin{aligned}
& \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[-\frac{i\left(b + \sqrt{-a^2 + b^2}\right)}{a}\right], \\
& i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \left. + \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2 + b^2}}}\right. \\
& \left. \sqrt{1 + \frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[\frac{i\left(b + \sqrt{-a^2 + b^2}\right)}{a}\right], i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \\
& \sqrt{\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}} \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e+fx)\right] + a \tan\left[\frac{1}{2}(e+fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}} \\
& \left(-\frac{\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]^2}{\left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2} + \frac{\sec\left[\frac{1}{2}(e+fx)\right]^2}{2\left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)}\right) + \\
& \left(\tan\left[\frac{1}{2}(e+fx)\right] \sqrt{\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}} \left(\frac{b \sec\left[\frac{1}{2}(e+fx)\right]^2 + a \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} - \right. \right. \\
& \left. \left. \frac{\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \left(a + 2b \tan\left[\frac{1}{2}(e+fx)\right] + a \tan\left[\frac{1}{2}(e+fx)\right]^2\right)}{\left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2}\right)\right) / \\
& \left(\sqrt{2} \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e+fx)\right] + a \tan\left[\frac{1}{2}(e+fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}}\right) - \left(\left(\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \left(a + 2b \cot\left[\frac{1}{2}(e+fx)\right] + a \cot\left[\frac{1}{2}(e+fx)\right]^2\right) + \right.\right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} i \left(b - \sqrt{-a^2+b^2} \right) \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \sqrt{\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}} \\
& \left(\text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] - \text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] \right) - \\
& \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \sqrt{\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}} \text{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, \right. \\
& \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] + \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \\
& \left. \sqrt{\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}} \text{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] \right) \\
& \left(\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^{3/2} \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \left(\frac{b \sec\left[\frac{1}{2}(e+fx)\right]^2 + a \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} - \right. \\
& \left. \frac{\sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \left(a + 2b \tan\left[\frac{1}{2}(e+fx)\right] + a \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)}{\left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2} \right) \Bigg) /
\end{aligned}$$

$$\left(\sqrt{2} \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \left(a + 2b \tan\left[\frac{1}{2}(e + fx)\right] + a \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e + fx)\right] + a \tan\left[\frac{1}{2}(e + fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e + fx)\right]^2}} \right) \right)$$

■ **Problem 1103: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx]^4 \csc[c + dx]^4 (a + b \sin[c + dx]) dx$$

Optimal (type 3, 114 leaves, 8 steps):

$$\frac{b \operatorname{ArcTanh}[\cos[c + dx]]}{16d} - \frac{a \cot[c + dx]^5}{5d} - \frac{a \cot[c + dx]^7}{7d} - \frac{b \cot[c + dx] \csc[c + dx]}{16d} + \frac{b \cot[c + dx] \csc[c + dx]^3}{8d} - \frac{b \cot[c + dx]^3 \csc[c + dx]^3}{6d}$$

Result (type 3, 239 leaves):

$$\frac{2a \cot[c + dx]}{35d} - \frac{b \csc\left[\frac{1}{2}(c + dx)\right]^2}{64d} + \frac{b \csc\left[\frac{1}{2}(c + dx)\right]^4}{64d} - \frac{b \csc\left[\frac{1}{2}(c + dx)\right]^6}{384d} - \frac{a \cot[c + dx] \csc[c + dx]^2}{35d} + \frac{8a \cot[c + dx] \csc[c + dx]^4}{35d} - \frac{a \cot[c + dx] \csc[c + dx]^6}{7d} - \frac{b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right]}{16d} + \frac{b \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right]}{16d} + \frac{b \sec\left[\frac{1}{2}(c + dx)\right]^2}{64d} - \frac{b \sec\left[\frac{1}{2}(c + dx)\right]^4}{64d} + \frac{b \sec\left[\frac{1}{2}(c + dx)\right]^6}{384d}$$

■ **Problem 1104: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx]^4 \csc[c + dx]^5 (a + b \sin[c + dx]) dx$$

Optimal (type 3, 136 leaves, 9 steps):

$$\frac{3a \operatorname{ArcTanh}[\cos[c + dx]]}{128d} - \frac{b \cot[c + dx]^5}{5d} - \frac{b \cot[c + dx]^7}{7d} - \frac{3a \cot[c + dx] \csc[c + dx]}{128d} - \frac{a \cot[c + dx] \csc[c + dx]^3}{64d} + \frac{a \cot[c + dx] \csc[c + dx]^5}{16d} - \frac{a \cot[c + dx]^3 \csc[c + dx]^5}{8d}$$

Result (type 3, 279 leaves):

$$\begin{aligned}
& - \frac{2 b \cot [c+d x]}{35 d} - \frac{3 a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{512 d} + \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4}{1024 d} + \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^6}{512 d} - \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^8}{2048 d} \\
& \frac{b \cot [c+d x] \operatorname{Csc}[c+d x]^2}{35 d} + \frac{8 b \cot [c+d x] \operatorname{Csc}[c+d x]^4}{35 d} - \frac{b \cot [c+d x] \operatorname{Csc}[c+d x]^6}{7 d} - \frac{3 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right]}{128 d} + \\
& \frac{3 a \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right]}{128 d} + \frac{3 a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{512 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4}{1024 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^6}{512 d} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^8}{2048 d}
\end{aligned}$$

■ **Problem 1111: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^4 (a+b \sin [c+d x])^2 dx$$

Optimal (type 3, 133 leaves, 13 steps):

$$\begin{aligned}
& a^2 x - \frac{3 b^2 x}{2} + \frac{3 a b \operatorname{ArcTanh}[\cos [c+d x]]}{d} - \frac{3 a b \cos [c+d x]}{d} + \frac{a^2 \cot [c+d x]}{d} - \\
& \frac{3 b^2 \cot [c+d x]}{2 d} + \frac{b^2 \cos [c+d x]^2 \cot [c+d x]}{2 d} - \frac{a b \cos [c+d x] \cot [c+d x]^2}{d} - \frac{a^2 \cot [c+d x]^3}{3 d}
\end{aligned}$$

Result (type 3, 293 leaves):

$$\begin{aligned}
& \frac{(2 a^2 - 3 b^2)(c+d x)}{2 d} - \frac{2 a b \cos [c+d x]}{d} + \frac{(4 a^2 \cos \left[\frac{1}{2}(c+d x)\right] - 3 b^2 \cos \left[\frac{1}{2}(c+d x)\right]) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]}{6 d} - \frac{a b \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{4 d} - \\
& \frac{a^2 \cot \left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{24 d} + \frac{3 a b \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right]}{d} - \frac{3 a b \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right]}{d} + \frac{a b \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{4 d} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right](-4 a^2 \sin \left[\frac{1}{2}(c+d x)\right] + 3 b^2 \sin \left[\frac{1}{2}(c+d x)\right])}{6 d} - \frac{b^2 \sin [2(c+d x)]}{4 d} + \frac{a^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{24 d}
\end{aligned}$$

■ **Problem 1122: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^4 \operatorname{Csc}[c+d x] (a+b \sin [c+d x])^3 dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$\begin{aligned}
& \frac{3}{2} b (2 a^2 - b^2) x - \frac{3 a (a^2 - 12 b^2) \operatorname{ArcTanh}[\cos [c+d x]]}{8 d} - \frac{b^2 (73 a^2 - 2 b^2) \cos [c+d x]}{8 a d} - \frac{13 b^3 \cos [c+d x] \sin [c+d x]}{4 d} + \\
& \frac{17 b \cot [c+d x] (a+b \sin [c+d x])^2}{8 d} + \frac{5 \cot [c+d x] \operatorname{Csc}[c+d x] (a+b \sin [c+d x])^3}{8 d} - \frac{\cot [c+d x] \operatorname{Csc}[c+d x]^3 (a+b \sin [c+d x])^4}{4 a d}
\end{aligned}$$

Result (type 3, 381 leaves):

$$\begin{aligned}
& - \frac{3b(-2a^2 + b^2)(c + dx)}{2d} - \frac{3ab^2 \operatorname{Cos}[c + dx]}{d} + \frac{(4a^2b \operatorname{Cos}[\frac{1}{2}(c + dx)] - b^3 \operatorname{Cos}[\frac{1}{2}(c + dx)]) \operatorname{Csc}[\frac{1}{2}(c + dx)]}{2d} + \\
& \frac{(5a^3 - 12ab^2) \operatorname{Csc}[\frac{1}{2}(c + dx)]^2}{32d} - \frac{a^2b \operatorname{Cot}[\frac{1}{2}(c + dx)] \operatorname{Csc}[\frac{1}{2}(c + dx)]^2}{8d} - \frac{a^3 \operatorname{Csc}[\frac{1}{2}(c + dx)]^4}{64d} - \frac{3(a^3 - 12ab^2) \operatorname{Log}[\operatorname{Cos}[\frac{1}{2}(c + dx)]]}{8d} + \\
& \frac{3(a^3 - 12ab^2) \operatorname{Log}[\operatorname{Sin}[\frac{1}{2}(c + dx)]]}{8d} + \frac{(-5a^3 + 12ab^2) \operatorname{Sec}[\frac{1}{2}(c + dx)]^2}{32d} + \frac{a^3 \operatorname{Sec}[\frac{1}{2}(c + dx)]^4}{64d} + \\
& \frac{\operatorname{Sec}[\frac{1}{2}(c + dx)](-4a^2b \operatorname{Sin}[\frac{1}{2}(c + dx)] + b^3 \operatorname{Sin}[\frac{1}{2}(c + dx)])}{2d} - \frac{b^3 \operatorname{Sin}[2(c + dx)]}{4d} + \frac{a^2b \operatorname{Sec}[\frac{1}{2}(c + dx)]^2 \operatorname{Tan}[\frac{1}{2}(c + dx)]}{8d}
\end{aligned}$$

■ **Problem 1135: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^4 \operatorname{Sin}[c + dx]^3}{(a + b \operatorname{Sin}[c + dx])^3} dx$$

Optimal (type 3, 331 leaves, 9 steps):

$$\begin{aligned}
& \frac{3(40a^4 - 24a^2b^2 + b^4)x}{8b^7} - \frac{3a(10a^4 - 11a^2b^2 + 2b^4) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}[\frac{1}{2}(c+dx)]}{\sqrt{a^2-b^2}}\right]}{b^7 \sqrt{a^2-b^2} d} + \\
& \frac{a(30a^2 - 13b^2) \operatorname{Cos}[c + dx]}{2b^6 d} - \frac{3(20a^2 - 7b^2) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{8b^5 d} + \frac{(10a^2 - 3b^2) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^2}{2ab^4 d} - \\
& \frac{(15a^2 - 4b^2) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^3}{4a^2b^3 d} - \frac{(a^2 - b^2) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^4}{2ab^2 d (a + b \operatorname{Sin}[c + dx])^2} + \frac{(7a^2 - 2b^2) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^4}{2a^2b^2 d (a + b \operatorname{Sin}[c + dx])}
\end{aligned}$$

Result (type 3, 1264 leaves):

$$\begin{aligned}
& \frac{1}{128b^3 d} \\
& 3 \left(-8(c + dx) + \frac{2a(8a^4 - 20a^2b^2 + 15b^4) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}[\frac{1}{2}(c+dx)]}{\sqrt{a^2-b^2}}\right]}{(a^2 - b^2)^{5/2}} + \frac{ab(4a^2 - 3b^2) \operatorname{Cos}[c + dx]}{(a - b)(a + b)(a + b \operatorname{Sin}[c + dx])^2} - \frac{3b(4a^4 - 7a^2b^2 + 2b^4) \operatorname{Cos}[c + dx]}{(a - b)^2 (a + b)^2 (a + b \operatorname{Sin}[c + dx])} \right) - \\
& 3 \left(\frac{6ab \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}[\frac{1}{2}(c+dx)]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{\operatorname{Cos}[c + dx](a(2a^2 + b^2) + b(a^2 + 2b^2) \operatorname{Sin}[c + dx])}{(a + b \operatorname{Sin}[c + dx])^2} \right) - \frac{1}{128(a - b)^2 (a + b)^2 d} - \frac{1}{128b^5 d}
\end{aligned}$$

$$\left(-24 (-8 a^2 + b^2) (c + d x) - \frac{6 a (64 a^6 - 168 a^4 b^2 + 140 a^2 b^4 - 35 b^6) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + 96 a b \operatorname{Cos}[c+d x] + \right.$$

$$\left. \frac{a b (-16 a^4 + 20 a^2 b^2 - 5 b^4) \operatorname{Cos}[c+d x]}{(a-b)(a+b)(a+b \operatorname{Sin}[c+d x])^2} + \frac{b (112 a^6 - 220 a^4 b^2 + 115 a^2 b^4 - 10 b^6) \operatorname{Cos}[c+d x]}{(a-b)^2 (a+b)^2 (a+b \operatorname{Sin}[c+d x])} - 8 b^2 \operatorname{Sin}[2(c+d x)] \right) -$$

$$\frac{1}{256 b^7 d} \left(\frac{12 a (640 a^8 - 1920 a^6 b^2 + 2016 a^4 b^4 - 840 a^2 b^6 + 105 b^8) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \right.$$

$$\left. \frac{1}{(a^2-b^2)^2 (a+b \operatorname{Sin}[c+d x])^2} (-3840 a^{10} (c+d x) + 7680 a^8 b^2 (c+d x) - 2976 a^6 b^4 (c+d x) - 1776 a^4 b^6 (c+d x) + \right.$$

$$960 a^2 b^8 (c+d x) - 48 b^{10} (c+d x) - 3840 a^9 b \operatorname{Cos}[c+d x] + 8640 a^7 b^3 \operatorname{Cos}[c+d x] - 5696 a^5 b^5 \operatorname{Cos}[c+d x] +$$

$$788 a^3 b^7 \operatorname{Cos}[c+d x] + 114 a b^9 \operatorname{Cos}[c+d x] + 1920 a^8 b^2 (c+d x) \operatorname{Cos}[2(c+d x)] - 4800 a^6 b^4 (c+d x) \operatorname{Cos}[2(c+d x)] +$$

$$3888 a^4 b^6 (c+d x) \operatorname{Cos}[2(c+d x)] - 1056 a^2 b^8 (c+d x) \operatorname{Cos}[2(c+d x)] + 48 b^{10} (c+d x) \operatorname{Cos}[2(c+d x)] + 320 a^7 b^3 \operatorname{Cos}[3(c+d x)] -$$

$$760 a^5 b^5 \operatorname{Cos}[3(c+d x)] + 560 a^3 b^7 \operatorname{Cos}[3(c+d x)] - 120 a b^9 \operatorname{Cos}[3(c+d x)] - 8 a^5 b^5 \operatorname{Cos}[5(c+d x)] + 16 a^3 b^7 \operatorname{Cos}[5(c+d x)] -$$

$$8 a b^9 \operatorname{Cos}[5(c+d x)] - 7680 a^9 b (c+d x) \operatorname{Sin}[c+d x] + 19200 a^7 b^3 (c+d x) \operatorname{Sin}[c+d x] - 15552 a^5 b^5 (c+d x) \operatorname{Sin}[c+d x] +$$

$$4224 a^3 b^7 (c+d x) \operatorname{Sin}[c+d x] - 192 a b^9 (c+d x) \operatorname{Sin}[c+d x] - 2880 a^8 b^2 \operatorname{Sin}[2(c+d x)] + 6880 a^6 b^4 \operatorname{Sin}[2(c+d x)] -$$

$$5182 a^4 b^6 \operatorname{Sin}[2(c+d x)] + 1221 a^2 b^8 \operatorname{Sin}[2(c+d x)] - 36 b^{10} \operatorname{Sin}[2(c+d x)] - 40 a^6 b^4 \operatorname{Sin}[4(c+d x)] + 88 a^4 b^6 \operatorname{Sin}[4(c+d x)] -$$

$$\left. \left. 56 a^2 b^8 \operatorname{Sin}[4(c+d x)] + 8 b^{10} \operatorname{Sin}[4(c+d x)] + 2 a^4 b^6 \operatorname{Sin}[6(c+d x)] - 4 a^2 b^8 \operatorname{Sin}[6(c+d x)] + 2 b^{10} \operatorname{Sin}[6(c+d x)] \right) \right)$$

■ **Problem 1136: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x]^4 \operatorname{Sin}[c+d x]^2}{(a+b \operatorname{Sin}[c+d x])^3} dx$$

Optimal (type 3, 284 leaves, 8 steps):

$$\frac{a \left(9 - \frac{20a^2}{b^2}\right) x}{2b^4} + \frac{(20a^4 - 19a^2b^2 + 2b^4) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{b^6 \sqrt{a^2-b^2} d} - \frac{(60a^2 - 17b^2) \operatorname{Cos}[c+dx]}{6b^5 d} + \frac{(5a^2 - b^2) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{ab^4 d} -$$

$$\frac{(20a^2 - 3b^2) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]^2}{6a^2 b^3 d} - \frac{(a^2 - b^2) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]^3}{2ab^2 d (a+b \operatorname{Sin}[c+dx])^2} + \frac{(6a^2 - b^2) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]^3}{2a^2 b^2 d (a+b \operatorname{Sin}[c+dx])^2}$$

Result (type 3, 1030 leaves):

$$\frac{1}{384d} \left(-\frac{1}{b^4} \left(-48a(c+dx) + \frac{6(16a^6 - 40a^4b^2 + 30a^2b^4 - 5b^6) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} - \right. \right.$$

$$\left. \left. \frac{16b \operatorname{Cos}[c+dx]}{(a-b)(a+b)(a+b \operatorname{Sin}[c+dx])^2} + \frac{ab(-40a^4 + 72a^2b^2 - 29b^4) \operatorname{Cos}[c+dx]}{(a-b)^2(a+b)^2(a+b \operatorname{Sin}[c+dx])} \right) + \right.$$

$$\left. 12 \left(\frac{2(2a^2 + b^2) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \frac{b \operatorname{Cos}[c+dx] (4a^2 - b^2 + 3ab \operatorname{Sin}[c+dx])}{(a-b)^2(a+b)^2(a+b \operatorname{Sin}[c+dx])^2} \right) + \right.$$

$$\left. 6 \left(-\frac{6b^2 \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{\operatorname{Cos}[c+dx] (-b(2a^2+b^2) + a(2a^2-5b^2) \operatorname{Sin}[c+dx])}{(a+b \operatorname{Sin}[c+dx])^2} \right) \right) -$$

$$\frac{1}{b^6} \left(-\frac{12(640a^8 - 1792a^6b^2 + 1680a^4b^4 - 560a^2b^6 + 35b^8) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \frac{1}{(a^2-b^2)^2(a+b \operatorname{Sin}[c+dx])^2} \right.$$

$$\left. (3840a^9(c+dx) - 6912a^7b^2(c+dx) + 1728a^5b^4(c+dx) + 1920a^3b^6(c+dx) - 576ab^8(c+dx) + 3840a^8b \operatorname{Cos}[c+dx] - \right.$$

$$\left. 7872a^6b^3 \operatorname{Cos}[c+dx] + 4256a^4b^5 \operatorname{Cos}[c+dx] - 172a^2b^7 \operatorname{Cos}[c+dx] - 70b^9 \operatorname{Cos}[c+dx] - 1920a^7b^2(c+dx) \operatorname{Cos}[2(c+dx)] + \right.$$

$$\left. 4416a^5b^4(c+dx) \operatorname{Cos}[2(c+dx)] - 3072a^3b^6(c+dx) \operatorname{Cos}[2(c+dx)] + 576ab^8(c+dx) \operatorname{Cos}[2(c+dx)] - \right.$$

$$\begin{aligned}
& 320 a^6 b^3 \cos[3(c+dx)] + 696 a^4 b^5 \cos[3(c+dx)] - 432 a^2 b^7 \cos[3(c+dx)] + 56 b^9 \cos[3(c+dx)] + 8 a^4 b^5 \cos[5(c+dx)] - \\
& 16 a^2 b^7 \cos[5(c+dx)] + 8 b^9 \cos[5(c+dx)] + 7680 a^8 b(c+dx) \sin[c+dx] - 17664 a^6 b^3(c+dx) \sin[c+dx] + \\
& 12288 a^4 b^5(c+dx) \sin[c+dx] - 2304 a^2 b^7(c+dx) \sin[c+dx] + 2880 a^7 b^2 \sin[2(c+dx)] - 6304 a^5 b^4 \sin[2(c+dx)] +
\end{aligned}$$

$$\begin{aligned}
& 4022 a^3 b^6 \sin[2(c+dx)] - 607 a b^8 \sin[2(c+dx)] + 40 a^5 b^4 \sin[4(c+dx)] - 80 a^3 b^6 \sin[4(c+dx)] + 40 a b^8 \sin[4(c+dx)]
\end{aligned}$$

■ **Problem 1145: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c+dx]^3 \cot[c+dx] \sqrt{a+b \sin[c+dx]} dx$$

Optimal (type 4, 338 leaves, 10 steps):

$$\begin{aligned}
& -\frac{2(8a^2 - 45b^2) \cos[c+dx] \sqrt{a+b \sin[c+dx]}}{105b^2 d} + \frac{8a \cos[c+dx] (a+b \sin[c+dx])^{3/2}}{35b^2 d} - \\
& \frac{2 \cos[c+dx] \sin[c+dx] (a+b \sin[c+dx])^{3/2}}{7bd} + \frac{2a(8a^2 - 51b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{a+b \sin[c+dx]}}{105b^3 d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} - \\
& \frac{2(8a^4 - 53a^2 b^2 - 60b^4) \operatorname{EllipticF}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{105b^3 d \sqrt{a+b \sin[c+dx]}} + \frac{2a \operatorname{EllipticPi}\left[2, \frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{d \sqrt{a+b \sin[c+dx]}}
\end{aligned}$$

Result (type 4, 435 leaves):

$$\frac{1}{210 b^2 d} \left(\frac{1}{b^2 \sqrt{-\frac{1}{a+b}}} 2 i (-8 a^2 + 51 b^2) \right. \\ \left. \left(-2 a (a-b) \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]} \right], \frac{a+b}{a-b} \right] + b \left(-2 a \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]} \right], \frac{a+b}{a-b} \right], \right. \right. \right. \\ \left. \left. \left. \frac{a+b}{a-b} \right] + b \text{EllipticPi} \left[\frac{a+b}{a}, i \text{ArcSinh} \left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]} \right], \frac{a+b}{a-b} \right] \right) \right) \text{Sec}[c+dx] \\ \frac{\sqrt{-\frac{b(-1+\sin[c+dx])}{a+b}} \sqrt{\frac{b(1+\sin[c+dx])}{-a+b}} - \frac{8 b (a^2 + 30 b^2) \text{EllipticF} \left[\frac{1}{4} (-2 c + \pi - 2 d x), \frac{2 b}{a+b} \right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \\ \frac{2 a (8 a^2 + 159 b^2) \text{EllipticPi} \left[2, \frac{1}{4} (-2 c + \pi - 2 d x), \frac{2 b}{a+b} \right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} + \\ \left. \left. \left. 2 \cos[c+dx] \sqrt{a+b \sin[c+dx]} (8 a^2 + 75 b^2 + 15 b^2 \cos[2(c+dx)] - 6 a b \sin[c+dx]) \right) \right) \right)$$

- **Problem 1146: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c+dx]^2 \cot[c+dx]^2 \sqrt{a+b \sin[c+dx]} dx$$

Optimal (type 4, 323 leaves, 10 steps):

$$\begin{aligned}
& \frac{(4a^2 + 15b^2) \cos[c + dx] \sqrt{a + b \sin[c + dx]}}{15abd} - \frac{2 \cos[c + dx] (a + b \sin[c + dx])^{3/2}}{5bd} - \\
& \frac{\cot[c + dx] (a + b \sin[c + dx])^{3/2}}{ad} - \frac{(4a^2 + 57b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{a + b \sin[c + dx]}}{15b^2 d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} + \\
& \frac{a(4a^2 + 11b^2) \operatorname{EllipticF}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{15b^2 d \sqrt{a + b \sin[c + dx]}} + \frac{b \operatorname{EllipticPi}\left[2, \frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{d \sqrt{a + b \sin[c + dx]}}
\end{aligned}$$

Result (type 4, 541 leaves):

$$\begin{aligned}
& \frac{1}{60 b d} \left(\frac{184 a b \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \right. \\
& \frac{2 \left(-4 a^2 - 27 b^2\right) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \left. \left(2 i \left(4 a^2 + 57 b^2\right) \cos [c+d x] \cos [2 (c+d x)] \right. \right. \\
& \left. \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sqrt{\frac{b-b \sin [c+d x]}{a+b}} \sqrt{-\frac{b+b \sin [c+d x]}{a-b}} \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin [c+d x]}^2 \left(-2 a^2 + b^2 + 4 a (a+b \sin [c+d x]) - 2 (a+b \sin [c+d x])^2\right) \right. \\
& \left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin [c+d x]) + (a+b \sin [c+d x])^2}{b^2}} \right) \right) + \\
& \frac{\sqrt{a+b \sin [c+d x]} \left(-\frac{2 a \cos [c+d x]}{15 b} - \cot [c+d x] - \frac{1}{5} \sin [2 (c+d x)]\right)}{d}
\end{aligned}$$

- **Problem 1147: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos [c+d x] \cot [c+d x]^3 \sqrt{a+b \sin [c+d x]} dx$$

Optimal (type 4, 345 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(8a^2 + 3b^2) \cos[c + dx] \sqrt{a + b \sin[c + dx]}}{12a^2 d} + \frac{b \cot[c + dx] (a + b \sin[c + dx])^{3/2}}{4a^2 d} - \\
& \frac{\cot[c + dx] \csc[c + dx] (a + b \sin[c + dx])^{3/2}}{2ad} + \frac{(8a^2 - 3b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{a + b \sin[c + dx]}}{12abd \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} - \\
& \frac{(8a^2 + 31b^2) \operatorname{EllipticF}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{12bd \sqrt{a + b \sin[c + dx]}} - \frac{(12a^2 + b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{4ad \sqrt{a + b \sin[c + dx]}}
\end{aligned}$$

Result (type 4, 547 leaves):

$$\begin{aligned}
& \frac{\left(-\frac{2}{3} \cos[c + dx] - \frac{b \cot[c + dx]}{4a} - \frac{1}{2} \cot[c + dx] \csc[c + dx]\right) \sqrt{a + b \sin[c + dx]}}{d} - \\
& \frac{1}{48ad} \left(\frac{136abd \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} - \frac{2(64a^2 + 9b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} - \right. \\
& \left. \left(2i(8a^2 - 3b^2) \cos[c + dx] \cos[2(c + dx)] \left(2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left(2a \operatorname{EllipticF}\left[\right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right]\right] \right) \right) \right) \\
& \left. \frac{\sqrt{\frac{b - b \sin[c + dx]}{a+b}} \sqrt{\frac{b + b \sin[c + dx]}{a-b}}}{\left(a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \sin[c + dx]^2} (-2a^2 + b^2 + 4a(a + b \sin[c + dx]) - 2(a + b \sin[c + dx])^2) \right)} \right) \\
& \left. \frac{\sqrt{-\frac{a^2 - b^2 - 2a(a + b \sin[c + dx]) + (a + b \sin[c + dx])^2}{b^2}}}{\left(\right)} \right)
\end{aligned}$$

■ **Problem 1148: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot[c + dx]^4 \sqrt{a + b \sin[c + dx]} \, dx$$

Optimal (type 4, 351 leaves, 10 steps):

$$\frac{(32 a^2 - 3 b^2) \operatorname{Cot}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{24 a^2 d} + \frac{b \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x] (a + b \operatorname{Sin}[c + d x])^{3/2}}{4 a^2 d} -$$

$$\frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2 (a + b \operatorname{Sin}[c + d x])^{3/2}}{3 a d} + \frac{(80 a^2 + 3 b^2) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{24 a^2 d \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}} -$$

$$\frac{(32 a^2 + b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{24 a d \sqrt{a + b \operatorname{Sin}[c + d x]}} - \frac{b (12 a^2 - b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{8 a^2 d \sqrt{a + b \operatorname{Sin}[c + d x]}}$$

Result (type 4, 593 leaves):

$$\begin{aligned}
& \left(\frac{(32 a^2 \cos [c+d x]+3 b^2 \cos [c+d x]) \operatorname{Csc}[c+d x]}{24 a^2} - \frac{b \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]}{12 a} - \frac{1}{3} \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^2 \right) \sqrt{a+b \sin [c+d x]} \\
& \frac{d}{96 a^2 d} \left(- \frac{2\left(96 a^3+4 a b^2\right) \operatorname{EllipticF}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \right. \\
& \frac{2\left(8 a^2 b+9 b^3\right) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \left. \left(2 i\left(-80 a^2 b-3 b^3\right) \cos [c+d x] \cos [2(c+d x)]\right) \right. \\
& \left. \left(2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right]+b\left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right]-b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \sqrt{\frac{b-b \sin [c+d x]}{a+b}} \sqrt{-\frac{b+b \sin [c+d x]}{a-b}} \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin [c+d x]^2}\left(-2 a^2+b^2+4 a(a+b \sin [c+d x])\right)-2(a+b \sin [c+d x])^2\right) \\
& \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \sin [c+d x])+(a+b \sin [c+d x])^2}{b^2}} \right)
\end{aligned}$$

■ **Problem 1149: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Cot}[c+d x]^4 \operatorname{Csc}[c+d x] \sqrt{a+b \sin [c+d x]} dx$$

Optimal (type 4, 412 leaves, 11 steps):

$$\begin{aligned}
& \frac{b (68 a^2 - 15 b^2) \operatorname{Cot}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{192 a^3 d} + \frac{5 (4 a^2 - b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{32 a^2 d} + \\
& \frac{5 b \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2 (a + b \operatorname{Sin}[c + d x])^{3/2}}{24 a^2 d} - \frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^3 (a + b \operatorname{Sin}[c + d x])^{3/2}}{4 a d} + \\
& \frac{b (68 a^2 - 15 b^2) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{192 a^3 d \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}} + \frac{b (196 a^2 + 5 b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{192 a^2 d \sqrt{a + b \operatorname{Sin}[c + d x]}} + \\
& \frac{(48 a^4 + 24 a^2 b^2 - 5 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{64 a^3 d \sqrt{a + b \operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 643 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(\frac{(68 a^2 b \cos[c+dx] - 15 b^3 \cos[c+dx]) \operatorname{Csc}[c+dx]}{192 a^3} + \right. \\
& \quad \left. \frac{5 (12 a^2 \cos[c+dx] + b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{96 a^2} - \frac{b \cot[c+dx] \operatorname{Csc}[c+dx]^2}{24 a} - \frac{1}{4} \cot[c+dx] \operatorname{Csc}[c+dx]^3 \right) \sqrt{a+b \sin[c+dx]} + \\
& \frac{1}{768 a^3 d} \left(- \frac{2 (528 a^3 b - 20 a b^3) \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \quad \left. \frac{2 (288 a^4 + 212 a^2 b^2 - 45 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \quad \left(2 i (-68 a^2 b^2 + 15 b^4) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \\
& \quad \left. \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \right. \right. \right. \\
& \quad \left. \left. \left. \frac{a+b}{a-b}\right] \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right. \\
& \quad \left. \left. \left. (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right) \right)
\end{aligned}$$

- **Problem 1150: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot[c+dx]^4 \operatorname{Csc}[c+dx]^2 \sqrt{a+b \sin[c+dx]} dx$$

Optimal (type 4, 484 leaves, 12 steps):

$$\begin{aligned}
& - \frac{(384 a^4 + 332 a^2 b^2 - 105 b^4) \operatorname{Cot}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{1920 a^4 d} + \frac{b (108 a^2 - 35 b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{960 a^3 d} + \\
& \frac{(96 a^2 - 35 b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2 \sqrt{a + b \operatorname{Sin}[c + d x]}}{240 a^2 d} + \frac{7 b \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^3 (a + b \operatorname{Sin}[c + d x])^{3/2}}{40 a^2 d} - \\
& \frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^4 (a + b \operatorname{Sin}[c + d x])^{3/2}}{5 a d} - \frac{(384 a^4 + 332 a^2 b^2 - 105 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{1920 a^4 d \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}} + \\
& \frac{(384 a^4 + 116 a^2 b^2 - 35 b^4) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{1920 a^3 d \sqrt{a + b \operatorname{Sin}[c + d x]}} + \\
& \frac{b (48 a^4 - 24 a^2 b^2 + 7 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{128 a^4 d \sqrt{a + b \operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 702 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(\frac{(-384 a^4 \cos[c+dx] - 332 a^2 b^2 \cos[c+dx] + 105 b^4 \cos[c+dx]) \operatorname{Csc}[c+dx]}{1920 a^4} + \frac{(108 a^2 b \cos[c+dx] - 35 b^3 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{960 a^3} \right. \\
& \quad \left. + \frac{(96 a^2 \cos[c+dx] + 7 b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^3}{240 a^2} - \frac{b \cot[c+dx] \operatorname{Csc}[c+dx]^3}{40 a} - \frac{1}{5} \cot[c+dx] \operatorname{Csc}[c+dx]^4 \right) \sqrt{a+b \sin[c+dx]} + \\
& \quad \frac{1}{7680 a^4 d} b \left(- \frac{2(-432 a^3 b + 140 a b^3) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \quad \left. \frac{2(1056 a^4 - 1052 a^2 b^2 + 315 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \quad \left(2 i (384 a^4 + 332 a^2 b^2 - 105 b^4) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \\
& \quad \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \right. \right. \\
& \quad \left. \left. \frac{a+b}{a-b}\right] \right) \left) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right. \\
& \quad \left. \left. (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right)
\end{aligned}$$

■ **Problem 1151: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 \sin[c+dx]^2 (a+b \sin[c+dx])^{3/2} dx$$

Optimal (type 4, 528 leaves, 11 steps):

$$\begin{aligned}
& \frac{8 (64 a^6 - 174 a^4 b^2 + 81 a^2 b^4 - 195 b^6) \operatorname{Cos}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{45 045 b^5 d} + \\
& \frac{16 a (32 a^4 - 47 a^2 b^2 - 27 b^4) \operatorname{Cos}[c + d x] (a + b \operatorname{Sin}[c + d x])^{3/2}}{45 045 b^5 d} - \frac{8 (160 a^4 - 375 a^2 b^2 + 117 b^4) \operatorname{Cos}[c + d x] (a + b \operatorname{Sin}[c + d x])^{5/2}}{45 045 b^5 d} + \\
& \frac{8 a (8 a^2 - 21 b^2) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x] (a + b \operatorname{Sin}[c + d x])^{5/2}}{1287 b^4 d} - \frac{2 (80 a^2 - 221 b^2) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]^2 (a + b \operatorname{Sin}[c + d x])^{5/2}}{2145 b^3 d} + \\
& \frac{4 a \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]^3 (a + b \operatorname{Sin}[c + d x])^{5/2}}{39 b^2 d} - \frac{2 \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]^4 (a + b \operatorname{Sin}[c + d x])^{5/2}}{15 b d} - \\
& \frac{16 a (32 a^6 - 111 a^4 b^2 + 102 a^2 b^4 - 471 b^6) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{45 045 b^6 d \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}}} + \\
& \frac{8 (64 a^8 - 238 a^6 b^2 + 255 a^4 b^4 - 276 a^2 b^6 + 195 b^8) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}}}{45 045 b^6 d \sqrt{a + b \operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 1987 leaves):

$$\begin{aligned}
& \frac{a \operatorname{EllipticE}\left[\frac{1}{4} (-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{8 d \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}}} - \frac{1}{24 d \sqrt{a + b \operatorname{Sin}[c + d x]}} \\
& \left(b \operatorname{Cos}[c + d x] (a + b \operatorname{Sin}[c + d x]) + a (a + b) \operatorname{EllipticE}\left[\frac{1}{4} (-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} - \right. \\
& \left. (a^2 - b^2) \operatorname{EllipticF}\left[\frac{1}{4} (-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} \right) + \\
& \left(a \left(-2 (4 a^3 + 4 a^2 b - 3 a b^2 - 3 b^3) \operatorname{EllipticE}\left[\frac{1}{4} (-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} + 8 a (a^2 - b^2) \operatorname{EllipticF}\left[\frac{1}{4} (-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \right. \right. \\
& \left. \left. \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} + 2 b \operatorname{Cos}[c + d x] (2 a^2 + 3 b^2 - 3 b^2 \operatorname{Cos}[2 (c + d x)] + 8 a b \operatorname{Sin}[c + d x]) \right) \right) / \left(480 b^2 d \sqrt{a + b \operatorname{Sin}[c + d x]} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(2 a (16 a^3 + 16 a^2 b + 3 a b^2 + 3 b^3) \operatorname{EllipticE}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} - \right. \\
& 2 (16 a^4 - a^2 b^2 - 15 b^4) \operatorname{EllipticF}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} + b \operatorname{Cos}[c + d x] \\
& \left. (-16 a^3 + 66 a b^2 - 36 a b^2 \operatorname{Cos}[2(c + d x)] + (-4 a^2 b + 75 b^3) \operatorname{Sin}[c + d x] - 15 b^3 \operatorname{Sin}[3(c + d x)]) \right) / (3360 b^2 d \sqrt{a + b \operatorname{Sin}[c + d x]}) + \\
& \frac{1}{2520 b^4 d \sqrt{a + b \operatorname{Sin}[c + d x]}} a \left(b (-32 a^3 b + 27 a b^3) \operatorname{EllipticF}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} - \right. \\
& (128 a^4 - 144 a^2 b^2 + 21 b^4) \left((a + b) \operatorname{EllipticE}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] - a \operatorname{EllipticF}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \right) \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} - \\
& \left. 2 b \operatorname{Cos}[c + d x] (a + b \operatorname{Sin}[c + d x]) (-32 a^3 + 22 a b^2 + 10 a b^2 \operatorname{Cos}[2(c + d x)] + b (24 a^2 - 49 b^2) \operatorname{Sin}[c + d x] + 35 b^3 \operatorname{Sin}[3(c + d x)]) \right) - \\
& \frac{1}{55440 b^4 d \sqrt{a + b \operatorname{Sin}[c + d x]}} \left(-2 a (1024 a^5 + 1024 a^4 b - 864 a^3 b^2 - 864 a^2 b^3 - 93 a b^4 - 93 b^5) \operatorname{EllipticE}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \right. \\
& \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} + 2 (1024 a^6 - 1120 a^4 b^2 + 51 a^2 b^4 + 45 b^6) \operatorname{EllipticF}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} - \\
& b \operatorname{Cos}[c + d x] (-1024 a^5 + 800 a^3 b^2 + 1034 a b^4 - 32 (2 a^3 b^2 + 57 a b^4) \operatorname{Cos}[2(c + d x)] + 700 a b^4 \operatorname{Cos}[4(c + d x)] - 256 a^4 b \operatorname{Sin}[c + d x] + \\
& \left. 164 a^2 b^3 \operatorname{Sin}[c + d x] + 1980 b^5 \operatorname{Sin}[c + d x] + 20 a^2 b^3 \operatorname{Sin}[3(c + d x)] - 1215 b^5 \operatorname{Sin}[3(c + d x)] + 315 b^5 \operatorname{Sin}[5(c + d x)]) \right) - \\
& \frac{1}{720720 b^6 d \sqrt{a + b \operatorname{Sin}[c + d x]}} a \left(-b (10240 a^5 b - 13504 a^3 b^3 + 3579 a b^5) \operatorname{EllipticF}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} - \right. \\
& (40960 a^6 - 65536 a^4 b^2 + 26508 a^2 b^4 - 1617 b^6) \\
& \left. \left((a + b) \operatorname{EllipticE}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] - a \operatorname{EllipticF}\left[\frac{1}{4}(-2 c + \pi - 2 d x), \frac{2 b}{a + b}\right] \right) \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}} + \right.
\end{aligned}$$

$$\begin{aligned}
& b (a + b \sin[c + dx]) \left(2a (10240a^4 - 13504a^2b^2 + 3579b^4) \cos[c + dx] - 10ab^2 (320a^2 - 257b^2) \cos[3(c + dx)] + 630ab^4 \cos[5(c + dx)] - \right. \\
& \left. 2b (3840a^4 - 4064a^2b^2 + 539b^4) \sin[2(c + dx)] + 70b^3 (20a^2 - 11b^2) \sin[4(c + dx)] + 3465b^5 \sin[6(c + dx)] \right) - \\
& \frac{1}{1441440b^6d\sqrt{a+b\sin[c+dx]}} \left(2a (32768a^7 + 32768a^6b - 47104a^5b^2 - 47104a^4b^3 + 13968a^3b^4 + 13968a^2b^5 + 711ab^6 + 711b^7) \right. \\
& \text{EllipticE} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}} - \\
& 2 (32768a^8 - 55296a^6b^2 + 23440a^4b^4 - 717a^2b^6 - 195b^8) \text{EllipticF} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}} + \\
& b \cos[c + dx] (-32768a^7 + 45056a^5b^2 - 12208a^3b^4 + 7018ab^6 - 16(128a^5b^2 - 124a^3b^4 + 969ab^6) \cos[2(c + dx)] + \\
& 112(2a^3b^4 + 137ab^6) \cos[4(c + dx)] - 6468ab^6 \cos[6(c + dx)] - 8192a^6b \sin[c + dx] + 10112a^4b^3 \sin[c + dx] - \\
& 1796a^2b^5 \sin[c + dx] + 15444b^7 \sin[c + dx] + 640a^4b^3 \sin[3(c + dx)] - 452a^2b^5 \sin[3(c + dx)] - \\
& \left. 15756b^7 \sin[3(c + dx)] - 84a^2b^5 \sin[5(c + dx)] + 10647b^7 \sin[5(c + dx)] - 3003b^7 \sin[7(c + dx)] \right)
\end{aligned}$$

■ **Problem 1153: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c + dx]^3 \cot[c + dx] (a + b \sin[c + dx])^{3/2} dx$$

Optimal (type 4, 390 leaves, 11 steps):

$$\begin{aligned}
& - \frac{2a(8a^2 - 87b^2) \cos[c + dx] \sqrt{a + b \sin[c + dx]}}{315b^2d} - \frac{2(8a^2 - 77b^2) \cos[c + dx] (a + b \sin[c + dx])^{3/2}}{315b^2d} + \frac{8a \cos[c + dx] (a + b \sin[c + dx])^{5/2}}{63b^2d} - \\
& \frac{2 \cos[c + dx] \sin[c + dx] (a + b \sin[c + dx])^{5/2}}{9bd} + \frac{2(8a^4 - 93a^2b^2 + 84b^4) \text{EllipticE} \left[\frac{1}{2} (c - \frac{\pi}{2} + dx), \frac{2b}{a+b} \right] \sqrt{a + b \sin[c + dx]}}{315b^3d \sqrt{\frac{a+b\sin[c+dx]}{a+b}}} - \\
& \frac{2a(8a^4 - 95a^2b^2 - 228b^4) \text{EllipticF} \left[\frac{1}{2} (c - \frac{\pi}{2} + dx), \frac{2b}{a+b} \right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{315b^3d \sqrt{a + b \sin[c + dx]}} + \frac{2a^2 \text{EllipticPi} \left[2, \frac{1}{2} (c - \frac{\pi}{2} + dx), \frac{2b}{a+b} \right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{d \sqrt{a + b \sin[c + dx]}}
\end{aligned}$$

Result (type 4, 611 leaves):

$$\begin{aligned}
& -\frac{1}{630 b^2 d} \left(\frac{2(-4 a^3 b - 624 a b^3) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \frac{2(-8 a^4 - 537 a^2 b^2 - 84 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \\
& \left(2 i (8 a^4 - 93 a^2 b^2 + 84 b^4) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \\
& \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - \right. \right. \\
& \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \Bigg) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \sin^2[c+dx]} (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \right. \\
& \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) + \frac{1}{d} \\
& \sqrt{a+b \sin[c+dx]} \left(\frac{a (8 a^2 + 303 b^2) \cos[c+dx]}{315 b^2} + \frac{5}{63} a \cos[3(c+dx)] + \frac{(-6 a^2 + 119 b^2) \sin[2(c+dx)]}{630 b} + \right. \\
& \left. \frac{1}{36} b \sin[4(c+dx)] \right)
\end{aligned}$$

■ **Problem 1154: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c+dx]^2 \cot[c+dx]^2 (a+b \sin[c+dx])^{3/2} dx$$

Optimal (type 4, 374 leaves, 11 steps):

$$\begin{aligned}
& \frac{(4a^2 + 65b^2) \cos[c + dx] \sqrt{a + b \sin[c + dx]}}{35bd} + \frac{(4a^2 + 35b^2) \cos[c + dx] (a + b \sin[c + dx])^{3/2}}{35abd} - \frac{2 \cos[c + dx] (a + b \sin[c + dx])^{5/2}}{7bd} - \\
& \frac{\cot[c + dx] (a + b \sin[c + dx])^{5/2}}{ad} - \frac{a(4a^2 + 167b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{a + b \sin[c + dx]}}{35b^2 d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} + \\
& \frac{(4a^4 + 61a^2b^2 + 40b^4) \operatorname{EllipticF}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{35b^2 d \sqrt{a + b \sin[c + dx]}} + \frac{3ab \operatorname{EllipticPi}\left[2, \frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{d \sqrt{a + b \sin[c + dx]}}
\end{aligned}$$

Result (type 4, 578 leaves):

$$\begin{aligned}
& \frac{1}{140 b d} \left(\frac{2 (-212 a^2 b + 80 b^3) \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{\sqrt{a+b \operatorname{Sin}[c+d x]}} - \right. \\
& \frac{2 (-4 a^3 + 43 a b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{\sqrt{a+b \operatorname{Sin}[c+d x]}} - \\
& \left. \left(2 i (4 a^3 + 167 a b^2) \operatorname{Cos}[c+d x] \operatorname{Cos}[2(c+d x)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[c+d x]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
& \left. \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[c+d x]}\right], \frac{a+b}{a-b}\right] - \right. \right. \right. \\
& \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sqrt{\frac{b-b \operatorname{Sin}[c+d x]}{a+b}} \sqrt{-\frac{b+b \operatorname{Sin}[c+d x]}{a-b}} \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\operatorname{Sin}[c+d x]^2} (-2 a^2 + b^2 + 4 a (a+b \operatorname{Sin}[c+d x]) - 2 (a+b \operatorname{Sin}[c+d x])^2) \right. \\
& \left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \operatorname{Sin}[c+d x]) + (a+b \operatorname{Sin}[c+d x])^2}{b^2}} \right) \right) + \\
& \frac{\sqrt{a+b \operatorname{Sin}[c+d x]} \left(\frac{(-4 a^2 + 55 b^2) \operatorname{Cos}[c+d x]}{70 b} + \frac{1}{14} b \operatorname{Cos}[3(c+d x)] - a \operatorname{Cot}[c+d x] - \frac{8}{35} a \operatorname{Sin}[2(c+d x)] \right)}{d}
\end{aligned}$$

- **Problem 1155: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Cos}[c+d x] \operatorname{Cot}[c+d x]^3 (a+b \operatorname{Sin}[c+d x])^{3/2} dx$$

Optimal (type 4, 383 leaves, 11 steps):

$$\begin{aligned}
& - \frac{(8a^2 - 15b^2) \cos[c+dx] \sqrt{a+b \sin[c+dx]}}{20ad} - \frac{(8a^2 - 5b^2) \cos[c+dx] (a+b \sin[c+dx])^{3/2}}{20a^2d} - \frac{b \cot[c+dx] (a+b \sin[c+dx])^{5/2}}{4a^2d} \\
& \frac{\cot[c+dx] \csc[c+dx] (a+b \sin[c+dx])^{5/2}}{2ad} + \frac{(8a^2 - 81b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{a+b \sin[c+dx]}}{20bd \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} \\
& \frac{a(8a^2 + 37b^2) \operatorname{EllipticF}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{20bd \sqrt{a+b \sin[c+dx]}} - \frac{3(4a^2 - b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{4d \sqrt{a+b \sin[c+dx]}}
\end{aligned}$$

Result (type 4, 434 leaves):

$$\begin{aligned}
& \frac{1}{80d} \left(\frac{1}{ab^2 \sqrt{-\frac{1}{a+b}}} 2i(-8a^2 + 81b^2) \right. \\
& \left(-2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left(-2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] + b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \operatorname{Sec}[c+dx] \\
& \frac{\sqrt{-\frac{b(-1 + \sin[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \sin[c+dx])}{a-b}} + \frac{472ab \operatorname{EllipticF}\left[\frac{1}{4}(-2c + \pi - 2dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}}}{2(112a^2 + 51b^2) \operatorname{EllipticPi}\left[2, \frac{1}{4}(-2c + \pi - 2dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} + \\
& \left. \frac{4 \cot[c+dx] \csc[c+dx] \sqrt{a+b \sin[c+dx]} (-18a + 8a \cos[2(c+dx)] - 31b \sin[c+dx] + 2b \sin[3(c+dx)])}{\sqrt{a+b \sin[c+dx]}} \right)
\end{aligned}$$

■ **Problem 1156: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot [c + d x]^4 (a + b \sin [c + d x])^{3/2} dx$$

Optimal (type 4, 386 leaves, 11 steps):

$$\begin{aligned} & - \frac{b (16 a^2 + b^2) \cos [c + d x] \sqrt{a + b \sin [c + d x]}}{8 a^2 d} + \frac{(32 a^2 + b^2) \cot [c + d x] (a + b \sin [c + d x])^{3/2}}{24 a^2 d} + \frac{b \cot [c + d x] \csc [c + d x] (a + b \sin [c + d x])^{5/2}}{12 a^2 d} \\ & - \frac{\cot [c + d x] \csc [c + d x]^2 (a + b \sin [c + d x])^{5/2}}{3 a d} + \frac{(32 a^2 - b^2) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{a + b \sin [c + d x]}}{8 a d \sqrt{\frac{a + b \sin [c + d x]}{a + b}}} \\ & - \frac{(16 a^2 + 21 b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \sin [c + d x]}{a + b}}}{8 d \sqrt{a + b \sin [c + d x]}} - \frac{b (36 a^2 + b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \sin [c + d x]}{a + b}}}{8 a d \sqrt{a + b \sin [c + d x]}} \end{aligned}$$

Result (type 4, 600 leaves):

$$\frac{1}{d} \left(-\frac{2}{3} b \cos[c+dx] + \frac{(32a^2 \cos[c+dx] - 3b^2 \cos[c+dx]) \csc[c+dx]}{24a} - \frac{7}{12} b \cot[c+dx] \csc[c+dx] - \frac{1}{3} a \cot[c+dx] \csc[c+dx]^2 \right)$$

$$\sqrt{a+b \sin[c+dx]} + \frac{1}{32ad}$$

$$\left(-\frac{2(32a^3 - 44ab^2) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \frac{2(-40a^2b - 3b^3) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} \right)$$

$$\left(2i(-32a^2b + b^3) \cos[c+dx] \cos[2(c+dx)] \left(2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left(2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \right)$$

$$\sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \left/ \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} (-2a^2 + b^2 + 4a(a+b \sin[c+dx]) - 2(a+b \sin[c+dx])^2) \right) \right.$$

$$\left. \sqrt{-\frac{a^2 - b^2 - 2a(a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right)$$

■ **Problem 1157: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot[c+dx]^4 \csc[c+dx] (a+b \sin[c+dx])^{3/2} dx$$

Optimal (type 4, 408 leaves, 11 steps):

$$\begin{aligned}
& \frac{b(68a^2 - 3b^2) \cot[c+dx] \sqrt{a+b \sin[c+dx]}}{64a^2 d} + \frac{(20a^2 - b^2) \cot[c+dx] \csc[c+dx] (a+b \sin[c+dx])^{3/2}}{32a^2 d} + \\
& \frac{b \cot[c+dx] \csc[c+dx]^2 (a+b \sin[c+dx])^{5/2}}{8a^2 d} - \frac{\cot[c+dx] \csc[c+dx]^3 (a+b \sin[c+dx])^{5/2}}{4ad} + \\
& \frac{b(236a^2 + 3b^2) \operatorname{EllipticE}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{a+b \sin[c+dx]}}{64a^2 d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} - \frac{b(20a^2 + b^2) \operatorname{EllipticF}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{64ad \sqrt{a+b \sin[c+dx]}} + \\
& \frac{3(16a^4 - 24a^2 b^2 + b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{64a^2 d \sqrt{a+b \sin[c+dx]}}
\end{aligned}$$

Result (type 4, 641 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(\frac{3 (36 a^2 b \cos[c+dx] + b^3 \cos[c+dx]) \operatorname{Csc}[c+dx]}{64 a^2} + \right. \\
& \quad \left. \frac{(20 a^2 \cos[c+dx] - b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{32 a} - \frac{3}{8} b \cot[c+dx] \operatorname{Csc}[c+dx]^2 - \frac{1}{4} a \cot[c+dx] \operatorname{Csc}[c+dx]^3 \right) \sqrt{a+b \sin[c+dx]} + \\
& \frac{1}{256 a^2 d} \left(- \frac{2 (432 a^3 b + 4 a b^3) \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \quad \left. \frac{2 (96 a^4 + 92 a^2 b^2 + 9 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \quad \left(2 i (-236 a^2 b^2 - 3 b^4) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \\
& \quad \left. \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \right. \right. \right. \\
& \quad \left. \left. \left. \frac{a+b}{a-b}\right] \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right. \\
& \quad \left. \left. \left. (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right) \right)
\end{aligned}$$

■ **Problem 1158: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot[c+dx]^4 \operatorname{Csc}[c+dx]^2 (a+b \sin[c+dx])^{3/2} dx$$

Optimal (type 4, 484 leaves, 12 steps):

$$\begin{aligned}
& - \frac{(128 a^4 - 116 a^2 b^2 + 15 b^4) \operatorname{Cot}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{640 a^3 d} + \frac{3 b (36 a^2 - 5 b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{320 a^2 d} + \\
& \frac{(32 a^2 - 5 b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2 (a + b \operatorname{Sin}[c + d x])^{3/2}}{80 a^2 d} + \frac{b \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^3 (a + b \operatorname{Sin}[c + d x])^{5/2}}{8 a^2 d} - \\
& \frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^4 (a + b \operatorname{Sin}[c + d x])^{5/2}}{5 a d} - \frac{(128 a^4 - 116 a^2 b^2 + 15 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{640 a^3 d \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}} + \\
& \frac{(128 a^4 + 692 a^2 b^2 + 5 b^4) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{640 a^2 d \sqrt{a + b \operatorname{Sin}[c + d x]}} + \\
& \frac{3 b (48 a^4 + 8 a^2 b^2 - b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{128 a^3 d \sqrt{a + b \operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 700 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(\frac{(-128 a^4 \cos[c+dx] + 116 a^2 b^2 \cos[c+dx] - 15 b^4 \cos[c+dx]) \operatorname{Csc}[c+dx]}{640 a^3} + \frac{(236 a^2 b \cos[c+dx] + 5 b^3 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{320 a^2} + \right. \\
& \left. \frac{(32 a^2 \cos[c+dx] - b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^3}{80 a} - \frac{11}{40} b \cot[c+dx] \operatorname{Csc}[c+dx]^3 - \frac{1}{5} a \cot[c+dx] \operatorname{Csc}[c+dx]^4 \right) \sqrt{a+b \sin[c+dx]} + \\
& \frac{1}{2560 a^3 d} b \left(- \frac{2 (1616 a^3 b - 20 a b^3) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \frac{2 (1312 a^4 + 356 a^2 b^2 - 45 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \left(2 i (128 a^4 - 116 a^2 b^2 + 15 b^4) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
& \left. \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right]\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b} \right) \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right. \\
& \left. \left. (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right)
\end{aligned}$$

■ **Problem 1159: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot[c+dx]^4 \operatorname{Csc}[c+dx]^3 (a+b \sin[c+dx])^{3/2} dx$$

Optimal (type 4, 551 leaves, 13 steps):

$$\begin{aligned}
& - \frac{b (2064 a^4 + 512 a^2 b^2 - 105 b^4) \operatorname{Cot}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{7680 a^4 d} - \frac{(240 a^4 - 168 a^2 b^2 + 35 b^4) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{3840 a^3 d} + \\
& \frac{b (156 a^2 - 35 b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2 \sqrt{a + b \operatorname{Sin}[c + d x]}}{960 a^2 d} + \frac{7 (4 a^2 - b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^3 (a + b \operatorname{Sin}[c + d x])^{3/2}}{96 a^2 d} + \\
& \frac{7 b \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^4 (a + b \operatorname{Sin}[c + d x])^{5/2}}{60 a^2 d} - \frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^5 (a + b \operatorname{Sin}[c + d x])^{5/2}}{6 a d} - \\
& \frac{b (2064 a^4 + 512 a^2 b^2 - 105 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{7680 a^4 d \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}} + \\
& \frac{b (2544 a^4 + 176 a^2 b^2 - 35 b^4) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{7680 a^3 d \sqrt{a + b \operatorname{Sin}[c + d x]}} + \\
& \frac{(64 a^6 + 144 a^4 b^2 - 36 a^2 b^4 + 7 b^6) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{512 a^4 d \sqrt{a + b \operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 771 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(\frac{(-2064 a^4 b \cos[c+dx] - 512 a^2 b^3 \cos[c+dx] + 105 b^5 \cos[c+dx]) \operatorname{Csc}[c+dx]}{7680 a^4} + \right. \\
& \frac{(-240 a^4 \cos[c+dx] + 168 a^2 b^2 \cos[c+dx] - 35 b^4 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{3840 a^3} + \frac{(436 a^2 b \cos[c+dx] + 7 b^3 \cos[c+dx]) \operatorname{Csc}[c+dx]^3}{960 a^2} + \\
& \left. \frac{(140 a^2 \cos[c+dx] - 3 b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^4}{480 a} - \frac{13}{60} b \cot[c+dx] \operatorname{Csc}[c+dx]^4 - \frac{1}{6} a \cot[c+dx] \operatorname{Csc}[c+dx]^5 \right) \sqrt{a+b \sin[c+dx]} + \\
& \frac{1}{30720 a^4 d} \left(- \frac{2 (960 a^5 b - 672 a^3 b^3 + 140 a b^5) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \frac{2 (1920 a^6 + 2256 a^4 b^2 - 1592 a^2 b^4 + 315 b^6) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \left(2 i (2064 a^4 b^2 + 512 a^2 b^4 - 105 b^6) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
& \left. \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] \right) \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right. \\
& \left. \left. \left. (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right) \right)
\end{aligned}$$

■ **Problem 1161: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c+dx]^3 \cot[c+dx] (a+b \sin[c+dx])^{5/2} dx$$

Optimal (type 4, 447 leaves, 12 steps):

$$\begin{aligned}
& - \frac{2 (8 a^4 - 141 a^2 b^2 + 36 b^4) \operatorname{Cos}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{693 b^2 d} - \frac{2 a (8 a^2 - 131 b^2) \operatorname{Cos}[c + d x] (a + b \operatorname{Sin}[c + d x])^{3/2}}{693 b^2 d} \\
& - \frac{2 (8 a^2 - 117 b^2) \operatorname{Cos}[c + d x] (a + b \operatorname{Sin}[c + d x])^{5/2}}{693 b^2 d} + \frac{8 a \operatorname{Cos}[c + d x] (a + b \operatorname{Sin}[c + d x])^{7/2}}{99 b^2 d} - \\
& \frac{2 \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x] (a + b \operatorname{Sin}[c + d x])^{7/2}}{11 b d} + \frac{2 a (8 a^4 - 147 a^2 b^2 + 444 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{693 b^3 d \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}}} - \\
& \frac{2 (8 a^6 - 149 a^4 b^2 - 516 a^2 b^4 - 36 b^6) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}}}{693 b^3 d \sqrt{a + b \operatorname{Sin}[c + d x]}} + \frac{2 a^3 \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}}}{d \sqrt{a + b \operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 654 leaves):

$$\begin{aligned}
& -\frac{1}{1386 b^2 d} \left(-\frac{2(-4 a^4 b - 1920 a^2 b^3 - 72 b^5) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} \right. \\
& \frac{2(-8 a^5 - 1239 a^3 b^2 - 444 a b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} \\
& \left. \left(2 i (8 a^5 - 147 a^3 b^2 + 444 a b^4) \cos[c+dx] \cos[2(c+dx)] \right. \right. \\
& \left. \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \Bigg) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \right. \\
& \left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right) + \frac{1}{d} \\
& \sqrt{a+b \sin[c+dx]} \left(-\frac{(-32 a^4 - 2886 a^2 b^2 + 117 b^4) \cos[c+dx]}{2772 b^2} + \frac{(452 a^2 - 279 b^2) \cos[3(c+dx)]}{5544} \right. \\
& \frac{1}{88} b^2 \cos[5(c+dx)] - \frac{a(6 a^2 - 569 b^2) \sin[2(c+dx)]}{1386 b} + \\
& \left. \frac{23}{396} a b \sin[4(c+dx)] \right)
\end{aligned}$$

■ **Problem 1162: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c+dx]^2 \cot[c+dx]^2 (a+b \sin[c+dx])^{5/2} dx$$

Optimal (type 4, 426 leaves, 12 steps):

$$\begin{aligned}
 & \frac{a (20 a^2 + 759 b^2) \cos [c + d x] \sqrt{a + b \sin [c + d x]}}{315 b d} + \frac{(20 a^2 + 469 b^2) \cos [c + d x] (a + b \sin [c + d x])^{3/2}}{315 b d} + \\
 & \frac{(4 a^2 + 63 b^2) \cos [c + d x] (a + b \sin [c + d x])^{5/2}}{63 a b d} - \frac{2 \cos [c + d x] (a + b \sin [c + d x])^{7/2}}{9 b d} - \\
 & \frac{\cot [c + d x] (a + b \sin [c + d x])^{7/2}}{a d} - \frac{(20 a^4 + 1689 a^2 b^2 - 168 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{a + b \sin [c + d x]}}{315 b^2 d \sqrt{\frac{a + b \sin [c + d x]}{a + b}}} + \\
 & \frac{a (20 a^4 + 739 a^2 b^2 + 816 b^4) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \sin [c + d x]}{a + b}}}{315 b^2 d \sqrt{a + b \sin [c + d x]}} + \frac{5 a^2 b \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \sin [c + d x]}{a + b}}}{d \sqrt{a + b \sin [c + d x]}}
 \end{aligned}$$

Result (type 4, 622 leaves):

$$\begin{aligned}
& \frac{1}{1260 b d} \left(- \frac{2 (-1900 a^3 b + 1968 a b^3) \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \right. \\
& \frac{2 \left(-20 a^4 + 1461 a^2 b^2 + 168 b^4\right) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \\
& \left. \left(2 i \left(20 a^4 + 1689 a^2 b^2 - 168 b^4\right) \cos [c+d x] \cos [2 (c+d x)] \right. \right. \\
& \left. \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sqrt{\frac{b-b \sin [c+d x]}{a+b}} \sqrt{-\frac{b+b \sin [c+d x]}{a-b}} \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin [c+d x]^2} \left(-2 a^2 + b^2 + 4 a (a+b \sin [c+d x]) - 2 (a+b \sin [c+d x])^2\right) \right. \\
& \left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin [c+d x]) + (a+b \sin [c+d x])^2}{b^2}} \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \sin [c+d x]} \left(-\frac{a \left(20 a^2 - 1101 b^2\right) \cos [c+d x]}{630 b} + \frac{19}{126} a b \cos [3 (c+d x)] - a^2 \cot [c+d x] - \right. \\
& \left. \frac{1}{630} \left(150 a^2 - 119 b^2\right) \sin [2 (c+d x)] + \frac{1}{36} b^2 \sin [4 (c+d x)] \right)
\end{aligned}$$

■ **Problem 1163: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos [c+d x] \cot [c+d x]^3 (a+b \sin [c+d x])^{5/2} d x$$

Optimal (type 4, 430 leaves, 12 steps):

$$\begin{aligned}
& - \frac{(8a^2 - 73b^2) \cos[c+dx] \sqrt{a+b \sin[c+dx]}}{28d} - \frac{(8a^2 - 35b^2) \cos[c+dx] (a+b \sin[c+dx])^{3/2}}{28ad} \\
& - \frac{(8a^2 - 21b^2) \cos[c+dx] (a+b \sin[c+dx])^{5/2}}{28a^2d} - \frac{3b \cot[c+dx] (a+b \sin[c+dx])^{7/2}}{4a^2d} \\
& - \frac{\cot[c+dx] \csc[c+dx] (a+b \sin[c+dx])^{7/2}}{2ad} + \frac{a(8a^2 - 247b^2) \operatorname{EllipticE}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{a+b \sin[c+dx]}}{28bd \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} \\
& - \frac{(8a^4 + 3a^2b^2 - 32b^4) \operatorname{EllipticF}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{28bd \sqrt{a+b \sin[c+dx]}} - \frac{3a(4a^2 - 5b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{4d \sqrt{a+b \sin[c+dx]}}
\end{aligned}$$

Result (type 4, 597 leaves):

$$\begin{aligned}
& \frac{1}{112 d} \\
& \left(\frac{2 (500 a^2 b - 64 b^3) \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} + \frac{2 (160 a^3 + 37 a b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} + \right. \\
& \left. \left(2 i (8 a^3 - 247 a b^2) \cos [c+d x] \cos [2 (c+d x)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[\right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right]\right] \right) \right) \\
& \left. \sqrt{\frac{b-b \sin [c+d x]}{a+b}} \sqrt{-\frac{b+b \sin [c+d x]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin [c+d x]^2} (-2 a^2 + b^2 + 4 a (a+b \sin [c+d x]) - 2 (a+b \sin [c+d x])^2) \right. \\
& \left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin [c+d x]) + (a+b \sin [c+d x])^2}{b^2}} \right) \right) + \\
& \frac{1}{d} \sqrt{a+b \sin [c+d x]} \left(-\frac{1}{14} (12 a^2 - 11 b^2) \cos [c+d x] + \frac{1}{14} b^2 \cos [3 (c+d x)] - \frac{9}{4} a b \cot [c+d x] - \right. \\
& \left. \frac{1}{2} a^2 \cot [c+d x] \operatorname{Csc}[c+d x] - \frac{3}{7} a b \sin [2 (c+d x)] \right)
\end{aligned}$$

■ **Problem 1164: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot [c+d x]^4 (a+b \sin [c+d x])^{5/2} dx$$

Optimal (type 4, 429 leaves, 12 steps):

$$\begin{aligned}
& - \frac{b (96 a^2 - 25 b^2) \cos [c + d x] \sqrt{a + b \sin [c + d x]}}{40 a d} - \frac{b (208 a^2 - 25 b^2) \cos [c + d x] (a + b \sin [c + d x])^{3/2}}{120 a^2 d} + \\
& \frac{(32 a^2 - 3 b^2) \cot [c + d x] (a + b \sin [c + d x])^{5/2}}{24 a^2 d} - \frac{b \cot [c + d x] \csc [c + d x] (a + b \sin [c + d x])^{7/2}}{12 a^2 d} - \\
& \frac{\cot [c + d x] \csc [c + d x]^2 (a + b \sin [c + d x])^{7/2}}{3 a d} + \frac{(176 a^2 - 167 b^2) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{a + b \sin [c + d x]}}{40 d \sqrt{\frac{a + b \sin [c + d x]}{a + b}}} - \\
& \frac{a (96 a^2 + 179 b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \sin [c + d x]}{a + b}}}{40 d \sqrt{a + b \sin [c + d x]}} - \frac{5 b (12 a^2 - b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \sin [c + d x]}{a + b}}}{8 d \sqrt{a + b \sin [c + d x]}}
\end{aligned}$$

Result (type 4, 615 leaves):

$$\begin{aligned}
& \frac{1}{160 d} \left(- \frac{2 (160 a^3 - 692 a b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \right. \\
& \frac{2 (-424 a^2 b - 117 b^3) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \\
& \left. \left(2 i (-176 a^2 b + 167 b^3) \cos [c+d x] \cos [2 (c+d x)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
& \left. \left. \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right] - \right. \right. \right. \\
& \left. \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right]\right) \right) \sqrt{\frac{b-b \sin [c+d x]}{a+b}} \sqrt{-\frac{b+b \sin [c+d x]}{a-b}} \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin [c+d x]}^2 (-2 a^2 + b^2 + 4 a (a+b \sin [c+d x]) - 2 (a+b \sin [c+d x])^2) \right. \\
& \left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin [c+d x]) + (a+b \sin [c+d x])^2}{b^2}} \right) \right) + \frac{1}{d} \\
& \sqrt{a+b \sin [c+d x]} \left(-\frac{22}{15} a b \cos [c+d x] + \frac{1}{24} (32 a^2 \cos [c+d x] - 33 b^2 \cos [c+d x]) \operatorname{Csc}[c+d x] - \right. \\
& \left. \frac{13}{12} a b \cot [c+d x] \operatorname{Csc}[c+d x] - \frac{1}{3} a^2 \cot [c+d x] \operatorname{Csc}[c+d x]^2 - \frac{1}{5} b^2 \sin [2 (c+d x)] \right)
\end{aligned}$$

■ **Problem 1165: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot [c+d x]^4 \operatorname{Csc}[c+d x] (a+b \sin [c+d x])^{5/2} dx$$

Optimal (type 4, 449 leaves, 12 steps):

$$\begin{aligned}
& - \frac{b^2 (196 a^2 + 5 b^2) \cos[c + dx] \sqrt{a + b \sin[c + dx]}}{64 a^2 d} + \frac{5 b (68 a^2 + b^2) \cot[c + dx] (a + b \sin[c + dx])^{3/2}}{192 a^2 d} + \\
& \frac{(60 a^2 + b^2) \cot[c + dx] \csc[c + dx] (a + b \sin[c + dx])^{5/2}}{96 a^2 d} + \frac{b \cot[c + dx] \csc[c + dx]^2 (a + b \sin[c + dx])^{7/2}}{24 a^2 d} - \\
& \frac{\cot[c + dx] \csc[c + dx]^3 (a + b \sin[c + dx])^{7/2}}{4 a d} + \frac{b (492 a^2 - 5 b^2) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{a + b \sin[c + dx]}}{64 a d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} - \\
& \frac{b (148 a^2 + 169 b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{64 d \sqrt{a + b \sin[c + dx]}} + \frac{(48 a^4 - 360 a^2 b^2 - 5 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{64 a d \sqrt{a + b \sin[c + dx]}}
\end{aligned}$$

Result (type 4, 655 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(-\frac{2}{3} b^2 \cos[c+dx] + \frac{5(116a^2b \cos[c+dx] - 3b^3 \cos[c+dx]) \operatorname{Csc}[c+dx]}{192a} + \frac{1}{96} (60a^2 \cos[c+dx] - 59b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^2 - \right. \\
& \left. \frac{17}{24} ab \cot[c+dx] \operatorname{Csc}[c+dx]^2 - \frac{1}{4} a^2 \cot[c+dx] \operatorname{Csc}[c+dx]^3 \right) \sqrt{a+b \sin[c+dx]} + \\
& \frac{1}{256ad} \left(-\frac{2(688a^3b - 348ab^3) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \frac{2(96a^4 - 228a^2b^2 - 15b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \left(2i(-492a^2b^2 + 5b^4) \cos[c+dx] \cos[2(c+dx)] \left(2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
& \left. \left. b \left(2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right. \\
& \left. \left. \left. (-2a^2 + b^2 + 4a(a+b \sin[c+dx]) - 2(a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2a(a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right) \right)
\end{aligned}$$

■ **Problem 1166: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot[c+dx]^4 \operatorname{Csc}[c+dx]^2 (a+b \sin[c+dx])^{5/2} dx$$

Optimal (type 4, 482 leaves, 12 steps):

$$\begin{aligned}
& - \frac{(128 a^4 - 580 a^2 b^2 + 15 b^4) \operatorname{Cot}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{640 a^2 d} + \frac{b (36 a^2 - b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x] (a + b \operatorname{Sin}[c + d x])^{3/2}}{64 a^2 d} + \\
& \frac{(32 a^2 - b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2 (a + b \operatorname{Sin}[c + d x])^{5/2}}{80 a^2 d} + \frac{3 b \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^3 (a + b \operatorname{Sin}[c + d x])^{7/2}}{40 a^2 d} - \\
& \frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^4 (a + b \operatorname{Sin}[c + d x])^{7/2}}{5 a d} - \frac{(128 a^4 - 2476 a^2 b^2 - 15 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{640 a^2 d \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}}} + \\
& \frac{(128 a^4 + 492 a^2 b^2 - 5 b^4) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}}}{640 a d \sqrt{a + b \operatorname{Sin}[c + d x]}} + \\
& \frac{3 b (80 a^4 - 40 a^2 b^2 + b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \operatorname{Sin}[c + d x]}{a + b}}}{128 a^2 d \sqrt{a + b \operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 700 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(\frac{(-128 a^4 \cos[c+dx] + 1196 a^2 b^2 \cos[c+dx] + 15 b^4 \cos[c+dx]) \operatorname{Csc}[c+dx]}{640 a^2} + \frac{(436 a^2 b \cos[c+dx] - 5 b^3 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{320 a} \right. \\
& \left. + \frac{1}{80} (32 a^2 \cos[c+dx] - 31 b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^3 - \frac{21}{40} a b \cot[c+dx] \operatorname{Csc}[c+dx]^3 - \frac{1}{5} a^2 \cot[c+dx] \operatorname{Csc}[c+dx]^4 \right) \\
& \sqrt{a+b \sin[c+dx]} + \frac{1}{2560 a^2 d} b \left(\frac{2 (5936 a^3 b + 20 a b^3) \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \frac{2 (2272 a^4 + 1276 a^2 b^2 + 45 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \left(2 i (128 a^4 - 2476 a^2 b^2 - 15 b^4) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
& \left. \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right]\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b} \right) \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right. \\
& \left. \left. (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right)
\end{aligned}$$

■ **Problem 1167: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot[c+dx]^4 \operatorname{Csc}[c+dx]^3 (a+b \sin[c+dx])^{5/2} dx$$

Optimal (type 4, 551 leaves, 13 steps):

$$\begin{aligned}
& - \frac{b (720 a^4 - 176 a^2 b^2 + 15 b^4) \operatorname{Cot}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{1536 a^3 d} - \\
& \frac{(16 a^4 - 56 a^2 b^2 + 5 b^4) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{256 a^2 d} + \frac{b (52 a^2 - 5 b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2 (a + b \operatorname{Sin}[c + d x])^{3/2}}{192 a^2 d} + \\
& \frac{(28 a^2 - 3 b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^3 (a + b \operatorname{Sin}[c + d x])^{5/2}}{96 a^2 d} + \frac{b \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^4 (a + b \operatorname{Sin}[c + d x])^{7/2}}{12 a^2 d} - \\
& \frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^5 (a + b \operatorname{Sin}[c + d x])^{7/2}}{6 a d} - \frac{b (720 a^4 - 176 a^2 b^2 + 15 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{1536 a^3 d \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}} + \\
& \frac{b (816 a^4 + 1696 a^2 b^2 + 5 b^4) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{1536 a^2 d \sqrt{a + b \operatorname{Sin}[c + d x]}} + \\
& \frac{(64 a^6 + 720 a^4 b^2 + 60 a^2 b^4 - 5 b^6) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{512 a^3 d \sqrt{a + b \operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 771 leaves):

$$\frac{1}{d} \left(\frac{(-720 a^4 b \cos[c+dx] + 176 a^2 b^3 \cos[c+dx] - 15 b^5 \cos[c+dx]) \operatorname{Csc}[c+dx]}{1536 a^3} + \frac{(-48 a^4 \cos[c+dx] + 600 a^2 b^2 \cos[c+dx] + 5 b^4 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{768 a^2} + \frac{(164 a^2 b \cos[c+dx] - b^3 \cos[c+dx]) \operatorname{Csc}[c+dx]^3}{192 a} + \frac{1}{96} (28 a^2 \cos[c+dx] - 27 b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^4 - \frac{5}{12} a b \cot[c+dx] \operatorname{Csc}[c+dx]^4 - \frac{1}{6} a^2 \cot[c+dx] \operatorname{Csc}[c+dx]^5 \right)$$

$$\sqrt{a+b \sin[c+dx]} + \frac{1}{6144 a^3 d} \left(- \frac{2 (192 a^5 b + 3744 a^3 b^3 - 20 a b^5) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \frac{2 (384 a^6 + 3600 a^4 b^2 + 536 a^2 b^4 - 45 b^6) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \left(2 i (720 a^4 b^2 - 176 a^2 b^4 + 15 b^6) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \sin[c+dx]^2} \right) \right)$$

$$(-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right)$$

■ **Problem 1171: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^3 \cot[c+dx]}{\sqrt{a+b \sin[c+dx]}} dx$$

Optimal (type 4, 288 leaves, 9 steps):

$$\frac{8 a \cos [c+d x] \sqrt{a+b \sin [c+d x]}}{15 b^2 d} - \frac{2 \cos [c+d x] \sin [c+d x] \sqrt{a+b \sin [c+d x]}}{5 b d} +$$

$$\frac{2\left(8 a^2-21 b^2\right) \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), \frac{2 b}{a+b}\right] \sqrt{a+b \sin [c+d x]}}{15 b^3 d \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}$$

$$\frac{2 a\left(8 a^2-23 b^2\right) \operatorname{EllipticF}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{15 b^3 d \sqrt{a+b \sin [c+d x]}} + \frac{2 \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{d \sqrt{a+b \sin [c+d x]}}$$

Result (type 4, 536 leaves):

$$-\frac{1}{30 b^2 d} \left(\frac{8 a b \operatorname{EllipticF}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \right.$$

$$\left. \frac{2\left(-8 a^2-9 b^2\right) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}} - \left(2 i\left(8 a^2-21 b^2\right) \cos [c+d x] \cos [2(c+d x)] \right) \right.$$

$$\left(2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \right. \right. \right.$$

$$\left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right] \right) \sqrt{\frac{b-b \sin [c+d x]}{a+b}} \sqrt{-\frac{b+b \sin [c+d x]}{a-b}} \right) /$$

$$\left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin [c+d x]^2} \left(-2 a^2+b^2+4 a(a+b \sin [c+d x]) - 2(a+b \sin [c+d x])^2\right) \right.$$

$$\left. \sqrt{-\frac{a^2-b^2-2 a(a+b \sin [c+d x])+(a+b \sin [c+d x])^2}{b^2}} \right) + \frac{\sqrt{a+b \sin [c+d x]}\left(\frac{8 a \cos [c+d x]}{15 b^2}-\frac{\sin [2(c+d x)]}{5 b}\right)}{d}$$

■ **Problem 1172: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^2 \cot [c+d x]^2}{\sqrt{a+b \sin [c+d x]}} dx$$

Optimal (type 4, 285 leaves, 9 steps):

$$\begin{aligned} & -\frac{2 \cos [c+d x] \sqrt{a+b \sin [c+d x]}}{3 b d}-\frac{\cot [c+d x] \sqrt{a+b \sin [c+d x]}}{a d}-\frac{\left(4 a^2+3 b^2\right) \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), \frac{2 b}{a+b}\right] \sqrt{a+b \sin [c+d x]}}{3 a b^2 d \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}+ \\ & \frac{\left(4 a^2-7 b^2\right) \operatorname{EllipticF}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{3 b^2 d \sqrt{a+b \sin [c+d x]}}-\frac{b \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{a d \sqrt{a+b \sin [c+d x]}} \end{aligned}$$

Result (type 4, 534 leaves):

$$\begin{aligned} & \frac{\left(-\frac{2 \cos [c+d x]}{3 b}-\frac{\cot [c+d x]}{a}\right) \sqrt{a+b \sin [c+d x]}}{d}+ \\ & \frac{1}{12 a b d}\left(\frac{40 a b \operatorname{EllipticF}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}}-\frac{2\left(-4 a^2-9 b^2\right) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(-c+\frac{\pi}{2}-d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{\sqrt{a+b \sin [c+d x]}}\right)- \\ & \left(2 i\left(4 a^2+3 b^2\right) \cos [c+d x] \cos [2(c+d x)]\left(2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right]+b\left(2 a \operatorname{EllipticF}\left[\right.\right.\right. \\ & \left.\left.\left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right]-b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+d x]}\right], \frac{a+b}{a-b}\right]\right)\right) \\ & \left.\sqrt{\frac{b-b \sin [c+d x]}{a+b}} \sqrt{\frac{b+b \sin [c+d x]}{a-b}}\right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin [c+d x]^2}\left(-2 a^2+b^2+4 a(a+b \sin [c+d x])\right)-2(a+b \sin [c+d x])^2\right) \\ & \left.\sqrt{-\frac{a^2-b^2-2 a(a+b \sin [c+d x])+(a+b \sin [c+d x])^2}{b^2}}\right) \end{aligned}$$

■ **Problem 1173: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx] \cot[c+dx]^3}{\sqrt{a+b\sin[c+dx]}} dx$$

Optimal (type 4, 307 leaves, 9 steps):

$$\frac{3 b \cot [c+d x] \sqrt{a+b \sin [c+d x]}}{4 a^2 d}-\frac{\cot [c+d x] \operatorname{Csc}[c+d x] \sqrt{a+b \sin [c+d x]}}{2 a d}+\frac{\left(8 a^2+3 b^2\right) \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), \frac{2 b}{a+b}\right] \sqrt{a+b \sin [c+d x]}}{4 a^2 b d \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}-\frac{\left(8 a^2+b^2\right) \operatorname{EllipticF}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{4 a b d \sqrt{a+b \sin [c+d x]}}-\frac{3\left(4 a^2-b^2\right) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+d x]}{a+b}}}{4 a^2 d \sqrt{a+b \sin [c+d x]}}$$

Result (type 4, 443 leaves):

$$\frac{1}{16 d} \left(\frac{1}{a^3 b^2 \sqrt{-\frac{1}{a+b}} (-2 + \operatorname{Csc}[c+dx])^2} \right. \\ \left. 2 i \left(8 a^2+3 b^2\right) \cos [2(c+dx)] \operatorname{Csc}[c+dx]^2 \left(2 a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+dx]}\right], \frac{a+b}{a-b}\right]+b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+dx]}\right], \frac{a+b}{a-b}\right]-b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin [c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \right) \\ \frac{\sec [c+dx] \sqrt{-\frac{b(-1+\sin [c+dx])}{a+b}} \sqrt{-\frac{b(1+\sin [c+dx])}{a-b}}-\frac{4 \cot [c+dx](-3 b+2 a \operatorname{Csc}[c+dx]) \sqrt{a+b \sin [c+dx]}}{a^2}}{a \sqrt{a+b \sin [c+dx]}}+\frac{2\left(16 a^2-9 b^2\right) \operatorname{EllipticPi}\left[2, \frac{1}{4}(-2 c+\pi-2 d x), \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sin [c+dx]}{a+b}}}{a^2 \sqrt{a+b \sin [c+dx]}}$$

■ **Problem 1174: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cot}[c + d x]^4}{\sqrt{a + b \text{Sin}[c + d x]}} dx$$

Optimal (type 4, 353 leaves, 10 steps):

$$\begin{aligned} & \frac{(32 a^2 - 15 b^2) \text{Cot}[c + d x] \sqrt{a + b \text{Sin}[c + d x]}}{24 a^3 d} + \frac{5 b \text{Cot}[c + d x] \text{Csc}[c + d x] \sqrt{a + b \text{Sin}[c + d x]}}{12 a^2 d} - \\ & \frac{\text{Cot}[c + d x] \text{Csc}[c + d x]^2 \sqrt{a + b \text{Sin}[c + d x]}}{3 a d} + \frac{(32 a^2 - 15 b^2) \text{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{a + b \text{Sin}[c + d x]}}{24 a^3 d \sqrt{\frac{a+b \text{Sin}[c+d x]}{a+b}}} + \\ & \frac{(16 a^2 + 5 b^2) \text{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \text{Sin}[c+d x]}{a+b}}}{24 a^2 d \sqrt{a + b \text{Sin}[c + d x]}} + \frac{b (12 a^2 - 5 b^2) \text{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \text{Sin}[c+d x]}{a+b}}}{8 a^3 d \sqrt{a + b \text{Sin}[c + d x]}} \end{aligned}$$

Result (type 4, 596 leaves):

$$\begin{aligned}
& \left(\frac{(32 a^2 \cos[c+dx] - 15 b^2 \cos[c+dx]) \csc[c+dx]}{24 a^3} + \frac{5 b \cot[c+dx] \csc[c+dx]}{12 a^2} - \frac{\cot[c+dx] \csc[c+dx]^2}{3 a} \right) \sqrt{a + b \sin[c + dx]} \\
& \frac{1}{96 a^3 d} \left(- \frac{2 (96 a^3 - 20 a b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} - \right. \\
& \frac{2 (104 a^2 b - 45 b^3) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} - \left. \left(2 i (-32 a^2 b + 15 b^3) \cos[c + dx] \cos[2(c + dx)] \right) \right. \\
& \left. \left(2 a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sqrt{\frac{b - b \sin[c + dx]}{a + b}} \sqrt{-\frac{b + b \sin[c + dx]}{a - b}} \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \sin[c + dx]}^2 (-2 a^2 + b^2 + 4 a (a + b \sin[c + dx]) - 2 (a + b \sin[c + dx])^2) \right. \\
& \left. \left. \sqrt{-\frac{a^2 - b^2 - 2 a (a + b \sin[c + dx]) + (a + b \sin[c + dx])^2}{b^2}} \right) \right)
\end{aligned}$$

■ **Problem 1175: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cot[c + dx]^4 \csc[c + dx]}{\sqrt{a + b \sin[c + dx]}} dx$$

Optimal (type 4, 412 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b (188 a^2 - 105 b^2) \operatorname{Cot}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{192 a^4 d} + \frac{5 (12 a^2 - 7 b^2) \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]}}{96 a^3 d} + \\
& \frac{7 b \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2 \sqrt{a + b \operatorname{Sin}[c + d x]}}{24 a^2 d} - \frac{\operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^3 \sqrt{a + b \operatorname{Sin}[c + d x]}}{4 a d} - \\
& \frac{b (188 a^2 - 105 b^2) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{a + b \operatorname{Sin}[c + d x]}}{192 a^4 d \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}} + \frac{b (68 a^2 - 35 b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{192 a^3 d \sqrt{a + b \operatorname{Sin}[c + d x]}} + \\
& \frac{(48 a^4 - 72 a^2 b^2 + 35 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[c+d x]}{a+b}}}{64 a^4 d \sqrt{a + b \operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 647 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(\frac{(-188 a^2 b \cos[c+dx] + 105 b^3 \cos[c+dx]) \operatorname{Csc}[c+dx]}{192 a^4} + \right. \\
& \left. \frac{5(12 a^2 \cos[c+dx] - 7 b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]^2}{96 a^3} + \frac{7 b \cot[c+dx] \operatorname{Csc}[c+dx]^2}{24 a^2} - \frac{\cot[c+dx] \operatorname{Csc}[c+dx]^3}{4 a} \right) \sqrt{a+b \sin[c+dx]} + \\
& \frac{1}{768 a^4 d} \left(- \frac{2(-240 a^3 b + 140 a b^3) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \frac{2(288 a^4 - 620 a^2 b^2 + 315 b^4) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \left. \left(2 i (188 a^2 b^2 - 105 b^4) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \right. \\
& \left. \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right. \\
& \left. \left. \left. (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right) \right)
\end{aligned}$$

■ **Problem 1179: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^3 \cot[c+dx]}{(a+b \sin[c+dx])^{3/2}} dx$$

Optimal (type 4, 296 leaves, 9 steps):

$$\frac{2(a^2 - b^2) \cos[c + dx]}{a b^2 d \sqrt{a + b \sin[c + dx]}} - \frac{2 \cos[c + dx] \sqrt{a + b \sin[c + dx]}}{3 b^2 d} - \frac{2(8a^2 - 3b^2) \operatorname{EllipticE}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{a + b \sin[c + dx]}}{3 a b^3 d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} +$$

$$\frac{2(8a^2 - 5b^2) \operatorname{EllipticF}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{3 b^3 d \sqrt{a + b \sin[c + dx]}} + \frac{2 \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{a d \sqrt{a + b \sin[c + dx]}}$$

Result (type 4, 565 leaves):

$$\frac{\sqrt{a + b \sin[c + dx]} \left(-\frac{2 \cos[c + dx]}{3 b^2} - \frac{2(a^2 \cos[c + dx] - b^2 \cos[c + dx])}{a b^2 (a + b \sin[c + dx])} \right)}{d} +$$

$$\frac{1}{6 a b^2 d} \left(\frac{8 a b \operatorname{EllipticF}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} - \frac{2(-8a^2 + 9b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} - \right.$$

$$\left. \left(2 i (8 a^2 - 3 b^2) \cos[c + dx] \cos[2(c + dx)] \left(2 a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]} \right], \frac{a+b}{a-b} \right] + b \left(2 a \operatorname{EllipticF}\left[\right. \right. \right. \right.$$

$$\left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]} \right], \frac{a+b}{a-b} \right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]} \right], \frac{a+b}{a-b} \right] \right) \right) \right)$$

$$\left. \left. \left. \left. \frac{\sqrt{\frac{b - b \sin[c + dx]}{a + b}} \sqrt{\frac{b + b \sin[c + dx]}{a - b}}}{\left(a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \sin[c + dx]^2} (-2a^2 + b^2 + 4a(a + b \sin[c + dx]) - 2(a + b \sin[c + dx])^2) \right)^2} \right. \right. \right. \right.$$

$$\left. \left. \left. \left. \frac{\sqrt{a^2 - b^2 - 2a(a + b \sin[c + dx]) + (a + b \sin[c + dx])^2}}{b^2} \right) \right) \right) \right)$$

■ **Problem 1180: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + dx]^2 \cot[c + dx]^2}{(a + b \sin[c + dx])^{3/2}} dx$$

Optimal (type 4, 294 leaves, 9 steps):

$$\frac{(2a^2 - 3b^2) \cos[c + dx]}{a^2 b d \sqrt{a + b \sin[c + dx]}} - \frac{\cot[c + dx]}{a d \sqrt{a + b \sin[c + dx]}} + \frac{(4a^2 - 3b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{a + b \sin[c + dx]}}{a^2 b^2 d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} -$$

$$\frac{(4a^2 - b^2) \operatorname{EllipticF}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{a b^2 d \sqrt{a + b \sin[c + dx]}} - \frac{3b \operatorname{EllipticPi}\left[2, \frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{a^2 d \sqrt{a + b \sin[c + dx]}}$$

Result (type 4, 563 leaves):

$$\frac{\sqrt{a + b \sin[c + dx]} \left(-\frac{\cot[c + dx]}{a^2} + \frac{2(a^2 \cos[c + dx] - b^2 \cos[c + dx])}{a^2 b (a + b \sin[c + dx])} \right)}{d} -$$

$$\frac{1}{4a^2 b d} \left(\frac{8ab \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} - \frac{2(-4a^2 + 9b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} - \right.$$

$$\left. \left(2i(4a^2 - 3b^2) \cos[c + dx] \cos[2(c + dx)] \left(2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left(2a \operatorname{EllipticF}\left[\right. \right. \right. \right.$$

$$\left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right] \right) \right) \right)$$

$$\left. \frac{\sqrt{\frac{b - b \sin[c + dx]}{a+b}} \sqrt{\frac{b + b \sin[c + dx]}{a-b}}}{\left(a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \sin^2[c + dx]} (-2a^2 + b^2 + 4a(a + b \sin[c + dx]) - 2(a + b \sin[c + dx])^2) \right)} \right)$$

$$\left. \frac{\sqrt{\frac{a^2 - b^2 - 2a(a + b \sin[c + dx]) + (a + b \sin[c + dx])^2}{b^2}}}{\left(\right)} \right)$$

■ **Problem 1181: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + dx] \cot^3[c + dx]}{(a + b \sin[c + dx])^{3/2}} dx$$

Optimal (type 4, 366 leaves, 10 steps):

$$\frac{(4a^2 - 5b^2) \cot[c+dx]}{2a^2bd\sqrt{a+b\sin[c+dx]}} - \frac{\cot[c+dx] \csc[c+dx]}{2ad\sqrt{a+b\sin[c+dx]}} -$$

$$\frac{(8a^2 - 15b^2) \cot[c+dx] \sqrt{a+b\sin[c+dx]}}{4a^3bd} - \frac{(8a^2 - 15b^2) \operatorname{EllipticE}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{a+b\sin[c+dx]}}{4a^3bd\sqrt{\frac{a+b\sin[c+dx]}{a+b}}} +$$

$$\frac{(8a^2 - 5b^2) \operatorname{EllipticF}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{4a^2bd\sqrt{a+b\sin[c+dx]}} - \frac{3(4a^2 - 5b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{4a^3d\sqrt{a+b\sin[c+dx]}}$$

Result (type 4, 579 leaves):

$$\frac{\sqrt{a+b\sin[c+dx]} \left(\frac{7b\cot[c+dx]}{4a^3} - \frac{\cot[c+dx] \csc[c+dx]}{2a^2} - \frac{2(a^2\cos[c+dx] - b^2\cos[c+dx])}{a^3(a+b\sin[c+dx])} \right)}{d} +$$

$$\frac{1}{16a^3d} \left(- \frac{40ab \operatorname{EllipticF}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{\sqrt{a+b\sin[c+dx]}} - \frac{2(-32a^2 + 45b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{\sqrt{a+b\sin[c+dx]}} - \right.$$

$$\left. \left(2i(8a^2 - 15b^2) \cos[c+dx] \cos[2(c+dx)] \left(2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\sin[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left(2a \operatorname{EllipticF}\left[\right. \right. \right. \right.$$

$$\left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b\sin[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) \right)$$

$$\left. \left. \left. \sqrt{\frac{b-b\sin[c+dx]}{a+b}} \sqrt{-\frac{b+b\sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} (-2a^2 + b^2 + 4a(a+b\sin[c+dx]) - 2(a+b\sin[c+dx])^2) \right. \right. \right.$$

$$\left. \left. \left. \sqrt{-\frac{a^2 - b^2 - 2a(a+b\sin[c+dx]) + (a+b\sin[c+dx])^2}{b^2}} \right) \right)$$

■ **Problem 1182: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cot}[c + d x]^4}{(a + b \text{Sin}[c + d x])^{3/2}} dx$$

Optimal (type 4, 416 leaves, 11 steps):

$$\begin{aligned} & \frac{(6 a^2 - 7 b^2) \text{Cot}[c + d x] \text{Csc}[c + d x]}{3 a^2 b d \sqrt{a + b \text{Sin}[c + d x]}} - \frac{\text{Cot}[c + d x] \text{Csc}[c + d x]^2}{3 a d \sqrt{a + b \text{Sin}[c + d x]}} + \frac{5 (16 a^2 - 21 b^2) \text{Cot}[c + d x] \sqrt{a + b \text{Sin}[c + d x]}}{24 a^4 d} - \\ & \frac{(24 a^2 - 35 b^2) \text{Cot}[c + d x] \text{Csc}[c + d x] \sqrt{a + b \text{Sin}[c + d x]}}{12 a^3 b d} + \frac{5 (16 a^2 - 21 b^2) \text{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{a + b \text{Sin}[c + d x]}}{24 a^4 d \sqrt{\frac{a + b \text{Sin}[c + d x]}{a + b}}} - \\ & \frac{(32 a^2 - 35 b^2) \text{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \text{Sin}[c + d x]}{a + b}}}{24 a^3 d \sqrt{a + b \text{Sin}[c + d x]}} + \frac{b (36 a^2 - 35 b^2) \text{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), \frac{2 b}{a + b}\right] \sqrt{\frac{a + b \text{Sin}[c + d x]}{a + b}}}{8 a^4 d \sqrt{a + b \text{Sin}[c + d x]}} \end{aligned}$$

Result (type 4, 636 leaves):

$$\begin{aligned}
& \frac{1}{d} \sqrt{a+b \sin[c+dx]} \left(\frac{(32 a^2 \cos[c+dx] - 57 b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]}{24 a^4} + \right. \\
& \quad \left. \frac{11 b \cot[c+dx] \operatorname{Csc}[c+dx]}{12 a^3} - \frac{\cot[c+dx] \operatorname{Csc}[c+dx]^2}{3 a^2} + \frac{2(a^2 b \cos[c+dx] - b^3 \cos[c+dx])}{a^4(a+b \sin[c+dx])} \right) + \\
& \frac{1}{96 a^4 d} \left(- \frac{2(96 a^3 - 140 a b^2) \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \quad \left. \frac{2(296 a^2 b - 315 b^3) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \right. \\
& \quad \left(2 i (-80 a^2 b + 105 b^3) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \\
& \quad \left. \left. b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \right. \right. \right. \\
& \quad \left. \left. \left. \frac{a+b}{a-b}\right] \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{-\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right. \\
& \quad \left. \left. \left. (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right) \right) \right)
\end{aligned}$$

■ **Problem 1183: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^4 \sin[c+dx]^3}{(a+b \sin[c+dx])^{5/2}} dx$$

Optimal (type 4, 469 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 (a^2 - b^2) \cos[c + dx] \sin[c + dx]^4}{3 a b^2 d (a + b \sin[c + dx])^{3/2}} + \frac{2 (13 a^2 - 5 b^2) \cos[c + dx] \sin[c + dx]^4}{3 a^2 b^2 d \sqrt{a + b \sin[c + dx]}} + \frac{128 a (40 a^2 - 19 b^2) \cos[c + dx] \sqrt{a + b \sin[c + dx]}}{315 b^6 d} - \\
& \frac{8 (480 a^2 - 203 b^2) \cos[c + dx] \sin[c + dx] \sqrt{a + b \sin[c + dx]}}{315 b^5 d} + \frac{4 (160 a^2 - 63 b^2) \cos[c + dx] \sin[c + dx]^2 \sqrt{a + b \sin[c + dx]}}{63 a b^4 d} - \\
& \frac{10 (8 a^2 - 3 b^2) \cos[c + dx] \sin[c + dx]^3 \sqrt{a + b \sin[c + dx]}}{9 a^2 b^3 d} + \frac{8 (1280 a^4 - 768 a^2 b^2 + 21 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{a + b \sin[c + dx]}}{315 b^7 d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} - \\
& \frac{8 a (1280 a^4 - 1088 a^2 b^2 + 123 b^4) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{315 b^7 d \sqrt{a + b \sin[c + dx]}}
\end{aligned}$$

Result (type 4, 1044 leaves):

$$\begin{aligned}
& \frac{1}{10080 d (a + b \sin[c + dx])^{3/2}} \\
& \left(315 \left(\frac{1}{(a-b)^2 b} \left((a^2 + 3b^2) \operatorname{EllipticE} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] + a(-a+b) \operatorname{EllipticF} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] \right) \left(\frac{a + b \sin[c + dx]}{a+b} \right)^{3/2} - \right. \\
& \quad \left. \frac{\cos[c + dx] (2a(a^2 + b^2) + b(a^2 + 3b^2) \sin[c + dx])}{(a^2 - b^2)^2} \right) + \frac{1}{b^3} 315 \left(1 / (a-b)^2 \left((32a^4 - 57a^2b^2 + 21b^4) \operatorname{EllipticE} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] + \right. \right. \\
& \quad \left. \left. a(-32a^3 + 32a^2b + 33ab^2 - 33b^3) \operatorname{EllipticF} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] \right) \left(\frac{a + b \sin[c + dx]}{a+b} \right)^{3/2} - \right. \\
& \quad \left. \left. \frac{b(4a(8a^4 - 13a^2b^2 + 3b^4) \cos[c + dx] + b(20a^4 - 33a^2b^2 + 9b^4) \sin[2(c + dx)])}{2(a^2 - b^2)^2} \right) - \frac{1}{b^5} \right. \\
& \quad \left. 21 \left(1 / (a-b)^2 \left((-2048a^6 + 4192a^4b^2 - 2355a^2b^4 + 231b^6) \operatorname{EllipticE} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] + \right. \right. \right. \\
& \quad \left. \left. a(2048a^5 - 2048a^4b - 2656a^3b^2 + 2656a^2b^3 + 603ab^4 - 603b^5) \operatorname{EllipticF} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] \right) \left(\frac{a + b \sin[c + dx]}{a+b} \right)^{3/2} + \right. \\
& \quad \left. \left. 1 / (a^2 - b^2)^2 b \cos[c + dx] (-64ab^2(a^2 - b^2)^2 \cos[2(c + dx)] + b(1280a^6 - 2536a^4b^2 + 1347a^2b^4 - 111b^6) \sin[c + dx] + \right. \right. \\
& \quad \left. \left. 2(512a^7 - 952a^5b^2 + 423a^3b^4 + 7ab^6 + 6b^3(a^2 - b^2)^2 \sin[3(c + dx)]) \right) \right) - \right. \\
& \quad \left. \frac{1}{b^7} 5(a + b \sin[c + dx]) \left(\frac{1}{(a-b)^2(a+b)^2} \left(-4b(-4096a^7b + 8960a^5b^3 - 5884a^3b^5 + 1041ab^7) \operatorname{EllipticF} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] + \right. \right. \right. \\
& \quad \left. \left. (65536a^8 - 161792a^6b^2 + 129664a^4b^4 - 35109a^2b^6 + 1617b^8) \right. \right. \\
& \quad \left. \left. \left((a+b) \operatorname{EllipticE} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] - a \operatorname{EllipticF} \left[\frac{1}{4} (-2c + \pi - 2dx), \frac{2b}{a+b} \right] \right) \right) \sqrt{\frac{a + b \sin[c + dx]}{a+b}} + \right. \\
& \quad \left. b(a + b \sin[c + dx]) \left(-128a(88a^2 - 27b^2) \cos[c + dx] + 416ab^2 \cos[3(c + dx)] + \frac{21a(64a^6 - 112a^4b^2 + 56a^2b^4 - 7b^6) \cos[c + dx]}{(a^2 - b^2)(a + b \sin[c + dx])^2} - \right. \right. \\
& \quad \left. \left. \frac{21(1088a^8 - 2576a^6b^2 + 1960a^4b^4 - 497a^2b^6 + 21b^8) \cos[c + dx]}{(a^2 - b^2)^2(a + b \sin[c + dx])} - 8b(-276a^2 + 35b^2) \sin[2(c + dx)] - 56b^3 \sin[4(c + dx)] \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 1186: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + dx]^3 \cot[c + dx]}{(a + b \sin[c + dx])^{5/2}} dx$$

Optimal (type 4, 313 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{2(a^2 - b^2) \cos[c + dx]}{3ab^2 d (a + b \sin[c + dx])^{3/2}} + \frac{2(5a^2 + 3b^2) \cos[c + dx]}{3a^2 b^2 d \sqrt{a + b \sin[c + dx]}} + \frac{2(8a^2 + 3b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{a + b \sin[c + dx]}}{3a^2 b^3 d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} \\
 & \frac{2(8a^2 + b^2) \operatorname{EllipticF}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{3ab^3 d \sqrt{a + b \sin[c + dx]}} + \frac{2 \operatorname{EllipticPi}\left[2, \frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{a^2 d \sqrt{a + b \sin[c + dx]}}
 \end{aligned}$$

Result (type 4, 599 leaves):

$$\begin{aligned}
 & \frac{\sqrt{a + b \sin[c + dx]} \left(-\frac{2(a^2 \cos[c + dx] - b^2 \cos[c + dx])}{3ab^2 (a + b \sin[c + dx])^2} + \frac{2(5a^2 \cos[c + dx] + 3b^2 \cos[c + dx])}{3a^2 b^2 (a + b \sin[c + dx])} \right)}{d} \\
 & \frac{1}{6a^2 b^2 d} \left(\frac{8ab \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} - \frac{2(-8a^2 - 9b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a + b \sin[c + dx]}} \right) \\
 & \left(2i(8a^2 + 3b^2) \cos[c + dx] \cos[2(c + dx)] \left(2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left(2a \operatorname{EllipticF}\left[\right. \right. \right. \right. \\
 & \quad \left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b \sin[c + dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \\
 & \left. \left. \left. \left. \sqrt{\frac{b - b \sin[c + dx]}{a+b}} \sqrt{-\frac{b + b \sin[c + dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \sin^2[c + dx]}^2 (-2a^2 + b^2 + 4a(a + b \sin[c + dx]) - 2(a + b \sin[c + dx])^2) \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sqrt{-\frac{a^2 - b^2 - 2a(a + b \sin[c + dx]) + (a + b \sin[c + dx])^2}{b^2}} \right) \right) \right) \right)
 \end{aligned}$$

■ **Problem 1187: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + dx]^2 \cot[c + dx]^2}{(a + b \sin[c + dx])^{5/2}} dx$$

Optimal (type 4, 346 leaves, 10 steps):

$$\frac{(2a^2 - 5b^2) \cos[c + dx]}{3a^2bd(a + b\sin[c + dx])^{3/2}} - \frac{\cot[c + dx]}{ad(a + b\sin[c + dx])^{3/2}} -$$

$$\frac{(4a^2 + 15b^2) \cos[c + dx]}{3a^3bd\sqrt{a + b\sin[c + dx]}} - \frac{(4a^2 + 15b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{a + b\sin[c + dx]}}{3a^3b^2d\sqrt{\frac{a+b\sin[c+dx]}{a+b}}} +$$

$$\frac{(4a^2 + 5b^2) \operatorname{EllipticF}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{3a^2b^2d\sqrt{a + b\sin[c + dx]}} - \frac{5b \operatorname{EllipticPi}\left[2, \frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{a^3d\sqrt{a + b\sin[c + dx]}}$$

Result (type 4, 609 leaves):

$$\frac{\sqrt{a + b\sin[c + dx]} \left(-\frac{\cot[c + dx]}{a^3} + \frac{2(a^2 \cos[c + dx] - b^2 \cos[c + dx])}{3a^2b(a + b\sin[c + dx])^2} - \frac{4(a^2 \cos[c + dx] + 3b^2 \cos[c + dx])}{3a^3b(a + b\sin[c + dx])} \right)}{d} +$$

$$\frac{1}{12a^3bd} \left(\frac{40ab \operatorname{EllipticF}\left[\frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{\sqrt{a + b\sin[c + dx]}} - \frac{2(-4a^2 - 45b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}(-c + \frac{\pi}{2} - dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b\sin[c+dx]}{a+b}}}{\sqrt{a + b\sin[c + dx]}} - \right.$$

$$\left. \left(2i(4a^2 + 15b^2) \cos[c + dx] \cos[2(c + dx)] \left(2a(a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b\sin[c + dx]}\right], \frac{a+b}{a-b}\right] + b \left(2a \operatorname{EllipticF}\left[\right. \right. \right. \right.$$

$$\left. \left. \left. i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b\sin[c + dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a + b\sin[c + dx]}\right], \frac{a+b}{a-b}\right] \right) \right) \right)$$

$$\left. \left. \left. \left. \sqrt{\frac{b - b\sin[c + dx]}{a+b}} \sqrt{\frac{b + b\sin[c + dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1 - \sin[c + dx]^2} (-2a^2 + b^2 + 4a(a + b\sin[c + dx]) - 2(a + b\sin[c + dx])^2) \right. \right. \right. \right.$$

$$\left. \left. \left. \left. \sqrt{-\frac{a^2 - b^2 - 2a(a + b\sin[c + dx]) + (a + b\sin[c + dx])^2}{b^2}} \right) \right) \right)$$

■ **Problem 1188: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + dx] \cot[c + dx]^3}{(a + b \sin[c + dx])^{5/2}} dx$$

Optimal (type 4, 407 leaves, 11 steps):

$$\frac{(4a^2 - 7b^2) \cot[c + dx]}{6a^2bd(a + b \sin[c + dx])^{3/2}} - \frac{\cot[c + dx] \csc[c + dx]}{2ad(a + b \sin[c + dx])^{3/2}} - \frac{(8a^2 - 105b^2) \cos[c + dx]}{12a^4d\sqrt{a + b \sin[c + dx]}} -$$

$$\frac{(8a^2 - 35b^2) \cot[c + dx]}{12a^3bd\sqrt{a + b \sin[c + dx]}} - \frac{(8a^2 - 105b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{a + b \sin[c + dx]}}{12a^4bd\sqrt{\frac{a+b \sin[c+dx]}{a+b}}} +$$

$$\frac{(8a^2 - 35b^2) \operatorname{EllipticF}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{12a^3bd\sqrt{a + b \sin[c + dx]}} - \frac{(12a^2 - 35b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2}(c - \frac{\pi}{2} + dx), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{4a^4d\sqrt{a + b \sin[c + dx]}}$$

Result (type 4, 622 leaves):

$$\frac{1}{d} \sqrt{a+b \sin[c+dx]} \left(\frac{11 b \cot[c+dx]}{4 a^4} - \frac{\cot[c+dx] \csc[c+dx]}{2 a^3} - \frac{2 (a^2 \cos[c+dx] - b^2 \cos[c+dx])}{3 a^3 (a+b \sin[c+dx])^2} - \frac{2 (a^2 \cos[c+dx] - 9 b^2 \cos[c+dx])}{3 a^4 (a+b \sin[c+dx])} \right) +$$

$$\frac{1}{48 a^4 d} \left(\frac{280 a b \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \frac{2 (-80 a^2 + 315 b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} \right) -$$

$$\left(2 i (8 a^2 - 105 b^2) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right]\right) \right) \right)$$

$$\left(\frac{\sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{\frac{b+b \sin[c+dx]}{a-b}}}{\left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} (-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2) \right)} \right)$$

$$\left(\sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}} \right)$$

■ **Problem 1189: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cot[c+dx]^4}{(a+b \sin[c+dx])^{5/2}} dx$$

Optimal (type 4, 458 leaves, 12 steps):

$$\frac{(2 a^2 - 3 b^2) \cot[c+dx] \csc[c+dx]}{3 a^2 b d (a+b \sin[c+dx])^{3/2}} - \frac{\cot[c+dx] \csc[c+dx]^2}{3 a d (a+b \sin[c+dx])^{3/2}} + \frac{b (32 a^2 - 105 b^2) \cos[c+dx]}{8 a^5 d \sqrt{a+b \sin[c+dx]}} + \frac{(16 a^2 - 35 b^2) \cot[c+dx]}{8 a^4 d \sqrt{a+b \sin[c+dx]}} -$$

$$\frac{(8 a^2 - 21 b^2) \cot[c+dx] \csc[c+dx]}{12 a^3 b d \sqrt{a+b \sin[c+dx]}} + \frac{(32 a^2 - 105 b^2) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{a+b \sin[c+dx]}}{8 a^5 d \sqrt{\frac{a+b \sin[c+dx]}{a+b}}} -$$

$$\frac{(16 a^2 - 35 b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{8 a^4 d \sqrt{a+b \sin[c+dx]}} + \frac{15 b (4 a^2 - 7 b^2) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{8 a^5 d \sqrt{a+b \sin[c+dx]}}$$

Result (type 4, 680 leaves) :

$$\frac{1}{d} \sqrt{a+b \sin[c+dx]} \left(\frac{(32 a^2 \cos[c+dx] - 123 b^2 \cos[c+dx]) \operatorname{Csc}[c+dx]}{24 a^5} + \frac{17 b \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{12 a^4} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{3 a^3} + \frac{2 (a^2 b \cos[c+dx] - b^3 \cos[c+dx])}{3 a^4 (a+b \sin[c+dx])^2} + \frac{8 (a^2 b \cos[c+dx] - 3 b^3 \cos[c+dx])}{3 a^5 (a+b \sin[c+dx])} \right) +$$

$$\frac{1}{32 a^5 d} \left(- \frac{2 (32 a^3 - 140 a b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \frac{2 (152 a^2 b - 315 b^3) \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(-c + \frac{\pi}{2} - dx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \sin[c+dx]}{a+b}}}{\sqrt{a+b \sin[c+dx]}} - \left(2 i (-32 a^2 b + 105 b^3) \cos[c+dx] \cos[2(c+dx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] + b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \sin[c+dx]}\right], \frac{a+b}{a-b}\right] \right) \right) \sqrt{\frac{b-b \sin[c+dx]}{a+b}} \sqrt{\frac{b+b \sin[c+dx]}{a-b}} \right) / \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\sin[c+dx]^2} \right) \right)$$

$$\left(-2 a^2 + b^2 + 4 a (a+b \sin[c+dx]) - 2 (a+b \sin[c+dx])^2 \right) \sqrt{-\frac{a^2 - b^2 - 2 a (a+b \sin[c+dx]) + (a+b \sin[c+dx])^2}{b^2}}$$

- **Problem 1190: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^4}{\sqrt{d \sin[e+fx]} (a+b \sin[e+fx])^{9/2}} dx$$

Optimal (type 4, 510 leaves, 8 steps) :

$$\frac{2 \operatorname{Cos}[e + f x]^3 \sqrt{d \operatorname{Sin}[e + f x]}}{7 a d f (a + b \operatorname{Sin}[e + f x])^{7/2}} + \frac{12 \operatorname{Cos}[e + f x] \sqrt{d \operatorname{Sin}[e + f x]}}{35 a^2 d f (a + b \operatorname{Sin}[e + f x])^{5/2}} +$$

$$\frac{8 (a^2 - 2 b^2) \operatorname{Cos}[e + f x] \sqrt{d \operatorname{Sin}[e + f x]}}{35 a^3 (a^2 - b^2) d f (a + b \operatorname{Sin}[e + f x])^{3/2}} + \frac{32 b (2 a^2 - b^2) \operatorname{Cos}[e + f x]}{35 a^3 (a^2 - b^2)^2 f \sqrt{d \operatorname{Sin}[e + f x]} \sqrt{a + b \operatorname{Sin}[e + f x]}} -$$

$$\left(\frac{32 b (2 a^2 - b^2) \sqrt{\frac{a (1 - \operatorname{Csc}[e + f x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Csc}[e + f x])}{a - b}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{d \operatorname{Sin}[e + f x]}}\right], -\frac{a + b}{a - b}\right] \operatorname{Tan}[e + f x] \right) /$$

$$\left(35 a^5 (a - b) (a + b)^{3/2} \sqrt{d} f \right) -$$

$$\left(\frac{8 (5 a^2 - 3 a b - 4 b^2) \sqrt{\frac{a (1 - \operatorname{Csc}[e + f x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Csc}[e + f x])}{a - b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{d \operatorname{Sin}[e + f x]}}\right], -\frac{a + b}{a - b}\right] \operatorname{Tan}[e + f x] \right) /$$

$$\left(35 a^4 (a - b) (a + b)^{3/2} \sqrt{d} f \right)$$

Result(type 4, 1670 leaves):

$$\frac{1}{f \sqrt{d \operatorname{Sin}[e + f x]}} \operatorname{Sin}[e + f x] \sqrt{a + b \operatorname{Sin}[e + f x]}$$

$$\left(-\frac{2 (a^2 \operatorname{Cos}[e + f x] - b^2 \operatorname{Cos}[e + f x])}{7 a b^2 (a + b \operatorname{Sin}[e + f x])^4} + \frac{4 (5 a^2 \operatorname{Cos}[e + f x] + 3 b^2 \operatorname{Cos}[e + f x])}{35 a^2 b^2 (a + b \operatorname{Sin}[e + f x])^3} - \frac{2 (5 a^4 \operatorname{Cos}[e + f x] - 9 a^2 b^2 \operatorname{Cos}[e + f x] + 8 b^4 \operatorname{Cos}[e + f x])}{35 a^3 b^2 (a^2 - b^2) (a + b \operatorname{Sin}[e + f x])^2} \right.$$

$$\left. + \frac{32 (2 a^2 b^2 \operatorname{Cos}[e + f x] - b^4 \operatorname{Cos}[e + f x])}{35 a^4 (a^2 - b^2)^2 (a + b \operatorname{Sin}[e + f x])} \right) + \frac{1}{35 a^4 (a - b)^2 (a + b)^2 f \sqrt{d \operatorname{Sin}[e + f x]}}$$

$$4 \sqrt{\operatorname{Sin}[e + f x]} \left(\left(4 a (5 a^4 - 9 a^2 b^2 + 4 b^4) \sqrt{\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-a + b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a + b \operatorname{Sin}[e + f x])}{a}}{\sqrt{2}}\right], -\frac{2 a}{-a + b}\right] \right.$$

$$\left. \operatorname{Sec}[e + f x] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Sin}[e + f x]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a + b \operatorname{Sin}[e + f x])}{a}} \right) /$$

$$\left((a + b) \sqrt{\operatorname{Sin}[e + f x]} \sqrt{a + b \operatorname{Sin}[e + f x]} \right) + 4 a (-8 a^3 b + 4 a b^3)$$

$$\left(\left(\sqrt{\frac{(a+b) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[efx])}}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \sec[efx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \sin[efx]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[efx])}{a}} \right) / \left((a+b) \sqrt{\sin[efx]} \right. \right.$$

$$\left. \left. \sqrt{a+b \sin[efx]} \right) - \left(\sqrt{\frac{(a+b) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-a+b}} \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[efx])}}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \right. \right.$$

$$\left. \left. \sec[efx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \sin[efx]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[efx])}{a}} \right) / \right. \right.$$

$$\left. \left. \left(b \sqrt{\sin[efx]} \sqrt{a+b \sin[efx]} \right) + 2(8a^2b^2 - 4b^4) \left(\frac{\cos[efx] \sqrt{a+b \sin[efx]}}{b \sqrt{\sin[efx]}} + \right. \right.$$

$$\left. \left. \left(i \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{Csc}[efx] \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{\sqrt{\sin[efx]}}\right], -\frac{2a}{-a-b}\right] \sqrt{a+b \sin[efx]} \right) / \right. \right.$$

$$\left. \left. \left(b \sqrt{\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Csc}[efx]} \sqrt{\frac{\operatorname{Csc}[efx] (a+b \sin[efx])}{a+b}} \right) + \right. \right.$$

$$\left. \left. \frac{1}{b} 2a \left(\left(a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[efx])}}{a}}}{\sqrt{2}}}\right], \right. \right. \right.$$

$$\begin{aligned}
& \left. \begin{aligned}
& -\frac{2a}{-a+b} \left] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Sin}[e+fx]}{a}} \right. \\
& \left. \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{a}} \right) \right/ \left((a+b) \sqrt{\operatorname{Sin}[e+fx]} \sqrt{a+b \operatorname{Sin}[e+fx]} \right) - \\
& \left(a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-a+b}} \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Sin}[e+fx]}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{a}} \right) \right/ \left(b \sqrt{\operatorname{Sin}[e+fx]} \sqrt{a+b \operatorname{Sin}[e+fx]} \right) \right) \right) \right)
\end{aligned}
\end{aligned}$$

■ **Problem 1208: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^5 \operatorname{Csc}[c+dx] (a+b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 86 leaves, 4 steps):

$$-\frac{a \operatorname{Csc}[c+dx]}{d} + \frac{b \operatorname{Csc}[c+dx]^2}{d} + \frac{2a \operatorname{Csc}[c+dx]^3}{3d} - \frac{b \operatorname{Csc}[c+dx]^4}{4d} - \frac{a \operatorname{Csc}[c+dx]^5}{5d} + \frac{b \operatorname{Log}[\operatorname{Sin}[c+dx]]}{d}$$

Result (type 3, 198 leaves):

$$\begin{aligned}
& -\frac{89a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{240d} + \frac{31a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{480d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{160d} + \frac{b \operatorname{Csc}[c+dx]^2}{d} - \frac{b \operatorname{Csc}[c+dx]^4}{4d} + \\
& \frac{b \operatorname{Log}[\operatorname{Sin}[c+dx]]}{d} - \frac{89a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{240d} + \frac{31a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{480d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{160d}
\end{aligned}$$

■ **Problem 1209: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^5 \operatorname{Csc}[c+dx]^2 (a+b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 61 leaves, 6 steps):

$$-\frac{a \operatorname{Cot}[c+dx]^6}{6d} - \frac{b \operatorname{Csc}[c+dx]}{d} + \frac{2b \operatorname{Csc}[c+dx]^3}{3d} - \frac{b \operatorname{Csc}[c+dx]^5}{5d}$$

Result (type 3, 173 leaves):

$$-\frac{89b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{240d} - \frac{a \operatorname{Cot}[c+dx]^6}{6d} + \frac{31b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{480d} - \frac{b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{160d}$$

$$-\frac{89b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{240d} + \frac{31b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{480d} - \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{160d}$$

■ **Problem 1210: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^5 \operatorname{Csc}[c+dx]^3 (a+b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 65 leaves, 6 steps):

$$-\frac{b \operatorname{Cot}[c+dx]^6}{6d} - \frac{a \operatorname{Csc}[c+dx]^3}{3d} + \frac{2a \operatorname{Csc}[c+dx]^5}{5d} - \frac{a \operatorname{Csc}[c+dx]^7}{7d}$$

Result (type 3, 233 leaves):

$$-\frac{103a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{3360d} - \frac{b \operatorname{Cot}[c+dx]^6}{6d} - \frac{103a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{6720d} +$$

$$\frac{9a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{1120d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6}{896d} - \frac{103a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{3360d} -$$

$$\frac{103a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{6720d} + \frac{9a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1120d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{896d}$$

■ **Problem 1211: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^5 \operatorname{Csc}[c+dx]^4 (a+b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 81 leaves, 7 steps):

$$-\frac{a \operatorname{Cot}[c+dx]^6}{6d} - \frac{a \operatorname{Cot}[c+dx]^8}{8d} - \frac{b \operatorname{Csc}[c+dx]^3}{3d} + \frac{2b \operatorname{Csc}[c+dx]^5}{5d} - \frac{b \operatorname{Csc}[c+dx]^7}{7d}$$

Result (type 3, 265 leaves):

$$\begin{aligned}
& - \frac{103 b \cot\left[\frac{1}{2}(c+dx)\right]}{3360 d} - \frac{103 b \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{6720 d} + \frac{9 b \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{1120 d} - \\
& \frac{b \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{896 d} - \frac{a \csc[c+dx]^4}{4 d} + \frac{a \csc[c+dx]^6}{3 d} - \frac{a \csc[c+dx]^8}{8 d} - \frac{103 b \tan\left[\frac{1}{2}(c+dx)\right]}{3360 d} - \\
& \frac{103 b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{6720 d} + \frac{9 b \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{1120 d} - \frac{b \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{896 d}
\end{aligned}$$

■ **Problem 1212: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^5 \csc[c+dx]^5 (a+b \sin[c+dx]) dx$$

Optimal (type 3, 81 leaves, 7 steps):

$$-\frac{b \cot[c+dx]^6}{6 d} - \frac{b \cot[c+dx]^8}{8 d} - \frac{a \csc[c+dx]^5}{5 d} + \frac{2 a \csc[c+dx]^7}{7 d} - \frac{a \csc[c+dx]^9}{9 d}$$

Result (type 3, 325 leaves):

$$\begin{aligned}
& - \frac{649 a \cot\left[\frac{1}{2}(c+dx)\right]}{80640 d} - \frac{649 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{161280 d} - \frac{31 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{53760 d} + \\
& \frac{37 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{32256 d} - \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^8}{4608 d} - \frac{b \csc[c+dx]^4}{4 d} + \\
& \frac{b \csc[c+dx]^6}{3 d} - \frac{b \csc[c+dx]^8}{8 d} - \frac{649 a \tan\left[\frac{1}{2}(c+dx)\right]}{80640 d} - \frac{649 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{161280 d} - \\
& \frac{31 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{53760 d} + \frac{37 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{32256 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{4608 d}
\end{aligned}$$

■ **Problem 1213: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^5 \csc[c+dx]^6 (a+b \sin[c+dx]) dx$$

Optimal (type 3, 97 leaves, 4 steps):

$$-\frac{b \csc[c+dx]^5}{5 d} - \frac{a \csc[c+dx]^6}{6 d} + \frac{2 b \csc[c+dx]^7}{7 d} + \frac{a \csc[c+dx]^8}{4 d} - \frac{b \csc[c+dx]^9}{9 d} - \frac{a \csc[c+dx]^{10}}{10 d}$$

Result (type 3, 325 leaves):

$$\begin{aligned}
& - \frac{649 b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{80640 d} - \frac{649 b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{161280 d} - \frac{31 b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{53760 d} + \\
& \frac{37 b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6}{32256 d} - \frac{b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^8}{4608 d} - \frac{a \operatorname{Csc}[c+dx]^6}{6 d} + \\
& \frac{a \operatorname{Csc}[c+dx]^8}{4 d} - \frac{a \operatorname{Csc}[c+dx]^{10}}{10 d} - \frac{649 b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{80640 d} - \frac{649 b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{161280 d} - \\
& \frac{31 b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{53760 d} + \frac{37 b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{32256 d} - \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{4608 d}
\end{aligned}$$

■ **Problem 1214: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+dx]^5 \operatorname{Csc}[c+dx]^7 (a+b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 97 leaves, 4 steps):

$$-\frac{b \operatorname{Csc}[c+dx]^6}{6 d} - \frac{a \operatorname{Csc}[c+dx]^7}{7 d} + \frac{b \operatorname{Csc}[c+dx]^8}{4 d} + \frac{2 a \operatorname{Csc}[c+dx]^9}{9 d} - \frac{b \operatorname{Csc}[c+dx]^{10}}{10 d} - \frac{a \operatorname{Csc}[c+dx]^{11}}{11 d}$$

Result (type 3, 385 leaves):

$$\begin{aligned}
& - \frac{1109 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{354816 d} - \frac{1109 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{709632 d} - \frac{13 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{29568 d} + \\
& \frac{173 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6}{1419264 d} + \frac{17 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^8}{101376 d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^{10}}{22528 d} - \frac{b \operatorname{Csc}[c+dx]^6}{6 d} + \\
& \frac{b \operatorname{Csc}[c+dx]^8}{4 d} - \frac{b \operatorname{Csc}[c+dx]^{10}}{10 d} - \frac{1109 a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{354816 d} - \frac{1109 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{709632 d} - \frac{13 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{29568 d} + \\
& \frac{173 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1419264 d} + \frac{17 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{101376 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^{10} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{22528 d}
\end{aligned}$$

■ **Problem 1232: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx] \operatorname{Cot}[c+dx]^4}{(a+b \operatorname{Sin}[c+dx])^2} dx$$

Optimal (type 3, 147 leaves, 4 steps):

$$\begin{aligned}
& \frac{(2 a^2 - 3 b^2) \operatorname{Csc}[c+dx]}{a^4 d} + \frac{b \operatorname{Csc}[c+dx]^2}{a^3 d} - \frac{\operatorname{Csc}[c+dx]^3}{3 a^2 d} + \\
& \frac{4 b (a^2 - b^2) \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^5 d} - \frac{4 b (a^2 - b^2) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{a^5 d} - \frac{(a^2 - b^2)^2}{a^4 b d (a+b \operatorname{Sin}[c+dx])}
\end{aligned}$$

Result (type 3, 304 leaves) :

$$\frac{(11 a^2 \cos[\frac{1}{2}(c+dx)] - 18 b^2 \cos[\frac{1}{2}(c+dx)]) \operatorname{Csc}[\frac{1}{2}(c+dx)]}{12 a^4 d} + \frac{b \operatorname{Csc}[\frac{1}{2}(c+dx)]^2}{4 a^3 d} - \frac{\operatorname{Cot}[\frac{1}{2}(c+dx)] \operatorname{Csc}[\frac{1}{2}(c+dx)]^2}{24 a^2 d} + \frac{4(a^2 b - b^3) \operatorname{Log}[\sin[c+dx]]}{a^5 d} - \frac{4(a^2 b - b^3) \operatorname{Log}[a+b \sin[c+dx]]}{a^5 d} + \frac{b \operatorname{Sec}[\frac{1}{2}(c+dx)]^2}{4 a^3 d} + \frac{\operatorname{Sec}[\frac{1}{2}(c+dx)] (11 a^2 \sin[\frac{1}{2}(c+dx)] - 18 b^2 \sin[\frac{1}{2}(c+dx)])}{12 a^4 d} - \frac{(a-b)^2 (a+b)^2}{a^4 b d (a+b \sin[c+dx])} - \frac{\operatorname{Sec}[\frac{1}{2}(c+dx)]^2 \operatorname{Tan}[\frac{1}{2}(c+dx)]}{24 a^2 d}$$

■ **Problem 1233: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^5}{(a+b \sin[c+dx])^2} dx$$

Optimal (type 3, 188 leaves, 3 steps) :

$$-\frac{4b(a^2-b^2) \operatorname{Csc}[c+dx]}{a^5 d} + \frac{(2a^2-3b^2) \operatorname{Csc}[c+dx]^2}{2a^4 d} + \frac{2b \operatorname{Csc}[c+dx]^3}{3a^3 d} - \frac{\operatorname{Csc}[c+dx]^4}{4a^2 d} + \frac{(a^4-6a^2b^2+5b^4) \operatorname{Log}[\sin[c+dx]]}{a^6 d} - \frac{(a^4-6a^2b^2+5b^4) \operatorname{Log}[a+b \sin[c+dx]]}{a^6 d} + \frac{(a^2-b^2)^2}{a^5 d (a+b \sin[c+dx])}$$

Result (type 3, 380 leaves) :

$$\frac{(-11 a^2 b \cos[\frac{1}{2}(c+dx)] + 12 b^3 \cos[\frac{1}{2}(c+dx)]) \operatorname{Csc}[\frac{1}{2}(c+dx)]}{6 a^5 d} + \frac{(7 a^2 - 12 b^2) \operatorname{Csc}[\frac{1}{2}(c+dx)]^2}{32 a^4 d} + \frac{b \operatorname{Cot}[\frac{1}{2}(c+dx)] \operatorname{Csc}[\frac{1}{2}(c+dx)]^2}{12 a^3 d} - \frac{\operatorname{Csc}[\frac{1}{2}(c+dx)]^4}{64 a^2 d} + \frac{(a^4 - 6 a^2 b^2 + 5 b^4) \operatorname{Log}[\sin[c+dx]]}{a^6 d} + \frac{(-a^4 + 6 a^2 b^2 - 5 b^4) \operatorname{Log}[a+b \sin[c+dx]]}{a^6 d} + \frac{(7 a^2 - 12 b^2) \operatorname{Sec}[\frac{1}{2}(c+dx)]^2}{32 a^4 d} - \frac{\operatorname{Sec}[\frac{1}{2}(c+dx)]^4}{64 a^2 d} + \frac{\operatorname{Sec}[\frac{1}{2}(c+dx)] (-11 a^2 b \sin[\frac{1}{2}(c+dx)] + 12 b^3 \sin[\frac{1}{2}(c+dx)])}{6 a^5 d} + \frac{(a-b)^2 (a+b)^2}{a^5 d (a+b \sin[c+dx])} + \frac{b \operatorname{Sec}[\frac{1}{2}(c+dx)]^2 \operatorname{Tan}[\frac{1}{2}(c+dx)]}{12 a^3 d}$$

■ **Problem 1235: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^5 \sin[c+dx]^n (a+b \sin[c+dx])^2 dx$$

Optimal (type 3, 170 leaves, 3 steps) :

$$\frac{a^2 \operatorname{Sin}[c+dx]^{1+n}}{d(1+n)} + \frac{2ab \operatorname{Sin}[c+dx]^{2+n}}{d(2+n)} - \frac{(2a^2 - b^2) \operatorname{Sin}[c+dx]^{3+n}}{d(3+n)} - \frac{4ab \operatorname{Sin}[c+dx]^{4+n}}{d(4+n)} + \frac{(a^2 - 2b^2) \operatorname{Sin}[c+dx]^{5+n}}{d(5+n)} + \frac{2ab \operatorname{Sin}[c+dx]^{6+n}}{d(6+n)} + \frac{b^2 \operatorname{Sin}[c+dx]^{7+n}}{d(7+n)}$$

Result (type 3, 648 leaves):

$$\frac{1}{32d(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)(7+n)} \operatorname{Sin}[c+dx]^{1+n} \left(119616a^2 + 9888b^2 + 164368a^2n + 20488b^2n + 89472a^2n^2 + 14576b^2n^2 + 25372a^2n^3 + 4654b^2n^3 + 4020a^2n^4 + 734b^2n^4 + 340a^2n^5 + 58b^2n^5 + 12a^2n^6 + 2b^2n^6 + (48 + 92n + 56n^2 + 13n^3 + n^4) (16a^2(7+n)^2 + b^2(-113 + 8n + n^2)) \operatorname{Cos}[2(c+dx)] + 2(144 + 324n + 260n^2 + 95n^3 + 16n^4 + n^5) (2a^2(7+n) - b^2(13+n)) \operatorname{Cos}[4(c+dx)] - 720b^2 \operatorname{Cos}[6(c+dx)] - 1764b^2n \operatorname{Cos}[6(c+dx)] - 1624b^2n^2 \operatorname{Cos}[6(c+dx)] - 735b^2n^3 \operatorname{Cos}[6(c+dx)] - 175b^2n^4 \operatorname{Cos}[6(c+dx)] - 21b^2n^5 \operatorname{Cos}[6(c+dx)] - b^2n^6 \operatorname{Cos}[6(c+dx)] + 73920ab \operatorname{Sin}[c+dx] + 135664abn \operatorname{Sin}[c+dx] + 81096abn^2 \operatorname{Sin}[c+dx] + 22304abn^3 \operatorname{Sin}[c+dx] + 3184abn^4 \operatorname{Sin}[c+dx] + 240abn^5 \operatorname{Sin}[c+dx] + 8abn^6 \operatorname{Sin}[c+dx] + 23520ab \operatorname{Sin}[3(c+dx)] + 53704abn \operatorname{Sin}[3(c+dx)] + 44460abn^2 \operatorname{Sin}[3(c+dx)] + 17392abn^3 \operatorname{Sin}[3(c+dx)] + 3432abn^4 \operatorname{Sin}[3(c+dx)] + 328abn^5 \operatorname{Sin}[3(c+dx)] + 12abn^6 \operatorname{Sin}[3(c+dx)] + 3360ab \operatorname{Sin}[5(c+dx)] + 8152abn \operatorname{Sin}[5(c+dx)] + 7396abn^2 \operatorname{Sin}[5(c+dx)] + 3280abn^3 \operatorname{Sin}[5(c+dx)] + 760abn^4 \operatorname{Sin}[5(c+dx)] + 88abn^5 \operatorname{Sin}[5(c+dx)] + 4abn^6 \operatorname{Sin}[5(c+dx)] \right)$$

■ **Problem 1236: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^5 \operatorname{Sin}[c+dx]^n (a + b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 123 leaves, 3 steps):

$$\frac{a \operatorname{Sin}[c+dx]^{1+n}}{d(1+n)} + \frac{b \operatorname{Sin}[c+dx]^{2+n}}{d(2+n)} - \frac{2a \operatorname{Sin}[c+dx]^{3+n}}{d(3+n)} - \frac{2b \operatorname{Sin}[c+dx]^{4+n}}{d(4+n)} + \frac{a \operatorname{Sin}[c+dx]^{5+n}}{d(5+n)} + \frac{b \operatorname{Sin}[c+dx]^{6+n}}{d(6+n)}$$

Result (type 3, 371 leaves):

$$\frac{1}{16d(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)} \operatorname{Sin}[c+dx]^{1+n} \left(8544a + 10520an + 4888a^2n^2 + 1114a^3n^3 + 128a^4n^4 + 6a^5n^5 + 8a(336 + 692n + 484n^2 + 147n^3 + 20n^4 + n^5) \operatorname{Cos}[2(c+dx)] + 2a(144 + 324n + 260n^2 + 95n^3 + 16n^4 + n^5) \operatorname{Cos}[4(c+dx)] + 2640b \operatorname{Sin}[c+dx] + 4468bn \operatorname{Sin}[c+dx] + 2258bn^2 \operatorname{Sin}[c+dx] + 474bn^3 \operatorname{Sin}[c+dx] + 46bn^4 \operatorname{Sin}[c+dx] + 2bn^5 \operatorname{Sin}[c+dx] + 840b \operatorname{Sin}[3(c+dx)] + 1798bn \operatorname{Sin}[3(c+dx)] + 1331bn^2 \operatorname{Sin}[3(c+dx)] + 431bn^3 \operatorname{Sin}[3(c+dx)] + 61bn^4 \operatorname{Sin}[3(c+dx)] + 3bn^5 \operatorname{Sin}[3(c+dx)] + 120b \operatorname{Sin}[5(c+dx)] + 274bn \operatorname{Sin}[5(c+dx)] + 225bn^2 \operatorname{Sin}[5(c+dx)] + 85bn^3 \operatorname{Sin}[5(c+dx)] + 15bn^4 \operatorname{Sin}[5(c+dx)] + bn^5 \operatorname{Sin}[5(c+dx)] \right)$$

■ **Problem 1237: Unable to integrate problem.**

$$\int \frac{\operatorname{Cos}[c+dx]^5 \operatorname{Sin}[c+dx]^n}{a + b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 5, 167 leaves, 5 steps):

$$-\frac{a(a^2 - 2b^2)\sin[c+dx]^{1+n}}{b^4 d(1+n)} + \frac{(a^2 - b^2)^2 \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{b\sin[c+dx]}{a}\right]\sin[c+dx]^{1+n}}{ab^4 d(1+n)} +$$

$$\frac{(a^2 - 2b^2)\sin[c+dx]^{2+n}}{b^3 d(2+n)} - \frac{a\sin[c+dx]^{3+n}}{b^2 d(3+n)} + \frac{\sin[c+dx]^{4+n}}{bd(4+n)}$$

Result (type 8, 31 leaves):

$$\int \frac{\cos[c+dx]^5 \sin[c+dx]^n}{a+b\sin[c+dx]} dx$$

■ **Problem 1238: Unable to integrate problem.**

$$\int \frac{\cos[c+dx]^5 \sin[c+dx]^n}{(a+b\sin[c+dx])^2} dx$$

Optimal (type 5, 191 leaves, 5 steps):

$$\frac{(3a^2 - 2b^2)\sin[c+dx]^{1+n}}{b^4 d(1+n)} + \frac{(a^2 - b^2)(b^2 n - a^2(4+n)) \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{b\sin[c+dx]}{a}\right]\sin[c+dx]^{1+n}}{a^2 b^4 d(1+n)} -$$

$$\frac{2a\sin[c+dx]^{2+n}}{b^3 d(2+n)} + \frac{\sin[c+dx]^{3+n}}{b^2 d(3+n)} + \frac{(a^2 - b^2)^2 \sin[c+dx]^{1+n}}{ab^4 d(a+b\sin[c+dx])}$$

Result (type 8, 31 leaves):

$$\int \frac{\cos[c+dx]^5 \sin[c+dx]^n}{(a+b\sin[c+dx])^2} dx$$

■ **Problem 1250: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^6 \operatorname{Csc}[c+dx] (a+b\sin[c+dx])^2 dx$$

Optimal (type 3, 175 leaves, 11 steps):

$$-2abx + \frac{5(a^2 - 6b^2) \operatorname{ArcTanh}[\cos[c+dx]]}{16d} + \frac{b^2 \cos[c+dx]}{d} - \frac{2ab \cot[c+dx]}{d} + \frac{2ab \cot[c+dx]^3}{3d} - \frac{2ab \cot[c+dx]^5}{5d} -$$

$$\frac{(11a^2 - 18b^2) \cot[c+dx] \operatorname{Csc}[c+dx]}{16d} + \frac{(13a^2 - 6b^2) \cot[c+dx] \operatorname{Csc}[c+dx]^3}{24d} - \frac{a^2 \cot[c+dx] \operatorname{Csc}[c+dx]^5}{6d}$$

Result (type 3, 384 leaves):

$$\frac{1}{1920 d} \left(-3840 a b c - 3840 a b d x + 1920 b^2 \cos[c + d x] - 2944 a b \cot\left[\frac{1}{2}(c + d x)\right] - 330 a^2 \csc\left[\frac{1}{2}(c + d x)\right]^2 + 540 b^2 \csc\left[\frac{1}{2}(c + d x)\right]^2 + \right. \\ \left. 60 a^2 \csc\left[\frac{1}{2}(c + d x)\right]^4 - 30 b^2 \csc\left[\frac{1}{2}(c + d x)\right]^4 - 5 a^2 \csc\left[\frac{1}{2}(c + d x)\right]^6 + 600 a^2 \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - 3600 b^2 \log\left[\cos\left[\frac{1}{2}(c + d x)\right]\right] - \right. \\ \left. 600 a^2 \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] + 3600 b^2 \log\left[\sin\left[\frac{1}{2}(c + d x)\right]\right] + 330 a^2 \sec\left[\frac{1}{2}(c + d x)\right]^2 - 540 b^2 \sec\left[\frac{1}{2}(c + d x)\right]^2 - \right. \\ \left. 60 a^2 \sec\left[\frac{1}{2}(c + d x)\right]^4 + 30 b^2 \sec\left[\frac{1}{2}(c + d x)\right]^4 + 5 a^2 \sec\left[\frac{1}{2}(c + d x)\right]^6 - 2624 a b \csc[c + d x]^3 \sin\left[\frac{1}{2}(c + d x)\right]^4 + \right. \\ \left. 768 a b \csc[c + d x]^5 \sin\left[\frac{1}{2}(c + d x)\right]^6 + 164 a b \csc\left[\frac{1}{2}(c + d x)\right]^4 \sin[c + d x] - 12 a b \csc\left[\frac{1}{2}(c + d x)\right]^6 \sin[c + d x] + 2944 a b \tan\left[\frac{1}{2}(c + d x)\right] \right)$$

■ **Problem 1256: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^6 \sin[c + d x]^3}{(a + b \sin[c + d x])^2} dx$$

Optimal (type 3, 525 leaves, 11 steps):

$$\frac{a (64 a^6 - 120 a^4 b^2 + 60 a^2 b^4 - 5 b^6) x}{8 b^9} - \frac{2 a^2 (8 a^2 - 3 b^2) (a^2 - b^2)^{3/2} \operatorname{ArcTan}\left[\frac{b + a \tan\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^2 - b^2}}\right]}{b^9 d} + \frac{(840 a^6 - 1435 a^4 b^2 + 588 a^2 b^4 - 15 b^6) \cos[c + d x]}{105 b^8 d} - \\ \frac{a (32 a^4 - 52 a^2 b^2 + 19 b^4) \cos[c + d x] \sin[c + d x]}{8 b^7 d} + \frac{(280 a^4 - 441 a^2 b^2 + 150 b^4) \cos[c + d x] \sin[c + d x]^2}{105 b^6 d} - \\ \frac{(24 a^4 - 37 a^2 b^2 + 12 b^4) \cos[c + d x] \sin[c + d x]^3}{12 a b^5 d} + \frac{(224 a^4 - 340 a^2 b^2 + 105 b^4) \cos[c + d x] \sin[c + d x]^4}{140 a^2 b^4 d} + \frac{\cos[c + d x] \sin[c + d x]^4}{4 a d (a + b \sin[c + d x])} - \\ \frac{3 b \cos[c + d x] \sin[c + d x]^5}{20 a^2 d (a + b \sin[c + d x])} - \frac{(20 a^4 - 30 a^2 b^2 + 9 b^4) \cos[c + d x] \sin[c + d x]^5}{15 a^2 b^3 d (a + b \sin[c + d x])} - \frac{4 a \cos[c + d x] \sin[c + d x]^6}{21 b^2 d (a + b \sin[c + d x])} + \frac{\cos[c + d x] \sin[c + d x]^7}{7 b d (a + b \sin[c + d x])}$$

Result (type 3, 1223 leaves):

$$\begin{aligned}
& \frac{3 \left(\frac{2b \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{\sqrt{a^2-b^2}} + \frac{a \operatorname{Cos} [c+dx]}{a+b \operatorname{Sin} [c+dx]} \right)}{128 (-a+b) (a+b) d} + \frac{8a (c+dx) - \frac{2(8a^4-12a^2b^2+3b^4) \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{(a^2-b^2)^{3/2}} + 4b \operatorname{Cos} [c+dx] + \frac{ab(4a^2-3b^2) \operatorname{Cos} [c+dx]}{(a-b)(a+b)(a+b \operatorname{Sin} [c+dx])}}{32b^3d} \\
& \frac{1}{1280b^7d} \left(\frac{30(384a^8-896a^6b^2+672a^4b^4-168a^2b^6+7b^8) \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{(a^2-b^2)^{3/2}} + \right. \\
& 60b(80a^4-48a^2b^2+3b^4) \operatorname{Cos} [c+dx] + 40b^3(-6a^2+b^2) \operatorname{Cos} [3(c+dx)] + 12b^5 \operatorname{Cos} [5(c+dx)] + \\
& \left. \frac{15ab(64a^6-112a^4b^2+56a^2b^4-7b^6) \operatorname{Cos} [c+dx]}{(a-b)(a+b)(a+b \operatorname{Sin} [c+dx])} - 120ab^2(8a^2-3b^2) \operatorname{Sin} [2(c+dx)] + 60ab^4 \operatorname{Sin} [4(c+dx)] \right) + \\
& \frac{1}{256} \left(-\frac{1}{b^9(a^2-b^2)^{3/2}d} 2(2048a^{10}-5760a^8b^2+5760a^6b^4-2400a^4b^6+360a^2b^8-9b^{10}) \right. \\
& \left. \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right] (b \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + a \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right])}{\sqrt{a^2-b^2}} \right] + \frac{1}{105b^9(-a^2+b^2)d(a+b \operatorname{Sin} [c+dx])} \right. \\
& \left(-215040a^{10}(c+dx) + 497280a^8b^2(c+dx) - 383040a^6b^4(c+dx) + 109200a^4b^6(c+dx) - 8400a^2b^8(c+dx) - 215040a^9b \operatorname{Cos} [c+dx] + \right. \\
& 470400a^7b^3 \operatorname{Cos} [c+dx] - 334320a^5b^5 \operatorname{Cos} [c+dx] + 84000a^3b^7 \operatorname{Cos} [c+dx] - 5145ab^9 \operatorname{Cos} [c+dx] - 8960a^7b^3 \operatorname{Cos} [3(c+dx)] + \\
& 17360a^5b^5 \operatorname{Cos} [3(c+dx)] - 9870a^3b^7 \operatorname{Cos} [3(c+dx)] + 1470ab^9 \operatorname{Cos} [3(c+dx)] + 672a^5b^5 \operatorname{Cos} [5(c+dx)] - \\
& 994a^3b^7 \operatorname{Cos} [5(c+dx)] + 322ab^9 \operatorname{Cos} [5(c+dx)] - 80a^3b^7 \operatorname{Cos} [7(c+dx)] + 80ab^9 \operatorname{Cos} [7(c+dx)] - 215040a^9b(c+dx) \operatorname{Sin} [c+dx] + \\
& 497280a^7b^3(c+dx) \operatorname{Sin} [c+dx] - 383040a^5b^5(c+dx) \operatorname{Sin} [c+dx] + 109200a^3b^7(c+dx) \operatorname{Sin} [c+dx] - 8400ab^9(c+dx) \operatorname{Sin} [c+dx] - \\
& 53760a^8b^2 \operatorname{Sin} [2(c+dx)] + 115360a^6b^4 \operatorname{Sin} [2(c+dx)] - 78400a^4b^6 \operatorname{Sin} [2(c+dx)] + 17430a^2b^8 \operatorname{Sin} [2(c+dx)] - \\
& 630b^{10} \operatorname{Sin} [2(c+dx)] + 2240a^6b^4 \operatorname{Sin} [4(c+dx)] - 3836a^4b^6 \operatorname{Sin} [4(c+dx)] + 1722a^2b^8 \operatorname{Sin} [4(c+dx)] - 126b^{10} \operatorname{Sin} [4(c+dx)] - \\
& \left. \left. 224a^4b^6 \operatorname{Sin} [6(c+dx)] + 278a^2b^8 \operatorname{Sin} [6(c+dx)] - 54b^{10} \operatorname{Sin} [6(c+dx)] + 30a^2b^8 \operatorname{Sin} [8(c+dx)] - 30b^{10} \operatorname{Sin} [8(c+dx)] \right) \right)
\end{aligned}$$

■ **Problem 1257: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos} [c+dx]^6 \operatorname{Sin} [c+dx]^2}{(a+b \operatorname{Sin} [c+dx])^2} dx$$

Optimal (type 3, 471 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(112 a^6 - 200 a^4 b^2 + 90 a^2 b^4 - 5 b^6) x}{16 b^8} + \frac{2 a (7 a^2 - 2 b^2) (a^2 - b^2)^{3/2} \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{b^8 d} - \frac{a (105 a^4 - 170 a^2 b^2 + 61 b^4) \operatorname{Cos}[c+d x]}{15 b^7 d} + \\
& \frac{(56 a^4 - 86 a^2 b^2 + 27 b^4) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{16 b^6 d} - \frac{(35 a^4 - 52 a^2 b^2 + 15 b^4) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^2}{15 a b^5 d} + \\
& \frac{(42 a^4 - 61 a^2 b^2 + 16 b^4) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^3}{24 a^2 b^4 d} + \frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^3}{3 a d (a+b \operatorname{Sin}[c+d x])} - \frac{b \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^4}{6 a^2 d (a+b \operatorname{Sin}[c+d x])} - \\
& \frac{(14 a^4 - 20 a^2 b^2 + 5 b^4) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^4}{10 a^2 b^3 d (a+b \operatorname{Sin}[c+d x])} - \frac{7 a \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^5}{30 b^2 d (a+b \operatorname{Sin}[c+d x])} + \frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^6}{6 b d (a+b \operatorname{Sin}[c+d x])}
\end{aligned}$$

Result (type 3, 1110 leaves):

$$\begin{aligned}
& 5 \left(\frac{2 a \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}} + \frac{b \operatorname{Cos}[c+d x]}{(a-b)(a+b)(a+b \operatorname{Sin}[c+d x])} \right) - 2 (c+d x) + \frac{2 a (2 a^2-3 b^2) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}} + \frac{b (-2 a^2+b^2) \operatorname{Cos}[c+d x]}{(a-b)(a+b)(a+b \operatorname{Sin}[c+d x])} \\
& \frac{1}{32 b^4 d} \left(-4 (-6 a^2 + b^2) (c+d x) - \frac{2 a (24 a^4 - 40 a^2 b^2 + 15 b^4) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}} + \right. \\
& \left. 16 a b \operatorname{Cos}[c+d x] + \frac{b (8 a^4 - 8 a^2 b^2 + b^4) \operatorname{Cos}[c+d x]}{(a-b)(a+b)(a+b \operatorname{Sin}[c+d x])} - 2 b^2 \operatorname{Sin}[2(c+d x)] \right) - \frac{1}{96 b^6 d} \\
& \left(-6 (80 a^4 - 48 a^2 b^2 + 3 b^4) (c+d x) + \frac{6 a (160 a^6 - 336 a^4 b^2 + 210 a^2 b^4 - 35 b^6) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}} - 48 a b (8 a^2 - 3 b^2) \operatorname{Cos}[c+d x] + \right. \\
& \left. 16 a b^3 \operatorname{Cos}[3(c+d x)] + \frac{3 b (-32 a^6 + 48 a^4 b^2 - 18 a^2 b^4 + b^6) \operatorname{Cos}[c+d x]}{(a-b)(a+b)(a+b \operatorname{Sin}[c+d x])} - 12 b^2 (-6 a^2 + b^2) \operatorname{Sin}[2(c+d x)] - 3 b^4 \operatorname{Sin}[4(c+d x)] \right) -
\end{aligned}$$

$$\frac{1}{1920 b^8 d} \left(- \frac{30 a (896 a^8 - 2304 a^6 b^2 + 2016 a^4 b^4 - 672 a^2 b^6 + 63 b^8) \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{(a^2 - b^2)^{3/2}} + \right.$$

$$\frac{1}{(a^2 - b^2) (a + b \operatorname{Sin}[c + dx])} (15 b (896 a^8 - 1744 a^6 b^2 + 1024 a^4 b^4 - 179 a^2 b^6 + 4 b^8) \operatorname{Cos}[c + dx] +$$

$$(a^2 - b^2) (13440 a^7 c - 14400 a^5 b^2 c + 3600 a^3 b^4 c - 120 a b^6 c + 13440 a^7 dx - 14400 a^5 b^2 dx + 3600 a^3 b^4 dx -$$

$$120 a b^6 dx + 10 (56 a^4 b^3 - 39 a^2 b^5 + 3 b^7) \operatorname{Cos}[3(c + dx)] + (-42 a^2 b^5 + 10 b^7) \operatorname{Cos}[5(c + dx)] + 5 b^7 \operatorname{Cos}[7(c + dx)] +$$

$$13440 a^6 b c \operatorname{Sin}[c + dx] - 14400 a^4 b^3 c \operatorname{Sin}[c + dx] + 3600 a^2 b^5 c \operatorname{Sin}[c + dx] - 120 b^7 c \operatorname{Sin}[c + dx] + 13440 a^6 b dx \operatorname{Sin}[c + dx] -$$

$$14400 a^4 b^3 dx \operatorname{Sin}[c + dx] + 3600 a^2 b^5 dx \operatorname{Sin}[c + dx] - 120 b^7 dx \operatorname{Sin}[c + dx] + 3360 a^5 b^2 \operatorname{Sin}[2(c + dx)] -$$

$$3040 a^3 b^4 \operatorname{Sin}[2(c + dx)] + 510 a b^6 \operatorname{Sin}[2(c + dx)] - 140 a^3 b^4 \operatorname{Sin}[4(c + dx)] + 66 a b^6 \operatorname{Sin}[4(c + dx)] + 14 a b^6 \operatorname{Sin}[6(c + dx)]) \left. \right)$$

■ **Problem 1266: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^6 \operatorname{Sin}[c + dx]^3}{(a + b \operatorname{Sin}[c + dx])^3} dx$$

Optimal (type 3, 536 leaves, 11 steps):

$$- \frac{(448 a^6 - 600 a^4 b^2 + 180 a^2 b^4 - 5 b^6) x}{16 b^9} + \frac{a \sqrt{a^2 - b^2} (56 a^4 - 47 a^2 b^2 + 6 b^4) \operatorname{ArcTan} \left[\frac{b+a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{b^9 d} -$$

$$\frac{a (840 a^4 - 985 a^2 b^2 + 213 b^4) \operatorname{Cos}[c + dx]}{30 b^8 d} + \frac{(224 a^4 - 244 a^2 b^2 + 43 b^4) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{16 b^7 d} -$$

$$\frac{(280 a^4 - 291 a^2 b^2 + 45 b^4) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^2}{30 a b^6 d} + \frac{(168 a^4 - 169 a^2 b^2 + 24 b^4) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^3}{24 a^2 b^5 d} +$$

$$\frac{\operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^4}{4 a d (a + b \operatorname{Sin}[c + dx])^2} - \frac{b \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^5}{10 a^2 d (a + b \operatorname{Sin}[c + dx])^2} - \frac{(56 a^4 - 60 a^2 b^2 + 9 b^4) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^5}{60 a^2 b^3 d (a + b \operatorname{Sin}[c + dx])^2} -$$

$$\frac{4 a \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^6}{15 b^2 d (a + b \operatorname{Sin}[c + dx])^2} + \frac{\operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^7}{6 b d (a + b \operatorname{Sin}[c + dx])^2} - \frac{(112 a^4 - 110 a^2 b^2 + 15 b^4) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]^4}{20 a^2 b^4 d (a + b \operatorname{Sin}[c + dx])}$$

Result (type 3, 2044 leaves):

$$\frac{1}{64 b^3 d}$$

$$\begin{aligned}
& \left(-8(c+dx) + \frac{2a(8a^4 - 20a^2b^2 + 15b^4) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \frac{ab(4a^2-3b^2) \operatorname{Cos}[c+dx]}{(a-b)(a+b)(a+b \operatorname{Sin}[c+dx])^2} - \frac{3b(4a^4-7a^2b^2+2b^4) \operatorname{Cos}[c+dx]}{(a-b)^2(a+b)^2(a+b \operatorname{Sin}[c+dx])} \right) \\
& - \frac{3 \left(\frac{6ab \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{\operatorname{Cos}[c+dx](a(2a^2+b^2)+b(a^2+2b^2) \operatorname{Sin}[c+dx])}{(a+b \operatorname{Sin}[c+dx])^2} \right)}{256(a-b)^2(a+b)^2d} \\
& - \frac{1}{1024b^7d} 3 \left(\frac{12a(640a^8-1920a^6b^2+2016a^4b^4-840a^2b^6+105b^8) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \right. \\
& \left. \frac{1}{(a^2-b^2)^2(a+b \operatorname{Sin}[c+dx])^2} (-3840a^{10}(c+dx) + 7680a^8b^2(c+dx) - 2976a^6b^4(c+dx) - 1776a^4b^6(c+dx) + \right. \\
& 960a^2b^8(c+dx) - 48b^{10}(c+dx) - 3840a^9b \operatorname{Cos}[c+dx] + 8640a^7b^3 \operatorname{Cos}[c+dx] - 5696a^5b^5 \operatorname{Cos}[c+dx] + \\
& 788a^3b^7 \operatorname{Cos}[c+dx] + 114ab^9 \operatorname{Cos}[c+dx] + 1920a^8b^2(c+dx) \operatorname{Cos}[2(c+dx)] - 4800a^6b^4(c+dx) \operatorname{Cos}[2(c+dx)] + \\
& 3888a^4b^6(c+dx) \operatorname{Cos}[2(c+dx)] - 1056a^2b^8(c+dx) \operatorname{Cos}[2(c+dx)] + 48b^{10}(c+dx) \operatorname{Cos}[2(c+dx)] + 320a^7b^3 \operatorname{Cos}[3(c+dx)] - \\
& 760a^5b^5 \operatorname{Cos}[3(c+dx)] + 560a^3b^7 \operatorname{Cos}[3(c+dx)] - 120ab^9 \operatorname{Cos}[3(c+dx)] - 8a^5b^5 \operatorname{Cos}[5(c+dx)] + 16a^3b^7 \operatorname{Cos}[5(c+dx)] - \\
& 8ab^9 \operatorname{Cos}[5(c+dx)] - 7680a^9b(c+dx) \operatorname{Sin}[c+dx] + 19200a^7b^3(c+dx) \operatorname{Sin}[c+dx] - 15552a^5b^5(c+dx) \operatorname{Sin}[c+dx] + \\
& 4224a^3b^7(c+dx) \operatorname{Sin}[c+dx] - 192ab^9(c+dx) \operatorname{Sin}[c+dx] - 2880a^8b^2 \operatorname{Sin}[2(c+dx)] + 6880a^6b^4 \operatorname{Sin}[2(c+dx)] - \\
& 5182a^4b^6 \operatorname{Sin}[2(c+dx)] + 1221a^2b^8 \operatorname{Sin}[2(c+dx)] - 36b^{10} \operatorname{Sin}[2(c+dx)] - 40a^6b^4 \operatorname{Sin}[4(c+dx)] + 88a^4b^6 \operatorname{Sin}[4(c+dx)] - \\
& \left. 56a^2b^8 \operatorname{Sin}[4(c+dx)] + 8b^{10} \operatorname{Sin}[4(c+dx)] + 2a^4b^6 \operatorname{Sin}[6(c+dx)] - 4a^2b^8 \operatorname{Sin}[6(c+dx)] + 2b^{10} \operatorname{Sin}[6(c+dx)] \right) + \\
& \frac{1}{256} \left(\frac{1}{b^9(a^2-b^2)^{5/2}d} a(14336a^{10} - 49280a^8b^2 + 63360a^6b^4 - 36960a^4b^6 + 9240a^2b^8 - 693b^{10}) \right. \\
& \left. \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right](b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{\sqrt{a^2-b^2}}\right] - \frac{1}{60b^9(-a^2+b^2)^2d(a+b \operatorname{Sin}[c+dx])^2} \right) \\
& (430080a^{12}(c+dx) - 1048320a^{10}b^2(c+dx) + 691200a^8b^4(c+dx) + 83040a^6b^6(c+dx) - 198000a^4b^8(c+dx) + 43200a^2b^{10}(c+dx) - \\
& 1200b^{12}(c+dx) + 430080a^{11}b \operatorname{Cos}[c+dx] - 1155840a^9b^3 \operatorname{Cos}[c+dx] + 1042880a^7b^5 \operatorname{Cos}[c+dx] - 332800a^5b^7 \operatorname{Cos}[c+dx] + \\
& 11060a^3b^9 \operatorname{Cos}[c+dx] + 4530ab^{11} \operatorname{Cos}[c+dx] - 215040a^{10}b^2(c+dx) \operatorname{Cos}[2(c+dx)] + 631680a^8b^4(c+dx) \operatorname{Cos}[2(c+dx)] - \\
& 661440a^6b^6(c+dx) \operatorname{Cos}[2(c+dx)] + 289200a^4b^8(c+dx) \operatorname{Cos}[2(c+dx)] - 45600a^2b^{10}(c+dx) \operatorname{Cos}[2(c+dx)] +
\end{aligned}$$

$$\begin{aligned}
& 1200 b^{12} (c+d x) \operatorname{Cos}[2(c+d x)] - 35840 a^9 b^3 \operatorname{Cos}[3(c+d x)] + 100800 a^7 b^5 \operatorname{Cos}[3(c+d x)] - 98424 a^5 b^7 \operatorname{Cos}[3(c+d x)] + \\
& 37808 a^3 b^9 \operatorname{Cos}[3(c+d x)] - 4344 a b^{11} \operatorname{Cos}[3(c+d x)] + 896 a^7 b^5 \operatorname{Cos}[5(c+d x)] - 2184 a^5 b^7 \operatorname{Cos}[5(c+d x)] + \\
& 1680 a^3 b^9 \operatorname{Cos}[5(c+d x)] - 392 a b^{11} \operatorname{Cos}[5(c+d x)] - 64 a^5 b^7 \operatorname{Cos}[7(c+d x)] + 128 a^3 b^9 \operatorname{Cos}[7(c+d x)] - 64 a b^{11} \operatorname{Cos}[7(c+d x)] + \\
& 860160 a^{11} b (c+d x) \operatorname{Sin}[c+d x] - 2526720 a^9 b^3 (c+d x) \operatorname{Sin}[c+d x] + 2645760 a^7 b^5 (c+d x) \operatorname{Sin}[c+d x] - \\
& 1156800 a^5 b^7 (c+d x) \operatorname{Sin}[c+d x] + 182400 a^3 b^9 (c+d x) \operatorname{Sin}[c+d x] - 4800 a b^{11} (c+d x) \operatorname{Sin}[c+d x] + 322560 a^{10} b^2 \operatorname{Sin}[2(c+d x)] - \\
& 911680 a^8 b^4 \operatorname{Sin}[2(c+d x)] + 903680 a^6 b^6 \operatorname{Sin}[2(c+d x)] - 362830 a^4 b^8 \operatorname{Sin}[2(c+d x)] + 49125 a^2 b^{10} \operatorname{Sin}[2(c+d x)] - \\
& 900 b^{12} \operatorname{Sin}[2(c+d x)] + 4480 a^8 b^4 \operatorname{Sin}[4(c+d x)] - 11816 a^6 b^6 \operatorname{Sin}[4(c+d x)] + 10392 a^4 b^8 \operatorname{Sin}[4(c+d x)] - \\
& 3256 a^2 b^{10} \operatorname{Sin}[4(c+d x)] + 200 b^{12} \operatorname{Sin}[4(c+d x)] - 224 a^6 b^6 \operatorname{Sin}[6(c+d x)] + 498 a^4 b^8 \operatorname{Sin}[6(c+d x)] - \\
& 324 a^2 b^{10} \operatorname{Sin}[6(c+d x)] + 50 b^{12} \operatorname{Sin}[6(c+d x)] + 20 a^4 b^8 \operatorname{Sin}[8(c+d x)] - 40 a^2 b^{10} \operatorname{Sin}[8(c+d x)] + 20 b^{12} \operatorname{Sin}[8(c+d x)] \Big)
\end{aligned}$$

■ **Problem 1267: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x]^6 \operatorname{Sin}[c+d x]^2}{(a+b \operatorname{Sin}[c+d x])^3} dx$$

Optimal (type 3, 485 leaves, 10 steps):

$$\begin{aligned}
& \frac{a(168 a^4 - 200 a^2 b^2 + 45 b^4) x}{8 b^8} - \frac{\sqrt{a^2 - b^2} (42 a^4 - 29 a^2 b^2 + 2 b^4) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2 - b^2}}\right]}{b^8 d} + \frac{(630 a^4 - 645 a^2 b^2 + 91 b^4) \operatorname{Cos}[c+d x]}{30 b^7 d} - \\
& \frac{(84 a^4 - 79 a^2 b^2 + 8 b^4) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{8 a b^6 d} + \frac{(210 a^4 - 187 a^2 b^2 + 15 b^4) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^2}{30 a^2 b^5 d} + \\
& \frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^3}{3 a d (a+b \operatorname{Sin}[c+d x])^2} - \frac{b \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^4}{12 a^2 d (a+b \operatorname{Sin}[c+d x])^2} - \frac{(63 a^4 - 60 a^2 b^2 + 5 b^4) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^4}{60 a^2 b^3 d (a+b \operatorname{Sin}[c+d x])^2} - \\
& \frac{7 a \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^5}{20 b^2 d (a+b \operatorname{Sin}[c+d x])^2} + \frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^6}{5 b d (a+b \operatorname{Sin}[c+d x])^2} - \frac{(63 a^4 - 54 a^2 b^2 + 4 b^4) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]^3}{12 a^2 b^4 d (a+b \operatorname{Sin}[c+d x])}
\end{aligned}$$

Result (type 3, 1913 leaves):

$$\begin{aligned}
& -\frac{1}{64 b^4 d} \left(-48 a (c+d x) + \frac{6(16 a^6 - 40 a^4 b^2 + 30 a^2 b^4 - 5 b^6) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2 - b^2}}\right]}{(a^2 - b^2)^{5/2}} - \right. \\
& \left. 16 b \operatorname{Cos}[c+d x] + \frac{b(8 a^4 - 8 a^2 b^2 + b^4) \operatorname{Cos}[c+d x]}{(a-b)(a+b)(a+b \operatorname{Sin}[c+d x])^2} + \frac{a b(-40 a^4 + 72 a^2 b^2 - 29 b^4) \operatorname{Cos}[c+d x]}{(a-b)^2 (a+b)^2 (a+b \operatorname{Sin}[c+d x])} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{5 \left(\frac{2(2a^2+b^2) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \frac{b \operatorname{Cos}[c+dx] (4a^2-b^2+3ab \operatorname{Sin}[c+dx])}{(a-b)^2 (a+b)^2 (a+b \operatorname{Sin}[c+dx])^2} \right)}{256 d} + \frac{-\frac{6b^2 \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{\operatorname{Cos}[c+dx] (-b(2a^2+b^2)+a(2a^2-5b^2) \operatorname{Sin}[c+dx])}{(a+b \operatorname{Sin}[c+dx])^2}}{64 (a-b)^2 (a+b)^2 d} - \\
& \frac{1}{384 b^6 d} \left(-\frac{12(640a^8 - 1792a^6 b^2 + 1680a^4 b^4 - 560a^2 b^6 + 35b^8) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \right. \\
& \left. \frac{1}{(a^2-b^2)^2 (a+b \operatorname{Sin}[c+dx])^2} (3840a^9 (c+dx) - 6912a^7 b^2 (c+dx) + 1728a^5 b^4 (c+dx) + 1920a^3 b^6 (c+dx) - 576ab^8 (c+dx) + \right. \\
& 3840a^8 b \operatorname{Cos}[c+dx] - 7872a^6 b^3 \operatorname{Cos}[c+dx] + 4256a^4 b^5 \operatorname{Cos}[c+dx] - 172a^2 b^7 \operatorname{Cos}[c+dx] - 70b^9 \operatorname{Cos}[c+dx] - \\
& 1920a^7 b^2 (c+dx) \operatorname{Cos}[2(c+dx)] + 4416a^5 b^4 (c+dx) \operatorname{Cos}[2(c+dx)] - 3072a^3 b^6 (c+dx) \operatorname{Cos}[2(c+dx)] + \\
& 576ab^8 (c+dx) \operatorname{Cos}[2(c+dx)] - 320a^6 b^3 \operatorname{Cos}[3(c+dx)] + 696a^4 b^5 \operatorname{Cos}[3(c+dx)] - 432a^2 b^7 \operatorname{Cos}[3(c+dx)] + 56b^9 \operatorname{Cos}[3(c+dx)] + \\
& 8a^4 b^5 \operatorname{Cos}[5(c+dx)] - 16a^2 b^7 \operatorname{Cos}[5(c+dx)] + 8b^9 \operatorname{Cos}[5(c+dx)] + 7680a^8 b (c+dx) \operatorname{Sin}[c+dx] - 17664a^6 b^3 (c+dx) \operatorname{Sin}[c+dx] + \\
& 12288a^4 b^5 (c+dx) \operatorname{Sin}[c+dx] - 2304a^2 b^7 (c+dx) \operatorname{Sin}[c+dx] + 2880a^7 b^2 \operatorname{Sin}[2(c+dx)] - 6304a^5 b^4 \operatorname{Sin}[2(c+dx)] + \\
& \left. 4022a^3 b^6 \operatorname{Sin}[2(c+dx)] - 607ab^8 \operatorname{Sin}[2(c+dx)] + 40a^5 b^4 \operatorname{Sin}[4(c+dx)] - 80a^3 b^6 \operatorname{Sin}[4(c+dx)] + 40ab^8 \operatorname{Sin}[4(c+dx)] \right) + \\
& \frac{1}{128} \left(-\frac{1}{b^8 (a^2-b^2)^{5/2} d} 3(1792a^{10} - 5760a^8 b^2 + 6720a^6 b^4 - 3360a^4 b^6 + 630a^2 b^8 - 21b^{10}) \right. \\
& \left. \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] (b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{\sqrt{a^2-b^2}}\right] - \right. \\
& \left. \frac{1}{20b^8 (-a^2+b^2)^2 d (a+b \operatorname{Sin}[c+dx])^2} (-53760a^{11} (c+dx) + 119040a^9 b^2 (c+dx) - 62400a^7 b^4 (c+dx) - 19680a^5 b^6 (c+dx) + \right. \\
& 19200a^3 b^8 (c+dx) - 2400ab^{10} (c+dx) - 53760a^{10} b \operatorname{Cos}[c+dx] + 132480a^8 b^3 \operatorname{Cos}[c+dx] - 103360a^6 b^5 \operatorname{Cos}[c+dx] + \\
& 23800a^4 b^7 \operatorname{Cos}[c+dx] + 1080a^2 b^9 \operatorname{Cos}[c+dx] - 210b^{11} \operatorname{Cos}[c+dx] + 26880a^9 b^2 (c+dx) \operatorname{Cos}[2(c+dx)] - \\
& 72960a^7 b^4 (c+dx) \operatorname{Cos}[2(c+dx)] + 67680a^5 b^6 (c+dx) \operatorname{Cos}[2(c+dx)] - 24000a^3 b^8 (c+dx) \operatorname{Cos}[2(c+dx)] + \\
& 2400ab^{10} (c+dx) \operatorname{Cos}[2(c+dx)] + 4480a^8 b^3 \operatorname{Cos}[3(c+dx)] - 11600a^6 b^5 \operatorname{Cos}[3(c+dx)] + 9928a^4 b^7 \operatorname{Cos}[3(c+dx)] - \\
& 2976a^2 b^9 \operatorname{Cos}[3(c+dx)] + 168b^{11} \operatorname{Cos}[3(c+dx)] - 112a^6 b^5 \operatorname{Cos}[5(c+dx)] + 248a^4 b^7 \operatorname{Cos}[5(c+dx)] - 160a^2 b^9 \operatorname{Cos}[5(c+dx)] + \\
& 24b^{11} \operatorname{Cos}[5(c+dx)] + 8a^4 b^7 \operatorname{Cos}[7(c+dx)] - 16a^2 b^9 \operatorname{Cos}[7(c+dx)] + 8b^{11} \operatorname{Cos}[7(c+dx)] - 107520a^{10} b (c+dx) \operatorname{Sin}[c+dx] + \\
& 291840a^8 b^3 (c+dx) \operatorname{Sin}[c+dx] - 270720a^6 b^5 (c+dx) \operatorname{Sin}[c+dx] + 96000a^4 b^7 (c+dx) \operatorname{Sin}[c+dx] - \\
& 9600a^2 b^9 (c+dx) \operatorname{Sin}[c+dx] - 40320a^9 b^2 \operatorname{Sin}[2(c+dx)] + 104960a^7 b^4 \operatorname{Sin}[2(c+dx)] - 91460a^5 b^6 \operatorname{Sin}[2(c+dx)] +
\end{aligned}$$

$$\left. \begin{aligned} & 29160 a^3 b^8 \sin[2(c+dx)] - 2325 a b^{10} \sin[2(c+dx)] - 560 a^7 b^4 \sin[4(c+dx)] + 1352 a^5 b^6 \sin[4(c+dx)] - \\ & 1024 a^3 b^8 \sin[4(c+dx)] + 232 a b^{10} \sin[4(c+dx)] + 28 a^5 b^6 \sin[6(c+dx)] - 56 a^3 b^8 \sin[6(c+dx)] + 28 a b^{10} \sin[6(c+dx)] \end{aligned} \right\}$$

■ **Problem 1268: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^6 \sin[c+dx]}{(a+b\sin[c+dx])^3} dx$$

Optimal (type 3, 237 leaves, 7 steps):

$$\begin{aligned} & -\frac{15(8a^4 - 8a^2b^2 + b^4)x}{8b^7} + \frac{15a(2a^4 - 3a^2b^2 + b^4) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{b^7 \sqrt{a^2-b^2} d} + \frac{\cos[c+dx]^5 (3a+b\sin[c+dx])}{4b^2 d (a+b\sin[c+dx])^2} + \\ & \frac{5 \cos[c+dx]^3 (4a^2 - b^2 + ab\sin[c+dx])}{4b^4 d (a+b\sin[c+dx])} - \frac{15 \cos[c+dx] (4a(2a^2 - b^2) - b(4a^2 - b^2) \sin[c+dx])}{8b^6 d} \end{aligned}$$

Result (type 3, 1250 leaves):

$$\begin{aligned} & \frac{1}{256d} \left(\frac{1}{b^3} 18 \left(-8(c+dx) + \frac{2a(8a^4 - 20a^2b^2 + 15b^4) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \frac{ab(4a^2 - 3b^2) \cos[c+dx]}{(a-b)(a+b)(a+b\sin[c+dx])^2} - \right. \right. \\ & \left. \left. \frac{3b(4a^4 - 7a^2b^2 + 2b^4) \cos[c+dx]}{(a-b)^2 (a+b)^2 (a+b\sin[c+dx])} - \frac{10 \left(\frac{6ab \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{\cos[c+dx] (a(2a^2+b^2) + b(a^2+2b^2) \sin[c+dx])}{(a+b\sin[c+dx])^2} \right)}{(a-b)^2 (a+b)^2} + \frac{1}{b^5} \right) \right. \\ & \left. 10 \left(-24(-8a^2 + b^2)(c+dx) - \frac{6a(64a^6 - 168a^4b^2 + 140a^2b^4 - 35b^6) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + 96ab \cos[c+dx] + \right. \right. \end{aligned}$$

$$\left. \begin{aligned}
& \frac{a b \left(-16 a^4 + 20 a^2 b^2 - 5 b^4 \right) \operatorname{Cos}[c+d x]}{(a-b)(a+b)(a+b \operatorname{Sin}[c+d x])^2} + \frac{b \left(112 a^6 - 220 a^4 b^2 + 115 a^2 b^4 - 10 b^6 \right) \operatorname{Cos}[c+d x]}{(a-b)^2(a+b)^2(a+b \operatorname{Sin}[c+d x])} - 8 b^2 \operatorname{Sin}[2(c+d x)] \right\} + \\
& \frac{1}{b^7} \left(\frac{12 a \left(640 a^8 - 1920 a^6 b^2 + 2016 a^4 b^4 - 840 a^2 b^6 + 105 b^8 \right) \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x) \right]}{\sqrt{a^2-b^2}} \right]}{(a^2-b^2)^{5/2}} + \right. \\
& \frac{1}{(a^2-b^2)^2(a+b \operatorname{Sin}[c+d x])^2} \left(-3840 a^{10}(c+d x) + 7680 a^8 b^2(c+d x) - 2976 a^6 b^4(c+d x) - 1776 a^4 b^6(c+d x) + \right. \\
& 960 a^2 b^8(c+d x) - 48 b^{10}(c+d x) - 3840 a^9 b \operatorname{Cos}[c+d x] + 8640 a^7 b^3 \operatorname{Cos}[c+d x] - 5696 a^5 b^5 \operatorname{Cos}[c+d x] + \\
& 788 a^3 b^7 \operatorname{Cos}[c+d x] + 114 a b^9 \operatorname{Cos}[c+d x] + 1920 a^8 b^2(c+d x) \operatorname{Cos}[2(c+d x)] - 4800 a^6 b^4(c+d x) \operatorname{Cos}[2(c+d x)] + \\
& 3888 a^4 b^6(c+d x) \operatorname{Cos}[2(c+d x)] - 1056 a^2 b^8(c+d x) \operatorname{Cos}[2(c+d x)] + 48 b^{10}(c+d x) \operatorname{Cos}[2(c+d x)] + 320 a^7 b^3 \operatorname{Cos}[3(c+d x)] - \\
& 760 a^5 b^5 \operatorname{Cos}[3(c+d x)] + 560 a^3 b^7 \operatorname{Cos}[3(c+d x)] - 120 a b^9 \operatorname{Cos}[3(c+d x)] - 8 a^5 b^5 \operatorname{Cos}[5(c+d x)] + 16 a^3 b^7 \operatorname{Cos}[5(c+d x)] - \\
& 8 a b^9 \operatorname{Cos}[5(c+d x)] - 7680 a^9 b(c+d x) \operatorname{Sin}[c+d x] + 19200 a^7 b^3(c+d x) \operatorname{Sin}[c+d x] - 15552 a^5 b^5(c+d x) \operatorname{Sin}[c+d x] + \\
& 4224 a^3 b^7(c+d x) \operatorname{Sin}[c+d x] - 192 a b^9(c+d x) \operatorname{Sin}[c+d x] - 2880 a^8 b^2 \operatorname{Sin}[2(c+d x)] + 6880 a^6 b^4 \operatorname{Sin}[2(c+d x)] - \\
& 5182 a^4 b^6 \operatorname{Sin}[2(c+d x)] + 1221 a^2 b^8 \operatorname{Sin}[2(c+d x)] - 36 b^{10} \operatorname{Sin}[2(c+d x)] - 40 a^6 b^4 \operatorname{Sin}[4(c+d x)] + 88 a^4 b^6 \operatorname{Sin}[4(c+d x)] - \\
& \left. \left. \left. 56 a^2 b^8 \operatorname{Sin}[4(c+d x)] + 8 b^{10} \operatorname{Sin}[4(c+d x)] + 2 a^4 b^6 \operatorname{Sin}[6(c+d x)] - 4 a^2 b^8 \operatorname{Sin}[6(c+d x)] + 2 b^{10} \operatorname{Sin}[6(c+d x)] \right) \right) \right)
\end{aligned} \right)$$

- **Problem 1276: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e+f x]^6}{\sqrt{d \operatorname{Sin}[e+f x]}(a+b \operatorname{Sin}[e+f x])^{13/2}} dx$$

Optimal (type 4, 712 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 \operatorname{Cos}[e + f x]^5 \sqrt{d \operatorname{Sin}[e + f x]}}{11 a d f (a + b \operatorname{Sin}[e + f x])^{11/2}} - \frac{20 (a^2 - b^2) \operatorname{Cos}[e + f x] \sqrt{d \operatorname{Sin}[e + f x]}}{99 a^2 b^2 d f (a + b \operatorname{Sin}[e + f x])^{9/2}} + \\
& \frac{80 (3 a^2 + 2 b^2) \operatorname{Cos}[e + f x] \sqrt{d \operatorname{Sin}[e + f x]}}{693 a^3 b^2 d f (a + b \operatorname{Sin}[e + f x])^{7/2}} - \frac{4 (5 a^4 - 17 a^2 b^2 + 16 b^4) \operatorname{Cos}[e + f x] \sqrt{d \operatorname{Sin}[e + f x]}}{231 a^4 b^2 (a^2 - b^2) d f (a + b \operatorname{Sin}[e + f x])^{5/2}} - \\
& \frac{8 (5 a^6 - 22 a^4 b^2 + 65 a^2 b^4 - 32 b^6) \operatorname{Cos}[e + f x] \sqrt{d \operatorname{Sin}[e + f x]}}{693 a^5 b^2 (a^2 - b^2)^2 d f (a + b \operatorname{Sin}[e + f x])^{3/2}} + \frac{16 b (93 a^4 - 93 a^2 b^2 + 32 b^4) \operatorname{Cos}[e + f x]}{693 a^5 (a^2 - b^2)^3 f \sqrt{d \operatorname{Sin}[e + f x]} \sqrt{a + b \operatorname{Sin}[e + f x]}} - \\
& \left(\frac{16 b (93 a^4 - 93 a^2 b^2 + 32 b^4) \sqrt{\frac{a (1 - \operatorname{Csc}[e + f x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Csc}[e + f x])}{a - b}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{d \operatorname{Sin}[e + f x]}}\right], -\frac{a + b}{a - b}\right] \operatorname{Tan}[e + f x]}{\right) / \\
& \left(\frac{693 a^7 (a - b)^2 (a + b)^{5/2} \sqrt{d} f}{16 (45 a^4 - 48 a^3 b - 69 a^2 b^2 + 24 a b^3 + 32 b^4) \sqrt{\frac{a (1 - \operatorname{Csc}[e + f x])}{a + b}} \right. \\
& \left. \sqrt{\frac{a (1 + \operatorname{Csc}[e + f x])}{a - b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{d \operatorname{Sin}[e + f x]}}\right], -\frac{a + b}{a - b}\right] \operatorname{Tan}[e + f x] \right) / \left(693 a^6 (a - b)^2 (a + b)^{5/2} \sqrt{d} f \right)
\end{aligned}$$

Result (type 4, 1906 leaves):

$$\begin{aligned}
& \frac{1}{f \sqrt{d \operatorname{Sin}[e + f x]}} \operatorname{Sin}[e + f x] \sqrt{a + b \operatorname{Sin}[e + f x]} \left(\frac{2 (a^4 \operatorname{Cos}[e + f x] - 2 a^2 b^2 \operatorname{Cos}[e + f x] + b^4 \operatorname{Cos}[e + f x])}{11 a b^4 (a + b \operatorname{Sin}[e + f x])^6} - \right. \\
& \frac{4 (18 a^4 \operatorname{Cos}[e + f x] - 13 a^2 b^2 \operatorname{Cos}[e + f x] - 5 b^4 \operatorname{Cos}[e + f x])}{99 a^2 b^4 (a + b \operatorname{Sin}[e + f x])^5} + \frac{4 (189 a^4 \operatorname{Cos}[e + f x] - 3 a^2 b^2 \operatorname{Cos}[e + f x] + 40 b^4 \operatorname{Cos}[e + f x])}{693 a^3 b^4 (a + b \operatorname{Sin}[e + f x])^4} - \\
& \frac{4 (42 a^6 \operatorname{Cos}[e + f x] - 37 a^4 b^2 \operatorname{Cos}[e + f x] - 17 a^2 b^4 \operatorname{Cos}[e + f x] + 16 b^6 \operatorname{Cos}[e + f x])}{231 a^4 b^4 (a^2 - b^2) (a + b \operatorname{Sin}[e + f x])^3} + \\
& \left. \frac{2 (63 a^8 \operatorname{Cos}[e + f x] - 146 a^6 b^2 \operatorname{Cos}[e + f x] + 151 a^4 b^4 \operatorname{Cos}[e + f x] - 260 a^2 b^6 \operatorname{Cos}[e + f x] + 128 b^8 \operatorname{Cos}[e + f x])}{693 a^5 b^4 (a^2 - b^2)^2 (a + b \operatorname{Sin}[e + f x])^2} - \frac{16 (93 a^4 b^2 \operatorname{Cos}[e + f x] - 93 a^2 b^4 \operatorname{Cos}[e + f x] + 32 b^6 \operatorname{Cos}[e + f x])}{693 a^6 (a^2 - b^2)^3 (a + b \operatorname{Sin}[e + f x])} \right) + \\
& \frac{1}{693 a^6 (a - b)^3 (a + b)^3 f \sqrt{d \operatorname{Sin}[e + f x]}} 8 \sqrt{\operatorname{Sin}[e + f x]}
\end{aligned}$$

$$\left(\left(4 a (45 a^6 - 114 a^4 b^2 + 101 a^2 b^4 - 32 b^6) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (a+b \operatorname{Sin}[e+f x])}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \right. \right.$$

$$\left. \left. \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Sin}[e+f x]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (a+b \operatorname{Sin}[e+f x])}{a}}}\right] \right) /$$

$$\left((a+b) \sqrt{\operatorname{Sin}[e+f x]} \sqrt{a+b \operatorname{Sin}[e+f x]} \right) + 4 a (-93 a^5 b + 93 a^3 b^3 - 32 a b^5)$$

$$\left(\left(\sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (a+b \operatorname{Sin}[e+f x])}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^4 \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Sin}[e+f x]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (a+b \operatorname{Sin}[e+f x])}{a}}}\right) / \left((a+b) \sqrt{\operatorname{Sin}[e+f x]} \right. \right.$$

$$\left. \left. \sqrt{a+b \operatorname{Sin}[e+f x]} \right) - \left(\sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{-a+b}} \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (a+b \operatorname{Sin}[e+f x])}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \right. \right.$$

$$\left. \left. \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Sin}[e+f x]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 (a+b \operatorname{Sin}[e+f x])}{a}}}\right] \right) /$$

$$\left(b \sqrt{\operatorname{Sin}[e+f x]} \sqrt{a+b \operatorname{Sin}[e+f x]} \right) + 2 (93 a^4 b^2 - 93 a^2 b^4 + 32 b^6) \left(\frac{\operatorname{Cos}[e+f x] \sqrt{a+b \operatorname{Sin}[e+f x]}}{b \sqrt{\operatorname{Sin}[e+f x]}} + \right.$$

$$\begin{aligned}
& \left(i \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \operatorname{Csc}[e + f x] \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\frac{\sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{\sin[e + f x]}} \right], -\frac{2a}{-a-b} \right] \sqrt{a + b \sin[e + f x]} \right) / \\
& \left(b \sqrt{\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Csc}[e + f x]} \sqrt{\frac{\operatorname{Csc}[e + f x] (a + b \sin[e + f x])}{a + b}} \right) + \\
& \frac{1}{b} 2a \left(\left(a \sqrt{\frac{(a + b) \cot \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{-a + b}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{\operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (a + b \sin[e + f x])}{a}}}{\sqrt{2}}}} \right], \right. \right. \\
& \left. \left. -\frac{2a}{-a + b} \right] \operatorname{Sec}[e + f x] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^4 \sqrt{-\frac{(a + b) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e + f x]}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{\operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (a + b \sin[e + f x])}{a}} \right) / \left((a + b) \sqrt{\sin[e + f x]} \sqrt{a + b \sin[e + f x]} \right) - \right. \\
& \left(a \sqrt{\frac{(a + b) \cot \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{-a + b}} \operatorname{EllipticPi} \left[-\frac{a}{b}, \operatorname{ArcSin} \left[\frac{\sqrt{\frac{\operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (a + b \sin[e + f x])}{a}}}{\sqrt{2}}}} \right], -\frac{2a}{-a + b} \right] \operatorname{Sec}[\right. \\
& \left. e + f x \right] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^4 \sqrt{-\frac{(a + b) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \sin[e + f x]}{a}} \right. \\
& \left. \left. \sqrt{\frac{\operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (a + b \sin[e + f x])}{a}} \right) / \left(b \sqrt{\sin[e + f x]} \sqrt{a + b \sin[e + f x]} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 1277: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b \sin[e + f x])^2}{(g \cos[e + f x])^{5/2} \sqrt{d \sin[e + f x]}} dx$$

Optimal (type 4, 159 leaves, 9 steps):

$$\frac{2(a^2 + b^2)\sqrt{d\sin[e+fx]}}{3dfg(g\cos[e+fx])^{3/2}} + \frac{4ab(d\sin[e+fx])^{3/2}}{3d^2fg(g\cos[e+fx])^{3/2}} + \frac{(2a^2 - b^2)\operatorname{EllipticF}\left[\frac{1}{4}(4e - \pi) + fx, 2\right]\sqrt{\sin[2e + 2fx]}}{3fg^2\sqrt{g\cos[e+fx]}\sqrt{d\sin[e+fx]}}$$

Result (type 5, 142 leaves):

$$\left(2 \left(-2(2a^2 - b^2)\cos[e+fx]^2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+fx]^2\right] + \right. \right. \\ \left. \left. (\sin[e+fx]^2)^{1/4} (a^2 + b^2 + (2a^2 - b^2)\cos[e+fx]^2 + 2ab\sin[e+fx]) \right) \tan[e+fx] \right) / \\ \left(3fg^2\sqrt{g\cos[e+fx]}\sqrt{d\sin[e+fx]}(\sin[e+fx]^2)^{1/4} \right)$$

■ **Problem 1278: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b\sin[e+fx])^2}{(g\cos[e+fx])^{7/2}\sqrt{d\sin[e+fx]}} dx$$

Optimal (type 4, 193 leaves, 6 steps):

$$\frac{8a^2\sqrt{d\sin[e+fx]}}{5dfg^3\sqrt{g\cos[e+fx]}} + \frac{8ab(d\sin[e+fx])^{3/2}}{5d^2fg^3\sqrt{g\cos[e+fx]}} + \\ \frac{2\sqrt{d\sin[e+fx]}(a + b\sin[e+fx])^2}{5dfg(g\cos[e+fx])^{5/2}} - \frac{8ab\sqrt{g\cos[e+fx]}\operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right]\sqrt{d\sin[e+fx]}}{5dfg^4\sqrt{\sin[2e + 2fx]}}$$

Result (type 5, 156 leaves):

$$\left(4ab\operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \cos[e+fx]^2\right]\sin[2(e+fx)]^2 + \right. \\ \left. 3\sec[e+fx](\sin[e+fx]^2)^{3/4}(6a^2 + b^2 + (4a^2 - b^2)\cos[2(e+fx)] + 6ab\sin[e+fx] + 2ab\sin[3(e+fx)])\tan[e+fx] \right) / \\ \left(15fg^3\sqrt{g\cos[e+fx]}\sqrt{d\sin[e+fx]}(\sin[e+fx]^2)^{3/4} \right)$$

■ **Problem 1292: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[c+dx]^2 \csc[c+dx]^2}{a + b\sin[c+dx]} dx$$

Optimal (type 3, 153 leaves, 9 steps):

$$\frac{2 b^2 \sqrt{a^2 - b^2} \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{a^4 d} - \frac{b(a^2 - 2 b^2) \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{2 a^4 d} +$$

$$\frac{(a^2 - 3 b^2) \operatorname{Cot}[c+dx]}{3 a^3 d} + \frac{b \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{2 a^2 d} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{3 a d}$$

Result (type 3, 351 leaves):

$$\frac{2 b^2 \sqrt{a^2 - b^2} \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{a^2-b^2}}\right]}{a^4 d} + \frac{(a^2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - 3 b^2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]}{6 a^3 d} +$$

$$\frac{b \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{8 a^2 d} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{24 a d} + \frac{(-a^2 b + 2 b^3) \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]]}{2 a^4 d} + \frac{(a^2 b - 2 b^3) \operatorname{Log}[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]]}{2 a^4 d} -$$

$$\frac{b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{8 a^2 d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(-a^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 3 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}{6 a^3 d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24 a d}$$

■ **Problem 1293: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^2 \operatorname{Csc}[c+dx]^3}{a + b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 194 leaves, 10 steps):

$$\frac{2 b^3 \sqrt{a^2 - b^2} \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{a^5 d} + \frac{(a^4 + 4 a^2 b^2 - 8 b^4) \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{8 a^5 d} -$$

$$\frac{b(a^2 - 3 b^2) \operatorname{Cot}[c+dx]}{3 a^4 d} + \frac{(a^2 - 4 b^2) \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{8 a^3 d} + \frac{b \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{3 a^2 d} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{4 a d}$$

Result (type 3, 430 leaves):

$$\frac{2 b^3 \sqrt{a^2 - b^2} \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{a^2-b^2}}\right]}{a^5 d} + \frac{(-a^2 b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 3 b^3 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]}{6 a^4 d} +$$

$$\frac{(a^2 - 4 b^2) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{32 a^3 d} + \frac{b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{24 a^2 d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{64 a d} +$$

$$\frac{(a^4 + 4 a^2 b^2 - 8 b^4) \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]]}{8 a^5 d} + \frac{(-a^4 - 4 a^2 b^2 + 8 b^4) \operatorname{Log}[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]]}{8 a^5 d} + \frac{(-a^2 + 4 b^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{32 a^3 d} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}{64 a d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(a^2 b \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 3 b^3 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}{6 a^4 d} - \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24 a^2 d}$$

■ **Problem 1294: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + dx]^2 \text{Csc}[c + dx]^4}{a + b \text{Sin}[c + dx]} dx$$

Optimal (type 3, 238 leaves, 11 steps):

$$\begin{aligned} & - \frac{2 b^4 \sqrt{a^2 - b^2} \text{ArcTan}\left[\frac{b + a \text{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 - b^2}}\right]}{a^6 d} - \frac{b (a^4 + 4 a^2 b^2 - 8 b^4) \text{ArcTanh}[\text{Cos}[c + dx]]}{8 a^6 d} + \frac{(2 a^4 + 5 a^2 b^2 - 15 b^4) \text{Cot}[c + dx]}{15 a^5 d} - \\ & \frac{b (a^2 - 4 b^2) \text{Cot}[c + dx] \text{Csc}[c + dx]}{8 a^4 d} + \frac{(a^2 - 5 b^2) \text{Cot}[c + dx] \text{Csc}[c + dx]^2}{15 a^3 d} + \frac{b \text{Cot}[c + dx] \text{Csc}[c + dx]^3}{4 a^2 d} - \frac{\text{Cot}[c + dx] \text{Csc}[c + dx]^4}{5 a d} \end{aligned}$$

Result (type 3, 506 leaves):

$$\begin{aligned} & \frac{1}{960 a^6 d} \left(-1920 b^4 \sqrt{a^2 - b^2} \text{ArcTan}\left[\frac{b + a \text{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 - b^2}}\right] + 32 (2 a^5 + 5 a^3 b^2 - 15 a b^4) \text{Cot}\left[\frac{1}{2}(c + dx)\right] - \right. \\ & 30 a^4 b \text{Csc}\left[\frac{1}{2}(c + dx)\right]^2 + 120 a^2 b^3 \text{Csc}\left[\frac{1}{2}(c + dx)\right]^2 + 15 a^4 b \text{Csc}\left[\frac{1}{2}(c + dx)\right]^4 - 120 a^4 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] - \\ & 480 a^2 b^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] + 960 b^5 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] + 120 a^4 b \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \\ & 480 a^2 b^3 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - 960 b^5 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + 30 a^4 b \text{Sec}\left[\frac{1}{2}(c + dx)\right]^2 - 120 a^2 b^3 \text{Sec}\left[\frac{1}{2}(c + dx)\right]^2 - \\ & 15 a^4 b \text{Sec}\left[\frac{1}{2}(c + dx)\right]^4 - 16 a^5 \text{Csc}[c + dx]^3 \text{Sin}\left[\frac{1}{2}(c + dx)\right]^4 + 320 a^3 b^2 \text{Csc}[c + dx]^3 \text{Sin}\left[\frac{1}{2}(c + dx)\right]^4 + \\ & a^5 \text{Csc}\left[\frac{1}{2}(c + dx)\right]^4 \text{Sin}[c + dx] - 20 a^3 b^2 \text{Csc}\left[\frac{1}{2}(c + dx)\right]^4 \text{Sin}[c + dx] - 3 a^5 \text{Csc}\left[\frac{1}{2}(c + dx)\right]^6 \text{Sin}[c + dx] - \\ & \left. 64 a^5 \text{Tan}\left[\frac{1}{2}(c + dx)\right] - 160 a^3 b^2 \text{Tan}\left[\frac{1}{2}(c + dx)\right] + 480 a b^4 \text{Tan}\left[\frac{1}{2}(c + dx)\right] + 6 a^5 \text{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \text{Tan}\left[\frac{1}{2}(c + dx)\right] \right) \end{aligned}$$

■ **Problem 1307: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + dx]^4}{a + b \text{Sin}[c + dx]} dx$$

Optimal (type 3, 154 leaves, 7 steps):

$$\begin{aligned} & \frac{2 (a^2 - b^2)^{3/2} \text{ArcTan}\left[\frac{b + a \text{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 - b^2}}\right]}{a^4 d} - \frac{b (3 a^2 - 2 b^2) \text{ArcTanh}[\text{Cos}[c + dx]]}{2 a^4 d} + \\ & \frac{(4 a^2 - 3 b^2) \text{Cot}[c + dx]}{3 a^3 d} + \frac{b \text{Cot}[c + dx] \text{Csc}[c + dx]}{2 a^2 d} - \frac{\text{Cot}[c + dx] \text{Csc}[c + dx]^2}{3 a d} \end{aligned}$$

Result (type 3, 350 leaves) :

$$\frac{2 (a^2 - b^2)^{3/2} \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{a^2 - b^2}}\right]}{a^4 d} + \frac{\left(4 a^2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - 3 b^2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]}{6 a^3 d} +$$

$$\frac{b \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{8 a^2 d} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{24 a d} + \frac{(-3 a^2 b + 2 b^3) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{2 a^4 d} + \frac{(3 a^2 b - 2 b^3) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{2 a^4 d} -$$

$$\frac{b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{8 a^2 d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(-4 a^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 3 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}{6 a^3 d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24 a d}$$

■ **Problem 1308: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^4 \operatorname{Csc}[c+dx]}{a+b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 198 leaves, 8 steps) :

$$\frac{2 b (a^2 - b^2)^{3/2} \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2 - b^2}}\right]}{a^5 d} - \frac{(3 a^4 - 12 a^2 b^2 + 8 b^4) \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{8 a^5 d} -$$

$$\frac{b (4 a^2 - 3 b^2) \operatorname{Cot}[c+dx]}{3 a^4 d} + \frac{(5 a^2 - 4 b^2) \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{8 a^3 d} + \frac{b \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{3 a^2 d} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{4 a d}$$

Result (type 3, 433 leaves) :

$$\frac{2 b (a^2 - b^2)^{3/2} \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{a^2 - b^2}}\right]}{a^5 d} + \frac{\left(-4 a^2 b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 3 b^3 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]}{6 a^4 d} +$$

$$\frac{(5 a^2 - 4 b^2) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{32 a^3 d} + \frac{b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{24 a^2 d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{64 a d} +$$

$$\frac{(-3 a^4 + 12 a^2 b^2 - 8 b^4) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{8 a^5 d} + \frac{(3 a^4 - 12 a^2 b^2 + 8 b^4) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8 a^5 d} + \frac{(-5 a^2 + 4 b^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{32 a^3 d} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}{64 a d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(4 a^2 b \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 3 b^3 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}{6 a^4 d} - \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24 a^2 d}$$

■ **Problem 1309: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^4 \operatorname{Csc}[c+dx]^2}{a+b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 244 leaves, 9 steps) :

$$\frac{2 b^2 (a^2 - b^2)^{3/2} \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{a^6 d} + \frac{b (3 a^4 - 12 a^2 b^2 + 8 b^4) \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{8 a^6 d} - \frac{(3 a^4 - 20 a^2 b^2 + 15 b^4) \operatorname{Cot}[c+d x]}{15 a^5 d} -$$

$$\frac{b (5 a^2 - 4 b^2) \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]}{8 a^4 d} + \frac{(6 a^2 - 5 b^2) \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^2}{15 a^3 d} + \frac{b \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^3}{4 a^2 d} - \frac{\operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^4}{5 a d}$$

Result (type 3, 507 leaves):

$$\frac{1}{960 a^6 d} \left(1920 b^2 (a^2 - b^2)^{3/2} \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right] - 32 (3 a^5 - 20 a^3 b^2 + 15 a b^4) \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] - \right.$$

$$150 a^4 b \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 + 120 a^2 b^3 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 + 15 a^4 b \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4 + 360 a^4 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] -$$

$$1440 a^2 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] + 960 b^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] - 360 a^4 b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] +$$

$$1440 a^2 b^3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - 960 b^5 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 150 a^4 b \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 - 120 a^2 b^3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 -$$

$$15 a^4 b \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 - 336 a^5 \operatorname{Csc}[c+d x]^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 + 320 a^3 b^2 \operatorname{Csc}[c+d x]^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^4 +$$

$$21 a^5 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Sin}[c+d x] - 20 a^3 b^2 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Sin}[c+d x] - 3 a^5 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^6 \operatorname{Sin}[c+d x] +$$

$$96 a^5 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 640 a^3 b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 480 a b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 6 a^5 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \left. \right)$$

■ **Problem 1316: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x] \operatorname{Cot}[c+d x]^4}{a+b \operatorname{Sin}[c+d x]} dx$$

Optimal (type 3, 120 leaves, 4 steps):

$$\frac{(2 a^2 - b^2) \operatorname{Csc}[c+d x]}{a^3 d} + \frac{b \operatorname{Csc}[c+d x]^2}{2 a^2 d} - \frac{\operatorname{Csc}[c+d x]^3}{3 a d} + \frac{b (2 a^2 - b^2) \operatorname{Log}[\operatorname{Sin}[c+d x]]}{a^4 d} + \frac{(a^2 - b^2)^2 \operatorname{Log}[a+b \operatorname{Sin}[c+d x]]}{a^4 b d}$$

Result (type 3, 275 leaves):

$$\frac{(11 a^2 \cos[\frac{1}{2}(c+dx)] - 6 b^2 \cos[\frac{1}{2}(c+dx)]) \operatorname{Csc}[\frac{1}{2}(c+dx)] - b \operatorname{Csc}[\frac{1}{2}(c+dx)]^2}{12 a^3 d} + \frac{b \operatorname{Csc}[\frac{1}{2}(c+dx)]^2}{8 a^2 d} -$$

$$\frac{\operatorname{Cot}[\frac{1}{2}(c+dx)] \operatorname{Csc}[\frac{1}{2}(c+dx)]^2}{24 a d} + \frac{(2 a^2 b - b^3) \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^4 d} + \frac{(a^4 - 2 a^2 b^2 + b^4) \operatorname{Log}[a + b \operatorname{Sin}[c+dx]]}{a^4 b d} +$$

$$\frac{b \operatorname{Sec}[\frac{1}{2}(c+dx)]^2}{8 a^2 d} + \frac{\operatorname{Sec}[\frac{1}{2}(c+dx)] (11 a^2 \operatorname{Sin}[\frac{1}{2}(c+dx)] - 6 b^2 \operatorname{Sin}[\frac{1}{2}(c+dx)])}{12 a^3 d} - \frac{\operatorname{Sec}[\frac{1}{2}(c+dx)]^2 \operatorname{Tan}[\frac{1}{2}(c+dx)]}{24 a d}$$

■ **Problem 1317: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^5}{a + b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 148 leaves, 3 steps):

$$-\frac{b(2a^2 - b^2) \operatorname{Csc}[c+dx]}{a^4 d} + \frac{(2a^2 - b^2) \operatorname{Csc}[c+dx]^2}{2a^3 d} + \frac{b \operatorname{Csc}[c+dx]^3}{3a^2 d} -$$

$$\frac{\operatorname{Csc}[c+dx]^4}{4a d} + \frac{(a^2 - b^2)^2 \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^5 d} - \frac{(a^2 - b^2)^2 \operatorname{Log}[a + b \operatorname{Sin}[c+dx]]}{a^5 d}$$

Result (type 3, 347 leaves):

$$\frac{(-11 a^2 b \cos[\frac{1}{2}(c+dx)] + 6 b^3 \cos[\frac{1}{2}(c+dx)]) \operatorname{Csc}[\frac{1}{2}(c+dx)]}{12 a^4 d} + \frac{(7 a^2 - 4 b^2) \operatorname{Csc}[\frac{1}{2}(c+dx)]^2}{32 a^3 d} + \frac{b \operatorname{Cot}[\frac{1}{2}(c+dx)] \operatorname{Csc}[\frac{1}{2}(c+dx)]^2}{24 a^2 d} -$$

$$\frac{\operatorname{Csc}[\frac{1}{2}(c+dx)]^4}{64 a d} + \frac{(a^4 - 2 a^2 b^2 + b^4) \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^5 d} + \frac{(-a^4 + 2 a^2 b^2 - b^4) \operatorname{Log}[a + b \operatorname{Sin}[c+dx]]}{a^5 d} + \frac{(7 a^2 - 4 b^2) \operatorname{Sec}[\frac{1}{2}(c+dx)]^2}{32 a^3 d} -$$

$$\frac{\operatorname{Sec}[\frac{1}{2}(c+dx)]^4}{64 a d} + \frac{\operatorname{Sec}[\frac{1}{2}(c+dx)] (-11 a^2 b \operatorname{Sin}[\frac{1}{2}(c+dx)] + 6 b^3 \operatorname{Sin}[\frac{1}{2}(c+dx)])}{12 a^4 d} + \frac{b \operatorname{Sec}[\frac{1}{2}(c+dx)]^2 \operatorname{Tan}[\frac{1}{2}(c+dx)]}{24 a^2 d}$$

■ **Problem 1318: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^5 \operatorname{Csc}[c+dx]}{a + b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 179 leaves, 4 steps):

$$-\frac{(a^2 - b^2)^2 \operatorname{Csc}[c+dx]}{a^5 d} - \frac{b(2a^2 - b^2) \operatorname{Csc}[c+dx]^2}{2a^4 d} + \frac{(2a^2 - b^2) \operatorname{Csc}[c+dx]^3}{3a^3 d} +$$

$$\frac{b \operatorname{Csc}[c+dx]^4}{4a^2 d} - \frac{\operatorname{Csc}[c+dx]^5}{5a d} - \frac{b(a^2 - b^2)^2 \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^6 d} + \frac{b(a^2 - b^2)^2 \operatorname{Log}[a + b \operatorname{Sin}[c+dx]]}{a^6 d}$$

Result (type 3, 492 leaves):

$$\begin{aligned}
& \frac{(-89 a^4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 220 a^2 b^2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - 120 b^4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]}{240 a^5 d} + \\
& \frac{(-7 a^2 b + 4 b^3) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{32 a^4 d} + \frac{(31 a^2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - 20 b^2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^3}{480 a^3 d} + \frac{b \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{64 a^2 d} - \\
& \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{160 a d} + \frac{(-a^4 b + 2 a^2 b^3 - b^5) \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^6 d} + \frac{(a^4 b - 2 a^2 b^3 + b^5) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{a^6 d} + \\
& \frac{(-7 a^2 b + 4 b^3) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{32 a^4 d} + \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}{64 a^2 d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (31 a^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 20 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{480 a^3 d} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] (-89 a^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 220 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 120 b^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{240 a^5 d} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{160 a d}
\end{aligned}$$

■ **Problem 1319: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^5 \operatorname{Csc}[c+dx]^2}{a+b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 212 leaves, 4 steps):

$$\begin{aligned}
& \frac{b(a^2 - b^2)^2 \operatorname{Csc}[c+dx]}{a^6 d} - \frac{(a^2 - b^2)^2 \operatorname{Csc}[c+dx]^2}{2 a^5 d} - \frac{b(2 a^2 - b^2) \operatorname{Csc}[c+dx]^3}{3 a^4 d} + \frac{(2 a^2 - b^2) \operatorname{Csc}[c+dx]^4}{4 a^3 d} + \\
& \frac{b \operatorname{Csc}[c+dx]^5}{5 a^2 d} - \frac{\operatorname{Csc}[c+dx]^6}{6 a d} + \frac{b^2 (a^2 - b^2)^2 \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^7 d} - \frac{b^2 (a^2 - b^2)^2 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{a^7 d}
\end{aligned}$$

Result (type 3, 493 leaves):

$$\begin{aligned}
& \frac{1}{1920 a^7 d} \left(8 (89 a^5 b - 220 a^3 b^3 + 120 a b^5) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] - 30 (5 a^6 - 14 a^4 b^2 + 8 a^2 b^4) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 + \right. \\
& 1920 a^4 b^2 \operatorname{Log}[\operatorname{Sin}[c+dx]] - 3840 a^2 b^4 \operatorname{Log}[\operatorname{Sin}[c+dx]] + 1920 b^6 \operatorname{Log}[\operatorname{Sin}[c+dx]] - 1920 a^4 b^2 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] + \\
& 3840 a^2 b^4 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] - 1920 b^6 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]] - 150 a^6 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + 420 a^4 b^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 - \\
& 240 a^2 b^4 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + 45 a^6 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 - 30 a^4 b^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 - 5 a^6 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6 - \\
& 992 a^5 b \operatorname{Csc}[c+dx]^3 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 + 640 a^3 b^3 \operatorname{Csc}[c+dx]^3 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^4 + 384 a^5 b \operatorname{Csc}[c+dx]^5 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^6 + \\
& a^5 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6 (-5 a + 6 b \operatorname{Sin}[c+dx]) + a^3 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 (45 a^3 - 30 a b^2 + (-62 a^2 b + 40 b^3) \operatorname{Sin}[c+dx]) + \\
& \left. 712 a^5 b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 1760 a^3 b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 960 a b^5 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)
\end{aligned}$$

■ **Problem 1327: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x] \cot [c+d x]^5}{a+b \sin [c+d x]} d x$$

Optimal (type 3, 195 leaves, 15 steps):

$$-\frac{x}{b} + \frac{2\left(a^2-b^2\right)^{5/2} \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{a^5 b d} - \frac{\left(15 a^4-20 a^2 b^2+8 b^4\right) \operatorname{ArcTanh}[\cos [c+d x]]}{8 a^5 d} +$$

$$\frac{b\left(-2 a^2+b^2\right) \cot [c+d x]}{a^4 d} + \frac{b \cot [c+d x]^3}{3 a^2 d} + \frac{\left(7 a^2-4 b^2\right) \cot [c+d x] \operatorname{Csc}[c+d x]}{8 a^3 d} - \frac{\cot [c+d x]^3 \operatorname{Csc}[c+d x]}{4 a d}$$

Result (type 3, 448 leaves):

$$-\frac{c+d x}{b d} + \frac{2\left(a^2-b^2\right)^{5/2} \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(b \cos \left[\frac{1}{2}(c+d x)\right]+a \sin \left[\frac{1}{2}(c+d x)\right]\right)}{\sqrt{a^2-b^2}}\right]}{a^5 b d} + \frac{\left(-7 a^2 b \cos \left[\frac{1}{2}(c+d x)\right]+3 b^3 \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]}{6 a^4 d} +$$

$$\frac{\left(9 a^2-4 b^2\right) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{32 a^3 d} + \frac{b \cot \left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{24 a^2 d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4}{64 a d} +$$

$$\frac{\left(-15 a^4+20 a^2 b^2-8 b^4\right) \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]\right]}{8 a^5 d} + \frac{\left(15 a^4-20 a^2 b^2+8 b^4\right) \operatorname{Log}\left[\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 a^5 d} + \frac{\left(-9 a^2+4 b^2\right) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{32 a^3 d} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4}{64 a d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(7 a^2 b \sin \left[\frac{1}{2}(c+d x)\right]-3 b^3 \sin \left[\frac{1}{2}(c+d x)\right]\right)}{6 a^4 d} - \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{24 a^2 d}$$

■ **Problem 1328: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot [c+d x]^6}{a+b \sin [c+d x]} d x$$

Optimal (type 3, 241 leaves, 9 steps):

$$-\frac{2\left(a^2-b^2\right)^{5/2} \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{a^6 d} + \frac{b\left(15 a^4-20 a^2 b^2+8 b^4\right) \operatorname{ArcTanh}[\cos [c+d x]]}{8 a^6 d} - \frac{\left(23 a^4-35 a^2 b^2+15 b^4\right) \cot [c+d x]}{15 a^5 d} +$$

$$\frac{b\left(-9 a^2+4 b^2\right) \cot [c+d x] \operatorname{Csc}[c+d x]}{8 a^4 d} + \frac{\left(11 a^2-5 b^2\right) \cot [c+d x] \operatorname{Csc}[c+d x]^2}{15 a^3 d} + \frac{b \cot [c+d x] \operatorname{Csc}[c+d x]^3}{4 a^2 d} - \frac{\cot [c+d x] \operatorname{Csc}[c+d x]^4}{5 a d}$$

Result (type 3, 504 leaves):

$$\frac{1}{960 a^6 d} \left(-1920 (a^2 - b^2)^{5/2} \operatorname{ArcTan}\left[\frac{b + a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 - b^2}}\right] - 32 (23 a^5 - 35 a^3 b^2 + 15 a b^4) \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right] - \right.$$

$$270 a^4 b \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2 + 120 a^2 b^3 \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2 + 15 a^4 b \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^4 + 1800 a^4 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]\right] -$$

$$2400 a^2 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]\right] + 960 b^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]\right] - 1800 a^4 b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] +$$

$$2400 a^2 b^3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - 960 b^5 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + 270 a^4 b \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 - 120 a^2 b^3 \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 -$$

$$15 a^4 b \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 - 656 a^5 \operatorname{Csc}[c + d x]^3 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]^4 + 320 a^3 b^2 \operatorname{Csc}[c + d x]^3 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]^4 +$$

$$41 a^5 \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sin}[c + d x] - 20 a^3 b^2 \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sin}[c + d x] - 3 a^5 \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^6 \operatorname{Sin}[c + d x] +$$

$$\left. 736 a^5 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 1120 a^3 b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 480 a b^4 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 6 a^5 \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right)$$

■ **Problem 1356: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c + d x] \operatorname{Sec}[c + d x]^4}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 194 leaves, 12 steps):

$$-\frac{2 b^5 \operatorname{ArcTan}\left[\frac{b + a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 - b^2}}\right]}{a (a^2 - b^2)^{5/2} d} - \frac{\operatorname{ArcTanh}[\operatorname{Cos}[c + d x]]}{a d} + \frac{\operatorname{Sec}[c + d x]}{a d} +$$

$$\frac{\operatorname{Sec}[c + d x]^3}{3 a d} + \frac{b \operatorname{Sec}[c + d x]^3 (b - a \operatorname{Sin}[c + d x])}{3 a (a^2 - b^2) d} - \frac{b \operatorname{Sec}[c + d x] (3 b^3 + a (2 a^2 - 5 b^2) \operatorname{Sin}[c + d x])}{3 a (a^2 - b^2)^2 d}$$

Result (type 3, 408 leaves):

$$\begin{aligned}
& \frac{2 b^5 \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(b \cos\left[\frac{1}{2}(c+d x)\right]+a \sin\left[\frac{1}{2}(c+d x)\right]\right)}{\sqrt{a^2-b^2}}\right]}{a\left(a^2-b^2\right)^{5 / 2} d}-\frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right]}{a d}+\frac{\operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{a d}+ \\
& \frac{1}{12(a+b) d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2}+\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6(a+b) d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3}- \\
& \frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6(a-b) d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3}+\frac{1}{12(a-b) d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2}+ \\
& \frac{-7 a \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+10 b \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6(a-b)^2 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}+\frac{7 a \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+10 b \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6(a+b)^2 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}
\end{aligned}$$

■ **Problem 1357: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c+d x]^2 \operatorname{Sec}[c+d x]^4}{a+b \operatorname{Sin}[c+d x]} d x$$

Optimal (type 3, 220 leaves, 15 steps):

$$\begin{aligned}
& \frac{2 b^6 \operatorname{ArcTan}\left[\frac{b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{a^2\left(a^2-b^2\right)^{5 / 2} d}+\frac{b \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{a^2 d}-\frac{\operatorname{Cot}[c+d x]}{a d}+ \\
& \frac{b\left(-a^2+2 b^2\right) \operatorname{Sec}[c+d x]}{\left(a^2-b^2\right)^2 d}+\frac{b \operatorname{Sec}[c+d x]^3(-a+b \operatorname{Sin}[c+d x])}{3 a\left(a^2-b^2\right) d}+\frac{\left(6 a^4-10 a^2 b^2+b^4\right) \operatorname{Tan}[c+d x]}{3 a\left(a^2-b^2\right)^2 d}+\frac{\operatorname{Tan}[c+d x]^3}{3 a d}
\end{aligned}$$

Result (type 3, 450 leaves):

$$\begin{aligned}
& \frac{2 b^6 \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(b \cos\left[\frac{1}{2}(c+d x)\right]+a \sin\left[\frac{1}{2}(c+d x)\right]\right)}{\sqrt{a^2-b^2}}\right]}{a^2\left(a^2-b^2\right)^{5 / 2} d}-\frac{\operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]}{2 a d}+\frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right]}{a^2 d}- \\
& \frac{b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{a^2 d}+\frac{1}{12(a+b) d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2}+\frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6(a+b) d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3}+ \\
& \frac{\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6(a-b) d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3}-\frac{1}{12(a-b) d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2}+ \\
& \frac{10 a \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]-13 b \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6(a-b)^2 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}+\frac{10 a \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+13 b \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6(a+b)^2 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}+\frac{\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2 a d}
\end{aligned}$$

■ **Problem 1358: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + d x]^3 \text{Sec}[c + d x]^4}{a + b \text{Sin}[c + d x]} dx$$

Optimal (type 3, 332 leaves, 20 steps):

$$\begin{aligned} & - \frac{2 b^7 \text{ArcTan}\left[\frac{b+a \text{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right]}{a^3 (a^2-b^2)^{5/2} d} - \frac{5 \text{ArcTanh}[\text{Cos}[c+d x]]}{2 a d} - \frac{b^2 \text{ArcTanh}[\text{Cos}[c+d x]]}{a^3 d} + \\ & \frac{b \text{Cot}[c+d x]}{a^2 d} + \frac{5 \text{Sec}[c+d x]}{2 a d} + \frac{b^2 \text{Sec}[c+d x]}{a^3 d} + \frac{5 \text{Sec}[c+d x]^3}{6 a d} + \frac{b^2 \text{Sec}[c+d x]^3}{3 a^3 d} - \frac{\text{Csc}[c+d x]^2 \text{Sec}[c+d x]^3}{2 a d} + \\ & \frac{b^3 \text{Sec}[c+d x]^3 (b-a \text{Sin}[c+d x])}{3 a^3 (a^2-b^2) d} - \frac{b^3 \text{Sec}[c+d x] (3 b^3 + a (2 a^2 - 5 b^2) \text{Sin}[c+d x])}{3 a^3 (a^2-b^2)^2 d} - \frac{2 b \text{Tan}[c+d x]}{a^2 d} - \frac{b \text{Tan}[c+d x]^3}{3 a^2 d} \end{aligned}$$

Result (type 3, 947 leaves):

$$\begin{aligned}
& 16 \left(\frac{a (13 a^2 - 19 b^2) \operatorname{Csc}[c + d x] (a + b \operatorname{Sin}[c + d x])}{96 (a^2 - b^2)^2 d (b + a \operatorname{Csc}[c + d x])} - \right. \\
& \frac{b^7 \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] (b \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + a \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])}{\sqrt{a^2 - b^2}}\right] \operatorname{Csc}[c + d x] (a + b \operatorname{Sin}[c + d x])}{8 a^3 (a^2 - b^2)^{5/2} d (b + a \operatorname{Csc}[c + d x])} + \frac{b \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}[c + d x] (a + b \operatorname{Sin}[c + d x])}{32 a^2 d (b + a \operatorname{Csc}[c + d x])} - \\
& \frac{\operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Csc}[c + d x] (a + b \operatorname{Sin}[c + d x])}{128 a d (b + a \operatorname{Csc}[c + d x])} + \frac{(-5 a^2 - 2 b^2) \operatorname{Csc}[c + d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sin}[c + d x])}{32 a^3 d (b + a \operatorname{Csc}[c + d x])} + \\
& \frac{(5 a^2 + 2 b^2) \operatorname{Csc}[c + d x] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sin}[c + d x])}{32 a^3 d (b + a \operatorname{Csc}[c + d x])} + \frac{\operatorname{Csc}[c + d x] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 (a + b \operatorname{Sin}[c + d x])}{128 a d (b + a \operatorname{Csc}[c + d x])} + \\
& \frac{\operatorname{Csc}[c + d x] (a + b \operatorname{Sin}[c + d x])}{192 (a + b) d (b + a \operatorname{Csc}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^2} + \frac{\operatorname{Csc}[c + d x] \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a + b \operatorname{Sin}[c + d x])}{96 (a + b) d (b + a \operatorname{Csc}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^3} - \\
& \frac{\operatorname{Csc}[c + d x] \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a + b \operatorname{Sin}[c + d x])}{96 (a - b) d (b + a \operatorname{Csc}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^3} + \frac{\operatorname{Csc}[c + d x] (a + b \operatorname{Sin}[c + d x])}{192 (a - b) d (b + a \operatorname{Csc}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^2} + \\
& \frac{\operatorname{Csc}[c + d x] (-13 a \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 16 b \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]) (a + b \operatorname{Sin}[c + d x])}{96 (a - b)^2 d (b + a \operatorname{Csc}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])} + \\
& \left. \frac{\operatorname{Csc}[c + d x] (13 a \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 16 b \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]) (a + b \operatorname{Sin}[c + d x])}{96 (a + b)^2 d (b + a \operatorname{Csc}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])} - \frac{b \operatorname{Csc}[c + d x] (a + b \operatorname{Sin}[c + d x]) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{32 a^2 d (b + a \operatorname{Csc}[c + d x])} \right)
\end{aligned}$$

■ **Problem 1359: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c + d x]^3 \operatorname{Tan}[c + d x]^5}{a + b \operatorname{Sin}[c + d x]} dx$$

Optimal (type 3, 240 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(35 a^2 + 57 a b + 24 b^2) \operatorname{Log}[1 - \operatorname{Sin}[c + d x]]}{16 (a + b)^3 d} + \frac{(35 a^2 - 57 a b + 24 b^2) \operatorname{Log}[1 + \operatorname{Sin}[c + d x]]}{16 (a - b)^3 d} - \frac{a^8 \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{b^3 (a^2 - b^2)^3 d} + \\
& \frac{a \operatorname{Sin}[c + d x]}{b^2 d} - \frac{\operatorname{Sin}[c + d x]^2}{2 b d} - \frac{\operatorname{Sec}[c + d x]^4 (b - a \operatorname{Sin}[c + d x])}{4 (a^2 - b^2) d} + \frac{\operatorname{Sec}[c + d x]^2 (4 b (4 a^2 - 3 b^2) - a (13 a^2 - 9 b^2) \operatorname{Sin}[c + d x])}{8 (a^2 - b^2)^2 d}
\end{aligned}$$

Result (type 3, 542 leaves) :

$$\frac{2 i (6 a^4 b - 8 a^2 b^3 + 3 b^5) (c + d x)}{(a - b)^3 (a + b)^3 d} + \frac{1}{8 (a + b)^3 d}$$

$$i (-35 a^2 - 57 a b - 24 b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c + d x] \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right) + \frac{1}{8 (a - b)^3 d}$$

$$i (35 a^2 - 57 a b + 24 b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c + d x] \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right) + \frac{\cos[2(c + d x)]}{4 b d} +$$

$$\frac{(-35 a^2 - 57 a b - 24 b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^2\right]}{16 (a + b)^3 d} + \frac{(35 a^2 - 57 a b + 24 b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^2\right]}{16 (a - b)^3 d} -$$

$$\frac{a^8 \operatorname{Log}[a + b \sin[c + d x]]}{b^3 (a^2 - b^2)^3 d} + \frac{1}{16 (a + b) d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{-13 a - 11 b}{16 (a + b)^2 d \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} -$$

$$\frac{1}{16 (a - b) d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{13 a - 11 b}{16 (a - b)^2 d \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{a \sin[c + d x]}{b^2 d}$$

■ **Problem 1360: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[c + d x]^2 \tan[c + d x]^5}{a + b \sin[c + d x]} dx$$

Optimal (type 3, 221 leaves, 6 steps) :

$$-\frac{(24 a^2 + 37 a b + 15 b^2) \operatorname{Log}[1 - \sin[c + d x]]}{16 (a + b)^3 d} - \frac{(24 a^2 - 37 a b + 15 b^2) \operatorname{Log}[1 + \sin[c + d x]]}{16 (a - b)^3 d} + \frac{a^7 \operatorname{Log}[a + b \sin[c + d x]]}{b^2 (a^2 - b^2)^3 d} -$$

$$\frac{\sin[c + d x]}{b d} + \frac{\sec[c + d x]^4 (a - b \sin[c + d x])}{4 (a^2 - b^2) d} - \frac{\sec[c + d x]^2 (4 a (3 a^2 - 2 b^2) - b (13 a^2 - 9 b^2) \sin[c + d x])}{8 (a^2 - b^2)^2 d}$$

Result (type 3, 522 leaves) :

$$\begin{aligned}
& - \frac{2i(3a^5 - 3a^3b^2 + ab^4)(c+dx)}{(a-b)^3(a+b)^3d} + \frac{1}{8(a+b)^3d} \\
& i(-24a^2 - 37ab - 15b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \\
& \frac{1}{8(a-b)^3d} i(-24a^2 + 37ab - 15b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \\
& \frac{(-24a^2 - 37ab - 15b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a+b)^3d} + \frac{(-24a^2 + 37ab - 15b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a-b)^3d} + \\
& \frac{a^7 \operatorname{Log}[a+b \sin[c+dx]]}{b^2(a^2-b^2)^3d} + \frac{1}{16(a+b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{-11a-9b}{16(a+b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{1}{16(a-b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{-11a+9b}{16(a-b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{\sin[c+dx]}{bd}
\end{aligned}$$

■ **Problem 1361: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[c+dx] \tan[c+dx]^5}{a+b \sin[c+dx]} dx$$

Optimal (type 3, 208 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(15a^2 + 21ab + 8b^2) \operatorname{Log}[1 - \sin[c+dx]]}{16(a+b)^3d} + \frac{(15a^2 - 21ab + 8b^2) \operatorname{Log}[1 + \sin[c+dx]]}{16(a-b)^3d} - \\
& \frac{a^6 \operatorname{Log}[a+b \sin[c+dx]]}{b(a^2-b^2)^3d} - \frac{\operatorname{Sec}[c+dx]^4(b-a \sin[c+dx])}{4(a^2-b^2)d} + \frac{\operatorname{Sec}[c+dx]^2(4b(3a^2-2b^2) - a(9a^2-5b^2) \sin[c+dx])}{8(a^2-b^2)^2d}
\end{aligned}$$

Result (type 3, 508 leaves):

$$\frac{2i(3a^4b - 3a^2b^3 + b^5)(c+dx)}{(a-b)^3(a+b)^3d} + \frac{1}{8(a+b)^3d}$$

$$i(-15a^2 - 21ab - 8b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] +$$

$$\frac{1}{8(a-b)^3d} i(15a^2 - 21ab + 8b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] +$$

$$\frac{(-15a^2 - 21ab - 8b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a+b)^3d} + \frac{(15a^2 - 21ab + 8b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a-b)^3d} -$$

$$\frac{a^6 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{b(a^2-b^2)^3d} + \frac{1}{16(a+b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{-9a-7b}{16(a+b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} -$$

$$\frac{1}{16(a-b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{9a-7b}{16(a-b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}$$

■ **Problem 1367: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx]^5}{a+b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 233 leaves, 4 steps):

$$-\frac{(8a^2 + 21ab + 15b^2) \operatorname{Log}[1 - \operatorname{Sin}[c+dx]]}{16(a+b)^3d} + \frac{\operatorname{Log}[\operatorname{Sin}[c+dx]]}{ad} - \frac{(8a^2 - 21ab + 15b^2) \operatorname{Log}[1 + \operatorname{Sin}[c+dx]]}{16(a-b)^3d} + \frac{b^6 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{a(a^2-b^2)^3d} +$$

$$\frac{1}{16(a+b)d(1 - \operatorname{Sin}[c+dx])^2} + \frac{5a+7b}{16(a+b)^2d(1 - \operatorname{Sin}[c+dx])} + \frac{1}{16(a-b)d(1 + \operatorname{Sin}[c+dx])^2} + \frac{5a-7b}{16(a-b)^2d(1 + \operatorname{Sin}[c+dx])}$$

Result (type 3, 521 leaves):

$$\begin{aligned}
& - \frac{2i(a^5 - 3a^3b^2 + 3ab^4)(c+dx)}{(a-b)^3(a+b)^3d} + \frac{1}{8(a+b)^3d} \\
& i(-8a^2 - 21ab - 15b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \\
& \frac{1}{8(a-b)^3d} i(-8a^2 + 21ab - 15b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \\
& \frac{(-8a^2 - 21ab - 15b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a+b)^3d} + \\
& \frac{(-8a^2 + 21ab - 15b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a-b)^3d} + \frac{\operatorname{Log}[\operatorname{Sin}[c+dx]]}{ad} + \frac{b^6 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{a(a^2-b^2)^3d} + \\
& \frac{1}{16(a+b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{5a+7b}{16(a+b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{1}{16(a-b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{5a-7b}{16(a-b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}
\end{aligned}$$

■ **Problem 1368: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c+dx]^2 \operatorname{Sec}[c+dx]^5}{a+b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 250 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\operatorname{Csc}[c+dx]}{ad} - \frac{(15a^2 + 37ab + 24b^2) \operatorname{Log}[1 - \operatorname{Sin}[c+dx]]}{16(a+b)^3d} - \frac{b \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^2d} + \\
& \frac{(15a^2 - 37ab + 24b^2) \operatorname{Log}[1 + \operatorname{Sin}[c+dx]]}{16(a-b)^3d} - \frac{b^7 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{a^2(a^2-b^2)^3d} + \frac{1}{16(a+b)d(1-\operatorname{Sin}[c+dx])^2} + \\
& \frac{7a+9b}{16(a+b)^2d(1-\operatorname{Sin}[c+dx])} - \frac{1}{16(a-b)d(1+\operatorname{Sin}[c+dx])^2} - \frac{7a-9b}{16(a-b)^2d(1+\operatorname{Sin}[c+dx])}
\end{aligned}$$

Result (type 3, 565 leaves):

$$\begin{aligned}
& \frac{2i(a^4b - 3a^2b^3 + 3b^5)(c+dx)}{(a-b)^3(a+b)^3d} + \frac{1}{8(a+b)^3d} \\
& i(-15a^2 - 37ab - 24b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \\
& \frac{1}{8(a-b)^3d} i(15a^2 - 37ab + 24b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] - \\
& \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{2ad} + \frac{(-15a^2 - 37ab - 24b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a+b)^3d} + \\
& \frac{(15a^2 - 37ab + 24b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a-b)^3d} - \frac{b \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^2d} - \frac{b^7 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{a^2(a^2-b^2)^3d} + \\
& \frac{1}{16(a+b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{7a+9b}{16(a+b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \\
& \frac{1}{16(a-b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{-7a+9b}{16(a-b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2ad}
\end{aligned}$$

■ **Problem 1369: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c+dx]^3 \operatorname{Sec}[c+dx]^5}{a+b \operatorname{Sin}[c+dx]} dx$$

Optimal (type 3, 274 leaves, 4 steps):

$$\begin{aligned}
& \frac{b \operatorname{Csc}[c+dx]}{a^2d} - \frac{\operatorname{Csc}[c+dx]^2}{2ad} - \frac{(24a^2 + 57ab + 35b^2) \operatorname{Log}[1 - \operatorname{Sin}[c+dx]]}{16(a+b)^3d} + \frac{(3a^2 + b^2) \operatorname{Log}[\operatorname{Sin}[c+dx]]}{a^3d} - \\
& \frac{(24a^2 - 57ab + 35b^2) \operatorname{Log}[1 + \operatorname{Sin}[c+dx]]}{16(a-b)^3d} + \frac{b^8 \operatorname{Log}[a+b \operatorname{Sin}[c+dx]]}{a^3(a^2-b^2)^3d} + \frac{1}{16(a+b)d(1-\operatorname{Sin}[c+dx])^2} + \\
& \frac{9a+11b}{16(a+b)^2d(1-\operatorname{Sin}[c+dx])} + \frac{1}{16(a-b)d(1+\operatorname{Sin}[c+dx])^2} + \frac{9a-11b}{16(a-b)^2d(1+\operatorname{Sin}[c+dx])}
\end{aligned}$$

Result (type 3, 618 leaves):

$$\begin{aligned}
& - \frac{2i(3a^5 - 8a^3b^2 + 6ab^4)(c+dx)}{(a-b)^3(a+b)^3d} + \frac{1}{8(a+b)^3d} \\
& i(-24a^2 - 57ab - 35b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right) + \\
& \frac{1}{8(a-b)^3d} i(-24a^2 + 57ab - 35b^2) \operatorname{ArcTan}\left[\operatorname{Csc}[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right) + \\
& \frac{b \cot\left[\frac{1}{2}(c+dx)\right] - \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{2a^2d} - \frac{(-24a^2 - 57ab - 35b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{8ad} + \frac{(-24a^2 + 57ab - 35b^2) \operatorname{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right]}{16(a+b)^3d} + \\
& \frac{(3a^2 + b^2) \operatorname{Log}[\sin[c+dx]]}{a^3d} + \frac{b^8 \operatorname{Log}[a+b \sin[c+dx]]}{a^3(a^2 - b^2)^3d} - \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{8ad} + \frac{1}{16(a+b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{9a+11b}{16(a+b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{1}{16(a-b)d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{9a-11b}{16(a-b)^2d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{b \tan\left[\frac{1}{2}(c+dx)\right]}{2a^2d}
\end{aligned}$$

- **Problem 1370: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \cos[e+fx]} \sin[e+fx]^4}{a+b \sin[e+fx]} dx$$

Optimal (type 4, 500 leaves, 21 steps):

$$\begin{aligned}
& \frac{a^4 \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{9/2} (-a^2+b^2)^{1/4} f} - \frac{a^4 \sqrt{g} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{9/2} (-a^2+b^2)^{1/4} f} - \frac{2 a^2 (g \cos[e+fx])^{3/2}}{3 b^3 f g} \\
& - \frac{2 (g \cos[e+fx])^{3/2}}{3 b f g} + \frac{2 (g \cos[e+fx])^{7/2}}{7 b f g^3} - \frac{2 a^3 \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[\frac{1}{2} (e+fx), 2\right]}{b^4 f \sqrt{\cos[e+fx]}} \\
& + \frac{4 a \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[\frac{1}{2} (e+fx), 2\right]}{5 b^2 f \sqrt{\cos[e+fx]}} + \frac{a^5 g \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (e+fx), 2\right]}{b^5 \left(b - \sqrt{-a^2+b^2}\right) f \sqrt{g \cos[e+fx]}} \\
& + \frac{a^5 g \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (e+fx), 2\right]}{b^5 \left(b + \sqrt{-a^2+b^2}\right) f \sqrt{g \cos[e+fx]}} + \frac{2 a (g \cos[e+fx])^{3/2} \operatorname{Sin}[e+fx]}{5 b^2 f g}
\end{aligned}$$

Result (type 6, 1210 leaves):

$$\begin{aligned}
& - \frac{1}{5 b^3 f \sqrt{\cos[e+fx]}} a \sqrt{g \cos[e+fx]} \\
& \left(\frac{1}{6 \sqrt{1 - \cos[e+fx]^2} (a + b \operatorname{Sin}[e+fx])} a b \left(a + b \sqrt{1 - \cos[e+fx]^2} \right) \left(- \left(56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \cos[e+fx]^{3/2} \right) / \left(\sqrt{1 - \cos[e+fx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] - \right. \right. \right. \\
& \left. \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \right) \cos[e+fx]^2 \left(a^2 + b^2 (-1 + \cos[e+fx]^2) \right) \right) - \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} \\
& \left. \left. \left. \left. \left. (3 + 3 i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx] \right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx] \right] \right) \right) \right) \right) \\
& \operatorname{Sin}[e+fx] - \frac{1}{(1 - \cos[e+fx]^2) (a + b \operatorname{Sin}[e+fx])} 2 (5 a^2 + 2 b^2) \left(a + b \sqrt{1 - \cos[e+fx]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(7b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e + fx]^2, \frac{b^2 \cos[e + fx]^2}{-a^2 + b^2} \right] \cos[e + fx]^{3/2} \sqrt{1 - \cos[e + fx]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e + fx]^2, \frac{b^2 \cos[e + fx]^2}{-a^2 + b^2} \right] + 2 \left(2b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[e + fx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[e + fx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + fx]^2, \frac{b^2 \cos[e + fx]^2}{-a^2 + b^2} \right] \right) \cos[e + fx]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + fx]^2)) \right) + \frac{1}{4\sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + fx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + fx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + fx]} + b \cos[e + fx] \right] - \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + fx]} + b \cos[e + fx] \right] \right) \right) \sin[e + fx]^2 + \\
& \frac{\sqrt{g \cos[e + fx]} \left(-\frac{(28a^2 + 19b^2) \cos[e + fx]}{42b^3} + \frac{\cos[3(e + fx)]}{14b} + \frac{a \sin[2(e + fx)]}{5b^2} \right)}{f}
\end{aligned}$$

■ **Problem 1371: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \cos[e + fx]} \sin[e + fx]^3}{a + b \sin[e + fx]} dx$$

Optimal (type 4, 448 leaves, 18 steps):

$$\begin{aligned}
& -\frac{a^3 \sqrt{g} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \cos[e + fx]}}{(a^2 + b^2)^{1/4} \sqrt{g}} \right]}{b^{7/2} (a^2 + b^2)^{1/4} f} + \frac{a^3 \sqrt{g} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \cos[e + fx]}}{(a^2 + b^2)^{1/4} \sqrt{g}} \right]}{b^{7/2} (a^2 + b^2)^{1/4} f} + \frac{2a (g \cos[e + fx])^{3/2}}{3b^2 f g} + \frac{2a^2 \sqrt{g \cos[e + fx]} \operatorname{EllipticE} \left[\frac{1}{2} (e + fx), 2 \right]}{b^3 f \sqrt{\cos[e + fx]}} + \\
& \frac{4 \sqrt{g \cos[e + fx]} \operatorname{EllipticE} \left[\frac{1}{2} (e + fx), 2 \right]}{5b f \sqrt{\cos[e + fx]}} - \frac{a^4 g \sqrt{\cos[e + fx]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + fx), 2 \right]}{b^4 (b - \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + fx]}} - \\
& \frac{a^4 g \sqrt{\cos[e + fx]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + fx), 2 \right]}{b^4 (b + \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + fx]}} - \frac{2 (g \cos[e + fx])^{3/2} \sin[e + fx]}{5b f g}
\end{aligned}$$

Result (type 6, 1183 leaves):

$$\begin{aligned}
& \frac{1}{5 b^2 f \sqrt{\cos[e+f x]}} \sqrt{g \cos[e+f x]} \left(\frac{1}{6 \sqrt{1-\cos[e+f x]^2} (a+b \sin[e+f x])} \right. \\
& a b \left(a+b \sqrt{1-\cos[e+f x]^2} \right) \left(- \left(56 a (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+f x]^2, \frac{b^2 \cos[e+f x]^2}{-a^2+b^2} \right] \cos[e+f x]^{3/2} \right) / \right. \\
& \left(\sqrt{1-\cos[e+f x]^2} \left(7 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+f x]^2, \frac{b^2 \cos[e+f x]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[e+f x]^2, \frac{b^2 \cos[e+f x]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e+f x]^2, \frac{b^2 \cos[e+f x]^2}{-a^2+b^2} \right] \right) \cos[e+f x]^2 \right) \\
& \left. (a^2+b^2 (-1+\cos[e+f x]^2)) \right) - \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+f x]}}{(-a^2+b^2)^{1/4}} \right] - \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+f x]}}{(-a^2+b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+f x]} + i b \cos[e+f x] \right] + \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+f x]} + i b \cos[e+f x] \right] \right) \left. \right) \sin[e+f x] - \\
& \frac{1}{(1-\cos[e+f x]^2) (a+b \sin[e+f x])} 2 (5 a^2+2 b^2) \left(a+b \sqrt{1-\cos[e+f x]^2} \right) \\
& \left(\left(7 b (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e+f x]^2, \frac{b^2 \cos[e+f x]^2}{-a^2+b^2} \right] \cos[e+f x]^{3/2} \sqrt{1-\cos[e+f x]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e+f x]^2, \frac{b^2 \cos[e+f x]^2}{-a^2+b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[e+f x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[e+f x]^2}{-a^2+b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e+f x]^2, \frac{b^2 \cos[e+f x]^2}{-a^2+b^2} \right] \right) \cos[e+f x]^2 \right) \\
& \left. (a^2+b^2 (-1+\cos[e+f x]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e+f x]}}{(a^2-b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e+f x]}}{(a^2-b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+f x]} + b \cos[e+f x] \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+f x]} + b \cos[e+f x] \right] \right) \left. \right) \sin[e+f x]^2 + \frac{\sqrt{g \cos[e+f x]} \left(\frac{2 a \cos[e+f x]}{3 b^2} - \frac{\sin[2(e+f x)]}{5 b} \right)}{f}
\end{aligned}$$

■ **Problem 1372: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{g \cos[e + f x]} \sin[e + f x]^2}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 369 leaves, 15 steps):

$$\frac{a^2 \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{b^{5/2} (-a^2 + b^2)^{1/4} f} - \frac{a^2 \sqrt{g} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{b^{5/2} (-a^2 + b^2)^{1/4} f} - \frac{2 (g \cos[e + f x])^{3/2}}{3 b f g} - \frac{2 a \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2} (e + f x), 2\right]}{b^2 f \sqrt{\cos[e + f x]}} +$$

$$\frac{a^3 g \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b^3 (b - \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}} + \frac{a^3 g \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b^3 (b + \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}}$$

Result (type 6, 608 leaves):

$$-\frac{2 \cos[e + f x] \sqrt{g \cos[e + f x]}}{3 b f} + \frac{1}{b f \sqrt{\cos[e + f x]} (1 - \cos[e + f x]^2) (a + b \sin[e + f x])} 2 a \sqrt{g \cos[e + f x]} (a + b \sqrt{1 - \cos[e + f x]^2})$$

$$\left(\left(7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^{3/2} \sqrt{1 - \cos[e + f x]^2} \right) / \right.$$

$$\left. \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + 2 \right. \right. \right.$$

$$\left. \left. \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \right. \right.$$

$$\left. \left. \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}}$$

$$a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + \right. \right.$$

$$\left. \left. b \cos[e + f x] - \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x]\right] \right) \right) \sin[e + f x]^2$$

■ **Problem 1373: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{g \cos[e + f x]} \sin[e + f x]}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 341 leaves, 12 steps):

$$\begin{aligned}
& - \frac{a \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{3/2} (-a^2+b^2)^{1/4} f} + \frac{a \sqrt{g} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{3/2} (-a^2+b^2)^{1/4} f} + \frac{2 \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[\frac{1}{2}(e+fx), 2\right]}{b f \sqrt{\cos[e+fx]}} \\
& - \frac{a^2 g \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^2 \left(b-\sqrt{-a^2+b^2}\right) f \sqrt{g \cos[e+fx]}} - \frac{a^2 g \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^2 \left(b+\sqrt{-a^2+b^2}\right) f \sqrt{g \cos[e+fx]}}
\end{aligned}$$

Result (type 6, 548 leaves):

$$\begin{aligned}
& - \frac{1}{f \sqrt{\cos[e+fx]} (a+b \sin[e+fx])} 2 \sqrt{g \cos[e+fx]} \left(a+b \sqrt{\sin[e+fx]^2}\right) \\
& \left(\frac{1}{4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4}} a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] + \operatorname{Log}\left[\right. \right. \\
& \quad \left. \left. \sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx] \right] - \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx] \right] \right) + \\
& \left(7 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \cos[e+fx]^{3/2} \sqrt{\sin[e+fx]^2} \right) / \\
& \left(3 (a^2-b^2+b^2 \cos[e+fx]^2) \left(-7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right) \left. \right)
\end{aligned}$$

■ **Problem 1374: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{g \cos[e+fx]} \operatorname{Csc}[e+fx]}{a+b \sin[e+fx]} dx$$

Optimal (type 4, 355 leaves, 16 steps):

$$\begin{aligned}
& \frac{\sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{a f} - \frac{\sqrt{b} \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{a (-a^2+b^2)^{1/4} f} - \frac{\sqrt{g} \operatorname{ArcTanh}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{a f} + \frac{\sqrt{b} \sqrt{g} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{a (-a^2+b^2)^{1/4} f} \\
& - \frac{g \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{\left(b-\sqrt{-a^2+b^2}\right) f \sqrt{g \cos[e+fx]}} - \frac{g \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{\left(b+\sqrt{-a^2+b^2}\right) f \sqrt{g \cos[e+fx]}}
\end{aligned}$$

Result (type 6, 700 leaves) :

$$\begin{aligned}
 & - \frac{1}{f \sqrt{\cos[e+fx]} (1 - \cos[e+fx]^2) (b + a \operatorname{Csc}[e+fx])} - 2 \sqrt{g \cos[e+fx]} (-1 + \cos[e+fx]^2) \left(a + b \sqrt{1 - \cos[e+fx]^2} \right) \\
 & \operatorname{Csc}[e+fx] \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2} \right] \cos[e+fx]^{3/2} \right) / \left(3 \sqrt{1 - \cos[e+fx]^2} \right) \right. \\
 & \left. \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
 & \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2} \right] \right) \cos[e+fx]^2 \right) (a^2 + b^2 (-1 + \cos[e+fx]^2)) \right) + \\
 & \frac{1}{8 a (a^2 - b^2)^{1/4}} \left(2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e+fx]}}{(a^2 - b^2)^{1/4}} \right] - 2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e+fx]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
 & 4 (a^2 - b^2)^{1/4} \operatorname{ArcTan} \left[\sqrt{\cos[e+fx]} \right] + 2 (a^2 - b^2)^{1/4} \operatorname{Log} \left[1 - \sqrt{\cos[e+fx]} \right] - \\
 & 2 (a^2 - b^2)^{1/4} \operatorname{Log} \left[1 + \sqrt{\cos[e+fx]} \right] - \sqrt{2} \sqrt{b} \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx] \right] + \\
 & \left. \left. \left. \sqrt{2} \sqrt{b} \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx] \right] \right) \right) \right)
 \end{aligned}$$

■ **Problem 1375: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \cos[e+fx]} \operatorname{Csc}[e+fx]^2}{a + b \sin[e+fx]} dx$$

Optimal (type 4, 433 leaves, 19 steps) :

$$\begin{aligned}
 & - \frac{b \sqrt{g} \operatorname{ArcTan} \left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}} \right]}{a^2 f} + \frac{b^{3/2} \sqrt{g} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a^2 (-a^2 + b^2)^{1/4} f} + \frac{b \sqrt{g} \operatorname{ArcTanh} \left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}} \right]}{a^2 f} - \\
 & \frac{b^{3/2} \sqrt{g} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a^2 (-a^2 + b^2)^{1/4} f} - \frac{(g \cos[e+fx])^{3/2} \operatorname{Csc}[e+fx]}{a f g} - \frac{\sqrt{g \cos[e+fx]} \operatorname{EllipticE} \left[\frac{1}{2} (e+fx), 2 \right]}{a f \sqrt{\cos[e+fx]}} + \\
 & \frac{b g \sqrt{\cos[e+fx]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e+fx), 2 \right]}{a (b - \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e+fx]}} + \frac{b g \sqrt{\cos[e+fx]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e+fx), 2 \right]}{a (b + \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e+fx]}}
 \end{aligned}$$

Result (type 6, 2385 leaves) :

$$\begin{aligned}
& - \frac{\sqrt{g \cos[e + f x]} \cot[e + f x]}{a f} + \\
& \frac{1}{4 a f \sqrt{\cos[e + f x]}} \sqrt{g \cos[e + f x]} \left(- \frac{1}{6 \sqrt{1 - \cos[e + f x]^2} (b + a \csc[e + f x])} a \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \left(- \left(56 a (a^2 - b^2) \right. \right. \right. \\
& \quad \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \Big/ \left(\sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \right. \right. \right. \\
& \quad \left. \left. \left. \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) \right) - \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} \\
& \quad (3 + 3 i) \left(2 \text{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \text{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - \text{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \\
& \quad \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] + \text{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \right) \Big) \Big) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (-1 + 2 \cos[e + f x]^2) (b + a \csc[e + f x])} 2 b (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \quad \cos[2 (e + f x)] \csc[e + f x] \\
& \left(\frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \text{ArcTan} \left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \text{ArcTan} \left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} \right) + \\
& \frac{\text{ArcTan} \left[\sqrt{\cos[e + f x]} \right]}{2 a} - \left(7 b (a^2 - b^2) \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) \Big/ \\
& \left(3 \sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \quad \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \left(22 b (a^2 - b^2) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{7/2} \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(7 \sqrt{1 - \cos[e + f x]^2} \left(11 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - \right. \right. \\
& \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) \right) + \frac{\log \left[1 - \sqrt{\cos[e + f x]} \right]}{4 a} - \\
& \frac{\log \left[1 + \sqrt{\cos[e + f x]} \right]}{4 a} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \log \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} - \\
& \left. \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \log \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} \right) + \\
& \frac{1}{(1 - \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 10 b (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \operatorname{Csc}[e + f x] \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) / \right. \\
& \quad \left(3 \sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \quad \left. \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) \right) + \frac{1}{8 a (a^2 - b^2)^{1/4}} \left(2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \right. \\
& \quad 2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + 4 (a^2 - b^2)^{1/4} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] + 2 (a^2 - b^2)^{1/4} \log \left[1 - \sqrt{\cos[e + f x]} \right] - \\
& \quad 2 (a^2 - b^2)^{1/4} \log \left[1 + \sqrt{\cos[e + f x]} \right] - \sqrt{2} \sqrt{b} \log \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \\
& \quad \left. \left. \left. \sqrt{2} \sqrt{b} \log \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 1376: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \cos[e + f x]} \operatorname{Csc}[e + f x]^3}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 544 leaves, 25 steps):

$$\frac{\sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}}\right]}{4 a f} + \frac{b^2 \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}}\right]}{a^3 f} - \frac{b^{5/2} \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a^3 (-a^2 + b^2)^{1/4} f} -$$

$$\frac{\sqrt{g} \operatorname{ArcTanh}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}}\right]}{4 a f} - \frac{b^2 \sqrt{g} \operatorname{ArcTanh}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}}\right]}{a^3 f} + \frac{b^{5/2} \sqrt{g} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a^3 (-a^2 + b^2)^{1/4} f} +$$

$$\frac{b (g \cos[e + f x])^{3/2} \operatorname{Csc}[e + f x]}{a^2 f g} - \frac{(g \cos[e + f x])^{3/2} \operatorname{Csc}[e + f x]^2}{2 a f g} + \frac{b \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2} (e + f x), 2\right]}{a^2 f \sqrt{\cos[e + f x]}} -$$

$$\frac{b^2 g \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{a^2 (b - \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}} - \frac{b^2 g \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{a^2 (b + \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}}$$

Result (type 6, 2419 leaves):

$$\frac{\sqrt{g \cos[e + f x]} \left(\frac{b \operatorname{Cot}[e + f x]}{a^2} - \frac{\operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]}{2 a} \right)}{f} -$$

$$\frac{1}{4 a^2 f \sqrt{\cos[e + f x]}} \sqrt{g \cos[e + f x]} \left(- \frac{1}{4 \sqrt{1 - \cos[e + f x]^2} (b + a \operatorname{Csc}[e + f x])} a b \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \right.$$

$$\left. - \left(56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^{3/2} \right) / \left(\sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \right. \right. \right.$$

$$\left. \left. \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \right. \right.$$

$$\left. \left. \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}}$$

$$(3 + 3 i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b}\right] \right)$$

$$\begin{aligned}
& \left. \left((-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right) + \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \right) \Bigg) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (-1 + 2 \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 2 b^2 (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \cos[2(e + f x)] \\
& \operatorname{Csc}[e + f x] \left(\frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} \right) + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} + \\
& \frac{\operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right]}{2 a} - \left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \left(22 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{7/2} \right) / \\
& \left(7 \sqrt{1 - \cos[e + f x]^2} \left(11 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - \right. \right. \\
& \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) \Bigg) + \frac{\operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right]}{4 a} - \\
& \frac{\operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right]}{4 a} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} - \\
& \left. \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 2 (-a^2 - 5 b^2) (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \operatorname{Csc}[e + f x]
\end{aligned}$$

$$\begin{aligned}
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) / \right. \\
& \left(3 \sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{1}{8 a (a^2 - b^2)^{1/4}} \left(2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \right. \\
& 2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + 4 (a^2 - b^2)^{1/4} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] + 2 (a^2 - b^2)^{1/4} \operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right] - \\
& 2 (a^2 - b^2)^{1/4} \operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right] - \sqrt{2} \sqrt{b} \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \\
& \left. \left. \left. \left. \left. \sqrt{2} \sqrt{b} \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right] \right] \right] \right] \right)
\end{aligned}$$

- **Problem 1377: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{3/2} \sin[e + f x]^3}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 621 leaves, 24 steps):

$$\begin{aligned}
& \frac{a^3 (-a^2 + b^2)^{1/4} g^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{9/2} f} + \frac{a^3 (-a^2 + b^2)^{1/4} g^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{9/2} f} - \\
& \frac{2 a^3 g \sqrt{g \cos[e+fx]}}{b^4 f} + \frac{2 a (g \cos[e+fx])^{5/2}}{5 b^2 f g} - \frac{2 a^4 g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2} (e+fx), 2\right]}{b^5 f \sqrt{g \cos[e+fx]}} + \\
& \frac{2 a^2 g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2} (e+fx), 2\right]}{3 b^3 f \sqrt{g \cos[e+fx]}} + \frac{4 g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2} (e+fx), 2\right]}{21 b f \sqrt{g \cos[e+fx]}} + \\
& \frac{a^4 (a^2 - b^2) g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (e+fx), 2\right]}{b^5 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}} + \frac{a^4 (a^2 - b^2) g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (e+fx), 2\right]}{b^5 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}} + \\
& \frac{2 a^2 g \sqrt{g \cos[e+fx]} \sin[e+fx]}{3 b^3 f} + \frac{4 g \sqrt{g \cos[e+fx]} \sin[e+fx]}{21 b f} - \frac{2 (g \cos[e+fx])^{5/2} \sin[e+fx]}{7 b f g}
\end{aligned}$$

Result (type 6, 2191 leaves):

$$\begin{aligned}
& - \frac{1}{420 b^3 f \cos[e+fx]^{3/2}} (g \cos[e+fx])^{3/2} \\
& \left(- \frac{1}{\sqrt{1 - \cos[e+fx]^2} (a+b \sin[e+fx])} 2 (70 a^3 - 19 a b^2) \left(a + b \sqrt{1 - \cos[e+fx]^2}\right) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2\right], \right. \right. \right. \\
& \left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \sqrt{\cos[e+fx]} \right) / \left(\sqrt{1 - \cos[e+fx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2\right], \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right) - \right. \right. \\
& \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \cos[e+fx]^2 \right) (a^2 + b^2 (-1 + \cos[e+fx]^2)) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8}\right) \sqrt{b} \\
& \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2 + b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \right. \right. \\
& \left. \left. \sqrt{\cos[e+fx]} + i b \cos[e+fx] \right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx] \right] \right) \right) \sin[e+fx] +
\end{aligned}$$

$$\frac{1}{\sqrt{1 - \cos[e + f x]^2} (-1 + 2 \cos[e + f x]^2) (a + b \sin[e + f x])} (210 a^3 - 21 a b^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \cos[2 (e + f x)]$$

$$\left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - \left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} + \frac{4 \sqrt{\cos[e + f x]}}{b} \right) +$$

$$\left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \right) / \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \right. \right.$$

$$\operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right.$$

$$\left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \left. \right) -$$

$$\left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{5/2} \right) / \left(5 \sqrt{1 - \cos[e + f x]^2} \left(9 (a^2 - b^2) \right. \right.$$

$$\operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right.$$

$$\left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \left. \right) +$$

$$\frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] -$$

$$\frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \left. \right) \sin[e + f x] -$$

$$\frac{1}{(1 - \cos[e + f x]^2) (a + b \sin[e + f x])} 2 (-98 a^2 b - 40 b^3) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right)$$

$$\left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \sqrt{1 - \cos[e + f x]^2} \right) / \right.$$

$$\left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \right. \right. \right.$$

$$\left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \left. \right)$$

$$\left. \left((a^2 + b^2 (-1 + \cos[e + f x])^2) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \right. \\ \left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \right. \right. \\ \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \sin[e + f x]^2 + \\ \frac{(g \cos[e + f x])^{3/2} \sec[e + f x] \left(\frac{a \cos[2(e + f x)]}{5 b^2} + \frac{(28 a^2 + 5 b^2) \sin[e + f x]}{42 b^3} - \frac{\sin[3(e + f x)]}{14 b} \right)}{f}$$

- **Problem 1378: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{3/2} \sin[e + f x]^2}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 514 leaves, 20 steps):

$$\frac{a^2 (-a^2 + b^2)^{1/4} g^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right] - a^2 (-a^2 + b^2)^{1/4} g^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{b^{7/2} f} + \\ \frac{2 a^2 g \sqrt{g \cos[e + f x]}}{b^3 f} - \frac{2 (g \cos[e + f x])^{5/2}}{5 b f g} + \frac{2 a^3 g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{b^4 f \sqrt{g \cos[e + f x]}} - \\ \frac{2 a g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{3 b^2 f \sqrt{g \cos[e + f x]}} - \frac{a^3 (a^2 - b^2) g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{b^4 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2} \right) \right) f \sqrt{g \cos[e + f x]}} - \\ \frac{a^3 (a^2 - b^2) g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{b^4 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2} \right) \right) f \sqrt{g \cos[e + f x]}} - \frac{2 a g \sqrt{g \cos[e + f x]} \sin[e + f x]}{3 b^2 f}$$

Result (type 6, 2153 leaves):

$$\frac{(g \cos[e + f x])^{3/2} \sec[e + f x] \left(-\frac{\cos[2(e + f x)]}{5 b} - \frac{2 a \sin[e + f x]}{3 b^2} \right)}{f} +$$

$$\begin{aligned}
& \frac{1}{60 b^2 f \cos[e + f x]^{3/2}} (g \cos[e + f x])^{3/2} \left(- \frac{1}{\sqrt{1 - \cos[e + f x]^2} (a + b \sin[e + f x])} 2 (10 a^2 + 3 b^2) (a + b \sqrt{1 - \cos[e + f x]^2}) \right. \\
& \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \right) / \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) + \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \\
& \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] \right) + \\
& \left. \left. \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] - \right. \right. \\
& \left. \left. \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \right) \right) \sin[e + f x] + \\
& \frac{1}{\sqrt{1 - \cos[e + f x]^2} (-1 + 2 \cos[e + f x]^2) (a + b \sin[e + f x])} (30 a^2 - 3 b^2) (a + b \sqrt{1 - \cos[e + f x]^2}) \cos[2 (e + f x)] \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} \right) + \\
& \frac{4 \sqrt{\cos[e + f x]}}{b} + \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \right) / \\
& \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& (a^2 + b^2 (-1 + \cos[e + f x]^2)) - \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{5/2} \right) / \\
& \left(5 \sqrt{1 - \cos[e + f x]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \\
& \quad \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \Bigg) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \Bigg) \sin[e + f x] + \\
& \frac{1}{(1 - \cos[e + f x]^2) (a + b \sin[e + f x])} 28 a b \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \sqrt{1 - \cos[e + f x]^2} \right) / \right. \\
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \\
& \left. \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \right) \sin[e + f x]^2 \Bigg)
\end{aligned}$$

- **Problem 1379: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{3/2} \sin[e + f x]}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 426 leaves, 13 steps):

$$\frac{a (-a^2 + b^2)^{1/4} g^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{5/2} f} + \frac{a (-a^2 + b^2)^{1/4} g^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{5/2} f} -$$

$$\frac{2 (3 a^2 - b^2) g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2} (e+fx), 2\right]}{3 b^3 f \sqrt{g \cos[e+fx]}} + \frac{a^2 (a^2 - b^2) g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (e+fx), 2\right]}{b^3 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}} +$$

$$\frac{a^2 (a^2 - b^2) g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (e+fx), 2\right]}{b^3 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}} - \frac{2 g \sqrt{g \cos[e+fx]} (3 a - b \sin[e+fx])}{3 b^2 f}$$

Result (type 6, 2109 leaves) :

$$-\frac{1}{6 b f \cos[e+fx]^{3/2}} (g \cos[e+fx])^{3/2}$$

$$\left(-\frac{1}{\sqrt{1-\cos[e+fx]^2} (a+b \sin[e+fx])} 2 a \left(a + b \sqrt{1-\cos[e+fx]^2} \right) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right], \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right) \right. \right.$$

$$\left. \sqrt{\cos[e+fx]} \right) / \left(\sqrt{1-\cos[e+fx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - \right. \right.$$

$$2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \right.$$

$$\left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \left(a^2 + b^2 (-1 + \cos[e+fx]^2) \right) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b}$$

$$\left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4}\right] \right.$$

$$\left. \left. \sqrt{\cos[e+fx]} + i b \cos[e+fx] \right) - \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx] \right] \right) \sin[e+fx] +$$

$$\frac{1}{\sqrt{1-\cos[e+fx]^2} (-1+2 \cos[e+fx]^2) (a+b \sin[e+fx])} 3 a \left(a + b \sqrt{1-\cos[e+fx]^2} \right) \cos[2 (e+fx)]$$

$$\left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2} (-a^2 + b^2)^{3/4}} + \frac{4 \sqrt{\cos[e+fx]}}{b} + \right.$$

$$\begin{aligned}
& \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \right) / \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \\
& \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{5/2} \right) / \left(5 \sqrt{1 - \cos[e + f x]^2} \left(9 (a^2 - b^2) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \Bigg) \sin[e + f x] + \\
& \frac{1}{(1 - \cos[e + f x]^2) (a + b \sin[e + f x])} 4 b \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \sqrt{1 - \cos[e + f x]^2} \right) / \right. \\
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \right.
\end{aligned}$$

$$\left. \left. \left. \text{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Cos}[e + f x]} + b \text{Cos}[e + f x] \right] \right] \right] \right) \text{Sin}[e + f x]^2 + \frac{2 (g \text{Cos}[e + f x])^{3/2} \text{Tan}[e + f x]}{3 b f}$$

▪ **Problem 1380: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \text{Cos}[e + f x])^{3/2} \text{Csc}[e + f x]}{a + b \text{Sin}[e + f x]} dx$$

Optimal (type 4, 439 leaves, 21 steps) :

$$\begin{aligned} & - \frac{g^{3/2} \text{ArcTan} \left[\frac{\sqrt{g \text{Cos}[e+f x]}}{\sqrt{g}} \right]}{a f} + \frac{(-a^2 + b^2)^{1/4} g^{3/2} \text{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \text{Cos}[e+f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a \sqrt{b} f} - \frac{g^{3/2} \text{ArcTanh} \left[\frac{\sqrt{g \text{Cos}[e+f x]}}{\sqrt{g}} \right]}{a f} + \\ & \frac{(-a^2 + b^2)^{1/4} g^{3/2} \text{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \text{Cos}[e+f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a \sqrt{b} f} - \frac{2 g^2 \sqrt{\text{Cos}[e + f x]} \text{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{b f \sqrt{g \text{Cos}[e + f x]}} + \\ & \frac{(a^2 - b^2) g^2 \sqrt{\text{Cos}[e + f x]} \text{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{b \left(a^2 - b \left(b - \sqrt{-a^2 + b^2} \right) \right) f \sqrt{g \text{Cos}[e + f x]}} + \frac{(a^2 - b^2) g^2 \sqrt{\text{Cos}[e + f x]} \text{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{b \left(a^2 - b \left(b + \sqrt{-a^2 + b^2} \right) \right) f \sqrt{g \text{Cos}[e + f x]}} \end{aligned}$$

Result (type 6, 700 leaves) :

$$\begin{aligned}
& - \frac{1}{f \cos[e + f x]^{3/2} (1 - \cos[e + f x]^2) (b + a \csc[e + f x])} 2 (g \cos[e + f x])^{3/2} (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \csc[e + f x] \left(\left(9 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{5/2} \right) / \left(5 \sqrt{1 - \cos[e + f x]^2} \right. \right. \\
& \left. \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \\
& \frac{1}{8 a \sqrt{b}} \left(2 \sqrt{2} (a^2 - b^2)^{1/4} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - 2 \sqrt{2} (a^2 - b^2)^{1/4} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 4 \sqrt{b} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] - 2 \sqrt{b} \operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right] + 2 \sqrt{b} \operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right] + \\
& \sqrt{2} (a^2 - b^2)^{1/4} \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] - \\
& \left. \left. \left. \sqrt{2} (a^2 - b^2)^{1/4} \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \right)
\end{aligned}$$

■ **Problem 1381: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{3/2} \csc[e + f x]^2}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 469 leaves, 24 steps):

$$\begin{aligned}
& \frac{b g^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}} \right]}{a^2 f} - \frac{\sqrt{b} (-a^2 + b^2)^{1/4} g^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a^2 f} + \frac{b g^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}} \right]}{a^2 f} - \\
& \frac{\sqrt{b} (-a^2 + b^2)^{1/4} g^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a^2 f} - \frac{g \sqrt{g \cos[e + f x]} \csc[e + f x]}{a f} + \frac{g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{a f \sqrt{g \cos[e + f x]}} - \\
& \frac{(a^2 - b^2) g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{a \left(a^2 - b \left(b - \sqrt{-a^2 + b^2} \right) \right) f \sqrt{g \cos[e + f x]}} - \frac{(a^2 - b^2) g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{a \left(a^2 - b \left(b + \sqrt{-a^2 + b^2} \right) \right) f \sqrt{g \cos[e + f x]}}
\end{aligned}$$

Result (type 6, 2381 leaves):

$$- \frac{1}{4 a f \cos[e + f x]^{3/2}} (g \cos[e + f x])^{3/2}$$

$$\begin{aligned}
& \left(-\frac{1}{\sqrt{1-\cos[e+fx]^2} (b+a \operatorname{Csc}[e+fx])} 4a \left(a+b\sqrt{1-\cos[e+fx]^2} \right) \left(\left(5a(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right. \right. \right. \\
& \left. \left. \left. \sqrt{\cos[e+fx]} \right) / \left(\sqrt{1-\cos[e+fx]^2} \left(5(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - \right. \right. \right. \\
& \left. \left. \left. 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \right) \cos[e+fx]^2 \right) (a^2+b^2(-1+\cos[e+fx]^2)) \right) - \frac{1}{(-a^2+b^2)^{3/4}} \\
& \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b} \right. \right. \\
& \left. \left. (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + ib \cos[e+fx] \right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + ib \cos[e+fx] \right] \right) \left. \right) - \\
& \frac{1}{(1-\cos[e+fx]^2)(-1+2\cos[e+fx]^2)(b+a \operatorname{Csc}[e+fx])} b(-1+\cos[e+fx]^2) \left(a+b\sqrt{1-\cos[e+fx]^2} \right) \cos[2(e+fx)] \\
& \operatorname{Csc}[e+fx] \left(\frac{(a^2-b^2)^{1/4}(-2a^2+b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2}(a^2-b^2)^{1/4}+2\sqrt{b}\sqrt{\cos[e+fx]}}{\sqrt{2}(a^2-b^2)^{1/4}}\right]}{\sqrt{2}a\sqrt{b}(-a^2+b^2)} + \frac{(a^2-b^2)^{1/4}(-2a^2+b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2}(a^2-b^2)^{1/4}+2\sqrt{b}\sqrt{\cos[e+fx]}}{\sqrt{2}(a^2-b^2)^{1/4}}\right]}{\sqrt{2}a\sqrt{b}(-a^2+b^2)} - \right. \\
& \frac{\operatorname{ArcTan}\left[\sqrt{\cos[e+fx]}\right]}{a} - \left(10b(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \sqrt{\cos[e+fx]} \right) / \\
& \left(\sqrt{1-\cos[e+fx]^2} \left(5(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right) \\
& \left. (a^2+b^2(-1+\cos[e+fx]^2)) \right) + \left(36b(a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \cos[e+fx]^{5/2} \right) / \\
& \left(5\sqrt{1-\cos[e+fx]^2} \left(9(a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \right. \right. \\
& \left. \left. \left(2b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \cos[e+fx]^2 \left(a^2+b^2 (-1+\cos[e+fx]^2) \right) \right) + \frac{\log\left[1-\sqrt{\cos[e+fx]}\right]}{2a} - \right. \\
& \frac{\log\left[1+\sqrt{\cos[e+fx]}\right]}{2a} - \frac{(a^2-b^2)^{1/4} (-2a^2+b^2) \log\left[\sqrt{a^2-b^2} - \sqrt{2}\sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx]\right]}{2\sqrt{2} a \sqrt{b} (-a^2+b^2)} + \\
& \left. \frac{(a^2-b^2)^{1/4} (-2a^2+b^2) \log\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx]\right]}{2\sqrt{2} a \sqrt{b} (-a^2+b^2)} \right) - \\
& \frac{1}{(1-\cos[e+fx]^2)(b+a \csc[e+fx])} 6b(-1+\cos[e+fx]^2) \left(a+b\sqrt{1-\cos[e+fx]^2} \right) \csc[e+fx] \\
& \left(\left(5b(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \sqrt{\cos[e+fx]} \right) / \left(\sqrt{1-\cos[e+fx]^2} \left(5(a^2-b^2) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right) (a^2+b^2(-1+\cos[e+fx]^2)) \right) - \\
& \frac{1}{8a(a^2-b^2)^{3/4}} \left(-2\sqrt{2} b^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] + 2\sqrt{2} b^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] + \right. \\
& 4(a^2-b^2)^{3/4} \operatorname{ArcTan}\left[\sqrt{\cos[e+fx]}\right] - 2(a^2-b^2)^{3/4} \log\left[1-\sqrt{\cos[e+fx]}\right] + \\
& 2(a^2-b^2)^{3/4} \log\left[1+\sqrt{\cos[e+fx]}\right] - \sqrt{2} b^{3/2} \log\left[\sqrt{a^2-b^2} - \sqrt{2}\sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx]\right] + \\
& \left. \left. \left. \left. \sqrt{2} b^{3/2} \log\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx]\right] \right) \right) \right) \right) - \frac{(g \cos[e+fx])^{3/2} \csc[e+fx] \sec[e+fx]}{af}
\end{aligned}$$

■ **Problem 1382: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e+fx])^{3/2} \csc[e+fx]^3}{a+b \sin[e+fx]} dx$$

Optimal (type 4, 574 leaves, 30 steps):

$$\begin{aligned}
& \frac{g^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{4 a f} - \frac{b^2 g^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{a^3 f} + \frac{b^{3/2} (-a^2+b^2)^{1/4} g^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{a^3 f} + \\
& \frac{g^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{4 a f} - \frac{b^2 g^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{a^3 f} + \frac{b^{3/2} (-a^2+b^2)^{1/4} g^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{a^3 f} + \\
& \frac{b g \sqrt{g \cos[e+fx]} \operatorname{Csc}[e+fx]}{a^2 f} - \frac{g \sqrt{g \cos[e+fx]} \operatorname{Csc}[e+fx]^2}{2 a f} - \frac{b g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2}(e+fx), 2\right]}{a^2 f \sqrt{g \cos[e+fx]}} + \\
& \frac{b (a^2-b^2) g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{a^2 \left(a^2-b \left(b-\sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}} + \frac{b (a^2-b^2) g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{a^2 \left(a^2-b \left(b+\sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}}
\end{aligned}$$

Result (type 6, 2411 leaves):

$$\begin{aligned}
& \frac{1}{4 a^2 f \cos[e+fx]^{3/2}} \\
& (g \cos[e+fx])^{3/2} \left(-\frac{1}{\sqrt{1-\cos[e+fx]^2} (b+a \operatorname{Csc}[e+fx])} \left(\left(5 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2\right], \right. \right. \right. \\
& \left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \sqrt{\cos[e+fx]} \right) / \left(\sqrt{1-\cos[e+fx]^2} \left(5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \right. \right. \right. \\
& \left. \left. \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right) \cos[e+fx]^2 \right) (a^2+b^2 (-1+\cos[e+fx]^2)) \right) - \frac{1}{(-a^2+b^2)^{3/4}} \\
& \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \\
& \left. \left. (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx] \right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx] \right] \right) \left. \right) - \\
& \frac{1}{(1-\cos[e+fx]^2) (-1+2 \cos[e+fx]^2) (b+a \operatorname{Csc}[e+fx])} b^2 (-1+\cos[e+fx]^2) \left(a+b \sqrt{1-\cos[e+fx]^2} \right) \\
& \operatorname{Cos}[2(e+fx)] \operatorname{Csc}[e+fx]
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{(a^2 - b^2)^{1/4} (-2a^2 + b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2}(a^2 - b^2)^{1/4} + 2\sqrt{b}\sqrt{\cos[e+fx]}}{\sqrt{2}(a^2 - b^2)^{1/4}}\right]}{\sqrt{2}a\sqrt{b}(-a^2 + b^2)} + \frac{(a^2 - b^2)^{1/4} (-2a^2 + b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2}(a^2 - b^2)^{1/4} + 2\sqrt{b}\sqrt{\cos[e+fx]}}{\sqrt{2}(a^2 - b^2)^{1/4}}\right]}{\sqrt{2}a\sqrt{b}(-a^2 + b^2)} - \right. \\
& \frac{\operatorname{ArcTan}\left[\sqrt{\cos[e+fx]}\right]}{a} - \left(10b(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \sqrt{\cos[e+fx]} \right) / \\
& \left(\sqrt{1 - \cos[e+fx]^2} \left(5(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right) \cos[e+fx]^2 \right) \\
& \left. (a^2 + b^2(-1 + \cos[e+fx]^2)) \right) + \left(36b(a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \cos[e+fx]^{5/2} \right) / \\
& \left(5\sqrt{1 - \cos[e+fx]^2} \left(9(a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] - \right. \right. \\
& \left. \left. 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right) \cos[e+fx]^2 (a^2 + b^2(-1 + \cos[e+fx]^2)) \right) + \frac{\operatorname{Log}\left[1 - \sqrt{\cos[e+fx]}\right]}{2a} - \\
& \frac{\operatorname{Log}\left[1 + \sqrt{\cos[e+fx]}\right]}{2a} - \frac{(a^2 - b^2)^{1/4} (-2a^2 + b^2) \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx]\right]}{2\sqrt{2}a\sqrt{b}(-a^2 + b^2)} + \\
& \left. \frac{(a^2 - b^2)^{1/4} (-2a^2 + b^2) \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx]\right]}{2\sqrt{2}a\sqrt{b}(-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \cos[e+fx]^2)(b + a \operatorname{Csc}[e+fx])} 2(-a^2 + 3b^2)(-1 + \cos[e+fx]^2) \left(a + b\sqrt{1 - \cos[e+fx]^2} \right) \operatorname{Csc}[e+fx] \\
& \left(\left(5b(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \sqrt{\cos[e+fx]} \right) / \right. \\
& \left(\sqrt{1 - \cos[e+fx]^2} \left(5(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right) \cos[e+fx]^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(- \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) / \left(\sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \\
& \quad \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \\
& \quad \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] + \right. \\
& \quad \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \right) \left. \right) \sin[e + f x] - \\
& \quad \frac{1}{(1 - \cos[e + f x]^2) (a + b \sin[e + f x])} 2 (15 a^4 - 9 a^2 b^2 - 2 b^4) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \quad \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \sqrt{1 - \cos[e + f x]^2} \right) / \right. \\
& \quad \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \quad \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \quad 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] - \\
& \quad \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \sin[e + f x]^2 + \\
& \quad \frac{1}{f} (g \cos[e + f x])^{5/2} \sec[e + f x]^2 \left(-\frac{a (28 a^2 - 9 b^2) \cos[e + f x]}{42 b^4} + \frac{a \cos[3 (e + f x)]}{14 b^2} - \right. \\
& \quad \left. \frac{(-18 a^2 + b^2) \sin[2 (e + f x)]}{90 b^3} \right) -
\end{aligned}$$

$$\left. \frac{\sin[4(e+fx)]}{36b} \right)$$

■ **Problem 1384: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e+fx])^{5/2} \sin[e+fx]^2}{a+b \sin[e+fx]} dx$$

Optimal (type 4, 501 leaves, 20 steps):

$$\frac{a^2 (-a^2+b^2)^{3/4} g^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{9/2} f} - \frac{a^2 (-a^2+b^2)^{3/4} g^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{9/2} f} +$$

$$\frac{2 a^2 g (g \cos[e+fx])^{3/2}}{3 b^3 f} - \frac{2 (g \cos[e+fx])^{7/2}}{7 b f g} + \frac{2 a^3 g^2 \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[\frac{1}{2}(e+fx), 2\right]}{b^4 f \sqrt{\cos[e+fx]}} -$$

$$\frac{6 a g^2 \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[\frac{1}{2}(e+fx), 2\right]}{5 b^2 f \sqrt{\cos[e+fx]}} - \frac{a^3 (a^2-b^2) g^3 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^5 (b-\sqrt{-a^2+b^2}) f \sqrt{g \cos[e+fx]}} -$$

$$\frac{a^3 (a^2-b^2) g^3 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^5 (b+\sqrt{-a^2+b^2}) f \sqrt{g \cos[e+fx]}} - \frac{2 a g (g \cos[e+fx])^{3/2} \sin[e+fx]}{5 b^2 f}$$

Result (type 6, 1218 leaves):

$$\frac{1}{5 b^3 f \cos[e+fx]^{5/2}} a (g \cos[e+fx])^{5/2} \left(\frac{1}{6 \sqrt{1-\cos[e+fx]^2} (a+b \sin[e+fx])} \right.$$

$$a b \left(a+b \sqrt{1-\cos[e+fx]^2} \right) \left(- \left(56 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \cos[e+fx]^{3/2} \right) / \right.$$

$$\left(\sqrt{1-\cos[e+fx]^2} \left(7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right.$$

$$\left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right)$$

$$\left. \left. (a^2+b^2 (-1+\cos[e+fx]^2)) \right) - \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - \right.$$

$$\begin{aligned}
& 2 \operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]} + i b \cos[e+fx]\right] + \\
& \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]} + i b \cos[e+fx]\right] \Bigg) \Bigg) \operatorname{Sin}[e+fx] - \\
& \frac{1}{(1-\cos[e+fx]^2)(a+b\operatorname{Sin}[e+fx])} 2(5a^2-3b^2)\left(a+b\sqrt{1-\cos[e+fx]^2}\right) \\
& \left(\left(7b(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] \cos[e+fx]^{3/2}\sqrt{1-\cos[e+fx]^2} \right) / \right. \\
& \left. \left(3 \left(-7(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] + 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[e+fx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2\cos[e+fx]^2}{-a^2+b^2} \right) + (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right) \right. \\
& \left. (a^2+b^2(-1+\cos[e+fx]^2)) \right) \Bigg) + \frac{1}{4\sqrt{2}b^{3/2}(a^2-b^2)^{1/4}} a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] + \right. \\
& \left. 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} - \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\cos[e+fx]} + b\cos[e+fx]\right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\cos[e+fx]} + b\cos[e+fx]\right] \right) \Bigg) \Bigg) \operatorname{Sin}[e+fx]^2 + \\
& \frac{(g\cos[e+fx])^{5/2}\operatorname{Sec}[e+fx]^2 \left(-\frac{(-28a^2+9b^2)\cos[e+fx]}{42b^3} - \frac{\cos[3(e+fx)]}{14b} - \frac{a\sin[2(e+fx)]}{5b^2} \right)}{f}
\end{aligned}$$

- **Problem 1385: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g\cos[e+fx])^{5/2}\operatorname{Sin}[e+fx]}{a+b\operatorname{Sin}[e+fx]} dx$$

Optimal (type 4, 413 leaves, 13 steps):

$$\begin{aligned}
& - \frac{a (-a^2 + b^2)^{3/4} g^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{b^{7/2} f} + \frac{a (-a^2 + b^2)^{3/4} g^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{b^{7/2} f} - \\
& \frac{2 (5 a^2 - 3 b^2) g^2 \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2} (e + f x), 2\right]}{5 b^3 f \sqrt{\cos[e + f x]}} + \frac{a^2 (a^2 - b^2) g^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b^4 (b - \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}} + \\
& \frac{a^2 (a^2 - b^2) g^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b^4 (b + \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}} - \frac{2 g (g \cos[e + f x])^{3/2} (5 a - 3 b \sin[e + f x])}{15 b^2 f}
\end{aligned}$$

Result (type 6, 1191 leaves):

$$\begin{aligned}
& - \frac{1}{5 b^2 f \operatorname{Cos}[e+f x]^{5/2}} (g \operatorname{Cos}[e+f x])^{5/2} \\
& \left(\frac{1}{6 \sqrt{1-\operatorname{Cos}[e+f x]^2} (a+b \operatorname{Sin}[e+f x])} a b \left(a+b \sqrt{1-\operatorname{Cos}[e+f x]^2} \right) \left(- \left(56 a \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e+f x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Cos}[e+f x]^{3/2} \right) / \left(\sqrt{1-\operatorname{Cos}[e+f x]^2} \left(7 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2} \right] - \right. \right. \right. \\
& \left. \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2} \right] + \left(-a^2+b^2 \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2} \right] \right) \operatorname{Cos}[e+f x]^2 \left(a^2+b^2 \left(-1+\operatorname{Cos}[e+f x]^2 \right) \right) \right) - \frac{1}{\sqrt{b} \left(-a^2+b^2 \right)^{1/4}} \\
& \left. \left. \left. \left(3+3 i \right) \left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[e+f x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] - 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[e+f x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \right. \right. \\
& \left. \left. \left. \left(-a^2+b^2 \right)^{1/4} \sqrt{\operatorname{Cos}[e+f x]} + i b \operatorname{Cos}[e+f x] \right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\operatorname{Cos}[e+f x]} + i b \operatorname{Cos}[e+f x] \right] \right) \right) \right) \\
& \operatorname{Sin}[e+f x] - \frac{1}{\left(1-\operatorname{Cos}[e+f x]^2 \right) \left(a+b \operatorname{Sin}[e+f x] \right)} 2 \left(5 a^2-3 b^2 \right) \left(a+b \sqrt{1-\operatorname{Cos}[e+f x]^2} \right) \\
& \left(\left(7 b \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2} \right] \operatorname{Cos}[e+f x]^{3/2} \sqrt{1-\operatorname{Cos}[e+f x]^2} \right) / \left(3 \left(-7 \left(a^2-b^2 \right) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2} \right] \right) \operatorname{Cos}[e+f x]^2 \left(a^2+b^2 \left(-1+\operatorname{Cos}[e+f x]^2 \right) \right) \right) \right) + \\
& \frac{1}{4 \sqrt{2} b^{3/2} \left(a^2-b^2 \right)^{1/4}} a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e+f x]}}{\left(a^2-b^2 \right)^{1/4}} \right] + 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e+f x]}}{\left(a^2-b^2 \right)^{1/4}} \right] + \operatorname{Log}\left[\sqrt{a^2-b^2} - \right. \right. \\
& \left. \left. \sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{1/4} \sqrt{\operatorname{Cos}[e+f x]} + b \operatorname{Cos}[e+f x] \right] - \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} \left(a^2-b^2 \right)^{1/4} \sqrt{\operatorname{Cos}[e+f x]} + b \operatorname{Cos}[e+f x] \right] \right) \right) \\
& \left. \left. \left. \operatorname{Sin}[e+f x]^2 \right) + \frac{\left(g \operatorname{Cos}[e+f x] \right)^{5/2} \operatorname{Sec}[e+f x]^2 \left(-\frac{2 a \operatorname{Cos}[e+f x]}{3 b^2} + \frac{\operatorname{Sin}[2(e+f x)]}{5 b} \right)}{f} \right)
\end{aligned}$$

- **Problem 1386: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{5/2} \operatorname{Csc}[e + f x]}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 425 leaves, 21 steps):

$$\frac{g^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}}\right]}{a f} - \frac{(-a^2 + b^2)^{3/4} g^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a b^{3/2} f} - \frac{g^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}}\right]}{a f} +$$

$$\frac{(-a^2 + b^2)^{3/4} g^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a b^{3/2} f} - \frac{2 g^2 \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2}(e + f x), 2\right]}{b f \sqrt{\cos[e + f x]}} +$$

$$\frac{(a^2 - b^2) g^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2}(e + f x), 2\right]}{b^2 (b - \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}} + \frac{(a^2 - b^2) g^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2}(e + f x), 2\right]}{b^2 (b + \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}}$$

Result (type 6, 1818 leaves):

$$\frac{1}{2 f \cos[e + f x]^{5/2}}$$

$$(g \cos[e + f x])^{5/2} \left(-\frac{1}{(1 - \cos[e + f x]^2) (-1 + 2 \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 2 (-1 + \cos[e + f x]^2) (a + b \sqrt{1 - \cos[e + f x]^2}) \cos[2(e + f x)] \right.$$

$$\left. \operatorname{Csc}[e + f x] \left(\frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} \right. \right.$$

$$\left. \frac{\operatorname{ArcTan}\left[\sqrt{\cos[e + f x]}\right]}{2 a} - \left(7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^{3/2} \right) /$$

$$\left(3 \sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right.$$

$$\left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right)$$

$$\left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \left(22 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^{7/2} \right) /$$

$$\begin{aligned}
& \left(7 \sqrt{1 - \cos[e + f x]^2} \left(11 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - \right. \right. \\
& \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) \right) + \frac{\log \left[1 - \sqrt{\cos[e + f x]} \right]}{4 a} - \\
& \frac{\log \left[1 + \sqrt{\cos[e + f x]} \right]}{4 a} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \log \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} - \\
& \left. \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \log \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 2 (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \operatorname{Csc}[e + f x] \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) / \right. \\
& \quad \left(3 \sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \right) \\
& \quad \left. \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) \right) + \frac{1}{8 a (a^2 - b^2)^{1/4}} \left(2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \right. \\
& \quad 2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + 4 (a^2 - b^2)^{1/4} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] + 2 (a^2 - b^2)^{1/4} \log \left[1 - \sqrt{\cos[e + f x]} \right] - \\
& \quad 2 (a^2 - b^2)^{1/4} \log \left[1 + \sqrt{\cos[e + f x]} \right] - \sqrt{2} \sqrt{b} \log \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \\
& \quad \left. \left. \left. \sqrt{2} \sqrt{b} \log \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 1387: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{5/2} \operatorname{Csc}[e + f x]^2}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 462 leaves, 24 steps):

$$\begin{aligned} & - \frac{b g^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}}\right]}{a^2 f} + \frac{(-a^2 + b^2)^{3/4} g^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a^2 \sqrt{b} f} + \frac{b g^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}}\right]}{a^2 f} - \\ & \frac{(-a^2 + b^2)^{3/4} g^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a^2 \sqrt{b} f} - \frac{g (g \cos[e + f x])^{3/2} \operatorname{Csc}[e + f x]}{a f} - \frac{g^2 \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2} (e + f x), 2\right]}{a f \sqrt{\cos[e + f x]}} - \\ & \frac{(a^2 - b^2) g^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{a b \left(b - \sqrt{-a^2 + b^2}\right) f \sqrt{g \cos[e + f x]}} - \frac{(a^2 - b^2) g^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{a b \left(b + \sqrt{-a^2 + b^2}\right) f \sqrt{g \cos[e + f x]}} \end{aligned}$$

Result (type 6, 2391 leaves):

$$\begin{aligned} & \frac{1}{4 a f \cos[e + f x]^{5/2}} (g \cos[e + f x])^{5/2} \\ & \left(- \frac{1}{2 \sqrt{1 - \cos[e + f x]^2} (b + a \operatorname{Csc}[e + f x])} a \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \left(- \left(56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2\right], \right. \right. \right. \\ & \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right) \cos[e + f x]^{3/2} \right) / \left(\sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] - \right. \right. \\ & \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \right. \\ & \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \left. \right) - \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} \\ & (3 + 3 i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \\ & \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \right) \left. \right) - \\ & \frac{1}{(1 - \cos[e + f x]^2) (-1 + 2 \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 2 b (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \end{aligned}$$

$$\text{Cos}[2(e + f x)] \text{Csc}[e + f x]$$

$$\left(\frac{(a^2 - b^2)^{3/4} (-2a^2 + b^2) \text{ArcTan}\left[\frac{-\sqrt{2}(a^2 - b^2)^{1/4} + 2\sqrt{b}\sqrt{\text{Cos}[e + f x]}}{\sqrt{2}(a^2 - b^2)^{1/4}}\right]}{2\sqrt{2}ab^{3/2}(-a^2 + b^2)} + \frac{(a^2 - b^2)^{3/4} (-2a^2 + b^2) \text{ArcTan}\left[\frac{\sqrt{2}(a^2 - b^2)^{1/4} + 2\sqrt{b}\sqrt{\text{Cos}[e + f x]}}{\sqrt{2}(a^2 - b^2)^{1/4}}\right]}{2\sqrt{2}ab^{3/2}(-a^2 + b^2)} + \right.$$

$$\frac{\text{ArcTan}\left[\sqrt{\text{Cos}[e + f x]}\right]}{2a} - \left(7b(a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \text{Cos}[e + f x]^{3/2} \right) /$$

$$\left(3\sqrt{1 - \text{Cos}[e + f x]^2} \left(7(a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] - 2 \left(2b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right.$$

$$\left. \left. \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \right) \text{Cos}[e + f x]^2 \right)$$

$$(a^2 + b^2(-1 + \text{Cos}[e + f x]^2)) + \left(22b(a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \text{Cos}[e + f x]^{7/2} \right) /$$

$$\left(7\sqrt{1 - \text{Cos}[e + f x]^2} \left(11(a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] - \right.$$

$$\left. 2 \left(2b^2 \text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right.$$

$$\left. \left. \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \right) \text{Cos}[e + f x]^2 \right) (a^2 + b^2(-1 + \text{Cos}[e + f x]^2)) \left. \right) + \frac{\text{Log}\left[1 - \sqrt{\text{Cos}[e + f x]}\right]}{4a} -$$

$$\frac{\text{Log}\left[1 + \sqrt{\text{Cos}[e + f x]}\right]}{4a} + \frac{(a^2 - b^2)^{3/4} (-2a^2 + b^2) \text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4}\sqrt{\text{Cos}[e + f x]} + b\text{Cos}[e + f x]\right]}{4\sqrt{2}ab^{3/2}(-a^2 + b^2)} -$$

$$\left. \frac{(a^2 - b^2)^{3/4} (-2a^2 + b^2) \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4}\sqrt{\text{Cos}[e + f x]} + b\text{Cos}[e + f x]\right]}{4\sqrt{2}ab^{3/2}(-a^2 + b^2)} \right) +$$

$$\frac{1}{(1 - \text{Cos}[e + f x]^2)(b + a\text{Csc}[e + f x])} 10b(-1 + \text{Cos}[e + f x]^2) \left(a + b\sqrt{1 - \text{Cos}[e + f x]^2} \right) \text{Csc}[e + f x]$$

$$\left(\left(7b(a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \text{Cos}[e + f x]^{3/2} \right) / \right.$$

$$\left. \left(3\sqrt{1 - \text{Cos}[e + f x]^2} \left(7(a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] - 2 \left(2b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right.$$

$$\begin{aligned}
& \left(-\frac{1}{4\sqrt{1-\cos[e+fx]^2}(b+a\csc[e+fx])} ab\left(a+b\sqrt{1-\cos[e+fx]^2}\right) \left(-\left(56a(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2\right], \right. \right. \right. \\
& \left. \left. \left. \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right)\cos[e+fx]^{3/2}\right) \left(\sqrt{1-\cos[e+fx]^2} \left(7(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2\right], \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right) - \right. \right. \\
& \left. \left. 2\left(2b^2\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right]\right)\cos[e+fx]^2\right) \left(a^2+b^2(-1+\cos[e+fx]^2) \right) \right) - \frac{1}{\sqrt{b}(-a^2+b^2)^{1/4}} \\
& (3+3i) \left(2\operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - 2\operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b} \right. \right. \\
& \left. \left. (-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]} + ib\cos[e+fx]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]} + ib\cos[e+fx]\right] \right) \left. \right) - \\
& \frac{1}{(1-\cos[e+fx]^2)(-1+2\cos[e+fx]^2)(b+a\csc[e+fx])} 2b^2(-1+\cos[e+fx]^2) \left(a+b\sqrt{1-\cos[e+fx]^2} \right) \\
& \cos[2(e+fx)]\csc[e+fx] \\
& \left(\frac{(a^2-b^2)^{3/4}(-2a^2+b^2)\operatorname{ArcTan}\left[\frac{-\sqrt{2}(a^2-b^2)^{1/4}+2\sqrt{b}\sqrt{\cos[e+fx]}}{\sqrt{2}(a^2-b^2)^{1/4}}\right]}{2\sqrt{2}ab^{3/2}(-a^2+b^2)} + \frac{(a^2-b^2)^{3/4}(-2a^2+b^2)\operatorname{ArcTan}\left[\frac{\sqrt{2}(a^2-b^2)^{1/4}+2\sqrt{b}\sqrt{\cos[e+fx]}}{\sqrt{2}(a^2-b^2)^{1/4}}\right]}{2\sqrt{2}ab^{3/2}(-a^2+b^2)} + \right. \\
& \frac{\operatorname{ArcTan}\left[\sqrt{\cos[e+fx]}\right]}{2a} - \left(7b(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right]\cos[e+fx]^{3/2}\right) \left. \right) \left. \right) / \\
& \left(3\sqrt{1-\cos[e+fx]^2} \left(7(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] - 2\left(2b^2\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] \right)\cos[e+fx]^2 \right) \\
& \left(a^2+b^2(-1+\cos[e+fx]^2) \right) \left. \right) + \left(22b(a^2-b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right]\cos[e+fx]^{7/2}\right) \left. \right) / \\
& \left(7\sqrt{1-\cos[e+fx]^2} \left(11(a^2-b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] - 2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 b^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) + \frac{\operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right]}{4 a} - \\
& \frac{\operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right]}{4 a} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} - \\
& \left. \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 2 (3 a^2 - 5 b^2) (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \operatorname{Csc}[e + f x] \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) / \left(3 \sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \\
& \frac{1}{8 a (a^2 - b^2)^{1/4}} \left(2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - 2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \quad 4 (a^2 - b^2)^{1/4} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] + 2 (a^2 - b^2)^{1/4} \operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right] - \\
& \quad 2 (a^2 - b^2)^{1/4} \operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right] - \sqrt{2} \sqrt{b} \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \\
& \quad \left. \left. \left. \sqrt{2} \sqrt{b} \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \right) + \\
& \frac{(g \cos[e + f x])^{5/2} \left(\frac{b \operatorname{Cot}[e + f x]}{a^2} - \frac{\operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]}{2 a} \right) \operatorname{Sec}[e + f x]^2}{f}
\end{aligned}$$

■ **Problem 1389:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sin[e + f x]^4}{\sqrt{g \cos[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 509 leaves, 23 steps):

$$\begin{aligned} & - \frac{a^4 \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{b^{7/2} (-a^2 + b^2)^{3/4} f \sqrt{g}} - \frac{a^4 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{b^{7/2} (-a^2 + b^2)^{3/4} f \sqrt{g}} - \frac{2 a^2 \sqrt{g \cos[e + f x]}}{b^3 f g} - \\ & \frac{2 \sqrt{g \cos[e + f x]}}{b f g} + \frac{2 (g \cos[e + f x])^{5/2}}{5 b f g^3} - \frac{2 a^3 \sqrt{\cos[e + f x]} \operatorname{EllipticF}\left[\frac{1}{2} (e + f x), 2\right]}{b^4 f \sqrt{g \cos[e + f x]}} - \\ & \frac{4 a \sqrt{\cos[e + f x]} \operatorname{EllipticF}\left[\frac{1}{2} (e + f x), 2\right]}{3 b^2 f \sqrt{g \cos[e + f x]}} + \frac{a^5 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b^4 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) f \sqrt{g \cos[e + f x]}} + \\ & \frac{a^5 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b^4 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) f \sqrt{g \cos[e + f x]}} + \frac{2 a \sqrt{g \cos[e + f x]} \sin[e + f x]}{3 b^2 f g} \end{aligned}$$

Result (type 6, 2153 leaves):

$$\begin{aligned} & \frac{\cos[e + f x] \left(\frac{\cos[2(e + f x)]}{5b} + \frac{2a \sin[e + f x]}{3b^2}\right)}{f \sqrt{g \cos[e + f x]}} - \\ & \frac{1}{60 b^2 f \sqrt{g \cos[e + f x]}} \sqrt{\cos[e + f x]} \left(- \frac{1}{\sqrt{1 - \cos[e + f x]^2} (a + b \sin[e + f x])} - 2 (10 a^2 - 27 b^2) \left(a + b \sqrt{1 - \cos[e + f x]^2}\right) \right. \\ & \left. \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \sqrt{\cos[e + f x]} \right) / \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \right. \right. \right. \right. \\ & \left. \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \right) \right) \right. \\ & \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) \right) - \\ & \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] \right) + \end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
& \text{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\text{Cos}[e+fx]} + i b \text{Cos}[e+fx]\right] - \\
& \text{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\text{Cos}[e+fx]} + i b \text{Cos}[e+fx]\right] \Bigg) \text{Sin}[e+fx] + \\
& \frac{1}{\sqrt{1-\text{Cos}[e+fx]^2}(-1+2\text{Cos}[e+fx]^2)(a+b\text{Sin}[e+fx])} (30a^2+27b^2)\left(a+b\sqrt{1-\text{Cos}[e+fx]^2}\right)\text{Cos}[2(e+fx)] \\
& \left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(-2a^2+b^2)\text{ArcTan}\left[1-\frac{(1+i)\sqrt{b}\sqrt{\text{Cos}[e+fx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2}(-a^2+b^2)^{3/4}} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)(-2a^2+b^2)\text{ArcTan}\left[1+\frac{(1+i)\sqrt{b}\sqrt{\text{Cos}[e+fx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2}(-a^2+b^2)^{3/4}} \right) + \\
& \frac{4\sqrt{\text{Cos}[e+fx]}}{b} + \left(10a(a^2-b^2)\text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e+fx]^2, \frac{b^2\text{Cos}[e+fx]^2}{-a^2+b^2}\right]\sqrt{\text{Cos}[e+fx]}\right) / \\
& \left(\sqrt{1-\text{Cos}[e+fx]^2}\left(5(a^2-b^2)\text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e+fx]^2, \frac{b^2\text{Cos}[e+fx]^2}{-a^2+b^2}\right] - 2\left(2b^2\text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \left. \left. \left. \text{Cos}[e+fx]^2, \frac{b^2\text{Cos}[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2)\text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \text{Cos}[e+fx]^2, \frac{b^2\text{Cos}[e+fx]^2}{-a^2+b^2}\right]\right)\text{Cos}[e+fx]^2\right) \right) \\
& \left.(a^2+b^2(-1+\text{Cos}[e+fx]^2)\right) - \left(36a(a^2-b^2)\text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Cos}[e+fx]^2, \frac{b^2\text{Cos}[e+fx]^2}{-a^2+b^2}\right]\text{Cos}[e+fx]^{5/2}\right) / \\
& \left(5\sqrt{1-\text{Cos}[e+fx]^2}\left(9(a^2-b^2)\text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Cos}[e+fx]^2, \frac{b^2\text{Cos}[e+fx]^2}{-a^2+b^2}\right] - \right. \right. \\
& \left. \left. 2\left(2b^2\text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \text{Cos}[e+fx]^2, \frac{b^2\text{Cos}[e+fx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2)\text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \text{Cos}[e+fx]^2, \frac{b^2\text{Cos}[e+fx]^2}{-a^2+b^2}\right]\right)\text{Cos}[e+fx]^2\right)\right)(a^2+b^2(-1+\text{Cos}[e+fx]^2)) \Bigg) + \\
& \frac{1}{b^{3/2}(-a^2+b^2)^{3/4}}\left(\frac{1}{4}-\frac{i}{4}\right)(-2a^2+b^2)\text{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\text{Cos}[e+fx]} + i b \text{Cos}[e+fx]\right] - \\
& \frac{1}{b^{3/2}(-a^2+b^2)^{3/4}}\left(\frac{1}{4}-\frac{i}{4}\right)(-2a^2+b^2)\text{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\text{Cos}[e+fx]} + i b \text{Cos}[e+fx]\right] \Bigg) \text{Sin}[e+fx] + \\
& \frac{1}{(1-\text{Cos}[e+fx]^2)(a+b\text{Sin}[e+fx])} 28ab\left(a+b\sqrt{1-\text{Cos}[e+fx]^2}\right)
\end{aligned}
\end{aligned}$$

$$\left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \sqrt{1 - \cos[e + f x]^2} \right) / \right. \\ \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \right. \right. \\ \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right) + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \\ (a^2 + b^2 (-1 + \cos[e + f x]^2)) \left. \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\ \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \right. \\ \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \sin[e + f x]^2$$

■ **Problem 1390: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]^3}{\sqrt{g \cos[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 457 leaves, 19 steps):

$$\frac{a^3 \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{b^{5/2} (-a^2 + b^2)^{3/4} f \sqrt{g}} + \frac{a^3 \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{b^{5/2} (-a^2 + b^2)^{3/4} f \sqrt{g}} + \frac{2 a \sqrt{g \cos[e + f x]}}{b^2 f g} + \frac{2 a^2 \sqrt{\cos[e + f x]} \operatorname{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{b^3 f \sqrt{g \cos[e + f x]}} + \\ \frac{4 \sqrt{\cos[e + f x]} \operatorname{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{3 b f \sqrt{g \cos[e + f x]}} - \frac{a^4 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{b^3 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2} \right) \right) f \sqrt{g \cos[e + f x]}} - \\ \frac{a^4 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{b^3 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2} \right) \right) f \sqrt{g \cos[e + f x]}} - \frac{2 \sqrt{g \cos[e + f x]} \sin[e + f x]}{3 b f g}$$

Result (type 6, 2115 leaves):

$$-\frac{2 \cos[e + f x] \sin[e + f x]}{3 b f \sqrt{g \cos[e + f x]}} +$$

$$\begin{aligned}
& \frac{1}{6 b f \sqrt{g \cos [e+f x]}} \sqrt{\cos [e+f x]} \left(-\frac{1}{\sqrt{1-\cos [e+f x]^2} (a+b \sin [e+f x])} 2 a \left(a+b \sqrt{1-\cos [e+f x]^2} \right) \left(\left(5 a \left(a^2-b^2 \right) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \sqrt{\cos [e+f x]}\right) / \left(\sqrt{1-\cos [e+f x]^2} \left(5 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. 1, \frac{5}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \right) - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \right) + \right. \\
& \quad \left. \left. \left. \left(-a^2+b^2 \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \right) \cos [e+f x]^2 \right) \left(a^2+b^2 \left(-1+\cos [e+f x]^2 \right) \right) \right) - \\
& \quad \frac{1}{\left(-a^2+b^2 \right)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos [e+f x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos [e+f x]}}{\left(-a^2+b^2 \right)^{1/4}} \right] \right) + \\
& \quad \left(\operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [e+f x]} + i b \cos [e+f x] \right] - \right. \\
& \quad \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} \left(-a^2+b^2 \right)^{1/4} \sqrt{\cos [e+f x]} + i b \cos [e+f x] \right] \right) \left. \right) \sin [e+f x] + \\
& \quad \frac{1}{\sqrt{1-\cos [e+f x]^2} \left(-1+2 \cos [e+f x]^2 \right) (a+b \sin [e+f x])} 3 a \left(a+b \sqrt{1-\cos [e+f x]^2} \right) \cos [2(e+f x)] \\
& \quad \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) \left(-2 a^2+b^2 \right) \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos [e+f x]}}{\left(-a^2+b^2 \right)^{1/4}} \right]}{b^{3/2} \left(-a^2+b^2 \right)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) \left(-2 a^2+b^2 \right) \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos [e+f x]}}{\left(-a^2+b^2 \right)^{1/4}} \right]}{b^{3/2} \left(-a^2+b^2 \right)^{3/4}} \right) + \\
& \quad \frac{4 \sqrt{\cos [e+f x]}}{b} + \left(10 a \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \sqrt{\cos [e+f x]}\right) / \\
& \quad \left(\sqrt{1-\cos [e+f x]^2} \left(5 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \right) + \left(-a^2+b^2 \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \right) \cos [e+f x]^2 \right) \\
& \quad \left. \left(a^2+b^2 \left(-1+\cos [e+f x]^2 \right) \right) \right) - \left(36 a \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \cos [e+f x]^{5/2} \right) / \\
& \quad \left(5 \sqrt{1-\cos [e+f x]^2} \left(9 \left(a^2-b^2 \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] - \right. \right. \\
& \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left((-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \right) \operatorname{Sin}[e + f x] - \\
& \frac{1}{(1 - \cos[e + f x]^2) (a + b \operatorname{Sin}[e + f x])} 8 b \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \sqrt{1 - \cos[e + f x]^2} \right) / \right. \\
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \right. \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \operatorname{Sin}[e + f x]^2 \right)
\end{aligned}$$

- **Problem 1391: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^2}{\sqrt{g \cos[e + f x]} (a + b \operatorname{Sin}[e + f x])} dx$$

Optimal (type 4, 380 leaves, 15 steps):

$$\begin{aligned}
& \frac{a^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{3/2} (-a^2+b^2)^{3/4} f \sqrt{g}} - \frac{a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{3/2} (-a^2+b^2)^{3/4} f \sqrt{g}} - \frac{2 \sqrt{g \cos[e+fx]}}{b f g} - \frac{2 a \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2}(e+fx), 2\right]}{b^2 f \sqrt{g \cos[e+fx]}} + \\
& \frac{a^3 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{-2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^2 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}} + \frac{a^3 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{-2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^2 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}}
\end{aligned}$$

Result (type 6, 1526 leaves):

$$\begin{aligned}
& \frac{1}{2 f \sqrt{g \cos[e+fx]}} \sqrt{\cos[e+fx]} \\
& \left(-\frac{1}{\sqrt{1-\cos[e+fx]^2} (a+b \sin[e+fx])} 2 \left(a+b \sqrt{1-\cos[e+fx]^2} \right) \left(\left(5 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right. \right. \right. \\
& \left. \left. \left. \frac{\sqrt{\cos[e+fx]}}{\sqrt{1-\cos[e+fx]^2}} \right) / \left(\sqrt{1-\cos[e+fx]^2} \left(5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right) \right) \\
& \left. \left. \left. \left. (a^2+b^2 (-1+\cos[e+fx]^2)) \right) - \frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - \right. \right. \right. \\
& \left. \left. \left. 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx]\right] - \right. \right. \\
& \left. \left. \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx]\right] \right) \right) \right) \right) \sin[e+fx] - \\
& \frac{1}{\sqrt{1-\cos[e+fx]^2} (-1+2 \cos[e+fx]^2) (a+b \sin[e+fx])} \left(a+b \sqrt{1-\cos[e+fx]^2} \right) \cos[2(e+fx)] \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2+b^2) \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2} (-a^2+b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2+b^2) \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right]}{b^{3/2} (-a^2+b^2)^{3/4}} \right) + \\
& \frac{4 \sqrt{\cos[e+fx]}}{b} + \left(10 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \sqrt{\cos[e+fx]} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \quad \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{5/2} \right) / \\
& \left(5 \sqrt{1 - \cos[e + f x]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \right. \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \quad \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{\left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} \\
& \quad \left. \frac{\left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} \right) \operatorname{Sin}[e + f x] \Bigg)
\end{aligned}$$

■ **Problem 1392: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sin}[e + f x]}{\sqrt{g \cos[e + f x]} (a + b \operatorname{Sin}[e + f x])} dx$$

Optimal (type 4, 352 leaves, 12 steps):

$$\begin{aligned}
& \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{\sqrt{b} (-a^2 + b^2)^{3/4} f \sqrt{g}} + \frac{a \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{\sqrt{b} (-a^2 + b^2)^{3/4} f \sqrt{g}} + \frac{2 \sqrt{\cos[e + f x]} \operatorname{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{b f \sqrt{g \cos[e + f x]}} - \\
& \frac{a^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{b (a^2 - b^2 + b \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}} - \frac{a^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{b (a^2 - b (b + \sqrt{-a^2 + b^2})) f \sqrt{g \cos[e + f x]}}
\end{aligned}$$

Result (type 6, 573 leaves):

$$\begin{aligned}
& - \frac{1}{f \sqrt{g \cos[e + f x]} (1 - \cos[e + f x]^2) (a + b \sin[e + f x])} - 2 \sqrt{\cos[e + f x]} \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \sqrt{1 - \cos[e + f x]^2} \right) / \right. \\
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) \left. \right) + \\
& \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \right. \\
& \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \left. \right) \sin[e + f x]^2
\end{aligned}$$

■ **Problem 1393: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Csc}[e + f x]}{\sqrt{g \cos[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 369 leaves, 16 steps):

$$\begin{aligned}
& - \frac{\operatorname{ArcTan} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}} \right]}{a f \sqrt{g}} + \frac{b^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a (-a^2 + b^2)^{3/4} f \sqrt{g}} - \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}} \right]}{a f \sqrt{g}} + \frac{b^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a (-a^2 + b^2)^{3/4} f \sqrt{g}} - \\
& \frac{b \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{(a^2 - b (b - \sqrt{-a^2 + b^2})) f \sqrt{g \cos[e + f x]}} - \frac{b \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{(a^2 - b (b + \sqrt{-a^2 + b^2})) f \sqrt{g \cos[e + f x]}}
\end{aligned}$$

Result (type 6, 698 leaves):

$$\begin{aligned}
& - \frac{1}{f \sqrt{g \cos[e + f x]} (1 - \cos[e + f x]^2) (b + a \csc[e + f x])} - 2 \sqrt{\cos[e + f x]} (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \csc[e + f x] \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \right) / \left(\sqrt{1 - \cos[e + f x]^2} \right. \right. \\
& \left. \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \\
& \frac{1}{8 a (a^2 - b^2)^{3/4}} \left(-2 \sqrt{2} b^{3/2} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + 2 \sqrt{2} b^{3/2} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 4 (a^2 - b^2)^{3/4} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] - 2 (a^2 - b^2)^{3/4} \operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right] + \\
& 2 (a^2 - b^2)^{3/4} \operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right] - \sqrt{2} b^{3/2} \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \\
& \left. \left. \left. \sqrt{2} b^{3/2} \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \right)
\end{aligned}$$

■ **Problem 1394: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[e + f x]^2}{\sqrt{g \cos[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 448 leaves, 19 steps):

$$\begin{aligned}
& \frac{b \operatorname{ArcTan} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}} \right]}{a^2 f \sqrt{g}} - \frac{b^{5/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a^2 (-a^2 + b^2)^{3/4} f \sqrt{g}} + \frac{b \operatorname{ArcTanh} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}} \right]}{a^2 f \sqrt{g}} - \\
& \frac{b^{5/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a^2 (-a^2 + b^2)^{3/4} f \sqrt{g}} - \frac{\sqrt{g \cos[e + f x]} \csc[e + f x]}{a f g} + \frac{\sqrt{\cos[e + f x]} \operatorname{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{a f \sqrt{g \cos[e + f x]}} + \\
& \frac{b^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{a (a^2 - b^2 + b \sqrt{-a^2 + b^2}) f \sqrt{g \cos[e + f x]}} + \frac{b^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{a (a^2 - b (b + \sqrt{-a^2 + b^2})) f \sqrt{g \cos[e + f x]}}
\end{aligned}$$

Result (type 6, 2375 leaves):

$$\begin{aligned}
& - \frac{\text{Cot}[e + f x]}{a f \sqrt{g \text{Cos}[e + f x]}} - \frac{1}{4 a f \sqrt{g \text{Cos}[e + f x]}} \sqrt{\text{Cos}[e + f x]} \\
& \left(\frac{1}{\sqrt{1 - \text{Cos}[e + f x]^2} (b + a \text{Csc}[e + f x])} - 4 a \left(a + b \sqrt{1 - \text{Cos}[e + f x]^2} \right) \left(\left(5 a (a^2 - b^2) \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right. \right. \right. \\
& \left. \left. \left. \sqrt{\text{Cos}[e + f x]} \right) / \left(\sqrt{1 - \text{Cos}[e + f x]^2} \left(5 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. 2, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right) \text{Cos}[e + f x]^2 \right) \right. \\
& \left. \left. (a^2 + b^2 (-1 + \text{Cos}[e + f x]^2)) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \text{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \\
& \left. 2 \text{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] + \text{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Cos}[e + f x]} + i b \text{Cos}[e + f x] \right] - \right. \\
& \left. \left. \text{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Cos}[e + f x]} + i b \text{Cos}[e + f x] \right] \right) \right) - \\
& \frac{1}{(1 - \text{Cos}[e + f x]^2) (-1 + 2 \text{Cos}[e + f x]^2) (b + a \text{Csc}[e + f x])} b (-1 + \text{Cos}[e + f x]^2) \left(a + b \sqrt{1 - \text{Cos}[e + f x]^2} \right) \text{Cos}[2 (e + f x)] \\
& \text{Csc}[e + f x] \left(\frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \text{ArcTan} \left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{\sqrt{2} a \sqrt{b} (-a^2 + b^2)} + \frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \text{ArcTan} \left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{\sqrt{2} a \sqrt{b} (-a^2 + b^2)} - \right. \\
& \frac{\text{ArcTan} \left[\sqrt{\text{Cos}[e + f x]} \right]}{a} - \left(10 b (a^2 - b^2) \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\text{Cos}[e + f x]} \right) / \\
& \left(\sqrt{1 - \text{Cos}[e + f x]^2} \left(5 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right) \text{Cos}[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \text{Cos}[e + f x]^2)) \right) + \left(36 b (a^2 - b^2) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] \text{Cos}[e + f x]^{5/2} \right) / \\
& \left(5 \sqrt{1 - \text{Cos}[e + f x]^2} \left(9 (a^2 - b^2) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) \left. + \frac{\operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right]}{2 a} - \right. \\
& \frac{\operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right]}{2 a} - \frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{2 \sqrt{2} a \sqrt{b} (-a^2 + b^2)} + \\
& \left. \frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{2 \sqrt{2} a \sqrt{b} (-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 6 b (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \operatorname{Csc}[e + f x] \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \right) / \right. \\
& \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \frac{1}{8 a (a^2 - b^2)^{3/4}} \left(-2 \sqrt{2} b^{3/2} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \sqrt{2} b^{3/2} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + 4 (a^2 - b^2)^{3/4} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] - 2 (a^2 - b^2)^{3/4} \operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right] + \\
& 2 (a^2 - b^2)^{3/4} \operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right] - \sqrt{2} b^{3/2} \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \\
& \left. \left. \left. \sqrt{2} b^{3/2} \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 1395: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + f x]^3}{\sqrt{g \cos[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 557 leaves, 25 steps):

$$\begin{aligned} & - \frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{4 a f \sqrt{g}} - \frac{b^2 \operatorname{ArcTan}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{a^3 f \sqrt{g}} + \frac{b^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{a^3 (-a^2+b^2)^{3/4} f \sqrt{g}} - \frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{4 a f \sqrt{g}} - \frac{b^2 \operatorname{ArcTanh}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{a^3 f \sqrt{g}} + \\ & \frac{b^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{a^3 (-a^2+b^2)^{3/4} f \sqrt{g}} + \frac{b \sqrt{g \cos[e+fx]} \operatorname{Csc}[e+fx]}{a^2 f g} - \frac{\sqrt{g \cos[e+fx]} \operatorname{Csc}[e+fx]^2}{2 a f g} - \frac{b \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2}(e+fx), 2\right]}{a^2 f \sqrt{g \cos[e+fx]}} \\ & \frac{b^3 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{a^2 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}} - \frac{b^3 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{a^2 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) f \sqrt{g \cos[e+fx]}} \end{aligned}$$

Result (type 6, 2411 leaves):

$$\begin{aligned} & \frac{\operatorname{Cos}[e+fx] \left(\frac{b \operatorname{Csc}[e+fx]}{a^2} - \frac{\operatorname{Csc}[e+fx]^2}{2a}\right)}{f \sqrt{g \cos[e+fx]}} + \\ & \frac{1}{4 a^2 f \sqrt{g \cos[e+fx]}} \sqrt{\cos[e+fx]} \left(- \frac{1}{\sqrt{1 - \cos[e+fx]^2} (b + a \operatorname{Csc}[e+fx])} - 2 a b \left(a + b \sqrt{1 - \cos[e+fx]^2}\right) \right. \\ & \left. \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \sqrt{\cos[e+fx]} \right) / \left(\sqrt{1 - \cos[e+fx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \right. \right. \right. \right. \right. \\ & \left. \left. \left. \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \right. \right. \right. \right. \\ & \left. \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right) (a^2 + b^2 (-1 + \cos[e+fx]^2)) \right) - \frac{1}{(-a^2+b^2)^{3/4}} \right. \\ & \left. \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \right. \\ & \left. \left. \left. (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx]\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx]\right] \right) \right) - \\ & \frac{1}{(1 - \cos[e+fx]^2) (-1 + 2 \cos[e+fx]^2) (b + a \operatorname{Csc}[e+fx])} b^2 (-1 + \cos[e+fx]^2) \left(a + b \sqrt{1 - \cos[e+fx]^2}\right) \cos[2(e+fx)] \end{aligned}$$

$$\begin{aligned}
& \text{Csc}[e + f x] \left[\frac{\left((a^2 - b^2)^{1/4} (-2 a^2 + b^2) \text{ArcTan}\left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{\sqrt{2} a \sqrt{b} (-a^2 + b^2)} \right) + \frac{\left((a^2 - b^2)^{1/4} (-2 a^2 + b^2) \text{ArcTan}\left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{\sqrt{2} a \sqrt{b} (-a^2 + b^2)} \right)}{\sqrt{2} a \sqrt{b} (-a^2 + b^2)} - \right. \\
& \frac{\text{ArcTan}\left[\sqrt{\text{Cos}[e + f x]}\right]}{a} - \left(10 b (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \sqrt{\text{Cos}[e + f x]}\right) / \\
& \left(\sqrt{1 - \text{Cos}[e + f x]^2} \left(5 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \left. \left. \left. \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \right) \text{Cos}[e + f x]^2 \right) \right) \\
& \left. (a^2 + b^2 (-1 + \text{Cos}[e + f x]^2)) \right) + \left(36 b (a^2 - b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \text{Cos}[e + f x]^{5/2} \right) / \\
& \left(5 \sqrt{1 - \text{Cos}[e + f x]^2} \left(9 (a^2 - b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] - \right. \right. \\
& \left. \left. 2 \left(2 b^2 \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \right) \text{Cos}[e + f x]^2 \right) (a^2 + b^2 (-1 + \text{Cos}[e + f x]^2)) \right) + \frac{\text{Log}\left[1 - \sqrt{\text{Cos}[e + f x]}\right]}{2 a} - \\
& \frac{\text{Log}\left[1 + \sqrt{\text{Cos}[e + f x]}\right]}{2 a} - \frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Cos}[e + f x]} + b \text{Cos}[e + f x]\right]}{2 \sqrt{2} a \sqrt{b} (-a^2 + b^2)} + \\
& \left. \frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Cos}[e + f x]} + b \text{Cos}[e + f x]\right]}{2 \sqrt{2} a \sqrt{b} (-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \text{Cos}[e + f x]^2) (b + a \text{Csc}[e + f x])} - 2 (3 a^2 + 3 b^2) (-1 + \text{Cos}[e + f x]^2) \left(a + b \sqrt{1 - \text{Cos}[e + f x]^2} \right) \text{Csc}[e + f x] \\
& \left(\left(5 b (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \sqrt{\text{Cos}[e + f x]}\right) / \right. \\
& \left(\sqrt{1 - \text{Cos}[e + f x]^2} \left(5 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] \right) \text{Cos}[e + f x]^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \left(2b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + \right. \\
& \left. (-a^2+b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \cos[e+fx]^2 \right) (a^2+b^2 (-1+\cos[e+fx]^2)) \Bigg) - \\
& \frac{1}{\sqrt{b} (-a^2+b^2)^{1/4}} (3+3i) \left(2 \text{ArcTan}\left[1 - \frac{(1+i)\sqrt{b}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \text{ArcTan}\left[1 + \frac{(1+i)\sqrt{b}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - \right. \\
& \left. \text{Log}\left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]} + ib\cos[e+fx]\right] + \right. \\
& \left. \text{Log}\left[\sqrt{-a^2+b^2} + (1+i)\sqrt{b}(-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]} + ib\cos[e+fx]\right] \right) \Bigg) \sin[e+fx] - \\
& \frac{1}{(1-\cos[e+fx]^2)(a+b\sin[e+fx])} 2(a^2-2b^2) \left(a+b\sqrt{1-\cos[e+fx]^2} \right) \\
& \left(\left(7b(a^2-b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \cos[e+fx]^{3/2} \sqrt{1-\cos[e+fx]^2} \right) / \right. \\
& \left(3 \left(-7(a^2-b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + 2 \left(2b^2 \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \cos[e+fx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (a^2-b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right) \right. \\
& \left. (a^2+b^2(-1+\cos[e+fx]^2)) \right) + \frac{1}{4\sqrt{2}b^{3/2}(a^2-b^2)^{1/4}} a \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] + \right. \\
& \left. 2 \text{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] + \text{Log}\left[\sqrt{a^2-b^2} - \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\cos[e+fx]} + b\cos[e+fx]\right] - \right. \\
& \left. \left. \text{Log}\left[\sqrt{a^2-b^2} + \sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\cos[e+fx]} + b\cos[e+fx]\right] \right) \right) \Bigg) \sin[e+fx]^2 \Bigg)
\end{aligned}$$

- **Problem 1397: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]^3}{(g\cos[e+fx])^{3/2}(a+b\sin[e+fx])} dx$$

Optimal (type 4, 509 leaves, 18 steps):

$$\begin{aligned}
& - \frac{a^3 \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{3/2} (-a^2+b^2)^{5/4} f g^{3/2}} + \frac{a^3 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{3/2} (-a^2+b^2)^{5/4} f g^{3/2}} + \frac{2 a}{(a^2-b^2) f g \sqrt{g \cos[e+fx]}} - \\
& \frac{2 a^2 \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[\frac{1}{2}(e+fx), 2\right]}{b (a^2-b^2) f g^2 \sqrt{\cos[e+fx]}} + \frac{4 b \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[\frac{1}{2}(e+fx), 2\right]}{(a^2-b^2) f g^2 \sqrt{\cos[e+fx]}} + \\
& \frac{a^4 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^2 (a^2-b^2) \left(b-\sqrt{-a^2+b^2}\right) f g \sqrt{g \cos[e+fx]}} + \frac{a^4 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^2 (a^2-b^2) \left(b+\sqrt{-a^2+b^2}\right) f g \sqrt{g \cos[e+fx]}} - \frac{2 b \sin[e+fx]}{(a^2-b^2) f g \sqrt{g \cos[e+fx]}}
\end{aligned}$$

Result (type 6, 1187 leaves):

$$\begin{aligned}
& \frac{2 \operatorname{Cos}[e + f x] (a - b \operatorname{Sin}[e + f x])}{(a^2 - b^2) f (g \operatorname{Cos}[e + f x])^{3/2}} - \\
& \frac{1}{(a - b) (a + b) f (g \operatorname{Cos}[e + f x])^{3/2}} \operatorname{Cos}[e + f x]^{3/2} \left(- \frac{1}{6 \sqrt{1 - \operatorname{Cos}[e + f x]^2} (a + b \operatorname{Sin}[e + f x])} a b \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \right. \\
& \left(- \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^{3/2} \right) / \left(\sqrt{1 - \operatorname{Cos}[e + f x]^2} \left(7 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right) \operatorname{Cos}[e + f x]^2 \right) \left(a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2) \right) \right) - \\
& \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] + \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] \right) \left. \right) \operatorname{Sin}[e + f x] - \\
& \frac{1}{(1 - \operatorname{Cos}[e + f x]^2) (a + b \operatorname{Sin}[e + f x])} 2 (a^2 - 2 b^2) \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^{3/2} \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right) \operatorname{Cos}[e + f x]^2 \right) \\
& \left. \left(a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2) \right) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] - \right. \\
& \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] \right) \right) \left. \right) \operatorname{Sin}[e + f x]^2 \left. \right)
\end{aligned}$$

- **Problem 1398: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]^2}{(g \cos[e + f x])^{3/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 453 leaves, 15 steps):

$$\frac{a^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right] - a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{\sqrt{b} (-a^2 + b^2)^{5/4} f g^{3/2}} - \frac{2 b}{\sqrt{b} (-a^2 + b^2)^{5/4} f g^{3/2}} - \frac{2 a \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) f g^2 \sqrt{\cos[e + f x]}} -$$

$$\frac{a^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b (a^2 - b^2) \left(b - \sqrt{-a^2 + b^2}\right) f g \sqrt{g \cos[e + f x]}} - \frac{a^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b (a^2 - b^2) \left(b + \sqrt{-a^2 + b^2}\right) f g \sqrt{g \cos[e + f x]}} + \frac{2 a \sin[e + f x]}{(a^2 - b^2) f g \sqrt{g \cos[e + f x]}}$$

Result (type 6, 1180 leaves):

$$\begin{aligned}
& \frac{2 \operatorname{Cos}[e + f x] (-b + a \operatorname{Sin}[e + f x])}{(a^2 - b^2) f (g \operatorname{Cos}[e + f x])^{3/2}} - \\
& \frac{1}{(a - b) (a + b) f (g \operatorname{Cos}[e + f x])^{3/2}} a \operatorname{Cos}[e + f x]^{3/2} \left(\frac{1}{6 \sqrt{1 - \operatorname{Cos}[e + f x]^2} (a + b \operatorname{Sin}[e + f x])} a \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \right. \\
& \left. \left(- \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^{3/2} \right) / \left(\sqrt{1 - \operatorname{Cos}[e + f x]^2} \left(7 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2)) \right) \right) - \\
& \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] + \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] \right) \left. \right) \operatorname{Sin}[e + f x] - \frac{1}{(1 - \operatorname{Cos}[e + f x]^2) (a + b \operatorname{Sin}[e + f x])} \\
& 2 b \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^{3/2} \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) / \right. \\
& \left. \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right) \operatorname{Cos}[e + f x]^2 \right) \right. \\
& \left. (a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] \right) \left. \right) \operatorname{Sin}[e + f x]^2 \left. \right)
\end{aligned}$$

- **Problem 1399: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]}{(g \cos[e + f x])^{3/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 413 leaves, 13 steps):

$$\begin{aligned} & - \frac{a \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{(-a^2 + b^2)^{5/4} f g^{3/2}} + \frac{a \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{(-a^2 + b^2)^{5/4} f g^{3/2}} + \frac{2 b \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[\frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) f g^2 \sqrt{\cos[e + f x]}} + \\ & \frac{a^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) (b - \sqrt{-a^2 + b^2}) f g \sqrt{g \cos[e + f x]}} + \frac{a^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) (b + \sqrt{-a^2 + b^2}) f g \sqrt{g \cos[e + f x]}} + \frac{2 (a - b \sin[e + f x])}{(a^2 - b^2) f g \sqrt{g \cos[e + f x]}} \end{aligned}$$

Result (type 6, 1178 leaves):

$$\begin{aligned}
& \frac{2 \operatorname{Cos}[e + f x] (a - b \operatorname{Sin}[e + f x])}{(a^2 - b^2) f (g \operatorname{Cos}[e + f x])^{3/2}} + \\
& \frac{1}{(a - b) (a + b) f (g \operatorname{Cos}[e + f x])^{3/2}} b \operatorname{Cos}[e + f x]^{3/2} \left(\frac{1}{6 \sqrt{1 - \operatorname{Cos}[e + f x]^2} (a + b \operatorname{Sin}[e + f x])} a \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \right. \\
& \left. - \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^{3/2} \right) / \left(\sqrt{1 - \operatorname{Cos}[e + f x]^2} \left(7 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2)) \right) \right) - \\
& \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} (3 + 3i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] + \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] \right) \left. \right) \operatorname{Sin}[e + f x] - \frac{1}{(1 - \operatorname{Cos}[e + f x]^2) (a + b \operatorname{Sin}[e + f x])} \\
& 2 b \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^{3/2} \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) / \right. \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right) \operatorname{Cos}[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] \right) \left. \right) \operatorname{Sin}[e + f x]^2 \left. \right)
\end{aligned}$$

- **Problem 1400: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[e + f x]}{(g \text{Cos}[e + f x])^{3/2} (a + b \text{Sin}[e + f x])} dx$$

Optimal (type 4, 507 leaves, 21 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{g \text{Cos}[e + f x]}}{\sqrt{g}}\right]}{a f g^{3/2}} - \frac{b^{5/2} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a (-a^2 + b^2)^{5/4} f g^{3/2}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{g \text{Cos}[e + f x]}}{\sqrt{g}}\right]}{a f g^{3/2}} + \frac{b^{5/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a (-a^2 + b^2)^{5/4} f g^{3/2}} +$$

$$\frac{2}{a f g \sqrt{g \text{Cos}[e + f x]}} + \frac{2 b \sqrt{g \text{Cos}[e + f x]} \text{EllipticE}\left[\frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) f g^2 \sqrt{\text{Cos}[e + f x]}} + \frac{b^2 \sqrt{\text{Cos}[e + f x]} \text{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) (b - \sqrt{-a^2 + b^2}) f g \sqrt{g \text{Cos}[e + f x]}} +$$

$$\frac{b^2 \sqrt{\text{Cos}[e + f x]} \text{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) (b + \sqrt{-a^2 + b^2}) f g \sqrt{g \text{Cos}[e + f x]}} + \frac{2 b (b - a \text{Sin}[e + f x])}{a (a^2 - b^2) f g \sqrt{g \text{Cos}[e + f x]}}$$

Result (type 6, 2424 leaves):

$$\frac{1}{2 (a - b) (a + b) f (g \text{Cos}[e + f x])^{3/2}}$$

$$\text{Cos}[e + f x]^{3/2} \left(- \frac{1}{3 \sqrt{1 - \text{Cos}[e + f x]^2} (b + a \text{Csc}[e + f x])} a b (a + b \sqrt{1 - \text{Cos}[e + f x]^2}) \left(- \left(56 a (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Cos}[e + f x]^2\right], \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] - \right. \right.$$

$$\frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \text{Cos}[e + f x]^{3/2} \left. \left/ \left(\sqrt{1 - \text{Cos}[e + f x]^2} \left(7 (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Cos}[e + f x]^2\right], \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] - \right. \right.$$

$$2 \left(2 b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right.$$

$$\left. \left. \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right) \text{Cos}[e + f x]^2 (a^2 + b^2 (-1 + \text{Cos}[e + f x]^2)) \right) - \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}}$$

$$(3 + 3 i) \left(2 \text{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \text{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] - \text{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right.$$

$$\left. \left. (-a^2 + b^2)^{1/4} \sqrt{\text{Cos}[e + f x]} + i b \text{Cos}[e + f x] \right] + \text{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Cos}[e + f x]} + i b \text{Cos}[e + f x] \right] \right) \right) -$$

$$\begin{aligned}
& \frac{1}{(1 - \cos[e + f x])^2 (-1 + 2 \cos[e + f x]) (b + a \csc[e + f x])} 2 b^2 (-1 + \cos[e + f x])^2 \left(a + b \sqrt{1 - \cos[e + f x]} \right) \\
& \cos[2(e + f x)] \csc[e + f x] \\
& \left(\frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} + \right. \\
& \frac{\operatorname{ArcTan}\left[\sqrt{\cos[e + f x]}\right]}{2 a} - \left(7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 - \cos[e + f x]}^2 \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x])^2) \right) + \left(22 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^{7/2} \right) / \\
& \left(7 \sqrt{1 - \cos[e + f x]}^2 \left(11 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] - 2 \right. \right. \\
& \left. \left. \left(2 b^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x])^2) \right) + \frac{\operatorname{Log}\left[1 - \sqrt{\cos[e + f x]}\right]}{4 a} - \\
& \frac{\operatorname{Log}\left[1 + \sqrt{\cos[e + f x]}\right]}{4 a} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x]\right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} - \\
& \left. \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x]\right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \cos[e + f x])^2 (b + a \csc[e + f x])} 2 (-2 a^2 + b^2) (-1 + \cos[e + f x])^2 \left(a + b \sqrt{1 - \cos[e + f x]} \right) \csc[e + f x] \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^{3/2} \right) / \left(3 \sqrt{1 - \cos[e + f x]}^2 \left(7 (a^2 - b^2) \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + \right. \\
& \left. (-a^2+b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \cos[e+fx]^2 \right) (a^2+b^2 (-1+\cos[e+fx]^2)) \Bigg) + \\
& \frac{1}{8 a (a^2-b^2)^{1/4}} \left(2 \sqrt{2} \sqrt{b} \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] - 2 \sqrt{2} \sqrt{b} \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e+fx]}}{(a^2-b^2)^{1/4}}\right] + \right. \\
& 4 (a^2-b^2)^{1/4} \text{ArcTan}\left[\sqrt{\cos[e+fx]}\right] + 2 (a^2-b^2)^{1/4} \text{Log}\left[1 - \sqrt{\cos[e+fx]}\right] - \\
& 2 (a^2-b^2)^{1/4} \text{Log}\left[1 + \sqrt{\cos[e+fx]}\right] - \sqrt{2} \sqrt{b} \text{Log}\left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx]\right] + \\
& \left. \left. \sqrt{2} \sqrt{b} \text{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\cos[e+fx]} + b \cos[e+fx]\right] \right) \right) \Bigg) + \frac{2 \cos[e+fx] (a-b \sin[e+fx])}{(a^2-b^2) f (g \cos[e+fx])^{3/2}}
\end{aligned}$$

■ **Problem 1401: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[e+fx]^2}{(g \cos[e+fx])^{3/2} (a+b \sin[e+fx])} dx$$

Optimal (type 4, 627 leaves, 25 steps):

$$\begin{aligned}
& -\frac{b \text{ArcTan}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{a^2 f g^{3/2}} + \frac{b^{7/2} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{a^2 (-a^2+b^2)^{5/4} f g^{3/2}} + \frac{b \text{ArcTanh}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g}}\right]}{a^2 f g^{3/2}} - \frac{b^{7/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{a^2 (-a^2+b^2)^{5/4} f g^{3/2}} \\
& - \frac{2 b}{a^2 f g \sqrt{g \cos[e+fx]}} - \frac{\csc[e+fx]}{a f g \sqrt{g \cos[e+fx]}} - \frac{3 \sqrt{g \cos[e+fx]} \text{EllipticE}\left[\frac{1}{2} (e+fx), 2\right]}{a f g^2 \sqrt{\cos[e+fx]}} \\
& - \frac{2 b^2 \sqrt{g \cos[e+fx]} \text{EllipticE}\left[\frac{1}{2} (e+fx), 2\right]}{a (a^2-b^2) f g^2 \sqrt{\cos[e+fx]}} - \frac{b^3 \sqrt{\cos[e+fx]} \text{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} (e+fx), 2\right]}{a (a^2-b^2) (b-\sqrt{-a^2+b^2}) f g \sqrt{g \cos[e+fx]}} \\
& + \frac{b^3 \sqrt{\cos[e+fx]} \text{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} (e+fx), 2\right]}{a (a^2-b^2) (b+\sqrt{-a^2+b^2}) f g \sqrt{g \cos[e+fx]}} + \frac{3 \sin[e+fx]}{a f g \sqrt{g \cos[e+fx]}} - \frac{2 b^2 (b-a \sin[e+fx])}{a^2 (a^2-b^2) f g \sqrt{g \cos[e+fx]}}
\end{aligned}$$

Result (type 6, 2469 leaves):

$$-\frac{1}{4 a (a-b) (a+b) f (g \cos[e+fx])^{3/2}}$$

$$\begin{aligned}
& \cos[e + f x]^{3/2} \left(\frac{1}{12 \sqrt{1 - \cos[e + f x]^2} (b + a \csc[e + f x])} (6 a^3 + 2 a b^2) (a + b \sqrt{1 - \cos[e + f x]^2}) \left(- \left(56 a (a^2 - b^2) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) / \left(\sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \frac{1}{\sqrt{b} (-a^2 + b^2)^{1/4}} \\
& \quad (3 + 3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \\
& \quad \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \right) \left. \right) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (-1 + 2 \cos[e + f x]^2) (b + a \csc[e + f x])} 2 (-3 a^2 b + b^3) (-1 + \cos[e + f x]^2) (a + b \sqrt{1 - \cos[e + f x]^2}) \\
& \quad \cos[2 (e + f x)] \csc[e + f x] \\
& \left(\frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\cos[e + f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{2 \sqrt{2} a b^{3/2} (-a^2 + b^2)} + \right. \\
& \quad \frac{\operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right]}{2 a} - \left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) / \\
& \quad \left(3 \sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \quad (a^2 + b^2 (-1 + \cos[e + f x]^2)) \left. \right) + \left(22 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{7/2} \right) / \\
& \quad \left(7 \sqrt{1 - \cos[e + f x]^2} \left(11 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 b^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) + \frac{\operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right]}{4 a} - \\
& \frac{\operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right]}{4 a} + \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} - \\
& \left. \frac{(a^2 - b^2)^{3/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{4 \sqrt{2} a b^{3/2} (-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 2 (7 a^2 b - 5 b^3) (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \operatorname{Csc}[e + f x] \\
& \left(\left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{3/2} \right) / \left(3 \sqrt{1 - \cos[e + f x]^2} \left(7 (a^2 - b^2) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \\
& \frac{1}{8 a (a^2 - b^2)^{1/4}} \left(2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - 2 \sqrt{2} \sqrt{b} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \quad 4 (a^2 - b^2)^{1/4} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] + 2 (a^2 - b^2)^{1/4} \operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right] - 2 (a^2 - b^2)^{1/4} \operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right] - \\
& \quad \left. \sqrt{2} \sqrt{b} \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \sqrt{2} \sqrt{b} \right. \\
& \quad \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \left. \right) + \frac{\cos[e + f x]^2 \left(-\frac{\operatorname{Cot}[e + f x]}{a} + \frac{2 \operatorname{Sec}[e + f x] (-b + a \operatorname{Sin}[e + f x])}{a^2 - b^2} \right)}{f (g \cos[e + f x])^{3/2}}
\end{aligned}$$

■ **Problem 1402: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]^4}{(g \cos[e + f x])^{5/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 601 leaves, 22 steps):

$$\begin{aligned}
& - \frac{a^4 \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{3/2} (-a^2+b^2)^{7/4} f g^{5/2}} - \frac{a^4 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{b^{3/2} (-a^2+b^2)^{7/4} f g^{5/2}} - \frac{2b}{3(a^2-b^2) f g (g \cos[e+fx])^{3/2}} + \\
& \frac{2a^2 \sqrt{g \cos[e+fx]}}{b(a^2-b^2) f g^3} - \frac{2b \sqrt{g \cos[e+fx]}}{(a^2-b^2) f g^3} - \frac{4a \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2}(e+fx), 2\right]}{3(a^2-b^2) f g^2 \sqrt{g \cos[e+fx]}} + \\
& \frac{2a^3 \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2}(e+fx), 2\right]}{b^2(a^2-b^2) f g^2 \sqrt{g \cos[e+fx]}} - \frac{a^5 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^2(a^2-b^2) \left(a^2-b \left(b-\sqrt{-a^2+b^2}\right)\right) f g^2 \sqrt{g \cos[e+fx]}} - \\
& \frac{a^5 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{b^2(a^2-b^2) \left(a^2-b \left(b+\sqrt{-a^2+b^2}\right)\right) f g^2 \sqrt{g \cos[e+fx]}} + \frac{2a \sin[e+fx]}{3(a^2-b^2) f g (g \cos[e+fx])^{3/2}}
\end{aligned}$$

Result (type 6, 2158 leaves):

$$\begin{aligned}
& \frac{2 \cos[e+fx] (-b+a \sin[e+fx])}{3(a^2-b^2) f (g \cos[e+fx])^{5/2}} + \\
& \frac{1}{6(a-b)(a+b) f (g \cos[e+fx])^{5/2}} \operatorname{Cos}[e+fx]^{5/2} \left(- \frac{1}{\sqrt{1-\cos[e+fx]^2} (a+b \sin[e+fx])} - 2(-7a^2+3b^2) \left(a+b \sqrt{1-\cos[e+fx]^2}\right) \right. \\
& \left. \left(\left(5a(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \sqrt{\cos[e+fx]} \right) / \left(\sqrt{1-\cos[e+fx]^2} \left(5(a^2-b^2) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) + \right. \right. \\
& \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right) \left(a^2+b^2 (-1+\cos[e+fx]^2) \right) \right) - \\
& \frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}}\right] + \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx]\right] - \right. \\
& \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]} + i b \cos[e+fx]\right] \right) \left. \right) \sin[e+fx] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{1 - \cos[e + f x]^2} (-1 + 2 \cos[e + f x]^2) (a + b \sin[e + f x])} (3 a^2 - 3 b^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \cos[2 (e + f x)] \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} \right) + \\
& \frac{4 \sqrt{\cos[e + f x]}}{b} + \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \right) / \\
& \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^{5/2} \right) / \\
& \left(5 \sqrt{1 - \cos[e + f x]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - \right. \right. \\
& \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] - \\
& \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \right) \sin[e + f x] - \\
& \frac{1}{(1 - \cos[e + f x]^2) (a + b \sin[e + f x])} 4 a b \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \sqrt{1 - \cos[e + f x]^2} \right) / \right. \\
& \left. \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \right. \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^2 \\ & \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}}\right] + \right. \\ & \left. 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x]\right] + \right. \\ & \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x]\right] \right) \right) \sin[e + f x]^2 \end{aligned}$$

■ **Problem 1403: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]^3}{(g \cos[e + f x])^{5/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 528 leaves, 18 steps):

$$\begin{aligned} & \frac{a^3 \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{\sqrt{b} (-a^2 + b^2)^{7/4} f g^{5/2}} + \frac{a^3 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{\sqrt{b} (-a^2 + b^2)^{7/4} f g^{5/2}} + \frac{2 a}{3 (a^2 - b^2) f g (g \cos[e + f x])^{3/2}} - \frac{2 a^2 \sqrt{\cos[e + f x]} \operatorname{EllipticF}\left[\frac{1}{2} (e + f x), 2\right]}{b (a^2 - b^2) f g^2 \sqrt{g \cos[e + f x]}} + \\ & \frac{4 b \sqrt{\cos[e + f x]} \operatorname{EllipticF}\left[\frac{1}{2} (e + f x), 2\right]}{3 (a^2 - b^2) f g^2 \sqrt{g \cos[e + f x]}} + \frac{a^4 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b (a^2 - b^2) \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) f g^2 \sqrt{g \cos[e + f x]}} + \\ & \frac{a^4 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{b (a^2 - b^2) \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) f g^2 \sqrt{g \cos[e + f x]}} - \frac{2 b \sin[e + f x]}{3 (a^2 - b^2) f g (g \cos[e + f x])^{3/2}} \end{aligned}$$

Result (type 6, 1193 leaves):

$$\begin{aligned} & \frac{2 \cos[e + f x] (a - b \sin[e + f x])}{3 (a^2 - b^2) f (g \cos[e + f x])^{5/2}} - \\ & \frac{1}{3 (a - b) (a + b) f (g \cos[e + f x])^{5/2}} \cos[e + f x]^{5/2} \left(\frac{1}{\sqrt{1 - \cos[e + f x]^2} (a + b \sin[e + f x])} - 4 a b \left(a + b \sqrt{1 - \cos[e + f x]^2}\right) \right) \\ & \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \sqrt{\cos[e + f x]} \right) \right) / \end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \\
& \quad \left. 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] - \right. \\
& \quad \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]} + i b \cos[e + f x] \right] \right) \left. \right) \sin[e + f x] - \\
& \frac{1}{(1 - \cos[e + f x]^2) (a + b \sin[e + f x])} 2 (3 a^2 - 2 b^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \sqrt{1 - \cos[e + f x]^2} \right) / \right. \\
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \cos[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& \quad \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \right. \\
& \quad \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \left. \right) \sin[e + f x]^2 \left. \right)
\end{aligned}$$

- **Problem 1404: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]^2}{(g \cos[e + f x])^{5/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 468 leaves, 15 steps):

$$\begin{aligned}
& - \frac{a^2 \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{(-a^2+b^2)^{7/4} f g^{5/2}} - \frac{a^2 \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e+fx]}}{(-a^2+b^2)^{1/4} \sqrt{g}}\right]}{(-a^2+b^2)^{7/4} f g^{5/2}} - \frac{2b}{3(a^2-b^2) f g (g \cos[e+fx])^{3/2}} + \\
& \frac{2a \sqrt{\cos[e+fx]} \operatorname{EllipticF}\left[\frac{1}{2}(e+fx), 2\right]}{3(a^2-b^2) f g^2 \sqrt{g \cos[e+fx]}} - \frac{a^3 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{(a^2-b^2) \left(a^2-b \left(b-\sqrt{-a^2+b^2}\right)\right) f g^2 \sqrt{g \cos[e+fx]}} - \\
& \frac{a^3 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}(e+fx), 2\right]}{(a^2-b^2) \left(a^2-b \left(b+\sqrt{-a^2+b^2}\right)\right) f g^2 \sqrt{g \cos[e+fx]}} + \frac{2a \sin[e+fx]}{3(a^2-b^2) f g (g \cos[e+fx])^{3/2}}
\end{aligned}$$

Result (type 6, 1184 leaves):

$$\begin{aligned}
& \frac{2 \operatorname{Cos}[e + f x] (-b + a \operatorname{Sin}[e + f x])}{3 (a^2 - b^2) f (g \operatorname{Cos}[e + f x])^{5/2}} - \\
& \frac{1}{3 (a - b) (a + b) f (g \operatorname{Cos}[e + f x])^{5/2}} a \operatorname{Cos}[e + f x]^{5/2} \left(- \frac{1}{\sqrt{1 - \operatorname{Cos}[e + f x]^2} (a + b \operatorname{Sin}[e + f x])} 4 a \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \right. \\
& \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\operatorname{Cos}[e + f x]} \right) / \left(\sqrt{1 - \operatorname{Cos}[e + f x]^2} \left(5 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2)) \right) \right) - \\
& \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] + \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] \right) \left. \right) \operatorname{Sin}[e + f x] + \frac{1}{(1 - \operatorname{Cos}[e + f x]^2) (a + b \operatorname{Sin}[e + f x])} \\
& 2 b \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\operatorname{Cos}[e + f x]} \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) / \right. \\
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \operatorname{Cos}[e + f x]^2 \right) \right) \operatorname{Cos}[e + f x]^2 \right) \\
& (a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2)) \left. \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] + \\
& \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] \right) \left. \right) \operatorname{Sin}[e + f x]^2 \left. \right)
\end{aligned}$$

- **Problem 1405: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]}{(g \cos[e + f x])^{5/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 432 leaves, 13 steps):

$$\frac{a b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{(-a^2 + b^2)^{7/4} f g^{5/2}} + \frac{a b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{(-a^2 + b^2)^{7/4} f g^{5/2}} -$$

$$\frac{2 b \sqrt{\cos[e + f x]} \operatorname{EllipticF}\left[\frac{1}{2} (e + f x), 2\right]}{3 (a^2 - b^2) f g^2 \sqrt{g \cos[e + f x]}} + \frac{a^2 b \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) f g^2 \sqrt{g \cos[e + f x]}} +$$

$$\frac{a^2 b \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) f g^2 \sqrt{g \cos[e + f x]}} + \frac{2 (a - b \sin[e + f x])}{3 (a^2 - b^2) f g (g \cos[e + f x])^{3/2}}$$

Result (type 6, 1183 leaves):

$$\begin{aligned}
& \frac{2 \operatorname{Cos}[e + f x] (a - b \operatorname{Sin}[e + f x])}{3 (a^2 - b^2) f (g \operatorname{Cos}[e + f x])^{5/2}} + \\
& \frac{1}{3 (a - b) (a + b) f (g \operatorname{Cos}[e + f x])^{5/2}} b \operatorname{Cos}[e + f x]^{5/2} \left(- \frac{1}{\sqrt{1 - \operatorname{Cos}[e + f x]^2} (a + b \operatorname{Sin}[e + f x])} 4 a \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \right. \\
& \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\operatorname{Cos}[e + f x]} \right) / \left(\sqrt{1 - \operatorname{Cos}[e + f x]^2} \left(5 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right) \operatorname{Cos}[e + f x]^2 \right) (a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2)) \right) - \\
& \frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}} \right] + \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] - \right. \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + i b \operatorname{Cos}[e + f x] \right] \right) \left. \right) \operatorname{Sin}[e + f x] + \frac{1}{(1 - \operatorname{Cos}[e + f x]^2) (a + b \operatorname{Sin}[e + f x])} \\
& 2 b \left(a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\operatorname{Cos}[e + f x]} \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) / \right. \\
& \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right) \operatorname{Cos}[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2)) \right) + \frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Cos}[e + f x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] + \\
& \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Cos}[e + f x]} + b \operatorname{Cos}[e + f x] \right] \right) \left. \right) \operatorname{Sin}[e + f x]^2 \left. \right)
\end{aligned}$$

- **Problem 1406: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[e + f x]}{(g \text{Cos}[e + f x])^{5/2} (a + b \text{Sin}[e + f x])} dx$$

Optimal (type 4, 527 leaves, 21 steps):

$$\begin{aligned} & - \frac{\text{ArcTan}\left[\frac{\sqrt{g \text{Cos}[e + f x]}}{\sqrt{g}}\right]}{a f g^{5/2}} + \frac{b^{7/2} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{g \text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a (-a^2 + b^2)^{7/4} f g^{5/2}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{g \text{Cos}[e + f x]}}{\sqrt{g}}\right]}{a f g^{5/2}} + \frac{b^{7/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{g \text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}}\right]}{a (-a^2 + b^2)^{7/4} f g^{5/2}} + \\ & \frac{2}{3 a f g (g \text{Cos}[e + f x])^{3/2}} - \frac{2 b \sqrt{\text{Cos}[e + f x]} \text{EllipticF}\left[\frac{1}{2} (e + f x), 2\right]}{3 (a^2 - b^2) f g^2 \sqrt{g \text{Cos}[e + f x]}} + \frac{b^3 \sqrt{\text{Cos}[e + f x]} \text{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) f g^2 \sqrt{g \text{Cos}[e + f x]}} + \\ & \frac{b^3 \sqrt{\text{Cos}[e + f x]} \text{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2\right]}{(a^2 - b^2) \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) f g^2 \sqrt{g \text{Cos}[e + f x]}} + \frac{2 b (b - a \text{Sin}[e + f x])}{3 a (a^2 - b^2) f g (g \text{Cos}[e + f x])^{3/2}} \end{aligned}$$

Result (type 6, 2418 leaves):

$$\begin{aligned} & \frac{1}{6 (a - b) (a + b) f (g \text{Cos}[e + f x])^{5/2}} \\ & \text{Cos}[e + f x]^{5/2} \left(- \frac{1}{\sqrt{1 - \text{Cos}[e + f x]^2} (b + a \text{Csc}[e + f x])} 8 a b \left(a + b \sqrt{1 - \text{Cos}[e + f x]^2}\right) \left(\left(5 a (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2\right], \right. \right. \right. \\ & \left. \left. \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\text{Cos}[e + f x]} \right) / \left(\sqrt{1 - \text{Cos}[e + f x]^2} \left(5 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] - 2 \right. \right. \\ & \left. \left(2 b^2 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \right. \right. \right. \\ & \left. \left. \left. \text{Cos}[e + f x]^2, \frac{b^2 \text{Cos}[e + f x]^2}{-a^2 + b^2} \right] \right) \text{Cos}[e + f x]^2 \left(a^2 + b^2 (-1 + \text{Cos}[e + f x]^2)\right) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \\ & \left. \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \text{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \text{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\text{Cos}[e + f x]}}{(-a^2 + b^2)^{1/4}}\right] + \text{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \right. \\ & \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\text{Cos}[e + f x]} + i b \text{Cos}[e + f x] \right] - \text{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Cos}[e + f x]} + i b \text{Cos}[e + f x] \right] \right) \right) \end{aligned}$$

$$\begin{aligned}
& \frac{1}{(1 - \cos[e + f x]^2) (-1 + 2 \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} b^2 (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \\
& \operatorname{Cos}[2(e + f x)] \operatorname{Csc}[e + f x] \\
& \left(\frac{(a^2 - b^2)^{1/4} (-2a^2 + b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2}(a^2 - b^2)^{1/4} + 2\sqrt{b}\sqrt{\cos[e + f x]}}{\sqrt{2}(a^2 - b^2)^{1/4}}\right]}{\sqrt{2}a\sqrt{b}(-a^2 + b^2)} + \frac{(a^2 - b^2)^{1/4} (-2a^2 + b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2}(a^2 - b^2)^{1/4} + 2\sqrt{b}\sqrt{\cos[e + f x]}}{\sqrt{2}(a^2 - b^2)^{1/4}}\right]}{\sqrt{2}a\sqrt{b}(-a^2 + b^2)} \right) - \\
& \frac{\operatorname{ArcTan}\left[\sqrt{\cos[e + f x]}\right]}{a} - \left(10b(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \sqrt{\cos[e + f x]} \right) / \\
& \left(\sqrt{1 - \cos[e + f x]^2} \left(5(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] - 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \left(36b(a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^{5/2} \right) / \\
& \left(5\sqrt{1 - \cos[e + f x]^2} \left(9(a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] - \right. \right. \\
& \left. \left. 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \right. \right. \right. \\
& \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) + \frac{\operatorname{Log}\left[1 - \sqrt{\cos[e + f x]}\right]}{2a} - \\
& \frac{\operatorname{Log}\left[1 + \sqrt{\cos[e + f x]}\right]}{2a} - \frac{(a^2 - b^2)^{1/4} (-2a^2 + b^2) \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x]\right]}{2\sqrt{2}a\sqrt{b}(-a^2 + b^2)} + \\
& \left. \frac{(a^2 - b^2)^{1/4} (-2a^2 + b^2) \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x]\right]}{2\sqrt{2}a\sqrt{b}(-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 2(6a^2 - 7b^2) (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \operatorname{Csc}[e + f x] \\
& \left(\left(5b(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \sqrt{\cos[e + f x]} \right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \frac{1}{8 a (a^2 - b^2)^{3/4}} \left(-2 \sqrt{2} b^{3/2} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \sqrt{2} b^{3/2} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + 4 (a^2 - b^2)^{3/4} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] - 2 (a^2 - b^2)^{3/4} \operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right] + \\
& 2 (a^2 - b^2)^{3/4} \operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right] - \sqrt{2} b^{3/2} \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \\
& \left. \left. \left. \left. \sqrt{2} b^{3/2} \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right] \right) \right) \right) + \frac{2 \cos[e + f x] (a - b \sin[e + f x])}{3 (a^2 - b^2) f (g \cos[e + f x])^{5/2}}
\end{aligned}$$

■ **Problem 1407: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[e + f x]^2}{(g \cos[e + f x])^{5/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 651 leaves, 25 steps):

$$\begin{aligned}
& \frac{b \operatorname{ArcTan} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}} \right]}{a^2 f g^{5/2}} - \frac{b^{9/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a^2 (-a^2 + b^2)^{7/4} f g^{5/2}} + \frac{b \operatorname{ArcTanh} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g}} \right]}{a^2 f g^{5/2}} - \frac{b^{9/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{g \cos[e + f x]}}{(-a^2 + b^2)^{1/4} \sqrt{g}} \right]}{a^2 (-a^2 + b^2)^{7/4} f g^{5/2}} - \\
& \frac{2 b}{3 a^2 f g (g \cos[e + f x])^{3/2}} - \frac{\csc[e + f x]}{a f g (g \cos[e + f x])^{3/2}} + \frac{5 \sqrt{\cos[e + f x]} \operatorname{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{3 a f g^2 \sqrt{g \cos[e + f x]}} + \\
& \frac{2 b^2 \sqrt{\cos[e + f x]} \operatorname{EllipticF} \left[\frac{1}{2} (e + f x), 2 \right]}{3 a (a^2 - b^2) f g^2 \sqrt{g \cos[e + f x]}} - \frac{b^4 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{a (a^2 - b^2) \left(a^2 - b \left(b - \sqrt{-a^2 + b^2} \right) \right) f g^2 \sqrt{g \cos[e + f x]}} - \\
& \frac{b^4 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} (e + f x), 2 \right]}{a (a^2 - b^2) \left(a^2 - b \left(b + \sqrt{-a^2 + b^2} \right) \right) f g^2 \sqrt{g \cos[e + f x]}} + \frac{5 \sin[e + f x]}{3 a f g (g \cos[e + f x])^{3/2}} - \frac{2 b^2 (b - a \sin[e + f x])}{3 a^2 (a^2 - b^2) f g (g \cos[e + f x])^{3/2}}
\end{aligned}$$

Result (type 6, 2465 leaves):

1

$$12 a (a-b) (a+b) f (g \operatorname{Cos}[e+f x])^{5/2}$$

$$\operatorname{Cos}[e+f x]^{5/2} \left(-\frac{1}{\sqrt{1-\operatorname{Cos}[e+f x]^2} (b+a \operatorname{Csc}[e+f x])} - 2 (10 a^3 - 18 a b^2) (a+b \sqrt{1-\operatorname{Cos}[e+f x]^2}) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Cos}[e+f x]} \right) / \left(\sqrt{1-\operatorname{Cos}[e+f x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \right. \right. \right. \right. \\ \left. \left. \left. - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \right) + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \right) \operatorname{Cos}[e+f x]^2 (a^2 + b^2 (-1 + \operatorname{Cos}[e+f x]^2)) \right) \right) - \frac{1}{(-a^2 + b^2)^{3/4}} \\ \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[e+f x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Cos}[e+f x]}}{(-a^2 + b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \\ \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e+f x]} + i b \operatorname{Cos}[e+f x] \right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Cos}[e+f x]} + i b \operatorname{Cos}[e+f x] \right] \right) \right) \right)$$

$$\frac{1}{(1 - \operatorname{Cos}[e+f x]^2) (-1 + 2 \operatorname{Cos}[e+f x]^2) (b+a \operatorname{Csc}[e+f x])} (-5 a^2 b + 3 b^3) (-1 + \operatorname{Cos}[e+f x]^2) (a+b \sqrt{1-\operatorname{Cos}[e+f x]^2}) \\ \operatorname{Cos}[2(e+f x)] \operatorname{Csc}[e+f x]$$

$$\left(\frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\operatorname{Cos}[e+f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{\sqrt{2} a \sqrt{b} (-a^2 + b^2)} + \frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2 \sqrt{b} \sqrt{\operatorname{Cos}[e+f x]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{\sqrt{2} a \sqrt{b} (-a^2 + b^2)} - \right.$$

$$\left. \frac{\operatorname{ArcTan}\left[\sqrt{\operatorname{Cos}[e+f x]}\right]}{a} - \left(10 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Cos}[e+f x]} \right) / \right.$$

$$\left(\sqrt{1-\operatorname{Cos}[e+f x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \right. \right. \right. \\ \left. \left. \left. + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \right) \operatorname{Cos}[e+f x]^2 \right) \right)$$

$$\left. \left(a^2 + b^2 (-1 + \operatorname{Cos}[e+f x]^2) \right) \right) + \left(36 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \operatorname{Cos}[e+f x]^{5/2} \right) /$$

$$\left(5 \sqrt{1-\operatorname{Cos}[e+f x]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] - \right.$$

$$\begin{aligned}
& 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) + \frac{\operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right]}{2 a} - \\
& \frac{\operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right]}{2 a} - \frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{2 \sqrt{2} a \sqrt{b} (-a^2 + b^2)} + \\
& \left. \frac{(a^2 - b^2)^{1/4} (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right]}{2 \sqrt{2} a \sqrt{b} (-a^2 + b^2)} \right) - \\
& \frac{1}{(1 - \cos[e + f x]^2) (b + a \operatorname{Csc}[e + f x])} 2 (-7 a^2 b + 9 b^3) (-1 + \cos[e + f x]^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \operatorname{Csc}[e + f x] \\
& \left(\left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \right) / \right. \\
& \left(\sqrt{1 - \cos[e + f x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \frac{1}{8 a (a^2 - b^2)^{3/4}} \left(-2 \sqrt{2} b^{3/2} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + \right. \\
& 2 \sqrt{2} b^{3/2} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\cos[e + f x]}}{(a^2 - b^2)^{1/4}} \right] + 4 (a^2 - b^2)^{3/4} \operatorname{ArcTan} \left[\sqrt{\cos[e + f x]} \right] - 2 (a^2 - b^2)^{3/4} \operatorname{Log} \left[1 - \sqrt{\cos[e + f x]} \right] + \\
& 2 (a^2 - b^2)^{3/4} \operatorname{Log} \left[1 + \sqrt{\cos[e + f x]} \right] - \sqrt{2} b^{3/2} \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] + \sqrt{2} b^{3/2} \\
& \left. \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\cos[e + f x]} + b \cos[e + f x] \right] \right) \right) \right) + \frac{\cos[e + f x]^3 \left(-\frac{\operatorname{Csc}[e + f x]}{a} + \frac{2 \operatorname{Sec}[e + f x]^2 (-b + a \operatorname{Sin}[e + f x])}{3 (a^2 - b^2)} \right)}{f (g \cos[e + f x])^{5/2}}
\end{aligned}$$

■ **Problem 1408: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \cos[e + f x]} (d \operatorname{Sin}[e + f x])^{5/2}}{a + b \operatorname{Sin}[e + f x]} dx$$

Optimal (type 4, 926 leaves, 31 steps):

$$\begin{aligned}
 & \frac{a^2 d^{5/2} \sqrt{g} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d \sin[e+fx]}}\right]}{\sqrt{2} b^3 f} + \frac{d^{5/2} \sqrt{g} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d \sin[e+fx]}}\right]}{4 \sqrt{2} b f} - \frac{a^2 d^{5/2} \sqrt{g} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d \sin[e+fx]}}\right]}{\sqrt{2} b^3 f} \\
 & \frac{d^{5/2} \sqrt{g} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d \sin[e+fx]}}\right]}{4 \sqrt{2} b f} - \frac{a^2 d^{5/2} \sqrt{g} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d \sin[e+fx]}}\right]}{2 \sqrt{2} b^3 f} \\
 & \frac{d^{5/2} \sqrt{g} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d \sin[e+fx]}}\right]}{8 \sqrt{2} b f} + \frac{a^2 d^{5/2} \sqrt{g} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d \sin[e+fx]}}\right]}{2 \sqrt{2} b^3 f} + \\
 & \frac{d^{5/2} \sqrt{g} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d \sin[e+fx]}}\right]}{8 \sqrt{2} b f} - \frac{2 \sqrt{2} a^3 d^3 \sqrt{g} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{b^3 \sqrt{-a+b} \sqrt{a+b} f \sqrt{d \sin[e+fx]}} + \\
 & \frac{2 \sqrt{2} a^3 d^3 \sqrt{g} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{b^3 \sqrt{-a+b} \sqrt{a+b} f \sqrt{d \sin[e+fx]}} - \\
 & \frac{d^2 (g \cos[e+fx])^{3/2} \sqrt{d \sin[e+fx]}}{2 b f g} - \frac{a d^2 \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{d \sin[e+fx]}}{b^2 f \sqrt{\sin[2e+2fx]}}
 \end{aligned}$$

Result (type 6, 2886 leaves):

$$\begin{aligned}
 & - \frac{\sqrt{g \cos[e+fx]} \cot[e+fx] \operatorname{Csc}[e+fx] (d \sin[e+fx])^{5/2}}{2 b f} + \\
 & \frac{1}{4 b f \sqrt{\cos[e+fx]} \sin[e+fx]^{5/2}} \sqrt{g \cos[e+fx]} (d \sin[e+fx])^{5/2} \left(- \left(\left(14 b (a^2 - b^2) \cos[e+fx]^{3/2} (a+b \sqrt{1 - \cos[e+fx]^2}) \right) \right. \right. \\
 & \left. \left(\left(b \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \sqrt{1 - \cos[e+fx]^2} \right) / \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \right. \right. \right. \\
 & \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{4}, \right. \right. \right. \\
 & \left. \left. 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right) \cos[e+fx]^2 \right) + \left(a \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right) /
 \end{aligned}$$

$$\begin{aligned}
& \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(-4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \\
& \left. \sin[e + f x]^{3/2} \right) / \left(3 (1 - \cos[e + f x]^2) (a^2 + b^2 (-1 + \cos[e + f x]^2)) (a + b \sin[e + f x]) \right) - \\
& \left(2 a \sqrt{\tan[e + f x]} \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + \right. \right. \right. \\
& \left. \left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] - \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] + \right. \right. \\
& \left. \left. \left. \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) \right) + \right. \\
& \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \right) \right) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(4 a \cos[2 (e + f x)] \sqrt{\tan[e + f x]} \right. \\
& \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \\
& \left(-\frac{a \operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right) + \\
& \left. \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{a \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[e + f x]} + \operatorname{Tan}[e + f x]\right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e + f x]} - \sqrt{a^2 - b^2} \operatorname{Tan}[e + f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \\
& \frac{(2 a^2 - b^2) \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e + f x]} + \sqrt{a^2 - b^2} \operatorname{Tan}[e + f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\operatorname{Tan}[e + f x]^{3/2}}{b \sqrt{1 + \operatorname{Tan}[e + f x]^2}} + \\
& \left(7 a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \operatorname{Tan}[e + f x]^{3/2}\right) / \\
& \left(b \sqrt{1 + \operatorname{Tan}[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] \operatorname{Tan}[e + f x]^2\right) (-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2))\right) - \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \operatorname{Tan}[e + f x]^{3/2}\right) / \left(3 \sqrt{1 + \operatorname{Tan}[e + f x]^2}\right. \\
& \quad \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + 2\left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2,\right.\right.\right. \\
& \quad \left.\left.\left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right]\right) \operatorname{Tan}[e + f x]^2 \\
& \quad \left.(-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2))\right) + \left(11 a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right]\right. \\
& \quad \left.\operatorname{Tan}[e + f x]^{7/2}\right) / \left(7 b \sqrt{1 + \operatorname{Tan}[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] \operatorname{Tan}[e + f x]^2\right) (-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2))\right) - \\
& \left(11 a^2 b \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \operatorname{Tan}[e + f x]^{7/2}\right) / \\
& \left(7 \sqrt{1 + \operatorname{Tan}[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + 2\right.\right.
\end{aligned}$$

$$\left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \Bigg) \Bigg/ \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right)$$

■ **Problem 1409: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{g \cos[e + f x]} (d \sin[e + f x])^{3/2}}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 578 leaves, 19 steps):

$$\begin{aligned} & - \frac{a d^{3/2} \sqrt{g} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{d \sin[e + f x]}} \right]}{\sqrt{2} b^2 f} + \frac{a d^{3/2} \sqrt{g} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{d \sin[e + f x]}} \right]}{\sqrt{2} b^2 f} + \\ & \frac{a d^{3/2} \sqrt{g} \operatorname{Log} \left[\sqrt{g} + \sqrt{g} \cot[e + f x] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{d \sin[e + f x]}} \right]}{2 \sqrt{2} b^2 f} - \frac{a d^{3/2} \sqrt{g} \operatorname{Log} \left[\sqrt{g} + \sqrt{g} \cot[e + f x] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{d \sin[e + f x]}} \right]}{2 \sqrt{2} b^2 f} + \\ & - \frac{2 \sqrt{2} a^2 d^2 \sqrt{g} \operatorname{EllipticPi} \left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{b^2 \sqrt{-a+b} \sqrt{a+b} f \sqrt{d \sin[e + f x]}} - \\ & \frac{2 \sqrt{2} a^2 d^2 \sqrt{g} \operatorname{EllipticPi} \left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{b^2 \sqrt{-a+b} \sqrt{a+b} f \sqrt{d \sin[e + f x]}} + \frac{d \sqrt{g \cos[e + f x]} \operatorname{EllipticE} \left[e - \frac{\pi}{4} + f x, 2 \right] \sqrt{d \sin[e + f x]}}{b f \sqrt{\sin[2e + 2fx]}} \end{aligned}$$

Result (type 6, 520 leaves):

$$\frac{1}{3 f (-a + b \sin[e + f x]) (a + b \sin[e + f x])^2} 14 (a^2 - b^2) \sqrt{g \cos[e + f x]} \cot[e + f x] (d \sin[e + f x])^{3/2} \left(a + b \sqrt{\sin[e + f x]^2} \right) \\ \left(\left(a \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) / \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \\ \left. \left(-4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \right) \\ \cos[e + f x]^2 \left. + \left(b \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{3}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\sin[e + f x]^2} \right) / \right. \\ \left. \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{3}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{3}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \\ \left. \left. 3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \right)$$

■ **Problem 1410: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 509 leaves, 16 steps):

$$\frac{\sqrt{d} \sqrt{g} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{d \sin[e + f x]}} \right]}{\sqrt{2} b f} - \frac{\sqrt{d} \sqrt{g} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{d \sin[e + f x]}} \right]}{\sqrt{2} b f} - \frac{\sqrt{d} \sqrt{g} \operatorname{Log} \left[\sqrt{g} + \sqrt{g} \cot[e + f x] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{d \sin[e + f x]}} \right]}{2 \sqrt{2} b f} + \\ \frac{\sqrt{d} \sqrt{g} \operatorname{Log} \left[\sqrt{g} + \sqrt{g} \cot[e + f x] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{d \sin[e + f x]}} \right]}{2 \sqrt{2} b f} - \frac{2 \sqrt{2} a d \sqrt{g} \operatorname{EllipticPi} \left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{b \sqrt{-a+b} \sqrt{a+b} f \sqrt{d \sin[e + f x]}} + \\ \frac{2 \sqrt{2} a d \sqrt{g} \operatorname{EllipticPi} \left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{b \sqrt{-a+b} \sqrt{a+b} f \sqrt{d \sin[e + f x]}}$$

Result (type 4, 201 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{2} d \sqrt{g \cos[e + f x]} \left(\text{EllipticPi} \left[-i, -\text{ArcSin} \left[\sqrt{\tan \left[\frac{1}{2} (e + f x) \right]} \right], -1 \right] + \text{EllipticPi} \left[i, -\text{ArcSin} \left[\sqrt{\tan \left[\frac{1}{2} (e + f x) \right]} \right], -1 \right] - \right. \\
& \quad \left. \text{EllipticPi} \left[\frac{a}{-b + \sqrt{-a^2 + b^2}}, -\text{ArcSin} \left[\sqrt{\tan \left[\frac{1}{2} (e + f x) \right]} \right], -1 \right] - \text{EllipticPi} \left[-\frac{a}{b + \sqrt{-a^2 + b^2}}, -\text{ArcSin} \left[\sqrt{\tan \left[\frac{1}{2} (e + f x) \right]} \right], -1 \right] \right) \\
& \quad \left. \sqrt{\tan \left[\frac{1}{2} (e + f x) \right]} \right) / \left(b f \sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \sqrt{d \sin[e + f x]} \right)
\end{aligned}$$

- **Problem 1411: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \cos[e + f x]}}{\sqrt{d \sin[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 208 leaves, 5 steps):

$$\frac{2 \sqrt{2} \sqrt{g} \text{EllipticPi} \left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \text{ArcSin} \left[\frac{\sqrt{g \cos[e+f x]}}{\sqrt{g} \sqrt{1+\sin[e+f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{\sqrt{-a+b} \sqrt{a+b} f \sqrt{d \sin[e + f x]}} - \frac{2 \sqrt{2} \sqrt{g} \text{EllipticPi} \left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \text{ArcSin} \left[\frac{\sqrt{g \cos[e+f x]}}{\sqrt{g} \sqrt{1+\sin[e+f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{\sqrt{-a+b} \sqrt{a+b} f \sqrt{d \sin[e + f x]}}$$

Result (type 6, 577 leaves):

$$\frac{1}{f \sqrt{d \sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2}} 2 \sqrt{g \cos[e + f x]} \sec[e + f x]^2 \sqrt{\tan[e + f x]} \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] - \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] + \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) + \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) / \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \right)$$

■ **Problem 1412: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \cos[e + f x]}}{(d \sin[e + f x])^{3/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 320 leaves, 9 steps):

$$\frac{2 (g \cos[e + f x])^{3/2}}{a d f g \sqrt{d \sin[e + f x]}} - \frac{2 \sqrt{2} b \sqrt{g} \operatorname{EllipticPi} \left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{a \sqrt{-a+b} \sqrt{a+b} d f \sqrt{d \sin[e + f x]}} + \frac{2 \sqrt{2} b \sqrt{g} \operatorname{EllipticPi} \left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{a \sqrt{-a+b} \sqrt{a+b} d f \sqrt{d \sin[e + f x]}} - \frac{2 \sqrt{g \cos[e + f x]} \operatorname{EllipticE} \left[e - \frac{\pi}{4} + f x, 2 \right] \sqrt{d \sin[e + f x]}}{a d^2 f \sqrt{\sin[2e + 2fx]}}$$

Result (type 6, 2881 leaves):

$$-\frac{2 \cos[e + f x] \sqrt{g \cos[e + f x]} \sin[e + f x]}{a f (d \sin[e + f x])^{3/2}} + \frac{1}{a f \sqrt{\cos[e + f x]} (d \sin[e + f x])^{3/2}} \sqrt{g \cos[e + f x]} \sin[e + f x]^{3/2} \left(\left(28 a (a^2 - b^2) \cos[e + f x]^{3/2} \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \right) \right)$$

$$\begin{aligned}
& \left(\left(b \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \sqrt{1-\cos[e+fx]^2} \right) / \left(-7(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + \left(4b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \right) \cos[e+fx]^2 + \left(a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \right) \Big) / \\
& \left(7(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + \left(-4b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \right) \cos[e+fx]^2 \Big) \sin[e+fx]^{3/2} \Big) / \\
& \left(3(1-\cos[e+fx]^2)(a^2+b^2(-1+\cos[e+fx]^2))(a+b\sin[e+fx]) \right) - \left(4b\sqrt{\tan[e+fx]} \left(b\tan[e+fx] + a\sqrt{1+\tan[e+fx]^2} \right) \right) \\
& \left(\frac{1}{4\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}} \left(-2\operatorname{ArcTan} \left[1 - \frac{\sqrt{2}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]}}{\sqrt{a}} \right] + 2\operatorname{ArcTan} \left[1 + \frac{\sqrt{2}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]}}{\sqrt{a}} \right] \right) - \right. \\
& \quad \left. \operatorname{Log} \left[-a + \sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]} - \sqrt{a^2-b^2}\tan[e+fx] \right] + \operatorname{Log} \left[a + \sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]} + \right. \right. \\
& \quad \left. \left. \sqrt{a^2-b^2}\tan[e+fx] \right] \right) + \left(7a^2b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \tan[e+fx]^{3/2} \right) \Big) / \\
& \left(3\sqrt{1+\tan[e+fx]^2} \left(-7a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + 2 \right. \right. \\
& \quad \left. \left((a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \left(-b^2 \tan[e+fx]^2 + a^2(1+\tan[e+fx]^2) \right) \right) \Big) \Big) / \\
& \left(\cos[e+fx]^{3/2} \sqrt{\sin[e+fx]} (a+b\sin[e+fx]) (1+\tan[e+fx]^2)^{3/2} \right) + \left(2b\cos[2(e+fx)] \sqrt{\tan[e+fx]} \right) \\
& \left(b\tan[e+fx] + a\sqrt{1+\tan[e+fx]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{a \operatorname{ArcTan}\left[\frac{-\sqrt{2}+2\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{2}+2\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} + \frac{(2a^2-b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2}\sqrt{a}+2(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} \right) + \\
& \frac{(2a^2-b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{a}+2(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \frac{a \operatorname{Log}\left[1-\sqrt{2}\sqrt{\operatorname{Tan}[e+fx]}+\operatorname{Tan}[e+fx]\right]}{2\sqrt{2}b^2} - \\
& \frac{a \operatorname{Log}\left[1+\sqrt{2}\sqrt{\operatorname{Tan}[e+fx]}+\operatorname{Tan}[e+fx]\right]}{2\sqrt{2}b^2} - \frac{(2a^2-b^2) \operatorname{Log}\left[-a+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}-\sqrt{a^2-b^2}\operatorname{Tan}[e+fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \\
& \frac{(2a^2-b^2) \operatorname{Log}\left[a+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}+\sqrt{a^2-b^2}\operatorname{Tan}[e+fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \frac{\operatorname{Tan}[e+fx]^{3/2}}{b\sqrt{1+\operatorname{Tan}[e+fx]^2}} + \\
& \left(7a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \operatorname{Tan}[e+fx]^{3/2} \right) / \\
& \left(b\sqrt{1+\operatorname{Tan}[e+fx]^2} \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \right. \right. \\
& \quad \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] \operatorname{Tan}[e+fx]^2 \right) (-b^2 \operatorname{Tan}[e+fx]^2 + a^2(1+\operatorname{Tan}[e+fx]^2)) \right) \right) - \\
& \left(7a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \operatorname{Tan}[e+fx]^{3/2} \right) / \left(3\sqrt{1+\operatorname{Tan}[e+fx]^2} \right. \\
& \quad \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \right) \\
& \quad \left. (-b^2 \operatorname{Tan}[e+fx]^2 + a^2(1+\operatorname{Tan}[e+fx]^2)) \right) + \left(11a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \right. \\
& \quad \left. \operatorname{Tan}[e+fx]^{7/2} \right) / \left(7b\sqrt{1+\operatorname{Tan}[e+fx]^2} \left(-11a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \right. \right. \\
& \quad \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \Big) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \Big) - \\
& \left(11 a^2 b \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{7/2} \right) / \\
& \left(7 \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left((a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) \Big) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right) \Big)
\end{aligned}$$

■ **Problem 1413: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \cos[e + f x]}}{(d \sin[e + f x])^{5/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 366 leaves, 11 steps):

$$\begin{aligned}
& -\frac{2 (g \cos[e + f x])^{3/2}}{3 a d f g (d \sin[e + f x])^{3/2}} + \frac{2 b (g \cos[e + f x])^{3/2}}{a^2 d^2 f g \sqrt{d \sin[e + f x]}} + \frac{2 \sqrt{2} b^2 \sqrt{g} \operatorname{EllipticPi} \left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{a^2 \sqrt{-a+b} \sqrt{a+b} d^2 f \sqrt{d \sin[e + f x]}} - \\
& \frac{2 \sqrt{2} b^2 \sqrt{g} \operatorname{EllipticPi} \left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{a^2 \sqrt{-a+b} \sqrt{a+b} d^2 f \sqrt{d \sin[e + f x]}} + \frac{2 b \sqrt{g \cos[e + f x]} \operatorname{EllipticE} \left[e - \frac{\pi}{4} + f x, 2 \right] \sqrt{d \sin[e + f x]}}{a^2 d^3 f \sqrt{\sin[2 e + 2 f x]}}
\end{aligned}$$

Result (type 6, 2907 leaves):

$$\frac{\sqrt{g \cos[e + f x]} \left(\frac{2 b \cot[e + f x]}{a^2} - \frac{2 \cot[e + f x] \operatorname{Csc}[e + f x]}{3 a} \right) \sin[e + f x]^3}{f (d \sin[e + f x])^{5/2}} -$$

$$\frac{1}{a^2 f \sqrt{\cos[e + f x]} (d \sin[e + f x])^{5/2}} b \sqrt{g \cos[e + f x]} \sin[e + f x]^{5/2} \left(\left(28 a (a^2 - b^2) \cos[e + f x]^{3/2} (a + b \sqrt{1 - \cos[e + f x]^2}) \right) \right)$$

$$\begin{aligned}
& \left(\left(b \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \sqrt{1-\cos[e+fx]^2} \right) / \left(-7(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + \left(4b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \right) \cos[e+fx]^2 + \left(a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \right) \Big) / \\
& \left(7(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + \left(-4b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] + (a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2} \right] \right) \cos[e+fx]^2 \Big) \sin[e+fx]^{3/2} \Big) / \\
& \left(3(1-\cos[e+fx]^2)(a^2+b^2(-1+\cos[e+fx]^2))(a+b\sin[e+fx]) \right) - \left(4b\sqrt{\tan[e+fx]} \left(b\tan[e+fx] + a\sqrt{1+\tan[e+fx]^2} \right) \right) \\
& \left(\frac{1}{4\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}} \left(-2\operatorname{ArcTan} \left[1 - \frac{\sqrt{2}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]}}{\sqrt{a}} \right] + 2\operatorname{ArcTan} \left[1 + \frac{\sqrt{2}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]}}{\sqrt{a}} \right] \right) - \right. \\
& \quad \left. \operatorname{Log} \left[-a + \sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]} - \sqrt{a^2-b^2}\tan[e+fx] \right] + \operatorname{Log} \left[a + \sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\tan[e+fx]} + \right. \right. \\
& \quad \left. \left. \sqrt{a^2-b^2}\tan[e+fx] \right] \right) + \left(7a^2b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \tan[e+fx]^{3/2} \right) \Big) / \\
& \left(3\sqrt{1+\tan[e+fx]^2} \left(-7a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + 2 \right. \right. \\
& \quad \left. \left((a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \left(-b^2 \tan[e+fx]^2 + a^2(1+\tan[e+fx]^2) \right) \right) \Big) \Big) / \\
& \left(\cos[e+fx]^{3/2} \sqrt{\sin[e+fx]} (a+b\sin[e+fx]) (1+\tan[e+fx]^2)^{3/2} \right) + \left(2b\cos[2(e+fx)] \sqrt{\tan[e+fx]} \right) \\
& \left(b\tan[e+fx] + a\sqrt{1+\tan[e+fx]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{a \operatorname{ArcTan}\left[\frac{-\sqrt{2}+2\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{2}+2\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} + \frac{(2a^2-b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2}\sqrt{a}+2(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} \right) + \\
& \frac{(2a^2-b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{a}+2(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \frac{a \operatorname{Log}\left[1-\sqrt{2}\sqrt{\operatorname{Tan}[e+fx]}+\operatorname{Tan}[e+fx]\right]}{2\sqrt{2}b^2} - \\
& \frac{a \operatorname{Log}\left[1+\sqrt{2}\sqrt{\operatorname{Tan}[e+fx]}+\operatorname{Tan}[e+fx]\right]}{2\sqrt{2}b^2} - \frac{(2a^2-b^2) \operatorname{Log}\left[-a+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}-\sqrt{a^2-b^2}\operatorname{Tan}[e+fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \\
& \frac{(2a^2-b^2) \operatorname{Log}\left[a+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}+\sqrt{a^2-b^2}\operatorname{Tan}[e+fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \frac{\operatorname{Tan}[e+fx]^{3/2}}{b\sqrt{1+\operatorname{Tan}[e+fx]^2}} + \\
& \left(7a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \operatorname{Tan}[e+fx]^{3/2} \right) / \\
& \left(b\sqrt{1+\operatorname{Tan}[e+fx]^2} \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \right. \right. \\
& \quad \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] \operatorname{Tan}[e+fx]^2 \right) (-b^2 \operatorname{Tan}[e+fx]^2 + a^2(1+\operatorname{Tan}[e+fx]^2)) \right) \right) - \\
& \left(7a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \operatorname{Tan}[e+fx]^{3/2} \right) / \left(3\sqrt{1+\operatorname{Tan}[e+fx]^2} \right. \\
& \quad \left. \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \right) \right) \\
& \quad \left. (-b^2 \operatorname{Tan}[e+fx]^2 + a^2(1+\operatorname{Tan}[e+fx]^2)) \right) + \left(11a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \right. \\
& \quad \left. \operatorname{Tan}[e+fx]^{7/2} \right) / \left(7b\sqrt{1+\operatorname{Tan}[e+fx]^2} \left(-11a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \right. \right. \\
& \quad \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2 \Big) \operatorname{Tan}[e+f x]^2 \left(-b^2 \operatorname{Tan}[e+f x]^2+a^2(1+\operatorname{Tan}[e+f x]^2)\right) \Big) - \\
& \left(11 a^2 b \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2}\right] \operatorname{Tan}[e+f x]^{7/2}\right) / \\
& \left(7 \sqrt{1+\operatorname{Tan}[e+f x]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right]+2\right.\right. \\
& \left.\left.2\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right]+a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right. \\
& \left.\left.\left.-\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Tan}[e+f x]^2 \left(-b^2 \operatorname{Tan}[e+f x]^2+a^2(1+\operatorname{Tan}[e+f x]^2)\right)\right)\right) \Big) \Big) / \\
& \left(\operatorname{Cos}[e+f x]^{3/2} \sqrt{\operatorname{Sin}[e+f x]}(a+b \operatorname{Sin}[e+f x])\left(-1+\operatorname{Tan}[e+f x]^2\right) \sqrt{1+\operatorname{Tan}[e+f x]^2}\right) \Big)
\end{aligned}$$

■ **Problem 1414: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \operatorname{Cos}[e+f x]}}{(d \operatorname{Sin}[e+f x])^{7/2}(a+b \operatorname{Sin}[e+f x])} dx$$

Optimal (type 4, 513 leaves, 16 steps):

$$\begin{aligned}
& -\frac{2(g \operatorname{Cos}[e+f x])^{3/2}}{5 a d f g(d \operatorname{Sin}[e+f x])^{5/2}} + \frac{2 b(g \operatorname{Cos}[e+f x])^{3/2}}{3 a^2 d^2 f g(d \operatorname{Sin}[e+f x])^{3/2}} - \frac{4(g \operatorname{Cos}[e+f x])^{3/2}}{5 a d^3 f g \sqrt{d \operatorname{Sin}[e+f x]}} - \\
& \frac{2 b^2(g \operatorname{Cos}[e+f x])^{3/2}}{a^3 d^3 f g \sqrt{d \operatorname{Sin}[e+f x]}} - \frac{2 \sqrt{2} b^3 \sqrt{g} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \operatorname{Cos}[e+f x]}}{\sqrt{g} \sqrt{1+\operatorname{Sin}[e+f x]}}\right], -1\right] \sqrt{\operatorname{Sin}[e+f x]}}{a^3 \sqrt{-a+b} \sqrt{a+b} d^3 f \sqrt{d \operatorname{Sin}[e+f x]}} + \\
& \frac{2 \sqrt{2} b^3 \sqrt{g} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \operatorname{Cos}[e+f x]}}{\sqrt{g} \sqrt{1+\operatorname{Sin}[e+f x]}}\right], -1\right] \sqrt{\operatorname{Sin}[e+f x]}}{a^3 \sqrt{-a+b} \sqrt{a+b} d^3 f \sqrt{d \operatorname{Sin}[e+f x]}} - \\
& \frac{4 \sqrt{g \operatorname{Cos}[e+f x]} \operatorname{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{d \operatorname{Sin}[e+f x]}}{5 a d^4 f \sqrt{\operatorname{Sin}[2 e+2 f x]}} - \frac{2 b^2 \sqrt{g \operatorname{Cos}[e+f x]} \operatorname{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{d \operatorname{Sin}[e+f x]}}{a^3 d^4 f \sqrt{\operatorname{Sin}[2 e+2 f x]}}
\end{aligned}$$

Result (type 6, 2987 leaves):

$$\begin{aligned}
& \frac{1}{f (d \operatorname{Sin}[e + f x])^{7/2}} \\
& \sqrt{g \operatorname{Cos}[e + f x]} \left(-\frac{2 (2 a^2 \operatorname{Cos}[e + f x] + 5 b^2 \operatorname{Cos}[e + f x]) \operatorname{Csc}[e + f x]}{5 a^3} + \frac{2 b \operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]}{3 a^2} - \frac{2 \operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]^2}{5 a} \right) \operatorname{Sin}[e + f x]^4 - \\
& \frac{1}{5 a^3 f \sqrt{\operatorname{Cos}[e + f x]} (d \operatorname{Sin}[e + f x])^{7/2}} \sqrt{g \operatorname{Cos}[e + f x]} \operatorname{Sin}[e + f x]^{7/2} \\
& \left(-\left(\left(14 (a^2 - b^2) (4 a^3 + 10 a b^2) \operatorname{Cos}[e + f x]^{3/2} (a + b \sqrt{1 - \operatorname{Cos}[e + f x]^2}) \left(\left(b \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right] \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{1 - \operatorname{Cos}[e + f x]^2} \right) / \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right] \right) \operatorname{Cos}[e + f x]^2 \right) + \\
& \left(a \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right] \right) / \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \operatorname{Cos}[e + f x]^2, \right. \right. \\
& \left. \left. \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2} \right] + \left(-4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right] + \right. \\
& \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \operatorname{Cos}[e + f x]^2, \frac{b^2 \operatorname{Cos}[e + f x]^2}{-a^2 + b^2}\right] \operatorname{Cos}[e + f x]^2 \right) \right) \operatorname{Sin}[e + f x]^{3/2} \Big) / \\
& \left(3 (1 - \operatorname{Cos}[e + f x]^2) (a^2 + b^2 (-1 + \operatorname{Cos}[e + f x]^2)) (a + b \operatorname{Sin}[e + f x]) \right) + \left(2 (2 a^2 b + 10 b^3) \sqrt{\operatorname{Tan}[e + f x]} \right. \\
& \left. (b \operatorname{Tan}[e + f x] + a \sqrt{1 + \operatorname{Tan}[e + f x]^2}) \right) \\
& \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e + f x]}}{\sqrt{a}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e + f x]}}{\sqrt{a}}\right] - \right. \right. \\
& \left. \left. \operatorname{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e + f x]} - \sqrt{a^2 - b^2} \operatorname{Tan}[e + f x]\right] + \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e + f x]} + \right. \right. \\
& \left. \left. \sqrt{a^2 - b^2} \operatorname{Tan}[e + f x]\right] \right) + \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] \operatorname{Tan}[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \operatorname{Tan}[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + 2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \\
& \quad \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \Big) \Big) \Big) \Big) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(2 (-2 a^2 b - 5 b^3) \cos[2 (e + f x)] \right. \\
& \quad \left. \sqrt{\tan[e + f x]} \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \right. \\
& \quad \left(-\frac{a \operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right. \\
& \quad \left. + \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} \right. \\
& \quad \left. - \frac{a \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right. \\
& \quad \left. + \frac{(2 a^2 - b^2) \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x] \right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\tan[e + f x]^{3/2}}{b \sqrt{1 + \tan[e + f x]^2}} \right. \\
& \quad \left. \left(7 a^4 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{3/2} \right) \Big) \Big) \Big) / \\
& \left(b \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \Big) - \\
& \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{3/2} \right) \Big) \Big) / \left(3 \sqrt{1 + \tan[e + f x]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \\
& \left. \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) + \left(11 a^4 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right. \\
& \left. \tan[e + f x]^{7/2} \right) / \left(7 b \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \quad \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) - \\
& \left(11 a^2 b \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{7/2} \right) / \\
& \left(7 \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \quad \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right)
\end{aligned}$$

- **Problem 1415: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \cos[e + f x]}}{(d \sin[e + f x])^{9/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 598 leaves, 19 steps):

$$\begin{aligned}
& - \frac{2 (g \cos[e + f x])^{3/2}}{7 a d f g (d \sin[e + f x])^{7/2}} + \frac{2 b (g \cos[e + f x])^{3/2}}{5 a^2 d^2 f g (d \sin[e + f x])^{5/2}} - \frac{8 (g \cos[e + f x])^{3/2}}{21 a d^3 f g (d \sin[e + f x])^{3/2}} - \frac{2 b^2 (g \cos[e + f x])^{3/2}}{3 a^3 d^3 f g (d \sin[e + f x])^{3/2}} + \\
& \frac{4 b (g \cos[e + f x])^{3/2}}{5 a^2 d^4 f g \sqrt{d \sin[e + f x]}} + \frac{2 b^3 (g \cos[e + f x])^{3/2}}{a^4 d^4 f g \sqrt{d \sin[e + f x]}} + \frac{2 \sqrt{2} b^4 \sqrt{g} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+f x]}}{\sqrt{g} \sqrt{1+\sin[e+f x]}}\right], -1\right] \sqrt{\sin[e + f x]}}{a^4 \sqrt{-a+b} \sqrt{a+b} d^4 f \sqrt{d \sin[e + f x]}} - \\
& \frac{2 \sqrt{2} b^4 \sqrt{g} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+f x]}}{\sqrt{g} \sqrt{1+\sin[e+f x]}}\right], -1\right] \sqrt{\sin[e + f x]}}{a^4 \sqrt{-a+b} \sqrt{a+b} d^4 f \sqrt{d \sin[e + f x]}} + \\
& \frac{4 b \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{d \sin[e + f x]}}{5 a^2 d^5 f \sqrt{\sin[2 e + 2 f x]}} + \frac{2 b^3 \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{d \sin[e + f x]}}{a^4 d^5 f \sqrt{\sin[2 e + 2 f x]}}
\end{aligned}$$

Result (type 6, 3029 leaves):

$$\begin{aligned}
& \frac{1}{f (d \sin[e + f x])^{9/2}} \sqrt{g \cos[e + f x]} \left(\frac{2 (2 a^2 b \cos[e + f x] + 5 b^3 \cos[e + f x]) \operatorname{Csc}[e + f x]}{5 a^4} - \right. \\
& \left. \frac{2 (4 a^2 \cos[e + f x] + 7 b^2 \cos[e + f x]) \operatorname{Csc}[e + f x]^2}{21 a^3} + \frac{2 b \cot[e + f x] \operatorname{Csc}[e + f x]^2}{5 a^2} - \frac{2 \cot[e + f x] \operatorname{Csc}[e + f x]^3}{7 a} \right) \sin[e + f x]^5 + \\
& \frac{1}{5 a^4 f \sqrt{\cos[e + f x]} (d \sin[e + f x])^{9/2}} b \sqrt{g \cos[e + f x]} \sin[e + f x]^{9/2} \\
& \left(- \left(\left(14 (a^2 - b^2) (4 a^3 + 10 a b^2) \cos[e + f x]^{3/2} (a + b \sqrt{1 - \cos[e + f x]^2}) \left(\left(b \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{1 - \cos[e + f x]^2} \right) / \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right) + \right. \\
& \left. \left(a \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) / \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(-4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right) \right) \sin[e + f x]^{3/2} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(3 (1 - \cos[e + f x])^2 (a^2 + b^2 (-1 + \cos[e + f x])) (a + b \sin[e + f x]) \right) + \left(2 (2 a^2 b + 10 b^3) \sqrt{\tan[e + f x]} \right. \\
& \left. (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left[-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] \right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] + \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \right. \right. \right. \\
& \quad \left. \left. \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) + \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) \Big/ \\
& \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \quad \left. \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \Big/ \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(2 (-2 a^2 b - 5 b^3) \cos[2 (e + f x)] \right. \\
& \left. \sqrt{\tan[e + f x]} (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right. \\
& \left. \left(-\frac{a \operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right) + \right. \\
& \quad \left. \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} \right. \\
& \quad \left. \frac{a \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \right. \\
& \quad \left. \frac{a \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \frac{(2a^2 - b^2) \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + fx]} + \sqrt{a^2 - b^2} \tan[e + fx]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\tan[e + fx]^{3/2}}{b \sqrt{1 + \tan[e + fx]^2}} + \\
& \left(7 a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx]^{3/2}\right) / \\
& \left(b \sqrt{1 + \tan[e + fx]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \tan[e + fx]^2\right) (-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2))\right) - \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx]^{3/2}\right) / \left(3 \sqrt{1 + \tan[e + fx]^2}\right. \\
& \quad \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + fx]^2,\right.\right.\right. \\
& \quad \left.\left.\left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right]\right) \tan[e + fx]^2 \\
& \quad \left.(-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2))\right) + \left(11 a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right]\right. \\
& \quad \left.\tan[e + fx]^{7/2}\right) / \left(7 b \sqrt{1 + \tan[e + fx]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \tan[e + fx]^2\right) (-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2))\right) - \\
& \left(11 a^2 b \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx]^{7/2}\right) / \\
& \left(7 \sqrt{1 + \tan[e + fx]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right.
\end{aligned}$$

$$\left. \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) \right) /$$

$$\left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right)$$

- **Problem 1416: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{3/2} (d \sin[e + f x])^{3/2}}{a + b \sin[e + f x]} dx$$

Optimal (type 4, 982 leaves, 31 steps):

$$\begin{aligned}
& \frac{3 d^{3/2} g^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{4 \sqrt{2} b f} + \frac{(a^2 - b^2) d^{3/2} g^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{\sqrt{2} b^3 f} - \\
& \frac{3 d^{3/2} g^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{4 \sqrt{2} b f} - \frac{(a^2 - b^2) d^{3/2} g^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{\sqrt{2} b^3 f} + \\
& \frac{2 \sqrt{2} a \sqrt{-a^2 + b^2} d^{3/2} g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b - \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{1 + \cos[e+fx]}}\right], -1\right]}{b^3 f \sqrt{g \cos[e+fx]}} - \\
& \frac{2 \sqrt{2} a \sqrt{-a^2 + b^2} d^{3/2} g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b + \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{1 + \cos[e+fx]}}\right], -1\right]}{b^3 f \sqrt{g \cos[e+fx]}} - \\
& \frac{3 d^{3/2} g^{3/2} \operatorname{Log}\left[\sqrt{d} - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \tan[e+fx]\right]}{8 \sqrt{2} b f} - \frac{(a^2 - b^2) d^{3/2} g^{3/2} \operatorname{Log}\left[\sqrt{d} - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \tan[e+fx]\right]}{2 \sqrt{2} b^3 f} + \\
& \frac{3 d^{3/2} g^{3/2} \operatorname{Log}\left[\sqrt{d} + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \tan[e+fx]\right]}{8 \sqrt{2} b f} + \frac{(a^2 - b^2) d^{3/2} g^{3/2} \operatorname{Log}\left[\sqrt{d} + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \tan[e+fx]\right]}{2 \sqrt{2} b^3 f} - \\
& \frac{a d g \sqrt{g \cos[e+fx]} \sqrt{d \sin[e+fx]}}{b^2 f} + \frac{g \sqrt{g \cos[e+fx]} (d \sin[e+fx])^{3/2}}{2 b f} + \frac{a d^2 g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{\sin[2e + 2fx]}}{2 b^2 f \sqrt{g \cos[e+fx]} \sqrt{d \sin[e+fx]}}
\end{aligned}$$

Result (type 6, 2370 leaves):

$$\begin{aligned}
& \frac{(g \cos[e+fx])^{3/2} \operatorname{Sec}[e+fx] (d \sin[e+fx])^{3/2}}{2 b f} - \\
& \frac{1}{4 b f \cos[e+fx]^{3/2} \sin[e+fx]^{3/2}} (g \cos[e+fx])^{3/2} (d \sin[e+fx])^{3/2} \left(\frac{1}{(1 - \cos[e+fx]^2) (a^2 + b^2 (-1 + \cos[e+fx]^2)) (a + b \sin[e+fx])} \right. \\
& \left. 10 b (a^2 - b^2) \sqrt{\cos[e+fx]} \left(a + b \sqrt{1 - \cos[e+fx]^2} \right) \left(\left(b \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{3}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \sqrt{1 - \cos[e+fx]^2} \right) \right) \right) / \\
& \left(-5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{3}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{3}{4}, 2, \frac{9}{4}, \cos[e+fx]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^2 \Bigg) + \\
& \left(a \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) / \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \\
& \left. \left(-4 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{4}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^2 \right) \right) \sin[e + f x]^{5/2} + \\
& \left(4 a \cos[2 (e + f x)] \sqrt{\sin[e + f x]} \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \left(-\frac{a \operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} - \right. \right. \\
& \left. \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} + \frac{\sqrt{a} (2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} b^2 (a^2 - b^2)^{3/4}} + \right. \\
& \left. \frac{\sqrt{a} (2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} b^2 (a^2 - b^2)^{3/4}} - \frac{a \log \left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} + \right. \\
& \left. \frac{a \log \left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} + \frac{\sqrt{a} (2 a^2 - b^2) \log \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right]}{4 \sqrt{2} b^2 (a^2 - b^2)^{3/4}} - \right. \\
& \left. \frac{\sqrt{a} (2 a^2 - b^2) \log \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x] \right]}{4 \sqrt{2} b^2 (a^2 - b^2)^{3/4}} - \frac{\sqrt{\tan[e + f x]}}{b \sqrt{1 + \tan[e + f x]^2}} - \right. \\
& \left. \left(5 a^4 \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \sqrt{\tan[e + f x]} \right) / \left(b \sqrt{1 + \tan[e + f x]^2} \right. \\
& \left. \left(-5 a^2 \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\tan[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \\
& \left. (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) + \left(9 a^4 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\tan[e+fx]^{5/2} \right) / \left(5 b \sqrt{1 + \tan[e+fx]^2} \left(-9 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + 2 \right. \right. \right. \\
& \left. \left((a^2 - b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \right. \right. \right. \\
& \left. \left. -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \left(-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2) \right) \right) \right) / \\
& \left(\cos[e+fx]^{5/2} (a + b \sin[e+fx]) \sqrt{\tan[e+fx]} (-1 + \tan[e+fx]^2) \sqrt{1 + \tan[e+fx]^2} \right) + \\
& \left(2 a \sqrt{\sin[e+fx]} \left(b \tan[e+fx] + a \sqrt{1 + \tan[e+fx]^2} \right) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} (a^2 - b^2)^{3/4}} \sqrt{a} \left[-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]}}{\sqrt{a}} \right] + \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} \right. \right. \right. \right. \\
& \left. \left. (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]} - \sqrt{a^2 - b^2} \tan[e+fx] \right] - \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]} + \sqrt{a^2 - b^2} \tan[e+fx] \right] \right) \right) + \\
& \left(9 a^2 b \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \tan[e+fx]^{5/2} \right) / \left(5 \sqrt{1 + \tan[e+fx]^2} \right. \\
& \left(-9 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\tan[e+fx]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \\
& \left. \left. \left. \left. \left(-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2) \right) \right) \right) \right) / \left(\cos[e+fx]^{5/2} (a + b \sin[e+fx]) \sqrt{\tan[e+fx]} (1 + \tan[e+fx]^2)^{3/2} \right) \right)
\end{aligned}$$

■ **Problem 1417: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \cos[e+fx])^{3/2} \sqrt{d \sin[e+fx]}}{a + b \sin[e+fx]} dx$$

Optimal (type 4, 611 leaves, 19 steps):

$$\begin{aligned}
& - \frac{a \sqrt{d} g^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{\sqrt{2} b^2 f} + \frac{a \sqrt{d} g^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{\sqrt{2} b^2 f} - \\
& \frac{2 \sqrt{2} \sqrt{-a^2 + b^2} \sqrt{d} g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b - \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{1 + \cos[e+fx]}}\right], -1\right]}{b^2 f \sqrt{g \cos[e+fx]}} + \\
& \frac{2 \sqrt{2} \sqrt{-a^2 + b^2} \sqrt{d} g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b + \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{1 + \cos[e+fx]}}\right], -1\right]}{b^2 f \sqrt{g \cos[e+fx]}} + \\
& \frac{a \sqrt{d} g^{3/2} \operatorname{Log}\left[\sqrt{d} - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \tan[e+fx]\right]}{2 \sqrt{2} b^2 f} - \frac{a \sqrt{d} g^{3/2} \operatorname{Log}\left[\sqrt{d} + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \tan[e+fx]\right]}{2 \sqrt{2} b^2 f} + \\
& \frac{g \sqrt{g \cos[e+fx]} \sqrt{d \sin[e+fx]}}{b f} - \frac{d g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{\sin[2e + 2fx]}}{2 b f \sqrt{g \cos[e+fx]} \sqrt{d \sin[e+fx]}}
\end{aligned}$$

Result (type 6, 999 leaves):

$$\begin{aligned}
& - \left(\left((g \cos[e + f x])^{3/2} (a + b \sqrt{1 - \cos[e + f x]^2}) \left(- \left(25 (a^2 - b^2)^2 \operatorname{AppellF1} \left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) / \right. \right. \\
& \quad \left(b \left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(4 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, \cos[e + f x]^2, \right. \right. \right. \\
& \quad \quad \left. \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 3 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \right) + \\
& \quad \left(18 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^2 \sqrt{1 - \cos[e + f x]^2} \right) / \\
& \quad \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \\
& \quad \left(-4 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{4}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{5}{4}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \\
& \quad \cos[e + f x]^2 \left. + \left(-5 \left(-4 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. 3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right. \right. \\
& \quad \left. \left. (-1 + \cos[e + f x]^2) (a^2 + b^2 (-1 + \cos[e + f x]^2)) - 9 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right. \right. \\
& \quad \left. \left. (a^4 (-5 + 3 \cos[e + f x]^2) + b^4 (-5 + 9 \cos[e + f x]^2 - 5 \cos[e + f x]^4) + a^2 b^2 (10 - 12 \cos[e + f x]^2 + 5 \cos[e + f x]^4)) \right) \right) / \\
& \quad \left(b \left(-9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(4 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}, \cos[e + f x]^2, \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + 3 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \right) \\
& \quad \left. \sqrt{d \sin[e + f x]} \tan[e + f x] \right) / \left(5 f (1 - \cos[e + f x]^2)^{3/2} (a^2 + b^2 (-1 + \cos[e + f x]^2)) (a + b \sin[e + f x]) \right)
\end{aligned}$$

■ **Problem 1418: Result unnecessarily involves higher level functions.**

$$\int \frac{(g \cos[e + f x])^{3/2}}{\sqrt{d \sin[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 577 leaves, 18 steps):

$$\begin{aligned}
& \frac{g^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{\sqrt{2} b \sqrt{d} f} - \frac{g^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{\sqrt{2} b \sqrt{d} f} + \\
& \frac{2 \sqrt{2} \sqrt{-a^2+b^2} g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{1+\cos[e+fx]}}\right], -1\right]}{a b \sqrt{d} f \sqrt{g \cos[e+fx]}} - \\
& \frac{2 \sqrt{2} \sqrt{-a^2+b^2} g^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{1+\cos[e+fx]}}\right], -1\right]}{a b \sqrt{d} f \sqrt{g \cos[e+fx]}} - \\
& \frac{g^{3/2} \operatorname{Log}\left[\sqrt{d} - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \operatorname{Tan}[e+fx]\right]}{2 \sqrt{2} b \sqrt{d} f} + \\
& \frac{g^{3/2} \operatorname{Log}\left[\sqrt{d} + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \operatorname{Tan}[e+fx]\right]}{2 \sqrt{2} b \sqrt{d} f} + \frac{g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{\sin[2e+2fx]}}{a f \sqrt{g \cos[e+fx]} \sqrt{d \sin[e+fx]}}
\end{aligned}$$

Result (type 6, 520 leaves):

$$\begin{aligned}
& \frac{1}{5 f \sqrt{d \sin[e+fx]} (-a+b \sin[e+fx]) (a+b \sin[e+fx])^2} 18 (a^2-b^2) (g \cos[e+fx])^{3/2} \cot[e+fx] \left(a+b \sqrt{\sin[e+fx]^2}\right) \\
& \left(\left(a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) / \left(9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + \right. \right. \\
& \left. \left(-4 b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + 3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \right. \\
& \left. \cos[e+fx]^2 \right) + \left(b \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \sqrt{\sin[e+fx]^2} \right) / \\
& \left(-9 (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{4}, 2, \frac{13}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + \right. \right. \\
& \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{4}, 1, \frac{13}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right)
\end{aligned}$$

■ **Problem 1419: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e+fx])^{3/2}}{(d \sin[e+fx])^{3/2} (a+b \sin[e+fx])} dx$$

Optimal (type 4, 321 leaves, 8 steps) :

$$\frac{2\sqrt{2}\sqrt{-a^2+b^2}g^2\sqrt{\cos[e+fx]}\operatorname{EllipticPi}\left[-\frac{a}{b-\sqrt{-a^2+b^2}},\operatorname{ArcSin}\left[\frac{\sqrt{d}\sin[e+fx]}{\sqrt{d}\sqrt{1+\cos[e+fx]}}\right],-1\right]}{a^2d^{3/2}f\sqrt{g\cos[e+fx]}} +$$

$$\frac{2\sqrt{2}\sqrt{-a^2+b^2}g^2\sqrt{\cos[e+fx]}\operatorname{EllipticPi}\left[-\frac{a}{b+\sqrt{-a^2+b^2}},\operatorname{ArcSin}\left[\frac{\sqrt{d}\sin[e+fx]}{\sqrt{d}\sqrt{1+\cos[e+fx]}}\right],-1\right]}{a^2d^{3/2}f\sqrt{g\cos[e+fx]}} -$$

$$\frac{2g\sqrt{g\cos[e+fx]}}{adf\sqrt{d}\sin[e+fx]} - \frac{bg^2\operatorname{EllipticF}\left[e-\frac{\pi}{4}+fx,2\right]\sqrt{\sin[2e+2fx]}}{a^2df\sqrt{g\cos[e+fx]}\sqrt{d}\sin[e+fx]}$$

Result (type 6, 1287 leaves) :

$$\frac{2(g\cos[e+fx])^{3/2}\tan[e+fx]}{af(d\sin[e+fx])^{3/2}} -$$

$$\frac{1}{af\cos[e+fx]^{3/2}(d\sin[e+fx])^{3/2}}(g\cos[e+fx])^{3/2}\sin[e+fx]^{3/2}\left(-\frac{1}{(1-\cos[e+fx])^{1/4}(a+b\sin[e+fx])}2b\left(a+b\sqrt{1-\cos[e+fx]^2}\right)\right.$$

$$\left.\left(\left(5a(a^2-b^2)\operatorname{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},\cos[e+fx]^2,\frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right]\sqrt{\cos[e+fx]}\right)/\left((1-\cos[e+fx])^{3/4}\left(5(a^2-b^2)\right.\right.\right.$$

$$\left.\left.\operatorname{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},\cos[e+fx]^2,\frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right]+(-4b^2\operatorname{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},\cos[e+fx]^2,\frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right])+\right.\right.$$

$$\left.\left.3(a^2-b^2)\operatorname{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},\cos[e+fx]^2,\frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right]\cos[e+fx]^2\right)(a^2+b^2(-1+\cos[e+fx]^2))\right)-$$

$$\frac{1}{\sqrt{a}(-a^2+b^2)^{3/4}}\left(\frac{1}{8}-\frac{i}{8}\right)b\left(2\operatorname{ArcTan}\left[1-\frac{(1+i)\sqrt{a}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}(-1+\cos[e+fx])^{1/4}}\right]-2\operatorname{ArcTan}\left[1+\frac{(1+i)\sqrt{a}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}(-1+\cos[e+fx])^{1/4}}\right]\right)+$$

$$\operatorname{Log}\left[\sqrt{-a^2+b^2}+\frac{ia\cos[e+fx]}{\sqrt{-1+\cos[e+fx]^2}}-\frac{(1+i)\sqrt{a}(-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]}}{(-1+\cos[e+fx])^{1/4}}\right]-$$

$$\operatorname{Log}\left[\sqrt{-a^2+b^2}+\frac{ia\cos[e+fx]}{\sqrt{-1+\cos[e+fx]^2}}+\frac{(1+i)\sqrt{a}(-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]}}{(-1+\cos[e+fx])^{1/4}}\right]\right)\sqrt{\sin[e+fx]} +$$

$$\begin{aligned}
& \left(2 a \sqrt{\sin[e+f x]} \left(b \tan[e+f x] + a \sqrt{1+\tan[e+f x]^2} \right) \left(\frac{1}{4 \sqrt{2} (a^2-b^2)^{3/4}} \sqrt{a} \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a^2-b^2)^{1/4} \sqrt{\tan[e+f x]}}{\sqrt{a}}\right] \right) + \right. \right. \\
& \quad \left. \left. 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a^2-b^2)^{1/4} \sqrt{\tan[e+f x]}}{\sqrt{a}}\right] + \operatorname{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2-b^2)^{1/4} \sqrt{\tan[e+f x]} - \sqrt{a^2-b^2} \tan[e+f x]\right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2-b^2)^{1/4} \sqrt{\tan[e+f x]} + \sqrt{a^2-b^2} \tan[e+f x]\right] \right) \right) + \\
& \left(9 a^2 b \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+f x]^2\right] \tan[e+f x]^{5/2} \right) / \left(5 \sqrt{1+\tan[e+f x]^2} \right. \\
& \quad \left(-9 a^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+f x]^2\right] + 2 \left(2 (a^2-b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\tan[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \tan[e+f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\tan[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+f x]^2\right] \right) \tan[e+f x]^2 \right) \\
& \quad \left. \left. \left. (-b^2 \tan[e+f x]^2 + a^2 (1+\tan[e+f x]^2)) \right) \right) \right) / \left(\cos[e+f x]^{5/2} (a+b \sin[e+f x]) \sqrt{\tan[e+f x]} (1+\tan[e+f x]^2)^{3/2} \right)
\end{aligned}$$

■ **Problem 1420: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e+f x])^{3/2}}{(d \sin[e+f x])^{5/2} (a+b \sin[e+f x])} dx$$

Optimal (type 4, 435 leaves, 12 steps):

$$\begin{aligned}
& \frac{2 \sqrt{2} b \sqrt{-a^2+b^2} g^2 \sqrt{\cos[e+f x]} \operatorname{EllipticPi}\left[-\frac{a}{b-\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e+f x]}{\sqrt{d} \sqrt{1+\cos[e+f x]}}\right], -1\right]}{a^3 d^{5/2} f \sqrt{g \cos[e+f x]}} \\
& - \frac{2 \sqrt{2} b \sqrt{-a^2+b^2} g^2 \sqrt{\cos[e+f x]} \operatorname{EllipticPi}\left[-\frac{a}{b+\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e+f x]}{\sqrt{d} \sqrt{1+\cos[e+f x]}}\right], -1\right]}{a^3 d^{5/2} f \sqrt{g \cos[e+f x]}} - \frac{2 g \sqrt{g \cos[e+f x]}}{3 a d f (d \sin[e+f x])^{3/2}} + \\
& \frac{2 b g \sqrt{g \cos[e+f x]}}{a^2 d^2 f \sqrt{d} \sin[e+f x]} + \frac{2 g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2 e + 2 f x]}}{3 a d^2 f \sqrt{g \cos[e+f x]} \sqrt{d} \sin[e+f x]} - \frac{(a^2-b^2) g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2 e + 2 f x]}}{a^3 d^2 f \sqrt{g \cos[e+f x]} \sqrt{d} \sin[e+f x]}
\end{aligned}$$

Result (type 6, 1330 leaves):

$$\frac{(g \cos[e+f x])^{3/2} \left(\frac{2 b \operatorname{Csc}[e+f x]}{a^2} - \frac{2 \operatorname{Csc}[e+f x]^2}{3 a} \right) \sin[e+f x]^2 \tan[e+f x]}{f (d \sin[e+f x])^{5/2}}$$

$$\begin{aligned}
& \frac{1}{3 a^2 f \operatorname{Cos}[e+f x]^{3/2} (d \operatorname{Sin}[e+f x])^{5/2}} (g \operatorname{Cos}[e+f x])^{3/2} \operatorname{Sin}[e+f x]^{5/2} \left(-\frac{1}{(1-\operatorname{Cos}[e+f x]^2)^{1/4} (a+b \operatorname{Sin}[e+f x])} \right. \\
& 2 (a^2-3 b^2) \left(a+b \sqrt{1-\operatorname{Cos}[e+f x]^2} \right) \left(\left(5 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Cos}[e+f x]} \right) / \right. \\
& \left((1-\operatorname{Cos}[e+f x]^2)^{3/4} \left(5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] + \left(-4 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] + 3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2+b^2}\right] \right) \operatorname{Cos}[e+f x]^2 \right) \\
& \left. \left. \left. (a^2+b^2 (-1+\operatorname{Cos}[e+f x]^2)) \right) \right) - \frac{1}{\sqrt{a} (-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) b \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{a} \sqrt{\operatorname{Cos}[e+f x]}}{(-a^2+b^2)^{1/4} (-1+\operatorname{Cos}[e+f x]^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[\right. \right. \\
& \left. \left. 1 + \frac{(1+i) \sqrt{a} \sqrt{\operatorname{Cos}[e+f x]}}{(-a^2+b^2)^{1/4} (-1+\operatorname{Cos}[e+f x]^2)^{1/4}} \right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + \frac{i a \operatorname{Cos}[e+f x]}{\sqrt{-1+\operatorname{Cos}[e+f x]^2}} - \frac{(1+i) \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Cos}[e+f x]}}{(-1+\operatorname{Cos}[e+f x]^2)^{1/4}} \right] - \right. \\
& \left. \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + \frac{i a \operatorname{Cos}[e+f x]}{\sqrt{-1+\operatorname{Cos}[e+f x]^2}} + \frac{(1+i) \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Cos}[e+f x]}}{(-1+\operatorname{Cos}[e+f x]^2)^{1/4}} \right] \right) \right) \sqrt{\operatorname{Sin}[e+f x]} - \\
& \left(4 a b \sqrt{\operatorname{Sin}[e+f x]} \left(b \operatorname{Tan}[e+f x] + a \sqrt{1+\operatorname{Tan}[e+f x]^2} \right) \left(\frac{1}{4 \sqrt{2} (a^2-b^2)^{3/4}} \sqrt{a} \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a^2-b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{a}} \right] + \right. \right. \\
& \left. \left. 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a^2-b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{a}} \right] + \operatorname{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2-b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]} - \sqrt{a^2-b^2} \operatorname{Tan}[e+f x] \right] - \right. \right. \\
& \left. \left. \left. \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2-b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]} + \sqrt{a^2-b^2} \operatorname{Tan}[e+f x] \right] \right) \right) + \\
& \left(9 a^2 b \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Tan}[e+f x]^{5/2} \right) / \left(5 \sqrt{1+\operatorname{Tan}[e+f x]^2} \right. \\
& \left(-9 a^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] + 2 \left(2 (a^2-b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \left. \left. \left. \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) \\
& \left. \left. \left. (-b^2 \operatorname{Tan}[e+f x]^2 + a^2 (1+\operatorname{Tan}[e+f x]^2)) \right) \right) \right) / \left(\operatorname{Cos}[e+f x]^{5/2} (a+b \operatorname{Sin}[e+f x]) \sqrt{\operatorname{Tan}[e+f x]} (1+\operatorname{Tan}[e+f x]^2)^{3/2} \right)
\end{aligned}$$

Problem 1421: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(g \cos[e + f x])^{3/2}}{(d \sin[e + f x])^{7/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 525 leaves, 15 steps):

$$\frac{2\sqrt{2} b^2 \sqrt{-a^2 + b^2} g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[-\frac{a}{b - \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e + f x]}{\sqrt{d} \sqrt{1 + \cos[e + f x]}}\right], -1\right]}{a^4 d^{7/2} f \sqrt{g \cos[e + f x]}} +$$

$$\frac{2\sqrt{2} b^2 \sqrt{-a^2 + b^2} g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[-\frac{a}{b + \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e + f x]}{\sqrt{d} \sqrt{1 + \cos[e + f x]}}\right], -1\right]}{a^4 d^{7/2} f \sqrt{g \cos[e + f x]}} -$$

$$\frac{2g \sqrt{g \cos[e + f x]}}{5 a d f (d \sin[e + f x])^{5/2}} + \frac{2 b g \sqrt{g \cos[e + f x]}}{3 a^2 d^2 f (d \sin[e + f x])^{3/2}} - \frac{8 g \sqrt{g \cos[e + f x]}}{5 a d^3 f \sqrt{d} \sin[e + f x]} + \frac{2 (a^2 - b^2) g \sqrt{g \cos[e + f x]}}{a^3 d^3 f \sqrt{d} \sin[e + f x]} -$$

$$\frac{2 b g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2 e + 2 f x]}}{3 a^2 d^3 f \sqrt{g \cos[e + f x]} \sqrt{d} \sin[e + f x]} + \frac{b (a^2 - b^2) g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2 e + 2 f x]}}{a^4 d^3 f \sqrt{g \cos[e + f x]} \sqrt{d} \sin[e + f x]}$$

Result (type 6, 1357 leaves):

$$\frac{(g \cos[e + f x])^{3/2} \left(\frac{2(a^2 - 5b^2) \operatorname{Csc}[e + f x]}{5a^3} + \frac{2b \operatorname{Csc}[e + f x]^2}{3a^2} - \frac{2 \operatorname{Csc}[e + f x]^3}{5a} \right) \sin[e + f x]^3 \tan[e + f x]}{f (d \sin[e + f x])^{7/2}} +$$

$$\frac{1}{3 a^3 f \cos[e + f x]^{3/2} (d \sin[e + f x])^{7/2}} b (g \cos[e + f x])^{3/2} \sin[e + f x]^{7/2}$$

$$\left(-\frac{1}{(1 - \cos[e + f x])^{1/4} (a + b \sin[e + f x])} 2 (a^2 - 3 b^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \right. \right.$$

$$\left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \sqrt{\cos[e + f x]} \right) \right) / \left((1 - \cos[e + f x])^{3/4} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + \right. \right.$$

$$\left. \left(-4 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + 3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right) \left(a^2 + b^2 (-1 + \cos[e + f x]^2) \right) \right) - \frac{1}{\sqrt{a} (-a^2 + b^2)^{3/4}}$$

$$\begin{aligned}
& \left(\frac{1}{8} - \frac{i}{8} \right) b \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{a} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4} (-1+\cos[e+fx])^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{a} \sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4} (-1+\cos[e+fx])^{1/4}} \right] \right) + \\
& \operatorname{Log} \left[\sqrt{-a^2+b^2} + \frac{i a \cos[e+fx]}{\sqrt{-1+\cos[e+fx]^2}} - \frac{(1+i) \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]}}{(-1+\cos[e+fx])^{1/4}} \right] - \\
& \operatorname{Log} \left[\sqrt{-a^2+b^2} + \frac{i a \cos[e+fx]}{\sqrt{-1+\cos[e+fx]^2}} + \frac{(1+i) \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\cos[e+fx]}}{(-1+\cos[e+fx])^{1/4}} \right] \Bigg) \sqrt{\sin[e+fx]} - \\
& \left(4 a b \sqrt{\sin[e+fx]} \left(b \tan[e+fx] + a \sqrt{1+\tan[e+fx]^2} \right) \left(\frac{1}{4 \sqrt{2} (a^2-b^2)^{3/4}} \sqrt{a} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2-b^2)^{1/4} \sqrt{\tan[e+fx]}}{\sqrt{a}} \right] \right) + \right. \right. \\
& \left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2-b^2)^{1/4} \sqrt{\tan[e+fx]}}{\sqrt{a}} \right] + \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2-b^2)^{1/4} \sqrt{\tan[e+fx]} - \sqrt{a^2-b^2} \tan[e+fx] \right] - \right. \right. \\
& \left. \left. \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2-b^2)^{1/4} \sqrt{\tan[e+fx]} + \sqrt{a^2-b^2} \tan[e+fx] \right] \right) \right) + \\
& \left(9 a^2 b \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \tan[e+fx]^{5/2} \right) / \left(5 \sqrt{1+\tan[e+fx]^2} \right. \\
& \left(-9 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + 2 \left(2 (a^2-b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\tan[e+fx]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \\
& \left. \left. \left. (-b^2 \tan[e+fx]^2 + a^2 (1+\tan[e+fx]^2)) \right) \right) \right) / \left(\cos[e+fx]^{5/2} (a+b \sin[e+fx]) \sqrt{\tan[e+fx]} (1+\tan[e+fx]^2)^{3/2} \right)
\end{aligned}$$

■ **Problem 1422: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e+fx])^{3/2}}{(d \sin[e+fx])^{9/2} (a+b \sin[e+fx])} dx$$

Optimal (type 4, 688 leaves, 20 steps):

$$\begin{aligned}
& \frac{2 \sqrt{2} b^3 \sqrt{-a^2 + b^2} g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[-\frac{a}{b - \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e + f x]}{\sqrt{d} \sqrt{1 + \cos[e + f x]}}\right], -1\right]}{a^5 d^{9/2} f \sqrt{g \cos[e + f x]}} - \\
& \frac{2 \sqrt{2} b^3 \sqrt{-a^2 + b^2} g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[-\frac{a}{b + \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e + f x]}{\sqrt{d} \sqrt{1 + \cos[e + f x]}}\right], -1\right]}{a^5 d^{9/2} f \sqrt{g \cos[e + f x]}} - \\
& \frac{2 g \sqrt{g \cos[e + f x]}}{7 a d f (d \sin[e + f x])^{7/2}} + \frac{2 b g \sqrt{g \cos[e + f x]}}{5 a^2 d^2 f (d \sin[e + f x])^{5/2}} - \frac{4 g \sqrt{g \cos[e + f x]}}{7 a d^3 f (d \sin[e + f x])^{3/2}} + \frac{2 (a^2 - b^2) g \sqrt{g \cos[e + f x]}}{3 a^3 d^3 f (d \sin[e + f x])^{3/2}} + \\
& \frac{8 b g \sqrt{g \cos[e + f x]}}{5 a^2 d^4 f \sqrt{d \sin[e + f x]}} - \frac{2 b (a^2 - b^2) g \sqrt{g \cos[e + f x]}}{a^4 d^4 f \sqrt{d \sin[e + f x]}} + \frac{4 g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2 e + 2 f x]}}{7 a d^4 f \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}} - \\
& \frac{2 (a^2 - b^2) g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2 e + 2 f x]}}{3 a^3 d^4 f \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}} - \frac{b^2 (a^2 - b^2) g^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2 e + 2 f x]}}{a^5 d^4 f \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}}
\end{aligned}$$

Result (type 6, 1402 leaves):

$$\begin{aligned}
& \frac{1}{f (d \sin[e + f x])^{9/2}} \\
& (g \cos[e + f x])^{3/2} \left(-\frac{2 b (a^2 - 5 b^2) \operatorname{Csc}[e + f x]}{5 a^4} + \frac{2 (a^2 - 7 b^2) \operatorname{Csc}[e + f x]^2}{21 a^3} + \frac{2 b \operatorname{Csc}[e + f x]^3}{5 a^2} - \frac{2 \operatorname{Csc}[e + f x]^4}{7 a} \right) \sin[e + f x]^4 \tan[e + f x] - \\
& \frac{1}{21 a^4 f \cos[e + f x]^{3/2} (d \sin[e + f x])^{9/2}} \\
& (g \cos[e + f x])^{3/2} \sin[e + f x]^{9/2} \left(-\frac{1}{(1 - \cos[e + f x]^2)^{1/4} (a + b \sin[e + f x])} 2 (2 a^4 + 7 a^2 b^2 - 21 b^4) (a + b \sqrt{1 - \cos[e + f x]^2}) \right. \\
& \left. \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \sqrt{\cos[e + f x]} \right) / \left((1 - \cos[e + f x]^2)^{3/4} \left(5 (a^2 - b^2) \right. \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) + \left(-4 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) + \right. \right. \\
& \left. \left. \left. 3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right) (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{a} (-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) b \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{a} \sqrt{\cos[e+fx]}}{(-a^2 + b^2)^{1/4} (-1 + \cos[e+fx])^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{a} \sqrt{\cos[e+fx]}}{(-a^2 + b^2)^{1/4} (-1 + \cos[e+fx])^{1/4}} \right] \right) + \\
& \operatorname{Log} \left[\sqrt{-a^2 + b^2} + \frac{i a \cos[e+fx]}{\sqrt{-1 + \cos[e+fx]^2}} - \frac{(1+i) \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\cos[e+fx]}}{(-1 + \cos[e+fx])^{1/4}} \right] - \\
& \operatorname{Log} \left[\sqrt{-a^2 + b^2} + \frac{i a \cos[e+fx]}{\sqrt{-1 + \cos[e+fx]^2}} + \frac{(1+i) \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\cos[e+fx]}}{(-1 + \cos[e+fx])^{1/4}} \right] \Bigg) \sqrt{\sin[e+fx]} + \\
& \left(2 (2 a^3 b - 14 a b^3) \sqrt{\sin[e+fx]} \left(b \tan[e+fx] + a \sqrt{1 + \tan[e+fx]^2} \right) \left(\frac{1}{4 \sqrt{2} (a^2 - b^2)^{3/4}} \right. \right. \\
& \left. \left. \sqrt{a} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]}}{\sqrt{a}} \right] \right) + \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \right. \right. \right. \\
& \left. \left. \left. \sqrt{\tan[e+fx]} - \sqrt{a^2 - b^2} \tan[e+fx] \right] - \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]} + \sqrt{a^2 - b^2} \tan[e+fx] \right] \right) \right) + \\
& \left(9 a^2 b \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \tan[e+fx]^{5/2} \right) / \left(5 \sqrt{1 + \tan[e+fx]^2} \right. \\
& \left(-9 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\tan[e+fx]^2, \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \\
& \left. \left. \left. (-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2)) \right) \right) \right) / \left(\cos[e+fx]^{5/2} (a + b \sin[e+fx]) \sqrt{\tan[e+fx]} (1 + \tan[e+fx]^2)^{3/2} \right)
\end{aligned}$$

■ **Problem 1423: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e+fx])^{5/2} \sqrt{d \sin[e+fx]}}{a + b \sin[e+fx]} dx$$

Optimal (type 4, 936 leaves, 31 steps):

$$\begin{aligned}
& \frac{\sqrt{d} g^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d} \sin[e+fx]}\right]}{4 \sqrt{2} b f} - \frac{(a^2 - b^2) \sqrt{d} g^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d} \sin[e+fx]}\right]}{\sqrt{2} b^3 f} + \\
& \frac{\sqrt{d} g^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d} \sin[e+fx]}\right]}{4 \sqrt{2} b f} + \frac{(a^2 - b^2) \sqrt{d} g^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d} \sin[e+fx]}\right]}{\sqrt{2} b^3 f} + \\
& \frac{\sqrt{d} g^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d} \sin[e+fx]}\right]}{8 \sqrt{2} b f} + \frac{(a^2 - b^2) \sqrt{d} g^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d} \sin[e+fx]}\right]}{2 \sqrt{2} b^3 f} - \\
& \frac{\sqrt{d} g^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d} \sin[e+fx]}\right]}{8 \sqrt{2} b f} - \frac{(a^2 - b^2) \sqrt{d} g^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d} \sin[e+fx]}\right]}{2 \sqrt{2} b^3 f} - \\
& \frac{2 \sqrt{2} a \sqrt{-a+b} \sqrt{a+b} d g^{5/2} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{b^3 f \sqrt{d} \sin[e+fx]} + \\
& \frac{2 \sqrt{2} a \sqrt{-a+b} \sqrt{a+b} d g^{5/2} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{b^3 f \sqrt{d} \sin[e+fx]} + \\
& \frac{g (g \cos[e+fx])^{3/2} \sqrt{d} \sin[e+fx]}{2 b f} + \frac{a g^2 \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{d} \sin[e+fx]}{b^2 f \sqrt{\sin[2e+2fx]}}
\end{aligned}$$

Result (type 6, 2878 leaves):

$$\begin{aligned}
& \frac{(g \cos[e+fx])^{5/2} \operatorname{Sec}[e+fx] \sqrt{d} \sin[e+fx]}{2 b f} - \\
& \frac{1}{4 b f \cos[e+fx]^{5/2} \sqrt{\sin[e+fx]}} (g \cos[e+fx])^{5/2} \sqrt{d} \sin[e+fx] \left(\frac{1}{(1 - \cos[e+fx])^2 (a^2 + b^2 (-1 + \cos[e+fx])^2) (a + b \sin[e+fx])} \right. \\
& \left. 14 b (a^2 - b^2) \cos[e+fx]^{3/2} \left(a + b \sqrt{1 - \cos[e+fx]^2} \right) \left(\left(b \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \sqrt{1 - \cos[e+fx]^2} \right) \right) \right) / \\
& \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos[e+fx]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^2 \Bigg) + \\
& \left(a \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) / \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + \right. \\
& \left. \left(-4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \cos[e + f x]^2 \right) \right) \sin[e + f x]^{3/2} - \\
& \left(2 a \sqrt{\tan[e + f x]} \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}}\right] \right) + \right. \right. \\
& \left. \left. 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}}\right] - \operatorname{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x]\right] + \right. \right. \\
& \left. \left. \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x]\right] \right) \right) + \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \tan[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + 2 \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \\
& \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \Bigg) \Bigg) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(4 a \cos[2 (e + f x)] \sqrt{\tan[e + f x]} \right. \\
& \left. \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \right. \\
& \left. \left(-\frac{a \operatorname{ArcTan}\left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}}\right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{(2a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{a} + 2(a^2 - b^2)^{1/4}\sqrt{\operatorname{Tan}[e + fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}\sqrt{a}b^2(a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log}\left[1 - \sqrt{2}\sqrt{\operatorname{Tan}[e + fx]} + \operatorname{Tan}[e + fx]\right]}{2\sqrt{2}b^2} - \\
& \frac{a \operatorname{Log}\left[1 + \sqrt{2}\sqrt{\operatorname{Tan}[e + fx]} + \operatorname{Tan}[e + fx]\right]}{2\sqrt{2}b^2} - \frac{(2a^2 - b^2) \operatorname{Log}\left[-a + \sqrt{2}\sqrt{a}(a^2 - b^2)^{1/4}\sqrt{\operatorname{Tan}[e + fx]} - \sqrt{a^2 - b^2}\operatorname{Tan}[e + fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2 - b^2)^{1/4}} + \\
& \frac{(2a^2 - b^2) \operatorname{Log}\left[a + \sqrt{2}\sqrt{a}(a^2 - b^2)^{1/4}\sqrt{\operatorname{Tan}[e + fx]} + \sqrt{a^2 - b^2}\operatorname{Tan}[e + fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2 - b^2)^{1/4}} + \frac{\operatorname{Tan}[e + fx]^{3/2}}{b\sqrt{1 + \operatorname{Tan}[e + fx]^2}} + \\
& \left(7a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + fx]^2, \frac{(-a^2 + b^2)\operatorname{Tan}[e + fx]^2}{a^2}\right] \operatorname{Tan}[e + fx]^{3/2}\right) / \\
& \left(b\sqrt{1 + \operatorname{Tan}[e + fx]^2} \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\operatorname{Tan}[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left. \left(2(a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\operatorname{Tan}[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4},\right.\right.\right.\right. \\
& \quad \left.\left.\left. -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\operatorname{Tan}[e + fx]^2\right]\right) \operatorname{Tan}[e + fx]^2\right) \left(-b^2 \operatorname{Tan}[e + fx]^2 + a^2(1 + \operatorname{Tan}[e + fx]^2)\right)\right) - \\
& \left(7a^2b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + fx]^2, \frac{(-a^2 + b^2)\operatorname{Tan}[e + fx]^2}{a^2}\right] \operatorname{Tan}[e + fx]^{3/2}\right) / \left(3\sqrt{1 + \operatorname{Tan}[e + fx]^2}\right. \\
& \quad \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\operatorname{Tan}[e + fx]^2\right] + 2\left(2(a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e + fx]^2,\right.\right.\right. \\
& \quad \left.\left.\left(-1 + \frac{b^2}{a^2}\right)\operatorname{Tan}[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\operatorname{Tan}[e + fx]^2\right]\right) \operatorname{Tan}[e + fx]^2\right) \\
& \quad \left(-b^2 \operatorname{Tan}[e + fx]^2 + a^2(1 + \operatorname{Tan}[e + fx]^2)\right)\right) + \left(11a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + fx]^2, \frac{(-a^2 + b^2)\operatorname{Tan}[e + fx]^2}{a^2}\right]\right. \\
& \quad \left.\operatorname{Tan}[e + fx]^{7/2}\right) / \left(7b\sqrt{1 + \operatorname{Tan}[e + fx]^2} \left(-11a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\operatorname{Tan}[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left(2(a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\operatorname{Tan}[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right. \\
& \quad \left.\left.\left. -\operatorname{Tan}[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\operatorname{Tan}[e + fx]^2\right]\right) \operatorname{Tan}[e + fx]^2\right) \left(-b^2 \operatorname{Tan}[e + fx]^2 + a^2(1 + \operatorname{Tan}[e + fx]^2)\right)\right) - \\
& \left(11a^2b \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + fx]^2, \frac{(-a^2 + b^2)\operatorname{Tan}[e + fx]^2}{a^2}\right] \operatorname{Tan}[e + fx]^{7/2}\right) /
\end{aligned}$$

$$\left(7 \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\ \left. \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \right. \\ \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \Bigg) / \\ \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right)$$

- **Problem 1424: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{5/2}}{\sqrt{d \sin[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 572 leaves, 19 steps):

$$\frac{a g^{5/2} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{d \sin[e + f x]}} \right]}{\sqrt{2} b^2 \sqrt{d} f} - \frac{a g^{5/2} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{d \sin[e + f x]}} \right]}{\sqrt{2} b^2 \sqrt{d} f} - \\ \frac{a g^{5/2} \operatorname{Log} \left[\sqrt{g} + \sqrt{g} \cot[e + f x] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{d \sin[e + f x]}} \right]}{2 \sqrt{2} b^2 \sqrt{d} f} + \frac{a g^{5/2} \operatorname{Log} \left[\sqrt{g} + \sqrt{g} \cot[e + f x] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{d \sin[e + f x]}} \right]}{2 \sqrt{2} b^2 \sqrt{d} f} + \\ \frac{2 \sqrt{2} \sqrt{-a + b} \sqrt{a + b} g^{5/2} \operatorname{EllipticPi} \left[-\frac{\sqrt{-a + b}}{\sqrt{a + b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{b^2 f \sqrt{d \sin[e + f x]}} - \\ \frac{2 \sqrt{2} \sqrt{-a + b} \sqrt{a + b} g^{5/2} \operatorname{EllipticPi} \left[\frac{\sqrt{-a + b}}{\sqrt{a + b}}, \operatorname{ArcSin} \left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}} \right], -1 \right] \sqrt{\sin[e + f x]}}{b^2 f \sqrt{d \sin[e + f x]}} - \\ \frac{g^2 \sqrt{g \cos[e + f x]} \operatorname{EllipticE} \left[e - \frac{\pi}{4} + f x, 2 \right] \sqrt{d \sin[e + f x]}}{b d f \sqrt{\sin[2 e + 2 f x]}}$$

Result (type 6, 2299 leaves):

$$\begin{aligned}
& \frac{1}{2 f \operatorname{Cos}[e+f x]^{5/2} \sqrt{d} \operatorname{Sin}[e+f x]} \\
& (g \operatorname{Cos}[e+f x])^{5/2} \sqrt{\operatorname{Sin}[e+f x]} \left(\left(2 \sqrt{\operatorname{Tan}[e+f x]} \left(b \operatorname{Tan}[e+f x] + a \sqrt{1 + \operatorname{Tan}[e+f x]^2} \right) \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \right. \right. \right. \\
& \left. \left. \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{a}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{a}}\right] - \operatorname{Log}\left[-a + \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]} - \sqrt{a^2 - b^2} \operatorname{Tan}[e+f x] \right] + \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]} + \sqrt{a^2 - b^2} \operatorname{Tan}[e+f x] \right] \right) \right) + \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Tan}[e+f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \operatorname{Tan}[e+f x]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] + \right. \right. \\
& \left. \left. 2 \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e+f x]^2\right] \operatorname{Tan}[e+f x]^2 \right) \left(-b^2 \operatorname{Tan}[e+f x]^2 + a^2 (1 + \operatorname{Tan}[e+f x]^2) \right) \right) \right) / \\
& \left(\operatorname{Cos}[e+f x]^{3/2} \sqrt{\operatorname{Sin}[e+f x]} (a + b \operatorname{Sin}[e+f x]) (1 + \operatorname{Tan}[e+f x]^2)^{3/2} \right) + \left(2 \operatorname{Cos}[2(e+f x)] \sqrt{\operatorname{Tan}[e+f x]} \right. \\
& \left. (b \operatorname{Tan}[e+f x] + a \sqrt{1 + \operatorname{Tan}[e+f x]^2}) \right) \\
& \left(-\frac{a \operatorname{ArcTan}\left[\frac{-\sqrt{2} + 2 \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{2} + 2 \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{2} \sqrt{a}}\right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \right. \\
& \left. \frac{(2 a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{2} \sqrt{a}}\right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[e+f x]} + \operatorname{Tan}[e+f x]\right]}{2 \sqrt{2} b^2} - \right. \\
& \left. \frac{a \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[e+f x]} + \operatorname{Tan}[e+f x]\right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]} - \sqrt{a^2 - b^2} \operatorname{Tan}[e+f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{(2a^2 - b^2) \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e + f x]} + \sqrt{a^2 - b^2} \operatorname{Tan}[e + f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\operatorname{Tan}[e + f x]^{3/2}}{b \sqrt{1 + \operatorname{Tan}[e + f x]^2}} + \\
& \left(7 a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \operatorname{Tan}[e + f x]^{3/2}\right) / \\
& \left(b \sqrt{1 + \operatorname{Tan}[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right]\right) \operatorname{Tan}[e + f x]^2 \left(-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)\right)\right) - \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \operatorname{Tan}[e + f x]^{3/2}\right) / \left(3 \sqrt{1 + \operatorname{Tan}[e + f x]^2}\right. \\
& \quad \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right]\right) \operatorname{Tan}[e + f x]^2 \right) \\
& \quad \left(-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)\right) \left. \right) + \left(11 a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \right. \\
& \quad \left. \operatorname{Tan}[e + f x]^{7/2}\right) / \left(7 b \sqrt{1 + \operatorname{Tan}[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right]\right) \operatorname{Tan}[e + f x]^2 \left(-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)\right)\right) - \\
& \left(11 a^2 b \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \operatorname{Tan}[e + f x]^{7/2}\right) / \\
& \left(7 \sqrt{1 + \operatorname{Tan}[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right]\right) \operatorname{Tan}[e + f x]^2 \left(-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)\right)\right) -
\end{aligned}$$

$$\left. \left(-\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \bigg) \bigg) /$$

$$\left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right)$$

■ **Problem 1425: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{5/2}}{(d \sin[e + f x])^{3/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 616 leaves, 20 steps):

$$-\frac{g^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{d \sin[e + f x]}}\right]}{\sqrt{2} b d^{3/2} f} + \frac{g^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{d \sin[e + f x]}}\right]}{\sqrt{2} b d^{3/2} f} +$$

$$\frac{g^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e + f x] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{d \sin[e + f x]}}\right]}{2 \sqrt{2} b d^{3/2} f} - \frac{g^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e + f x] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e + f x]}}{\sqrt{d \sin[e + f x]}}\right]}{2 \sqrt{2} b d^{3/2} f} -$$

$$\frac{2 g (g \cos[e + f x])^{3/2}}{a d f \sqrt{d \sin[e + f x]}} - \frac{2 \sqrt{2} \sqrt{-a + b} \sqrt{a + b} g^{5/2} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a + b}}{\sqrt{a + b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}}\right], -1\right] \sqrt{\sin[e + f x]}}{a b d f \sqrt{d \sin[e + f x]}} +$$

$$\frac{2 \sqrt{2} \sqrt{-a + b} \sqrt{a + b} g^{5/2} \operatorname{EllipticPi}\left[\frac{\sqrt{-a + b}}{\sqrt{a + b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}}\right], -1\right] \sqrt{\sin[e + f x]}}{a b d f \sqrt{d \sin[e + f x]}} -$$

$$\frac{2 g^2 \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{d \sin[e + f x]}}{a d^2 f \sqrt{\sin[2 e + 2 f x]}}$$

Result (type 6, 2873 leaves):

$$-\frac{2 (g \cos[e + f x])^{5/2} \tan[e + f x]}{a f (d \sin[e + f x])^{3/2}} +$$

$$\frac{1}{a f \cos[e + f x]^{5/2} (d \sin[e + f x])^{3/2}} (g \cos[e + f x])^{5/2} \sin[e + f x]^{3/2} \left(\frac{1}{(1 - \cos[e + f x]^2) (a^2 + b^2 (-1 + \cos[e + f x]^2)) (a + b \sin[e + f x])} \right)$$

$$\begin{aligned}
& 14 a (a^2 - b^2) \cos[e + f x]^{3/2} \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \left(\left(b \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{1 - \cos[e + f x]^2} \right) / \right. \\
& \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) + \\
& \left(a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) / \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \\
& \left(-4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \left. \right) \sin[e + f x]^{3/2} - \\
& \left(4 b \sqrt{\tan[e + f x]} \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + \right. \right. \right. \\
& \left. \left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] - \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] + \right. \right. \\
& \left. \left. \left. \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) \right) + \right. \\
& \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \\
& \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) \right) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(2 b \cos[2(e + f x)] \sqrt{\tan[e + f x]} \right. \\
& \left. \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{a \operatorname{ArcTan}\left[\frac{-\sqrt{2}+2\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{2}+2\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} + \frac{(2a^2-b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2}\sqrt{a}+2(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} \right) + \\
& \frac{(2a^2-b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{a}+2(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \frac{a \operatorname{Log}\left[1-\sqrt{2}\sqrt{\operatorname{Tan}[e+fx]}+\operatorname{Tan}[e+fx]\right]}{2\sqrt{2}b^2} - \\
& \frac{a \operatorname{Log}\left[1+\sqrt{2}\sqrt{\operatorname{Tan}[e+fx]}+\operatorname{Tan}[e+fx]\right]}{2\sqrt{2}b^2} - \frac{(2a^2-b^2) \operatorname{Log}\left[-a+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}-\sqrt{a^2-b^2}\operatorname{Tan}[e+fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \\
& \frac{(2a^2-b^2) \operatorname{Log}\left[a+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}+\sqrt{a^2-b^2}\operatorname{Tan}[e+fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \frac{\operatorname{Tan}[e+fx]^{3/2}}{b\sqrt{1+\operatorname{Tan}[e+fx]^2}} + \\
& \left(7a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \operatorname{Tan}[e+fx]^{3/2} \right) / \\
& \left(b\sqrt{1+\operatorname{Tan}[e+fx]^2} \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \right. \right. \\
& \quad \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] \operatorname{Tan}[e+fx]^2 \right) (-b^2 \operatorname{Tan}[e+fx]^2 + a^2(1+\operatorname{Tan}[e+fx]^2)) \right) \right) - \\
& \left(7a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \operatorname{Tan}[e+fx]^{3/2} \right) / \left(3\sqrt{1+\operatorname{Tan}[e+fx]^2} \right. \\
& \quad \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \right) \\
& \quad \left. (-b^2 \operatorname{Tan}[e+fx]^2 + a^2(1+\operatorname{Tan}[e+fx]^2)) \right) + \left(11a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \right. \\
& \quad \left. \operatorname{Tan}[e+fx]^{7/2} \right) / \left(7b\sqrt{1+\operatorname{Tan}[e+fx]^2} \left(-11a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \right. \right. \\
& \quad \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2 \Big) \tan[e+fx]^2 \left(-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2)\right) \Big) - \\
& \left(11 a^2 b \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e+fx]^2, \frac{(-a^2+b^2)\tan[e+fx]^2}{a^2}\right] \tan[e+fx]^{7/2} \right) / \\
& \left(7 \sqrt{1 + \tan[e+fx]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2\right] + 2 \right. \right. \\
& \left. \left((a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e+fx]^2\right] \right) \tan[e+fx]^2 \left(-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2)\right) \right) \right) / \\
& \left(\cos[e+fx]^{3/2} \sqrt{\sin[e+fx]} (a + b \sin[e+fx]) (-1 + \tan[e+fx]^2) \sqrt{1 + \tan[e+fx]^2} \right)
\end{aligned}$$

■ **Problem 1426: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e+fx])^{5/2}}{(d \sin[e+fx])^{5/2} (a + b \sin[e+fx])} dx$$

Optimal (type 4, 359 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 g (g \cos[e+fx])^{3/2}}{3 a d f (d \sin[e+fx])^{3/2}} + \frac{2 b g (g \cos[e+fx])^{3/2}}{a^2 d^2 f \sqrt{d \sin[e+fx]}} + \\
& \frac{2 \sqrt{2} \sqrt{-a+b} \sqrt{a+b} g^{5/2} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{a^2 d^2 f \sqrt{d \sin[e+fx]}} - \\
& \frac{2 \sqrt{2} \sqrt{-a+b} \sqrt{a+b} g^{5/2} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{a^2 d^2 f \sqrt{d \sin[e+fx]}} + \\
& \frac{2 b g^2 \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{d \sin[e+fx]}}{a^2 d^3 f \sqrt{\sin[2e + 2fx]}}
\end{aligned}$$

Result (type 6, 2923 leaves):

$$\begin{aligned}
& \frac{(g \cos[e + f x])^{5/2} \left(\frac{2b \cot[e + f x]}{a^2} - \frac{2 \cot[e + f x] \csc[e + f x]}{3a} \right) \sin[e + f x] \tan[e + f x]^2}{f (d \sin[e + f x])^{5/2}} \\
& \frac{1}{a^2 f \cos[e + f x]^{5/2} (d \sin[e + f x])^{5/2}} (g \cos[e + f x])^{5/2} \sin[e + f x]^{5/2} \\
& \left(\left(28 a b (a^2 - b^2) \cos[e + f x]^{3/2} \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \left(\left(b \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{1 - \cos[e + f x]^2} \right) \right) \right) \right) / \\
& \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \\
& \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right) + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 + \\
& \left(a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) / \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \\
& \left. \left(-4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \sin[e + f x]^{3/2} / \\
& (3 (1 - \cos[e + f x]^2) (a^2 + b^2 (-1 + \cos[e + f x]^2)) (a + b \sin[e + f x])) + \left(2 (a^2 - 2 b^2) \sqrt{\tan[e + f x]} \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \right) \\
& \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] \right) - \right. \\
& \left. \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] + \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \right. \right. \\
& \left. \left. \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) + \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right) \tan[e + f x]^2 \right) \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(2 b^2 \cos[2(e + f x)] \sqrt{\tan[e + f x]} \right. \\
& \left. (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right) \\
& \left(-\frac{a \operatorname{ArcTan}\left[\frac{-\sqrt{2} + 2\sqrt{\tan[e + f x]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{2} + 2\sqrt{\tan[e + f x]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2} \sqrt{a} + 2(a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}}\right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right) + \\
& \frac{(2 a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a} + 2(a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}}\right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x]\right]}{2 \sqrt{2} b^2} - \\
& \frac{a \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x]\right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \\
& \frac{(2 a^2 - b^2) \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\tan[e + f x]^{3/2}}{b \sqrt{1 + \tan[e + f x]^2}} + \\
& \left(7 a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2}\right] \tan[e + f x]^{3/2} \right) / \\
& \left(b \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + 2 \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \right) - \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2}\right] \tan[e + f x]^{3/2} \right) / \left(3 \sqrt{1 + \tan[e + f x]^2} \right. \\
& \quad \left. \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \right) \tan[e + f x]^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) + \left(11 a^4 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right. \\
& \left. \tan[e + f x]^{7/2} \right) / \left(7 b \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) - \\
& \left(11 a^2 b \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{7/2} \right) / \\
& \left(7 \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right)
\end{aligned}$$

- **Problem 1427: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{5/2}}{(d \sin[e + f x])^{7/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 519 leaves, 15 steps):

$$\begin{aligned}
& - \frac{2 g (g \cos [e+f x])^{3 / 2}}{5 a d f (d \sin [e+f x])^{5 / 2}} + \frac{2 b g (g \cos [e+f x])^{3 / 2}}{3 a^2 d^2 f (d \sin [e+f x])^{3 / 2}} - \frac{4 g (g \cos [e+f x])^{3 / 2}}{5 a d^3 f \sqrt{d \sin [e+f x]}} + \frac{2 (a^2-b^2) g (g \cos [e+f x])^{3 / 2}}{a^3 d^3 f \sqrt{d \sin [e+f x]}} - \\
& \frac{2 \sqrt{2} b \sqrt{-a+b} \sqrt{a+b} g^{5 / 2} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos [e+f x]}}{\sqrt{g} \sqrt{1+\sin [e+f x]}}\right], -1\right] \sqrt{\sin [e+f x]}}{a^3 d^3 f \sqrt{d \sin [e+f x]}} + \\
& \frac{2 \sqrt{2} b \sqrt{-a+b} \sqrt{a+b} g^{5 / 2} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos [e+f x]}}{\sqrt{g} \sqrt{1+\sin [e+f x]}}\right], -1\right] \sqrt{\sin [e+f x]}}{a^3 d^3 f \sqrt{d \sin [e+f x]}} - \\
& \frac{4 g^2 \sqrt{g \cos [e+f x]} \operatorname{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{d \sin [e+f x]}}{5 a d^4 f \sqrt{\sin [2 e+2 f x]}} + \frac{2 (a^2-b^2) g^2 \sqrt{g \cos [e+f x]} \operatorname{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{d \sin [e+f x]}}{a^3 d^4 f \sqrt{\sin [2 e+2 f x]}}
\end{aligned}$$

Result (type 6, 2995 leaves):

$$\begin{aligned}
& \frac{1}{f (d \sin [e+f x])^{7 / 2}} \\
& (g \cos [e+f x])^{5 / 2} \left(\frac{2 (3 a^2 \cos [e+f x] - 5 b^2 \cos [e+f x]) \operatorname{Csc}[e+f x]}{5 a^3} + \frac{2 b \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]}{3 a^2} - \frac{2 \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2}{5 a} \right) \\
& \sin [e+f x]^2 \tan [e+f x]^2 + \frac{1}{5 a^3 f \cos [e+f x]^{5 / 2} (d \sin [e+f x])^{7 / 2}} \\
& (g \cos [e+f x])^{5 / 2} \sin [e+f x]^{7 / 2} \left(- \left(\left(14 (a^2-b^2) (6 a^3 - 10 a b^2) \cos [e+f x]^{3 / 2} (a+b \sqrt{1-\cos [e+f x]^2}) \right. \right. \right. \\
& \left. \left. \left(\left(b \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \sqrt{1-\cos [e+f x]^2} \right) / \left(-7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. 1, \frac{11}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \right) \cos [e+f x]^2 + \left(a \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \right) / \right. \\
& \left. \left(7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] + \left(-4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos [e+f x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \right) \cos [e+f x]^2 \right) \left. \right) \sin [e+f x]^{3 / 2} /
\end{aligned}$$

$$\begin{aligned}
& \left(3 (1 - \cos[e + f x])^2 (a^2 + b^2 (-1 + \cos[e + f x])) (a + b \sin[e + f x]) \right) + \left(2 (8 a^2 b - 10 b^3) \sqrt{\tan[e + f x]} \right. \\
& \left. (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right) \\
& \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left[-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] \right] - \right. \\
& \left. \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] + \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \right. \right. \\
& \left. \left. \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) + \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(2 (-3 a^2 b + 5 b^3) \cos[2 (e + f x)] \right. \\
& \left. \sqrt{\tan[e + f x]} (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right) \\
& \left(-\frac{a \operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right) + \\
& \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} - \\
& \frac{a \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{(2a^2 - b^2) \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + fx]} + \sqrt{a^2 - b^2} \tan[e + fx]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\tan[e + fx]^{3/2}}{b \sqrt{1 + \tan[e + fx]^2}} + \\
& \left(7 a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx]^{3/2}\right) / \\
& \left(b \sqrt{1 + \tan[e + fx]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \tan[e + fx]^2\right) (-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2))\right) - \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx]^{3/2}\right) / \left(3 \sqrt{1 + \tan[e + fx]^2}\right. \\
& \quad \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + fx]^2,\right.\right.\right. \\
& \quad \left.\left.\left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right]\right) \tan[e + fx]^2 \\
& \quad \left.(-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2))\right) + \left(11 a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right]\right. \\
& \quad \left.\tan[e + fx]^{7/2}\right) / \left(7 b \sqrt{1 + \tan[e + fx]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \tan[e + fx]^2\right) (-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2))\right) - \\
& \left(11 a^2 b \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx]^{7/2}\right) / \\
& \left(7 \sqrt{1 + \tan[e + fx]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right.
\end{aligned}$$

$$\left. \left. \left. -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + fx]^2 \right) \tan[e + fx]^2 \left(-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2) \right) \right) \right) \left. \right) /$$

$$\left(\cos[e + fx]^{3/2} \sqrt{\sin[e + fx]} (a + b \sin[e + fx]) (-1 + \tan[e + fx]^2) \sqrt{1 + \tan[e + fx]^2} \right)$$

■ **Problem 1428: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + fx])^{5/2}}{(d \sin[e + fx])^{9/2} (a + b \sin[e + fx])} dx$$

Optimal (type 4, 612 leaves, 18 steps):

$$\begin{aligned} & -\frac{2g(g \cos[e + fx])^{3/2}}{7adf(d \sin[e + fx])^{7/2}} + \frac{2bg(g \cos[e + fx])^{3/2}}{5a^2d^2f(d \sin[e + fx])^{5/2}} - \frac{8g(g \cos[e + fx])^{3/2}}{21ad^3f(d \sin[e + fx])^{3/2}} + \\ & \frac{2(a^2 - b^2)g(g \cos[e + fx])^{3/2}}{3a^3d^3f(d \sin[e + fx])^{3/2}} + \frac{4bg(g \cos[e + fx])^{3/2}}{5a^2d^4f\sqrt{d \sin[e + fx]}} - \frac{2b(a^2 - b^2)g(g \cos[e + fx])^{3/2}}{a^4d^4f\sqrt{d \sin[e + fx]}} + \\ & \frac{2\sqrt{2}b^2\sqrt{-a+b}\sqrt{a+b}g^{5/2} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e + fx]}}{\sqrt{g \sqrt{1+\sin[e + fx]}}}\right], -1\right] \sqrt{\sin[e + fx]}}{a^4d^4f\sqrt{d \sin[e + fx]}} - \\ & \frac{2\sqrt{2}b^2\sqrt{-a+b}\sqrt{a+b}g^{5/2} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e + fx]}}{\sqrt{g \sqrt{1+\sin[e + fx]}}}\right], -1\right] \sqrt{\sin[e + fx]}}{a^4d^4f\sqrt{d \sin[e + fx]}} + \\ & \frac{4bg^2\sqrt{g \cos[e + fx]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{d \sin[e + fx]}}{5a^2d^5f\sqrt{\sin[2e + 2fx]}} - \frac{2b(a^2 - b^2)g^2\sqrt{g \cos[e + fx]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{d \sin[e + fx]}}{a^4d^5f\sqrt{\sin[2e + 2fx]}} \end{aligned}$$

Result (type 6, 3037 leaves):

$$\frac{1}{f(d \sin[e + fx])^{9/2}} (g \cos[e + fx])^{5/2} \left(-\frac{2(3a^2b \cos[e + fx] - 5b^3 \cos[e + fx]) \operatorname{Csc}[e + fx]}{5a^4} + \frac{2(3a^2 \cos[e + fx] - 7b^2 \cos[e + fx]) \operatorname{Csc}[e + fx]^2}{21a^3} + \frac{2b \operatorname{Cot}[e + fx] \operatorname{Csc}[e + fx]^2}{5a^2} - \frac{2 \operatorname{Cot}[e + fx] \operatorname{Csc}[e + fx]^3}{7a} \right) \sin[e + fx]^3 \tan[e + fx]^2 -$$

$$\begin{aligned}
& \frac{1}{5 a^4 f \operatorname{Cos}[e+f x]^{5/2} (d \operatorname{Sin}[e+f x])^{9/2}} b (g \operatorname{Cos}[e+f x])^{5/2} \operatorname{Sin}[e+f x]^{9/2} \\
& \left(- \left(\left(14 (a^2 - b^2) (6 a^3 - 10 a b^2) \operatorname{Cos}[e+f x]^{3/2} \left(a + b \sqrt{1 - \operatorname{Cos}[e+f x]^2} \right) \left(\left(b \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2 + b^2} \right] \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{1 - \operatorname{Cos}[e+f x]^2} \right) \right) / \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2 + b^2} \right] + \left(4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2 + b^2} \right] \right) \operatorname{Cos}[e+f x]^2 \right) + \\
& \left(a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2 + b^2} \right] \right) / \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \operatorname{Cos}[e+f x]^2, \right. \right. \\
& \left. \left. \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2 + b^2} \right] + \left(-4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2 + b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \operatorname{Cos}[e+f x]^2, \frac{b^2 \operatorname{Cos}[e+f x]^2}{-a^2 + b^2} \right] \right) \operatorname{Cos}[e+f x]^2 \right) \operatorname{Sin}[e+f x]^{3/2} \right) / \\
& \left(3 (1 - \operatorname{Cos}[e+f x]^2) (a^2 + b^2 (-1 + \operatorname{Cos}[e+f x]^2)) (a + b \operatorname{Sin}[e+f x]) \right) + \left(2 (8 a^2 b - 10 b^3) \sqrt{\operatorname{Tan}[e+f x]} \right. \\
& \left. (b \operatorname{Tan}[e+f x] + a \sqrt{1 + \operatorname{Tan}[e+f x]^2}) \right) \\
& \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]}}{\sqrt{a}} \right] - \right. \right. \\
& \left. \left. \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]} - \sqrt{a^2 - b^2} \operatorname{Tan}[e+f x] \right] + \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[e+f x]} + \right. \right. \right. \\
& \left. \left. \sqrt{a^2 - b^2} \operatorname{Tan}[e+f x] \right] \right) + \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \operatorname{Tan}[e+f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + 2 \right. \right. \\
& \left. \left((a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^2 \right) (-b^2 \operatorname{Tan}[e+f x]^2 + a^2 (1 + \operatorname{Tan}[e+f x]^2)) \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(2 (-3 a^2 b + 5 b^3) \cos[2 (e + f x)] \right. \\
& \left. \sqrt{\tan[e + f x]} \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \right. \\
& \left(-\frac{a \operatorname{ArcTan}\left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}}\right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right. \\
& \left. + \frac{(2 a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}}\right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x]\right]}{2 \sqrt{2} b^2} - \right. \\
& \left. \frac{a \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x]\right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \right. \\
& \left. \frac{(2 a^2 - b^2) \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\tan[e + f x]^{3/2}}{b \sqrt{1 + \tan[e + f x]^2}} + \right. \\
& \left(7 a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2}\right] \tan[e + f x]^{3/2} \right) / \\
& \left(b \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + 2 \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \right) - \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2}\right] \tan[e + f x]^{3/2} \right) / \left(3 \sqrt{1 + \tan[e + f x]^2} \right. \\
& \left. \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \right) \tan[e + f x]^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) + \left(11 a^4 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right. \\
& \left. \tan[e + f x]^{7/2} \right) / \left(7 b \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) - \\
& \left(11 a^2 b \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{7/2} \right) / \\
& \left(7 \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right)
\end{aligned}$$

- **Problem 1429: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{5/2}}{(d \sin[e + f x])^{11/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 822 leaves, 24 steps):

$$\begin{aligned}
& - \frac{2 g (g \cos [e+f x])^{3 / 2}}{9 a d f (d \sin [e+f x])^{9 / 2}} + \frac{2 b g (g \cos [e+f x])^{3 / 2}}{7 a^2 d^2 f (d \sin [e+f x])^{7 / 2}} - \frac{4 g (g \cos [e+f x])^{3 / 2}}{15 a d^3 f (d \sin [e+f x])^{5 / 2}} + \frac{2 (a^2-b^2) g (g \cos [e+f x])^{3 / 2}}{5 a^3 d^3 f (d \sin [e+f x])^{5 / 2}} + \\
& \frac{8 b g (g \cos [e+f x])^{3 / 2}}{21 a^2 d^4 f (d \sin [e+f x])^{3 / 2}} - \frac{2 b (a^2-b^2) g (g \cos [e+f x])^{3 / 2}}{3 a^4 d^4 f (d \sin [e+f x])^{3 / 2}} - \frac{8 g (g \cos [e+f x])^{3 / 2}}{15 a d^5 f \sqrt{d \sin [e+f x]}} + \frac{4 (a^2-b^2) g (g \cos [e+f x])^{3 / 2}}{5 a^3 d^5 f \sqrt{d \sin [e+f x]}} + \\
& \frac{2 b^2 (a^2-b^2) g (g \cos [e+f x])^{3 / 2}}{a^5 d^5 f \sqrt{d \sin [e+f x]}} - \frac{2 \sqrt{2} b^3 \sqrt{-a+b} \sqrt{a+b} g^{5 / 2} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos [e+f x]}}{\sqrt{g} \sqrt{1+\sin [e+f x]}}\right], -1\right] \sqrt{\sin [e+f x]}}{a^5 d^5 f \sqrt{d \sin [e+f x]}} + \\
& \frac{2 \sqrt{2} b^3 \sqrt{-a+b} \sqrt{a+b} g^{5 / 2} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos [e+f x]}}{\sqrt{g} \sqrt{1+\sin [e+f x]}}\right], -1\right] \sqrt{\sin [e+f x]}}{a^5 d^5 f \sqrt{d \sin [e+f x]}} - \\
& \frac{8 g^2 \sqrt{g \cos [e+f x]} \operatorname{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{d \sin [e+f x]}}{15 a d^6 f \sqrt{\sin [2 e+2 f x]}} + \frac{4 (a^2-b^2) g^2 \sqrt{g \cos [e+f x]} \operatorname{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{d \sin [e+f x]}}{5 a^3 d^6 f \sqrt{\sin [2 e+2 f x]}} + \\
& \frac{2 b^2 (a^2-b^2) g^2 \sqrt{g \cos [e+f x]} \operatorname{EllipticE}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{d \sin [e+f x]}}{a^5 d^6 f \sqrt{\sin [2 e+2 f x]}}
\end{aligned}$$

Result (type 6, 3111 leaves):

$$\begin{aligned}
& \frac{1}{f (d \sin [e+f x])^{11 / 2}} (g \cos [e+f x])^{5 / 2} \\
& \left(\frac{2 (2 a^4 \cos [e+f x] + 9 a^2 b^2 \cos [e+f x] - 15 b^4 \cos [e+f x]) \operatorname{Csc}[e+f x]}{15 a^5} - \frac{2 (3 a^2 b \cos [e+f x] - 7 b^3 \cos [e+f x]) \operatorname{Csc}[e+f x]^2}{21 a^4} + \right. \\
& \left. \frac{2 (a^2 \cos [e+f x] - 3 b^2 \cos [e+f x]) \operatorname{Csc}[e+f x]^3}{15 a^3} + \frac{2 b \cot [e+f x] \operatorname{Csc}[e+f x]^3}{7 a^2} - \frac{2 \cot [e+f x] \operatorname{Csc}[e+f x]^4}{9 a} \right) \\
& \sin [e+f x]^4 \tan [e+f x]^2 + \frac{1}{15 a^5 f \cos [e+f x]^{5 / 2} (d \sin [e+f x])^{11 / 2}} \\
& (g \cos [e+f x])^{5 / 2} \sin [e+f x]^{11 / 2} \left[- \left(\left(14 (a^2-b^2) (4 a^5 + 18 a^3 b^2 - 30 a b^4) \cos [e+f x]^{3 / 2} (a+b \sqrt{1-\cos [e+f x]^2}) \right) \right. \right. \\
& \left. \left(\left(b \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] \sqrt{1-\cos [e+f x]^2} \right) / \left(-7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos [e+f x]^2, \frac{b^2 \cos [e+f x]^2}{-a^2+b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{4}, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \cos[e + f x]^2 \Bigg) + \left(a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) / \\
& \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(-4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \sin[e + f x]^{3/2} \Bigg) / \\
& \left(3 (1 - \cos[e + f x]^2) (a^2 + b^2 (-1 + \cos[e + f x]^2)) (a + b \sin[e + f x]) \right) \Bigg) + \left(2 (2 a^4 b + 24 a^2 b^3 - 30 b^5) \sqrt{\tan[e + f x]} \right. \\
& \left. (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right. \\
& \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] - \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] + \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) \right) + \\
& \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) / \left(3 \sqrt{1 + \tan[e + f x]^2} \right. \\
& \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \\
& \left. \left. \left. (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \right) \right) \Bigg) / \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \\
& \left(2 (-2 a^4 b - 9 a^2 b^3 + 15 b^5) \cos[2 (e + f x)] \sqrt{\tan[e + f x]} (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right. \\
& \left(-\frac{a \operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right. \\
& \left. \left. \left. \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{a \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x]\right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \\
& \frac{(2 a^2 - b^2) \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\tan[e + f x]^{3/2}}{b \sqrt{1 + \tan[e + f x]^2}} + \\
& \left(7 a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2}\right] \tan[e + f x]^{3/2}\right) / \\
& \left(b \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \tan[e + f x]^2\right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2))\right) - \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2}\right] \tan[e + f x]^{3/2}\right) / \left(3 \sqrt{1 + \tan[e + f x]^2}\right. \\
& \quad \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + 2\left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2,\right.\right.\right. \\
& \quad \left.\left.\left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right]\right) \tan[e + f x]^2 \\
& \quad \left.(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2))\right) + \left(11 a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2}\right]\right. \\
& \quad \left.\tan[e + f x]^{7/2}\right) / \left(7 b \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] \tan[e + f x]^2\right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2))\right) - \\
& \left(11 a^2 b \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2}\right] \tan[e + f x]^{7/2}\right) / \\
& \left(7 \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + f x]^2\right] + 2\right.\right.
\end{aligned}$$

$$\left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \Bigg) \Bigg) /$$

$$\left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right)$$

■ **Problem 1430: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(d \sin[e + f x])^{5/2}}{\sqrt{g \cos[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 616 leaves, 19 steps):

$$\frac{a d^{5/2} \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e + f x]}}{\sqrt{d} \sqrt{g \cos[e + f x]}} \right]}{\sqrt{2} b^2 f \sqrt{g}} - \frac{a d^{5/2} \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e + f x]}}{\sqrt{d} \sqrt{g \cos[e + f x]}} \right]}{\sqrt{2} b^2 f \sqrt{g}} -$$

$$\frac{2 \sqrt{2} a^2 d^{5/2} \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[-\frac{a}{b \sqrt{-a^2 + b^2}}, \operatorname{ArcSin} \left[\frac{\sqrt{d \sin[e + f x]}}{\sqrt{d} \sqrt{1 + \cos[e + f x]}} \right], -1 \right]}{b^2 \sqrt{-a^2 + b^2} f \sqrt{g \cos[e + f x]}} +$$

$$\frac{2 \sqrt{2} a^2 d^{5/2} \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[-\frac{a}{b \sqrt{-a^2 + b^2}}, \operatorname{ArcSin} \left[\frac{\sqrt{d \sin[e + f x]}}{\sqrt{d} \sqrt{1 + \cos[e + f x]}} \right], -1 \right]}{b^2 \sqrt{-a^2 + b^2} f \sqrt{g \cos[e + f x]}} - \frac{a d^{5/2} \operatorname{Log} \left[\sqrt{d} - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e + f x]}}{\sqrt{g \cos[e + f x]}} + \sqrt{d} \tan[e + f x] \right]}{2 \sqrt{2} b^2 f \sqrt{g}} +$$

$$\frac{a d^{5/2} \operatorname{Log} \left[\sqrt{d} + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e + f x]}}{\sqrt{g \cos[e + f x]}} + \sqrt{d} \tan[e + f x] \right]}{2 \sqrt{2} b^2 f \sqrt{g}} - \frac{d^2 \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}}{b f g} + \frac{d^3 \operatorname{EllipticF} \left[e - \frac{\pi}{4} + f x, 2 \right] \sqrt{\sin[2 e + 2 f x]}}{2 b f \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}}$$

Result (type 6, 1790 leaves):

$$\frac{1}{2 f \sqrt{g \cos[e + f x]} \sin[e + f x]^{5/2}}$$

$$\begin{aligned}
& \sqrt{\cos[e+fx]} (d \sin[e+fx])^{5/2} \left(- \left(\left(2 \cos[2(e+fx)] \sqrt{\sin[e+fx]} \left(b \tan[e+fx] + a \sqrt{1 + \tan[e+fx]^2} \right) \right) \left(- \frac{a \operatorname{ArcTan}\left[\frac{-\sqrt{2} + 2\sqrt{\tan[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} \right) \right. \right. \\
& \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{2} + 2\sqrt{\tan[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} + \frac{\sqrt{a} (2a^2 - b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2}\sqrt{a} + 2(a^2 - b^2)^{1/4}\sqrt{\tan[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2} b^2 (a^2 - b^2)^{3/4}} + \\
& \frac{\sqrt{a} (2a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{a} + 2(a^2 - b^2)^{1/4}\sqrt{\tan[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2} b^2 (a^2 - b^2)^{3/4}} - \frac{a \log\left[1 - \sqrt{2}\sqrt{\tan[e+fx]} + \tan[e+fx]\right]}{2\sqrt{2} b^2} + \\
& \frac{a \log\left[1 + \sqrt{2}\sqrt{\tan[e+fx]} + \tan[e+fx]\right]}{2\sqrt{2} b^2} + \frac{\sqrt{a} (2a^2 - b^2) \log\left[-a + \sqrt{2}\sqrt{a} (a^2 - b^2)^{1/4}\sqrt{\tan[e+fx]} - \sqrt{a^2 - b^2} \tan[e+fx]\right]}{4\sqrt{2} b^2 (a^2 - b^2)^{3/4}} - \\
& \frac{\sqrt{a} (2a^2 - b^2) \log\left[a + \sqrt{2}\sqrt{a} (a^2 - b^2)^{1/4}\sqrt{\tan[e+fx]} + \sqrt{a^2 - b^2} \tan[e+fx]\right]}{4\sqrt{2} b^2 (a^2 - b^2)^{3/4}} - \frac{\sqrt{\tan[e+fx]}}{b\sqrt{1 + \tan[e+fx]^2}} - \\
& \left(5a^4 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\tan[e+fx]^2, \frac{(-a^2 + b^2)\tan[e+fx]^2}{a^2}\right] \sqrt{\tan[e+fx]} \right) / \\
& \left(b\sqrt{1 + \tan[e+fx]^2} \left(-5a^2 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\tan[e+fx]^2\right] + \right. \right. \\
& \left. \left. 2 \left(2(a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\tan[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\tan[e+fx]^2\right) \tan[e+fx]^2 \right) (-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2)) \right) \right) + \\
& \left(9a^4 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, \frac{(-a^2 + b^2)\tan[e+fx]^2}{a^2}\right] \tan[e+fx]^{5/2} \right) / \\
& \left(5b\sqrt{1 + \tan[e+fx]^2} \left(-9a^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\tan[e+fx]^2\right] + \right. \right. \\
& \left. \left. 2 \left(2(a^2 - b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\tan[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan[e+fx]^2, \left(-1 + \frac{b^2}{a^2}\right)\tan[e+fx]^2\right) \tan[e+fx]^2 \right) (-b^2 \tan[e+fx]^2 + a^2 (1 + \tan[e+fx]^2)) \right) \right) \right) /
\end{aligned}$$

$$\left(\left(\cos[e + f x]^{5/2} (a + b \sin[e + f x]) \sqrt{\tan[e + f x]} (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right) + \left(2 \sqrt{\sin[e + f x]} \right. \right.$$

$$\left. \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \right.$$

$$\left. \left(\frac{1}{4 \sqrt{2} (a^2 - b^2)^{3/4}} \sqrt{a} \left[-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + \right. \right.$$

$$\left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] - \right. \right.$$

$$\left. \left. \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) \right.$$

$$\left. \left(9 a^2 b \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{5/2} \right) / \left(5 \sqrt{1 + \tan[e + f x]^2} \right. \right.$$

$$\left. \left(-9 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\tan[e + f x]^2, \right. \right. \right.$$

$$\left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right)$$

$$\left. \left. \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) / \left(\cos[e + f x]^{5/2} (a + b \sin[e + f x]) \sqrt{\tan[e + f x]} (1 + \tan[e + f x]^2)^{3/2} \right)$$

■ **Problem 1431: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \sin[e + f x])^{3/2}}{\sqrt{g \cos[e + f x]} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 508 leaves, 15 steps):

$$\begin{aligned}
& - \frac{d^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{\sqrt{2} b f \sqrt{g}} + \frac{d^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{d} \sqrt{g \cos[e+fx]}}\right]}{\sqrt{2} b f \sqrt{g}} + \\
& \frac{2 \sqrt{2} a d^{3/2} \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b-\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e+fx]}{\sqrt{d} \sqrt{1+\cos[e+fx]}}\right], -1\right]}{b \sqrt{-a^2+b^2} f \sqrt{g \cos[e+fx]}} - \\
& \frac{2 \sqrt{2} a d^{3/2} \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b+\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e+fx]}{\sqrt{d} \sqrt{1+\cos[e+fx]}}\right], -1\right]}{b \sqrt{-a^2+b^2} f \sqrt{g \cos[e+fx]}} + \\
& \frac{d^{3/2} \operatorname{Log}\left[\sqrt{d} - \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \operatorname{Tan}[e+fx]\right]}{2 \sqrt{2} b f \sqrt{g}} - \frac{d^{3/2} \operatorname{Log}\left[\sqrt{d} + \frac{\sqrt{2} \sqrt{g} \sqrt{d \sin[e+fx]}}{\sqrt{g \cos[e+fx]}} + \sqrt{d} \operatorname{Tan}[e+fx]\right]}{2 \sqrt{2} b f \sqrt{g}}
\end{aligned}$$

Result(type 6, 518 leaves):

$$\begin{aligned}
& \frac{1}{f \sqrt{g \cos[e+fx]} (-a+b \sin[e+fx]) (a+b \sin[e+fx])^2} 10 (a^2-b^2) \cot[e+fx] (d \sin[e+fx])^{3/2} \left(a+b \sqrt{\sin[e+fx]^2}\right) \\
& \left(\left(a \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) / \left(5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + \right. \right. \\
& \left. \left(-4 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{4}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \right. \\
& \left. \cos[e+fx]^2 \right) + \left(b \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{3}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \sqrt{\sin[e+fx]^2} \right) / \\
& \left(-5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{3}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{3}{4}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] + \right. \right. \\
& \left. \left. 3 (a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \right)
\end{aligned}$$

■ **Problem 1433: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{g \cos[e+fx]} \sqrt{d \sin[e+fx]} (a+b \sin[e+fx])} dx$$

Optimal (type 4, 273 leaves, 7 steps):

$$\frac{2\sqrt{2} b \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d}\sin[e+fx]}{\sqrt{d}\sqrt{1+\cos[e+fx]}}\right], -1\right]}{a\sqrt{-a^2+b^2}\sqrt{d}f\sqrt{g\cos[e+fx]}} - \frac{2\sqrt{2} b \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d}\sin[e+fx]}{\sqrt{d}\sqrt{1+\cos[e+fx]}}\right], -1\right]}{a\sqrt{-a^2+b^2}\sqrt{d}f\sqrt{g\cos[e+fx]}} + \frac{\operatorname{EllipticF}\left[e-\frac{\pi}{4}+fx, 2\right] \sqrt{\sin[2e+2fx]}}{af\sqrt{g\cos[e+fx]}\sqrt{d}\sin[e+fx]}$$

Result(type 6, 664 leaves):

$$\begin{aligned} & - \left(\left(2\sqrt{\cos[e+fx]} \left(a + b\sqrt{1-\cos[e+fx]^2} \right) \right. \right. \\ & \left. \left(5a(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] \sqrt{\cos[e+fx]} \right) / \left((1-\cos[e+fx]^2)^{3/4} \right. \right. \\ & \left. \left(5(a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] + \left(-4b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] \right) + \right. \right. \\ & \left. \left. 3(a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2\cos[e+fx]^2}{-a^2+b^2}\right] \right) \cos[e+fx]^2 \left(a^2 + b^2(-1+\cos[e+fx]^2) \right) \right) - \\ & \left. \frac{1}{\sqrt{a}(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) b \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{a}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}(-1+\cos[e+fx]^2)^{1/4}} \right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{a}\sqrt{\cos[e+fx]}}{(-a^2+b^2)^{1/4}(-1+\cos[e+fx]^2)^{1/4}} \right] \right) + \right. \\ & \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + \frac{ia\cos[e+fx]}{\sqrt{-1+\cos[e+fx]^2}} - \frac{(1+i)\sqrt{a}(-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]}}{(-1+\cos[e+fx]^2)^{1/4}} \right] - \right. \\ & \left. \operatorname{Log}\left[\sqrt{-a^2+b^2} + \frac{ia\cos[e+fx]}{\sqrt{-1+\cos[e+fx]^2}} + \frac{(1+i)\sqrt{a}(-a^2+b^2)^{1/4}\sqrt{\cos[e+fx]}}{(-1+\cos[e+fx]^2)^{1/4}} \right] \right) \left. \left. \right) \right) \operatorname{Sin}[e+fx] \Big/ \\ & \left(f\sqrt{g\cos[e+fx]}(1-\cos[e+fx]^2)^{1/4}\sqrt{d}\sin[e+fx] + (a+b\sin[e+fx]) \right) \end{aligned}$$

■ **Problem 1434: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{g\cos[e+fx]}(d\sin[e+fx])^{3/2}(a+b\sin[e+fx])} dx$$

Optimal(type 4, 320 leaves, 9 steps):

$$\begin{aligned}
& \frac{2\sqrt{2} b^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b-\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d}\sin[e+fx]}{\sqrt{d}\sqrt{1+\cos[e+fx]}}\right], -1\right]}{a^2 \sqrt{-a^2+b^2} d^{3/2} f \sqrt{g \cos[e+fx]}} + \\
& \frac{2\sqrt{2} b^2 \sqrt{\cos[e+fx]} \operatorname{EllipticPi}\left[-\frac{a}{b+\sqrt{-a^2+b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d}\sin[e+fx]}{\sqrt{d}\sqrt{1+\cos[e+fx]}}\right], -1\right]}{a^2 \sqrt{-a^2+b^2} d^{3/2} f \sqrt{g \cos[e+fx]}} - \\
& \frac{2\sqrt{g \cos[e+fx]}}{a d f g \sqrt{d \sin[e+fx]}} - \frac{b \operatorname{EllipticF}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{\sin[2e+2fx]}}{a^2 d f \sqrt{g \cos[e+fx]} \sqrt{d \sin[e+fx]}}
\end{aligned}$$

Result (type 6, 715 leaves):

$$\begin{aligned}
& - \frac{2 \cos[e+fx] \sin[e+fx]}{a f \sqrt{g \cos[e+fx]} (d \sin[e+fx])^{3/2}} + \\
& \left(2 b \sqrt{\cos[e+fx]} \left(a + b \sqrt{1 - \cos[e+fx]^2} \right) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \sqrt{\cos[e+fx]} \right) / \right. \\
& \left((1 - \cos[e+fx]^2)^{3/4} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + \right. \right. \\
& \left. \left(-4 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + 3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right) \right. \\
& \left. \left. \cos[e+fx]^2 \right) (a^2 + b^2 (-1 + \cos[e+fx]^2)) \right) - \frac{1}{\sqrt{a} (-a^2 + b^2)^{3/4}} \\
& \left(\frac{1}{8} - \frac{i}{8} \right) b \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i)\sqrt{a}\sqrt{\cos[e+fx]}}{(-a^2 + b^2)^{1/4} (-1 + \cos[e+fx]^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i)\sqrt{a}\sqrt{\cos[e+fx]}}{(-a^2 + b^2)^{1/4} (-1 + \cos[e+fx]^2)^{1/4}}\right] \right) + \\
& \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \frac{i a \cos[e+fx]}{\sqrt{-1 + \cos[e+fx]^2}} - \frac{(1+i)\sqrt{a}(-a^2 + b^2)^{1/4}\sqrt{\cos[e+fx]}}{(-1 + \cos[e+fx]^2)^{1/4}}\right] - \\
& \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \frac{i a \cos[e+fx]}{\sqrt{-1 + \cos[e+fx]^2}} + \frac{(1+i)\sqrt{a}(-a^2 + b^2)^{1/4}\sqrt{\cos[e+fx]}}{(-1 + \cos[e+fx]^2)^{1/4}}\right] \left. \right) \sin[e+fx]^2 \Big/ \\
& \left(a f \sqrt{g \cos[e+fx]} (1 - \cos[e+fx]^2)^{1/4} (d \sin[e+fx])^{3/2} (a + b \sin[e+fx]) \right)
\end{aligned}$$

- **Problem 1435: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{g \cos[e + f x]} (d \sin[e + f x])^{5/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 424 leaves, 13 steps):

$$\frac{2 \sqrt{2} b^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[-\frac{a}{b - \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e + f x]}{\sqrt{d} \sqrt{1 + \cos[e + f x]}}\right], -1\right]}{a^3 \sqrt{-a^2 + b^2} d^{5/2} f \sqrt{g \cos[e + f x]}}$$

$$- \frac{2 \sqrt{2} b^3 \sqrt{\cos[e + f x]} \operatorname{EllipticPi}\left[-\frac{a}{b + \sqrt{-a^2 + b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{d} \sin[e + f x]}{\sqrt{d} \sqrt{1 + \cos[e + f x]}}\right], -1\right]}{a^3 \sqrt{-a^2 + b^2} d^{5/2} f \sqrt{g \cos[e + f x]}} - \frac{2 \sqrt{g \cos[e + f x]}}{3 a d f g (d \sin[e + f x])^{3/2}} +$$

$$\frac{2 b \sqrt{g \cos[e + f x]}}{a^2 d^2 f g \sqrt{d \sin[e + f x]}} + \frac{2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2 e + 2 f x]}}{3 a d^2 f \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}} + \frac{b^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2 e + 2 f x]}}{a^3 d^2 f \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}}$$

Result (type 6, 1332 leaves):

$$\frac{\cos[e + f x] \left(\frac{2 b \operatorname{Csc}[e + f x]}{a^2} - \frac{2 \operatorname{Csc}[e + f x]^2}{3 a} \right) \sin[e + f x]^3}{f \sqrt{g \cos[e + f x]} (d \sin[e + f x])^{5/2}} +$$

$$\frac{1}{3 a^2 f \sqrt{g \cos[e + f x]} (d \sin[e + f x])^{5/2}} \sqrt{\cos[e + f x]} \sin[e + f x]^{5/2} \left(-\frac{1}{(1 - \cos[e + f x])^{1/4} (a + b \sin[e + f x])} \right.$$

$$2 (2 a^2 + 3 b^2) \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \sqrt{\cos[e + f x]} \right) / \right.$$

$$\left. \left((1 - \cos[e + f x])^{3/4} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + \left(-4 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, \right. \right. \right. \right.$$

$$\left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right) + 3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right)$$

$$\left. (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) - \frac{1}{\sqrt{a} (-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) b \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4} (-1 + \cos[e + f x])^{1/4}}\right] - 2 \operatorname{ArcTan}\left[\right. \right.$$

$$\left. \left. 1 + \frac{(1 + i) \sqrt{a} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4} (-1 + \cos[e + f x])^{1/4}} \right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \frac{i a \cos[e + f x]}{\sqrt{-1 + \cos[e + f x]^2}} - \frac{(1 + i) \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]}}{(-1 + \cos[e + f x])^{1/4}} \right] -$$

$$\begin{aligned}
 & \left. \left. \left. \left. \left. \left. \text{Log} \left[\sqrt{-a^2 + b^2} + \frac{i a \text{Cos}[e + f x]}{\sqrt{-1 + \text{Cos}[e + f x]^2}} + \frac{(1 + i) \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\text{Cos}[e + f x]}}{(-1 + \text{Cos}[e + f x]^2)^{1/4}} \right] \right] \right] \right] \right] \right] \sqrt{\text{Sin}[e + f x]} + \\
 & \left(4 a b \sqrt{\text{Sin}[e + f x]} \left(b \text{Tan}[e + f x] + a \sqrt{1 + \text{Tan}[e + f x]^2} \right) \left(\frac{1}{4 \sqrt{2} (a^2 - b^2)^{3/4}} \sqrt{a} \left(-2 \text{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\text{Tan}[e + f x]}}{\sqrt{a}} \right] \right) + \right. \right. \\
 & \left. 2 \text{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\text{Tan}[e + f x]}}{\sqrt{a}} \right] \right) + \text{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\text{Tan}[e + f x]} - \sqrt{a^2 - b^2} \text{Tan}[e + f x] \right] - \\
 & \left. \left. \left. \left. \left. \left. \text{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\text{Tan}[e + f x]} + \sqrt{a^2 - b^2} \text{Tan}[e + f x] \right] \right] \right] \right] \right] \right] + \\
 & \left(9 a^2 b \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e + f x]^2 \right] \text{Tan}[e + f x]^{5/2} \right) / \left(5 \sqrt{1 + \text{Tan}[e + f x]^2} \right. \\
 & \left(-9 a^2 \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e + f x]^2 \right] + 2 \left(2 (a^2 - b^2) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\text{Tan}[e + f x]^2, \right. \right. \right. \\
 & \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e + f x]^2 \right] + a^2 \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \text{Tan}[e + f x]^2 \right] \right) \text{Tan}[e + f x]^2 \right) \\
 & \left. \left. \left. \left. \left. \left. (-b^2 \text{Tan}[e + f x]^2 + a^2 (1 + \text{Tan}[e + f x]^2)) \right) \right) \right) \right) \right) / \left(\text{Cos}[e + f x]^{5/2} (a + b \text{Sin}[e + f x]) \sqrt{\text{Tan}[e + f x]} (1 + \text{Tan}[e + f x]^2)^{3/2} \right)
 \end{aligned}$$

■ Problem 1436: Attempted integration timed out after 120 seconds.

$$\int \frac{(d \text{Sin}[e + f x])^{5/2}}{(g \text{Cos}[e + f x])^{3/2} (a + b \text{Sin}[e + f x])} dx$$

Optimal (type 4, 1064 leaves, 31 steps):

$$\begin{aligned}
& - \frac{a^2 d^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d \sin[e+fx]}}\right]}{\sqrt{2} b (a^2 - b^2) f g^{3/2}} + \frac{b d^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d \sin[e+fx]}}\right]}{\sqrt{2} (a^2 - b^2) f g^{3/2}} + \frac{a^2 d^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d \sin[e+fx]}}\right]}{\sqrt{2} b (a^2 - b^2) f g^{3/2}} - \\
& \frac{b d^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{d \sin[e+fx]}}\right]}{\sqrt{2} (a^2 - b^2) f g^{3/2}} + \frac{a^2 d^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d \sin[e+fx]}}\right]}{2 \sqrt{2} b (a^2 - b^2) f g^{3/2}} - \\
& \frac{b d^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] - \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d \sin[e+fx]}}\right]}{2 \sqrt{2} (a^2 - b^2) f g^{3/2}} - \frac{a^2 d^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d \sin[e+fx]}}\right]}{2 \sqrt{2} b (a^2 - b^2) f g^{3/2}} + \\
& \frac{b d^{5/2} \operatorname{Log}\left[\sqrt{g} + \sqrt{g} \cot[e+fx] + \frac{\sqrt{2} \sqrt{d} \sqrt{g \cos[e+fx]}}{\sqrt{d \sin[e+fx]}}\right]}{2 \sqrt{2} (a^2 - b^2) f g^{3/2}} - \frac{2 \sqrt{2} a^3 d^3 \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{b (-a+b)^{3/2} (a+b)^{3/2} f g^{3/2} \sqrt{d \sin[e+fx]}} + \\
& \frac{2 \sqrt{2} a^3 d^3 \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{b (-a+b)^{3/2} (a+b)^{3/2} f g^{3/2} \sqrt{d \sin[e+fx]}} - \frac{2 b d^2 \sqrt{d \sin[e+fx]}}{(a^2 - b^2) f g \sqrt{g \cos[e+fx]}} + \\
& \frac{2 a d (d \sin[e+fx])^{3/2}}{(a^2 - b^2) f g \sqrt{g \cos[e+fx]}} - \frac{2 a d^2 \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{d \sin[e+fx]}}{(a^2 - b^2) f g^2 \sqrt{\sin[2e+2fx]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

- **Problem 1437: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(d \sin[e+fx])^{3/2}}{(g \cos[e+fx])^{3/2} (a+b \sin[e+fx])} dx$$

Optimal (type 4, 379 leaves, 10 steps):

$$\frac{2\sqrt{2} a^2 d^2 \text{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \text{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{(-a+b)^{3/2} (a+b)^{3/2} f g^{3/2} \sqrt{d \sin[e+fx]}} -$$

$$\frac{2\sqrt{2} a^2 d^2 \text{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \text{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{(-a+b)^{3/2} (a+b)^{3/2} f g^{3/2} \sqrt{d \sin[e+fx]}} + \frac{2 a d \sqrt{d \sin[e+fx]}}{(a^2 - b^2) f g \sqrt{g \cos[e+fx]}} -$$

$$\frac{2 b (d \sin[e+fx])^{3/2}}{(a^2 - b^2) f g \sqrt{g \cos[e+fx]}} + \frac{2 b d \sqrt{g \cos[e+fx]} \text{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{d \sin[e+fx]}}{(a^2 - b^2) f g^2 \sqrt{\sin[2e+2fx]}}$$

Result (type 6, 2915 leaves):

$$\frac{2 \cot[e+fx] (d \sin[e+fx])^{3/2} (a - b \sin[e+fx])}{(a^2 - b^2) f (g \cos[e+fx])^{3/2}} -$$

$$\frac{1}{(a-b)(a+b) f (g \cos[e+fx])^{3/2} \sin[e+fx]^{3/2}} \cos[e+fx]^{3/2} (d \sin[e+fx])^{3/2} \left(\left(28 a b (a^2 - b^2) \cos[e+fx]^{3/2} \left(a + b \sqrt{1 - \cos[e+fx]^2} \right) \right. \right.$$

$$\left. \left(\left(b \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \sqrt{1 - \cos[e+fx]^2} \right) / \left(-7 (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right. \right.$$

$$\left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2} \right) + \left(4 b^2 \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right) \cos[e+fx]^2 \right) + \left(a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right) /$$

$$\left(7 (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] + \left(-4 b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right. \right.$$

$$\left. \left. \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2} \right) + (a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e+fx]^2, \frac{b^2 \cos[e+fx]^2}{-a^2 + b^2}\right] \right) \cos[e+fx]^2 \left. \right) \sin[e+fx]^{3/2} /$$

$$(3 (1 - \cos[e+fx]^2) (a^2 + b^2 (-1 + \cos[e+fx]^2)) (a + b \sin[e+fx])) + \left(2 (a^2 - b^2) \sqrt{\tan[e+fx]} \left(b \tan[e+fx] + a \sqrt{1 + \tan[e+fx]^2} \right) \right.$$

$$\left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]}}{\sqrt{a}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]}}{\sqrt{a}}\right] \right) -$$

$$\text{Log}\left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]} - \sqrt{a^2 - b^2} \tan[e+fx]\right] + \text{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e+fx]} +$$

$$\begin{aligned}
& \left. \sqrt{a^2 - b^2} \tan[e + f x] \right) + \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) \right) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(2 b^2 \cos[2(e + f x)] \sqrt{\tan[e + f x]} \right. \\
& \quad \left. \left(b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2} \right) \right. \\
& \quad \left(-\frac{a \operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \right. \\
& \quad \left. \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} - \right. \\
& \quad \left. \frac{a \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \right. \\
& \quad \left. \frac{(2 a^2 - b^2) \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \sqrt{a^2 - b^2} \tan[e + f x] \right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\tan[e + f x]^{3/2}}{b \sqrt{1 + \tan[e + f x]^2}} + \right. \\
& \quad \left. \left(7 a^4 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{3/2} \right) / \right. \\
& \quad \left. \left(b \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \Big) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \Big) - \\
& \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{3/2} \right) / \left(3 \sqrt{1 + \tan[e + f x]^2} \right. \\
& \left. \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \right. \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \\
& \left. \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) + \left(11 a^4 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \right. \\
& \left. \tan[e + f x]^{7/2} \right) / \left(7 b \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right) \right) \tan[e + f x]^2 \right) \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \Big) - \\
& \left(11 a^2 b \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{7/2} \right) / \\
& \left(7 \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right) \right) \tan[e + f x]^2 \right) \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \Big) \Big) \Big) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right) \Big)
\end{aligned}$$

■ **Problem 1438: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sqrt{d \sin[e + f x]}}{(g \cos[e + f x])^{3/2} (a + b \sin[e + f x])} dx$$

Optimal (type 4, 374 leaves, 11 steps):

$$\frac{2\sqrt{2}abd \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{(-a+b)^{3/2} (a+b)^{3/2} f g^{3/2} \sqrt{d \sin[e+fx]}} +$$

$$\frac{2\sqrt{2}abd \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{(-a+b)^{3/2} (a+b)^{3/2} f g^{3/2} \sqrt{d \sin[e+fx]}} - \frac{2b \sqrt{d \sin[e+fx]}}{(a^2-b^2) f g \sqrt{g \cos[e+fx]}} +$$

$$\frac{2a(d \sin[e+fx])^{3/2}}{(a^2-b^2) d f g \sqrt{g \cos[e+fx]}} - \frac{2a \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{d \sin[e+fx]}}{(a^2-b^2) f g^2 \sqrt{\sin[2e+2fx]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 1439: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{(g \cos[e+fx])^{3/2} \sqrt{d \sin[e+fx]} (a+b \sin[e+fx])} dx$$

Optimal (type 4, 380 leaves, 11 steps):

$$\frac{2\sqrt{2}b^2 \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{(-a+b)^{3/2} (a+b)^{3/2} f g^{3/2} \sqrt{d \sin[e+fx]}} -$$

$$\frac{2\sqrt{2}b^2 \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e+fx]}}{\sqrt{g} \sqrt{1+\sin[e+fx]}}\right], -1\right] \sqrt{\sin[e+fx]}}{(-a+b)^{3/2} (a+b)^{3/2} f g^{3/2} \sqrt{d \sin[e+fx]}} + \frac{2a \sqrt{d \sin[e+fx]}}{(a^2-b^2) d f g \sqrt{g \cos[e+fx]}} -$$

$$\frac{2b(d \sin[e+fx])^{3/2}}{(a^2-b^2) d^2 f g \sqrt{g \cos[e+fx]}} + \frac{2b \sqrt{g \cos[e+fx]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + fx, 2\right] \sqrt{d \sin[e+fx]}}{(a^2-b^2) d f g^2 \sqrt{\sin[2e+2fx]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 1440: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(g \cos[e+fx])^{3/2} (d \sin[e+fx])^{3/2} (a+b \sin[e+fx])} dx$$

Optimal (type 4, 568 leaves, 16 steps):

$$\begin{aligned}
 & - \frac{2 a}{(a^2 - b^2) d f g \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}} + \frac{2 b^2 (g \cos[e + f x])^{3/2}}{a (a^2 - b^2) d f g^3 \sqrt{d \sin[e + f x]}} - \\
 & \frac{2 \sqrt{2} b^3 \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}}\right], -1\right] \sqrt{\sin[e + f x]}}{a (-a+b)^{3/2} (a+b)^{3/2} d f g^{3/2} \sqrt{d \sin[e + f x]}} + \\
 & \frac{2 \sqrt{2} b^3 \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{g \cos[e + f x]}}{\sqrt{g} \sqrt{1 + \sin[e + f x]}}\right], -1\right] \sqrt{\sin[e + f x]}}{a (-a+b)^{3/2} (a+b)^{3/2} d f g^{3/2} \sqrt{d \sin[e + f x]}} - \frac{2 b \sqrt{d \sin[e + f x]}}{(a^2 - b^2) d^2 f g \sqrt{g \cos[e + f x]}} + \frac{4 a (d \sin[e + f x])^{3/2}}{(a^2 - b^2) d^3 f g \sqrt{g \cos[e + f x]}} - \\
 & \frac{4 a \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{d \sin[e + f x]}}{(a^2 - b^2) d^2 f g^2 \sqrt{\sin[2 e + 2 f x]}} + \frac{2 b^2 \sqrt{g \cos[e + f x]} \operatorname{EllipticE}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{d \sin[e + f x]}}{a (a^2 - b^2) d^2 f g^2 \sqrt{\sin[2 e + 2 f x]}}
 \end{aligned}$$

Result (type 6, 2968 leaves):

$$\begin{aligned}
 & \frac{\cos[e + f x]^2 \sin[e + f x]^2 \left(-\frac{2 \cot[e + f x]}{a} + \frac{2 \sec[e + f x] (-b + a \sin[e + f x])}{a^2 - b^2}\right)}{f (g \cos[e + f x])^{3/2} (d \sin[e + f x])^{3/2}} - \\
 & \frac{1}{a (a - b) (a + b) f (g \cos[e + f x])^{3/2} (d \sin[e + f x])^{3/2}} \cos[e + f x]^{3/2} \sin[e + f x]^{3/2} \\
 & \left(- \left(\left(14 (a^2 - b^2) (4 a^3 - 2 a b^2) \cos[e + f x]^{3/2} \left(a + b \sqrt{1 - \cos[e + f x]^2} \right) \left(\left(b \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \sqrt{1 - \cos[e + f x]^2} \right) / \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \right. \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) + \\
 & \left(a \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) / \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \right. \right. \\
 & \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(-4 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \\
 & \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2}\right] \right) \cos[e + f x]^2 \right) \sin[e + f x]^{3/2} \Big) /
 \end{aligned}$$

$$\begin{aligned}
& \left(3 (1 - \cos[e + f x])^2 (a^2 + b^2 (-1 + \cos[e + f x])) (a + b \sin[e + f x]) \right) + \left(2 (2 a^2 b - 2 b^3) \sqrt{\tan[e + f x]} \right. \\
& \left. (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right. \\
& \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left[-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] \right] - \right. \\
& \left. \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] + \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \right. \right. \\
& \left. \left. \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) + \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(2 (-2 a^2 b + b^3) \cos[2 (e + f x)] \right) \\
& \sqrt{\tan[e + f x]} (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \\
& \left(-\frac{a \operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{2} + 2 \sqrt{\tan[e + f x]}}{\sqrt{2}} \right]}{\sqrt{2} b^2} + \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} \right) + \\
& \frac{(2 a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a} + 2 (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{2} \sqrt{a}} \right]}{2 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{a \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} - \\
& \frac{a \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan[e + f x]} + \tan[e + f x] \right]}{2 \sqrt{2} b^2} - \frac{(2 a^2 - b^2) \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{(2a^2 - b^2) \operatorname{Log}\left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + fx]} + \sqrt{a^2 - b^2} \tan[e + fx]\right]}{4 \sqrt{2} \sqrt{a} b^2 (a^2 - b^2)^{1/4}} + \frac{\tan[e + fx]^{3/2}}{b \sqrt{1 + \tan[e + fx]^2}} + \\
& \left(7 a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx]^{3/2}\right) / \\
& \left(b \sqrt{1 + \tan[e + fx]^2} \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \tan[e + fx]^2\right) (-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2))\right) - \\
& \left(7 a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx]^{3/2}\right) / \left(3 \sqrt{1 + \tan[e + fx]^2}\right. \\
& \quad \left(-7 a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\left(2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + fx]^2,\right.\right.\right. \\
& \quad \left.\left.\left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right]\right) \tan[e + fx]^2 \\
& \quad \left.(-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2))\right) + \left(11 a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right]\right. \\
& \quad \left.\tan[e + fx]^{7/2}\right) / \left(7 b \sqrt{1 + \tan[e + fx]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right. \\
& \quad \left.\left.\left.-\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] \tan[e + fx]^2\right) (-b^2 \tan[e + fx]^2 + a^2 (1 + \tan[e + fx]^2))\right) - \\
& \left(11 a^2 b \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \frac{(-a^2 + b^2) \tan[e + fx]^2}{a^2}\right] \tan[e + fx]^{7/2}\right) / \\
& \left(7 \sqrt{1 + \tan[e + fx]^2} \left(-11 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + 2\right.\right. \\
& \quad \left.\left.2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + fx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[e + fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4},\right.\right.\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\sqrt{1 - \cos[e + f x]^2} \right) / \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{4}, 2, \frac{11}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) + \\
& \left(a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) / \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{4}, 1, \frac{7}{4}, \cos[e + f x]^2, \right. \right. \\
& \quad \left. \left. \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(-4 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{4}, 2, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{5}{4}, 1, \frac{11}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 \right) \left. \right) \sin[e + f x]^{3/2} / \\
& \left(3 (1 - \cos[e + f x]^2) (a^2 + b^2 (-1 + \cos[e + f x]^2)) (a + b \sin[e + f x]) \right) + \left(2 (2 a^2 b - 2 b^3) \sqrt{\tan[e + f x]} \right. \\
& \left. (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right) \\
& \left(\frac{1}{4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4}} \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]}}{\sqrt{a}} \right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log} \left[-a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} - \sqrt{a^2 - b^2} \tan[e + f x] \right] + \operatorname{Log} \left[a + \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan[e + f x]} + \right. \right. \right. \\
& \quad \left. \left. \sqrt{a^2 - b^2} \tan[e + f x] \right] \right) + \left(7 a^2 b \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^{3/2} \right) / \\
& \left(3 \sqrt{1 + \tan[e + f x]^2} \left(-7 a^2 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \quad \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \\
& \quad \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (1 + \tan[e + f x]^2)^{3/2} \right) + \left(2 (-2 a^2 b + b^3) \cos[2 (e + f x)] \right. \\
& \left. \sqrt{\tan[e + f x]} (b \tan[e + f x] + a \sqrt{1 + \tan[e + f x]^2}) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{a \operatorname{ArcTan}\left[\frac{-\sqrt{2}+2\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} - \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{2}+2\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}}\right]}{\sqrt{2} b^2} + \frac{(2a^2-b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2}\sqrt{a}+2(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} \right) + \\
& \frac{(2a^2-b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{a}+2(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}}{\sqrt{2}\sqrt{a}}\right]}{2\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \frac{a \operatorname{Log}\left[1-\sqrt{2}\sqrt{\operatorname{Tan}[e+fx]}+\operatorname{Tan}[e+fx]\right]}{2\sqrt{2}b^2} - \\
& \frac{a \operatorname{Log}\left[1+\sqrt{2}\sqrt{\operatorname{Tan}[e+fx]}+\operatorname{Tan}[e+fx]\right]}{2\sqrt{2}b^2} - \frac{(2a^2-b^2) \operatorname{Log}\left[-a+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}-\sqrt{a^2-b^2}\operatorname{Tan}[e+fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \\
& \frac{(2a^2-b^2) \operatorname{Log}\left[a+\sqrt{2}\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\operatorname{Tan}[e+fx]}+\sqrt{a^2-b^2}\operatorname{Tan}[e+fx]\right]}{4\sqrt{2}\sqrt{a}b^2(a^2-b^2)^{1/4}} + \frac{\operatorname{Tan}[e+fx]^{3/2}}{b\sqrt{1+\operatorname{Tan}[e+fx]^2}} + \\
& \left(7a^4 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \operatorname{Tan}[e+fx]^{3/2} \right) / \\
& \left(b\sqrt{1+\operatorname{Tan}[e+fx]^2} \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \right. \right. \\
& \quad \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] \operatorname{Tan}[e+fx]^2 \right) (-b^2 \operatorname{Tan}[e+fx]^2 + a^2(1+\operatorname{Tan}[e+fx]^2)) \right) \right) - \\
& \left(7a^2 b \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \operatorname{Tan}[e+fx]^{3/2} \right) / \left(3\sqrt{1+\operatorname{Tan}[e+fx]^2} \right. \\
& \quad \left(-7a^2 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] \right) \operatorname{Tan}[e+fx]^2 \right) \\
& \quad \left. (-b^2 \operatorname{Tan}[e+fx]^2 + a^2(1+\operatorname{Tan}[e+fx]^2)) \right) + \left(11a^4 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[e+fx]^2}{a^2}\right] \right. \\
& \quad \left. \operatorname{Tan}[e+fx]^{7/2} \right) / \left(7b\sqrt{1+\operatorname{Tan}[e+fx]^2} \left(-11a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + 2 \right. \right. \\
& \quad \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[e+fx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[e+fx]^2\right] + a^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \Big) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \Big) - \\
& \left(11 a^2 b \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x]^{7/2} \right) / \\
& \left(7 \sqrt{1 + \tan[e + f x]^2} \left(-11 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 \right. \right. \\
& \left. \left((a^2 - b^2) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right) \tan[e + f x]^2 \left(-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2) \right) \right) \right) \Big) / \\
& \left(\cos[e + f x]^{3/2} \sqrt{\sin[e + f x]} (a + b \sin[e + f x]) (-1 + \tan[e + f x]^2) \sqrt{1 + \tan[e + f x]^2} \right) \Big)
\end{aligned}$$

■ **Problem 1442: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e + f x])^{3/2}}{\sqrt{d \sin[e + f x]} (a + b \sin[e + f x])^2} dx$$

Optimal (type 4, 331 leaves, 8 steps):

$$\begin{aligned}
& \frac{\sqrt{2} b g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[-\frac{a}{b - \sqrt{-a^2 + b^2}}, \operatorname{ArcSin} \left[\frac{\sqrt{d \sin[e + f x]}}{\sqrt{d} \sqrt{1 + \cos[e + f x]}} \right], -1 \right]}{a^2 \sqrt{-a^2 + b^2} \sqrt{d} f \sqrt{g \cos[e + f x]}} - \\
& \frac{\sqrt{2} b g^2 \sqrt{\cos[e + f x]} \operatorname{EllipticPi} \left[-\frac{a}{b + \sqrt{-a^2 + b^2}}, \operatorname{ArcSin} \left[\frac{\sqrt{d \sin[e + f x]}}{\sqrt{d} \sqrt{1 + \cos[e + f x]}} \right], -1 \right]}{a^2 \sqrt{-a^2 + b^2} \sqrt{d} f \sqrt{g \cos[e + f x]}} + \\
& \frac{g \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}}{a d f (a + b \sin[e + f x])} + \frac{g^2 \operatorname{EllipticF} \left[e - \frac{\pi}{4} + f x, 2 \right] \sqrt{\sin[2 e + 2 f x]}}{2 a^2 f \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}}
\end{aligned}$$

Result (type 6, 717 leaves):

$$\begin{aligned}
& - \left((g \cos[e + f x])^{3/2} (a + b \sqrt{1 - \cos[e + f x]^2}) \right. \\
& \left. \left(\left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \sqrt{\cos[e + f x]} \right) / \left((1 - \cos[e + f x]^2)^{3/4} \left(5 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \left(-4 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. 3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, \cos[e + f x]^2, \frac{b^2 \cos[e + f x]^2}{-a^2 + b^2} \right] \right) \cos[e + f x]^2 (a^2 + b^2 (-1 + \cos[e + f x]^2)) \right) \right) - \\
& \frac{1}{\sqrt{a} (-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) b \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4} (-1 + \cos[e + f x]^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{a} \sqrt{\cos[e + f x]}}{(-a^2 + b^2)^{1/4} (-1 + \cos[e + f x]^2)^{1/4}} \right] \right) + \\
& \operatorname{Log} \left[\sqrt{-a^2 + b^2} + \frac{i a \cos[e + f x]}{\sqrt{-1 + \cos[e + f x]^2}} - \frac{(1 + i) \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]}}{(-1 + \cos[e + f x]^2)^{1/4}} \right] - \\
& \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + \frac{i a \cos[e + f x]}{\sqrt{-1 + \cos[e + f x]^2}} + \frac{(1 + i) \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\cos[e + f x]}}{(-1 + \cos[e + f x]^2)^{1/4}} \right] \right) \sin[e + f x] \Bigg) / \\
& \left(a f \cos[e + f x]^{3/2} (1 - \cos[e + f x]^2)^{1/4} \sqrt{d \sin[e + f x]} (a + b \sin[e + f x]) \right) \Bigg) + \\
& \frac{(g \cos[e + f x])^{3/2} \tan[e + f x]}{a f \sqrt{d \sin[e + f x]} (a + b \sin[e + f x])}
\end{aligned}$$

■ **Problem 1449: Result more than twice size of optimal antiderivative.**

$$\int \csc[c + d x]^3 \sec[c + d x]^2 (a + b \sin[c + d x]) dx$$

Optimal (type 3, 75 leaves, 8 steps):

$$-\frac{3 a \operatorname{ArcTanh}[\cos[c + d x]]}{2 d} - \frac{b \operatorname{Cot}[c + d x]}{d} + \frac{3 a \sec[c + d x]}{2 d} - \frac{a \csc[c + d x]^2 \sec[c + d x]}{2 d} + \frac{b \tan[c + d x]}{d}$$

Result (type 3, 172 leaves):

$$\begin{aligned}
& - \frac{2 b \cot [2 (c+d x)]}{d} - \frac{a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{8 d} - \frac{3 a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \frac{3 a \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \\
& \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{8 d} + \frac{a \sin\left[\frac{1}{2}(c+d x)\right]}{d\left(\cos\left[\frac{1}{2}(c+d x)\right] - \sin\left[\frac{1}{2}(c+d x)\right]\right)} - \frac{a \sin\left[\frac{1}{2}(c+d x)\right]}{d\left(\cos\left[\frac{1}{2}(c+d x)\right] + \sin\left[\frac{1}{2}(c+d x)\right]\right)}
\end{aligned}$$

■ **Problem 1455: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+d x]^3 \operatorname{Sec}[c+d x]^2 (a+b \sin [c+d x])^2 dx$$

Optimal (type 3, 100 leaves, 10 steps):

$$-\frac{(3 a^2+2 b^2) \operatorname{ArcTanh}[\cos [c+d x]]}{2 d}-\frac{2 a b \cot [c+d x]}{d}+\frac{(3 a^2+2 b^2) \operatorname{Sec}[c+d x]}{2 d}-\frac{a^2 \operatorname{Csc}[c+d x]^2 \operatorname{Sec}[c+d x]}{2 d}+\frac{2 a b \tan [c+d x]}{d}$$

Result (type 3, 238 leaves):

$$\begin{aligned}
& \frac{1}{2 d\left(\operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2-\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right)} \operatorname{Csc}[c+d x]^4 \\
& \left(2 a^2+4 b^2-2\left(3 a^2+2 b^2\right) \cos [2(c+d x)]+3 a^2 \cos [3(c+d x)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]\right]+2 b^2 \cos [3(c+d x)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]\right]-\right. \\
& \left.\left(3 a^2+2 b^2\right) \cos [c+d x]\left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]\right]-\operatorname{Log}\left[\sin\left[\frac{1}{2}(c+d x)\right]\right]\right)-3 a^2 \cos [3(c+d x)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+d x)\right]\right]-\right. \\
& \left.2 b^2 \cos [3(c+d x)] \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+d x)\right]\right]+8 a b \sin [c+d x]-8 a b \sin [3(c+d x)]\right)
\end{aligned}$$

■ **Problem 1462: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+d x]^3 \operatorname{Sec}[c+d x]^2 (a+b \sin [c+d x])^3 dx$$

Optimal (type 3, 132 leaves, 14 steps):

$$\begin{aligned}
& -\frac{3 a^3 \operatorname{ArcTanh}[\cos [c+d x]]}{2 d}-\frac{3 a b^2 \operatorname{ArcTanh}[\cos [c+d x]]}{d}-\frac{3 a^2 b \cot [c+d x]}{d}+ \\
& \frac{3 a^3 \operatorname{Sec}[c+d x]}{2 d}+\frac{3 a b^2 \operatorname{Sec}[c+d x]}{d}-\frac{a^3 \operatorname{Csc}[c+d x]^2 \operatorname{Sec}[c+d x]}{2 d}+\frac{3 a^2 b \tan [c+d x]}{d}+\frac{b^3 \tan [c+d x]}{d}
\end{aligned}$$

Result (type 3, 267 leaves):

$$\frac{1}{2 d \left(\operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 - \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \right)} \operatorname{Csc}[c+d x]^4$$

$$\left(2 a^3 + 12 a b^2 - 6 \left(a^3 + 2 a b^2 \right) \operatorname{Cos}[2(c+d x)] + 3 a^3 \operatorname{Cos}[3(c+d x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] + 6 a b^2 \operatorname{Cos}[3(c+d x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] \right) -$$

$$3 a \left(a^2 + 2 b^2 \right) \operatorname{Cos}[c+d x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) - 3 a^3 \operatorname{Cos}[3(c+d x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] -$$

$$6 a b^2 \operatorname{Cos}[3(c+d x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 12 a^2 b \operatorname{Sin}[c+d x] + 6 b^3 \operatorname{Sin}[c+d x] - 12 a^2 b \operatorname{Sin}[3(c+d x)] - 2 b^3 \operatorname{Sin}[3(c+d x)] \right)$$

- **Problem 1479: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+f x]^2 (a+b \operatorname{Sin}[e+f x])^{3/2}}{\sqrt{d} \operatorname{Sin}[e+f x]} dx$$

Optimal (type 4, 312 leaves, ? steps):

$$\frac{\operatorname{Sec}[e+f x] (b+a \operatorname{Sin}[e+f x]) \sqrt{a+b \operatorname{Sin}[e+f x]}}{f \sqrt{d} \operatorname{Sin}[e+f x]}$$

$$\frac{(a+b)^{3/2} \sqrt{-\frac{a(-1+\operatorname{Csc}[e+f x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Csc}[e+f x])}{a-b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{d} \operatorname{Sin}[e+f x]}\right], -\frac{a+b}{a-b}\right] \operatorname{Tan}[e+f x]}{\sqrt{d} f}$$

$$\left(b(a+b) \sqrt{-\frac{a(-1+\operatorname{Csc}[e+f x])}{a+b}} \sqrt{\frac{b+a \operatorname{Csc}[e+f x]}{-a+b}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{b+a \operatorname{Csc}[e+f x]}{a-b}}\right], \frac{-a+b}{a+b}\right] (1+\operatorname{Sin}[e+f x]) \operatorname{Tan}[e+f x] \right) /$$

$$\left(f \sqrt{\frac{a(1+\operatorname{Csc}[e+f x])}{a-b}} \sqrt{d} \operatorname{Sin}[e+f x] \sqrt{a+b \operatorname{Sin}[e+f x]} \right)$$

Result (type 4, 9055 leaves):

$$-\frac{1}{f \sqrt{d} \operatorname{Sin}[e+f x]} 2 a^2 \sqrt{\operatorname{Sin}[e+f x]}$$

$$\left(\left(\sqrt{\operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Sin}[e+f x]}}{a}}}{\sqrt{2}}\right], \frac{2 a}{a+b}\right] \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \right)$$

$$\left(\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 \operatorname{Sin}[e+fx]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (a+b \operatorname{Sin}[e+fx])}{a}} \right) / \left(\sqrt{\operatorname{Sin}[e+fx]} \sqrt{a+b \operatorname{Sin}[e+fx]} \right) -$$

$$\left(b \sqrt{\operatorname{Cot}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2} \operatorname{EllipticPi}\left[\frac{a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 \operatorname{Sin}[e+fx]}{a}}}{\sqrt{2}}}\right], \frac{2a}{a+b}\right] \operatorname{Sec}[e+fx]$$

$$\operatorname{Sin}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 \operatorname{Sin}[e+fx]}{a}}$$

$$\left(\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}(-e+\frac{\pi}{2}-fx)\right]^2 (a+b \operatorname{Sin}[e+fx])}{a}} \right) / \left((a+b) \sqrt{\operatorname{Sin}[e+fx]} \sqrt{a+b \operatorname{Sin}[e+fx]} \right) -$$

$$\left(b \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Sin}[e+fx]} \left(\sqrt{2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \sqrt{\frac{a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} - \right.$$

$$\left. \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} (a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2)} \sqrt{2} \left(\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a+2b \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]+a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) +$$

$$\frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} i \left(b-\sqrt{-a^2+b^2} \right) \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}}$$

$$\left(\text{EllipticE} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) -$$

$$\frac{1}{\sqrt{\text{Tan} \left[\frac{1}{2} (e+f x) \right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \text{Cot} \left[\frac{1}{2} (e+f x) \right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{a \text{Cot} \left[\frac{1}{2} (e+f x) \right]}{b-\sqrt{-a^2+b^2}}} \text{EllipticPi} \left[-\frac{i (b+\sqrt{-a^2+b^2})}{a},$$

$$i \text{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] + \frac{1}{\sqrt{\text{Tan} \left[\frac{1}{2} (e+f x) \right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \text{Cot} \left[\frac{1}{2} (e+f x) \right]}{b+\sqrt{-a^2+b^2}}}$$

$$\sqrt{\frac{a \text{Cot} \left[\frac{1}{2} (e+f x) \right]}{b-\sqrt{-a^2+b^2}}} \text{EllipticPi} \left[\frac{i (b+\sqrt{-a^2+b^2})}{a}, i \text{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right)$$

$$\left(\frac{\text{Tan} \left[\frac{1}{2} (e+f x) \right]}{1+\text{Tan} \left[\frac{1}{2} (e+f x) \right]^2} \right)^{3/2} \left(1+\text{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right)^2 \sqrt{\frac{a+2 b \text{Tan} \left[\frac{1}{2} (e+f x) \right] + a \text{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{1+\text{Tan} \left[\frac{1}{2} (e+f x) \right]^2}} \right) /$$

$$\left(f \sqrt{d \text{Sin} [e+f x]} \left(\frac{\text{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \sqrt{\frac{\text{Tan} \left[\frac{1}{2} (e+f x) \right]}{1+\text{Tan} \left[\frac{1}{2} (e+f x) \right]^2}} \sqrt{\frac{a+2 b \text{Tan} \left[\frac{1}{2} (e+f x) \right] + a \text{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{1+\text{Tan} \left[\frac{1}{2} (e+f x) \right]^2}}}{\sqrt{2}} + \right.$$

$$\left. \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} (a+2 b \text{Tan} \left[\frac{1}{2} (e+f x) \right] + a \text{Tan} \left[\frac{1}{2} (e+f x) \right]^2)^2} \sqrt{2} \left(\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a+2 b \text{Cot} \left[\frac{1}{2} (e+f x) \right] + a \text{Cot} \left[\frac{1}{2} (e+f x) \right]^2 \right) + \right.$$

$$\begin{aligned}
& \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} i \left(b - \sqrt{-a^2+b^2} \right) \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \sqrt{\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}} \\
& \left(\text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] - \text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] \right) - \\
& \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \sqrt{\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}} \text{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, \right. \\
& \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] + \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2+b^2} + a \cot\left[\frac{1}{2}(e+fx)\right]}{b + \sqrt{-a^2+b^2}}} \\
& \left. \sqrt{\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}} \text{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}} \right] \right) \\
& \left(b \sec\left[\frac{1}{2}(e+fx)\right]^2 + a \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2} \right)^{3/2} \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \\
& \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e+fx)\right] + a \tan\left[\frac{1}{2}(e+fx)\right]^2}{1 + \tan\left[\frac{1}{2}(e+fx)\right]^2}} - \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a + 2b \tan\left[\frac{1}{2}(e+fx)\right] + a \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)}
\end{aligned}$$

$$\begin{aligned}
& 2\sqrt{2} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \left[\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a+2b \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right. \\
& \left. i \left(b-\sqrt{-a^2+b^2} \right) \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \right. \\
& \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) - \right. \\
& \left. \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[-\frac{i \left(b+\sqrt{-a^2+b^2} \right)}{a}, \right. \right. \\
& \left. \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \right. \\
& \left. \left. \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[\frac{i \left(b+\sqrt{-a^2+b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) \right. \\
& \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^{3/2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \sqrt{\frac{a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} - \right. \\
& \left. \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{2} \left(\frac{i a \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{4 \left(1 - i \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}} - \frac{i a \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{4 \left(1 + i \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}} \right) + \\
& \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(-b \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 - a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right] \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \frac{1}{4 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}} \\
& i \left(b - \sqrt{-a^2+b^2} \right) \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \\
& \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{4 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \\
& \operatorname{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 - \\
& \frac{1}{4 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2+b^2} \right)}{a}, \right. \\
& \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 - \left(i a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\text{EllipticE} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \text{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) / \\
& \left(4 \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e+f x) \right]}{b-\sqrt{-a^2+b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]} \right) - \left(i a (b-\sqrt{-a^2+b^2}) \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e+f x) \right]}{b-\sqrt{-a^2+b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e+f x) \right]^2 \right. \\
& \left. \left(\text{EllipticE} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \text{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) / \\
& \left(4 (b+\sqrt{-a^2+b^2}) \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e+f x) \right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]} \right) + \\
& \left(a^2 \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e+f x) \right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{EllipticPi} \left[-\frac{i (b+\sqrt{-a^2+b^2})}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \right. \right. \\
& \left. \left. \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) / \left(4 (b-\sqrt{-a^2+b^2}) \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e+f x) \right]}{b-\sqrt{-a^2+b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]} \right) + \\
& \left(a^2 \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e+f x) \right]}{b-\sqrt{-a^2+b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{EllipticPi} \left[-\frac{i (b+\sqrt{-a^2+b^2})}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(4 \left(b + \sqrt{-a^2 + b^2} \right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]} \right) - \\
& \left(a^2 \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2 + b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}\right], \right. \right. \\
& \left. \left. \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) / \left(4 \left(b - \sqrt{-a^2 + b^2} \right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]} \right) - \\
& \left(a^2 \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2 + b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) / \\
& \left(4 \left(b + \sqrt{-a^2 + b^2} \right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]} \right) + \\
& \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} i \left(b - \sqrt{-a^2 + b^2} \right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}} \\
& \left(\frac{i \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{4 \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^{3/2}} - \frac{i \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{4 \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^{3/2}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^{3/2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \sqrt{\frac{a+2b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} + \\
& \frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \sqrt{\frac{a+2b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)} \right)}{\sqrt{2} \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}}} \\
& \frac{1}{\sqrt{2} \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a+2b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)} \\
& 3 \left(\frac{\sqrt{a}}{b+\sqrt{-a^2+b^2}} \left(a+2b\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]+a\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right. \\
& \left. i \left(b-\sqrt{-a^2+b^2} \right) \sqrt{\frac{b+\sqrt{-a^2+b^2}+a\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \right. \\
& \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) - \\
& \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[-\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}, \right.
\end{aligned}$$

$$\begin{aligned}
& i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \Bigg] + \frac{1}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} a \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \\
& \left. \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi} \left[\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) \\
& \sqrt{\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)^2 \sqrt{\frac{a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} \\
& \left(- \frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)^2} \right) + \\
& \left(\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \sqrt{\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} \left(\frac{b \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 + a \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2} - \right. \right. \\
& \left. \left. \frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \left(a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)^2} \right) \right) \Bigg/ \\
& \left(\sqrt{2} \sqrt{\frac{a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} \right) - \left(\left(\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \left(a + 2 b \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) + \right. \\
& \left. \frac{1}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} i \left(b - \sqrt{-a^2 + b^2} \right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \right)
\end{aligned}$$

$$\left(\text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) -$$

$$\frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \text{EllipticPi}\left[-\frac{i(b+\sqrt{-a^2+b^2})}{a}\right],$$

$$i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \text{EllipticPi}\left[\frac{i(b+\sqrt{-a^2+b^2})}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right)$$

$$\left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^{3/2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \left(\frac{b \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2+a \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} - \right.$$

$$\left. \frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] (a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2)}{(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2)^2} \right) \left/ \left(\sqrt{2} \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \right) \right)$$

$$\left(a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \sqrt{\frac{a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \right) \left. \right) + \frac{(a+b \operatorname{Sin}[e+fx])^{3/2} \operatorname{Tan}[e+fx]}{f \sqrt{d} \operatorname{Sin}[e+fx]}$$

■ **Problem 1480:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sec[e + f x]^4 (a + b \sin[e + f x])^{5/2}}{\sqrt{d \sin[e + f x]}} dx$$

Optimal (type 4, 366 leaves, ? steps):

$$\frac{5 a \sec[e + f x] (b + a \sin[e + f x]) \sqrt{a + b \sin[e + f x]}}{6 f \sqrt{d \sin[e + f x]}} + \frac{\sec[e + f x]^3 \sqrt{d \sin[e + f x]} (a + b \sin[e + f x])^{5/2}}{3 d f} - \frac{1}{6 \sqrt{d} f}$$

$$5 a (a + b)^{3/2} \sqrt{-\frac{a (-1 + \csc[e + f x])}{a + b}} \sqrt{\frac{a (1 + \csc[e + f x])}{a - b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{d \sin[e + f x]}}\right], -\frac{a + b}{a - b}\right] \tan[e + f x] -$$

$$\left(5 a b (a + b) \sqrt{-\frac{a (-1 + \csc[e + f x])}{a + b}} \sqrt{\frac{b + a \csc[e + f x]}{-a + b}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{b + a \csc[e + f x]}{a - b}}\right], \frac{-a + b}{a + b}\right] (1 + \sin[e + f x]) \tan[e + f x]\right) /$$

$$\left(6 f \sqrt{\frac{a (1 + \csc[e + f x])}{a - b}} \sqrt{d \sin[e + f x]} \sqrt{a + b \sin[e + f x]}\right)$$

Result (type 4, 9121 leaves):

$$-\frac{1}{3 f \sqrt{d \sin[e + f x]}} 5 a^3 \sqrt{\sin[e + f x]}$$

$$\left(\left(\sqrt{\cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sin[e + f x]}}{a}}\right], \frac{2 a}{a + b}\right] \sec[e + f x] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^4\right.\right.$$

$$\left.\left.\sqrt{-\frac{(a + b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sin[e + f x]}{a}} \sqrt{\frac{\csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a + b \sin[e + f x])}{a}}\right) / \left(\sqrt{\sin[e + f x]} \sqrt{a + b \sin[e + f x]}\right) -$$

$$\left(b \sqrt{\cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \operatorname{EllipticPi}\left[\frac{a}{a + b}, \operatorname{ArcSin}\left[\frac{\sqrt{-(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \sin[e + f x]}}{a}}\right], \frac{2 a}{a + b}\right] \sec[e + f x]\right)$$

$$\begin{aligned}
& \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \sin[efx]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b\sin[efx])}{a}} \right) \\
& \left. \left((a+b)\sqrt{\sin[efx]}\sqrt{a+b\sin[efx]} \right) + \frac{1}{f\sqrt{d}\sin[efx]} \sin[efx]\sqrt{a+b\sin[efx]} \right. \\
& \left. \left(\frac{1}{3}\operatorname{Sec}[efx]^3(a^2+b^2+2ab\sin[efx]) + \frac{1}{6}\operatorname{Sec}[efx](5a^2-2b^2+5ab\sin[efx]) \right) - \right. \\
& \left. \left(5ab\sin[efx]\sqrt{a+b\sin[efx]} \left[\sqrt{2}\tan\left[\frac{1}{2}(efx)\right] \sqrt{\frac{\tan\left[\frac{1}{2}(efx)\right]}{1+\tan\left[\frac{1}{2}(efx)\right]^2}} \sqrt{\frac{a+2b\tan\left[\frac{1}{2}(efx)\right]+a\tan\left[\frac{1}{2}(efx)\right]^2}{1+\tan\left[\frac{1}{2}(efx)\right]^2}} \right. \right. \right. \\
& \left. \left. \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}} \left(a+2b\tan\left[\frac{1}{2}(efx)\right]+a\tan\left[\frac{1}{2}(efx)\right]^2 \right) \sqrt{2} \left[\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a+2b\cot\left[\frac{1}{2}(efx)\right]+a\cot\left[\frac{1}{2}(efx)\right]^2 \right) \right. \right. \right. \\
& \left. \left. \frac{1}{\sqrt{\tan\left[\frac{1}{2}(efx)\right]}} i \left(b-\sqrt{-a^2+b^2} \right) \sqrt{\frac{b+\sqrt{-a^2+b^2}+a\cot\left[\frac{1}{2}(efx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{a\cot\left[\frac{1}{2}(efx)\right]}{1+\frac{a\cot\left[\frac{1}{2}(efx)\right]}{b-\sqrt{-a^2+b^2}}}} \right. \right. \\
& \left. \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(efx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(efx)\right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) - \right. \\
& \left. \frac{1}{\sqrt{\tan\left[\frac{1}{2}(efx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a\cot\left[\frac{1}{2}(efx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{a\cot\left[\frac{1}{2}(efx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[-\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a} \right], \right.
\end{aligned}$$

$$\begin{aligned}
& i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \Bigg] + \frac{1}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} a \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \\
& \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi} \left[\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \\
& \left. \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2} \right)^{3/2} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)^2 \sqrt{\frac{a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} \right) \Bigg] / \\
& \left(6 f \sqrt{d} \operatorname{Sin}[e + f x] \left(\frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \sqrt{\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} \sqrt{\frac{a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}}}{\sqrt{2}} \right) + \right. \\
& \left. \frac{1}{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \left(a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)^2} \sqrt{2} \left(\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \left(a + 2 b \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]^2 \right) + \right. \right. \\
& \left. \frac{1}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} i \left(b - \sqrt{-a^2 + b^2} \right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \right) \\
& \left. \left(\operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] - \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[-\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}\right], \\
& i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \Bigg] + \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \\
& \left. \sqrt{1+\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}\right], i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \\
& \left(b \sec\left[\frac{1}{2}(e+fx)\right]\right)^2 + a \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \left(\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1+\tan\left[\frac{1}{2}(e+fx)\right]^2}\right)^{3/2} \left(1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \\
& \sqrt{\frac{a+2b \tan\left[\frac{1}{2}(e+fx)\right]+a \tan\left[\frac{1}{2}(e+fx)\right]^2}{1+\tan\left[\frac{1}{2}(e+fx)\right]^2}} - \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\left(a+2b \tan\left[\frac{1}{2}(e+fx)\right]+a \tan\left[\frac{1}{2}(e+fx)\right]^2\right)} \\
& 2\sqrt{2} \sec\left[\frac{1}{2}(e+fx)\right]^2 \left(\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\left(a+2b \cot\left[\frac{1}{2}(e+fx)\right]+a \cot\left[\frac{1}{2}(e+fx)\right]^2\right) + \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right) \\
& i\left(b-\sqrt{-a^2+b^2}\right) \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \\
& \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a\cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a\cot\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[-\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}\right], \\
& i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \left. + \frac{1}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2}+a\cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \right. \\
& \left. \sqrt{1+\frac{a\cot\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}\right], i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \\
& \tan\left[\frac{1}{2}(e+fx)\right] \left(\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1+\tan\left[\frac{1}{2}(e+fx)\right]^2}\right)^{3/2} \left(1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right) \sqrt{\frac{a+2b\tan\left[\frac{1}{2}(e+fx)\right]+a\tan\left[\frac{1}{2}(e+fx)\right]^2}{1+\tan\left[\frac{1}{2}(e+fx)\right]^2}} - \\
& \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}} \left(a+2b\tan\left[\frac{1}{2}(e+fx)\right]+a\tan\left[\frac{1}{2}(e+fx)\right]^2\right) \\
& \sqrt{2} \left[\frac{i a \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{b+\sqrt{-a^2+b^2}+a\cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{4\left(1-i\cot\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{1+\frac{a\cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}} - \frac{i a \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{b+\sqrt{-a^2+b^2}+a\cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{4\left(1+i\cot\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{1+\frac{a\cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}} \right] + \\
& \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(-b\operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 - a\cot\left[\frac{1}{2}(e+fx)\right] \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2\right) - \frac{1}{4\tan\left[\frac{1}{2}(e+fx)\right]^{3/2}} \\
& i\left(b-\sqrt{-a^2+b^2}\right) \sqrt{\frac{b+\sqrt{-a^2+b^2}+a\cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{1+\frac{a\cot\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}}
\end{aligned}$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right)$$

$$\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + \frac{1}{4 \text{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{a \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}}$$

$$\text{EllipticPi}\left[-\frac{i(b+\sqrt{-a^2+b^2})}{a}, i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 -$$

$$\frac{1}{4 \text{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{a \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \text{EllipticPi}\left[\frac{i(b+\sqrt{-a^2+b^2})}{a},$$

$$i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 - \left(i a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \text{Csc}\left[\frac{1}{2}(e+fx)\right]^2$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) /$$

$$\left(4 \sqrt{\frac{a \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \sqrt{\text{Tan}\left[\frac{1}{2}(e+fx)\right]} - \left(i a (b-\sqrt{-a^2+b^2}) \sqrt{\frac{a \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}} \text{Csc}\left[\frac{1}{2}(e+fx)\right]^2$$

$$\begin{aligned}
& \left(\text{EllipticE} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] - \text{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) / \\
& \left(4 (b+\sqrt{-a^2+b^2}) \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot} \left[\frac{1}{2} (e+fx) \right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]} \right) + \\
& \left(a^2 \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot} \left[\frac{1}{2} (e+fx) \right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{EllipticPi} \left[-\frac{i (b+\sqrt{-a^2+b^2})}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}} \right], \right. \right. \\
& \left. \left. \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) / \left(4 (b-\sqrt{-a^2+b^2}) \sqrt{1+\frac{a \operatorname{Cot} \left[\frac{1}{2} (e+fx) \right]}{b-\sqrt{-a^2+b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]} \right) + \\
& \left(a^2 \sqrt{\frac{a \operatorname{Cot} \left[\frac{1}{2} (e+fx) \right]}{b-\sqrt{-a^2+b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{EllipticPi} \left[-\frac{i (b+\sqrt{-a^2+b^2})}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}} \right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) / \\
& \left(4 (b+\sqrt{-a^2+b^2}) \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot} \left[\frac{1}{2} (e+fx) \right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]} \right) - \\
& \left(a^2 \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Cot} \left[\frac{1}{2} (e+fx) \right]}{b+\sqrt{-a^2+b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{EllipticPi} \left[\frac{i (b+\sqrt{-a^2+b^2})}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}} \right], \right. \right. \\
& \left. \left. \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}}{\left(4 \left(b - \sqrt{-a^2 + b^2} \right) \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right)} \right) - \\
& \left(a^2 \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{EllipticPi} \left[\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) / \\
& \left(4 \left(b + \sqrt{-a^2 + b^2} \right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]} \right) + \\
& \frac{1}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} i \left(b - \sqrt{-a^2 + b^2} \right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \\
& \left(\frac{i \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}{4 \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^{3/2}} - \frac{i \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}{4 \sqrt{1 + \frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^{3/2}} \right) \\
& \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2} \right)^{3/2} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)^2 \sqrt{\frac{a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} + \\
& \frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \sqrt{\frac{a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} \left(- \frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)^2} \right)}{\sqrt{2} \sqrt{\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}}}
\end{aligned}$$

1

$$\sqrt{2} \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \left(a + 2b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^2 \right)$$

$$3 \left[\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \left(a + 2b \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right] + a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]^2 \right) + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} \right]$$

$$i \left(b - \sqrt{-a^2 + b^2} \right) \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b - \sqrt{-a^2 + b^2}}}$$

$$\left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right) -$$

$$\frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b + \sqrt{-a^2 + b^2}}} \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, \right.$$

$$\left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} a \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b + \sqrt{-a^2 + b^2}}}$$

$$\left. \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \right)$$

$$\sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^2}} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^2 \right)^2 \sqrt{\frac{a + 2b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^2}}$$

$$\begin{aligned}
& \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)} \right) + \\
& \left(\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \left(\frac{b \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + a \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} - \right. \right. \\
& \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2} \right) \right) / \\
& \left(\sqrt{2} \sqrt{\frac{a+2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \right) - \left(\left(\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \left(a+2b \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2\right) + \right. \right. \\
& \left. \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} i \left(b-\sqrt{-a^2+b^2}\right) \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}}} \right. \\
& \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \right) - \\
& \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}}} \operatorname{EllipticPi}\left[-\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}\right], \\
& i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] + \frac{1}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} a \sqrt{\frac{b+\sqrt{-a^2+b^2} + a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}
\end{aligned}$$

$$\left(\sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b - \sqrt{-a^2+b^2}}}\right) \operatorname{EllipticPi}\left[\frac{i\left(b + \sqrt{-a^2+b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b + \sqrt{-a^2+b^2}}{b - \sqrt{-a^2+b^2}}\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right)^{3/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \left(\frac{b \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 + a \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} - \frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(a + 2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2} \right) \left(\sqrt{2} \sqrt{\frac{a}{b + \sqrt{-a^2+b^2}}} \left(a + 2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \sqrt{\frac{a + 2b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \right) \right)$$

■ **Problem 1483: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sin[c + dx]) \tan[c + dx]^5 dx$$

Optimal (type 3, 116 leaves, 7 steps):

$$\frac{(8a + 15b) \operatorname{Log}[1 - \sin[c + dx]]}{16d} - \frac{(8a - 15b) \operatorname{Log}[1 + \sin[c + dx]]}{16d} - \frac{15b \sin[c + dx]}{8d} - \frac{(4a + 5b \sin[c + dx]) \tan[c + dx]^2}{8d} + \frac{(a + b \sin[c + dx]) \tan[c + dx]^4}{4d}$$

Result (type 3, 246 leaves):

$$\frac{a \operatorname{Log}[\cos[c + dx]]}{d} - \frac{15b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} + \frac{15b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} - \frac{a \operatorname{Sec}[c + dx]^2}{d} + \frac{a \operatorname{Sec}[c + dx]^4}{4d} + \frac{b}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} - \frac{9b}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{b}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{9b}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{b \sin[c + dx]}{d}$$

■ **Problem 1484: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx] (a + b \sin[c + dx]) \tan[c + dx]^4 dx$$

Optimal (type 3, 103 leaves, 7 steps):

$$\frac{3 a \operatorname{ArcTanh}[\sin[c + dx]]}{8 d} - \frac{b \operatorname{Log}[\cos[c + dx]]}{d} - \frac{3 a \sec[c + dx] \tan[c + dx]}{8 d} - \frac{b \tan[c + dx]^2}{2 d} + \frac{a \sec[c + dx] \tan[c + dx]^3}{4 d} + \frac{b \tan[c + dx]^4}{4 d}$$

Result (type 3, 234 leaves):

$$\begin{aligned} & - \frac{b \operatorname{Log}[\cos[c + dx]]}{d} - \frac{3 a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} \\ & - \frac{b \sec[c + dx]^2}{d} + \frac{b \sec[c + dx]^4}{4 d} + \frac{a}{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} - \frac{5 a}{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} \\ & - \frac{a}{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{5 a}{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} \end{aligned}$$

■ **Problem 1486: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^3 (a + b \sin[c + dx]) \tan[c + dx]^2 dx$$

Optimal (type 3, 74 leaves, 6 steps):

$$\frac{a \operatorname{ArcTanh}[\sin[c + dx]]}{8 d} - \frac{a \sec[c + dx] \tan[c + dx]}{8 d} + \frac{a \sec[c + dx]^3 \tan[c + dx]}{4 d} + \frac{b \tan[c + dx]^4}{4 d}$$

Result (type 3, 207 leaves):

$$\begin{aligned} & \frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} - \frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \\ & - \frac{a}{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} - \frac{a}{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} \\ & - \frac{a}{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{a}{16 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{b \tan[c + dx]^4}{4 d} \end{aligned}$$

■ **Problem 1487: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^4 (a + b \sin[c + dx]) \tan[c + dx] dx$$

Optimal (type 3, 74 leaves, 6 steps):

$$\frac{b \operatorname{ArcTanh}[\sin[c + dx]]}{8 d} + \frac{a \sec[c + dx]^4}{4 d} - \frac{b \sec[c + dx] \tan[c + dx]}{8 d} + \frac{b \sec[c + dx]^3 \tan[c + dx]}{4 d}$$

Result (type 3, 207 leaves) :

$$\frac{b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} - \frac{b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} +$$

$$\frac{a \operatorname{Sec}[c+dx]^4}{4d} + \frac{b}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{b}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}$$

$$\frac{b}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{b}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}$$

■ **Problem 1488: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx]^5 (a+b \sin[c+dx]) dx$$

Optimal (type 3, 99 leaves, 8 steps) :

$$\frac{3b \operatorname{ArcTanh}[\sin[c+dx]]}{8d} + \frac{a \operatorname{Log}[\tan[c+dx]]}{d} + \frac{3b \operatorname{Sec}[c+dx] \tan[c+dx]}{8d} + \frac{b \operatorname{Sec}[c+dx]^3 \tan[c+dx]}{4d} + \frac{a \tan[c+dx]^2}{d} + \frac{a \tan[c+dx]^4}{4d}$$

Result (type 3, 248 leaves) :

$$-\frac{a \operatorname{Log}[\cos[c+dx]]}{d} - \frac{3b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{3b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} +$$

$$\frac{a \operatorname{Log}[\sin[c+dx]]}{d} + \frac{a \operatorname{Sec}[c+dx]^2}{2d} + \frac{a \operatorname{Sec}[c+dx]^4}{4d} + \frac{b}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} +$$

$$\frac{3b}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{b}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{3b}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}$$

■ **Problem 1489: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^2 \operatorname{Sec}[c+dx]^5 (a+b \sin[c+dx]) dx$$

Optimal (type 3, 115 leaves, 10 steps) :

$$\frac{15a \operatorname{ArcTanh}[\sin[c+dx]]}{8d} - \frac{15a \operatorname{Csc}[c+dx]}{8d} + \frac{b \operatorname{Log}[\tan[c+dx]]}{d} +$$

$$\frac{5a \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx]^2}{8d} + \frac{a \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx]^4}{4d} + \frac{b \tan[c+dx]^2}{d} + \frac{b \tan[c+dx]^4}{4d}$$

Result (type 3, 284 leaves) :

$$\begin{aligned}
& - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{2d} - \frac{b \operatorname{Log}[\operatorname{Cos}[c+dx]]}{d} - \frac{15a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{15a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \\
& \frac{b \operatorname{Log}[\operatorname{Sin}[c+dx]]}{d} + \frac{b \operatorname{Sec}[c+dx]^2}{2d} + \frac{b \operatorname{Sec}[c+dx]^4}{4d} + \frac{a}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{7a}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - \\
& \frac{a}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{7a}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2d}
\end{aligned}$$

■ **Problem 1490: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^3 \operatorname{Sec}[c+dx]^5 (a+b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 135 leaves, 10 steps):

$$\begin{aligned}
& \frac{15b \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{8d} - \frac{a \operatorname{Cot}[c+dx]^2}{2d} - \frac{15b \operatorname{Csc}[c+dx]}{8d} + \frac{3a \operatorname{Log}[\operatorname{Tan}[c+dx]]}{d} + \\
& \frac{5b \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx]^2}{8d} + \frac{b \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx]^4}{4d} + \frac{3a \operatorname{Tan}[c+dx]^2}{2d} + \frac{a \operatorname{Tan}[c+dx]^4}{4d}
\end{aligned}$$

Result (type 3, 298 leaves):

$$\begin{aligned}
& - \frac{b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{2d} - \frac{a \operatorname{Csc}[c+dx]^2}{2d} - \frac{3a \operatorname{Log}[\operatorname{Cos}[c+dx]]}{d} - \frac{15b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \\
& \frac{15b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{3a \operatorname{Log}[\operatorname{Sin}[c+dx]]}{d} + \frac{a \operatorname{Sec}[c+dx]^2}{d} + \frac{a \operatorname{Sec}[c+dx]^4}{4d} + \\
& \frac{b}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{7b}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - \\
& \frac{b}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{7b}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2d}
\end{aligned}$$

■ **Problem 1491: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^4 \operatorname{Sec}[c+dx]^5 (a+b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 155 leaves, 11 steps):

$$\begin{aligned}
& \frac{35a \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{8d} - \frac{b \operatorname{Cot}[c+dx]^2}{2d} - \frac{35a \operatorname{Csc}[c+dx]}{8d} - \frac{35a \operatorname{Csc}[c+dx]^3}{24d} + \\
& \frac{3b \operatorname{Log}[\operatorname{Tan}[c+dx]]}{d} + \frac{7a \operatorname{Csc}[c+dx]^3 \operatorname{Sec}[c+dx]^2}{8d} + \frac{a \operatorname{Csc}[c+dx]^3 \operatorname{Sec}[c+dx]^4}{4d} + \frac{3b \operatorname{Tan}[c+dx]^2}{2d} + \frac{b \operatorname{Tan}[c+dx]^4}{4d}
\end{aligned}$$

Result (type 3, 358 leaves) :

$$\begin{aligned} & - \frac{19 a \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]}{12 d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{24 d} - \frac{b \operatorname{Csc}[c+d x]^2}{2 d} - \\ & \frac{3 b \operatorname{Log}[\operatorname{Cos}[c+d x]]}{d} - \frac{35 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{35 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\ & \frac{3 b \operatorname{Log}[\operatorname{Sin}[c+d x]]}{d} + \frac{b \operatorname{Sec}[c+d x]^2}{d} + \frac{b \operatorname{Sec}[c+d x]^4}{4 d} + \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \\ & \frac{11 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \\ & \frac{11 a}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{19 a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{12 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{24 d} \end{aligned}$$

■ **Problem 1495: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^2 (a+b \operatorname{Sin}[c+d x])^2 \operatorname{Tan}[c+d x]^3 dx$$

Optimal (type 3, 116 leaves, 7 steps) :

$$\begin{aligned} & - \frac{b(3 a+4 b) \operatorname{Log}[1-\operatorname{Sin}[c+d x]]}{8 d} + \frac{(3 a-4 b) b \operatorname{Log}[1+\operatorname{Sin}[c+d x]]}{8 d} + \\ & \frac{\operatorname{Sec}[c+d x]^4 (a+b \operatorname{Sin}[c+d x])^2}{4 d} - \frac{\operatorname{Sec}[c+d x]^2 (a+b \operatorname{Sin}[c+d x]) (2 a+3 b \operatorname{Sin}[c+d x])}{4 d} \end{aligned}$$

Result (type 3, 264 leaves) :

$$\begin{aligned} & - \frac{b^2 \operatorname{Log}[\operatorname{Cos}[c+d x]]}{d} - \frac{3 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{4 d} + \frac{3 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{4 d} - \\ & \frac{b^2 \operatorname{Sec}[c+d x]^2}{d} + \frac{b^2 \operatorname{Sec}[c+d x]^4}{4 d} + \frac{a b}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{5 a b}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \\ & \frac{a b}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{5 a b}{8 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{a^2 \operatorname{Tan}[c+d x]^4}{4 d} \end{aligned}$$

■ **Problem 1496: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^3 (a+b \operatorname{Sin}[c+d x])^2 \operatorname{Tan}[c+d x]^2 dx$$

Optimal (type 3, 93 leaves, 5 steps) :

$$-\frac{(a^2 - 3b^2) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} - \frac{\sec[c + dx]^2 (4ab + (a^2 + 3b^2) \sin[c + dx])}{8d} + \frac{\sec[c + dx]^3 (a + b \sin[c + dx])^2 \tan[c + dx]}{4d}$$

Result (type 3, 216 leaves):

$$\frac{1}{16d} \left(2(a^2 - 3b^2) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c + dx) \right] - \sin \left[\frac{1}{2}(c + dx) \right] \right] - \right. \\ \left. 2(a^2 - 3b^2) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c + dx) \right] + \sin \left[\frac{1}{2}(c + dx) \right] \right] + \frac{(a + b)^2}{\left(\cos \left[\frac{1}{2}(c + dx) \right] - \sin \left[\frac{1}{2}(c + dx) \right] \right)^4} - \right. \\ \left. \frac{a^2 + 6ab + 5b^2}{\left(\cos \left[\frac{1}{2}(c + dx) \right] - \sin \left[\frac{1}{2}(c + dx) \right] \right)^2} - \frac{(a - b)^2}{\left(\cos \left[\frac{1}{2}(c + dx) \right] + \sin \left[\frac{1}{2}(c + dx) \right] \right)^4} + \frac{a^2 - 6ab + 5b^2}{\left(\cos \left[\frac{1}{2}(c + dx) \right] + \sin \left[\frac{1}{2}(c + dx) \right] \right)^2} \right)$$

■ **Problem 1497: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^4 (a + b \sin[c + dx])^2 \tan[c + dx] dx$$

Optimal (type 3, 72 leaves, 6 steps):

$$-\frac{ab \operatorname{ArcTanh}[\sin[c + dx]]}{4d} + \frac{\sec[c + dx]^4 (a + b \sin[c + dx])^2}{4d} - \frac{\sec[c + dx]^2 (b^2 + ab \sin[c + dx])}{4d}$$

Result (type 3, 200 leaves):

$$\frac{1}{16d} \left(4ab \operatorname{Log} \left[\cos \left[\frac{1}{2}(c + dx) \right] - \sin \left[\frac{1}{2}(c + dx) \right] \right] - 4ab \operatorname{Log} \left[\cos \left[\frac{1}{2}(c + dx) \right] + \sin \left[\frac{1}{2}(c + dx) \right] \right] + \frac{(a + b)^2}{\left(\cos \left[\frac{1}{2}(c + dx) \right] - \sin \left[\frac{1}{2}(c + dx) \right] \right)^4} + \right. \\ \left. \frac{a^2 - 2ab - 3b^2}{\left(\cos \left[\frac{1}{2}(c + dx) \right] - \sin \left[\frac{1}{2}(c + dx) \right] \right)^2} + \frac{(a - b)^2}{\left(\cos \left[\frac{1}{2}(c + dx) \right] + \sin \left[\frac{1}{2}(c + dx) \right] \right)^4} + \frac{a^2 + 2ab - 3b^2}{\left(\cos \left[\frac{1}{2}(c + dx) \right] + \sin \left[\frac{1}{2}(c + dx) \right] \right)^2} \right)$$

■ **Problem 1500: Result more than twice size of optimal antiderivative.**

$$\int \csc[c + dx]^3 \sec[c + dx]^5 (a + b \sin[c + dx])^2 dx$$

Optimal (type 3, 185 leaves, 6 steps):

$$-\frac{2ab \csc[c + dx]}{d} - \frac{a^2 \csc[c + dx]^2}{2d} - \frac{(12a^2 + 15ab + 4b^2) \operatorname{Log}[1 - \sin[c + dx]]}{8d} + \frac{(3a^2 + b^2) \operatorname{Log}[\sin[c + dx]]}{d} - \\ \frac{(12a^2 - 15ab + 4b^2) \operatorname{Log}[1 + \sin[c + dx]]}{8d} + \frac{\sec[c + dx]^4 (a^2 + b^2 + 2ab \sin[c + dx])}{4d} + \frac{\sec[c + dx]^2 (2(2a^2 + b^2) + 7ab \sin[c + dx])}{4d}$$

Result (type 3, 717 leaves):

$$\begin{aligned}
& - \frac{a b \cot\left[\frac{1}{2}(c+dx)\right] (b+a \operatorname{Csc}[c+dx])^2 \sin[c+dx]^2}{d(a+b \sin[c+dx])^2} - \frac{a^2 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 (b+a \operatorname{Csc}[c+dx])^2 \sin[c+dx]^2}{8 d(a+b \sin[c+dx])^2} + \\
& \frac{(-12 a^2 - 15 a b - 4 b^2) (b+a \operatorname{Csc}[c+dx])^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \sin[c+dx]^2}{4 d(a+b \sin[c+dx])^2} + \\
& \frac{(-12 a^2 + 15 a b - 4 b^2) (b+a \operatorname{Csc}[c+dx])^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \sin[c+dx]^2}{4 d(a+b \sin[c+dx])^2} + \\
& \frac{(3 a^2 + b^2) (b+a \operatorname{Csc}[c+dx])^2 \operatorname{Log}[\sin[c+dx]] \sin[c+dx]^2}{d(a+b \sin[c+dx])^2} - \frac{a^2 (b+a \operatorname{Csc}[c+dx])^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx]^2}{8 d(a+b \sin[c+dx])^2} + \\
& \frac{(a^2 + 2 a b + b^2) (b+a \operatorname{Csc}[c+dx])^2 \sin[c+dx]^2}{16 d\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 (a+b \sin[c+dx])^2} + \\
& \frac{(9 a^2 + 14 a b + 5 b^2) (b+a \operatorname{Csc}[c+dx])^2 \sin[c+dx]^2}{16 d\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 (a+b \sin[c+dx])^2} + \frac{(a^2 - 2 a b + b^2) (b+a \operatorname{Csc}[c+dx])^2 \sin[c+dx]^2}{16 d\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4 (a+b \sin[c+dx])^2} + \\
& \frac{(9 a^2 - 14 a b + 5 b^2) (b+a \operatorname{Csc}[c+dx])^2 \sin[c+dx]^2}{16 d\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2 (a+b \sin[c+dx])^2} - \frac{a b (b+a \operatorname{Csc}[c+dx])^2 \sin[c+dx]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{d(a+b \sin[c+dx])^2}
\end{aligned}$$

■ **Problem 1505: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^4 (a+b \sin[c+dx])^3 \tan[c+dx] dx$$

Optimal (type 3, 90 leaves, 5 steps):

$$-\frac{3 b (a^2 - b^2) \operatorname{ArcTanh}[\sin[c+dx]]}{8 d} + \frac{\sec[c+dx]^4 (a+b \sin[c+dx])^3}{4 d} - \frac{3 \sec[c+dx]^2 (a+b \sin[c+dx]) (b^2 + a b \sin[c+dx])}{8 d}$$

Result (type 3, 212 leaves):

$$\begin{aligned}
& \frac{1}{16 d} \left(6 b (a^2 - b^2) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \right. \\
& 6 b (-a^2 + b^2) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + \frac{(a+b)^3}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \\
& \left. \frac{(a-5 b) (a+b)^2}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{(a-b)^3}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{(a-b)^2 (a+5 b)}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)
\end{aligned}$$

■ **Problem 1508: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c + dx]^3 \text{Sec}[c + dx]^5 (a + b \text{Sin}[c + dx])^3 dx$$

Optimal (type 3, 221 leaves, 6 steps):

$$\begin{aligned} & - \frac{3 a^2 b \text{Csc}[c + dx]}{d} - \frac{a^3 \text{Csc}[c + dx]^2}{2d} - \frac{3(a+b)(8a^2 + 7ab + b^2) \text{Log}[1 - \text{Sin}[c + dx]]}{16d} + \\ & \frac{3a(a^2 + b^2) \text{Log}[\text{Sin}[c + dx]]}{d} - \frac{3(a-b)(8a^2 - 7ab + b^2) \text{Log}[1 + \text{Sin}[c + dx]]}{16d} + \\ & \frac{b^2 \text{Sec}[c + dx]^4 \left(a \left(3 + \frac{a^2}{b^2} \right) + \left(1 + \frac{3a^2}{b^2} \right) b \text{Sin}[c + dx] \right)}{4d} + \frac{b^2 \text{Sec}[c + dx]^2 \left(4a \left(3 + \frac{2a^2}{b^2} \right) + 3 \left(1 + \frac{7a^2}{b^2} \right) b \text{Sin}[c + dx] \right)}{8d} \end{aligned}$$

Result (type 3, 772 leaves):

$$\begin{aligned} & - \frac{3 a^2 b \text{Cot}\left[\frac{1}{2}(c + dx)\right] (b + a \text{Csc}[c + dx])^3 \text{Sin}[c + dx]^3}{2d(a + b \text{Sin}[c + dx])^3} - \frac{a^3 \text{Csc}\left[\frac{1}{2}(c + dx)\right]^2 (b + a \text{Csc}[c + dx])^3 \text{Sin}[c + dx]^3}{8d(a + b \text{Sin}[c + dx])^3} \\ & \frac{3(8a^3 + 15a^2b + 8ab^2 + b^3)(b + a \text{Csc}[c + dx])^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[c + dx]^3}{8d(a + b \text{Sin}[c + dx])^3} - \\ & \frac{3(8a^3 - 15a^2b + 8ab^2 - b^3)(b + a \text{Csc}[c + dx])^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sin}[c + dx]^3}{8d(a + b \text{Sin}[c + dx])^3} + \\ & \frac{3(a^3 + ab^2)(b + a \text{Csc}[c + dx])^3 \text{Log}[\text{Sin}[c + dx]] \text{Sin}[c + dx]^3}{d(a + b \text{Sin}[c + dx])^3} - \\ & \frac{a^3 (b + a \text{Csc}[c + dx])^3 \text{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \text{Sin}[c + dx]^3}{8d(a + b \text{Sin}[c + dx])^3} + \frac{(a^3 + 3a^2b + 3ab^2 + b^3)(b + a \text{Csc}[c + dx])^3 \text{Sin}[c + dx]^3}{16d(\text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right])^4 (a + b \text{Sin}[c + dx])^3} + \\ & \frac{3(3a^3 + 7a^2b + 5ab^2 + b^3)(b + a \text{Csc}[c + dx])^3 \text{Sin}[c + dx]^3}{16d(\text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right])^2 (a + b \text{Sin}[c + dx])^3} + \frac{(a^3 - 3a^2b + 3ab^2 - b^3)(b + a \text{Csc}[c + dx])^3 \text{Sin}[c + dx]^3}{16d(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right])^4 (a + b \text{Sin}[c + dx])^3} \\ & \frac{3(3a^3 - 7a^2b + 5ab^2 - b^3)(b + a \text{Csc}[c + dx])^3 \text{Sin}[c + dx]^3}{16d(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right])^2 (a + b \text{Sin}[c + dx])^3} - \frac{3a^2b(b + a \text{Csc}[c + dx])^3 \text{Sin}[c + dx]^3 \text{Tan}\left[\frac{1}{2}(c + dx)\right]}{2d(a + b \text{Sin}[c + dx])^3} \end{aligned}$$

■ **Problem 1509: Unable to integrate problem.**

$$\int \text{Sec}[c + dx]^5 \text{Sin}[c + dx]^n (a + b \text{Sin}[c + dx])^4 dx$$

Optimal (type 5, 295 leaves, 6 steps):

$$\begin{aligned}
& - \frac{1}{8d(1+n)} (6a^2b^2(1-n^2) - a^4(3-4n+n^2) - b^4(3+4n+n^2)) \operatorname{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, \sin[c+dx]^2\right] \sin[c+dx]^{1+n} - \\
& \frac{abn(a^2(2-n) - b^2(2+n)) \operatorname{Hypergeometric2F1}\left[1, \frac{2+n}{2}, \frac{4+n}{2}, \sin[c+dx]^2\right] \sin[c+dx]^{2+n}}{2d(2+n)} + \\
& \frac{\operatorname{Sec}[c+dx]^4 \sin[c+dx]^{1+n} (a^4 + 6a^2b^2 + b^4 + 4ab(a^2 + b^2) \sin[c+dx])}{4d} + \frac{1}{8d} \\
& \operatorname{Sec}[c+dx]^2 \sin[c+dx]^{1+n} (a^4(3-n) - 6a^2b^2(1+n) - b^4(5+n) + 4ab(a^2(2-n) - b^2(2+n)) \sin[c+dx])
\end{aligned}$$

Result (type 8, 31 leaves):

$$\int \operatorname{Sec}[c+dx]^5 \sin[c+dx]^n (a+b \sin[c+dx])^4 dx$$

■ **Problem 1513: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^5 \sin[c+dx]^n}{a+b \sin[c+dx]} dx$$

Optimal (type 5, 360 leaves, 10 steps):

$$\begin{aligned}
& \frac{(3a^2 - 9ab + 8b^2) \operatorname{Hypergeometric2F1}[1, 1+n, 2+n, -\sin[c+dx]] \sin[c+dx]^{1+n}}{16(a-b)^3 d(1+n)} + \\
& \frac{(3a^2 + 9ab + 8b^2) \operatorname{Hypergeometric2F1}[1, 1+n, 2+n, \sin[c+dx]] \sin[c+dx]^{1+n}}{16(a+b)^3 d(1+n)} - \\
& \frac{b^5 \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{b \sin[c+dx]}{a}\right] \sin[c+dx]^{1+n}}{a(a^2 - b^2)^3 d(1+n)} + \frac{(3a-5b) \operatorname{Hypergeometric2F1}[2, 1+n, 2+n, -\sin[c+dx]] \sin[c+dx]^{1+n}}{16(a-b)^2 d(1+n)} + \\
& \frac{(3a+5b) \operatorname{Hypergeometric2F1}[2, 1+n, 2+n, \sin[c+dx]] \sin[c+dx]^{1+n}}{16(a+b)^2 d(1+n)} + \\
& \frac{\operatorname{Hypergeometric2F1}[3, 1+n, 2+n, -\sin[c+dx]] \sin[c+dx]^{1+n}}{8(a-b)d(1+n)} + \frac{\operatorname{Hypergeometric2F1}[3, 1+n, 2+n, \sin[c+dx]] \sin[c+dx]^{1+n}}{8(a+b)d(1+n)}
\end{aligned}$$

Result (type 6, 16959 leaves):

$$\left(\operatorname{Sec}[c+dx]^5 \sin[c+dx]^n \right. \\
\left. - \left(\left(a^3(3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, \frac{1}{2}(-1+n), 1, \frac{3+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2)\tan[c+dx]^2}{a^2}\right] \tan[c+dx]^2 \left(\frac{\tan[c+dx]}{\sqrt{1+\tan[c+dx]^2}} \right)^{-1+n} \right) \right) / \right. \\
\left. \left((1+n) \left(-a^2(3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, \frac{1}{2}(-1+n), 1, \frac{3+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + \right) \right) \right)$$

$$\begin{aligned}
& \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 2, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \right. \right. \\
& \quad \left. \left. \frac{1+n}{2}, 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \Big) + \\
& \left(a^2 b (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^2 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^n \right) / \\
& \left((2+n) \left(-a^2 (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 2, \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{2+n}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) - \\
& \left(2 a^3 (5+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^4 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right) / \\
& \left((3+n) \left(-a^2 (5+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 2, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{1+n}{2}, 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) + \\
& \left(2 a^2 b (6+n) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^4 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^n \right) / \\
& \left((4+n) \left(-a^2 (6+n) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 2, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{2+n}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(a^3 (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^6 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right) / \\
& \left((5+n) \left(-a^2 (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7+n}{2}, \frac{1}{2} (-1+n), 2, \frac{9+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{7+n}{2}, \right. \right. \right. \\
& \quad \quad \left. \left. \frac{1+n}{2}, 1, \frac{9+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) + \\
& \left(a^2 b (8+n) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^6 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^n \right) / \\
& \left((6+n) \left(-a^2 (8+n) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{8+n}{2}, \frac{n}{2}, 2, 5 + \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{8+n}{2}, \frac{2+n}{2}, 1, 5 + \right. \right. \right. \\
& \quad \quad \left. \left. \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) \right) / \\
& \left(d (a+b \operatorname{Sin}[c+dx]) \left(\left(a^3 (3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, \frac{1}{2} (-1+n), 1, \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \right. \right. \right. \\
& \quad \left. \left. \operatorname{Tan}[c+dx]^2 (2 a^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - 2 b^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]) \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right) \right) / \\
& \left((1+n) \left(-a^2 (3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, \frac{1}{2} (-1+n), 1, \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 2, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \right. \right. \right. \\
& \quad \quad \left. \left. \frac{1+n}{2}, 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right)^2 \right) - \\
& \left(a^2 b (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \left(2 a^2 \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] - 2 b^2 \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \right) \left(\frac{\operatorname{Tan}[c+d x]}{\sqrt{1+\operatorname{Tan}[c+d x]^2}} \right)^n / \\
& \left((2+n) \left(-a^2 (4+n) \operatorname{AppellF1}\left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \right. \right. \\
& \quad \left. \left(2 \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{4+n}{2}, \frac{n}{2}, 2, \frac{6+n}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + a^2 n \operatorname{AppellF1}\left[\frac{4+n}{2}, \frac{2+n}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{6+n}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] \right) \operatorname{Tan}[c+d x]^2 \right) \left(-b^2 \operatorname{Tan}[c+d x]^2 + a^2 \left(1+\operatorname{Tan}[c+d x]^2\right)\right)^2 \right) + \\
& \left(2 a^3 (5+n) \operatorname{AppellF1}\left[\frac{3+n}{2}, \frac{1}{2}(-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+d x]^2, \frac{\left(-a^2+b^2\right) \operatorname{Tan}[c+d x]^2}{a^2}\right] \operatorname{Tan}[c+d x]^4 \right. \\
& \quad \left. \left(2 a^2 \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] - 2 b^2 \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \right) \left(\frac{\operatorname{Tan}[c+d x]}{\sqrt{1+\operatorname{Tan}[c+d x]^2}} \right)^{-1+n} \right) / \\
& \left((3+n) \left(-a^2 (5+n) \operatorname{AppellF1}\left[\frac{3+n}{2}, \frac{1}{2}(-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \right. \right. \\
& \quad \left. \left(2 \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5+n}{2}, \frac{1}{2}(-1+n), 2, \frac{7+n}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + a^2 (-1+n) \operatorname{AppellF1}\left[\frac{5+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1+n}{2}, 1, \frac{7+n}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] \right) \operatorname{Tan}[c+d x]^2 \right) \left(-b^2 \operatorname{Tan}[c+d x]^2 + a^2 \left(1+\operatorname{Tan}[c+d x]^2\right)\right)^2 \right) - \\
& \left(2 a^2 b (6+n) \operatorname{AppellF1}\left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\operatorname{Tan}[c+d x]^2, \frac{\left(-a^2+b^2\right) \operatorname{Tan}[c+d x]^2}{a^2}\right] \operatorname{Tan}[c+d x]^4 \right. \\
& \quad \left. \left(2 a^2 \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] - 2 b^2 \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \right) \left(\frac{\operatorname{Tan}[c+d x]}{\sqrt{1+\operatorname{Tan}[c+d x]^2}} \right)^n \right) / \\
& \left((4+n) \left(-a^2 (6+n) \operatorname{AppellF1}\left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + \right. \right. \\
& \quad \left. \left(2 \left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{6+n}{2}, \frac{n}{2}, 2, \frac{8+n}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] + a^2 n \operatorname{AppellF1}\left[\frac{6+n}{2}, \frac{2+n}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{8+n}{2}, -\operatorname{Tan}[c+d x]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+d x]^2\right] \right) \operatorname{Tan}[c+d x]^2 \right) \left(-b^2 \operatorname{Tan}[c+d x]^2 + a^2 \left(1+\operatorname{Tan}[c+d x]^2\right)\right)^2 \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(a^3 (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^6 \right. \\
& \quad \left. (2 a^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - 2 b^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]) \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right) / \\
& \left((5+n) \left(-a^2 (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7+n}{2}, \frac{1}{2} (-1+n), 2, \frac{9+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{7+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1+n}{2}, 1, \frac{9+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \right) (-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2))^2 \right) - \\
& \left(a^2 b (8+n) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^6 \right. \\
& \quad \left. (2 a^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - 2 b^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]) \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^n \right) / \\
& \left((6+n) \left(-a^2 (8+n) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{8+n}{2}, \frac{n}{2}, 2, 5 + \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{8+n}{2}, \frac{2+n}{2}, 1, 5 + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \right) (-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2))^2 \right) - \\
& \left(2 a^3 (3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, \frac{1}{2} (-1+n), 1, \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \right. \\
& \quad \left. \operatorname{Tan}[c+dx] \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right) / \\
& \left((1+n) \left(-a^2 (3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, \frac{1}{2} (-1+n), 1, \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 2, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{1+n}{2}, 1, \frac{5+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right) \tan[c+dx]^2 \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2)\right) \Bigg) - \\
& \left(a^3 (3+n) \tan[c+dx]^2 \left(-\frac{1}{3+n} (-1+n) (1+n) \operatorname{AppellF1} \left[1 + \frac{1+n}{2}, 1 + \frac{1}{2} (-1+n), 1, 1 + \frac{3+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}[c+dx]^2 \tan[c+dx] + \frac{1}{a^2 (3+n)} 2 (-a^2+b^2) (1+n) \operatorname{AppellF1} \left[1 + \frac{1+n}{2}, \frac{1}{2} (-1+n), 2, 1 + \frac{3+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \left(\frac{\tan[c+dx]}{\sqrt{1 + \tan[c+dx]^2}} \right)^{-1+n} \right) \Bigg) / \\
& \left((1+n) \left(-a^2 (3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, \frac{1}{2} (-1+n), 1, \frac{3+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 2, \frac{5+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1+n}{2}, 1, \frac{5+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right) \tan[c+dx]^2 \right) \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2)\right) \right) \Bigg) + \\
& \left(2 a^2 b (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \left(\frac{\tan[c+dx]}{\sqrt{1 + \tan[c+dx]^2}} \right)^n \right) \Bigg) / \\
& \left((2+n) \left(-a^2 (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 2, \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{2+n}{2}, 1, \frac{6+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right) \tan[c+dx]^2 \right) \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2)\right) \right) \Bigg) + \left(a^2 b (4+n) \tan[c+dx]^2 \right. \\
& \quad \left. \left(-\frac{1}{4+n} n (2+n) \operatorname{AppellF1} \left[1 + \frac{2+n}{2}, 1 + \frac{n}{2}, 1, 1 + \frac{4+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] + \right. \right. \\
& \quad \left. \left. \frac{1}{a^2 (4+n)} 2 (-a^2+b^2) (2+n) \operatorname{AppellF1} \left[1 + \frac{2+n}{2}, \frac{n}{2}, 2, 1 + \frac{4+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \Bigg) \\
& \left(\frac{\tan[c+dx]}{\sqrt{1 + \tan[c+dx]^2}} \right)^n \Bigg) / \left((2+n) \left(-a^2 (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 2, \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{2+n}{2}, 1, \right. \right. \\
& \quad \left. \left. \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) - \\
& \left(8 a^3 (5+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \right. \\
& \quad \left. \operatorname{Tan}[c+dx]^3 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right) / \\
& \left((3+n) \left(-a^2 (5+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 2, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{1+n}{2}, 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) - \\
& \left(2 a^3 (5+n) \operatorname{Tan}[c+dx]^4 \left(-\frac{1}{5+n} (-1+n) (3+n) \operatorname{AppellF1} \left[1 + \frac{3+n}{2}, 1 + \frac{1}{2} (-1+n), 1, 1 + \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \frac{1}{a^2 (5+n)} 2 (-a^2+b^2) (3+n) \operatorname{AppellF1} \left[1 + \frac{3+n}{2}, \frac{1}{2} (-1+n), \right. \right. \\
& \quad \left. \left. 2, 1 + \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right) / \\
& \left((3+n) \left(-a^2 (5+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 2, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{1+n}{2}, 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) + \\
& \left(8 a^2 b (6+n) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^3 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^n \right) /
\end{aligned}$$

$$\begin{aligned}
& \left((4+n) \left(-a^2 (6+n) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right. \right. \\
& \quad \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 2, \frac{8+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{2+n}{2}, 1, \frac{8+n}{2}, \right. \right. \\
& \quad \left. \left. -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2) \right) \right) + \left(2 a^2 b (6+n) \tan[c+dx]^4 \right. \\
& \quad \left(-\frac{1}{6+n} n (4+n) \operatorname{AppellF1} \left[1 + \frac{4+n}{2}, 1 + \frac{n}{2}, 1, 1 + \frac{6+n}{2}, -\tan[c+dx]^2, \frac{(-a^2 + b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] + \right. \\
& \quad \left. \frac{1}{a^2 (6+n)} 2 (-a^2 + b^2) (4+n) \operatorname{AppellF1} \left[1 + \frac{4+n}{2}, \frac{n}{2}, 2, 1 + \frac{6+n}{2}, -\tan[c+dx]^2, \frac{(-a^2 + b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \\
& \quad \left(\frac{\tan[c+dx]}{\sqrt{1 + \tan[c+dx]^2}} \right)^n \Bigg/ \left((4+n) \left(-a^2 (6+n) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right. \right. \\
& \quad \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 2, \frac{8+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{2+n}{2}, 1, \right. \right. \\
& \quad \left. \left. \frac{8+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2) \right) \Bigg) - \\
& \quad \left(6 a^3 (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\tan[c+dx]^2, \frac{(-a^2 + b^2) \tan[c+dx]^2}{a^2} \right] \operatorname{Sec}[c+dx]^2 \right. \\
& \quad \left. \tan[c+dx]^5 \left(\frac{\tan[c+dx]}{\sqrt{1 + \tan[c+dx]^2}} \right)^{-1+n} \right) \Bigg/ \\
& \quad \left((5+n) \left(-a^2 (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right. \right. \\
& \quad \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7+n}{2}, \frac{1}{2} (-1+n), 2, \frac{9+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{7+n}{2}, \right. \right. \\
& \quad \left. \left. \frac{1+n}{2}, 1, \frac{9+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2) \right) \Bigg) - \\
& \quad \left(a^3 (7+n) \tan[c+dx]^6 \left(-\frac{1}{7+n} (-1+n) (5+n) \operatorname{AppellF1} \left[1 + \frac{5+n}{2}, 1 + \frac{1}{2} (-1+n), 1, 1 + \frac{7+n}{2}, -\tan[c+dx]^2, \frac{(-a^2 + b^2) \tan[c+dx]^2}{a^2} \right] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \frac{1}{a^2(7+n)} 2(-a^2+b^2)(5+n) \operatorname{AppellF1}\left[1+\frac{5+n}{2}, \frac{1}{2}(-1+n), 2, 1+\frac{7+n}{2}, \right. \\
& \left. -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[c+dx]^2}{a^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}}\right)^{-1+n} \Big/ \\
& \left((5+n) \left(-a^2(7+n) \operatorname{AppellF1}\left[\frac{5+n}{2}, \frac{1}{2}(-1+n), 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[c+dx]^2\right] + \right. \right. \\
& \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7+n}{2}, \frac{1}{2}(-1+n), 2, \frac{9+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[c+dx]^2\right] + a^2(-1+n) \operatorname{AppellF1}\left[\frac{7+n}{2}, \right. \right. \right. \\
& \left. \left. \frac{1+n}{2}, 1, \frac{9+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[c+dx]^2\right]\right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2(1+\operatorname{Tan}[c+dx]^2)\right) \Big) + \\
& \left(6a^2b(8+n) \operatorname{AppellF1}\left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[c+dx]^2}{a^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^5 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}}\right)^n \Big/ \right. \\
& \left. \left((6+n) \left(-a^2(8+n) \operatorname{AppellF1}\left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[c+dx]^2\right] + \right. \right. \\
& \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{8+n}{2}, \frac{n}{2}, 2, 5+\frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[c+dx]^2\right] + a^2n \operatorname{AppellF1}\left[\frac{8+n}{2}, \frac{2+n}{2}, 1, 5+\frac{n}{2}, \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[c+dx]^2\right]\right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2(1+\operatorname{Tan}[c+dx]^2)\right) \Big) + \left(a^2b(8+n) \operatorname{Tan}[c+dx]^6 \right. \\
& \left. \left(-\frac{1}{8+n}n(6+n) \operatorname{AppellF1}\left[1+\frac{6+n}{2}, 1+\frac{n}{2}, 1, 1+\frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[c+dx]^2}{a^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \right. \right. \\
& \left. \left. \frac{1}{a^2(8+n)} 2(-a^2+b^2)(6+n) \operatorname{AppellF1}\left[1+\frac{6+n}{2}, \frac{n}{2}, 2, 1+\frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2)\operatorname{Tan}[c+dx]^2}{a^2}\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right. \\
& \left. \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}}\right)^n \Big/ \left((6+n) \left(-a^2(8+n) \operatorname{AppellF1}\left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[c+dx]^2\right] + \right. \right. \right. \\
& \left. \left(2(a^2-b^2) \operatorname{AppellF1}\left[\frac{8+n}{2}, \frac{n}{2}, 2, 5+\frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[c+dx]^2\right] + a^2n \operatorname{AppellF1}\left[\frac{8+n}{2}, \frac{2+n}{2}, 1, 5+\frac{n}{2}, \right. \right. \right. \\
& \left. \left. \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right)\operatorname{Tan}[c+dx]^2\right]\right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2(1+\operatorname{Tan}[c+dx]^2)\right) \Big) -
\end{aligned}$$

$$\begin{aligned}
& \left(a^3 (-1+n) (3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, \frac{1}{2} (-1+n), 1, \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^2 \right. \\
& \quad \left. \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-2+n} \left(-\frac{\operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^2}{(1+\operatorname{Tan}[c+dx]^2)^{3/2}} + \frac{\operatorname{Sec}[c+dx]^2}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right) \right) / \\
& \quad \left((1+n) \left(-a^2 (3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, \frac{1}{2} (-1+n), 1, \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 2, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1+n}{2}, 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Tan}[c+dx]^2 \right) \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1+\operatorname{Tan}[c+dx]^2) \right) \right) \right) + \\
& \quad \left(a^2 b n (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^2 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right. \\
& \quad \left. \left(-\frac{\operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^2}{(1+\operatorname{Tan}[c+dx]^2)^{3/2}} + \frac{\operatorname{Sec}[c+dx]^2}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right) \right) / \\
& \quad \left((2+n) \left(-a^2 (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 2, \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{2+n}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Tan}[c+dx]^2 \right) \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1+\operatorname{Tan}[c+dx]^2) \right) \right) \right) - \\
& \quad \left(2 a^3 (-1+n) (5+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^4 \right. \\
& \quad \left. \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-2+n} \left(-\frac{\operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^2}{(1+\operatorname{Tan}[c+dx]^2)^{3/2}} + \frac{\operatorname{Sec}[c+dx]^2}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right) \right) / \\
& \quad \left((3+n) \left(-a^2 (5+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 2, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{1+n}{2}, 1, \frac{7+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right) \tan[c+dx]^2 \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2)\right) \Bigg) + \\
& \left(2 a^2 b n (6+n) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \tan[c+dx]^4 \right. \\
& \quad \left. \left(\frac{\tan[c+dx]}{\sqrt{1+\tan[c+dx]^2}} \right)^{-1+n} \left(-\frac{\sec[c+dx]^2 \tan[c+dx]^2}{(1+\tan[c+dx]^2)^{3/2}} + \frac{\sec[c+dx]^2}{\sqrt{1+\tan[c+dx]^2}} \right) \right) / \\
& \left((4+n) \left(-a^2 (6+n) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 2, \frac{8+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{2+n}{2}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{8+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2)\right) \right) - \\
& \left(a^3 (-1+n) (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \tan[c+dx]^6 \right. \\
& \quad \left. \left(\frac{\tan[c+dx]}{\sqrt{1+\tan[c+dx]^2}} \right)^{-2+n} \left(-\frac{\sec[c+dx]^2 \tan[c+dx]^2}{(1+\tan[c+dx]^2)^{3/2}} + \frac{\sec[c+dx]^2}{\sqrt{1+\tan[c+dx]^2}} \right) \right) / \\
& \left((5+n) \left(-a^2 (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7+n}{2}, \frac{1}{2} (-1+n), 2, \frac{9+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{7+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1+n}{2}, 1, \frac{9+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2)\right) \right) \Bigg) + \\
& \left(a^2 b n (8+n) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \tan[c+dx]^6 \left(\frac{\tan[c+dx]}{\sqrt{1+\tan[c+dx]^2}} \right)^{-1+n} \right. \\
& \quad \left. \left(-\frac{\sec[c+dx]^2 \tan[c+dx]^2}{(1+\tan[c+dx]^2)^{3/2}} + \frac{\sec[c+dx]^2}{\sqrt{1+\tan[c+dx]^2}} \right) \right) / \\
& \left((6+n) \left(-a^2 (8+n) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \tan[c+dx]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{8+n}{2}, \frac{n}{2}, 2, 5 + \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{8+n}{2}, \frac{2+n}{2}, 1, 5 + \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) + \\
& \left(a^3 (3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, \frac{1}{2} (-1+n), 1, \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^2 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right. \\
& \left(2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 2, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \\
& \left. a^2 (-1+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1+n}{2}, 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - a^2 (3+n) \\
& \left(-\frac{1}{3+n} (-1+n) (1+n) \operatorname{AppellF1} \left[1 + \frac{1+n}{2}, 1 + \frac{1}{2} (-1+n), 1, 1 + \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \right. \\
& \left. \operatorname{Tan}[c+dx] + \frac{1}{3+n} 2 \left(-1 + \frac{b^2}{a^2} \right) (1+n) \operatorname{AppellF1} \left[1 + \frac{1+n}{2}, \frac{1}{2} (-1+n), 2, 1 + \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \\
& \left. \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + \operatorname{Tan}[c+dx]^2 \left(2 (a^2 - b^2) \left(-\frac{1}{5+n} (-1+n) (3+n) \operatorname{AppellF1} \left[1 + \frac{3+n}{2}, 1 + \frac{1}{2} (-1+n), 2, 1 + \frac{5+n}{2}, \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \frac{1}{5+n} 4 \left(-1 + \frac{b^2}{a^2} \right) (3+n) \operatorname{AppellF1} \left[1 + \frac{3+n}{2}, \frac{1}{2} (-1+n), \right. \right. \\
& \left. \left. 3, 1 + \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + a^2 (-1+n) \left(\frac{1}{5+n} 2 \left(-1 + \frac{b^2}{a^2} \right) (3+n) \right. \\
& \left. \operatorname{AppellF1} \left[1 + \frac{3+n}{2}, \frac{1+n}{2}, 2, 1 + \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \frac{1}{5+n} (1+n) \right. \\
& \left. \left. (3+n) \operatorname{AppellF1} \left[1 + \frac{3+n}{2}, 1 + \frac{1+n}{2}, 1, 1 + \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) \Big) \Big) / \\
& \left((1+n) \left(-a^2 (3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, \frac{1}{2} (-1+n), 1, \frac{3+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 2, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \right. \right. \right. \\
& \left. \left. \frac{1+n}{2}, 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(a^2 b (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \tan[c+dx]^2 \left(\frac{\tan[c+dx]}{\sqrt{1+\tan[c+dx]^2}} \right)^n \right. \\
& \left(2 \left(2 (a^2-b^2) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 2, \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right. \right. \\
& \left. \left. a^2 n \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{2+n}{2}, 1, \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \sec[c+dx]^2 \tan[c+dx] - \right. \\
& a^2 (4+n) \left(-\frac{1}{4+n} n (2+n) \operatorname{AppellF1} \left[1 + \frac{2+n}{2}, 1 + \frac{n}{2}, 1, 1 + \frac{4+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \sec[c+dx]^2 \tan[c+dx] + \right. \\
& \left. \frac{1}{4+n} 2 \left(-1 + \frac{b^2}{a^2} \right) (2+n) \operatorname{AppellF1} \left[1 + \frac{2+n}{2}, \frac{n}{2}, 2, 1 + \frac{4+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \sec[c+dx]^2 \tan[c+dx] \right) + \\
& \tan[c+dx]^2 \left(2 (a^2-b^2) \left(-\frac{1}{6+n} n (4+n) \operatorname{AppellF1} \left[1 + \frac{4+n}{2}, 1 + \frac{n}{2}, 2, 1 + \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right. \right. \\
& \left. \left. \sec[c+dx]^2 \tan[c+dx] + \frac{1}{6+n} 4 \left(-1 + \frac{b^2}{a^2} \right) (4+n) \operatorname{AppellF1} \left[1 + \frac{4+n}{2}, \frac{n}{2}, 3, 1 + \frac{6+n}{2}, \right. \right. \right. \\
& \left. \left. -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \sec[c+dx]^2 \tan[c+dx] \right) + a^2 n \left(\frac{1}{6+n} 2 \left(-1 + \frac{b^2}{a^2} \right) (4+n) \right. \\
& \left. \operatorname{AppellF1} \left[1 + \frac{4+n}{2}, \frac{2+n}{2}, 2, 1 + \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \sec[c+dx]^2 \tan[c+dx] - \frac{1}{6+n} (2+n) \right. \\
& \left. \left. (4+n) \operatorname{AppellF1} \left[1 + \frac{4+n}{2}, 1 + \frac{2+n}{2}, 1, 1 + \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \sec[c+dx]^2 \tan[c+dx] \right) \right) \right) / \\
& \left((2+n) \left(-a^2 (4+n) \operatorname{AppellF1} \left[\frac{2+n}{2}, \frac{n}{2}, 1, \frac{4+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2-b^2) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 2, \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{2+n}{2}, 1, \right. \right. \right. \\
& \left. \left. \frac{6+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] \right) \tan[c+dx]^2 \left(-b^2 \tan[c+dx]^2 + a^2 (1 + \tan[c+dx]^2) \right) \right) + \\
& \left(2 a^3 (5+n) \operatorname{AppellF1} \left[\frac{3+n}{2}, \frac{1}{2} (-1+n), 1, \frac{5+n}{2}, -\tan[c+dx]^2, \frac{(-a^2+b^2) \tan[c+dx]^2}{a^2} \right] \tan[c+dx]^4 \right. \\
& \left. \left(\frac{\tan[c+dx]}{\sqrt{1+\tan[c+dx]^2}} \right)^{-1+n} \left(2 \left(2 (a^2-b^2) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 2, \frac{7+n}{2}, -\tan[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[c+dx]^2 \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& a^2 (-1+n) \operatorname{AppellF1}\left[\frac{5+n}{2}, \frac{1+n}{2}, 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \\
& a^2 (5+n) \left(-\frac{1}{5+n} (-1+n) (3+n) \operatorname{AppellF1}\left[1+\frac{3+n}{2}, 1+\frac{1}{2}(-1+n), 1, 1+\frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \\
& \quad \left. \left. \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \frac{1}{5+n} 2 \left(-1+\frac{b^2}{a^2}\right) (3+n) \right. \\
& \quad \left. \operatorname{AppellF1}\left[1+\frac{3+n}{2}, \frac{1}{2}(-1+n), 2, 1+\frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + \\
& \operatorname{Tan}[c+dx]^2 \left(2 (a^2-b^2) \left(-\frac{1}{7+n} (-1+n) (5+n) \operatorname{AppellF1}\left[1+\frac{5+n}{2}, 1+\frac{1}{2}(-1+n), 2, 1+\frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \frac{1}{7+n} 4 \left(-1+\frac{b^2}{a^2}\right) (5+n) \operatorname{AppellF1}\left[1+\frac{5+n}{2}, \frac{1}{2}(-1+n), 3, \right. \right. \\
& \quad \left. \left. 1+\frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + a^2 (-1+n) \left(\frac{1}{7+n} 2 \left(-1+\frac{b^2}{a^2}\right) (5+n) \right. \\
& \quad \left. \operatorname{AppellF1}\left[1+\frac{5+n}{2}, \frac{1+n}{2}, 2, 1+\frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \frac{1}{7+n} (1+n) \right. \\
& \quad \left. \left. (5+n) \operatorname{AppellF1}\left[1+\frac{5+n}{2}, 1+\frac{1+n}{2}, 1, 1+\frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) \right) \Bigg) / \\
& \left((3+n) \left(-a^2 (5+n) \operatorname{AppellF1}\left[\frac{3+n}{2}, \frac{1}{2}(-1+n), 1, \frac{5+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] + \right. \right. \\
& \quad \left. \left(2 (a^2-b^2) \operatorname{AppellF1}\left[\frac{5+n}{2}, \frac{1}{2}(-1+n), 2, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] + a^2 (-1+n) \operatorname{AppellF1}\left[\frac{5+n}{2}, \right. \right. \right. \\
& \quad \left. \left. \frac{1+n}{2}, 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Tan}[c+dx]^2 \right)^2 (-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2)) \right) \Bigg) - \\
& \left(2 a^2 b (6+n) \operatorname{AppellF1}\left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2+b^2) \operatorname{Tan}[c+dx]^2}{a^2}\right] \operatorname{Tan}[c+dx]^4 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right)^n \right. \\
& \quad \left. \left(2 \left(2 (a^2-b^2) \operatorname{AppellF1}\left[\frac{6+n}{2}, \frac{n}{2}, 2, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] + \right. \right. \right. \\
& \quad \left. \left. a^2 n \operatorname{AppellF1}\left[\frac{6+n}{2}, \frac{2+n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \\
& \quad \left. a^2 (6+n) \left(-\frac{1}{6+n} n (4+n) \operatorname{AppellF1}\left[1+\frac{4+n}{2}, 1+\frac{n}{2}, 1, 1+\frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1+\frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{6+n} 2 \left(-1 + \frac{b^2}{a^2} \right) (4+n) \operatorname{AppellF1} \left[1 + \frac{4+n}{2}, \frac{n}{2}, 2, 1 + \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \Bigg) + \\
& \operatorname{Tan}[c+dx]^2 \left(2 (a^2 - b^2) \left(-\frac{1}{8+n} n (6+n) \operatorname{AppellF1} \left[1 + \frac{6+n}{2}, 1 + \frac{n}{2}, 2, 1 + \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right. \right. \\
& \quad \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \frac{1}{8+n} 4 \left(-1 + \frac{b^2}{a^2} \right) (6+n) \operatorname{AppellF1} \left[1 + \frac{6+n}{2}, \frac{n}{2}, 3, 1 + \frac{8+n}{2}, \right. \\
& \quad \left. \left. -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + a^2 n \left(\frac{1}{8+n} 2 \left(-1 + \frac{b^2}{a^2} \right) (6+n) \right. \\
& \quad \left. \operatorname{AppellF1} \left[1 + \frac{6+n}{2}, \frac{2+n}{2}, 2, 1 + \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \frac{1}{8+n} (2+n) \right. \\
& \quad \left. \left. (6+n) \operatorname{AppellF1} \left[1 + \frac{6+n}{2}, 1 + \frac{2+n}{2}, 1, 1 + \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) \Bigg) \Bigg) / \\
& \left((4+n) \left(-a^2 (6+n) \operatorname{AppellF1} \left[\frac{4+n}{2}, \frac{n}{2}, 1, \frac{6+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 2, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + a^2 n \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{2+n}{2}, 1, \right. \right. \\
& \quad \left. \left. \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \Bigg) + \\
& \left(a^3 (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c+dx]^2}{a^2} \right] \operatorname{Tan}[c+dx]^6 \left(\frac{\operatorname{Tan}[c+dx]}{\sqrt{1 + \operatorname{Tan}[c+dx]^2}} \right)^{-1+n} \right. \\
& \quad \left(2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7+n}{2}, \frac{1}{2} (-1+n), 2, \frac{9+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] + \right. \right. \\
& \quad \left. \left. a^2 (-1+n) \operatorname{AppellF1} \left[\frac{7+n}{2}, \frac{1+n}{2}, 1, \frac{9+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \\
& \quad \left. a^2 (7+n) \left(-\frac{1}{7+n} (-1+n) (5+n) \operatorname{AppellF1} \left[1 + \frac{5+n}{2}, 1 + \frac{1}{2} (-1+n), 1, 1 + \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \frac{1}{7+n} 2 \left(-1 + \frac{b^2}{a^2} \right) (5+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[1 + \frac{5+n}{2}, \frac{1}{2} (-1+n), 2, 1 + \frac{7+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + \right. \\
& \quad \left. \operatorname{Tan}[c+dx]^2 \left(2 (a^2 - b^2) \left(-\frac{1}{9+n} (-1+n) (7+n) \operatorname{AppellF1} \left[1 + \frac{7+n}{2}, 1 + \frac{1}{2} (-1+n), 2, 1 + \frac{9+n}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \frac{1}{9+n} 4 \left(-1 + \frac{b^2}{a^2} \right) (7+n) \operatorname{AppellF1} \left[1 + \frac{7+n}{2}, \frac{1}{2} (-1+n), 3, \right. \\
& \left. 1 + \frac{9+n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \left. \right) + a^2 (-1+n) \left(\frac{1}{9+n} 2 \left(-1 + \frac{b^2}{a^2} \right) (7+n) \right. \\
& \left. \operatorname{AppellF1} \left[1 + \frac{7+n}{2}, \frac{1+n}{2}, 2, 1 + \frac{9+n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] - \frac{1}{9+n} (1+n) \right. \\
& \left. (7+n) \operatorname{AppellF1} \left[1 + \frac{7+n}{2}, 1 + \frac{1+n}{2}, 1, 1 + \frac{9+n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) \left. \right) \Bigg) / \\
& \left((5+n) \left(-a^2 (7+n) \operatorname{AppellF1} \left[\frac{5+n}{2}, \frac{1}{2} (-1+n), 1, \frac{7+n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7+n}{2}, \frac{1}{2} (-1+n), 2, \frac{9+n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + a^2 (-1+n) \operatorname{AppellF1} \left[\frac{7+n}{2}, \right. \right. \right. \\
& \left. \left. \frac{1+n}{2}, 1, \frac{9+n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Tan}[c + d x]^2 \left. \right)^2 \left(-b^2 \operatorname{Tan}[c + d x]^2 + a^2 (1 + \operatorname{Tan}[c + d x]^2) \right) \Bigg) - \\
& \left(a^2 b (8+n) \operatorname{AppellF1} \left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c + d x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[c + d x]^2}{a^2} \right] \operatorname{Tan}[c + d x]^6 \left(\frac{\operatorname{Tan}[c + d x]}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} \right)^n \right. \\
& \left. \left(2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{8+n}{2}, \frac{n}{2}, 2, 5 + \frac{n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] + \right. \right. \right. \\
& \left. \left. a^2 n \operatorname{AppellF1} \left[\frac{8+n}{2}, \frac{2+n}{2}, 1, 5 + \frac{n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] - \right. \\
& \left. a^2 (8+n) \left(-\frac{1}{8+n} n (6+n) \operatorname{AppellF1} \left[1 + \frac{6+n}{2}, 1 + \frac{n}{2}, 1, 1 + \frac{8+n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \right. \right. \\
& \left. \left. \frac{1}{8+n} 2 \left(-1 + \frac{b^2}{a^2} \right) (6+n) \operatorname{AppellF1} \left[1 + \frac{6+n}{2}, \frac{n}{2}, 2, 1 + \frac{8+n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) + \right. \\
& \left. \operatorname{Tan}[c + d x]^2 \left(2 (a^2 - b^2) \left(-\frac{1}{2 (5 + \frac{n}{2})} n (8+n) \operatorname{AppellF1} \left[1 + \frac{8+n}{2}, 1 + \frac{n}{2}, 2, 6 + \frac{n}{2}, -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \right. \right. \right. \\
& \left. \left. \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] + \frac{1}{5 + \frac{n}{2}} 2 \left(-1 + \frac{b^2}{a^2} \right) (8+n) \operatorname{AppellF1} \left[1 + \frac{8+n}{2}, \frac{n}{2}, 3, 6 + \frac{n}{2}, \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}[c + d x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[c + d x]^2 \right] \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) + a^2 n \left(\frac{1}{5 + \frac{n}{2}} \left(-1 + \frac{b^2}{a^2} \right) (8+n) \right. \right.
\end{aligned}$$

$$\begin{aligned} & \operatorname{AppellF1}\left[1 + \frac{8+n}{2}, \frac{2+n}{2}, 2, 6 + \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \frac{1}{2\left(5 + \frac{n}{2}\right)} \\ & (2+n)(8+n) \operatorname{AppellF1}\left[1 + \frac{8+n}{2}, 1 + \frac{2+n}{2}, 1, 6 + \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right] \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\ & \left((6+n) \left(-a^2 (8+n) \operatorname{AppellF1}\left[\frac{6+n}{2}, \frac{n}{2}, 1, \frac{8+n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] + \right. \right. \\ & \left(2(a^2 - b^2) \operatorname{AppellF1}\left[\frac{8+n}{2}, \frac{n}{2}, 2, 5 + \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] + a^2 n \operatorname{AppellF1}\left[\frac{8+n}{2}, \frac{2+n}{2}, 1, \right. \right. \\ & \left. \left. 5 + \frac{n}{2}, -\operatorname{Tan}[c+dx]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[c+dx]^2\right] \right) \operatorname{Tan}[c+dx]^2 \left(-b^2 \operatorname{Tan}[c+dx]^2 + a^2 (1 + \operatorname{Tan}[c+dx]^2) \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \end{aligned}$$

■ **Problem 1514: Unable to integrate problem.**

$$\int \operatorname{Sec}[c+dx]^5 \operatorname{Sin}[c+dx]^n (a+b \operatorname{Sin}[c+dx])^p dx$$

Optimal (type 6, 487 leaves, 17 steps):

$$\begin{aligned} & \frac{1}{16 d (1+n)} {}_3 \operatorname{AppellF1}\left[1+n, -p, 1, 2+n, -\frac{b \operatorname{Sin}[c+dx]}{a}, -\operatorname{Sin}[c+dx]\right] \operatorname{Sin}[c+dx]^{1+n} (a+b \operatorname{Sin}[c+dx])^p \left(1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right)^{-p} + \\ & \frac{1}{16 d (1+n)} {}_3 \operatorname{AppellF1}\left[1+n, -p, 1, 2+n, -\frac{b \operatorname{Sin}[c+dx]}{a}, \operatorname{Sin}[c+dx]\right] \operatorname{Sin}[c+dx]^{1+n} (a+b \operatorname{Sin}[c+dx])^p \left(1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right)^{-p} + \\ & \frac{1}{16 d (1+n)} {}_3 \operatorname{AppellF1}\left[1+n, -p, 2, 2+n, -\frac{b \operatorname{Sin}[c+dx]}{a}, -\operatorname{Sin}[c+dx]\right] \operatorname{Sin}[c+dx]^{1+n} (a+b \operatorname{Sin}[c+dx])^p \left(1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right)^{-p} + \\ & \frac{1}{16 d (1+n)} {}_3 \operatorname{AppellF1}\left[1+n, -p, 2, 2+n, -\frac{b \operatorname{Sin}[c+dx]}{a}, \operatorname{Sin}[c+dx]\right] \operatorname{Sin}[c+dx]^{1+n} (a+b \operatorname{Sin}[c+dx])^p \left(1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right)^{-p} + \\ & \frac{1}{8 d (1+n)} \operatorname{AppellF1}\left[1+n, -p, 3, 2+n, -\frac{b \operatorname{Sin}[c+dx]}{a}, -\operatorname{Sin}[c+dx]\right] \operatorname{Sin}[c+dx]^{1+n} (a+b \operatorname{Sin}[c+dx])^p \left(1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right)^{-p} + \\ & \frac{1}{8 d (1+n)} \operatorname{AppellF1}\left[1+n, -p, 3, 2+n, -\frac{b \operatorname{Sin}[c+dx]}{a}, \operatorname{Sin}[c+dx]\right] \operatorname{Sin}[c+dx]^{1+n} (a+b \operatorname{Sin}[c+dx])^p \left(1 + \frac{b \operatorname{Sin}[c+dx]}{a}\right)^{-p} \end{aligned}$$

Result (type 8, 31 leaves):

$$\int \operatorname{Sec}[c+dx]^5 \operatorname{Sin}[c+dx]^n (a+b \operatorname{Sin}[c+dx])^p dx$$

■ **Problem 1515: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]^6 (a+b \operatorname{Sin}[e+fx])^{9/2}}{\sqrt{d \operatorname{Sin}[e+fx]}} dx$$

Optimal (type 4, 502 leaves, ? steps):

$$\begin{aligned}
 & - \frac{3 a b (-2 a^2 + b^2) \operatorname{Cos}[e + f x] \sqrt{a + b \operatorname{Sin}[e + f x]}}{5 f \sqrt{d} \operatorname{Sin}[e + f x]} + \\
 & \frac{\operatorname{Sec}[e + f x]^5 \sqrt{d} \operatorname{Sin}[e + f x] (a + b \operatorname{Sin}[e + f x])^{9/2}}{5 d f} - \frac{1}{20 d f} 3 a \operatorname{Sec}[e + f x]^3 \sqrt{d} \operatorname{Sin}[e + f x] \sqrt{a + b \operatorname{Sin}[e + f x]} \\
 & (-a (7 a^2 + b^2) + 2 b (-7 a^2 + b^2) \operatorname{Sin}[e + f x] + 5 a (a^2 - b^2) \operatorname{Sin}[e + f x]^2 + (8 a^2 b - 4 b^3) \operatorname{Sin}[e + f x]^3) - \frac{1}{20 \sqrt{d} f} 3 a (a + b)^{3/2} (5 a^2 + 3 a b - 4 b^2) \\
 & \sqrt{-\frac{a (-1 + \operatorname{Csc}[e + f x])}{a + b}} \sqrt{\frac{a (1 + \operatorname{Csc}[e + f x])}{a - b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{d} \operatorname{Sin}[e + f x]}\right], -\frac{a + b}{a - b}\right] \operatorname{Tan}[e + f x] - \\
 & \frac{1}{5 d f \sqrt{a + b \operatorname{Sin}[e + f x]}} 3 b (2 a^4 - 3 a^2 b^2 + b^4) \sqrt{-\frac{a (-1 + \operatorname{Csc}[e + f x])}{a + b}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{b + a \operatorname{Csc}[e + f x]}{a - b}}\right], 1 - \frac{2 a}{a + b}\right] \\
 & \sqrt{d} \operatorname{Sin}[e + f x] \sqrt{-\frac{a \operatorname{Csc}[e + f x]^2 (1 + \operatorname{Sin}[e + f x]) (a + b \operatorname{Sin}[e + f x])}{(a - b)^2}} \operatorname{Tan}[e + f x]
 \end{aligned}$$

Result (type 4, 1600 leaves):

$$\begin{aligned}
 & \frac{1}{f \sqrt{d} \operatorname{Sin}[e + f x]} \\
 & \operatorname{Sin}[e + f x] \sqrt{a + b \operatorname{Sin}[e + f x]} \left(\frac{1}{20} \operatorname{Sec}[e + f x] (15 a^4 - 15 a^2 b^2 + 4 b^4 + 24 a^3 b \operatorname{Sin}[e + f x] - 12 a b^3 \operatorname{Sin}[e + f x]) + \frac{1}{10} \operatorname{Sec}[e + f x]^3 \right. \\
 & \left. (3 a^4 - 3 a^2 b^2 - 4 b^4 + 9 a^3 b \operatorname{Sin}[e + f x] - 5 a b^3 \operatorname{Sin}[e + f x]) + \frac{1}{5} \operatorname{Sec}[e + f x]^5 (a^4 + 6 a^2 b^2 + b^4 + 4 a^3 b \operatorname{Sin}[e + f x] + 4 a b^3 \operatorname{Sin}[e + f x]) \right) + \\
 & \frac{1}{40 f \sqrt{d} \operatorname{Sin}[e + f x]} 3 a \sqrt{\operatorname{Sin}[e + f x]} \left(\left(4 a (5 a^4 - 9 a^2 b^2 + 4 b^4) \sqrt{\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^2}{-a + b}} \operatorname{EllipticF}\left[\right. \right. \right. \\
 & \left. \left. \left. \operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^2 (a + b \operatorname{Sin}[e + f x])}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a + b}\right] \operatorname{Sec}[e + f x] \operatorname{Sin}\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^4 \sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Sin}[e + f x]}{a}} \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b\sin[efx])}{a}} \right) / \left((a+b)\sqrt{\sin[efx]}\sqrt{a+b\sin[efx]} \right) + 4a(-8a^3b+4ab^3) \\
& \left(\left(\sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b\sin[efx])}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\sin[efx]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b\sin[efx])}{a}} \right) / \left((a+b)\sqrt{\sin[efx]} \right. \right. \\
& \left. \left. \sqrt{a+b\sin[efx]} \right) - \left(\sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-a+b}} \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b\sin[efx])}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \right. \right. \\
& \left. \left. \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{-\frac{(a+b)\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\sin[efx]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2(a+b\sin[efx])}{a}} \right) / \right. \right. \\
& \left. \left. \left(b\sqrt{\sin[efx]}\sqrt{a+b\sin[efx]} \right) \right) + 2(8a^2b^2-4b^4) \left(\frac{\cos[efx]\sqrt{a+b\sin[efx]}}{b\sqrt{\sin[efx]}} + \right. \right. \\
& \left. \left. \left(i\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{Csc}[efx] \operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}{\sqrt{\sin[efx]}}\right], -\frac{2a}{-a-b}\right] \sqrt{a+b\sin[efx]} \right) / \right. \right. \\
& \left. \left. \left(b\sqrt{\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\operatorname{Csc}[efx]} \sqrt{\frac{\operatorname{Csc}[efx](a+b\sin[efx])}{a+b}} \right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b} 2a \left(\left(a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}}{a}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. -\frac{2a}{-a+b}\right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Sin}[e+fx]}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{a}} \right) / \left((a+b) \sqrt{\operatorname{Sin}[e+fx]} \sqrt{a+b \operatorname{Sin}[e+fx]} \right) - \right. \\
& \left(a \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-a+b}} \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}}{a}}}{\sqrt{2}}}\right], -\frac{2a}{-a+b}\right] \operatorname{Sec}[\right. \\
& \left. \left. e+fx\right] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Sin}[e+fx]}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{a}} \right) / \left(b \sqrt{\operatorname{Sin}[e+fx]} \sqrt{a+b \operatorname{Sin}[e+fx]} \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 1516: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^2 (a+b \operatorname{Sin}[e+fx])^2 (c+d \operatorname{Sin}[e+fx])^{4/3} dx$$

Optimal (type 6, 458 leaves, 11 steps):

$$\frac{9 (64 a b c d - 26 a^2 d^2 - b^2 (18 c^2 - 13 d^2)) \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^{7/3}}{2080 d^3 f} - \frac{9 b (3 b c - 2 a d) \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] (c + d \operatorname{Sin}[e + f x])^{7/3}}{208 d^2 f} + \frac{3 \operatorname{Cos}[e + f x] (a + b \operatorname{Sin}[e + f x])^2 (c + d \operatorname{Sin}[e + f x])^{7/3}}{16 d f}$$

$$\left(3 (c + d)^2 (208 a^2 c d^2 - 64 a b d (3 c^2 - 5 d^2) + b^2 c (54 c^2 + d^2)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{7}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sin}[e + f x]), \frac{d (1 - \operatorname{Sin}[e + f x])}{c + d}\right] \right)$$

$$\operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^{1/3} \Big/ \left(1040 \sqrt{2} d^4 f \sqrt{1 + \operatorname{Sin}[e + f x]} \left(\frac{c + d \operatorname{Sin}[e + f x]}{c + d} \right)^{1/3} \right) -$$

$$\left(3 (c - d) (c + d)^2 (192 a b c d - 208 a^2 d^2 - b^2 (54 c^2 + 91 d^2)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sin}[e + f x]), \frac{d (1 - \operatorname{Sin}[e + f x])}{c + d}\right] \right)$$

$$\operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^{1/3} \Big/ \left(1040 \sqrt{2} d^4 f \sqrt{1 + \operatorname{Sin}[e + f x]} \left(\frac{c + d \operatorname{Sin}[e + f x]}{c + d} \right)^{1/3} \right)$$

Result (type 6, 3522 leaves):

$$\frac{1}{455 f} 513 a b c \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c + d \operatorname{Sin}[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \operatorname{Sin}[e + f x]}{(-1 - \frac{c}{d}) d}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{\frac{-d - d \operatorname{Sin}[e + f x]}{c - d}} \sqrt{\frac{d - d \operatorname{Sin}[e + f x]}{c + d}} (c + d \operatorname{Sin}[e + f x])^{1/3} + \frac{1}{7280 d^3 f}$$

$$81 b^2 c^4 \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c + d \operatorname{Sin}[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \operatorname{Sin}[e + f x]}{(-1 - \frac{c}{d}) d}\right] \operatorname{Sec}[e + f x] \sqrt{\frac{-d - d \operatorname{Sin}[e + f x]}{c - d}} \sqrt{\frac{d - d \operatorname{Sin}[e + f x]}{c + d}}$$

$$(c + d \operatorname{Sin}[e + f x])^{1/3} - \frac{1}{455 d^2 f} 18 a b c^3 \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c + d \operatorname{Sin}[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \operatorname{Sin}[e + f x]}{(-1 - \frac{c}{d}) d}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{\frac{-d - d \operatorname{Sin}[e + f x]}{c - d}} \sqrt{\frac{d - d \operatorname{Sin}[e + f x]}{c + d}} (c + d \operatorname{Sin}[e + f x])^{1/3} + \frac{1}{35 d f}$$

$$54 a^2 c^2 \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c + d \operatorname{Sin}[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \operatorname{Sin}[e + f x]}{(-1 - \frac{c}{d}) d}\right] \operatorname{Sec}[e + f x] \sqrt{\frac{-d - d \operatorname{Sin}[e + f x]}{c - d}} \sqrt{\frac{d - d \operatorname{Sin}[e + f x]}{c + d}}$$

$$(c + d \operatorname{Sin}[e + f x])^{1/3} + \frac{1}{14560 d f} 5211 b^2 c^2 \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c + d \operatorname{Sin}[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \operatorname{Sin}[e + f x]}{(-1 - \frac{c}{d}) d}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{\frac{-d - d \operatorname{Sin}[e + f x]}{c - d}} \sqrt{\frac{d - d \operatorname{Sin}[e + f x]}{c + d}} (c + d \operatorname{Sin}[e + f x])^{1/3} + \frac{1}{40 f}$$

$$\begin{aligned}
& 9 a^2 d \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(1-\frac{c}{d}\right) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(-1-\frac{c}{d}\right) d}\right] \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} \\
& (c+d \operatorname{Sin}[e+f x])^{1/3} + \frac{1}{640 f} 63 b^2 d \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(1-\frac{c}{d}\right) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(-1-\frac{c}{d}\right) d}\right] \\
& \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{1/3} + \frac{1}{65 f} \\
& 9 a b c^2 \left(-1/d^2 3 c \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(1-\frac{c}{d}\right) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(-1-\frac{c}{d}\right) d}\right] \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \right. \\
& \left. \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{1/3} + 1/(4 d^2) 3 \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(1-\frac{c}{d}\right) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(-1-\frac{c}{d}\right) d}\right] \right. \\
& \left. \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{4/3} \right) + \frac{1}{7280 d^3 f} \\
& 81 b^2 c^5 \left(-1/d^2 3 c \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(1-\frac{c}{d}\right) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(-1-\frac{c}{d}\right) d}\right] \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \right. \\
& \left. \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{1/3} + 1/(4 d^2) 3 \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(1-\frac{c}{d}\right) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(-1-\frac{c}{d}\right) d}\right] \right. \\
& \left. \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{4/3} \right) - \frac{1}{455 d^2 f} \\
& 18 a b c^4 \left(-1/d^2 3 c \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(1-\frac{c}{d}\right) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(-1-\frac{c}{d}\right) d}\right] \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \right. \\
& \left. \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{1/3} + 1/(4 d^2) 3 \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(1-\frac{c}{d}\right) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{\left(-1-\frac{c}{d}\right) d}\right] \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{Sec}[e+fx] \sqrt{\frac{-d-d\sin[e+fx]}{c-d}} \sqrt{\frac{d-d\sin[e+fx]}{c+d}} (c+d\sin[e+fx])^{4/3} \right) + \frac{1}{70df} \\
& 3a^2c^3 \left(-1/d^2 {}_3F_4 \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d\sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d\sin[e+fx]}{(-1-\frac{c}{d})d} \right] \operatorname{Sec}[e+fx] \sqrt{\frac{-d-d\sin[e+fx]}{c-d}} \right. \\
& \quad \left. \sqrt{\frac{d-d\sin[e+fx]}{c+d}} (c+d\sin[e+fx])^{1/3} + 1/(4d^2) {}_3F_4 \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c+d\sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d\sin[e+fx]}{(-1-\frac{c}{d})d} \right] \right. \\
& \quad \left. \operatorname{Sec}[e+fx] \sqrt{\frac{-d-d\sin[e+fx]}{c-d}} \sqrt{\frac{d-d\sin[e+fx]}{c+d}} (c+d\sin[e+fx])^{4/3} \right) - \frac{1}{1040df} \\
& 21b^2c^3 \left(-1/d^2 {}_3F_4 \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d\sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d\sin[e+fx]}{(-1-\frac{c}{d})d} \right] \operatorname{Sec}[e+fx] \sqrt{\frac{-d-d\sin[e+fx]}{c-d}} \right. \\
& \quad \left. \sqrt{\frac{d-d\sin[e+fx]}{c+d}} (c+d\sin[e+fx])^{1/3} + 1/(4d^2) {}_3F_4 \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c+d\sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d\sin[e+fx]}{(-1-\frac{c}{d})d} \right] \right. \\
& \quad \left. \operatorname{Sec}[e+fx] \sqrt{\frac{-d-d\sin[e+fx]}{c-d}} \sqrt{\frac{d-d\sin[e+fx]}{c+d}} (c+d\sin[e+fx])^{4/3} \right) + \frac{1}{280f} \\
& 153a^2cd \left(-1/d^2 {}_3F_4 \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d\sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d\sin[e+fx]}{(-1-\frac{c}{d})d} \right] \operatorname{Sec}[e+fx] \sqrt{\frac{-d-d\sin[e+fx]}{c-d}} \right. \\
& \quad \left. \sqrt{\frac{d-d\sin[e+fx]}{c+d}} (c+d\sin[e+fx])^{1/3} + 1/(4d^2) {}_3F_4 \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c+d\sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d\sin[e+fx]}{(-1-\frac{c}{d})d} \right] \right. \\
& \quad \left. \operatorname{Sec}[e+fx] \sqrt{\frac{-d-d\sin[e+fx]}{c-d}} \sqrt{\frac{d-d\sin[e+fx]}{c+d}} (c+d\sin[e+fx])^{4/3} \right) + \frac{1}{58240f} \\
& 9603b^2cd \left(-1/d^2 {}_3F_4 \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d\sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d\sin[e+fx]}{(-1-\frac{c}{d})d} \right] \operatorname{Sec}[e+fx] \sqrt{\frac{-d-d\sin[e+fx]}{c-d}} \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{1/3} + 1 / (4 d^2)^3 \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{(1-\frac{c}{d}) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{(-1-\frac{c}{d}) d}\right] \\
& \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{4/3} + \frac{1}{91 f} \\
24 a b d^2 & \left(-1 / d^2 3 c \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{(1-\frac{c}{d}) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{(-1-\frac{c}{d}) d}\right] \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \right. \\
& \left. \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{1/3} + 1 / (4 d^2)^3 \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c+d \operatorname{Sin}[e+f x]}{(1-\frac{c}{d}) d}, -\frac{c+d \operatorname{Sin}[e+f x]}{(-1-\frac{c}{d}) d}\right] \right. \\
& \left. \operatorname{Sec}[e+f x] \sqrt{\frac{-d-d \operatorname{Sin}[e+f x]}{c-d}} \sqrt{\frac{d-d \operatorname{Sin}[e+f x]}{c+d}} (c+d \operatorname{Sin}[e+f x])^{4/3} + \frac{1}{f} \right) \\
& (c+d \operatorname{Sin}[e+f x])^{1/3} \left(-1 / (58240 d^3)^3 (-216 b^2 c^4 + 768 a b c^3 d - 832 a^2 c^2 d^2 + 332 b^2 c^2 d^2 + 7232 a b c d^3 + 2912 a^2 d^4 + 1729 b^2 d^4) \operatorname{Cos}[e+f x] - \right. \\
& \left. \frac{3 (8 b^2 c^2 + 896 a b c d + 416 a^2 d^2 + 117 b^2 d^2) \operatorname{Cos}[3 (e+f x)]}{16640 d} + \frac{3}{256} b^2 d \operatorname{Cos}[5 (e+f x)] + \right. \\
& \left. \frac{3 (-18 b^2 c^3 + 64 a b c^2 d + 1144 a^2 c d^2 + 23 b^2 c d^2 + 80 a b d^3) \operatorname{Sin}[2 (e+f x)]}{14560 d^2} - \frac{3 b (17 b c + 32 a d) \operatorname{Sin}[4 (e+f x)]}{1664} \right)
\end{aligned}$$

■ **Problem 1517: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e+f x]^2 (a+b \operatorname{Sin}[e+f x]) (c+d \operatorname{Sin}[e+f x])^{4/3} dx$$

Optimal (type 6, 341 leaves, 10 steps):

$$\begin{aligned}
& - \frac{3(6bc - 13ad) \cos[e+fx] (c+d \sin[e+fx])^{7/3}}{130d^2f} + \frac{3b \cos[e+fx] \sin[e+fx] (c+d \sin[e+fx])^{7/3}}{13df} + \\
& \left(3(c+d)^2 (6bc^2 - 13acd - 10bd^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{7}{3}, \frac{3}{2}, \frac{1}{2} (1 - \sin[e+fx]), \frac{d(1 - \sin[e+fx])}{c+d} \right] \cos[e+fx] (c+d \sin[e+fx])^{1/3} \right) / \\
& \left(65\sqrt{2} d^3 f \sqrt{1 + \sin[e+fx]} \left(\frac{c+d \sin[e+fx]}{c+d} \right)^{1/3} \right) - \\
& \left(3(c-d)(c+d)^2 (6bc - 13ad) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2} (1 - \sin[e+fx]), \frac{d(1 - \sin[e+fx])}{c+d} \right] \cos[e+fx] (c+d \sin[e+fx])^{1/3} \right) / \\
& \left(65\sqrt{2} d^3 f \sqrt{1 + \sin[e+fx]} \left(\frac{c+d \sin[e+fx]}{c+d} \right)^{1/3} \right)
\end{aligned}$$

Result (type 6, 2110 leaves):

$$\begin{aligned}
& \frac{1}{910f} 513bc \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d \sin[e+fx]}{(-1-\frac{c}{d})d} \right] \sec[e+fx] \sqrt{\frac{-d-d \sin[e+fx]}{c-d}} \\
& \sqrt{\frac{d-d \sin[e+fx]}{c+d}} (c+d \sin[e+fx])^{1/3} - \frac{1}{455d^2f} 9b^3c^3 \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d \sin[e+fx]}{(-1-\frac{c}{d})d} \right] \\
& \sec[e+fx] \sqrt{\frac{-d-d \sin[e+fx]}{c-d}} \sqrt{\frac{d-d \sin[e+fx]}{c+d}} (c+d \sin[e+fx])^{1/3} + \frac{1}{35df} \\
& 54a^2c^2 \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d \sin[e+fx]}{(-1-\frac{c}{d})d} \right] \sec[e+fx] \sqrt{\frac{-d-d \sin[e+fx]}{c-d}} \\
& \sqrt{\frac{d-d \sin[e+fx]}{c+d}} (c+d \sin[e+fx])^{1/3} + \frac{1}{40f} 9ad \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d \sin[e+fx]}{(-1-\frac{c}{d})d} \right] \\
& \sec[e+fx] \sqrt{\frac{-d-d \sin[e+fx]}{c-d}} \sqrt{\frac{d-d \sin[e+fx]}{c+d}} (c+d \sin[e+fx])^{1/3} + \frac{1}{130f} \\
& 9bc^2 \left(-1/d^2 3c \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c+d \sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d \sin[e+fx]}{(-1-\frac{c}{d})d} \right] \sec[e+fx] \sqrt{\frac{-d-d \sin[e+fx]}{c-d}} \right. \\
& \left. \sqrt{\frac{d-d \sin[e+fx]}{c+d}} (c+d \sin[e+fx])^{1/3} + 1/(4d^2) 3 \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c+d \sin[e+fx]}{(1-\frac{c}{d})d}, -\frac{c+d \sin[e+fx]}{(-1-\frac{c}{d})d} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{Sec}[e + f x] \sqrt{\frac{-d - d \sin[e + f x]}{c - d}} \sqrt{\frac{d - d \sin[e + f x]}{c + d}} (c + d \sin[e + f x])^{4/3} \right) - \frac{1}{455 d^2 f} \\
9 b c^4 & \left(-1/d^2 3 c \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c + d \sin[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \sin[e + f x]}{(-1 - \frac{c}{d}) d} \right] \text{Sec}[e + f x] \sqrt{\frac{-d - d \sin[e + f x]}{c - d}} \right. \\
& \left. \sqrt{\frac{d - d \sin[e + f x]}{c + d}} (c + d \sin[e + f x])^{1/3} + 1/(4 d^2) 3 \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c + d \sin[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \sin[e + f x]}{(-1 - \frac{c}{d}) d} \right] \right. \\
& \left. \text{Sec}[e + f x] \sqrt{\frac{-d - d \sin[e + f x]}{c - d}} \sqrt{\frac{d - d \sin[e + f x]}{c + d}} (c + d \sin[e + f x])^{4/3} \right) + \frac{1}{70 d f} \\
3 a c^3 & \left(-1/d^2 3 c \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c + d \sin[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \sin[e + f x]}{(-1 - \frac{c}{d}) d} \right] \text{Sec}[e + f x] \sqrt{\frac{-d - d \sin[e + f x]}{c - d}} \right. \\
& \left. \sqrt{\frac{d - d \sin[e + f x]}{c + d}} (c + d \sin[e + f x])^{1/3} + 1/(4 d^2) 3 \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c + d \sin[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \sin[e + f x]}{(-1 - \frac{c}{d}) d} \right] \right. \\
& \left. \text{Sec}[e + f x] \sqrt{\frac{-d - d \sin[e + f x]}{c - d}} \sqrt{\frac{d - d \sin[e + f x]}{c + d}} (c + d \sin[e + f x])^{4/3} \right) + \frac{1}{280 f} \\
153 a c d & \left(-1/d^2 3 c \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c + d \sin[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \sin[e + f x]}{(-1 - \frac{c}{d}) d} \right] \text{Sec}[e + f x] \sqrt{\frac{-d - d \sin[e + f x]}{c - d}} \right. \\
& \left. \sqrt{\frac{d - d \sin[e + f x]}{c + d}} (c + d \sin[e + f x])^{1/3} + 1/(4 d^2) 3 \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c + d \sin[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \sin[e + f x]}{(-1 - \frac{c}{d}) d} \right] \right. \\
& \left. \text{Sec}[e + f x] \sqrt{\frac{-d - d \sin[e + f x]}{c - d}} \sqrt{\frac{d - d \sin[e + f x]}{c + d}} (c + d \sin[e + f x])^{4/3} \right) + \frac{1}{91 f} \\
12 b d^2 & \left(-1/d^2 3 c \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, -\frac{c + d \sin[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \sin[e + f x]}{(-1 - \frac{c}{d}) d} \right] \text{Sec}[e + f x] \sqrt{\frac{-d - d \sin[e + f x]}{c - d}} \right.
\end{aligned}$$

$$\sqrt{\frac{d - d \sin[e + f x]}{c + d}} (c + d \sin[e + f x])^{1/3} + 1 / (4 d^2)^3 \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, -\frac{c + d \sin[e + f x]}{(1 - \frac{c}{d}) d}, -\frac{c + d \sin[e + f x]}{(-1 - \frac{c}{d}) d}\right]$$

$$\operatorname{Sec}[e + f x] \sqrt{\frac{-d - d \sin[e + f x]}{c - d}} \sqrt{\frac{d - d \sin[e + f x]}{c + d}} (c + d \sin[e + f x])^{4/3} + \frac{1}{f}$$

$$(c + d \sin[e + f x])^{1/3} \left(-\frac{3 (12 b c^3 - 26 a c^2 d + 113 b c d^2 + 91 a d^3) \cos[e + f x]}{1820 d^2} - \frac{3}{520} (14 b c + 13 a d) \cos[3 (e + f x)] + \right.$$

$$\left. \frac{3 (4 b c^2 + 143 a c d + 5 b d^2) \sin[2 (e + f x)]}{1820 d} - \frac{3}{104} b d \sin[4 (e + f x)] \right)$$

■ **Problem 1518: Result more than twice size of optimal antiderivative.**

$$\int \cos[e + f x]^2 (c + d \sin[e + f x])^{4/3} dx$$

Optimal (type 6, 125 leaves, 2 steps):

$$\frac{3 \operatorname{AppellF1}\left[\frac{7}{3}, -\frac{1}{2}, -\frac{1}{2}, \frac{10}{3}, \frac{c + d \sin[e + f x]}{c - d}, \frac{c + d \sin[e + f x]}{c + d}\right] \cos[e + f x] (c + d \sin[e + f x])^{7/3}}{7 d f \sqrt{1 - \frac{c + d \sin[e + f x]}{c - d}} \sqrt{1 - \frac{c + d \sin[e + f x]}{c + d}}}$$

Result (type 6, 301 leaves):

$$-\frac{1}{1120 d^3 f} 3 \operatorname{Sec}[e + f x] (c + d \sin[e + f x])^{1/3}$$

$$\left(12 (4 c^4 + 3 c^2 d^2 - 7 d^4) \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{c + d \sin[e + f x]}{c - d}, \frac{c + d \sin[e + f x]}{c + d}\right] \sqrt{\frac{d (-1 + \sin[e + f x])}{c + d}} \sqrt{\frac{d (1 + \sin[e + f x])}{c - d}} - \right.$$

$$\left. 3 c (4 c^2 + 51 d^2) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, \frac{1}{2}, \frac{7}{3}, \frac{c + d \sin[e + f x]}{c - d}, \frac{c + d \sin[e + f x]}{c + d}\right] \sqrt{\frac{d (-1 + \sin[e + f x])}{c + d}} \right.$$

$$\left. \sqrt{-\frac{d (1 + \sin[e + f x])}{c - d}} (c + d \sin[e + f x]) + 4 d^2 \cos[e + f x]^2 (-4 c^2 + 7 d^2 + 14 d^2 \cos[2 (e + f x)] - 44 c d \sin[e + f x]) \right)$$

■ **Problem 1523: Unable to integrate problem.**

$$\int \cos[e + f x]^2 (a + b \sin[e + f x])^2 (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 552 leaves, 11 steps):

$$\frac{(2a^2d^2(3+n) - 4abcd(4+n) + b^2(6c^2 - d^2(3+n))) \cos[e+fx] (c+d \sin[e+fx])^{1+n}}{d^3 f (2+n) (3+n) (4+n)} -$$

$$\frac{b(3bc - 2ad) \cos[e+fx] \sin[e+fx] (c+d \sin[e+fx])^{1+n}}{d^2 f (3+n) (4+n)} + \frac{\cos[e+fx] (a+b \sin[e+fx])^2 (c+d \sin[e+fx])^{1+n}}{df (4+n)} -$$

$$\left(\sqrt{2} (c+d) (a^2cd^2(12+7n+n^2) - 2abd(4+n)(2c^2 - d^2(2+n)) + b^2c(6c^2 - d^2(3-n-n^2))) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1-n, \frac{3}{2}, \frac{1}{2}(1 - \sin[e+fx])\right], \right.$$

$$\left. \frac{d(1 - \sin[e+fx])}{c+d} \right] \cos[e+fx] (c+d \sin[e+fx])^n \left(\frac{c+d \sin[e+fx]}{c+d} \right)^{-n} \Big/ \left(d^4 f (2+n) (3+n) (4+n) \sqrt{1 + \sin[e+fx]} \right) -$$

$$\left(\sqrt{2} (c^2 - d^2) (4abcd(4+n) - a^2d^2(12+7n+n^2) - b^2(6c^2 + d^2(3+4n+n^2))) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -n, \frac{3}{2}, \frac{1}{2}(1 - \sin[e+fx])\right], \right.$$

$$\left. \frac{d(1 - \sin[e+fx])}{c+d} \right] \cos[e+fx] (c+d \sin[e+fx])^n \left(\frac{c+d \sin[e+fx]}{c+d} \right)^{-n} \Big/ \left(d^4 f (2+n) (3+n) (4+n) \sqrt{1 + \sin[e+fx]} \right)$$

Result (type 8, 35 leaves):

$$\int \cos[e+fx]^2 (a+b \sin[e+fx])^2 (c+d \sin[e+fx])^n dx$$

■ **Problem 1524: Unable to integrate problem.**

$$\int \cos[e+fx]^2 (a+b \sin[e+fx]) (c+d \sin[e+fx])^n dx$$

Optimal (type 6, 375 leaves, 10 steps):

$$- \frac{(2bc - ad(3+n)) \cos[e+fx] (c+d \sin[e+fx])^{1+n}}{d^2 f (2+n) (3+n)} + \frac{b \cos[e+fx] \sin[e+fx] (c+d \sin[e+fx])^{1+n}}{df (3+n)} -$$

$$\left(\sqrt{2} (c+d) (acd(3+n) - b(2c^2 - d^2(2+n))) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1-n, \frac{3}{2}, \frac{1}{2}(1 - \sin[e+fx])\right], \frac{d(1 - \sin[e+fx])}{c+d} \right]$$

$$\cos[e+fx] (c+d \sin[e+fx])^n \left(\frac{c+d \sin[e+fx]}{c+d} \right)^{-n} \Big/ \left(d^3 f (2+n) (3+n) \sqrt{1 + \sin[e+fx]} \right) -$$

$$\left(\sqrt{2} (c^2 - d^2) (2bc - ad(3+n)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -n, \frac{3}{2}, \frac{1}{2}(1 - \sin[e+fx])\right], \frac{d(1 - \sin[e+fx])}{c+d} \right]$$

$$\cos[e+fx] (c+d \sin[e+fx])^n \left(\frac{c+d \sin[e+fx]}{c+d} \right)^{-n} \Big/ \left(d^3 f (2+n) (3+n) \sqrt{1 + \sin[e+fx]} \right)$$

Result (type 8, 33 leaves):

$$\int \cos[e+fx]^2 (a+b \sin[e+fx]) (c+d \sin[e+fx])^n dx$$

■ **Problem 1525: Unable to integrate problem.**

$$\int \cos[e+fx]^2 (c+d \sin[e+fx])^n dx$$

Optimal (type 6, 127 leaves, 2 steps) :

$$\frac{\text{AppellF1}\left[1+n, -\frac{1}{2}, -\frac{1}{2}, 2+n, \frac{c+d\sin[ex+f x]}{c-d}, \frac{c+d\sin[ex+f x]}{c+d}\right] \cos[ex+f x] (c+d\sin[ex+f x])^{1+n}}{d f (1+n) \sqrt{1-\frac{c+d\sin[ex+f x]}{c-d}} \sqrt{1-\frac{c+d\sin[ex+f x]}{c+d}}}$$

Result (type 8, 23 leaves) :

$$\int \cos[ex+f x]^2 (c+d\sin[ex+f x])^n dx$$

■ **Problem 1532: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx] (a+b\sin[c+dx]) (A+B\sin[c+dx]) dx$$

Optimal (type 3, 64 leaves, 5 steps) :

$$-\frac{(a+b)(A+B)\log[1-\sin[c+dx]]}{2d} + \frac{(a-b)(A-B)\log[1+\sin[c+dx]]}{2d} - \frac{bB\sin[c+dx]}{d}$$

Result (type 3, 172 leaves) :

$$-\frac{Ab\log[\cos[c+dx]]}{d} - \frac{aB\log[\cos[c+dx]]}{d} - \frac{aA\log\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right]}{d} + \frac{aA\log\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]+\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right]}{d} - \frac{bB\log\left[\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right]}{d} + \frac{bB\log\left[\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right]}{d} - \frac{bB\sin[c+dx]}{d}$$

■ **Problem 1533: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^3 (a+b\sin[c+dx]) (A+B\sin[c+dx]) dx$$

Optimal (type 3, 59 leaves, 3 steps) :

$$\frac{(aA-bB)\text{ArcTanh}[\sin[c+dx]]}{2d} + \frac{\sec[c+dx]^2 (Ab+aB+(aA+bB)\sin[c+dx])}{2d}$$

Result (type 3, 141 leaves) :

$$\frac{1}{4d} \left(2(-aA+bB)\log\left[\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right] + 2(aA-bB)\log\left[\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right] + \frac{(a+b)(A+B)}{\left(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{(a-b)(A-B)}{\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 1534: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^5 (a+b\sin[c+dx]) (A+B\sin[c+dx]) dx$$

Optimal (type 3, 88 leaves, 4 steps):

$$\frac{(3 a A - b B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{\operatorname{Sec}[c + d x]^4 (A b + a B + (a A + b B) \operatorname{Sin}[c + d x])}{4 d} + \frac{(3 a A - b B) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d}$$

Result (type 3, 220 leaves):

$$\frac{1}{16 d} \left(2 (-3 a A + b B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\ \left. 2 (3 a A - b B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \frac{(a + b)(A + B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \right. \\ \left. \frac{b(A - B) + a(3 A + B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{(a - b)(A - B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{a(-3 A + B) + b(A + B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} \right)$$

■ **Problem 1535: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^7 (a + b \operatorname{Sin}[c + d x]) (A + B \operatorname{Sin}[c + d x]) dx$$

Optimal (type 3, 118 leaves, 5 steps):

$$\frac{(5 a A - b B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{16 d} + \frac{\operatorname{Sec}[c + d x]^6 (A b + a B + (a A + b B) \operatorname{Sin}[c + d x])}{6 d} + \\ \frac{(5 a A - b B) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{16 d} + \frac{(5 a A - b B) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{24 d}$$

Result (type 3, 297 leaves):

$$\frac{1}{96 d} \left(6 (-5 a A + b B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 6 (5 a A - b B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\ \frac{2(a + b)(A + B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^6} + \frac{3(A b + a(2 A + B))}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{3(b(A - B) + a(5 A + B))}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \\ \left. \frac{2(a - b)(A - B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^6} + \frac{3(A b + a(-2 A + B))}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{3(a(-5 A + B) + b(A + B))}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} \right)$$

■ **Problem 1542: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^5 (a + b \operatorname{Sin}[c + d x])^2 (A + B \operatorname{Sin}[c + d x]) dx$$

Optimal (type 3, 122 leaves, 4 steps):

$$\frac{(3a^2A - Ab^2 - 2abB) \operatorname{ArcTanh}[\sin[c+dx]]}{8d} + \frac{\sec[c+dx]^4 (B + A \sin[c+dx]) (a + b \sin[c+dx])^2}{4d} + \frac{\sec[c+dx]^2 (2b(2aA - bB) + (3a^2A + Ab^2 - 2abB) \sin[c+dx])}{8d}$$

Result (type 3, 255 leaves):

$$\frac{1}{16d} \left(2(-3a^2A + Ab^2 + 2abB) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right] + 2(3a^2A - Ab^2 - 2abB) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right] + \frac{(a+b)^2(A+B)}{\left(\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right)^4} + \frac{(a+b)(a(3A+B) - b(A+3B))}{\left(\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right)^2} - \frac{(a-b)^2(A-B)}{\left(\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right)^4} - \frac{(a-b)(3aA + Ab - aB - 3bB)}{\left(\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right)^2} \right)$$

■ **Problem 1543: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^7 (a + b \sin[c+dx])^2 (A + B \sin[c+dx]) dx$$

Optimal (type 3, 160 leaves, 5 steps):

$$\frac{(5a^2A - Ab^2 - 2abB) \operatorname{ArcTanh}[\sin[c+dx]]}{16d} + \frac{\sec[c+dx]^6 (B + A \sin[c+dx]) (a + b \sin[c+dx])^2}{6d} + \frac{\sec[c+dx]^4 (2b(4aA - bB) + (5a^2A + 3Ab^2 - 2abB) \sin[c+dx])}{24d} + \frac{(5a^2A - Ab^2 - 2abB) \sec[c+dx] \tan[c+dx]}{16d}$$

Result (type 3, 459 leaves):

$$\frac{(-5a^2A + Ab^2 + 2abB) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right]}{16d} + \frac{(5a^2A - Ab^2 - 2abB) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right]}{16d} + \frac{a^2A + 2aAb + Ab^2 + a^2B + 2abB + b^2B}{48d \left(\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right)^6} + \frac{2a^2A + 2aAb + a^2B - b^2B}{32d \left(\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right)^4} + \frac{5a^2A + 2aAb - Ab^2 + a^2B - 2abB - b^2B}{32d \left(\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right)^2} + \frac{-a^2A + 2aAb - Ab^2 + a^2B - 2abB + b^2B}{48d \left(\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right)^6} + \frac{-2a^2A + 2aAb + a^2B - b^2B}{32d \left(\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right)^4} + \frac{-5a^2A + 2aAb + Ab^2 + a^2B + 2abB - b^2B}{32d \left(\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right)^2}$$

■ **Problem 1557: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^3 (A + B \sin[c+dx])}{(a + b \sin[c+dx])^2} dx$$

Optimal (type 3, 228 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(aA + 3Ab + 2bB) \operatorname{Log}[1 - \operatorname{Sin}[c + dx]]}{4(a+b)^3 d} + \frac{(aA - 3Ab + 2bB) \operatorname{Log}[1 + \operatorname{Sin}[c + dx]]}{4(a-b)^3 d} + \\
& \frac{b^2 (4aAb - 3a^2 B - b^2 B) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]]}{(a^2 - b^2)^3 d} - \frac{b(a^2 A + 3Ab^2 - 4abB)}{2(a^2 - b^2)^2 d (a + b \operatorname{Sin}[c + dx])} - \frac{\operatorname{Sec}[c + dx]^2 (Ab - aB - (aA - bB) \operatorname{Sin}[c + dx])}{2(a^2 - b^2) d (a + b \operatorname{Sin}[c + dx])}
\end{aligned}$$

Result (type 3, 347 leaves):

$$\begin{aligned}
& \frac{1}{4d} \left(\frac{8ib^2 (-4aAb + 3a^2 B + b^2 B) (c + dx)}{(a-b)^3 (a+b)^3} + \frac{2i(aA - 3Ab + 2bB) \operatorname{ArcTan}[\operatorname{Cot}[c + dx]]}{(a-b)^3} - \right. \\
& \frac{2i(aA + 3Ab + 2bB) \operatorname{ArcTan}[\operatorname{Cot}[c + dx]]}{(a+b)^3} - \frac{(aA + 3Ab + 2bB) \operatorname{Log}\left[\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2\right]}{(a+b)^3} + \\
& \frac{(aA - 3Ab + 2bB) \operatorname{Log}\left[\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2\right]}{(a-b)^3} + \frac{4b^2 (-4aAb + 3a^2 B + b^2 B) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]]}{(-a^2 + b^2)^3} + \\
& \left. \frac{A+B}{(a+b)^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{-A+B}{(a-b)^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{4b^2 (-Ab + aB)}{(a-b)^2 (a+b)^2 (a + b \operatorname{Sin}[c + dx])} \right)
\end{aligned}$$

■ **Problem 1558: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + dx]^5 (A + B \operatorname{Sin}[c + dx])}{(a + b \operatorname{Sin}[c + dx])^2} dx$$

Optimal (type 3, 372 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(3a^2 A + 2ab(6A + B) + b^2(15A + 8B)) \operatorname{Log}[1 - \operatorname{Sin}[c + dx]]}{16(a+b)^4 d} + \frac{(3a^2 A + b^2(15A - 8B) - 2ab(6A - B)) \operatorname{Log}[1 + \operatorname{Sin}[c + dx]]}{16(a-b)^4 d} - \\
& \frac{b^4 (6aAb - 5a^2 B - b^2 B) \operatorname{Log}[a + b \operatorname{Sin}[c + dx]]}{(a^2 - b^2)^4 d} - \frac{b(3a^4 A - 12a^2 Ab^2 - 15Ab^4 + 2a^3 bB + 22ab^3 B)}{8(a^2 - b^2)^3 d (a + b \operatorname{Sin}[c + dx])} - \\
& \frac{\operatorname{Sec}[c + dx]^4 (Ab - aB - (aA - bB) \operatorname{Sin}[c + dx])}{4(a^2 - b^2) d (a + b \operatorname{Sin}[c + dx])} + \frac{\operatorname{Sec}[c + dx]^2 (b(a^2 A + 5Ab^2 - 6abB) + (3a^3 A - 9aAb^2 + 2a^2 bB + 4b^3 B) \operatorname{Sin}[c + dx])}{8(a^2 - b^2)^2 d (a + b \operatorname{Sin}[c + dx])}
\end{aligned}$$

Result (type 3, 642 leaves):

$$\begin{aligned}
& \frac{1}{64 d} \left(- \frac{128 i b^4 (-6 a A b + 5 a^2 B + b^2 B) (c + d x)}{(a - b)^4 (a + b)^4} + \right. \\
& \frac{8 i (3 a^2 A + b^2 (15 A - 8 B) + 2 a b (-6 A + B)) \operatorname{ArcTan}[\operatorname{Cot}[c + d x]]}{(a - b)^4} - \frac{8 i (3 a^2 A + 2 a b (6 A + B) + b^2 (15 A + 8 B)) \operatorname{ArcTan}[\operatorname{Cot}[c + d x]]}{(a + b)^4} - \\
& \frac{4 (3 a^2 A + 2 a b (6 A + B) + b^2 (15 A + 8 B)) \operatorname{Log}\left[\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2\right]}{(a + b)^4} + \\
& \frac{4 (3 a^2 A + b^2 (15 A - 8 B) + 2 a b (-6 A + B)) \operatorname{Log}\left[\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2\right]}{(a - b)^4} + \\
& \frac{64 b^4 (-6 a A b + 5 a^2 B + b^2 B) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{(a^2 - b^2)^4} + \frac{1}{(a^2 - b^2)^3 (a + b \operatorname{Sin}[c + d x])} \\
& \left. \operatorname{Sec}[c + d x]^4 (-21 a^4 A b + 84 a^2 A b^3 + 9 A b^5 + 16 a^5 B - 62 a^3 b^2 B - 26 a b^4 B - 8 b (a^4 A - 8 a^2 A b^2 - 5 A b^4 + 4 a^3 b B + 8 a b^3 B) \operatorname{Cos}[2(c + d x)] + \right. \\
& b (-3 a^4 A + 12 a^2 A b^2 + 15 A b^4 - 2 a^3 b B - 22 a b^3 B) \operatorname{Cos}[4(c + d x)] + 22 a^5 A \operatorname{Sin}[c + d x] - 56 a^3 A b^2 \operatorname{Sin}[c + d x] + \\
& 34 a A b^4 \operatorname{Sin}[c + d x] - 12 a^4 b B \operatorname{Sin}[c + d x] + 36 a^2 b^3 B \operatorname{Sin}[c + d x] - 24 b^5 B \operatorname{Sin}[c + d x] + 6 a^5 A \operatorname{Sin}[3(c + d x)] - \\
& \left. 24 a^3 A b^2 \operatorname{Sin}[3(c + d x)] + 18 a A b^4 \operatorname{Sin}[3(c + d x)] + 4 a^4 b B \operatorname{Sin}[3(c + d x)] + 4 a^2 b^3 B \operatorname{Sin}[3(c + d x)] - 8 b^5 B \operatorname{Sin}[3(c + d x)] \right)
\end{aligned}$$

■ **Problem 1559: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^7 (A + B \operatorname{Sin}[c + d x])}{(a + b \operatorname{Sin}[c + d x])^2} dx$$

Optimal (type 3, 550 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(5 a^3 A + a^2 b (25 A + 2 B) + a b^2 (47 A + 10 B) + b^3 (35 A + 16 B)) \operatorname{Log}[1 - \operatorname{Sin}[c + d x]]}{32 (a + b)^5 d} + \\
& \frac{(5 a^3 A - b^3 (35 A - 16 B) + a b^2 (47 A - 10 B) - a^2 (25 A b - 2 b B)) \operatorname{Log}[1 + \operatorname{Sin}[c + d x]]}{32 (a - b)^5 d} + \frac{b^6 (8 a A b - 7 a^2 B - b^2 B) \operatorname{Log}[a + b \operatorname{Sin}[c + d x]]}{(a^2 - b^2)^5 d} - \\
& \frac{b (5 a^6 A - 23 a^4 A b^2 + 47 a^2 A b^4 + 35 A b^6 + 2 a^5 b B - 12 a^3 b^3 B - 54 a b^5 B)}{16 (a^2 - b^2)^4 d (a + b \operatorname{Sin}[c + d x])} - \frac{\operatorname{Sec}[c + d x]^6 (A b - a B - (a A - b B) \operatorname{Sin}[c + d x])}{6 (a^2 - b^2) d (a + b \operatorname{Sin}[c + d x])} + \\
& \frac{\operatorname{Sec}[c + d x]^4 (b (a^2 A + 7 A b^2 - 8 a b B) + (5 a^3 A - 13 a A b^2 + 2 a^2 b B + 6 b^3 B) \operatorname{Sin}[c + d x])}{24 (a^2 - b^2)^2 d (a + b \operatorname{Sin}[c + d x])} + \\
& \frac{(\operatorname{Sec}[c + d x]^2 (b (5 a^4 A - 18 a^2 A b^2 - 35 A b^4 + 2 a^3 b B + 46 a b^3 B) + 3 (5 a^5 A - 18 a^3 A b^2 + 29 a A b^4 + 2 a^4 b B - 10 a^2 b^3 B - 8 b^5 B) \operatorname{Sin}[c + d x]))}{(48 (a^2 - b^2)^3 d (a + b \operatorname{Sin}[c + d x]))}
\end{aligned}$$

Result (type 3, 1293 leaves):

$$\begin{aligned}
& \frac{2i(-8aAb^7 + 7a^2b^6B + b^8B)(c+dx)}{(a-b)^5(a+b)^5d} + \frac{1}{16(a+b)^5d}i(-5a^3A - 25a^2Ab - 47aAb^2 - 35Ab^3 - 2a^2bB - 10ab^2B - 16b^3B) \\
& \text{ArcTan}\left[\text{Csc}[c+dx]\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \\
& \frac{1}{16(a-b)^5d}i(5a^3A - 25a^2Ab + 47aAb^2 - 35Ab^3 + 2a^2bB - 10ab^2B + 16b^3B) \\
& \text{ArcTan}\left[\text{Csc}[c+dx]\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)\right] + \frac{1}{32(a+b)^5d} \\
& (-5a^3A - 25a^2Ab - 47aAb^2 - 35Ab^3 - 2a^2bB - 10ab^2B - 16b^3B) \text{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right] + \frac{1}{32(a-b)^5d} \\
& (5a^3A - 25a^2Ab + 47aAb^2 - 35Ab^3 + 2a^2bB - 10ab^2B + 16b^3B) \text{Log}\left[\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right] + \\
& \frac{(8aAb^7 - 7a^2b^6B - b^8B) \text{Log}[a+b\text{Sin}[c+dx]]}{(a^2-b^2)^5d} + \frac{1}{1536(a^2-b^2)^4d(a+b\text{Sin}[c+dx])} \\
& \text{Sec}[c+dx]^6(-314a^6Ab + 1342a^4Ab^3 - 2798a^2Ab^5 - 150Ab^7 + 256a^7B - 1060a^5b^2B + 2168a^3b^4B + 556ab^6B - 113a^6Ab\text{Cos}[2(c+dx)] + \\
& 827a^4Ab^3\text{Cos}[2(c+dx)] - 2803a^2Ab^5\text{Cos}[2(c+dx)] - 791Ab^7\text{Cos}[2(c+dx)] - 314a^5b^2B\text{Cos}[2(c+dx)] + 1756a^3b^4B \\
& \text{Cos}[2(c+dx)] + 1438ab^6B\text{Cos}[2(c+dx)] - 70a^6Ab\text{Cos}[4(c+dx)] + 322a^4Ab^3\text{Cos}[4(c+dx)] - 914a^2Ab^5\text{Cos}[4(c+dx)] - \\
& 490Ab^7\text{Cos}[4(c+dx)] - 28a^5b^2B\text{Cos}[4(c+dx)] + 392a^3b^4B\text{Cos}[4(c+dx)] + 788ab^6B\text{Cos}[4(c+dx)] - 15a^6Ab\text{Cos}[6(c+dx)] + \\
& 69a^4Ab^3\text{Cos}[6(c+dx)] - 141a^2Ab^5\text{Cos}[6(c+dx)] - 105Ab^7\text{Cos}[6(c+dx)] - 6a^5b^2B\text{Cos}[6(c+dx)] + 36a^3b^4B\text{Cos}[6(c+dx)] + \\
& 162ab^6B\text{Cos}[6(c+dx)] + 396a^7A\text{Sin}[c+dx] - 1412a^5Ab^2\text{Sin}[c+dx] + 1828a^3Ab^4\text{Sin}[c+dx] - 812aAb^6\text{Sin}[c+dx] - \\
& 200a^6bB\text{Sin}[c+dx] + 656a^4b^3B\text{Sin}[c+dx] - 904a^2b^5B\text{Sin}[c+dx] + 448b^7B\text{Sin}[c+dx] + 170a^7A\text{Sin}[3(c+dx)] - \\
& 782a^5Ab^2\text{Sin}[3(c+dx)] + 1342a^3Ab^4\text{Sin}[3(c+dx)] - 730aAb^6\text{Sin}[3(c+dx)] + 68a^6bB\text{Sin}[3(c+dx)] - 184a^4b^3B\text{Sin}[3(c+dx)] - \\
& 124a^2b^5B\text{Sin}[3(c+dx)] + 240b^7B\text{Sin}[3(c+dx)] + 30a^7A\text{Sin}[5(c+dx)] - 138a^5Ab^2\text{Sin}[5(c+dx)] + 282a^3Ab^4\text{Sin}[5(c+dx)] - \\
& 174aAb^6\text{Sin}[5(c+dx)] + 12a^6bB\text{Sin}[5(c+dx)] - 72a^4b^3B\text{Sin}[5(c+dx)] + 12a^2b^5B\text{Sin}[5(c+dx)] + 48b^7B\text{Sin}[5(c+dx)])
\end{aligned}$$

■ **Problem 1561: Result more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e+fx])^p}{(a+b\sin[e+fx])(c+d\sin[e+fx])} dx$$

Optimal (type 6, 330 leaves, 4 steps):

$$\begin{aligned}
& -\frac{1}{(bc-ad)f(1-p)} g \text{AppellF1}\left[1-p, \frac{1-p}{2}, \frac{1-p}{2}, 2-p, \frac{a+b}{a+b\sin[e+fx]}, \frac{a-b}{a+b\sin[e+fx]}\right] \\
& (g \cos[e+fx])^{-1+p} \left(-\frac{b(1-\sin[e+fx])}{a+b\sin[e+fx]}\right)^{\frac{1-p}{2}} \left(\frac{b(1+\sin[e+fx])}{a+b\sin[e+fx]}\right)^{\frac{1-p}{2}} + \frac{1}{(bc-ad)f(1-p)} \\
& g \text{AppellF1}\left[1-p, \frac{1-p}{2}, \frac{1-p}{2}, 2-p, \frac{c+d}{c+d\sin[e+fx]}, \frac{c-d}{c+d\sin[e+fx]}\right] (g \cos[e+fx])^{-1+p} \left(-\frac{d(1-\sin[e+fx])}{c+d\sin[e+fx]}\right)^{\frac{1-p}{2}} \left(\frac{d(1+\sin[e+fx])}{c+d\sin[e+fx]}\right)^{\frac{1-p}{2}}
\end{aligned}$$

Result (type 6, 9347 leaves):

$$\begin{aligned}
& - \left(\left(a^2 b^2 (g \cos[e + f x])^p \tan[e + f x] (1 + \tan[e + f x]^2)^{-p/2} \left(- \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) / \right. \right. \\
& \quad \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) + \\
& \quad \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x] \right) / \left(\sqrt{1 + \tan[e + f x]^2} \right. \\
& \quad \left. \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[e + f x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \right) \Bigg) / \\
& \left((b c - a d) (-b c + a d) f (a + b \sin[e + f x]) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2)) \right) \\
& \left(- \frac{1}{(b c - a d) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2))^2} a^2 b \tan[e + f x] (2 a^2 \sec[e + f x]^2 \tan[e + f x] - 2 b^2 \sec[e + f x]^2 \tan[e + f x]) \right. \\
& \quad (1 + \tan[e + f x]^2)^{-p/2} \left(- \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) / \right. \\
& \quad \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) + \\
& \quad \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[e + f x] \right) / \left(\sqrt{1 + \tan[e + f x]^2} \left(-4 a^2 \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad \left. \left. \left. 1, \frac{1+p}{2}, 1, 2, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \tan[e + f x]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \right) \Bigg) - \\
& \frac{1}{(b c - a d) (-b^2 \tan[e + f x]^2 + a^2 (1 + \tan[e + f x]^2))} a^2 b p \sec[e + f x]^2 \tan[e + f x]^2 (1 + \tan[e + f x]^2)^{-1-\frac{p}{2}} \\
& \left(- \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Tan}[e+f x] \right) / \left(\sqrt{1+\operatorname{Tan}[e+f x]^2} \left(-4 a^2 \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad 1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \right. \right. \\
& \quad \left. \left. \operatorname{Tan}[e+f x]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \left. \right) \left. \right) + \\
& \frac{1}{(b c - a d) \left(-b^2 \operatorname{Tan}[e+f x]^2 + a^2 (1 + \operatorname{Tan}[e+f x]^2) \right)} a^2 b \operatorname{Sec}[e+f x]^2 (1 + \operatorname{Tan}[e+f x]^2)^{-p/2} \\
& \left(- \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) / \right. \\
& \quad \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Tan}[e+f x] \right) / \left(\sqrt{1+\operatorname{Tan}[e+f x]^2} \left(-4 a^2 \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad 1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \right. \right. \\
& \quad \left. \left. \operatorname{Tan}[e+f x]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \left. \right) \left. \right) + \\
& \frac{1}{(b c - a d) \left(-b^2 \operatorname{Tan}[e+f x]^2 + a^2 (1 + \operatorname{Tan}[e+f x]^2) \right)} a^2 b \operatorname{Tan}[e+f x] (1 + \operatorname{Tan}[e+f x]^2)^{-p/2} \\
& \left(- \left(3 a \left(-\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 + \frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \right. \right. \right. \\
& \quad \left. \left. \frac{2}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) / \right. \\
& \quad \left(-3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 - \\
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]^2 \right) / \\
& \left((1 + \operatorname{Tan}[e + f x]^2)^{3/2} \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left. a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) + \\
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \right) / \\
& \left(\sqrt{1 + \operatorname{Tan}[e + f x]^2} \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left. a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) + \\
& \left(2 b \operatorname{Tan}[e + f x] \left(1/a^2 (-a^2 + b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \right. \right. \\
& \left. \left. \operatorname{Tan}[e + f x] - \frac{1}{2} (1+p) \operatorname{AppellF1} \left[2, 1 + \frac{1+p}{2}, 1, 3, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \right) / \\
& \left(\sqrt{1 + \operatorname{Tan}[e + f x]^2} \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left. a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) + \\
& \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \left(2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Sec}[e + f x]^2 \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \tan[ex] - 3a^2 \left(-\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 + \frac{p}{2}, 1, \frac{5}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] + \right. \\
& \quad \left. \frac{2}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] \right) + \tan[ex]^2 \\
& \left(2(a^2 - b^2) \left(-\frac{3}{5} p \operatorname{AppellF1} \left[\frac{5}{2}, 1 + \frac{p}{2}, 2, \frac{7}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] + \right. \right. \\
& \quad \left. \frac{12}{5} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{p}{2}, 3, \frac{7}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] \right) + \\
& \quad a^2 p \left(\frac{6}{5} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{2+p}{2}, 2, \frac{7}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] - \right. \\
& \quad \left. \left. \frac{3}{5} (2+p) \operatorname{AppellF1} \left[\frac{5}{2}, 1 + \frac{2+p}{2}, 1, \frac{7}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] \right) \right) \right) / \\
& \left(-3a^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] + \left(2(a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[ex]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] + a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \right) \tan[ex]^2 \right)^2 - \\
& \left(2b \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[ex]^2, \frac{(-a^2 + b^2) \tan[ex]^2}{a^2} \right] \tan[ex] \left(2 \left(2(a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, \right. \right. \right. \right. \\
& \quad \left. \left. -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] + a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \right) \right) \\
& \sec[ex]^2 \tan[ex] - 4a^2 \left(\left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \right. \\
& \quad \left. \tan[ex] - \frac{1}{2} (1+p) \operatorname{AppellF1} \left[2, 1 + \frac{1+p}{2}, 1, 3, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] \right) + \\
& \tan[ex]^2 \left(2(a^2 - b^2) \left(\frac{8}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[3, \frac{1+p}{2}, 3, 4, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] \right. \right. \\
& \quad \left. \left. + \frac{2}{3} (1+p) \operatorname{AppellF1} \left[3, 1 + \frac{1+p}{2}, 2, 4, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] \right) + \right. \\
& \quad \left. a^2 (1+p) \left(\frac{4}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[3, \frac{3+p}{2}, 2, 4, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] - \right. \right. \\
& \quad \left. \left. \frac{2}{3} (3+p) \operatorname{AppellF1} \left[3, 1 + \frac{3+p}{2}, 1, 4, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] \sec[ex]^2 \tan[ex] \right) \right) \right) / \\
& \left(\sqrt{1 + \tan[ex]^2} \left(-4a^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[ex]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[ex]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \\
& \quad \left. a^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{b^2}{a^2} \right) \operatorname{Tan}[e+fx]^2 \right] \operatorname{Tan}[e+fx]^2 \right) \left. \right) \left. \right) \left. \right) \left. \right) - \\
& \left(c^2 d^2 (g \operatorname{Cos}[e+fx])^p \operatorname{Tan}[e+fx] (1 + \operatorname{Tan}[e+fx]^2)^{-p/2} \left(- \left(3 c \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+fx]^2 \right] \right) / \right. \right. \\
& \quad \left(-3 c^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \\
& \quad \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \\
& \quad \left. c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \right) + \\
& \left. \left(2 d \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e+fx]^2, \frac{(-c^2 + d^2) \operatorname{Tan}[e+fx]^2}{c^2} \right] \operatorname{Tan}[e+fx] \right) / \right. \\
& \quad \left(\sqrt{1 + \operatorname{Tan}[e+fx]^2} \right. \\
& \quad \left(-4 c^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \\
& \quad \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \right. \\
& \quad \left. c^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \left. \right) \left. \right) \left. \right) \left. \right) / \\
& \left((bc - ad) (-bc + ad) f (c + d \operatorname{Sin}[e+fx]) (-d^2 \operatorname{Tan}[e+fx]^2 + c^2 (1 + \operatorname{Tan}[e+fx]^2)) \right) \\
& \left(- \frac{1}{(-bc + ad) (-d^2 \operatorname{Tan}[e+fx]^2 + c^2 (1 + \operatorname{Tan}[e+fx]^2))^2} \right. \\
& \quad c^2 d \operatorname{Tan}[e+fx] \\
& \quad (2 c^2 \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - 2 d^2 \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]) \\
& \quad (1 + \operatorname{Tan}[e+fx]^2)^{-p/2} \\
& \quad \left(- \left(3 c \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+fx]^2 \right] \right) / \right. \\
& \quad \left(-3 c^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+fx]^2 \right] + \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+fx]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 + c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Tan}[e + f x]^2 \right) + \\
& \left(2 d \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \frac{(-c^2 + d^2) \operatorname{Tan}[e + f x]^2}{c^2} \right] \operatorname{Tan}[e + f x] \right) / \left(\sqrt{1 + \operatorname{Tan}[e + f x]^2} \right. \\
& \left. \left(-4 c^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + c^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) \Bigg) - \\
& \frac{1}{(-b c + a d) (-d^2 \operatorname{Tan}[e + f x]^2 + c^2 (1 + \operatorname{Tan}[e + f x]^2))} c^2 d p \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]^2 (1 + \operatorname{Tan}[e + f x]^2)^{-1-\frac{p}{2}} \\
& \left(- \left(3 c \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) / \right. \\
& \left(-3 c^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) + \\
& \left(2 d \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \frac{(-c^2 + d^2) \operatorname{Tan}[e + f x]^2}{c^2} \right] \operatorname{Tan}[e + f x] \right) / \left(\sqrt{1 + \operatorname{Tan}[e + f x]^2} \right. \\
& \left. \left(-4 c^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + c^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) \Bigg) + \\
& \frac{1}{(-b c + a d) (-d^2 \operatorname{Tan}[e + f x]^2 + c^2 (1 + \operatorname{Tan}[e + f x]^2))} c^2 d \operatorname{Sec}[e + f x]^2 (1 + \operatorname{Tan}[e + f x]^2)^{-p/2} \\
& \left(- \left(3 c \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) / \right. \\
& \left(-3 c^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right) + \\
& \left(2 d \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \frac{(-c^2 + d^2) \operatorname{Tan}[e + f x]^2}{c^2} \right] \operatorname{Tan}[e + f x] \right) / \left(\sqrt{1 + \operatorname{Tan}[e + f x]^2} \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-4 c^2 \operatorname{AppellF1}\left[1, \frac{1+p}{2}, 1, 2, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] + \left(2\left(c^2-d^2\right) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\tan[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] + c^2(1+p) \operatorname{AppellF1}\left[2, \frac{3+p}{2}, 1, 3, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right]\right) \tan[e+f x]^2 \Bigg) + \\
& \frac{1}{(-b c+a d)\left(-d^2 \tan[e+f x]^2+c^2\left(1+\tan[e+f x]^2\right)\right)} c^2 d \tan[e+f x]\left(1+\tan[e+f x]^2\right)^{-p / 2} \\
& \left(-3 c\left(-\frac{1}{3} p \operatorname{AppellF1}\left[\frac{3}{2}, 1+\frac{p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \tan[e+f x] + \right. \right. \\
& \quad \left. \left. \frac{2}{3}\left(-1+\frac{d^2}{c^2}\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \tan[e+f x]\right)\right) / \\
& \left(-3 c^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] + \left(2\left(c^2-d^2\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] + c^2 p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right]\right) \tan[e+f x]^2 - \\
& \left(2 d \operatorname{AppellF1}\left[1, \frac{1+p}{2}, 1, 2, -\tan[e+f x]^2, \frac{\left(-c^2+d^2\right) \tan[e+f x]^2}{c^2}\right] \operatorname{Sec}[e+f x]^2 \tan[e+f x]^2\right) / \\
& \left(\left(1+\tan[e+f x]^2\right)^{3 / 2}\left(-4 c^2 \operatorname{AppellF1}\left[1, \frac{1+p}{2}, 1, 2, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] + \right. \right. \\
& \quad \left. \left(2\left(c^2-d^2\right) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] + \right. \right. \\
& \quad \left. \left. c^2(1+p) \operatorname{AppellF1}\left[2, \frac{3+p}{2}, 1, 3, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right]\right) \tan[e+f x]^2\right) + \\
& \left(2 d \operatorname{AppellF1}\left[1, \frac{1+p}{2}, 1, 2, -\tan[e+f x]^2, \frac{\left(-c^2+d^2\right) \tan[e+f x]^2}{c^2}\right] \operatorname{Sec}[e+f x]^2\right) / \left(\sqrt{1+\tan[e+f x]^2}\right) \\
& \left(-4 c^2 \operatorname{AppellF1}\left[1, \frac{1+p}{2}, 1, 2, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] + \left(2\left(c^2-d^2\right) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\tan[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right] + c^2(1+p) \operatorname{AppellF1}\left[2, \frac{3+p}{2}, 1, 3, -\tan[e+f x]^2, \left(-1+\frac{d^2}{c^2}\right) \tan[e+f x]^2\right]\right) \tan[e+f x]^2 \Bigg) + \\
& \left(2 d \tan[e+f x]\left(1 / c^2\left(-c^2+d^2\right) \operatorname{AppellF1}\left[2, \frac{1+p}{2}, 2, 3, -\tan[e+f x]^2, \frac{\left(-c^2+d^2\right) \tan[e+f x]^2}{c^2}\right] \operatorname{Sec}[e+f x]^2 \right. \right. \\
& \quad \left. \left. \tan[e+f x] - \frac{1}{2}(1+p) \operatorname{AppellF1}\left[2, 1+\frac{1+p}{2}, 1, 3, -\tan[e+f x]^2, \frac{\left(-c^2+d^2\right) \tan[e+f x]^2}{c^2}\right] \operatorname{Sec}[e+f x]^2 \tan[e+f x]\right)\right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{1 + \tan[e + f x]^2} \left(-4 c^2 \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] + \right. \\
& \quad \left. \left. c^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) + \\
& \left(3 c \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \left(2 \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] + c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \right) \sec[e + f x]^2 \right. \right. \\
& \quad \tan[e + f x] - 3 c^2 \left(-\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 + \frac{p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \right. \\
& \quad \left. \frac{2}{3} \left(-1 + \frac{d^2}{c^2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) + \tan[e + f x]^2 \\
& \quad \left(2 (c^2 - d^2) \left(-\frac{3}{5} p \operatorname{AppellF1} \left[\frac{5}{2}, 1 + \frac{p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \right. \right. \\
& \quad \left. \frac{12}{5} \left(-1 + \frac{d^2}{c^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{p}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) + \\
& \quad \left. c^2 p \left(\frac{6}{5} \left(-1 + \frac{d^2}{c^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{2+p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \quad \left. \left. \frac{3}{5} (2+p) \operatorname{AppellF1} \left[\frac{5}{2}, 1 + \frac{2+p}{2}, 1, \frac{7}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \right) \Big/ \\
& \left(-3 c^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] + \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] + c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2+p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right)^2 - \\
& \left(2 d \operatorname{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[e + f x]^2, \frac{(-c^2 + d^2) \tan[e + f x]^2}{c^2} \right] \tan[e + f x] \left(2 \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] + c^2 (1+p) \operatorname{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \right) \right. \\
& \quad \left. \sec[e + f x]^2 \tan[e + f x] - 4 c^2 \left(\left(-1 + \frac{d^2}{c^2} \right) \operatorname{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \right. \right. \\
& \quad \left. \left. \tan[e + f x] - \frac{1}{2} (1+p) \operatorname{AppellF1} \left[2, 1 + \frac{1+p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(\tan[e+fx]^2 \left(2(c^2-d^2) \left(\frac{8}{3} \left(-1 + \frac{d^2}{c^2} \right) \text{AppellF1} \left[3, \frac{1+p}{2}, 3, 4, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{2}{3} (1+p) \text{AppellF1} \left[3, 1 + \frac{1+p}{2}, 2, 4, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right) + \right. \right. \\
& \quad \left. \left. c^2 (1+p) \left(\frac{4}{3} \left(-1 + \frac{d^2}{c^2} \right) \text{AppellF1} \left[3, \frac{3+p}{2}, 2, 4, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{2}{3} (3+p) \text{AppellF1} \left[3, 1 + \frac{3+p}{2}, 1, 4, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) \right) \Bigg) / \\
& \left(\sqrt{1 + \tan[e+fx]^2} \left(-4c^2 \text{AppellF1} \left[1, \frac{1+p}{2}, 1, 2, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] + \right. \right. \\
& \quad \left. \left(2(c^2-d^2) \text{AppellF1} \left[2, \frac{1+p}{2}, 2, 3, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] + \right. \right. \\
& \quad \left. \left. c^2 (1+p) \text{AppellF1} \left[2, \frac{3+p}{2}, 1, 3, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] \right) \tan[e+fx]^2 \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 1562: Result more than twice size of optimal antiderivative.**

$$\int \frac{(g \cos[e+fx])^p}{(a+b \sin[e+fx]) (c+d \sin[e+fx])^2} dx$$

Optimal (type 6, 508 leaves, 5 steps):

$$\begin{aligned}
& - \frac{1}{(bc-ad)^2 f (1-p)} b g \text{AppellF1} \left[1-p, \frac{1-p}{2}, \frac{1-p}{2}, 2-p, \frac{a+b}{a+b \sin[e+fx]}, \frac{a-b}{a+b \sin[e+fx]} \right] \\
& (g \cos[e+fx])^{-1+p} \left(- \frac{b(1-\sin[e+fx])}{a+b \sin[e+fx]} \right)^{\frac{1-p}{2}} \left(\frac{b(1+\sin[e+fx])}{a+b \sin[e+fx]} \right)^{\frac{1-p}{2}} + \frac{1}{(bc-ad)^2 f (1-p)} \\
& b g \text{AppellF1} \left[1-p, \frac{1-p}{2}, \frac{1-p}{2}, 2-p, \frac{c+d}{c+d \sin[e+fx]}, \frac{c-d}{c+d \sin[e+fx]} \right] (g \cos[e+fx])^{-1+p} \left(- \frac{d(1-\sin[e+fx])}{c+d \sin[e+fx]} \right)^{\frac{1-p}{2}} \\
& \left(\frac{d(1+\sin[e+fx])}{c+d \sin[e+fx]} \right)^{\frac{1-p}{2}} + \left(g \text{AppellF1} \left[2-p, \frac{1-p}{2}, \frac{1-p}{2}, 3-p, \frac{c+d}{c+d \sin[e+fx]}, \frac{c-d}{c+d \sin[e+fx]} \right] \right) \\
& (g \cos[e+fx])^{-1+p} \left(- \frac{d(1-\sin[e+fx])}{c+d \sin[e+fx]} \right)^{\frac{1-p}{2}} \left(\frac{d(1+\sin[e+fx])}{c+d \sin[e+fx]} \right)^{\frac{1-p}{2}} \Bigg) / ((bc-ad) f (2-p) (c+d \sin[e+fx]))
\end{aligned}$$

Result (type 6, 23548 leaves): Display of huge result suppressed!

■ **Problem 1563: Result more than twice size of optimal antiderivative.**

$$\int \frac{(g \operatorname{Sec}[e + f x])^p}{(a + b \operatorname{Sin}[e + f x]) (c + d \operatorname{Sin}[e + f x])} dx$$

Optimal (type 6, 308 leaves, 5 steps):

$$\begin{aligned} & -\frac{1}{(bc - ad) f (1+p)} \operatorname{AppellF1}\left[1+p, \frac{1+p}{2}, \frac{1+p}{2}, 2+p, \frac{a+b}{a+b \operatorname{Sin}[e+fx]}, \frac{a-b}{a+b \operatorname{Sin}[e+fx]}\right] \\ & \operatorname{Sec}[e+fx] (g \operatorname{Sec}[e+fx])^p \left(-\frac{b(1-\operatorname{Sin}[e+fx])}{a+b \operatorname{Sin}[e+fx]} \right)^{\frac{1+p}{2}} \left(\frac{b(1+\operatorname{Sin}[e+fx])}{a+b \operatorname{Sin}[e+fx]} \right)^{\frac{1+p}{2}} + \\ & \frac{1}{(bc - ad) f (1+p)} \operatorname{AppellF1}\left[1+p, \frac{1+p}{2}, \frac{1+p}{2}, 2+p, \frac{c+d}{c+d \operatorname{Sin}[e+fx]}, \frac{c-d}{c+d \operatorname{Sin}[e+fx]}\right] \operatorname{Sec}[e+fx] \\ & (g \operatorname{Sec}[e+fx])^p \left(-\frac{d(1-\operatorname{Sin}[e+fx])}{c+d \operatorname{Sin}[e+fx]} \right)^{\frac{1+p}{2}} \left(\frac{d(1+\operatorname{Sin}[e+fx])}{c+d \operatorname{Sin}[e+fx]} \right)^{\frac{1+p}{2}} \end{aligned}$$

Result (type 6, 9549 leaves):

$$\begin{aligned} & -\left(\left(a^2 b^2 (g \operatorname{Sec}[e + f x])^p \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{p/2} \left(\left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] \right) / \right. \right. \right. \\ & \left(3 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + \left(a^2 p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \\ & \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + 2(-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] \right) \operatorname{Tan}[e + f x]^2 \right) + \\ & \left(2 b \operatorname{AppellF1}\left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \frac{(-a^2 + b^2) \operatorname{Tan}[e + f x]^2}{a^2}\right] \operatorname{Tan}[e + f x] \right) / \left(\sqrt{1 + \operatorname{Tan}[e + f x]^2} \right) \\ & \left(-4 a^2 \operatorname{AppellF1}\left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] + \left(2(a^2 - b^2) \operatorname{AppellF1}\left[2, \frac{1-p}{2}, 2, 3, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \\ & \left. \left. \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] - a^2(-1+p) \operatorname{AppellF1}\left[2, \frac{3-p}{2}, 1, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) / \\ & \left((bc - ad) (-bc + ad) f (a + b \operatorname{Sin}[e + f x]) (-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2)) \right. \\ & \left. \left(-\frac{1}{(bc - ad) (-b^2 \operatorname{Tan}[e + f x]^2 + a^2 (1 + \operatorname{Tan}[e + f x]^2))^2} a^2 b \operatorname{Tan}[e + f x] (2 a^2 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - 2 b^2 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]) \right. \right. \\ & \left. \left. (1 + \operatorname{Tan}[e + f x]^2)^{p/2} \left(\left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{b^2}{a^2}\right) \operatorname{Tan}[e + f x]^2\right] \right) / \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Tan}[e+f x] \right) / \left(\sqrt{1+\operatorname{Tan}[e+f x]^2} \left(-4 a^2 \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad \left. \left. \left. 1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(2 (a^2-b^2) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}[e+f x]^2 \right] - a^2 (-1+p) \operatorname{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^2 \right) \right) \right) + \\
& \frac{1}{(b c-a d) \left(-b^2 \operatorname{Tan}[e+f x]^2+a^2 \left(1+\operatorname{Tan}[e+f x]^2 \right) \right)} a^2 b \operatorname{Tan}[e+f x] \left(1+\operatorname{Tan}[e+f x]^2 \right)^{p/2} \\
& \left(\left(3 a \left(\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1-\frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \right. \right. \right. \\
& \quad \left. \left. \frac{2}{3} \left(-1+\frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) / \\
& \left(3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, 1-\frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + 2 \left(-a^2+b^2 \right) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]^2 \right) / \\
& \left(\left(1+\operatorname{Tan}[e+f x]^2 \right)^{3/2} \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left(2 \left(a^2-b^2 \right) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left. a^2 (-1+p) \operatorname{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) + \\
& \left(2 b \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \frac{(-a^2+b^2) \operatorname{Tan}[e+f x]^2}{a^2} \right] \operatorname{Sec}[e+f x]^2 \right) / \\
& \left(\sqrt{1+\operatorname{Tan}[e+f x]^2} \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left(2 \left(a^2-b^2 \right) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left. a^2 (-1+p) \operatorname{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\operatorname{Tan}[e+f x]^2, \left(-1+\frac{b^2}{a^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(2 b \tan[e + f x] \left(-\frac{1}{2} (1 - p) \operatorname{AppellF1} \left[2, 1 + \frac{1 - p}{2}, 1, 3, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \sec[e + f x]^2 \tan[e + f x] + \right. \right. \\
& \quad \left. \left. 1 / a^2 (-a^2 + b^2) \operatorname{AppellF1} \left[2, \frac{1 - p}{2}, 2, 3, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) / \\
& \left(\sqrt{1 + \tan[e + f x]^2} \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1 - p}{2}, 1, 2, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1 - p}{2}, 2, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \\
& \quad \left. \left. a^2 (-1 + p) \operatorname{AppellF1} \left[2, \frac{3 - p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \tan[e + f x]^2 \right) \right) - \\
& \left(3 a \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \left(2 \left(a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \sec[e + f x]^2 \right. \right. \\
& \quad \left. \tan[e + f x] + 3 a^2 \left(\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \right. \right. \\
& \quad \left. \left. \frac{2}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) + \tan[e + f x]^2 \right. \right. \\
& \quad \left(a^2 p \left(\frac{6}{5} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, 1 - \frac{p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \quad \left. \frac{6}{5} \left(1 - \frac{p}{2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, 2 - \frac{p}{2}, 1, \frac{7}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) + \\
& \quad \left. 2 (-a^2 + b^2) \left(\frac{3}{5} p \operatorname{AppellF1} \left[\frac{5}{2}, 1 - \frac{p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \right. \right. \\
& \quad \left. \left. \frac{12}{5} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{p}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \right) \right) / \\
& \left(3 a^2 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \left(a^2 p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \right) \\
& \quad \left. \tan[e + f x]^2 \right)^2 - \left(2 b \operatorname{AppellF1} \left[1, \frac{1 - p}{2}, 1, 2, -\tan[e + f x]^2, \frac{(-a^2 + b^2) \tan[e + f x]^2}{a^2} \right] \tan[\right.
\end{aligned}$$

$$\begin{aligned}
& e + f x \left[2 \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \\
& \quad \left. \left. a^2 (-1+p) \operatorname{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \sec[e + f x]^2 \tan[e + f x] - \right. \\
& \quad \left. 4 a^2 \left(-\frac{1}{2} (1-p) \operatorname{AppellF1} \left[2, 1 + \frac{1-p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \right. \right. \\
& \quad \left. \left. \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) + \tan[e + f x]^2 \right. \\
& \quad \left. \left(2 (a^2 - b^2) \left(-\frac{2}{3} (1-p) \operatorname{AppellF1} \left[3, 1 + \frac{1-p}{2}, 2, 4, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \right. \right. \right. \\
& \quad \left. \left. \frac{8}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[3, \frac{1-p}{2}, 3, 4, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) - \right. \\
& \quad \left. a^2 (-1+p) \left(-\frac{2}{3} (3-p) \operatorname{AppellF1} \left[3, 1 + \frac{3-p}{2}, 1, 4, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] + \right. \right. \\
& \quad \left. \left. \frac{4}{3} \left(-1 + \frac{b^2}{a^2} \right) \operatorname{AppellF1} \left[3, \frac{3-p}{2}, 2, 4, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \right) \right) \Big/ \\
& \left(\sqrt{1 + \tan[e + f x]^2} \left(-4 a^2 \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] + \right. \right. \\
& \quad \left. \left(2 (a^2 - b^2) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] - \right. \right. \\
& \quad \left. \left. a^2 (-1+p) \operatorname{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{b^2}{a^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \right) \Big/ \\
& \left(c^2 d^2 (g \sec[e + f x])^p \tan[e + f x] (1 + \tan[e + f x]^2)^{p/2} \left(\left(3 c \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \right) \right) \Big/ \right. \\
& \quad \left(3 c^2 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] + \right. \\
& \quad \left(c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] + \right. \\
& \quad \left. 2 (-c^2 + d^2) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) + \\
& \quad \left(2 d \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\tan[e + f x]^2, \frac{(-c^2 + d^2) \tan[e + f x]^2}{c^2} \right] \tan[e + f x] \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{1 + \tan[e + f x]^2} \right. \\
& \left(-4 c^2 \operatorname{AppellF1}\left[1, \frac{1-p}{2}, 1, 2, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] + \right. \\
& \left. \left(2 (c^2 - d^2) \operatorname{AppellF1}\left[2, \frac{1-p}{2}, 2, 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] - \right. \right. \\
& \left. \left. c^2 (-1+p) \operatorname{AppellF1}\left[2, \frac{3-p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] \right) \tan[e + f x]^2 \right) \right) \Big/ \\
& \left((b c - a d) (-b c + a d) f (c + d \sin[e + f x]) (-d^2 \tan[e + f x]^2 + c^2 (1 + \tan[e + f x]^2)) \right) \\
& \left(-\frac{1}{(-b c + a d) (-d^2 \tan[e + f x]^2 + c^2 (1 + \tan[e + f x]^2))^2} \right. \\
& \left. c^2 d \tan[e + f x] \right. \\
& \left. (2 c^2 \sec[e + f x]^2 \tan[e + f x] - 2 d^2 \sec[e + f x]^2 \tan[e + f x]) \right. \\
& \left. (1 + \tan[e + f x]^2)^{p/2} \right. \\
& \left(\left(3 c \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] \right) \Big/ \right. \\
& \left(3 c^2 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] + \left(c^2 p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] + 2 (-c^2 + d^2) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] \right) \tan[e + f x]^2 \right) + \\
& \left. \left(2 d \operatorname{AppellF1}\left[1, \frac{1-p}{2}, 1, 2, -\tan[e + f x]^2, \frac{(-c^2 + d^2) \tan[e + f x]^2}{c^2}\right] \tan[e + f x] \right) \Big/ \right. \\
& \left(\sqrt{1 + \tan[e + f x]^2} \left(-4 c^2 \operatorname{AppellF1}\left[1, \frac{1-p}{2}, 1, 2, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] + \right. \right. \\
& \left. \left(2 (c^2 - d^2) \operatorname{AppellF1}\left[2, \frac{1-p}{2}, 2, 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] - \right. \right. \\
& \left. \left. c^2 (-1+p) \operatorname{AppellF1}\left[2, \frac{3-p}{2}, 1, 3, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] \right) \tan[e + f x]^2 \right) \right) \Big) + \\
& \frac{1}{(-b c + a d) (-d^2 \tan[e + f x]^2 + c^2 (1 + \tan[e + f x]^2))} c^2 d p \sec[e + f x]^2 \tan[e + f x]^2 (1 + \tan[e + f x]^2)^{-1 + \frac{p}{2}} \\
& \left(\left(3 c \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \left(-1 + \frac{d^2}{c^2}\right) \tan[e + f x]^2\right] \right) \Big/ \right.
\end{aligned}$$

$$\begin{aligned}
& \left(3 c^2 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] + 2 \left(-c^2 + d^2 \right) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \left(2 d \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \frac{\left(-c^2 + d^2 \right) \operatorname{Tan}[e+f x]^2}{c^2} \right] \operatorname{Tan}[e+f x] \right) / \\
& \left(\sqrt{1 + \operatorname{Tan}[e+f x]^2} \left(-4 c^2 \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left(2 \left(c^2 - d^2 \right) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left. c^2 \left(-1 + p \right) \operatorname{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) \Bigg) + \\
& \frac{1}{(-b c + a d) \left(-d^2 \operatorname{Tan}[e+f x]^2 + c^2 \left(1 + \operatorname{Tan}[e+f x]^2 \right) \right)} c^2 d \operatorname{Sec}[e+f x]^2 \left(1 + \operatorname{Tan}[e+f x]^2 \right)^{p/2} \\
& \left(\left(3 c \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) / \right. \\
& \quad \left(3 c^2 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \left(c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] + 2 \left(-c^2 + d^2 \right) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \quad \left(2 d \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \frac{\left(-c^2 + d^2 \right) \operatorname{Tan}[e+f x]^2}{c^2} \right] \operatorname{Tan}[e+f x] \right) / \\
& \quad \left(\sqrt{1 + \operatorname{Tan}[e+f x]^2} \left(-4 c^2 \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] + \right. \right. \\
& \quad \left. \left(2 \left(c^2 - d^2 \right) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left. c^2 \left(-1 + p \right) \operatorname{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) \Bigg) + \\
& \frac{1}{(-b c + a d) \left(-d^2 \operatorname{Tan}[e+f x]^2 + c^2 \left(1 + \operatorname{Tan}[e+f x]^2 \right) \right)} c^2 d \operatorname{Tan}[e+f x] \left(1 + \operatorname{Tan}[e+f x]^2 \right)^{p/2} \\
& \left(\left(3 c \left(\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \frac{2}{3} \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \frac{d^2}{c^2} \right) \text{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \Big) \Big) / \\
& \left(3 c^2 \text{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] + \left(c^2 p \text{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\text{Tan}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] + 2 (-c^2 + d^2) \text{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] \right) \text{Tan}[e + f x]^2 \right) - \\
& \left(2 d \text{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\text{Tan}[e + f x]^2, \frac{(-c^2 + d^2) \text{Tan}[e + f x]^2}{c^2} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x]^2 \right) / \\
& \left((1 + \text{Tan}[e + f x]^2)^{3/2} \left(-4 c^2 \text{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left(2 (c^2 - d^2) \text{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] - \right. \right. \\
& \left. \left. c^2 (-1 + p) \text{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] \right) \text{Tan}[e + f x]^2 \right) \Big) + \\
& \left(2 d \text{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\text{Tan}[e + f x]^2, \frac{(-c^2 + d^2) \text{Tan}[e + f x]^2}{c^2} \right] \text{Sec}[e + f x]^2 \right) / \left(\sqrt{1 + \text{Tan}[e + f x]^2} \right. \\
& \left(-4 c^2 \text{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] + \left(2 (c^2 - d^2) \text{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\text{Tan}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] - c^2 (-1 + p) \text{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] \right) \text{Tan}[e + f x]^2 \Big) \Big) + \\
& \left(2 d \text{Tan}[e + f x] \left(-\frac{1}{2} (1-p) \text{AppellF1} \left[2, 1 + \frac{1-p}{2}, 1, 3, -\text{Tan}[e + f x]^2, \frac{(-c^2 + d^2) \text{Tan}[e + f x]^2}{c^2} \right] \text{Sec}[e + f x]^2 \right. \right. \\
& \left. \left. \text{Tan}[e + f x] + 1 / c^2 (-c^2 + d^2) \text{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\text{Tan}[e + f x]^2, \frac{(-c^2 + d^2) \text{Tan}[e + f x]^2}{c^2} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) \right) \Big) \Big) / \\
& \left(\sqrt{1 + \text{Tan}[e + f x]^2} \left(-4 c^2 \text{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] + \right. \right. \\
& \left. \left(2 (c^2 - d^2) \text{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] - \right. \right. \\
& \left. \left. c^2 (-1 + p) \text{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] \right) \text{Tan}[e + f x]^2 \right) \Big) - \\
& \left(3 c \text{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\text{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \text{Tan}[e + f x]^2 \right] \left(2 \left(c^2 p \text{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\text{Tan}[e + f x]^2, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 + 2 (-c^2 + d^2) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Sec}[e + f x]^2 \\
& \operatorname{Tan}[e + f x] + 3 c^2 \left(\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \right. \\
& \left. \frac{2}{3} \left(-1 + \frac{d^2}{c^2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) + \operatorname{Tan}[e + f x]^2 \\
& \left(c^2 p \left(\frac{6}{5} \left(-1 + \frac{d^2}{c^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, 1 - \frac{p}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \right. \\
& \left. \left. \frac{6}{5} \left(1 - \frac{p}{2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, 2 - \frac{p}{2}, 1, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) + \right. \\
& \left. 2 (-c^2 + d^2) \left(\frac{3}{5} p \operatorname{AppellF1} \left[\frac{5}{2}, 1 - \frac{p}{2}, 2, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \right. \right. \\
& \left. \left. \frac{12}{5} \left(-1 + \frac{d^2}{c^2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{p}{2}, 3, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \right) \right) \Bigg) / \\
& \left(3 c^2 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{p}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + \left(c^2 p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{p}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] + 2 (-c^2 + d^2) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{p}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Tan}[e + f x]^2 \right)^2 - \\
& \left(2 d \operatorname{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\operatorname{Tan}[e + f x]^2, \frac{(-c^2 + d^2) \operatorname{Tan}[e + f x]^2}{c^2} \right] \operatorname{Tan}[e + f x] \right. \\
& \left. \left(2 \left(2 (c^2 - d^2) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] - \right. \right. \right. \\
& \left. \left. c^2 (-1+p) \operatorname{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \right) \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - 4 c^2 \right. \\
& \left. \left(-\frac{1}{2} (1-p) \operatorname{AppellF1} \left[2, 1 + \frac{1-p}{2}, 1, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \right. \right. \\
& \left. \left. \left(-1 + \frac{d^2}{c^2} \right) \operatorname{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) + \operatorname{Tan}[e + f x]^2 \right. \\
& \left. \left(2 (c^2 - d^2) \left(-\frac{2}{3} (1-p) \operatorname{AppellF1} \left[3, 1 + \frac{1-p}{2}, 2, 4, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \right. \right. \right. \\
& \left. \left. \frac{8}{3} \left(-1 + \frac{d^2}{c^2} \right) \operatorname{AppellF1} \left[3, \frac{1-p}{2}, 3, 4, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) - \right. \\
& \left. c^2 (-1+p) \left(-\frac{2}{3} (3-p) \operatorname{AppellF1} \left[3, 1 + \frac{3-p}{2}, 1, 4, -\operatorname{Tan}[e + f x]^2, \left(-1 + \frac{d^2}{c^2} \right) \operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \right. \right.
\end{aligned}$$

$$\left. \left. \left. \frac{4}{3} \left(-1 + \frac{d^2}{c^2} \right) \text{AppellF1} \left[3, \frac{3-p}{2}, 2, 4, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] \sec[e+fx]^2 \tan[e+fx] \right] \right) \right) \left. \left. \left. \sqrt{1 + \tan[e+fx]^2} \left(-4 c^2 \text{AppellF1} \left[1, \frac{1-p}{2}, 1, 2, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] + \right. \right. \right. \right.$$

$$\left. \left. \left. \left. 2 (c^2 - d^2) \text{AppellF1} \left[2, \frac{1-p}{2}, 2, 3, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] - \right. \right. \right. \right.$$

$$\left. \left. \left. \left. c^2 (-1 + p) \text{AppellF1} \left[2, \frac{3-p}{2}, 1, 3, -\tan[e+fx]^2, \left(-1 + \frac{d^2}{c^2} \right) \tan[e+fx]^2 \right] \tan[e+fx]^2 \right] \right) \right) \right) \right)$$

Test results for the 51 problems in "4.1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n.m"

■ Problem 8: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[e+fx]^4 (a + a \sin[e+fx])^2 (c - c \sin[e+fx]) \, dx$$

Optimal (type 3, 61 leaves, 6 steps):

$$\frac{a^2 c \text{ArcTanh}[\cos[e+fx]]}{2 f} - \frac{a^2 c \cot[e+fx]^3}{3 f} - \frac{a^2 c \cot[e+fx] \text{Csc}[e+fx]}{2 f}$$

Result (type 3, 172 leaves):

$$a^2 c \left(\frac{\cot\left[\frac{1}{2}(e+fx)\right]}{6 f} - \frac{\text{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{8 f} - \frac{\cot\left[\frac{1}{2}(e+fx)\right] \text{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{24 f} + \right.$$

$$\left. \frac{\text{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right]\right]}{2 f} - \frac{\text{Log}\left[\sin\left[\frac{1}{2}(e+fx)\right]\right]}{2 f} + \frac{\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{8 f} - \frac{\tan\left[\frac{1}{2}(e+fx)\right]}{6 f} + \frac{\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]}{24 f} \right)$$

■ Problem 9: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[e+fx]^5 (a + a \sin[e+fx])^2 (c - c \sin[e+fx]) \, dx$$

Optimal (type 3, 86 leaves, 11 steps):

$$\frac{a^2 c \text{ArcTanh}[\cos[e+fx]]}{8 f} - \frac{a^2 c \cot[e+fx]^3}{3 f} + \frac{a^2 c \cot[e+fx] \text{Csc}[e+fx]}{8 f} - \frac{a^2 c \cot[e+fx] \text{Csc}[e+fx]^3}{4 f}$$

Result (type 3, 179 leaves):

$$\frac{a^2 c \cot [e+f x]}{3 f} + \frac{a^2 c \csc \left[\frac{1}{2}(e+f x)\right]^2}{32 f} - \frac{a^2 c \csc \left[\frac{1}{2}(e+f x)\right]^4}{64 f} - \frac{a^2 c \cot [e+f x] \csc [e+f x]^2}{3 f} +$$

$$\frac{a^2 c \log \left[\cos \left[\frac{1}{2}(e+f x)\right]\right]}{8 f} - \frac{a^2 c \log \left[\sin \left[\frac{1}{2}(e+f x)\right]\right]}{8 f} - \frac{a^2 c \sec \left[\frac{1}{2}(e+f x)\right]^2}{32 f} + \frac{a^2 c \sec \left[\frac{1}{2}(e+f x)\right]^4}{64 f}$$

■ **Problem 13: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc [e+f x] \sqrt{a+a \sin [e+f x]}}{c-c \sin [e+f x]} dx$$

Optimal (type 3, 69 leaves, 5 steps):

$$-\frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [e+f x]}{\sqrt{a+a \sin [e+f x]}}\right]}{c f} + \frac{2 \sec [e+f x] \sqrt{a+a \sin [e+f x]}}{c f}$$

Result (type 3, 157 leaves):

$$\frac{1}{c f} \sec [e+f x] \left(2 + \cos \left[\frac{1}{2}(e+f x)\right] \left(-\log \left[1 + \cos \left[\frac{1}{2}(e+f x)\right] - \sin \left[\frac{1}{2}(e+f x)\right]\right] + \log \left[1 - \cos \left[\frac{1}{2}(e+f x)\right] + \sin \left[\frac{1}{2}(e+f x)\right]\right]\right) + \right.$$

$$\left. \left(\log \left[1 + \cos \left[\frac{1}{2}(e+f x)\right] - \sin \left[\frac{1}{2}(e+f x)\right]\right] - \log \left[1 - \cos \left[\frac{1}{2}(e+f x)\right] + \sin \left[\frac{1}{2}(e+f x)\right]\right]\right) \sin \left[\frac{1}{2}(e+f x)\right] \sqrt{a(1 + \sin [e+f x])}\right)$$

■ **Problem 14: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\csc [e+f x]}{\sqrt{a+a \sin [e+f x]} (c-c \sin [e+f x])} dx$$

Optimal (type 3, 120 leaves, 8 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [e+f x]}{\sqrt{a+a \sin [e+f x]}}\right]}{\sqrt{a} c f} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos [e+f x]}{\sqrt{2} \sqrt{a+a \sin [e+f x]}}\right]}{\sqrt{2} \sqrt{a} c f} + \frac{\sec [e+f x] \sqrt{a+a \sin [e+f x]}}{a c f}$$

Result (type 3, 234 leaves):

$$\frac{1}{c f (-1 + \sin [e+f x]) \sqrt{a(1 + \sin [e+f x])}}$$

$$\cos [e+f x] \left(-1 + \cos \left[\frac{1}{2}(e+f x)\right] \left(\log \left[1 + \cos \left[\frac{1}{2}(e+f x)\right] - \sin \left[\frac{1}{2}(e+f x)\right]\right] - \log \left[1 - \cos \left[\frac{1}{2}(e+f x)\right] + \sin \left[\frac{1}{2}(e+f x)\right]\right]\right) + \right.$$

$$\left.(1+i)(-1)^{3/4} \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan \left[\frac{1}{4}(e+f x)\right]\right)\right] \left(\cos \left[\frac{1}{2}(e+f x)\right] - \sin \left[\frac{1}{2}(e+f x)\right]\right) - \right.$$

$$\left.\log \left[1 + \cos \left[\frac{1}{2}(e+f x)\right] - \sin \left[\frac{1}{2}(e+f x)\right]\right] \sin \left[\frac{1}{2}(e+f x)\right] + \log \left[1 - \cos \left[\frac{1}{2}(e+f x)\right] + \sin \left[\frac{1}{2}(e+f x)\right]\right] \sin \left[\frac{1}{2}(e+f x)\right]\right)$$

- **Problem 15: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{g \sin[e + f x]} \sqrt{a + a \sin[e + f x]}}{c - c \sin[e + f x]} dx$$

Optimal (type 3, 103 leaves, 6 steps):

$$\frac{2 \sqrt{a} \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{g} \cos[e + f x]}{\sqrt{g \sin[e + f x]} \sqrt{a + a \sin[e + f x]}}\right]}{c f} + \frac{2 \operatorname{Sec}[e + f x] \sqrt{g \sin[e + f x]} \sqrt{a + a \sin[e + f x]}}{c f}$$

Result (type 3, 180 leaves):

$$\left(2 e^{i(e+fx)} \left(2 \sqrt{-1 + e^{2i(e+fx)}} + (1 + i e^{i(e+fx)}) \operatorname{ArcTan}\left[\frac{1}{\sqrt{-1 + e^{2i(e+fx)}}}\right] - (-i + e^{i(e+fx)}) \operatorname{Log}\left[e^{i(e+fx)} + \sqrt{-1 + e^{2i(e+fx)}}\right] \right) \right) / \left(c \sqrt{-1 + e^{2i(e+fx)}} (1 + e^{2i(e+fx)}) f \right)$$

$$\sqrt{g \sin[e + f x]} \sqrt{a (1 + \sin[e + f x])}$$

- **Problem 17: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \sin[e + f x]}}{\sqrt{a + a \sin[e + f x]} (c - c \sin[e + f x])} dx$$

Optimal (type 3, 114 leaves, 6 steps):

$$\frac{\sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{g} \cos[e + f x]}{\sqrt{2} \sqrt{g \sin[e + f x]} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{2} \sqrt{a} c f} + \frac{\operatorname{Sec}[e + f x] \sqrt{g \sin[e + f x]} \sqrt{a + a \sin[e + f x]}}{a c f}$$

Result (type 4, 232 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{g \sin[e+fx]} \right. \\
\left. \left(1 - \left(\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{1}{4}(e+fx)\right]}}\right]}, -1\right] + \text{EllipticPi}\left[1-\sqrt{2}, -\text{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{1}{4}(e+fx)\right]}}\right]}, -1\right] + \right. \right. \right. \\
\left. \left. \left. \text{EllipticPi}\left[1+\sqrt{2}, -\text{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{1}{4}(e+fx)\right]}}\right]}, -1\right] \right) \sec\left[\frac{1}{4}(e+fx)\right]^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \\
\left(\sqrt{1 - \cot\left[\frac{1}{4}(e+fx)\right]^2} \tan\left[\frac{1}{4}(e+fx)\right]^{3/2} \right) \left. \right) / \left(f \sqrt{a(1+\sin[e+fx])} (c - c \sin[e+fx]) \right)$$

■ **Problem 18: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{g \sin[e+fx]} \sqrt{a+a \sin[e+fx]} (c - c \sin[e+fx])} dx$$

Optimal (type 3, 118 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a}\sqrt{g}\cos[e+fx]}{\sqrt{2}\sqrt{g\sin[e+fx]}\sqrt{a+a\sin[e+fx]}}\right]}{\sqrt{2}\sqrt{a}cf\sqrt{g}} + \frac{\sec[e+fx]\sqrt{g\sin[e+fx]}\sqrt{a+a\sin[e+fx]}}{acfg}$$

Result (type 4, 234 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{g \sin[e+fx]} \right. \\ \left. \left(1 + \left(\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{1}{4}(e+fx)\right]}}\right]}, -1\right] + \text{EllipticPi}\left[1 - \sqrt{2}, -\text{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{1}{4}(e+fx)\right]}}\right]}, -1\right] + \right. \right. \right. \\ \left. \left. \left. \text{EllipticPi}\left[1 + \sqrt{2}, -\text{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{1}{4}(e+fx)\right]}}\right]}, -1\right] \right) \sec\left[\frac{1}{4}(e+fx)\right]^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \\ \left(\sqrt{1 - \cot\left[\frac{1}{4}(e+fx)\right]^2} \tan\left[\frac{1}{4}(e+fx)\right]^{3/2} \right) \left. \right) / \left(fg \sqrt{a(1 + \sin[e+fx])} (c - c \sin[e+fx]) \right)$$

■ **Problem 20: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\csc[e+fx] \sqrt{a+a \sin[e+fx]}}{\sqrt{c-c \sin[e+fx]}} dx$$

Optimal (type 3, 102 leaves, 6 steps):

$$-\frac{a \cos[e+fx] \log[1 - \sin[e+fx]]}{f \sqrt{a+a \sin[e+fx]} \sqrt{c-c \sin[e+fx]}} + \frac{\log[\sin[e+fx]] \sec[e+fx] \sqrt{a+a \sin[e+fx]} \sqrt{c-c \sin[e+fx]}}{cf}$$

Result (type 3, 144 leaves):

$$\left(\sqrt{2} (-i + e^{i(e+fx)}) \left(2 \text{ArcTan}\left[e^{i(e+fx)}\right] + i \left(\log[1 - e^{2i(e+fx)}] - \log[1 + e^{2i(e+fx)}] \right) \right) \sqrt{a(1 + \sin[e+fx])} \right) / \\ \left(\sqrt{i c e^{-i(e+fx)} (-i + e^{i(e+fx)})^2 (i + e^{i(e+fx)})} f \right)$$

■ **Problem 21: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\csc[e+fx] \sqrt{c-c \sin[e+fx]}}{\sqrt{a+a \sin[e+fx]}} dx$$

Optimal (type 3, 100 leaves, 6 steps) :

$$-\frac{c \cos[e + f x] \log[1 + \sin[e + f x]]}{f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} + \frac{\log[\sin[e + f x]] \sec[e + f x] \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}}{a f}$$

Result (type 3, 145 leaves) :

$$\frac{\sqrt{2} \left(i + e^{i(e+fx)} \right) \left(2 \operatorname{ArcTan}\left[e^{i(e+fx)} \right] - i \left(\log\left[1 - e^{2i(e+fx)} \right] - \log\left[1 + e^{2i(e+fx)} \right] \right) \right) \sqrt{c - c \sin[e + f x]}}{\left(-i + e^{i(e+fx)} \right) \sqrt{-i a e^{-i(e+fx)} \left(i + e^{i(e+fx)} \right)^2 f}}$$

■ **Problem 22: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + f x]}{\sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 46 leaves, 3 steps) :

$$\frac{\cos[e + f x] \log[\tan[e + f x]]}{f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 96 leaves) :

$$-\left(\cos[e + f x] \left(\log\left[\cos\left[\frac{1}{2}(e + f x) \right] - \sin\left[\frac{1}{2}(e + f x) \right] \right] + \log\left[\cos\left[\frac{1}{2}(e + f x) \right] + \sin\left[\frac{1}{2}(e + f x) \right] \right] - \log[\sin[e + f x]] \right) \right) / \left(f \sqrt{a(1 + \sin[e + f x])} \sqrt{c - c \sin[e + f x]} \right)$$

■ **Problem 23: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Csc}[e + f x] \sqrt{a + a \sin[e + f x]}}{c + d \sin[e + f x]} dx$$

Optimal (type 3, 105 leaves, 5 steps) :

$$-\frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{a + a \sin[e + f x]}} \right]}{c f} + \frac{2 \sqrt{a} \sqrt{d} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}} \right]}{c \sqrt{c + d} f}$$

Result (type 7, 746 leaves) :

$$\begin{aligned}
& - \frac{1}{c \sqrt{c+d} f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)} \\
& \left(\frac{1}{8} - \frac{i}{8} \right) \left((4+4i) \sqrt{c+d} \left(\log\left[1 + \cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right] - \log\left[1 - \cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right] \right) \right) + \\
& \sqrt{d} \operatorname{RootSum}\left[-d + 2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, \frac{1}{-id - c e^{ie} \#1^2} \right. \\
& \left. \left((1+i)d \sqrt{e^{-ie}} f x - (2-2i)d \sqrt{e^{-ie}} \log\left[e^{\frac{ifx}{2}} - \#1\right] - i\sqrt{d} \sqrt{c+d} f x \#1 + 2\sqrt{d} \sqrt{c+d} \log\left[e^{\frac{ifx}{2}} - \#1\right] \#1 + \frac{(1-i) c f x \#1^2}{\sqrt{e^{-ie}}} + \right. \right. \\
& \left. \left. \frac{(2+2i)c \log\left[e^{\frac{ifx}{2}} - \#1\right] \#1^2}{\sqrt{e^{-ie}}} - \sqrt{d} \sqrt{c+d} e^{ie} f x \#1^3 - 2i\sqrt{d} \sqrt{c+d} e^{ie} \log\left[e^{\frac{ifx}{2}} - \#1\right] \#1^3 \right) \& \right] \left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right] \right) + \\
& \sqrt{d} \operatorname{RootSum}\left[-d + 2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, \frac{1}{d - ic e^{ie} \#1^2} \left((1-i)d \sqrt{e^{-ie}} f x + (2+2i)d \sqrt{e^{-ie}} \log\left[e^{\frac{ifx}{2}} - \#1\right] + \right. \right. \\
& \left. \left. \sqrt{d} \sqrt{c+d} f x \#1 + 2i\sqrt{d} \sqrt{c+d} \log\left[e^{\frac{ifx}{2}} - \#1\right] \#1 - \frac{(1+i) c f x \#1^2}{\sqrt{e^{-ie}}} + \frac{(2-2i)c \log\left[e^{\frac{ifx}{2}} - \#1\right] \#1^2}{\sqrt{e^{-ie}}} - \right. \right. \\
& \left. \left. i\sqrt{d} \sqrt{c+d} e^{ie} f x \#1^3 + 2\sqrt{d} \sqrt{c+d} e^{ie} \log\left[e^{\frac{ifx}{2}} - \#1\right] \#1^3 \right) \& \right] \left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right] \right) \sqrt{a(1 + \sin[e+fx])}
\end{aligned}$$

■ **Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+fx]}{\sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])} dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$- \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{a} c f} + \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{a} (c-d) f} - \frac{2 d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{c+d} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{a} c (c-d) \sqrt{c+d} f}$$

Result (type 3, 331 leaves):

$$\begin{aligned}
& - \frac{1}{c(c-d)\sqrt{c+d}f\sqrt{a(1+\sin[efx])}} \\
& \left((2+2i)(-1)^{3/4}c\sqrt{c+d}\operatorname{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left[\frac{1}{4}(efx)\right]\right)\right] + (c-d)\sqrt{c+d}\operatorname{Log}\left[1+\cos\left[\frac{1}{2}(efx)\right]-\sin\left[\frac{1}{2}(efx)\right]\right] \right) - \\
& c\sqrt{c+d}\operatorname{Log}\left[1-\cos\left[\frac{1}{2}(efx)\right]+\sin\left[\frac{1}{2}(efx)\right]\right] + d\sqrt{c+d}\operatorname{Log}\left[1-\cos\left[\frac{1}{2}(efx)\right]+\sin\left[\frac{1}{2}(efx)\right]\right] + \\
& d^{3/2}\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(efx)\right]^2\left(\sqrt{c+d}+\sqrt{d}\cos\left[\frac{1}{2}(efx)\right]-\sqrt{d}\sin\left[\frac{1}{2}(efx)\right]\right)\right] - \\
& d^{3/2}\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(efx)\right]^2\left(\sqrt{c+d}-\sqrt{d}\cos\left[\frac{1}{2}(efx)\right]+\sqrt{d}\sin\left[\frac{1}{2}(efx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(efx)\right]+\sin\left[\frac{1}{2}(efx)\right]\right)
\end{aligned}$$

- **Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g\sin[efx]}\sqrt{a+a\sin[efx]}}{c+d\sin[efx]} dx$$

Optimal (type 3, 149 leaves, 5 steps):

$$\frac{2\sqrt{a}\sqrt{g}\operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{g}\cos[efx]}{\sqrt{g\sin[efx]}\sqrt{a+a\sin[efx]}}\right]}{df} + \frac{2\sqrt{a}\sqrt{c}\sqrt{g}\operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos[efx]}{\sqrt{c+d}\sqrt{g\sin[efx]}\sqrt{a+a\sin[efx]}}\right]}{d\sqrt{c+d}f}$$

Result (type 3, 908 leaves):

$$\begin{aligned}
& \left(\frac{1}{2}+\frac{i}{2}\right)\left(-2\sqrt{c^2-d^2}\sqrt{-c+\sqrt{c^2-d^2}}\sqrt{c+\sqrt{c^2-d^2}}\operatorname{ArcTan}\left[\frac{1}{\sqrt{-1+\cos[2(efx)]+i\sin[2(efx)]}}\right] + \right. \\
& \left. \sqrt{2}\sqrt{c}\sqrt{c+\sqrt{c^2-d^2}}(-c+d+\sqrt{c^2-d^2}) \right. \\
& \left. \operatorname{Log}\left[d^2\sqrt{c^2-d^2}e^{-ie}\left(i\sqrt{2}d-\sqrt{2}ce^{i(efx)}+\sqrt{2}\sqrt{c^2-d^2}e^{i(efx)}-2i\sqrt{c}\sqrt{-c+\sqrt{c^2-d^2}}\sqrt{-1+e^{2i(efx)}}\right)f\right] \right) / \\
& \left(2c^{3/2}\sqrt{-c+\sqrt{c^2-d^2}}(-c+d+\sqrt{c^2-d^2})(-c+\sqrt{c^2-d^2}+ie^{i(efx)})\right) + i\sqrt{-c+\sqrt{c^2-d^2}}(c-d+\sqrt{c^2-d^2}) \\
& \operatorname{Log}\left[i d^2 e^{-ie}\left(-i\sqrt{2}d\sqrt{c^2-d^2}+\sqrt{2}c^2e^{i(efx)}-\sqrt{2}d^2e^{i(efx)}+\sqrt{2}c\sqrt{c^2-d^2}e^{i(efx)}+2\sqrt{c}\sqrt{c^2-d^2}\right)\right]
\end{aligned}$$

$$\begin{aligned}
 & \left. \left(\frac{\sqrt{c + \sqrt{c^2 - d^2}} \sqrt{-1 + e^{2i(e+fx)}}}{f} \right) / \left(2 c^{3/2} \sqrt{c + \sqrt{c^2 - d^2}} (c - d + \sqrt{c^2 - d^2}) (c + \sqrt{c^2 - d^2} - i d e^{i(e+fx)}) \right) \right) - \\
 & \left. 2 i \sqrt{c^2 - d^2} \sqrt{-c + \sqrt{c^2 - d^2}} \sqrt{c + \sqrt{c^2 - d^2}} \operatorname{Log} \left[\operatorname{Cos}[e + f x] + i \operatorname{Sin}[e + f x] + \sqrt{-1 + \operatorname{Cos}[2(e + f x)]} + i \operatorname{Sin}[2(e + f x)] \right] \right) \\
 & \left(\operatorname{Cos} \left[\frac{1}{2}(e + f x) \right] + i \operatorname{Sin} \left[\frac{1}{2}(e + f x) \right] \right) \\
 & \sqrt{g \operatorname{Sin}[e + f x]} \\
 & \left. \sqrt{a(1 + \operatorname{Sin}[e + f x])} \right) / \left(d \right. \\
 & \sqrt{c^2 - d^2} \\
 & \sqrt{-c + \sqrt{c^2 - d^2}} \\
 & \sqrt{c + \sqrt{c^2 - d^2}} \\
 & f \\
 & \left. \left(\operatorname{Cos} \left[\frac{1}{2}(e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2}(e + f x) \right] \right) \right. \\
 & \left. \sqrt{-1 + \operatorname{Cos}[2(e + f x)] + i \operatorname{Sin}[2(e + f x)]} \right)
 \end{aligned}$$

- **Problem 26: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \operatorname{Sin}[e + f x]}}{\sqrt{g \operatorname{Sin}[e + f x]} (c + d \operatorname{Sin}[e + f x])} dx$$

Optimal (type 3, 83 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{c} \sqrt{g} \operatorname{Cos}[e + f x]}{\sqrt{c + d} \sqrt{g \operatorname{Sin}[e + f x]} \sqrt{a + a \operatorname{Sin}[e + f x]}} \right]}{\sqrt{c} \sqrt{c + d} f \sqrt{g}}$$

Result (type 3, 748 leaves):

$$\begin{aligned}
& \left(\left(\frac{1}{2} + \frac{i}{2} \right) \left(i \sqrt{c + \sqrt{c^2 - d^2}} \left(-c + d + \sqrt{c^2 - d^2} \right) \operatorname{Log} \left[\left(d e^{-i e} \left(\sqrt{2} d \sqrt{c^2 - d^2} - i \sqrt{2} c^2 e^{i(e+f x)} + i \sqrt{2} d^2 e^{i(e+f x)} + i \sqrt{2} c \sqrt{c^2 - d^2} e^{i(e+f x)} - \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 2 \sqrt{c} \sqrt{c^2 - d^2} \sqrt{-c + \sqrt{c^2 - d^2}} \sqrt{-1 + e^{2 i(e+f x)}} \right) f \right] / \left(\sqrt{c} \sqrt{-c + \sqrt{c^2 - d^2}} \left(-c + d + \sqrt{c^2 - d^2} \right) \left(-c + \sqrt{c^2 - d^2} + i d e^{i(e+f x)} \right) \right) \right) \right) - \\
& \quad \sqrt{-c + \sqrt{c^2 - d^2}} \left(c - d + \sqrt{c^2 - d^2} \right) \operatorname{Log} \left[\left(d e^{-i e} \left(-i \sqrt{2} d \sqrt{c^2 - d^2} + \sqrt{2} c^2 e^{i(e+f x)} - \sqrt{2} d^2 e^{i(e+f x)} + \sqrt{2} c \sqrt{c^2 - d^2} e^{i(e+f x)} + \right. \right. \right. \\
& \quad \left. \left. \left. \left. 2 \sqrt{c} \sqrt{c^2 - d^2} \sqrt{c + \sqrt{c^2 - d^2}} \sqrt{-1 + e^{2 i(e+f x)}} \right) f \right] / \left(\sqrt{c} \sqrt{c + \sqrt{c^2 - d^2}} \left(c - d + \sqrt{c^2 - d^2} \right) \left(c + \sqrt{c^2 - d^2} - i d e^{i(e+f x)} \right) \right) \right) \right) \\
& \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] - i \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{a(1 + \operatorname{Sin}[e + f x])} \sqrt{-1 + \operatorname{Cos}[2(e + f x)] + i \operatorname{Sin}[2(e + f x)]} / \left(\sqrt{2} \right. \\
& \quad \sqrt{c} \\
& \quad \sqrt{c^2 - d^2} \\
& \quad \sqrt{-c + \sqrt{c^2 - d^2}} \\
& \quad \sqrt{c + \sqrt{c^2 - d^2}} \\
& \quad f \\
& \quad \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \\
& \quad \left. \sqrt{g \operatorname{Sin}[e + f x]} \right)
\end{aligned}$$

- **Problem 27: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \operatorname{Sin}[e + f x]}}{\sqrt{a + a \operatorname{Sin}[e + f x]} (c + d \operatorname{Sin}[e + f x])} dx$$

Optimal (type 3, 166 leaves, 5 steps):

$$\frac{\sqrt{2} \sqrt{g} \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{g} \operatorname{Cos}[e + f x]}{\sqrt{2} \sqrt{g \operatorname{Sin}[e + f x]} \sqrt{a + a \operatorname{Sin}[e + f x]}} \right]}{\sqrt{a} (c - d) f} - \frac{2 \sqrt{c} \sqrt{g} \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{c} \sqrt{g} \operatorname{Cos}[e + f x]}{\sqrt{c + d} \sqrt{g \operatorname{Sin}[e + f x]} \sqrt{a + a \operatorname{Sin}[e + f x]}} \right]}{\sqrt{a} (c - d) \sqrt{c + d} f}$$

Result (type 4, 61 316 leaves): Display of huge result suppressed!

- **Problem 28: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{g \sin[e + f x]} \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])} dx$$

Optimal (type 3, 168 leaves, 5 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{g} \cos[e + f x]}{\sqrt{2} \sqrt{g \sin[e + f x]} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} (c - d) f \sqrt{g}} + \frac{2 d \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{c} \sqrt{g} \cos[e + f x]}{\sqrt{c + d} \sqrt{g \sin[e + f x]} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} \sqrt{c} (c - d) \sqrt{c + d} f \sqrt{g}}$$

Result (type 4, 99997 leaves): Display of huge result suppressed!

- **Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + f x] \sqrt{a + b \sin[e + f x]}}{c + c \sin[e + f x]} dx$$

Optimal (type 4, 238 leaves, 9 steps):

$$\frac{\operatorname{EllipticE}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2b}{a+b}\right] \sqrt{a + b \sin[e + f x]}}{c f \sqrt{\frac{a + b \sin[e + f x]}{a + b}}} - \frac{(a - b) \operatorname{EllipticF}\left[\frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a + b \sin[e + f x]}{a + b}}}{c f \sqrt{a + b \sin[e + f x]}} +$$

$$\frac{2 a \operatorname{EllipticPi}\left[2, \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a + b \sin[e + f x]}{a + b}}}{c f \sqrt{a + b \sin[e + f x]}} + \frac{\cos[e + f x] \sqrt{a + b \sin[e + f x]}}{f (c + c \sin[e + f x])}$$

Result (type 4, 611 leaves):

$$\begin{aligned}
& - \frac{2 \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{a+b \operatorname{Sin}[e+fx]}}{f(c+c \operatorname{Sin}[e+fx])} + \\
& \frac{1}{4f(c+c \operatorname{Sin}[e+fx])} \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^2 \left(- \frac{4b \operatorname{EllipticF}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}{\sqrt{a+b \operatorname{Sin}[e+fx]}} + \right. \\
& \left. \frac{2(-4a-b) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}{\sqrt{a+b \operatorname{Sin}[e+fx]}} + 2ib \operatorname{Cos}[e+fx] \operatorname{Cos}[2(e+fx)] \right. \\
& \left. \left(2a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[e+fx]}\right], \frac{a+b}{a-b}\right] + b \left(2a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[e+fx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{a+b}{a-b}\right] - b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[e+fx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sqrt{\frac{b-b \operatorname{Sin}[e+fx]}{a+b}} \sqrt{-\frac{b+b \operatorname{Sin}[e+fx]}{a-b}} \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\operatorname{Sin}[e+fx]^2} \left(-2a^2+b^2+4a(a+b \operatorname{Sin}[e+fx]) - 2(a+b \operatorname{Sin}[e+fx])^2\right) \right. \\
& \left. \sqrt{-\frac{a^2-b^2-2a(a+b \operatorname{Sin}[e+fx])+(a+b \operatorname{Sin}[e+fx])^2}{b^2}} + \frac{2 \operatorname{Cot}[e+fx] \sqrt{a+b \operatorname{Sin}[e+fx]} \operatorname{Sin}[2(e+fx)]}{1-\operatorname{Sin}[e+fx]^2} \right)
\end{aligned}$$

- **Problem 30: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+fx]}{\sqrt{a+b \operatorname{Sin}[e+fx]}(c+c \operatorname{Sin}[e+fx])} dx$$

Optimal (type 4, 246 leaves, 9 steps):

$$\begin{aligned}
& \frac{\text{EllipticE}\left[\frac{1}{2}\left(e - \frac{\pi}{2} + f x\right), \frac{2b}{a+b}\right] \sqrt{a + b \sin[e + f x]}}{(a - b) c f \sqrt{\frac{a + b \sin[e + f x]}{a + b}}} - \frac{\text{EllipticF}\left[\frac{1}{2}\left(e - \frac{\pi}{2} + f x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a + b \sin[e + f x]}{a + b}}}{c f \sqrt{a + b \sin[e + f x]}} + \\
& \frac{2 \text{EllipticPi}\left[2, \frac{1}{2}\left(e - \frac{\pi}{2} + f x\right), \frac{2b}{a+b}\right] \sqrt{\frac{a + b \sin[e + f x]}{a + b}}}{c f \sqrt{a + b \sin[e + f x]}} + \frac{\cos[e + f x] \sqrt{a + b \sin[e + f x]}}{(a - b) f (c + c \sin[e + f x])}
\end{aligned}$$

Result (type 4, 625 leaves):

$$\begin{aligned}
& - \frac{2 \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right) \sqrt{a+b \operatorname{Sin}[e+fx]}}{(a-b) f (c+c \operatorname{Sin}[e+fx])} - \\
& \frac{1}{4(a-b) f (c+c \operatorname{Sin}[e+fx])} \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right)^2 \\
& \left(\frac{4 b \operatorname{EllipticF}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}{\sqrt{a+b \operatorname{Sin}[e+fx]}} - \frac{2(-4a+3b) \operatorname{EllipticPi}\left[2, \frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right), \frac{2b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}{\sqrt{a+b \operatorname{Sin}[e+fx]}} \right) - \\
& \left(2 i b \operatorname{Cos}[e+fx] \operatorname{Cos}[2(e+fx)] \left(2 a (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[e+fx]}\right], \frac{a+b}{a-b}\right] + \right. \right. \\
& \quad b \left(2 a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[e+fx]}\right], \frac{a+b}{a-b}\right] - \right. \\
& \quad \left. \left. b \operatorname{EllipticPi}\left[\frac{a+b}{a}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{1}{a+b}} \sqrt{a+b \operatorname{Sin}[e+fx]}\right], \frac{a+b}{a-b}\right]\right) \right) \sqrt{\frac{b-b \operatorname{Sin}[e+fx]}{a+b}} \sqrt{-\frac{b+b \operatorname{Sin}[e+fx]}{a-b}} \right) / \\
& \left(a \sqrt{-\frac{1}{a+b}} \sqrt{1-\operatorname{Sin}[e+fx]^2} \left(-2 a^2+b^2+4 a(a+b \operatorname{Sin}[e+fx]) - 2(a+b \operatorname{Sin}[e+fx])^2\right) \right. \\
& \quad \left. \sqrt{-\frac{a^2-b^2-2 a(a+b \operatorname{Sin}[e+fx])+(a+b \operatorname{Sin}[e+fx])^2}{b^2}} - \frac{2 \operatorname{Cot}[e+fx] \sqrt{a+b \operatorname{Sin}[e+fx]} \operatorname{Sin}[2(e+fx)]}{1-\operatorname{Sin}[e+fx]^2} \right)
\end{aligned}$$

- **Problem 31: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \operatorname{Sin}[e+fx]} \sqrt{a+b \operatorname{Sin}[e+fx]}}{c+c \operatorname{Sin}[e+fx]} dx$$

Optimal (type 4, 267 leaves, 3 steps):

$$\frac{1}{\sqrt{a+b} c f} 2 \sqrt{g} \operatorname{EllipticPi}\left[\frac{b}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{g \sin[e+f x]}}{\sqrt{g} \sqrt{a+b \sin[e+f x]}}\right], -\frac{a-b}{a+b}\right] \operatorname{Sec}[e+f x] \sqrt{\frac{a(1-\sin[e+f x])}{a+b \sin[e+f x]}}$$

$$\sqrt{\frac{a(1+\sin[e+f x])}{a+b \sin[e+f x]}} (a+b \sin[e+f x]) + \frac{g \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\cos[e+f x]}{1+\sin[e+f x]}\right], -\frac{a-b}{a+b}\right] \sqrt{\frac{\sin[e+f x]}{1+\sin[e+f x]}} \sqrt{a+b \sin[e+f x]}}{c f \sqrt{g \sin[e+f x]} \sqrt{\frac{a+b \sin[e+f x]}{(a+b)(1+\sin[e+f x])}}}$$

Result (type 4, 10621 leaves):

$$\frac{2 \sin\left[\frac{1}{2}(e+f x)\right] \left(\cos\left[\frac{1}{2}(e+f x)\right] + \sin\left[\frac{1}{2}(e+f x)\right]\right) \sqrt{g \sin[e+f x]} \sqrt{a+b \sin[e+f x]}}{f(c+c \sin[e+f x])} +$$

$$\left(\left(\cos\left[\frac{1}{2}(e+f x)\right] + \sin\left[\frac{1}{2}(e+f x)\right] \right)^2 \sqrt{g \sin[e+f x]} \right.$$

$$\left. - \frac{a \sqrt{\sin[e+f x]}}{2 \sqrt{a+b \sin[e+f x]}} + \frac{b \sqrt{\sin[e+f x]}}{2 \sqrt{a+b \sin[e+f x]}} - \frac{b \cos\left[\frac{3}{2}(e+f x)\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \sqrt{\sin[e+f x]}}{2 \sqrt{a+b \sin[e+f x]}} - \right.$$

$$\left. \frac{b \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \sqrt{\sin[e+f x]} \sin\left[\frac{3}{2}(e+f x)\right]}{2 \sqrt{a+b \sin[e+f x]}} + \frac{a \sqrt{\sin[e+f x]} \tan\left[\frac{1}{2}(e+f x)\right]}{2 \sqrt{a+b \sin[e+f x]}} - \frac{b \sqrt{\sin[e+f x]} \tan\left[\frac{1}{2}(e+f x)\right]}{2 \sqrt{a+b \sin[e+f x]}} \right)$$

$$\left(-\sqrt{2} \left(1 + \tan\left[\frac{1}{2}(e+f x)\right] \right) \sqrt{\frac{\tan\left[\frac{1}{2}(e+f x)\right]}{1 + \tan\left[\frac{1}{2}(e+f x)\right]^2}} \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e+f x)\right] + a \tan\left[\frac{1}{2}(e+f x)\right]^2}{1 + \tan\left[\frac{1}{2}(e+f x)\right]^2}} + \right.$$

$$\left. \frac{1}{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e+f x)\right] + a \tan\left[\frac{1}{2}(e+f x)\right]^2}{1 + \tan\left[\frac{1}{2}(e+f x)\right]^2}}} \sqrt{2} \cot\left[\frac{1}{2}(e+f x)\right] \sqrt{\frac{\tan\left[\frac{1}{2}(e+f x)\right]}{1 + \tan\left[\frac{1}{2}(e+f x)\right]^2}} \right)$$

$$\left(a \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} + 2b \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] + a \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^2 - \right.$$

$$\left. i \left(-b + \sqrt{-a^2 + b^2}\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^{3/2} \right.$$

$$\left. \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} + i \left(a - b + \sqrt{-a^2 + b^2}\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticF}\left[\right.$$

$$\left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^{3/2} \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} + \right.$$

$$\left. 2b \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2 + b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^{3/2} \right.$$

$$\left. \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} - 2b \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + fx)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2 + b^2}\right)}{a}, \right.$$

$$\left. \left. \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^{3/2} \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}} \right] \right) \right)$$

$$\left(f \sqrt{\sin[e + f x]} (c + c \sin[e + f x]) \right) \left(\frac{\sec\left[\frac{1}{2}(e + f x)\right]^2 \sqrt{\frac{\tan\left[\frac{1}{2}(e + f x)\right]}{1 + \tan\left[\frac{1}{2}(e + f x)\right]^2}} \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e + f x)\right] + a \tan\left[\frac{1}{2}(e + f x)\right]^2}{1 + \tan\left[\frac{1}{2}(e + f x)\right]^2}}}{\sqrt{2}} \right) -$$

$$\frac{1}{\sqrt{2} \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \sqrt{\frac{a + 2b \tan\left[\frac{1}{2}(e + f x)\right] + a \tan\left[\frac{1}{2}(e + f x)\right]^2}{1 + \tan\left[\frac{1}{2}(e + f x)\right]^2}}} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2 \sqrt{\frac{\tan\left[\frac{1}{2}(e + f x)\right]}{1 + \tan\left[\frac{1}{2}(e + f x)\right]^2}}$$

$$\left(a \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} + 2b \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \tan\left[\frac{1}{2}(e + f x)\right] + a \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \tan\left[\frac{1}{2}(e + f x)\right]^2 -$$

$$i \left(-b + \sqrt{-a^2 + b^2} \right) \sqrt{1 + \frac{a \cot\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e + f x)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \tan\left[\frac{1}{2}(e + f x)\right]^{3/2}$$

$$\sqrt{\frac{a + b \tan\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \tan\left[\frac{1}{2}(e + f x)\right]}{b \tan\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \tan\left[\frac{1}{2}(e + f x)\right]}} + i \left(a - b + \sqrt{-a^2 + b^2} \right) \sqrt{1 + \frac{a \cot\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticF}\left[$$

$$i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e + f x)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \tan\left[\frac{1}{2}(e + f x)\right]^{3/2} \sqrt{\frac{a + b \tan\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \tan\left[\frac{1}{2}(e + f x)\right]}{b \tan\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \tan\left[\frac{1}{2}(e + f x)\right]}} +$$

$$2b \sqrt{1 + \frac{a \cot\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e + f x)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \tan\left[\frac{1}{2}(e + f x)\right]^{3/2}$$

$$\begin{aligned}
& \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} - 2b \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[\frac{i\left(b + \sqrt{-a^2 + b^2}\right)}{a}, \right. \\
& \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^{3/2} \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}\right] - \\
& \frac{1}{\sqrt{2} \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}}} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right) \sqrt{\frac{a + 2b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}} \\
& \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{2\left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right)^2}\right) + \\
& \frac{1}{\sqrt{2} \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}} \sqrt{\frac{a + 2b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}} \\
& \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \left(a \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} + 2b \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] + a \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 - \right. \\
& \left. i\left(-b + \sqrt{-a^2 + b^2}\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^{3/2}\right] \right. \\
& \left. \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} + i\left(a - b + \sqrt{-a^2 + b^2}\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticF}\left[\right.
\end{aligned}$$

$$\begin{aligned}
& i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^{3/2} \sqrt{\frac{a + b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] - \sqrt{-a^2 + b^2} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] - \sqrt{-a^2 + b^2} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} + \\
& 2 b \sqrt{\frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi} \left[-\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \\
& \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^{3/2} \sqrt{\frac{a + b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] - \sqrt{-a^2 + b^2} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] - \sqrt{-a^2 + b^2} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} - \\
& 2 b \sqrt{\frac{a \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi} \left[\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \right] \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^{3/2} \\
& \sqrt{\frac{a + b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] - \sqrt{-a^2 + b^2} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] - \sqrt{-a^2 + b^2} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}} \left(-\frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)} \right) - \\
& \left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \sqrt{\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} \left(\frac{b \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 + a \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2} - \right. \right. \\
& \left. \left. \frac{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \left(a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)^2} \right) \right) / \\
& \left(\sqrt{2} \sqrt{\frac{a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} \right) - \frac{1}{\sqrt{2} \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \left(\frac{a + 2 b \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2} \right)^{3/2}}
\end{aligned}$$

$$\begin{aligned}
& \cot\left[\frac{1}{2}(e+fx)\right] \sqrt{\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1+\tan\left[\frac{1}{2}(e+fx)\right]^2}} \left(a \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} + 2b \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& a \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \tan\left[\frac{1}{2}(e+fx)\right]^2 - i(-b+\sqrt{-a^2+b^2}) \sqrt{1+\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \text{EllipticE}\left[\right. \\
& \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \tan\left[\frac{1}{2}(e+fx)\right]^{3/2} \sqrt{\frac{a+b \tan\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(e+fx)\right]}{b \tan\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(e+fx)\right]}} + \right. \\
& \left. i(a-b+\sqrt{-a^2+b^2}) \sqrt{1+\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \tan\left[\frac{1}{2}(e+fx)\right]^{3/2} \right. \\
& \left. \sqrt{\frac{a+b \tan\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(e+fx)\right]}{b \tan\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(e+fx)\right]}} + 2b \sqrt{1+\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \text{EllipticPi}\left[-\frac{i(b+\sqrt{-a^2+b^2})}{a}, \right. \\
& \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \tan\left[\frac{1}{2}(e+fx)\right]^{3/2} \sqrt{\frac{a+b \tan\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(e+fx)\right]}{b \tan\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \tan\left[\frac{1}{2}(e+fx)\right]}} - \right. \\
& \left. 2b \sqrt{1+\frac{a \cot\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \text{EllipticPi}\left[\frac{i(b+\sqrt{-a^2+b^2})}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\tan\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}} \right] \tan\left[\frac{1}{2}(e+fx)\right]^{3/2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}}{1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \right. \\
& \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] \left(a+2 b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]+a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right)^2} \right) + \\
& \frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \sqrt{\frac{a+2 b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]+a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}} \sqrt{2} \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right] \sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}} \\
& \left(b \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 + a \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \left. \frac{\left(a-b+\sqrt{-a^2+b^2}\right) \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}}{4 \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]}{b-\sqrt{-a^2+b^2}}}} \right. \\
& \left. \frac{i b \sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}}{2\left(1-i \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]\right) \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]}{b-\sqrt{-a^2+b^2}}}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{i b \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}{2 \left(1 + i \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}}} - \frac{1}{4} \left(-b + \sqrt{-a^2 + b^2}\right) \sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}} \\
& \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b - \sqrt{-a^2 + b^2}}} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} - \\
& \frac{3}{4} i \left(-b + \sqrt{-a^2 + b^2}\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]} \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} + \\
& \frac{3}{4} i \left(a - b + \sqrt{-a^2 + b^2}\right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]} \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} + \\
& \frac{3}{2} b \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2 + b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}\right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}}\right]
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]} \sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} - \\
& \frac{3}{2} b \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} \operatorname{EllipticPi}\left[\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]} \sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}} + \\
& \left(i a\left(-b+\sqrt{-a^2+b^2}\right) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}\right. \\
& \left.\sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right) / \left(4\left(b+\sqrt{-a^2+b^2}\right) \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}\right) - \\
& \left(i a\left(a-b+\sqrt{-a^2+b^2}\right) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}\right. \\
& \left.\sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right) / \left(4\left(b+\sqrt{-a^2+b^2}\right) \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}\right) -
\end{aligned}$$

$$\left(a b \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{EllipticPi}\left[-\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \right. \\
\left. \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2} \sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right) / \left(2\left(b+\sqrt{-a^2+b^2}\right) \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\right) + \\
\left(a b \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{EllipticPi}\left[\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \right. \\
\left. \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2} \sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right) / \left(2\left(b+\sqrt{-a^2+b^2}\right) \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\right) - \\
\left(i\left(-b+\sqrt{-a^2+b^2}\right) \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\right) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2} \\
\left(\frac{\frac{1}{2} b \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 - \frac{1}{2} \sqrt{-a^2+b^2} \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]} - \left(\frac{1}{2} b \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 - \frac{1}{2} \sqrt{-a^2+b^2} \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \right) \right. \\
\left. \left. \left(a+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] \right) \right) / \left(b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] \right)^2 \right) /
\right)$$

$$\begin{aligned}
& \left(2 \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right) + \left(i \left(a - b + \sqrt{-a^2 + b^2} \right) \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticF}\left[\right. \right. \\
& \left. \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right]^{3/2} \left(\frac{\frac{1}{2} b \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 - \frac{1}{2} \sqrt{-a^2 + b^2} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]} \right. \right. \\
& \left. \left. \left(\left(\frac{1}{2} b \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 - \frac{1}{2} \sqrt{-a^2 + b^2} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right) \left(a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right) \right) \right) / \right. \\
& \left. \left(b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right)^2 \right) \left. \right) / \left(2 \sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right) + \\
& \left(b \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[-\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b + \sqrt{-a^2 + b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right], \frac{b + \sqrt{-a^2 + b^2}}{b - \sqrt{-a^2 + b^2}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right]^{3/2} \right. \\
& \left. \left(\frac{\frac{1}{2} b \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 - \frac{1}{2} \sqrt{-a^2 + b^2} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]} - \left(\left(\frac{1}{2} b \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 - \frac{1}{2} \sqrt{-a^2 + b^2} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right. \right. \right. \\
& \left. \left. \left(a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right) \right) \right) / \left(b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right)^2 \right) \left. \right) / \\
& \left(\sqrt{\frac{a + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \sqrt{-a^2 + b^2} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}} \right) - \left(b \sqrt{1 + \frac{a \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \operatorname{EllipticPi}\left[\frac{i \left(b + \sqrt{-a^2 + b^2} \right)}{a}, \right. \right.
\end{aligned}$$

$$i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2} \left(\frac{\frac{1}{2}b \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 - \frac{1}{2}\sqrt{-a^2+b^2} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]} - \left(\frac{\frac{1}{2}b \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 - \frac{1}{2}\sqrt{-a^2+b^2} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{\left(a+b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}\right)\right) / \left(\left(b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^2\right) / \left(\sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right)$$

■ **Problem 32: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sqrt{a+b \operatorname{Sin}[e+fx]}}{\sqrt{g \operatorname{Sin}[e+fx]} (c+c \operatorname{Sin}[e+fx])} dx$$

Optimal (type 4, 116 leaves, 1 step):

$$\frac{\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Sin}[e+fx]}\right], -\frac{a-b}{a+b}\right] \sqrt{\frac{\operatorname{Sin}[e+fx]}{1+\operatorname{Sin}[e+fx]}} \sqrt{a+b \operatorname{Sin}[e+fx]}}{c f \sqrt{g \operatorname{Sin}[e+fx]} \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{(a+b)(1+\operatorname{Sin}[e+fx])}}}}$$

Result (type 1, 1 leaves):

???

■ **Problem 33: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sqrt{g \operatorname{Sin}[e+fx]}}{\sqrt{a+b \operatorname{Sin}[e+fx]} (c+c \operatorname{Sin}[e+fx])} dx$$

Optimal (type 4, 252 leaves, 3 steps):

$$\frac{g \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\cos[e+fx]}{1+\sin[e+fx]}\right], -\frac{a-b}{a+b}\right] \sqrt{\frac{\sin[e+fx]}{1+\sin[e+fx]}} \sqrt{a+b \sin[e+fx]}}{(a-b) c f \sqrt{g \sin[e+fx]} \sqrt{\frac{a+b \sin[e+fx]}{(a+b)(1+\sin[e+fx])}}}$$

$$\frac{2 \sqrt{a+b} \sqrt{g} \sqrt{\frac{a(1-\operatorname{Csc}[e+fx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Csc}[e+fx])}{a-b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{g} \sqrt{a+b \sin[e+fx]}}{\sqrt{a+b} \sqrt{g \sin[e+fx]}}\right], -\frac{a+b}{a-b}\right] \operatorname{Tan}[e+fx]}{(a-b) c f}$$

Result (type 1, 1 leaves):

???

■ **Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{g \sin[e+fx]} \sqrt{a+b \sin[e+fx]} (c+c \sin[e+fx])} dx$$

Optimal (type 4, 256 leaves, 3 steps):

$$\frac{\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\cos[e+fx]}{1+\sin[e+fx]}\right], -\frac{a-b}{a+b}\right] \sqrt{\frac{\sin[e+fx]}{1+\sin[e+fx]}} \sqrt{a+b \sin[e+fx]}}{(a-b) c f \sqrt{g \sin[e+fx]} \sqrt{\frac{a+b \sin[e+fx]}{(a+b)(1+\sin[e+fx])}}}$$

$$+$$

$$\frac{2 b \sqrt{a+b} \sqrt{\frac{a(1-\operatorname{Csc}[e+fx])}{a+b}} \sqrt{\frac{a(1+\operatorname{Csc}[e+fx])}{a-b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{g} \sqrt{a+b \sin[e+fx]}}{\sqrt{a+b} \sqrt{g \sin[e+fx]}}\right], -\frac{a+b}{a-b}\right] \operatorname{Tan}[e+fx]}{a(a-b) c f \sqrt{g}}$$

Result (type 4, 1662 leaves):

$$\frac{2 \sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right) \sin[e+fx] \sqrt{a+b \sin[e+fx]}}{(a-b) f \sqrt{g \sin[e+fx]} (c+c \sin[e+fx])} +$$

$$\frac{1}{2(a-b) f \sqrt{g \sin[e+fx]} (c+c \sin[e+fx])} \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2 \sqrt{\sin[e+fx]}$$

$$\left(\left(4 a (a-b) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \sin[e+fx])}}{a}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sec}[e+fx] \right)$$

$$\begin{aligned}
& \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \sin[efx]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{a}} \right) / \\
& \left((a+b) \sqrt{\sin[efx]} \sqrt{a+b \sin[efx]} \right) + \frac{2 a \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{\sin[efx]}}{\sqrt{a+b \sin[efx]}}\right] \cos[efx]^2}{\sqrt{b} (1 - \sin[efx]^2)} + \\
& 4 a^2 \left(\left(\sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{a}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sec}[efx] \right. \right. \\
& \left. \left. \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \sin[efx]}{a}} \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{a}} \right) / \right. \\
& \left. \left((a+b) \sqrt{\sin[efx]} \sqrt{a+b \sin[efx]} \right) - \left(\sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-a+b}} \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{a}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \sin[efx]}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{a}} \right) / \left(b \sqrt{\sin[efx]} \sqrt{a+b \sin[efx]} \right) \right) - \\
& 2 b \left(\frac{\cos[efx] \sqrt{a+b \sin[efx]}}{b \sqrt{\sin[efx]}} + \left(i \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{Csc}[efx] \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{\sin[efx]}}\right], -\frac{2 a}{-a-b}\right] \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{a+b \sin[e+f x]} \right) / \left(b \sqrt{\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Csc}[e+f x]} \sqrt{\frac{\operatorname{Csc}[e+f x](a+b \sin[e+f x])}{a+b}} \right) + \\
& \frac{1}{b} 2 a \left(\left(a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-a+b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sec}\left[\right. \right. \\
& \left. \left. e+f x\right] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \sin[e+f x]}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{a}} \right) / \left((a+b) \sqrt{\sin[e+f x]} \sqrt{a+b \sin[e+f x]} \right) - \right. \\
& \left. \left(a \sqrt{\frac{(a+b) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-a+b}} \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{a}}}{\sqrt{2}}}\right], -\frac{2 a}{-a+b}\right] \operatorname{Sec}\left[\right. \right. \\
& \left. \left. e+f x\right] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \sin[e+f x]}{a}} \right. \right. \\
& \left. \left. \sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{a}} \right) / \left(b \sqrt{\sin[e+f x]} \sqrt{a+b \sin[e+f x]} \right) \right) \right) + \\
& \left. \frac{2 b \cot[e+f x] \left(-\frac{a \operatorname{Log}\left[b \sqrt{\sin[e+f x]}+\sqrt{b} \sqrt{a+b \sin[e+f x]}\right]}{2 b^{3/2}} + \frac{\sqrt{\sin[e+f x]} \sqrt{a+b \sin[e+f x]}}{2 b} \right) \sin[2(e+f x)]}{1-\sin[e+f x]^2} \right)
\end{aligned}$$

■ **Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+f x] \sqrt{a+a \sin[e+f x]} \sqrt{c+d \sin[e+f x]} dx$$

Optimal (type 3, 123 leaves, 5 steps) :

$$\frac{2\sqrt{a}\sqrt{d}\operatorname{ArcTan}\left[\frac{\sqrt{a}\sqrt{d}\cos[e+fx]}{\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{f} - \frac{2\sqrt{a}\sqrt{c}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c}\cos[e+fx]}{\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{f}$$

Result (type 3, 567 leaves) :

$$\begin{aligned} & - \left(\left(\left(\sqrt{c} \operatorname{Log}\left[\frac{1}{c^{3/2}(1+e^{i(e+fx)})}\right] \left(\frac{1}{2} + \frac{i}{2}\right) e^{-\frac{ie}{2}} \left(-\sqrt{2}c(-1+e^{i(e+fx)}) - i\sqrt{2}d(1+e^{i(e+fx)}) + 2i\sqrt{c}\sqrt{2ce^{i(e+fx)} - id(-1+e^{2i(e+fx)})}\right) \right] f \right) + \right. \\ & \quad \left. \sqrt{c} \operatorname{Log}\left[\frac{1}{c^{3/2}(-1+e^{i(e+fx)})}\right] \left(\frac{1}{2} + \frac{i}{2}\right) e^{-\frac{ie}{2}} \left(-i\sqrt{2}d(-1+e^{i(e+fx)}) + \sqrt{2}c(1+e^{i(e+fx)}) + 2\sqrt{c}\sqrt{2ce^{i(e+fx)} - id(-1+e^{2i(e+fx)})}\right) \right] f \right) - \\ & \quad \left. i\sqrt{d} \left[\operatorname{Log}\left[\frac{2e^{-\frac{1}{2}i(e+2fx)}\left((-1)^{3/4}d + (-1)^{1/4}ce^{i(e+fx)} + i\sqrt{d}\sqrt{2ce^{i(e+fx)} - id(-1+e^{2i(e+fx)})}\right)}{d^{3/2}}\right] f \right] - \right. \\ & \quad \left. \operatorname{Log}\left[\frac{1}{\sqrt{d}}(1+i)\sqrt{2}\left(c-id\cos[e+fx] + d\sin[e+fx] + (1-i)\sqrt{d}\sqrt{(\cos[e+fx] + i\sin[e+fx])(c+d\sin[e+fx])}\right)\right] \right] \right) \\ & \quad \left(\cos\left[\frac{1}{2}(e+fx)\right] + i\sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{a(1+\sin[e+fx])}\sqrt{c+d\sin[e+fx]} \Bigg/ \\ & \quad \left(f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{(\cos[e+fx] + i\sin[e+fx])(c+d\sin[e+fx])} \right) \end{aligned}$$

■ **Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+fx]\sqrt{a+a\sin[e+fx]}}{\sqrt{c+d\sin[e+fx]}} dx$$

Optimal (type 3, 61 leaves, 2 steps) :

$$\frac{2\sqrt{a}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c}\cos[e+fx]}{\sqrt{a+a\sin[e+fx]}\sqrt{c+d\sin[e+fx]}}\right]}{\sqrt{c}f}$$

Result (type 3, 367 leaves) :

$$\begin{aligned}
& - \frac{1}{\sqrt{c} f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c+d \sin[e+fx]}} \\
& \left(\operatorname{Log}\left[-\frac{1}{\sqrt{c} (1+e^{i(e+fx)})} (1+i) e^{\frac{ie}{2}} \left(\sqrt{2} c (-1+e^{i(e+fx)}) + i \sqrt{2} d (1+e^{i(e+fx)}) - 2i \sqrt{c} \sqrt{2c e^{i(e+fx)} - i d (-1+e^{2i(e+fx)})} \right) f \right] + \right. \\
& \left. \operatorname{Log}\left[\frac{1}{\sqrt{c} (-1+e^{i(e+fx)})} (1+i) e^{\frac{ie}{2}} \left(-i \sqrt{2} d (-1+e^{i(e+fx)}) + \sqrt{2} c (1+e^{i(e+fx)}) + 2 \sqrt{c} \sqrt{2c e^{i(e+fx)} - i d (-1+e^{2i(e+fx)})} \right) f \right] \right) \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] - i \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{a(1+\sin[e+fx])} \sqrt{(\cos[e+fx] + i \sin[e+fx]) (c+d \sin[e+fx])}
\end{aligned}$$

■ **Problem 37: Humongous result has more than 200000 leaves.**

$$\int \frac{\operatorname{Csc}[e+fx] \sqrt{c+d \sin[e+fx]}}{\sqrt{a+a \sin[e+fx]}} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$-\frac{2 \sqrt{c} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c} \cos[e+fx]}{\sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{\sqrt{a} f} + \frac{\sqrt{2} \sqrt{c-d} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{\sqrt{a} f}$$

Result (type ?, 472502 leaves): Display of huge result suppressed!

■ **Problem 38: Humongous result has more than 200000 leaves.**

$$\int \frac{\operatorname{Csc}[e+fx]}{\sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c} \cos[e+fx]}{\sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{\sqrt{a} \sqrt{c} f} + \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c-d} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]} \sqrt{c+d \sin[e+fx]}}\right]}{\sqrt{a} \sqrt{c-d} f}$$

Result (type ?, 309693 leaves): Display of huge result suppressed!

■ **Problem 40: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Csc}[e+fx] \sqrt{c+d \sin[e+fx]}}{a+b \sin[e+fx]} dx$$

Optimal (type 4, 154 leaves, 5 steps):

$$\frac{2c \operatorname{EllipticPi}\left[2, \frac{1}{2}(e-\frac{\pi}{2}+fx), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{af \sqrt{c+d \sin[e+fx]}} - \frac{2(bc-ad) \operatorname{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(e-\frac{\pi}{2}+fx), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{a(a+b) f \sqrt{c+d \sin[e+fx]}}$$

Result (type 4, 179 leaves) :

$$\frac{1}{a \sqrt{-\frac{1}{c+d}} f} - 2i \left(\text{EllipticPi}\left[\frac{c+d}{c}, i \text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] - \text{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] \right)$$

$$\text{Sec}[e+fx] \sqrt{-\frac{d(-1+\sin[e+fx])}{c+d}} \sqrt{\frac{d(1+\sin[e+fx])}{-c+d}}$$

■ **Problem 41: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Csc}[e+fx]}{(a+b \sin[e+fx]) \sqrt{c+d \sin[e+fx]}} dx$$

Optimal (type 4, 146 leaves, 5 steps) :

$$\frac{2 \text{EllipticPi}\left[2, \frac{1}{2} \left(e - \frac{\pi}{2} + fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{af \sqrt{c+d \sin[e+fx]}} - \frac{2b \text{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2} \left(e - \frac{\pi}{2} + fx\right), \frac{2d}{c+d}\right] \sqrt{\frac{c+d \sin[e+fx]}{c+d}}}{a(a+b) f \sqrt{c+d \sin[e+fx]}}$$

Result (type 4, 203 leaves) :

$$-\frac{1}{ac \sqrt{-\frac{1}{c+d}} (bc-ad) f} \left((-bc+ad) \text{EllipticPi}\left[\frac{c+d}{c}, i \text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] + \right.$$

$$\left. bc \text{EllipticPi}\left[\frac{b(c+d)}{bc-ad}, i \text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}} \sqrt{c+d \sin[e+fx]}\right], \frac{c+d}{c-d}\right] \right) \text{Sec}[e+fx] \sqrt{-\frac{d(-1+\sin[e+fx])}{c+d}} \sqrt{\frac{d(1+\sin[e+fx])}{c-d}}$$

■ **Problem 42: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \sin[e+fx]} \sqrt{a+b \sin[e+fx]}}{c+d \sin[e+fx]} dx$$

Optimal (type 4, 254 leaves, 3 steps) :

$$\frac{1}{df} 2\sqrt{a+b} \sqrt{g} \sqrt{\frac{a(1-\text{Csc}[e+fx])}{a+b}} \sqrt{\frac{a(1+\text{Csc}[e+fx])}{a-b}} \text{EllipticPi}\left[\frac{a+b}{b}, \text{ArcSin}\left[\frac{\sqrt{g}\sqrt{a+b}\text{Sin}[e+fx]}{\sqrt{a+b}\sqrt{g}\text{Sin}[e+fx]}\right], -\frac{a+b}{a-b}\right] \text{Tan}[e+fx] - \left(2(bc-ad)\sqrt{-\text{Cot}[e+fx]^2} \sqrt{\frac{b+a\text{Csc}[e+fx]}{a+b}} \text{EllipticPi}\left[\frac{2c}{c+d}, \text{ArcSin}\left[\frac{\sqrt{1-\text{Csc}[e+fx]}}{\sqrt{2}}\right], \frac{2a}{a+b}\right] \sqrt{g}\text{Sin}[e+fx] \text{Tan}[e+fx]\right) / (d(c+d)f\sqrt{a+b}\text{Sin}[e+fx])$$

Result (type 4, 75407 leaves) : Display of huge result suppressed!

■ **Problem 43: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b}\text{Sin}[e+fx]}{\sqrt{g}\text{Sin}[e+fx] (c+d\text{Sin}[e+fx])} dx$$

Optimal (type 4, 250 leaves, 3 steps) :

$$\frac{2\sqrt{a+b} \sqrt{\frac{a(1-\text{Csc}[e+fx])}{a+b}} \sqrt{\frac{a(1+\text{Csc}[e+fx])}{a-b}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{g}\sqrt{a+b}\text{Sin}[e+fx]}{\sqrt{a+b}\sqrt{g}\text{Sin}[e+fx]}\right], -\frac{a+b}{a-b}\right] \text{Tan}[e+fx]}{cf\sqrt{g}} + \left(2(bc-ad)\sqrt{-\text{Cot}[e+fx]^2} \sqrt{\frac{b+a\text{Csc}[e+fx]}{a+b}} \text{EllipticPi}\left[\frac{2c}{c+d}, \text{ArcSin}\left[\frac{\sqrt{1-\text{Csc}[e+fx]}}{\sqrt{2}}\right], \frac{2a}{a+b}\right] \sqrt{g}\text{Sin}[e+fx] \text{Tan}[e+fx]\right) / (c(c+d)fg\sqrt{a+b}\text{Sin}[e+fx])$$

Result (type 4, 45019 leaves) : Display of huge result suppressed!

■ **Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g}\text{Sin}[e+fx]}{\sqrt{a+b}\text{Sin}[e+fx] (c+d\text{Sin}[e+fx])} dx$$

Optimal (type 4, 114 leaves, 1 step) :

$$\left(2\sqrt{-\text{Cot}[e+fx]^2} \sqrt{\frac{b+a\text{Csc}[e+fx]}{a+b}} \text{EllipticPi}\left[\frac{2c}{c+d}, \text{ArcSin}\left[\frac{\sqrt{1-\text{Csc}[e+fx]}}{\sqrt{2}}\right], \frac{2a}{a+b}\right] \sqrt{g}\text{Sin}[e+fx] \text{Tan}[e+fx]\right) / ((c+d)f\sqrt{a+b}\text{Sin}[e+fx])$$

Result (type 4, 3427 leaves) :

$$\left(a \sqrt{-a^2 + b^2} \left(a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(-d + \sqrt{-c^2 + d^2} \right) \right) \text{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a d + a \sqrt{-c^2 + d^2}}, \right. \right.$$

$$\left. \left. \text{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}}, \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] + \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \right. \right.$$

$$\left. \left. \text{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a \left(d + \sqrt{-c^2 + d^2} \right)}, \text{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}}, \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] \right] \right) \right)$$

$$\left. \left. \sqrt{\text{Sin}[e + f x]} \sqrt{g \text{Sin}[e + f x]} \sqrt{\frac{a \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 (a + b \text{Sin}[e + f x])}{a^2 - b^2}} \right) \right) /$$

$$\left(\left(b + \sqrt{-a^2 + b^2} \right)^2 (b c - a d) \sqrt{-c^2 + d^2} f (a + b \text{Sin}[e + f x]) (c + d \text{Sin}[e + f x]) \sqrt{-\frac{a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \right)$$

$$\left(\left(a^2 \sqrt{-a^2 + b^2} \left(a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(-d + \sqrt{-c^2 + d^2} \right) \right) \text{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a d + a \sqrt{-c^2 + d^2}}, \right. \right.$$

$$\begin{aligned}
& \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right] + \left(-ac + \left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right) \text{EllipticPi}\left[\right. \\
& \left.\frac{2\sqrt{-a^2+b^2}c}{bc+\sqrt{-a^2+b^2}c-a\left(d+\sqrt{-c^2+d^2}\right)}, \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sqrt{\text{Sin}[e+fx]} \\
& \left.\sqrt{\frac{a \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 (a+b \text{Sin}[e+fx])}{a^2-b^2}}\right) / \left(4\left(b+\sqrt{-a^2+b^2}\right)^3 (bc-ad) \sqrt{-c^2+d^2} \sqrt{a+b \text{Sin}[e+fx]} \left(\frac{a \tan\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}\right)^{3/2}\right) - \\
& \left(a b \sqrt{-a^2+b^2} \text{Cos}[e+fx]\right) \left(\left(ac + \left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right) \text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}c}{bc+\sqrt{-a^2+b^2}c-ad+a\sqrt{-c^2+d^2}}\right], \right. \\
& \left.\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right] + \left(-ac + \left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right) \right. \\
& \left.\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}c}{bc+\sqrt{-a^2+b^2}c-a\left(d+\sqrt{-c^2+d^2}\right)}, \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]\right] \sqrt{\text{Sin}[e+fx]}
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{\frac{a \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 (a+b \sin [e+f x])}{a^2-b^2}} \right) / \left(2\left(b+\sqrt{-a^2+b^2}\right)^2 (b c-a d) \sqrt{-c^2+d^2} (a+b \sin [e+f x])^{3/2} \sqrt{-\frac{a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}} \right) + \\
& \left(a \sqrt{-a^2+b^2} \cos [e+f x] \left(a c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right) \right) \operatorname{EllipticPi}\left[\frac{2 \sqrt{-a^2+b^2} c}{b c+\sqrt{-a^2+b^2} c-a d+a \sqrt{-c^2+d^2}}\right], \right. \\
& \left. \operatorname{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2 \sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right] + \left(-a c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right) \right) \\
& \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{-a^2+b^2} c}{b c+\sqrt{-a^2+b^2} c-a\left(d+\sqrt{-c^2+d^2}\right)}\right], \operatorname{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2 \sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right] \right) \\
& \left. \sqrt{\frac{a \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 (a+b \sin [e+f x])}{a^2-b^2}} \right) / \left(2\left(b+\sqrt{-a^2+b^2}\right)^2 (b c-a d) \sqrt{-c^2+d^2} \sqrt{\sin [e+f x]} \right) + \\
& \sqrt{a+b \sin [e+f x]} \sqrt{-\frac{a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}} +
\end{aligned}$$

$$\left(a \sqrt{-a^2 + b^2} \left(a c + (b + \sqrt{-a^2 + b^2}) (-d + \sqrt{-c^2 + d^2}) \right) \text{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a d + a \sqrt{-c^2 + d^2}}, \right. \right.$$

$$\left. \text{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}}, \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] + (-a c + (b + \sqrt{-a^2 + b^2}) (d + \sqrt{-c^2 + d^2})) \right.$$

$$\left. \left. \text{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a (d + \sqrt{-c^2 + d^2})}, \text{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}}, \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] \right] \right)$$

$$\left. \sqrt{\text{Sin}[e + f x]} \left(\frac{a b \text{Cos}[e + f x] \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2}{a^2 - b^2} + \frac{a \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 (a + b \text{Sin}[e + f x]) \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{a^2 - b^2} \right) \right) /$$

$$\left(2 (b + \sqrt{-a^2 + b^2})^2 (b c - a d) \sqrt{-c^2 + d^2} \sqrt{a + b \text{Sin}[e + f x]} \sqrt{\frac{a \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 (a + b \text{Sin}[e + f x])}{a^2 - b^2}} \sqrt{\frac{a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \right) +$$

$$\left(a \sqrt{-a^2 + b^2} \sqrt{\text{Sin}[e + f x]} \sqrt{\frac{a \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 (a + b \text{Sin}[e + f x])}{a^2 - b^2}} \right)$$

$$\left(\left(a (a c + (b + \sqrt{-a^2 + b^2}) (-d + \sqrt{-c^2 + d^2})) \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right) / \left(4 \sqrt{2} \sqrt{-a^2 + b^2} \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}} \right) \right)$$

$$\begin{aligned}
& \left(\sqrt{1 - \frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{2\sqrt{-a^2 + b^2}}} \sqrt{1 - \frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \left(1 - \frac{c \left(b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right)}{b c + \sqrt{-a^2 + b^2} c - a d + a \sqrt{-c^2 + d^2}}\right) \right) + \\
& \left(a \left(-a c + \left(b + \sqrt{-a^2 + b^2}\right) \left(d + \sqrt{-c^2 + d^2}\right) \right) \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right) / \left(4 \sqrt{2} \sqrt{-a^2 + b^2} \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{-a^2 + b^2}}} \right) \\
& \left(\sqrt{1 - \frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{2\sqrt{-a^2 + b^2}}} \sqrt{1 - \frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \left(1 - \frac{c \left(b + \sqrt{-a^2 + b^2} + a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right)}{b c + \sqrt{-a^2 + b^2} c - a \left(d + \sqrt{-c^2 + d^2}\right)}\right) \right) \Bigg) / \\
& \left(\left(b + \sqrt{-a^2 + b^2} \right)^2 (b c - a d) \sqrt{-c^2 + d^2} \sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{-\frac{a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{b + \sqrt{-a^2 + b^2}}} \right) \Bigg)
\end{aligned}$$

■ **Problem 45: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{g \operatorname{Sin}[e + f x]} \sqrt{a + b \operatorname{Sin}[e + f x]} (c + d \operatorname{Sin}[e + f x])} dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$\frac{2 \sqrt{a + b} \sqrt{\frac{a(1 - \operatorname{Csc}[e + f x])}{a + b}} \sqrt{\frac{a(1 + \operatorname{Csc}[e + f x])}{a - b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{g} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{g \operatorname{Sin}[e + f x]}}\right], -\frac{a + b}{a - b}\right] \operatorname{Tan}[e + f x]}{a c f \sqrt{g}}$$

$$\left(2 d \sqrt{-\operatorname{Cot}[e + f x]^2} \sqrt{\frac{b + a \operatorname{Csc}[e + f x]}{a + b}} \operatorname{EllipticPi}\left[\frac{2 c}{c + d}, \operatorname{ArcSin}\left[\frac{\sqrt{1 - \operatorname{Csc}[e + f x]}}{\sqrt{2}}\right], \frac{2 a}{a + b}\right] \sqrt{g \operatorname{Sin}[e + f x]} \operatorname{Tan}[e + f x] \right) / \\
(c (c + d) f g \sqrt{a + b \operatorname{Sin}[e + f x]})$$

Result (type 4, 4935 leaves):

$$- \left(\left(4 \sqrt{-a^2 + b^2} \cos \left[\frac{1}{2} (e + f x) \right] \right)^4 - 2 \left(b + \sqrt{-a^2 + b^2} \right) (b c - a d) \sqrt{-c^2 + d^2} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] \right) -$$

$$a d \left(a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(-d + \sqrt{-c^2 + d^2} \right) \right) \operatorname{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a d + a \sqrt{-c^2 + d^2}}, \right.$$

$$\left. \operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] + \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right)$$

$$\left. \operatorname{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a \left(d + \sqrt{-c^2 + d^2} \right)}, \operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] \right)$$

$$\sqrt{\frac{a \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 (a + b \operatorname{Sin}[e + f x])}{a^2 - b^2} \left(-\frac{a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}} \right)^{3/2}} \left(a^2 c (-b c + a d) \sqrt{-c^2 + d^2} f \operatorname{Sin}[e + f x]^{3/2} \right)$$

$$\sqrt{g \operatorname{Sin}[e + f x]} (a + b \operatorname{Sin}[e + f x]) (c + d \operatorname{Sin}[e + f x])$$

$$\left(\left(3 \sqrt{-a^2 + b^2} \cos \left[\frac{1}{2} (e + f x) \right] \right)^2 \left(-2 \left(b + \sqrt{-a^2 + b^2} \right) (b c - a d) \sqrt{-c^2 + d^2} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] - \right.$$

$$a d \left(a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(-d + \sqrt{-c^2 + d^2} \right) \right) \operatorname{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a d + a \sqrt{-c^2 + d^2}}, \right.$$

$$\operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] + \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right)$$

$$\left. \operatorname{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a \left(d + \sqrt{-c^2 + d^2} \right)}, \operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] \right)$$

$$\left. \sqrt{\frac{a \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 (a + b \operatorname{Sin}[e + f x])}{a^2 - b^2}} \sqrt{\frac{a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \right) / \left(a \left(b + \sqrt{-a^2 + b^2} \right) c (-b c + a d) \right)$$

$$\sqrt{-c^2 + d^2} \operatorname{Sin}[e + f x]^{3/2} \sqrt{a + b \operatorname{Sin}[e + f x]} \Big) +$$

$$\frac{1}{a^2 c (-b c + a d) \sqrt{-c^2 + d^2} \operatorname{Sin}[e + f x]^{3/2} (a + b \operatorname{Sin}[e + f x])^{3/2}} 2 b \sqrt{-a^2 + b^2} \cos \left[\frac{1}{2} (e + f x) \right]^4 \cos[e + f x]$$

$$\begin{aligned}
& \left(-2 \left(b + \sqrt{-a^2 + b^2} \right) (bc - ad) \sqrt{-c^2 + d^2} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] - \right. \\
& a d \left(a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(-d + \sqrt{-c^2 + d^2} \right) \right) \operatorname{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{bc + \sqrt{-a^2 + b^2} c - ad + a \sqrt{-c^2 + d^2}}, \operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \right. \\
& \left. \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] + \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \operatorname{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{bc + \sqrt{-a^2 + b^2} c - a \left(d + \sqrt{-c^2 + d^2} \right)}, \right. \\
& \left. \operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] \left. \right) \sqrt{\frac{a \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 (a + b \operatorname{Sin}[e + f x])}{a^2 - b^2}} \left(-\frac{a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}} \right)^{3/2} + \\
& \frac{1}{a^2 c (-bc + ad) \sqrt{-c^2 + d^2} \operatorname{Sin}[e + f x]^{5/2} \sqrt{a + b \operatorname{Sin}[e + f x]}} 6 \sqrt{-a^2 + b^2} \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^4 \operatorname{Cos}[e + f x] \\
& \left(-2 \left(b + \sqrt{-a^2 + b^2} \right) (bc - ad) \sqrt{-c^2 + d^2} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] - \right.
\end{aligned}$$

$$\begin{aligned}
& a d \left(a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(-d + \sqrt{-c^2 + d^2} \right) \right) \text{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a d + a \sqrt{-c^2 + d^2}}, \text{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right] \right], \\
& \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \left] + \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \text{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a \left(d + \sqrt{-c^2 + d^2} \right)}, \right. \\
& \left. \text{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] \right) \sqrt{\frac{a \sec \left[\frac{1}{2} (e + f x) \right]^2 (a + b \sin[e + f x])}{a^2 - b^2} \left(-\frac{a \tan \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}} \right)^{3/2}} + \\
& \frac{1}{a^2 c (-b c + a d) \sqrt{-c^2 + d^2} \sin[e + f x]^{3/2} \sqrt{a + b \sin[e + f x]}} 8 \sqrt{-a^2 + b^2} \cos \left[\frac{1}{2} (e + f x) \right]^3 \\
& \left(-2 \left(b + \sqrt{-a^2 + b^2} \right) (b c - a d) \sqrt{-c^2 + d^2} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] - \right. \\
& \left. a d \left(a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(-d + \sqrt{-c^2 + d^2} \right) \right) \text{EllipticPi} \left[\frac{2 \sqrt{-a^2 + b^2} c}{b c + \sqrt{-a^2 + b^2} c - a d + a \sqrt{-c^2 + d^2}}, \right. \right. \\
& \left. \left. \text{ArcSin} \left[\frac{\sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-a^2 + b^2}}{b + \sqrt{-a^2 + b^2}} \right] + \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \text{EllipticPi} \left[\right. \right.
\end{aligned}$$

$$\left. \left. \left. \frac{2\sqrt{-a^2+b^2}c}{bc+\sqrt{-a^2+b^2}c-a(d+\sqrt{-c^2+d^2})}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\tan\left[\frac{1}{2}(e+fx)\right]}}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}} \right] \right) \right)$$

$$\sin\left[\frac{1}{2}(e+fx)\right] \sqrt{\frac{a\sec\left[\frac{1}{2}(e+fx)\right]^2(a+b\sin[e+fx])}{a^2-b^2}} \left(\frac{a\tan\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}\right)^{3/2} -$$

$$\left(2\sqrt{-a^2+b^2}\cos\left[\frac{1}{2}(e+fx)\right]^4 \left(-2(b+\sqrt{-a^2+b^2})(bc-ad)\sqrt{-c^2+d^2}\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\tan\left[\frac{1}{2}(e+fx)\right]}}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right] - \right.$$

$$\left. ad \left(ac + (b+\sqrt{-a^2+b^2})(-d+\sqrt{-c^2+d^2}) \right) \operatorname{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}c}{bc+\sqrt{-a^2+b^2}c-ad+a\sqrt{-c^2+d^2}}\right], \right.$$

$$\left. \operatorname{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\tan\left[\frac{1}{2}(e+fx)\right]}}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right] + (-ac + (b+\sqrt{-a^2+b^2})(d+\sqrt{-c^2+d^2}))$$

$$\left. \left. \left. \operatorname{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}c}{bc+\sqrt{-a^2+b^2}c-a(d+\sqrt{-c^2+d^2})}\right], \operatorname{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\tan\left[\frac{1}{2}(e+fx)\right]}}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}} \right] \right) \right)$$

$$\left. \left(-\frac{a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}} \right)^{3/2} \left(\frac{ab \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{a^2-b^2} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 (a+b \operatorname{Sin}[e+fx]) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{a^2-b^2} \right) \right) /$$

$$\left(a^2 c (-bc+ad) \sqrt{-c^2+d^2} \operatorname{Sin}[e+fx]^{3/2} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{\frac{a \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 (a+b \operatorname{Sin}[e+fx])}{a^2-b^2}} \right) -$$

$$\frac{1}{a^2 c (-bc+ad) \sqrt{-c^2+d^2} \operatorname{Sin}[e+fx]^{3/2} \sqrt{a+b \operatorname{Sin}[e+fx]}}$$

$$4 \sqrt{-a^2+b^2} \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^4 \sqrt{\frac{a \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 (a+b \operatorname{Sin}[e+fx])}{a^2-b^2}} \left(-\frac{a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}} \right)^{3/2}$$

$$\left(-\left(a \left(b+\sqrt{-a^2+b^2} \right) (bc-ad) \sqrt{-c^2+d^2} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) / \left(2\sqrt{2} \sqrt{-a^2+b^2} \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-a^2+b^2}}} \right) \right)$$

$$\sqrt{1 - \frac{b+\sqrt{-a^2+b^2}+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{2\sqrt{-a^2+b^2}}} \sqrt{1 - \frac{b+\sqrt{-a^2+b^2}+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}} -$$

$$ad \left(a \left(ac + \left(b+\sqrt{-a^2+b^2} \right) \left(-d+\sqrt{-c^2+d^2} \right) \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right) / \left(4\sqrt{2} \sqrt{-a^2+b^2} \sqrt{\frac{b+\sqrt{-a^2+b^2}+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-a^2+b^2}}} \right)$$

$$\sqrt{1 - \frac{b+\sqrt{-a^2+b^2}+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{2\sqrt{-a^2+b^2}}} \sqrt{1 - \frac{b+\sqrt{-a^2+b^2}+a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\left(\frac{c \left(b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)}{b c + \sqrt{-a^2 + b^2} c - a d + a \sqrt{-c^2 + d^2}} \right) + \left(a \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right) /$$

$$\left(4 \sqrt{2} \sqrt{-a^2 + b^2} \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}}} \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{2 \sqrt{-a^2 + b^2}}} \right.$$

$$\left. \sqrt{\frac{b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{b + \sqrt{-a^2 + b^2}}} \left(\frac{c \left(b + \sqrt{-a^2 + b^2} + a \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right)}{b c + \sqrt{-a^2 + b^2} c - a \left(d + \sqrt{-c^2 + d^2} \right)} \right) \right) \right)$$

- **Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]}}{a + b \operatorname{Sin}[e + f x]} dx$$

Optimal (type 4, 254 leaves, 3 steps):

$$\frac{1}{b f} 2 \sqrt{c + d} \sqrt{g} \sqrt{\frac{c (1 - \operatorname{Csc}[e + f x])}{c + d}} \sqrt{\frac{c (1 + \operatorname{Csc}[e + f x])}{c - d}} \operatorname{EllipticPi} \left[\frac{c + d}{d}, \operatorname{ArcSin} \left[\frac{\sqrt{g} \sqrt{c + d \operatorname{Sin}[e + f x]}}{\sqrt{c + d} \sqrt{g \operatorname{Sin}[e + f x]}} \right], -\frac{c + d}{c - d} \right] \operatorname{Tan}[e + f x] +$$

$$\left(2 (b c - a d) \sqrt{-\operatorname{Cot}[e + f x]^2} \sqrt{\frac{d + c \operatorname{Csc}[e + f x]}{c + d}} \operatorname{EllipticPi} \left[\frac{2 a}{a + b}, \operatorname{ArcSin} \left[\frac{\sqrt{1 - \operatorname{Csc}[e + f x]}}{\sqrt{2}} \right], \frac{2 c}{c + d} \right] \sqrt{g \operatorname{Sin}[e + f x]} \operatorname{Tan}[e + f x] \right) /$$

$$(b (a + b) f \sqrt{c + d \operatorname{Sin}[e + f x]})$$

Result (type 4, 75413 leaves): Display of huge result suppressed!

- **Problem 47: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{g \operatorname{Sin}[e + f x]}}{(a + b \operatorname{Sin}[e + f x]) \sqrt{c + d \operatorname{Sin}[e + f x]}} dx$$

Optimal (type 4, 114 leaves, 1 step):

$$\left(2 \sqrt{-\cot[e + f x]^2} \sqrt{\frac{d + c \operatorname{Csc}[e + f x]}{c + d}} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \operatorname{ArcSin}\left[\frac{\sqrt{1 - \operatorname{Csc}[e + f x]}}{\sqrt{2}}\right], \frac{2 c}{c + d}\right] \sqrt{g \operatorname{Sin}[e + f x] \operatorname{Tan}[e + f x]} \right) /$$

$$\left((a + b) f \sqrt{c + d \operatorname{Sin}[e + f x]} \right)$$

Result (type 4, 3429 leaves):

$$- \left(\left(c \sqrt{-c^2 + d^2} \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \operatorname{EllipticPi}\left[\frac{2 a \sqrt{-c^2 + d^2}}{-b c - \sqrt{-a^2 + b^2} c + a \left(d + \sqrt{-c^2 + d^2} \right)}, \right. \right.$$

$$\left. \operatorname{ArcSin}\left[\frac{\sqrt{\frac{d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}{\sqrt{-c^2 + d^2}}}, \frac{2 \sqrt{-c^2 + d^2}}{d + \sqrt{-c^2 + d^2}} \right] + \left(a c + \left(-b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{2 a \sqrt{-c^2 + d^2}}{-b c + \sqrt{-a^2 + b^2} c + a \left(d + \sqrt{-c^2 + d^2} \right)}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}}{\sqrt{-c^2 + d^2}}}, \frac{2 \sqrt{-c^2 + d^2}}{d + \sqrt{-c^2 + d^2}} \right] \right) \right)$$

$$\left. \sqrt{\operatorname{Sin}[e + f x]} \sqrt{g \operatorname{Sin}[e + f x]} \sqrt{\frac{c \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 (c + d \operatorname{Sin}[e + f x])}{c^2 - d^2}} \right) /$$

$$\left(\sqrt{-a^2 + b^2} (b c - a d) \left(d + \sqrt{-c^2 + d^2} \right)^2 f (a + b \operatorname{Sin}[e + f x]) (c + d \operatorname{Sin}[e + f x]) \sqrt{\frac{c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{d + \sqrt{-c^2 + d^2}}} \right)$$

$$\begin{aligned}
& \left(\left(\left(c^2 \sqrt{-c^2 + d^2} \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \text{EllipticPi} \left[\frac{2 a \sqrt{-c^2 + d^2}}{-b c - \sqrt{-a^2 + b^2} c + a \left(d + \sqrt{-c^2 + d^2} \right)}, \right. \right. \right. \\
& \left. \left. \left. \text{ArcSin} \left[\frac{\sqrt{\frac{d + \sqrt{-c^2 + d^2} + c \text{Tan} \left[\frac{1}{2} (e + f x) \right]}}{\sqrt{-c^2 + d^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-c^2 + d^2}}{d + \sqrt{-c^2 + d^2}} \right] + \left(a c + \left(-b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \text{EllipticPi} \left[\right. \right. \\
& \left. \left. \left. \frac{2 a \sqrt{-c^2 + d^2}}{-b c + \sqrt{-a^2 + b^2} c + a \left(d + \sqrt{-c^2 + d^2} \right)} \right], \text{ArcSin} \left[\frac{\sqrt{\frac{d + \sqrt{-c^2 + d^2} + c \text{Tan} \left[\frac{1}{2} (e + f x) \right]}}{\sqrt{-c^2 + d^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-c^2 + d^2}}{d + \sqrt{-c^2 + d^2}} \right] \right) \\
& \left. \left. \left. \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \sqrt{\text{Sin}[e + f x]} \sqrt{\frac{c \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 (c + d \text{Sin}[e + f x])}{c^2 - d^2}} \right] \right) \right) / \\
& \left(4 \sqrt{-a^2 + b^2} (b c - a d) \left(d + \sqrt{-c^2 + d^2} \right)^3 \sqrt{c + d \text{Sin}[e + f x]} \left(-\frac{c \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{d + \sqrt{-c^2 + d^2}} \right)^{3/2} \right) + \\
& \left(c d \sqrt{-c^2 + d^2} \text{Cos}[e + f x] \left(-a c + \left(b + \sqrt{-a^2 + b^2} \right) \left(d + \sqrt{-c^2 + d^2} \right) \right) \text{EllipticPi} \left[\frac{2 a \sqrt{-c^2 + d^2}}{-b c - \sqrt{-a^2 + b^2} c + a \left(d + \sqrt{-c^2 + d^2} \right)}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right] + \left(a c + \left(-b + \sqrt{-a^2+b^2}\right) \left(d + \sqrt{-c^2+d^2}\right)\right) \\
& \left. \text{EllipticPi}\left[\frac{2 a \sqrt{-c^2+d^2}}{-b c + \sqrt{-a^2+b^2} c + a \left(d + \sqrt{-c^2+d^2}\right)}, \text{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right] \right\} \sqrt{\sin[e+fx]} \\
& \left. \sqrt{\frac{c \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 (c+d \sin[e+fx])}{c^2-d^2}}\right] / \left(2 \sqrt{-a^2+b^2} (b c - a d) \left(d + \sqrt{-c^2+d^2}\right)^2 (c+d \sin[e+fx])^{3/2} \sqrt{-\frac{c \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{d+\sqrt{-c^2+d^2}}}\right) - \\
& \left(c \sqrt{-c^2+d^2} \cos[e+fx] \right) \left(-a c + \left(b + \sqrt{-a^2+b^2}\right) \left(d + \sqrt{-c^2+d^2}\right) \right) \text{EllipticPi}\left[\frac{2 a \sqrt{-c^2+d^2}}{-b c - \sqrt{-a^2+b^2} c + a \left(d + \sqrt{-c^2+d^2}\right)}, \right. \\
& \left. \text{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right] + \left(a c + \left(-b + \sqrt{-a^2+b^2}\right) \left(d + \sqrt{-c^2+d^2}\right)\right) \\
& \left. \text{EllipticPi}\left[\frac{2 a \sqrt{-c^2+d^2}}{-b c + \sqrt{-a^2+b^2} c + a \left(d + \sqrt{-c^2+d^2}\right)}, \text{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right], \frac{2\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right] \right)
\end{aligned}$$

$$\left. \sqrt{\frac{c \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 (c+d \operatorname{Sin}[e+f x])}{c^2-d^2}} \right) / \left(2 \sqrt{-a^2+b^2} (b c-a d) \left(d+\sqrt{-c^2+d^2}\right)^2 \sqrt{\operatorname{Sin}[e+f x]} \right.$$

$$\left. \sqrt{c+d \operatorname{Sin}[e+f x]} \sqrt{\frac{c \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{d+\sqrt{-c^2+d^2}}} \right) -$$

$$\left(c \sqrt{-c^2+d^2} \left(-a c + \left(b + \sqrt{-a^2+b^2}\right) \left(d + \sqrt{-c^2+d^2}\right) \right) \operatorname{EllipticPi}\left[\frac{2 a \sqrt{-c^2+d^2}}{-b c - \sqrt{-a^2+b^2} c + a \left(d + \sqrt{-c^2+d^2}\right)}, \right.$$

$$\left. \operatorname{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}} \right] + \left(a c + \left(-b + \sqrt{-a^2+b^2}\right) \left(d + \sqrt{-c^2+d^2}\right) \right)$$

$$\left. \operatorname{EllipticPi}\left[\frac{2 a \sqrt{-c^2+d^2}}{-b c + \sqrt{-a^2+b^2} c + a \left(d + \sqrt{-c^2+d^2}\right)}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}} \right], \frac{2 \sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}} \right] \right)$$

$$\left. \sqrt{\operatorname{Sin}[e+f x]} \left(\frac{c d \operatorname{Cos}[e+f x] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{c^2-d^2} + \frac{c \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 (c+d \operatorname{Sin}[e+f x]) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{c^2-d^2} \right) \right) /$$

$$\begin{aligned}
& \left(2 \sqrt{-a^2 + b^2} (bc - ad) (d + \sqrt{-c^2 + d^2})^2 \sqrt{c + d \sin[ex]} \sqrt{\frac{c \operatorname{Sec}\left[\frac{1}{2}(ex)\right]^2 (c + d \sin[ex])}{c^2 - d^2}} \sqrt{\frac{c \operatorname{Tan}\left[\frac{1}{2}(ex)\right]}{d + \sqrt{-c^2 + d^2}}} \right) - \\
& \left(c \sqrt{-c^2 + d^2} \sqrt{\sin[ex]} \sqrt{\frac{c \operatorname{Sec}\left[\frac{1}{2}(ex)\right]^2 (c + d \sin[ex])}{c^2 - d^2}} \right) \\
& \left(\left(c \left(-ac + (b + \sqrt{-a^2 + b^2}) (d + \sqrt{-c^2 + d^2}) \right) \operatorname{Sec}\left[\frac{1}{2}(ex)\right]^2 \right) / \left(4 \sqrt{2} \sqrt{-c^2 + d^2} \sqrt{\frac{d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(ex)\right]}{\sqrt{-c^2 + d^2}}} \right) \right. \\
& \left. \sqrt{1 - \frac{d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(ex)\right]}{2 \sqrt{-c^2 + d^2}}} \sqrt{1 - \frac{d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(ex)\right]}{d + \sqrt{-c^2 + d^2}}} \right. \\
& \left. \left(1 - \frac{a \left(d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(ex)\right] \right)}{-bc - \sqrt{-a^2 + b^2} c + a \left(d + \sqrt{-c^2 + d^2} \right)} \right) \right) + \left(c \left(ac + (-b + \sqrt{-a^2 + b^2}) (d + \sqrt{-c^2 + d^2}) \right) \operatorname{Sec}\left[\frac{1}{2}(ex)\right]^2 \right) / \\
& \left(4 \sqrt{2} \sqrt{-c^2 + d^2} \sqrt{\frac{d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(ex)\right]}{\sqrt{-c^2 + d^2}}} \sqrt{1 - \frac{d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(ex)\right]}{2 \sqrt{-c^2 + d^2}}} \right. \\
& \left. \sqrt{1 - \frac{d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(ex)\right]}{d + \sqrt{-c^2 + d^2}}} \left(1 - \frac{a \left(d + \sqrt{-c^2 + d^2} + c \operatorname{Tan}\left[\frac{1}{2}(ex)\right] \right)}{-bc + \sqrt{-a^2 + b^2} c + a \left(d + \sqrt{-c^2 + d^2} \right)} \right) \right) / \\
& \left(\sqrt{-a^2 + b^2} (bc - ad) (d + \sqrt{-c^2 + d^2})^2 \sqrt{c + d \sin[ex]} \sqrt{\frac{c \operatorname{Tan}\left[\frac{1}{2}(ex)\right]}{d + \sqrt{-c^2 + d^2}}} \right) \right) \right)
\end{aligned}$$

■ Problem 48: Unable to integrate problem.

$$\int \text{Csc}[e + f x] \sqrt{a + b \text{Sin}[e + f x]} \sqrt{c + d \text{Sin}[e + f x]} dx$$

Optimal (type 4, 391 leaves, 3 steps):

$$-\frac{1}{\sqrt{a+b} f} 2 \sqrt{c+d} \text{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \text{Sin}[e+fx]}{\sqrt{c+d} \sqrt{a+b \text{Sin}[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\text{Sin}[e+fx])}{(c+d)(a+b \text{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\text{Sin}[e+fx])}{(c-d)(a+b \text{Sin}[e+fx])}} (a+b \text{Sin}[e+fx]) +$$

$$\frac{1}{\sqrt{a+b} f} 2 \sqrt{c+d} \text{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \text{Sin}[e+fx]}{\sqrt{c+d} \sqrt{a+b \text{Sin}[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \text{Sec}[e+fx]$$

$$\sqrt{-\frac{(bc-ad)(1-\text{Sin}[e+fx])}{(c+d)(a+b \text{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\text{Sin}[e+fx])}{(c-d)(a+b \text{Sin}[e+fx])}} (a+b \text{Sin}[e+fx])$$

Result (type 8, 37 leaves):

$$\int \text{Csc}[e + f x] \sqrt{a + b \text{Sin}[e + f x]} \sqrt{c + d \text{Sin}[e + f x]} dx$$

■ **Problem 49: Unable to integrate problem.**

$$\int \frac{\text{Csc}[e + f x] \sqrt{a + b \text{Sin}[e + f x]}}{\sqrt{c + d \text{Sin}[e + f x]}} dx$$

Optimal (type 4, 198 leaves, 1 step):

$$-\frac{1}{\sqrt{a+b} c f} 2 \sqrt{c+d} \text{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \text{Sin}[e+fx]}{\sqrt{c+d} \sqrt{a+b \text{Sin}[e+fx]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\text{Sec}[e+fx] \sqrt{-\frac{(bc-ad)(1-\text{Sin}[e+fx])}{(c+d)(a+b \text{Sin}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\text{Sin}[e+fx])}{(c-d)(a+b \text{Sin}[e+fx])}} (a+b \text{Sin}[e+fx])$$

Result (type 8, 37 leaves):

$$\int \frac{\text{Csc}[e + f x] \sqrt{a + b \text{Sin}[e + f x]}}{\sqrt{c + d \text{Sin}[e + f x]}} dx$$

■ **Problem 50: Unable to integrate problem.**

$$\int \frac{\text{Csc}[e + f x]}{\sqrt{a + b \text{Sin}[e + f x]} \sqrt{c + d \text{Sin}[e + f x]}} dx$$

Optimal (type 4, 398 leaves, 3 steps):

$$\begin{aligned}
& -\frac{1}{a\sqrt{a+b}cf} 2\sqrt{c+d} \operatorname{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sqrt{c+d}\sin[e+fx]}{\sqrt{c+d}\sqrt{a+b}\sin[e+fx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(c+d)(a+b)\sin[e+fx]}} \sqrt{\frac{(bc-ad)(1+\sin[e+fx])}{(c-d)(a+b)\sin[e+fx]}} (a+b\sin[e+fx]) - \\
& \frac{1}{a\sqrt{c+d}(bc-ad)f} 2b\sqrt{a+b} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{a+b}\sin[e+fx]}{\sqrt{a+b}\sqrt{c+d}\sin[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \operatorname{Sec}[e+fx] \\
& \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d)\sin[e+fx]}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d)\sin[e+fx]}} (c+d\sin[e+fx])
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{\operatorname{Csc}[e+fx]}{\sqrt{a+b}\sin[e+fx]\sqrt{c+d}\sin[e+fx]} dx$$

■ **Problem 51: Result more than twice size of optimal antiderivative.**

$$\int (a+a\sin[e+fx])^m (A+B\sin[e+fx])^p (c-c\sin[e+fx])^n dx$$

Optimal (type 6, 157 leaves, 4 steps):

$$\begin{aligned}
& \frac{1}{af(1+2m)} 2^{\frac{1}{2}+n} \operatorname{AppellF1}\left[\frac{1}{2}+m, \frac{1}{2}-n, -p, \frac{3}{2}+m, \frac{1}{2}(1+\sin[e+fx]), -\frac{B(1+\sin[e+fx])}{A-B}\right] \\
& \operatorname{Sec}[e+fx] (1-\sin[e+fx])^{\frac{1}{2}-n} (a+a\sin[e+fx])^{1+m} (A+B\sin[e+fx])^p \left(\frac{A+B\sin[e+fx]}{A-B}\right)^{-p} (c-c\sin[e+fx])^n
\end{aligned}$$

Result (type 6, 417 leaves):

$$\left(2 (A+B) (3+2n) \operatorname{AppellF1}\left[\frac{1}{2}+n, \frac{1}{2}-m, -p, \frac{3}{2}+n, \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2, \frac{2B \sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{A+B}\right] \left(\cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^2\right)^{-\frac{1}{2}+m} \right. \\ \left. \cot\left[\frac{1}{4}(2e+\pi+2fx)\right] (a(1+\sin[e+fx]))^m (A+B \sin[e+fx])^p (c-c \sin[e+fx])^n \left(\sin\left[\frac{1}{4}(2e+\pi+2fx)\right]^2\right)^{\frac{1}{2}-m}\right) / \\ \left(f(1+2n) \left(- (A+B) (3+2n) \operatorname{AppellF1}\left[\frac{1}{2}+n, \frac{1}{2}-m, -p, \frac{3}{2}+n, \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2, \frac{2B \sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{A+B}\right] \right) + \right. \\ \left(4Bp \operatorname{AppellF1}\left[\frac{3}{2}+n, \frac{1}{2}-m, 1-p, \frac{5}{2}+n, \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2, \frac{2B \sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{A+B}\right] \right) + \\ \left. (A+B) (-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}+n, \frac{3}{2}-m, -p, \frac{5}{2}+n, \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2, \frac{2B \sin\left[\frac{1}{4}(2e-\pi+2fx)\right]^2}{A+B}\right] \right) \cos\left[\frac{1}{4}(2e+\pi+2fx)\right]^2 \right)$$

Test results for the 358 problems in "4.1.3.1 (a+b sin)^m (c+d sin)^n (A+B sin).m"

■ Problem 1: Unable to integrate problem.

$$\int (d \sin[e+fx])^n (a+a \sin[e+fx])^3 (A+B \sin[e+fx]) dx$$

Optimal (type 5, 373 leaves, 7 steps):

$$- \frac{a^3 (B(27+14n+2n^2) + A(28+15n+2n^2)) \cos[e+fx] (d \sin[e+fx])^{1+n}}{df(2+n)(3+n)(4+n)} + \\ \left(a^3 (B(15+19n+4n^2) + A(20+21n+4n^2)) \cos[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \sin[e+fx]^2\right] (d \sin[e+fx])^{1+n} \right) / \\ \left(df(1+n)(2+n)(4+n) \sqrt{\cos[e+fx]^2} \right) + \\ \left(a^3 (B(9+4n) + A(11+4n)) \cos[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \sin[e+fx]^2\right] (d \sin[e+fx])^{2+n} \right) / \\ \left(d^2 f(2+n)(3+n) \sqrt{\cos[e+fx]^2} \right) - \frac{aB \cos[e+fx] (d \sin[e+fx])^{1+n} (a+a \sin[e+fx])^2}{df(4+n)} - \\ \frac{(A(4+n) + B(6+n)) \cos[e+fx] (d \sin[e+fx])^{1+n} (a^3 + a^3 \sin[e+fx])}{df(3+n)(4+n)}$$

Result (type 9, 68520 leaves): Display of huge result suppressed!

■ **Problem 2: Unable to integrate problem.**

$$\int (\text{d Sin}[e + f x])^n (a + a \text{Sin}[e + f x])^2 (A + B \text{Sin}[e + f x]) dx$$

Optimal (type 5, 277 leaves, 6 steps):

$$\begin{aligned} & - \frac{a^2 (A (3+n) + B (4+n)) \text{Cos}[e + f x] (\text{d Sin}[e + f x])^{1+n}}{d f (2+n) (3+n)} + \\ & \left(a^2 (2 B (1+n) + A (3+2n)) \text{Cos}[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \text{Sin}[e + f x]^2\right] (\text{d Sin}[e + f x])^{1+n} \right) / \\ & \left(d f (1+n) (2+n) \sqrt{\text{Cos}[e + f x]^2} \right) + \\ & \left(a^2 (2 A (3+n) + B (5+2n)) \text{Cos}[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \text{Sin}[e + f x]^2\right] (\text{d Sin}[e + f x])^{2+n} \right) / \\ & \left(d^2 f (2+n) (3+n) \sqrt{\text{Cos}[e + f x]^2} \right) - \frac{B \text{Cos}[e + f x] (\text{d Sin}[e + f x])^{1+n} (a^2 + a^2 \text{Sin}[e + f x])}{d f (3+n)} \end{aligned}$$

Result (type 9, 25571 leaves): Display of huge result suppressed!

■ **Problem 3: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (\text{d Sin}[e + f x])^n (a + a \text{Sin}[e + f x]) (A + B \text{Sin}[e + f x]) dx$$

Optimal (type 5, 191 leaves, 5 steps):

$$\begin{aligned} & - \frac{a B \text{Cos}[e + f x] (\text{d Sin}[e + f x])^{1+n}}{d f (2+n)} + \frac{a (B (1+n) + A (2+n)) \text{Cos}[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \text{Sin}[e + f x]^2\right] (\text{d Sin}[e + f x])^{1+n}}{d f (1+n) (2+n) \sqrt{\text{Cos}[e + f x]^2}} + \\ & \frac{a (A + B) \text{Cos}[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \text{Sin}[e + f x]^2\right] (\text{d Sin}[e + f x])^{2+n}}{d^2 f (2+n) \sqrt{\text{Cos}[e + f x]^2}} \end{aligned}$$

Result (type 5, 392 leaves):

$$\begin{aligned}
& - \frac{1}{f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^2} \\
& 2^{-2-n} a e^{i f n x} \left(1 - e^{2 i (e + f x)} \right)^{-n} \left(-i e^{-i (e + f x)} \left(-1 + e^{2 i (e + f x)} \right) \right)^n \left(\frac{2 (A + B) e^{-i (e + f (1+n) x)} \operatorname{Hypergeometric2F1} \left[\frac{1}{2} (-1 - n), -n, \frac{1-n}{2}, e^{2 i (e + f x)} \right]}{1 + n} \right. \\
& \left. \frac{2 (A + B) e^{i (e - f (-1+n) x)} \operatorname{Hypergeometric2F1} \left[\frac{1-n}{2}, -n, \frac{3-n}{2}, e^{2 i (e + f x)} \right]}{-1 + n} + \right. \\
& \left. i \left(\frac{B e^{-i (2e + f (2+n) x)} \operatorname{Hypergeometric2F1} \left[-1 - \frac{n}{2}, -n, -\frac{n}{2}, e^{2 i (e + f x)} \right]}{2 + n} + \frac{1}{(-2 + n) n} e^{-i f n x} \left(B e^{2 i (e + f x)} n \operatorname{Hypergeometric2F1} \left[1 - \frac{n}{2}, -n, 2 - \frac{n}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. e^{2 i (e + f x)} \right] - 2 (2 A + B) (-2 + n) \operatorname{Hypergeometric2F1} \left[-n, -\frac{n}{2}, 1 - \frac{n}{2}, e^{2 i (e + f x)} \right] \right) \right) \right) \left. \right) \left. \right) \sin[e + f x]^{-n} (d \sin[e + f x])^n (1 + \sin[e + f x])
\end{aligned}$$

■ **Problem 4: Unable to integrate problem.**

$$\int \frac{(d \sin[e + f x])^n (A + B \sin[e + f x])}{a + a \sin[e + f x]} dx$$

Optimal (type 5, 202 leaves, 4 steps):

$$\begin{aligned}
& \frac{(B - A n + B n) \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \sin[e + f x]^2 \right] (d \sin[e + f x])^{1+n}}{a d f (1 + n) \sqrt{\cos[e + f x]^2}} + \\
& \frac{(A - B) (1 + n) \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \sin[e + f x]^2 \right] (d \sin[e + f x])^{2+n}}{a d^2 f (2 + n) \sqrt{\cos[e + f x]^2}} + \frac{(A - B) \cos[e + f x] (d \sin[e + f x])^{1+n}}{d f (a + a \sin[e + f x])}
\end{aligned}$$

Result (type 8, 35 leaves):

$$\int \frac{(d \sin[e + f x])^n (A + B \sin[e + f x])}{a + a \sin[e + f x]} dx$$

■ **Problem 5: Unable to integrate problem.**

$$\int \frac{(d \sin[e + f x])^n (A + B \sin[e + f x])}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 5, 279 leaves, 5 steps):

$$\begin{aligned}
& - \left(n (A - 2 A n + 2 B (1 + n)) \operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \operatorname{Sin}[e + f x]^2 \right] (d \operatorname{Sin}[e + f x])^{1+n} \right) / \left(3 a^2 d f (1 + n) \sqrt{\operatorname{Cos}[e + f x]^2} \right) + \\
& \left((1 + n) (B + 2 A (1 - n) + 2 B n) \operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \operatorname{Sin}[e + f x]^2 \right] (d \operatorname{Sin}[e + f x])^{2+n} \right) / \\
& \left(3 a^2 d^2 f (2 + n) \sqrt{\operatorname{Cos}[e + f x]^2} \right) + \frac{(B + 2 A (1 - n) + 2 B n) \operatorname{Cos}[e + f x] (d \operatorname{Sin}[e + f x])^{1+n}}{3 a^2 d f (1 + \operatorname{Sin}[e + f x])} + \frac{(A - B) \operatorname{Cos}[e + f x] (d \operatorname{Sin}[e + f x])^{1+n}}{3 d f (a + a \operatorname{Sin}[e + f x])^2}
\end{aligned}$$

Result (type 8, 35 leaves):

$$\int \frac{(d \operatorname{Sin}[e + f x])^n (A + B \operatorname{Sin}[e + f x])}{(a + a \operatorname{Sin}[e + f x])^2} dx$$

■ **Problem 6: Unable to integrate problem.**

$$\int \frac{(d \operatorname{Sin}[e + f x])^n (A + B \operatorname{Sin}[e + f x])}{(a + a \operatorname{Sin}[e + f x])^3} dx$$

Optimal (type 5, 362 leaves, 6 steps):

$$\begin{aligned}
& - \left(n (B (3 - n - 4 n^2) + A (2 - 9 n + 4 n^2)) \operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \operatorname{Sin}[e + f x]^2 \right] (d \operatorname{Sin}[e + f x])^{1+n} \right) / \\
& \left(15 a^3 d f (1 + n) \sqrt{\operatorname{Cos}[e + f x]^2} \right) + \\
& \left((1 - n) (1 + n) (7 A + 3 B - 4 A n + 4 B n) \operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \operatorname{Sin}[e + f x]^2 \right] (d \operatorname{Sin}[e + f x])^{2+n} \right) / \\
& \left(15 a^3 d^2 f (2 + n) \sqrt{\operatorname{Cos}[e + f x]^2} \right) + \frac{(A - B) \operatorname{Cos}[e + f x] (d \operatorname{Sin}[e + f x])^{1+n}}{5 d f (a + a \operatorname{Sin}[e + f x])^3} + \\
& \frac{(A (5 - 2 n) + 2 B n) \operatorname{Cos}[e + f x] (d \operatorname{Sin}[e + f x])^{1+n}}{15 a d f (a + a \operatorname{Sin}[e + f x])^2} + \frac{(1 - n) (7 A + 3 B - 4 A n + 4 B n) \operatorname{Cos}[e + f x] (d \operatorname{Sin}[e + f x])^{1+n}}{15 d f (a^3 + a^3 \operatorname{Sin}[e + f x])}
\end{aligned}$$

Result (type 8, 35 leaves):

$$\int \frac{(d \operatorname{Sin}[e + f x])^n (A + B \operatorname{Sin}[e + f x])}{(a + a \operatorname{Sin}[e + f x])^3} dx$$

■ **Problem 7: Result more than twice size of optimal antiderivative.**

$$\int (d \operatorname{Sin}[e + f x])^n (a + a \operatorname{Sin}[e + f x])^{5/2} (A + B \operatorname{Sin}[e + f x]) dx$$

Optimal (type 5, 336 leaves, 6 steps):

$$\begin{aligned}
& - \left(2 a^3 \left(2 B \left(115 + 203 n + 104 n^2 + 16 n^3 \right) + A \left(301 + 478 n + 224 n^2 + 32 n^3 \right) \right) \cos[e + f x] \right. \\
& \quad \left. \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -n, \frac{3}{2}, 1 - \sin[e + f x] \right] \sin[e + f x]^{-n} (d \sin[e + f x])^n \right) / \left(f (3 + 2 n) (5 + 2 n) (7 + 2 n) \sqrt{a + a \sin[e + f x]} \right) - \\
& \quad \frac{2 a^3 \left(2 B \left(35 + 23 n + 4 n^2 \right) + A \left(77 + 50 n + 8 n^2 \right) \right) \cos[e + f x] (d \sin[e + f x])^{1+n}}{d f (3 + 2 n) (5 + 2 n) (7 + 2 n) \sqrt{a + a \sin[e + f x]}} - \\
& \quad \frac{2 a^2 \left(2 B (5 + n) + A (7 + 2 n) \right) \cos[e + f x] (d \sin[e + f x])^{1+n} \sqrt{a + a \sin[e + f x]}}{d f (5 + 2 n) (7 + 2 n)} - \\
& \quad \frac{2 a B \cos[e + f x] (d \sin[e + f x])^{1+n} (a + a \sin[e + f x])^{3/2}}{d f (7 + 2 n)}
\end{aligned}$$

Result (type 5, 791 leaves):

$$\begin{aligned}
& \frac{1}{f \sqrt{\sec\left[\frac{1}{2}(e+fx)\right]^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^5}} 2^{1+n} \sec\left[\frac{1}{2}(e+fx)\right] \sin[e+fx]^{-n} (d \sin[e+fx])^n (a(1+\sin[e+fx]))^{5/2} \\
& \tan\left[\frac{1}{2}(e+fx)\right] \left(\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{1+\tan\left[\frac{1}{2}(e+fx)\right]} \right)^n \left(1+\tan\left[\frac{1}{2}(e+fx)\right]\right)^2 \left(\frac{\text{A Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{9}{2}+n, \frac{3+n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]}{1+n} \right) + \\
& \frac{(5A+2B) \text{Hypergeometric2F1}\left[1+\frac{n}{2}, \frac{9}{2}+n, 2+\frac{n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]}{2+n} + \\
& \frac{11A \text{Hypergeometric2F1}\left[\frac{3+n}{2}, \frac{9}{2}+n, \frac{5+n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2}{3+n} + \\
& \frac{10B \text{Hypergeometric2F1}\left[\frac{3+n}{2}, \frac{9}{2}+n, \frac{5+n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2}{3+n} + \\
& \frac{5(3A+4B) \text{Hypergeometric2F1}\left[2+\frac{n}{2}, \frac{9}{2}+n, 3+\frac{n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^3}{4+n} + \\
& \frac{15A \text{Hypergeometric2F1}\left[\frac{9}{2}+n, \frac{5+n}{2}, \frac{7+n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^4}{5+n} + \\
& \frac{20B \text{Hypergeometric2F1}\left[\frac{9}{2}+n, \frac{5+n}{2}, \frac{7+n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^4}{5+n} + \\
& \frac{11A \text{Hypergeometric2F1}\left[3+\frac{n}{2}, \frac{9}{2}+n, 4+\frac{n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^5}{6+n} + \\
& \frac{10B \text{Hypergeometric2F1}\left[3+\frac{n}{2}, \frac{9}{2}+n, 4+\frac{n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^5}{6+n} + \\
& \frac{5A \text{Hypergeometric2F1}\left[\frac{9}{2}+n, \frac{7+n}{2}, \frac{9+n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^6}{7+n} + \\
& \frac{2B \text{Hypergeometric2F1}\left[\frac{9}{2}+n, \frac{7+n}{2}, \frac{9+n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^6}{7+n} + \\
& \frac{A \text{Hypergeometric2F1}\left[4+\frac{n}{2}, \frac{9}{2}+n, 5+\frac{n}{2}, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^7}{8+n}
\end{aligned}$$

■ **Problem 8: Result more than twice size of optimal antiderivative.**

$$\int (d \sin[e+fx])^n (a+a \sin[e+fx])^{3/2} (A+B \sin[e+fx]) dx$$

Optimal (type 5, 229 leaves, 5 steps) :

$$-\left(2 a^2 (2 B (9+13 n+4 n^2)+A (25+30 n+8 n^2)) \operatorname{Cos}[e+f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2},-n,\frac{3}{2},1-\operatorname{Sin}[e+f x]\right] \operatorname{Sin}[e+f x]^{-n} (d \operatorname{Sin}[e+f x])^n\right) /$$

$$\left(f(3+2 n)(5+2 n) \sqrt{a+a \operatorname{Sin}[e+f x]}\right) -$$

$$\frac{2 a^2 (2 B (3+n)+A (5+2 n)) \operatorname{Cos}[e+f x] (d \operatorname{Sin}[e+f x])^{1+n}}{d f(3+2 n)(5+2 n) \sqrt{a+a \operatorname{Sin}[e+f x]}} - \frac{2 a B \operatorname{Cos}[e+f x] (d \operatorname{Sin}[e+f x])^{1+n} \sqrt{a+a \operatorname{Sin}[e+f x]}}{d f(5+2 n)}$$

Result (type 5, 575 leaves) :

$$\frac{1}{f \sqrt{\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^3}} 2^{1+n} \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \operatorname{Sin}[e+f x]^{-n} (d \operatorname{Sin}[e+f x])^n (a(1+\operatorname{Sin}[e+f x]))^{3/2}$$

$$\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}\right)^n \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right)^n \left(\frac{A \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{7}{2}+n, \frac{3+n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right]}{1+n} +\right.$$

$$\frac{(3 A+2 B) \operatorname{Hypergeometric2F1}\left[1+\frac{n}{2}, \frac{7}{2}+n, 2+\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{2+n} +$$

$$\frac{4 A \operatorname{Hypergeometric2F1}\left[\frac{3+n}{2}, \frac{7}{2}+n, \frac{5+n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{3+n} +$$

$$\frac{6 B \operatorname{Hypergeometric2F1}\left[\frac{3+n}{2}, \frac{7}{2}+n, \frac{5+n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{3+n} +$$

$$\frac{2(2 A+3 B) \operatorname{Hypergeometric2F1}\left[2+\frac{n}{2}, \frac{7}{2}+n, 3+\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^3}{4+n} +$$

$$\frac{3 A \operatorname{Hypergeometric2F1}\left[\frac{7}{2}+n, \frac{5+n}{2}, \frac{7+n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^4}{5+n} +$$

$$\frac{2 B \operatorname{Hypergeometric2F1}\left[\frac{7}{2}+n, \frac{5+n}{2}, \frac{7+n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^4}{5+n} +$$

$$\left. \frac{A \operatorname{Hypergeometric2F1}\left[3+\frac{n}{2}, \frac{7}{2}+n, 4+\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^5}{6+n}\right)$$

- **Problem 9: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (d \operatorname{Sin}[e+f x])^n \sqrt{a+a \operatorname{Sin}[e+f x]} (A+B \operatorname{Sin}[e+f x]) dx$$

Optimal (type 5, 137 leaves, 4 steps) :

$$- \left(2a (2B (1+n) + A (3+2n)) \operatorname{Cos}[e+fx] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -n, \frac{3}{2}, 1 - \operatorname{Sin}[e+fx] \right] \operatorname{Sin}[e+fx]^{-n} (d \operatorname{Sin}[e+fx])^n \right) /$$

$$\left(f (3+2n) \sqrt{a+a \operatorname{Sin}[e+fx]} \right) - \frac{2aB \operatorname{Cos}[e+fx] (d \operatorname{Sin}[e+fx])^{1+n}}{df (3+2n) \sqrt{a+a \operatorname{Sin}[e+fx]}}$$

Result (type 5, 409 leaves):

$$- \frac{1}{\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right] + \operatorname{Sin} \left[\frac{1}{2} (e+fx) \right]} (1+i) 2^{-2-n} e^{-\frac{3ie}{2}+ifnx} (1 - e^{2i(e+fx)})^{-n}$$

$$(-i e^{-i(e+fx)} (-1 + e^{2i(e+fx)}))^n \left(\frac{2B e^{-\frac{1}{2}if(3+2n)x} \operatorname{Hypergeometric2F1} \left[\frac{1}{4} (-3-2n), -n, \frac{1}{4} (1-2n), e^{2i(e+fx)} \right]}{f(3+2n)} + \right.$$

$$2e^i e \left(- \frac{i(2A+B) e^{-\frac{1}{2}if(1+2n)x} \operatorname{Hypergeometric2F1} \left[\frac{1}{4} (-1-2n), -n, \frac{1}{4} (3-2n), e^{2i(e+fx)} \right]}{f+2fn} + \frac{1}{f(-3+2n)(-1+2n)} \right.$$

$$e^{\frac{1}{2}i(2e+fx(1-2n)x)} \left(-(2A+B)(-3+2n) \operatorname{Hypergeometric2F1} \left[\frac{1}{4} (1-2n), -n, \frac{1}{4} (5-2n), e^{2i(e+fx)} \right] + iB e^{i(e+fx)} (-1+2n) \right.$$

$$\left. \left. \left. \operatorname{Hypergeometric2F1} \left[\frac{1}{4} (3-2n), -n, \frac{1}{4} (7-2n), e^{2i(e+fx)} \right] \right) \right) \right) \operatorname{Sin}[e+fx]^{-n} (d \operatorname{Sin}[e+fx])^n \sqrt{a(1+\operatorname{Sin}[e+fx])}$$

■ **Problem 10: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Sin}[e+fx])^n (A+B \operatorname{Sin}[e+fx])}{\sqrt{a+a \operatorname{Sin}[e+fx]}} dx$$

Optimal (type 6, 152 leaves, 9 steps):

$$- \frac{1}{f \sqrt{a+a \operatorname{Sin}[e+fx]}} (A-B) \operatorname{AppellF1} \left[\frac{1}{2}, -n, 1, \frac{3}{2}, 1 - \operatorname{Sin}[e+fx], \frac{1}{2} (1 - \operatorname{Sin}[e+fx]) \right] \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^{-n} (d \operatorname{Sin}[e+fx])^n -$$

$$\frac{2B \operatorname{Cos}[e+fx] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -n, \frac{3}{2}, 1 - \operatorname{Sin}[e+fx] \right] \operatorname{Sin}[e+fx]^{-n} (d \operatorname{Sin}[e+fx])^n}{f \sqrt{a+a \operatorname{Sin}[e+fx]}}$$

Result (type 6, 818 leaves):

$$\begin{aligned}
& \frac{1}{f \sqrt{a(1+\sin[ex+fx])}} \operatorname{Sec}[ex+fx] \sin[ex+fx]^{-n} (d \sin[ex+fx])^n \\
& (1+\sin[ex+fx])^2 \left(B \sin[ex+fx]^n \left(\left(4 a \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\sin[ex+fx]), 1+\sin[ex+fx]\right], 1+\sin[ex+fx]\right) \right) / \right. \\
& \left(8 a \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\sin[ex+fx]), 1+\sin[ex+fx]\right] + a \left(-4 n \operatorname{AppellF1}\left[2, \frac{1}{2}, 1-n, 3, \frac{1}{2}(1+\sin[ex+fx]), 1+\right. \right. \\
& \left. \left. \sin[ex+fx]\right] + \operatorname{AppellF1}\left[2, \frac{3}{2}, -n, 3, \frac{1}{2}(1+\sin[ex+fx]), 1+\sin[ex+fx]\right] \right) (1+\sin[ex+fx]) \right) + \\
& \left((-1+2n) \operatorname{AppellF1}\left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\sin[ex+fx]}, \frac{1}{1+\sin[ex+fx]} \right] (-1+\sin[ex+fx]) \right) / \\
& \left((1+2n) \left(2 \left(n \operatorname{AppellF1}\left[\frac{1}{2}-n, -\frac{1}{2}, 1-n, \frac{3}{2}-n, \frac{2}{1+\sin[ex+fx]}, \frac{1}{1+\sin[ex+fx]} \right] + \operatorname{AppellF1}\left[\frac{1}{2}-n, \frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\sin[ex+fx]}, \right. \right. \right. \\
& \left. \left. \frac{1}{1+\sin[ex+fx]} \right] \right) + (-1+2n) \operatorname{AppellF1}\left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\sin[ex+fx]}, \frac{1}{1+\sin[ex+fx]} \right] (1+\sin[ex+fx]) \right) \right) \right) + \\
& A \left(\left(4 \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\sin[ex+fx]), 1+\sin[ex+fx]\right] (-\sin[ex+fx])^{-n} (-\sin[ex+fx]^2)^n \right) / \right. \\
& \left(8 \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\sin[ex+fx]), 1+\sin[ex+fx]\right] - \left(4 n \operatorname{AppellF1}\left[2, \frac{1}{2}, 1-n, 3, \frac{1}{2}(1+\sin[ex+fx]), 1+\sin[ex+fx]\right] - \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[2, \frac{3}{2}, -n, 3, \frac{1}{2}(1+\sin[ex+fx]), 1+\sin[ex+fx]\right] \right) (1+\sin[ex+fx]) \right) - \\
& \left((-1+2n) \operatorname{AppellF1}\left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\sin[ex+fx]}, \frac{1}{1+\sin[ex+fx]} \right] (-1+\sin[ex+fx]) \sin[ex+fx]^n \right) / \\
& \left((1+2n) \left(2 \left(n \operatorname{AppellF1}\left[\frac{1}{2}-n, -\frac{1}{2}, 1-n, \frac{3}{2}-n, \frac{2}{1+\sin[ex+fx]}, \frac{1}{1+\sin[ex+fx]} \right] + \operatorname{AppellF1}\left[\frac{1}{2}-n, \frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\sin[ex+fx]}, \right. \right. \right. \\
& \left. \left. \frac{1}{1+\sin[ex+fx]} \right] \right) + (-1+2n) \operatorname{AppellF1}\left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\sin[ex+fx]}, \frac{1}{1+\sin[ex+fx]} \right] (1+\sin[ex+fx]) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 11: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \sin[ex+fx])^n (A+B \sin[ex+fx])}{(a+a \sin[ex+fx])^{3/2}} dx$$

Optimal (type 6, 226 leaves, 10 steps):

$$\frac{(A-B) \cos[e+fx] (d \sin[e+fx])^{1+n}}{2df(a+a \sin[e+fx])^{3/2}} - \frac{1}{4af\sqrt{a+a \sin[e+fx]}}$$

$$(A-4An+B(3+4n)) \operatorname{AppellF1}\left[\frac{1}{2}, -n, 1, \frac{3}{2}, 1-\sin[e+fx], \frac{1}{2}(1-\sin[e+fx])\right] \cos[e+fx] \sin[e+fx]^{-n} (d \sin[e+fx])^n -$$

$$\frac{1}{2af\sqrt{a+a \sin[e+fx]}} (A-B) (1+2n) \cos[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, 1-\sin[e+fx]\right] \sin[e+fx]^{-n} (d \sin[e+fx])^n$$

Result (type 6, 1568 leaves):

$$\left(B \cos[e+fx] \sin[e+fx]^{1+n} (d \sin[e+fx])^n (1+\sin[e+fx]) \right.$$

$$\left. \left(\frac{-a+a(1+\sin[e+fx])}{a} \right)^{-n} \left(\left(4a \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\sin[e+fx]), 1+\sin[e+fx]\right] (1+\sin[e+fx]) \right) \right) /$$

$$\left(8a \operatorname{AppellF1}\left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\sin[e+fx]), 1+\sin[e+fx]\right] + a \left(-4n \operatorname{AppellF1}\left[2, \frac{1}{2}, 1-n, 3, \frac{1}{2}(1+\sin[e+fx]), 1+\sin[e+fx]\right] + \operatorname{AppellF1}\left[2, \frac{3}{2}, -n, 3, \frac{1}{2}(1+\sin[e+fx]), 1+\sin[e+fx]\right] \right) (1+\sin[e+fx]) \right) -$$

$$\left((-1+2n) \operatorname{AppellF1}\left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]}\right] (1+\sin[e+fx]) (-2a+a(1+\sin[e+fx])) \right) /$$

$$\left((1+2n) \left(2a \left(n \operatorname{AppellF1}\left[\frac{1}{2}-n, -\frac{1}{2}, 1-n, \frac{3}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]}\right] + \operatorname{AppellF1}\left[\frac{1}{2}-n, \frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]}\right] \right) \right. \right.$$

$$\left. \left. + a (-1+2n) \operatorname{AppellF1}\left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]}\right] (1+\sin[e+fx]) \right) \right) +$$

$$\left(2(-3+2n) \operatorname{AppellF1}\left[\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]}\right] (-2a+a(1+\sin[e+fx])) \right) / \left((-1+2n) \right.$$

$$\left. \left(2a \left(n \operatorname{AppellF1}\left[\frac{3}{2}-n, -\frac{1}{2}, 1-n, \frac{5}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]}\right] + \operatorname{AppellF1}\left[\frac{3}{2}-n, \frac{1}{2}, -n, \frac{5}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]}\right] \right) \right. \right.$$

$$\left. \left. + a (-3+2n) \operatorname{AppellF1}\left[\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\sin[e+fx]}, \frac{1}{1+\sin[e+fx]}\right] (1+\sin[e+fx]) \right) \right) \right) /$$

$$\left(2f\sqrt{a(1+\sin[e+fx])} (-a+a(1+\sin[e+fx])) \sqrt{\frac{2a^2(1+\sin[e+fx]) - a^2(1+\sin[e+fx])^2}{a^2}} \right.$$

$$\left. \sqrt{1 - \frac{(-a+a(1+\sin[e+fx]))^2}{a^2}} \right) +$$

$$\left(A \cos[e+fx] (d \sin[e+fx])^n (1+\sin[e+fx]) \left(\frac{-a+a(1+\sin[e+fx])}{a} \right)^{-n} \right)$$

$$\begin{aligned}
& \left(\left(4 a^2 \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), 1 + \sin[e + f x] \right] (-\sin[e + f x])^{-n} (1 + \sin[e + f x]) \left(-\frac{(a - a (1 + \sin[e + f x]))^2}{a^2} \right)^n \right) / \right. \\
& \left(8 a \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), 1 + \sin[e + f x] \right] + a \left(-4 n \operatorname{AppellF1} \left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2} (1 + \sin[e + f x]), \right. \right. \right. \\
& \quad \left. \left. \left. 1 + \sin[e + f x] \right] + \operatorname{AppellF1} \left[2, \frac{3}{2}, -n, 3, \frac{1}{2} (1 + \sin[e + f x]), 1 + \sin[e + f x] \right] \right) (1 + \sin[e + f x]) \right) - \left(a (-1 + 2 n) \right. \\
& \quad \left. \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{1}{1 + \sin[e + f x]} \right] \sin[e + f x]^n (1 + \sin[e + f x]) (-2 a + a (1 + \sin[e + f x])) \right) / \\
& \left((1 + 2 n) \left(2 a \left(n \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{1}{1 + \sin[e + f x]} \right] + \operatorname{AppellF1} \left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{1 + \sin[e + f x]} \right] \right) + a (-1 + 2 n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{1}{1 + \sin[e + f x]} \right] (1 + \sin[e + f x]) \right) \right) - \\
& \left(2 a (-3 + 2 n) \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{1}{1 + \sin[e + f x]} \right] \sin[e + f x]^n (-2 a + a (1 + \sin[e + f x])) \right) / \left((-1 + 2 n) \right. \\
& \quad \left(2 a \left(n \operatorname{AppellF1} \left[\frac{3}{2} - n, -\frac{1}{2}, 1 - n, \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{1}{1 + \sin[e + f x]} \right] + \operatorname{AppellF1} \left[\frac{3}{2} - n, \frac{1}{2}, -n, \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{1 + \sin[e + f x]} \right] \right) + a (-3 + 2 n) \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{1}{1 + \sin[e + f x]} \right] (1 + \sin[e + f x]) \right) \right) \right) / \\
& \left(2 a^2 f \sqrt{a (1 + \sin[e + f x])} \sqrt{\frac{2 a^2 (1 + \sin[e + f x]) - a^2 (1 + \sin[e + f x])^2}{a^2}} \sqrt{1 - \frac{(-a + a (1 + \sin[e + f x]))^2}{a^2}} \right)
\end{aligned}$$

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int (d \sin[e + f x])^n (a + a \sin[e + f x])^m (A + B \sin[e + f x]) dx$$

Optimal (type 6, 221 leaves, 9 steps):

$$\begin{aligned}
& -\frac{1}{f} 2^{\frac{3}{2}+m} B \operatorname{AppellF1} \left[\frac{1}{2}, -n, -\frac{1}{2} - m, \frac{3}{2}, 1 - \sin[e + f x], \frac{1}{2} (1 - \sin[e + f x]) \right] \\
& \quad \cos[e + f x] \sin[e + f x]^{-n} (d \sin[e + f x])^n (1 + \sin[e + f x])^{-\frac{1}{2}-m} (a + a \sin[e + f x])^m - \\
& \quad \frac{1}{f} 2^{\frac{1}{2}+m} (A - B) \operatorname{AppellF1} \left[\frac{1}{2}, -n, \frac{1}{2} - m, \frac{3}{2}, 1 - \sin[e + f x], \frac{1}{2} (1 - \sin[e + f x]) \right] \cos[e + f x] \\
& \quad \sin[e + f x]^{-n} (d \sin[e + f x])^n (1 + \sin[e + f x])^{-\frac{1}{2}-m} (a + a \sin[e + f x])^m
\end{aligned}$$

Result (type 6, 5918 leaves):

$$-\left(\left(6 \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1-2m} \left(\sec \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^{-2m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] (d \sin[e + f x])^n \right)$$

$$\frac{3}{5} (1-n) \operatorname{AppellF1}\left[\frac{5}{2}, 2-n, 2+m+n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2$$

$$\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] + (2+m+n) \left(-\frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1-n, 3+m+n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{3}{5} (3+m+n) \operatorname{AppellF1}\left[\frac{5}{2}, -n, 4+m+n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) \right) \right) /$$

$$\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -n, 2+m+n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] - 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2}, 1-n, 2+m+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + (2+m+n) \operatorname{AppellF1}\left[\frac{3}{2}, -n, 3+m+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) \right) \right)$$

■ **Problem 13: Unable to integrate problem.**

$$\int (d \operatorname{Sin}[e+fx])^n (a - a \operatorname{Sin}[e+fx]) (a + a \operatorname{Sin}[e+fx])^m dx$$

Optimal (type 6, 114 leaves, 4 steps):

$$\left(\operatorname{AppellF1}\left[1+n, -\frac{1}{2}, \frac{1}{2}-m, 2+n, \operatorname{Sin}[e+fx], -\operatorname{Sin}[e+fx]\right] \operatorname{Sec}[e+fx] \right. \\ \left. (d \operatorname{Sin}[e+fx])^{1+n} (1 + \operatorname{Sin}[e+fx])^{\frac{1}{2}-m} (a - a \operatorname{Sin}[e+fx]) (a + a \operatorname{Sin}[e+fx])^m\right) / (df(1+n) \sqrt{1 - \operatorname{Sin}[e+fx]})$$

Result (type 8, 36 leaves):

$$\int (d \operatorname{Sin}[e+fx])^n (a - a \operatorname{Sin}[e+fx]) (a + a \operatorname{Sin}[e+fx])^m dx$$

■ **Problem 14: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sin}[c+dx]^n (a + a \operatorname{Sin}[c+dx])^{-2-n} (-1-n - (-2-n) \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 37 leaves, 1 step):

$$\frac{\operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]^{1+n} (a + a \operatorname{Sin}[c+dx])^{-2-n}}{d}$$

Result (type 3, 107 leaves):

$$-\frac{1}{d} 2^n \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \\ \left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] \left(-\operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{3}{4}(c+dx)\right] \right) \right)^n (1 + \operatorname{Cos}[c+dx] - \operatorname{Sin}[c+dx]) (a(1 + \operatorname{Sin}[c+dx]))^{-2-n}$$

■ **Problem 21: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x]) (A + B \operatorname{Sin}[e + f x])}{c - c \operatorname{Sin}[e + f x]} dx$$

Optimal (type 3, 56 leaves, 4 steps):

$$-\frac{a(A+2B)x}{c} + \frac{aB \operatorname{Cos}[e+fx]}{cf} + \frac{2a(A+B) \operatorname{Cos}[e+fx]}{f(c-c \operatorname{Sin}[e+fx])}$$

Result (type 3, 125 leaves):

$$\left(a \left(- (A+2B)x + \frac{B \operatorname{Cos}[e] \operatorname{Cos}[fx]}{f} - \frac{B \operatorname{Sin}[e] \operatorname{Sin}[fx]}{f} + \frac{4(A+B) \operatorname{Sin}\left[\frac{fx}{2}\right]}{f \left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right)} \right) (1 + \operatorname{Sin}[e+fx]) \right) / \\ \left(c \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right) \right)^2$$

■ **Problem 22: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x]) (A + B \operatorname{Sin}[e + f x])}{(c - c \operatorname{Sin}[e + f x])^2} dx$$

Optimal (type 3, 72 leaves, 4 steps):

$$\frac{aBx}{c^2} - \frac{a(A+7B) \operatorname{Cos}[e+fx]}{3c^2 f (1 - \operatorname{Sin}[e+fx])} + \frac{2a(A+B) \operatorname{Cos}[e+fx]}{3f(c-c \operatorname{Sin}[e+fx])^2}$$

Result (type 3, 160 leaves):

$$-\left(a \left(-9Bfx \operatorname{Cos}\left[\frac{fx}{2}\right] - 6(A+3B) \operatorname{Cos}\left[e + \frac{fx}{2}\right] + 2A \operatorname{Cos}\left[e + \frac{3fx}{2}\right] + 14B \operatorname{Cos}\left[e + \frac{3fx}{2}\right] + 3Bfx \operatorname{Cos}\left[2e + \frac{3fx}{2}\right] + 24B \operatorname{Sin}\left[\frac{fx}{2}\right] + \right. \\ \left. 9Bfx \operatorname{Sin}\left[e + \frac{fx}{2}\right] + 3Bfx \operatorname{Sin}\left[e + \frac{3fx}{2}\right] \right) \right) / \left(6c^2 f \left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right) \right)^3$$

■ **Problem 32: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x])^2 (A + B \operatorname{Sin}[e + f x])}{(c - c \operatorname{Sin}[e + f x])^2} dx$$

Optimal (type 3, 109 leaves, 5 steps):

$$\frac{a^2(A+4B)x}{c^2} - \frac{a^2(A+4B) \operatorname{Cos}[e+fx]}{c^2 f} + \frac{a^2(A+B)c^2 \operatorname{Cos}[e+fx]^5}{3f(c-c \operatorname{Sin}[e+fx])^4} - \frac{2a^2(A+4B) \operatorname{Cos}[e+fx]^3}{3f(c-c \operatorname{Sin}[e+fx])^2}$$

Result (type 3, 238 leaves) :

$$\frac{1}{3 f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 (c - c \sin[e + f x])^2}$$

$$a^2 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(4 (A + B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) + \right.$$

$$3 (A + 4 B) (e + f x) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^3 - 3 B \cos[e + f x] \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^3 +$$

$$\left. 8 (A + B) \sin \left[\frac{1}{2} (e + f x) \right] - 8 (2 A + 5 B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 \sin \left[\frac{1}{2} (e + f x) \right] \right) (1 + \sin[e + f x])^2$$

■ **Problem 33: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^2 (A + B \sin[e + f x])}{(c - c \sin[e + f x])^3} dx$$

Optimal (type 3, 112 leaves, 5 steps) :

$$-\frac{a^2 B x}{c^3} + \frac{a^2 (A + B) c^2 \cos[e + f x]^5}{5 f (c - c \sin[e + f x])^5} - \frac{2 a^2 B \cos[e + f x]^3}{3 f (c - c \sin[e + f x])^3} + \frac{2 a^2 B \cos[e + f x]}{f (c^3 - c^3 \sin[e + f x])}$$

Result (type 3, 278 leaves) :

$$\frac{1}{15 f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 (c - c \sin[e + f x])^3}$$

$$a^2 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(12 (A + B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) - 4 (3 A + 8 B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^3 - \right.$$

$$15 B (e + f x) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^5 + 24 (A + B) \sin \left[\frac{1}{2} (e + f x) \right] - 8 (3 A + 8 B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2$$

$$\left. \sin \left[\frac{1}{2} (e + f x) \right] + 2 (3 A + 43 B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 \sin \left[\frac{1}{2} (e + f x) \right] \right) (1 + \sin[e + f x])^2$$

■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^2 (A + B \sin[e + f x])}{(c - c \sin[e + f x])^4} dx$$

Optimal (type 3, 75 leaves, 3 steps) :

$$\frac{a^2 (A + B) c^2 \cos[e + f x]^5}{7 f (c - c \sin[e + f x])^6} + \frac{a^2 (A - 6 B) c \cos[e + f x]^5}{35 f (c - c \sin[e + f x])^5}$$

Result (type 3, 191 leaves) :

$$\begin{aligned}
& - \left(a^2 \left(-35 (A + 4 B) \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + 7 (2 A + 13 B) \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] + 35 B \operatorname{Cos} \left[\frac{5}{2} (e + f x) \right] + \right. \right. \\
& \quad A \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] - 6 B \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] - 70 A \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + 70 B \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] - 35 A \operatorname{Sin} \left[\frac{3}{2} (e + f x) \right] + \\
& \quad \left. \left. 35 B \operatorname{Sin} \left[\frac{3}{2} (e + f x) \right] + 7 A \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] - 7 B \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] \right) \right) / \left(140 c^4 f \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] - \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) \right)^7
\end{aligned}$$

■ **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x])^2 (A + B \operatorname{Sin}[e + f x])}{(c - c \operatorname{Sin}[e + f x])^5} dx$$

Optimal (type 3, 115 leaves, 4 steps):

$$\frac{a^2 (A + B) c^2 \operatorname{Cos}[e + f x]^5}{9 f (c - c \operatorname{Sin}[e + f x])^7} + \frac{a^2 (2 A - 7 B) c \operatorname{Cos}[e + f x]^5}{63 f (c - c \operatorname{Sin}[e + f x])^6} + \frac{a^2 (2 A - 7 B) \operatorname{Cos}[e + f x]^5}{315 f (c - c \operatorname{Sin}[e + f x])^5}$$

Result (type 3, 261 leaves):

$$\begin{aligned}
& - \frac{1}{2520 c^5 f \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right)^4 (-1 + \operatorname{Sin}[e + f x])^5} \\
& \quad a^2 \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] - \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right) (1 + \operatorname{Sin}[e + f x])^2 \left(315 (2 A + 3 B) \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] - 63 (4 A + 11 B) \operatorname{Cos} \left[\frac{3}{2} (e + f x) \right] - \right. \\
& \quad \left. 315 B \operatorname{Cos} \left[\frac{5}{2} (e + f x) \right] - 18 A \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] + 63 B \operatorname{Cos} \left[\frac{7}{2} (e + f x) \right] + 882 A \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + 63 B \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + \right. \\
& \quad \left. 420 A \operatorname{Sin} \left[\frac{3}{2} (e + f x) \right] + 105 B \operatorname{Sin} \left[\frac{3}{2} (e + f x) \right] - 72 A \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] - 63 B \operatorname{Sin} \left[\frac{5}{2} (e + f x) \right] + 2 A \operatorname{Sin} \left[\frac{9}{2} (e + f x) \right] - 7 B \operatorname{Sin} \left[\frac{9}{2} (e + f x) \right] \right)
\end{aligned}$$

■ **Problem 38: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sin}[e + f x])^3 (A + B \operatorname{Sin}[e + f x]) (c - c \operatorname{Sin}[e + f x])^6 dx$$

Optimal (type 3, 265 leaves, 9 steps):

$$\begin{aligned}
& \frac{11}{256} a^3 (10 A - 3 B) c^6 x + \frac{11 a^3 (10 A - 3 B) c^6 \operatorname{Cos}[e + f x]^7}{560 f} + \frac{11 a^3 (10 A - 3 B) c^6 \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{256 f} + \\
& \frac{11 a^3 (10 A - 3 B) c^6 \operatorname{Cos}[e + f x]^3 \operatorname{Sin}[e + f x]}{384 f} + \frac{11 a^3 (10 A - 3 B) c^6 \operatorname{Cos}[e + f x]^5 \operatorname{Sin}[e + f x]}{480 f} - \frac{a^3 B \operatorname{Cos}[e + f x]^7 (c^2 - c^2 \operatorname{Sin}[e + f x])^3}{10 f} + \\
& \frac{a^3 (10 A - 3 B) \operatorname{Cos}[e + f x]^7 (c^3 - c^3 \operatorname{Sin}[e + f x])^2}{90 f} + \frac{11 a^3 (10 A - 3 B) \operatorname{Cos}[e + f x]^7 (c^6 - c^6 \operatorname{Sin}[e + f x])}{720 f}
\end{aligned}$$

Result (type 3, 1033 leaves):

$$\begin{aligned}
& \frac{11 (10 A - 3 B) (e + f x) (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6}{256 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} + \\
& \frac{(33 A - 19 B) \cos[e + f x] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6}{128 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} + \\
& \frac{(29 A - 15 B) \cos[3 (e + f x)] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6}{192 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} + \\
& \frac{(3 A - B) \cos[5 (e + f x)] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6}{64 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} + \\
& \frac{(9 A + 5 B) \cos[7 (e + f x)] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6}{1792 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} - \\
& \frac{(A - 3 B) \cos[9 (e + f x)] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6}{2304 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} + \\
& \frac{(144 A - 25 B) (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6 \sin[2 (e + f x)]}{512 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} + \\
& \frac{(6 A + 7 B) (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6 \sin[4 (e + f x)]}{256 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} - \\
& \frac{(32 A - 51 B) (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6 \sin[6 (e + f x)]}{3072 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} - \\
& \frac{(6 A - 5 B) (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6 \sin[8 (e + f x)]}{2048 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} - \\
& \frac{B (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^6 \sin[10 (e + f x)]}{5120 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^{12} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6}
\end{aligned}$$

■ **Problem 46: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^3 (A + B \sin[e + f x])}{(c - c \sin[e + f x])^3} dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$-\frac{a^3 (A + 6 B) x}{c^3} + \frac{a^3 (A + 6 B) \cos[e + f x]}{c^3 f} + \frac{a^3 (A + B) c^3 \cos[e + f x]^7}{5 f (c - c \sin[e + f x])^6} - \frac{2 a^3 (A + 6 B) c \cos[e + f x]^5}{15 f (c - c \sin[e + f x])^4} + \frac{2 a^3 (A + 6 B) c^3 \cos[e + f x]^3}{3 f (c^3 - c^3 \sin[e + f x])^2}$$

Result (type 3, 316 leaves) :

$$\frac{1}{15 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c - c \sin[e+fx])^3}$$

$$a^3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(24(A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) - 4(11A+21B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 - \right.$$

$$15(A+6B)(e+fx) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 + 15B \cos[e+fx] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 +$$

$$48(A+B) \sin\left[\frac{1}{2}(e+fx)\right] - 8(11A+21B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] +$$

$$\left. 4(23A+93B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 \sin\left[\frac{1}{2}(e+fx)\right] \right) (1 + \sin[e+fx])^3$$

■ **Problem 47: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e+fx])^3 (A + B \sin[e+fx])}{(c - c \sin[e+fx])^4} dx$$

Optimal (type 3, 151 leaves, 6 steps) :

$$\frac{a^3 B x}{c^4} + \frac{a^3 (A+B) c^3 \cos[e+fx]^7}{7 f (c - c \sin[e+fx])^7} - \frac{2 a^3 B c \cos[e+fx]^5}{5 f (c - c \sin[e+fx])^5} + \frac{2 a^3 B c^2 \cos[e+fx]^3}{3 f (c^2 - c^2 \sin[e+fx])^3} - \frac{2 a^3 B \cos[e+fx]}{f (c^4 - c^4 \sin[e+fx])}$$

Result (type 3, 356 leaves) :

$$\frac{1}{105 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c - c \sin[e+fx])^4} a^3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)$$

$$\left(120(A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) - 12(15A+29B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 + \right.$$

$$2(45A+199B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 + 105B(e+fx) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 + 240(A+B) \sin\left[\frac{1}{2}(e+fx)\right] -$$

$$24(15A+29B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] + 4(45A+199B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4$$

$$\left. \sin\left[\frac{1}{2}(e+fx)\right] - 2(15A+337B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 \sin\left[\frac{1}{2}(e+fx)\right] \right) (1 + \sin[e+fx])^3$$

■ **Problem 48: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e+fx])^3 (A + B \sin[e+fx])}{(c - c \sin[e+fx])^5} dx$$

Optimal (type 3, 77 leaves, 3 steps) :

$$\frac{a^3 (A+B) c^3 \cos[e+fx]^7}{9f(c-c\sin[e+fx])^8} + \frac{a^3 (A-8B) c^2 \cos[e+fx]^7}{63f(c-c\sin[e+fx])^7}$$

Result (type 3, 283 leaves):

$$\frac{1}{504c^5f\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^6(-1+\sin[e+fx])^5}$$

$$a^3\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)(1+\sin[e+fx])^3\left(315(A-B)\cos\left[\frac{1}{2}(e+fx)\right] - 189(A-B)\cos\left[\frac{3}{2}(e+fx)\right] - 63A\cos\left[\frac{5}{2}(e+fx)\right] + 63B\cos\left[\frac{5}{2}(e+fx)\right] + 9A\cos\left[\frac{7}{2}(e+fx)\right] - 9B\cos\left[\frac{7}{2}(e+fx)\right] + 189A\sin\left[\frac{1}{2}(e+fx)\right] + 693B\sin\left[\frac{1}{2}(e+fx)\right] + 105A\sin\left[\frac{3}{2}(e+fx)\right] + 483B\sin\left[\frac{3}{2}(e+fx)\right] - 27A\sin\left[\frac{5}{2}(e+fx)\right] - 225B\sin\left[\frac{5}{2}(e+fx)\right] - 63B\sin\left[\frac{7}{2}(e+fx)\right] - A\sin\left[\frac{9}{2}(e+fx)\right] + 8B\sin\left[\frac{9}{2}(e+fx)\right]\right)$$

■ **Problem 49: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\sin[e+fx])^3(A+B\sin[e+fx])}{(c-c\sin[e+fx])^6} dx$$

Optimal (type 3, 118 leaves, 4 steps):

$$\frac{a^3(A+B)c^3\cos[e+fx]^7}{11f(c-c\sin[e+fx])^9} + \frac{a^3(2A-9B)c^2\cos[e+fx]^7}{99f(c-c\sin[e+fx])^8} + \frac{a^3(2A-9B)c\cos[e+fx]^7}{693f(c-c\sin[e+fx])^7}$$

Result (type 3, 313 leaves):

$$\frac{1}{11088c^6f\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^6(-1+\sin[e+fx])^6}$$

$$a^3\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)(1+\sin[e+fx])^3\left(462(11A+3B)\cos\left[\frac{1}{2}(e+fx)\right] - 594(5A+2B)\cos\left[\frac{3}{2}(e+fx)\right] - 924A\cos\left[\frac{5}{2}(e+fx)\right] - 693B\cos\left[\frac{5}{2}(e+fx)\right] + 110A\cos\left[\frac{7}{2}(e+fx)\right] + 198B\cos\left[\frac{7}{2}(e+fx)\right] - 2A\cos\left[\frac{11}{2}(e+fx)\right] + 9B\cos\left[\frac{11}{2}(e+fx)\right] + 4158A\sin\left[\frac{1}{2}(e+fx)\right] + 5544B\sin\left[\frac{1}{2}(e+fx)\right] + 2310A\sin\left[\frac{3}{2}(e+fx)\right] + 4158B\sin\left[\frac{3}{2}(e+fx)\right] - 594A\sin\left[\frac{5}{2}(e+fx)\right] - 2178B\sin\left[\frac{5}{2}(e+fx)\right] - 693B\sin\left[\frac{7}{2}(e+fx)\right] - 22A\sin\left[\frac{9}{2}(e+fx)\right] + 99B\sin\left[\frac{9}{2}(e+fx)\right]\right)$$

■ **Problem 50: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\sin[e+fx])^3(A+B\sin[e+fx])}{(c-c\sin[e+fx])^7} dx$$

Optimal (type 3, 156 leaves, 5 steps):

$$\frac{a^3 (A+B) c^3 \cos[e+fx]^7}{13 f (c - c \sin[e+fx])^{10}} + \frac{a^3 (3A-10B) c^2 \cos[e+fx]^7}{143 f (c - c \sin[e+fx])^9} + \frac{2 a^3 (3A-10B) c \cos[e+fx]^7}{1287 f (c - c \sin[e+fx])^8} + \frac{2 a^3 (3A-10B) \cos[e+fx]^7}{9009 f (c - c \sin[e+fx])^7}$$

Result (type 3, 352 leaves):

$$\frac{1}{144144 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c - c \sin[e+fx])^7} \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (a + a \sin[e+fx])^3 \left(54054 A \cos\left[\frac{1}{2}(e+fx)\right] + 30030 B \cos\left[\frac{1}{2}(e+fx)\right] - 30888 A \cos\left[\frac{3}{2}(e+fx)\right] - 23166 B \cos\left[\frac{3}{2}(e+fx)\right] - 9009 A \cos\left[\frac{5}{2}(e+fx)\right] - 12012 B \cos\left[\frac{5}{2}(e+fx)\right] + 858 A \cos\left[\frac{7}{2}(e+fx)\right] + 3146 B \cos\left[\frac{7}{2}(e+fx)\right] - 39 A \cos\left[\frac{11}{2}(e+fx)\right] + 130 B \cos\left[\frac{11}{2}(e+fx)\right] + 48906 A \sin\left[\frac{1}{2}(e+fx)\right] + 47190 B \sin\left[\frac{1}{2}(e+fx)\right] + 27027 A \sin\left[\frac{3}{2}(e+fx)\right] + 36036 B \sin\left[\frac{3}{2}(e+fx)\right] - 6864 A \sin\left[\frac{5}{2}(e+fx)\right] - 19162 B \sin\left[\frac{5}{2}(e+fx)\right] - 6006 B \sin\left[\frac{7}{2}(e+fx)\right] - 234 A \sin\left[\frac{9}{2}(e+fx)\right] + 780 B \sin\left[\frac{9}{2}(e+fx)\right] + 3 A \sin\left[\frac{13}{2}(e+fx)\right] - 10 B \sin\left[\frac{13}{2}(e+fx)\right] \right)$$

■ **Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \sin[e+fx]) (c - c \sin[e+fx])}{a + a \sin[e+fx]} dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{(A-2B)cx}{a} + \frac{Bc \cos[e+fx]}{af} - \frac{2(A-B)c \cos[e+fx]}{f(a+a \sin[e+fx])}$$

Result (type 3, 127 leaves):

$$\left(\left(- (A-2B)x + \frac{B \cos[e] \cos[fx]}{f} - \frac{B \sin[e] \sin[fx]}{f} + \frac{4(A-B) \sin\left[\frac{fx}{2}\right]}{f \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)} \right) (c - c \sin[e+fx]) \right) / \left(a \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \right)^2$$

■ **Problem 63: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \sin[e+fx]) (c - c \sin[e+fx])^2}{(a + a \sin[e+fx])^2} dx$$

Optimal (type 3, 108 leaves, 5 steps):

$$\frac{(A-4B)c^2x}{a^2} + \frac{(A-4B)c^2 \cos[e+fx]}{a^2 f} - \frac{a^2(A-B)c^2 \cos[e+fx]^5}{3f(a+a \sin[e+fx])^4} + \frac{2(A-4B)c^2 \cos[e+fx]^3}{3f(a+a \sin[e+fx])^2}$$

Result (type 3, 234 leaves) :

$$\frac{1}{3 a^2 f \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 (1 + \sin [e + f x])^2}$$

$$\left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(8 (A - B) \sin \left[\frac{1}{2} (e + f x) \right] - 4 (A - B) \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) - \right.$$

$$8 (2 A - 5 B) \sin \left[\frac{1}{2} (e + f x) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 + 3 (A - 4 B) (e + f x) \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^3 -$$

$$\left. 3 B \cos [e + f x] \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^3 \right) (c - c \sin [e + f x])^2$$

■ **Problem 64: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin [e + f x]) (c - c \sin [e + f x])}{(a + a \sin [e + f x])^2} dx$$

Optimal (type 3, 72 leaves, 4 steps) :

$$-\frac{B c x}{a^2} + \frac{(A - 7 B) c \cos [e + f x]}{3 a^2 f (1 + \sin [e + f x])} - \frac{2 (A - B) c \cos [e + f x]}{3 f (a + a \sin [e + f x])^2}$$

Result (type 3, 156 leaves) :

$$\left(c \left(-9 B f x \cos \left[\frac{f x}{2} \right] - 6 (A - 3 B) \cos \left[e + \frac{f x}{2} \right] + 2 A \cos \left[e + \frac{3 f x}{2} \right] - 14 B \cos \left[e + \frac{3 f x}{2} \right] + 3 B f x \cos \left[2 e + \frac{3 f x}{2} \right] + 24 B \sin \left[\frac{f x}{2} \right] - \right.$$

$$\left. 9 B f x \sin \left[e + \frac{f x}{2} \right] - 3 B f x \sin \left[e + \frac{3 f x}{2} \right] \right) \left/ \left(6 a^2 f \left(\cos \left[\frac{e}{2} \right] + \sin \left[\frac{e}{2} \right] \right) \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^3 \right) \right.$$

■ **Problem 67: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin [e + f x]}{(a + a \sin [e + f x])^2 (c - c \sin [e + f x])^3} dx$$

Optimal (type 3, 93 leaves, 4 steps) :

$$\frac{(A + B) \sec [e + f x]^3}{5 a^2 f (c^3 - c^3 \sin [e + f x])} + \frac{(4 A - B) \tan [e + f x]}{5 a^2 c^3 f} + \frac{(4 A - B) \tan [e + f x]^3}{15 a^2 c^3 f}$$

Result (type 3, 237 leaves) :

$$\frac{1}{960 a^2 c^3 f (-1 + \sin [e + f x])^3 (1 + \sin [e + f x])^2}$$

$$\left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(-240 B + 54 (A + B) \cos [e + f x] - 32 (4 A - B) \cos [2 (e + f x)] + \right.$$

$$18 A \cos [3 (e + f x)] + 18 B \cos [3 (e + f x)] - 64 A \cos [4 (e + f x)] + 16 B \cos [4 (e + f x)] - 384 A \sin [e + f x] + 96 B \sin [e + f x] -$$

$$\left. 18 A \sin [2 (e + f x)] - 18 B \sin [2 (e + f x)] - 128 A \sin [3 (e + f x)] + 32 B \sin [3 (e + f x)] - 9 A \sin [4 (e + f x)] - 9 B \sin [4 (e + f x)] \right)$$

■ **Problem 68: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^2 (c - c \sin[e + f x])^4} dx$$

Optimal (type 3, 135 leaves, 5 steps):

$$\frac{(A + B) \operatorname{Sec}[e + f x]^3}{7 a^2 f (c^2 - c^2 \sin[e + f x])^2} + \frac{(5 A - 2 B) \operatorname{Sec}[e + f x]^3}{35 a^2 f (c^4 - c^4 \sin[e + f x])} + \frac{4 (5 A - 2 B) \operatorname{Tan}[e + f x]}{35 a^2 c^4 f} + \frac{4 (5 A - 2 B) \operatorname{Tan}[e + f x]^3}{105 a^2 c^4 f}$$

Result (type 3, 285 leaves):

$$\frac{1}{13440 a^2 c^4 f (-1 + \sin[e + f x])^4 (1 + \sin[e + f x])^2} \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(-2688 B + 42 (25 A + 4 B) \cos[e + f x] - 512 (5 A - 2 B) \cos[2(e + f x)] + 225 A \cos[3(e + f x)] + 36 B \cos[3(e + f x)] - 1280 A \cos[4(e + f x)] + 512 B \cos[4(e + f x)] - 75 A \cos[5(e + f x)] - 12 B \cos[5(e + f x)] - 4480 A \sin[e + f x] + 1792 B \sin[e + f x] - 600 A \sin[2(e + f x)] - 96 B \sin[2(e + f x)] - 960 A \sin[3(e + f x)] + 384 B \sin[3(e + f x)] - 300 A \sin[4(e + f x)] - 48 B \sin[4(e + f x)] + 320 A \sin[5(e + f x)] - 128 B \sin[5(e + f x)] \right)$$

■ **Problem 72: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c - c \sin[e + f x])^3}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$-\frac{(A - 6 B) c^3 x}{a^3} - \frac{(A - 6 B) c^3 \cos[e + f x]}{a^3 f} - \frac{a^3 (A - B) c^3 \cos[e + f x]^7}{5 f (a + a \sin[e + f x])^6} + \frac{2 a (A - 6 B) c^3 \cos[e + f x]^5}{15 f (a + a \sin[e + f x])^4} - \frac{2 a^3 (A - 6 B) c^3 \cos[e + f x]^3}{3 f (a^3 + a^3 \sin[e + f x])^2}$$

Result (type 3, 308 leaves):

$$\frac{1}{15 a^3 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^6 (1 + \sin[e + f x])^3} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(48 (A - B) \sin\left[\frac{1}{2}(e + f x)\right] - 24 (A - B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) - 8 (11 A - 21 B) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + 4 (11 A - 21 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 + 4 (23 A - 93 B) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 - 15 (A - 6 B) (e + f x) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 + 15 B \cos[e + f x] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \right) (c - c \sin[e + f x])^3$$

■ **Problem 73: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c - c \sin[e + f x])^2}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 3, 110 leaves, 5 steps):

$$\frac{B c^2 x}{a^3} - \frac{a^2 (A - B) c^2 \cos[e + f x]^5}{5 f (a + a \sin[e + f x])^5} - \frac{2 B c^2 \cos[e + f x]^3}{3 f (a + a \sin[e + f x])^3} + \frac{2 B c^2 \cos[e + f x]}{f (a^3 + a^3 \sin[e + f x])}$$

Result (type 3, 272 leaves):

$$\frac{1}{15 a^3 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 (1 + \sin[e + f x])^3} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(24 (A - B) \sin\left[\frac{1}{2}(e + f x)\right] - 12 (A - B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) - 8 (3 A - 8 B) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + 4 (3 A - 8 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 + 2 (3 A - 43 B) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 + 15 B (e + f x) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 \right) (c - c \sin[e + f x])^2$$

■ **Problem 76: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^3 (c - c \sin[e + f x])^2} dx$$

Optimal (type 3, 90 leaves, 4 steps):

$$-\frac{(A - B) \sec[e + f x]^3}{5 c^2 f (a^3 + a^3 \sin[e + f x])} + \frac{(4 A + B) \tan[e + f x]}{5 a^3 c^2 f} + \frac{(4 A + B) \tan[e + f x]^3}{15 a^3 c^2 f}$$

Result (type 3, 237 leaves):

$$\frac{1}{960 a^3 c^2 f (-1 + \sin[e + f x])^2 (1 + \sin[e + f x])^3} \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(240 B + 54 (A - B) \cos[e + f x] - 32 (4 A + B) \cos[2(e + f x)] + 18 A \cos[3(e + f x)] - 18 B \cos[3(e + f x)] - 64 A \cos[4(e + f x)] - 16 B \cos[4(e + f x)] + 384 A \sin[e + f x] + 96 B \sin[e + f x] + 18 A \sin[2(e + f x)] - 18 B \sin[2(e + f x)] + 128 A \sin[3(e + f x)] + 32 B \sin[3(e + f x)] + 9 A \sin[4(e + f x)] - 9 B \sin[4(e + f x)] \right)$$

■ **Problem 78: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^3 (c - c \sin[e + f x])^4} dx$$

Optimal (type 3, 121 leaves, 4 steps):

$$\frac{(A+B) \operatorname{Sec}[e+fx]^5}{7a^3 f (c^4 - c^4 \operatorname{Sin}[e+fx])} + \frac{(6A-B) \operatorname{Tan}[e+fx]}{7a^3 c^4 f} + \frac{2(6A-B) \operatorname{Tan}[e+fx]^3}{21a^3 c^4 f} + \frac{(6A-B) \operatorname{Tan}[e+fx]^5}{35a^3 c^4 f}$$

Result (type 3, 325 leaves):

$$\frac{1}{53760a^3 c^4 f (-1 + \operatorname{Sin}[e+fx])^4 (1 + \operatorname{Sin}[e+fx])^3} \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right) \\ (-8960B + 1500(A+B) \operatorname{Cos}[e+fx] - 640(6A-B) \operatorname{Cos}[2(e+fx)] + 750A \operatorname{Cos}[3(e+fx)] + 750B \operatorname{Cos}[3(e+fx)] - 3072A \operatorname{Cos}[4(e+fx)] + \\ 512B \operatorname{Cos}[4(e+fx)] + 150A \operatorname{Cos}[5(e+fx)] + 150B \operatorname{Cos}[5(e+fx)] - 768A \operatorname{Cos}[6(e+fx)] + 128B \operatorname{Cos}[6(e+fx)] - \\ 15360A \operatorname{Sin}[e+fx] + 2560B \operatorname{Sin}[e+fx] - 375A \operatorname{Sin}[2(e+fx)] - 375B \operatorname{Sin}[2(e+fx)] - 7680A \operatorname{Sin}[3(e+fx)] + 1280B \operatorname{Sin}[3(e+fx)] - \\ 300A \operatorname{Sin}[4(e+fx)] - 300B \operatorname{Sin}[4(e+fx)] - 1536A \operatorname{Sin}[5(e+fx)] + 256B \operatorname{Sin}[5(e+fx)] - 75A \operatorname{Sin}[6(e+fx)] - 75B \operatorname{Sin}[6(e+fx)])$$

■ **Problem 79: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sin}[e+fx]}{(a + a \operatorname{Sin}[e+fx])^3 (c - c \operatorname{Sin}[e+fx])^5} dx$$

Optimal (type 3, 162 leaves, 5 steps):

$$\frac{(A+B) \operatorname{Sec}[e+fx]^5}{9a^3 c^3 f (c - c \operatorname{Sin}[e+fx])^2} + \frac{(7A-2B) \operatorname{Sec}[e+fx]^5}{63a^3 f (c^5 - c^5 \operatorname{Sin}[e+fx])} + \frac{2(7A-2B) \operatorname{Tan}[e+fx]}{21a^3 c^5 f} + \frac{4(7A-2B) \operatorname{Tan}[e+fx]^3}{63a^3 c^5 f} + \frac{2(7A-2B) \operatorname{Tan}[e+fx]^5}{105a^3 c^5 f}$$

Result (type 3, 373 leaves):

$$\frac{1}{1290240a^3 c^5 f (-1 + \operatorname{Sin}[e+fx])^5 (1 + \operatorname{Sin}[e+fx])^3} \\ \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \right) (-184320B + 1125(49A + 13B) \operatorname{Cos}[e+fx] - \\ 20480(7A-2B) \operatorname{Cos}[2(e+fx)] + 23275A \operatorname{Cos}[3(e+fx)] + 6175B \operatorname{Cos}[3(e+fx)] - 114688A \operatorname{Cos}[4(e+fx)] + 32768B \operatorname{Cos}[4(e+fx)] + \\ 1225A \operatorname{Cos}[5(e+fx)] + 325B \operatorname{Cos}[5(e+fx)] - 28672A \operatorname{Cos}[6(e+fx)] + 8192B \operatorname{Cos}[6(e+fx)] - 1225A \operatorname{Cos}[7(e+fx)] - \\ 325B \operatorname{Cos}[7(e+fx)] - 322560A \operatorname{Sin}[e+fx] + 92160B \operatorname{Sin}[e+fx] - 24500A \operatorname{Sin}[2(e+fx)] - 6500B \operatorname{Sin}[2(e+fx)] - \\ 136192A \operatorname{Sin}[3(e+fx)] + 38912B \operatorname{Sin}[3(e+fx)] - 19600A \operatorname{Sin}[4(e+fx)] - 5200B \operatorname{Sin}[4(e+fx)] - 7168A \operatorname{Sin}[5(e+fx)] + \\ 2048B \operatorname{Sin}[5(e+fx)] - 4900A \operatorname{Sin}[6(e+fx)] - 1300B \operatorname{Sin}[6(e+fx)] + 7168A \operatorname{Sin}[7(e+fx)] - 2048B \operatorname{Sin}[7(e+fx)])$$

■ **Problem 85: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sin}[e+fx]) (A + B \operatorname{Sin}[e+fx])}{\sqrt{c - c \operatorname{Sin}[e+fx]}} dx$$

Optimal (type 3, 122 leaves, 5 steps):

$$\frac{2\sqrt{2} a (A+B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \operatorname{Cos}[e+fx]}{\sqrt{2} \sqrt{c-c \operatorname{Sin}[e+fx]}}\right]}{\sqrt{c} f} - \frac{2a(3A+5B) \operatorname{Cos}[e+fx]}{3f \sqrt{c-c \operatorname{Sin}[e+fx]}} + \frac{2aB \operatorname{Cos}[e+fx] \sqrt{c-c \operatorname{Sin}[e+fx]}}{3cf}$$

Result (type 3, 200 leaves):

$$- \left(a \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(3(2A+3B)\sqrt{c} - B\sqrt{c} \cos[2(e+fx)] + \right. \right. \\ \left. \left. 2(3A+5B)\sqrt{c} \sin[e+fx] - 6i\sqrt{2}(A+B) \operatorname{Log}\left[\frac{2(-i\sqrt{2}\sqrt{c} + \sqrt{-c(1+\sin[e+fx])})}{\sqrt{c-c\sin[e+fx]}} \right] \sqrt{-c(1+\sin[e+fx])} \right) \right) / \\ \left(3\sqrt{c}f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c-c\sin[e+fx]} \right)$$

■ **Problem 86: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+a\sin[e+fx])(A+B\sin[e+fx])}{(c-c\sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 115 leaves, 5 steps):

$$- \frac{a(A+5B) \operatorname{ArcTanh}\left[\frac{\sqrt{c}\cos[e+fx]}{\sqrt{2}\sqrt{c-c\sin[e+fx]}}\right]}{\sqrt{2}c^{3/2}f} + \frac{a(A+B)\cos[e+fx]}{f(c-c\sin[e+fx])^{3/2}} + \frac{2aB\cos[e+fx]}{cf\sqrt{c-c\sin[e+fx]}}$$

Result (type 3, 218 leaves):

$$\left(a(-1+\sin[e+fx])(1+\sin[e+fx]) \left(i\sqrt{2}(A+5B) \operatorname{Log}\left[\frac{2(-i\sqrt{2}\sqrt{c} + \sqrt{-c(1+\sin[e+fx])})}{\sqrt{c-c\sin[e+fx]}} \right] \operatorname{Sec}[e+fx] \sqrt{-c(1+\sin[e+fx])} - \right. \right. \\ \left. \left. \frac{2\sqrt{c}(\cos[\frac{1}{2}(e+fx)] + \sin[\frac{1}{2}(e+fx)])(A+3B-2B\sin[e+fx])}{(\cos[\frac{1}{2}(e+fx)] - \sin[\frac{1}{2}(e+fx)])^3} \right) \right) / \\ \left(2c^{3/2}f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sqrt{c-c\sin[e+fx]} \right)$$

■ **Problem 87: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+a\sin[e+fx])(A+B\sin[e+fx])}{(c-c\sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$- \frac{a(A-7B) \operatorname{ArcTanh}\left[\frac{\sqrt{c}\cos[e+fx]}{\sqrt{2}\sqrt{c-c\sin[e+fx]}}\right]}{8\sqrt{2}c^{5/2}f} + \frac{a(A+B)\cos[e+fx]}{2f(c-c\sin[e+fx])^{5/2}} - \frac{a(A+9B)\cos[e+fx]}{8cf(c-c\sin[e+fx])^{3/2}}$$

Result (type 3, 223 leaves):

$$\left(a (-1 + \sin[e + f x]) (1 + \sin[e + f x]) \left(i \sqrt{2} (A - 7 B) \operatorname{Log} \left[\frac{2 \left(-i \sqrt{2} \sqrt{c} + \sqrt{-c (1 + \sin[e + f x])} \right)}{\sqrt{c - c \sin[e + f x]}} \right] \operatorname{Sec}[e + f x] \sqrt{-c (1 + \sin[e + f x])} - \frac{2 \sqrt{c} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) (3 A - 5 B + (A + 9 B) \sin[e + f x])}{\left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^5} \right) \right) / \left(16 c^{5/2} f \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 \sqrt{c - c \sin[e + f x]} \right)$$

- **Problem 88: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x]) (A + B \sin[e + f x])}{(c - c \sin[e + f x])^{7/2}} dx$$

Optimal (type 3, 163 leaves, 6 steps):

$$- \frac{a (A - 3 B) \operatorname{ArcTanh} \left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}} \right]}{32 \sqrt{2} c^{7/2} f} + \frac{a (A + B) \cos[e + f x]}{3 f (c - c \sin[e + f x])^{7/2}} - \frac{a (A + 13 B) \cos[e + f x]}{24 c f (c - c \sin[e + f x])^{5/2}} - \frac{a (A - 3 B) \cos[e + f x]}{32 c^2 f (c - c \sin[e + f x])^{3/2}}$$

Result (type 3, 796 leaves):

$$\begin{aligned}
& a \left(\left(i (A - 3B) \operatorname{Cos}[e + f x] \operatorname{Log} \left[\frac{2 \left(-i \sqrt{2} \sqrt{c} + \sqrt{-2c - c(-1 + \operatorname{Sin}[e + f x])} \right)}{\sqrt{-c(-1 + \operatorname{Sin}[e + f x])}} \right] \right. \right. \\
& \left. \left. \sqrt{-2c - c(-1 + \operatorname{Sin}[e + f x])} (-1 + \operatorname{Sin}[e + f x]) (1 + \operatorname{Sin}[e + f x]) \right) / \left(32 \sqrt{2} c^{7/2} f \left(\operatorname{Cos} \left[\frac{e}{2} + \frac{f x}{2} \right] + \operatorname{Sin} \left[\frac{e}{2} + \frac{f x}{2} \right] \right)^2 \right. \right. \\
& \left. \left. \sqrt{1 - \frac{(c + c(-1 + \operatorname{Sin}[e + f x]))^2}{c^2}} \sqrt{\frac{-2c^2(-1 + \operatorname{Sin}[e + f x]) - c^2(-1 + \operatorname{Sin}[e + f x])^2}{c^2}} \sqrt{-c(-1 + \operatorname{Sin}[e + f x])} \right) + \right. \\
& \left. \frac{1}{\left(\operatorname{Cos} \left[\frac{e}{2} + \frac{f x}{2} \right] + \operatorname{Sin} \left[\frac{e}{2} + \frac{f x}{2} \right] \right)^2} \left(\frac{2 \left(A \operatorname{Sin} \left[\frac{f x}{2} \right] + B \operatorname{Sin} \left[\frac{f x}{2} \right] \right)}{3 c^4 f \left(\operatorname{Cos} \left[\frac{e}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} \right] \right) \left(\operatorname{Cos} \left[\frac{e}{2} + \frac{f x}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} + \frac{f x}{2} \right] \right)^7} + \right. \\
& \frac{A \operatorname{Cos} \left[\frac{e}{2} \right] + B \operatorname{Cos} \left[\frac{e}{2} \right] + A \operatorname{Sin} \left[\frac{e}{2} \right] + B \operatorname{Sin} \left[\frac{e}{2} \right]}{3 c^4 f \left(\operatorname{Cos} \left[\frac{e}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} \right] \right) \left(\operatorname{Cos} \left[\frac{e}{2} + \frac{f x}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} + \frac{f x}{2} \right] \right)^6} + \frac{-A \operatorname{Sin} \left[\frac{f x}{2} \right] - 13 B \operatorname{Sin} \left[\frac{f x}{2} \right]}{12 c^4 f \left(\operatorname{Cos} \left[\frac{e}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} \right] \right) \left(\operatorname{Cos} \left[\frac{e}{2} + \frac{f x}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} + \frac{f x}{2} \right] \right)^5} + \\
& \frac{-A \operatorname{Cos} \left[\frac{e}{2} \right] - 13 B \operatorname{Cos} \left[\frac{e}{2} \right] - A \operatorname{Sin} \left[\frac{e}{2} \right] - 13 B \operatorname{Sin} \left[\frac{e}{2} \right]}{24 c^4 f \left(\operatorname{Cos} \left[\frac{e}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} \right] \right) \left(\operatorname{Cos} \left[\frac{e}{2} + \frac{f x}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} + \frac{f x}{2} \right] \right)^4} + \frac{-A \operatorname{Sin} \left[\frac{f x}{2} \right] + 3 B \operatorname{Sin} \left[\frac{f x}{2} \right]}{16 c^4 f \left(\operatorname{Cos} \left[\frac{e}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} \right] \right) \left(\operatorname{Cos} \left[\frac{e}{2} + \frac{f x}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} + \frac{f x}{2} \right] \right)^3} + \\
& \left. \frac{-A \operatorname{Cos} \left[\frac{e}{2} \right] + 3 B \operatorname{Cos} \left[\frac{e}{2} \right] - A \operatorname{Sin} \left[\frac{e}{2} \right] + 3 B \operatorname{Sin} \left[\frac{e}{2} \right]}{32 c^4 f \left(\operatorname{Cos} \left[\frac{e}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} \right] \right) \left(\operatorname{Cos} \left[\frac{e}{2} + \frac{f x}{2} \right] - \operatorname{Sin} \left[\frac{e}{2} + \frac{f x}{2} \right] \right)^2} \right) (1 + \operatorname{Sin}[e + f x]) \sqrt{c - c \operatorname{Sin}[e + f x]} \Big)
\end{aligned}$$

■ **Problem 89: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sin}[e + f x])^2 (A + B \operatorname{Sin}[e + f x]) (c - c \operatorname{Sin}[e + f x])^{7/2} dx$$

Optimal (type 3, 210 leaves, 6 steps):

$$\begin{aligned}
& \frac{256 a^2 (13 A - 3 B) c^6 \operatorname{Cos}[e + f x]^5}{15015 f (c - c \operatorname{Sin}[e + f x])^{5/2}} + \frac{64 a^2 (13 A - 3 B) c^5 \operatorname{Cos}[e + f x]^5}{3003 f (c - c \operatorname{Sin}[e + f x])^{3/2}} + \frac{8 a^2 (13 A - 3 B) c^4 \operatorname{Cos}[e + f x]^5}{429 f \sqrt{c - c \operatorname{Sin}[e + f x]}} + \\
& \frac{2 a^2 (13 A - 3 B) c^3 \operatorname{Cos}[e + f x]^5 \sqrt{c - c \operatorname{Sin}[e + f x]}}{143 f} - \frac{2 a^2 B c^2 \operatorname{Cos}[e + f x]^5 (c - c \operatorname{Sin}[e + f x])^{3/2}}{13 f}
\end{aligned}$$

Result (type 3, 1355 leaves):

$$\frac{(7 A - 2 B) \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] (a + a \operatorname{Sin}[e + f x])^2 (c - c \operatorname{Sin}[e + f x])^{7/2}}{8 f \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] - \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right)^7 \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] \right)^4} -$$

$$\begin{aligned}
& \frac{(4A+B) \cos\left[\frac{3}{2}(e+fx)\right] (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2}}{32f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} + \\
& \frac{(22A-7B) \cos\left[\frac{5}{2}(e+fx)\right] (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2}}{160f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} + \\
& \frac{(A-4B) \cos\left[\frac{7}{2}(e+fx)\right] (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2}}{112f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} + \\
& \frac{A \cos\left[\frac{9}{2}(e+fx)\right] (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2}}{48f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} + \\
& \frac{(2A-3B) \cos\left[\frac{11}{2}(e+fx)\right] (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2}}{352f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} + \\
& \frac{B \cos\left[\frac{13}{2}(e+fx)\right] (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2}}{416f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} + \\
& \frac{(7A-2B) \sin\left[\frac{1}{2}(e+fx)\right] (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2}}{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} + \\
& \frac{(4A+B) (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2} \sin\left[\frac{3}{2}(e+fx)\right]}{32f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} + \\
& \frac{(22A-7B) (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2} \sin\left[\frac{5}{2}(e+fx)\right]}{160f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} - \\
& \frac{(A-4B) (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2} \sin\left[\frac{7}{2}(e+fx)\right]}{112f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} + \\
& \frac{A (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2} \sin\left[\frac{9}{2}(e+fx)\right]}{48f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} - \\
& \frac{(2A-3B) (a+a\sin[e+fx])^2 (c-c\sin[e+fx])^{7/2} \sin\left[\frac{11}{2}(e+fx)\right]}{352f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4} +
\end{aligned}$$

$$\frac{B (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{7/2} \sin\left[\frac{13}{2} (e + f x)\right]}{416 f \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right]\right)^7 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right)^4}$$

■ **Problem 90: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^2 (A + B \sin[e + f x]) (c - c \sin[e + f x])^{5/2} dx$$

Optimal (type 3, 167 leaves, 5 steps):

$$\frac{64 a^2 (11 A - B) c^5 \cos[e + f x]^5}{3465 f (c - c \sin[e + f x])^{5/2}} + \frac{16 a^2 (11 A - B) c^4 \cos[e + f x]^5}{693 f (c - c \sin[e + f x])^{3/2}} + \frac{2 a^2 (11 A - B) c^3 \cos[e + f x]^5}{99 f \sqrt{c - c \sin[e + f x]}} - \frac{2 a^2 B c^2 \cos[e + f x]^5 \sqrt{c - c \sin[e + f x]}}{11 f}$$

Result (type 3, 1173 leaves):

■ **Problem 91: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^2 (A + B \sin[e + f x]) (c - c \sin[e + f x])^{3/2} dx$$

Optimal (type 3, 120 leaves, 4 steps):

$$\frac{8 a^2 (9 A + B) c^4 \cos[e + f x]^5}{315 f (c - c \sin[e + f x])^{5/2}} + \frac{2 a^2 (9 A + B) c^3 \cos[e + f x]^5}{63 f (c - c \sin[e + f x])^{3/2}} - \frac{2 a^2 B c^2 \cos[e + f x]^5}{9 f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 955 leaves):

$$\begin{aligned} & \frac{3 A \cos\left[\frac{1}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \\ & \frac{(3 A + B) \cos\left[\frac{3}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2}}{12 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{(A - B) \cos\left[\frac{5}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2}}{20 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \\ & \frac{(2 A + B) \cos\left[\frac{7}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2}}{56 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \\ & \frac{B \cos\left[\frac{9}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2}}{72 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{3 A \sin\left[\frac{1}{2}(e + f x)\right] (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{(3 A + B) (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2} \sin\left[\frac{3}{2}(e + f x)\right]}{12 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{(A - B) (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2} \sin\left[\frac{5}{2}(e + f x)\right]}{20 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} + \\ & \frac{(2 A + B) (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2} \sin\left[\frac{7}{2}(e + f x)\right]}{56 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} - \\ & \frac{B (a + a \sin[e + f x])^2 (c - c \sin[e + f x])^{3/2} \sin\left[\frac{9}{2}(e + f x)\right]}{72 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^4} \end{aligned}$$

■ **Problem 93: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^2 (A + B \sin[e + f x])}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 161 leaves, 6 steps) :

$$\frac{4 \sqrt{2} a^2 (A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{\sqrt{c} f} - \frac{2 a^2 B c^2 \cos[e + f x]^5}{5 f (c - c \sin[e + f x])^{5/2}} - \frac{2 a^2 (A + B) c \cos[e + f x]^3}{3 f (c - c \sin[e + f x])^{3/2}} - \frac{4 a^2 (A + B) \cos[e + f x]}{f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 175 leaves) :

$$\begin{aligned} & - \left(a^2 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) (1 + \sin[e + f x])^2 \left((120 + 120 i) (-1)^{1/4} (A + B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \right) + \right. \\ & \quad \left. \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) (70 A + 79 B - 3 B \cos[2(e + f x)] + 2(5 A + 11 B) \sin[e + f x]) \right) \Big/ \\ & \quad \left(15 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 \sqrt{c - c \sin[e + f x]} \right) \end{aligned}$$

■ **Problem 94: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^2 (A + B \sin[e + f x])}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 176 leaves, 6 steps) :

$$- \frac{\sqrt{2} a^2 (3 A + 7 B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{c^{3/2} f} + \frac{a^2 (A + B) c^2 \cos[e + f x]^5}{2 f (c - c \sin[e + f x])^{7/2}} + \frac{a^2 (3 A + 7 B) \cos[e + f x]^3}{6 f (c - c \sin[e + f x])^{3/2}} + \frac{a^2 (3 A + 7 B) \cos[e + f x]}{c f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 355 leaves) :

$$\frac{1}{3 f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 (c - c \sin[e + f x])^{3/2}}$$

$$a^2 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) (1 + \sin[e + f x])^2 \left(6 (A + B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) + \right.$$

$$(6 + 6 i) (-1)^{1/4} (3 A + 7 B) \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 +$$

$$3 (2 A + 7 B) \cos \left[\frac{1}{2} (e + f x) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 - B \cos \left[\frac{3}{2} (e + f x) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 +$$

$$12 (A + B) \sin \left[\frac{1}{2} (e + f x) \right] + 3 (2 A + 7 B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 \sin \left[\frac{1}{2} (e + f x) \right] +$$

$$B \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 \sin \left[\frac{3}{2} (e + f x) \right]$$

■ **Problem 95: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^2 (A + B \sin[e + f x])}{(c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 175 leaves, 6 steps):

$$\frac{3 a^2 (A + 9 B) \operatorname{ArcTanh} \left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}} \right]}{4 \sqrt{2} c^{5/2} f} + \frac{a^2 (A + B) c^2 \cos[e + f x]^5}{4 f (c - c \sin[e + f x])^{9/2}} - \frac{a^2 (A + 9 B) \cos[e + f x]^3}{8 f (c - c \sin[e + f x])^{5/2}} - \frac{3 a^2 (A + 9 B) \cos[e + f x]}{8 c^2 f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 344 leaves):

$$\frac{1}{4 f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 (c - c \sin[e + f x])^{5/2}}$$

$$a^2 \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \left(4 (A + B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) - (5 A + 13 B) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \right)^3 -$$

$$(3 + 3 i) (-1)^{1/4} (A + 9 B) \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \left(1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 -$$

$$8 B \cos \left[\frac{1}{2} (e + f x) \right] \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 + 8 (A + B) \sin \left[\frac{1}{2} (e + f x) \right] - 2 (5 A + 13 B)$$

$$\left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2 \sin \left[\frac{1}{2} (e + f x) \right] - 8 B \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^4 \sin \left[\frac{1}{2} (e + f x) \right] (1 + \sin[e + f x])^2$$

■ **Problem 96: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^2 (A + B \sin[e + f x])}{(c - c \sin[e + f x])^{7/2}} dx$$

Optimal (type 3, 175 leaves, 6 steps) :

$$\frac{a^2 (A - 11 B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{16 \sqrt{2} c^{7/2} f} + \frac{a^2 (A+B) c^2 \cos[e+fx]^5}{6 f (c-c \sin[e+fx])^{11/2}} + \frac{a^2 (A-11 B) \cos[e+fx]^3}{24 f (c-c \sin[e+fx])^{7/2}} - \frac{a^2 (A-11 B) \cos[e+fx]}{16 c^2 f (c-c \sin[e+fx])^{3/2}}$$

Result (type 3, 342 leaves) :

$$\frac{1}{48 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 (c-c \sin[e+fx])^{7/2}}$$

$$a^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(32 (A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) - 4 (7A+19 B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \right)^3 +$$

$$3 (A+21 B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 - (3+3i) (-1)^{1/4} (A-11 B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right]$$

$$\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 + 64 (A+B) \sin\left[\frac{1}{2}(e+fx)\right] - 8 (7A+19 B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] +$$

$$6 (A+21 B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 \sin\left[\frac{1}{2}(e+fx)\right] \left(1 + \sin[e+fx]\right)^2$$

■ **Problem 97: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e+fx])^2 (A + B \sin[e+fx])}{(c - c \sin[e+fx])^{9/2}} dx$$

Optimal (type 3, 222 leaves, 7 steps) :

$$\frac{a^2 (3A - 13 B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{256 \sqrt{2} c^{9/2} f} + \frac{a^2 (A+B) c^2 \cos[e+fx]^5}{8 f (c-c \sin[e+fx])^{13/2}} +$$

$$\frac{a^2 (3A - 13 B) \cos[e+fx]^3}{48 f (c-c \sin[e+fx])^{9/2}} - \frac{a^2 (3A - 13 B) \cos[e+fx]}{64 c^2 f (c-c \sin[e+fx])^{5/2}} + \frac{a^2 (3A - 13 B) \cos[e+fx]}{256 c^3 f (c-c \sin[e+fx])^{3/2}}$$

Result (type 3, 357 leaves) :

1

$$\begin{aligned}
& 6144 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 (c - c \sin[e+fx])^{9/2} \\
& a^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (1 + \sin[e+fx])^2 \left(2013 A \cos\left[\frac{1}{2}(e+fx)\right] + 1517 B \cos\left[\frac{1}{2}(e+fx)\right] - 999 A \cos\left[\frac{3}{2}(e+fx)\right] - \right. \\
& \quad 791 B \cos\left[\frac{3}{2}(e+fx)\right] - 69 A \cos\left[\frac{5}{2}(e+fx)\right] - 725 B \cos\left[\frac{5}{2}(e+fx)\right] - 9 A \cos\left[\frac{7}{2}(e+fx)\right] + 39 B \cos\left[\frac{7}{2}(e+fx)\right] - \\
& \quad \left. (24 + 24i) (-1)^{1/4} (3A - 13B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^8 + \\
& 2013 A \sin\left[\frac{1}{2}(e+fx)\right] + 1517 B \sin\left[\frac{1}{2}(e+fx)\right] + 999 A \sin\left[\frac{3}{2}(e+fx)\right] + 791 B \sin\left[\frac{3}{2}(e+fx)\right] - \\
& 69 A \sin\left[\frac{5}{2}(e+fx)\right] - 725 B \sin\left[\frac{5}{2}(e+fx)\right] + 9 A \sin\left[\frac{7}{2}(e+fx)\right] - 39 B \sin\left[\frac{7}{2}(e+fx)\right] \Big)
\end{aligned}$$

■ **Problem 98: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e+fx])^3 (A + B \sin[e+fx]) (c - c \sin[e+fx])^{7/2} dx$$

Optimal (type 3, 210 leaves, 6 steps):

$$\begin{aligned}
& \frac{256 a^3 (15 A - B) c^7 \cos[e+fx]^7}{45 045 f (c - c \sin[e+fx])^{7/2}} + \frac{64 a^3 (15 A - B) c^6 \cos[e+fx]^7}{6435 f (c - c \sin[e+fx])^{5/2}} + \\
& \frac{8 a^3 (15 A - B) c^5 \cos[e+fx]^7}{715 f (c - c \sin[e+fx])^{3/2}} + \frac{2 a^3 (15 A - B) c^4 \cos[e+fx]^7}{195 f \sqrt{c - c \sin[e+fx]}} - \frac{2 a^3 B c^3 \cos[e+fx]^7 \sqrt{c - c \sin[e+fx]}}{15 f}
\end{aligned}$$

Result (type 3, 1569 leaves):

$$\begin{aligned}
& \frac{5 (8 A - B) \cos\left[\frac{1}{2}(e+fx)\right] (a + a \sin[e+fx])^3 (c - c \sin[e+fx])^{7/2}}{64 f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6} - \\
& \frac{5 (6 A + B) \cos\left[\frac{3}{2}(e+fx)\right] (a + a \sin[e+fx])^3 (c - c \sin[e+fx])^{7/2}}{192 f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6} + \\
& \frac{3 (10 A - 3 B) \cos\left[\frac{5}{2}(e+fx)\right] (a + a \sin[e+fx])^3 (c - c \sin[e+fx])^{7/2}}{320 f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6} - \\
& \frac{3 (4 A + 3 B) \cos\left[\frac{7}{2}(e+fx)\right] (a + a \sin[e+fx])^3 (c - c \sin[e+fx])^{7/2}}{448 f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6} +
\end{aligned}$$

$$\begin{aligned}
& \frac{(12A - 5B) \cos\left[\frac{9}{2}(e + fx)\right] (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2}}{576 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} - \\
& \frac{(2A + 5B) \cos\left[\frac{11}{2}(e + fx)\right] (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2}}{704 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} + \\
& \frac{(2A - B) \cos\left[\frac{13}{2}(e + fx)\right] (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2}}{832 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} - \\
& \frac{B \cos\left[\frac{15}{2}(e + fx)\right] (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2}}{960 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} + \\
& \frac{5(8A - B) \sin\left[\frac{1}{2}(e + fx)\right] (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2}}{64 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} + \\
& \frac{5(6A + B) (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2} \sin\left[\frac{3}{2}(e + fx)\right]}{192 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} + \\
& \frac{3(10A - 3B) (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2} \sin\left[\frac{5}{2}(e + fx)\right]}{320 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} + \\
& \frac{3(4A + 3B) (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2} \sin\left[\frac{7}{2}(e + fx)\right]}{448 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} + \\
& \frac{(12A - 5B) (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2} \sin\left[\frac{9}{2}(e + fx)\right]}{576 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} + \\
& \frac{(2A + 5B) (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2} \sin\left[\frac{11}{2}(e + fx)\right]}{704 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} + \\
& \frac{(2A - B) (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2} \sin\left[\frac{13}{2}(e + fx)\right]}{832 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} + \\
& \frac{B (a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{7/2} \sin\left[\frac{15}{2}(e + fx)\right]}{960 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^7 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6}
\end{aligned}$$

■ **Problem 99: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^3 (A + B \sin[e + f x]) (c - c \sin[e + f x])^{5/2} dx$$

Optimal (type 3, 161 leaves, 5 steps):

$$\frac{64 a^3 (13 A + B) c^6 \cos[e + f x]^7}{9009 f (c - c \sin[e + f x])^{7/2}} + \frac{16 a^3 (13 A + B) c^5 \cos[e + f x]^7}{1287 f (c - c \sin[e + f x])^{5/2}} + \frac{2 a^3 (13 A + B) c^4 \cos[e + f x]^7}{143 f (c - c \sin[e + f x])^{3/2}} - \frac{2 a^3 B c^3 \cos[e + f x]^7}{13 f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 1351 leaves):

$$\begin{aligned} & \frac{5 A \cos\left[\frac{1}{2}(e + f x)\right] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}}{8 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} - \\ & \frac{5 (4 A + B) \cos\left[\frac{3}{2}(e + f x)\right] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}}{96 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} + \\ & \frac{(2 A - B) \cos\left[\frac{5}{2}(e + f x)\right] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}}{32 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} - \\ & \frac{(5 A + 2 B) \cos\left[\frac{7}{2}(e + f x)\right] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}}{112 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} + \\ & \frac{(A - 2 B) \cos\left[\frac{9}{2}(e + f x)\right] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}}{144 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} - \\ & \frac{(2 A + B) \cos\left[\frac{11}{2}(e + f x)\right] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}}{352 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} - \\ & \frac{B \cos\left[\frac{13}{2}(e + f x)\right] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}}{416 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} + \\ & \frac{5 A \sin\left[\frac{1}{2}(e + f x)\right] (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}}{8 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} + \\ & \frac{5 (4 A + B) (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2} \sin\left[\frac{3}{2}(e + f x)\right]}{96 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} + \\ & \frac{(2 A - B) (a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2} \sin\left[\frac{5}{2}(e + f x)\right]}{32 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6} + \end{aligned}$$

$$\frac{(5A + 2B)(a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{5/2} \sin\left[\frac{7}{2}(e + fx)\right]}{112f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} +$$

$$\frac{(A - 2B)(a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{5/2} \sin\left[\frac{9}{2}(e + fx)\right]}{144f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} +$$

$$\frac{(2A + B)(a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{5/2} \sin\left[\frac{11}{2}(e + fx)\right]}{352f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6} -$$

$$\frac{B(a + a \sin[e + fx])^3 (c - c \sin[e + fx])^{5/2} \sin\left[\frac{13}{2}(e + fx)\right]}{416f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)^5 \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)^6}$$

■ **Problem 100: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^3 (A + B \sin[e + fx]) (c - c \sin[e + fx])^{3/2} dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$\frac{8a^3(11A + 3B)c^5 \cos[e + fx]^7}{693f(c - c \sin[e + fx])^{7/2}} + \frac{2a^3(11A + 3B)c^4 \cos[e + fx]^7}{99f(c - c \sin[e + fx])^{5/2}} - \frac{2a^3Bc^3 \cos[e + fx]^7}{11f(c - c \sin[e + fx])^{3/2}}$$

Result (type 3, 1157 leaves):

■ **Problem 102: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^3 (A + B \sin[e + f x])}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 200 leaves, 7 steps):

$$\frac{8 \sqrt{2} a^3 (A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{\sqrt{c} f} - \frac{2 a^3 B c^3 \cos[e + f x]^7}{7 f (c - c \sin[e + f x])^{7/2}} - \frac{2 a^3 (A + B) c^2 \cos[e + f x]^5}{5 f (c - c \sin[e + f x])^{5/2}} - \frac{4 a^3 (A + B) c \cos[e + f x]^3}{3 f (c - c \sin[e + f x])^{3/2}} - \frac{8 a^3 (A + B) \cos[e + f x]}{f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 193 leaves):

$$-\frac{1}{420 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^6 \sqrt{c - c \sin[e + f x]}} + a^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right) (1 + \sin[e + f x])^3 \left((6720 + 6720 i) (-1)^{1/4} (A + B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right]\right) - 2 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right) \left(-2086 A - 2236 B + 6(7 A + 22 B) \cos[2(e + f x)] - (448 A + 673 B) \sin[e + f x] + 15 B \sin[3(e + f x)]\right)$$

■ **Problem 103: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^3 (A + B \sin[e + f x])}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 218 leaves, 7 steps):

$$-\frac{2 \sqrt{2} a^3 (5 A + 9 B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{c^{3/2} f} + \frac{a^3 (A + B) c^3 \cos[e + f x]^7}{2 f (c - c \sin[e + f x])^{9/2}} + \frac{a^3 (5 A + 9 B) c \cos[e + f x]^5}{10 f (c - c \sin[e + f x])^{5/2}} + \frac{a^3 (5 A + 9 B) \cos[e + f x]^3}{3 f (c - c \sin[e + f x])^{3/2}} + \frac{2 a^3 (5 A + 9 B) \cos[e + f x]}{c f \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 444 leaves):

1

$$30 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c - c \sin[e+fx])^{3/2}$$

$$\begin{aligned} & a^3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (1 + \sin[e+fx])^3 \left(120(A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) + \right. \\ & \quad \left. (120 + 120i) (-1)^{1/4} (5A+9B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 + \\ & \quad 30(9A+20B) \cos\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 - 5(2A+9B) \cos\left[\frac{3}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 - \\ & \quad 3B \cos\left[\frac{5}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 + 240(A+B) \sin\left[\frac{1}{2}(e+fx)\right] + \\ & \quad 30(9A+20B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] + \\ & \quad \left. 5(2A+9B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{3}{2}(e+fx)\right] - 3B \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{5}{2}(e+fx)\right] \right) \end{aligned}$$

■ **Problem 104: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e+fx])^3 (A + B \sin[e+fx])}{(c - c \sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 225 leaves, 7 steps):

$$\begin{aligned} & \frac{5 a^3 (3 A + 11 B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{2 \sqrt{2} c^{5/2} f} + \frac{a^3 (A+B) c^3 \cos[e+fx]^7}{4 f (c-c \sin[e+fx])^{11/2}} - \\ & \frac{a^3 (3 A + 11 B) c \cos[e+fx]^5}{8 f (c-c \sin[e+fx])^{7/2}} - \frac{5 a^3 (3 A + 11 B) \cos[e+fx]^3}{24 c f (c-c \sin[e+fx])^{3/2}} - \frac{5 a^3 (3 A + 11 B) \cos[e+fx]}{4 c^2 f \sqrt{c-c \sin[e+fx]}} \end{aligned}$$

Result (type 3, 434 leaves):

$$\frac{1}{6 f \left(\cos\left[\frac{1}{2}(e+f x)\right] + \sin\left[\frac{1}{2}(e+f x)\right] \right)^6 (c - c \sin[e+f x])^{5/2}} a^3 \left(\cos\left[\frac{1}{2}(e+f x)\right] - \sin\left[\frac{1}{2}(e+f x)\right] \right)$$

$$(1 + \sin[e+f x])^3 \left(12(A+B) \left(\cos\left[\frac{1}{2}(e+f x)\right] - \sin\left[\frac{1}{2}(e+f x)\right] \right) - 3(9A+17B) \left(\cos\left[\frac{1}{2}(e+f x)\right] - \sin\left[\frac{1}{2}(e+f x)\right] \right)^3 - \right.$$

$$\left. (15+15i)(-1)^{1/4}(3A+11B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4}\left(1 + \tan\left[\frac{1}{4}(e+f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+f x)\right] - \sin\left[\frac{1}{2}(e+f x)\right] \right)^4 - \right.$$

$$6(2A+11B) \cos\left[\frac{1}{2}(e+f x)\right] \left(\cos\left[\frac{1}{2}(e+f x)\right] - \sin\left[\frac{1}{2}(e+f x)\right] \right)^4 + 2B \cos\left[\frac{3}{2}(e+f x)\right] \left(\cos\left[\frac{1}{2}(e+f x)\right] - \sin\left[\frac{1}{2}(e+f x)\right] \right)^4 +$$

$$24(A+B) \sin\left[\frac{1}{2}(e+f x)\right] - 6(9A+17B) \left(\cos\left[\frac{1}{2}(e+f x)\right] - \sin\left[\frac{1}{2}(e+f x)\right] \right)^2 \sin\left[\frac{1}{2}(e+f x)\right] -$$

$$6(2A+11B) \left(\cos\left[\frac{1}{2}(e+f x)\right] - \sin\left[\frac{1}{2}(e+f x)\right] \right)^4 \sin\left[\frac{1}{2}(e+f x)\right] - 2B \left(\cos\left[\frac{1}{2}(e+f x)\right] - \sin\left[\frac{1}{2}(e+f x)\right] \right)^4 \sin\left[\frac{3}{2}(e+f x)\right] \Big)$$

■ **Problem 105: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e+f x])^3 (A + B \sin[e+f x])}{(c - c \sin[e+f x])^{7/2}} dx$$

Optimal (type 3, 217 leaves, 7 steps):

$$-\frac{5 a^3 (A+13 B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+f x]}{\sqrt{2} \sqrt{c-c \sin[e+f x]}}\right]}{8 \sqrt{2} c^{7/2} f} + \frac{a^3 (A+B) c^3 \cos[e+f x]^7}{6 f (c-c \sin[e+f x])^{13/2}} -$$

$$\frac{a^3 (A+13 B) c \cos[e+f x]^5}{24 f (c-c \sin[e+f x])^{9/2}} + \frac{5 a^3 (A+13 B) \cos[e+f x]^3}{48 c f (c-c \sin[e+f x])^{5/2}} + \frac{5 a^3 (A+13 B) \cos[e+f x]}{16 c^3 f \sqrt{c-c \sin[e+f x]}}$$

Result (type 3, 910 leaves):

$$\begin{aligned}
& \frac{4(A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 (a+a\sin[e+fx])^3}{3f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c-c\sin[e+fx])^{7/2}} + \frac{(-13A-25B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 (a+a\sin[e+fx])^3}{6f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c-c\sin[e+fx])^{7/2}} + \\
& \frac{(11A+47B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (a+a\sin[e+fx])^3}{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^6 (c-c\sin[e+fx])^{7/2}} + \\
& \left(\left(\frac{5}{8} + \frac{5i}{8} \right) (-1)^{1/4} (A+13B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \operatorname{Sec}\left[\frac{1}{4}(e+fx) \right] \left(\cos\left[\frac{1}{4}(e+fx) \right] + \sin\left[\frac{1}{4}(e+fx) \right] \right) \right] \right) \\
& \left(\cos\left[\frac{1}{2}(e+fx) \right] - \sin\left[\frac{1}{2}(e+fx) \right] \right)^7 (a+a\sin[e+fx])^3 \Big/ \left(f \left(\cos\left[\frac{1}{2}(e+fx) \right] + \sin\left[\frac{1}{2}(e+fx) \right] \right)^6 (c-c\sin[e+fx])^{7/2} \right) + \\
& \frac{2B \cos\left[\frac{1}{2}(e+fx) \right] \left(\cos\left[\frac{1}{2}(e+fx) \right] - \sin\left[\frac{1}{2}(e+fx) \right] \right)^7 (a+a\sin[e+fx])^3}{f \left(\cos\left[\frac{1}{2}(e+fx) \right] + \sin\left[\frac{1}{2}(e+fx) \right] \right)^6 (c-c\sin[e+fx])^{7/2}} + \\
& \frac{2B \left(\cos\left[\frac{1}{2}(e+fx) \right] - \sin\left[\frac{1}{2}(e+fx) \right] \right)^7 \sin\left[\frac{1}{2}(e+fx) \right] (a+a\sin[e+fx])^3}{f \left(\cos\left[\frac{1}{2}(e+fx) \right] + \sin\left[\frac{1}{2}(e+fx) \right] \right)^6 (c-c\sin[e+fx])^{7/2}} + \\
& \left(\left(\cos\left[\frac{1}{2}(e+fx) \right] - \sin\left[\frac{1}{2}(e+fx) \right] \right)^3 \left(-13A \sin\left[\frac{1}{2}(e+fx) \right] - 25B \sin\left[\frac{1}{2}(e+fx) \right] \right) (a+a\sin[e+fx])^3 \right) \Big/ \\
& \left(3f \left(\cos\left[\frac{1}{2}(e+fx) \right] + \sin\left[\frac{1}{2}(e+fx) \right] \right)^6 (c-c\sin[e+fx])^{7/2} \right) + \\
& \frac{8 \left(\cos\left[\frac{1}{2}(e+fx) \right] - \sin\left[\frac{1}{2}(e+fx) \right] \right) \left(A \sin\left[\frac{1}{2}(e+fx) \right] + B \sin\left[\frac{1}{2}(e+fx) \right] \right) (a+a\sin[e+fx])^3}{3f \left(\cos\left[\frac{1}{2}(e+fx) \right] + \sin\left[\frac{1}{2}(e+fx) \right] \right)^6 (c-c\sin[e+fx])^{7/2}} + \\
& \frac{\left(\cos\left[\frac{1}{2}(e+fx) \right] - \sin\left[\frac{1}{2}(e+fx) \right] \right)^5 \left(11A \sin\left[\frac{1}{2}(e+fx) \right] + 47B \sin\left[\frac{1}{2}(e+fx) \right] \right) (a+a\sin[e+fx])^3}{4f \left(\cos\left[\frac{1}{2}(e+fx) \right] + \sin\left[\frac{1}{2}(e+fx) \right] \right)^6 (c-c\sin[e+fx])^{7/2}}
\end{aligned}$$

■ **Problem 106: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+a\sin[e+fx])^3 (A+B\sin[e+fx])}{(c-c\sin[e+fx])^{9/2}} dx$$

Optimal (type 3, 217 leaves, 7 steps):

$$\begin{aligned}
& - \frac{5a^3 (A-15B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c\sin[e+fx]}} \right]}{128 \sqrt{2} c^{9/2} f} + \frac{a^3 (A+B) c^3 \cos[e+fx]^7}{8f (c-c\sin[e+fx])^{15/2}} + \\
& \frac{a^3 (A-15B) c \cos[e+fx]^5}{48f (c-c\sin[e+fx])^{11/2}} - \frac{5a^3 (A-15B) \cos[e+fx]^3}{192cf (c-c\sin[e+fx])^{7/2}} + \frac{5a^3 (A-15B) \cos[e+fx]}{128c^3 f (c-c\sin[e+fx])^{3/2}}
\end{aligned}$$

Result (type 3, 431 leaves) :

$$\left(\left(\frac{5}{128} + \frac{5i}{128} \right) (-1)^{1/4} (A - 15B) \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \operatorname{Sec} \left[\frac{1}{4} (e + f x) \right] \left(\cos \left[\frac{1}{4} (e + f x) \right] + \sin \left[\frac{1}{4} (e + f x) \right] \right) \right] \right)$$

$$\left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^9 (a + a \sin[e + f x])^3 \Big/ \left(f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right) \right)^6 (c - c \sin[e + f x])^{9/2} +$$

$$\frac{1}{3072 f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^6 (c - c \sin[e + f x])^{9/2}} \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) (a + a \sin[e + f x])^3$$

$$\left(1765 A \cos \left[\frac{1}{2} (e + f x) \right] + 405 B \cos \left[\frac{1}{2} (e + f x) \right] - 895 A \cos \left[\frac{3}{2} (e + f x) \right] - 2703 B \cos \left[\frac{3}{2} (e + f x) \right] - 397 A \cos \left[\frac{5}{2} (e + f x) \right] + 579 B \right.$$

$$\cos \left[\frac{5}{2} (e + f x) \right] + 15 A \cos \left[\frac{7}{2} (e + f x) \right] + 543 B \cos \left[\frac{7}{2} (e + f x) \right] + 1765 A \sin \left[\frac{1}{2} (e + f x) \right] + 405 B \sin \left[\frac{1}{2} (e + f x) \right] + 895 A \sin \left[\frac{3}{2} (e + f x) \right] +$$

$$\left. 2703 B \sin \left[\frac{3}{2} (e + f x) \right] - 397 A \sin \left[\frac{5}{2} (e + f x) \right] + 579 B \sin \left[\frac{5}{2} (e + f x) \right] - 15 A \sin \left[\frac{7}{2} (e + f x) \right] - 543 B \sin \left[\frac{7}{2} (e + f x) \right] \right)$$

■ **Problem 107: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sin[e + f x])^3 (A + B \sin[e + f x])}{(c - c \sin[e + f x])^{11/2}} dx$$

Optimal (type 3, 266 leaves, 8 steps) :

$$-\frac{a^3 (3A - 17B) \operatorname{ArcTanh} \left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}} \right]}{512 \sqrt{2} c^{11/2} f} + \frac{a^3 (A + B) c^3 \cos[e + f x]^7}{10 f (c - c \sin[e + f x])^{17/2}} + \frac{a^3 (3A - 17B) c \cos[e + f x]^5}{80 f (c - c \sin[e + f x])^{13/2}} -$$

$$\frac{a^3 (3A - 17B) \cos[e + f x]^3}{96 c f (c - c \sin[e + f x])^{9/2}} + \frac{a^3 (3A - 17B) \cos[e + f x]}{128 c^3 f (c - c \sin[e + f x])^{5/2}} - \frac{a^3 (3A - 17B) \cos[e + f x]}{512 c^4 f (c - c \sin[e + f x])^{3/2}}$$

Result (type 3, 485 leaves) :

$$\begin{aligned}
& \left(\left(\frac{1}{512} + \frac{i}{512} \right) (-1)^{1/4} (3A - 17B) \operatorname{ArcTan} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \operatorname{Sec} \left[\frac{1}{4} (e + f x) \right] \left(\cos \left[\frac{1}{4} (e + f x) \right] + \sin \left[\frac{1}{4} (e + f x) \right] \right) \right] \right) \\
& \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^{11} (a + a \sin[e + f x])^3 \Big/ \\
& \left(f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^6 (c - c \sin[e + f x])^{11/2} \right) + \left(\left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) (a + a \sin[e + f x])^3 \right. \\
& \left(56370 A \cos \left[\frac{1}{2} (e + f x) \right] + 38970 B \cos \left[\frac{1}{2} (e + f x) \right] - 31140 A \cos \left[\frac{3}{2} (e + f x) \right] - 38580 B \cos \left[\frac{3}{2} (e + f x) \right] - 10404 A \cos \left[\frac{5}{2} (e + f x) \right] - \right. \\
& \left. 12724 B \cos \left[\frac{5}{2} (e + f x) \right] + 435 A \cos \left[\frac{7}{2} (e + f x) \right] + 7775 B \cos \left[\frac{7}{2} (e + f x) \right] - 45 A \cos \left[\frac{9}{2} (e + f x) \right] + 255 B \cos \left[\frac{9}{2} (e + f x) \right] + \right. \\
& \left. 56370 A \sin \left[\frac{1}{2} (e + f x) \right] + 38970 B \sin \left[\frac{1}{2} (e + f x) \right] + 31140 A \sin \left[\frac{3}{2} (e + f x) \right] + 38580 B \sin \left[\frac{3}{2} (e + f x) \right] - 10404 A \sin \left[\frac{5}{2} (e + f x) \right] - \right. \\
& \left. 12724 B \sin \left[\frac{5}{2} (e + f x) \right] - 435 A \sin \left[\frac{7}{2} (e + f x) \right] - 7775 B \sin \left[\frac{7}{2} (e + f x) \right] - 45 A \sin \left[\frac{9}{2} (e + f x) \right] + 255 B \sin \left[\frac{9}{2} (e + f x) \right] \Big) \Big/ \\
& \left(122880 f \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)^6 (c - c \sin[e + f x])^{11/2} \right)
\end{aligned}$$

■ **Problem 108: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c - c \sin[e + f x])^{7/2}}{a + a \sin[e + f x]} dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\begin{aligned}
& - \frac{128 (7A - 9B) c^4 \cos[e + f x]}{35 a f \sqrt{c - c \sin[e + f x]}} - \frac{32 (7A - 9B) c^3 \cos[e + f x] \sqrt{c - c \sin[e + f x]}}{35 a f} - \frac{12 (7A - 9B) c^2 \cos[e + f x] (c - c \sin[e + f x])^{3/2}}{35 a f} \\
& \frac{(7A - 9B) c \cos[e + f x] (c - c \sin[e + f x])^{5/2}}{7 a f} - \frac{(A - B) \operatorname{Sec}[e + f x] (c - c \sin[e + f x])^{9/2}}{a c f}
\end{aligned}$$

Result (type 3, 864 leaves):

$$\begin{aligned}
& \frac{16 (A - B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) (c - c \sin[e + f x])^{7/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a + a \sin[e + f x])} - \\
& \frac{(76 A - 111 B) \cos\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 (c - c \sin[e + f x])^{7/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a + a \sin[e + f x])} - \\
& \frac{(6 A - 13 B) \cos\left[\frac{3}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 (c - c \sin[e + f x])^{7/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a + a \sin[e + f x])} + \\
& \frac{(2 A - 9 B) \cos\left[\frac{5}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 (c - c \sin[e + f x])^{7/2}}{20 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a + a \sin[e + f x])} - \\
& \frac{B \cos\left[\frac{7}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 (c - c \sin[e + f x])^{7/2}}{28 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a + a \sin[e + f x])} - \\
& \frac{(76 A - 111 B) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 (c - c \sin[e + f x])^{7/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a + a \sin[e + f x])} + \\
& \frac{(6 A - 13 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 (c - c \sin[e + f x])^{7/2} \sin\left[\frac{3}{2}(e + f x)\right]}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a + a \sin[e + f x])} + \\
& \frac{(2 A - 9 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 (c - c \sin[e + f x])^{7/2} \sin\left[\frac{5}{2}(e + f x)\right]}{20 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a + a \sin[e + f x])} + \\
& \frac{B \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 (c - c \sin[e + f x])^{7/2} \sin\left[\frac{7}{2}(e + f x)\right]}{28 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a + a \sin[e + f x])}
\end{aligned}$$

■ **Problem 112: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x]) \sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 91 leaves, 4 steps):

$$\frac{(A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{\sqrt{2} a \sqrt{c} f} - \frac{(A - B) \operatorname{Sec}[e + f x] \sqrt{c - c \sin[e + f x]}}{a c f}$$

Result (type 3, 140 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \right. \\ \left. \left(-A+B - (1+i)(-1)^{1/4}(A+B)\operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4}\left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \\ \left(af(1 + \sin[e+fx])\sqrt{c - c\sin[e+fx]} \right)$$

- **Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + fx]}{(a + a \sin[e + fx]) (c - c \sin[e + fx])^{3/2}} dx$$

Optimal (type 3, 136 leaves, 5 steps):

$$\frac{(3A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c\sin[e+fx]}}\right]}{4\sqrt{2} a c^{3/2} f} + \frac{(3A - B) \cos[e + fx]}{4 a f (c - c \sin[e + fx])^{3/2}} - \frac{(A - B) \sec[e + fx]}{a c f \sqrt{c - c \sin[e + fx]}}$$

Result (type 3, 284 leaves):

$$\frac{1}{4 a f (1 + \sin[e + fx]) (c - c \sin[e + fx])^{3/2}} \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right) \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right) \\ \left(2(-A + B) \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^2 + (A + B) \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right) \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right) \right) - \\ (1 + i)(-1)^{1/4} (3A - B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4}\left(1 + \tan\left[\frac{1}{4}(e + fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^2 \\ \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right) + 2(A + B) \sin\left[\frac{1}{2}(e + fx)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)$$

- **Problem 114: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + fx]}{(a + a \sin[e + fx]) (c - c \sin[e + fx])^{5/2}} dx$$

Optimal (type 3, 180 leaves, 6 steps):

$$\frac{3(5A - 3B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c\sin[e+fx]}}\right]}{32\sqrt{2} a c^{5/2} f} + \frac{3(5A - 3B) \cos[e + fx]}{32 a c f (c - c \sin[e + fx])^{3/2}} + \frac{(A + B) \sec[e + fx]}{4 a c f (c - c \sin[e + fx])^{3/2}} - \frac{(5A - 3B) \sec[e + fx]}{8 a c^2 f \sqrt{c - c \sin[e + fx]}}$$

Result (type 3, 404 leaves):

$$\frac{1}{32 a f (1 + \sin[e + f x]) (c - c \sin[e + f x])^{5/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \\
\left(8(-A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 + 4(A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) + \\
(7A - B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) - \\
(3 + 3i) (-1)^{1/4} (5A - 3B) \operatorname{ArcTan}\left[\frac{1}{2} + \frac{i}{2}\right] (-1)^{1/4} \left(1 + \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 \\
\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) + 8(A + B) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) + \\
2(7A - B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \right)$$

■ **Problem 115: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c - c \sin[e + f x])^{9/2}}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 3, 242 leaves, 7 steps):

$$\frac{2048 (7A - 13B) c^4 \operatorname{Sec}[e + f x] \sqrt{c - c \sin[e + f x]}}{105 a^2 f} - \frac{512 (7A - 13B) c^3 \operatorname{Sec}[e + f x] (c - c \sin[e + f x])^{3/2}}{105 a^2 f} - \\
\frac{64 (7A - 13B) c^2 \operatorname{Sec}[e + f x] (c - c \sin[e + f x])^{5/2}}{105 a^2 f} - \frac{16 (7A - 13B) c \operatorname{Sec}[e + f x] (c - c \sin[e + f x])^{7/2}}{105 a^2 f} - \\
\frac{(7A - 13B) \operatorname{Sec}[e + f x] (c - c \sin[e + f x])^{9/2}}{21 a^2 f} - \frac{(A - B) \operatorname{Sec}[e + f x]^3 (c - c \sin[e + f x])^{13/2}}{3 a^2 c^2 f}$$

Result (type 3, 953 leaves):

$$\begin{aligned}
& - \frac{32 (A - B) \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right) (c - c \sin[e + fx])^{9/2}}{3 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2} + \frac{32 (2A - 3B) \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^3 (c - c \sin[e + fx])^{9/2}}{f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2} + \\
& \frac{(164A - 351B) \cos\left[\frac{1}{2}(e + fx)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^4 (c - c \sin[e + fx])^{9/2}}{4 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2} + \\
& \frac{(26A - 83B) \cos\left[\frac{3}{2}(e + fx)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^4 (c - c \sin[e + fx])^{9/2}}{12 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2} - \\
& \frac{(2A - 13B) \cos\left[\frac{5}{2}(e + fx)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^4 (c - c \sin[e + fx])^{9/2}}{20 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2} + \\
& \frac{B \cos\left[\frac{7}{2}(e + fx)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^4 (c - c \sin[e + fx])^{9/2}}{28 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2} + \\
& \frac{(164A - 351B) \sin\left[\frac{1}{2}(e + fx)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^4 (c - c \sin[e + fx])^{9/2}}{4 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2} - \\
& \frac{(26A - 83B) \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^4 (c - c \sin[e + fx])^{9/2} \sin\left[\frac{3}{2}(e + fx)\right]}{12 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2} - \\
& \frac{(2A - 13B) \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^4 (c - c \sin[e + fx])^{9/2} \sin\left[\frac{5}{2}(e + fx)\right]}{20 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2} - \\
& \frac{B \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^4 (c - c \sin[e + fx])^{9/2} \sin\left[\frac{7}{2}(e + fx)\right]}{28 f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^9 (a + a \sin[e + fx])^2}
\end{aligned}$$

■ **Problem 120: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + fx]}{(a + a \sin[e + fx])^2 \sqrt{c - c \sin[e + fx]}} dx$$

Optimal (type 3, 135 leaves, 5 steps):

$$\frac{(A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + fx]}{\sqrt{2} \sqrt{c - c \sin[e + fx]}}\right]}{2 \sqrt{2} a^2 \sqrt{c} f} - \frac{(A + B) \operatorname{Sec}[e + fx] \sqrt{c - c \sin[e + fx]}}{2 a^2 c f} - \frac{(A - B) \operatorname{Sec}[e + fx]^3 (c - c \sin[e + fx])^{3/2}}{3 a^2 c^2 f}$$

Result (type 3, 176 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \right. \\ \left. \left(2(-A+B) - 3(A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 - (3+3i)(-1)^{1/4}(A+B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \right) \right. \\ \left. \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \right) \Big/ \left(6a^2 f (1 + \sin[e+fx])^2 \sqrt{c - c \sin[e+fx]} \right)$$

■ **Problem 121: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + fx]}{(a + a \sin[e + fx])^2 (c - c \sin[e + fx])^{3/2}} dx$$

Optimal (type 3, 175 leaves, 6 steps):

$$\frac{(5A+B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{8\sqrt{2} a^2 c^{3/2} f} + \frac{(5A+B) \cos[e+fx]}{8a^2 f (c - c \sin[e+fx])^{3/2}} - \frac{(5A+B) \sec[e+fx]}{6a^2 c f \sqrt{c - c \sin[e+fx]}} - \frac{(A-B) \sec[e+fx]^3 \sqrt{c - c \sin[e+fx]}}{3a^2 c^2 f}$$

Result (type 3, 300 leaves):

$$\frac{1}{24a^2 f (1 + \sin[e + fx])^2 (c - c \sin[e + fx])^{3/2}} \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right) \\ \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right) \left(-12A \cos[e + fx]^2 + 4(-A + B) \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^2 + \right. \\ \left. 3(A + B) \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right) \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^3 - \right. \\ \left. (3 + 3i)(-1)^{1/4} (5A + B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e + fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^2 \right) \\ \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^3 + 6(A + B) \sin\left[\frac{1}{2}(e + fx)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right] \right)^3 \Big/ \left(6a^2 f (1 + \sin[e + fx])^2 \sqrt{c - c \sin[e + fx]} \right)$$

■ **Problem 122: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + fx]}{(a + a \sin[e + fx])^2 (c - c \sin[e + fx])^{5/2}} dx$$

Optimal (type 3, 225 leaves, 7 steps):

$$\frac{5(7A-B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{64\sqrt{2} a^2 c^{5/2} f} + \frac{5(7A-B) \cos[e+fx]}{64a^2 c f (c - c \sin[e+fx])^{3/2}} + \\ \frac{(7A-B) \sec[e+fx]}{24a^2 c f (c - c \sin[e+fx])^{3/2}} - \frac{5(7A-B) \sec[e+fx]}{48a^2 c^2 f \sqrt{c - c \sin[e+fx]}} - \frac{(A-B) \sec[e+fx]^3}{3a^2 c^2 f \sqrt{c - c \sin[e+fx]}}$$

Result (type 3, 430 leaves) :

$$\frac{1}{192 a^2 f (1 + \sin[e + f x])^2 (c - c \sin[e + f x])^{5/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \\ \left(3 (11 A + 3 B) \cos[e + f x]^3 + 16 (-A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 + 24 (-3 A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 \right. \\ \left. \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + 12 (A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 - \right. \\ \left. (15 + 15 i) (-1)^{1/4} (7 A - B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 \right. \\ \left. \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 + 24 (A + B) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 + \right. \\ \left. 6 (11 A + 3 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \right)$$

■ **Problem 123: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c - c \sin[e + f x])^{9/2}}{(a + a \sin[e + f x])^3} dx$$

Optimal (type 3, 242 leaves, 7 steps) :

$$-\frac{2048 (A - 3 B) c^3 \operatorname{Sec}[e + f x]^3 (c - c \sin[e + f x])^{3/2}}{15 a^3 f} + \frac{512 (A - 3 B) c^2 \operatorname{Sec}[e + f x]^3 (c - c \sin[e + f x])^{5/2}}{5 a^3 f} - \\ \frac{64 (A - 3 B) c \operatorname{Sec}[e + f x]^3 (c - c \sin[e + f x])^{7/2}}{5 a^3 f} - \frac{16 (A - 3 B) \operatorname{Sec}[e + f x]^3 (c - c \sin[e + f x])^{9/2}}{15 a^3 f} - \\ \frac{(A - 3 B) \operatorname{Sec}[e + f x]^3 (c - c \sin[e + f x])^{11/2}}{5 a^3 c f} - \frac{(A - B) \operatorname{Sec}[e + f x]^5 (c - c \sin[e + f x])^{15/2}}{5 a^3 c^3 f}$$

Result (type 3, 840 leaves) :

$$\begin{aligned}
& - \frac{32 (A - B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) (c - c \sin[e + f x])^{9/2}}{5 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a + a \sin[e + f x])^3} + \frac{32 (2 A - 3 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 (c - c \sin[e + f x])^{9/2}}{3 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a + a \sin[e + f x])^3} \\
& - \frac{16 (3 A - 7 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{9/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a + a \sin[e + f x])^3} \\
& - \frac{(15 A - 56 B) \cos\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^6 (c - c \sin[e + f x])^{9/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a + a \sin[e + f x])^3} \\
& - \frac{(2 A - 15 B) \cos\left[\frac{3}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^6 (c - c \sin[e + f x])^{9/2}}{6 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a + a \sin[e + f x])^3} \\
& - \frac{B \cos\left[\frac{5}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^6 (c - c \sin[e + f x])^{9/2}}{10 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a + a \sin[e + f x])^3} \\
& + \frac{(15 A - 56 B) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^6 (c - c \sin[e + f x])^{9/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a + a \sin[e + f x])^3} \\
& - \frac{(2 A - 15 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^6 (c - c \sin[e + f x])^{9/2} \sin\left[\frac{3}{2}(e + f x)\right]}{6 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a + a \sin[e + f x])^3} \\
& - \frac{B \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^6 (c - c \sin[e + f x])^{9/2} \sin\left[\frac{5}{2}(e + f x)\right]}{10 f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a + a \sin[e + f x])^3}
\end{aligned}$$

■ **Problem 128: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^3 \sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 3, 174 leaves, 6 steps):

$$\frac{(A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{4 \sqrt{2} a^3 \sqrt{c} f} - \frac{(A + B) \operatorname{Sec}[e + f x] \sqrt{c - c \sin[e + f x]}}{4 a^3 c f} - \frac{(A + B) \operatorname{Sec}[e + f x]^3 (c - c \sin[e + f x])^{3/2}}{6 a^3 c^2 f} - \frac{(A - B) \operatorname{Sec}[e + f x]^5 (c - c \sin[e + f x])^{5/2}}{5 a^3 c^3 f}$$

Result (type 3, 204 leaves):

$$\frac{1}{60 a^3 f (1 + \sin[e + f x])^3 \sqrt{c - c \sin[e + f x]}} \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \\ \left(12(-A + B) - 10(A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 - 15(A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 - \right. \\ \left. (15 + 15i)(-1)^{1/4}(A + B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4}\left(1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5$$

■ **Problem 129: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 224 leaves, 7 steps):

$$\frac{(7A + 3B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e + f x]}{\sqrt{2} \sqrt{c - c \sin[e + f x]}}\right]}{16 \sqrt{2} a^3 c^{3/2} f} + \frac{(7A + 3B) \cos[e + f x]}{16 a^3 f (c - c \sin[e + f x])^{3/2}} - \frac{(7A + 3B) \sec[e + f x]}{12 a^3 c f \sqrt{c - c \sin[e + f x]}} - \\ \frac{(7A + 3B) \sec[e + f x]^3 \sqrt{c - c \sin[e + f x]}}{30 a^3 c^2 f} - \frac{(A - B) \sec[e + f x]^5 (c - c \sin[e + f x])^{3/2}}{5 a^3 c^3 f}$$

Result (type 3, 357 leaves):

$$\frac{1}{240 a^3 f (1 + \sin[e + f x])^3 (c - c \sin[e + f x])^{3/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \\ \left(-40A \cos[e + f x]^2 + 24(-A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 - 30(3A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \right. \\ \left. \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^4 + 15(A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 - \right. \\ \left. (15 + 15i)(-1)^{1/4}(7A + 3B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{1/4}\left(1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \\ \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 + 30(A + B) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5$$

■ **Problem 130: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^3 (c - c \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 258 leaves, 8 steps):

$$\frac{7(9A+B) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \cos[e+fx]}{\sqrt{2} \sqrt{c-c \sin[e+fx]}}\right]}{128 \sqrt{2} a^3 c^{5/2} f} + \frac{7(9A+B) \cos[e+fx]}{128 a^3 c f (c-c \sin[e+fx])^{3/2}} + \frac{7(9A+B) \operatorname{Sec}[e+fx]}{240 a^3 c f (c-c \sin[e+fx])^{3/2}} -$$

$$\frac{7(9A+B) \operatorname{Sec}[e+fx]}{96 a^3 c^2 f \sqrt{c-c \sin[e+fx]}} - \frac{(9A+B) \operatorname{Sec}[e+fx]^3}{30 a^3 c^2 f \sqrt{c-c \sin[e+fx]}} - \frac{(A-B) \operatorname{Sec}[e+fx]^5 \sqrt{c-c \sin[e+fx]}}{5 a^3 c^3 f}$$

Result (type 3, 479 leaves):

$$\frac{1}{1920 a^3 f (1 + \sin[e+fx])^3 (c - c \sin[e+fx])^{5/2}} \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)$$

$$\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(-720 A \cos[e+fx]^4 + 96(-A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 + \right.$$

$$80(-3A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 + 60(A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)$$

$$\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 + 15(15A+7B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 -$$

$$(105 + 105i) (-1)^{1/4} (9A+B) \operatorname{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \left(1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^4$$

$$\left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 + 120(A+B) \sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 +$$

$$30(15A+7B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 \Bigg)$$

■ **Problem 135: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+a \sin[e+fx]} (A+B \sin[e+fx])}{\sqrt{c-c \sin[e+fx]}} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{a(A+B) \cos[e+fx] \operatorname{Log}[1 - \sin[e+fx]]}{f \sqrt{a+a \sin[e+fx]} \sqrt{c-c \sin[e+fx]}} + \frac{aB \cos[e+fx] \sqrt{c-c \sin[e+fx]}}{c f \sqrt{a+a \sin[e+fx]}}$$

Result (type 3, 133 leaves):

$$-\left(\left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{a(1 + \sin[e+fx])} \right.$$

$$\left. \left(2i(A+B) \operatorname{ArcTan}\left[e^{i(e+fx)}\right] + (A+B) (-ifx + \operatorname{Log}[1 + e^{2i(e+fx)}]) + B \sin[e+fx] \right) \right) /$$

$$\left(f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \sqrt{c-c \sin[e+fx]} \right)$$

■ **Problem 136: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a + a \sin[e + f x]} (A + B \sin[e + f x])}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 99 leaves, 5 steps):

$$\frac{a (A + B) \cos[e + f x]}{f \sqrt{a + a \sin[e + f x]} (c - c \sin[e + f x])^{3/2}} + \frac{a B \cos[e + f x] \log[1 - \sin[e + f x]]}{c f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 177 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \sqrt{a(1 + \sin[e + f x])} \right. \\ \left. (-A - B + i B f x - B \log[1 + e^{2i(e + f x)}] + 2i B \operatorname{ArcTan}[e^{i(e + f x)}](-1 + \sin[e + f x]) + B(-i f x + \log[1 + e^{2i(e + f x)}]) \sin[e + f x]) \right) / \\ \left(c f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) (-1 + \sin[e + f x]) \sqrt{c - c \sin[e + f x]} \right)$$

■ **Problem 158: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^{5/2} (A + B \sin[e + f x])}{(c - c \sin[e + f x])^{11/2}} dx$$

Optimal (type 3, 146 leaves, 3 steps):

$$\frac{(A + B) \cos[e + f x] (a + a \sin[e + f x])^{5/2}}{10 f (c - c \sin[e + f x])^{11/2}} + \frac{(A - 4 B) \cos[e + f x] (a + a \sin[e + f x])^{5/2}}{40 c f (c - c \sin[e + f x])^{9/2}} + \frac{(A - 4 B) \cos[e + f x] (a + a \sin[e + f x])^{5/2}}{240 c^2 f (c - c \sin[e + f x])^{7/2}}$$

Result (type 3, 348 leaves):

$$\frac{4 (A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) (a(1 + \sin[e + f x]))^{5/2}}{5 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{11/2}} + \frac{(-A - 2 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 (a(1 + \sin[e + f x]))^{5/2}}{f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{11/2}} \\ - \frac{(A + 5 B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (a(1 + \sin[e + f x]))^{5/2}}{3 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{11/2}} - \frac{B \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a(1 + \sin[e + f x]))^{5/2}}{2 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (c - c \sin[e + f x])^{11/2}}$$

■ **Problem 160: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^{7/2} (A + B \sin[e + f x]) (c - c \sin[e + f x])^{9/2} dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$\frac{a^4 (9A - B) \cos[e + fx] (c - c \sin[e + fx])^{9/2}}{315 f \sqrt{a + a \sin[e + fx]}} - \frac{a^3 (9A - B) \cos[e + fx] \sqrt{a + a \sin[e + fx]} (c - c \sin[e + fx])^{9/2}}{126 f} - \frac{a^2 (9A - B) \cos[e + fx] (a + a \sin[e + fx])^{3/2} (c - c \sin[e + fx])^{9/2}}{84 f} - \frac{a (9A - B) \cos[e + fx] (a + a \sin[e + fx])^{5/2} (c - c \sin[e + fx])^{9/2}}{72 f} - \frac{B \cos[e + fx] (a + a \sin[e + fx])^{7/2} (c - c \sin[e + fx])^{9/2}}{9 f}$$

Result (type 3, 870 leaves):

$$\frac{7 (A - B) \cos[2 (e + fx)] (a (1 + \sin[e + fx]))^{7/2} (c - c \sin[e + fx])^{9/2}}{128 f \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^9 \left(\cos\left[\frac{1}{2} (e + fx)\right] + \sin\left[\frac{1}{2} (e + fx)\right] \right)^7} + \frac{7 (A - B) \cos[4 (e + fx)] (a (1 + \sin[e + fx]))^{7/2} (c - c \sin[e + fx])^{9/2}}{256 f \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^9 \left(\cos\left[\frac{1}{2} (e + fx)\right] + \sin\left[\frac{1}{2} (e + fx)\right] \right)^7} + \frac{(A - B) \cos[6 (e + fx)] (a (1 + \sin[e + fx]))^{7/2} (c - c \sin[e + fx])^{9/2}}{128 f \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^9 \left(\cos\left[\frac{1}{2} (e + fx)\right] + \sin\left[\frac{1}{2} (e + fx)\right] \right)^7} + \frac{(A - B) \cos[8 (e + fx)] (a (1 + \sin[e + fx]))^{7/2} (c - c \sin[e + fx])^{9/2}}{1024 f \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^9 \left(\cos\left[\frac{1}{2} (e + fx)\right] + \sin\left[\frac{1}{2} (e + fx)\right] \right)^7} + \frac{7 (10A - B) \sin[e + fx] (a (1 + \sin[e + fx]))^{7/2} (c - c \sin[e + fx])^{9/2}}{128 f \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^9 \left(\cos\left[\frac{1}{2} (e + fx)\right] + \sin\left[\frac{1}{2} (e + fx)\right] \right)^7} + \frac{7A (a (1 + \sin[e + fx]))^{7/2} (c - c \sin[e + fx])^{9/2} \sin[3 (e + fx)]}{64 f \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^9 \left(\cos\left[\frac{1}{2} (e + fx)\right] + \sin\left[\frac{1}{2} (e + fx)\right] \right)^7} + \frac{(7A + 2B) (a (1 + \sin[e + fx]))^{7/2} (c - c \sin[e + fx])^{9/2} \sin[5 (e + fx)]}{320 f \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^9 \left(\cos\left[\frac{1}{2} (e + fx)\right] + \sin\left[\frac{1}{2} (e + fx)\right] \right)^7} + \frac{(4A + 5B) (a (1 + \sin[e + fx]))^{7/2} (c - c \sin[e + fx])^{9/2} \sin[7 (e + fx)]}{1792 f \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^9 \left(\cos\left[\frac{1}{2} (e + fx)\right] + \sin\left[\frac{1}{2} (e + fx)\right] \right)^7} + \frac{B (a (1 + \sin[e + fx]))^{7/2} (c - c \sin[e + fx])^{9/2} \sin[9 (e + fx)]}{2304 f \left(\cos\left[\frac{1}{2} (e + fx)\right] - \sin\left[\frac{1}{2} (e + fx)\right] \right)^9 \left(\cos\left[\frac{1}{2} (e + fx)\right] + \sin\left[\frac{1}{2} (e + fx)\right] \right)^7}$$

■ **Problem 170: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^{7/2} (A + B \sin[e + fx])}{(c - c \sin[e + fx])^{11/2}} dx$$

Optimal (type 3, 96 leaves, 2 steps):

$$\frac{(A+B) \cos[e+fx] (a+a \sin[e+fx])^{7/2}}{10 f (c-c \sin[e+fx])^{11/2}} + \frac{(A-9B) \cos[e+fx] (a+a \sin[e+fx])^{7/2}}{80 c f (c-c \sin[e+fx])^{9/2}}$$

Result (type 3, 434 leaves):

$$\begin{aligned} & \frac{8(A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (a(1+\sin[e+fx]))^{7/2}}{5 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{11/2}} + \\ & \frac{(-3A-5B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{11/2}} + \\ & \frac{2(A+3B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{11/2}} + \\ & \frac{(-A-7B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (a(1+\sin[e+fx]))^{7/2}}{2 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{11/2}} + \frac{B \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^9 (a(1+\sin[e+fx]))^{7/2}}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{11/2}} \end{aligned}$$

■ **Problem 171: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \sin[e+fx])^{7/2} (A+B \sin[e+fx])}{(c-c \sin[e+fx])^{13/2}} dx$$

Optimal (type 3, 146 leaves, 3 steps):

$$\frac{(A+B) \cos[e+fx] (a+a \sin[e+fx])^{7/2}}{12 f (c-c \sin[e+fx])^{13/2}} + \frac{(A-5B) \cos[e+fx] (a+a \sin[e+fx])^{7/2}}{60 c f (c-c \sin[e+fx])^{11/2}} + \frac{(A-5B) \cos[e+fx] (a+a \sin[e+fx])^{7/2}}{480 c^2 f (c-c \sin[e+fx])^{9/2}}$$

Result (type 3, 442 leaves):

$$\begin{aligned} & \frac{4(A+B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right) (a(1+\sin[e+fx]))^{7/2}}{3 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{13/2}} - \\ & \frac{4(3A+5B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (a(1+\sin[e+fx]))^{7/2}}{5 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{13/2}} + \\ & \frac{3(A+3B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5 (a(1+\sin[e+fx]))^{7/2}}{2 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{13/2}} + \\ & \frac{(-A-7B) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (a(1+\sin[e+fx]))^{7/2}}{3 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{13/2}} + \frac{B \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^9 (a(1+\sin[e+fx]))^{7/2}}{2 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^7 (c-c \sin[e+fx])^{13/2}} \end{aligned}$$

■ **Problem 172: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^{7/2} (A + B \sin[e + f x])}{(c - c \sin[e + f x])^{15/2}} dx$$

Optimal (type 3, 202 leaves, 4 steps):

$$\frac{(A + B) \cos[e + f x] (a + a \sin[e + f x])^{7/2}}{14 f (c - c \sin[e + f x])^{15/2}} + \frac{(3A - 11B) \cos[e + f x] (a + a \sin[e + f x])^{7/2}}{168 c f (c - c \sin[e + f x])^{13/2}} +$$

$$\frac{(3A - 11B) \cos[e + f x] (a + a \sin[e + f x])^{7/2}}{840 c^2 f (c - c \sin[e + f x])^{11/2}} + \frac{(3A - 11B) \cos[e + f x] (a + a \sin[e + f x])^{7/2}}{6720 c^3 f (c - c \sin[e + f x])^{9/2}}$$

Result (type 3, 442 leaves):

$$\frac{8 (A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) (a (1 + \sin[e + f x]))^{7/2}}{7 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (c - c \sin[e + f x])^{15/2}} -$$

$$\frac{2 (3A + 5B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 (a (1 + \sin[e + f x]))^{7/2}}{3 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (c - c \sin[e + f x])^{15/2}} +$$

$$\frac{6 (A + 3B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^5 (a (1 + \sin[e + f x]))^{7/2}}{5 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (c - c \sin[e + f x])^{15/2}} +$$

$$\frac{(-A - 7B) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (a (1 + \sin[e + f x]))^{7/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (c - c \sin[e + f x])^{15/2}} + \frac{B \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right)^9 (a (1 + \sin[e + f x]))^{7/2}}{3 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^7 (c - c \sin[e + f x])^{15/2}}$$

■ **Problem 176: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \sin[e + f x]) \sqrt{c - c \sin[e + f x]}}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 96 leaves, 5 steps):

$$\frac{(A - B) c \cos[e + f x] \log[1 + \sin[e + f x]]}{f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}} - \frac{B \cos[e + f x] \sqrt{c - c \sin[e + f x]}}{f \sqrt{a + a \sin[e + f x]}}$$

Result (type 3, 136 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(-2i (A - B) \operatorname{ArcTan}\left[e^{i(e + f x)}\right] + (A - B) \left(-i f x + \log[1 + e^{2i(e + f x)}] \right) + B \sin[e + f x] \right) \sqrt{c - c \sin[e + f x]} \right) /$$

$$\left(f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \sqrt{a (1 + \sin[e + f x])} \right)$$

■ **Problem 183: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \sin[e + f x]) \sqrt{c - c \sin[e + f x]}}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{(A - B) c \cos[e + f x]}{f (a + a \sin[e + f x])^{3/2} \sqrt{c - c \sin[e + f x]}} + \frac{B c \cos[e + f x] \operatorname{Log}[1 + \sin[e + f x]]}{a f \sqrt{a + a \sin[e + f x]} \sqrt{c - c \sin[e + f x]}}$$

Result (type 3, 161 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \sqrt{c - c \sin[e + f x]} \right. \\ \left. (-A + B - i B f x + B \operatorname{Log}[1 + e^{2i(e+f x)}] + B(-i f x + \operatorname{Log}[1 + e^{2i(e+f x)}]) \sin[e + f x] - 2i B \operatorname{ArcTan}[e^{i(e+f x)}] (1 + \sin[e + f x])) \right) / \\ \left(f \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) (a (1 + \sin[e + f x]))^{3/2} \right)$$

■ **Problem 195: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) (c - c \sin[e + f x])^n dx$$

Optimal (type 5, 174 leaves, 5 steps):

$$\frac{1}{f (1 + 2m) (1 + m + n)} 2^{\frac{1}{2} + n} c (B(m - n) + A(1 + m + n)) \cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(1 + 2m), \frac{1}{2}(1 - 2n), \frac{1}{2}(3 + 2m), \frac{1}{2}(1 + \sin[e + f x])\right] \\ (1 - \sin[e + f x])^{\frac{1}{2} - n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1 + n} - \frac{B \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n}{f (1 + m + n)}$$

Result (type 6, 15882 leaves):

$$-\left(\left(4^{1+n} (3 + 2n) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-2n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n \right. \right. \\ \left. \left(A \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2n} + B \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2n} \sin[e + f x] \right) \right. \\ \left. \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]}{1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2n} \left(\frac{1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2m} \right. \\ \left. \left(- \left(\left(\operatorname{AppellF1}\left[\frac{1}{2} + n, -2m, 1 + 2(m + n), \frac{3}{2} + n, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^2 \right) \right) / \right. \\ \left. \left(-(3 + 2n) \operatorname{AppellF1}\left[\frac{1}{2} + n, -2m, 1 + 2(m + n), \frac{3}{2} + n, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right)$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}+n, -2m, 3+2(m+n), \frac{5}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \\
& \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - (3+2n) \left(-\frac{1}{\frac{3}{2}+n} m \left(\frac{1}{2}+n\right) \text{AppellF1}\left[\frac{3}{2}+n, 1-2m, 2(1+m+n), \frac{5}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{\frac{3}{2}+n} \left(\frac{1}{2}+n\right) (1+m+n) \text{AppellF1}\left[\frac{3}{2}+n, -2m, \right. \right. \\
& \quad \left. \left. 1+2(1+m+n), \frac{5}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) + \\
& 4 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(m \left(-\frac{1}{\frac{5}{2}+n} \left(\frac{3}{2}+n\right) (1+m+n) \text{AppellF1}\left[\frac{5}{2}+n, 1-2m, 1+2(1+m+n), \frac{7}{2}+n, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \right. \\
& \quad \left. \frac{1}{2\left(\frac{5}{2}+n\right)} (1-2m) \left(\frac{3}{2}+n\right) \text{AppellF1}\left[\frac{5}{2}+n, 2-2m, 2(1+m+n), \frac{7}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) + (1+m+n) \\
& \left(-\frac{1}{\frac{5}{2}+n} m \left(\frac{3}{2}+n\right) \text{AppellF1}\left[\frac{5}{2}+n, 1-2m, 3+2(m+n), \frac{7}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \\
& \quad \left. \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{2\left(\frac{5}{2}+n\right)} \left(\frac{3}{2}+n\right) (3+2(m+n)) \text{AppellF1}\left[\frac{5}{2}+n, -2m, 4+2(m+n), \right. \right. \\
& \quad \left. \left. \frac{7}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \Bigg) / \\
& \left(-(3+2n) \text{AppellF1}\left[\frac{1}{2}+n, -2m, 2(1+m+n), \frac{3}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \\
& \quad \left. 4 \left(m \text{AppellF1}\left[\frac{3}{2}+n, 1-2m, 2(1+m+n), \frac{5}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + (1+m+n) \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{3}{2}+n, -2m, 3+2(m+n), \frac{5}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^2 + \\
& \left(A \text{AppellF1}\left[\frac{1}{2}+n, -2m, 1+2(m+n), \frac{3}{2}+n, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \left(1 + \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^2 \right)^2
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}+n, -2m, 2(1+m+n), \frac{5}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - \\
& \left(8 \text{B AppellF1}\left[\frac{1}{2}+n, -2m, 3+2(m+n), \frac{3}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \\
& \left. - \left(2m \text{AppellF1}\left[\frac{3}{2}+n, 1-2m, 3+2(m+n), \frac{5}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right. \\
& \left. \left. (3+2m+2n) \text{AppellF1}\left[\frac{3}{2}+n, -2m, 2(2+m+n), \frac{5}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + (3+2n) \left(-\frac{1}{\frac{3}{2}+n} m \left(\frac{1}{2}+n\right) \text{AppellF1}\left[\frac{3}{2}+n, 1-2m, 3+2(m+n), \right. \right. \right. \\
& \left. \left. \frac{5}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{2\left(\frac{3}{2}+n\right)} \right. \right. \\
& \left. \left. \left(\frac{1}{2}+n\right) (3+2(m+n)) \text{AppellF1}\left[\frac{3}{2}+n, -2m, 4+2(m+n), \frac{5}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - 2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(2m \left(-\frac{1}{2\left(\frac{5}{2}+n\right)} \right. \right. \right. \\
& \left. \left. \left(\frac{3}{2}+n\right) (3+2(m+n)) \text{AppellF1}\left[\frac{5}{2}+n, 1-2m, 4+2(m+n), \frac{7}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \frac{1}{2\left(\frac{5}{2}+n\right)} (1-2m) \left(\frac{3}{2}+n\right) \text{AppellF1}\left[\frac{5}{2}+n, 2-2m, 3+2(m+n), \frac{7}{2}+ \right. \right. \\
& \left. \left. n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) + (3+2m+2n) \\
& \left. \left(-\frac{1}{\frac{5}{2}+n} m \left(\frac{3}{2}+n\right) \text{AppellF1}\left[\frac{5}{2}+n, 1-2m, 2(2+m+n), \frac{7}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{\frac{5}{2}+n} \left(\frac{3}{2}+n\right) (2+m+n) \text{AppellF1}\left[\frac{5}{2}+n, -2m, 1+2(2+m+n), \right. \right. \\
& \left. \left. \frac{7}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \right) \Big/ \\
& \left. \left((3+2n) \text{AppellF1}\left[\frac{1}{2}+n, -2m, 3+2(m+n), \frac{3}{2}+n, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] - \right. \right.
\end{aligned}$$

$$2 \left(2 m \operatorname{AppellF1} \left[\frac{3}{2} + n, 1 - 2 m, 3 + 2 (m + n), \frac{5}{2} + n, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + (3 + 2 m + 2 n) \operatorname{AppellF1} \left[\frac{3}{2} + n, -2 m, 2 (2 + m + n), \frac{5}{2} + n, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^2 \right)$$

- **Problem 196: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) (c - c \sin[e + f x])^3 dx$$

Optimal (type 5, 145 leaves, 5 steps):

$$\frac{1}{7 f (4 + m)} 2^{\frac{1}{2} + m} a^4 c^3 (B (3 - m) - A (4 + m)) \operatorname{Cos}[e + f x]^7 \operatorname{Hypergeometric2F1} \left[\frac{7}{2}, \frac{1}{2} - m, \frac{9}{2}, \frac{1}{2} (1 - \sin[e + f x]) \right] (1 + \sin[e + f x])^{\frac{1}{2} - m} (a + a \sin[e + f x])^{-4 + m} - \frac{a^3 B c^3 \operatorname{Cos}[e + f x]^7 (a + a \sin[e + f x])^{-3 + m}}{f (4 + m)}$$

Result (type 6, 31 879 leaves): Display of huge result suppressed!

- **Problem 197: Attempted integration timed out after 120 seconds.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) (c - c \sin[e + f x])^2 dx$$

Optimal (type 5, 145 leaves, 5 steps):

$$\frac{1}{5 f (3 + m)} 2^{\frac{1}{2} + m} a^3 c^2 (B (2 - m) - A (3 + m)) \operatorname{Cos}[e + f x]^5 \operatorname{Hypergeometric2F1} \left[\frac{5}{2}, \frac{1}{2} - m, \frac{7}{2}, \frac{1}{2} (1 - \sin[e + f x]) \right] (1 + \sin[e + f x])^{\frac{1}{2} - m} (a + a \sin[e + f x])^{-3 + m} - \frac{a^2 B c^2 \operatorname{Cos}[e + f x]^5 (a + a \sin[e + f x])^{-2 + m}}{f (3 + m)}$$

Result (type 1, 1 leaves):

???

- **Problem 198: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) (c - c \sin[e + f x]) dx$$

Optimal (type 5, 139 leaves, 5 steps):

$$\frac{1}{3 f (2 + m)} 2^{\frac{1}{2} + m} a^2 c (B (1 - m) - A (2 + m)) \operatorname{Cos}[e + f x]^3 \operatorname{Hypergeometric2F1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{5}{2}, \frac{1}{2} (1 - \sin[e + f x]) \right] (1 + \sin[e + f x])^{\frac{1}{2} - m} (a + a \sin[e + f x])^{-2 + m} - \frac{a B c \operatorname{Cos}[e + f x]^3 (a + a \sin[e + f x])^{-1 + m}}{f (2 + m)}$$

Result (type 5, 460 leaves):

$$\frac{1}{f \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^2}$$

$$\frac{i 4^{-1-m} c e^{i f m x} \left(1 + i e^{-i (e+f x)} \right)^{-2m} \left(e^{-\frac{1}{4} i (2e+\pi+2fx)} \left(i + e^{i (e+f x)} \right) \right)^{2m} \left(-\frac{i B e^{-i (2e+f (2+m) x)} \text{Hypergeometric2F1}[-2-m, -2m, -1-m, -i e^{-i (e+f x)}]}{2+m} \right. +$$

$$\frac{2 (-i A + B) e^{-i (e+f (1+m) x)} \text{Hypergeometric2F1}[-1-m, -2m, -m, -i e^{-i (e+f x)}]}{1+m} +$$

$$\frac{2 i A e^{i (e-f (-1+m) x)} \text{Hypergeometric2F1}[1-m, -2m, 2-m, -i e^{-i (e+f x)}]}{-1+m} +$$

$$\frac{2 B e^{i (e-f (-1+m) x)} \text{Hypergeometric2F1}[1-m, -2m, 2-m, -i e^{-i (e+f x)}]}{-1+m} +$$

$$\left. \frac{i B e^{2 i e - i f (-2+m) x} \text{Hypergeometric2F1}[2-m, -2m, 3-m, -i e^{-i (e+f x)}]}{-2+m} + \frac{4 A e^{-i f m x} \text{Hypergeometric2F1}[-2m, -m, 1-m, -i e^{-i (e+f x)}]}{m} \right)$$

$$(-1 + \sin[e + f x]) (a (1 + \sin[e + f x]))^m \sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^{-2m}$$

- **Problem 199: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) dx$$

Optimal (type 5, 117 leaves, 3 steps):

$$\frac{B \cos[e + f x] (a + a \sin[e + f x])^m}{f (1+m)} - \frac{1}{f (1+m)}$$

$$2^{\frac{1}{2}+m} (A + A m + B m) \cos[e + f x] \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]) \right] (1 + \sin[e + f x])^{-\frac{1}{2}-m} (a + a \sin[e + f x])^m$$

Result (type 5, 295 leaves):

$$-\frac{1}{f} (a (1 + \sin[e + f x]))^m \left(\frac{1}{-1+m^2} 2^{-1-2m} B e^{-i (e+f x)} \left(1 + i e^{-i (e+f x)} \right)^{-2m} \left(e^{-\frac{1}{4} i (2e+\pi+2fx)} \left(i + e^{i (e+f x)} \right) \right)^{2m} \right.$$

$$\left. \left(e^{2 i (e+f x)} (-1+m) \text{Hypergeometric2F1}[-1-m, -2m, -m, -i e^{-i (e+f x)}] - (1+m) \text{Hypergeometric2F1}[1-m, -2m, 2-m, -i e^{-i (e+f x)}] \right) + \right.$$

$$\left. \left(2 \sqrt{2} A \cos \left[\frac{1}{4} (2e - \pi + 2fx) \right] \right)^{1+2m} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} + m, \frac{3}{2} + m, \sin \left[\frac{1}{4} (2e + \pi + 2fx) \right] \right]^2 \sin \left[\frac{1}{4} (2e - \pi + 2fx) \right] \right) /$$

$$\left((1+2m) \sqrt{1 - \sin[e + f x]} \right) \sin \left[\frac{1}{4} (2e + \pi + 2fx) \right]^{-2m}$$

- **Problem 200: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m (A + B \sin[e + f x])}{c - c \sin[e + f x]} dx$$

Optimal (type 5, 123 leaves, 5 steps) :

$$\frac{1}{c f m} 2^{\frac{1}{2}+m} (B + A m + B m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - m, \frac{1}{2}, \frac{1}{2} (1 - \sin[e + f x])\right] \text{Sec}[e + f x] (1 + \sin[e + f x])^{\frac{1}{2}-m} (a + a \sin[e + f x])^m - \frac{B \text{Sec}[e + f x] (a + a \sin[e + f x])^{1+m}}{a c f m}$$

Result (type 6, 8388 leaves) :

$$\begin{aligned} & - \left(\left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} \cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 (a + a \sin[e + f x])^m \right. \right. \\ & \left. \left(\frac{A \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m}}{\left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right]\right)^2} + \frac{B \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \sin[e + f x]}{\left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right]\right)^2} \right) \right. \\ & \left. \left(\frac{1 - \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2m} \left(- \left((A + B) \text{AppellF1}\left[-\frac{1}{2}, -2m, 2m, \frac{1}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \right) \right) / \\ & \left(\text{AppellF1}\left[-\frac{1}{2}, -2m, 2m, \frac{1}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \\ & 4m \left(\text{AppellF1}\left[\frac{1}{2}, 1 - 2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\ & \left. \left. \text{AppellF1}\left[\frac{1}{2}, -2m, 1 + 2m, \frac{3}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \\ & \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(\left(3 (A + B) \text{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \right) / \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \right. \\ & 4m \left(\text{AppellF1}\left[\frac{3}{2}, 1 - 2m, 2m, \frac{5}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\ & \left. \left. \text{AppellF1}\left[\frac{3}{2}, -2m, 1 + 2m, \frac{5}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) - \\ & \left(8 B \text{AppellF1}\left[\frac{1}{2}, -2m, 1 + 2m, \frac{3}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) / \\ & \left(\left(1 + \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left(\text{AppellF1}\left[\frac{1}{2}, -2m, 1 + 2m, \frac{3}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] - \frac{2}{3} \right. \right. \\ & \left. \left. \left(2m \text{AppellF1}\left[\frac{3}{2}, 1 - 2m, 1 + 2m, \frac{5}{2}, \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. (1 + 2m) \text{AppellF1}\left[\frac{3}{2}, -2m, 2 + 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right) \right) \Bigg/ \\
& \left(2f(c - c \sin[e + fx]) \left(-\frac{1}{8} \text{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right)^2 \left(\frac{1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right)^{2m} \left(- \left(\left((A + B) \text{AppellF1}\left[-\frac{1}{2}, -2m, 2m, \frac{1}{2}, \right. \right. \right. \right. \right. \\
& \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \Bigg/ \left(\text{AppellF1}\left[-\frac{1}{2}, -2m, 2m, \frac{1}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - 4m \left(\text{AppellF1}\left[\frac{1}{2}, 1 - 2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right] + \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{1}{2}, -2m, 1 + 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) + \\
& \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \left(\left(3(A + B) \text{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) \Bigg/ \right. \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, -2m, 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right] - \right. \\
& \left. 4m \left(\text{AppellF1}\left[\frac{3}{2}, 1 - 2m, 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right] \right) + \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{3}{2}, -2m, 1 + 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) - \right. \\
& \left(8B \text{AppellF1}\left[\frac{1}{2}, -2m, 1 + 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) \Bigg/ \\
& \left(\left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \left(\text{AppellF1}\left[\frac{1}{2}, -2m, 1 + 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) - \right. \right. \\
& \left. \left. \frac{2}{3} \left(2m \text{AppellF1}\left[\frac{3}{2}, 1 - 2m, 1 + 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) + \right. \right. \\
& \left. \left. (1 + 2m) \text{AppellF1}\left[\frac{3}{2}, -2m, 2 + 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) \Bigg) \Bigg) + \\
& m \text{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(\frac{1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right)^{-1+2m} \left(- \frac{\text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)}{2 \left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^2} - \right. \\
& \left. \frac{\text{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]}{2 \left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)} \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& 1 + 2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \Bigg) + \\
& (1 + 2m) \left(-\frac{3}{5} m \operatorname{AppellF1}\left[\frac{5}{2}, 1 - 2m, 2 + 2m, \frac{7}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \\
& \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{3}{10} (2 + 2m) \operatorname{AppellF1}\left[\frac{5}{2}, -2m, 3 + 2m, \frac{7}{2}, \right. \\
& \left. \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \Bigg) \Bigg) \Bigg) / \\
& \left(\left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \left(\operatorname{AppellF1}\left[\frac{1}{2}, -2m, 1 + 2m, \frac{3}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \right. \right. \\
& \left. \left. \frac{2}{3} \left(2m \operatorname{AppellF1}\left[\frac{3}{2}, 1 - 2m, 1 + 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + (1 + 2m) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, -2m, 2 + 2m, \frac{5}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^m (A + B \sin[e + fx])}{(c - c \sin[e + fx])^2} dx$$

Optimal (type 5, 148 leaves, 5 steps):

$$\begin{aligned}
& \frac{1}{3 a c^2 f (1 - m)} 2^{\frac{1}{2} + m} (A (1 - m) - B (2 + m)) \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{2} - m, -\frac{1}{2}, \frac{1}{2} (1 - \sin[e + fx])\right] \\
& \operatorname{Sec}[e + fx]^3 (1 + \sin[e + fx])^{\frac{1}{2} - m} (a + a \sin[e + fx])^{1 + m} + \frac{B \operatorname{Sec}[e + fx]^3 (a + a \sin[e + fx])^{2 + m}}{a^2 c^2 f (1 - m)}
\end{aligned}$$

Result (type 6, 15419 leaves):

$$\begin{aligned}
& - \left(\left(\operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} \operatorname{Cot}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^3 \left(\operatorname{Cos}\left[\frac{1}{2}(e + fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + fx)\right] \right)^4 (a + a \sin[e + fx])^m \right. \right. \\
& \left. \left(\frac{A \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2m}}{\left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] - \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] \right)^4} + \frac{B \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{2m} \operatorname{Sin}[e + fx]}{\left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] - \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right)\right] \right)^4} \right) \right. \\
& \left. \left(\frac{1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right)^{2m} \left(- \left(\left((A + B) \operatorname{AppellF1}\left[-\frac{3}{2}, -2m, 2m, -\frac{1}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \right) \right) \Bigg) / \\
& \left(\operatorname{AppellF1}\left[-\frac{3}{2}, -2m, 2m, -\frac{1}{2}, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right.
\end{aligned}$$

Problem 202: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sin[e + f x])^m (A + B \sin[e + f x])}{(c - c \sin[e + f x])^3} dx$$

Optimal (type 5, 148 leaves, 5 steps):

$$\frac{1}{5 a^2 c^3 f (2 - m)} 2^{\frac{1}{2} + m} (A (2 - m) - B (3 + m)) \text{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{1}{2} - m, -\frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x])\right] \\ \text{Sec}[e + f x]^5 (1 + \sin[e + f x])^{\frac{1}{2} - m} (a + a \sin[e + f x])^{2+m} + \frac{B \text{Sec}[e + f x]^5 (a + a \sin[e + f x])^{3+m}}{a^3 c^3 f (2 - m)}$$

Result (type 6, 34716 leaves): Display of huge result suppressed!

■ **Problem 203: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m (A + B \sin[e + f x])}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 5, 118 leaves, 4 steps):

$$-\frac{2 B \cos[e + f x] (a + a \sin[e + f x])^m}{f (1 + 2 m) \sqrt{c - c \sin[e + f x]}} + \frac{(A + B) \cos[e + f x] \text{Hypergeometric2F1}\left[1, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x])\right] (a + a \sin[e + f x])^m}{f (1 + 2 m) \sqrt{c - c \sin[e + f x]}}$$

Result (type 6, 7013 leaves):

$$-\left(\left(\sqrt{2} \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right]\right) (a + a \sin[e + f x])^m\right.\right. \\ \left.\left(\frac{A \cos\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^{2m}}{\cos\left[\frac{\pi}{4} + \frac{1}{2} (e - \frac{\pi}{2} + f x)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} (e - \frac{\pi}{2} + f x)\right]} + \frac{B \cos\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]^{2m} \sin[e + f x]}{\cos\left[\frac{\pi}{4} + \frac{1}{2} (e - \frac{\pi}{2} + f x)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} (e - \frac{\pi}{2} + f x)\right]}\right) \\ \left(2 B \left(\cos\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right] - \cos\left[\frac{1}{2} (-e + \frac{\pi}{2} - f x)\right]\right)^{-2m} + \left(A (1 + m) \text{AppellF1}\left[1 + 2 m, 2 m, 1, 2 + 2 m, \right.\right. \\ \left.\frac{1}{2} \left(1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right), 1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right] \cot\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2 \left(1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right)\right) \Bigg) / \\ \left(-2 (1 + m) \text{AppellF1}\left[1 + 2 m, 2 m, 1, 2 + 2 m, \frac{1}{2} \left(1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right), 1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right] + \right. \\ \left.\left(\text{AppellF1}\left[2 + 2 m, 2 m, 2, 3 + 2 m, \frac{1}{2} \left(1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right), 1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right] + m \right.\right. \\ \left.\left.\text{AppellF1}\left[2 + 2 m, 1 + 2 m, 1, 3 + 2 m, \frac{1}{2} \left(1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right), 1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right]\right) \left(-1 + \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right)\right) + \\ \left(B (1 + m) \text{AppellF1}\left[1 + 2 m, 2 m, 1, 2 + 2 m, \frac{1}{2} \left(1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right), 1 - \tan\left[\frac{1}{4} (-e + \frac{\pi}{2} - f x)\right]^2\right]\right)$$

$$\begin{aligned}
& 2, 4 + 2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] + \\
& m \left(-\frac{1}{2(3+2m)} (2+2m) \operatorname{AppellF1} \left[3+2m, 1+2m, 2, 4+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right. \\
& \quad \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] - \frac{1}{4(3+2m)} (1+2m) (2+2m) \operatorname{AppellF1} \left[3+2m, 2+2m, 1, 4+2m, \right. \\
& \quad \left. \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \left. \right) \Bigg) / \\
& \left(-2(1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right. \\
& \quad \left(\operatorname{AppellF1} \left[2+2m, 2m, 2, 3+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + m \operatorname{AppellF1} \left[2+2m, \right. \right. \\
& \quad \left. \left. 1+2m, 1, 3+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \left(-1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 204: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + fx]) (c + c \sin[e + fx])^m}{\sqrt{a - a \sin[e + fx]}} dx$$

Optimal (type 5, 118 leaves, 4 steps):

$$-\frac{2B \cos[e + fx] (c + c \sin[e + fx])^m}{f(1+2m)\sqrt{a - a \sin[e + fx]}} + \frac{(A+B) \cos[e + fx] \operatorname{Hypergeometric2F1} \left[1, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + fx]) \right] (c + c \sin[e + fx])^m}{f(1+2m)\sqrt{a - a \sin[e + fx]}}$$

Result (type 6, 7013 leaves):

$$\begin{aligned}
& - \left(\left(\sqrt{2} \left(\cos \left[\frac{1}{2} (e + fx) \right] - \sin \left[\frac{1}{2} (e + fx) \right] \right) (c + c \sin[e + fx])^m \right. \right. \\
& \quad \left(\frac{A \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^{2m}}{\cos \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] - \sin \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right]} + \frac{B \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^{2m} \sin[e + fx]}{\cos \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right] - \sin \left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + fx \right) \right]} \right) \\
& \quad \left(2B \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right] - \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^{-2m} \right) + \left(A(1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \right. \right. \right. \\
& \quad \left. \left. \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) \Bigg) / \\
& \quad \left(-2(1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 1, 3 + 2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \right) + \\
& \left(-1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \left(-\frac{1}{3 + 2m} (2 + 2m) \operatorname{AppellF1} \left[3 + 2m, 2m, 3, 4 + 2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \right. \right. \\
& \left. \left. \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] - \frac{1}{2(3 + 2m)} m (2 + 2m) \operatorname{AppellF1} \left[3 + 2m, 1 + 2m, \right. \right. \\
& \left. \left. 2, 4 + 2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] + \right. \\
& \left. m \left(-\frac{1}{2(3 + 2m)} (2 + 2m) \operatorname{AppellF1} \left[3 + 2m, 1 + 2m, 2, 4 + 2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right. \right. \\
& \left. \left. \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] - \frac{1}{4(3 + 2m)} (1 + 2m) (2 + 2m) \operatorname{AppellF1} \left[3 + 2m, 2 + 2m, 1, 4 + 2m, \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \right) \right) \right) / \\
& \left(-2(1 + m) \operatorname{AppellF1} \left[1 + 2m, 2m, 1, 2 + 2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right. \\
& \left. \left(\operatorname{AppellF1} \left[2 + 2m, 2m, 2, 3 + 2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + m \operatorname{AppellF1} \left[2 + 2m, \right. \right. \right. \\
& \left. \left. \left. 1 + 2m, 1, 3 + 2m, \frac{1}{2} \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right), 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \left(-1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 205: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m (A + B \sin[e + fx]) (c - c \sin[e + fx])^{5/2} dx$$

Optimal (type 3, 275 leaves, 4 steps):

$$\begin{aligned}
& \frac{64 c^3 (B (5 - 2m) - A (7 + 2m)) \cos[e + fx] (a + a \sin[e + fx])^m}{f (5 + 2m) (7 + 2m) (3 + 8m + 4m^2) \sqrt{c - c \sin[e + fx]}} - \\
& \frac{16 c^2 (B (5 - 2m) - A (7 + 2m)) \cos[e + fx] (a + a \sin[e + fx])^m \sqrt{c - c \sin[e + fx]}}{f (7 + 2m) (15 + 16m + 4m^2)} - \\
& \frac{2c (B (5 - 2m) - A (7 + 2m)) \cos[e + fx] (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{3/2}}{f (5 + 2m) (7 + 2m)} - \frac{2B \cos[e + fx] (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{5/2}}{f (7 + 2m)}
\end{aligned}$$

Result (type 3, 667 leaves):

$$\frac{1}{f \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^5} (a (1 + \sin[e + f x]))^m (c - c \sin[e + f x])^{5/2}$$

$$\left(\left((2100 A - 1575 B + 1272 A m - 110 B m + 304 A m^2 - 68 B m^2 + 32 A m^3 - 8 B m^3) \left(\left(\frac{1}{8} + \frac{i}{8} \right) \cos \left[\frac{1}{2} (e + f x) \right] + \left(\frac{1}{8} - \frac{i}{8} \right) \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) \right) /$$

$$\left((1 + 2 m) (3 + 2 m) (5 + 2 m) (7 + 2 m) \right) + \left((2100 A - 1575 B + 1272 A m - 110 B m + 304 A m^2 - 68 B m^2 + 32 A m^3 - 8 B m^3) \right.$$

$$\left. \left(\left(\frac{1}{8} - \frac{i}{8} \right) \cos \left[\frac{1}{2} (e + f x) \right] + \left(\frac{1}{8} + \frac{i}{8} \right) \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \left((1 + 2 m) (3 + 2 m) (5 + 2 m) (7 + 2 m) \right) +$$

$$\left((350 A - 385 B + 184 A m - 104 B m + 24 A m^2 - 12 B m^2) \left(\left(\frac{1}{8} - \frac{i}{8} \right) \cos \left[\frac{3}{2} (e + f x) \right] - \left(\frac{1}{8} + \frac{i}{8} \right) \sin \left[\frac{3}{2} (e + f x) \right] \right) \right) / \left((3 + 2 m) (5 + 2 m) (7 + 2 m) \right) +$$

$$\left((350 A - 385 B + 184 A m - 104 B m + 24 A m^2 - 12 B m^2) \left(\left(\frac{1}{8} + \frac{i}{8} \right) \cos \left[\frac{3}{2} (e + f x) \right] - \left(\frac{1}{8} - \frac{i}{8} \right) \sin \left[\frac{3}{2} (e + f x) \right] \right) \right) / \left((3 + 2 m) (5 + 2 m) (7 + 2 m) \right) +$$

$$\frac{(14 A - 35 B + 4 A m - 6 B m) \left(\left(-\frac{1}{8} + \frac{i}{8} \right) \cos \left[\frac{5}{2} (e + f x) \right] - \left(\frac{1}{8} + \frac{i}{8} \right) \sin \left[\frac{5}{2} (e + f x) \right] \right)}{(5 + 2 m) (7 + 2 m)} +$$

$$\frac{(14 A - 35 B + 4 A m - 6 B m) \left(\left(-\frac{1}{8} - \frac{i}{8} \right) \cos \left[\frac{5}{2} (e + f x) \right] - \left(\frac{1}{8} - \frac{i}{8} \right) \sin \left[\frac{5}{2} (e + f x) \right] \right)}{(5 + 2 m) (7 + 2 m)} +$$

$$\left. \frac{\left(\frac{1}{8} - \frac{i}{8} \right) B \cos \left[\frac{7}{2} (e + f x) \right] - \left(\frac{1}{8} + \frac{i}{8} \right) B \sin \left[\frac{7}{2} (e + f x) \right]}{7 + 2 m} + \frac{\left(\frac{1}{8} + \frac{i}{8} \right) B \cos \left[\frac{7}{2} (e + f x) \right] - \left(\frac{1}{8} - \frac{i}{8} \right) B \sin \left[\frac{7}{2} (e + f x) \right]}{7 + 2 m} \right)$$

■ **Problem 208: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m (A + B \sin[e + f x])}{\sqrt{c - c \sin[e + f x]}} dx$$

Optimal (type 5, 118 leaves, 4 steps):

$$-\frac{2 B \cos[e + f x] (a + a \sin[e + f x])^m}{f (1 + 2 m) \sqrt{c - c \sin[e + f x]}} + \frac{(A + B) \cos[e + f x] \text{Hypergeometric2F1}\left[1, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x])\right] (a + a \sin[e + f x])^m}{f (1 + 2 m) \sqrt{c - c \sin[e + f x]}}$$

Result (type 6, 7013 leaves):

$$-\left(\left(\sqrt{2} \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) (a + a \sin[e + f x])^m \right.$$

$$\left. \left(\frac{A \cos \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^{2m}}{\cos \left[\frac{\pi}{4} + \frac{1}{2} (e - \frac{\pi}{2} + f x) \right] - \sin \left[\frac{\pi}{4} + \frac{1}{2} (e - \frac{\pi}{2} + f x) \right]} + \frac{B \cos \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^{2m} \sin[e + f x]}{\cos \left[\frac{\pi}{4} + \frac{1}{2} (e - \frac{\pi}{2} + f x) \right] - \sin \left[\frac{\pi}{4} + \frac{1}{2} (e - \frac{\pi}{2} + f x) \right]} \right) \right)$$

$$\begin{aligned}
& m \operatorname{AppellF1}\left[2+2 m, 1+2 m, 1, 3+2 m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \\
& \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-2(1+m)\left(-\frac{1}{2(2+2 m)}(1+2 m) \operatorname{AppellF1}\left[2+2 m, 2 m, 2, 3+2 m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\right.\right.\right. \\
& \left.\left.\left.\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{2(2+2 m)} m(1+2 m) \operatorname{AppellF1}\left[2+2 m, 1+2 m, \right.\right.\right. \\
& \left.\left.\left.1, 3+2 m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)+ \\
& \left(-1+\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\left(-\frac{1}{3+2 m}(2+2 m) \operatorname{AppellF1}\left[3+2 m, 2 m, 3, 4+2 m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\right.\right.\right. \\
& \left.\left.\left.\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{2(3+2 m)} m(2+2 m) \operatorname{AppellF1}\left[3+2 m, 1+2 m, \right.\right.\right. \\
& \left.\left.\left.2, 4+2 m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)+ \\
& m\left(-\frac{1}{2(3+2 m)}(2+2 m) \operatorname{AppellF1}\left[3+2 m, 1+2 m, 2, 4+2 m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{4(3+2 m)}(1+2 m)(2+2 m) \operatorname{AppellF1}\left[3+2 m, 2+2 m, 1, 4+2 m, \right.\right. \\
& \left.\left.\frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\right) / \\
& \left(-2(1+m) \operatorname{AppellF1}\left[1+2 m, 2 m, 1, 2+2 m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+ \right. \\
& \left.\left(\operatorname{AppellF1}\left[2+2 m, 2 m, 2, 3+2 m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+m \operatorname{AppellF1}\left[2+2 m, \right.\right.\right. \\
& \left.\left.\left.1+2 m, 1, 3+2 m, \frac{1}{2}\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right), 1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right)\left(-1+\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 209: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sin}[e+f x])^m (A+B \operatorname{Sin}[e+f x])}{(c-c \operatorname{Sin}[e+f x])^{3/2}} dx$$

Optimal (type 5, 134 leaves, 4 steps):

$$\frac{(A+B) \cos[e+fx] (a+a \sin[e+fx])^m}{2f(c-c \sin[e+fx])^{3/2}} +$$

$$\left(\frac{(A(1-2m) - B(3+2m)) \cos[e+fx] \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\sin[e+fx])\right] (a+a \sin[e+fx])^m}{(4cf(1+2m)\sqrt{c-c \sin[e+fx]})} \right) /$$

Result (type 6, 14818 leaves):

$$\begin{aligned} & - \left(\left(\cos\left[\frac{1}{2}(-e + \frac{\pi}{2} - fx)\right] \right)^{-2m} \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3 (a+a \sin[e+fx])^m \right. \\ & \left. \left(\frac{A \cos\left[\frac{1}{2}(-e + \frac{\pi}{2} - fx)\right]^{2m}}{\left(\cos\left[\frac{\pi}{4} + \frac{1}{2}(e - \frac{\pi}{2} + fx)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2}(e - \frac{\pi}{2} + fx)\right]\right)^3} + \frac{B \cos\left[\frac{1}{2}(-e + \frac{\pi}{2} - fx)\right]^{2m} \sin[e+fx]}{\left(\cos\left[\frac{\pi}{4} + \frac{1}{2}(e - \frac{\pi}{2} + fx)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2}(e - \frac{\pi}{2} + fx)\right]\right)^3} \right) \right. \\ & \left. \left(\frac{1 - \tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2}{1 + \tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2} \right)^{2m} \left(- \left(A \operatorname{AppellF1}\left[1, -2m, 2m, 2, \cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \right) \right) / \right. \\ & \left(-m \left(\operatorname{AppellF1}\left[2, 1-2m, 2m, 3, \cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \right) + \right. \\ & \quad \left. \operatorname{AppellF1}\left[2, -2m, 1+2m, 3, \cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \right) + \\ & \quad \left. \operatorname{AppellF1}\left[1, -2m, 2m, 2, \cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2 \right) - \\ & \left(B \operatorname{AppellF1}\left[1, -2m, 2m, 2, \cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \right) / \\ & \left(-m \left(\operatorname{AppellF1}\left[2, 1-2m, 2m, 3, \cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \right) + \right. \\ & \quad \left. \operatorname{AppellF1}\left[2, -2m, 1+2m, 3, \cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \right) + \\ & \quad \left. \operatorname{AppellF1}\left[1, -2m, 2m, 2, \cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\cot\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2 \right) + \\ & \left(A \operatorname{AppellF1}\left[1, -2m, 2m, 2, \tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2 \right) / \\ & \left(\operatorname{AppellF1}\left[1, -2m, 2m, 2, \tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] - \right. \\ & \quad \left. m \left(\operatorname{AppellF1}\left[2, 1-2m, 2m, 3, \tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2, -\tan\left[\frac{1}{4}(-e + \frac{\pi}{2} - fx)\right]^2\right] \right) + \right. \end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[2, -2m, 1+2m, 3, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 + \\
& \left(\text{B AppellF1}\left[1, -2m, 2m, 2, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right) / \\
& \left(\text{AppellF1}\left[1, -2m, 2m, 2, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] - \right. \\
& m \left(\text{AppellF1}\left[2, 1-2m, 2m, 3, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \left. \text{AppellF1}\left[2, -2m, 1+2m, 3, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right) - \\
& \left(4A(1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \\
& \left. \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \left(-1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\right) / \\
& \left((1+2m) \left(-2(1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\
& \left. \left(\text{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + m \text{AppellF1}\left[2+2m, \right. \right. \\
& \left. \left. 1+2m, 1, 3+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \left(-1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\right) \right) + \\
& \left(12B(1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right. \\
& \left. \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \left(-1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\right) / \\
& \left((1+2m) \left(-2(1+m) \text{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\
& \left. \left(\text{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + m \text{AppellF1}\left[2+2m, 1+2m, \right. \right. \\
& \left. \left. 1, 3+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \left(-1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\right) \right) \right) / \left(8\sqrt{2} f \right. \\
& \left. (c - c \sin[e + fx])^{3/2} \left(\frac{1}{4\sqrt{2}} m \left(\frac{1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}\right)^{-1+2m} \left(-\frac{\sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)}{2 \left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^2}\right) - \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left(\frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) / \\ & \left((1+2m) \left(-2(1+m) \operatorname{AppellF1}\left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\ & \left. \left(\operatorname{AppellF1}\left[2+2m, 2m, 2, 3+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + m \operatorname{AppellF1}\left[2+2m, 1+2m, \right. \right. \right. \\ & \left. \left. \left. 1, 3+2m, \frac{1}{2} - \frac{1}{2} \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, 1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \right) \left(-1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) \right) \right) \end{aligned}$$

- **Problem 210: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^m (A + B \sin[e + fx])}{(c - c \sin[e + fx])^{5/2}} dx$$

Optimal (type 5, 134 leaves, 4 steps):

$$\begin{aligned} & \frac{(A + B) \cos[e + fx] (a + a \sin[e + fx])^m}{4 f (c - c \sin[e + fx])^{5/2}} + \\ & \left((A(3 - 2m) - B(5 + 2m)) \cos[e + fx] \operatorname{Hypergeometric2F1}\left[2, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + fx])\right] (a + a \sin[e + fx])^m \right) / \\ & \left(16 c^2 f (1 + 2m) \sqrt{c - c \sin[e + fx]} \right) \end{aligned}$$

Result (type 6, 28451 leaves): Display of huge result suppressed!

- **Problem 214: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m (A + B \sin[e + fx]) (c - c \sin[e + fx])^{-1-m} dx$$

Optimal (type 5, 163 leaves, 5 steps):

$$\begin{aligned} & \frac{(A + B) \cos[e + fx] (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{-1-m}}{f (1 + 2m)} - \frac{1}{f (1 + 2m)} 2^{\frac{1}{2}-m} B \cos[e + fx] \\ & \operatorname{Hypergeometric2F1}\left[\frac{1}{2} (1 + 2m), \frac{1}{2} (1 + 2m), \frac{1}{2} (3 + 2m), \frac{1}{2} (1 + \sin[e + fx])\right] (1 - \sin[e + fx])^{\frac{1}{2}+m} (a + a \sin[e + fx])^m (c - c \sin[e + fx])^{-1-m} \end{aligned}$$

Result (type 6, 6197 leaves):

$$- \left(\left(2^{-1-3m} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \right)^{-2m} \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right] \right)^{-2(-1-m)} (a + a \sin[e + fx])^m \right)$$

$$\begin{aligned}
& (c - c \operatorname{Sin}[e + f x])^{-1-m} \left(A \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^{-2-2m} + \right. \\
& B \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Sin}[e + f x] \left(\operatorname{Cos}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \operatorname{Sin}\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^{-2-2m} \left(\frac{1}{1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2m} \\
& \left(\frac{\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]}{1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{-2m} \left(\left(8 B (-3 + 2 m) \operatorname{AppellF1}\left[\frac{1}{2} - m, -2 m, 1, \frac{3}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(1 - \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{2m} \right) / \right. \\
& \left((-1 + 2 m) \left(1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left((-3 + 2 m) \operatorname{AppellF1}\left[\frac{1}{2} - m, -2 m, 1, \frac{3}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\
& \left. \left. 2 \left(2 m \operatorname{AppellF1}\left[\frac{3}{2} - m, 1 - 2 m, 1, \frac{5}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2} - m, -2 m, 2, \frac{5}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) - \\
& \frac{1}{-1 + 4 m^2} (A + B) \left((-1 + 2 m) \operatorname{Hypergeometric2F1}\left[-\frac{1}{2} - m, -2 m, \frac{1}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \\
& \left. (1 + 2 m) \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - m, -2 m, \frac{3}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left. \right) / \\
& \left(f \left(-2^{-3-3m} \operatorname{Csc}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(\frac{1}{1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2m} \left(\frac{\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]}{1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{-2m} \left(\left(8 B (-3 + 2 m) \operatorname{AppellF1}\left[\frac{1}{2} - m, \right. \right. \right. \right. \\
& \left. \left. \left. -2 m, 1, \frac{3}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(1 - \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{2m} \right) \right) / \right. \\
& \left((-1 + 2 m) \left(1 + \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left((-3 + 2 m) \operatorname{AppellF1}\left[\frac{1}{2} - m, -2 m, 1, \frac{3}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + 2 \left(2 m \operatorname{AppellF1}\left[\frac{3}{2} - m, 1 - 2 m, 1, \frac{5}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} - m, -2 m, 2, \frac{5}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) \right) - \\
& \frac{1}{-1 + 4 m^2} (A + B) \left((-1 + 2 m) \operatorname{Hypergeometric2F1}\left[-\frac{1}{2} - m, -2 m, \frac{1}{2} - m, \operatorname{Tan}\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 3, \frac{7}{2} - m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \\
& 2 m \left(-\frac{1}{2\left(\frac{5}{2} - m\right)}\left(\frac{3}{2} - m\right) \operatorname{AppellF1}\left[\frac{5}{2} - m, 1 - 2 m, 2, \frac{7}{2} - m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{2\left(\frac{5}{2} - m\right)}(1 - 2 m)\left(\frac{3}{2} - m\right) \operatorname{AppellF1}\left[\frac{5}{2} - m, 2 - 2 m, 1, \frac{7}{2} - m, \right.\right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)\right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left((-1 + 2 m) \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right) \left((-3 + 2 m) \operatorname{AppellF1}\left[\frac{1}{2} - m, -2 m, 1, \frac{3}{2} - m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right.\right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + 2 \left(2 m \operatorname{AppellF1}\left[\frac{3}{2} - m, 1 - 2 m, 1, \frac{5}{2} - m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.\right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} - m, -2 m, 2, \frac{5}{2} - m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^2 \right) \right) - \\
& \frac{1}{-1 + 4 m^2} (A + B) \left(\frac{1}{2}(1 + 2 m) \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - m, -2 m, \frac{3}{2} - m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right. \\
& \left. \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] + \frac{1}{2}\left(-\frac{1}{2} - m\right) (-1 + 2 m) \operatorname{Csc}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2} - m, \right.\right. \right. \\
& \left. \left. \left. -2 m, \frac{1}{2} - m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{2 m}\right) + \frac{1}{2}\left(\frac{1}{2} - m\right) (1 + 2 m) \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \right. \\
& \left. \left. \left. \left. \left. \left. \frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2} - m, -2 m, \frac{3}{2} - m, \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{2 m}\right)\right)\right)\right)\right)\right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 215: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) (c - c \sin[e + f x])^{-m} dx$$

Optimal (type 5, 158 leaves, 5 steps):

$$\frac{1}{f(1+2m)} 2^{\frac{1}{2}-m} c (A+2Bm) \operatorname{Cos}[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(1+2m), \frac{1}{2}(1+2m), \frac{1}{2}(3+2m), \frac{1}{2}(1+\sin[e+fx])\right] \\
(1-\sin[e+fx])^{\frac{1}{2}+m} (a+a \sin[e+fx])^m (c-c \sin[e+fx])^{-1-m} - \frac{B \operatorname{Cos}[e+fx] (a+a \sin[e+fx])^m (c-c \sin[e+fx])^{-m}}{f}$$

Result (type 6, 15390 leaves):

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}-m, -2m, 2, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 + \\
& \left(\text{B AppellF1}\left[\frac{1}{2}-m, -2m, 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \left(1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^2 \right. \\
& \left. \left(\left(2m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}-m, -2m, \right. \right. \right. \\
& \left. \left. \left. 2, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \right. \\
& \left. (-3+2m) \left(-\frac{1}{\frac{3}{2}-m} \left(\frac{1}{2}-m\right) m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{2\left(\frac{3}{2}-m\right)} \left(\frac{1}{2}-m\right) \text{AppellF1}\left[\frac{3}{2}-m, -2m, 2, \frac{5}{2}-m, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) + \\
& 2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(-\frac{1}{\frac{5}{2}-m} \left(\frac{3}{2}-m\right) m \text{AppellF1}\left[\frac{5}{2}-m, 1-2m, 2, \frac{7}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{\frac{5}{2}-m} \left(\frac{3}{2}-m\right) \text{AppellF1}\left[\frac{5}{2}-m, -2m, \right. \right. \\
& \left. \left. 3, \frac{7}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \right. \\
& \left. 2m \left(-\frac{1}{2\left(\frac{5}{2}-m\right)} \left(\frac{3}{2}-m\right) \text{AppellF1}\left[\frac{5}{2}-m, 1-2m, 2, \frac{7}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \frac{1}{2\left(\frac{5}{2}-m\right)} (1-2m) \left(\frac{3}{2}-m\right) \text{AppellF1}\left[\frac{5}{2}-m, 2-2m, 1, \frac{7}{2}-m, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \right) \Big/ \\
& \left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2m, 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \\
& \left. 2 \left(2m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}-m, -2m, 3, \frac{5}{2}-m, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 + \\
& \left(8 \text{B AppellF1}\left[\frac{1}{2}-m, -2m, 3, \frac{3}{2}-m, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \\
& \left. \left(\left(2m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 3, \frac{5}{2}-m, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + 3 \text{AppellF1}\left[\frac{3}{2}-m, -2m, \right. \right. \right. \\
& \left. \left. \left. 4, \frac{5}{2}-m, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \right. \right. \\
& \left. \left. (-3+2m) \left(-\frac{1}{\frac{3}{2}-m}\left(\frac{1}{2}-m\right)m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 3, \frac{5}{2}-m, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{2\left(\frac{3}{2}-m\right)}3\left(\frac{1}{2}-m\right) \text{AppellF1}\left[\frac{3}{2}-m, -2m, 4, \frac{5}{2}-m, \right. \right. \right. \\
& \left. \left. \left. \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) + \\
& 2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(2m \left(-\frac{1}{2\left(\frac{5}{2}-m\right)}3\left(\frac{3}{2}-m\right) \text{AppellF1}\left[\frac{5}{2}-m, 1-2m, 4, \frac{7}{2}-m, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + \frac{1}{2\left(\frac{5}{2}-m\right)}(1-2m)\left(\frac{3}{2}-m\right) \text{AppellF1}\left[\frac{5}{2}-m, \right. \right. \right. \\
& \left. \left. \left. 2-2m, 3, \frac{7}{2}-m, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) + \\
& 3 \left(-\frac{1}{\frac{5}{2}-m}\left(\frac{3}{2}-m\right)m \text{AppellF1}\left[\frac{5}{2}-m, 1-2m, 4, \frac{7}{2}-m, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right. \\
& \left. \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{\frac{5}{2}-m}2\left(\frac{3}{2}-m\right) \text{AppellF1}\left[\frac{5}{2}-m, -2m, 5, \frac{7}{2}-m, \right. \right. \right. \\
& \left. \left. \left. \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \text{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \Bigg) \Bigg) / \\
& \left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2m, 3, \frac{3}{2}-m, \text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\text{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right.
\end{aligned}$$

$$2 \left(2 m \operatorname{AppellF1} \left[\frac{3}{2} - m, 1 - 2 m, 3, \frac{5}{2} - m, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \right. \\ \left. 3 \operatorname{AppellF1} \left[\frac{3}{2} - m, -2 m, 4, \frac{5}{2} - m, \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^2 \right)$$

- **Problem 216: Attempted integration timed out after 120 seconds.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) (c - c \sin[e + f x])^{1-m} dx$$

Optimal (type 5, 170 leaves, 5 steps):

$$\frac{1}{f(1+2m)} 2^{\frac{1}{2}-m} c^2 (2A - B(1-2m)) \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}(-1+2m), \frac{1}{2}(1+2m), \frac{1}{2}(3+2m), \frac{1}{2}(1+\sin[e + f x]) \right] \\ (1 - \sin[e + f x])^{\frac{1}{2}+m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1-m} - \frac{B \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{1-m}}{2f}$$

Result (type 1, 1 leaves):

???

- **Problem 217: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) (c - c \sin[e + f x])^{2-m} dx$$

Optimal (type 5, 173 leaves, 5 steps):

$$\frac{1}{3f(1+2m)} 2^{\frac{5}{2}-m} c^3 (3A - 2B(1-m)) \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}(-3+2m), \frac{1}{2}(1+2m), \frac{1}{2}(3+2m), \frac{1}{2}(1+\sin[e + f x]) \right] \\ (1 - \sin[e + f x])^{\frac{1}{2}+m} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1-m} - \frac{B \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{2-m}}{3f}$$

Result (type 6, 37061 leaves): Display of huge result suppressed!

- **Problem 232: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + d x]^5 (a + a \sin[c + d x])^3 (A - A \sin[c + d x]) dx$$

Optimal (type 3, 86 leaves, 10 steps):

$$\frac{5 a^3 A \operatorname{ArcTanh}[\cos[c + d x]]}{8 d} - \frac{2 a^3 A \operatorname{Cot}[c + d x]^3}{3 d} - \frac{3 a^3 A \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]}{8 d} - \frac{a^3 A \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^3}{4 d}$$

Result (type 3, 210 leaves):

$$a^3 A \left(\frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{3d} - \frac{3 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{32d} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{12d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{64d} + \frac{5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} - \frac{5 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{3 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{32d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}{64d} - \frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{3d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{12d} \right)$$

■ **Problem 233: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^6 (a+a \operatorname{Sin}[c+dx])^3 (A-A \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 105 leaves, 12 steps):

$$\frac{a^3 A \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{4d} - \frac{2 a^3 A \operatorname{Cot}[c+dx]^3}{3d} - \frac{a^3 A \operatorname{Cot}[c+dx]^5}{5d} + \frac{a^3 A \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{4d} - \frac{a^3 A \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{2d}$$

Result (type 3, 268 leaves):

$$a^3 A \left(\frac{7 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{30d} + \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{16d} - \frac{19 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{480d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{32d} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{160d} + \frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{4d} - \frac{\operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{4d} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{16d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}{32d} - \frac{7 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{30d} + \frac{19 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{480d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{160d} \right)$$

■ **Problem 234: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^7 (a+a \operatorname{Sin}[c+dx])^3 (A-A \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 130 leaves, 12 steps):

$$\frac{3 a^3 A \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{16d} - \frac{2 a^3 A \operatorname{Cot}[c+dx]^3}{3d} - \frac{2 a^3 A \operatorname{Cot}[c+dx]^5}{5d} + \frac{3 a^3 A \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{16d} - \frac{5 a^3 A \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^3}{24d} - \frac{a^3 A \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^5}{6d}$$

Result (type 3, 306 leaves):

$$a^3 A \left(\frac{2 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{15d} + \frac{3 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{64d} + \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{240d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{64d} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{80d} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6}{384d} + \frac{3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{16d} - \frac{3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{16d} - \frac{3 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{64d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}{64d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6}{384d} - \frac{2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{15d} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{240d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{80d} \right)$$

■ **Problem 236: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c+dx]^3 (A - A \operatorname{Sin}[c+dx])}{(a + a \operatorname{Sin}[c+dx])^3} dx$$

Optimal (type 3, 103 leaves, 9 steps):

$$\frac{4Ax}{a^3} + \frac{A \operatorname{Cos}[c+dx]}{a^3 d} + \frac{2A \operatorname{Cos}[c+dx]}{5a^3 d (1 + \operatorname{Sin}[c+dx])^3} - \frac{31A \operatorname{Cos}[c+dx]}{15a^3 d (1 + \operatorname{Sin}[c+dx])^2} + \frac{104A \operatorname{Cos}[c+dx]}{15a^3 d (1 + \operatorname{Sin}[c+dx])}$$

Result (type 3, 228 leaves):

$$\frac{1}{120 a^3 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^5} A \left(-1200 dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 1665 \operatorname{Cos}\left[c + \frac{dx}{2}\right] - 1675 \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + 600 dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 120 dx \operatorname{Cos}\left[2c + \frac{5dx}{2}\right] + 75 \operatorname{Cos}\left[3c + \frac{5dx}{2}\right] + 15 \operatorname{Cos}\left[3c + \frac{7dx}{2}\right] + 2495 \operatorname{Sin}\left[\frac{dx}{2}\right] - 1200 dx \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 600 dx \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 405 \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 491 \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 120 dx \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] + 15 \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] \right)$$

■ **Problem 237: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c+dx]^2 (A - A \operatorname{Sin}[c+dx])}{(a + a \operatorname{Sin}[c+dx])^3} dx$$

Optimal (type 3, 89 leaves, 8 steps):

$$-\frac{Ax}{a^3} - \frac{2A \operatorname{Cos}[c+dx]}{5a^3 d (1 + \operatorname{Sin}[c+dx])^3} + \frac{7A \operatorname{Cos}[c+dx]}{5a^3 d (1 + \operatorname{Sin}[c+dx])^2} - \frac{13A \operatorname{Cos}[c+dx]}{5a^3 d (1 + \operatorname{Sin}[c+dx])}$$

Result (type 3, 189 leaves):

$$\left(A \left(-50 d x \cos \left[\frac{d x}{2} \right] + 110 \cos \left[c + \frac{d x}{2} \right] - 90 \cos \left[c + \frac{3 d x}{2} \right] + 25 d x \cos \left[2 c + \frac{3 d x}{2} \right] + 5 d x \cos \left[2 c + \frac{5 d x}{2} \right] + 150 \sin \left[\frac{d x}{2} \right] - 50 d x \sin \left[c + \frac{d x}{2} \right] - 25 d x \sin \left[c + \frac{3 d x}{2} \right] + 40 \sin \left[2 c + \frac{3 d x}{2} \right] - 26 \sin \left[2 c + \frac{5 d x}{2} \right] + 5 d x \sin \left[3 c + \frac{5 d x}{2} \right] \right) / \left(20 a^3 d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^5 \right)$$

■ **Problem 240: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc [c + d x] (A - A \sin [c + d x])}{(a + a \sin [c + d x])^3} dx$$

Optimal (type 3, 98 leaves, 9 steps):

$$-\frac{A \operatorname{ArcTanh}[\cos [c + d x]]}{a^3 d} + \frac{2 A \cos [c + d x]}{5 a^3 d (1 + \sin [c + d x])^3} + \frac{3 A \cos [c + d x]}{5 a^3 d (1 + \sin [c + d x])^2} + \frac{8 A \cos [c + d x]}{5 a^3 d (1 + \sin [c + d x])}$$

Result (type 3, 313 leaves):

$$\left(\left(\left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right) \left(2 \cos \left[\frac{c}{2} \right] - 2 \sin \left[\frac{c}{2} \right] + 3 \cos \left[\frac{c}{2} \right] \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2 - 3 \sin \left[\frac{c}{2} \right] \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2 - 5 \log \left[\cos \left[\frac{1}{2} (c + d x) \right] \right] \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^4 + 5 \log \left[\sin \left[\frac{1}{2} (c + d x) \right] \right] \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^4 \right) + 2 \sin \left[\frac{d x}{2} \right] (-17 + 4 \cos [2 (c + d x)] - 19 \sin [c + d x]) \right) (A - A \sin [c + d x]) / \left(5 a^3 d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^2 \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^5 \right)$$

■ **Problem 241: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc [c + d x]^2 (A - A \sin [c + d x])}{(a + a \sin [c + d x])^3} dx$$

Optimal (type 3, 113 leaves, 15 steps):

$$\frac{4 A \operatorname{ArcTanh}[\cos [c + d x]]}{a^3 d} - \frac{A \cot [c + d x]}{a^3 d} - \frac{2 A \cot [c + d x]}{5 a^3 d (1 + \csc [c + d x])^3} + \frac{31 A \cot [c + d x]}{15 a^3 d (1 + \csc [c + d x])^2} - \frac{104 A \cot [c + d x]}{15 a^3 d (1 + \csc [c + d x])}$$

Result (type 3, 252 leaves):

$$\frac{1}{a^3} A \left(-\frac{\text{Cot}\left[\frac{1}{2}(c+dx)\right]}{2d} + \frac{4 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{d} - \frac{4 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{d} + \frac{4 \text{Sin}\left[\frac{1}{2}(c+dx)\right]}{5d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{2}{5d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{38 \text{Sin}\left[\frac{1}{2}(c+dx)\right]}{15d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{19}{15d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{158 \text{Sin}\left[\frac{1}{2}(c+dx)\right]}{15d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{\text{Tan}\left[\frac{1}{2}(c+dx)\right]}{2d} \right)$$

■ **Problem 242: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c+dx]^3 (A - A \text{Sin}[c+dx])}{(a + a \text{Sin}[c+dx])^3} dx$$

Optimal (type 3, 138 leaves, 13 steps):

$$-\frac{19 A \text{ArcTanh}[\text{Cos}[c+dx]]}{2 a^3 d} + \frac{4 A \text{Cot}[c+dx]}{a^3 d} - \frac{A \text{Cot}[c+dx] \text{Csc}[c+dx]}{2 a^3 d} + \frac{2 A \text{Cos}[c+dx]}{5 a^3 d (1 + \text{Sin}[c+dx])^3} + \frac{29 A \text{Cos}[c+dx]}{15 a^3 d (1 + \text{Sin}[c+dx])^2} + \frac{164 A \text{Cos}[c+dx]}{15 a^3 d (1 + \text{Sin}[c+dx])}$$

Result (type 3, 290 leaves):

$$\frac{1}{a^3} A \left(\frac{2 \text{Cot}\left[\frac{1}{2}(c+dx)\right]}{d} - \frac{\text{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{8d} - \frac{19 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{19 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{8d} - \frac{4 \text{Sin}\left[\frac{1}{2}(c+dx)\right]}{5d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \frac{2}{5d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{58 \text{Sin}\left[\frac{1}{2}(c+dx)\right]}{15d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{29}{15d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{328 \text{Sin}\left[\frac{1}{2}(c+dx)\right]}{15d \left(\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} - \frac{2 \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{d} \right)$$

■ **Problem 243: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c+dx]^4 (A - A \text{Sin}[c+dx])}{(a + a \text{Sin}[c+dx])^3} dx$$

Optimal (type 3, 153 leaves, 15 steps):

$$\frac{18 A \text{ArcTanh}[\text{Cos}[c+dx]]}{a^3 d} - \frac{10 A \text{Cot}[c+dx]}{a^3 d} - \frac{A \text{Cot}[c+dx]^3}{3 a^3 d} + \frac{2 A \text{Cot}[c+dx] \text{Csc}[c+dx]}{a^3 d} - \frac{2 A \text{Cos}[c+dx]}{5 a^3 d (1 + \text{Sin}[c+dx])^3} - \frac{13 A \text{Cos}[c+dx]}{5 a^3 d (1 + \text{Sin}[c+dx])^2} - \frac{93 A \text{Cos}[c+dx]}{5 a^3 d (1 + \text{Sin}[c+dx])}$$

Result (type 3, 348 leaves) :

$$\frac{1}{a^3} A \left(-\frac{29 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{6d} + \frac{\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{2d} - \frac{\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{24d} + \frac{18 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right]}{d} - \frac{18 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{d} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2d} + \frac{4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{5d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} - \frac{2}{5d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{26 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{5d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{13}{5d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{186 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{5d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{29 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{6d} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24d} \right)$$

■ **Problem 248: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x]) (A + B \operatorname{Sin}[e + f x])}{c + d \operatorname{Sin}[e + f x]} dx$$

Optimal (type 3, 98 leaves, 6 steps) :

$$-\frac{a(Bc - (A+B)d)x}{d^2} + \frac{2a(c-d)(Bc - Ad) \operatorname{ArcTan}\left[\frac{d+c \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c^2-d^2}}\right]}{d^2 \sqrt{c^2-d^2} f} - \frac{aB \operatorname{Cos}[e + f x]}{df}$$

Result (type 3, 196 leaves) :

$$\left(a \left(A dx + B(-c+d)x - \frac{Bd \operatorname{Cos}[e] \operatorname{Cos}[fx]}{f} + \frac{2(c-d)(Bc - Ad) \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{fx}{2}\right] (\operatorname{Cos}[e] - i \operatorname{Sin}[e]) (d \operatorname{Cos}\left[e + \frac{fx}{2}\right] + c \operatorname{Sin}\left[\frac{fx}{2}\right])}{\sqrt{c^2-d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}}\right]}{\sqrt{c^2-d^2} f \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}} \right) (\operatorname{Cos}[e] - i \operatorname{Sin}[e]) + \frac{Bd \operatorname{Sin}[e] \operatorname{Sin}[fx]}{f} (1 + \operatorname{Sin}[e + f x]) \right) / \left(d^2 \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right)^2 \right)$$

■ **Problem 249: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x]) (A + B \operatorname{Sin}[e + f x])}{(c + d \operatorname{Sin}[e + f x])^2} dx$$

Optimal (type 3, 124 leaves, 6 steps) :

$$\frac{a B x}{d^2} + \frac{2 a \left((A + B) (c - d) d^2 - B c (c^2 - d^2) \right) \text{ArcTan}\left[\frac{d + c \text{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{c^2 - d^2}}\right]}{d^2 (c^2 - d^2)^{3/2} f} + \frac{a (B c - A d) \text{Cos}[e + f x]}{d (c + d) f (c + d \text{Sin}[e + f x])}$$

Result (type 3, 217 leaves):

$$\left(a (1 + \text{Sin}[e + f x]) \left(B x + \frac{2 (A d^2 - B (c^2 + c d - d^2)) \text{ArcTan}\left[\frac{\text{Sec}\left[\frac{f x}{2}\right] (\text{Cos}[e] - i \text{Sin}[e]) (d \text{Cos}\left[e + \frac{f x}{2}\right] + c \text{Sin}\left[\frac{f x}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2}}\right]}{(c + d) \sqrt{c^2 - d^2} f \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2}} \right) + \frac{(-B c + A d) \text{Csc}[e] (c \text{Cos}[e] + d \text{Sin}[f x])}{(c + d) f (c + d \text{Sin}[e + f x])} \right) \right) / \left(d^2 \left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right] \right)^2 \right)$$

■ **Problem 250: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \text{Sin}[e + f x]) (A + B \text{Sin}[e + f x])}{(c + d \text{Sin}[e + f x])^3} dx$$

Optimal (type 3, 176 leaves, 7 steps):

$$\frac{a (2 A c + B c - A d - 2 B d) \text{ArcTan}\left[\frac{d + c \text{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{c^2 - d^2}}\right]}{(c + d) (c^2 - d^2)^{3/2} f} + \frac{a (B c - A d) \text{Cos}[e + f x]}{2 d (c + d) f (c + d \text{Sin}[e + f x])^2} - \frac{a (A (c - 2 d) d + B (c^2 + 2 c d - 2 d^2)) \text{Cos}[e + f x]}{2 (c - d) d (c + d)^2 f (c + d \text{Sin}[e + f x])}$$

Result (type 3, 345 leaves):

$$\left(a (1 + \text{Sin}[e + f x]) \left(\frac{4 (2 A c + B c - A d - 2 B d) \text{ArcTan}\left[\frac{\text{Sec}\left[\frac{f x}{2}\right] (\text{Cos}[e] - i \text{Sin}[e]) (d \text{Cos}\left[e + \frac{f x}{2}\right] + c \text{Sin}\left[\frac{f x}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2}}\right]}{(c + d) \sqrt{c^2 - d^2} f \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2}} \right) (\text{Cos}[e] - i \text{Sin}[e]) + \frac{1}{d^2 (c + d \text{Sin}[e + f x])^2} \left((2 c^2 + d^2) (A (c - 2 d) d + B (c^2 + 2 c d - 2 d^2)) \text{Cot}[e] + d \text{Csc}[e] (-d (A (c - 2 d) d + B (c^2 + 2 c d - 2 d^2)) \text{Cos}[e + 2 f x] + (B c (2 c^2 + 6 c d - 5 d^2) - A d (-4 c^2 + 6 c d + d^2)) \text{Sin}[f x] + (A d^2 (-2 c + d) + B c (2 c^2 + 2 c d - 3 d^2)) \text{Sin}[2 e + f x]) \right) \right) \right) / \left(4 (c - d) (c + d)^2 f \left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right] \right)^2 \right)$$

■ **Problem 264: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^3 (A + B \sin[e + f x])}{(c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 305 leaves, 8 steps):

$$\begin{aligned} & - \frac{a^3 (3 B c - A d - 3 B d) x}{d^4} - \frac{a^3 (c - d) (A d (2 c^2 + 6 c d + 7 d^2) - 3 B (2 c^3 + 4 c^2 d + c d^2 - 2 d^3)) \operatorname{ArcTan}\left[\frac{d + c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{c^2 - d^2}}\right]}{d^4 (c + d)^2 \sqrt{c^2 - d^2} f} \\ & + \frac{a^3 (3 B c (2 c + 3 d) - A d (2 c + 5 d)) \cos[e + f x]}{2 d^3 (c + d)^2 f} + \frac{a (B c - A d) \cos[e + f x] (a + a \sin[e + f x])^2}{2 d (c + d) f (c + d \sin[e + f x])^2} \\ & - \frac{(A d (c + 4 d) - B (3 c^2 + 4 c d - 2 d^2)) \cos[e + f x] (a^3 + a^3 \sin[e + f x])}{2 d^2 (c + d)^2 f (c + d \sin[e + f x])} \end{aligned}$$

Result (type 3, 830 leaves):

$$\begin{aligned} & \frac{1}{4 d^4 (c + d)^2 f \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^6} \\ & + a^3 (1 + \sin[e + f x])^3 \left(\frac{4 (c - d) (-A d (2 c^2 + 6 c d + 7 d^2) + 3 B (2 c^3 + 4 c^2 d + c d^2 - 2 d^3)) \operatorname{ArcTan}\left[\frac{d + c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{c^2 - d^2}}\right]}{\sqrt{c^2 - d^2}} + \right. \\ & \left. \frac{1}{(c + d \sin[e + f x])^2} \left(-12 B c^5 e + 4 A c^4 d e - 12 B c^4 d e + 8 A c^3 d^2 e + 6 B c^3 d^2 e + 6 A c^2 d^3 e + 6 B c^2 d^3 e + 4 A c d^4 e + 6 B c d^4 e + 2 A d^5 e + 6 B d^5 e - \right. \right. \\ & 12 B c^5 f x + 4 A c^4 d f x - 12 B c^4 d f x + 8 A c^3 d^2 f x + 6 B c^3 d^2 f x + 6 A c^2 d^3 f x + 6 B c^2 d^3 f x + 4 A c d^4 f x + 6 B c d^4 f x + 2 A d^5 f x + \\ & 6 B d^5 f x - d \left(2 A d (-2 c^3 - 4 c^2 d + 5 c d^2 + d^3) + B (12 c^4 + 12 c^3 d - 9 c^2 d^2 + 4 c d^3 + d^4) \right) \cos[e + f x] - 2 d^2 (c + d)^2 (-3 B c + A d + 3 B d) \\ & (e + f x) \cos[2 (e + f x)] + B c^2 d^3 \cos[3 (e + f x)] + 2 B c d^4 \cos[3 (e + f x)] + B d^5 \cos[3 (e + f x)] - 24 B c^4 d e \sin[e + f x] + \\ & 8 A c^3 d^2 e \sin[e + f x] - 24 B c^3 d^2 e \sin[e + f x] + 16 A c^2 d^3 e \sin[e + f x] + 24 B c^2 d^3 e \sin[e + f x] + 8 A c d^4 e \sin[e + f x] + \\ & 24 B c d^4 e \sin[e + f x] - 24 B c^4 d f x \sin[e + f x] + 8 A c^3 d^2 f x \sin[e + f x] - 24 B c^3 d^2 f x \sin[e + f x] + 16 A c^2 d^3 f x \sin[e + f x] + \\ & 24 B c^2 d^3 f x \sin[e + f x] + 8 A c d^4 f x \sin[e + f x] + 24 B c d^4 f x \sin[e + f x] - 9 B c^3 d^2 \sin[2 (e + f x)] + 3 A c^2 d^3 \sin[2 (e + f x)] - \\ & \left. \left. 9 B c^2 d^3 \sin[2 (e + f x)] + 3 A c d^4 \sin[2 (e + f x)] + 4 B c d^4 \sin[2 (e + f x)] - 6 A d^5 \sin[2 (e + f x)] - 2 B d^5 \sin[2 (e + f x)] \right) \right) \end{aligned}$$

■ **Problem 265: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^3}{a + a \sin[e + f x]} dx$$

Optimal (type 3, 220 leaves, 3 steps):

$$\frac{(3 A d (2 c^2 - 2 c d + d^2) + B (2 c^3 - 6 c^2 d + 9 c d^2 - 3 d^3)) x}{2 a} + \frac{2 d (3 A (c^2 - 3 c d + d^2) - B (7 c^2 - 9 c d + 4 d^2)) \operatorname{Cos}[e + f x]}{3 a f} +$$

$$\frac{d^2 (6 A c - 11 B c - 9 A d + 9 B d) \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{6 a f} + \frac{(3 A - 4 B) d \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^2}{3 a f} - \frac{(A - B) \operatorname{Cos}[e + f x] (c + d \operatorname{Sin}[e + f x])^3}{f (a + a \operatorname{Sin}[e + f x])}$$

Result (type 3, 788 leaves):

$$\frac{1}{24 a f (1 + \operatorname{Sin}[e + f x])} \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] \right) \left(3 (4 A d (6 c^2 (e + f x) - 3 c d (1 + 2 e + 2 f x)) + d^2 (1 + 3 e + 3 f x)) + \right.$$

$$B (8 c^3 (e + f x) - 12 c^2 d (1 + 2 e + 2 f x) + 12 c d^2 (1 + 3 e + 3 f x) - d^3 (7 + 12 e + 12 f x)) \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] +$$

$$9 d (A d (-4 c + d) + B (-4 c^2 + 3 c d - 2 d^2)) \operatorname{Cos}\left[\frac{3}{2} (e + f x)\right] + 9 B c d^2 \operatorname{Cos}\left[\frac{5}{2} (e + f x)\right] + 3 A d^3 \operatorname{Cos}\left[\frac{5}{2} (e + f x)\right] - 2 B d^3 \operatorname{Cos}\left[\frac{5}{2} (e + f x)\right] +$$

$$B d^3 \operatorname{Cos}\left[\frac{7}{2} (e + f x)\right] + 48 A c^3 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 48 B c^3 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 144 A c^2 d \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 180 B c^2 d \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] +$$

$$180 A c d^2 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 180 B c d^2 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 60 A d^3 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 69 B d^3 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 24 B c^3 e \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] +$$

$$72 A c^2 d e \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 72 B c^2 d e \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 72 A c d^2 e \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 108 B c d^2 e \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] +$$

$$36 A d^3 e \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 36 B d^3 e \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 24 B c^3 f x \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 72 A c^2 d f x \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] -$$

$$72 B c^2 d f x \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 72 A c d^2 f x \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 108 B c d^2 f x \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 36 A d^3 f x \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] -$$

$$36 B d^3 f x \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 36 B c^2 d \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] - 36 A c d^2 \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] + 27 B c d^2 \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] + 9 A d^3 \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] -$$

$$18 B d^3 \operatorname{Sin}\left[\frac{3}{2} (e + f x)\right] - 9 B c d^2 \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] - 3 A d^3 \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] + 2 B d^3 \operatorname{Sin}\left[\frac{5}{2} (e + f x)\right] + B d^3 \operatorname{Sin}\left[\frac{7}{2} (e + f x)\right] \left. \right)$$

■ **Problem 268: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sin}[e + f x]}{a + a \operatorname{Sin}[e + f x]} dx$$

Optimal (type 3, 35 leaves, 2 steps):

$$\frac{B x}{a} - \frac{(A - B) \operatorname{Cos}[e + f x]}{f (a + a \operatorname{Sin}[e + f x])}$$

Result (type 3, 79 leaves):

$$\frac{1}{a f (1 + \operatorname{Sin}[e + f x])} \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] \right) \left(B (e + f x) \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + (2 A + B (-2 + e + f x)) \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] \right)$$

■ **Problem 272: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^3}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 3, 228 leaves, 3 steps):

$$\frac{d (2 A (3 c - 2 d) d + B (6 c^2 - 12 c d + 7 d^2)) x}{2 a^2} + \frac{2 d (A (c^2 + 6 c d - 5 d^2) + B (2 c^2 - 15 c d + 8 d^2)) \cos[e + f x]}{3 a^2 f} + \frac{d^2 (B (4 c - 21 d) + 2 A (c + 6 d)) \cos[e + f x] \sin[e + f x]}{6 a^2 f} - \frac{(2 B (c - 4 d) + A (c + 5 d)) \cos[e + f x] (c + d \sin[e + f x])^2}{3 a^2 f (1 + \sin[e + f x])} - \frac{(A - B) \cos[e + f x] (c + d \sin[e + f x])^3}{3 f (a + a \sin[e + f x])^2}$$

Result (type 3, 1032 leaves):

$$\frac{1}{48 f (a + a \sin[e + f x])^2} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(\begin{aligned} & \left(48 B c^3 \cos\left[\frac{1}{2} (e + f x)\right] + 144 A c^2 d \cos\left[\frac{1}{2} (e + f x)\right] - 288 B c^2 d \cos\left[\frac{1}{2} (e + f x)\right] - 288 A c d^2 \cos\left[\frac{1}{2} (e + f x)\right] + 360 B c d^2 \cos\left[\frac{1}{2} (e + f x)\right] + \right. \\ & 120 A d^3 \cos\left[\frac{1}{2} (e + f x)\right] - 147 B d^3 \cos\left[\frac{1}{2} (e + f x)\right] + 216 B c^2 d (e + f x) \cos\left[\frac{1}{2} (e + f x)\right] + 216 A c d^2 (e + f x) \cos\left[\frac{1}{2} (e + f x)\right] - \\ & 432 B c d^2 (e + f x) \cos\left[\frac{1}{2} (e + f x)\right] - 144 A d^3 (e + f x) \cos\left[\frac{1}{2} (e + f x)\right] + 252 B d^3 (e + f x) \cos\left[\frac{1}{2} (e + f x)\right] - 16 A c^3 \cos\left[\frac{3}{2} (e + f x)\right] - \\ & 32 B c^3 \cos\left[\frac{3}{2} (e + f x)\right] - 96 A c^2 d \cos\left[\frac{3}{2} (e + f x)\right] + 240 B c^2 d \cos\left[\frac{3}{2} (e + f x)\right] + 240 A c d^2 \cos\left[\frac{3}{2} (e + f x)\right] - 492 B c d^2 \cos\left[\frac{3}{2} (e + f x)\right] - \\ & 164 A d^3 \cos\left[\frac{3}{2} (e + f x)\right] + 239 B d^3 \cos\left[\frac{3}{2} (e + f x)\right] - 72 B c^2 d (e + f x) \cos\left[\frac{3}{2} (e + f x)\right] - 72 A c d^2 (e + f x) \cos\left[\frac{3}{2} (e + f x)\right] + \\ & 144 B c d^2 (e + f x) \cos\left[\frac{3}{2} (e + f x)\right] + 48 A d^3 (e + f x) \cos\left[\frac{3}{2} (e + f x)\right] - 84 B d^3 (e + f x) \cos\left[\frac{3}{2} (e + f x)\right] + 36 B c d^2 \cos\left[\frac{5}{2} (e + f x)\right] + \\ & 12 A d^3 \cos\left[\frac{5}{2} (e + f x)\right] - 15 B d^3 \cos\left[\frac{5}{2} (e + f x)\right] + 3 B d^3 \cos\left[\frac{7}{2} (e + f x)\right] + 48 A c^3 \sin\left[\frac{1}{2} (e + f x)\right] + 48 B c^3 \sin\left[\frac{1}{2} (e + f x)\right] + \\ & 144 A c^2 d \sin\left[\frac{1}{2} (e + f x)\right] - 432 B c^2 d \sin\left[\frac{1}{2} (e + f x)\right] - 432 A c d^2 \sin\left[\frac{1}{2} (e + f x)\right] + 792 B c d^2 \sin\left[\frac{1}{2} (e + f x)\right] + \\ & 264 A d^3 \sin\left[\frac{1}{2} (e + f x)\right] - 381 B d^3 \sin\left[\frac{1}{2} (e + f x)\right] + 216 B c^2 d (e + f x) \sin\left[\frac{1}{2} (e + f x)\right] + 216 A c d^2 (e + f x) \sin\left[\frac{1}{2} (e + f x)\right] - \\ & 432 B c d^2 (e + f x) \sin\left[\frac{1}{2} (e + f x)\right] - 144 A d^3 (e + f x) \sin\left[\frac{1}{2} (e + f x)\right] + 252 B d^3 (e + f x) \sin\left[\frac{1}{2} (e + f x)\right] - \\ & 108 B c d^2 \sin\left[\frac{3}{2} (e + f x)\right] - 36 A d^3 \sin\left[\frac{3}{2} (e + f x)\right] + 63 B d^3 \sin\left[\frac{3}{2} (e + f x)\right] + 72 B c^2 d (e + f x) \sin\left[\frac{3}{2} (e + f x)\right] + \\ & 72 A c d^2 (e + f x) \sin\left[\frac{3}{2} (e + f x)\right] - 144 B c d^2 (e + f x) \sin\left[\frac{3}{2} (e + f x)\right] - 48 A d^3 (e + f x) \sin\left[\frac{3}{2} (e + f x)\right] + \\ & \left. 84 B d^3 (e + f x) \sin\left[\frac{3}{2} (e + f x)\right] - 36 B c d^2 \sin\left[\frac{5}{2} (e + f x)\right] - 12 A d^3 \sin\left[\frac{5}{2} (e + f x)\right] + 15 B d^3 \sin\left[\frac{5}{2} (e + f x)\right] + 3 B d^3 \sin\left[\frac{7}{2} (e + f x)\right] \right) \end{aligned} \right)$$

■ **Problem 273: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^2}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 3, 132 leaves, 5 steps):

$$\frac{d (2 B (c - d) + A d) x}{a^2} + \frac{(A - 4 B) d^2 \cos[e + f x]}{3 a^2 f} - \frac{(c - d) (2 B (c - 3 d) + A (c + 3 d)) \cos[e + f x]}{3 a^2 f (1 + \sin[e + f x])} - \frac{(A - B) \cos[e + f x] (c + d \sin[e + f x])^2}{3 f (a + a \sin[e + f x])^2}$$

Result (type 3, 338 leaves):

$$\frac{1}{12 a^2 f (1 + \sin[e + f x])^2} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(6 (A d (4 c + d (-4 + 3 e + 3 f x)) + B (2 c^2 + d^2 (5 - 6 e - 6 f x) + 2 c d (-4 + 3 e + 3 f x))) \cos\left[\frac{1}{2}(e + f x)\right] - (B (8 c^2 + d^2 (41 - 12 e - 12 f x) + 4 c d (-10 + 3 e + 3 f x)) + 2 A (2 c^2 + 8 c d + d^2 (-10 + 3 e + 3 f x))) \cos\left[\frac{3}{2}(e + f x)\right] + 3 B d^2 \cos\left[\frac{5}{2}(e + f x)\right] + 6 (2 A c^2 + 2 B c^2 + 4 A c d - 12 B c d - 6 A d^2 + 9 B d^2 + 8 B c d e + 4 A d^2 e - 8 B d^2 e + 8 B c d f x + 4 A d^2 f x - 8 B d^2 f x - 2 d (-2 B c (e + f x) - A d (e + f x) + 2 B d (1 + e + f x)) \cos[e + f x] - B d^2 \cos[2(e + f x)]) \sin\left[\frac{1}{2}(e + f x)\right] \right)$$

■ **Problem 274: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 3, 85 leaves, 4 steps):

$$\frac{B d x}{a^2} - \frac{(A c + 2 B c + 2 A d - 5 B d) \cos[e + f x]}{3 a^2 f (1 + \sin[e + f x])} - \frac{(A - B) (c - d) \cos[e + f x]}{3 f (a + a \sin[e + f x])^2}$$

Result (type 3, 180 leaves):

$$\frac{1}{3 a^2 f (1 + \sin[e + f x])^2} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(2 (A - B) (c - d) \sin\left[\frac{1}{2}(e + f x)\right] - (A - B) (c - d) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) + 2 (A c + 2 B c + 2 A d - 5 B d) \sin\left[\frac{1}{2}(e + f x)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + 3 B d (e + f x) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3$$

■ **Problem 278: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^2 (c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 386 leaves, 8 steps):

$$\frac{d \left(A d \left(12 c^2 + 16 c d + 7 d^2 \right) - B \left(6 c^3 + 12 c^2 d + 13 c d^2 + 4 d^3 \right) \right) \operatorname{ArcTan} \left[\frac{d+c \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{c^2-d^2}} \right]}{a^2 (c-d)^4 (c+d)^2 \sqrt{c^2-d^2} f} - \frac{d \left(A \left(2 c^2 - 16 c d - 21 d^2 \right) + B \left(4 c^2 + 19 c d + 12 d^2 \right) \right) \operatorname{Cos}[e+f x]}{6 a^2 (c-d)^3 (c+d) f (c+d \operatorname{Sin}[e+f x])^2} - \frac{(A c + 2 B c - 8 A d + 5 B d) \operatorname{Cos}[e+f x]}{3 a^2 (c-d)^2 f (1 + \operatorname{Sin}[e+f x]) (c+d \operatorname{Sin}[e+f x])^2} - \frac{(A-B) \operatorname{Cos}[e+f x]}{3 (c-d) f (a+a \operatorname{Sin}[e+f x])^2 (c+d \operatorname{Sin}[e+f x])^2} - \frac{d \left(A \left(2 c^3 - 16 c^2 d - 59 c d^2 - 32 d^3 \right) + B \left(4 c^3 + 37 c^2 d + 44 c d^2 + 20 d^3 \right) \right) \operatorname{Cos}[e+f x]}{6 a^2 (c-d)^4 (c+d)^2 f (c+d \operatorname{Sin}[e+f x])}$$

Result (type 3, 1522 leaves):

$$- \left(d \left(6 B c^3 - 12 A c^2 d + 12 B c^2 d - 16 A c d^2 + 13 B c d^2 - 7 A d^3 + 4 B d^3 \right) \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \left(d \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] + c \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] \right)}{\sqrt{c^2-d^2}} \right] \right) \left(\operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] \right)^4 \Bigg/ \left((c-d)^4 (c+d)^2 \sqrt{c^2-d^2} f (a+a \operatorname{Sin}[e+f x])^2 \right) + \frac{1}{48 (c-d)^4 (c+d)^2 f (a+a \operatorname{Sin}[e+f x])^2 (c+d \operatorname{Sin}[e+f x])^2} \left(\operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] + \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] \right) \left(48 B c^5 \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] - 96 A c^4 d \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] + 240 B c^4 d \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] - 524 A c^3 d^2 \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] + 536 B c^3 d^2 \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] - 776 A c^2 d^3 \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] + 701 B c^2 d^3 \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] - 487 A c d^4 \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] + 400 B c d^4 \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] - 112 A d^5 \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] + 70 B d^5 \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right] - 16 A c^5 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] - 32 B c^5 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] + 80 A c^4 d \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] - 224 B c^4 d \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] + 536 A c^3 d^2 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] - 728 B c^3 d^2 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] + 1028 A c^2 d^3 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] - 893 B c^2 d^3 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] + 695 A c d^4 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] - 482 B c d^4 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] + 134 A d^5 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] - 98 B d^5 \operatorname{Cos} \left[\frac{3}{2} (e+f x) \right] + 24 B c^3 d^2 \operatorname{Cos} \left[\frac{5}{2} (e+f x) \right] - 12 A c^2 d^3 \operatorname{Cos} \left[\frac{5}{2} (e+f x) \right] + 21 B c^2 d^3 \operatorname{Cos} \left[\frac{5}{2} (e+f x) \right] - 15 A c d^4 \operatorname{Cos} \left[\frac{5}{2} (e+f x) \right] - 18 B c d^4 \operatorname{Cos} \left[\frac{5}{2} (e+f x) \right] + 6 A d^5 \operatorname{Cos} \left[\frac{5}{2} (e+f x) \right] - 6 B d^5 \operatorname{Cos} \left[\frac{5}{2} (e+f x) \right] + 4 A c^3 d^2 \operatorname{Cos} \left[\frac{7}{2} (e+f x) \right] + 8 B c^3 d^2 \operatorname{Cos} \left[\frac{7}{2} (e+f x) \right] - 32 A c^2 d^3 \operatorname{Cos} \left[\frac{7}{2} (e+f x) \right] + 59 B c^2 d^3 \operatorname{Cos} \left[\frac{7}{2} (e+f x) \right] - 97 A c d^4 \operatorname{Cos} \left[\frac{7}{2} (e+f x) \right] + 76 B c d^4 \operatorname{Cos} \left[\frac{7}{2} (e+f x) \right] - 52 A d^5 \operatorname{Cos} \left[\frac{7}{2} (e+f x) \right] + 34 B d^5 \operatorname{Cos} \left[\frac{7}{2} (e+f x) \right] + 48 A c^5 \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] + 48 B c^5 \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] - 224 A c^4 d \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] + 416 B c^4 d \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] - 872 A c^3 d^2 \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] + 992 B c^3 d^2 \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] - 1144 A c^2 d^3 \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] + 967 B c^2 d^3 \operatorname{Sin} \left[\frac{1}{2} (e+f x) \right] -$$

$$\begin{aligned}
& 685 A c d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 496 B c d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 168 A d^5 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 126 B d^5 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 48 B c^4 d \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - \\
& 132 A c^3 d^2 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 96 B c^3 d^2 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 204 A c^2 d^3 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 207 B c^2 d^3 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - \\
& 165 A c d^4 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 174 B c d^4 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 66 A d^5 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 42 B d^5 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 16 A c^4 d \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - \\
& 32 B c^4 d \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + 116 A c^3 d^2 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 224 B c^3 d^2 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + 412 A c^2 d^3 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - \\
& 409 B c^2 d^3 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + 403 A c d^4 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 286 B c d^4 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + 114 A d^5 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 78 B d^5 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + \\
& 15 B c^2 d^3 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] - 21 A c d^4 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] + 12 B c d^4 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] - 12 A d^5 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] + 6 B d^5 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] \Big)
\end{aligned}$$

■ **Problem 280: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A+B \operatorname{Sin}[e+f x])(c+d \operatorname{Sin}[e+f x])^2}{(a+a \operatorname{Sin}[e+f x])^3} dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$\begin{aligned}
& \frac{B d^2 x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d)) \operatorname{Cos}[e+f x]}{15 a f (a+a \operatorname{Sin}[e+f x])^2} - \\
& \frac{(B(3c^2+14cd-29d^2)+2A(c^2+3cd+2d^2)) \operatorname{Cos}[e+f x]}{15 f (a^3+a^3 \operatorname{Sin}[e+f x])} - \frac{(A-B) \operatorname{Cos}[e+f x](c+d \operatorname{Sin}[e+f x])^2}{5 f (a+a \operatorname{Sin}[e+f x])^3}
\end{aligned}$$

Result (type 3, 514 leaves):

$$\begin{aligned}
& \frac{1}{60 a^3 f (1+\operatorname{Sin}[e+f x])^3} \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right) \\
& \left(30(2Ad(c+d)+B(c^2+4cd+d^2(-9+5e+5fx))) \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - 5(4A(c^2+3cd+2d^2)+B(6c^2+16cd+d^2(-46+15e+15fx))) \right. \\
& \quad \left. \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] - 15Bd^2e \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] - 15Bd^2fx \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] + 40Ac^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 30Bc^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \quad \left. 60Acd \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 160Bcd \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 80Ad^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 370Bd^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 150Bd^2e \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + \right. \\
& \quad \left. 150Bd^2fx \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 60Bcd \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 30Ad^2 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 90Bd^2 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + \right. \\
& \quad \left. 75Bd^2e \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 75Bd^2fx \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 4Ac^2 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 6Bc^2 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 12Acd \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - \right. \\
& \quad \left. 28Bcd \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 14Ad^2 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + 64Bd^2 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 15Bd^2e \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 15Bd^2fx \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] \right)
\end{aligned}$$

■ **Problem 283: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^3 (c + d \sin[e + f x])} dx$$

Optimal (type 3, 229 leaves, 7 steps):

$$\frac{2 d^2 (B c - A d) \operatorname{ArcTan}\left[\frac{d + c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{c^2 - d^2}}\right]}{a^3 (c - d)^3 \sqrt{c^2 - d^2} f} - \frac{(A - B) \cos[e + f x]}{5 (c - d) f (a + a \sin[e + f x])^3} -$$

$$\frac{(2 A c + 3 B c - 7 A d + 2 B d) \cos[e + f x]}{15 a (c - d)^2 f (a + a \sin[e + f x])^2} - \frac{(B (3 c^2 - 16 c d - 2 d^2) + A (2 c^2 - 9 c d + 22 d^2)) \cos[e + f x]}{15 (c - d)^3 f (a^3 + a^3 \sin[e + f x])}$$

Result (type 3, 502 leaves):

$$\frac{1}{30 a^3 (c - d)^3 f (1 + \sin[e + f x])^3}$$

$$\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(15 B c^2 \cos\left[\frac{1}{2}(e + f x)\right] - 15 A c d \cos\left[\frac{1}{2}(e + f x)\right] - 75 B c d \cos\left[\frac{1}{2}(e + f x)\right] + 75 A d^2 \cos\left[\frac{1}{2}(e + f x)\right] - \right.$$

$$10 A c^2 \cos\left[\frac{3}{2}(e + f x)\right] - 15 B c^2 \cos\left[\frac{3}{2}(e + f x)\right] + 45 A c d \cos\left[\frac{3}{2}(e + f x)\right] + 65 B c d \cos\left[\frac{3}{2}(e + f x)\right] - 95 A d^2 \cos\left[\frac{3}{2}(e + f x)\right] +$$

$$10 B d^2 \cos\left[\frac{3}{2}(e + f x)\right] + 20 A c^2 \sin\left[\frac{1}{2}(e + f x)\right] + 15 B c^2 \sin\left[\frac{1}{2}(e + f x)\right] - 75 A c d \sin\left[\frac{1}{2}(e + f x)\right] - 85 B c d \sin\left[\frac{1}{2}(e + f x)\right] +$$

$$145 A d^2 \sin\left[\frac{1}{2}(e + f x)\right] - 20 B d^2 \sin\left[\frac{1}{2}(e + f x)\right] - \frac{60 d^2 (-B c + A d) \operatorname{ArcTan}\left[\frac{d + c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{c^2 - d^2}}\right] (\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right])^5}{\sqrt{c^2 - d^2}} -$$

$$15 B c d \sin\left[\frac{3}{2}(e + f x)\right] + 15 A d^2 \sin\left[\frac{3}{2}(e + f x)\right] - 2 A c^2 \sin\left[\frac{5}{2}(e + f x)\right] - 3 B c^2 \sin\left[\frac{5}{2}(e + f x)\right] +$$

$$\left. 9 A c d \sin\left[\frac{5}{2}(e + f x)\right] + 16 B c d \sin\left[\frac{5}{2}(e + f x)\right] - 22 A d^2 \sin\left[\frac{5}{2}(e + f x)\right] + 2 B d^2 \sin\left[\frac{5}{2}(e + f x)\right] \right)$$

■ **Problem 284: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^3 (c + d \sin[e + f x])^2} dx$$

Optimal (type 3, 381 leaves, 8 steps) :

$$\frac{2 d^2 (A d (4 c + 3 d) - B (3 c^2 + 3 c d + d^2)) \operatorname{ArcTan}\left[\frac{d+c \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c^2-d^2}}\right]}{a^3 (c-d)^4 (c+d) \sqrt{c^2-d^2} f} - \frac{d (B (3 c^3 - 23 c^2 d - 63 c d^2 - 22 d^3) + A (2 c^3 - 12 c^2 d + 43 c d^2 + 72 d^3)) \operatorname{Cos}[e+f x]}{15 a^3 (c-d)^4 (c+d) f (c+d \operatorname{Sin}[e+f x])} - \frac{(A-B) \operatorname{Cos}[e+f x]}{5 (c-d) f (a+a \operatorname{Sin}[e+f x])^3 (c+d \operatorname{Sin}[e+f x])} - \frac{(2 A c + 3 B c - 9 A d + 4 B d) \operatorname{Cos}[e+f x]}{15 a (c-d)^2 f (a+a \operatorname{Sin}[e+f x])^2 (c+d \operatorname{Sin}[e+f x])} - \frac{(B (3 c^2 - 23 c d - 15 d^2) + A (2 c^2 - 12 c d + 45 d^2)) \operatorname{Cos}[e+f x]}{15 (c-d)^3 f (a^3 + a^3 \operatorname{Sin}[e+f x]) (c+d \operatorname{Sin}[e+f x])}$$

Result (type 3, 1253 leaves) :

$$\left(2d^2 (3Bc^2 - 4Acd + 3Bcd - 3Ad^2 + Bd^2) \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{1}{2} (e + fx) \right] (d \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] + c \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right])}{\sqrt{c^2 - d^2}} \right] \right. \\ \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] \right)^6 \right) / \left((c-d)^4 (c+d) \sqrt{c^2 - d^2} f (a + a \operatorname{Sin}[e + fx])^3 \right) + \\ \frac{1}{120 (c-d)^4 (c+d) f (a + a \operatorname{Sin}[e + fx])^3 (c+d \operatorname{Sin}[e + fx])} \left(\operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] + \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] \right) \\ \left(60Bc^4 \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] - 80Ac^3 d \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] - 390Bc^3 d \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] + 540Ac^2 d^2 \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] - 1090Bc^2 d^2 \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] + \right. \\ 1430Ac d^3 \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] - 885Bc d^3 \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] + 735Ad^4 \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] - 320Bd^4 \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right] - 40Ac^4 \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] - \\ 60Bc^4 \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] + 196Ac^3 d \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] + 304Bc^3 d \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] - 476Ac^2 d^2 \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] + 1076Bc^2 d^2 \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] - \\ 1546Ac d^3 \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] + 1181Bc d^3 \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] - 969Ad^4 \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] + 334Bd^4 \operatorname{Cos} \left[\frac{3}{2} (e + fx) \right] + \\ 60Bc^2 d^2 \operatorname{Cos} \left[\frac{5}{2} (e + fx) \right] - 90Ac d^3 \operatorname{Cos} \left[\frac{5}{2} (e + fx) \right] + 15Bc d^3 \operatorname{Cos} \left[\frac{5}{2} (e + fx) \right] - 15Ad^4 \operatorname{Cos} \left[\frac{5}{2} (e + fx) \right] + 30Bd^4 \operatorname{Cos} \left[\frac{5}{2} (e + fx) \right] + \\ 4Ac^3 d \operatorname{Cos} \left[\frac{7}{2} (e + fx) \right] + 6Bc^3 d \operatorname{Cos} \left[\frac{7}{2} (e + fx) \right] - 24Ac^2 d^2 \operatorname{Cos} \left[\frac{7}{2} (e + fx) \right] - 46Bc^2 d^2 \operatorname{Cos} \left[\frac{7}{2} (e + fx) \right] + 86Ac d^3 \operatorname{Cos} \left[\frac{7}{2} (e + fx) \right] - \\ 111Bc d^3 \operatorname{Cos} \left[\frac{7}{2} (e + fx) \right] + 129Ad^4 \operatorname{Cos} \left[\frac{7}{2} (e + fx) \right] - 44Bd^4 \operatorname{Cos} \left[\frac{7}{2} (e + fx) \right] + 80Ac^4 \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] + 60Bc^4 \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] - \\ 340Ac^3 d \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] - 440Bc^3 d \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] + 820Ac^2 d^2 \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] - 1520Bc^2 d^2 \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] + \\ 2140Ac d^3 \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] - 1435Bc d^3 \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] + 975Ad^4 \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] - 340Bd^4 \operatorname{Sin} \left[\frac{1}{2} (e + fx) \right] - 90Bc^3 d \operatorname{Sin} \left[\frac{3}{2} (e + fx) \right] + \\ 120Ac^2 d^2 \operatorname{Sin} \left[\frac{3}{2} (e + fx) \right] - 390Bc^2 d^2 \operatorname{Sin} \left[\frac{3}{2} (e + fx) \right] + 540Ac d^3 \operatorname{Sin} \left[\frac{3}{2} (e + fx) \right] - 315Bc d^3 \operatorname{Sin} \left[\frac{3}{2} (e + fx) \right] + \\ 285Ad^4 \operatorname{Sin} \left[\frac{3}{2} (e + fx) \right] - 150Bd^4 \operatorname{Sin} \left[\frac{3}{2} (e + fx) \right] - 8Ac^4 \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] - 12Bc^4 \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] + 28Ac^3 d \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] + \\ 62Bc^3 d \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] - 52Ac^2 d^2 \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] + 362Bc^2 d^2 \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] - 568Ac d^3 \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] + \\ 553Bc d^3 \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] - 555Ad^4 \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] + 190Bd^4 \operatorname{Sin} \left[\frac{5}{2} (e + fx) \right] - 15Bc d^3 \operatorname{Sin} \left[\frac{7}{2} (e + fx) \right] + 15Ad^4 \operatorname{Sin} \left[\frac{7}{2} (e + fx) \right] \left. \right)$$

■ **Problem 290: Result is not expressed in closed-form.**

$$\int \frac{\sqrt{a + a \operatorname{Sin}[e + fx]} (A + B \operatorname{Sin}[e + fx])}{c + d \operatorname{Sin}[e + fx]} dx$$

Optimal (type 3, 100 leaves, 3 steps) :

$$\frac{2 \sqrt{a} (B c - A d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+f x]}{\sqrt{c+d} \sqrt{a+a \sin[e+f x]}}\right]}{d^{3/2} \sqrt{c+d} f} - \frac{2 a B \cos[e+f x]}{d f \sqrt{a+a \sin[e+f x]}}$$

Result (type 7, 903 leaves) :

$$\frac{1}{d^{3/2} \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)}$$

$$\left(\frac{1}{2} + \frac{i}{2} \right) \left(-\frac{(2-2i)B\sqrt{d}\cos\left[\frac{fx}{2}\right]\left(\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]\right)}{f} + \frac{1}{\sqrt{c+d}\left(\cos[e] + i(-1+\sin[e])\right)\sqrt{\cos[e] - i\sin[e]}} \right)$$

$$(-Bc+Ad)\left(\cos\left[\frac{e}{2}\right] + i\sin\left[\frac{e}{2}\right]\right) \left((-1+i)x\cos[e] + \frac{1}{4f}\text{RootSum}\left[-d+2ic e^{ie}\#1^2+d e^{2ie}\#1^4\ \&, \right. \right.$$

$$\left. \left. \frac{1}{(d-ic e^{ie}\#1^2)} \left((1+i)d\sqrt{e^{-ie}}fx - (2-2i)d\sqrt{e^{-ie}}\text{Log}\left[e^{\frac{ifx}{2}}-\#1\right] - i\sqrt{d}\sqrt{c+d}fx\#1 + 2\sqrt{d}\sqrt{c+d}\text{Log}\left[e^{\frac{ifx}{2}}-\#1\right]\#1 + \right. \right.$$

$$\left. \left. \frac{(1-i)cfx\#1^2}{\sqrt{e^{-ie}}} + \frac{(2+2i)c\text{Log}\left[e^{\frac{ifx}{2}}-\#1\right]\#1^2}{\sqrt{e^{-ie}}} - \sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3 - 2i\sqrt{d}\sqrt{c+d}e^{ie}\text{Log}\left[e^{\frac{ifx}{2}}-\#1\right]\#1^3 \right) \& \right)$$

$$\left. \left. \left(\cos[e] + i(-1+\sin[e])\sqrt{\cos[e] - i\sin[e]} + (1+i)x\sin[e] \right) + \frac{1}{\sqrt{c+d}\left(\cos[e] + i(-1+\sin[e])\right)\sqrt{\cos[e] - i\sin[e]}} \right)$$

$$(-Bc+Ad)\left(\cos\left[\frac{e}{2}\right] + i\sin\left[\frac{e}{2}\right]\right) \left((1-i)x\cos[e] - (1+i)x\sin[e] + \frac{1}{4f}\text{RootSum}\left[-d+2ic e^{ie}\#1^2+d e^{2ie}\#1^4\ \&, \right. \right.$$

$$\left. \left. \frac{1}{(d-ic e^{ie}\#1^2)} \left((1-i)d\sqrt{e^{-ie}}fx + (2+2i)d\sqrt{e^{-ie}}\text{Log}\left[e^{\frac{ifx}{2}}-\#1\right] + \sqrt{d}\sqrt{c+d}fx\#1 + 2i\sqrt{d}\sqrt{c+d}\text{Log}\left[e^{\frac{ifx}{2}}-\#1\right]\#1 - \right. \right.$$

$$\left. \left. \frac{(1+i)cfx\#1^2}{\sqrt{e^{-ie}}} + \frac{(2-2i)c\text{Log}\left[e^{\frac{ifx}{2}}-\#1\right]\#1^2}{\sqrt{e^{-ie}}} - i\sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3 + 2\sqrt{d}\sqrt{c+d}e^{ie}\text{Log}\left[e^{\frac{ifx}{2}}-\#1\right]\#1^3 \right) \& \right)$$

$$\left. \left. \sqrt{\cos[e] - i\sin[e]}(-1-i\cos[e] + \sin[e]) \right) + \frac{(2-2i)B\sqrt{d}\left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]\right)\sin\left[\frac{fx}{2}\right]}{f} \sqrt{a(1+\sin[e+fx])}$$

■ **Problem 291: Result is not expressed in closed-form.**

$$\int \frac{\sqrt{a+a\sin[e+fx]}(A+B\sin[e+fx])}{(c+d\sin[e+fx])^2} dx$$

Optimal (type 3, 126 leaves, 3 steps):

$$-\frac{\sqrt{a} (A d + B (c + 2 d)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c+d} \sqrt{a+a \sin[e+f x]}}\right]}{d^{3/2} (c+d)^{3/2} f} + \frac{a (B c - A d) \cos[e + f x]}{d (c+d) f \sqrt{a+a \sin[e+f x]} (c+d \sin[e+f x])}$$

Result (type 7, 901 leaves):

$$\frac{1}{d^{3/2} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)}$$

$$\left(\frac{1}{4} + \frac{i}{4} \right) \sqrt{a (1 + \sin[e + f x])} \left(\frac{1}{(c+d)^{3/2} (\cos[e] + i (-1 + \sin[e])) \sqrt{\cos[e] - i \sin[e]}} (A d + B (c + 2 d)) \left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right] \right) \right.$$

$$\left. \left((-1 + i) x \cos[e] + \frac{1}{4 f} \operatorname{RootSum}\left[-d + 2 i c e^{i e} \#1^2 + d e^{2 i e} \#1^4 \&, 1 / (d - i c e^{i e} \#1^2)\right] \left((1 + i) d \sqrt{e^{-i e}} f x - (2 - 2 i) d \sqrt{e^{-i e}} \operatorname{Log}\left[e^{\frac{i f x}{2}} - \#1\right] - \right. \right.$$

$$i \sqrt{d} \sqrt{c+d} f x \#1 + 2 \sqrt{d} \sqrt{c+d} \operatorname{Log}\left[e^{\frac{i f x}{2}} - \#1\right] \#1 + \frac{(1 - i) c f x \#1^2}{\sqrt{e^{-i e}}} + \frac{(2 + 2 i) c \operatorname{Log}\left[e^{\frac{i f x}{2}} - \#1\right] \#1^2}{\sqrt{e^{-i e}}} - \sqrt{d} \sqrt{c+d} e^{i e} f x \#1^3 -$$

$$\left. \left. 2 i \sqrt{d} \sqrt{c+d} e^{i e} \operatorname{Log}\left[e^{\frac{i f x}{2}} - \#1\right] \#1^3 \right) \& \right) (\cos[e] + i (-1 + \sin[e])) \sqrt{\cos[e] - i \sin[e]} + (1 + i) x \sin[e] \left. \right)$$

$$\frac{1}{(c+d)^{3/2} (\cos[e] + i (-1 + \sin[e])) \sqrt{\cos[e] - i \sin[e]}} (A d + B (c + 2 d)) \left(\cos\left[\frac{e}{2}\right] + i \sin\left[\frac{e}{2}\right] \right)$$

$$\left((1 - i) x \cos[e] - (1 + i) x \sin[e] + \frac{1}{4 f} \operatorname{RootSum}\left[-d + 2 i c e^{i e} \#1^2 + d e^{2 i e} \#1^4 \&, \right.$$

$$1 / (d - i c e^{i e} \#1^2) \left((1 - i) d \sqrt{e^{-i e}} f x + (2 + 2 i) d \sqrt{e^{-i e}} \operatorname{Log}\left[e^{\frac{i f x}{2}} - \#1\right] + \sqrt{d} \sqrt{c+d} f x \#1 + 2 i \sqrt{d} \sqrt{c+d} \operatorname{Log}\left[e^{\frac{i f x}{2}} - \#1\right] \#1 - \right.$$

$$\left. \left. \frac{(1 + i) c f x \#1^2}{\sqrt{e^{-i e}}} + \frac{(2 - 2 i) c \operatorname{Log}\left[e^{\frac{i f x}{2}} - \#1\right] \#1^2}{\sqrt{e^{-i e}}} - i \sqrt{d} \sqrt{c+d} e^{i e} f x \#1^3 + 2 \sqrt{d} \sqrt{c+d} e^{i e} \operatorname{Log}\left[e^{\frac{i f x}{2}} - \#1\right] \#1^3 \right) \& \right)$$

$$\left. \sqrt{\cos[e] - i \sin[e]} (-1 - i \cos[e] + \sin[e]) \right) - \frac{(2 - 2 i) \sqrt{d} (-B c + A d) \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)}{(c+d) f (c+d \sin[e + f x])}$$

■ **Problem 292: Result is not expressed in closed-form.**

$$\int \frac{\sqrt{a + a \sin[e + f x]} (A + B \sin[e + f x])}{(c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 192 leaves, 4 steps):

$$-\frac{\sqrt{a} (3 A d + B (c + 4 d)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{4 d^{3/2} (c + d)^{5/2} f} + \frac{a (B c - A d) \cos[e + f x]}{2 d (c + d) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^2} - \frac{a (3 A d + B (c + 4 d)) \cos[e + f x]}{4 d (c + d)^2 f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])}$$

Result (type 7, 967 leaves):

$$\begin{aligned}
& \frac{1}{d^{3/2} \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)} \\
& \left(\frac{1}{16} + \frac{i}{16} \right) \sqrt{a(1+\sin[e+fx])} \left(\frac{1}{(c+d)^{5/2} (\cos[e] + i(-1+\sin[e])) \sqrt{\cos[e] - i\sin[e]}} (3Ad+B(c+4d)) \left(\cos\left[\frac{e}{2}\right] + i\sin\left[\frac{e}{2}\right] \right) \right. \\
& \left. \left((-1+i)x\cos[e] + \frac{1}{4f} \text{RootSum}\left[-d+2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, 1/(d-ic e^{ie} \#1^2)\right] \left((1+i)d\sqrt{e^{-ie}}fx - (2-2i)d\sqrt{e^{-ie}} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] - \right. \right. \right. \\
& \left. \left. \left. i\sqrt{d}\sqrt{c+d}fx\#1 + 2\sqrt{d}\sqrt{c+d} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right]\#1 + \frac{(1-i)cfx\#1^2}{\sqrt{e^{-ie}}} + \frac{(2+2i)c \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right]\#1^2}{\sqrt{e^{-ie}}} - \sqrt{d}\sqrt{c+d} e^{ie}fx\#1^3 - \right. \right. \right. \\
& \left. \left. \left. 2i\sqrt{d}\sqrt{c+d} e^{ie} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right]\#1^3 \right) \& \right) (\cos[e] + i(-1+\sin[e])) \sqrt{\cos[e] - i\sin[e]} + (1+i)x\sin[e] \right) + \\
& \frac{1}{(c+d)^{5/2} (\cos[e] + i(-1+\sin[e])) \sqrt{\cos[e] - i\sin[e]}} (3Ad+B(c+4d)) \left(\cos\left[\frac{e}{2}\right] + i\sin\left[\frac{e}{2}\right] \right) \\
& \left((1-i)x\cos[e] - (1+i)x\sin[e] + \frac{1}{4f} \text{RootSum}\left[-d+2ic e^{ie} \#1^2 + d e^{2ie} \#1^4 \&, 1/(d-ic e^{ie} \#1^2)\right] \left((1-i)d\sqrt{e^{-ie}}fx + (2+2i)d \right. \right. \\
& \left. \left. \sqrt{e^{-ie}} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right] + \sqrt{d}\sqrt{c+d}fx\#1 + 2i\sqrt{d}\sqrt{c+d} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right]\#1 - \frac{(1+i)cfx\#1^2}{\sqrt{e^{-ie}}} + \frac{(2-2i)c \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right]\#1^2}{\sqrt{e^{-ie}}} - \right. \right. \\
& \left. \left. \left. i\sqrt{d}\sqrt{c+d} e^{ie}fx\#1^3 + 2\sqrt{d}\sqrt{c+d} e^{ie} \text{Log}\left[e^{\frac{ifx}{2}} - \#1\right]\#1^3 \right) \& \right) \sqrt{\cos[e] - i\sin[e]} (-1-i\cos[e] + \sin[e]) \right) - \\
& \frac{(4-4i)\sqrt{d}(-Bc+Ad) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)}{(c+d)f(c+d\sin[e+fx])^2} - \frac{(2-2i)\sqrt{d}(3Ad+B(c+4d)) \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)}{(c+d)^2f(c+d\sin[e+fx])} \right)
\end{aligned}$$

■ **Problem 293: Result more than twice size of optimal antiderivative.**

$$\int (a+a\sin[e+fx])^{3/2} (A+B\sin[e+fx]) (c+d\sin[e+fx])^3 dx$$

Optimal (type 3, 374 leaves, 6 steps):

$$\begin{aligned}
& \frac{4 a^2 (c+d) (15 c^2+10 c d+7 d^2) (11 A (c-17 d) d-3 B (c^2-9 c d+56 d^2)) \operatorname{Cos}[e+f x]}{3465 d^2 f \sqrt{a+a \operatorname{Sin}[e+f x]}} + \\
& \frac{8 a (5 c-d) (c+d) (11 A (c-17 d) d-3 B (c^2-9 c d+56 d^2)) \operatorname{Cos}[e+f x] \sqrt{a+a \operatorname{Sin}[e+f x]}}{3465 d f} + \\
& \frac{4 (c+d) (11 A (c-17 d) d-3 B (c^2-9 c d+56 d^2)) \operatorname{Cos}[e+f x] (a+a \operatorname{Sin}[e+f x])^{3/2}}{1155 f} + \\
& \frac{2 a^2 (11 A (c-17 d) d-3 B (c^2-9 c d+56 d^2)) \operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^3}{693 d^2 f \sqrt{a+a \operatorname{Sin}[e+f x]}} + \\
& \frac{2 a^2 (3 B (c-4 d)-11 A d) \operatorname{Cos}[e+f x] (c+d \operatorname{Sin}[e+f x])^4}{99 d^2 f \sqrt{a+a \operatorname{Sin}[e+f x]}} - \frac{2 a B \operatorname{Cos}[e+f x] \sqrt{a+a \operatorname{Sin}[e+f x]} (c+d \operatorname{Sin}[e+f x])^4}{11 d f}
\end{aligned}$$

Result (type 3, 1101 leaves):

1

$$55\,440 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^3$$

$$\begin{aligned} & (a(1+\sin[e+fx]))^{3/2} \left(-166\,320 A c^3 \cos\left[\frac{1}{2}(e+fx)\right] - 110\,880 B c^3 \cos\left[\frac{1}{2}(e+fx)\right] - 332\,640 A c^2 d \cos\left[\frac{1}{2}(e+fx)\right] - \right. \\ & 291\,060 B c^2 d \cos\left[\frac{1}{2}(e+fx)\right] - 291\,060 A c d^2 \cos\left[\frac{1}{2}(e+fx)\right] - 249\,480 B c d^2 \cos\left[\frac{1}{2}(e+fx)\right] - 83\,160 A d^3 \cos\left[\frac{1}{2}(e+fx)\right] - \\ & 76\,230 B d^3 \cos\left[\frac{1}{2}(e+fx)\right] - 18\,480 A c^3 \cos\left[\frac{3}{2}(e+fx)\right] - 27\,720 B c^3 \cos\left[\frac{3}{2}(e+fx)\right] - 83\,160 A c^2 d \cos\left[\frac{3}{2}(e+fx)\right] - \\ & 69\,300 B c^2 d \cos\left[\frac{3}{2}(e+fx)\right] - 69\,300 A c d^2 \cos\left[\frac{3}{2}(e+fx)\right] - 69\,300 B c d^2 \cos\left[\frac{3}{2}(e+fx)\right] - 23\,100 A d^3 \cos\left[\frac{3}{2}(e+fx)\right] - \\ & 20\,790 B d^3 \cos\left[\frac{3}{2}(e+fx)\right] + 55\,440 B c^3 \cos\left[\frac{5}{2}(e+fx)\right] + 16\,632 A c^2 d \cos\left[\frac{5}{2}(e+fx)\right] + 24\,948 B c^2 d \cos\left[\frac{5}{2}(e+fx)\right] + \\ & 24\,948 A c d^2 \cos\left[\frac{5}{2}(e+fx)\right] + 24\,948 B c d^2 \cos\left[\frac{5}{2}(e+fx)\right] + 8316 A d^3 \cos\left[\frac{5}{2}(e+fx)\right] + 9009 B d^3 \cos\left[\frac{5}{2}(e+fx)\right] + \\ & 5940 B c^2 d \cos\left[\frac{7}{2}(e+fx)\right] + 5940 A c d^2 \cos\left[\frac{7}{2}(e+fx)\right] + 8910 B c d^2 \cos\left[\frac{7}{2}(e+fx)\right] + 2970 A d^3 \cos\left[\frac{7}{2}(e+fx)\right] + \\ & 3465 B d^3 \cos\left[\frac{7}{2}(e+fx)\right] - 2310 B c d^2 \cos\left[\frac{9}{2}(e+fx)\right] - 770 A d^3 \cos\left[\frac{9}{2}(e+fx)\right] - 1155 B d^3 \cos\left[\frac{9}{2}(e+fx)\right] - \\ & 315 B d^3 \cos\left[\frac{11}{2}(e+fx)\right] + 166\,320 A c^3 \sin\left[\frac{1}{2}(e+fx)\right] + 110\,880 B c^3 \sin\left[\frac{1}{2}(e+fx)\right] + 332\,640 A c^2 d \sin\left[\frac{1}{2}(e+fx)\right] + \\ & 291\,060 B c^2 d \sin\left[\frac{1}{2}(e+fx)\right] + 291\,060 A c d^2 \sin\left[\frac{1}{2}(e+fx)\right] + 249\,480 B c d^2 \sin\left[\frac{1}{2}(e+fx)\right] + 83\,160 A d^3 \sin\left[\frac{1}{2}(e+fx)\right] + \\ & 76\,230 B d^3 \sin\left[\frac{1}{2}(e+fx)\right] - 18\,480 A c^3 \sin\left[\frac{3}{2}(e+fx)\right] - 27\,720 B c^3 \sin\left[\frac{3}{2}(e+fx)\right] - 83\,160 A c^2 d \sin\left[\frac{3}{2}(e+fx)\right] - \\ & 69\,300 B c^2 d \sin\left[\frac{3}{2}(e+fx)\right] - 69\,300 A c d^2 \sin\left[\frac{3}{2}(e+fx)\right] - 69\,300 B c d^2 \sin\left[\frac{3}{2}(e+fx)\right] - 23\,100 A d^3 \sin\left[\frac{3}{2}(e+fx)\right] - \\ & 20\,790 B d^3 \sin\left[\frac{3}{2}(e+fx)\right] - 55\,440 B c^3 \sin\left[\frac{5}{2}(e+fx)\right] - 16\,632 A c^2 d \sin\left[\frac{5}{2}(e+fx)\right] - 24\,948 B c^2 d \sin\left[\frac{5}{2}(e+fx)\right] - \\ & 24\,948 A c d^2 \sin\left[\frac{5}{2}(e+fx)\right] - 24\,948 B c d^2 \sin\left[\frac{5}{2}(e+fx)\right] - 8316 A d^3 \sin\left[\frac{5}{2}(e+fx)\right] - 9009 B d^3 \sin\left[\frac{5}{2}(e+fx)\right] + \\ & 5940 B c^2 d \sin\left[\frac{7}{2}(e+fx)\right] + 5940 A c d^2 \sin\left[\frac{7}{2}(e+fx)\right] + 8910 B c d^2 \sin\left[\frac{7}{2}(e+fx)\right] + 2970 A d^3 \sin\left[\frac{7}{2}(e+fx)\right] + \\ & 3465 B d^3 \sin\left[\frac{7}{2}(e+fx)\right] + 2310 B c d^2 \sin\left[\frac{9}{2}(e+fx)\right] + 770 A d^3 \sin\left[\frac{9}{2}(e+fx)\right] + 1155 B d^3 \sin\left[\frac{9}{2}(e+fx)\right] - 315 B d^3 \sin\left[\frac{11}{2}(e+fx)\right] \left. \right) \end{aligned}$$

■ **Problem 297: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e+fx])^{3/2} (A + B \sin[e+fx])}{c + d \sin[e+fx]} dx$$

Optimal (type 3, 153 leaves, 4 steps) :

$$-\frac{2 a^{3/2} (c-d) (B c-A d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos [e+f x]}{\sqrt{c+d} \sqrt{a+a \sin [e+f x]}}\right]}{d^{5/2} \sqrt{c+d} f} + \frac{2 a^2 (3 B c-3 A d-4 B d) \cos [e+f x]}{3 d^2 f \sqrt{a+a \sin [e+f x]}} - \frac{2 a B \cos [e+f x] \sqrt{a+a \sin [e+f x]}}{3 d f}$$

Result (type 3, 356 leaves) :

$$\frac{1}{6 d^{5/2} f \left(\cos \left[\frac{1}{2}(e+f x)\right] + \sin \left[\frac{1}{2}(e+f x)\right]\right)^3} (a(1+\sin [e+f x]))^{3/2} \\ \left(-6 \sqrt{d}(-2 B c+2 A d+3 B d) \cos \left[\frac{1}{2}(e+f x)\right] - 2 B d^{3/2} \cos \left[\frac{3}{2}(e+f x)\right] - \frac{1}{\sqrt{c+d}} 3(c-d)(B c-A d) \left(e+f x - 2 \log \left[\sec \left[\frac{1}{4}(e+f x)\right]^2\right]\right) + \right. \\ \left. 2 \log \left[-\sec \left[\frac{1}{4}(e+f x)\right]^2 \left(c+d + \sqrt{d} \sqrt{c+d} \cos \left[\frac{1}{2}(e+f x)\right] - \sqrt{d} \sqrt{c+d} \sin \left[\frac{1}{2}(e+f x)\right]\right)\right]\right) + \frac{1}{\sqrt{c+d}} 3(c-d)(B c-A d) \\ \left(e+f x - 2 \log \left[\sec \left[\frac{1}{4}(e+f x)\right]^2\right] + 2 \log \left[(c+d) \sec \left[\frac{1}{4}(e+f x)\right]^2 + \sqrt{d} \sqrt{c+d} \left(-1 + 2 \tan \left[\frac{1}{4}(e+f x)\right] + \tan \left[\frac{1}{4}(e+f x)\right]^2\right)\right]\right) + \\ \left. 6 \sqrt{d}(-2 B c+2 A d+3 B d) \sin \left[\frac{1}{2}(e+f x)\right] - 2 B d^{3/2} \sin \left[\frac{3}{2}(e+f x)\right]\right)$$

■ **Problem 300: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+a \sin [e+f x])^{5/2} (A+B \sin [e+f x]) (c+d \sin [e+f x])^3 dx$$

Optimal (type 3, 534 leaves, 7 steps) :

$$-\frac{(4 a^3 (c+d) (15 c^2+10 c d+7 d^2) (13 A d (3 c^2-38 c d+355 d^2) - B (15 c^3-150 c^2 d+799 c d^2-4184 d^3)) \cos [e+f x]) /}{(45045 d^3 f \sqrt{a+a \sin [e+f x]}) - \frac{1}{45045 d^2 f}} \\ 8 a^2 (5 c-d) (c+d) (13 A d (3 c^2-38 c d+355 d^2) - B (15 c^3-150 c^2 d+799 c d^2-4184 d^3)) \cos [e+f x] \sqrt{a+a \sin [e+f x]} - \\ \frac{1}{15015 d f} 4 a (c+d) (13 A d (3 c^2-38 c d+355 d^2) - B (15 c^3-150 c^2 d+799 c d^2-4184 d^3)) \cos [e+f x] (a+a \sin [e+f x])^{3/2} - \\ (2 a^3 (13 A d (3 c^2-38 c d+355 d^2) - B (15 c^3-150 c^2 d+799 c d^2-4184 d^3)) \cos [e+f x] (c+d \sin [e+f x])^3) / (9009 d^3 f \sqrt{a+a \sin [e+f x]}) - \\ \frac{2 a^3 (15 B c^2-39 A c d-75 B c d+299 A d^2+280 B d^2) \cos [e+f x] (c+d \sin [e+f x])^4}{1287 d^3 f \sqrt{a+a \sin [e+f x]}} + \\ \frac{2 a^2 (5 B c-13 A d-16 B d) \cos [e+f x] \sqrt{a+a \sin [e+f x]} (c+d \sin [e+f x])^4}{143 d^2 f} - \frac{2 a B \cos [e+f x] (a+a \sin [e+f x])^{3/2} (c+d \sin [e+f x])^4}{13 d f}$$

Result (type 3, 1565 leaves) :

$$\begin{aligned}
& \frac{B d^3 \operatorname{Cos}\left[\frac{13}{2}(e+f x)\right] (a(1+\operatorname{Sin}[e+f x]))^{5/2}}{416 f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^5} + \\
& \left((40 A c^3 + 30 B c^3 + 90 A c^2 d + 78 B c^2 d + 78 A c d^2 + 69 B c d^2 + 23 A d^3 + 21 B d^3) \left(\left(-\frac{1}{16} - \frac{i}{16} \right) \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \left(\frac{1}{16} - \frac{i}{16} \right) \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right) \right. \\
& \quad \left. (a(1+\operatorname{Sin}[e+f x]))^{5/2} \right) / \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^5 \right) + \\
& \left((40 A c^3 + 30 B c^3 + 90 A c^2 d + 78 B c^2 d + 78 A c d^2 + 69 B c d^2 + 23 A d^3 + 21 B d^3) \left(\left(-\frac{1}{16} + \frac{i}{16} \right) \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \left(\frac{1}{16} + \frac{i}{16} \right) \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right) \right. \\
& \quad \left. (a(1+\operatorname{Sin}[e+f x]))^{5/2} \right) / \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^5 \right) + \\
& \left((80 A c^3 + 88 B c^3 + 264 A c^2 d + 240 B c^2 d + 240 A c d^2 + 228 B c d^2 + 76 A d^3 + 71 B d^3) (a(1+\operatorname{Sin}[e+f x]))^{5/2} \right. \\
& \quad \left. \left(\left(-\frac{1}{192} + \frac{i}{192} \right) \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] - \left(\frac{1}{192} + \frac{i}{192} \right) \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] \right) \right) / \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^5 \right) + \\
& \left((80 A c^3 + 88 B c^3 + 264 A c^2 d + 240 B c^2 d + 240 A c d^2 + 228 B c d^2 + 76 A d^3 + 71 B d^3) (a(1+\operatorname{Sin}[e+f x]))^{5/2} \right. \\
& \quad \left. \left(\left(-\frac{1}{192} - \frac{i}{192} \right) \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] - \left(\frac{1}{192} - \frac{i}{192} \right) \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] \right) \right) / \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^5 \right) + \\
& \left((16 A c^3 + 40 B c^3 + 120 A c^2 d + 144 B c^2 d + 144 A c d^2 + 150 B c d^2 + 50 A d^3 + 51 B d^3) (a(1+\operatorname{Sin}[e+f x]))^{5/2} \right. \\
& \quad \left. \left(\left(\frac{1}{320} - \frac{i}{320} \right) \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] - \left(\frac{1}{320} + \frac{i}{320} \right) \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] \right) \right) / \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^5 \right) + \\
& \left((16 A c^3 + 40 B c^3 + 120 A c^2 d + 144 B c^2 d + 144 A c d^2 + 150 B c d^2 + 50 A d^3 + 51 B d^3) (a(1+\operatorname{Sin}[e+f x]))^{5/2} \right. \\
& \quad \left. \left(\left(\frac{1}{320} + \frac{i}{320} \right) \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] - \left(\frac{1}{320} - \frac{i}{320} \right) \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] \right) \right) / \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^5 \right) + \\
& \left((4 B c^3 + 12 A c^2 d + 30 B c^2 d + 30 A c d^2 + 39 B c d^2 + 13 A d^3 + 15 B d^3) (a(1+\operatorname{Sin}[e+f x]))^{5/2} \right. \\
& \quad \left. \left(\left(\frac{1}{224} + \frac{i}{224} \right) \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] + \left(\frac{1}{224} - \frac{i}{224} \right) \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] \right) \right) / \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^5 \right) + \\
& \left((4 B c^3 + 12 A c^2 d + 30 B c^2 d + 30 A c d^2 + 39 B c d^2 + 13 A d^3 + 15 B d^3) (a(1+\operatorname{Sin}[e+f x]))^{5/2} \right. \\
& \quad \left. \left(\left(\frac{1}{224} - \frac{i}{224} \right) \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] + \left(\frac{1}{224} + \frac{i}{224} \right) \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] \right) \right) / \left(f \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right)^5 \right) +
\end{aligned}$$

$$\begin{aligned}
& \left((6 B c^2 + 6 A c d + 15 B c d + 5 A d^2 + 7 B d^2) (a (1 + \sin[e + f x]))^{5/2} \left(\left(-\frac{1}{288} - \frac{i}{288} \right) d \cos\left[\frac{9}{2} (e + f x)\right] + \left(\frac{1}{288} - \frac{i}{288} \right) d \sin\left[\frac{9}{2} (e + f x)\right] \right) \right) / \\
& \left(f \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \right)^5 + \\
& \left((6 B c^2 + 6 A c d + 15 B c d + 5 A d^2 + 7 B d^2) (a (1 + \sin[e + f x]))^{5/2} \left(\left(-\frac{1}{288} + \frac{i}{288} \right) d \cos\left[\frac{9}{2} (e + f x)\right] + \left(\frac{1}{288} + \frac{i}{288} \right) d \sin\left[\frac{9}{2} (e + f x)\right] \right) \right) / \\
& \left(f \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \right)^5 + \\
& \left((6 B c + 2 A d + 5 B d) (a (1 + \sin[e + f x]))^{5/2} \left(\left(-\frac{1}{704} + \frac{i}{704} \right) d^2 \cos\left[\frac{11}{2} (e + f x)\right] - \left(\frac{1}{704} + \frac{i}{704} \right) d^2 \sin\left[\frac{11}{2} (e + f x)\right] \right) \right) / \\
& \left(f \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \right)^5 + \\
& \left((6 B c + 2 A d + 5 B d) (a (1 + \sin[e + f x]))^{5/2} \left(\left(-\frac{1}{704} - \frac{i}{704} \right) d^2 \cos\left[\frac{11}{2} (e + f x)\right] - \left(\frac{1}{704} - \frac{i}{704} \right) d^2 \sin\left[\frac{11}{2} (e + f x)\right] \right) \right) / \\
& \left(f \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \right)^5 - \frac{B d^3 (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{13}{2} (e + f x)\right]}{416 f \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^5}
\end{aligned}$$

■ **Problem 301: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^{5/2} (A + B \sin[e + f x]) (c + d \sin[e + f x])^2 dx$$

Optimal (type 3, 429 leaves, 6 steps):

$$\begin{aligned}
& - \frac{2 a^3 (15 c^2 + 10 c d + 7 d^2) (11 A d (c^2 - 10 c d + 73 d^2) - B (5 c^3 - 40 c^2 d + 169 c d^2 - 710 d^3)) \cos[e + f x]}{3465 d^3 f \sqrt{a + a \sin[e + f x]}} - \\
& \frac{1}{3465 d^2 f} 4 a^2 (5 c - d) (11 A d (c^2 - 10 c d + 73 d^2) - B (5 c^3 - 40 c^2 d + 169 c d^2 - 710 d^3)) \cos[e + f x] \sqrt{a + a \sin[e + f x]} - \\
& \frac{2 a (11 A d (c^2 - 10 c d + 73 d^2) - B (5 c^3 - 40 c^2 d + 169 c d^2 - 710 d^3)) \cos[e + f x] (a + a \sin[e + f x])^{3/2}}{1155 d f} + \\
& \frac{2 a^3 (11 A (3 c - 19 d) d - B (15 c^2 - 65 c d + 194 d^2)) \cos[e + f x] (c + d \sin[e + f x])^3}{693 d^3 f \sqrt{a + a \sin[e + f x]}} + \\
& \frac{2 a^2 (5 B c - 11 A d - 14 B d) \cos[e + f x] \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^3}{99 d^2 f} - \frac{2 a B \cos[e + f x] (a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])^3}{11 d f}
\end{aligned}$$

Result (type 3, 891 leaves):

1

$$55\,440 f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5$$

$$\begin{aligned} & (a(1 + \sin[e+fx]))^{5/2} \left(-277\,200 A c^2 \cos\left[\frac{1}{2}(e+fx)\right] - 207\,900 B c^2 \cos\left[\frac{1}{2}(e+fx)\right] - 415\,800 A c d \cos\left[\frac{1}{2}(e+fx)\right] - \right. \\ & 360\,360 B c d \cos\left[\frac{1}{2}(e+fx)\right] - 180\,180 A d^2 \cos\left[\frac{1}{2}(e+fx)\right] - 159\,390 B d^2 \cos\left[\frac{1}{2}(e+fx)\right] - 46\,200 A c^2 \cos\left[\frac{3}{2}(e+fx)\right] - \\ & 50\,820 B c^2 \cos\left[\frac{3}{2}(e+fx)\right] - 101\,640 A c d \cos\left[\frac{3}{2}(e+fx)\right] - 92\,400 B c d \cos\left[\frac{3}{2}(e+fx)\right] - 46\,200 A d^2 \cos\left[\frac{3}{2}(e+fx)\right] - \\ & 43\,890 B d^2 \cos\left[\frac{3}{2}(e+fx)\right] + 55\,440 A c^2 \cos\left[\frac{5}{2}(e+fx)\right] + 13\,860 B c^2 \cos\left[\frac{5}{2}(e+fx)\right] + 27\,720 A c d \cos\left[\frac{5}{2}(e+fx)\right] + \\ & 33\,264 B c d \cos\left[\frac{5}{2}(e+fx)\right] + 16\,632 A d^2 \cos\left[\frac{5}{2}(e+fx)\right] + 17\,325 B d^2 \cos\left[\frac{5}{2}(e+fx)\right] + 1980 B c^2 \cos\left[\frac{7}{2}(e+fx)\right] + \\ & 3960 A c d \cos\left[\frac{7}{2}(e+fx)\right] + 9900 B c d \cos\left[\frac{7}{2}(e+fx)\right] + 4950 A d^2 \cos\left[\frac{7}{2}(e+fx)\right] + 6435 B d^2 \cos\left[\frac{7}{2}(e+fx)\right] - 1540 B c d \cos\left[\frac{9}{2}(e+fx)\right] - \\ & 770 A d^2 \cos\left[\frac{9}{2}(e+fx)\right] - 1925 B d^2 \cos\left[\frac{9}{2}(e+fx)\right] - 315 B d^2 \cos\left[\frac{11}{2}(e+fx)\right] + 277\,200 A c^2 \sin\left[\frac{1}{2}(e+fx)\right] + \\ & 207\,900 B c^2 \sin\left[\frac{1}{2}(e+fx)\right] + 415\,800 A c d \sin\left[\frac{1}{2}(e+fx)\right] + 360\,360 B c d \sin\left[\frac{1}{2}(e+fx)\right] + 180\,180 A d^2 \sin\left[\frac{1}{2}(e+fx)\right] + \\ & 159\,390 B d^2 \sin\left[\frac{1}{2}(e+fx)\right] - 46\,200 A c^2 \sin\left[\frac{3}{2}(e+fx)\right] - 50\,820 B c^2 \sin\left[\frac{3}{2}(e+fx)\right] - 101\,640 A c d \sin\left[\frac{3}{2}(e+fx)\right] - \\ & 92\,400 B c d \sin\left[\frac{3}{2}(e+fx)\right] - 46\,200 A d^2 \sin\left[\frac{3}{2}(e+fx)\right] - 43\,890 B d^2 \sin\left[\frac{3}{2}(e+fx)\right] - 55\,440 A c^2 \sin\left[\frac{5}{2}(e+fx)\right] - \\ & 13\,860 B c^2 \sin\left[\frac{5}{2}(e+fx)\right] - 27\,720 A c d \sin\left[\frac{5}{2}(e+fx)\right] - 33\,264 B c d \sin\left[\frac{5}{2}(e+fx)\right] - 16\,632 A d^2 \sin\left[\frac{5}{2}(e+fx)\right] - \\ & 17\,325 B d^2 \sin\left[\frac{5}{2}(e+fx)\right] + 1980 B c^2 \sin\left[\frac{7}{2}(e+fx)\right] + 3960 A c d \sin\left[\frac{7}{2}(e+fx)\right] + 9900 B c d \sin\left[\frac{7}{2}(e+fx)\right] + 4950 A d^2 \sin\left[\frac{7}{2}(e+fx)\right] + \\ & 6435 B d^2 \sin\left[\frac{7}{2}(e+fx)\right] + 1540 B c d \sin\left[\frac{9}{2}(e+fx)\right] + 770 A d^2 \sin\left[\frac{9}{2}(e+fx)\right] + 1925 B d^2 \sin\left[\frac{9}{2}(e+fx)\right] - 315 B d^2 \sin\left[\frac{11}{2}(e+fx)\right] \left. \right) \end{aligned}$$

■ **Problem 302: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e+fx])^{5/2} (A + B \sin[e+fx]) (c + d \sin[e+fx]) dx$$

Optimal (type 3, 212 leaves, 6 steps):

$$\frac{64 a^3 (21 A c + 15 B c + 15 A d + 13 B d) \cos[e + f x]}{315 f \sqrt{a + a \sin[e + f x]}} - \frac{16 a^2 (21 A c + 15 B c + 15 A d + 13 B d) \cos[e + f x] \sqrt{a + a \sin[e + f x]}}{315 f}$$

$$\frac{2 a (21 A c + 15 B c + 15 A d + 13 B d) \cos[e + f x] (a + a \sin[e + f x])^{3/2}}{105 f} -$$

$$\frac{2 (9 B c + 9 A d - 2 B d) \cos[e + f x] (a + a \sin[e + f x])^{5/2}}{63 f} - \frac{2 B d \cos[e + f x] (a + a \sin[e + f x])^{7/2}}{9 a f}$$

Result (type 3, 673 leaves):

$$\frac{(20 A c + 15 B c + 15 A d + 13 B d) \cos\left[\frac{1}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} -$$

$$\frac{(10 A c + 11 B c + 11 A d + 10 B d) \cos\left[\frac{3}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{12 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} +$$

$$\frac{(2 A c + 5 B c + 5 A d + 6 B d) \cos\left[\frac{5}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{20 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} + \frac{(2 B c + 2 A d + 5 B d) \cos\left[\frac{7}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{56 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} -$$

$$\frac{B d \cos\left[\frac{9}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{72 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} + \frac{(20 A c + 15 B c + 15 A d + 13 B d) \sin\left[\frac{1}{2}(e + f x)\right] (a (1 + \sin[e + f x]))^{5/2}}{4 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} -$$

$$\frac{(10 A c + 11 B c + 11 A d + 10 B d) (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{3}{2}(e + f x)\right]}{12 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} - \frac{(2 A c + 5 B c + 5 A d + 6 B d) (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{5}{2}(e + f x)\right]}{20 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} +$$

$$\frac{(2 B c + 2 A d + 5 B d) (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{7}{2}(e + f x)\right]}{56 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5} + \frac{B d (a (1 + \sin[e + f x]))^{5/2} \sin\left[\frac{9}{2}(e + f x)\right]}{72 f \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5}$$

■ **Problem 304: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^{5/2} (A + B \sin[e + f x])}{c + d \sin[e + f x]} dx$$

Optimal (type 3, 218 leaves, 5 steps):

$$\frac{2 a^{5/2} (c - d)^2 (B c - A d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{d^{7/2} \sqrt{c + d} f} + \frac{2 a^3 (5 A (3 c - 7 d) d - B (15 c^2 - 35 c d + 32 d^2)) \cos[e + f x]}{15 d^3 f \sqrt{a + a \sin[e + f x]}} +$$

$$\frac{2 a^2 (5 B c - 5 A d - 8 B d) \cos[e + f x] \sqrt{a + a \sin[e + f x]}}{15 d^2 f} - \frac{2 a B \cos[e + f x] (a + a \sin[e + f x])^{3/2}}{5 d f}$$

Result (type 3, 450 leaves):

$$\frac{1}{30 d^{7/2} f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^5}$$

$$\left(a(1 + \sin[e+fx]) \right)^{5/2} \left(-30 \sqrt{d} (Ad(-2c+5d) + B(2c^2 - 5cd + 5d^2)) \cos\left[\frac{1}{2}(e+fx)\right] - 5d^{3/2} (-2Bc + 2Ad + 5Bd) \cos\left[\frac{3}{2}(e+fx)\right] + \right.$$

$$\left. 3Bd^{5/2} \cos\left[\frac{5}{2}(e+fx)\right] + \frac{1}{\sqrt{c+d}} 15(c-d)^2 (Bc - Ad) \left(e+fx - 2 \log\left[\sec\left[\frac{1}{4}(e+fx)\right]\right]^2 \right) + \right.$$

$$\left. 2 \log\left[-\sec\left[\frac{1}{4}(e+fx)\right]\right]^2 \left(c+d + \sqrt{d} \sqrt{c+d} \cos\left[\frac{1}{2}(e+fx)\right] - \sqrt{d} \sqrt{c+d} \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) - \frac{1}{\sqrt{c+d}} 15(c-d)^2 (Bc - Ad)$$

$$\left(e+fx - 2 \log\left[\sec\left[\frac{1}{4}(e+fx)\right]\right]^2 \right) + 2 \log\left[(c+d) \sec\left[\frac{1}{4}(e+fx)\right]^2 + \sqrt{d} \sqrt{c+d} \left(-1 + 2 \tan\left[\frac{1}{4}(e+fx)\right] + \tan\left[\frac{1}{4}(e+fx)\right]^2 \right) \right] \right) +$$

$$30 \sqrt{d} (Ad(-2c+5d) + B(2c^2 - 5cd + 5d^2)) \sin\left[\frac{1}{2}(e+fx)\right] - 5d^{3/2} (-2Bc + 2Ad + 5Bd) \sin\left[\frac{3}{2}(e+fx)\right] - 3Bd^{5/2} \sin\left[\frac{5}{2}(e+fx)\right] \Bigg)$$

■ **Problem 307: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \sin[e+fx]) (c + d \sin[e+fx])^3}{\sqrt{a + a \sin[e+fx]}} dx$$

Optimal (type 3, 284 leaves, 7 steps):

$$\frac{\sqrt{2} (A - B) (c - d)^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{a} f} - \frac{4 (7Ad(21c^2 - 12cd + 7d^2) + B(36c^3 - 63c^2d + 144cd^2 - 37d^3)) \cos[e+fx]}{105f \sqrt{a+a \sin[e+fx]}}$$

$$\frac{2d(7A(9c-d)d + B(24c^2 - 15cd + 31d^2)) \cos[e+fx] \sqrt{a+a \sin[e+fx]}}{105af}$$

$$\frac{2(6Bc + 7Ad - Bd) \cos[e+fx] (c + d \sin[e+fx])^2}{35f \sqrt{a+a \sin[e+fx]}} - \frac{2B \cos[e+fx] (c + d \sin[e+fx])^3}{7f \sqrt{a+a \sin[e+fx]}}$$

Result (type 3, 375 leaves):

$$\frac{1}{420 f \sqrt{a (1 + \sin[e + f x])}} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left((840 + 840 i) (-1)^{3/4} (A - B) (c - d)^3 \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \right) -$$

$$105 (4 A d (6 c^2 - 3 c d + 2 d^2) + B (8 c^3 - 12 c^2 d + 24 c d^2 - 5 d^3)) \cos\left[\frac{1}{2}(e + f x)\right] -$$

$$35 d (2 A (6 c - d) d + B (12 c^2 - 6 c d + 5 d^2)) \cos\left[\frac{3}{2}(e + f x)\right] + 21 d^2 (6 B c + 2 A d - B d) \cos\left[\frac{5}{2}(e + f x)\right] +$$

$$15 B d^3 \cos\left[\frac{7}{2}(e + f x)\right] + 105 (4 A d (6 c^2 - 3 c d + 2 d^2) + B (8 c^3 - 12 c^2 d + 24 c d^2 - 5 d^3)) \sin\left[\frac{1}{2}(e + f x)\right] -$$

$$35 d (2 A (6 c - d) d + B (12 c^2 - 6 c d + 5 d^2)) \sin\left[\frac{3}{2}(e + f x)\right] + 21 d^2 (-2 A d + B (-6 c + d)) \sin\left[\frac{5}{2}(e + f x)\right] + 15 B d^3 \sin\left[\frac{7}{2}(e + f x)\right]$$

■ **Problem 308: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^2}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$-\frac{\sqrt{2} (A - B) (c - d)^2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} f} - \frac{4 (5 A (3 c - d) d + B (6 c^2 - 7 c d + 7 d^2)) \cos[e + f x]}{15 f \sqrt{a + a \sin[e + f x]}}$$

$$\frac{2 d (4 B c + 5 A d - B d) \cos[e + f x] \sqrt{a + a \sin[e + f x]}}{15 a f} - \frac{2 B \cos[e + f x] (c + d \sin[e + f x])^2}{5 f \sqrt{a + a \sin[e + f x]}}$$

Result (type 3, 246 leaves):

$$\frac{1}{30 f \sqrt{a (1 + \sin[e + f x])}} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left((60 + 60 i) (-1)^{3/4} (A - B) (c - d)^2 \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \right) -$$

$$30 (A (4 c - d) d + 2 B (c^2 - c d + d^2)) \cos\left[\frac{1}{2}(e + f x)\right] + 5 d (-2 A d + B (-4 c + d)) \cos\left[\frac{3}{2}(e + f x)\right] + 3 B d^2 \cos\left[\frac{5}{2}(e + f x)\right] +$$

$$30 (A (4 c - d) d + 2 B (c^2 - c d + d^2)) \sin\left[\frac{1}{2}(e + f x)\right] + 5 d (-2 A d + B (-4 c + d)) \sin\left[\frac{3}{2}(e + f x)\right] - 3 B d^2 \sin\left[\frac{5}{2}(e + f x)\right]$$

■ **Problem 309: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 130 leaves, 5 steps) :

$$\frac{\sqrt{2} (A - B) (c - d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} f} - \frac{2 (3 B c + 3 A d - 2 B d) \cos[e + f x]}{3 f \sqrt{a + a \sin[e + f x]}} - \frac{2 B d \cos[e + f x] \sqrt{a + a \sin[e + f x]}}{3 a f}$$

Result (type 3, 135 leaves) :

$$-\frac{1}{3 f \sqrt{a (1 + \sin[e + f x])}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left((-6 - 6 i) (-1)^{3/4} (A - B) (c - d) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \right) + 2 \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right) (3 B c + 3 A d - B d + B d \sin[e + f x])$$

■ **Problem 310: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + f x]}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 3, 79 leaves, 3 steps) :

$$\frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} f} - \frac{2 B \cos[e + f x]}{f \sqrt{a + a \sin[e + f x]}}$$

Result (type 3, 106 leaves) :

$$\frac{1}{f \sqrt{a (1 + \sin[e + f x])}} 2 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left((1 + i) (-1)^{3/4} (A - B) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \right) + B \left(-\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)$$

■ **Problem 311: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + f x]}{\sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])} dx$$

Optimal (type 3, 136 leaves, 5 steps) :

$$\frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} (c - d) f} - \frac{2 (B c - A d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{\sqrt{a} (c - d) \sqrt{d} \sqrt{c + d} f}$$

Result (type 3, 238 leaves) :

$$\frac{1}{(c-d) \sqrt{d} \sqrt{c+d} f \sqrt{a} (1 + \sin[e + f x])} (-1)^{3/4} \left((2 + 2i) (A - B) \sqrt{d} \sqrt{c+d} \operatorname{ArcTanh} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \left(-1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] \right) +$$

$$(-1)^{1/4} (Bc - Ad) \left(\operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{4} (e + f x) \right] \right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos \left[\frac{1}{2} (e + f x) \right] - \sqrt{d} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) -$$

$$\operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{4} (e + f x) \right] \right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos \left[\frac{1}{2} (e + f x) \right] + \sqrt{d} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)$$

■ **Problem 312: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + f x]}{\sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^2} dx$$

Optimal (type 3, 207 leaves, 6 steps):

$$-\frac{\sqrt{2} (A - B) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}} \right]}{\sqrt{a} (c - d)^2 f} +$$

$$\frac{(Ad (3c + d) - B(c^2 + cd + 2d^2)) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c+d} \sqrt{a + a \sin[e + f x]}} \right]}{\sqrt{a} (c - d)^2 \sqrt{d} (c + d)^{3/2} f} - \frac{(Bc - Ad) \cos[e + f x]}{(c^2 - d^2) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])}$$

Result (type 3, 374 leaves):

$$\frac{1}{4 (c - d)^2 f \sqrt{a} (1 + \sin[e + f x])} \left(\cos \left[\frac{1}{2} (e + f x) \right] + \sin \left[\frac{1}{2} (e + f x) \right] \right)$$

$$\left((8 + 8i) (-1)^{3/4} (A - B) \operatorname{ArcTanh} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \left(-1 + \tan \left[\frac{1}{4} (e + f x) \right] \right) \right] \right) - \frac{1}{\sqrt{d} (c + d)^{3/2}} (-Ad (3c + d) + B(c^2 + cd + 2d^2))$$

$$\left(e + f x - 2 \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{4} (e + f x) \right] \right]^2 \right) + 2 \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{4} (e + f x) \right] \right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos \left[\frac{1}{2} (e + f x) \right] - \sqrt{d} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) +$$

$$\frac{1}{\sqrt{d} (c + d)^{3/2}} (-Ad (3c + d) + B(c^2 + cd + 2d^2))$$

$$\left(e + f x - 2 \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{4} (e + f x) \right] \right]^2 \right) + 2 \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{4} (e + f x) \right] \right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos \left[\frac{1}{2} (e + f x) \right] + \sqrt{d} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) -$$

$$\frac{4 (c - d) (Bc - Ad) \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)}{(c + d) (c + d \sin[e + f x])}$$

■ **Problem 313: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{\sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 309 leaves, 7 steps) :

$$\frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right] (Ad (15 c^2 + 10 c d + 7 d^2) - B (3 c^3 + 6 c^2 d + 19 c d^2 + 4 d^3)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e+fx]}{\sqrt{c+d} \sqrt{a+a \sin[e+fx]}}\right]}{\sqrt{a} (c-d)^3 f} + \frac{4 \sqrt{a} (c-d)^3 \sqrt{d} (c+d)^{5/2} f}{(B c - A d) \cos[e+fx] + \frac{(Ad (7 c + d) - B (3 c^2 + c d + 4 d^2)) \cos[e+fx]}{2 (c^2 - d^2) f \sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])^2} + \frac{(Ad (7 c + d) - B (3 c^2 + c d + 4 d^2)) \cos[e+fx]}{4 (c^2 - d^2)^2 f \sqrt{a+a \sin[e+fx]} (c+d \sin[e+fx])}}$$

Result (type 3, 847 leaves) :

$$\begin{aligned} & \left((2 + 2 i) (A - B) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4} (e + f x)\right] \left(\cos\left[\frac{1}{4} (e + f x)\right] - \sin\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right) \right) / \\ & \left(((-1)^{1/4} c^3 - 3 (-1)^{1/4} c^2 d + 3 (-1)^{1/4} c d^2 - (-1)^{1/4} d^3) f \sqrt{a (1 + \sin[e + f x])} \right) - \\ & \left(-A d (15 c^2 + 10 c d + 7 d^2) + B (3 c^3 + 6 c^2 d + 19 c d^2 + 4 d^3) \right) \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]^2\right] + \right. \\ & \quad \left. 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right) / \\ & \left(16 (c-d)^3 \sqrt{d} (c+d)^{5/2} f \sqrt{a (1 + \sin[e + f x])} \right) + \left(-A d (15 c^2 + 10 c d + 7 d^2) + B (3 c^3 + 6 c^2 d + 19 c d^2 + 4 d^3) \right) \\ & \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]^2\right] + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right)\right] \right) \\ & \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right) / \left(16 (c-d)^3 \sqrt{d} (c+d)^{5/2} f \sqrt{a (1 + \sin[e + f x])} \right) + \\ & \left(\left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right) \left(-B c \cos\left[\frac{1}{2} (e + f x)\right] + A d \cos\left[\frac{1}{2} (e + f x)\right] + B c \sin\left[\frac{1}{2} (e + f x)\right] - A d \sin\left[\frac{1}{2} (e + f x)\right]\right) \right) / \\ & \left(2 (c-d) (c+d) f \sqrt{a (1 + \sin[e + f x])} (c+d \sin[e + f x])^2 \right) + \\ & \left(\left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right) \left(-3 B c^2 \cos\left[\frac{1}{2} (e + f x)\right] + 7 A c d \cos\left[\frac{1}{2} (e + f x)\right] - B c d \cos\left[\frac{1}{2} (e + f x)\right] + \right. \right. \\ & \quad \left. \left. A d^2 \cos\left[\frac{1}{2} (e + f x)\right] - 4 B d^2 \cos\left[\frac{1}{2} (e + f x)\right] + 3 B c^2 \sin\left[\frac{1}{2} (e + f x)\right] - 7 A c d \sin\left[\frac{1}{2} (e + f x)\right] + B c d \sin\left[\frac{1}{2} (e + f x)\right] - \right. \right. \\ & \quad \left. \left. A d^2 \sin\left[\frac{1}{2} (e + f x)\right] + 4 B d^2 \sin\left[\frac{1}{2} (e + f x)\right]\right) \right) / \left(4 (c-d)^2 (c+d)^2 f \sqrt{a (1 + \sin[e + f x])} (c+d \sin[e + f x]) \right) \end{aligned}$$

■ **Problem 314: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^3}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 283 leaves, 7 steps) :

$$\frac{(c-d)^2 (3B(c-5d) + A(c+11d)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{2\sqrt{2} a^{3/2} f} +$$

$$\frac{d(15Ac^2 - 99Bc^2 - 120Acd + 168Bcd + 65Ad^2 - 93Bd^2) \cos[e+fx]}{15af \sqrt{a+a \sin[e+fx]}} + \frac{d^2(15Ac - 51Bc - 35Ad + 39Bd) \cos[e+fx] \sqrt{a+a \sin[e+fx]}}{30a^2 f} +$$

$$\frac{(5A-9B)d \cos[e+fx] (c+d \sin[e+fx])^2}{10af \sqrt{a+a \sin[e+fx]}} - \frac{(A-B) \cos[e+fx] (c+d \sin[e+fx])^3}{2f(a+a \sin[e+fx])^{3/2}}$$

Result (type 3, 684 leaves):

$$\frac{1}{60f(a(1+\sin[e+fx]))^{3/2}} \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(-30Ac^3 \cos\left[\frac{1}{2}(e+fx)\right] + 30Bc^3 \cos\left[\frac{1}{2}(e+fx)\right] + 90Ac^2d \cos\left[\frac{1}{2}(e+fx)\right] - 270Bc^2d \cos\left[\frac{1}{2}(e+fx)\right] - \right.$$

$$270Ac d^2 \cos\left[\frac{1}{2}(e+fx)\right] + 330Bc d^2 \cos\left[\frac{1}{2}(e+fx)\right] + 110Ad^3 \cos\left[\frac{1}{2}(e+fx)\right] - 165Bd^3 \cos\left[\frac{1}{2}(e+fx)\right] -$$

$$180Bc^2d \cos\left[\frac{3}{2}(e+fx)\right] - 180Ac d^2 \cos\left[\frac{3}{2}(e+fx)\right] + 210Bc d^2 \cos\left[\frac{3}{2}(e+fx)\right] + 70Ad^3 \cos\left[\frac{3}{2}(e+fx)\right] -$$

$$123Bd^3 \cos\left[\frac{3}{2}(e+fx)\right] + 30Bc d^2 \cos\left[\frac{5}{2}(e+fx)\right] + 10Ad^3 \cos\left[\frac{5}{2}(e+fx)\right] - 9Bd^3 \cos\left[\frac{5}{2}(e+fx)\right] + 3Bd^3 \cos\left[\frac{7}{2}(e+fx)\right] +$$

$$30Ac^3 \sin\left[\frac{1}{2}(e+fx)\right] - 30Bc^3 \sin\left[\frac{1}{2}(e+fx)\right] - 90Ac^2d \sin\left[\frac{1}{2}(e+fx)\right] + 270Bc^2d \sin\left[\frac{1}{2}(e+fx)\right] + 270Ac d^2 \sin\left[\frac{1}{2}(e+fx)\right] -$$

$$330Bc d^2 \sin\left[\frac{1}{2}(e+fx)\right] - 110Ad^3 \sin\left[\frac{1}{2}(e+fx)\right] + 165Bd^3 \sin\left[\frac{1}{2}(e+fx)\right] + (30+30i)(-1)^{3/4}(c-d)^2$$

$$(3B(c-5d) + A(c+11d)) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2 -$$

$$180Bc^2d \sin\left[\frac{3}{2}(e+fx)\right] - 180Ac d^2 \sin\left[\frac{3}{2}(e+fx)\right] + 210Bc d^2 \sin\left[\frac{3}{2}(e+fx)\right] + 70Ad^3 \sin\left[\frac{3}{2}(e+fx)\right] -$$

$$123Bd^3 \sin\left[\frac{3}{2}(e+fx)\right] - 30Bc d^2 \sin\left[\frac{5}{2}(e+fx)\right] - 10Ad^3 \sin\left[\frac{5}{2}(e+fx)\right] + 9Bd^3 \sin\left[\frac{5}{2}(e+fx)\right] + 3Bd^3 \sin\left[\frac{7}{2}(e+fx)\right] \Big)$$

■ **Problem 315: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \sin[e+fx])(c+d \sin[e+fx])^2}{(a+a \sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 203 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(c-d)(Ac+3Bc+7Ad-11Bd) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{2\sqrt{2} a^{3/2} f} + \frac{d(3Ac-15Bc-9Ad+13Bd) \cos[e+fx]}{3af \sqrt{a+a \sin[e+fx]}} + \\
& \frac{(3A-7B)d^2 \cos[e+fx] \sqrt{a+a \sin[e+fx]}}{6a^2 f} - \frac{(A-B) \cos[e+fx] (c+d \sin[e+fx])^2}{2f(a+a \sin[e+fx])^{3/2}}
\end{aligned}$$

Result (type 3, 357 leaves):

$$\begin{aligned}
& \frac{1}{6f(a(1+\sin[e+fx]))^{3/2}} \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(6(A-B)(c-d)^2 \sin\left[\frac{1}{2}(e+fx)\right] - 3(A-B)(c-d)^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& (3+3i)(-1)^{3/4}(c-d)(Ac+3Bc+7Ad-11Bd) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 + \\
& 6d(-4Bc-2Ad+3Bd) \cos\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 - \\
& 2Bd^2 \cos\left[\frac{3}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 - 6d(-4Bc-2Ad+3Bd) \sin\left[\frac{1}{2}(e+fx)\right] \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 - 2Bd^2 \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 \sin\left[\frac{3}{2}(e+fx)\right]
\end{aligned}$$

■ **Problem 316: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \sin[e+fx])(c+d \sin[e+fx])}{(a+a \sin[e+fx])^{3/2}} dx$$

Optimal (type 3, 133 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(Ac+3Bc+3Ad-7Bd) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{2\sqrt{2} a^{3/2} f} - \frac{(A-B)(c-d) \cos[e+fx]}{2f(a+a \sin[e+fx])^{3/2}} - \frac{2Bd \cos[e+fx]}{af \sqrt{a+a \sin[e+fx]}}
\end{aligned}$$

Result (type 3, 246 leaves):

$$\begin{aligned}
& \frac{1}{2f(a(1+\sin[e+fx]))^{3/2}} \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(2(A-B)(c-d) \sin\left[\frac{1}{2}(e+fx)\right] - (A-B)(c-d) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& (1+i)(-1)^{3/4}(Ac+3Bc+3Ad-7Bd) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 - \\
& 4Bd \cos\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2 + 4Bd \sin\left[\frac{1}{2}(e+fx)\right] \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^2
\end{aligned}$$

■ **Problem 317: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 87 leaves, 3 steps):

$$-\frac{(A + 3 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{2 \sqrt{2} a^{3/2} f} - \frac{(A - B) \cos[e + f x]}{2 f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 150 leaves):

$$\left(\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(2(A - B) \sin\left[\frac{1}{2}(e + f x)\right] + (-A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) + (1 + i) (-1)^{3/4} (A + 3 B) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \right) \right) / (2 f (a (1 + \sin[e + f x]))^{3/2})$$

■ **Problem 318: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])} dx$$

Optimal (type 3, 187 leaves, 6 steps):

$$-\frac{(A(c - 5d) + B(3c + d)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{2 \sqrt{2} a^{3/2} (c - d)^2 f} + \frac{2 \sqrt{d} (Bc - Ad) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{a^{3/2} (c - d)^2 \sqrt{c + d} f} - \frac{(A - B) \cos[e + f x]}{2 (c - d) f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 419 leaves):

$$\frac{1}{2 (c - d)^2 f (a (1 + \sin[e + f x]))^{3/2}} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(2(A - B)(c - d) \sin\left[\frac{1}{2}(e + f x)\right] + (-A + B)(c - d) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) + (1 + i) (-1)^{3/4} (A(c - 5d) + B(3c + d)) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + \frac{1}{\sqrt{c + d}} \sqrt{d} (Bc - Ad) \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 \left(\sqrt{c + d} + \sqrt{d} \cos\left[\frac{1}{2}(e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 + \frac{1}{\sqrt{c + d}} \sqrt{d} (-Bc + Ad) \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 \left(\sqrt{c + d} - \sqrt{d} \cos\left[\frac{1}{2}(e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2 \right)$$

■ **Problem 319: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])^2} dx$$

Optimal (type 3, 292 leaves, 7 steps):

$$\frac{(A c + 3 B c - 9 A d + 5 B d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right] - \sqrt{d} (A d (5 c + 3 d) - B (3 c^2 + 3 c d + 2 d^2)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{2 \sqrt{2} a^{3/2} (c - d)^3 f} - \frac{a^{3/2} (c - d)^3 (c + d)^{3/2} f}{(A - B) \cos[e + f x]} + \frac{d (B (3 c + d) - A (c + 3 d)) \cos[e + f x]}{2 a (c - d)^2 (c + d) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])}$$

Result (type 3, 745 leaves):

$$\begin{aligned} & \frac{(-A + B) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^2}{2 (c - d)^2 f (a (1 + \sin[e + f x]))^{3/2}} + \\ & \left((1 + i) (A c + 3 B c - 9 A d + 5 B d) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right] \left(\cos\left[\frac{1}{4}(e + f x)\right] - \sin\left[\frac{1}{4}(e + f x)\right] \right) \right] \right. \\ & \quad \left. \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \right) / \left((2 (-1)^{1/4} c^3 - 6 (-1)^{1/4} c^2 d + 6 (-1)^{1/4} c d^2 - 2 (-1)^{1/4} d^3) f (a (1 + \sin[e + f x]))^{3/2} \right) + \\ & \left(\sqrt{d} (-A d (5 c + 3 d) + B (3 c^2 + 3 c d + 2 d^2)) \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 \right) + \right. \\ & \quad \left. 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 \left(\sqrt{c + d} + \sqrt{d} \cos\left[\frac{1}{2}(e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 / \\ & (4 (c - d)^3 (c + d)^{3/2} f (a (1 + \sin[e + f x]))^{3/2}) + \left(\sqrt{d} (A d (5 c + 3 d) - B (3 c^2 + 3 c d + 2 d^2)) \right. \\ & \quad \left. \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]\right]^2 \left(\sqrt{c + d} - \sqrt{d} \cos\left[\frac{1}{2}(e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) \right) \\ & \quad \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 / (4 (c - d)^3 (c + d)^{3/2} f (a (1 + \sin[e + f x]))^{3/2}) + \\ & \frac{\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) (A \sin\left[\frac{1}{2}(e + f x)\right] - B \sin\left[\frac{1}{2}(e + f x)\right])}{(c - d)^2 f (a (1 + \sin[e + f x]))^{3/2}} + \\ & \left(\left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right)^3 \left(B c d \cos\left[\frac{1}{2}(e + f x)\right] - A d^2 \cos\left[\frac{1}{2}(e + f x)\right] - B c d \sin\left[\frac{1}{2}(e + f x)\right] + A d^2 \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) / \\ & ((c - d)^2 (c + d) f (a (1 + \sin[e + f x]))^{3/2} (c + d \sin[e + f x])) \end{aligned}$$

- **Problem 320: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 402 leaves, 8 steps):

$$\begin{aligned} & - \frac{(A (c - 13 d) + 3 B (c + 3 d)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{2 \sqrt{2} a^{3/2} (c - d)^4 f} \\ & - \frac{\sqrt{d} (A d (35 c^2 + 42 c d + 19 d^2) - 3 B (5 c^3 + 10 c^2 d + 13 c d^2 + 4 d^3)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{4 a^{3/2} (c - d)^4 (c + d)^{5/2} f} \\ & + \frac{(A - B) \cos[e + f x]}{2 (c - d) f (a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])^2} + \frac{d (B (2 c + d) - A (c + 2 d)) \cos[e + f x]}{2 a (c - d)^2 (c + d) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^2} \\ & + \frac{d (3 B (3 c^2 + 3 c d + 2 d^2) - A (2 c^2 + 15 c d + 7 d^2)) \cos[e + f x]}{4 a (c - d)^3 (c + d)^2 f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])} \end{aligned}$$

Result (type 3, 1395 leaves):

$$\begin{aligned}
& \left((1+i)(Ac+3Bc-13Ad+9Bd) \right. \\
& \quad \left. \text{ArcTanh} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \text{Sec} \left[\frac{1}{4} (e+fx) \right] \left(\cos \left[\frac{1}{4} (e+fx) \right] - \sin \left[\frac{1}{4} (e+fx) \right] \right) \right] \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right)^3 \right) / \\
& \quad \left((2(-1)^{1/4}c^4 - 8(-1)^{1/4}c^3d + 12(-1)^{1/4}c^2d^2 - 8(-1)^{1/4}cd^3 + 2(-1)^{1/4}d^4) f(a(1+\sin[e+fx]))^{3/2} + \right. \\
& \quad \left. \sqrt{d}(-Ad(35c^2+42cd+19d^2) + 3B(5c^3+10c^2d+13cd^2+4d^3)) \right. \\
& \quad \left. \left(e+fx - 2 \log \left[\text{Sec} \left[\frac{1}{4} (e+fx) \right] \right]^2 + 2 \log \left[\text{Sec} \left[\frac{1}{4} (e+fx) \right] \left(\sqrt{c+d} + \sqrt{d} \cos \left[\frac{1}{2} (e+fx) \right] - \sqrt{d} \sin \left[\frac{1}{2} (e+fx) \right] \right) \right] \right) \right) \\
& \quad \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right)^3 \Big/ (16(c-d)^4(c+d)^{5/2} f(a(1+\sin[e+fx]))^{3/2}) - \\
& \quad \left(\sqrt{d}(-Ad(35c^2+42cd+19d^2) + 3B(5c^3+10c^2d+13cd^2+4d^3)) \left(e+fx - 2 \log \left[\text{Sec} \left[\frac{1}{4} (e+fx) \right] \right]^2 + \right. \right. \\
& \quad \left. \left. 2 \log \left[\text{Sec} \left[\frac{1}{4} (e+fx) \right] \left(\sqrt{c+d} - \sqrt{d} \cos \left[\frac{1}{2} (e+fx) \right] + \sqrt{d} \sin \left[\frac{1}{2} (e+fx) \right] \right) \right] \right) \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right)^3 \right) / \\
& \quad \frac{1}{(16(c-d)^4(c+d)^{5/2} f(a(1+\sin[e+fx]))^{3/2}) + 16(c-d)^3(c+d)^2 f(a(1+\sin[e+fx]))^{3/2}(c+d \sin[e+fx])^2} \\
& \quad \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right) \left(-8Ac^4 \cos \left[\frac{1}{2} (e+fx) \right] + 8Bc^4 \cos \left[\frac{1}{2} (e+fx) \right] - 8Ac^3d \cos \left[\frac{1}{2} (e+fx) \right] + \right. \\
& \quad 26Bc^3d \cos \left[\frac{1}{2} (e+fx) \right] - 22Ac^2d^2 \cos \left[\frac{1}{2} (e+fx) \right] + 6Bc^2d^2 \cos \left[\frac{1}{2} (e+fx) \right] - 10Ac^3d^3 \cos \left[\frac{1}{2} (e+fx) \right] + \\
& \quad 4Bcd^3 \cos \left[\frac{1}{2} (e+fx) \right] + 4Bd^4 \cos \left[\frac{1}{2} (e+fx) \right] - 8Ac^3d \cos \left[\frac{3}{2} (e+fx) \right] + 26Bc^3d \cos \left[\frac{3}{2} (e+fx) \right] - 40Ac^2d^2 \cos \left[\frac{3}{2} (e+fx) \right] + \\
& \quad 31Bc^2d^2 \cos \left[\frac{3}{2} (e+fx) \right] - 25Ac^3d \cos \left[\frac{3}{2} (e+fx) \right] + 13Bcd^3 \cos \left[\frac{3}{2} (e+fx) \right] + Ad^4 \cos \left[\frac{3}{2} (e+fx) \right] + 2Bd^4 \cos \left[\frac{3}{2} (e+fx) \right] + \\
& \quad 2Ac^2d^2 \cos \left[\frac{5}{2} (e+fx) \right] - 9Bc^2d^2 \cos \left[\frac{5}{2} (e+fx) \right] + 15Ac^3d \cos \left[\frac{5}{2} (e+fx) \right] - 9Bcd^3 \cos \left[\frac{5}{2} (e+fx) \right] + 7Ad^4 \cos \left[\frac{5}{2} (e+fx) \right] - \\
& \quad 6Bd^4 \cos \left[\frac{5}{2} (e+fx) \right] + 8Ac^4 \sin \left[\frac{1}{2} (e+fx) \right] - 8Bc^4 \sin \left[\frac{1}{2} (e+fx) \right] + 8Ac^3d \sin \left[\frac{1}{2} (e+fx) \right] - 26Bc^3d \sin \left[\frac{1}{2} (e+fx) \right] + \\
& \quad 22Ac^2d^2 \sin \left[\frac{1}{2} (e+fx) \right] - 6Bc^2d^2 \sin \left[\frac{1}{2} (e+fx) \right] + 10Ac^3d \sin \left[\frac{1}{2} (e+fx) \right] - 4Bcd^3 \sin \left[\frac{1}{2} (e+fx) \right] - 4Bd^4 \sin \left[\frac{1}{2} (e+fx) \right] - \\
& \quad 8Ac^3d \sin \left[\frac{3}{2} (e+fx) \right] + 26Bc^3d \sin \left[\frac{3}{2} (e+fx) \right] - 40Ac^2d^2 \sin \left[\frac{3}{2} (e+fx) \right] + 31Bc^2d^2 \sin \left[\frac{3}{2} (e+fx) \right] - \\
& \quad 25Ac^3d \sin \left[\frac{3}{2} (e+fx) \right] + 13Bcd^3 \sin \left[\frac{3}{2} (e+fx) \right] + Ad^4 \sin \left[\frac{3}{2} (e+fx) \right] + 2Bd^4 \sin \left[\frac{3}{2} (e+fx) \right] - 2Ac^2d^2 \sin \left[\frac{5}{2} (e+fx) \right] + \\
& \quad \left. 9Bc^2d^2 \sin \left[\frac{5}{2} (e+fx) \right] - 15Ac^3d \sin \left[\frac{5}{2} (e+fx) \right] + 9Bcd^3 \sin \left[\frac{5}{2} (e+fx) \right] - 7Ad^4 \sin \left[\frac{5}{2} (e+fx) \right] + 6Bd^4 \sin \left[\frac{5}{2} (e+fx) \right] \right)
\end{aligned}$$

■ **Problem 321: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^3}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 308 leaves, 7 steps):

$$\begin{aligned} & - \frac{(c - d) (B (5 c^2 + 62 c d - 163 d^2) + 3 A (c^2 + 6 c d + 25 d^2)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} f} + \\ & \frac{d (A (9 c^2 + 36 c d - 93 d^2) + B (15 c^2 - 228 c d + 197 d^2)) \cos[e + f x]}{24 a^2 f \sqrt{a + a \sin[e + f x]}} + \frac{d^2 (9 A c + 15 B c + 39 A d - 95 B d) \cos[e + f x] \sqrt{a + a \sin[e + f x]}}{48 a^3 f} - \\ & \frac{(3 A c + 5 B c + 9 A d - 17 B d) \cos[e + f x] (c + d \sin[e + f x])^2}{16 a f (a + a \sin[e + f x])^{3/2}} - \frac{(A - B) \cos[e + f x] (c + d \sin[e + f x])^3}{4 f (a + a \sin[e + f x])^{5/2}} \end{aligned}$$

Result (type 3, 523 leaves):

$$\begin{aligned} & \frac{1}{48 f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \\ & \left(24 (A - B) (c - d)^3 \sin\left[\frac{1}{2} (e + f x)\right] - 12 (A - B) (c - d)^3 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) + 6 (c - d)^2 (B (5 c - 29 d) + 3 A (c + 7 d)) \right. \\ & \left. \sin\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - 3 (c - d)^2 (B (5 c - 29 d) + 3 A (c + 7 d)) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^3 + \right. \\ & \left. (3 + 3 i) (-1)^{3/4} (c - d) (B (5 c^2 + 62 c d - 163 d^2) + 3 A (c^2 + 6 c d + 25 d^2)) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \right) \\ & \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 - 16 B d^3 \cos\left[\frac{3}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 + \\ & (24 + 24 i) d^2 (-6 B c - 2 A d + 5 B d) \left(\cos\left[\frac{1}{2} (e + f x)\right] + i \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 + \\ & (24 + 24 i) d^2 (6 B c + 2 A d - 5 B d) \left(i \cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 - \\ & 16 B d^3 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 \sin\left[\frac{3}{2} (e + f x)\right] \end{aligned}$$

■ **Problem 322: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^2}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 219 leaves, 6 steps):

$$\frac{(B(5c^2 + 38cd - 75d^2) + A(3c^2 + 10cd + 19d^2)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{16\sqrt{2} a^{5/2} f} - \frac{(c-d)(3Ac + 5Bc + 5Ad - 13Bd) \cos[e+fx]}{16af(a+a \sin[e+fx])^{3/2}} + \frac{(A-9B)d^2 \cos[e+fx]}{4a^2 f \sqrt{a+a \sin[e+fx]}} - \frac{(A-B) \cos[e+fx] (c+d \sin[e+fx])^2}{4f(a+a \sin[e+fx])^{5/2}}$$

Result (type 3, 544 leaves):

$$\frac{1}{32f(a(1+\sin[e+fx]))^{5/2}} \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right) \left(-11Ac^2 \cos\left[\frac{1}{2}(e+fx)\right] + 3Bc^2 \cos\left[\frac{1}{2}(e+fx)\right] + 6Ac d \cos\left[\frac{1}{2}(e+fx)\right] + 10Bc d \cos\left[\frac{1}{2}(e+fx)\right] + 5Ad^2 \cos\left[\frac{1}{2}(e+fx)\right] - 45Bd^2 \cos\left[\frac{1}{2}(e+fx)\right] - 3Ac^2 \cos\left[\frac{3}{2}(e+fx)\right] - 5Bc^2 \cos\left[\frac{3}{2}(e+fx)\right] - 10Ac d \cos\left[\frac{3}{2}(e+fx)\right] + 26Bc d \cos\left[\frac{3}{2}(e+fx)\right] + 13Ad^2 \cos\left[\frac{3}{2}(e+fx)\right] - 69Bd^2 \cos\left[\frac{3}{2}(e+fx)\right] + 16Bd^2 \cos\left[\frac{5}{2}(e+fx)\right] + 11Ac^2 \sin\left[\frac{1}{2}(e+fx)\right] - 3Bc^2 \sin\left[\frac{1}{2}(e+fx)\right] - 6Ac d \sin\left[\frac{1}{2}(e+fx)\right] - 10Bc d \sin\left[\frac{1}{2}(e+fx)\right] - 5Ad^2 \sin\left[\frac{1}{2}(e+fx)\right] + 45Bd^2 \sin\left[\frac{1}{2}(e+fx)\right] + (2+2i)(-1)^{3/4} (B(5c^2 + 38cd - 75d^2) + A(3c^2 + 10cd + 19d^2)) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4}(e+fx)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right] \right)^4 - 3Ac^2 \sin\left[\frac{3}{2}(e+fx)\right] - 5Bc^2 \sin\left[\frac{3}{2}(e+fx)\right] - 10Ac d \sin\left[\frac{3}{2}(e+fx)\right] + 26Bc d \sin\left[\frac{3}{2}(e+fx)\right] + 13Ad^2 \sin\left[\frac{3}{2}(e+fx)\right] - 69Bd^2 \sin\left[\frac{3}{2}(e+fx)\right] - 16Bd^2 \sin\left[\frac{5}{2}(e+fx)\right] \right)$$

■ **Problem 323: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(A+B \sin[e+fx])(c+d \sin[e+fx])}{(a+a \sin[e+fx])^{5/2}} dx$$

Optimal (type 3, 151 leaves, 5 steps):

$$\frac{(3Ac + 5Bc + 5Ad + 19Bd) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{2} \sqrt{a+a \sin[e+fx]}}\right]}{16\sqrt{2} a^{5/2} f} - \frac{(A-B)(c-d) \cos[e+fx]}{4f(a+a \sin[e+fx])^{5/2}} - \frac{(3Ac + 5Bc + 5Ad - 13Bd) \cos[e+fx]}{16af(a+a \sin[e+fx])^{3/2}}$$

Result (type 3, 267 leaves):

$$\frac{1}{16 f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(8 (A - B) (c - d) \sin\left[\frac{1}{2} (e + f x)\right] - 4 (A - B) (c - d) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) + 2 (3 A c + 5 B c + 5 A d - 13 B d) \sin\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - (3 A c + 5 B c + 5 A d - 13 B d) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^3 + (1 + i) (-1)^{3/4} (3 A c + 5 B c + 5 A d + 19 B d) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 \right)$$

■ **Problem 324: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^{5/2}} dx$$

Optimal (type 3, 126 leaves, 4 steps):

$$-\frac{(3 A + 5 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} f} - \frac{(A - B) \cos[e + f x]}{4 f (a + a \sin[e + f x])^{5/2}} - \frac{(3 A + 5 B) \cos[e + f x]}{16 a f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 227 leaves):

$$\frac{1}{16 f (a (1 + \sin[e + f x]))^{5/2}} \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(8 (A - B) \sin\left[\frac{1}{2} (e + f x)\right] + 4 (-A + B) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) + 2 (3 A + 5 B) \sin\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - (3 A + 5 B) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^3 + (1 + i) (-1)^{3/4} (3 A + 5 B) \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 \right)$$

■ **Problem 325: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])} dx$$

Optimal (type 3, 261 leaves, 7 steps):

$$-\frac{(B (5 c^2 - 34 c d - 3 d^2) + A (3 c^2 - 14 c d + 43 d^2)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} (c - d)^3 f} - \frac{2 d^{3/2} (B c - A d) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c + d} \sqrt{a + a \sin[e + f x]}}\right]}{a^{5/2} (c - d)^3 \sqrt{c + d} f} - \frac{(A - B) \cos[e + f x]}{4 (c - d) f (a + a \sin[e + f x])^{5/2}} - \frac{(3 A c + 5 B c - 11 A d + 3 B d) \cos[e + f x]}{16 a (c - d)^2 f (a + a \sin[e + f x])^{3/2}}$$

Result (type 3, 550 leaves):

$$\begin{aligned}
& \frac{1}{16 (c-d)^3 f (a (1 + \sin[e + f x]))^{5/2}} \\
& \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) \left(8 (A-B) (c-d)^2 \sin\left[\frac{1}{2} (e + f x)\right] + 4 (-A+B) (c-d)^2 \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right) + \right. \\
& \quad \left. 2 (c-d) (3 A c + 5 B c - 11 A d + 3 B d) \sin\left[\frac{1}{2} (e + f x)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^2 - \right. \\
& \quad \left. (c-d) (3 A c + 5 B c - 11 A d + 3 B d) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^3 + (1+i) (-1)^{3/4} (B (5 c^2 - 34 c d - 3 d^2) + A (3 c^2 - 14 c d + 43 d^2)) \right. \\
& \quad \left. \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \left(-1 + \tan\left[\frac{1}{4} (e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 + \frac{1}{\sqrt{c+d}} \right. \\
& \quad \left. 8 d^{3/2} (-B c + A d) \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]^2\right] + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right)\right] \right) \right. \\
& \quad \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 + \frac{1}{\sqrt{c+d}} 8 d^{3/2} (B c - A d) \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]^2\right] + \right. \\
& \quad \left. \left. 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (e + f x)\right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos\left[\frac{1}{2} (e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2} (e + f x)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right] \right)^4 \right)
\end{aligned}$$

■ **Problem 326: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])^2} dx$$

Optimal (type 3, 395 leaves, 8 steps):

$$\begin{aligned}
& \frac{(B (5 c^2 - 58 c d - 43 d^2) + A (3 c^2 - 22 c d + 115 d^2)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} (c-d)^4 f} + \\
& \frac{d^{3/2} (A d (7 c + 5 d) - B (5 c^2 + 5 c d + 2 d^2)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c+d} \sqrt{a + a \sin[e + f x]}}\right]}{a^{5/2} (c-d)^4 (c+d)^{3/2} f} - \frac{(A-B) \cos[e + f x]}{4 (c-d) f (a + a \sin[e + f x])^{5/2} (c+d \sin[e + f x])} - \\
& \frac{(3 A c + 5 B c - 15 A d + 7 B d) \cos[e + f x]}{16 a (c-d)^2 f (a + a \sin[e + f x])^{3/2} (c+d \sin[e + f x])} - \frac{d (A (3 c^2 - 16 c d - 35 d^2) + B (5 c^2 + 32 c d + 11 d^2)) \cos[e + f x]}{16 a^2 (c-d)^3 (c+d) f \sqrt{a + a \sin[e + f x]} (c+d \sin[e + f x])}
\end{aligned}$$

Result (type 3, 1318 leaves):

$$\begin{aligned}
& \left((1+i) (3Ac^2 + 5Bc^2 - 22Acd - 58Bcd + 115Ad^2 - 43Bd^2) \right. \\
& \quad \text{ArcTanh} \left[\left(\frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \text{Sec} \left[\frac{1}{4} (e+fx) \right] \left(\cos \left[\frac{1}{4} (e+fx) \right] - \sin \left[\frac{1}{4} (e+fx) \right] \right) \right] \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right)^5 \right] / \\
& \quad \left((16(-1)^{1/4}c^4 - 64(-1)^{1/4}c^3d + 96(-1)^{1/4}c^2d^2 - 64(-1)^{1/4}cd^3 + 16(-1)^{1/4}d^4) f (a(1+\sin[e+fx]))^{5/2} + \right. \\
& \quad \left. d^{3/2} (Ad(7c+5d) - B(5c^2+5cd+2d^2)) \left(e+fx - 2 \log \left[\text{Sec} \left[\frac{1}{4} (e+fx) \right] \right]^2 + \right. \right. \\
& \quad \left. \left. 2 \log \left[\text{Sec} \left[\frac{1}{4} (e+fx) \right] \right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos \left[\frac{1}{2} (e+fx) \right] - \sqrt{d} \sin \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right)^5 \right] / \\
& \quad \left(4(c-d)^4 (c+d)^{3/2} f (a(1+\sin[e+fx]))^{5/2} + \left(d^{3/2} (-Ad(7c+5d) + B(5c^2+5cd+2d^2)) \right) \right. \\
& \quad \left(e+fx - 2 \log \left[\text{Sec} \left[\frac{1}{4} (e+fx) \right] \right]^2 + 2 \log \left[\text{Sec} \left[\frac{1}{4} (e+fx) \right] \right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos \left[\frac{1}{2} (e+fx) \right] + \sqrt{d} \sin \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) \\
& \quad \left. \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right)^5 \right] / \left(4(c-d)^4 (c+d)^{3/2} f (a(1+\sin[e+fx]))^{5/2} + \right. \\
& \quad \left. \frac{1}{64(c-d)^3 (c+d) f (a(1+\sin[e+fx]))^{5/2} (c+d \sin[e+fx])} \left(\cos \left[\frac{1}{2} (e+fx) \right] + \sin \left[\frac{1}{2} (e+fx) \right] \right) \right) \\
& \quad \left(-22Ac^3 \cos \left[\frac{1}{2} (e+fx) \right] + 6Bc^3 \cos \left[\frac{1}{2} (e+fx) \right] + 40Ac^2d \cos \left[\frac{1}{2} (e+fx) \right] - 40Bc^2d \cos \left[\frac{1}{2} (e+fx) \right] + 54Ac^2d^2 \cos \left[\frac{1}{2} (e+fx) \right] - \right. \\
& \quad 70Bcd^2 \cos \left[\frac{1}{2} (e+fx) \right] + 24Ad^3 \cos \left[\frac{1}{2} (e+fx) \right] + 8Bd^3 \cos \left[\frac{1}{2} (e+fx) \right] - 6Ac^3 \cos \left[\frac{3}{2} (e+fx) \right] - 10Bc^3 \cos \left[\frac{3}{2} (e+fx) \right] + \\
& \quad 21Ac^2d \cos \left[\frac{3}{2} (e+fx) \right] - 29Bc^2d \cos \left[\frac{3}{2} (e+fx) \right] + 54Ac^2d^2 \cos \left[\frac{3}{2} (e+fx) \right] - 86Bcd^2 \cos \left[\frac{3}{2} (e+fx) \right] + \\
& \quad 75Ad^3 \cos \left[\frac{3}{2} (e+fx) \right] - 19Bd^3 \cos \left[\frac{3}{2} (e+fx) \right] + 3Ac^2d \cos \left[\frac{5}{2} (e+fx) \right] + 5Bc^2d \cos \left[\frac{5}{2} (e+fx) \right] - 16Ac^2d^2 \cos \left[\frac{5}{2} (e+fx) \right] + \\
& \quad 32Bcd^2 \cos \left[\frac{5}{2} (e+fx) \right] - 35Ad^3 \cos \left[\frac{5}{2} (e+fx) \right] + 11Bd^3 \cos \left[\frac{5}{2} (e+fx) \right] + 22Ac^3 \sin \left[\frac{1}{2} (e+fx) \right] - 6Bc^3 \sin \left[\frac{1}{2} (e+fx) \right] - \\
& \quad 40Ac^2d \sin \left[\frac{1}{2} (e+fx) \right] + 40Bc^2d \sin \left[\frac{1}{2} (e+fx) \right] - 54Ac^2d^2 \sin \left[\frac{1}{2} (e+fx) \right] + 70Bcd^2 \sin \left[\frac{1}{2} (e+fx) \right] - 24Ad^3 \sin \left[\frac{1}{2} (e+fx) \right] - \\
& \quad 8Bd^3 \sin \left[\frac{1}{2} (e+fx) \right] - 6Ac^3 \sin \left[\frac{3}{2} (e+fx) \right] - 10Bc^3 \sin \left[\frac{3}{2} (e+fx) \right] + 21Ac^2d \sin \left[\frac{3}{2} (e+fx) \right] - 29Bc^2d \sin \left[\frac{3}{2} (e+fx) \right] + \\
& \quad 54Ac^2d^2 \sin \left[\frac{3}{2} (e+fx) \right] - 86Bcd^2 \sin \left[\frac{3}{2} (e+fx) \right] + 75Ad^3 \sin \left[\frac{3}{2} (e+fx) \right] - 19Bd^3 \sin \left[\frac{3}{2} (e+fx) \right] - 3Ac^2d \sin \left[\frac{5}{2} (e+fx) \right] - \\
& \quad \left. 5Bc^2d \sin \left[\frac{5}{2} (e+fx) \right] + 16Ac^2d^2 \sin \left[\frac{5}{2} (e+fx) \right] - 32Bcd^2 \sin \left[\frac{5}{2} (e+fx) \right] + 35Ad^3 \sin \left[\frac{5}{2} (e+fx) \right] - 11Bd^3 \sin \left[\frac{5}{2} (e+fx) \right] \right) \left. \right)
\end{aligned}$$

■ **Problem 327: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sin[e + f x]}{(a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])^3} dx$$

Optimal (type 3, 519 leaves, 9 steps):

$$\begin{aligned} & \frac{(B(5c^2 - 82cd - 115d^2) + 3A(c^2 - 10cd + 73d^2)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{2} \sqrt{a + a \sin[e + f x]}}\right]}{16\sqrt{2} a^{5/2} (c - d)^5 f} + \\ & \frac{d^{3/2} (3Ad(21c^2 + 30cd + 13d^2) - B(35c^3 + 70c^2d + 67cd^2 + 20d^3)) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{d} \cos[e + f x]}{\sqrt{c+d} \sqrt{a + a \sin[e + f x]}}\right]}{4a^{5/2} (c - d)^5 (c + d)^{5/2} f} - \\ & \frac{(A - B) \cos[e + f x]}{4(c - d) f (a + a \sin[e + f x])^{5/2} (c + d \sin[e + f x])^2} - \frac{(3Ac + 5Bc - 19Ad + 11Bd) \cos[e + f x]}{16a(c - d)^2 f (a + a \sin[e + f x])^{3/2} (c + d \sin[e + f x])^2} - \\ & \frac{d(A(3c^2 - 20cd - 31d^2) + B(5c^2 + 28cd + 15d^2)) \cos[e + f x]}{16a^2(c - d)^3 (c + d) f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])^2} - \\ & \frac{d(3A(c^3 - 7c^2d - 37cd^2 - 21d^3) + B(5c^3 + 73c^2d + 79cd^2 + 35d^3)) \cos[e + f x]}{16a^2(c - d)^4 (c + d)^2 f \sqrt{a + a \sin[e + f x]} (c + d \sin[e + f x])} \end{aligned}$$

Result (type 3, 2103 leaves):

$$\begin{aligned} & \left((1 + i) (3Ac^2 + 5Bc^2 - 30Acd - 82Bcd + 219Ad^2 - 115Bd^2) \right. \\ & \quad \left. \operatorname{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right] \left(\cos\left[\frac{1}{4}(e + f x)\right] - \sin\left[\frac{1}{4}(e + f x)\right]\right)\right] \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5\right] / \\ & \quad \left((16(-1)^{1/4} c^5 - 80(-1)^{1/4} c^4 d + 160(-1)^{1/4} c^3 d^2 - 160(-1)^{1/4} c^2 d^3 + 80(-1)^{1/4} c d^4 - 16(-1)^{1/4} d^5) f (a(1 + \sin[e + f x]))^{5/2} - \right. \\ & \quad \left. d^{3/2} (-3Ad(21c^2 + 30cd + 13d^2) + B(35c^3 + 70c^2d + 67cd^2 + 20d^3)) \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2\right] + \right. \right. \\ & \quad \left. \left. 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \left(\sqrt{c+d} + \sqrt{d} \cos\left[\frac{1}{2}(e + f x)\right] - \sqrt{d} \sin\left[\frac{1}{2}(e + f x)\right]\right)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 \right) / \\ & \quad (16(c - d)^5 (c + d)^{5/2} f (a(1 + \sin[e + f x]))^{5/2}) + \left(d^{3/2} (-3Ad(21c^2 + 30cd + 13d^2) + B(35c^3 + 70c^2d + 67cd^2 + 20d^3)) \right. \\ & \quad \left. \left(e + f x - 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2\right] + 2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \left(\sqrt{c+d} - \sqrt{d} \cos\left[\frac{1}{2}(e + f x)\right] + \sqrt{d} \sin\left[\frac{1}{2}(e + f x)\right]\right)\right] \right) \right) \\ & \quad \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right)^5 / (16(c - d)^5 (c + d)^{5/2} f (a(1 + \sin[e + f x]))^{5/2}) + \\ & \quad \frac{1}{128(c - d)^4 (c + d)^2 f (a(1 + \sin[e + f x]))^{5/2} (c + d \sin[e + f x])^2} \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right) \end{aligned}$$

$$\begin{aligned}
& \left(-44 A c^5 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + 12 B c^5 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + 84 A c^4 d \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - 116 B c^4 d \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + 249 A c^3 d^2 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \right. \\
& 433 B c^3 d^2 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + 385 A c^2 d^3 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - 277 B c^2 d^3 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + 239 A c d^4 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \\
& 95 B c d^4 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + 47 A d^5 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - 51 B d^5 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - 12 A c^5 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] - 20 B c^5 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] + \\
& 40 A c^4 d \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] - 104 B c^4 d \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] + 261 A c^3 d^2 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] - 581 B c^3 d^2 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] + \\
& 781 A c^2 d^3 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] - 665 B c^2 d^3 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] + 579 A c d^4 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] - 299 B c d^4 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] + \\
& 79 A d^5 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] - 59 B d^5 \operatorname{Cos}\left[\frac{3}{2}(e+f x)\right] + 12 A c^4 d \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] + 20 B c^4 d \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] - 73 A c^3 d^2 \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] + \\
& 217 B c^3 d^2 \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] - 353 A c^2 d^3 \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] + 397 B c^2 d^3 \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] - 419 A c d^4 \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] + \\
& 251 B c d^4 \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] - 127 A d^5 \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] + 75 B d^5 \operatorname{Cos}\left[\frac{5}{2}(e+f x)\right] + 3 A c^3 d^2 \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] + 5 B c^3 d^2 \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] - \\
& 21 A c^2 d^3 \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] + 73 B c^2 d^3 \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] - 111 A c d^4 \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] + 79 B c d^4 \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] - 63 A d^5 \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] + \\
& 35 B d^5 \operatorname{Cos}\left[\frac{7}{2}(e+f x)\right] + 44 A c^5 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 12 B c^5 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 84 A c^4 d \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 116 B c^4 d \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - \\
& 249 A c^3 d^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 433 B c^3 d^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 385 A c^2 d^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 277 B c^2 d^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - \\
& 239 A c d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 95 B c d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 47 A d^5 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 51 B d^5 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 12 A c^5 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - \\
& 20 B c^5 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 40 A c^4 d \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 104 B c^4 d \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 261 A c^3 d^2 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 581 B c^3 d^2 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + \\
& 781 A c^2 d^3 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 665 B c^2 d^3 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 579 A c d^4 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 299 B c d^4 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] + 79 A d^5 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - \\
& 59 B d^5 \operatorname{Sin}\left[\frac{3}{2}(e+f x)\right] - 12 A c^4 d \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 20 B c^4 d \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + 73 A c^3 d^2 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 217 B c^3 d^2 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + \\
& 353 A c^2 d^3 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 397 B c^2 d^3 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + 419 A c d^4 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 251 B c d^4 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + \\
& 127 A d^5 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] - 75 B d^5 \operatorname{Sin}\left[\frac{5}{2}(e+f x)\right] + 3 A c^3 d^2 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] + 5 B c^3 d^2 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] - 21 A c^2 d^3 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] + \\
& \left. 73 B c^2 d^3 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] - 111 A c d^4 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] + 79 B c d^4 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] - 63 A d^5 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] + 35 B d^5 \operatorname{Sin}\left[\frac{7}{2}(e+f x)\right] \right)
\end{aligned}$$

■ **Problem 328: Unable to integrate problem.**

$$\int (a + a \sin[e + f x])^2 (A + B \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 221 leaves, 7 steps):

$$-\frac{1}{f \sqrt{1 + \sin[e + f x]}} 8 \sqrt{2} a^2 B \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{5}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right]$$

$$\cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n} - \frac{1}{f \sqrt{1 + \sin[e + f x]}}$$

$$4 \sqrt{2} a^2 (A - B) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 8, 37 leaves):

$$\int (a + a \sin[e + f x])^2 (A + B \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

■ **Problem 329: Unable to integrate problem.**

$$\int (a + a \sin[e + f x]) (A + B \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 217 leaves, 8 steps):

$$-\frac{1}{f \sqrt{1 + \sin[e + f x]}} 4 \sqrt{2} a B \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right]$$

$$\cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n} - \frac{1}{f \sqrt{1 + \sin[e + f x]}}$$

$$2 \sqrt{2} a (A - B) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d}\right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d}\right)^{-n}$$

Result (type 8, 35 leaves):

$$\int (a + a \sin[e + f x]) (A + B \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

■ **Problem 330: Unable to integrate problem.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^n}{a + a \sin[e + f x]} dx$$

Optimal (type 6, 221 leaves, 7 steps):

$$\begin{aligned}
& - \frac{1}{a f \sqrt{1 + \sin[e + f x]}} \sqrt{2} \text{BAppellF1} \left[\frac{1}{2}, \frac{1}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d} \right] \\
& \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d} \right)^{-n} - \frac{1}{\sqrt{2} a f \sqrt{1 + \sin[e + f x]}} \\
& (A - B) \text{AppellF1} \left[\frac{1}{2}, \frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d} \right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d} \right)^{-n}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^n}{a + a \sin[e + f x]} dx$$

■ **Problem 331: Unable to integrate problem.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^2} dx$$

Optimal (type 6, 223 leaves, 7 steps):

$$\begin{aligned}
& - \left(\text{BAppellF1} \left[\frac{1}{2}, \frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d} \right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d} \right)^{-n} \right) / \\
& \left(\sqrt{2} a^2 f \sqrt{1 + \sin[e + f x]} \right) - \\
& \left((A - B) \text{AppellF1} \left[\frac{1}{2}, \frac{5}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{d (1 - \sin[e + f x])}{c + d} \right] \cos[e + f x] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d} \right)^{-n} \right) / \\
& \left(2 \sqrt{2} a^2 f \sqrt{1 + \sin[e + f x]} \right)
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^n}{(a + a \sin[e + f x])^2} dx$$

■ **Problem 333: Unable to integrate problem.**

$$\int \sqrt{a + a \sin[e + f x]} (A + B \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

Optimal (type 5, 167 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 a B \cos[e + f x] (c + d \sin[e + f x])^{1+n}}{d f (3 + 2 n) \sqrt{a + a \sin[e + f x]}} - \\
& \left(2 a (A d (3 + 2 n) - B (c - 2 d (1 + n))) \cos[e + f x] \text{Hypergeometric2F1} \left[\frac{1}{2}, -n, \frac{3}{2}, \frac{d (1 - \sin[e + f x])}{c + d} \right] (c + d \sin[e + f x])^n \right. \\
& \left. \left(\frac{c + d \sin[e + f x]}{c + d} \right)^{-n} \right) / \left(d f (3 + 2 n) \sqrt{a + a \sin[e + f x]} \right)
\end{aligned}$$

Result (type 8, 39 leaves):

$$\int \sqrt{a + a \sin[e + f x]} (A + B \sin[e + f x]) (c + d \sin[e + f x])^n dx$$

■ **Problem 334: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^n}{\sqrt{a + a \sin[e + f x]}} dx$$

Optimal (type 6, 220 leaves, 7 steps):

$$\begin{aligned} & - \left((A - B) \operatorname{AppellF1} \left[1 + n, \frac{1}{2}, 1, 2 + n, \frac{c + d \sin[e + f x]}{c + d}, \frac{c + d \sin[e + f x]}{c - d} \right] \cos[e + f x] \sqrt{\frac{d(1 - \sin[e + f x])}{c + d}} (c + d \sin[e + f x])^{1+n} \right) / \\ & \frac{\left((c - d) f (1 + n) (1 - \sin[e + f x]) \sqrt{a + a \sin[e + f x]} \right) - 2 B \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -n, \frac{3}{2}, \frac{d(1 - \sin[e + f x])}{c + d} \right] (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c + d} \right)^{-n}}{f \sqrt{a + a \sin[e + f x]}} \end{aligned}$$

Result (type 6, 1282 leaves):

$$\begin{aligned} & \frac{1}{a} \left(\left(a^2 B \cos[e + f x] \sin[e + f x] (1 + \sin[e + f x])^2 (c + d \sin[e + f x])^{2n} \left(c + \frac{d(-a + a(1 + \sin[e + f x]))}{a} \right)^{-n} \right. \right. \\ & \left(\left(4 a (c - d) \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), -\frac{a d (1 + \sin[e + f x])}{a c - a d} \right] \right) / \left(8 a (c - d) \right. \right. \\ & \left. \left. \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), -\frac{a d (1 + \sin[e + f x])}{a c - a d} \right] + a \left(4 d n \operatorname{AppellF1} \left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2} (1 + \sin[e + f x]), \right. \right. \right. \right. \\ & \left. \left. \left. -\frac{a d (1 + \sin[e + f x])}{a c - a d} \right] + (c - d) \operatorname{AppellF1} \left[2, \frac{3}{2}, -n, 3, \frac{1}{2} (1 + \sin[e + f x]), -\frac{a d (1 + \sin[e + f x])}{a c - a d} \right] \right) (1 + \sin[e + f x]) \right) + \\ & \left(d (-1 + 2 n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] (-2 a + a (1 + \sin[e + f x])) \right) / \\ & \left((1 + 2 n) \left(2 a \left((-c + d) n \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] + \right. \right. \right. \\ & \left. \left. \left. d \operatorname{AppellF1} \left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] \right) \right) + \\ & \left. \left. \left. a d (-1 + 2 n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] (1 + \sin[e + f x]) \right) \right) \right) / \\ & \left(f \sqrt{a (1 + \sin[e + f x])} (-a + a (1 + \sin[e + f x])) \sqrt{\frac{2 a^2 (1 + \sin[e + f x]) - a^2 (1 + \sin[e + f x])^2}{a^2}} \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \frac{(-a + a(1 + \sin[ex]))^2}{a^2}} + \\
& \left(a^2 A \cos[ex] (1 + \sin[ex])^2 (c + d \sin[ex])^{2n} \left(c + \frac{d(-a + a(1 + \sin[ex]))}{a} \right)^{-n} \right. \\
& \left(\left(4(c-d) \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1 + \sin[ex]), -\frac{ad(1 + \sin[ex])}{ac - ad} \right] \right) / \right. \\
& \left(8a(c-d) \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1 + \sin[ex]), -\frac{ad(1 + \sin[ex])}{ac - ad} \right] + \right. \\
& a \left(4dn \operatorname{AppellF1} \left[2, \frac{1}{2}, 1-n, 3, \frac{1}{2}(1 + \sin[ex]), -\frac{ad(1 + \sin[ex])}{ac - ad} \right] + \right. \\
& \left. \left. (c-d) \operatorname{AppellF1} \left[2, \frac{3}{2}, -n, 3, \frac{1}{2}(1 + \sin[ex]), -\frac{ad(1 + \sin[ex])}{ac - ad} \right] \right) (1 + \sin[ex]) \right) - \\
& \left. \left(d(-1 + 2n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[ex]}, \frac{-c + d}{d(1 + \sin[ex])} \right] (-2a + a(1 + \sin[ex])) \right) \right) / \\
& \left(a(1 + 2n) \left(2a \left((-c + d)n \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, 1-n, \frac{3}{2} - n, \frac{2}{1 + \sin[ex]}, \frac{-c + d}{d(1 + \sin[ex])} \right] + \right. \right. \right. \\
& \left. \left. d \operatorname{AppellF1} \left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[ex]}, \frac{-c + d}{d(1 + \sin[ex])} \right] \right) + \right. \\
& \left. \left. ad(-1 + 2n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[ex]}, \frac{-c + d}{d(1 + \sin[ex])} \right] (1 + \sin[ex]) \right) \right) \right) / \\
& \left(f \sqrt{a(1 + \sin[ex])} \sqrt{\frac{2a^2(1 + \sin[ex]) - a^2(1 + \sin[ex])^2}{a^2}} \sqrt{1 - \frac{(-a + a(1 + \sin[ex]))^2}{a^2}} \right)
\end{aligned}$$

■ **Problem 335: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[ex]) (c + d \sin[ex])^n}{(a + a \sin[ex])^{3/2}} dx$$

Optimal (type 6, 269 leaves, 7 steps):

$$\begin{aligned}
& - \left(B \operatorname{AppellF1} \left[1+n, \frac{1}{2}, 1, 2+n, \frac{c+d \operatorname{Sin}[e+f x]}{c+d}, \frac{c+d \operatorname{Sin}[e+f x]}{c-d} \right] \operatorname{Cos}[e+f x] \sqrt{\frac{d(1-\operatorname{Sin}[e+f x])}{c+d}} (c+d \operatorname{Sin}[e+f x])^{1+n} \right) / \\
& \quad \left(a(c-d) f(1+n)(1-\operatorname{Sin}[e+f x]) \sqrt{a+a \operatorname{Sin}[e+f x]} \right) + \\
& \left((A-B) d \operatorname{AppellF1} \left[1+n, \frac{1}{2}, 2, 2+n, \frac{c+d \operatorname{Sin}[e+f x]}{c+d}, \frac{c+d \operatorname{Sin}[e+f x]}{c-d} \right] \operatorname{Cos}[e+f x] \sqrt{\frac{d(1-\operatorname{Sin}[e+f x])}{c+d}} (c+d \operatorname{Sin}[e+f x])^{1+n} \right) / \\
& \quad \left((c-d)^2 f(1+n)(a-a \operatorname{Sin}[e+f x]) \sqrt{a+a \operatorname{Sin}[e+f x]} \right)
\end{aligned}$$

Result (type 6, 1854 leaves):

$$\begin{aligned}
& \left(B \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x] (1+\operatorname{Sin}[e+f x]) (c+d \operatorname{Sin}[e+f x])^{2n} \left(c + \frac{d(-a+a(1+\operatorname{Sin}[e+f x]))}{a} \right)^{-n} \right. \\
& \quad \left(\left(4 a(c-d) \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\operatorname{Sin}[e+f x]), -\frac{ad(1+\operatorname{Sin}[e+f x])}{ac-ad} \right] (1+\operatorname{Sin}[e+f x]) \right) / \right. \\
& \quad \left(8 a(c-d) \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2}(1+\operatorname{Sin}[e+f x]), -\frac{ad(1+\operatorname{Sin}[e+f x])}{ac-ad} \right] + a \left(4 d n \operatorname{AppellF1} \left[2, \frac{1}{2}, 1-n, 3, \frac{1}{2}(1+\operatorname{Sin}[e+f x]), \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{ad(1+\operatorname{Sin}[e+f x])}{ac-ad} \right] + (c-d) \operatorname{AppellF1} \left[2, \frac{3}{2}, -n, 3, \frac{1}{2}(1+\operatorname{Sin}[e+f x]), -\frac{ad(1+\operatorname{Sin}[e+f x])}{ac-ad} \right] \right) (1+\operatorname{Sin}[e+f x]) \right) - \\
& \quad \left(d(-1+2n) \operatorname{AppellF1} \left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\operatorname{Sin}[e+f x]}, \frac{-c+d}{d(1+\operatorname{Sin}[e+f x])} \right] (1+\operatorname{Sin}[e+f x]) (-2a+a(1+\operatorname{Sin}[e+f x])) \right) / \\
& \quad \left((1+2n) \left(2 a \left((-c+d) n \operatorname{AppellF1} \left[\frac{1}{2}-n, -\frac{1}{2}, 1-n, \frac{3}{2}-n, \frac{2}{1+\operatorname{Sin}[e+f x]}, \frac{-c+d}{d(1+\operatorname{Sin}[e+f x])} \right] + \right. \right. \\
& \quad \left. \left. d \operatorname{AppellF1} \left[\frac{1}{2}-n, \frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\operatorname{Sin}[e+f x]}, \frac{-c+d}{d(1+\operatorname{Sin}[e+f x])} \right] \right) + \right. \\
& \quad \left. a d(-1+2n) \operatorname{AppellF1} \left[-\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{1}{2}-n, \frac{2}{1+\operatorname{Sin}[e+f x]}, \frac{-c+d}{d(1+\operatorname{Sin}[e+f x])} \right] (1+\operatorname{Sin}[e+f x]) \right) \right) + \\
& \quad \left(2 d(-3+2n) \operatorname{AppellF1} \left[\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\operatorname{Sin}[e+f x]}, \frac{-c+d}{d(1+\operatorname{Sin}[e+f x])} \right] (-2a+a(1+\operatorname{Sin}[e+f x])) \right) / \\
& \quad \left((-1+2n) \left(2 a \left((-c+d) n \operatorname{AppellF1} \left[\frac{3}{2}-n, -\frac{1}{2}, 1-n, \frac{5}{2}-n, \frac{2}{1+\operatorname{Sin}[e+f x]}, \frac{-c+d}{d(1+\operatorname{Sin}[e+f x])} \right] + \right. \right. \\
& \quad \left. \left. d \operatorname{AppellF1} \left[\frac{3}{2}-n, \frac{1}{2}, -n, \frac{5}{2}-n, \frac{2}{1+\operatorname{Sin}[e+f x]}, \frac{-c+d}{d(1+\operatorname{Sin}[e+f x])} \right] \right) + \right. \\
& \quad \left. a d(-3+2n) \operatorname{AppellF1} \left[\frac{1}{2}-n, -\frac{1}{2}, -n, \frac{3}{2}-n, \frac{2}{1+\operatorname{Sin}[e+f x]}, \frac{-c+d}{d(1+\operatorname{Sin}[e+f x])} \right] (1+\operatorname{Sin}[e+f x]) \right) \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(2 f \sqrt{a (1 + \sin[e + f x])} (-a + a (1 + \sin[e + f x])) \sqrt{\frac{2 a^2 (1 + \sin[e + f x]) - a^2 (1 + \sin[e + f x])^2}{a^2}} \right. \\
& \left. \sqrt{1 - \frac{(-a + a (1 + \sin[e + f x]))^2}{a^2}} \right) + \left(A \right. \\
& \cos[e + f x] \\
& (1 + \sin[e + f x]) \\
& (c + d \sin[e + f x])^{2n} \\
& \left(c + \frac{d (-a + a (1 + \sin[e + f x]))}{a} \right)^{-n} \\
& \left(\left(4 a^2 (c - d) \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), -\frac{a d (1 + \sin[e + f x])}{a c - a d} \right] (1 + \sin[e + f x]) \right) / \right. \\
& \left(8 a (c - d) \operatorname{AppellF1} \left[1, \frac{1}{2}, -n, 2, \frac{1}{2} (1 + \sin[e + f x]), -\frac{a d (1 + \sin[e + f x])}{a c - a d} \right] + a \left(4 d n \operatorname{AppellF1} \left[2, \frac{1}{2}, 1 - n, 3, \frac{1}{2} (1 + \sin[e + f x]), \right. \right. \right. \\
& \left. \left. \left. -\frac{a d (1 + \sin[e + f x])}{a c - a d} \right] + (c - d) \operatorname{AppellF1} \left[2, \frac{3}{2}, -n, 3, \frac{1}{2} (1 + \sin[e + f x]), -\frac{a d (1 + \sin[e + f x])}{a c - a d} \right] \right) (1 + \sin[e + f x]) \right) - \\
& \left(a d (-1 + 2 n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] (1 + \sin[e + f x]) (-2 a + a (1 + \sin[e + f x])) \right) / \\
& \left((1 + 2 n) \left(2 a \left((-c + d) n \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, 1 - n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] \right) + \right. \\
& \left. d \operatorname{AppellF1} \left[\frac{1}{2} - n, \frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] \right) + \\
& \left. a d (-1 + 2 n) \operatorname{AppellF1} \left[-\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{1}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] (1 + \sin[e + f x]) \right) \left. \right) - \\
& \left(2 a d (-3 + 2 n) \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] (-2 a + a (1 + \sin[e + f x])) \right) / \\
& \left((-1 + 2 n) \left(2 a \left((-c + d) n \operatorname{AppellF1} \left[\frac{3}{2} - n, -\frac{1}{2}, 1 - n, \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] \right) + \right. \\
& \left. d \operatorname{AppellF1} \left[\frac{3}{2} - n, \frac{1}{2}, -n, \frac{5}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] \right) + \\
& \left. a d (-3 + 2 n) \operatorname{AppellF1} \left[\frac{1}{2} - n, -\frac{1}{2}, -n, \frac{3}{2} - n, \frac{2}{1 + \sin[e + f x]}, \frac{-c + d}{d (1 + \sin[e + f x])} \right] (1 + \sin[e + f x]) \right) \left. \right) \left. \right) / \\
& \left(2 a^2 f \sqrt{a (1 + \sin[e + f x])} \sqrt{\frac{2 a^2 (1 + \sin[e + f x]) - a^2 (1 + \sin[e + f x])^2}{a^2}} \sqrt{1 - \frac{(-a + a (1 + \sin[e + f x]))^2}{a^2}} \right)
\end{aligned}$$

Problem 336: Result more than twice size of optimal antiderivative.

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) (c + d \sin[e + f x])^2 dx$$

Optimal (type 5, 351 leaves, 6 steps):

$$\frac{1}{f(1+m)(2+m)(3+m)}$$

$$\left(d(A d(3+m) + B(2c + d m)) - 2(2+m)(A c d(3+m) + B(c^2 + d^2 + c d m)) \right) \cos[e + f x] (a + a \sin[e + f x])^m - \frac{1}{f(1+m)(2+m)(3+m)}$$

$$2^{\frac{1}{2}+m} (A(3+m)(2c d m(2+m) + d^2(1+m+m^2) + c^2(2+3m+m^2)) + B(d^2 m(5+3m+m^2) + c^2 m(6+5m+m^2) + 2c d(3+4m+4m^2+m^3)))$$

$$\cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2}(1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{2}-m} (a + a \sin[e + f x])^m -$$

$$\frac{d(A d(3+m) + B(2c + d m)) \cos[e + f x] (a + a \sin[e + f x])^{1+m}}{a f(2+m)(3+m)} - \frac{B \cos[e + f x] (a + a \sin[e + f x])^m (c + d \sin[e + f x])^2}{f(3+m)}$$

Result (type 5, 23845 leaves): Display of huge result suppressed!

■ **Problem 338: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) dx$$

Optimal (type 5, 117 leaves, 3 steps):

$$-\frac{B \cos[e + f x] (a + a \sin[e + f x])^m}{f(1+m)} - \frac{1}{f(1+m)}$$

$$2^{\frac{1}{2}+m} (A + A m + B m) \cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2}(1 - \sin[e + f x])\right] (1 + \sin[e + f x])^{-\frac{1}{2}-m} (a + a \sin[e + f x])^m$$

Result (type 5, 295 leaves):

$$-\frac{1}{f} (a(1 + \sin[e + f x]))^m \left(\frac{1}{-1+m^2} 2^{-1-2m} B e^{-i(e+fx)} (1 + i e^{-i(e+fx)})^{-2m} \left(e^{-\frac{1}{4}i(2e+\pi+2fx)} (i + e^{i(e+fx)}) \right)^{2m} \right.$$

$$\left. (e^{2i(e+fx)} (-1+m) \operatorname{Hypergeometric2F1}[1-m, -2m, -m, -i e^{-i(e+fx)}] - (1+m) \operatorname{Hypergeometric2F1}[1-m, -2m, 2-m, -i e^{-i(e+fx)}]) + \right.$$

$$\left. \left(2\sqrt{2} A \cos\left[\frac{1}{4}(2e-\pi+2fx)\right]^{1+2m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}+m, \frac{3}{2}+m, \sin\left[\frac{1}{4}(2e+\pi+2fx)\right]^2\right] \sin\left[\frac{1}{4}(2e-\pi+2fx)\right] \right) \right/$$

$$\left((1+2m) \sqrt{1 - \sin[e + f x]} \right) \sin\left[\frac{1}{4}(2e+\pi+2fx)\right]^{-2m}$$

■ **Problem 339: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m (A + B \sin[e + f x])}{c + d \sin[e + f x]} dx$$

Optimal (type 6, 191 leaves, 6 steps):

$$\begin{aligned}
& - \left(\sqrt{2} (B c - A d) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, 1, \frac{3}{2} + m, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), -\frac{d (1 + \operatorname{Sin}[e + f x])}{c - d} \right] \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^m \right) / \\
& \left((c - d) d f (1 + 2 m) \sqrt{1 - \operatorname{Sin}[e + f x]} \right) - \frac{1}{d f} \\
& 2^{\frac{1}{2} + m} B \operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sin}[e + f x]) \right] (1 + \operatorname{Sin}[e + f x])^{-\frac{1}{2} - m} (a + a \operatorname{Sin}[e + f x])^m
\end{aligned}$$

Result (type 6, 1022 leaves):

$$\begin{aligned}
& -\frac{1}{f} \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{-2 m} \\
& \left(-\left(\left(6 A(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, 1, \frac{3}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{-1+2 m}\right.\right.\right. \\
& \quad \left.\left.\left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{\frac{1}{2}-m} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\left(1-\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-\frac{1}{2}+m}\right)\right) / \\
& \quad \left(\left(c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, 1, \frac{3}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right)+\right. \\
& \quad \left(-4 d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, 2, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]+(c+d)(-1+2 m)\right. \\
& \quad \left.\left.\operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, 1, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\right)\right) + \\
& B\left(-\left(2 \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{1+2 m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}(1+2 m), \frac{1}{2}(3+2 m), \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right) / \right. \\
& \quad \left(\left(d(1+2 m) \sqrt{\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}\right)+\left(6 c(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, 1, \frac{3}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right)\right) \\
& \quad \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{-1+2 m}\left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{\frac{1}{2}-m} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\left(1-\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-\frac{1}{2}+m}\right) / \\
& \quad \left(d\left(c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, 1, \frac{3}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right)+\right. \\
& \quad \left(-4 d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, 2, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]+(c+d)(-1+2 m) \operatorname{AppellF1}\left[\frac{3}{2},\right.\right. \\
& \quad \left.\left.\frac{3}{2}-m, 1, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\right)\right)\right) (a+a \operatorname{Sin}[e+f x])^m
\end{aligned}$$

■ **Problem 340: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m (A + B \sin[e + f x])}{(c + d \sin[e + f x])^2} dx$$

Optimal (type 6, 293 leaves, 7 steps):

$$\begin{aligned} & \left(\sqrt{2} (A d (c (1 - m) - d m) - B (d^2 - c^2 m - c d m)) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, 1, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \right. \\ & \quad \left. \cos[e + f x] (a + a \sin[e + f x])^m \right) / \left((c - d)^2 d (c + d) f (1 + 2 m) \sqrt{1 - \sin[e + f x]} \right) + \frac{1}{d (c^2 - d^2) f} \\ & 2^{\frac{1}{2} + m} (B c - A d) m \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]) \right] (1 + \sin[e + f x])^{-\frac{1}{2} - m} (a + a \sin[e + f x])^m - \\ & \quad \frac{(B c - A d) \cos[e + f x] (a + a \sin[e + f x])^m}{(c^2 - d^2) f (c + d \sin[e + f x])} \end{aligned}$$

Result (type 6, 1332 leaves):

$$\begin{aligned} & -\frac{1}{f} \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-2m} \\ & \left(- \left(\left(6 A (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 2, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \right. \right. \\ & \quad \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2} - m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2} + m} \right) / \right. \\ & \quad \left. \left(\left(c + d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^2 \left(-3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 2, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right)^2 + \right. \right. \\ & \quad \left. \left(-8 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 3, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] + (c + d) (-1 + 2 m) \right. \right. \\ & \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, 2, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) + \\ & B \left(- \left(\left(6 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 1, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \right. \right. \\ & \quad \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2} - m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2} + m} \right) / \right. \end{aligned}$$

$$\begin{aligned}
& \left(d \left(c + d - 2 d \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^2 \right) \left(-3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 1, \frac{3}{2}, \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) + \right. \\
& \left(-4 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 2, \frac{5}{2}, \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) + (c + d) (-1 + 2 m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, 1, \frac{5}{2}, \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) + \\
& \left(6 c (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 2, \frac{3}{2}, \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2 m} \right. \\
& \left. \left(\operatorname{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^{\frac{1}{2}-m} \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^{-\frac{1}{2}+m} \right) / \\
& \left(d \left(c + d - 2 d \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^2 \right) \left(-3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 2, \frac{3}{2}, \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) + \right. \\
& \left(-8 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 3, \frac{5}{2}, \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) + (c + d) (-1 + 2 m) \operatorname{AppellF1} \left[\frac{3}{2}, \right. \\
& \left. \left. \frac{3}{2} - m, 2, \frac{5}{2}, \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \operatorname{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) \left(a + a \operatorname{Sin} [e + f x] \right)^m
\end{aligned}$$

■ **Problem 341: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin} [e + f x])^m (A + B \operatorname{Sin} [e + f x])}{(c + d \operatorname{Sin} [e + f x])^3} dx$$

Optimal (type 6, 467 leaves, 8 steps):

$$\begin{aligned}
& \left((B (2 d^3 m + c^3 (1 - m) m + 2 c^2 d (1 - m) m - c d^2 (3 - 3 m + m^2)) - A d (2 c d (2 - m) m - c^2 (2 - 3 m + m^2) - d^2 (1 - m + m^2))) \right. \\
& \quad \left. \text{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, 1, \frac{3}{2} + m, \frac{1}{2} (1 + \text{Sin}[e + f x]), -\frac{d (1 + \text{Sin}[e + f x])}{c - d} \right] \text{Cos}[e + f x] (a + a \text{Sin}[e + f x])^m \right] / \\
& \quad \left(\sqrt{2} (c - d)^3 d (c + d)^2 f (1 + 2 m) \sqrt{1 - \text{Sin}[e + f x]} \right) - \frac{1}{d (c^2 - d^2)^2 f} 2^{-\frac{1}{2} + m} m (A d (c (3 - m) - d m) - B (2 d^2 + c^2 (1 - m) - c d m)) \\
& \quad \text{Cos}[e + f x] \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2} (1 - \text{Sin}[e + f x]) \right] (1 + \text{Sin}[e + f x])^{-\frac{1}{2} - m} (a + a \text{Sin}[e + f x])^m - \\
& \quad \frac{(B c - A d) \text{Cos}[e + f x] (a + a \text{Sin}[e + f x])^m}{2 (c^2 - d^2) f (c + d \text{Sin}[e + f x])^2} + \frac{(A d (c (3 - m) - d m) - B (2 d^2 + c^2 (1 - m) - c d m)) \text{Cos}[e + f x] (a + a \text{Sin}[e + f x])^m}{2 (c^2 - d^2)^2 f (c + d \text{Sin}[e + f x])}
\end{aligned}$$

Result (type 6, 1332 leaves):

$$\begin{aligned}
& -\frac{1}{f} \text{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-2m} \\
& \left(- \left(\left(6 A (c + d) \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 3, \frac{3}{2}, \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \text{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \right. \\
& \quad \left. \left. \left(\text{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2} - m} \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2} + m} \right) / \right. \\
& \quad \left(\left(c + d - 2 d \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^3 \left(-3 (c + d) \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 3, \frac{3}{2}, \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) + \right. \\
& \quad \left(-12 d \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 4, \frac{5}{2}, \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] + (c + d) (-1 + 2 m) \right. \\
& \quad \left. \left. \left. \left. \text{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, 3, \frac{5}{2}, \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) + \\
& \quad B \left(- \left(\left(6 (c + d) \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 2, \frac{3}{2}, \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \text{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left(\text{Cos} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2} - m} \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \text{Sin} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2} + m} \right) / \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(d \left(c + d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^2 \right)^2 \left(-3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 2, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) + \\
& \left(-8 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 3, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) + (c + d) (-1 + 2 m) \\
& \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, 2, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) + \\
& \left(6 c (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 3, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2 m} \right. \\
& \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^{\frac{1}{2}-m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^{-\frac{1}{2}+m} \right) / \\
& \left(d \left(c + d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^2 \right)^3 \left(-3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, 3, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) + \right. \\
& \left(-12 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 4, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) + (c + d) (-1 + 2 m) \operatorname{AppellF1} \left[\frac{3}{2}, \right. \\
& \left. \frac{3}{2} - m, 3, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) (a + a \sin [e + f x])^m
\end{aligned}$$

■ **Problem 342: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin [e + f x])^m (A + B \sin [e + f x]) (c + d \sin [e + f x])^{3/2} dx$$

Optimal (type 6, 284 leaves, 9 steps):

$$\left(\sqrt{2} (A - B) (c - d) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, -\frac{3}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c - d} \right] \right. \\ \left. \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{c + d \sin[e + f x]} \right) / \left(f (1 + 2m) \sqrt{1 - \sin[e + f x]} \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right) + \\ \left(\sqrt{2} B (c - d) \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, -\frac{3}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c - d} \right] \cos[e + f x] \right. \\ \left. (a + a \sin[e + f x])^{1+m} \sqrt{c + d \sin[e + f x]} \right) / \left(a f (3 + 2m) \sqrt{1 - \sin[e + f x]} \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right)$$

Result (type 6, 4033 leaves):

$$-\frac{1}{f} \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-2m} \\ \left(- \left(\left(3 B d (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2} - m, -\frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{3+2m} \right. \right. \\ \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2} + \frac{1}{2} (-4-2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{3}{2} + m} \sqrt{c + d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} \right) / \\ \left(-3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2} - m, -\frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] + \right. \\ \left. \left(2 d \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2} - m, \frac{1}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] + \right. \\ \left. \left. (c + d) (3 + 2m) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - m, -\frac{1}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\ \left(6 B c (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - m, -\frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{1+2m} \right. \\ \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2} + \frac{1}{2} (-2-2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2} + m} \sqrt{c + d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} \right) / \right)$$

$$\begin{aligned}
& (c+d) (-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) + \\
& \left(3Bd(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-1+2m} \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2} \right) / \right. \\
& \quad \left. \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] - \right. \right. \\
& \quad \left. \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \right. \\
& \quad \left. \left. (c+d) (-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) + \right. \\
& \quad \left. \left(10Bd(c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{1+2m} \right. \right. \\
& \quad \left. \left. \left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{\frac{1}{2}(-1-2m)} \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^3 \left(1 - \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2} \right) / \right. \\
& \quad \left. \left(-5(c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \right. \\
& \quad \left. \left(2d \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}-m, \frac{1}{2}, \frac{7}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \right. \\
& \quad \left. \left. (c+d) (1+2m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{7}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) + \right. \\
& \quad \left. \left(10Bc(c+d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-1+2m} \right. \right.
\end{aligned}$$

$$\left(2 d \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{9}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + (c+d)(-1+2m) \right. \\ \left. \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{9}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) \right) (a+a \sin[e+f x])^m$$

■ **Problem 343: Result more than twice size of optimal antiderivative.**

$$\int (a+a \sin[e+f x])^m (A+B \sin[e+f x]) \sqrt{c+d \sin[e+f x]} dx$$

Optimal (type 6, 274 leaves, 9 steps):

$$\left(\sqrt{2} (A-B) \operatorname{AppellF1}\left[\frac{1}{2}+m, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1+\sin[e+f x]), -\frac{d(1+\sin[e+f x])}{c-d}\right] \cos[e+f x] (a+a \sin[e+f x])^m \sqrt{c+d \sin[e+f x]} \right) / \\ \left(f(1+2m) \sqrt{1-\sin[e+f x]} \sqrt{\frac{c+d \sin[e+f x]}{c-d}} \right) + \\ \left(\sqrt{2} B \operatorname{AppellF1}\left[\frac{3}{2}+m, \frac{1}{2}, -\frac{1}{2}, \frac{5}{2}, \frac{1}{2}(1+\sin[e+f x]), -\frac{d(1+\sin[e+f x])}{c-d}\right] \cos[e+f x] (a+a \sin[e+f x])^{1+m} \sqrt{c+d \sin[e+f x]} \right) / \\ \left(a f(3+2m) \sqrt{1-\sin[e+f x]} \sqrt{\frac{c+d \sin[e+f x]}{c-d}} \right)$$

Result (type 6, 1364 leaves):

$$-\frac{1}{f} \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{-2m} \\ \left(\left(6 A(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{-1+2m} \right. \right. \\ \left. \left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(1-\sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d-2 d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2} \right) / \\ \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] - \right. \\ \left. \left(2 d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + \right. \right.$$

$$\begin{aligned}
& (c+d) (-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) + \\
& B \left[\left(\left(6(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{3}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-1+2m} \right. \right. \right. \\
& \quad \left. \left. \left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{-\frac{1}{2}+m} \left(c+d - 2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{3/2} \right) \right] / \\
& \quad \left(d \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{3}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \right. \\
& \quad \left. \left(6d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + (c+d) (-1+2m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -\frac{3}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) \Bigg) - \\
& \quad \left(6c(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-1+2m} \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2} \right) \right] / \\
& \quad \left(d \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] - \right. \right. \\
& \quad \left. \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + (c+d) (-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
& \quad \left. \left. \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) \Bigg) \Bigg) (a + a \sin[e + fx])^m
\end{aligned}$$

■ **Problem 344: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m (A + B \sin[e + f x])}{\sqrt{c + d \sin[e + f x]}} dx$$

Optimal (type 6, 274 leaves, 9 steps):

$$\left(\sqrt{2} (A - B) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c - d} \right] \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right) /$$

$$\left(f (1 + 2m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) +$$

$$\left(\sqrt{2} B \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c - d} \right] \cos[e + f x] (a + a \sin[e + f x])^{1+m} \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right) /$$

$$\left(a f (3 + 2m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right)$$

Result (type 6, 1363 leaves):

$$-\frac{1}{f} \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-2m}$$

$$\left(- \left(\left(6 A (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \right.$$

$$\left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}-m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \right) /$$

$$\left(\sqrt{c + d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} \left(-3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right. \right.$$

$$\left. \left(-2 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) + (c + d) (-1 + 2m) \right.$$

$$\left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, \frac{1}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) +$$

$$B \left(\left(6 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -\frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \right.$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{-\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \Bigg/ \\
& \left(d \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right) - \right. \\
& \quad \left. \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right) + \right. \\
& \quad \left. (c+d) (-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \Bigg) + \\
& \left(6c(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-1+2m} \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{-\frac{1}{2}+m} \right) \Bigg/ \\
& \left(d \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right) + \right. \\
& \quad \left. \left(-2d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, \frac{3}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right) + (c+d) (-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
& \quad \left. \left. \frac{3}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \Bigg) \Bigg) \Bigg) (a + a \sin[e + fx])^m
\end{aligned}$$

■ **Problem 345: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^m (A + B \sin[e + fx])}{(c + d \sin[e + fx])^{3/2}} dx$$

Optimal (type 6, 288 leaves, 9 steps):

$$\left(\sqrt{2} (A-B) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c-d} \right] \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{\frac{c + d \sin[e + f x]}{c-d}} \right) /$$

$$\left((c-d) f (1+2m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) +$$

$$\left(\sqrt{2} B \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, \frac{3}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c-d} \right] \cos[e + f x] (a + a \sin[e + f x])^{1+m} \sqrt{\frac{c + d \sin[e + f x]}{c-d}} \right) /$$

$$\left(a (c-d) f (3+2m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right)$$

Result (type 6, 1362 leaves):

$$-\frac{1}{f} \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-2m}$$

$$\left(- \left(\left(6 A (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \right.$$

$$\left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}-m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \right) /$$

$$\left(\left(c+d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{3/2} \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right. \right.$$

$$\left. \left(-6 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{5}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (-1+2m) \right. \right.$$

$$\left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, \frac{3}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) +$$

$$B \left(- \left(\left(6 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \right.$$

$$\left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}-m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \right) /$$

$$\left(d \sqrt{c+d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right. \right. +$$

$$\begin{aligned}
& \left(-2 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] + (c + d) (-1 + 2 m) \right. \\
& \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, \frac{1}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) + \\
& \left(6 c (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2 m} \right. \\
& \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}-m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \right) / \\
& \left(d \left(c + d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{3/2} \left(-3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] + \right. \right. \\
& \left. \left(-6 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, \frac{5}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] + (c + d) (-1 + 2 m) \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \\
& \left. \left. \frac{3}{2} - m, \frac{3}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) \left(a + a \sin [e + f x] \right)^m
\end{aligned}$$

■ **Problem 346: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin [e + f x])^m (A + B \sin [e + f x]) (c + d \sin [e + f x])^n dx$$

Optimal (type 6, 270 leaves, 9 steps):

$$\begin{aligned}
& \left(\sqrt{2} (A - B) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, -n, \frac{3}{2} + m, \frac{1}{2} (1 + \sin [e + f x]), -\frac{d (1 + \sin [e + f x])}{c - d} \right] \right. \\
& \left. \cos [e + f x] (a + a \sin [e + f x])^m (c + d \sin [e + f x])^n \left(\frac{c + d \sin [e + f x]}{c - d} \right)^{-n} \right) / \left(f (1 + 2 m) \sqrt{1 - \sin [e + f x]} \right) + \\
& \left(\sqrt{2} B \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, -n, \frac{5}{2} + m, \frac{1}{2} (1 + \sin [e + f x]), -\frac{d (1 + \sin [e + f x])}{c - d} \right] \cos [e + f x] (a + a \sin [e + f x])^{1+m} \right. \\
& \left. (c + d \sin [e + f x])^n \left(\frac{c + d \sin [e + f x]}{c - d} \right)^{-n} \right) / \left(a f (3 + 2 m) \sqrt{1 - \sin [e + f x]} \right)
\end{aligned}$$

Result (type 6, 1375 leaves):

$$-\frac{1}{f} \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-2 m}$$

$$\left(d \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -n, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - \right. \right. \\ \left. \left. \left(4 d n \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 1 - n, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (-1+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \right. \\ \left. \left. \left. \frac{3}{2} - m, -n, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) (a + a \sin[e + f x])^m$$

■ **Problem 347: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x]) (c + d \sin[e + f x])^{-1-m} dx$$

Optimal (type 6, 277 leaves, 7 steps):

$$- \frac{1}{(c+d) f} 2^{\frac{1}{2}+m} a (A-B) \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{(c-d)(1-\sin[e + f x])}{2(c+d \sin[e + f x])} \right] \\ (a + a \sin[e + f x])^{-1+m} \left(\frac{(c+d)(1+\sin[e + f x])}{c+d \sin[e + f x]} \right)^{\frac{1}{2}-m} (c+d \sin[e + f x])^{-m} + \\ \left(\sqrt{2} B \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, 1 + m, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d(1+\sin[e + f x])}{c-d} \right] \cos[e + f x] \right. \\ \left. (a + a \sin[e + f x])^{1+m} (c+d \sin[e + f x])^{-m} \left(\frac{c+d \sin[e + f x]}{c-d} \right)^m \right) / (a(c-d) f (3+2m) \sqrt{1-\sin[e + f x]})$$

Result (type 6, 1020 leaves):

$$\begin{aligned}
& -\frac{1}{f} \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{-2 m} \\
& \left(\frac{1}{c+d} {}_2 F_1\left[\frac{1}{2}, \frac{1}{2}-m, \frac{3}{2}, \frac{(c-d) \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}\right]\right)^{\frac{1}{2}-m} \\
& \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(1-\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{-\frac{1}{2}+m} \left(-\frac{(c+d)\left(-1+\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)}{c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}\right)^{\frac{1}{2}-m} \left(c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-m} + \\
& B\left(-\frac{1}{d(c+d)} {}_2 F_1\left[\frac{1}{2}, \frac{1}{2}-m, \frac{3}{2}, \frac{(c-d) \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}\right]\right)^{\frac{1}{2}-m} \\
& \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-m, \frac{3}{2}, \frac{(c-d) \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}\right] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \\
& \left(1-\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{-\frac{1}{2}+m} \left(-\frac{(c+d)\left(-1+\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)}{c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}\right)^{\frac{1}{2}-m} \left(c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-m} + \\
& \left(6(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, m, \frac{3}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{-1+2 m}\right. \\
& \left.\left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{\frac{1}{2}-m} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(1-\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{-\frac{1}{2}+m} \left(c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-m}\right) / \\
& \left(d\left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, m, \frac{3}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]-\right.\right. \\
& \left.\left(-4 d m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, 1+m, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]+(c+d)(-1+2 m) \operatorname{AppellF1}\left[\frac{3}{2},\right.\right. \\
& \left.\left.\frac{3}{2}-m, m, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\right)\right) (a+a \operatorname{Sin}[e+f x])^m
\end{aligned}$$

■ **Problem 348: Unable to integrate problem.**

$$\int (a - a \sin[e + f x]) (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n dx$$

Optimal (type 6, 132 leaves, 4 steps):

$$\frac{1}{f(1+2m)} 2\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2} + m, -\frac{1}{2}, -n, \frac{3}{2} + m, \frac{1}{2}(1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c-d}\right] \\ \sec[e + f x] \sqrt{1 - \sin[e + f x]} (a + a \sin[e + f x])^{1+m} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c-d}\right)^{-n}$$

Result (type 8, 38 leaves):

$$\int (a - a \sin[e + f x]) (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n dx$$

■ **Problem 349: Unable to integrate problem.**

$$\int (a - a \sin[e + f x]) (a + a \sin[e + f x])^m (c + d \sin[e + f x])^{-1-m} dx$$

Optimal (type 6, 139 leaves, 4 steps):

$$\frac{1}{(c-d)f(1+2m)} 2\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2} + m, -\frac{1}{2}, 1+m, \frac{3}{2} + m, \frac{1}{2}(1 + \sin[e + f x]), -\frac{d(1 + \sin[e + f x])}{c-d}\right] \\ \sec[e + f x] \sqrt{1 - \sin[e + f x]} (a + a \sin[e + f x])^{1+m} (c + d \sin[e + f x])^{-m} \left(\frac{c + d \sin[e + f x]}{c-d}\right)^m$$

Result (type 8, 42 leaves):

$$\int (a - a \sin[e + f x]) (a + a \sin[e + f x])^m (c + d \sin[e + f x])^{-1-m} dx$$

■ **Problem 353: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B \sin[e + f x]) (c + d \sin[e + f x])^{3/2}}{(a + b \sin[e + f x])^{3/2}} dx$$

Optimal (type 4, 840 leaves, 7 steps):

$$\begin{aligned}
& \left((c-d) \sqrt{c+d} (2Ab^2c - 2abBc - 2aAbd + 3a^2Bd - b^2Bd) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]} \right], \frac{(a-b)(c+d)}{(a+b)(c-d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \left. \sqrt{-\frac{(bc-ad)(1-\sin[e+fx])}{(c+d)(a+b\sin[e+fx])}} \sqrt{\frac{(bc-ad)(1+\sin[e+fx])}{(c-d)(a+b\sin[e+fx])}} (a+b\sin[e+fx]) \right] / \left((a-b)b^2\sqrt{a+b} (bc-ad)f \right) + \frac{1}{b^3\sqrt{a+b}f} \\
& \sqrt{c+d} (3bBc + 2Abd - 3aBd) \operatorname{EllipticPi} \left[\frac{b(c+d)}{(a+b)d}, \operatorname{ArcSin} \left[\frac{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]}{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]} \right], \frac{(a-b)(c+d)}{(a+b)(c-d)} \right] \operatorname{Sec}[e+fx] \\
& \sqrt{-\frac{(bc-ad)(1-\sin[e+fx])}{(c+d)(a+b\sin[e+fx])}} \sqrt{\frac{(bc-ad)(1+\sin[e+fx])}{(c-d)(a+b\sin[e+fx])}} (a+b\sin[e+fx]) + \frac{2(Ab-aB)(bc-ad)\cos[e+fx]\sqrt{c+d}\sin[e+fx]}{b(a^2-b^2)f\sqrt{a+b}\sin[e+fx]} - \\
& \frac{(2Ab(bc-ad) - B(2abc - 3a^2d + b^2d)) \cos[e+fx] \sqrt{c+d} \sin[e+fx]}{b(a^2-b^2)f\sqrt{a+b}\sin[e+fx]} + \frac{1}{(a-b)b^3\sqrt{c+d}f} \\
& \sqrt{a+b} (2Ab(b(c-2d) + ad) - B(3a^2d - 6abd + b^2(2c+d))) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b} \sin[e+fx]}{\sqrt{a+b} \sqrt{c+d} \sin[e+fx]} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \\
& \operatorname{Sec}[e+fx] \sqrt{\frac{(bc-ad)(1-\sin[e+fx])}{(a+b)(c+d\sin[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\sin[e+fx])}{(a-b)(c+d\sin[e+fx])}} (c+d\sin[e+fx])
\end{aligned}$$

Result (type 4, 2012 leaves):

$$\begin{aligned}
& - \frac{2(Ab^2c \cos[e+fx] - abBc \cos[e+fx] - aAbd \cos[e+fx] + a^2Bd \cos[e+fx]) \sqrt{c+d} \sin[e+fx]}{b(-a^2+b^2)f\sqrt{a+b}\sin[e+fx]} + \\
& \frac{1}{2(a-b)b(a+b)f} \left(- \left(4(-bc+ad)(2aAbc^2 - 2b^2Bc^2 - 2Ab^2cd + 2abBcd + a^2Bd^2 - b^2Bd^2) \sqrt{\frac{(c+d) \operatorname{Cot} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2}{-c+d}} \right. \right. \\
& \left. \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{-\frac{(a+b) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 (c+d) \sin[e+fx]}{-bc+ad}} \right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx \right) \right]^4 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \sqrt{\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) / \\
& \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - 4(-bc+ad) (2Ab^2c^2 - 2abBc^2 + 4a^2Bcd - 4b^2Bcd - 2Ab^2d^2 + 2abBd^2) \\
& \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right) \right. \\
& \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \\
& \left. \sqrt{\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \right) \\
& \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \\
& \left. \sqrt{\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) + \\
& 2(-2Ab^2cd + 2abBcd + 2aAbd^2 - 3a^2Bd^2 + b^2Bd^2) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{\frac{a+b \sin[efx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \sin[efx]} \right) / \\
& \left(bd \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{a+b \sin[efx]}} \sqrt{a+b \sin[efx]} \sqrt{\frac{a+b \sin[efx]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[efx])}{(c+d)(a+b \sin[efx])}} \right) - \\
& \frac{1}{bd} 2(-bc+ad) \left((a+b)c+ad \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}\right], \frac{1}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) - \\
& \left((bc+ad) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}\right], \frac{1}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) \right) \right) \right)
\end{aligned}$$

■ Problem 354: Result more than twice size of optimal antiderivative.

$$\int \frac{(A + B \sin[e + f x]) \sqrt{c + d \sin[e + f x]}}{(a + b \sin[e + f x])^{3/2}} dx$$

Optimal (type 4, 630 leaves, 5 steps):

$$\left(2 (A b - a B) (c - d) \sqrt{c + d} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}} \right], \frac{(a - b) (c + d)}{(a + b) (c - d)} \right] \operatorname{Sec}[e + f x] \right. \\ \left. \sqrt{-\frac{(b c - a d) (1 - \sin[e + f x])}{(c + d) (a + b \sin[e + f x])}} \sqrt{\frac{(b c - a d) (1 + \sin[e + f x])}{(c - d) (a + b \sin[e + f x])}} (a + b \sin[e + f x]) \right] / \left((a - b) b \sqrt{a + b} (b c - a d) f \right) + \\ \left(2 \sqrt{a + b} (A b - a B) (c - d) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}} \right], \frac{(a + b) (c - d)}{(a - b) (c + d)} \right] \operatorname{Sec}[e + f x] \right. \\ \left. \sqrt{\frac{(b c - a d) (1 - \sin[e + f x])}{(a + b) (c + d \sin[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \sin[e + f x])}{(a - b) (c + d \sin[e + f x])}} (c + d \sin[e + f x]) \right] / \left((a - b) b \sqrt{c + d} (b c - a d) f \right) + \\ \frac{1}{b^2 \sqrt{c + d} f} 2 \sqrt{a + b} B \operatorname{EllipticPi} \left[\frac{(a + b) d}{b (c + d)}, \operatorname{ArcSin} \left[\frac{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}} \right], \frac{(a + b) (c - d)}{(a - b) (c + d)} \right] \operatorname{Sec}[e + f x] \\ \left. \sqrt{\frac{(b c - a d) (1 - \sin[e + f x])}{(a + b) (c + d \sin[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \sin[e + f x])}{(a - b) (c + d \sin[e + f x])}} (c + d \sin[e + f x]) \right]$$

Result (type 4, 1871 leaves):

$$-\frac{2 (-A b \cos[e + f x] + a B \cos[e + f x]) \sqrt{c + d \sin[e + f x]}}{(a^2 - b^2) f \sqrt{a + b \sin[e + f x]}} + \frac{1}{(a - b) (a + b) f} \\ \left(- \left(4 (A A c - b B c) (-b c + a d) \sqrt{\frac{(c + d) \operatorname{Cot} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{-c + d}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{(a + b) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (c + d \sin[e + f x])}{-b c + a d}}}{\sqrt{2}}} \right], \frac{2 (-b c + a d)}{(a + b) (-c + d)} \right] \right. \right. \\ \left. \left. \operatorname{Sec}[e + f x] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^4 \sqrt{\frac{(c + d) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (a + b \sin[e + f x])}{-b c + a d}} \right] \right)$$

$$\begin{aligned}
& \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \\
& 4(-bc+ad)(Abc-aBc+aAd-bBd) \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}}}\right. \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) - \right. \\
& \left. \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \right. \\
& \left. \left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}}}\right] \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right) / \right. \\
& \left. \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) + 2(-Abd+aBd) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} + \right. \right.
\end{aligned}$$

$$\left(\sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]}{\sqrt{\frac{a+b \sin[efx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \sqrt{c+d \sin[efx]} \right) /$$

$$\left(bd \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{a+b \sin[efx]}} \sqrt{a+b \sin[efx]} \sqrt{\frac{a+b \sin[efx]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[efx])}{(c+d)(a+b \sin[efx])}} \right) -$$

$$\frac{1}{bd} 2(-bc+ad) \left((a+b)c+ad \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}\right], \sqrt{2}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) -$$

$$\left((bc+ad) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}}}\right], \sqrt{2}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \operatorname{Sec}[efx] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \sin[efx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \sin[efx])}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \sin[efx]} \sqrt{c+d \sin[efx]} \right) \right) \right)$$

■ Problem 355: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \sin[e + f x]}{(a + b \sin[e + f x])^{3/2} \sqrt{c + d \sin[e + f x]}} dx$$

Optimal (type 4, 417 leaves, 3 steps):

$$\left(2 (A b - a B) (c - d) \sqrt{c + d} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}}{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}} \right], \frac{(a - b)(c + d)}{(a + b)(c - d)} \right] \operatorname{Sec}[e + f x] \right. \\ \left. \sqrt{-\frac{(bc - ad)(1 - \sin[e + f x])}{(c + d)(a + b \sin[e + f x])}} \sqrt{\frac{(bc - ad)(1 + \sin[e + f x])}{(c - d)(a + b \sin[e + f x])}} (a + b \sin[e + f x]) \right] / \left((a - b) \sqrt{a + b} (bc - ad)^2 f \right) + \\ \left(2 \sqrt{a + b} (A - B) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c + d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{c + d \sin[e + f x]}} \right], \frac{(a + b)(c - d)}{(a - b)(c + d)} \right] \operatorname{Sec}[e + f x] \sqrt{\frac{(bc - ad)(1 - \sin[e + f x])}{(a + b)(c + d \sin[e + f x])}} \right. \\ \left. \sqrt{-\frac{(bc - ad)(1 + \sin[e + f x])}{(a - b)(c + d \sin[e + f x])}} (c + d \sin[e + f x]) \right] / \left((a - b) \sqrt{c + d} (bc - ad) f \right)$$

Result (type 4, 1919 leaves):

$$-\frac{2 (A b^2 \cos[e + f x] - a b B \cos[e + f x]) \sqrt{c + d \sin[e + f x]}}{(a^2 - b^2) (-bc + ad) f \sqrt{a + b \sin[e + f x]}} + \\ \frac{1}{(a - b)(a + b)(-bc + ad) f} \left(- \left(4 (-bc + ad) (-a A b c + b^2 B c + a^2 A d - A b^2 d) \sqrt{\frac{(c + d) \operatorname{Cot} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{-c + d}} \right. \right. \\ \left. \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{(a + b) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (c + d \sin[e + f x])}{-bc + ad}}}{\sqrt{2}}} \right], \frac{2(-bc + ad)}{(a + b)(-c + d)} \right] \operatorname{Sec}[e + f x] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^4 \right. \right. \\ \left. \left. \sqrt{\frac{(c + d) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (a + b \sin[e + f x])}{-bc + ad}} \sqrt{-\frac{(a + b) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 (c + d \sin[e + f x])}{-bc + ad}} \right] / \right. \\ \left. \left((a + b)(c + d) \sqrt{a + b \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) - 4 (-bc + ad) (-A b^2 c + a b B c - a A b d + a^2 B d) \right)$$

$$\left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \right. \right.$$

$$\left. \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+fx] \right.$$

$$\left. \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (a+b) \operatorname{Sin}[e+fx]}{-bc+ad}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d) \operatorname{Sin}[e+fx]}{-bc+ad}} \right) /$$

$$\left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) + 2(Ab^2d - abBd) \left(\frac{\operatorname{Cos}[e+fx] \sqrt{c+d \operatorname{Sin}[e+fx]}}{d \sqrt{a+b \operatorname{Sin}[e+fx]}} +$$

$$\left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]}}{\sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) /$$

$$\left(bd \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{a+b \operatorname{Sin}[e+fx]}} \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{\frac{a+b \operatorname{Sin}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(c+d \operatorname{Sin}[e+fx])}{(c+d)(a+b \operatorname{Sin}[e+fx])}} \right) -$$

$$\frac{1}{bd} 2 (-bc + ad) \left(\left((a+b) c + ad \right) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right. \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) -$$

$$\left((bc+ad) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (a+b \operatorname{Sin}[e+fx])}{-bc+ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}} \right) / \left((a+b)d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right) \right)$$

■ **Problem 356: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sin}[e + fx]}{(a + b \operatorname{Sin}[e + fx])^{3/2} (c + d \operatorname{Sin}[e + fx])^{3/2}} dx$$

Optimal (type 4, 544 leaves, 4 steps):

$$\frac{2 b (A b - a B) \operatorname{Cos}[e + f x]}{(a^2 - b^2) (b c - a d) f \sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]}} -$$

$$\left(2 (A (a^2 d^2 + b^2 (c^2 - 2 d^2)) - B (a^2 c d - b^2 c d + a b (c^2 - d^2))) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{c + d \operatorname{Sin}[e + f x]}}\right], \frac{(a + b) (c - d)}{(a - b) (c + d)}\right] \operatorname{Sec}[e + f x] \right.$$

$$\left. \sqrt{\frac{(b c - a d) (1 - \operatorname{Sin}[e + f x])}{(a + b) (c + d \operatorname{Sin}[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Sin}[e + f x])}{(a - b) (c + d \operatorname{Sin}[e + f x])}} (c + d \operatorname{Sin}[e + f x])} \right) / \left(\sqrt{a + b} (c - d) \sqrt{c + d} (b c - a d)^3 f \right) +$$

$$\left(2 (A b c + b B c - a A d - 2 A b d + a B d) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{c + d \operatorname{Sin}[e + f x]}}\right], \frac{(a + b) (c - d)}{(a - b) (c + d)}\right] \operatorname{Sec}[e + f x] \right.$$

$$\left. \sqrt{\frac{(b c - a d) (1 - \operatorname{Sin}[e + f x])}{(a + b) (c + d \operatorname{Sin}[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Sin}[e + f x])}{(a - b) (c + d \operatorname{Sin}[e + f x])}} (c + d \operatorname{Sin}[e + f x])} \right) / \left(\sqrt{a + b} (c - d) \sqrt{c + d} (b c - a d)^2 f \right)$$

Result (type 4, 2236 leaves):

$$\frac{\sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]} \left(\frac{2 (A b^3 \operatorname{Cos}[e + f x] - a b^2 B \operatorname{Cos}[e + f x])}{(a^2 - b^2) (-b c + a d)^2 (a + b \operatorname{Sin}[e + f x])} - \frac{2 (B c d^2 \operatorname{Cos}[e + f x] - A d^3 \operatorname{Cos}[e + f x])}{(b c - a d)^2 (c^2 - d^2) (c + d \operatorname{Sin}[e + f x])} \right)}{f} +$$

$$\frac{1}{(a - b) (a + b) (c - d) (c + d) (-b c + a d)^2 f} \left[-\frac{1}{(a + b) (c + d) \sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]}} \right.$$

$$4 (-b c + a d) (a A b^2 c^3 - b^3 B c^3 - 2 a^2 A b c^2 d + 2 A b^3 c^2 d + a^3 A c d^2 - 2 a A b^2 c d^2 + b^3 B c d^2 + 2 a^2 A b d^3 - 2 A b^3 d^3 - a^3 B d^3 + a b^2 B d^3)$$

$$\left. \sqrt{\frac{(c + d) \operatorname{Cot}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-c + d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c + d \operatorname{Sin}[e + f x])}{-b c + a d}}}{\sqrt{2}}}\right], \frac{2 (-b c + a d)}{(a + b) (-c + d)}\right] \operatorname{Sec}[e + f x] \right.$$

$$\left. \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \sqrt{\frac{(c + d) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a + b \operatorname{Sin}[e + f x])}{-b c + a d}} \sqrt{-\frac{(a + b) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c + d \operatorname{Sin}[e + f x])}{-b c + a d}} \right] -$$

$$4 (-b c + a d) (A b^3 c^3 - a b^2 B c^3 + a A b^2 c^2 d - 2 a^2 b B c^2 d + b^3 B c^2 d + a^2 A b c d^2 - 2 A b^3 c d^2 - a^3 B c d^2 + 2 a b^2 B c d^2 + a^3 A d^3 -$$

$$\begin{aligned}
& 2 a A b^2 d^3 + a^2 b B d^3 \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d) \operatorname{Sin}[e+f x]}}{-b c+a d}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \right. \right. \\
& \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b) \operatorname{Sin}[e+f x]}{-b c+a d}} \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d) \operatorname{Sin}[e+f x]}{-b c+a d}} \right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]} \right) - \right. \\
& \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d) \operatorname{Sin}[e+f x]}}{-b c+a d}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \right) \\
& \operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b) \operatorname{Sin}[e+f x]}{-b c+a d}} \\
& \left. \left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (c+d) \operatorname{Sin}[e+f x]}{-b c+a d}} \right) / \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]} \right) \right) + \\
& 2(-A b^3 c^2 d + a b^2 B c^2 d + a^2 b B c d^2 - b^3 B c d^2 - a^2 A b d^3 + 2 A b^3 d^3 - a b^2 B d^3) \left(\frac{\operatorname{Cos}[e+f x] \sqrt{c+d \operatorname{Sin}[e+f x]}}{d \sqrt{a+b \operatorname{Sin}[e+f x]}} + \right. \\
& \left. \left(\sqrt{\frac{a-b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{\sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}\right], \frac{2(-b c+a d)}{(a-b)(c+d)}\right] \sqrt{c+d \operatorname{Sin}[e+f x]} \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(b d \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{a+b \sin[e+f x]}} \sqrt{a+b \sin[e+f x]} \sqrt{\frac{a+b \sin[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \right) - \\
& \frac{1}{b d} 2(-b c+a d) \left(\left((a+b) c+a d \right) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) - \\
& \left((b c+a d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(-b c+a d)}{(a+b)(-c+d)} \right] \operatorname{Sec}[e+f x] \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b \sin[e+f x])}{-b c+a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d \sin[e+f x])}{-b c+a d}} \right) / \left((a+b) d \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) \right) \right)
\end{aligned}$$

■ **Problem 357: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \sin[e+f x]}{(a+b \sin[e+f x])^{3/2} (c+d \sin[e+f x])^{5/2}} dx$$

Optimal (type 4, 858 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 b (A b - a B) \operatorname{Cos}[e + f x]}{(a^2 - b^2) (b c - a d) f \sqrt{a + b \operatorname{Sin}[e + f x]} (c + d \operatorname{Sin}[e + f x])^{3/2}} + \\
& \frac{2 d (A (a^2 d^2 + b^2 (3 c^2 - 4 d^2)) - B (a^2 c d - b^2 c d + 3 a b (c^2 - d^2))) \operatorname{Cos}[e + f x] \sqrt{a + b \operatorname{Sin}[e + f x]}}{3 (a^2 - b^2) (b c - a d)^2 (c^2 - d^2) f (c + d \operatorname{Sin}[e + f x])^{3/2}} + \\
& \frac{1}{3 \sqrt{a + b} (c - d)^2 (c + d)^{3/2} (b c - a d)^4 f} 2 (B (2 a^2 b c d (3 c^2 - d^2) - 2 b^3 c d (3 c^2 - d^2) - a^3 d^2 (c^2 + 3 d^2) + a b^2 (3 c^4 - 5 c^2 d^2 + 6 d^4)) + \\
& A (4 a^3 c d^3 - 4 a b^2 c d^3 - a^2 b d^2 (9 c^2 - 5 d^2) - b^3 (3 c^4 - 15 c^2 d^2 + 8 d^4))) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{c + d \operatorname{Sin}[e + f x]}}\right], \frac{(a + b) (c - d)}{(a - b) (c + d)}\right] \\
& \operatorname{Sec}[e + f x] \sqrt{\frac{(b c - a d) (1 - \operatorname{Sin}[e + f x])}{(a + b) (c + d \operatorname{Sin}[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Sin}[e + f x])}{(a - b) (c + d \operatorname{Sin}[e + f x])}} (c + d \operatorname{Sin}[e + f x]) - \\
& \left(2 (B (a^2 d^2 (c + 3 d) - b^2 c (3 c^2 + 3 c d - 2 d^2) - 6 a b d (c^2 - d^2)) - A (a^2 d^2 (3 c + d) - 6 a b d (c^2 - d^2) + b^2 (3 c^3 - 9 c^2 d - 6 c d^2 + 8 d^3)))\right) \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{a + b \operatorname{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{c + d \operatorname{Sin}[e + f x]}}\right], \frac{(a + b) (c - d)}{(a - b) (c + d)}\right] \operatorname{Sec}[e + f x] \sqrt{\frac{(b c - a d) (1 - \operatorname{Sin}[e + f x])}{(a + b) (c + d \operatorname{Sin}[e + f x])}} \\
& \left.\sqrt{-\frac{(b c - a d) (1 + \operatorname{Sin}[e + f x])}{(a - b) (c + d \operatorname{Sin}[e + f x])}} (c + d \operatorname{Sin}[e + f x])\right) / \left(3 \sqrt{a + b} (c - d)^2 (c + d)^{3/2} (b c - a d)^3 f\right)
\end{aligned}$$

Result (type 4, 2807 leaves):

$$\begin{aligned}
& \frac{1}{f} \sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]} \\
& \left(-\frac{2 (A b^4 \operatorname{Cos}[e + f x] - a b^3 B \operatorname{Cos}[e + f x])}{(a^2 - b^2) (-b c + a d)^3 (a + b \operatorname{Sin}[e + f x])} + \frac{2 (-B c d^2 \operatorname{Cos}[e + f x] + A d^3 \operatorname{Cos}[e + f x])}{3 (b c - a d)^2 (c^2 - d^2) (c + d \operatorname{Sin}[e + f x])^2} - (2 (6 b B c^3 d^2 \operatorname{Cos}[e + f x] - 9 A b c^2 d^3 \operatorname{Cos}[e + f x] - \right. \\
& \left. a B c^2 d^3 \operatorname{Cos}[e + f x] + 4 a A c d^4 \operatorname{Cos}[e + f x] - 2 b B c d^4 \operatorname{Cos}[e + f x] + 5 A b d^5 \operatorname{Cos}[e + f x] - 3 a B d^5 \operatorname{Cos}[e + f x])) / \right. \\
& \left. (3 (b c - a d)^3 (c^2 - d^2)^2 (c + d \operatorname{Sin}[e + f x]))\right) + \frac{1}{3 (a - b) (a + b) (c - d)^2 (c + d)^2 (-b c + a d)^3 f} \\
& \left(-\frac{1}{(a + b) (c + d) \sqrt{a + b \operatorname{Sin}[e + f x]} \sqrt{c + d \operatorname{Sin}[e + f x]}} 4 (-b c + a d) (-3 a A b^3 c^5 + 3 b^4 B c^5 + 9 a^2 A b^2 c^4 d - 9 A b^4 c^4 d - 9 a^3 A b c^3 d^2 + \right. \\
& \left. 15 a A b^3 c^3 d^2 - a^2 b^2 B c^3 d^2 - 5 b^4 B c^3 d^2 + 3 a^4 A c^2 d^3 - 20 a^2 A b^2 c^2 d^3 + 17 A b^4 c^2 d^3 + 10 a^3 b B c^2 d^3 - 10 a b^3 B c^2 d^3 + \right.
\end{aligned}$$

$$5 a^3 A b c d^4 - 8 a A b^3 c d^4 - 4 a^4 B c d^4 + 5 a^2 b^2 B c d^4 + 2 b^4 B c d^4 + a^4 A d^5 + 7 a^2 A b^2 d^5 - 8 A b^4 d^5 - 6 a^3 b B d^5 + 6 a b^3 B d^5)$$

$$\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d) \operatorname{Sin}[e+f x]}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right] \operatorname{Sec}[e+f x]$$

$$\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b) \operatorname{Sin}[e+f x]}{-b c+a d}} \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d) \operatorname{Sin}[e+f x]}{-b c+a d}} -$$

$$4(-b c+a d)\left(-3 A b^4 c^5+3 a b^3 B c^5-3 a A b^3 c^4 d+9 a^2 b^2 B c^4 d-6 b^4 B c^4 d-9 a^2 A b^2 c^3 d^2+15 A b^4 c^3 d^2+5 a^3 b B c^3 d^2-11 a b^3 B c^3 d^2-5 a^3 A b c^2 d^3+11 a A b^3 c^2 d^3-a^4 B c^2 d^3-7 a^2 b^2 B c^2 d^3+2 b^4 B c^2 d^3+4 a^4 A c d^4+a^2 A b^2 c d^4-8 A b^4 c d^4-5 a^3 b B c d^4+8 a b^3 B c d^4+5 a^3 A b d^5-8 a A b^3 d^5-3 a^4 B d^5+6 a^2 b^2 B d^5\right)$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d) \operatorname{Sin}[e+f x]}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right]\right)$$

$$\operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b) \operatorname{Sin}[e+f x]}{-b c+a d}}$$

$$\left.\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d) \operatorname{Sin}[e+f x]}{-b c+a d}}\right) / \left((a+b)(c+d) \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}\right) -$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-b c+a d}{(a+b) d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d) \operatorname{Sin}[e+f x]}{-b c+a d}}}{\sqrt{2}}}\right], \frac{2(-b c+a d)}{(a+b)(-c+d)}\right]\right)$$

$$\operatorname{Sec}[e+f x] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{\frac{(c+d) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(a+b) \operatorname{Sin}[e+f x]}{-b c+a d}}$$

$$\left.\sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2(c+d) \operatorname{Sin}[e+f x]}{-b c+a d}}\right) / \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+f x]} \sqrt{c+d \operatorname{Sin}[e+f x]}\right) +$$

$$2 (3 A b^4 c^4 d - 3 a b^3 B c^4 d - 6 a^2 b^2 B c^3 d^2 + 6 b^4 B c^3 d^2 + 9 a^2 A b^2 c^2 d^3 - 15 A b^4 c^2 d^3 + a^3 b B c^2 d^3 + 5 a b^3 B c^2 d^3 - 4 a^3 A b c d^4 +$$

$$4 a A b^3 c d^4 + 2 a^2 b^2 B c d^4 - 2 b^4 B c d^4 - 5 a^2 A b^2 d^5 + 8 A b^4 d^5 + 3 a^3 b B d^5 - 6 a b^3 B d^5) \left(\frac{\cos[e + f x] \sqrt{c + d \sin[e + f x]}}{d \sqrt{a + b \sin[e + f x]}} + \right.$$

$$\left. \left(\sqrt{\frac{a-b}{a+b}} (a+b) \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{a-b}{a+b}} \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]}{\sqrt{\frac{a+b \sin[e+f x]}{a+b}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)} \sqrt{c+d \sin[e+f x]}\right] \right) / \right.$$

$$\left. \left(b d \sqrt{\frac{(a+b) \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{a+b \sin[e+f x]}} \sqrt{a+b \sin[e+f x]} \sqrt{\frac{a+b \sin[e+f x]}{a+b}} \sqrt{\frac{(a+b)(c+d \sin[e+f x])}{(c+d)(a+b \sin[e+f x])}} \right) - \right.$$

$$\frac{1}{bd} 2(-bc+ad) \left(\left((a+b)c+ad \right) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-c+d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c+d \sin[e+f x])}{-bc+ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \sec[e+f x] \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a+b \sin[e+f x])}{-bc+ad}}$$

$$\left. \sqrt{-\frac{(a+b) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c+d \sin[e+f x])}{-bc+ad}} \right) / \left((a+b)(c+d) \sqrt{a+b \sin[e+f x]} \sqrt{c+d \sin[e+f x]} \right) -$$

$$\left((bc+ad) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{-c+d}} \operatorname{EllipticPi}\left[\frac{-bc+ad}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (c+d \sin[e+f x])}{-bc+ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2(-bc+ad)}{(a+b)(-c+d)} \right] \sec[e+f x] \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^4 \sqrt{\frac{(c+d) \csc\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 (a+b \sin[e+f x])}{-bc+ad}}$$

$$\left. \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 (c+d \operatorname{Sin}[e+fx])}{-bc+ad}}}\right/ \left. \left((a+b) d \sqrt{a+b \operatorname{Sin}[e+fx]} \sqrt{c+d \operatorname{Sin}[e+fx]} \right) \right. \left. \left. \right. \right)$$

Test results for the 19 problems in "4.1.4.1 (a+b sin)^m (A+B sin+C sin^2).m"

- **Problem 2: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sin}[e+fx]^5 (6-7 \operatorname{Sin}[e+fx]^2) dx$$

Optimal (type 3, 18 leaves, 1 step):

$$\frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^6}{f}$$

Result (type 3, 59 leaves):

$$\frac{5 \operatorname{Cos}[e+fx]}{64 f} - \frac{9 \operatorname{Cos}[3(e+fx)]}{64 f} + \frac{5 \operatorname{Cos}[5(e+fx)]}{64 f} - \frac{\operatorname{Cos}[7(e+fx)]}{64 f}$$

- **Problem 3: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sin}[e+fx]^4 (5-6 \operatorname{Sin}[e+fx]^2) dx$$

Optimal (type 3, 18 leaves, 1 step):

$$\frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^5}{f}$$

Result (type 3, 39 leaves):

$$\frac{24 e + 5 \operatorname{Sin}[2(e+fx)] - 4 \operatorname{Sin}[4(e+fx)] + \operatorname{Sin}[6(e+fx)]}{32 f}$$

- **Problem 4: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sin}[e+fx]^3 (4-5 \operatorname{Sin}[e+fx]^2) dx$$

Optimal (type 3, 18 leaves, 1 step):

$$\frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^4}{f}$$

Result (type 3, 44 leaves):

$$\frac{\cos[e + f x]}{8 f} - \frac{3 \cos[3(e + f x)]}{16 f} + \frac{\cos[5(e + f x)]}{16 f}$$

- **Problem 6: Result more than twice size of optimal antiderivative.**

$$\int \sin[e + f x] (2 - 3 \sin[e + f x]^2) dx$$

Optimal (type 3, 18 leaves, 1 step):

$$\frac{\cos[e + f x] \sin[e + f x]^2}{f}$$

Result (type 3, 51 leaves):

$$-\frac{2 \cos[e] \cos[f x]}{f} + \frac{9 \cos[e + f x]}{4 f} - \frac{\cos[3(e + f x)]}{4 f} + \frac{2 \sin[e] \sin[f x]}{f}$$

- **Problem 7: Result more than twice size of optimal antiderivative.**

$$\int (1 - 2 \sin[e + f x]^2) dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$\frac{\cos[e + f x] \sin[e + f x]}{f}$$

Result (type 3, 33 leaves):

$$\frac{\cos[2 f x] \sin[2 e]}{2 f} + \frac{\cos[2 e] \sin[2 f x]}{2 f}$$

- **Problem 8: Result more than twice size of optimal antiderivative.**

$$\int -\sin[e + f x] dx$$

Optimal (type 3, 10 leaves, 1 step):

$$\frac{\cos[e + f x]}{f}$$

Result (type 3, 22 leaves):

$$\frac{\cos[e] \cos[f x]}{f} - \frac{\sin[e] \sin[f x]}{f}$$

- **Problem 10: Result more than twice size of optimal antiderivative.**

$$\int \csc[e + f x]^3 (-2 + \sin[e + f x]^2) dx$$

Optimal (type 3, 16 leaves, 1 step):

$$\frac{\text{Cot}[e + f x] \text{Csc}[e + f x]}{f}$$

Result (type 3, 107 leaves) :

$$\frac{\text{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{4 f} - \frac{\text{Log}\left[\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]\right]}{f} + \frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right]}{f} + \frac{\text{Log}\left[\text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right]}{f} - \frac{\text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{f} - \frac{\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{4 f}$$

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[e + f x]^5 (-4 + 3 \text{Sin}[e + f x]^2) dx$$

Optimal (type 3, 18 leaves, 1 step) :

$$\frac{\text{Cot}[e + f x] \text{Csc}[e + f x]^3}{f}$$

Result (type 3, 39 leaves) :

$$\frac{\text{Csc}\left[\frac{1}{2}(e + f x)\right]^4}{16 f} - \frac{\text{Sec}\left[\frac{1}{2}(e + f x)\right]^4}{16 f}$$

■ **Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \text{Sin}[e + f x])^m (A + C \text{Sin}[e + f x]^2) dx$$

Optimal (type 5, 171 leaves, 4 steps) :

$$\frac{C \text{Cos}[e + f x] (a + a \text{Sin}[e + f x])^m}{f (2 + 3 m + m^2)} - \frac{1}{f (1 + m) (2 + m)} 2^{\frac{1}{2} + m} (C (1 + m + m^2) + A (2 + 3 m + m^2)) \text{Cos}[e + f x]$$

$$\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2} (1 - \text{Sin}[e + f x])\right] (1 + \text{Sin}[e + f x])^{-\frac{1}{2} - m} (a + a \text{Sin}[e + f x])^m - \frac{C \text{Cos}[e + f x] (a + a \text{Sin}[e + f x])^{1+m}}{a f (2 + m)}$$

Result (type 5, 398 leaves) :

$$-\frac{1}{2 f} (a (1 + \text{Sin}[e + f x]))^m \left(-\frac{1}{-4 + m^2} i 2^{-1-2m} C e^{-2i(e+fx)} (1 + i e^{-i(e+fx)})^{-2m} \left(e^{-\frac{1}{4}i(2e+\pi+2fx)} (i + e^{i(e+fx)}) \right)^{2m} \right.$$

$$\left. \left(e^{4i(e+fx)} (-2 + m) \text{Hypergeometric2F1}\left[-2 - m, -2 m, -1 - m, -i e^{-i(e+fx)}\right] + (2 + m) \text{Hypergeometric2F1}\left[2 - m, -2 m, 3 - m, -i e^{-i(e+fx)}\right] \right) + \right.$$

$$\left. \left(4 \sqrt{2} A \text{Cos}\left[\frac{1}{4}(2e - \pi + 2fx)\right] \right)^{1+2m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{3}{2} + m, \text{Sin}\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2\right] \text{Sin}\left[\frac{1}{4}(2e - \pi + 2fx)\right] \right) /$$

$$\left((1 + 2m) \sqrt{1 - \text{Sin}[e + f x]} \right) +$$

$$\left(2 \sqrt{2} C \text{Cos}\left[\frac{1}{4}(2e - \pi + 2fx)\right] \right)^{1+2m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{3}{2} + m, \text{Sin}\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2\right] \text{Sin}\left[\frac{1}{4}(2e - \pi + 2fx)\right] \right) /$$

$$\left((1 + 2m) \sqrt{1 - \text{Sin}[e + f x]} \right) \text{Sin}\left[\frac{1}{4}(2e + \pi + 2fx)\right]^{-2m}$$

■ **Problem 14: Unable to integrate problem.**

$$\int (a + b \sin[e + f x])^m (A - A \sin[e + f x]^2) dx$$

Optimal (type 6, 211 leaves, 7 steps):

$$\frac{1}{f \sqrt{1 + \sin[e + f x]}} 4 \sqrt{2} A \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{b (1 - \sin[e + f x])}{a + b}\right]$$

$$\cos[e + f x] (a + b \sin[e + f x])^m \left(\frac{a + b \sin[e + f x]}{a + b}\right)^{-m} - \frac{1}{f \sqrt{1 + \sin[e + f x]}}$$

$$4 \sqrt{2} A \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{b (1 - \sin[e + f x])}{a + b}\right] \cos[e + f x] (a + b \sin[e + f x])^m \left(\frac{a + b \sin[e + f x]}{a + b}\right)^{-m}$$

Result (type 8, 28 leaves):

$$\int (a + b \sin[e + f x])^m (A - A \sin[e + f x]^2) dx$$

■ **Problem 15: Unable to integrate problem.**

$$\int (a + b \sin[e + f x])^m (A + C \sin[e + f x]^2) dx$$

Optimal (type 6, 286 leaves, 8 steps):

$$-\frac{C \cos[e + f x] (a + b \sin[e + f x])^{1+m}}{b f (2+m)} +$$

$$\left(\sqrt{2} a (a + b) C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1 - m, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{b (1 - \sin[e + f x])}{a + b}\right] \cos[e + f x] (a + b \sin[e + f x])^m \left(\frac{a + b \sin[e + f x]}{a + b}\right)^{-m}\right) /$$

$$\left(b^2 f (2+m) \sqrt{1 + \sin[e + f x]}\right) - \left(\sqrt{2} (a^2 C + b^2 (C (1+m) + A (2+m))) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \sin[e + f x]), \frac{b (1 - \sin[e + f x])}{a + b}\right]\right)$$

$$\cos[e + f x] (a + b \sin[e + f x])^m \left(\frac{a + b \sin[e + f x]}{a + b}\right)^{-m} / \left(b^2 f (2+m) \sqrt{1 + \sin[e + f x]}\right)$$

Result (type 8, 27 leaves):

$$\int (a + b \sin[e + f x])^m (A + C \sin[e + f x]^2) dx$$

■ **Problem 17: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (A + B \sin[e + f x] + C \sin[e + f x]^2) dx$$

Optimal (type 5, 184 leaves, 4 steps):

$$\frac{(C - B(2 + m)) \cos[e + fx] (a + a \sin[e + fx])^m}{f(1 + m)(2 + m)} - \frac{1}{f(1 + m)(2 + m)} 2^{\frac{1}{2} + m} (Bm(2 + m) + C(1 + m + m^2) + A(2 + 3m + m^2)) \cos[e + fx]$$

$$\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2}(1 - \sin[e + fx])\right] (1 + \sin[e + fx])^{-\frac{1}{2} - m} (a + a \sin[e + fx])^m - \frac{C \cos[e + fx] (a + a \sin[e + fx])^{1 + m}}{af(2 + m)}$$

Result (type 5, 558 leaves):

$$-\frac{1}{2f} (a(1 + \sin[e + fx]))^m \left(\frac{1}{-1 + m^2} 4^{-m} B e^{-i(e + fx)} (1 + i e^{-i(e + fx)})^{-2m} \left(e^{-\frac{1}{4}i(2e + \pi + 2fx)} (i + e^{i(e + fx)}) \right)^{2m} \right.$$

$$\left. (e^{2i(e + fx)} (-1 + m) \text{Hypergeometric2F1}[-1 - m, -2m, -m, -i e^{-i(e + fx)}] - (1 + m) \text{Hypergeometric2F1}[1 - m, -2m, 2 - m, -i e^{-i(e + fx)}]) - \right.$$

$$\frac{1}{-4 + m^2} i 2^{-1 - 2m} C e^{-2i(e + fx)} (1 + i e^{-i(e + fx)})^{-2m} \left(e^{-\frac{1}{4}i(2e + \pi + 2fx)} (i + e^{i(e + fx)}) \right)^{2m}$$

$$\left. (e^{4i(e + fx)} (-2 + m) \text{Hypergeometric2F1}[-2 - m, -2m, -1 - m, -i e^{-i(e + fx)}] + (2 + m) \text{Hypergeometric2F1}[2 - m, -2m, 3 - m, -i e^{-i(e + fx)}]) + \right.$$

$$\left(4\sqrt{2} A \cos\left[\frac{1}{4}(2e - \pi + 2fx)\right]^{1 + 2m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{3}{2} + m, \sin\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2\right] \sin\left[\frac{1}{4}(2e - \pi + 2fx)\right] \right) /$$

$$\left((1 + 2m) \sqrt{1 - \sin[e + fx]} \right) +$$

$$\left(2\sqrt{2} C \cos\left[\frac{1}{4}(2e - \pi + 2fx)\right]^{1 + 2m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{3}{2} + m, \sin\left[\frac{1}{4}(2e + \pi + 2fx)\right]^2\right] \sin\left[\frac{1}{4}(2e - \pi + 2fx)\right] \right) /$$

$$\left((1 + 2m) \sqrt{1 - \sin[e + fx]} \right) \left. \sin\left[\frac{1}{4}(2e + \pi + 2fx)\right]^{-2m} \right)$$

■ **Problem 18: Unable to integrate problem.**

$$\int (a + b \sin[e + fx])^m (A + (A + C) \sin[e + fx] + C \sin[e + fx]^2) dx$$

Optimal (type 6, 215 leaves, 7 steps):

$$-\frac{1}{f\sqrt{1 + \sin[e + fx]}} 4\sqrt{2} C \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -m, \frac{3}{2}, \frac{1}{2}(1 - \sin[e + fx]), \frac{b(1 - \sin[e + fx])}{a + b}\right]$$

$$\cos[e + fx] (a + b \sin[e + fx])^m \left(\frac{a + b \sin[e + fx]}{a + b} \right)^{-m} - \frac{1}{f\sqrt{1 + \sin[e + fx]}}$$

$$2\sqrt{2} (A - C) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2}(1 - \sin[e + fx]), \frac{b(1 - \sin[e + fx])}{a + b}\right] \cos[e + fx] (a + b \sin[e + fx])^m \left(\frac{a + b \sin[e + fx]}{a + b} \right)^{-m}$$

Result (type 8, 37 leaves):

$$\int (a + b \sin[e + fx])^m (A + (A + C) \sin[e + fx] + C \sin[e + fx]^2) dx$$

■ **Problem 19: Unable to integrate problem.**

$$\int (a + b \sin[e + fx])^m (A + B \sin[e + fx] + C \sin[e + fx]^2) dx$$

Optimal (type 6, 304 leaves, 8 steps) :

$$\begin{aligned}
 & - \frac{C \operatorname{Cos}[e + f x] (a + b \operatorname{Sin}[e + f x])^{1+m}}{b f (2+m)} + \\
 & \left(\sqrt{2} (a+b) (a C - b B (2+m)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1-m, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sin}[e + f x]), \frac{b (1 - \operatorname{Sin}[e + f x])}{a+b}\right] \operatorname{Cos}[e + f x] \right. \\
 & \quad \left. (a + b \operatorname{Sin}[e + f x])^m \left(\frac{a + b \operatorname{Sin}[e + f x]}{a+b} \right)^{-m} \right) / \left(b^2 f (2+m) \sqrt{1 + \operatorname{Sin}[e + f x]} \right) - \\
 & \left(\sqrt{2} (a^2 C + b^2 C (1+m) + A b^2 (2+m) - a b B (2+m)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sin}[e + f x]), \frac{b (1 - \operatorname{Sin}[e + f x])}{a+b}\right] \right. \\
 & \quad \left. \operatorname{Cos}[e + f x] (a + b \operatorname{Sin}[e + f x])^m \left(\frac{a + b \operatorname{Sin}[e + f x]}{a+b} \right)^{-m} \right) / \left(b^2 f (2+m) \sqrt{1 + \operatorname{Sin}[e + f x]} \right)
 \end{aligned}$$

Result (type 8, 35 leaves) :

$$\int (a + b \operatorname{Sin}[e + f x])^m (A + B \operatorname{Sin}[e + f x] + C \operatorname{Sin}[e + f x]^2) dx$$

Test results for the 34 problems in "4.1.4.2 (a+b sin)^m (c+d sin)^n (A+B sin+C sin^2).m"

- **Problem 1: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sin}[e + f x])^m (c - c \operatorname{Sin}[e + f x])^{5/2} (A + C \operatorname{Sin}[e + f x]^2) dx$$

Optimal (type 3, 384 leaves, 5 steps) :

$$\begin{aligned}
 & \frac{64 c^3 (C (39 - 16 m + 4 m^2) + A (63 + 32 m + 4 m^2)) \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^m}{f (5+2 m) (7+2 m) (9+2 m) (3+8 m+4 m^2) \sqrt{c - c \operatorname{Sin}[e + f x]}} + \\
 & \frac{16 c^2 (C (39 - 16 m + 4 m^2) + A (63 + 32 m + 4 m^2)) \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^m \sqrt{c - c \operatorname{Sin}[e + f x]}}{f (7+2 m) (9+2 m) (15+16 m+4 m^2)} + \\
 & \frac{2 c (C (39 - 16 m + 4 m^2) + A (63 + 32 m + 4 m^2)) \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^m (c - c \operatorname{Sin}[e + f x])^{3/2}}{f (5+2 m) (7+2 m) (9+2 m)} - \\
 & \frac{4 C (1+2 m) \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^m (c - c \operatorname{Sin}[e + f x])^{5/2}}{f (7+2 m) (9+2 m)} + \frac{2 C \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^m (c - c \operatorname{Sin}[e + f x])^{7/2}}{c f (9+2 m)}
 \end{aligned}$$

Result (type 3, 899 leaves) :

$$\begin{aligned}
& \frac{1}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^5} \\
& (a(1+\sin[e+fx]))^m (c - c \sin[e+fx])^{5/2} \left(\left((18900A + 12285C + 15648Am + 648Cm + 5280Am^2 + 1416Cm^2 + 896Am^3 + 224Cm^3 + 64Am^4 + 16Cm^4) \right. \right. \\
& \left. \left. \left(\left(\frac{1}{8} + \frac{i}{8} \right) \cos\left[\frac{1}{2}(e+fx)\right] + \left(\frac{1}{8} - \frac{i}{8} \right) \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) / ((1+2m)(3+2m)(5+2m)(7+2m)(9+2m)) + \right. \\
& \left((18900A + 12285C + 15648Am + 648Cm + 5280Am^2 + 1416Cm^2 + 896Am^3 + 224Cm^3 + 64Am^4 + 16Cm^4) \right. \\
& \left. \left(\left(\frac{1}{8} - \frac{i}{8} \right) \cos\left[\frac{1}{2}(e+fx)\right] + \left(\frac{1}{8} + \frac{i}{8} \right) \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) / ((1+2m)(3+2m)(5+2m)(7+2m)(9+2m)) + \\
& \left((1575A + 1575C + 1178Am + 414Cm + 292Am^2 + 100Cm^2 + 24Am^3 + 8Cm^3) \left(\left(\frac{1}{4} - \frac{i}{4} \right) \cos\left[\frac{3}{2}(e+fx)\right] - \left(\frac{1}{4} + \frac{i}{4} \right) \sin\left[\frac{3}{2}(e+fx)\right] \right) \right) / \\
& ((3+2m)(5+2m)(7+2m)(9+2m)) + \\
& \left((1575A + 1575C + 1178Am + 414Cm + 292Am^2 + 100Cm^2 + 24Am^3 + 8Cm^3) \left(\left(\frac{1}{4} + \frac{i}{4} \right) \cos\left[\frac{3}{2}(e+fx)\right] - \left(\frac{1}{4} - \frac{i}{4} \right) \sin\left[\frac{3}{2}(e+fx)\right] \right) \right) / \\
& ((3+2m)(5+2m)(7+2m)(9+2m)) + \frac{(63A + 189C + 32Am + 44Cm + 4Am^2 + 4Cm^2) \left(\left(-\frac{1}{4} + \frac{i}{4} \right) \cos\left[\frac{5}{2}(e+fx)\right] - \left(\frac{1}{4} + \frac{i}{4} \right) \sin\left[\frac{5}{2}(e+fx)\right] \right)}{(5+2m)(7+2m)(9+2m)} + \\
& \frac{(63A + 189C + 32Am + 44Cm + 4Am^2 + 4Cm^2) \left(\left(-\frac{1}{4} - \frac{i}{4} \right) \cos\left[\frac{5}{2}(e+fx)\right] - \left(\frac{1}{4} - \frac{i}{4} \right) \sin\left[\frac{5}{2}(e+fx)\right] \right)}{(5+2m)(7+2m)(9+2m)} + \\
& \frac{(15+2m) \left(\left(-\frac{3}{16} - \frac{3i}{16} \right) C \cos\left[\frac{7}{2}(e+fx)\right] + \left(\frac{3}{16} - \frac{3i}{16} \right) C \sin\left[\frac{7}{2}(e+fx)\right] \right)}{(7+2m)(9+2m)} + \\
& \frac{(15+2m) \left(\left(-\frac{3}{16} + \frac{3i}{16} \right) C \cos\left[\frac{7}{2}(e+fx)\right] + \left(\frac{3}{16} + \frac{3i}{16} \right) C \sin\left[\frac{7}{2}(e+fx)\right] \right)}{(7+2m)(9+2m)} + \\
& \frac{\left(\frac{1}{16} + \frac{i}{16} \right) C \cos\left[\frac{9}{2}(e+fx)\right] + \left(\frac{1}{16} - \frac{i}{16} \right) C \sin\left[\frac{9}{2}(e+fx)\right]}{9+2m} + \frac{\left(\frac{1}{16} - \frac{i}{16} \right) C \cos\left[\frac{9}{2}(e+fx)\right] + \left(\frac{1}{16} + \frac{i}{16} \right) C \sin\left[\frac{9}{2}(e+fx)\right]}{9+2m} \left. \right)
\end{aligned}$$

■ **Problem 4: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + a \sin[e+fx])^m (A + C \sin[e+fx])^2}{\sqrt{c - c \sin[e+fx]}} dx$$

Optimal (type 5, 123 leaves, 4 steps):

$$\frac{(A+C) \cos[e+fx] \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\sin[e+fx])\right] (a+a \sin[e+fx])^m}{f(1+2m) \sqrt{c-c \sin[e+fx]}} - \frac{2C \cos[e+fx] (a+a \sin[e+fx])^{1+m}}{af(3+2m) \sqrt{c-c \sin[e+fx]}}$$

Result (type 1, 1 leaves) :

???

- **Problem 5: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m (A + C \sin[e + f x]^2)}{(c - c \sin[e + f x])^{3/2}} dx$$

Optimal (type 5, 202 leaves, 5 steps) :

$$\frac{(A + C) \cos[e + f x] (a + a \sin[e + f x])^{1+m}}{4 a f (c - c \sin[e + f x])^{3/2}} + \frac{(A + 2 A m + C (9 + 2 m)) \cos[e + f x] (a + a \sin[e + f x])^m}{4 c f (1 + 2 m) \sqrt{c - c \sin[e + f x]}} +$$

$$\left(\frac{(A (1 - 2 m) - C (7 + 2 m)) \cos[e + f x] \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x])\right] (a + a \sin[e + f x])^m}{(4 c f (1 + 2 m) \sqrt{c - c \sin[e + f x]})} \right) /$$

Result (type 6, 16031 leaves) :

$$- \left(\left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} \left(\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right] \right)^3 (a + a \sin[e + f x])^m \right. \right.$$

$$\left. \left(- \frac{2 A \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m}}{\left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^3} - \frac{C \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m}}{\left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^3} \right. \right.$$

$$\left. \left. \frac{C \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[2 \left(-e + \frac{\pi}{2} - f x\right)\right]}{\left(\cos\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] - \sin\left[\frac{\pi}{4} + \frac{1}{2} \left(e - \frac{\pi}{2} + f x\right)\right] \right)^3} \right) \left(\frac{1 - \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{2m} \right)$$

$$- \left(A \operatorname{AppellF1}\left[1, -2 m, 2 m, 2, \cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) / \left(-m \left(\operatorname{AppellF1}\left[2, 1 - 2 m, 2 m, 3, \cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right. \right.$$

$$\left. \left. + \operatorname{AppellF1}\left[2, -2 m, 1 + 2 m, 3, \cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) + \right.$$

$$\left. \operatorname{AppellF1}\left[1, -2 m, 2 m, 2, \cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \tan\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) -$$

$$\left(C \operatorname{AppellF1}\left[1, -2 m, 2 m, 2, \cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\cot\left[\frac{1}{4} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) /$$

$$\begin{aligned}
& \left((1+2m) \left(-2(1+m) \operatorname{AppellF1} \left[1+2m, 2m, 1, 2+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right. \\
& \quad \left(\operatorname{AppellF1} \left[2+2m, 2m, 2, 3+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + m \operatorname{AppellF1} \left[2+2m, \right. \right. \\
& \quad \left. \left. 1+2m, 1, 3+2m, \frac{1}{2} - \frac{1}{2} \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, 1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \left(-1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \right) \Bigg) - \\
& \frac{32 C \left(1 - \left(\frac{1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2}{1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2} \right)^{-2m} + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \left(-1 - \left(\frac{1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2}{1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2} \right)^{-2m} \right) \right)}{(1+2m) \left(1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)} \Bigg) \Bigg/ \left(8 \sqrt{2} f (c - c \sin[e + fx])^{3/2} \right. \\
& \left. \left(-\frac{1}{2\sqrt{2}} m \left(\frac{1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2}{1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2} \right)^{-1+2m} \left(-\frac{\sec \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right] \left(1 - \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)}{2 \left(1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)^2} - \right. \right. \right. \\
& \left. \left. \frac{\sec \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]}{2 \left(1 + \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right)} \right) \right) \left(-\left(\operatorname{AppellF1} \left[1, -2m, 2m, 2, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \right) \Bigg) \Bigg/ \\
& \left(-m \left(\operatorname{AppellF1} \left[2, 1-2m, 2m, 3, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \right. \\
& \quad \left. \operatorname{AppellF1} \left[2, -2m, 1+2m, 3, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) + \\
& \quad \operatorname{AppellF1} \left[1, -2m, 2m, 2, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) - \left(C \operatorname{AppellF1} \left[1, \right. \right. \\
& \quad \left. \left. -2m, 2m, 2, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \Bigg) \Bigg/ \left(-m \left(\operatorname{AppellF1} \left[2, 1-2m, 2m, 3, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] + \operatorname{AppellF1} \left[2, -2m, 1+2m, 3, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \right) \right) + \\
& \quad \operatorname{AppellF1} \left[1, -2m, 2m, 2, \cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\cot \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) + \\
& \left(A \operatorname{AppellF1} \left[1, -2m, 2m, 2, \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2, -\tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right] \tan \left[\frac{1}{4} \left(-e + \frac{\pi}{2} - fx \right) \right]^2 \right) \Bigg) \Bigg/
\end{aligned}$$

$$\begin{aligned}
& \left(16 C \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \frac{\left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-2m}}{\left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)} + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right. \right. \\
& \left. \left. \left(-1 - \frac{\left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-2m}}{\left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)} \right) \right) \right) / \left((1 + 2m) \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^2 \right) - \\
& \frac{1}{(1 + 2m) \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)} 32 C \left(\frac{1}{2} \operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(-1 - \frac{\left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-2m}}{\left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)} \right) \right) + \\
& 2 m \left(\frac{1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{\left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)} \right)^{-1-2m} \left(- \frac{\operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)}{2 \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^2} - \right. \\
& \left. \frac{\operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]}{2 \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)} + 2 m \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \\
& \left(\frac{1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{\left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)} \right)^{-1-2m} \left(- \frac{\operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)}{2 \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^2} - \right. \\
& \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]}{2 \left(1 + \operatorname{Tan}\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)} \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 6: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x])^m (A + C \operatorname{Sin}[e + f x]^2)}{(c - c \operatorname{Sin}[e + f x])^{5/2}} dx$$

Optimal (type 5, 207 leaves, 5 steps) :

$$\begin{aligned}
& \frac{(A + C) \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^{1+m}}{8 a f (c - c \operatorname{Sin}[e + f x])^{5/2}} + \frac{(A (5 - 2 m) - C (11 + 2 m)) \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^m}{16 c f (c - c \operatorname{Sin}[e + f x])^{3/2}} + \\
& \left((A (3 - 8 m + 4 m^2) + C (19 + 24 m + 4 m^2)) \operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \operatorname{Sin}[e + f x])\right] (a + a \operatorname{Sin}[e + f x])^m \right) / \\
& \left(32 c^2 f (1 + 2 m) \sqrt{c - c \operatorname{Sin}[e + f x]} \right)
\end{aligned}$$

Result (type 6, 27269 leaves) : Display of huge result suppressed!

■ **Problem 8: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n (A + C \sin[e + f x]^2) dx$$

Optimal (type 5, 257 leaves, 6 steps):

$$\left(2^{\frac{1}{2}+n} c (C (1+2m) (m-n) + (1+m+n) (C (1-m+n) + A (2+m+n))) \cos[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2} (1+2m), \frac{1}{2} (1-2n), \frac{1}{2} (3+2m), \frac{1}{2} (1+\sin[e + f x])\right] (1-\sin[e + f x])^{\frac{1}{2}-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1+n} \right) / (f (1+2m) (1+m+n) (2+m+n)) - \frac{C (1+2m) \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n}{f (1+m+n) (2+m+n)} + \frac{C \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{1+n}}{c f (2+m+n)}$$

Result (type 6, 25546 leaves): Display of huge result suppressed!

■ **Problem 9: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n (A + C \sin[e + f x]^2) dx$$

Optimal (type 6, 366 leaves, 10 steps):

$$-\frac{C \cos[e + f x] (a + a \sin[e + f x])^m (c + d \sin[e + f x])^{1+n}}{d f (2+m+n)} + \left(\sqrt{2} (c (C + 2 C m) + d (C (1-m+n) + A (2+m+n))) \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, -n, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d}\right] \cos[e + f x] (a + a \sin[e + f x])^m (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c - d}\right)^{-n} \right) / (d f (1+2m) (2+m+n) \sqrt{1 - \sin[e + f x]}) + \left(\sqrt{2} C (d m - c (1+m)) \operatorname{AppellF1}\left[\frac{3}{2} + m, \frac{1}{2}, -n, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d}\right] \cos[e + f x] (a + a \sin[e + f x])^{1+m} (c + d \sin[e + f x])^n \left(\frac{c + d \sin[e + f x]}{c - d}\right)^{-n} \right) / (a d f (3+2m) (2+m+n) \sqrt{1 - \sin[e + f x]})$$

Result (type 6, 2255 leaves):

$$-\frac{1}{2f} \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-2m} \left(- \left(\left(6 C (c + d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2} - m, -n, \frac{3}{2}, \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2 d \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c + d}\right] \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{3+2m} \left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{\frac{1}{2} + \frac{1}{2} (-4-2m)} \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{\frac{3}{2}+m} \left(c + d - 2 d \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^n \right) / \right)$$

$$\begin{aligned}
& (c+d) (-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -n, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) + \\
& \left(20 C (c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -n, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{1+2m} \right. \\
& \left. \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{\frac{1}{2}(-1-2m)} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^3 \left(1 - \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{\frac{1}{2}+m} \left(c+d - 2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^n \right) / \\
& \left(-5 (c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -n, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) + \\
& \left(4 d n \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}-m, 1-n, \frac{7}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) + \\
& (c+d) (1+2m) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}-m, -n, \frac{7}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) - \\
& \left(14 C (c+d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}-m, -n, \frac{7}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-1+2m} \right. \\
& \left. \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{\frac{1}{2}(1-2m)} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^5 \left(1 - \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{-\frac{1}{2}+m} \left(c+d - 2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^n \right) / \\
& \left(5 \left(-7 (c+d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}-m, -n, \frac{7}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) + \right. \\
& \left. \left(4 d n \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2}-m, 1-n, \frac{9}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) + (c+d) (-1+2m) \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}-m, -n, \frac{9}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) \right) (a+a \operatorname{Sin}[e+fx])^m
\end{aligned}$$

■ **Problem 10: Result more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sin}[e+fx])^m (c+d \operatorname{Sin}[e+fx])^{-2-m} (A+C \operatorname{Sin}[e+fx]^2) dx$$

Optimal (type 6, 392 leaves, 8 steps):

$$\frac{(c^2 C + A d^2) \operatorname{Cos}[e + f x] (a + a \operatorname{Sin}[e + f x])^m (c + d \operatorname{Sin}[e + f x])^{-1-m}}{d (c^2 - d^2) f (1 + m)}$$

$$\frac{1}{(c - d) d (c + d)^2 f (1 + m)} 2^{\frac{1}{2}+m} a (c (A + C) d (1 + m) + d^2 (C - A m + C m) - c^2 (C + 2 C m)) \operatorname{Cos}[e + f x]$$

$$\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{(c - d) (1 - \operatorname{Sin}[e + f x])}{2 (c + d \operatorname{Sin}[e + f x])}\right] (a + a \operatorname{Sin}[e + f x])^{-1+m} \left(\frac{(c + d) (1 + \operatorname{Sin}[e + f x])}{c + d \operatorname{Sin}[e + f x]}\right)^{\frac{1}{2}-m} (c + d \operatorname{Sin}[e + f x])^{-m} +$$

$$\left(\sqrt{2} \operatorname{CAppellF1}\left[\frac{3}{2} + m, \frac{1}{2}, 1 + m, \frac{5}{2} + m, \frac{1}{2} (1 + \operatorname{Sin}[e + f x]), -\frac{d (1 + \operatorname{Sin}[e + f x])}{c - d}\right] \operatorname{Cos}[e + f x]\right.$$

$$\left. (a + a \operatorname{Sin}[e + f x])^{1+m} (c + d \operatorname{Sin}[e + f x])^{-m} \left(\frac{c + d \operatorname{Sin}[e + f x]}{c - d}\right)^m\right) / \left(a (c - d) d f (3 + 2 m) \sqrt{1 - \operatorname{Sin}[e + f x]}\right)$$

Result (type 6, 7642 leaves):

$$- \left(\left(2 \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-1-2m} \operatorname{Sin}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] (a + a \operatorname{Sin}[e + f x])^m \right. \right.$$

$$\left. \left(2 A \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} (c + d \operatorname{Sin}[e + f x])^{-2-m} + C \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} (c + d \operatorname{Sin}[e + f x])^{-2-m} + \right. \right.$$

$$\left. \left. C \operatorname{Cos}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Cos}\left[2 \left(-e + \frac{\pi}{2} - f x\right)\right] (c + d \operatorname{Sin}[e + f x])^{-2-m} \right) \left(1 + \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-m} \right.$$

$$\left. \left(c + \frac{d - d \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{-m} \left(\frac{A (c + d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right]}{c - d} \right) \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right)^m \right. -$$

$$\frac{c^2 C (c + d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right)^m}{(c - d) d^2} +$$

$$\frac{2 c C (c + d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right)^m}{(c - d) d} -$$

$$\frac{2 c^2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2 + m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right)^m}{(c - d) d} -$$

$$\begin{aligned}
& \frac{2 A d \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1+\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m}{c-d} \\
& \left(3 C (c+d)^3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] \right) / \\
& \left(d^2 \left(1+\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right) \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left((c+d) \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] + (c-d) m \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 \right) \right) \right) / \\
& \left((c+d)^2 f \left(-\frac{1}{(c+d)^2} 4 m \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right)^{-m} \left(-\frac{d \operatorname{Sec}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2} - \right. \right. \right. \\
& \quad \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right] \left(d-d \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right)^2} \right) \left(c + \frac{d-d \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2} \right)^{-1-m} \right. \\
& \quad \left. \left(\frac{1}{c-d} A (c+d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1+\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m - \right. \right. \\
& \quad \left. \frac{1}{(c-d) d^2} c^2 C (c+d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1+\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m + \right. \\
& \quad \left. \frac{1}{(c-d) d} 2 c C (c+d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1+\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m - \right. \\
& \quad \left. \left. \left. \left. \left. \right) \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{2 c^2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right]}{(c-d) d} \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m \\
& \frac{2 A d \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right]}{c-d} \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m \\
& \left(3 C (c+d)^3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] / \right. \\
& \left. \left(d^2 \left(1 + \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right) \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] + \right. \right. \right. \\
& \left. \left. 2 \left((c+d) \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] + (c-d) m \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right)\right)\right) \\
& \frac{1}{(c+d)^2} 4 m \operatorname{Sec}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 \left(1 + \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right)^{-1-m} \left(c + \frac{d-d \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}\right)^{-m} \\
& \left(\frac{A (c+d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right]}{c-d} \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m - \frac{1}{(c-d) d^2} \right. \\
& \left. c^2 C (c+d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m + \frac{1}{(c-d) d} \right. \\
& \left. 2 c C (c+d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 c^2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1+\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m}{(c-d) d} \\
& \frac{2 A d \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1+\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m}{c-d} \\
& \left(3 C (c+d)^3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right]\right) / \\
& \left(d^2 \left(1+\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right) \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] + \right. \right. \\
& \left. \left. 2 \left((c+d) \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. (c-d) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right)\right) \right) + \\
& \frac{1}{(c+d)^2} 2 \operatorname{Sec}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 \left(1+\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right)^{-m} \left(c+\frac{d-d \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}\right)^{-m} \\
& \left(\frac{A (c+d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1+\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m}{c-d} - \frac{1}{(c-d) d^2} \right. \\
& \left. c^2 C (c+d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1+\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m + \frac{1}{(c-d) d} \right. \\
& \left. 2 c C (c+d) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \left(1+\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 c^2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right]}{(c-d) d} \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m \\
& \frac{2 A d \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right]}{c-d} \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^m \\
& \left(3 C (c+d)^3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right]\right) / \\
& \left(d^2 \left(1 + \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right) \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] + \right. \right. \\
& \left. \left. 2 \left((c+d) \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. (c-d) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right)\right) + \\
& \frac{1}{(c+d)^2} 4 \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2\right)^{-m} \left(c + \frac{d-d \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}\right)^{-m} \\
& \left(A m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right] \right. \\
& \left.\left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^{-1+m} - \frac{1}{d^2} c^2 C m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right] \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right)^{-1+m} + \frac{1}{d} \right. \\
& \left. 2 c C m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e+\frac{\pi}{2}-f x)\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \right)^{-1+m} - \frac{1}{d(c+d)} {}_2F_1\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \right)^{-1+m} - \frac{1}{c+d} {}_2F_1\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \right)^{-1+m} + \\
& \left(3 C (c+d)^3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \right. \\
& \left. \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] \right) / \left(d^2 \left(1 + \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \right)^2 \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2\right] \right. \right. \\
& \left. \left. - \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \right) + 2 \left((c+d) \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2\right] \right. \right. \\
& \left. \left. (c-d) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \right) \right) - \\
& \left(3 C (c+d)^3 \left(-\frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2\right] \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] - \frac{1}{3(c+d)} (c-d) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. \left. -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] \right) \right) / \\
& \left(d^2 \left(1 + \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \right) \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2\right] \right. \right. \\
& \left. \left. + 2 \left((c+d) \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2\right] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (c-d) m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2 \Bigg) - \\
& \frac{1}{(c-d) d} c^2 C \operatorname{Csc} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \operatorname{Sec} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^m \\
& \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right] + \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^{-2-m} \right) - \\
& \frac{1}{c-d} A d \operatorname{Csc} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \operatorname{Sec} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^m \\
& \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right] + \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^{-2-m} \right) + \\
& \frac{1}{2(c-d)} A (c+d) \operatorname{Csc} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \operatorname{Sec} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^m \\
& \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right] + \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^{-1-m} \right) - \\
& \frac{1}{2(c-d) d^2} c^2 C (c+d) \operatorname{Csc} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \operatorname{Sec} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^m \\
& \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right] + \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^{-1-m} \right) + \\
& \frac{1}{(c-d) d} c C (c+d) \operatorname{Csc} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \operatorname{Sec} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right] \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^m \\
& \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right] + \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d} \right)^{-1-m} \right) + \\
& \left(3 C (c+d)^3 \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - f x) \right]^2 \right] \right)
\end{aligned}$$

$$\text{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^2\right)^2\right)^2\right)^2\right)^2\right)$$

■ **Problem 11: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[ex + fx])^m (c + d \sin[ex + fx])^{3/2} (A + C \sin[ex + fx]^2) dx$$

Optimal (type 6, 385 leaves, 10 steps):

$$\begin{aligned} & -\frac{2C \cos[ex + fx] (a + a \sin[ex + fx])^m (c + d \sin[ex + fx])^{5/2}}{df(7+2m)} + \\ & \left(\sqrt{2} (c-d) (2c(C+2Cm) + d(C(5-2m) + A(7+2m))) \text{AppellF1}\left[\frac{1}{2}+m, \frac{1}{2}, -\frac{3}{2}, \frac{3}{2}+m, \frac{1}{2}(1+\sin[ex + fx]), -\frac{d(1+\sin[ex + fx])}{c-d}\right] \right. \\ & \quad \left. \cos[ex + fx] (a + a \sin[ex + fx])^m \sqrt{c+d \sin[ex + fx]} \right) / \left(df(1+2m)(7+2m) \sqrt{1-\sin[ex + fx]} \sqrt{\frac{c+d \sin[ex + fx]}{c-d}} \right) + \\ & \left(2\sqrt{2} C(c-d)(dm - c(1+m)) \text{AppellF1}\left[\frac{3}{2}+m, \frac{1}{2}, -\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}(1+\sin[ex + fx]), -\frac{d(1+\sin[ex + fx])}{c-d}\right] \cos[ex + fx] \right. \\ & \quad \left. (a + a \sin[ex + fx])^{1+m} \sqrt{c+d \sin[ex + fx]} \right) / \left(adf(3+2m)(7+2m) \sqrt{1-\sin[ex + fx]} \sqrt{\frac{c+d \sin[ex + fx]}{c-d}} \right) \end{aligned}$$

Result (type 6, 5809 leaves):

$$\begin{aligned} & \frac{1}{2f} \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-2m} \\ & \left(\left(3Cd(c+d) \text{AppellF1}\left[\frac{1}{2}, -\frac{5}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{5+2m} \right. \right. \\ & \quad \left. \left(\cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{\frac{1}{2}+\frac{1}{2}(-6-2m)} \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{\frac{5}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2} \right) \right) / \\ & \left(-3(c+d) \text{AppellF1}\left[\frac{1}{2}, -\frac{5}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) + \end{aligned}$$

$$\begin{aligned}
& \left(9 c d (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{1+2m} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2} + \frac{1}{2} (-2-2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}+m} \sqrt{c+d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} \right) / \\
& \quad \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \quad \left. \left(2 d \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \quad \left. \left. (c+d) (1+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) - \\
& \quad \left(12 A c (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}-m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \sqrt{c+d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} \right) / \\
& \quad \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - \right. \\
& \quad \left. \left(2 d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \quad \left. \left. (c+d) (-1+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) - \\
& \quad \left(6 c C (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{-\frac{1}{2}+m} \sqrt{c+d-2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \Big/ \\
& \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] - \\
& \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \\
& (c+d)(-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \Big) - \\
& \left(25c d (c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{3+2m} \\
& \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^{\frac{1}{2}(-3-2m)} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^3 \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{\frac{3}{2}+m} \sqrt{c+d-2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \Big/ \\
& \left(-5(c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \\
& \left(2d \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{3}{2}-m, \frac{1}{2}, \frac{7}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \\
& (c+d)(3+2m) \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{7}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \Big) - \\
& \left(20c c (c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{1+2m} \\
& \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^{\frac{1}{2}(-1-2m)} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^3 \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)^{\frac{1}{2}+m} \sqrt{c+d-2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \Big/ \\
& \left(-5(c+d) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] +
\end{aligned}$$

$$\left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{\frac{1}{2}(1-2m)} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^7 \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right) /$$

$$\left(7 \left(-9(c+d) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{9}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \right. \right.$$

$$\left. \left(2d \operatorname{AppellF1}\left[\frac{9}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{11}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + (c+d)(-1+2m) \right. \right.$$

$$\left. \left. \operatorname{AppellF1}\left[\frac{9}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{11}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) \left. \right) \left. \right) (a + a \sin[e + fx])^m$$

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + fx])^m \sqrt{c+d \sin[e + fx]} (A + C \sin[e + fx]^2) dx$$

Optimal (type 6, 375 leaves, 10 steps):

$$-\frac{2C \cos[e + fx] (a + a \sin[e + fx])^m (c+d \sin[e + fx])^{3/2}}{df(5+2m)} +$$

$$\left(\sqrt{2} (2c(C+2Cm) + d(C(3-2m) + A(5+2m))) \operatorname{AppellF1}\left[\frac{1}{2}+m, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}+m, \frac{1}{2}(1+\sin[e + fx]), -\frac{d(1+\sin[e + fx])}{c-d}\right] \right.$$

$$\left. \cos[e + fx] (a + a \sin[e + fx])^m \sqrt{c+d \sin[e + fx]} \right) / \left(df(1+2m)(5+2m) \sqrt{1-\sin[e + fx]} \sqrt{\frac{c+d \sin[e + fx]}{c-d}} \right) +$$

$$\left(2\sqrt{2} C(dm - c(1+m)) \operatorname{AppellF1}\left[\frac{3}{2}+m, \frac{1}{2}, -\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}(1+\sin[e + fx]), -\frac{d(1+\sin[e + fx])}{c-d}\right] \cos[e + fx] \right.$$

$$\left. (a + a \sin[e + fx])^{1+m} \sqrt{c+d \sin[e + fx]} \right) / \left(adf(3+2m)(5+2m) \sqrt{1-\sin[e + fx]} \sqrt{\frac{c+d \sin[e + fx]}{c-d}} \right)$$

Result (type 6, 2250 leaves):

$$\frac{1}{2f} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m}$$

$$\left(\left(6C(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{3+2m} \right. \right.$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{\frac{1}{2} + \frac{1}{2}(-4-2m)} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{\frac{3}{2}+m} \sqrt{c+d-2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right) / \\
& \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \right. \\
& \left. \left. (c+d)(3+2m) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) - \\
& \left(12A(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-1+2m} \right. \\
& \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d-2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right) / \\
& \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - \right. \\
& \left. \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \right. \\
& \left. \left. (c+d)(-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) - \\
& \left(6C(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-1+2m} \right. \\
& \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d-2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right) / \\
& \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d\sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - \right.
\end{aligned}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sin[e + f x])^m (A + C \sin[e + f x]^2)}{\sqrt{c + d \sin[e + f x]}} dx$$

Optimal (type 6, 365 leaves, 10 steps):

$$\begin{aligned} & - \frac{2 C \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{c + d \sin[e + f x]}}{d f (3 + 2 m)} + \\ & \left(\sqrt{2} (2 c (C + 2 C m) + d (C - 2 C m + A (3 + 2 m))) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \right. \\ & \quad \left. \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right) / \left(d f (1 + 2 m) (3 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) - \\ & \left(2 \sqrt{2} C (c + c m - d m) \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \cos[e + f x] \right. \\ & \quad \left. (a + a \sin[e + f x])^{1+m} \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right) / \left(a d f (3 + 2 m)^2 \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) \end{aligned}$$

Result (type 6, 11762 leaves):

$$\begin{aligned} & - \left(\left(2 (c + d) \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1-2m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] (a + a \sin[e + f x])^m \right. \right. \\ & \quad \left. \left(-\frac{2 A \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m}}{\sqrt{c + d \sin[e + f x]}} - \frac{C \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m}}{\sqrt{c + d \sin[e + f x]}} - \frac{C \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m} \cos \left[2 \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{c + d \sin[e + f x]}} \right) \right. \\ & \quad \left. \left(1 + \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^{-2m} \sqrt{\frac{c + d + c \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1 + \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right. \\ & \quad \left. \left(\left(9 (A (c - 3 d) + C (-3 c + d)) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{3}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \right) \right) / \end{aligned}$$

$$\begin{aligned}
& \left(3 (c-d)^2 f \left(-\frac{1}{3 (c-d)^2} 4 (c+d) (-2-m) \operatorname{Sec}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(1 + \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-3-m} \right. \right. \\
& \sqrt{\frac{c+d + c \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 - d \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \\
& \left. \left(\left(9 (A (c-3d) + C (-3c+d)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) / \right. \right. \\
& \left. \left(3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) + \right. \\
& \left. \left((c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - (c+d) (5+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \\
& \left. \left. \frac{7}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 - \left(36 (c^2 C + A d^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) / \left(\left(c+d + c \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 - \right. \right. \right. \\
& \left. \left. \left. d \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) + \right. \right. \\
& \left. \left((c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + (c+d) (5+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) + \\
& \left. \left(5 (A+C) (c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(5 (c+d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left((c-d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{7}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - (c+d) (5+2m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{5}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{7}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \Bigg) - \\
& \frac{1}{3 (c-d)^2} 2 (c+d) \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-2m} \sqrt{\frac{c+d+c \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right. \\
& \left(\left(9 (A (c-3d) + C (-3c+d)) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \right) / \\
& \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left((c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - (c+d) (5+2m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \Bigg) - \\
& \left(36 (c^2 C + A d^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \\
& \left(\left(c+d+c \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right. \\
& \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left. (c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (5+2m) \right) \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{AppellF1} \left[\frac{3}{2}, \frac{7}{2} + m, \frac{1}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) + \\
& \left(5 (A+C) (c-d) \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \\
& \left(5 (c+d) \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left((c-d) \text{AppellF1} \left[\frac{5}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{7}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - (c+d) (5+2m) \right. \right. \\
& \left. \left. \text{AppellF1} \left[\frac{5}{2}, \frac{7}{2} + m, -\frac{1}{2}, \frac{7}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \frac{1}{3 (c-d)^2 \sqrt{\frac{c+d+c \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1+\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}}} 2 (c+d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 + \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-2-m} \\
& \left(\frac{c \text{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - d \text{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{1 + \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} - \left(\text{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^2 \right. \\
& \left. \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(c+d+c \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) / \left(1 + \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^2 \right) \\
& \left(\left(9 (A (c-3d) + C (-3c+d)) \text{AppellF1} \left[\frac{1}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{3}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \right. \\
& \left. \left(3 (c+d) \text{AppellF1} \left[\frac{1}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{3}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \left. \left. \left((c-d) \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - (c+d) (5+2m) \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1} \left[\frac{3}{2}, \frac{7}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(36 (c^2 C + A d^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \\
& \left(\left(c+d+c \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right. \\
& \left. \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \left. \left((c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (5+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{2} + m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) / \\
& \left(5 (A+C) (c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \\
& \left(5 (c+d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left((c-d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{7}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - (c+d) (5+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{5}{2}, \frac{7}{2} + m, -\frac{1}{2}, \frac{7}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \frac{1}{3 (c-d)^2} 4 (c+d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-2m} \sqrt{\frac{c+d+c \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \\
& \left(\left(9 (A (c-3d) + C (-3c+d)) \left(\frac{1}{6 (c+d)} (c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \right. \right. \\
& \left. \left. \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{3} \left(\frac{5}{2} + m \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{2} + m, -\frac{1}{2}, \frac{5}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]\right) \Bigg) \Bigg) / \\
& \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left((c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] - (c+d)(5+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) + \right. \\
& \left. \left(36(c^2c+Ad^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \right. \\
& \left. \left(c \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - d \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \Bigg) / \\
& \left(\left(c+d+c \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - d \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^2 \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \right. \right. \right. \\
& \left. \left. \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left. \left((c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + (c+d)(5+2m) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) \right) - \\
& \left(36(c^2c+Ad^2) \left(-\frac{1}{6(c+d)} (c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{3}\left(\frac{5}{2}+m\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \\
& \left. \left. -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \Bigg) / \left(\left(c+d+c \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \Bigg) - \\
& \left(9(A(c-3d)+C(-3c+d)) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \right. \\
& \left. \left(\left((c-d) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] - (c+d)(5+2m) \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right. \right. \\
& \left. \left. \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] + 3(c+d) \left(\frac{1}{6(c+d)}(c-d) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] - \frac{1}{3}\left(\frac{5}{2}+m\right) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, \right. \right. \right. \\
& \left. \left. \left. -\frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \right) \right) \Bigg) + \\
& \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \left((c-d) \left(-\frac{1}{10(c+d)} 3(c-d) \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] - \frac{3}{5}\left(\frac{5}{2}+m\right) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, \frac{1}{2}, \right. \right. \right. \\
& \left. \left. \left. \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \right) \right) - \\
& (c+d)(5+2m) \left(\frac{1}{10(c+d)} 3(c-d) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] - \frac{3}{5}\left(\frac{7}{2}+m\right) \text{AppellF1}\left[\frac{5}{2}, \frac{9}{2}+m, -\frac{1}{2}, \right. \right. \right. \\
& \left. \left. \left. \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \right) \right) \Bigg) \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& - \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] - \frac{5}{7} \left(\frac{5}{2} + m\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{2} + m, \frac{1}{2}, \right. \\
& \left. \frac{9}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]\right] - \\
& (c+d)(5+2m) \left(\frac{1}{14(c+d)} 5(c-d) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{2} + m, \frac{1}{2}, \frac{9}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, \right. \right. \\
& \left. \left. -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] - \frac{5}{7} \left(\frac{7}{2} + m\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{9}{2} + m, -\frac{1}{2}, \right. \right. \right. \\
& \left. \left. \frac{9}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]\right]\right) \right) / \\
& \left(5(c+d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \right] + \right. \\
& \left. \left((c-d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \right] - (c+d)(5+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2} + m, -\frac{1}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \right] \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 14: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x])^m (A + C \operatorname{Sin}[e + f x]^2)}{(c + d \operatorname{Sin}[e + f x])^{3/2}} dx$$

Optimal (type 6, 413 leaves, 10 steps):

$$\frac{2 (c^2 C + A d^2) \cos[e + f x] (a + a \sin[e + f x])^m}{d (c^2 - d^2) f \sqrt{c + d \sin[e + f x]}} +$$

$$\left(\sqrt{2} (c (A + C) d - d^2 (A - C + 4 A m) - 2 c^2 (C + 2 C m)) \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d}\right] \right)$$

$$\cos[e + f x] (a + a \sin[e + f x])^m \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \Big/ \left(d (c^2 - d^2) f (1 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) +$$

$$\left(\sqrt{2} (2 c^2 C (1 + m) + d^2 (A - C + 2 A m)) \operatorname{AppellF1}\left[\frac{3}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d}\right] \cos[e + f x] \right)$$

$$(a + a \sin[e + f x])^{1+m} \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \Big/ \left(a d (c^2 - d^2) f (3 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right)$$

Result (type 6, 19675 leaves):

$$- \left(\left(6 (c + d) \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-1-2m} \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] (a + a \sin[e + f x])^m \right. \right.$$

$$\left. \left(-\frac{2 A \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m}}{(c + d \sin[e + f x])^{3/2}} - \frac{C \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m}}{(c + d \sin[e + f x])^{3/2}} - \frac{C \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \cos\left[2 \left(-e + \frac{\pi}{2} - f x\right)\right]}{(c + d \sin[e + f x])^{3/2}} \right) \right.$$

$$\left. \left(1 + \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \right)^{-1-m} \sqrt{\frac{c + d + c \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 - d \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \right.$$

$$\left. \left(\left((A + C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2} + m, -\frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c - d) \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c + d}\right] \right) \Big/ \right.$$

$$\left. \left(3 (c + d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2} + m, -\frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c - d) \tan\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c + d}\right] \right) + \right.$$

$$\begin{aligned}
& \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left(3 (c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (3+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) - \\
& \left(A d^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \\
& \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left(3 (c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (3+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) / \\
& \left((c-d)^2 f \left(-\frac{1}{(c-d)^2} 12 (c+d) (-1-m) \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-2-m} \right. \right. \\
& \left. \left. \sqrt{\frac{c+d+c \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2-d \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1+\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right. \right. \\
& \left. \left. \left((A+C) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) - \\
& \left(c^2 C \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) / \\
& \left(-3(c+d) \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) + \\
& \left(3(c-d) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) + (c+d)(3+2m) \\
& \left. \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) - \\
& \left(A d^2 \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) / \\
& \left(-3(c+d) \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) + \\
& \left(3(c-d) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) + (c+d)(3+2m) \\
& \left. \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) \right) \right) - \\
& \frac{1}{(c-d)^2} 6(c+d) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \left(1+\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-1-m} \sqrt{\frac{c+d+c\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2-d\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{1+\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}} \\
& \left(\left((A+C) \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) / \right. \\
& \left. \left(3(c+d) \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(c^2 \text{C AppellF1} \left[\frac{1}{2}, \frac{3}{2} + m, \frac{3}{2}, \frac{3}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \\
& \left(-3 (c+d) \text{AppellF1} \left[\frac{1}{2}, \frac{3}{2} + m, \frac{3}{2}, \frac{3}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left(3 (c-d) \text{AppellF1} \left[\frac{3}{2}, \frac{3}{2} + m, \frac{5}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (3+2m) \right. \right. \\
& \left. \left. \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, \frac{3}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) - \\
& \left(A d^2 \text{AppellF1} \left[\frac{1}{2}, \frac{3}{2} + m, \frac{3}{2}, \frac{3}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \\
& \left(-3 (c+d) \text{AppellF1} \left[\frac{1}{2}, \frac{3}{2} + m, \frac{3}{2}, \frac{3}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left(3 (c-d) \text{AppellF1} \left[\frac{3}{2}, \frac{3}{2} + m, \frac{5}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (3+2m) \right. \right. \\
& \left. \left. \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, \frac{3}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) - \\
& \frac{1}{(c-d)^2 \sqrt{\frac{c+d+c \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1+\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}}} 6 (c+d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 + \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-1-m} \\
& \left(\frac{c \text{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - d \text{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]}{1 + \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} - \right. \\
& \left. \left(\text{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(c+d+c \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) / \right. \\
& \left. \left(1 + \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
& (c+d)(3+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 + \frac{1}{(c+d+c\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2-d\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2)^2} \\
4 & \left(\left(c \operatorname{C AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right. \right. \\
& \left. \left(c \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - d \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) / \\
& \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \\
& \left. (c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + (c+d)(3+2m) \right. \\
& \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 + \\
& \left(a d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right. \\
& \left. \left(c \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - d \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) / \\
& \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \\
& \left. (c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + (c+d)(3+2m) \right. \\
& \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{1}{10(c+d)} 3(c-d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{3}{5}\left(\frac{5}{2}+m\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{7}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \\
& \quad \left. \left((c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + (c+d)(3+2m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^2 + \\
& \left(c^2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \left(\left(3(c-d) \operatorname{AppellF1}\left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + (c+d)(3+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \right. \\
& \quad \left. 3(c+d) \left(-\frac{1}{2(c+d)} (c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{1}{3}\left(\frac{3}{2}+m\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) + \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \\
& \left. \left(3(c-d) \left(-\frac{1}{2(c+d)} 3(c-d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}+m, \frac{7}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{3}{5}\left(\frac{3}{2} + m\right) \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2} + m, \frac{5}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right. \right. \right. \\
& \left. \left. \left. - \frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + (c+d)(3+2m) \right. \\
& \left. \left(-\frac{1}{10(c+d)} {}_9 \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2} + m, \frac{5}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{3}{5}\left(\frac{5}{2} + m\right) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2} + m, \frac{3}{2}, \frac{7}{2}, \right. \right. \right. \\
& \left. \left. \left. -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \right) / \\
& \left(-3(c+d) \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2} + m, \frac{3}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left(3(c-d) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2} + m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + (c+d)(3+2m) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2} + m, \frac{3}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \\
& \left(\text{Ad}^2 \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2} + m, \frac{3}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \left(\left(3(c-d) \text{AppellF1}\left[\right. \right. \right. \right. \\
& \left. \left. \left. \frac{3}{2}, \frac{3}{2} + m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + (c+d)(3+2m) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2} + m, \right. \right. \right. \\
& \left. \left. \left. \frac{3}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \right. \\
& \left. 3(c+d) \left(-\frac{1}{2(c+d)} (c-d) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2} + m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d) \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{3}\left(\frac{3}{2} + m\right) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2} + m, \frac{3}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d} \left] \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] \right) + \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \\
& \left(3(c-d) \left(-\frac{1}{2(c+d)} 3(c-d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}+m, \frac{7}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}\right] \right. \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] - \frac{3}{5}\left(\frac{3}{2}+m\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}+m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, \right. \\
& \quad \left. \left. -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] \right) \right) + (c+d)(3+2m) \\
& \left(-\frac{1}{10(c+d)} 9(c-d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}+m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}\right] \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] - \frac{3}{5}\left(\frac{5}{2}+m\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, \frac{3}{2}, \frac{7}{2}, \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right] \right) \right) \right) \right) \right) \Big/ \\
& \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}\right] \right) + \left(3(c-d) \right. \\
& \quad \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}\right] \right) + (c+d)(3+2m) \operatorname{AppellF1}\left[\right. \\
& \quad \left. \left. \left. \left. \left. \left. \frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(-e + \frac{\pi}{2} - f x)\right]^2 \right) \right) \right) \right) \right) \right) \right) \Big/
\end{aligned}$$

■ **Problem 15: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sin}[e + f x])^m (A + C \operatorname{Sin}[e + f x])^2}{(c + d \operatorname{Sin}[e + f x])^{5/2}} dx$$

Optimal (type 6, 424 leaves, 10 steps):

$$\frac{2 (c^2 C + A d^2) \cos[e + f x] (a + a \sin[e + f x])^m}{3 d (c^2 - d^2) f (c + d \sin[e + f x])^{3/2}} +$$

$$\left(\sqrt{2} (3 c (A + C) d + d^2 (A + 3 C - 4 A m) - 2 c^2 (C + 2 C m)) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \right)$$

$$\cos[e + f x] (a + a \sin[e + f x])^m \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \Big/ \left(3 (c - d)^2 d (c + d) f (1 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) +$$

$$\left(\sqrt{2} (2 c^2 C (1 + m) - d^2 (A + 3 C - 2 A m)) \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, \frac{3}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \cos[e + f x] \right)$$

$$(a + a \sin[e + f x])^{1+m} \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \Big/ \left(3 a (c - d)^2 d (c + d) f (3 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right)$$

Result (type 6, 25117 leaves) : Display of huge result suppressed!

- **Problem 17: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n (A + B \sin[e + f x] + C \sin[e + f x]^2) dx$$

Optimal (type 5, 269 leaves, 6 steps) :

$$\left(2^{\frac{1}{2}+n} c ((1+m+n)(C(1-m+n)+A(2+m+n)) + (m-n)(C+2Cm+B(2+m+n))) \cos[e + f x] \operatorname{Hypergeometric2F1} \left[\frac{1}{2} (1 + 2 m), \frac{1}{2} (1 - 2 n), \right. \right.$$

$$\left. \frac{1}{2} (3 + 2 m), \frac{1}{2} (1 + \sin[e + f x]) \right] (1 - \sin[e + f x])^{\frac{1}{2}-n} (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{-1+n} \Big/ (f (1 + 2 m) (1 + m + n) (2 + m + n)) -$$

$$\frac{(C + 2 C m + B (2 + m + n)) \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^n}{f (1 + m + n) (2 + m + n)} + \frac{C \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{1+n}}{c f (2 + m + n)}$$

Result (type 6, 38254 leaves) : Display of huge result suppressed!

- **Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{5/2} (A + B \sin[e + f x] + C \sin[e + f x]^2) dx$$

Optimal (type 3, 435 leaves, 5 steps) :

$$\begin{aligned}
& - \frac{64 c^3 (B (45 - 8 m - 4 m^2) - C (39 - 16 m + 4 m^2) - A (63 + 32 m + 4 m^2)) \cos[e + f x] (a + a \sin[e + f x])^m}{f (5 + 2 m) (7 + 2 m) (9 + 2 m) (3 + 8 m + 4 m^2) \sqrt{c - c \sin[e + f x]}} - \\
& \left(\frac{16 c^2 (B (45 - 8 m - 4 m^2) - C (39 - 16 m + 4 m^2) - A (63 + 32 m + 4 m^2)) \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{c - c \sin[e + f x]}}{f (7 + 2 m) (9 + 2 m) (15 + 16 m + 4 m^2)} \right) - \\
& \frac{(2 c (B (45 - 8 m - 4 m^2) - C (39 - 16 m + 4 m^2) - A (63 + 32 m + 4 m^2)) \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{3/2})}{(f (5 + 2 m) (7 + 2 m) (9 + 2 m)) - \frac{2 (9 B + 2 C + 2 B m + 4 C m) \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{5/2}}{f (7 + 2 m) (9 + 2 m)} +} \\
& \frac{2 C \cos[e + f x] (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{7/2}}{c f (9 + 2 m)}
\end{aligned}$$

Result (type 3, 1029 leaves):

$$\frac{1}{f \left(\cos \left[\frac{1}{2} (e + f x) \right] - \sin \left[\frac{1}{2} (e + f x) \right] \right)^5} (a (1 + \sin[e + f x]))^m (c - c \sin[e + f x])^{5/2}$$

$$\left(\left((18900 A - 14175 B + 12285 C + 15648 A m - 4140 B m + 648 C m + 5280 A m^2 - 832 B m^2 + 1416 C m^2 + 896 A m^3 - 208 B m^3 + 224 C m^3 + 64 A m^4 - 16 B m^4 + 16 C m^4) \left(\left(\frac{1}{8} + \frac{i}{8} \right) \cos \left[\frac{1}{2} (e + f x) \right] + \left(\frac{1}{8} - \frac{i}{8} \right) \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \left((1 + 2 m) (3 + 2 m) (5 + 2 m) (7 + 2 m) (9 + 2 m) \right) + \right.$$

$$\left((18900 A - 14175 B + 12285 C + 15648 A m - 4140 B m + 648 C m + 5280 A m^2 - 832 B m^2 + 1416 C m^2 + 896 A m^3 - 208 B m^3 + 224 C m^3 + 64 A m^4 - 16 B m^4 + 16 C m^4) \left(\left(\frac{1}{8} - \frac{i}{8} \right) \cos \left[\frac{1}{2} (e + f x) \right] + \left(\frac{1}{8} + \frac{i}{8} \right) \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \left((1 + 2 m) (3 + 2 m) (5 + 2 m) (7 + 2 m) (9 + 2 m) \right) + \right.$$

$$\left((3150 A - 3465 B + 3150 C + 2356 A m - 1706 B m + 828 C m + 584 A m^2 - 316 B m^2 + 200 C m^2 + 48 A m^3 - 24 B m^3 + 16 C m^3) \left(\left(\frac{1}{8} - \frac{i}{8} \right) \cos \left[\frac{3}{2} (e + f x) \right] - \left(\frac{1}{8} + \frac{i}{8} \right) \sin \left[\frac{3}{2} (e + f x) \right] \right) \right) / \left((3 + 2 m) (5 + 2 m) (7 + 2 m) (9 + 2 m) \right) + \right.$$

$$\left((3150 A - 3465 B + 3150 C + 2356 A m - 1706 B m + 828 C m + 584 A m^2 - 316 B m^2 + 200 C m^2 + 48 A m^3 - 24 B m^3 + 16 C m^3) \left(\left(\frac{1}{8} + \frac{i}{8} \right) \cos \left[\frac{3}{2} (e + f x) \right] - \left(\frac{1}{8} - \frac{i}{8} \right) \sin \left[\frac{3}{2} (e + f x) \right] \right) \right) / \left((3 + 2 m) (5 + 2 m) (7 + 2 m) (9 + 2 m) \right) + \right.$$

$$\left((126 A - 315 B + 378 C + 64 A m - 124 B m + 88 C m + 8 A m^2 - 12 B m^2 + 8 C m^2) \left(\left(-\frac{1}{8} + \frac{i}{8} \right) \cos \left[\frac{5}{2} (e + f x) \right] - \left(\frac{1}{8} + \frac{i}{8} \right) \sin \left[\frac{5}{2} (e + f x) \right] \right) \right) / \left((5 + 2 m) (7 + 2 m) (9 + 2 m) \right) + \right.$$

$$\left((126 A - 315 B + 378 C + 64 A m - 124 B m + 88 C m + 8 A m^2 - 12 B m^2 + 8 C m^2) \left(\left(-\frac{1}{8} - \frac{i}{8} \right) \cos \left[\frac{5}{2} (e + f x) \right] - \left(\frac{1}{8} - \frac{i}{8} \right) \sin \left[\frac{5}{2} (e + f x) \right] \right) \right) / \left((5 + 2 m) (7 + 2 m) (9 + 2 m) \right) + \right.$$

$$\frac{(18 B - 45 C + 4 B m - 6 C m) \left(\left(\frac{1}{16} - \frac{i}{16} \right) \cos \left[\frac{7}{2} (e + f x) \right] - \left(\frac{1}{16} + \frac{i}{16} \right) \sin \left[\frac{7}{2} (e + f x) \right] \right)}{(7 + 2 m) (9 + 2 m)} +$$

$$\frac{(18 B - 45 C + 4 B m - 6 C m) \left(\left(\frac{1}{16} + \frac{i}{16} \right) \cos \left[\frac{7}{2} (e + f x) \right] - \left(\frac{1}{16} - \frac{i}{16} \right) \sin \left[\frac{7}{2} (e + f x) \right] \right)}{(7 + 2 m) (9 + 2 m)} +$$

$$\frac{\left(\frac{1}{16} + \frac{i}{16} \right) C \cos \left[\frac{9}{2} (e + f x) \right] + \left(\frac{1}{16} - \frac{i}{16} \right) C \sin \left[\frac{9}{2} (e + f x) \right]}{9 + 2 m} + \frac{\left(\frac{1}{16} - \frac{i}{16} \right) C \cos \left[\frac{9}{2} (e + f x) \right] + \left(\frac{1}{16} + \frac{i}{16} \right) C \sin \left[\frac{9}{2} (e + f x) \right]}{9 + 2 m} \right)$$

■ **Problem 19: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c - c \sin[e + f x])^{3/2} (A + B \sin[e + f x] + C \sin[e + f x]^2) dx$$

Optimal (type 3, 322 leaves, 4 steps):

$$\frac{8c^2 (B(21-8m-4m^2) - C(19-8m+4m^2) - A(35+24m+4m^2)) \cos[e+fx] (a+a\sin[e+fx])^m}{f(5+2m)(7+2m)(3+8m+4m^2)\sqrt{c-c\sin[e+fx]}}$$

$$\left(\frac{2c(B(21-8m-4m^2) - C(19-8m+4m^2) - A(35+24m+4m^2)) \cos[e+fx] (a+a\sin[e+fx])^m \sqrt{c-c\sin[e+fx]}}{(f(3+2m)(5+2m)(7+2m)) - \frac{2(7B+2C+2Bm+4Cm) \cos[e+fx] (a+a\sin[e+fx])^m (c-c\sin[e+fx])^{3/2}}{f(5+2m)(7+2m)}} + \frac{2C \cos[e+fx] (a+a\sin[e+fx])^m (c-c\sin[e+fx])^{5/2}}{cf(7+2m)} \right)$$

Result (type 3, 719 leaves):

$$\frac{1}{f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right] \right)^3}$$

$$(a(1+\sin[e+fx]))^m (c-c\sin[e+fx])^{3/2} \left(\left((1260A-840B+735C+1144Am-128Bm-18Cm+336Am^2+32Bm^2+100Cm^2+32Am^3+8Cm^3) \left(\left(\frac{1}{8} + \frac{i}{8} \right) \cos\left[\frac{1}{2}(e+fx)\right] + \left(\frac{1}{8} - \frac{i}{8} \right) \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) / ((1+2m)(3+2m)(5+2m)(7+2m)) + \left((1260A-840B+735C+1144Am-128Bm-18Cm+336Am^2+32Bm^2+100Cm^2+32Am^3+8Cm^3) \left(\left(\frac{1}{8} - \frac{i}{8} \right) \cos\left[\frac{1}{2}(e+fx)\right] + \left(\frac{1}{8} + \frac{i}{8} \right) \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) / ((1+2m)(3+2m)(5+2m)(7+2m)) + \left((140A-210B+175C+96Am-88Bm+16Cm+16Am^2-8Bm^2+4Cm^2) \left(\left(\frac{1}{8} - \frac{i}{8} \right) \cos\left[\frac{3}{2}(e+fx)\right] - \left(\frac{1}{8} + \frac{i}{8} \right) \sin\left[\frac{3}{2}(e+fx)\right] \right) \right) / ((3+2m)(5+2m)(7+2m)) + \left((140A-210B+175C+96Am-88Bm+16Cm+16Am^2-8Bm^2+4Cm^2) \left(\left(\frac{1}{8} + \frac{i}{8} \right) \cos\left[\frac{3}{2}(e+fx)\right] - \left(\frac{1}{8} - \frac{i}{8} \right) \sin\left[\frac{3}{2}(e+fx)\right] \right) \right) / ((3+2m)(5+2m)(7+2m)) + \frac{(14B-21C+4Bm-2Cm) \left(\left(\frac{1}{8} + \frac{i}{8} \right) \cos\left[\frac{5}{2}(e+fx)\right] + \left(\frac{1}{8} - \frac{i}{8} \right) \sin\left[\frac{5}{2}(e+fx)\right] \right)}{(5+2m)(7+2m)} + \frac{(14B-21C+4Bm-2Cm) \left(\left(\frac{1}{8} - \frac{i}{8} \right) \cos\left[\frac{5}{2}(e+fx)\right] + \left(\frac{1}{8} + \frac{i}{8} \right) \sin\left[\frac{5}{2}(e+fx)\right] \right)}{(5+2m)(7+2m)} + \frac{\left(-\frac{1}{8} - \frac{i}{8} \right) C \cos\left[\frac{7}{2}(e+fx)\right] + \left(\frac{1}{8} - \frac{i}{8} \right) C \sin\left[\frac{7}{2}(e+fx)\right]}{7+2m} + \frac{\left(-\frac{1}{8} + \frac{i}{8} \right) C \cos\left[\frac{7}{2}(e+fx)\right] + \left(\frac{1}{8} + \frac{i}{8} \right) C \sin\left[\frac{7}{2}(e+fx)\right]}{7+2m} \right)$$

■ Problem 21: Attempted integration timed out after 120 seconds.

$$\int \frac{(a+a\sin[e+fx])^m (A+B\sin[e+fx]+C\sin[e+fx]^2)}{\sqrt{c-c\sin[e+fx]}} dx$$

Optimal (type 5, 170 leaves, 5 steps) :

$$-\frac{2 B \cos [e+f x] (a+a \sin [e+f x])^m}{f(1+2 m) \sqrt{c-c \sin [e+f x]}} + \frac{(A+B+C) \cos [e+f x] \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\sin [e+f x])\right] (a+a \sin [e+f x])^m}{f(1+2 m) \sqrt{c-c \sin [e+f x]}} - \frac{2 C \cos [e+f x] (a+a \sin [e+f x])^{1+m}}{a f(3+2 m) \sqrt{c-c \sin [e+f x]}}$$

Result (type 1, 1 leaves) :

???

- **Problem 22: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \sin [e+f x])^m (A+B \sin [e+f x]+C \sin [e+f x]^2)}{(c-c \sin [e+f x])^{3/2}} dx$$

Optimal (type 5, 216 leaves, 5 steps) :

$$\frac{(A+B+C) \cos [e+f x] (a+a \sin [e+f x])^{1+m}}{4 a f(c-c \sin [e+f x])^{3/2}} + \frac{(A+B+2 A m+2 B m+C(9+2 m)) \cos [e+f x] (a+a \sin [e+f x])^m}{4 c f(1+2 m) \sqrt{c-c \sin [e+f x]}} + \frac{\left((A(1-2 m)-B(3+2 m)-C(7+2 m)) \cos [e+f x] \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\sin [e+f x])\right] (a+a \sin [e+f x])^m\right)}{\left(4 c f(1+2 m) \sqrt{c-c \sin [e+f x]}\right)}$$

Result (type 6, 23229 leaves) : Display of huge result suppressed!

- **Problem 23: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \sin [e+f x])^m (A+B \sin [e+f x]+C \sin [e+f x]^2)}{(c-c \sin [e+f x])^{5/2}} dx$$

Optimal (type 5, 230 leaves, 5 steps) :

$$\frac{(A+B+C) \cos [e+f x] (a+a \sin [e+f x])^{1+m}}{8 a f(c-c \sin [e+f x])^{5/2}} + \frac{(A(5-2 m)-B(3+2 m)-C(11+2 m)) \cos [e+f x] (a+a \sin [e+f x])^m}{16 c f(c-c \sin [e+f x])^{3/2}} - \frac{\left((B(5-8 m-4 m^2)-A(3-8 m+4 m^2)-C(19+24 m+4 m^2)) \cos [e+f x] \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\sin [e+f x])\right] (a+a \sin [e+f x])^m\right)}{\left(32 c^2 f(1+2 m) \sqrt{c-c \sin [e+f x]}\right)}$$

Result (type 6, 40823 leaves) : Display of huge result suppressed!

- **Problem 24: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a \sin [e+f x])^m (c-c \sin [e+f x])^{-2-m} (A+B \sin [e+f x]+C \sin [e+f x]^2) dx$$

Optimal (type 5, 232 leaves, 6 steps) :

$$-\frac{1}{f(3+2m)} 2^{-\frac{1}{2}-m} C \cos[e+fx]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(3+2m), \frac{1}{2}(3+2m), \frac{1}{2}(5+2m), \frac{1}{2}(1+\sin[e+fx])\right]$$

$$\frac{(1-\sin[e+fx])^{\frac{1}{2}+m} (a+a\sin[e+fx])^m (c-c\sin[e+fx])^{-2-m} + (A+B+C) \cos[e+fx] (a+a\sin[e+fx])^{1+m} (c-c\sin[e+fx])^{-2-m}}{2af(3+2m)} + \frac{(A-B+C) \cos[e+fx] (a+a\sin[e+fx])^m (c-c\sin[e+fx])^{-1-m}}{2cf(1+2m)}$$

Result (type 6, 7618 leaves) :

$$-\left(2^{-4-3m} \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{-2m} \cot\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^3 \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^{-2(-2-m)} (a+a\sin[e+fx])^m\right.$$

$$(c-c\sin[e+fx])^{-2m} \left(-2A \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{2m} \left(\cos\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right] - \sin\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right]\right)^{-4-2m} -$$

$$C \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{2m} \left(\cos\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right] - \sin\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right]\right)^{-4-2m} -$$

$$C \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{2m} \cos\left[2\left(-e+\frac{\pi}{2}-fx\right)\right] \left(\cos\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right] - \sin\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right]\right)^{-4-2m} -$$

$$2B \cos\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^{2m} \sin[e+fx] \left(\cos\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right] - \sin\left[\frac{\pi}{4}+\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right]\right)^{-4-2m} \left(\frac{1}{1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}\right)^{2m}$$

$$\left(\frac{\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]}{1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}\right)^{-2m} \left(-\frac{(A+B+C) \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}-m, -2m, -\frac{1}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]}{3+2m} - \frac{1}{1+2m}\right.$$

$$(3A-5B-13C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}-m, -2m, \frac{1}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - \left(64C(-3+2m) \operatorname{AppellF1}\left[\right.\right.$$

$$\left.\left.\frac{1}{2}-m, -2m, 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \left(1-\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{2m}\right) /$$

$$\left((-1+2m) \left(1+\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)\right) \left((-3+2m) \operatorname{AppellF1}\left[\frac{1}{2}-m, -2m, 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] +\right.$$

$$2\left(2m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2m, 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}-m,\right.$$

$$\left.-2m, 2, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right]) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) - \frac{1}{3-8m+4m^2}$$

$$\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \left((3A-5B-13C)(-3+2m) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}-m, -2m, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] +\right.$$

$$\begin{aligned}
& -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 + 2\left(2m \operatorname{AppellF1}\left[\frac{3}{2} - m, 1 - 2m, 1, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \left. \operatorname{AppellF1}\left[\frac{3}{2} - m, -2m, 2, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) - \frac{1}{3 - 8m + 4m^2} \\
& \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \left((3A - 5B - 13C) (-3 + 2m) \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - m, -2m, \frac{3}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \left. (A + B + C) (-1 + 2m) \operatorname{Hypergeometric2F1}\left[\frac{3}{2} - m, -2m, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) + \\
& 2^{-2-3m} m \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^3 \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right)^{2m} \left(\frac{\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right)^{-1-2m} \\
& \left(-\frac{\sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{2\left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^2} + \frac{\sec\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{4\left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)} \right) \\
& \left(-\frac{(A + B + C) \operatorname{Hypergeometric2F1}\left[-\frac{3}{2} - m, -2m, -\frac{1}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]}{3 + 2m} - \frac{1}{1 + 2m} \right. \\
& \left. (3A - 5B - 13C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{2} - m, -2m, \frac{1}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 - \left(64C (-3 + 2m) \operatorname{AppellF1}\left[\right. \right. \right. \\
& \left. \left. \frac{1}{2} - m, -2m, 1, \frac{3}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{2m} \right) \right) / \\
& \left((-1 + 2m) \left(1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right) \right) \left((-3 + 2m) \operatorname{AppellF1}\left[\frac{1}{2} - m, -2m, 1, \frac{3}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + 2\left(2m \operatorname{AppellF1}\left[\frac{3}{2} - m, 1 - 2m, 1, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2} - m, -2m, 2, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right]\right) \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) - \frac{1}{3 - 8m + 4m^2} \\
& \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \left((3A - 5B - 13C) (-3 + 2m) \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - m, -2m, \frac{3}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \right. \\
& \left. (A + B + C) (-1 + 2m) \operatorname{Hypergeometric2F1}\left[\frac{3}{2} - m, -2m, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) - \\
& 8^{-1-m} \cot\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^3 \left(\frac{1}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right)^{2m} \left(\frac{\tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]}{1 + \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right)^{-2m}
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}-m, -2m, 2, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) + \\
& \left(64 C m (-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2m, 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^5 \left(1 - \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{-1+2m}\right) / \left((-1+2m) \left(1 + \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)\right) \\
& \quad \left((-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2m, 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + 2 \right. \\
& \quad \left. \left(2m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \text{AppellF1}\left[\frac{3}{2}-m, -2m, 2, \frac{5}{2}-m, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) - \frac{1}{3-8m+4m^2} \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \\
& \quad \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^3 \left((3A-5B-13C) (-3+2m) \text{Hypergeometric2F1}\left[\frac{1}{2}-m, -2m, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \\
& \quad \left.(A+B+C) (-1+2m) \text{Hypergeometric2F1}\left[\frac{3}{2}-m, -2m, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right) - \\
& \quad \frac{1}{2(3+2m)} (A+B+C) \left(-\frac{3}{2}-m\right) \csc\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \\
& \quad \left(-\text{Hypergeometric2F1}\left[-\frac{3}{2}-m, -2m, -\frac{1}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \left(1 - \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{2m}\right) - \\
& \quad \frac{1}{2(1+2m)} (3A-5B-13C) \left(-\frac{1}{2}-m\right) \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] \\
& \quad \left(-\text{Hypergeometric2F1}\left[-\frac{1}{2}-m, -2m, \frac{1}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \left(1 - \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{2m}\right) + \\
& \quad \left(64 C (-3+2m) \text{AppellF1}\left[\frac{1}{2}-m, -2m, 1, \frac{3}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \right. \\
& \quad \left. \left(1 - \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{2m} \left(\left(2m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{3}{2}-m, -2m, 2, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right] \right) \sec\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right] + (-3+2m) \left(-\frac{1}{\frac{3}{2}-m}\right) \left(\frac{1}{2}-m\right) m \text{AppellF1}\left[\frac{3}{2}-m, 1-2m, 1, \frac{5}{2}-m, \tan\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{2\left(\frac{3}{2}-m\right)}\left(\frac{1}{2}-m\right) \operatorname{AppellF1}\left[\frac{3}{2}-m,-2 m, 2,\right. \\
& \left.\frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]+2 \\
& \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\left(-\frac{1}{\frac{5}{2}-m}\left(\frac{3}{2}-m\right) m \operatorname{AppellF1}\left[\frac{5}{2}-m, 1-2 m, 2,\frac{7}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,\right.\right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]-\frac{1}{\frac{5}{2}-m}\left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m,-2 m,\right. \\
& \left.3,\frac{7}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]+ \\
& 2 m\left(-\frac{1}{2\left(\frac{5}{2}-m\right)}\left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m, 1-2 m, 2,\frac{7}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]+\frac{1}{2\left(\frac{5}{2}-m\right)}(1-2 m)\left(\frac{3}{2}-m\right) \operatorname{AppellF1}\left[\frac{5}{2}-m, 2-2 m, 1,\frac{7}{2}-m,\right.\right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)\right) / \\
& \left((-1+2 m)\left(1+\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\left((-3+2 m) \operatorname{AppellF1}\left[\frac{1}{2}-m,-2 m, 1,\frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,\right.\right.\right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+2\left(2 m \operatorname{AppellF1}\left[\frac{3}{2}-m, 1-2 m, 1,\frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+ \right. \\
& \left. \operatorname{AppellF1}\left[\frac{3}{2}-m,-2 m, 2,\frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^2- \\
& \frac{1}{3-8 m+4 m^2} \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^4\left(\frac{1}{2}(A+B+C)(-1+2 m) \operatorname{Hypergeometric2F1}\left[\frac{3}{2}-m,-2 m,\frac{5}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]+\frac{1}{2}(3 A-5 B-13 C)\left(\frac{1}{2}-m\right)(-3+2 m) \operatorname{Csc}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right] \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) \\
& \left.\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right)\left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}-m,-2 m,\frac{3}{2}-m, \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right]+\left(1-\operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{2 m}\right)+ \\
& \frac{1}{2}(A+B+C)\left(\frac{3}{2}-m\right)(-1+2 m) \operatorname{Sec}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{4}\left(-e+\frac{\pi}{2}-f x\right)\right]
\end{aligned}$$

$$\left(-\text{Hypergeometric2F1}\left[\frac{3}{2} - m, -2m, \frac{5}{2} - m, \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right] + \left(1 - \tan\left[\frac{1}{4}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{2m}\right) \right) \right) \right) \right)$$

■ **Problem 25: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[ex + f])^m (c + d \sin[ex + f])^n (A + B \sin[ex + f] + C \sin[ex + f]^2) dx$$

Optimal (type 6, 383 leaves, 10 steps):

$$\begin{aligned} & -\frac{C \cos[ex + f] (a + a \sin[ex + f])^m (c + d \sin[ex + f])^{1+n}}{df(2+m+n)} + \\ & \left(\sqrt{2} (c(C+2Cm) + d(C(1-m+n) + A(2+m+n) - B(2+m+n))) \text{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, -n, \frac{3}{2} + m, \frac{1}{2}(1 + \sin[ex + f]), -\frac{d(1 + \sin[ex + f])}{c-d}\right] \right. \\ & \quad \left. \cos[ex + f] (a + a \sin[ex + f])^m (c + d \sin[ex + f])^n \left(\frac{c + d \sin[ex + f]}{c-d}\right)^{-n} \right) / (df(1+2m)(2+m+n)\sqrt{1 - \sin[ex + f]}) - \\ & \left(\sqrt{2} (cC(1+m) - d(Cm + B(2+m+n))) \text{AppellF1}\left[\frac{3}{2} + m, \frac{1}{2}, -n, \frac{5}{2} + m, \frac{1}{2}(1 + \sin[ex + f]), -\frac{d(1 + \sin[ex + f])}{c-d}\right] \right. \\ & \quad \left. \cos[ex + f] (a + a \sin[ex + f])^{1+m} (c + d \sin[ex + f])^n \left(\frac{c + d \sin[ex + f]}{c-d}\right)^{-n} \right) / (adf(3+2m)(2+m+n)\sqrt{1 - \sin[ex + f]}) \end{aligned}$$

Result (type 6, 3145 leaves):

$$\begin{aligned} & -\frac{1}{2f} \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-2m} \\ & \left(-\left(\left(6C(c+d) \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2} - m, -n, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{3+2m} \right. \right. \\ & \quad \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^{\frac{1}{2} + \frac{1}{2}(-4-2m)} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{\frac{3}{2}+m} \left(c + d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^n \right) / \right. \\ & \quad \left(-3(c+d) \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2} - m, -n, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \right. \\ & \quad \left(4dn \text{AppellF1}\left[\frac{3}{2}, -\frac{3}{2} - m, 1-n, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \right. \\ & \quad \left. \left. (c+d)(3+2m) \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} - m, -n, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right) - \end{aligned}$$

$$\begin{aligned}
& \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} - m, -n, \frac{3}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - \right. \\
& \left. \left(4 d n \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, 1 - n, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \left. \left. (c+d) (-1+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} - m, -n, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) + \\
& \left(20 C (c+d) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - m, -n, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{1+2m} \right. \\
& \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}(-1-2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^3 \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}+m} \left(c+d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^n \right) / \\
& \left(-5 (c+d) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - m, -n, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left(4 d n \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{1}{2} - m, 1 - n, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \left. \left. (c+d) (1+2m) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2} - m, -n, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) + \\
& \left(20 B (c+d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, -n, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \\
& \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}(1-2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^3 \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \left(c+d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^n \right) / \\
& \left(3 \left(-5 (c+d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - m, -n, \frac{5}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \left. \left. \left(4 d n \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2} - m, 1 - n, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. (c+d) (-1+2m) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{3}{2}-m, -n, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \left(14 C (c+d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}-m, -n, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \\
& \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}(1-2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^5 \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \left(c+d - 2d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^n \right) / \\
& \left(5 \left(-7 (c+d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}-m, -n, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) + \right. \\
& \left. \left(4 d n \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1}{2}-m, 1-n, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) + (c+d) (-1+2m) \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[\frac{7}{2}, \frac{3}{2}-m, -n, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) \left(a + a \sin[e + f x] \right)^m
\end{aligned}$$

■ **Problem 26: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c + d \sin[e + f x])^{-2m} (A + B \sin[e + f x] + C \sin[e + f x]^2) dx$$

Optimal (type 6, 410 leaves, 8 steps):

$$\begin{aligned}
& \frac{(c^2 C - B c d + A d^2) \cos[e + f x] (a + a \sin[e + f x])^m (c + d \sin[e + f x])^{-1-m}}{d (c^2 - d^2) f (1+m)} - \\
& \frac{1}{(c-d) d (c+d)^2 f (1+m)} 2^{\frac{1}{2}+m} a (c d (A + C + A m + B m + C m) - c^2 (C + 2 C m) - d^2 (A m + B (1+m) - C (1+m))) \cos[e + f x] \\
& \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2}-m, \frac{3}{2}, \frac{(c-d) (1 - \sin[e + f x])}{2 (c+d \sin[e + f x])} \right] (a + a \sin[e + f x])^{-1+m} \left(\frac{(c+d) (1 + \sin[e + f x])}{c+d \sin[e + f x]} \right)^{\frac{1}{2}-m} (c+d \sin[e + f x])^{-m} + \\
& \left(\sqrt{2} C \operatorname{AppellF1} \left[\frac{3}{2}+m, \frac{1}{2}, 1+m, \frac{5}{2}+m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c-d} \right] \cos[e + f x] \right. \\
& \left. (a + a \sin[e + f x])^{1+m} (c+d \sin[e + f x])^{-m} \left(\frac{c+d \sin[e + f x]}{c-d} \right)^m \right) / (a (c-d) d f (3+2m) \sqrt{1 - \sin[e + f x]})
\end{aligned}$$

Result (type 6, 5581 leaves):

$$\left(2 \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1-2m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] (a + a \sin[e + f x])^m \right)$$

$$\begin{aligned}
& \left(2 A \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} (c + d \operatorname{Sin}[e + f x])^{-2-m} + C \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} (c + d \operatorname{Sin}[e + f x])^{-2-m} + \right. \\
& \quad \left. C \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Cos}\left[2\left(-e + \frac{\pi}{2} - f x\right)\right] (c + d \operatorname{Sin}[e + f x])^{-2-m} + 2 B \operatorname{Cos}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{2m} \operatorname{Sin}[e + f x] (c + d \operatorname{Sin}[e + f x])^{-2-m} \right) \\
& \left(1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-m} \left(c + \frac{d - d \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{-m} \\
& \left(\frac{1}{c-d} \left((c+d) (c^2 C - 2 c C d + (-A+B) d^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \right. \\
& \quad \left. \left. 2 d (c^2 C - B c d + A d^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d} \right)^m + \right. \\
& \quad \left. \left(3 C (c+d)^3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) / \right. \\
& \quad \left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left((c+d) \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) + \right. \right. \\
& \quad \left. \left. (c-d) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) \right) / \right. \\
& \left(d^2 (c+d)^2 f \left(\frac{1}{d^2 (c+d)^2} 4 m \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{-m} \left(-\frac{d \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} - \right. \right. \right. \\
& \quad \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] (d - d \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2)}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^2} \right) \left(c + \frac{d - d \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right)^{-1-m} \right. \\
& \quad \left. \left(\frac{1}{c-d} \left((c+d) (c^2 C - 2 c C d + (-A+B) d^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2d (c^2 C - Bcd + Ad^2) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d} \right] \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d} \right)^m + \\
& \left(3C (c+d)^3 \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right] \right) / \\
& \left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right) \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right] + \right. \right. \\
& 2 \left((c+d) \operatorname{AppellF1} \left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right] + \right. \\
& \left. \left. (c-d) m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right) \Bigg) + \\
& \frac{1}{d^2 (c+d)^2} 4m \operatorname{Sec} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right)^{-1-m} \left(c + \frac{d - d \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2} \right)^{-m} \\
& \left(\frac{1}{c-d} \left((c+d) (c^2 C - 2cCd + (-A+B) d^2) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d} \right] + \right. \right. \\
& 2d (c^2 C - Bcd + Ad^2) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d} \right] \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d} \right)^m + \\
& \left(3C (c+d)^3 \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right] \right) / \\
& \left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right) \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right] + \right. \right. \\
& 2 \left((c+d) \operatorname{AppellF1} \left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right] + \right. \\
& \left. \left. (c-d) m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (-e + \frac{\pi}{2} - fx) \right]^2 \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{d^2 (c+d)^2} 2 \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \left(1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{-m} \left(c + \frac{d - d \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}\right)^{-m} \\
& \left(\frac{1}{c-d} \left((c+d) (c^2 C - 2 c C d + (-A+B) d^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right.\right. \\
& \quad \left.2 d (c^2 C - B c d + A d^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right]\right) \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right)^m \\
& \quad \left(3 C (c+d)^3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right]\right) / \\
& \quad \left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^2 \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.\right. \\
& \quad \left.2 \left((c+d) \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] + \right.\right. \\
& \quad \left.\left.(c-d) m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)\right) \Bigg) - \\
& \frac{1}{d^2 (c+d)^2} 4 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{-m} \left(c + \frac{d - d \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}\right)^{-m} \\
& \left(\frac{1}{c+d} \left((c+d) (c^2 C - 2 c C d + (-A+B) d^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right.\right. \\
& \quad \left.2 d (c^2 C - B c d + A d^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right]\right) \\
& \quad \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right)^{-1+m} - \\
& \quad \left(3 C (c+d)^3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \Big] - 3 (c+d) \left(-\frac{1}{3} \operatorname{AppellF1} \left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \\
& \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{3 (c+d)} (c-d) m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d}, \right. \\
& \left. \left. -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) + 2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left((c+d) \left(-\frac{6}{5} \operatorname{AppellF1} \left[\right. \right. \right. \\
& \left. \left. \left. \frac{5}{2}, m, 3, \frac{7}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \right. \right. \\
& \left. \left. \frac{1}{5 (c+d)} 3 (c-d) m \operatorname{AppellF1} \left[\frac{5}{2}, 1+m, 2, \frac{7}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right. \right. \\
& \left. \left. \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) + (c-d) m \left(-\frac{3}{5} \operatorname{AppellF1} \left[\frac{5}{2}, 1+m, 2, \frac{7}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d}, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{5 (c+d)} 3 (c-d) (1+m) \operatorname{AppellF1} \left[\frac{5}{2}, 2+m, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{7}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) \right) \Big] \Big] \Big] \Big] / \\
& \left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \right. \right. \\
& \left. \left. 2 \left((c+d) \operatorname{AppellF1} \left[\frac{3}{2}, m, 2, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] + \right. \right. \right. \\
& \left. \left. \left. (c-d) m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \Big] \Big] + \\
& \frac{1}{c-d} \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right)^m \left(d (c^2 C - B c d + A d^2) \operatorname{Csc} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right. \\
& \left. \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 2+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \left(1 + \frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right)^{-2-m} \right) \right) \Big] \Big]
\end{aligned}$$

$$\frac{1}{2} (c+d) (c^2 C - 2 c C d + (-A+B) d^2) \operatorname{Csc}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \operatorname{Sec}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, \frac{3}{2}, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \left(1 + \frac{(c-d) \operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right)^{-1-m} \right) \right)$$

■ **Problem 27: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sin[e + f x])^m (c + d \sin[e + f x])^{3/2} (A + B \sin[e + f x] + C \sin[e + f x]^2) dx$$

Optimal (type 6, 406 leaves, 10 steps):

$$\frac{2 C \cos[e + f x] (a + a \sin[e + f x])^m (c + d \sin[e + f x])^{5/2}}{d f (7 + 2 m)} + \left(\sqrt{2} (c-d) (2 c (C + 2 C m) - d (7 B - 5 C + 2 B m + 2 C m - A (7 + 2 m))) \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, -\frac{3}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c-d}\right] \right. \\ \left. \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{c + d \sin[e + f x]} \right) / \left(d f (1 + 2 m) (7 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{\frac{c + d \sin[e + f x]}{c-d}} \right) - \\ \left(\sqrt{2} (c-d) (2 c C (1 + m) - d (2 C m + B (7 + 2 m))) \operatorname{AppellF1}\left[\frac{3}{2} + m, \frac{1}{2}, -\frac{3}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c-d}\right] \right. \\ \left. \cos[e + f x] (a + a \sin[e + f x])^{1+m} \sqrt{c + d \sin[e + f x]} \right) / \left(a d f (3 + 2 m) (7 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{\frac{c + d \sin[e + f x]}{c-d}} \right)$$

Result (type 6, 8472 leaves):

$$\frac{1}{2 f} \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{-2 m} \left(\left(3 C d (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{5}{2} - m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2 d \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^{5+2 m} \right. \right. \\ \left. \left(\cos\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right)^{\frac{1}{2} + \frac{1}{2} (-6-2 m)} \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{\frac{5}{2} + m} \sqrt{c + d - 2 d \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right) / \\ \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{5}{2} - m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2 d \sin\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right.$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^{\frac{1}{2} + \frac{1}{2}(-2-2m)} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right) / \\
& \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \right. \right. \\
& \left. \left. (c+d)(1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) - \\
& \left(12Ac(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-1+2m} \right. \\
& \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right) / \\
& \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] - \right. \\
& \left. \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \right. \right. \\
& \left. \left. (c+d)(-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) - \\
& \left(6cC(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^{-1+2m} \right. \\
& \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2} \right) / \\
& \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(14 B d (c+d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}(1-2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^5 \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \sqrt{c+d-2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} \right) / \\
& \left(5 \left(-7 (c+d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \quad \left. \left(2 d \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \quad \left. \left. (c+d) (-1+2m) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \left(9 C d (c+d) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}(1-2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^7 \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \sqrt{c+d-2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} \right) / \\
& \left(7 \left(-9 (c+d) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \quad \left. \left(2 d \operatorname{AppellF1} \left[\frac{9}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{11}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (-1+2m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{9}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{11}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \right) (a+a \sin [e+f x])^m
\end{aligned}$$

■ **Problem 28: Result more than twice size of optimal antiderivative.**

$$\int (a+a \sin [e+f x])^m \sqrt{c+d \sin [e+f x]} (A+B \sin [e+f x]+C \sin [e+f x]^2) dx$$

Optimal (type 6, 396 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 C \operatorname{Cos}[e+f x] (a+a \operatorname{Sin}[e+f x])^m (c+d \operatorname{Sin}[e+f x])^{3/2}}{d f (5+2 m)} + \\
& \left(\sqrt{2} (2 c (C+2 C m)-d (5 B-3 C+2 B m+2 C m-A (5+2 m))) \operatorname{AppellF1}\left[\frac{1}{2}+m, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}+m, \frac{1}{2} (1+\operatorname{Sin}[e+f x]), -\frac{d (1+\operatorname{Sin}[e+f x])}{c-d}\right] \right. \\
& \left. \operatorname{Cos}[e+f x] (a+a \operatorname{Sin}[e+f x])^m \sqrt{c+d \operatorname{Sin}[e+f x]} \right) / \left(d f (1+2 m) (5+2 m) \sqrt{1-\operatorname{Sin}[e+f x]} \sqrt{\frac{c+d \operatorname{Sin}[e+f x]}{c-d}} \right) - \\
& \left(\sqrt{2} (2 c C (1+m)-d (2 C m+B (5+2 m))) \operatorname{AppellF1}\left[\frac{3}{2}+m, \frac{1}{2}, -\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2} (1+\operatorname{Sin}[e+f x]), -\frac{d (1+\operatorname{Sin}[e+f x])}{c-d}\right] \right. \\
& \left. \operatorname{Cos}[e+f x] (a+a \operatorname{Sin}[e+f x])^{1+m} \sqrt{c+d \operatorname{Sin}[e+f x]} \right) / \left(a d f (3+2 m) (5+2 m) \sqrt{1-\operatorname{Sin}[e+f x]} \sqrt{\frac{c+d \operatorname{Sin}[e+f x]}{c-d}} \right)
\end{aligned}$$

Result (type 6, 3138 leaves):

$$\begin{aligned}
& \frac{1}{2 f} \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{-2 m} \\
& \left(\left(6 C (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}-m, -\frac{1}{2}, \frac{3}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{3+2 m} \right. \right. \\
& \left. \left(\operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right)^{\frac{1}{2}+\frac{1}{2}(-4-2 m)} \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(1-\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{\frac{3}{2}+m} \sqrt{c+d-2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2} \right) / \\
& \left(-3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}-m, -\frac{1}{2}, \frac{3}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \right) + \\
& \left(2 d \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}-m, \frac{1}{2}, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \right) + \\
& \left. (c+d) (3+2 m) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \right) \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) + \\
& \left(12 B (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, \frac{2 d \operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \operatorname{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^{1+2 m} \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{\frac{1}{2} + \frac{1}{2}(-2-2m)} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right) / \\
& \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \right. \\
& \left. \left. (c+d)(1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) - \\
& \left(12A(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-1+2m} \right. \\
& \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right) / \\
& \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - \right. \\
& \left. \left(2d \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \right. \\
& \left. \left. (c+d)(-1+2m) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}-m, -\frac{1}{2}, \frac{5}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) - \\
& \left(6C(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^{-1+2m} \right. \\
& \left. \left(\cos\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{\frac{1}{2}-m} \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \left(1 - \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2\right)^{-\frac{1}{2}+m} \sqrt{c+d - 2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2} \right) / \\
& \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{3}{2}, \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, \frac{2d \sin\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(14 C (c+d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2} - m, -\frac{1}{2}, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1+2m} \right. \\
& \left. \left(\cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{\frac{1}{2}(1-2m)} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^5 \left(1 - \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-\frac{1}{2}+m} \sqrt{c+d - 2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2} \right) / \\
& \left(5 \left(-7 (c+d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2} - m, -\frac{1}{2}, \frac{7}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \right. \\
& \left. \left(2 d \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1}{2} - m, \frac{1}{2}, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (-1+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{7}{2}, \frac{3}{2} - m, -\frac{1}{2}, \frac{9}{2}, \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \frac{2 d \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) \left(a + a \sin[e + f x] \right)^m
\end{aligned}$$

■ **Problem 29: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m (A + B \sin[e + f x] + C \sin[e + f x]^2)}{\sqrt{c + d \sin[e + f x]}} dx$$

Optimal (type 6, 389 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 C \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{c + d \sin[e + f x]}}{d f (3 + 2 m)} + \\
& \left(\sqrt{2} (2 c (C + 2 C m) - d (3 B - C + 2 B m + 2 C m - A (3 + 2 m))) \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \right. \\
& \left. \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right) / \left(d f (1 + 2 m) (3 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) - \\
& \left(\sqrt{2} (2 c C (1 + m) - d (2 C m + B (3 + 2 m))) \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \right. \\
& \left. \cos[e + f x] (a + a \sin[e + f x])^{1+m} \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right) / \left(a d f (3 + 2 m)^2 \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right)
\end{aligned}$$

Result (type 6, 11893 leaves) :

$$\begin{aligned}
& - \left(\left(2 (c+d) \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1-2m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] (a + a \sin [e + f x])^m \right. \right. \\
& \left. \left(- \frac{2 A \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m}}{\sqrt{c+d} \sin [e + f x]} - \frac{C \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m}}{\sqrt{c+d} \sin [e + f x]} - \frac{C \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m} \cos \left[2 \left(-e + \frac{\pi}{2} - f x \right) \right]}{\sqrt{c+d} \sin [e + f x]} - \right. \right. \\
& \left. \left. \frac{2 B \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m} \sin [e + f x]}{\sqrt{c+d} \sin [e + f x]} \right) \left(1 + \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^{-2m} \sqrt{\frac{c+d+c \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1 + \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right. \\
& \left. \left(\left(9 (A (c-3d) + C (-3c+d) + B (c+d)) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \right. \right. \\
& \left. \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) + \right. \\
& \left. \left((c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) - \right. \\
& \left. \left. (c+d) (5+2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \right) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) - \\
& \left(36 (c^2 C - B c d + A d^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \\
& \left((c+d+c \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2) \right. \\
& \left. \left(-3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) + \right. \\
& \left. \left((c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) + (c+d) (5+2m) \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{AppellF1} \left[\frac{3}{2}, \frac{7}{2} + m, \frac{1}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) + \\
& \left(5 (A - B + C) (c - d) \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \\
& \left(5 (c + d) \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left((c - d) \text{AppellF1} \left[\frac{5}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{7}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - (c + d) (5 + 2 m) \right. \right. \\
& \left. \left. \text{AppellF1} \left[\frac{5}{2}, \frac{7}{2} + m, -\frac{1}{2}, \frac{7}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) / \\
& \left(3 (c - d)^2 f \left(-\frac{1}{3 (c - d)^2} 4 (c + d) (-2 - m) \text{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(1 + \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-3 - m} \right. \right. \\
& \left. \left. \sqrt{\frac{c + d + c \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1 + \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right) \right) / \\
& \left(9 (A (c - 3 d) + C (-3 c + d) + B (c + d)) \text{AppellF1} \left[\frac{1}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{3}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \\
& \left(3 (c + d) \text{AppellF1} \left[\frac{1}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{3}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left((c - d) \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{5}{2}, -\text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \text{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - (c + d) (5 + 2 m) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) - \\
& \left(36(c^2C - Bcd + Ad^2) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) / \\
& \left(\left(c+d+c\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - d\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right. \\
& \left. \left(-3(c+d) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) + \right. \\
& \left. \left((c-d) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) + (c+d)(5+2m) \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) + \\
& \left(5(A-B+C)(c-d) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \\
& \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) / \left(5(c+d) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) + \\
& \left((c-d) \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) - (c+d)(5+2m) \\
& \left. \left. \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) - \\
& \frac{1}{3(c-d)^2} 2(c+d) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(1 + \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{-2-m} \sqrt{\frac{c+d+c\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - d\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{1 + \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}} \\
& \left(\left(9(A(c-3d) + C(-3c+d) + B(c+d)) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3(c-d)^2 \sqrt{\frac{c+d+c \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2-d \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}}} 2(c+d) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^{-2-m} \\
& \left(\frac{c \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]-d \operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]}{1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2} - \left(\operatorname{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^2 \right. \\
& \quad \left. \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(c+d+c \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2-d \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) / \left(1+\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right)^2 \right) \\
& \left(\left(9(A(c-3d)+C(-3c+d)+B(c+d)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \right) / \right. \\
& \quad \left(3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + \right. \\
& \quad \left. \left((c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] - (c+d)(5+2m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \right) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) - \right. \\
& \quad \left. \left(36(c^2 C - B c d + A d^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \right) / \right. \\
& \quad \left(c+d+c \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2-d \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) \\
& \quad \left(-3(c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + \right. \\
& \quad \left. \left((c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + (c+d)(5+2m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \right) \tan\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(5 (A - B + C) (c - d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right. \\
& \left. \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \left(5 (c + d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right. + \\
& \left((c - d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{7}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] - (c + d) (5 + 2 m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{5}{2}, \frac{7}{2} + m, -\frac{1}{2}, \frac{7}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \frac{1}{3 (c - d)^2} 4 (c + d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \left(1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-2 - m} \sqrt{\frac{c + d + c \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \\
& \left(\left(9 (A (c - 3 d) + C (-3 c + d) + B (c + d)) \left(\frac{1}{6 (c + d)} (c - d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] - \frac{1}{3} \left(\frac{5}{2} + m \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{2} + m, -\frac{1}{2}, \right. \right. \right. \\
& \left. \left. \left. \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right) \right) / \\
& \left(3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right. + \\
& \left((c - d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] - (c + d) (5 + 2 m) \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{2} + m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) + \\
& \left(36 (c^2 C - B c d + A d^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\left]\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right] + \\
& \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\left((c-d)\left(-\frac{1}{10(c+d)}{}_3F_3\left[\frac{5}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{7}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right.\right.\right. \\
& \quad \left.\left.\left.-\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right]\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{3}{5}\left(\frac{5}{2}+m\right)\text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, \frac{1}{2}, \right.\right.\right. \\
& \quad \left.\left.\frac{7}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right]\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)\right) - \\
& (c+d)(5+2m)\left(\frac{1}{10(c+d)}{}_3F_3\left[\frac{5}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{7}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right.\right. \\
& \quad \left.\left.-\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right]\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{3}{5}\left(\frac{7}{2}+m\right)\text{AppellF1}\left[\frac{5}{2}, \frac{9}{2}+m, -\frac{1}{2}, \right.\right. \\
& \quad \left.\left.\frac{7}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right]\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)\right)\right) / \\
& \left(3(c+d)\text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left((c-d)\text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] - (c+d)(5+2m)\right.\right. \\
& \quad \left.\left.\text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right]\right)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)^2 + \\
& \left. 36(c^2c - Bcd + Ad^2)\text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{3}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) \\
& \left(\left((c-d)\text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + (c+d)(5+2m)\text{AppellF1}\left[\frac{3}{2}, \right.\right.\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{7}{2} + m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \\
& 3(c+d) \left(-\frac{1}{6(c+d)}(c-d)\operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2} + m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right) \\
& \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{1}{3}\left(\frac{5}{2} + m\right)\operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2} + m, \frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, \\
& -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \\
& \left((c-d) \left(-\frac{1}{10(c+d)}9(c-d)\operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2} + m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right) \right. \\
& \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{3}{5}\left(\frac{5}{2} + m\right)\operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2} + m, \frac{3}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, \\
& \left. -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) + (c+d)(5+2m) \\
& \left(-\frac{1}{10(c+d)}3(c-d)\operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2} + m, \frac{3}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right) \\
& \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] - \frac{3}{5}\left(\frac{7}{2} + m\right)\operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2} + m, \frac{1}{2}, \frac{7}{2}, \right. \\
& \left. -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right] \right) \right) \Bigg) / \\
& \left((c+d + c\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 - d\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2) \left(-3(c+d)\operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2} + m, \frac{1}{2}, \frac{3}{2}, \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right) \right) + \\
& \left((c-d)\operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2} + m, \frac{3}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right) + (c+d)(5+2m) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^2 - \\
& \left(5(A-B+C)(c-d) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right. \\
& \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left(\left((c-d) \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] - (c+d) \right. \right. \\
& \left. \left. (5+2m) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right. \\
& \left. \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] + 5(c+d) \left(\frac{1}{10(c+d)} 3(c-d) \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \right. \\
& \left. \left. - \frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{3}{5}\left(\frac{5}{2}+m\right) \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, \right. \right. \\
& \left. \left. - \frac{1}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) + \\
& \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \left((c-d) \left(-\frac{1}{14(c+d)} 5(c-d) \text{AppellF1}\left[\frac{7}{2}, \frac{5}{2}+m, \frac{3}{2}, \frac{9}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \right. \\
& \left. \left. - \frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{5}{7}\left(\frac{5}{2}+m\right) \text{AppellF1}\left[\frac{7}{2}, \frac{7}{2}+m, \frac{1}{2}, \right. \right. \\
& \left. \left. \frac{9}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) - \\
& (c+d)(5+2m) \left(\frac{1}{14(c+d)} 5(c-d) \text{AppellF1}\left[\frac{7}{2}, \frac{7}{2}+m, \frac{1}{2}, \frac{9}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, \right. \right. \\
& \left. \left. - \frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] - \frac{5}{7}\left(\frac{7}{2}+m\right) \text{AppellF1}\left[\frac{7}{2}, \frac{9}{2}+m, -\frac{1}{2}, \right. \right. \\
& \left. \left. \frac{9}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d} \right) \text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \right) \right) \Bigg) /
\end{aligned}$$

$$\left(5 (c+d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2}+m, -\frac{1}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \\ \left. \left((c-d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] - (c+d)(5+2m) \right. \right. \\ \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}+m, -\frac{1}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) \right)$$

■ **Problem 30: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + fx])^m (A + B \sin[e + fx] + C \sin[e + fx]^2)}{(c + d \sin[e + fx])^{3/2}} dx$$

Optimal (type 6, 433 leaves, 10 steps):

$$\frac{2 (c^2 C - B c d + A d^2) \cos[e + fx] (a + a \sin[e + fx])^m}{d (c^2 - d^2) f \sqrt{c + d \sin[e + fx]}}$$

$$\left[\sqrt{2} (d^2 (A + B - C + 4 A m) - c d (A + B + C + 4 B m) + 2 c^2 (C + 2 C m)) \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + fx]), -\frac{d (1 + \sin[e + fx])}{c - d}\right] \right]$$

$$\cos[e + fx] (a + a \sin[e + fx])^m \sqrt{\frac{c + d \sin[e + fx]}{c - d}} / \left(d (c^2 - d^2) f (1 + 2 m) \sqrt{1 - \sin[e + fx]} \sqrt{c + d \sin[e + fx]} \right) -$$

$$\left[\sqrt{2} (d (B c - A d) (1 + 2 m) + C (d^2 - 2 c^2 (1 + m))) \operatorname{AppellF1}\left[\frac{3}{2} + m, \frac{1}{2}, \frac{1}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + fx]), -\frac{d (1 + \sin[e + fx])}{c - d}\right] \right]$$

$$\cos[e + fx] (a + a \sin[e + fx])^{1+m} \sqrt{\frac{c + d \sin[e + fx]}{c - d}} / \left(a d (c^2 - d^2) f (3 + 2 m) \sqrt{1 - \sin[e + fx]} \sqrt{c + d \sin[e + fx]} \right)$$

Result (type 6, 31436 leaves): Display of huge result suppressed!

■ **Problem 31: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sin[e + f x])^m (A + B \sin[e + f x] + C \sin[e + f x]^2)}{(c + d \sin[e + f x])^{5/2}} dx$$

Optimal (type 6, 451 leaves, 10 steps):

$$\frac{2 (c^2 C - B c d + A d^2) \cos[e + f x] (a + a \sin[e + f x])^m}{3 d (c^2 - d^2) f (c + d \sin[e + f x])^{3/2}} + \left(\sqrt{2} (d^2 (A - 3 B + 3 C - 4 A m) + c d (3 A - B + 3 C + 4 B m) - 2 c^2 (C + 2 C m)) \right. \\ \left. \operatorname{AppellF1} \left[\frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \frac{3}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \cos[e + f x] (a + a \sin[e + f x])^m \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right) / \\ \left(3 (c - d)^2 d (c + d) f (1 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right) + \\ \left(\sqrt{2} (B c d (1 - 2 m) + 2 c^2 C (1 + m) - d^2 (A + 3 C - 2 A m)) \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, \frac{3}{2}, \frac{5}{2} + m, \frac{1}{2} (1 + \sin[e + f x]), -\frac{d (1 + \sin[e + f x])}{c - d} \right] \right. \\ \left. \cos[e + f x] (a + a \sin[e + f x])^{1+m} \sqrt{\frac{c + d \sin[e + f x]}{c - d}} \right) / \left(3 a (c - d)^2 d (c + d) f (3 + 2 m) \sqrt{1 - \sin[e + f x]} \sqrt{c + d \sin[e + f x]} \right)$$

Result (type 6, 12922 leaves):

$$- \left(\left(2 (c + d) \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{-1-2m} \sin \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] (a + a \sin[e + f x])^m \right. \right. \\ \left. \left(-\frac{2 A \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m}}{(c + d \sin[e + f x])^{5/2}} - \frac{C \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m}}{(c + d \sin[e + f x])^{5/2}} - \frac{C \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m} \cos \left[2 \left(-e + \frac{\pi}{2} - f x \right) \right]}{(c + d \sin[e + f x])^{5/2}} - \right. \right. \\ \left. \left. \frac{2 B \cos \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^{2m} \sin[e + f x]}{(c + d \sin[e + f x])^{5/2}} \right) \left(1 + \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right] \right)^{-m} \sqrt{\frac{c + d + c \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1 + \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \right. \\ \left. \left(\left(45 (A + B + C) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} + m, \frac{5}{2}, \frac{3}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) \right) / \right. \\ \left. \left(3 (c + d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} + m, \frac{5}{2}, \frac{3}{2}, -\tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c - d) \tan \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c + d} \right] \right) - \right.$$

$$\begin{aligned}
& \left(1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right)^{-m} \sqrt{\frac{c + d + c \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 - d \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}} \\
& \left(\left(45 (A + B + C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{5}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) / \\
& \left(3 (c+d) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{5}{2}, \frac{3}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] - \right. \\
& \left. \left(5 (c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + m, \frac{7}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + (c+d) (1+2m) \right. \\
& \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2} + m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) + \\
& \left(50 (-A + C) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) / \\
& \left(-5 (c+d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left(5 (c-d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2} + m, \frac{7}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + (c+d) (1+2m) \right. \\
& \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2} + m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2 \right) - \\
& \left(21 (A - B + C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2} + m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^4 \right) / \\
& \left(-7 (c+d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2} + m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left(5 (c-d) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2} + m, \frac{7}{2}, \frac{9}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]\right]^2, -\frac{(c-d) \operatorname{Tan}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - f x\right)\right]^2}{c+d}\right] + (c+d) (1+2m) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{AppellF1}\left[\frac{7}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{9}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) + \\
& \frac{1}{15\left(c+d+c\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2-d\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^3} 4(c+d)m\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \\
& \left(1+\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2\right)^{-1-m} \sqrt{\frac{c+d+c\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2-d\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{1+\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}} \\
& \left(\left(45(A+B+C)\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right)/\right. \\
& \left. \left(3(c+d)\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) - \right. \\
& \left. \left(5(c-d)\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) + (c+d)(1+2m) \right. \\
& \left. \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) + \\
& \left(50(-A+C)\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) / \\
& \left(-5(c+d)\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) + \\
& \left(5(c-d)\text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) + (c+d)(1+2m) \\
& \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \Bigg) - \\
& \left(21(A-B+C)\text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right]\right) \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& \left(-7 (c+d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left(5 (c-d) \operatorname{AppellF1} \left[\frac{7}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{9}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (1+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{7}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{9}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) - \\
& \frac{1}{15 \left(c+d+c \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^3} 2 (c+d) \operatorname{Sec} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right)^{-m} \\
& \sqrt{\frac{c+d+c \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 - d \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}} \\
& \left(\left(45 (A+B+C) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) / \right. \\
& \left. \left(3 (c+d) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{3}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] - \right. \right. \\
& \left. \left. \left(5 (c-d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (1+2m) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \right) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) \right) + \\
& \left(50 (-A+C) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2 \right) / \\
& \left(-5 (c+d) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + \right. \\
& \left. \left(5 (c-d) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{7}{2}, -\operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2, -\frac{(c-d) \operatorname{Tan} \left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right) \right]^2}{c+d} \right] + (c+d) (1+2m) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 - \\
& \left(21(A-B+C)\text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4\right) / \\
& \left(-7(c+d)\text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{7}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left(5(c-d)\text{AppellF1}\left[\frac{7}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{9}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + (c+d)(1+2m)\right.\right. \\
& \left.\left.\text{AppellF1}\left[\frac{7}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{9}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)\right) - \\
& \left(2(c+d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] \left(1+\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^{-m} \left(\left(c\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right] - d\right.\right.\right. \\
& \left.\left.\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right) / \left(1+\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) - \left(\text{Sec}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]\right.\right. \\
& \left.\left.\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right) \left(c+d+c\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 - d\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) / \left(1+\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right)^2\right) \\
& \left(\left(45(A+B+C)\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right]\right) / \right. \\
& \left. \left(3(c+d)\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{3}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] - \right.\right. \\
& \left. \left. \left(5(c-d)\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] + (c+d)(1+2m)\right.\right.\right. \\
& \left.\left.\left.\text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) + \right. \\
& \left.\left. \left(50(-A+C)\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{5}{2}, -\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2, -\frac{(c-d)\text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{c+d}\right] \text{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-5(c+d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left(5(c-d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + (c+d)(1+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) - \\
& \left(21(A-B+C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^4 \right) / \\
& \left(-7(c+d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{7}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + \right. \\
& \left. \left(5(c-d) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{9}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] + (c+d)(1+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{9}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right) \right) / \\
& \left(15 \left(c+d+c \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - d \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^3 \sqrt{\frac{c+d+c \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - d \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}} \right) - \\
& \frac{1}{15 \left(c+d+c \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - d \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^3} \\
& 4(c+d) \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 \right)^{-m} \\
& \sqrt{\frac{c+d+c \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2 - d \operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}} \\
& \left(\left(45(A+B+C) \left(-\frac{1}{6(c+d)} 5(c-d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{5}{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2, -\frac{(c-d)\operatorname{Tan}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right]^2}{c+d}\right] \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. \frac{9}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right) \right) \right) \Big/ \\
& \left(-5(c+d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{5}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) + \right. \\
& \left(5(c-d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{7}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) + (c+d)(1+2m) \right. \\
& \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{7}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right)^2 + \\
& \left(21(A-B+C) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{7}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^4 \right. \\
& \left(\left(5(c-d) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{9}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) + (c+d)(1+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{9}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \right) \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \right) \right. \\
& \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] - 7(c+d) \left(-\frac{1}{14(c+d)} 25(c-d) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{9}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \\
& \left. \left. -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{5}{7}\left(\frac{1}{2}+m\right) \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}+m, \frac{5}{2}, \right. \right. \\
& \left. \left. \frac{9}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] \right) \right) \Big/ \\
& \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \left(5(c-d) \left(-\frac{1}{18(c+d)} 49(c-d) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{1}{2}+m, \frac{9}{2}, \frac{11}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, \right. \right. \right. \\
& \left. \left. -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] \operatorname{Sec}\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2 \tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{7}{9}\left(\frac{1}{2}+m\right) \operatorname{AppellF1}\left[\frac{9}{2}, \frac{3}{2}+m, \frac{7}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{11}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\left]\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right) + \\
& (c+d)(1+2m)\left(-\frac{1}{18(c+d)}35(c-d)\operatorname{AppellF1}\left[\frac{9}{2}, \frac{3}{2}+m, \frac{7}{2}, \frac{11}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, \right. \\
& \quad \left.-\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right]\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right] - \frac{7}{9}\left(\frac{3}{2}+m\right)\operatorname{AppellF1}\left[\frac{9}{2}, \frac{5}{2}+m, \frac{5}{2}, \right. \\
& \quad \left.\frac{11}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right]\sec\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right)\right)\right) / \\
& \left(-7(c+d)\operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{7}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + \right. \\
& \quad \left. \left(5(c-d)\operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2}+m, \frac{7}{2}, \frac{9}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right] + (c+d)(1+2m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{7}{2}, \frac{3}{2}+m, \frac{5}{2}, \frac{9}{2}, -\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]\right]^2, -\frac{(c-d)\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2}{c+d}\right]\tan\left[\frac{1}{2}\left(-e + \frac{\pi}{2} - fx\right)\right]^2\right)\right)\right)
\end{aligned}$$

Test results for the 594 problems in "4.1.7 (d trig)^m (a+b (c sin)^n)^p.m"

- Problem 40: Result more than twice size of optimal antiderivative.

$$\int \frac{\csc[c+dx]^3}{a-a\sin[c+dx]^2} dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$-\frac{3\operatorname{ArcTanh}[\cos[c+dx]]}{2ad} + \frac{3\sec[c+dx]}{2ad} - \frac{\csc[c+dx]^2\sec[c+dx]}{2ad}$$

Result (type 3, 146 leaves):

$$\begin{aligned}
& \left(\csc[c+dx]^4\left(2-6\cos[2(c+dx)]+2\cos[3(c+dx)]+3\cos[3(c+dx)]\log\left[\cos\left[\frac{1}{2}(c+dx)\right]\right]-3\cos[3(c+dx)]\log\left[\sin\left[\frac{1}{2}(c+dx)\right]\right]\right) + \right. \\
& \quad \left.\cos[c+dx]\left(-2-3\log\left[\cos\left[\frac{1}{2}(c+dx)\right]\right]+3\log\left[\sin\left[\frac{1}{2}(c+dx)\right]\right]\right)\right) / \left(2ad\left(\csc\left[\frac{1}{2}(c+dx)\right]^2-\sec\left[\frac{1}{2}(c+dx)\right]^2\right)\right)
\end{aligned}$$

■ **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + dx]^5}{a - a \text{Sin}[c + dx]^2} dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$-\frac{15 \text{ArcTanh}[\text{Cos}[c + dx]]}{8 a d} + \frac{15 \text{Sec}[c + dx]}{8 a d} - \frac{5 \text{Csc}[c + dx]^2 \text{Sec}[c + dx]}{8 a d} - \frac{\text{Csc}[c + dx]^4 \text{Sec}[c + dx]}{4 a d}$$

Result (type 3, 194 leaves):

$$\frac{1}{a} \left(-\frac{7 \text{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{32 d} - \frac{\text{Csc}\left[\frac{1}{2}(c + dx)\right]^4}{64 d} - \frac{15 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \frac{15 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{8 d} + \frac{7 \text{Sec}\left[\frac{1}{2}(c + dx)\right]^2}{32 d} + \frac{\text{Sec}\left[\frac{1}{2}(c + dx)\right]^4}{64 d} + \frac{\text{Sin}\left[\frac{1}{2}(c + dx)\right]}{d \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} - \frac{\text{Sin}\left[\frac{1}{2}(c + dx)\right]}{d \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} \right)$$

■ **Problem 54: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + dx]^3}{(a - a \text{Sin}[c + dx]^2)^2} dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$-\frac{5 \text{ArcTanh}[\text{Cos}[c + dx]]}{2 a^2 d} + \frac{5 \text{Sec}[c + dx]}{2 a^2 d} + \frac{5 \text{Sec}[c + dx]^3}{6 a^2 d} - \frac{\text{Csc}[c + dx]^2 \text{Sec}[c + dx]^3}{2 a^2 d}$$

Result (type 3, 208 leaves):

$$\frac{1}{3 a^2 d \left(\text{Csc}\left[\frac{1}{2}(c + dx)\right]^2 - \text{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \right)^3} \left(2 \text{Csc}[c + dx]^8 \left(22 - 40 \text{Cos}[2(c + dx)] + 13 \text{Cos}[3(c + dx)] - 30 \text{Cos}[4(c + dx)] + 13 \text{Cos}[5(c + dx)] + 15 \text{Cos}[3(c + dx)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] + 15 \text{Cos}[5(c + dx)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] - 15 \text{Cos}[3(c + dx)] \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - 15 \text{Cos}[5(c + dx)] \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \text{Cos}[c + dx] \left(-26 - 30 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right]\right] + 30 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) \right) \right)$$

■ **Problem 66: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c + dx] (a + b \text{Sin}[c + dx]^2) dx$$

Optimal (type 3, 26 leaves, 2 steps):

$$-\frac{a \operatorname{ArcTanh}[\cos[c + dx]]}{d} - \frac{b \cos[c + dx]}{d}$$

Result (type 3, 63 leaves):

$$-\frac{b \cos[c] \cos[dx]}{d} - \frac{a \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \operatorname{Log}\left[\sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{b \sin[c] \sin[dx]}{d}$$

■ **Problem 67: Result more than twice size of optimal antiderivative.**

$$\int \csc[c + dx]^3 (a + b \sin[c + dx]^2) dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$-\frac{(a + 2b) \operatorname{ArcTanh}[\cos[c + dx]]}{2d} - \frac{a \operatorname{Cot}[c + dx] \csc[c + dx]}{2d}$$

Result (type 3, 118 leaves):

$$-\frac{a \csc\left[\frac{1}{2}(c + dx)\right]^2}{8d} - \frac{b \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} - \frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \frac{b \operatorname{Log}\left[\sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2}{8d}$$

■ **Problem 76: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + b \sin[x]^2)^3 dx$$

Optimal (type 3, 87 leaves, 2 steps):

$$\frac{1}{16} (2a + b) (8a^2 + 8ab + 5b^2) x - \frac{1}{48} b (64a^2 + 54ab + 15b^2) \cos[x] \sin[x] - \frac{5}{24} b^2 (2a + b) \cos[x] \sin[x]^3 - \frac{1}{6} b \cos[x] \sin[x] (a + b \sin[x]^2)^2$$

Result (type 3, 80 leaves):

$$\frac{1}{192} (12(2a + b) (8a^2 + 8ab + 5b^2) x + 9ib(4ia + (1 + 2i)b) (4a + (2 + i)b) \sin[2x] + 9b^2(2a + b) \sin[4x] - b^3 \sin[6x])$$

■ **Problem 78: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]^7}{a + b \sin[c + dx]^2} dx$$

Optimal (type 3, 106 leaves, 4 steps):

$$\frac{a^3 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \cos[c + dx]}{\sqrt{a + b}}\right]}{b^{7/2} \sqrt{a + b} d} - \frac{(a^2 - ab + b^2) \cos[c + dx]}{b^3 d} - \frac{(a - 2b) \cos[c + dx]^3}{3b^2 d} - \frac{\cos[c + dx]^5}{5bd}$$

Result (type 3, 180 leaves):

$$\frac{1}{240 \sqrt{-a-b} b^{7/2} d} \left(-240 a^3 \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{-a-b}} \right] - 240 a^3 \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{-a-b}} \right] - \right. \\ \left. 2 \sqrt{-a-b} \sqrt{b} \operatorname{Cos}[c + dx] (120 a^2 - 100 a b + 89 b^2 + 4 (5 a - 7 b) b \operatorname{Cos}[2 (c + dx)] + 3 b^2 \operatorname{Cos}[4 (c + dx)]) \right)$$

- **Problem 79: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[c + dx]^5}{a + b \operatorname{Sin}[c + dx]^2} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$- \frac{a^2 \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Cos}[c + dx]}{\sqrt{a+b}} \right]}{b^{5/2} \sqrt{a+b} d} + \frac{(a-b) \operatorname{Cos}[c + dx]}{b^2 d} + \frac{\operatorname{Cos}[c + dx]^3}{3 b d}$$

Result (type 3, 150 leaves):

$$\frac{1}{6 \sqrt{-a-b} b^{5/2} d} \left(6 a^2 \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{-a-b}} \right] + 6 a^2 \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{-a-b}} \right] + \sqrt{-a-b} \sqrt{b} \operatorname{Cos}[c + dx] (6 a - 5 b + b \operatorname{Cos}[2 (c + dx)]) \right)$$

- **Problem 80: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c + dx]^3}{a + b \operatorname{Sin}[c + dx]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$\frac{a \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Cos}[c + dx]}{\sqrt{a+b}} \right]}{b^{3/2} \sqrt{a+b} d} - \frac{\operatorname{Cos}[c + dx]}{b d}$$

Result (type 3, 125 leaves):

$$- \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{-a-b}} \right] + a \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{-a-b}} \right] + \sqrt{-a-b} \sqrt{b} \operatorname{Cos}[c + dx]}{\sqrt{-a-b} b^{3/2} d}$$

- **Problem 81: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c + dx]}{a + b \operatorname{Sin}[c + dx]^2} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{b} \cos[c+dx]}{\sqrt{a+b}}\right]}{\sqrt{b} \sqrt{a+b} d}$$

Result (type 3, 97 leaves):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right] + \text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{\sqrt{-a-b}\sqrt{b}d}$$

- **Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c+dx]}{a+b\sin[c+dx]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[\cos[c+dx]]}{ad} + \frac{\sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \cos[c+dx]}{\sqrt{a+b}}\right]}{a\sqrt{a+b}d}$$

Result (type 3, 143 leaves):

$$-\frac{1}{ad} \left(\frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{\sqrt{-a-b}} + \frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{\sqrt{-a-b}} + \text{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] - \text{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] \right)$$

- **Problem 83: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c+dx]^3}{a+b\sin[c+dx]^2} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$-\frac{(a-2b) \text{ArcTanh}[\cos[c+dx]]}{2a^2d} - \frac{b^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \cos[c+dx]}{\sqrt{a+b}}\right]}{a^2\sqrt{a+b}d} - \frac{\cot[c+dx] \csc[c+dx]}{2ad}$$

Result (type 3, 224 leaves):

$$- \left((2a + b - b \cos[2(c + dx)]) \operatorname{Csc}[c + dx]^2 \right. \\ \left. - 8b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} - i\sqrt{a} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a-b}}\right] - 8b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} + i\sqrt{a} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a-b}}\right] + \sqrt{-a-b} \left(a \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2 + \right. \right. \\ \left. \left. 4(a - 2b) \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right]\right) - a \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) / \left(16a^2 \sqrt{-a-b} d (b + a \operatorname{Csc}[c + dx]^2)\right)$$

■ **Problem 84: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c + dx]^5}{a + b \sin[c + dx]^2} dx$$

Optimal (type 3, 125 leaves, 6 steps):

$$-\frac{(3a^2 - 4ab + 8b^2) \operatorname{ArcTanh}[\cos[c + dx]]}{8a^3 d} + \frac{b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \cos[c + dx]}{\sqrt{a+b}}\right]}{a^3 \sqrt{a+b} d} - \frac{(3a - 4b) \cot[c + dx] \operatorname{Csc}[c + dx]}{8a^2 d} - \frac{\cot[c + dx] \operatorname{Csc}[c + dx]^3}{4ad}$$

Result (type 3, 657 leaves):

$$\frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] (\sqrt{b} \cos\left[\frac{1}{2}(c + dx)\right] - i\sqrt{a} \sin\left[\frac{1}{2}(c + dx)\right])}{\sqrt{-a-b}}\right] (-2a - b + b \cos[2(c + dx)]) \operatorname{Csc}[c + dx]^2}{2a^3 \sqrt{-a-b} d (b + a \operatorname{Csc}[c + dx]^2)} + \\ \frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] (\sqrt{b} \cos\left[\frac{1}{2}(c + dx)\right] + i\sqrt{a} \sin\left[\frac{1}{2}(c + dx)\right])}{\sqrt{-a-b}}\right] (-2a - b + b \cos[2(c + dx)]) \operatorname{Csc}[c + dx]^2}{2a^3 \sqrt{-a-b} d (b + a \operatorname{Csc}[c + dx]^2)} + \\ \frac{(3a - 4b) (-2a - b + b \cos[2(c + dx)]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Csc}[c + dx]^2}{64a^2 d (b + a \operatorname{Csc}[c + dx]^2)} + \frac{(-2a - b + b \cos[2(c + dx)]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^4 \operatorname{Csc}[c + dx]^2}{128ad (b + a \operatorname{Csc}[c + dx]^2)} + \\ \frac{(3a^2 - 4ab + 8b^2) (-2a - b + b \cos[2(c + dx)]) \operatorname{Csc}[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right]}{16a^3 d (b + a \operatorname{Csc}[c + dx]^2)} + \\ \frac{(-3a^2 + 4ab - 8b^2) (-2a - b + b \cos[2(c + dx)]) \operatorname{Csc}[c + dx]^2 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right]}{16a^3 d (b + a \operatorname{Csc}[c + dx]^2)} + \\ \frac{(-3a + 4b) (-2a - b + b \cos[2(c + dx)]) \operatorname{Csc}[c + dx]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2}{64a^2 d (b + a \operatorname{Csc}[c + dx]^2)} - \frac{(-2a - b + b \cos[2(c + dx)]) \operatorname{Csc}[c + dx]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4}{128ad (b + a \operatorname{Csc}[c + dx]^2)}$$

■ **Problem 94: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c+dx]^7}{(a+b\sin[c+dx]^2)^2} dx$$

Optimal (type 3, 128 leaves, 5 steps):

$$-\frac{a^2(5a+6b)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\cos[c+dx]}{\sqrt{a+b}}\right]}{2b^{7/2}(a+b)^{3/2}d} + \frac{(2a-b)\cos[c+dx]}{b^3d} + \frac{\cos[c+dx]^3}{3b^2d} + \frac{a^3\cos[c+dx]}{2b^3(a+b)d(a+b-b\cos[c+dx]^2)}$$

Result (type 3, 194 leaves):

$$\frac{1}{12b^{7/2}d} \left(\frac{6a^2(5a+6b)\operatorname{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{(-a-b)^{3/2}} - \frac{6a^2(5a+6b)\operatorname{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{(-a-b)^{3/2}} + \sqrt{b} \left(\cos[c+dx] \left(24a-9b + \frac{12a^3}{(a+b)(2a+b-b\cos[2(c+dx)])} \right) + b\cos[3(c+dx)] \right) \right)$$

■ **Problem 95: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c+dx]^5}{(a+b\sin[c+dx]^2)^2} dx$$

Optimal (type 3, 102 leaves, 5 steps):

$$\frac{a(3a+4b)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\cos[c+dx]}{\sqrt{a+b}}\right]}{2b^{5/2}(a+b)^{3/2}d} - \frac{\cos[c+dx]}{b^2d} - \frac{a^2\cos[c+dx]}{2b^2(a+b)d(a+b-b\cos[c+dx]^2)}$$

Result (type 3, 172 leaves):

$$\frac{1}{2b^{5/2}d} \left(\frac{a(3a+4b)\operatorname{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{(-a-b)^{3/2}} + \frac{a(3a+4b)\operatorname{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{(-a-b)^{3/2}} + 2\sqrt{b}\cos[c+dx] \left(-1 - \frac{a^2}{(a+b)(2a+b-b\cos[2(c+dx)])} \right) \right)$$

- **Problem 96: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c + dx]^3}{(a + b \sin[c + dx]^2)^2} dx$$

Optimal (type 3, 83 leaves, 3 steps):

$$-\frac{(a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \cos[c + dx]}{\sqrt{a+b}}\right]}{2b^{3/2} (a+b)^{3/2} d} + \frac{a \cos[c + dx]}{2b (a+b) d (a + b - b \cos[c + dx]^2)}$$

Result (type 3, 160 leaves):

$$\frac{(a+2b) \operatorname{ArcTan}\left[\frac{\sqrt{-b-i\sqrt{a}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{\sqrt{-a-b}} + \frac{(a+2b) \operatorname{ArcTan}\left[\frac{\sqrt{-b+i\sqrt{a}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{\sqrt{-a-b}} + \frac{2a\sqrt{b} \cos[c+dx]}{2a+b-b \cos[2(c+dx)]}$$

$$2b^{3/2} (a+b) d$$

- **Problem 97: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[c + dx]}{(a + b \sin[c + dx]^2)^2} dx$$

Optimal (type 3, 74 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \cos[c + dx]}{\sqrt{a+b}}\right]}{2\sqrt{b} (a+b)^{3/2} d} - \frac{\cos[c + dx]}{2(a+b) d (a + b - b \cos[c + dx]^2)}$$

Result (type 3, 149 leaves):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{-b-i\sqrt{a}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{\sqrt{-a-b} \sqrt{b}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{-b+i\sqrt{a}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{\sqrt{-a-b} \sqrt{b}} - \frac{2 \cos[c + dx]}{2a+b-b \cos[2(c+dx)]}$$

$$2(a+b) d$$

- **Problem 98: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Csc}[c + dx]}{(a + b \sin[c + dx]^2)^2} dx$$

Optimal (type 3, 103 leaves, 5 steps):

$$-\frac{\operatorname{ArcTanh}[\cos[c + dx]]}{a^2 d} + \frac{\sqrt{b} (3a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \cos[c + dx]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{3/2} d} + \frac{b \cos[c + dx]}{2a (a+b) d (a + b - b \cos[c + dx]^2)}$$

Result (type 3, 194 leaves):

$$\frac{1}{2 a^2 d} \left(\frac{\sqrt{b} (3 a + 2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{(-a-b)^{3/2}} + \frac{\sqrt{b} (3 a + 2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{(-a-b)^{3/2}} + \right. \\ \left. 2 \left(\frac{a b \operatorname{Cos}[c+dx]}{(a+b)(2a+b-b \operatorname{Cos}[2(c+dx)])} - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) \right)$$

- **Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c+dx]^3}{(a+b \operatorname{Sin}[c+dx]^2)^2} dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$-\frac{(a-4b) \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{2 a^3 d} - \frac{b^{3/2} (5 a + 4 b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Cos}[c+dx]}{\sqrt{a+b}}\right]}{2 a^3 (a+b)^{3/2} d} - \frac{b (a+2b) \operatorname{Cos}[c+dx]}{2 a^2 (a+b) d (a+b-b \operatorname{Cos}[c+dx]^2)} - \frac{\operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]}{2 a d (a+b-b \operatorname{Cos}[c+dx]^2)}$$

Result (type 3, 390 leaves):

$$\frac{1}{32 a^3 d (b+a \operatorname{Csc}[c+dx]^2)^2} (-2 a - b + b \operatorname{Cos}[2(c+dx)]) \operatorname{Csc}[c+dx]^3 \\ \left(\frac{8 a b^2 \operatorname{Cot}[c+dx]}{a+b} + \frac{4 b^{3/2} (5 a + 4 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{(-a-b)^{3/2}} (2 a + b - b \operatorname{Cos}[2(c+dx)]) \operatorname{Csc}[c+dx] \right. \\ \left. + \frac{4 b^{3/2} (5 a + 4 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-a-b}}\right]}{(-a-b)^{3/2}} (2 a + b - b \operatorname{Cos}[2(c+dx)]) \operatorname{Csc}[c+dx] \right. \\ \left. + a (2 a + b - b \operatorname{Cos}[2(c+dx)]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Csc}[c+dx] + 4 (a-4b) (2 a + b - b \operatorname{Cos}[2(c+dx)]) \operatorname{Csc}[c+dx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] - \right. \\ \left. 4 (a-4b) (2 a + b - b \operatorname{Cos}[2(c+dx)]) \operatorname{Csc}[c+dx] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - a (2 a + b - b \operatorname{Cos}[2(c+dx)]) \operatorname{Csc}[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)$$

- **Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[x]}{\sqrt{1+\operatorname{Sin}[x]^2}} dx$$

Optimal (type 3, 11 leaves, 2 steps) :

$$-\text{ArcSin}\left[\frac{\text{Cos}[x]}{\sqrt{2}}\right]$$

Result (type 3, 29 leaves) :

$$i \text{Log}\left[i \sqrt{2} \text{Cos}[x] + \sqrt{3 - \text{Cos}[2x]}\right]$$

- **Problem 114: Result unnecessarily involves imaginary or complex numbers.**

$$\int \text{Sin}[x] \sqrt{1 + \text{Sin}[x]^2} \, dx$$

Optimal (type 3, 30 leaves, 3 steps) :

$$-\text{ArcSin}\left[\frac{\text{Cos}[x]}{\sqrt{2}}\right] - \frac{1}{2} \text{Cos}[x] \sqrt{2 - \text{Cos}[x]^2}$$

Result (type 3, 53 leaves) :

$$-\frac{\text{Cos}[x] \sqrt{3 - \text{Cos}[2x]}}{2\sqrt{2}} + i \text{Log}\left[i \sqrt{2} \text{Cos}[x] + \sqrt{3 - \text{Cos}[2x]}\right]$$

- **Problem 115: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[7 + 3x]}{\sqrt{3 + \text{Sin}[7 + 3x]^2}} \, dx$$

Optimal (type 3, 15 leaves, 2 steps) :

$$-\frac{1}{3} \text{ArcSin}\left[\frac{1}{2} \text{Cos}[7 + 3x]\right]$$

Result (type 3, 39 leaves) :

$$\frac{1}{3} i \text{Log}\left[i \sqrt{2} \text{Cos}[7 + 3x] + \sqrt{7 - \text{Cos}[2(7 + 3x)]}\right]$$

- **Problem 119: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a - a \text{Sin}[x]^2}} \, dx$$

Optimal (type 3, 16 leaves, 3 steps) :

$$\frac{\text{ArcTanh}[\text{Sin}[x]] \text{Cos}[x]}{\sqrt{a \text{Cos}[x]^2}}$$

Result (type 3, 46 leaves) :

$$\frac{\cos[x] \left(-\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right)}{\sqrt{a \cos[x]^2}}$$

- **Problem 120: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a - a \sin[x]^2)^{3/2}} dx$$

Optimal (type 3, 42 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\sin[x]] \cos[x]}{2 a \sqrt{a \cos[x]^2}} + \frac{\tan[x]}{2 a \sqrt{a \cos[x]^2}}$$

Result (type 3, 91 leaves):

$$-\frac{1}{4 (a \cos[x]^2)^{3/2}} \cos[x] \left(\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \cos[2x] \left(\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - 2 \sin[x] \right)$$

- **Problem 172: Result unnecessarily involves higher level functions.**

$$\int \sin[e + f x]^5 (a + b \sin[e + f x]^2)^p dx$$

Optimal (type 5, 220 leaves, 5 steps):

$$\frac{(3a - 2b(2+p)) \cos[e + f x] (a + b - b \cos[e + f x]^2)^{1+p}}{b^2 f (3 + 2p) (5 + 2p)} - \frac{1}{b^2 f (3 + 2p) (5 + 2p)}$$

$$(3a^2 - 4ab(1+p) + 4b^2(2 + 3p + p^2)) \cos[e + f x] (a + b - b \cos[e + f x]^2)^p \left(1 - \frac{b \cos[e + f x]^2}{a + b} \right)^{-p}$$

$$\text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{b \cos[e + f x]^2}{a + b}\right] - \frac{\cos[e + f x] (a + b - b \cos[e + f x]^2)^{1+p} \sin[e + f x]^2}{b f (5 + 2p)}$$

Result (type 6, 184 leaves):

$$\left(4 a \text{AppellF1}\left[3, \frac{1}{2}, -p, 4, \sin[e + f x]^2, -\frac{b \sin[e + f x]^2}{a}\right] \sin[e + f x]^5 (a + b \sin[e + f x]^2)^p \tan[e + f x] \right) /$$

$$\left(3 f \left(8 a \text{AppellF1}\left[3, \frac{1}{2}, -p, 4, \sin[e + f x]^2, -\frac{b \sin[e + f x]^2}{a}\right] + \right. \right.$$

$$\left. \left(2 b p \text{AppellF1}\left[4, \frac{1}{2}, 1 - p, 5, \sin[e + f x]^2, -\frac{b \sin[e + f x]^2}{a}\right] + a \text{AppellF1}\left[4, \frac{3}{2}, -p, 5, \sin[e + f x]^2, -\frac{b \sin[e + f x]^2}{a}\right] \right) \sin[e + f x]^2 \right)$$

■ **Problem 173: Result unnecessarily involves higher level functions.**

$$\int \sin[e + f x]^3 (a + b \sin[e + f x]^2)^p dx$$

Optimal (type 5, 131 leaves, 4 steps):

$$-\frac{\cos[e + f x] (a + b - b \cos[e + f x]^2)^{1+p}}{b f (3 + 2 p)} + \frac{1}{b f (3 + 2 p)} \\ (a - 2 b (1 + p)) \cos[e + f x] (a + b - b \cos[e + f x]^2)^p \left(1 - \frac{b \cos[e + f x]^2}{a + b}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{b \cos[e + f x]^2}{a + b}\right]$$

Result (type 6, 184 leaves):

$$\left(3 a \text{AppellF1}\left[2, \frac{1}{2}, -p, 3, \sin[e + f x]^2, -\frac{b \sin[e + f x]^2}{a}\right] \sin[e + f x]^3 (a + b \sin[e + f x]^2)^p \tan[e + f x]\right) / \\ \left(2 f \left(6 a \text{AppellF1}\left[2, \frac{1}{2}, -p, 3, \sin[e + f x]^2, -\frac{b \sin[e + f x]^2}{a}\right] + \right. \right. \\ \left. \left. \left(2 b p \text{AppellF1}\left[3, \frac{1}{2}, 1 - p, 4, \sin[e + f x]^2, -\frac{b \sin[e + f x]^2}{a}\right] + a \text{AppellF1}\left[3, \frac{3}{2}, -p, 4, \sin[e + f x]^2, -\frac{b \sin[e + f x]^2}{a}\right]\right) \sin[e + f x]^2\right)\right)$$

■ **Problem 175: Unable to integrate problem.**

$$\int \csc[e + f x] (a + b \sin[e + f x]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$-\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, \cos[e + f x]^2, \frac{b \cos[e + f x]^2}{a + b}\right] \cos[e + f x] (a + b - b \cos[e + f x]^2)^p \left(1 - \frac{b \cos[e + f x]^2}{a + b}\right)^{-p}$$

Result (type 8, 23 leaves):

$$\int \csc[e + f x] (a + b \sin[e + f x]^2)^p dx$$

■ **Problem 176: Unable to integrate problem.**

$$\int \csc[e + f x]^3 (a + b \sin[e + f x]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$-\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, \cos[e + f x]^2, \frac{b \cos[e + f x]^2}{a + b}\right] \cos[e + f x] (a + b - b \cos[e + f x]^2)^p \left(1 - \frac{b \cos[e + f x]^2}{a + b}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \csc[e + f x]^3 (a + b \sin[e + f x]^2)^p dx$$

■ **Problem 177: Unable to integrate problem.**

$$\int \text{Csc}[e + f x]^5 (a + b \text{Sin}[e + f x]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$-\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 3, -p, \frac{3}{2}, \text{Cos}[e + f x]^2, \frac{b \text{Cos}[e + f x]^2}{a + b}\right] \text{Cos}[e + f x] (a + b - b \text{Cos}[e + f x]^2)^p \left(1 - \frac{b \text{Cos}[e + f x]^2}{a + b}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \text{Csc}[e + f x]^5 (a + b \text{Sin}[e + f x]^2)^p dx$$

■ **Problem 179: Result more than twice size of optimal antiderivative.**

$$\int \text{Sin}[e + f x]^2 (a + b \text{Sin}[e + f x]^2)^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{1}{3f} \text{AppellF1}\left[\frac{3}{2}, 2 + p, -p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{(a + b) \text{Tan}[e + f x]^2}{a}\right] \\ (\text{Sec}[e + f x]^2)^p (a + b \text{Sin}[e + f x]^2)^p \text{Tan}[e + f x]^3 \left(1 + \frac{(a + b) \text{Tan}[e + f x]^2}{a}\right)^{-p}$$

Result (type 6, 240 leaves):

$$-\frac{1}{b^2 f (1 + p) (2 + p)} 2^{-2-p} \sqrt{\frac{b \text{Cos}[e + f x]^2}{a + b}} (2a + b - b \text{Cos}[2(e + f x)])^{1+p} \\ \left(2a(2 + p) \text{AppellF1}\left[1 + p, \frac{1}{2}, \frac{1}{2}, 2 + p, \frac{2a + b - b \text{Cos}[2(e + f x)]}{2(a + b)}, \frac{2a + b - b \text{Cos}[2(e + f x)]}{2a}\right] - (1 + p) \text{AppellF1}\left[2 + p, \frac{1}{2}, \frac{1}{2}, 3 + p, \frac{2a + b - b \text{Cos}[2(e + f x)]}{2(a + b)}, \frac{2a + b - b \text{Cos}[2(e + f x)]}{2a}\right] (2a + b - b \text{Cos}[2(e + f x)])\right) \text{Csc}[2(e + f x)] \sqrt{-\frac{b \text{Sin}[e + f x]^2}{a}}$$

■ **Problem 180: Unable to integrate problem.**

$$\int \text{Csc}[e + f x]^2 (a + b \text{Sin}[e + f x]^2)^p dx$$

Optimal (type 6, 97 leaves, 3 steps):

$$-\frac{1}{f} \text{AppellF1}\left[-\frac{1}{2}, \frac{1}{2}, -p, \frac{1}{2}, \text{Sin}[e + f x]^2, -\frac{b \text{Sin}[e + f x]^2}{a}\right] \sqrt{\text{Cos}[e + f x]^2} \text{Csc}[e + f x] \text{Sec}[e + f x] (a + b \text{Sin}[e + f x]^2)^p \left(1 + \frac{b \text{Sin}[e + f x]^2}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \text{Csc}[e + f x]^2 (a + b \text{Sin}[e + f x]^2)^p dx$$

■ **Problem 181: Unable to integrate problem.**

$$\int \text{Csc}[e + f x]^4 (a + b \text{Sin}[e + f x]^2)^p dx$$

Optimal (type 6, 101 leaves, 3 steps):

$$-\frac{1}{3f}$$

$$\text{AppellF1}\left[-\frac{3}{2}, \frac{1}{2}, -p, -\frac{1}{2}, \text{Sin}[e + f x]^2, -\frac{b \text{Sin}[e + f x]^2}{a}\right] \sqrt{\text{Cos}[e + f x]^2} \text{Csc}[e + f x]^3 \text{Sec}[e + f x] (a + b \text{Sin}[e + f x]^2)^p \left(1 + \frac{b \text{Sin}[e + f x]^2}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \text{Csc}[e + f x]^4 (a + b \text{Sin}[e + f x]^2)^p dx$$

■ **Problem 182: Result is not expressed in closed-form.**

$$\int \frac{\text{Sin}[c + d x]^7}{a + b \text{Sin}[c + d x]^3} dx$$

Optimal (type 3, 335 leaves, 17 steps):

$$\frac{3x}{8b} + \frac{2(-1)^{2/3} a^{5/3} \text{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} b^{7/3} d} - \frac{2a^{5/3} \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3\sqrt{a^{2/3} - b^{2/3}} b^{7/3} d} +$$

$$\frac{2(-1)^{1/3} a^{5/3} \text{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} b^{7/3} d} + \frac{a \text{Cos}[c + d x]}{b^2 d} - \frac{3 \text{Cos}[c + d x] \text{Sin}[c + d x]}{8 b d} - \frac{\text{Cos}[c + d x] \text{Sin}[c + d x]^3}{4 b d}$$

Result (type 7, 219 leaves):

$$\frac{1}{96 b^2 d} \left(96 a \text{Cos}[c + d x] - 32 a^2 \text{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \right. \right.$$

$$\left. \left(-2 \text{ArcTan}\left[\frac{\text{Sin}[c + d x]}{\text{Cos}[c + d x] - \#1}\right] + i \text{Log}\left[1 - 2 \text{Cos}[c + d x] \#1 + \#1^2\right] + 2 \text{ArcTan}\left[\frac{\text{Sin}[c + d x]}{\text{Cos}[c + d x] - \#1}\right] \#1^2 - i \text{Log}\left[1 - 2 \text{Cos}[c + d x] \#1 + \#1^2\right] \#1^2 \right) /$$

$$\left. (b - 4 i a \#1 - 2 b \#1^2 + b \#1^4) \& \right] + 3 b (12 (c + d x) - 8 \text{Sin}[2 (c + d x)] + \text{Sin}[4 (c + d x)]) \left. \right)$$

■ **Problem 183: Result is not expressed in closed-form.**

$$\int \frac{\text{Sin}[c + d x]^5}{a + b \text{Sin}[c + d x]^3} dx$$

Optimal (type 3, 273 leaves, 15 steps):

$$\frac{x}{2b} - \frac{2a \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3\sqrt{a^{2/3}-b^{2/3}} b^{5/3}d} + \frac{2a \operatorname{ArcTanh}\left[\frac{b^{1/3}-(-1)^{1/3}a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right]}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}} b^{5/3}d} + \frac{2a \operatorname{ArcTanh}\left[\frac{b^{1/3}+(-1)^{2/3}a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{(-1)^{1/3}a^{2/3}+b^{2/3}}}\right]}{3\sqrt{(-1)^{1/3}a^{2/3}+b^{2/3}} b^{5/3}d} - \frac{\operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2bd}$$

Result (type 7, 255 leaves):

$$\frac{1}{12bd} \left(6(c+dx) - 2i a \operatorname{RootSum}\left[-ib + 3ib\#1^2 + 8a\#1^3 - 3ib\#1^4 + ib\#1^6 \&, \right. \right. \\ \left. \left. \left(2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] - 4 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^2 + 2i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^2 + \right. \right. \\ \left. \left. 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^4 - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^4 \right) / (b\#1 - 4ia\#1^2 - 2b\#1^3 + b\#1^5) \& \right] - 3 \operatorname{Sin}[2(c+dx)] \right)$$

■ **Problem 184: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c+dx]^3}{a+b \operatorname{Sin}[c+dx]^3} dx$$

Optimal (type 3, 259 leaves, 13 steps):

$$\frac{x}{b} - \frac{2a^{1/3} \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3\sqrt{a^{2/3}-b^{2/3}} bd} - \frac{2a^{1/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3}b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3}+(-1)^{1/3}b^{2/3}}}\right]}{3\sqrt{a^{2/3}+(-1)^{1/3}b^{2/3}} bd} + \frac{2a^{1/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3}(b^{1/3}+(-1)^{2/3}a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right])}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right]}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}} bd}$$

Result (type 7, 140 leaves):

$$\frac{1}{3bd} \left(3c + 3dx + 2i a \operatorname{RootSum}\left[-ib + 3ib\#1^2 + 8a\#1^3 - 3ib\#1^4 + ib\#1^6 \&, \right. \right. \\ \left. \left. \frac{2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1 - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1}{b - 4ia\#1 - 2b\#1^2 + b\#1^4} \& \right) \right)$$

■ **Problem 185: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c+dx]}{a+b \operatorname{Sin}[c+dx]^3} dx$$

Optimal (type 3, 267 leaves, 11 steps):

$$\frac{2(-1)^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3}b^{1/3}-a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right]}{3a^{1/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}} b^{1/3}d} - \frac{2 \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3a^{1/3}\sqrt{a^{2/3}-b^{2/3}} b^{1/3}d} + \frac{2(-1)^{1/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3}b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3}+(-1)^{1/3}b^{2/3}}}\right]}{3a^{1/3}\sqrt{a^{2/3}+(-1)^{1/3}b^{2/3}} b^{1/3}d}$$

Result (type 7, 172 leaves):

$$-\frac{1}{3d} \text{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \ \&, \right. \\ \left. \left(-2 \text{ArcTan}\left[\frac{\text{Sin}[c + d x]}{\text{Cos}[c + d x] - \#1}\right] + i \text{Log}\left[1 - 2 \text{Cos}[c + d x] \#1 + \#1^2\right] + 2 \text{ArcTan}\left[\frac{\text{Sin}[c + d x]}{\text{Cos}[c + d x] - \#1}\right] \#1^2 - i \text{Log}\left[1 - 2 \text{Cos}[c + d x] \#1 + \#1^2\right] \#1^2\right) / \right. \\ \left. (b - 4 i a \#1 - 2 b \#1^2 + b \#1^4) \ \&\right]$$

■ **Problem 186: Result is not expressed in closed-form.**

$$\int \frac{\text{Csc}[c + d x]}{a + b \text{Sin}[c + d x]^3} dx$$

Optimal (type 3, 264 leaves, 14 steps):

$$-\frac{2 b^{1/3} \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a \sqrt{a^{2/3} - b^{2/3}} d} - \frac{\text{ArcTanh}[\text{Cos}[c + d x]]}{a d} + \frac{2 b^{1/3} \text{ArcTanh}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}}\right]}{3 a \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} d} + \frac{2 b^{1/3} \text{ArcTanh}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}}\right]}{3 a \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} d}$$

Result (type 7, 264 leaves):

$$-\frac{1}{6 a d} \left(6 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right]\right] - 6 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + i b \text{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \ \&, \right. \right. \\ \left. \left(2 \text{ArcTan}\left[\frac{\text{Sin}[c + d x]}{\text{Cos}[c + d x] - \#1}\right] - i \text{Log}\left[1 - 2 \text{Cos}[c + d x] \#1 + \#1^2\right] - 4 \text{ArcTan}\left[\frac{\text{Sin}[c + d x]}{\text{Cos}[c + d x] - \#1}\right] \#1^2 + 2 i \text{Log}\left[1 - 2 \text{Cos}[c + d x] \#1 + \#1^2\right] \#1^2 + \right. \right. \\ \left. \left. 2 \text{ArcTan}\left[\frac{\text{Sin}[c + d x]}{\text{Cos}[c + d x] - \#1}\right] \#1^4 - i \text{Log}\left[1 - 2 \text{Cos}[c + d x] \#1 + \#1^2\right] \#1^4\right) / (b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5) \ \&\right)$$

■ **Problem 187: Result is not expressed in closed-form.**

$$\int \frac{\text{Csc}[c + d x]^3}{a + b \text{Sin}[c + d x]^3} dx$$

Optimal (type 3, 287 leaves, 15 steps):

$$-\frac{2 b \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{5/3} \sqrt{a^{2/3} - b^{2/3}} d} - \frac{2 b \text{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{5/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} + \\ \frac{2 b \text{ArcTan}\left[\frac{(-1)^{1/3} (b^{1/3} + (-1)^{2/3} a^{1/3} \text{Tan}\left[\frac{1}{2}(c + d x)\right])}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{5/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d} - \frac{\text{ArcTanh}[\text{Cos}[c + d x]]}{2 a d} - \frac{\text{Cot}[c + d x] \text{Csc}[c + d x]}{2 a d}$$

Result (type 7, 181 leaves):

$$\frac{1}{24 a d} \left(16 i b \operatorname{RootSum} \left[-b + 3 b \#1^2 - 8 i a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \frac{2 \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1 - i \operatorname{Log} \left[1 - 2 \cos [c+d x] \#1 + \#1^2 \right] \#1}{b - 4 i a \#1 - 2 b \#1^2 + b \#1^4} \& \right] - \right. \\ \left. 3 \left(\operatorname{Csc} \left[\frac{1}{2} (c+d x) \right] \right)^2 + 4 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+d x) \right] \right] - 4 \operatorname{Log} \left[\sin \left[\frac{1}{2} (c+d x) \right] \right] - \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right] \right)^2 \right)$$

■ **Problem 188: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Csc} [c+d x]^5}{a+b \sin [c+d x]^3} dx$$

Optimal (type 3, 344 leaves, 18 steps):

$$\frac{2 (-1)^{2/3} b^{5/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 a^{7/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d} - \frac{2 b^{5/3} \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 a^{7/3} \sqrt{a^{2/3} - b^{2/3}} d} + \frac{2 (-1)^{1/3} b^{5/3} \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 a^{7/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} - \frac{3 \operatorname{ArcTanh} [\cos [c+d x]]}{8 a d} + \frac{b \operatorname{Cot} [c+d x]}{a^2 d} - \frac{3 \operatorname{Cot} [c+d x] \operatorname{Csc} [c+d x]}{8 a d} - \frac{\operatorname{Cot} [c+d x] \operatorname{Csc} [c+d x]^3}{4 a d}$$

Result (type 7, 290 leaves):

$$\frac{1}{192 a^2 d} \left(-64 b^2 \operatorname{RootSum} \left[-b + 3 b \#1^2 - 8 i a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \right. \right. \\ \left. \left(-2 \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] + i \operatorname{Log} \left[1 - 2 \cos [c+d x] \#1 + \#1^2 \right] + 2 \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1^2 - i \operatorname{Log} \left[1 - 2 \cos [c+d x] \#1 + \#1^2 \right] \#1^2 \right) / \right. \\ \left. (b - 4 i a \#1 - 2 b \#1^2 + b \#1^4) \& \right] + 3 \left(32 b \operatorname{Cot} \left[\frac{1}{2} (c+d x) \right] - 6 a \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^2 - a \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^4 - \right. \\ \left. 24 a \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+d x) \right] \right] + 24 a \operatorname{Log} \left[\sin \left[\frac{1}{2} (c+d x) \right] \right] + 6 a \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^2 + a \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^4 - 32 b \tan \left[\frac{1}{2} (c+d x) \right] \right) \right)$$

■ **Problem 189: Result is not expressed in closed-form.**

$$\int \frac{\sin [c+d x]^6}{a+b \sin [c+d x]^3} dx$$

Optimal (type 3, 293 leaves, 15 steps):

$$\begin{aligned}
& -\frac{a x}{b^2} + \frac{2 a^{4/3} \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3 \sqrt{a^{2/3}-b^{2/3}} b^2 d} + \frac{2 a^{4/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}}}\right]}{3 \sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}} b^2 d} - \\
& \frac{2 a^{4/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3}\left(b^{1/3}+(-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)}{\sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}}}\right]}{3 \sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}} b^2 d} - \frac{\operatorname{Cos}[c+d x]}{b d} + \frac{\operatorname{Cos}[c+d x]^3}{3 b d}
\end{aligned}$$

Result (type 7, 164 leaves):

$$\begin{aligned}
& -\frac{1}{12 b^2 d} \left(12 a c + 12 a d x + 9 b \operatorname{Cos}[c+d x] - b \operatorname{Cos}[3(c+d x)] + \right. \\
& \left. 8 i a^2 \operatorname{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \frac{2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] \#1 - i \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] \#1}{b-4 i a \#1-2 b \#1^2+b \#1^4} \& \right] \right)
\end{aligned}$$

■ **Problem 190: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c+d x]^4}{a+b \operatorname{Sin}[c+d x]^3} dx$$

Optimal (type 3, 281 leaves, 14 steps):

$$\begin{aligned}
& -\frac{2(-1)^{2/3} a^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3}-a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}}}\right]}{3 \sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}} b^{4/3} d} + \\
& \frac{2 a^{2/3} \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3 \sqrt{a^{2/3}-b^{2/3}} b^{4/3} d} - \frac{2(-1)^{1/3} a^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}}}\right]}{3 \sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}} b^{4/3} d} - \frac{\operatorname{Cos}[c+d x]}{b d}
\end{aligned}$$

Result (type 7, 186 leaves):

$$\begin{aligned}
& \frac{1}{3 b d} \left(-3 \operatorname{Cos}[c+d x] + a \operatorname{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \right. \right. \\
& \left. \left(-2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] + i \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] + 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] \#1^2 - i \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] \#1^2 \right) / \right. \\
& \left. \left. (b-4 i a \#1-2 b \#1^2+b \#1^4) \& \right] \right)
\end{aligned}$$

■ **Problem 191: Result is not expressed in closed-form.**

$$\int \frac{\sin[c + dx]^2}{a + b \sin[c + dx]^3} dx$$

Optimal (type 3, 240 leaves, 11 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^{2/3} d} - \frac{2 \operatorname{ArcTanh}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} b^{2/3} d} - \frac{2 \operatorname{ArcTanh}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} b^{2/3} d}$$

Result (type 7, 231 leaves):

$$\frac{1}{6d} i \operatorname{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \right. \\ \left. \left(2 \operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#1}\right] - i \operatorname{Log}\left[1 - 2 \cos[c + dx] \#1 + \#1^2\right] - 4 \operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#1}\right] \#1^2 + 2 i \operatorname{Log}\left[1 - 2 \cos[c + dx] \#1 + \#1^2\right] \#1^2 + \right. \\ \left. 2 \operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#1}\right] \#1^4 - i \operatorname{Log}\left[1 - 2 \cos[c + dx] \#1 + \#1^2\right] \#1^4 \right) / (b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5) \& \left. \right]$$

■ **Problem 192: Result is not expressed in closed-form.**

$$\int \frac{1}{a + b \sin[c + dx]^3} dx$$

Optimal (type 3, 245 leaves, 11 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} d} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/3} (b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right])}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d}$$

Result (type 7, 126 leaves):

$$-\frac{1}{3d} 2 i \operatorname{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \frac{2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1 - i \operatorname{Log}\left[1 - 2 \cos[c + dx] \#1 + \#1^2\right] \#1}{b - 4 i a \#1 - 2 b \#1^2 + b \#1^4} \& \right]$$

■ **Problem 193: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Csc}[c + dx]^2}{a + b \sin[c + dx]^3} dx$$

Optimal (type 3, 281 leaves, 15 steps):

$$\begin{aligned}
& \frac{2 (-1)^{2/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{4/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d} + \\
& \frac{2 b^{2/3} \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{4/3} \sqrt{a^{2/3} - b^{2/3}} d} - \frac{2 (-1)^{1/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{4/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} - \frac{\operatorname{Cot}[c+dx]}{a d}
\end{aligned}$$

Result (type 7, 196 leaves):

$$\begin{aligned}
& \frac{1}{6 a d} \left(-3 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] + 2 b \operatorname{RootSum}\left[-b + 3 b \#1^2 - 8 i a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \right. \right. \\
& \left. \left. \left(-2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] + i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] + 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^2 - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^2 \right) / \right. \right. \\
& \left. \left. (b - 4 i a \#1 - 2 b \#1^2 + b \#1^4) \& \right] + 3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)
\end{aligned}$$

■ **Problem 194: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Csc}[c+dx]^4}{a + b \operatorname{Sin}[c+dx]^3} dx$$

Optimal (type 3, 296 leaves, 16 steps):

$$\begin{aligned}
& \frac{2 b^{4/3} \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^2 \sqrt{a^{2/3} - b^{2/3}} d} + \frac{b \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{a^2 d} - \\
& \frac{2 b^{4/3} \operatorname{ArcTanh}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}}\right]}{3 a^2 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} d} - \frac{2 b^{4/3} \operatorname{ArcTanh}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}}\right]}{3 a^2 \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} d} - \frac{\operatorname{Cot}[c+dx]}{a d} - \frac{\operatorname{Cot}[c+dx]^3}{3 a d}
\end{aligned}$$

Result (type 7, 333 leaves):

$$\frac{1}{24 a^2 d} \left(-8 a \operatorname{Cot} \left[\frac{1}{2} (c + d x) \right] + 24 b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] \right] - 24 b \operatorname{Log} \left[\operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] + 4 i b^2 \operatorname{RootSum} \left[-b + 3 b \#1^2 - 8 i a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \right. \right. \\ \left. \left. \left(2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] - i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] - 4 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] \#1^2 + 2 i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] \#1^2 + \right. \right. \right. \\ \left. \left. \left. 2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] \#1^4 - i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] \#1^4 \right) / (b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5) \& \right) + \right. \\ \left. 8 a \operatorname{Csc}[c + d x]^3 \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right]^4 - \frac{1}{2} a \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^4 \operatorname{Sin}[c + d x] + 8 a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right)$$

■ **Problem 195: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c + d x]^9}{a - b \operatorname{Sin}[c + d x]^4} dx$$

Optimal (type 3, 177 leaves, 6 steps):

$$-\frac{a^{3/2} \operatorname{ArcTan} \left[\frac{b^{1/4} \operatorname{Cos}[c + d x]}{\sqrt{\sqrt{a} - \sqrt{b}}} \right]}{2 \sqrt{\sqrt{a} - \sqrt{b}} b^{9/4} d} - \frac{a^{3/2} \operatorname{ArcTanh} \left[\frac{b^{1/4} \operatorname{Cos}[c + d x]}{\sqrt{\sqrt{a} + \sqrt{b}}} \right]}{2 \sqrt{\sqrt{a} + \sqrt{b}} b^{9/4} d} + \frac{(a + b) \operatorname{Cos}[c + d x]}{b^2 d} - \frac{2 \operatorname{Cos}[c + d x]^3}{3 b d} + \frac{\operatorname{Cos}[c + d x]^5}{5 b d}$$

Result (type 7, 228 leaves):

$$\frac{1}{120 b^2 d} \left(\operatorname{Cos}[c + d x] (120 a + 89 b - 28 b \operatorname{Cos}[2(c + d x)] + 3 b \operatorname{Cos}[4(c + d x)]) + \right. \\ \left. 60 i a^2 \operatorname{RootSum} \left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \left(-2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] \#1 + i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] \#1 + \right. \right. \right. \\ \left. \left. \left. 2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] \#1^3 - i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] \#1^3 \right) / (-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6) \& \right) \right)$$

■ **Problem 196: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c + d x]^7}{a - b \operatorname{Sin}[c + d x]^4} dx$$

Optimal (type 3, 148 leaves, 6 steps):

$$-\frac{a \operatorname{ArcTan} \left[\frac{b^{1/4} \operatorname{Cos}[c + d x]}{\sqrt{\sqrt{a} - \sqrt{b}}} \right]}{2 \sqrt{\sqrt{a} - \sqrt{b}} b^{7/4} d} + \frac{a \operatorname{ArcTanh} \left[\frac{b^{1/4} \operatorname{Cos}[c + d x]}{\sqrt{\sqrt{a} + \sqrt{b}}} \right]}{2 \sqrt{\sqrt{a} + \sqrt{b}} b^{7/4} d} + \frac{\operatorname{Cos}[c + d x]}{b d} - \frac{\operatorname{Cos}[c + d x]^3}{3 b d}$$

Result (type 7, 310 leaves):

$$\frac{1}{24 b d} \left(18 \operatorname{Cos}[c + d x] - 2 \operatorname{Cos}[3(c + d x)] - 3 i a \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \right. \\ \left. \left. \left(-2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] + i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] + 6 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^2 - \right. \right. \right. \\ \left. \left. \left. 3 i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^2 - 6 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^4 + 3 i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^4 + \right. \right. \right. \\ \left. \left. \left. 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^6 - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^6\right) / \left(-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7 \right) \& \right] \right)$$

■ **Problem 197: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c + d x]^5}{a - b \operatorname{Sin}[c + d x]^4} dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cos}[c + d x]}{\sqrt{\sqrt{a} - \sqrt{b}}}\right]}{2 \sqrt{\sqrt{a} - \sqrt{b}} b^{5/4} d} - \frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cos}[c + d x]}{\sqrt{\sqrt{a} + \sqrt{b}}}\right]}{2 \sqrt{\sqrt{a} + \sqrt{b}} b^{5/4} d} + \frac{\operatorname{Cos}[c + d x]}{b d}$$

Result (type 7, 198 leaves):

$$\frac{1}{2 b d} \left(2 \operatorname{Cos}[c + d x] + \right. \\ \left. i a \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \left(-2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1 + i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1 + \right. \right. \right. \\ \left. \left. \left. 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^3 - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^3\right) / \left(-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6 \right) \& \right] \right)$$

■ **Problem 198: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c + d x]^3}{a - b \operatorname{Sin}[c + d x]^4} dx$$

Optimal (type 3, 115 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cos}[c + d x]}{\sqrt{\sqrt{a} - \sqrt{b}}}\right]}{2 \sqrt{\sqrt{a} - \sqrt{b}} b^{3/4} d} + \frac{\operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cos}[c + d x]}{\sqrt{\sqrt{a} + \sqrt{b}}}\right]}{2 \sqrt{\sqrt{a} + \sqrt{b}} b^{3/4} d}$$

Result (type 7, 285 leaves):

$$\begin{aligned}
& -\frac{1}{8d} i \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \ \&, \right. \\
& \quad \left. \left(-2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] + i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] + 6 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^2 - \right. \right. \\
& \quad \quad \left. \left. 3 i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^2 - 6 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^4 + 3 i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^4 + \right. \right. \\
& \quad \quad \left. \left. 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^6 - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^6\right) / \left(-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7 \right) \ \& \left. \right]
\end{aligned}$$

■ **Problem 199: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c+d x]}{a - b \operatorname{Sin}[c+d x]^4} dx$$

Optimal (type 3, 125 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 \sqrt{a} \sqrt{\sqrt{a}-\sqrt{b}} b^{1/4} d} - \frac{\operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 \sqrt{a} \sqrt{\sqrt{a}+\sqrt{b}} b^{1/4} d}$$

Result (type 7, 183 leaves):

$$\begin{aligned}
& \frac{1}{2d} i \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \ \&, \right. \\
& \quad \left. \left(-2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1 + i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1 + 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^3 - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^3\right) / \right. \\
& \quad \quad \left. \left(-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6 \right) \ \& \left. \right]
\end{aligned}$$

■ **Problem 200: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Csc}[c+d x]}{a - b \operatorname{Sin}[c+d x]^4} dx$$

Optimal (type 3, 136 leaves, 7 steps):

$$-\frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a \sqrt{\sqrt{a}-\sqrt{b}} d} - \frac{\operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{a d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a \sqrt{\sqrt{a}+\sqrt{b}} d}$$

Result (type 7, 318 leaves):

$$\begin{aligned}
& -\frac{1}{8ad} \left(8 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] \right] - 8 \operatorname{Log} \left[\operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] + i b \operatorname{RootSum} \left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \right. \\
& \quad \left. \left. \left(-2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] + i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2 \right] + 6 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1^2 - \right. \right. \right. \\
& \quad \left. \left. \left. 3 i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2 \right] \#1^2 - 6 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1^4 + 3 i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2 \right] \#1^4 + \right. \right. \right. \\
& \quad \left. \left. \left. 2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1^6 - i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2 \right] \#1^6 \right) / \left(-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7 \right) \& \right] \right)
\end{aligned}$$

■ **Problem 201: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Csc}[c+dx]^3}{a-b\operatorname{Sin}[c+dx]^4} dx$$

Optimal (type 3, 184 leaves, 7 steps):

$$\begin{aligned}
& -\frac{b^{3/4} \operatorname{ArcTan} \left[\frac{b^{1/4} \operatorname{Cos}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}} \right]}{2 a^{3/2} \sqrt{\sqrt{a}-\sqrt{b}} d} - \frac{\operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{2 a d} - \frac{b^{3/4} \operatorname{ArcTanh} \left[\frac{b^{1/4} \operatorname{Cos}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}} \right]}{2 a^{3/2} \sqrt{\sqrt{a}+\sqrt{b}} d} - \frac{1}{4 a d (1-\operatorname{Cos}[c+dx])} + \frac{1}{4 a d (1+\operatorname{Cos}[c+dx])}
\end{aligned}$$

Result (type 7, 242 leaves):

$$\begin{aligned}
& \frac{1}{8ad} \left(-\operatorname{Csc} \left[\frac{1}{2} (c+dx) \right]^2 - 4 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] \right] + 4 \operatorname{Log} \left[\operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] + \right. \\
& \quad \left. 4 i b \operatorname{RootSum} \left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \left(-2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1 + i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2 \right] \#1 + \right. \right. \right. \\
& \quad \left. \left. \left. 2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1^3 - i \operatorname{Log} \left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2 \right] \#1^3 \right) / \left(-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6 \right) \& \right] + \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right)
\end{aligned}$$

■ **Problem 202: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Csc}[c+dx]^5}{a-b\operatorname{Sin}[c+dx]^4} dx$$

Optimal (type 3, 229 leaves, 7 steps):

$$\begin{aligned}
& -\frac{b^{5/4} \operatorname{ArcTan} \left[\frac{b^{1/4} \operatorname{Cos}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}} \right]}{2 a^2 \sqrt{\sqrt{a}-\sqrt{b}} d} - \frac{(3 a + 8 b) \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{8 a^2 d} + \frac{b^{5/4} \operatorname{ArcTanh} \left[\frac{b^{1/4} \operatorname{Cos}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}} \right]}{2 a^2 \sqrt{\sqrt{a}+\sqrt{b}} d} - \\
& \frac{1}{16 a d (1-\operatorname{Cos}[c+dx])^2} - \frac{3}{16 a d (1-\operatorname{Cos}[c+dx])} + \frac{1}{16 a d (1+\operatorname{Cos}[c+dx])^2} + \frac{3}{16 a d (1+\operatorname{Cos}[c+dx])}
\end{aligned}$$

Result (type 7, 409 leaves):

$$\begin{aligned} & \frac{1}{64 a^2 d} \left(-6 a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 - a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4 - 24 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] - 64 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right] \right) + \\ & 24 a \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 64 b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - 8 i b^2 \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \\ & \left. \left(-2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] + i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] + 6 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^2 - 3 i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^2 - \right. \right. \\ & \left. \left. 6 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^4 + 3 i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^4 + 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^6 - \right. \right. \\ & \left. \left. i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^6\right) / \left(-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7 \right) \& \right] + 6 a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 + a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \end{aligned}$$

■ **Problem 212: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c+d x]^9}{(a-b \operatorname{Sin}[c+d x]^4)^2} dx$$

Optimal (type 3, 236 leaves, 7 steps):

$$\frac{\sqrt{a} \left(5 \sqrt{a} - 6 \sqrt{b} \right) \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right] - \sqrt{a} \left(5 \sqrt{a} + 6 \sqrt{b} \right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 \left(\sqrt{a} - \sqrt{b} \right)^{3/2} b^{9/4} d} + \frac{\sqrt{a} \left(5 \sqrt{a} + 6 \sqrt{b} \right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right] - \sqrt{a} \left(5 \sqrt{a} - 6 \sqrt{b} \right) \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8 \left(\sqrt{a} + \sqrt{b} \right)^{3/2} b^{9/4} d} - \frac{\operatorname{Cos}[c+d x]}{b^2 d} - \frac{a \operatorname{Cos}[c+d x] \left(a+b-b \operatorname{Cos}[c+d x]^2 \right)}{4 (a-b) b^2 d \left(a-b+2 b \operatorname{Cos}[c+d x]^2 - b \operatorname{Cos}[c+d x]^4 \right)}$$

Result (type 7, 486 leaves):

$$\begin{aligned} & -\frac{1}{32 b^2 d} \left(32 \operatorname{Cos}[c+d x] + \frac{32 a \operatorname{Cos}[c+d x] \left(2 a+b-b \operatorname{Cos}[2(c+d x)] \right)}{(a-b) \left(8 a-3 b+4 b \operatorname{Cos}[2(c+d x)] - b \operatorname{Cos}[4(c+d x)] \right)} \right) + \\ & \frac{1}{a-b} i a \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \right. \\ & \left. \left(-2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] + i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] - 40 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^2 + \right. \right. \\ & \left. \left. 54 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^2 + 20 i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^2 - 27 i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^2 + \right. \right. \\ & \left. \left. 40 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^4 - 54 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^4 - 20 i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^4 + \right. \right. \\ & \left. \left. 27 i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^4 + 2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^6 - i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^6 \right) \& \right] \end{aligned}$$

■ **Problem 213: Result is not expressed in closed-form.**

$$\int \frac{\sin[c + dx]^7}{(a - b \sin[c + dx]^4)^2} dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$\frac{(3\sqrt{a} - 4\sqrt{b}) \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8(\sqrt{a}-\sqrt{b})^{3/2} b^{7/4} d} - \frac{(3\sqrt{a} + 4\sqrt{b}) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8(\sqrt{a}+\sqrt{b})^{3/2} b^{7/4} d} - \frac{a \cos[c+dx] (2 - \cos[c+dx]^2)}{4(a-b) b d (a-b+2b \cos[c+dx]^2 - b \cos[c+dx]^4)}$$

Result (type 7, 565 leaves):

$$\frac{1}{32(a-b) b d} \left(\frac{16a(-5 \cos[c+dx] + \cos[3(c+dx)])}{8a - 3b + 4b \cos[2(c+dx)] - b \cos[4(c+dx)]} - \frac{1}{-b \#1 - 8a \#1^3 + 3b \#1^5 - 3b \#1^7 + b \#1^9} \right. \\ \left. i \operatorname{RootSum}\left[b - 4b \#1^2 - 16a \#1^4 + 6b \#1^6 - 4b \#1^8 + b \#1^{10} \&, \frac{1}{-b \#1 - 8a \#1^3 + 3b \#1^5 - 3b \#1^7 + b \#1^9} \right. \right. \\ \left. \left(6a \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] - 8b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] - 3i a \operatorname{Log}[1 - 2 \cos[c+dx] \#1 + \#1^2] + 4i b \operatorname{Log}[1 - 2 \cos[c+dx] \#1 + \#1^2] - \right. \right. \\ \left. 10a \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^2 + 24b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^2 + 5i a \operatorname{Log}[1 - 2 \cos[c+dx] \#1 + \#1^2] \#1^2 - \right. \\ \left. 12i b \operatorname{Log}[1 - 2 \cos[c+dx] \#1 + \#1^2] \#1^2 + 10a \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^4 - 24b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^4 - \right. \\ \left. 5i a \operatorname{Log}[1 - 2 \cos[c+dx] \#1 + \#1^2] \#1^4 + 12i b \operatorname{Log}[1 - 2 \cos[c+dx] \#1 + \#1^2] \#1^4 - 6a \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^6 + \right. \\ \left. \left. 8b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^6 + 3i a \operatorname{Log}[1 - 2 \cos[c+dx] \#1 + \#1^2] \#1^6 - 4i b \operatorname{Log}[1 - 2 \cos[c+dx] \#1 + \#1^2] \#1^6 \right) \& \right) \left. \right)$$

■ **Problem 214: Result is not expressed in closed-form.**

$$\int \frac{\sin[c + dx]^5}{(a - b \sin[c + dx]^4)^2} dx$$

Optimal (type 3, 217 leaves, 5 steps):

$$\frac{(\sqrt{a} - 2\sqrt{b}) \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8\sqrt{a} (\sqrt{a}-\sqrt{b})^{3/2} b^{5/4} d} + \frac{(\sqrt{a} + 2\sqrt{b}) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8\sqrt{a} (\sqrt{a}+\sqrt{b})^{3/2} b^{5/4} d} - \frac{\cos[c+dx] (a + b - b \cos[c+dx]^2)}{4(a-b) b d (a-b+2b \cos[c+dx]^2 - b \cos[c+dx]^4)}$$

Result (type 7, 469 leaves):

$$\begin{aligned}
& - \frac{1}{32 (a-b) b d} \left(\frac{32 \cos [c+d x] (2 a+b-b \cos [2 (c+d x)])}{8 a-3 b+4 b \cos [2 (c+d x)]-b \cos [4 (c+d x)]} + \right. \\
& \quad \left. i \operatorname{RootSum}\left[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \frac{1}{-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7}\right. \right. \\
& \quad \left. \left. \left(-2 b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]+i b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right]-8 a \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^2+\right. \right. \right. \\
& \quad \left. \left. 22 b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^2+4 i a \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2-11 i b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2+\right. \right. \\
& \quad \left. \left. 8 a \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^4-22 b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^4-4 i a \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^4+\right. \right. \\
& \quad \left. \left. 11 i b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^4+2 b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^6-i b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^6\right) \& \right] \left. \right)
\end{aligned}$$

■ **Problem 215: Result is not expressed in closed-form.**

$$\int \frac{\sin [c+d x]^3}{(a-b \sin [c+d x]^4)^2} dx$$

Optimal (type 3, 186 leaves, 5 steps):

$$\begin{aligned}
& - \frac{\operatorname{ArcTan}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{a}-\sqrt{b}}\right]}{8 \sqrt{a} (\sqrt{a}-\sqrt{b})^{3/2} b^{3/4} d} + \frac{\operatorname{ArcTan}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{a}+\sqrt{b}}\right]}{8 \sqrt{a} (\sqrt{a}+\sqrt{b})^{3/2} b^{3/4} d} - \frac{\cos [c+d x] (2-\cos [c+d x])^2}{4 (a-b) d (a-b+2 b \cos [c+d x])^2-b \cos [c+d x]^4}
\end{aligned}$$

Result (type 7, 345 leaves):

$$\begin{aligned}
& \frac{1}{32 (a-b) d} \left(\frac{16 (-5 \cos [c+d x] + \cos [3 (c+d x)])}{8 a-3 b+4 b \cos [2 (c+d x)]-b \cos [4 (c+d x)]} - i \operatorname{RootSum}\left[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \right. \right. \\
& \quad \left. \left. \left(-2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]+i \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right]+14 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^2-\right. \right. \right. \\
& \quad \left. \left. 7 i \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2-14 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^4+7 i \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^4+\right. \right. \\
& \quad \left. \left. 2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^6-i \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^6\right) / \left(-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7\right) \& \right] \left. \right)
\end{aligned}$$

■ **Problem 216: Result is not expressed in closed-form.**

$$\int \frac{\sin [c+d x]}{(a-b \sin [c+d x]^4)^2} dx$$

Optimal (type 3, 221 leaves, 5 steps):

$$-\frac{(3\sqrt{a}-2\sqrt{b})\operatorname{ArcTan}\left[\frac{b^{1/4}\cos[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8a^{3/2}(\sqrt{a}-\sqrt{b})^{3/2}b^{1/4}d}-\frac{(3\sqrt{a}+2\sqrt{b})\operatorname{ArcTanh}\left[\frac{b^{1/4}\cos[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8a^{3/2}(\sqrt{a}+\sqrt{b})^{3/2}b^{1/4}d}-\frac{\cos[c+dx](a+b-b\cos[c+dx]^2)}{4a(a-b)d(a-b+2b\cos[c+dx]^2-b\cos[c+dx]^4)}$$

Result (type 7, 469 leaves):

$$-\frac{1}{32a(a-b)d}\left(\frac{32\cos[c+dx](2a+b-b\cos[2(c+dx)])}{8a-3b+4b\cos[2(c+dx)]-b\cos[4(c+dx)]}+\right. \\ \left.i\operatorname{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\ \&, \frac{1}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\right.\right. \\ \left.\left(-2b\operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right]+ib\operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right]+24a\operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right]\#1^2-\right. \\ \left.10b\operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right]\#1^2-12ia\operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right]\#1^2+5ib\operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right]\#1^2- \right. \\ \left.24a\operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right]\#1^4+10b\operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right]\#1^4+12ia\operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right]\#1^4- \right. \\ \left.5ib\operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right]\#1^4+2b\operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right]\#1^6-ib\operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right]\#1^6\right)\ \& \left. \right]$$

■ **Problem 217: Result is not expressed in closed-form.**

$$\int \frac{\csc[c+dx]}{(a-b\sin[c+dx]^4)^2} dx$$

Optimal (type 3, 325 leaves, 11 steps):

$$-\frac{b^{1/4}\operatorname{ArcTan}\left[\frac{b^{1/4}\cos[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8a^{3/2}(\sqrt{a}-\sqrt{b})^{3/2}d}-\frac{b^{1/4}\operatorname{ArcTan}\left[\frac{b^{1/4}\cos[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}d}-\frac{\operatorname{ArcTanh}[\cos[c+dx]]}{a^2d}+ \\ \frac{b^{1/4}\operatorname{ArcTanh}\left[\frac{b^{1/4}\cos[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8a^{3/2}(\sqrt{a}+\sqrt{b})^{3/2}d}+\frac{b^{1/4}\operatorname{ArcTanh}\left[\frac{b^{1/4}\cos[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}d}-\frac{b\cos[c+dx](2-\cos[c+dx]^2)}{4a(a-b)d(a-b+2b\cos[c+dx]^2-b\cos[c+dx]^4)}$$

Result (type 7, 600 leaves):

$$\frac{1}{32 a^2 d} \left(\frac{16 a b (-5 \cos [c+d x]+ \cos [3(c+d x)])}{(a-b)(8 a-3 b+4 b \cos [2(c+d x)]-b \cos [4(c+d x)])} - \right.$$

$$32 \log \left[\cos \left[\frac{1}{2}(c+d x) \right] \right] + 32 \log \left[\sin \left[\frac{1}{2}(c+d x) \right] \right] - \frac{1}{a-b} i b \operatorname{RootSum} \left[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \right.$$

$$\frac{1}{-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7} \left(-10 a \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1} \right] + 8 b \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1} \right] + \right.$$

$$5 i a \log \left[1-2 \cos [c+d x] \#1+\#1^2 \right] - 4 i b \log \left[1-2 \cos [c+d x] \#1+\#1^2 \right] + 38 a \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1} \right] \#1^2 -$$

$$24 b \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1} \right] \#1^2 - 19 i a \log \left[1-2 \cos [c+d x] \#1+\#1^2 \right] \#1^2 + 12 i b \log \left[1-2 \cos [c+d x] \#1+\#1^2 \right] \#1^2 -$$

$$38 a \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1} \right] \#1^4 + 24 b \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1} \right] \#1^4 + 19 i a \log \left[1-2 \cos [c+d x] \#1+\#1^2 \right] \#1^4 -$$

$$12 i b \log \left[1-2 \cos [c+d x] \#1+\#1^2 \right] \#1^4 + 10 a \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1} \right] \#1^6 - 8 b \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1} \right] \#1^6 -$$

$$\left. 5 i a \log \left[1-2 \cos [c+d x] \#1+\#1^2 \right] \#1^6 + 4 i b \log \left[1-2 \cos [c+d x] \#1+\#1^2 \right] \#1^6 \right) \& \left. \right)$$

■ **Problem 224: Result is not expressed in closed-form.**

$$\int \frac{\sin [c+d x]^9}{(a-b \sin [c+d x]^4)^3} dx$$

Optimal (type 3, 315 leaves, 6 steps):

$$\frac{(5 a-14 \sqrt{a} \sqrt{b}+12 b) \operatorname{ArcTan} \left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}} \right]}{64 \sqrt{a}(\sqrt{a}-\sqrt{b})^{5/2} b^{9/4} d} - \frac{(5 a+14 \sqrt{a} \sqrt{b}+12 b) \operatorname{ArcTanh} \left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}} \right]}{64 \sqrt{a}(\sqrt{a}+\sqrt{b})^{5/2} b^{9/4} d} -$$

$$\frac{a \cos [c+d x](a+b-b \cos [c+d x]^2)}{8(a-b) b^2 d(a-b+2 b \cos [c+d x]^2-b \cos [c+d x]^4)^2} + \frac{\cos [c+d x](9 a^2-11 a b-10 b^2-2(2 a-5 b) b \cos [c+d x]^2)}{32(a-b)^2 b^2 d(a-b+2 b \cos [c+d x]^2-b \cos [c+d x]^4)}$$

Result (type 7, 785 leaves):

$$\frac{1}{128 (a-b)^2 b^2 d} \left(-\frac{32 \cos [c+d x] \left(-9 a^2+13 a b+5 b^2+(2 a-5 b) b \cos [2(c+d x)]\right)}{8 a-3 b+4 b \cos [2(c+d x)]-b \cos [4(c+d x)]} - \right.$$

$$\frac{512 a(a-b) \cos [c+d x] \left(2 a+b-b \cos [2(c+d x)]\right)}{\left(-8 a+3 b-4 b \cos [2(c+d x)]+b \cos [4(c+d x)]\right)^2} + i \operatorname{RootSum}\left[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \right.$$

$$\frac{1}{-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7} \left(-4 a b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]+10 b^2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]+ \right.$$

$$2 i a b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right]-5 i b^2 \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right]-20 a^2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^2 +$$

$$56 a b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^2-78 b^2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^2+10 i a^2 \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2 -$$

$$28 i a b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2+39 i b^2 \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2+20 a^2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^4 -$$

$$56 a b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^4+78 b^2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^4-10 i a^2 \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^4 +$$

$$28 i a b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^4-39 i b^2 \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^4+4 a b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^6 -$$

$$\left. 10 b^2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^6-2 i a b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^6+5 i b^2 \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^6\right) \& \left. \right)$$

■ **Problem 225: Result is not expressed in closed-form.**

$$\int \frac{\sin [c+d x]^7}{(a-b \sin [c+d x]^4)^3} dx$$

Optimal (type 3, 290 leaves, 6 steps):

$$\frac{3\left(\sqrt{a}-2 \sqrt{b}\right) \operatorname{ArcTan}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]-3\left(\sqrt{a}+2 \sqrt{b}\right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 \sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2} b^{7/4} d}-\frac{3\left(\sqrt{a}+2 \sqrt{b}\right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 \sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2} b^{7/4} d}-$$

$$\frac{a \cos [c+d x]\left(2-\cos [c+d x]^2\right)}{8(a-b) b d\left(a-b+2 b \cos [c+d x]^2-b \cos [c+d x]^4\right)^2}+\frac{\cos [c+d x]\left(5 a-17 b-3(a-3 b) \cos [c+d x]^2\right)}{32(a-b)^2 b d\left(a-b+2 b \cos [c+d x]^2-b \cos [c+d x]^4\right)}$$

Result (type 7, 630 leaves):

$$\frac{1}{256 (a-b)^2 b d} \left(-\frac{32 \operatorname{Cos}[c+d x] (-7 a+25 b+3 (a-3 b) \operatorname{Cos}[2(c+d x)])}{8 a-3 b+4 b \operatorname{Cos}[2(c+d x)]-b \operatorname{Cos}[4(c+d x)]} + \frac{512 a(a-b) (-5 \operatorname{Cos}[c+d x]+\operatorname{Cos}[3(c+d x)])}{(-8 a+3 b-4 b \operatorname{Cos}[2(c+d x)]+b \operatorname{Cos}[4(c+d x)])^2} - \right.$$

$$3 i \operatorname{RootSum}\left[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \frac{1}{-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7}\right.$$

$$\left. \left(2 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right]-6 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right]-i a \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right]+3 i b \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right]-\right.$$

$$6 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] \#1^2+34 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] \#1^2+3 i a \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] \#1^2-$$

$$17 i b \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] \#1^2+6 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] \#1^4-34 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] \#1^4-$$

$$3 i a \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] \#1^4+17 i b \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] \#1^4-2 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] \#1^6+$$

$$\left. \left. 6 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] \#1^6+i a \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] \#1^6-3 i b \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] \#1^6\right) \&\right]$$

■ **Problem 226: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sin}[c+d x]^5}{(a-b \operatorname{Sin}[c+d x]^4)^3} dx$$

Optimal (type 3, 313 leaves, 6 steps):

$$\frac{(3 a-10 \sqrt{a} \sqrt{b}+4 b) \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a}-\sqrt{b})^{5/2} b^{5/4} d} + \frac{(3 a+10 \sqrt{a} \sqrt{b}+4 b) \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cos}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a}+\sqrt{b})^{5/2} b^{5/4} d} -$$

$$\frac{\operatorname{Cos}[c+d x] (a+b-b \operatorname{Cos}[c+d x]^2)}{8 (a-b) b d (a-b+2 b \operatorname{Cos}[c+d x]^2-b \operatorname{Cos}[c+d x]^4)^2} + \frac{\operatorname{Cos}[c+d x] (a^2-11 a b-2 b^2+2 b (2 a+b) \operatorname{Cos}[c+d x]^2)}{32 a (a-b)^2 b d (a-b+2 b \operatorname{Cos}[c+d x]^2-b \operatorname{Cos}[c+d x]^4)}$$

Result (type 7, 786 leaves):

$$\frac{1}{128 (a-b)^2 b d} \left(\frac{32 \cos [c+d x] \left(a^2 - 9 a b - b^2 + b (2 a + b) \cos [2 (c+d x)] \right)}{a (8 a - 3 b + 4 b \cos [2 (c+d x)] - b \cos [4 (c+d x)])} - \right.$$

$$\frac{512 (a-b) \cos [c+d x] (2 a + b - b \cos [2 (c+d x)])}{(-8 a + 3 b - 4 b \cos [2 (c+d x)] + b \cos [4 (c+d x)])^2} + \frac{1}{a} i \operatorname{RootSum} \left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right.$$

$$\left. \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(4 a b \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] + 2 b^2 \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] - \right.$$

$$2 i a b \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] - i b^2 \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] + 12 a^2 \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1^2 -$$

$$64 a b \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1^2 + 10 b^2 \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1^2 - 6 i a^2 \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] \#1^2 +$$

$$32 i a b \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] \#1^2 - 5 i b^2 \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] \#1^2 - 12 a^2 \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1^4 +$$

$$64 a b \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1^4 - 10 b^2 \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1^4 + 6 i a^2 \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] \#1^4 -$$

$$32 i a b \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] \#1^4 + 5 i b^2 \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] \#1^4 - 4 a b \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1^6 -$$

$$\left. \left. 2 b^2 \operatorname{ArcTan} \left[\frac{\sin [c+d x]}{\cos [c+d x] - \#1} \right] \#1^6 + 2 i a b \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] \#1^6 + i b^2 \operatorname{Log} [1 - 2 \cos [c+d x] \#1 + \#1^2] \#1^6 \right) \& \right]$$

■ **Problem 227: Result is not expressed in closed-form.**

$$\int \frac{\sin [c+d x]^3}{(a-b \sin [c+d x]^4)^3} dx$$

Optimal (type 3, 288 leaves, 6 steps):

$$-\frac{(5 \sqrt{a} - 2 \sqrt{b}) \operatorname{ArcTan} \left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a} - \sqrt{b}}} \right]}{64 a^{3/2} (\sqrt{a} - \sqrt{b})^{5/2} b^{3/4} d} + \frac{(5 \sqrt{a} + 2 \sqrt{b}) \operatorname{ArcTan} \left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a} + \sqrt{b}}} \right]}{64 a^{3/2} (\sqrt{a} + \sqrt{b})^{5/2} b^{3/4} d} -$$

$$\frac{\cos [c+d x] (2 - \cos [c+d x]^2)}{8 (a-b) d (a-b + 2 b \cos [c+d x]^2 - b \cos [c+d x]^4)^2} - \frac{\cos [c+d x] (11 a + b - (5 a + b) \cos [c+d x]^2)}{32 a (a-b)^2 d (a-b + 2 b \cos [c+d x]^2 - b \cos [c+d x]^4)^2}$$

Result (type 7, 631 leaves):

$$\frac{1}{256 (a-b)^2 d} \left(\frac{32 \cos [c+d x] (-17 a-b+(5 a+b) \cos [2(c+d x)])}{a(8 a-3 b+4 b \cos [2(c+d x)]-b \cos [4(c+d x)])} + \frac{512(a-b)(-5 \cos [c+d x]+\cos [3(c+d x)])}{(-8 a+3 b-4 b \cos [2(c+d x)]+b \cos [4(c+d x)])^2} + \frac{1}{a} \operatorname{RootSum}\left[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8\right] \&, \right. \\ \left. \frac{1}{-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7} \left(10 a \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]+2 b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]- \right. \\ \left. 5 i a \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right]-i b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right]-94 a \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^2+ \right. \\ \left. 10 b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^2+47 i a \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2-5 i b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2+ \right. \\ \left. 94 a \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^4-10 b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^4-47 i a \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^4+ \right. \\ \left. 5 i b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^4-10 a \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^6-2 b \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^6+ \right. \\ \left. 5 i a \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^6+i b \operatorname{Log}\left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^6\right) \& \left. \right)$$

■ **Problem 228: Result is not expressed in closed-form.**

$$\int \frac{\sin [c+d x]}{(a-b \sin [c+d x]^4)^3} dx$$

Optimal (type 3, 313 leaves, 6 steps):

$$\frac{3(7 a-10 \sqrt{a} \sqrt{b}+4 b) \operatorname{ArcTan}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64 a^{5/2}(\sqrt{a}-\sqrt{b})^{5/2} b^{1/4} d} - \frac{3(7 a+10 \sqrt{a} \sqrt{b}+4 b) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 a^{5/2}(\sqrt{a}+\sqrt{b})^{5/2} b^{1/4} d} \\ - \frac{\cos [c+d x](a+b-b \cos [c+d x]^2)}{8 a(a-b) d(a-b+2 b \cos [c+d x]^2-b \cos [c+d x]^4)^2} - \frac{\cos [c+d x]((7 a-3 b)(a+2 b)-6(2 a-b) b \cos [c+d x]^2)}{32 a^2(a-b)^2 d(a-b+2 b \cos [c+d x]^2-b \cos [c+d x]^4)^2}$$

Result (type 7, 784 leaves):

$$\begin{aligned}
& \frac{1}{128 a^2 (a-b)^2 d} \left(- \frac{32 \operatorname{Cos}[c+dx] (7 a^2 + 5 a b - 3 b^2 + 3 b (-2 a + b) \operatorname{Cos}[2(c+dx)])}{8 a - 3 b + 4 b \operatorname{Cos}[2(c+dx)] - b \operatorname{Cos}[4(c+dx)]} - \right. \\
& \frac{512 a (a-b) \operatorname{Cos}[c+dx] (2 a + b - b \operatorname{Cos}[2(c+dx)])}{(-8 a + 3 b - 4 b \operatorname{Cos}[2(c+dx)] + b \operatorname{Cos}[4(c+dx)])^2} + 3 i \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \\
& \left. \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(4 a b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] - 2 b^2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] - \right. \right. \\
& \left. 2 i a b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] + i b^2 \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] - 28 a^2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^2 + \right. \\
& \left. 24 a b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^2 - 10 b^2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^2 + 14 i a^2 \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^2 - \right. \\
& \left. 12 i a b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^2 + 5 i b^2 \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^2 + 28 a^2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^4 - \right. \\
& \left. 24 a b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^4 + 10 b^2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^4 - 14 i a^2 \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^4 + \right. \\
& \left. 12 i a b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^4 - 5 i b^2 \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^4 - 4 a b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^6 + \right. \\
& \left. 2 b^2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^6 + 2 i a b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^6 - i b^2 \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^6 \right) \& \left. \right)
\end{aligned}$$

■ **Problem 229: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Csc}[c+dx]}{(a-b \operatorname{Sin}[c+dx]^4)^3} dx$$

Optimal (type 3, 617 leaves, 16 steps):

$$\begin{aligned}
& \frac{(5\sqrt{a} - 2\sqrt{b}) b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64 a^{5/2} (\sqrt{a} - \sqrt{b})^{5/2} d} - \frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8 a^{5/2} (\sqrt{a} - \sqrt{b})^{3/2} d} \\
& - \frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a^3 \sqrt{\sqrt{a} - \sqrt{b}} d} - \frac{\operatorname{ArcTanh}[\cos[c+dx]]}{a^3 d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 a^{5/2} (\sqrt{a} + \sqrt{b})^{3/2} d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a^3 \sqrt{\sqrt{a} + \sqrt{b}} d} + \\
& \frac{(5\sqrt{a} + 2\sqrt{b}) b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 a^{5/2} (\sqrt{a} + \sqrt{b})^{5/2} d} - \frac{b \cos[c+dx] (2 - \cos[c+dx])^2}{8 a (a-b) d (a-b+2b \cos[c+dx])^2 - b \cos[c+dx]^4} \\
& - \frac{b \cos[c+dx] (2 - \cos[c+dx])^2}{4 a^2 (a-b) d (a-b+2b \cos[c+dx])^2 - b \cos[c+dx]^4} - \frac{b \cos[c+dx] (11a+b - (5a+b) \cos[c+dx])^2}{32 a^2 (a-b)^2 d (a-b+2b \cos[c+dx])^2 - b \cos[c+dx]^4}
\end{aligned}$$

Result (type 7, 920 leaves):

$$\begin{aligned}
& \frac{1}{256 a^3 d} \left(\frac{32 a b \cos[c+dx] (-41a+23b+(13a-7b) \cos[2(c+dx)])}{(a-b)^2 (8a-3b+4b \cos[2(c+dx)] - b \cos[4(c+dx)])} + \right. \\
& \frac{512 a^2 b (-5 \cos[c+dx] + \cos[3(c+dx)])}{(a-b) (-8a+3b-4b \cos[2(c+dx)] + b \cos[4(c+dx)])^2} - 256 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]\right] + 256 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right] - \\
& \frac{1}{(a-b)^2} i b \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \right. \\
& \left. \left(-90 a^2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] + 142 a b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] - 64 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] + \right. \\
& 45 i a^2 \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] - 71 i a b \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] + 32 i b^2 \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] + \\
& 398 a^2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^2 - 506 a b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^2 + 192 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^2 - \\
& 199 i a^2 \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^2 + 253 i a b \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^2 - 96 i b^2 \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^2 - \\
& 398 a^2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^4 + 506 a b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^4 - 192 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^4 + \\
& 199 i a^2 \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^4 - 253 i a b \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^4 + 96 i b^2 \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^4 + \\
& 90 a^2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^6 - 142 a b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^6 + 64 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^6 - \\
& \left. 45 i a^2 \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^6 + 71 i a b \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^6 - 32 i b^2 \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^6 \right) \& \left. \right)
\end{aligned}$$

■ **Problem 237: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{a + b \sin[x]^4} dx$$

Optimal (type 3, 487 leaves, 10 steps):

$$\begin{aligned} & - \frac{(\sqrt{a} + \sqrt{a+b}) \operatorname{ArcTan}\left[\frac{a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b} - \sqrt{2} (a+b)^{3/4} \tan[x]}{a^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}}\right] + (\sqrt{a} + \sqrt{a+b}) \operatorname{ArcTan}\left[\frac{a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b} + \sqrt{2} (a+b)^{3/4} \tan[x]}{a^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}}\right]}{2 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}} + \frac{(\sqrt{a} + \sqrt{a+b}) \operatorname{ArcTan}\left[\frac{a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b} + \sqrt{2} (a+b)^{3/4} \tan[x]}{a^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}}\right]}{2 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}} + \\ & \frac{(\sqrt{a} - \sqrt{a+b}) \operatorname{Log}\left[\sqrt{a} (a+b)^{1/4} - \sqrt{2} a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b} \tan[x] + (a+b)^{3/4} \tan[x]^2\right]}{4 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}} - \\ & \frac{(\sqrt{a} - \sqrt{a+b}) \operatorname{Log}\left[\sqrt{a} (a+b)^{1/4} + \sqrt{2} a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b} \tan[x] + (a+b)^{3/4} \tan[x]^2\right]}{4 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}} \end{aligned}$$

Result (type 3, 148 leaves):

$$\frac{1}{2a(a+b)} \left((\sqrt{a} - i\sqrt{b}) \sqrt{a+i\sqrt{a}\sqrt{b}} \operatorname{ArcTan}\left[\frac{\sqrt{a+i\sqrt{a}\sqrt{b}} \tan[x]}{\sqrt{a}}\right] - (\sqrt{a} + i\sqrt{b}) \sqrt{-a+i\sqrt{a}\sqrt{b}} \operatorname{ArcTanh}\left[\frac{\sqrt{-a+i\sqrt{a}\sqrt{b}} \tan[x]}{\sqrt{a}}\right] \right)$$

■ **Problem 238: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{1 + \sin[x]^4} dx$$

Optimal (type 3, 309 leaves, 10 steps):

$$\begin{aligned} & \frac{x}{2 \sqrt{-1+\sqrt{2}}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{-1+\sqrt{2}} - 2\sqrt{-1+\sqrt{2}} \cos[x]^2 - (-2+\sqrt{2}) \cos[x] \sin[x]}{2+\sqrt{1+\sqrt{2}} + (-2+\sqrt{2}) \cos[x]^2 - 2\sqrt{-1+\sqrt{2}} \cos[x] \sin[x]}\right]}{4 \sqrt{-1+\sqrt{2}}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{-1+\sqrt{2}} - 2\sqrt{-1+\sqrt{2}} \cos[x]^2 + (-2+\sqrt{2}) \cos[x] \sin[x]}{2+\sqrt{1+\sqrt{2}} + (-2+\sqrt{2}) \cos[x]^2 + 2\sqrt{-1+\sqrt{2}} \cos[x] \sin[x]}\right]}{4 \sqrt{-1+\sqrt{2}}} - \\ & \frac{1}{8} \sqrt{-1+\sqrt{2}} \operatorname{Log}\left[\sqrt{2} - 2\sqrt{-1+\sqrt{2}} \tan[x] + 2 \tan[x]^2\right] + \frac{1}{8} \sqrt{-1+\sqrt{2}} \operatorname{Log}\left[1 + \sqrt{2} (-1+\sqrt{2}) \tan[x] + \sqrt{2} \tan[x]^2\right] \end{aligned}$$

Result (type 3, 45 leaves):

$$\frac{\operatorname{ArcTan}\left[\sqrt{1-i} \tan[x]\right]}{2 \sqrt{1-i}} + \frac{\operatorname{ArcTan}\left[\sqrt{1+i} \tan[x]\right]}{2 \sqrt{1+i}}$$

■ **Problem 239: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sin[c + dx] \sqrt{a + b \sin^2[c + dx]} dx$$

Optimal (type 4, 477 leaves, 5 steps):

$$\begin{aligned} & - \frac{\cos[c + dx] \sqrt{a + b - 2b \cos[c + dx]^2 + b \cos^4[c + dx]}}{3d} + \frac{2\sqrt{b} \cos[c + dx] \sqrt{a + b - 2b \cos[c + dx]^2 + b \cos^4[c + dx]}}{3\sqrt{a+b} d \left(1 + \frac{\sqrt{b} \cos^2[c + dx]}{\sqrt{a+b}}\right)} \\ & \left(2b^{1/4} (a+b)^{3/4} \left(1 + \frac{\sqrt{b} \cos^2[c + dx]}{\sqrt{a+b}}\right) \sqrt{\frac{a + b - 2b \cos[c + dx]^2 + b \cos^4[c + dx]}{(a+b) \left(1 + \frac{\sqrt{b} \cos^2[c + dx]}{\sqrt{a+b}}\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c + dx]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] \right) / \\ & \left(3d \sqrt{a + b - 2b \cos[c + dx]^2 + b \cos^4[c + dx]} \right) + \left((a+b)^{3/4} (\sqrt{b} - \sqrt{a+b}) \left(1 + \frac{\sqrt{b} \cos^2[c + dx]}{\sqrt{a+b}}\right) \sqrt{\frac{a + b - 2b \cos[c + dx]^2 + b \cos^4[c + dx]}{(a+b) \left(1 + \frac{\sqrt{b} \cos^2[c + dx]}{\sqrt{a+b}}\right)^2}} \right. \\ & \left. \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c + dx]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] \right) / \left(3b^{1/4} d \sqrt{a + b - 2b \cos[c + dx]^2 + b \cos^4[c + dx]} \right) \end{aligned}$$

Result (type 4, 542 leaves):

$$\begin{aligned}
& - \frac{\cos [c+d x] \sqrt{8 a+3 b-4 b \cos [2(c+d x)]+b \cos [4(c+d x)]}}{6 \sqrt{2} d} + \\
& \frac{1}{3 d \sqrt{\sec [c+d x]^2(1+\tan [c+d x]^2)^{3 / 2}} \sqrt{\frac{b \tan [c+d x]^4+a(1+\tan [c+d x]^2)^2}{(1+\tan [c+d x]^2)^2}}} 2 \sec [c+d x] \\
& \left(a+2 a \tan [c+d x]^2+a \tan [c+d x]^4+b \tan [c+d x]^4 - \left(-i \sqrt{b} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i(a-i \sqrt{a} \sqrt{b}+a \tan [c+d x]^2+b \tan [c+d x]^2)}{\sqrt{a} \sqrt{b}}}}{\sqrt{2}}}\right], \frac{2 \sqrt{a}}{\sqrt{a}-i \sqrt{b}}\right] + \right. \\
& \left. (\sqrt{a}+i \sqrt{b}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i(a-i \sqrt{a} \sqrt{b}+a \tan [c+d x]^2+b \tan [c+d x]^2)}{\sqrt{a} \sqrt{b}}}}{\sqrt{2}}}\right], \frac{2 \sqrt{a}}{\sqrt{a}-i \sqrt{b}}\right] \right) \sqrt{\frac{(-i \sqrt{a}+\sqrt{b})(1+\tan [c+d x]^2)}{\sqrt{b}}} \\
& \left. \sqrt{\frac{i(a-i \sqrt{a} \sqrt{b}+a \tan [c+d x]^2+b \tan [c+d x]^2)}{\sqrt{a} \sqrt{b}}} (-i \sqrt{b} \tan [c+d x]^2+\sqrt{a}(1+\tan [c+d x]^2)) \right) / \\
& \left(\sqrt{-\frac{i(a+i \sqrt{a} \sqrt{b}+a \tan [c+d x]^2+b \tan [c+d x]^2)}{\sqrt{a} \sqrt{b}}} \right)
\end{aligned}$$

■ **Problem 240: Unable to integrate problem.**

$$\int \csc [c+d x] \sqrt{a+b \sin [c+d x]^4} dx$$

Optimal (type 4, 521 leaves, 8 steps):

$$\frac{\sqrt{-a} \operatorname{ArcTan}\left[\frac{\sqrt{-a} \cos[c+dx]}{\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}}\right]}{2d} + \frac{\sqrt{b} \cos[c+dx] \sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}}{\sqrt{a+b} d \left(1 + \frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right)}$$

$$\left(b^{1/4} (a+b)^{3/4} \left(1 + \frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b) \left(1 + \frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] \right) /$$

$$\left(d \sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \right) - \left((a+b)^{1/4} (\sqrt{b} - \sqrt{a+b})^2 \left(1 + \frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b) \left(1 + \frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{(\sqrt{b} + \sqrt{a+b})^2}{4\sqrt{b}\sqrt{a+b}}, 2 \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] \right) / \left(4 b^{1/4} d \sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}\right)$$

Result (type 8, 25 leaves):

$$\int \operatorname{Csc}[c+dx] \sqrt{a+b \sin[c+dx]^4} dx$$

■ **Problem 241: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sin[c+dx]^5}{\sqrt{a+b \sin[c+dx]^4}} dx$$

Optimal (type 4, 484 leaves, 5 steps):

$$\begin{aligned}
& - \frac{\cos [c+d x] \sqrt{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}}{3 b d} + \frac{2 \cos [c+d x] \sqrt{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}}{3 \sqrt{b} \sqrt{a+b} d \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right)} - \\
& \left(2 (a+b)^{3 / 4} \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1 / 4} \cos [c+d x]}{(a+b)^{1 / 4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] \right) / \\
& \left(3 b^{3 / 4} d \sqrt{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4} \right) + \\
& \left((a+b)^{1 / 4} (a-2 b+2 \sqrt{b} \sqrt{a+b}) \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right)^2}} \right. \\
& \left. \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1 / 4} \cos [c+d x]}{(a+b)^{1 / 4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] \right) / \left(6 b^{5 / 4} d \sqrt{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4} \right)
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 242: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin [c+d x]^3}{\sqrt{a+b \sin [c+d x]^4}} dx$$

Optimal (type 4, 431 leaves, 4 steps):

$$\frac{\cos[c+dx] \sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}}{\sqrt{b}\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right)}$$

$$\left((a+b)^{3/4} \left(1 + \frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}} \right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b)\left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] \right) /$$

$$\left(b^{3/4}d\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \right) - \left((a+b)^{1/4}(\sqrt{b}-\sqrt{a+b}) \left(1 + \frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}} \right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b)\left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \right)$$

$$\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] / \left(2b^{3/4}d\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \right)$$

Result (type 4, 35489 leaves) : Display of huge result suppressed!

■ **Problem 243: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin[c+dx]}{\sqrt{a+b\sin[c+dx]^4}} dx$$

Optimal (type 4, 171 leaves, 2 steps) :

$$- \left((a+b)^{1/4} \left(1 + \frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}} \right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b)\left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] \right) / \left(2b^{1/4}d\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \right)$$

Result (type 4, 13300 leaves) :

$$- \left(\left(8\sqrt{2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\left(\left(\operatorname{Root}\left[a+4a\#1+(6a+16b)\#1^2+4a\#1^3+a\#1^4\&, 2\right]-\operatorname{Root}\left[a+4a\#1+(6a+16b)\#1^2+4a\#1^3+a\#1^4\&, 4\right]\right)}\right]} \right) \right. \right. \\ \left. \left. \left(-\operatorname{Root}\left[a+4a\#1+(6a+16b)\#1^2+4a\#1^3+a\#1^4\&, 1\right] + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) /$$

$$\begin{aligned}
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right), \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
& \quad \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right) / \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
& \quad \left. \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right) \right) \\
& \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \\
& \text{Sin}[c + d x] \\
& \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \\
& \sqrt{\left(\left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \right. \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \\
& \sqrt{\left(\left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \right. \right. \\
& \quad \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \\
& \quad \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right)^2 \\
& \quad \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \left(\sqrt{8 a + 3 b - 4 b \text{Cos}[2 (c + d x)] + b \text{Cos}[4 (c + d x)]} \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \right. \right. \\
& \quad \left. \left. \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \\
& \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \Big/ \\
& \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right)^2 \right. \\
& \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/ \\
& \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] \right) \\
& \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right) \\
& \left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \sqrt{\frac{16 b \text{Tan}\left[\frac{1}{2} (c + d x)\right]^4 + a \left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^4}{\left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^4}} \right) \Big/ \\
& \left(8 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right)} \right]} \right] \right. \\
& \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/ \\
& \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right) \right. \\
& \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/ \\
& \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right] \right) \right. \\
& \left. \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right) \right) \Big/ \\
& \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right] \right) \right. \\
& \left. \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right) \right) \Big/ \\
& \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right) \\
& \text{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \text{Tan}\left[\frac{1}{2} (c + d x)\right] \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \\
& \sqrt{\left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] \right) \right)}
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \Big/ \\
& \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right] \right) \right. \\
& \quad \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/ \\
& \sqrt{\left(\left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] \right) \right. \right. \\
& \quad \left. \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right) \right. \\
& \quad \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right. \\
& \quad \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/ \\
& \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right)^2 \right. \\
& \quad \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/ \\
& \left(\left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] \right) \right. \\
& \quad \left. \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right) \right. \\
& \quad \left. \left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^3 \sqrt{\frac{16 b \text{Tan}\left[\frac{1}{2} (c + d x)\right]^4 + a \left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^4}{\left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^4}} \right) - \\
& \left(2 \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right) \right. \\
& \quad \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] \right) \right. \right. \right. \\
& \quad \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \\
\sqrt{\left(\left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \right. \right. \\
& \quad \left. \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right)^2 \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \right) \\
& \left(- \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \right. \\
& \quad \left. \left. \text{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \text{Tan}\left[\frac{1}{2} (c + d x)\right] \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \right) + \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \text{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \right. \\
& \quad \left. \text{Tan}\left[\frac{1}{2} (c + d x)\right] \right) / \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, \right. \right. \\
& \quad \left. \left. 4] \right) \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \\
& \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \right. \\
& \quad \left. \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \quad \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \left(-\text{Root}[\right. \right. \\
& \quad \left. \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \sqrt{\frac{16 b \text{Tan}\left[\frac{1}{2} (c + d x)\right]^4 + a \left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^4}{\left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^4}} \\
& \sqrt{\left(1 - \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \right. \\
& \quad \left. \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \quad \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \\
& \sqrt{\left(1 - \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \right. \\
& \quad \left. \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \quad \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \right) - \\
& \left(2 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \right) / \right. \\
& \quad \left. \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \right. \\
& \quad \left. \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \right) \right],
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \right. \right. \\
& \quad \left. \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/ \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right)^2 \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \right) \\
& \sqrt{\frac{16 b \text{Tan}\left[\frac{1}{2} (c + d x)\right]^4 + a \left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^4}{\left(1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^4}} + \left(2 \text{EllipticF}\left[\text{ArcSin}\left[\right. \right. \right. \\
& \sqrt{\left(\left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/ \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/ \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
& \quad \left. \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right) \Big/ \\
& \left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
& \quad \left. \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right) \Big/ \\
& \left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \\
& \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \\
& \sqrt{\left(\left(\left(\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \right. \right. \\
& \quad \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{\sqrt{-a} \cos[c+dx]}{\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}}\right]}{2\sqrt{-a}d} + \left(b^{1/4} (a+b)^{1/4} (\sqrt{b} - \sqrt{a+b}) \left(1 + \frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}} \right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b) \left(1 + \frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}} \right)^2}} \right. \\
& \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{\sqrt{b}}{\sqrt{a+b}} \right) \right] \right) / \left(2ad \sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \right) - \\
& \left((a+b)^{1/4} (\sqrt{b} - \sqrt{a+b})^2 \left(1 + \frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}} \right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b) \left(1 + \frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}} \right)^2}} \right. \\
& \left. \text{EllipticPi}\left[\frac{(\sqrt{b} + \sqrt{a+b})^2}{4\sqrt{b}\sqrt{a+b}}, 2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{\sqrt{b}}{\sqrt{a+b}} \right) \right] \right) / \left(4ab^{1/4}d \sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \right)
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\csc[c+dx]}{\sqrt{a+b\sin[c+dx]^4}} dx$$

■ **Problem 245: Attempted integration timed out after 120 seconds.**

$$\int \frac{\csc[c+dx]^3}{\sqrt{a+b\sin[c+dx]^4}} dx$$

Optimal (type 4, 776 leaves, 7 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{\sqrt{-a} \cos[c+dx]}{\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}}\right]}{4\sqrt{-a}d} - \frac{\sqrt{b} \cos[c+dx] \sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}}{2a\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right)} \\
& \frac{\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \cot[c+dx] \csc[c+dx]}{2ad} + \\
& \left(b^{1/4} (a+b)^{3/4} \left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b)\left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[\frac{b^{1/4}\cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] \right) / \\
& \left(2ad\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \right) - \left(b^{1/4} (a+b-\sqrt{b}\sqrt{a+b}) \left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b)\left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \right) \\
& \text{EllipticF}\left[2\text{ArcTan}\left[\frac{b^{1/4}\cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] / \left(2a(a+b)^{1/4}d\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \right) - \\
& \left((a+b)^{1/4} (\sqrt{b}-\sqrt{a+b})^2 \left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4}{(a+b)\left(1+\frac{\sqrt{b}\cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \right) \\
& \text{EllipticPi}\left[\frac{(\sqrt{b}+\sqrt{a+b})^2}{4\sqrt{b}\sqrt{a+b}}, 2\text{ArcTan}\left[\frac{b^{1/4}\cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right] / \left(8ab^{1/4}d\sqrt{a+b-2b\cos[c+dx]^2+b\cos[c+dx]^4} \right)
\end{aligned}$$

Result (type 1, 1 leaves):

???

- **Problem 246: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin[c+dx]^2}{\sqrt{a+b\sin[c+dx]^4}} dx$$

Optimal (type 4, 499 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b} \tan[c+dx]}{\sqrt{a+2a \tan[c+dx]^2+(a+b) \tan[c+dx]^4}}\right] \cos[c+dx]^2 \sqrt{a+2a \tan[c+dx]^2+(a+b) \tan[c+dx]^4}}{2\sqrt{b} d \sqrt{a+b} \sin[c+dx]^4}$$

$$\left[a^{1/4} (\sqrt{a} + \sqrt{a+b}) \cos[c+dx]^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{(a+b)^{1/4} \tan[c+dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a+b}}\right)\right]\right]$$

$$\left(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2 \right) \sqrt{\frac{a+2a \tan[c+dx]^2+(a+b) \tan[c+dx]^4}{(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2)^2}} \Big/ \left(2b(a+b)^{1/4} d \sqrt{a+b} \sin[c+dx]^4 \right) +$$

$$\left[(\sqrt{a} + \sqrt{a+b})^2 \cos[c+dx]^2 \text{EllipticPi}\left[-\frac{(\sqrt{a} - \sqrt{a+b})^2}{4\sqrt{a}\sqrt{a+b}}, 2 \text{ArcTan}\left[\frac{(a+b)^{1/4} \tan[c+dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a+b}}\right)\right]\right]$$

$$\left(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2 \right) \sqrt{\frac{a+2a \tan[c+dx]^2+(a+b) \tan[c+dx]^4}{(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2)^2}} \Big/ \left(4a^{1/4} b(a+b)^{1/4} d \sqrt{a+b} \sin[c+dx]^4 \right) +$$

Result (type 4, 287 leaves):

$$\begin{aligned} & - \left(2i \cos[c+dx]^2 \left[\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}} \tan[c+dx]}\right], \frac{\sqrt{a} + i\sqrt{b}}{\sqrt{a} - i\sqrt{b}}\right] - \right. \right. \\ & \quad \left. \left. \text{EllipticPi}\left[\frac{\sqrt{a}}{\sqrt{a} - i\sqrt{b}}, i \text{ArcSinh}\left[\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}} \tan[c+dx]}\right], \frac{\sqrt{a} + i\sqrt{b}}{\sqrt{a} - i\sqrt{b}}\right] \right] \sqrt{1 + \left(1 + \frac{i\sqrt{b}}{\sqrt{a}}\right) \tan[c+dx]^2} \right. \\ & \quad \left. \sqrt{2 + \left(2 - \frac{2i\sqrt{b}}{\sqrt{a}}\right) \tan[c+dx]^2} \right) \Big/ \left(\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}} \tan[c+dx]} d \sqrt{8a + 3b - 4b \cos[2(c+dx)] + b \cos[4(c+dx)]} \right) \end{aligned}$$

■ **Problem 247: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+b} \sin[c+dx]^4} dx$$

Optimal (type 4, 162 leaves, 2 steps):

$$\left(\cos[c+dx]^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{(a+b)^{1/4} \tan[c+dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a+b}}\right)\right] \right. \\ \left. \frac{(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2) \sqrt{\frac{a + 2a \tan[c+dx]^2 + (a+b) \tan[c+dx]^4}{(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2)^2}}}{2 a^{1/4} (a+b)^{1/4} d \sqrt{a+b} \sin[c+dx]^4} \right)$$

Result (type 4, 195 leaves):

$$- \left(2 i \cos[c+dx]^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}}} \tan[c+dx]\right], \frac{\sqrt{a} + i\sqrt{b}}{\sqrt{a} - i\sqrt{b}}\right] \sqrt{1 + \left(1 + \frac{i\sqrt{b}}{\sqrt{a}}\right) \tan[c+dx]^2} \right. \\ \left. \frac{\sqrt{2 + \left(2 - \frac{2i\sqrt{b}}{\sqrt{a}}\right) \tan[c+dx]^2}}{\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}}} d \sqrt{8a + 3b - 4b \cos[2(c+dx)] + b \cos[4(c+dx)]}} \right)$$

- **Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[c+dx]^2}{\sqrt{a+b \sin[c+dx]^4}} dx$$

Optimal (type 4, 493 leaves, 5 steps):

$$\begin{aligned}
& - \frac{\text{Cos}[c+dx]^2 \text{Cot}[c+dx] (a+2a \text{Tan}[c+dx]^2 + (a+b) \text{Tan}[c+dx]^4)}{ad \sqrt{a+b \text{Sin}[c+dx]^4}} + \\
& \frac{\sqrt{a+b} \text{Cos}[c+dx] \text{Sin}[c+dx] (a+2a \text{Tan}[c+dx]^2 + (a+b) \text{Tan}[c+dx]^4)}{ad \sqrt{a+b \text{Sin}[c+dx]^4} (\sqrt{a} + \sqrt{a+b} \text{Tan}[c+dx]^2)} - \\
& \left((a+b)^{1/4} \text{Cos}[c+dx]^2 \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{(a+b)^{1/4} \text{Tan}[c+dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a+b}}\right)\right] \right. \\
& \left. (\sqrt{a} + \sqrt{a+b} \text{Tan}[c+dx]^2) \sqrt{\frac{a+2a \text{Tan}[c+dx]^2 + (a+b) \text{Tan}[c+dx]^4}{(\sqrt{a} + \sqrt{a+b} \text{Tan}[c+dx]^2)^2}} \right) / (a^{3/4} d \sqrt{a+b \text{Sin}[c+dx]^4}) + \\
& \left((a+b + \sqrt{a} \sqrt{a+b}) \text{Cos}[c+dx]^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{(a+b)^{1/4} \text{Tan}[c+dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a+b}}\right)\right] (\sqrt{a} + \sqrt{a+b} \text{Tan}[c+dx]^2) \right. \\
& \left. \sqrt{\frac{a+2a \text{Tan}[c+dx]^2 + (a+b) \text{Tan}[c+dx]^4}{(\sqrt{a} + \sqrt{a+b} \text{Tan}[c+dx]^2)^2}} \right) / (2 a^{3/4} (a+b)^{3/4} d \sqrt{a+b \text{Sin}[c+dx]^4})
\end{aligned}$$

Result (type 4, 1403 leaves):

$$\begin{aligned}
& - \frac{\sqrt{8a+3b-4b \text{Cos}[2(c+dx)] + b \text{Cos}[4(c+dx)]} \text{Cot}[c+dx]}{2\sqrt{2} ad} + \\
& \left(i b \text{Cos}[c+dx]^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}}} \text{Tan}[c+dx]\right], \frac{\sqrt{a} + i\sqrt{b}}{\sqrt{a} - i\sqrt{b}}\right] \sqrt{1 + \left(1 - \frac{i\sqrt{b}}{\sqrt{a}}\right) \text{Tan}[c+dx]^2} \right. \\
& \left. \sqrt{1 + \left(1 + \frac{i\sqrt{b}}{\sqrt{a}}\right) \text{Tan}[c+dx]^2} \right) / \left(\sqrt{2} a \sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}}} d \sqrt{8a+3b-4b \text{Cos}[2(c+dx)] + b \text{Cos}[4(c+dx)]} \right) + \\
& \left(i \sqrt{2} b \text{Cos}[c+dx]^2 \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}}} \text{Tan}[c+dx]\right], \frac{\sqrt{a} + i\sqrt{b}}{\sqrt{a} - i\sqrt{b}}\right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{EllipticPi}\left[\frac{\sqrt{a}}{\sqrt{a}-i\sqrt{b}}, i \operatorname{ArcSinh}\left[\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan[c+dx]\right], \frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right] \sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} \\
& \sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} \Bigg/ \left(a \sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\, d \sqrt{8a+3b-4b\cos[2(c+dx)]+b\cos[4(c+dx)]}\right) - \\
& \frac{1}{4a \sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\, d (1+\tan[c+dx])^2 \sqrt{\frac{b\tan[c+dx]^4+a(1+\tan[c+dx])^2}{(1+\tan[c+dx])^2}}} \\
& \left(4a \sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan[c+dx] + 8a \sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan[c+dx]^3 + 4a \sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan[c+dx]^5 + 4 \sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\, b \tan[c+dx]^5 - \right. \\
& \left. 4ib \operatorname{EllipticPi}\left[\frac{\sqrt{a}}{\sqrt{a}-i\sqrt{b}}, i \operatorname{ArcSinh}\left[\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan[c+dx]\right], \frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right] \sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} \right. \\
& \left. \sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} - 4ib \operatorname{EllipticPi}\left[\frac{\sqrt{a}}{\sqrt{a}-i\sqrt{b}}, i \operatorname{ArcSinh}\left[\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan[c+dx]\right], \frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right] \right. \\
& \left. \tan[c+dx]^2 \sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} \sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} + \right. \\
& \left. 4\sqrt{a}(i\sqrt{a}+\sqrt{b}) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan[c+dx]\right], \frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right] (1+\tan[c+dx])^2 \sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} \right. \\
& \left. \sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} - (4\sqrt{a}-3i\sqrt{b})\sqrt{b} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan[c+dx]\right], \frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right] \right. \\
& \left. (1+\tan[c+dx])^2 \sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} \sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan[c+dx]^2} \right)
\end{aligned}$$

■ **Problem 249: Result is not expressed in closed-form.**

$$\int \frac{1}{a + b \sin[x]^5} dx$$

Optimal (type 3, 384 leaves, 17 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{b^{1/5} + a^{1/5} \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^{2/5} - b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} - b^{2/5}}} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{2/5} b^{1/5} + a^{1/5} \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^{2/5} - (-1)^{4/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} - (-1)^{4/5} b^{2/5}}} +$$

$$\frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{4/5} b^{1/5} + a^{1/5} \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^{2/5} + (-1)^{3/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} + (-1)^{3/5} b^{2/5}}} - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{3/5} (b^{1/5} + (-1)^{2/5} a^{1/5} \operatorname{Tan}\left[\frac{x}{2}\right])}{\sqrt{a^{2/5} + (-1)^{1/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} + (-1)^{1/5} b^{2/5}}} - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/5} (b^{1/5} + (-1)^{4/5} a^{1/5} \operatorname{Tan}\left[\frac{x}{2}\right])}{\sqrt{a^{2/5} - (-1)^{2/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} - (-1)^{2/5} b^{2/5}}}$$

Result (type 7, 149 leaves):

$$\frac{8}{5} i \operatorname{RootSum}\left[i b - 5 i b \#1^2 + 10 i b \#1^4 + 32 a \#1^5 - 10 i b \#1^6 + 5 i b \#1^8 - i b \#1^{10} \&, \frac{2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^3 - i \operatorname{Log}\left[1 - 2 \cos[x] \#1 + \#1^2\right] \#1^3}{b - 4 b \#1^2 + 16 i a \#1^3 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8} \&\right]$$

■ **Problem 250: Result is not expressed in closed-form.**

$$\int \frac{1}{a + b \sin[x]^6} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/3} + b^{1/3}} \operatorname{Tan}[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3} + b^{1/3}}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/3} - (-1)^{1/3} b^{1/3}} \operatorname{Tan}[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3} - (-1)^{1/3} b^{1/3}}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/3} + (-1)^{2/3} b^{1/3}} \operatorname{Tan}[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3} + (-1)^{2/3} b^{1/3}}}$$

Result (type 7, 148 leaves):

$$-\frac{8}{3} \operatorname{RootSum}\left[b - 6 b \#1 + 15 b \#1^2 - 64 a \#1^3 - 20 b \#1^3 + 15 b \#1^4 - 6 b \#1^5 + b \#1^6 \&, \frac{2 \operatorname{ArcTan}\left[\frac{\sin[2x]}{\cos[2x] - \#1}\right] \#1^2 - i \operatorname{Log}\left[1 - 2 \cos[2x] \#1 + \#1^2\right] \#1^2}{-b + 5 b \#1 - 32 a \#1^2 - 10 b \#1^2 + 10 b \#1^3 - 5 b \#1^4 + b \#1^5} \&\right]$$

■ **Problem 251: Result is not expressed in closed-form.**

$$\int \frac{1}{a + b \sin[x]^8} dx$$

Optimal (type 3, 245 leaves, 9 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{(-a)^{1/4} - b^{1/4}} \operatorname{Tan}[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4} - b^{1/4}}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{(-a)^{1/4} - i b^{1/4}} \operatorname{Tan}[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4} - i b^{1/4}}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{(-a)^{1/4} + i b^{1/4}} \operatorname{Tan}[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4} + i b^{1/4}}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{(-a)^{1/4} + b^{1/4}} \operatorname{Tan}[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4} + b^{1/4}}}$$

Result (type 7, 174 leaves):

$$8 \operatorname{RootSum}\left[b - 8 b \#1 + 28 b \#1^2 - 56 b \#1^3 + 256 a \#1^4 + 70 b \#1^4 - 56 b \#1^5 + 28 b \#1^6 - 8 b \#1^7 + b \#1^8 \&, \right. \\ \left. \frac{2 \operatorname{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right] \#1^3 - i \operatorname{Log}\left[1 - 2 \cos[2x] \#1 + \#1^2\right] \#1^3}{-b + 7 b \#1 - 21 b \#1^2 + 128 a \#1^3 + 35 b \#1^3 - 35 b \#1^4 + 21 b \#1^5 - 7 b \#1^6 + b \#1^7} \&\right]$$

■ **Problem 252: Result is not expressed in closed-form.**

$$\int \frac{1}{a - b \sin[x]^5} dx$$

Optimal (type 3, 379 leaves, 17 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{b^{1/5}-a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}-b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}-b^{2/5}}} - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{2/5} b^{1/5}-a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}-(-1)^{4/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}-(-1)^{4/5} b^{2/5}}} - \\ \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{4/5} b^{1/5}-a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}+(-1)^{3/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}+(-1)^{3/5} b^{2/5}}} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/5} b^{1/5}+a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}-(-1)^{2/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}-(-1)^{2/5} b^{2/5}}} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{3/5} b^{1/5}+a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}+(-1)^{1/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}+(-1)^{1/5} b^{2/5}}}$$

Result (type 7, 149 leaves):

$$-\frac{8}{5} i \operatorname{RootSum}\left[-i b + 5 i b \#1^2 - 10 i b \#1^4 + 32 a \#1^5 + 10 i b \#1^6 - 5 i b \#1^8 + i b \#1^{10} \&, \right. \\ \left. \frac{2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1^3 - i \operatorname{Log}\left[1 - 2 \cos[x] \#1 + \#1^2\right] \#1^3}{b - 4 b \#1^2 - 16 i a \#1^3 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8} \&\right]$$

■ **Problem 253: Result is not expressed in closed-form.**

$$\int \frac{1}{a - b \sin[x]^6} dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/3}-b^{1/3}} \tan[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-b^{1/3}}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/3}+(-1)^{1/3} b^{1/3}} \tan[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}+(-1)^{1/3} b^{1/3}}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/3}-(-1)^{2/3} b^{1/3}} \tan[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-(-1)^{2/3} b^{1/3}}}$$

Result (type 7, 148 leaves):

$$\frac{8}{3} \operatorname{RootSum}\left[b - 6 b \#1 + 15 b \#1^2 + 64 a \#1^3 - 20 b \#1^3 + 15 b \#1^4 - 6 b \#1^5 + b \#1^6 \&, \right. \\ \left. \frac{2 \operatorname{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right] \#1^2 - i \operatorname{Log}\left[1 - 2 \cos[2x] \#1 + \#1^2\right] \#1^2}{-b + 5 b \#1 + 32 a \#1^2 - 10 b \#1^2 + 10 b \#1^3 - 5 b \#1^4 + b \#1^5} \&\right]$$

■ **Problem 254: Result is not expressed in closed-form.**

$$\int \frac{1}{a - b \sin[x]^8} dx$$

Optimal (type 3, 213 leaves, 9 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}-b^{1/4}} \tan[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}-b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}-i b^{1/4}} \tan[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}-i b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}+i b^{1/4}} \tan[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}+i b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}+b^{1/4}} \tan[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}+b^{1/4}}}$$

Result (type 7, 174 leaves):

$$-8 \text{RootSum}\left[b - 8 b \#1 + 28 b \#1^2 - 56 b \#1^3 - 256 a \#1^4 + 70 b \#1^4 - 56 b \#1^5 + 28 b \#1^6 - 8 b \#1^7 + b \#1^8 \&, \right. \\ \left. \frac{2 \text{ArcTan}\left[\frac{\text{Sin}[2x]}{\text{Cos}[2x]-\#1}\right] \#1^3 - i \text{Log}\left[1 - 2 \text{Cos}[2x] \#1 + \#1^2\right] \#1^3}{-b + 7 b \#1 - 21 b \#1^2 - 128 a \#1^3 + 35 b \#1^3 - 35 b \#1^4 + 21 b \#1^5 - 7 b \#1^6 + b \#1^7} \&\right]$$

■ **Problem 255: Result is not expressed in closed-form.**

$$\int \frac{1}{1 + \text{Sin}[x]^5} dx$$

Optimal (type 3, 195 leaves, 15 steps):

$$\frac{2 \text{ArcTan}\left[\frac{(-1)^{2/5} + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{1 - (-1)^{4/5}}}\right]}{5 \sqrt{1 - (-1)^{4/5}}} + \frac{2 \text{ArcTan}\left[\frac{(-1)^{4/5} + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{1 + (-1)^{3/5}}}\right]}{5 \sqrt{1 + (-1)^{3/5}}} - \frac{2 \text{ArcTan}\left[\frac{(-1)^{3/5} (1 + (-1)^{2/5} \text{Tan}\left[\frac{x}{2}\right])}{\sqrt{1 + (-1)^{1/5}}}\right]}{5 \sqrt{1 + (-1)^{1/5}}} - \frac{2 \text{ArcTan}\left[\frac{(-1)^{1/5} (1 + (-1)^{4/5} \text{Tan}\left[\frac{x}{2}\right])}{\sqrt{1 - (-1)^{2/5}}}\right]}{5 \sqrt{1 - (-1)^{2/5}}} - \frac{\text{Cos}[x]}{5 (1 + \text{Sin}[x])}$$

Result (type 7, 411 leaves):

$$-\frac{1}{10} i \text{RootSum}\left[1 + 2 i \#1 - 8 \#1^2 - 14 i \#1^3 + 30 \#1^4 + 14 i \#1^5 - 8 \#1^6 - 2 i \#1^7 + \#1^8 \&, \right. \\ \left. \frac{1}{i - 8 \#1 - 21 i \#1^2 + 60 \#1^3 + 35 i \#1^4 - 24 \#1^5 - 7 i \#1^6 + 4 \#1^7} \left(-2 \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] + i \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] - 8 i \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \right. \right. \\ \left. \#1 - 4 \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1 + 30 \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^2 - 15 i \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^2 + 80 i \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^3 + \right. \\ \left. 40 \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^3 - 30 \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^4 + 15 i \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^4 - 8 i \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^5 - \right. \\ \left. 4 \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^5 + 2 \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^6 - i \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^6 \right) \&\right] + \frac{2 \text{Sin}\left[\frac{x}{2}\right]}{5 \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)}$$

■ **Problem 257: Result is not expressed in closed-form.**

$$\int \frac{1}{1 + \text{Sin}[x]^8} dx$$

Optimal (type 3, 218 leaves, 9 steps):

$$\frac{1}{8} \left(\sqrt{1 + \sqrt{4 - 2\sqrt{2}}} + \sqrt{2 + 2 \times 2^{1/4} + 2\sqrt{1 + \sqrt{2}}} + 2\sqrt{2 + \sqrt{2}} + \sqrt{1 + \sqrt{4 + 2\sqrt{2}}} \right) (x - \text{ArcTan}[\text{Tan}[x]]) +$$

$$\frac{\text{ArcTan}\left[\sqrt{1 - (-1)^{1/4}} \text{Tan}[x]\right]}{4\sqrt{1 - (-1)^{1/4}}} + \frac{\text{ArcTan}\left[\sqrt{1 + (-1)^{1/4}} \text{Tan}[x]\right]}{4\sqrt{1 + (-1)^{1/4}}} + \frac{\text{ArcTan}\left[\sqrt{1 - (-1)^{3/4}} \text{Tan}[x]\right]}{4\sqrt{1 - (-1)^{3/4}}} + \frac{\text{ArcTan}\left[\sqrt{1 + (-1)^{3/4}} \text{Tan}[x]\right]}{4\sqrt{1 + (-1)^{3/4}}}$$

Result (type 7, 141 leaves):

$$8 \text{RootSum}\left[1 - 8 \#1 + 28 \#1^2 - 56 \#1^3 + 326 \#1^4 - 56 \#1^5 + 28 \#1^6 - 8 \#1^7 + \#1^8 \&, \frac{2 \text{ArcTan}\left[\frac{\text{Sin}[2x]}{\text{Cos}[2x] - \#1}\right] \#1^3 - i \text{Log}\left[1 - 2 \text{Cos}[2x] \#1 + \#1^2\right] \#1^3}{-1 + 7 \#1 - 21 \#1^2 + 163 \#1^3 - 35 \#1^4 + 21 \#1^5 - 7 \#1^6 + \#1^7} \&\right]$$

■ **Problem 258: Result is not expressed in closed-form.**

$$\int \frac{1}{1 - \text{Sin}[x]^5} dx$$

Optimal (type 3, 187 leaves, 15 steps):

$$-\frac{2 \text{ArcTan}\left[\frac{(-1)^{2/5} - \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{1 - (-1)^{4/5}}}\right]}{5\sqrt{1 - (-1)^{4/5}}} - \frac{2 \text{ArcTan}\left[\frac{(-1)^{4/5} - \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{1 + (-1)^{3/5}}}\right]}{5\sqrt{1 + (-1)^{3/5}}} + \frac{2 \text{ArcTan}\left[\frac{(-1)^{1/5} + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{1 - (-1)^{2/5}}}\right]}{5\sqrt{1 - (-1)^{2/5}}} + \frac{2 \text{ArcTan}\left[\frac{(-1)^{3/5} + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{1 + (-1)^{1/5}}}\right]}{5\sqrt{1 + (-1)^{1/5}}} + \frac{\text{Cos}[x]}{5(1 - \text{Sin}[x])}$$

Result (type 7, 413 leaves):

$$\frac{1}{10} i \text{RootSum}\left[1 - 2 i \#1 - 8 \#1^2 + 14 i \#1^3 + 30 \#1^4 - 14 i \#1^5 - 8 \#1^6 + 2 i \#1^7 + \#1^8 \&, \frac{1}{-i - 8 \#1 + 21 i \#1^2 + 60 \#1^3 - 35 i \#1^4 - 24 \#1^5 + 7 i \#1^6 + 4 \#1^7} \left(-2 \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] + i \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] + 8 i \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1 + 4 \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1 + 30 \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^2 - 15 i \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^2 - 80 i \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^3 - 40 \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^3 - 30 \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^4 + 15 i \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^4 + 8 i \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^5 + 4 \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^5 + 2 \text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x] - \#1}\right] \#1^6 - i \text{Log}\left[1 - 2 \text{Cos}[x] \#1 + \#1^2\right] \#1^6 \right) \&\right] + \frac{2 \text{Sin}\left[\frac{x}{2}\right]}{5 \left(\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right)}$$

■ **Problem 265: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[x]}{a - a \text{Sin}[x]^2} dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[x]]}{a}$$

a

Result (type 3, 37 leaves) :

$$\frac{-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]}{a}$$

- **Problem 270: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[x]}{a - a \text{Sin}[x]^2} dx$$

Optimal (type 3, 22 leaves, 3 steps) :

$$\frac{\text{ArcTanh}[\text{Sin}[x]]}{2a} + \frac{\text{Sec}[x] \text{Tan}[x]}{2a}$$

Result (type 3, 45 leaves) :

$$\frac{-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sec}[x] \text{Tan}[x]}{2a}$$

- **Problem 276: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[x]^3}{(a - a \text{Sin}[x]^2)^2} dx$$

Optimal (type 3, 7 leaves, 2 steps) :

$$\frac{\text{ArcTanh}[\text{Sin}[x]]}{a^2}$$

Result (type 3, 37 leaves) :

$$\frac{-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]}{a^2}$$

- **Problem 277: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[x]}{(a - a \text{Sin}[x]^2)^2} dx$$

Optimal (type 3, 22 leaves, 3 steps) :

$$\frac{\text{ArcTanh}[\text{Sin}[x]]}{2a^2} + \frac{\text{Sec}[x] \text{Tan}[x]}{2a^2}$$

Result (type 3, 45 leaves) :

$$\frac{-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + \sec[x] \tan[x]}{2a^2}$$

■ **Problem 291: Result more than twice size of optimal antiderivative.**

$$\int \sec[e + fx]^6 (a + b \sin[e + fx]^2) dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$\frac{a \tan[e + fx]}{f} + \frac{(2a + b) \tan[e + fx]^3}{3f} + \frac{(a + b) \tan[e + fx]^5}{5f}$$

Result (type 3, 117 leaves):

$$\frac{8a \tan[e + fx]}{15f} - \frac{2b \tan[e + fx]}{15f} + \frac{4a \sec[e + fx]^2 \tan[e + fx]}{15f} - \frac{b \sec[e + fx]^2 \tan[e + fx]}{15f} + \frac{a \sec[e + fx]^4 \tan[e + fx]}{5f} + \frac{b \sec[e + fx]^4 \tan[e + fx]}{5f}$$

■ **Problem 292: Result more than twice size of optimal antiderivative.**

$$\int \sec[e + fx]^8 (a + b \sin[e + fx]^2) dx$$

Optimal (type 3, 72 leaves, 3 steps):

$$\frac{a \tan[e + fx]}{f} + \frac{(3a + b) \tan[e + fx]^3}{3f} + \frac{(3a + 2b) \tan[e + fx]^5}{5f} + \frac{(a + b) \tan[e + fx]^7}{7f}$$

Result (type 3, 161 leaves):

$$\frac{16a \tan[e + fx]}{35f} - \frac{8b \tan[e + fx]}{105f} + \frac{8a \sec[e + fx]^2 \tan[e + fx]}{35f} - \frac{4b \sec[e + fx]^2 \tan[e + fx]}{105f} + \frac{6a \sec[e + fx]^4 \tan[e + fx]}{35f} - \frac{b \sec[e + fx]^4 \tan[e + fx]}{35f} + \frac{a \sec[e + fx]^6 \tan[e + fx]}{7f} + \frac{b \sec[e + fx]^6 \tan[e + fx]}{7f}$$

■ **Problem 294: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[e + fx]^2 (a + b \sin[e + fx]^2)^2 dx$$

Optimal (type 3, 116 leaves, 5 steps):

$$\frac{1}{16} (8a^2 + 4ab + b^2) x + \frac{(8a^2 + 4ab + b^2) \cos[e + fx] \sin[e + fx]}{16f} - \frac{b(8a + 3b) \cos[e + fx]^3 \sin[e + fx]}{24f} - \frac{b \cos[e + fx]^5 \sin[e + fx] (a + (a + b) \tan[e + fx]^2)}{6f}$$

Result (type 3, 79 leaves):

$$\frac{1}{192 f} (12 ((2 - 2 i) a + b) ((2 + 2 i) a + b) (e + f x) + 3 (4 a - b) (4 a + b) \sin[2 (e + f x)] - 3 b (4 a + b) \sin[4 (e + f x)] + b^2 \sin[6 (e + f x)])$$

- **Problem 308: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[x]}{a + b \sin[x]^2} dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a} (a + b)} + \frac{\operatorname{ArcTanh}[\sin[x]]}{a + b}$$

Result (type 3, 96 leaves):

$$\frac{1}{2 \sqrt{a} (a + b)} \left(-\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \csc[x]}{\sqrt{b}}\right] + \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right] + 2 \sqrt{a} \left(-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) \right)$$

- **Problem 310: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[x]^3}{a + b \sin[x]^2} dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$\frac{b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a} (a + b)^2} + \frac{(a + 3 b) \operatorname{ArcTanh}[\sin[x]]}{2 (a + b)^2} + \frac{\sec[x] \tan[x]}{2 (a + b)}$$

Result (type 3, 147 leaves):

$$\frac{1}{4 (a + b)^2} \left(-\frac{2 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \csc[x]}{\sqrt{b}}\right]}{\sqrt{a}} + \frac{2 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a}} - \frac{2 (a + 3 b) \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + 2 (a + 3 b) \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right]}{\left(\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right)^2} + \frac{a + b}{\left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right)^2} \right)$$

- **Problem 312: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[x]^5}{a + b \sin[x]^2} dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a} (a+b)^3} + \frac{(3a^2 + 10ab + 15b^2) \operatorname{ArcTanh}[\sin[x]]}{8(a+b)^3} + \frac{(3a+7b) \sec[x] \tan[x]}{8(a+b)^2} + \frac{\sec[x]^3 \tan[x]}{4(a+b)}$$

Result (type 3, 214 leaves):

$$-\frac{1}{16(a+b)^3} \left(\frac{8b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \csc[x]}{\sqrt{b}}\right]}{\sqrt{a}} - \frac{8b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a}} + 2(3a^2 + 10ab + 15b^2) \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \right. \\ \left. 2(3a^2 + 10ab + 15b^2) \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - \frac{(a+b)^2}{(\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right])^4} + \frac{(a+b)^2}{(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right])^4} + \frac{(a+b)(3a+7b)}{(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right])^2} + \frac{(a+b)(3a+7b)}{-1 + \sin[x]} \right)$$

■ **Problem 373: Result unnecessarily involves higher level functions.**

$$\int \cos[e+fx]^5 (a+b \sin[e+fx]^2)^p dx$$

Optimal (type 5, 214 leaves, 5 steps):

$$-\frac{(3a+b(7+2p)) \sin[e+fx] (a+b \sin[e+fx]^2)^{1+p}}{b^2 f (3+2p) (5+2p)} - \\ \frac{\cos[e+fx]^2 \sin[e+fx] (a+b \sin[e+fx]^2)^{1+p}}{b f (5+2p)} + \frac{1}{b^2 f (3+2p) (5+2p)} (3a^2 + 2ab(5+2p) + b^2(15+16p+4p^2)) \\ \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \sin[e+fx]^2}{a}\right] \sin[e+fx] (a+b \sin[e+fx]^2)^p \left(1 + \frac{b \sin[e+fx]^2}{a}\right)^{-p}$$

Result (type 6, 191 leaves):

$$\left(3a \operatorname{AppellF1}\left[\frac{1}{2}, -2, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \cos[e+fx]^4 \sin[e+fx] (a+b \sin[e+fx]^2)^p \right) / \\ \left(f \left(3a \operatorname{AppellF1}\left[\frac{1}{2}, -2, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, -2, 1-p, \frac{5}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] - \right. \right. \right. \\ \left. \left. \left. 2a \operatorname{AppellF1}\left[\frac{3}{2}, -1, -p, \frac{5}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \right) \sin[e+fx]^2 \right) \right)$$

■ **Problem 374: Unable to integrate problem.**

$$\int \cos[e+fx]^3 (a+b \sin[e+fx]^2)^p dx$$

Optimal (type 5, 124 leaves, 4 steps):

$$-\frac{\sin[e+fx] (a+b\sin[e+fx]^2)^{1+p}}{bf(3+2p)} + \frac{1}{bf(3+2p)}$$

$$(a+b(3+2p)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b\sin[e+fx]^2}{a}\right] \sin[e+fx] (a+b\sin[e+fx]^2)^p \left(1 + \frac{b\sin[e+fx]^2}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \cos[e+fx]^3 (a+b\sin[e+fx]^2)^p dx$$

■ **Problem 376: Unable to integrate problem.**

$$\int \sec[e+fx] (a+b\sin[e+fx]^2)^p dx$$

Optimal (type 6, 76 leaves, 3 steps):

$$\frac{\operatorname{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b\sin[e+fx]^2}{a}\right] \sin[e+fx] (a+b\sin[e+fx]^2)^p \left(1 + \frac{b\sin[e+fx]^2}{a}\right)^{-p}}{f}$$

Result (type 8, 23 leaves):

$$\int \sec[e+fx] (a+b\sin[e+fx]^2)^p dx$$

■ **Problem 377: Unable to integrate problem.**

$$\int \sec[e+fx]^3 (a+b\sin[e+fx]^2)^p dx$$

Optimal (type 6, 76 leaves, 3 steps):

$$\frac{\operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b\sin[e+fx]^2}{a}\right] \sin[e+fx] (a+b\sin[e+fx]^2)^p \left(1 + \frac{b\sin[e+fx]^2}{a}\right)^{-p}}{f}$$

Result (type 8, 25 leaves):

$$\int \sec[e+fx]^3 (a+b\sin[e+fx]^2)^p dx$$

■ **Problem 378: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^4 (a+b\sin[e+fx]^2)^p dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b\sin[e+fx]^2}{a}\right] \sqrt{\cos[e+fx]^2} (a+b\sin[e+fx]^2)^p \left(1 + \frac{b\sin[e+fx]^2}{a}\right)^{-p} \tan[e+fx]$$

Result (type 6, 199 leaves):

$$\left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \cos[e+fx]^3 \sin[e+fx] (a+b \sin[e+fx]^2)^p \right) /$$

$$\left(f \left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] + \left(2 b p \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}, 1-p, \frac{5}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] - \right. \right. \right.$$

$$\left. \left. 3 a \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \right) \sin[e+fx]^2 \right) \right)$$

■ **Problem 379: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^2 (a+b \sin[e+fx]^2)^p dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \sqrt{\cos[e+fx]^2} (a+b \sin[e+fx]^2)^p \left(1 + \frac{b \sin[e+fx]^2}{a}\right)^{-p} \tan[e+fx]$$

Result (type 6, 195 leaves):

$$\left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] (a+b \sin[e+fx]^2)^p \sin[2(e+fx)] \right) /$$

$$\left(2 f \left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] + \left(2 b p \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] - \right. \right. \right.$$

$$\left. \left. a \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \right) \sin[e+fx]^2 \right) \right)$$

■ **Problem 381: Unable to integrate problem.**

$$\int \sec[e+fx]^2 (a+b \sin[e+fx]^2)^p dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \sqrt{\cos[e+fx]^2} (a+b \sin[e+fx]^2)^p \left(1 + \frac{b \sin[e+fx]^2}{a}\right)^{-p} \tan[e+fx]$$

Result (type 8, 25 leaves):

$$\int \sec[e+fx]^2 (a+b \sin[e+fx]^2)^p dx$$

■ **Problem 382: Unable to integrate problem.**

$$\int \sec[e+fx]^4 (a+b \sin[e+fx]^2)^p dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \sqrt{\cos[e+fx]^2} (a+b \sin[e+fx]^2)^p \left(1 + \frac{b \sin[e+fx]^2}{a}\right)^{-p} \tan[e+fx]$$

Result (type 8, 25 leaves):

$$\int \sec[e+fx]^4 (a+b \sin[e+fx]^2)^p dx$$

■ **Problem 383: Result is not expressed in closed-form.**

$$\int \frac{\cos[c+dx]^5}{a+b \sin[c+dx]^3} dx$$

Optimal (type 3, 219 leaves, 11 steps):

$$\frac{(a^{4/3} - b^{4/3}) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3} \sin[c+dx]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{5/3} d} + \frac{(a^{4/3} + b^{4/3}) \operatorname{Log}[a^{1/3} + b^{1/3} \sin[c+dx]]}{3 a^{2/3} b^{5/3} d} - \frac{(a^{4/3} + b^{4/3}) \operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} \sin[c+dx] + b^{2/3} \sin[c+dx]^2]}{6 a^{2/3} b^{5/3} d} - \frac{2 \operatorname{Log}[a+b \sin[c+dx]^3]}{3 b d} + \frac{\sin[c+dx]^2}{2 b d}$$

Result (type 7, 230 leaves):

$$\frac{1}{12 b d} \left(-3 \cos[2(c+dx)] + 24 \operatorname{Log}\left[\sec\left[\frac{1}{2}(c+dx)\right]^2\right] - 4 \operatorname{RootSum}\left[a + 3 a \#1^2 + 8 b \#1^3 + 3 a \#1^4 + a \#1^6 \&, \right. \right. \\ \left. \left. \left(-b \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c+dx)\right]\right] + 4 a \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c+dx)\right]\right] \#1 + 8 b \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c+dx)\right]\right] \#1^2 + 2 a \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c+dx)\right]\right] \right] \right. \\ \left. \#1^3 + b \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c+dx)\right]\right] \#1^4 + 2 a \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c+dx)\right]\right] \#1^5 \right) / \left(a \#1 + 4 b \#1^2 + 2 a \#1^3 + a \#1^5 \& \right)$$

■ **Problem 384: Result is not expressed in closed-form.**

$$\int \frac{\cos[c+dx]^3}{a+b \sin[c+dx]^3} dx$$

Optimal (type 3, 167 leaves, 9 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3} \sin[c+dx]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{1/3} d} + \frac{\operatorname{Log}[a^{1/3} + b^{1/3} \sin[c+dx]]}{3 a^{2/3} b^{1/3} d} - \frac{\operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} \sin[c+dx] + b^{2/3} \sin[c+dx]^2]}{6 a^{2/3} b^{1/3} d} - \frac{\operatorname{Log}[a+b \sin[c+dx]^3]}{3 b d}$$

Result (type 7, 216 leaves):

$$-\frac{1}{3bd} \left(-3 \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right] \right]^2 + \operatorname{RootSum} \left[a + 3a\#1^2 + 8b\#1^3 + 3a\#1^4 + a\#1^6 \&, \right. \right. \\ \left. \left. \left(-b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] + a \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1 + 4b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^2 + 2a \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^3 + \right. \right. \\ \left. \left. b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^4 + a \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^5 \right) / \left(a\#1 + 4b\#1^2 + 2a\#1^3 + a\#1^5 \& \right) \right]$$

■ **Problem 386: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sec}[c+dx]}{a+b\sin[c+dx]^3} dx$$

Optimal (type 3, 290 leaves, 11 steps):

$$-\frac{b^{1/3} (a^{4/3} - b^{4/3}) \operatorname{ArcTan} \left[\frac{a^{1/3} - 2b^{1/3} \sin[c+dx]}{\sqrt{3} a^{1/3}} \right]}{\sqrt{3} a^{2/3} (a^2 - b^2) d} - \frac{\operatorname{Log}[1 - \sin[c+dx]]}{2(a+b)d} + \frac{\operatorname{Log}[1 + \sin[c+dx]]}{2(a-b)d} - \\ \frac{b^{1/3} (a^{4/3} + b^{4/3}) \operatorname{Log}[a^{1/3} + b^{1/3} \sin[c+dx]]}{3 a^{2/3} (a^2 - b^2) d} + \frac{b^{1/3} (a^{4/3} + b^{4/3}) \operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} \sin[c+dx] + b^{2/3} \sin[c+dx]^2]}{6 a^{2/3} (a^2 - b^2) d} - \frac{b \operatorname{Log}[a + b \sin[c+dx]^3]}{3 (a^2 - b^2) d}$$

Result (type 7, 288 leaves):

$$\frac{1}{3(a-b)(a+b)d} \\ \left(3 \left(b \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right] \right]^2 + (-a+b) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] + (a+b) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] \right) - \\ b \operatorname{RootSum} \left[a + 3a\#1^2 + 8b\#1^3 + 3a\#1^4 + a\#1^6 \&, \right. \\ \left. \left(b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] - a \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1 + 4b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^2 + 4a \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^3 - \right. \right. \\ \left. \left. b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^4 + a \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^5 \right) / \left(a\#1 + 4b\#1^2 + 2a\#1^3 + a\#1^5 \& \right) \right]$$

■ **Problem 387: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sec}[c+dx]^3}{a+b\sin[c+dx]^3} dx$$

Optimal (type 3, 385 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b^{5/3} (2a^2 - 3a^{4/3}b^{2/3} + b^2) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3} \sin[c+dx]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} (a^2 - b^2)^2 d} - \frac{(a+4b) \operatorname{Log}[1 - \sin[c+dx]]}{4(a+b)^2 d} + \frac{(a-4b) \operatorname{Log}[1 + \sin[c+dx]]}{4(a-b)^2 d} + \\
& \frac{b^{5/3} (2a^2 + 3a^{4/3}b^{2/3} + b^2) \operatorname{Log}[a^{1/3} + b^{1/3} \sin[c+dx]]}{3a^{2/3} (a^2 - b^2)^2 d} - \frac{b^{5/3} (2a^2 + 3a^{4/3}b^{2/3} + b^2) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3} \sin[c+dx] + b^{2/3} \sin[c+dx]^2]}{6a^{2/3} (a^2 - b^2)^2 d} + \\
& \frac{b(a^2 + 2b^2) \operatorname{Log}[a + b \sin[c+dx]^3]}{3(a^2 - b^2)^2 d} + \frac{1}{4(a+b)d(1 - \sin[c+dx])} - \frac{1}{4(a-b)d(1 + \sin[c+dx])}
\end{aligned}$$

Result (type 7, 535 leaves):

$$\begin{aligned}
& \frac{1}{12d} \left(- \frac{6(a+4b) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{(a+b)^2} + \frac{6(a-4b) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{(a-b)^2} + \right. \\
& \left. \frac{1}{(a^2 - b^2)^2} 4b \left(-3(a^2 + 2b^2) \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right] + \operatorname{RootSum}\left[a + 3a\#1^2 + 8b\#1^3 + 3a\#1^4 + a\#1^6 \&, \right. \right. \right. \\
& \left. \frac{1}{a\#1 + 4b\#1^2 + 2a\#1^3 + a\#1^5} \left(2a^2b \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] + b^3 \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] + a^3 \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) \#1 - \right. \\
& \left. 4ab^2 \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \#1 + 4a^2b \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \#1^2 + 8b^3 \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \#1^2 + \right. \\
& \left. 2a^3 \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \#1^3 + 10ab^2 \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \#1^3 - 2a^2b \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \#1^4 - \right. \\
& \left. b^3 \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \#1^4 + a^3 \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \#1^5 + 2ab^2 \operatorname{Log}\left[-\#1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \#1^5 \right) \& \left. \right) + \\
& \left. \frac{3}{(a+b) \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{3}{(a-b) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)
\end{aligned}$$

■ **Problem 388: Result is not expressed in closed-form.**

$$\int \frac{\cos[c+dx]^4}{a+b \sin[c+dx]^3} dx$$

Optimal (type 3, 764 leaves, 38 steps):

$$\begin{aligned}
& - \frac{2 (-1)^{2/3} a^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} b^{4/3} d} + \frac{2 \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} d} + \frac{2 a^{2/3} \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^{4/3} d} \\
& - \frac{4 \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^{2/3} d} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} - \frac{2 (-1)^{1/3} a^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} b^{4/3} d} \\
& - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/3} (b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right])}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d} + \frac{4 \operatorname{ArcTanh}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} b^{2/3} d} + \frac{4 \operatorname{ArcTanh}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{1/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{-(-1)^{1/3} a^{2/3} + b^{2/3}} b^{2/3} d} - \frac{\operatorname{Cos}[c+dx]}{bd}
\end{aligned}$$

Result (type 7, 300 leaves):

$$\begin{aligned}
& - \frac{1}{3bd} \left(3 \operatorname{Cos}[c+dx] + i \operatorname{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \right. \right. \\
& \left. \left. \left(2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] - i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] - 2 i a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1 - \right. \right. \right. \\
& \left. \left. \left. a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1 + 2 i a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^3 + a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^3 + \right. \right. \right. \\
& \left. \left. \left. 2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1}\right] \#1^4 - i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^4 \right) \right) / (b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5 \&) \right)
\end{aligned}$$

■ **Problem 389: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Cos}[c+dx]^2}{a + b \operatorname{Sin}[c+dx]^3} dx$$

Optimal (type 3, 484 leaves, 24 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} d} - \frac{2 \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^{2/3} d} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} \\
& - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/3} (b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right])}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d} + \frac{2 \operatorname{ArcTanh}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} b^{2/3} d} + \frac{2 \operatorname{ArcTanh}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{1/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{-(-1)^{1/3} a^{2/3} + b^{2/3}} b^{2/3} d}
\end{aligned}$$

Result (type 7, 231 leaves):

$$-\frac{1}{6d} i \operatorname{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \right. \\ \left. \left(2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx]-\#1}\right] - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] + 4 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx]-\#1}\right] \#1^2 - 2 i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^2 + \right. \right. \\ \left. \left. 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx]-\#1}\right] \#1^4 - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1^4\right) / (b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5) \& \right]$$

■ **Problem 390: Result is not expressed in closed-form.**

$$\int \frac{1}{a + b \operatorname{Sin}[c + dx]^3} dx$$

Optimal (type 3, 245 leaves, 11 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} d} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/3} (b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right])}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d}$$

Result (type 7, 126 leaves):

$$-\frac{1}{3d} 2 i \operatorname{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \frac{2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx]-\#1}\right] \#1 - i \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+dx] \#1 + \#1^2\right] \#1}{b - 4 i a \#1 - 2 b \#1^2 + b \#1^4} \& \right]$$

■ **Problem 391: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sec}[c + dx]^2}{a + b \operatorname{Sin}[c + dx]^3} dx$$

Optimal (type 3, 299 leaves, ? steps):

$$\frac{2 (-1)^{2/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} - (-1)^{2/3} b^{2/3})^{3/2} d} - \frac{2 b^{2/3} \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} - b^{2/3})^{3/2} d} + \\ \frac{2 (-1)^{1/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} + (-1)^{1/3} b^{2/3})^{3/2} d} + \frac{\operatorname{Sec}[c + dx] (b - a \operatorname{Sin}[c + dx])}{(-a^2 + b^2) d}$$

Result (type 7, 432 leaves):

$$\begin{aligned}
& \left(-6 b + 6 b \operatorname{Cos}[c + d x] - \right. \\
& \quad i b \operatorname{Cos}[c + d x] \operatorname{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \frac{1}{b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5} \left(2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] - \right. \right. \\
& \quad \quad i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] + 4 i a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] \#1 + 2 a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] \#1 - \\
& \quad \quad 12 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] \#1^2 + 6 i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] \#1^2 - 4 i a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] \#1^3 - \\
& \quad \quad \left. \left. 2 a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] \#1^3 + 2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1} \right] \#1^4 - i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2 \right] \#1^4 \right) \& \right] + \\
& \quad \left. 6 a \operatorname{Sin}[c + d x] \right) / \left(6 (a - b) (a + b) d \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x) \right] \right) \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x) \right] \right) \right)
\end{aligned}$$

■ **Problem 392: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sec}[c + d x]^4}{a + b \operatorname{Sin}[c + d x]^3} dx$$

Optimal (type 3, 1093 leaves, ? steps):

$$\begin{aligned}
& - \frac{2 (-1)^{2/3} a^{2/3} b^{8/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} - \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 a^{2/3} b^{8/3} \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{\sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \\
& \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} - \frac{2 (-1)^{1/3} a^{2/3} b^{8/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} + \\
& \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} - \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x) \right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} - \\
& \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x) \right]}{\sqrt{-(-1)^{1/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{-(-1)^{1/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} + \frac{\operatorname{Cos}[c + d x]}{12 (a + b) d (1 - \operatorname{Sin}[c + d x])^2} + \frac{\operatorname{Cos}[c + d x]}{12 (a + b) d (1 - \operatorname{Sin}[c + d x])} + \\
& \frac{(a + 4 b) \operatorname{Cos}[c + d x]}{4 (a + b)^2 d (1 - \operatorname{Sin}[c + d x])} - \frac{\operatorname{Cos}[c + d x]}{12 (a - b) d (1 + \operatorname{Sin}[c + d x])^2} - \frac{(a - 4 b) \operatorname{Cos}[c + d x]}{4 (a - b)^2 d (1 + \operatorname{Sin}[c + d x])} - \frac{\operatorname{Cos}[c + d x]}{12 (a - b) d (1 + \operatorname{Sin}[c + d x])}
\end{aligned}$$

Result (type 7, 679 leaves):

$$\frac{1}{24 (a-b)^2 (a+b)^2 d} \left(4 i b^2 \text{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \frac{1}{b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5} \left(2 a^2 \text{ArcTan}\left[\frac{\text{Sin}[c+d x]}{\text{Cos}[c+d x] - \#1}\right] + 4 b^2 \text{ArcTan}\left[\frac{\text{Sin}[c+d x]}{\text{Cos}[c+d x] - \#1}\right] - i a^2 \text{Log}\left[1 - 2 \text{Cos}[c+d x] \#1 + \#1^2\right] - 2 i b^2 \text{Log}\left[1 - 2 \text{Cos}[c+d x] \#1 + \#1^2\right] + 12 i a b \text{ArcTan}\left[\frac{\text{Sin}[c+d x]}{\text{Cos}[c+d x] - \#1}\right] \#1 + 6 a b \text{Log}\left[1 - 2 \text{Cos}[c+d x] \#1 + \#1^2\right] \#1 - 20 a^2 \text{ArcTan}\left[\frac{\text{Sin}[c+d x]}{\text{Cos}[c+d x] - \#1}\right] \#1^2 - 16 b^2 \text{ArcTan}\left[\frac{\text{Sin}[c+d x]}{\text{Cos}[c+d x] - \#1}\right] \#1^2 + 10 i a^2 \text{Log}\left[1 - 2 \text{Cos}[c+d x] \#1 + \#1^2\right] \#1^2 + 8 i b^2 \text{Log}\left[1 - 2 \text{Cos}[c+d x] \#1 + \#1^2\right] \#1^2 - 12 i a b \text{ArcTan}\left[\frac{\text{Sin}[c+d x]}{\text{Cos}[c+d x] - \#1}\right] \#1^3 - 6 a b \text{Log}\left[1 - 2 \text{Cos}[c+d x] \#1 + \#1^2\right] \#1^3 + 2 a^2 \text{ArcTan}\left[\frac{\text{Sin}[c+d x]}{\text{Cos}[c+d x] - \#1}\right] \#1^4 + 4 b^2 \text{ArcTan}\left[\frac{\text{Sin}[c+d x]}{\text{Cos}[c+d x] - \#1}\right] \#1^4 - i a^2 \text{Log}\left[1 - 2 \text{Cos}[c+d x] \#1 + \#1^2\right] \#1^4 - 2 i b^2 \text{Log}\left[1 - 2 \text{Cos}[c+d x] \#1 + \#1^2\right] \#1^4 \right) \& \right] + \text{Sec}[c+d x]^3 \left(4 a^2 b + 32 b^3 - 3 b \left(5 a^2 + 13 b^2 \right) \text{Cos}[c+d x] + 12 b \left(a^2 + 2 b^2 \right) \text{Cos}[2(c+d x)] - 5 a^2 b \text{Cos}[3(c+d x)] - 13 b^3 \text{Cos}[3(c+d x)] + 12 a^3 \text{Sin}[c+d x] - 30 a b^2 \text{Sin}[c+d x] + 4 a^3 \text{Sin}[3(c+d x)] - 22 a b^2 \text{Sin}[3(c+d x)] \right) \right)$$

■ **Problem 393: Result is not expressed in closed-form.**

$$\int \frac{\text{Cos}[c+d x]^7}{(a+b \text{Sin}[c+d x]^3)^2} dx$$

Optimal (type 3, 288 leaves, 10 steps):

$$\frac{2 \left(2 a^2 + 3 a^{4/3} b^{2/3} + b^2 \right) \text{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} \text{Sin}[c+d x]}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3} b^{7/3} d} + \frac{2 \left(2 a^2 - 3 a^{4/3} b^{2/3} + b^2 \right) \text{Log}\left[a^{1/3} + b^{1/3} \text{Sin}[c+d x]\right]}{9 a^{5/3} b^{7/3} d} - \frac{\left(2 a^2 - 3 a^{4/3} b^{2/3} + b^2 \right) \text{Log}\left[a^{2/3} - a^{1/3} b^{1/3} \text{Sin}[c+d x] + b^{2/3} \text{Sin}[c+d x]^2\right]}{9 a^{5/3} b^{7/3} d} - \frac{\text{Sin}[c+d x]}{b^2 d} - \frac{\text{Sin}[c+d x] \left(a^2 - b^2 + 3 a b \text{Sin}[c+d x] + 3 b^2 \text{Sin}[c+d x]^2 \right)}{3 a b^2 d \left(a + b \text{Sin}[c+d x]^3 \right)}$$

Result (type 7, 490 leaves):

$$\begin{aligned}
& -\frac{1}{9b^2d} \left(\frac{1}{a} \operatorname{RootSum} \left[-ib + 3ib\#1^2 + 8a\#1^3 - 3ib\#1^4 + ib\#1^6 \ \&, \right. \right. \\
& \quad \frac{1}{b\#1 - 4ia\#1^2 - 2b\#1^3 + b\#1^5} \left(-6ab \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] + 3iab \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] + 8ia^2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \right) \\
& \quad \#1 + 4ib^2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1 + 4a^2 \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] \#1 + 2b^2 \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] \#1 + \\
& \quad 8ia^2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1^3 + 4ib^2 \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1^3 + 4a^2 \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] \#1^3 + \\
& \quad \left. 2b^2 \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] \#1^3 + 6ab \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1^4 - 3iab \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] \#1^4 \right) \ \& \left. \right) + \\
& \quad 9 \operatorname{Sin}[c+dx] - \frac{6(3ab + 3ab \operatorname{Cos}[2(c+dx)] - 2(a^2 - b^2) \operatorname{Sin}[c+dx])}{a(4a + 3b \operatorname{Sin}[c+dx] - b \operatorname{Sin}[3(c+dx)])} \Big)
\end{aligned}$$

■ **Problem 394: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Cos}[c+dx]^5}{(a+b \operatorname{Sin}[c+dx]^3)^2} dx$$

Optimal (type 3, 238 leaves, 8 steps):

$$\begin{aligned}
& -\frac{2(a^{4/3} + b^{4/3}) \operatorname{ArcTan} \left[\frac{a^{1/3} - 2b^{1/3} \operatorname{Sin}[c+dx]}{\sqrt{3} a^{1/3}} \right]}{3\sqrt{3} a^{5/3} b^{5/3} d} - \frac{2(a^{4/3} - b^{4/3}) \operatorname{Log} [a^{1/3} + b^{1/3} \operatorname{Sin}[c+dx]]}{9a^{5/3} b^{5/3} d} + \\
& \frac{(a^{4/3} - b^{4/3}) \operatorname{Log} [a^{2/3} - a^{1/3} b^{1/3} \operatorname{Sin}[c+dx] + b^{2/3} \operatorname{Sin}[c+dx]^2]}{9a^{5/3} b^{5/3} d} + \frac{\operatorname{Sin}[c+dx] (b - a \operatorname{Sin}[c+dx] - 2b \operatorname{Sin}[c+dx]^2)}{3abd (a+b \operatorname{Sin}[c+dx]^3)}
\end{aligned}$$

Result (type 7, 346 leaves):

$$\begin{aligned}
& \frac{1}{9abd} \left(i \operatorname{RootSum} \left[-ib + 3ib\#1^2 + 8a\#1^3 - 3ib\#1^4 + ib\#1^6 \ \&, \right. \right. \\
& \quad \left(-2ia \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] - a \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] - 4b \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1 + 2ib \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] \right) \\
& \quad \#1 - 4b \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1^3 + 2ib \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] \#1^3 + 2ia \operatorname{ArcTan} \left[\frac{\operatorname{Sin}[c+dx]}{\operatorname{Cos}[c+dx] - \#1} \right] \#1^4 + \\
& \quad \left. a \operatorname{Log} [1 - 2\operatorname{Cos}[c+dx] \#1 + \#1^2] \#1^4 \right) \Big/ (b\#1 - 4ia\#1^2 - 2b\#1^3 + b\#1^5) \ \& \left. \right) + \frac{6(3a + a \operatorname{Cos}[2(c+dx)] + 2b \operatorname{Sin}[c+dx])}{4a + 3b \operatorname{Sin}[c+dx] - b \operatorname{Sin}[3(c+dx)]} \Big)
\end{aligned}$$

■ **Problem 395: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Cos}[c+dx]^3}{(a+b \operatorname{Sin}[c+dx]^3)^2} dx$$

Optimal (type 3, 183 leaves, 9 steps) :

$$-\frac{2 \operatorname{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \operatorname{Sin}[c+d x]}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3} b^{1/3} d} + \frac{2 \operatorname{Log}\left[a^{1/3}+b^{1/3} \operatorname{Sin}[c+d x]\right]}{9 a^{5/3} b^{1/3} d} - \frac{\operatorname{Log}\left[a^{2/3}-a^{1/3} b^{1/3} \operatorname{Sin}[c+d x]+b^{2/3} \operatorname{Sin}[c+d x]^2\right]}{9 a^{5/3} b^{1/3} d} + \frac{a+b \operatorname{Sin}[c+d x]}{3 a b d (a+b \operatorname{Sin}[c+d x]^3)}$$

Result (type 7, 221 leaves) :

$$\frac{1}{9 a d} 2 \left(-i \operatorname{RootSum}\left[-i b+3 i b \#1^2+8 a \#1^3-3 i b \#1^4+i b \#1^6 \&, \right. \right. \\ \left. \left. \left(2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] - i \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] + 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\#1}\right] \#1^2 - i \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \#1+\#1^2\right] \#1^2 \right) / \right. \right. \\ \left. \left. (b-4 i a \#1-2 b \#1^2+b \#1^4) \& \right] + \frac{6(a+b \operatorname{Sin}[c+d x])}{b(4 a+3 b \operatorname{Sin}[c+d x]-b \operatorname{Sin}[3(c+d x)])} \right)$$

■ **Problem 397: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sec}[c+d x]}{(a+b \operatorname{Sin}[c+d x]^3)^2} dx$$

Optimal (type 3, 587 leaves, 18 steps) :

$$-\frac{b^{1/3}\left(a^{4/3}-2 b^{4/3}\right) \operatorname{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \operatorname{Sin}[c+d x]}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3}\left(a^2-b^2\right) d} - \frac{b^{1/3}\left(a^2-2 a^{2/3} b^{4/3}+b^2\right) \operatorname{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \operatorname{Sin}[c+d x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{1/3}\left(a^2-b^2\right)^2 d} - \\ \frac{\operatorname{Log}[1-\operatorname{Sin}[c+d x]]}{2(a+b)^2 d} + \frac{\operatorname{Log}[1+\operatorname{Sin}[c+d x]]}{2(a-b)^2 d} - \frac{b^{1/3}\left(a^{4/3}+2 b^{4/3}\right) \operatorname{Log}\left[a^{1/3}+b^{1/3} \operatorname{Sin}[c+d x]\right]}{9 a^{5/3}\left(a^2-b^2\right) d} - \\ \frac{b^{1/3}\left(a^2+2 a^{2/3} b^{4/3}+b^2\right) \operatorname{Log}\left[a^{1/3}+b^{1/3} \operatorname{Sin}[c+d x]\right]}{3 a^{1/3}\left(a^2-b^2\right)^2 d} + \frac{b^{1/3}\left(a^{4/3}+2 b^{4/3}\right) \operatorname{Log}\left[a^{2/3}-a^{1/3} b^{1/3} \operatorname{Sin}[c+d x]+b^{2/3} \operatorname{Sin}[c+d x]^2\right]}{18 a^{5/3}\left(a^2-b^2\right) d} + \\ \frac{b^{1/3}\left(a^2+2 a^{2/3} b^{4/3}+b^2\right) \operatorname{Log}\left[a^{2/3}-a^{1/3} b^{1/3} \operatorname{Sin}[c+d x]+b^{2/3} \operatorname{Sin}[c+d x]^2\right]}{6 a^{1/3}\left(a^2-b^2\right)^2 d} - \\ \frac{2 a b \operatorname{Log}\left[a+b \operatorname{Sin}[c+d x]^3\right]}{3\left(a^2-b^2\right)^2 d} + \frac{b(a-\operatorname{Sin}[c+d x])(b-a \operatorname{Sin}[c+d x])}{3 a\left(a^2-b^2\right) d(a+b \operatorname{Sin}[c+d x]^3)}$$

Result (type 7, 478 leaves) :

$$\frac{1}{9d} \left(-\frac{9 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right]}{(a+b)^2} + \frac{9 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right]}{(a-b)^2} + \right. \\ \left. \frac{1}{a(a^2-b^2)^2} 2b \left(9a^2 \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right] \right]^2 - \operatorname{RootSum} \left[a + 3a\#1^2 + 8b\#1^3 + 3a\#1^4 + a\#1^6 \&, \right. \right. \right. \\ \left. \left. \frac{1}{a\#1 + 4b\#1^2 + 2a\#1^3 + a\#1^5} \left(4a^2b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] - b^3 \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] - a^3 \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \right] \#1 - \right. \right. \\ \left. \left. 2ab^2 \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1 + 12a^2b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^2 + 10a^3 \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^3 + \right. \right. \\ \left. \left. 2ab^2 \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^3 - 4a^2b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^4 + b^3 \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^4 + \right. \right. \\ \left. \left. 3a^3 \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right] \#1^5 \right) \& \right] \right) - \frac{6b(-3a+a \operatorname{Cos}[2(c+dx)] + 2b \operatorname{Sin}[c+dx])}{a(a-b)(a+b)(4a+3b \operatorname{Sin}[c+dx] - b \operatorname{Sin}[3(c+dx)])} \left. \right)$$

■ **Problem 398: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sec}[c+dx]^3}{(a+b \operatorname{Sin}[c+dx])^2} dx$$

Optimal (type 3, 747 leaves, 18 steps):

$$\frac{b^{5/3} (4a^2 - 3a^{4/3}b^{2/3} + 2b^2) \operatorname{ArcTan} \left[\frac{a^{1/3} - 2b^{1/3} \operatorname{Sin}[c+dx]}{\sqrt{3} a^{1/3}} \right]}{3\sqrt{3} a^{5/3} (a^2 - b^2)^2 d} - \\ \frac{b^{5/3} (4a^{8/3} - 9a^2b^{2/3} + 8a^{2/3}b^2 - 3b^{8/3}) \operatorname{ArcTan} \left[\frac{a^{1/3} - 2b^{1/3} \operatorname{Sin}[c+dx]}{\sqrt{3} a^{1/3}} \right]}{\sqrt{3} a^{1/3} (a^2 - b^2)^3 d} - \frac{(a+7b) \operatorname{Log}[1 - \operatorname{Sin}[c+dx]]}{4(a+b)^3 d} + \frac{(a-7b) \operatorname{Log}[1 + \operatorname{Sin}[c+dx]]}{4(a-b)^3 d} + \\ \frac{b^{5/3} (4a^2 + 3a^{4/3}b^{2/3} + 2b^2) \operatorname{Log}[a^{1/3} + b^{1/3} \operatorname{Sin}[c+dx]]}{9a^{5/3} (a^2 - b^2)^2 d} + \frac{b^{5/3} (3b^{2/3} (3a^2 + b^2) + 4a^{2/3} (a^2 + 2b^2)) \operatorname{Log}[a^{1/3} + b^{1/3} \operatorname{Sin}[c+dx]]}{3a^{1/3} (a^2 - b^2)^3 d} - \\ \frac{b^{5/3} (4a^2 + 3a^{4/3}b^{2/3} + 2b^2) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3} \operatorname{Sin}[c+dx] + b^{2/3} \operatorname{Sin}[c+dx]^2]}{18a^{5/3} (a^2 - b^2)^2 d} - \\ \frac{b^{5/3} (3b^{2/3} (3a^2 + b^2) + 4a^{2/3} (a^2 + 2b^2)) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3} \operatorname{Sin}[c+dx] + b^{2/3} \operatorname{Sin}[c+dx]^2]}{6a^{1/3} (a^2 - b^2)^3 d} + \frac{2ab(a^2 + 5b^2) \operatorname{Log}[a+b \operatorname{Sin}[c+dx]^3]}{3(a^2 - b^2)^3 d} + \\ \frac{1}{4(a+b)^2 d (1 - \operatorname{Sin}[c+dx])} - \frac{1}{4(a-b)^2 d (1 + \operatorname{Sin}[c+dx])} - \frac{b(a(a^2 + 2b^2) - b \operatorname{Sin}[c+dx] (2a^2 + b^2 - 3ab \operatorname{Sin}[c+dx]))}{3a(a^2 - b^2)^2 d (a+b \operatorname{Sin}[c+dx]^3)}$$

Result (type 7, 773 leaves):

$$\begin{aligned}
& \frac{(-a - 7b) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2(a + b)^3 d} + \\
& \frac{(a - 7b) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2(a - b)^3 d} + \frac{1}{9a(a^2 - b^2)^3 d} 2b \left(-9a^2(a^2 + 5b^2) \operatorname{Log}\left[\sec\left[\frac{1}{2}(c + dx)\right]\right]^2\right) + \\
& \operatorname{RootSum}\left[a + 3a\#1^2 + 8b\#1^3 + 3a\#1^4 + a\#1^6 \&, \frac{1}{a\#1 + 4b\#1^2 + 2a\#1^3 + a\#1^5} \left(8a^4 b \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right]\right) + \right. \\
& 11a^2 b^3 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] - b^5 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] + 3a^5 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1 - \\
& 15a^3 b^2 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1 - 6ab^4 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1 + 12a^4 b \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^2 + \\
& 60a^2 b^3 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^2 + 6a^5 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^3 + 60a^3 b^2 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^3 + \\
& 6ab^4 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^3 - 8a^4 b \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^4 - 11a^2 b^3 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^4 + \\
& \left. b^5 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^4 + 3a^5 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^5 + 15a^3 b^2 \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(c + dx)\right]\right] \#1^5 \&\right) + \\
& \frac{1}{4(a + b)^2 d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{1}{4(a - b)^2 d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \\
& \frac{2(-2a^3 b - 7ab^3 + 3ab^3 \cos[2(c + dx)] + 4a^2 b^2 \sin[c + dx] + 2b^4 \sin[c + dx])}{3a(a - b)^2(a + b)^2 d(-4a - 3b \sin[c + dx] + b \sin[3(c + dx)])}
\end{aligned}$$

■ **Problem 404: Result is not expressed in closed-form.**

$$\int \frac{\cos[c + dx]^7}{a - b \sin[c + dx]^4} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$\frac{(\sqrt{a} + \sqrt{b})^3 \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c + dx]}{a^{1/4}}\right]}{2a^{3/4} b^{7/4} d} - \frac{(\sqrt{a} - \sqrt{b})^3 \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c + dx]}{a^{1/4}}\right]}{2a^{3/4} b^{7/4} d} - \frac{3 \sin[c + dx]}{bd} + \frac{\sin[c + dx]^3}{3bd}$$

Result (type 7, 524 leaves):

$$\frac{1}{24 b d} \left(3 \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(-2 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] - 6 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] + i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] + 3 i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] - 22 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^2 - 2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^2 + 11 i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^2 + i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^2 - 22 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^4 - 2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^4 + 11 i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^4 + i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^4 - 2 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^6 - 6 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^6 + i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^6 + 3 i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^6 \right) \& \right] - 2 (33 \operatorname{Sin}[c + d x] + \operatorname{Sin}[3 (c + d x)]) \right)$$

■ **Problem 405: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Cos}[c + d x]^5}{a - b \operatorname{Sin}[c + d x]^4} dx$$

Optimal (type 3, 113 leaves, 6 steps):

$$\frac{(\sqrt{a} + \sqrt{b})^2 \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Sin}[c + d x]}{a^{1/4}}\right]}{2 a^{3/4} b^{5/4} d} + \frac{(a - 2 \sqrt{a} \sqrt{b} + b) \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Sin}[c + d x]}{a^{1/4}}\right]}{2 a^{3/4} b^{5/4} d} - \frac{\operatorname{Sin}[c + d x]}{b d}$$

Result (type 7, 411 leaves):

$$-\frac{1}{4 b d} \left(\operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] - i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] + 4 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^2 + 2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^2 - 2 i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^2 - i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^2 + 4 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^4 + 2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^4 - 2 i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^4 - i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^4 + 2 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c + d x]}{\operatorname{Cos}[c + d x] - \#1}\right] \#1^6 - i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c + d x] \#1 + \#1^2\right] \#1^6 \right) \& \right] + 4 \operatorname{Sin}[c + d x] \right)$$

■ **Problem 406: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Cos}[c + d x]^3}{a - b \operatorname{Sin}[c + d x]^4} dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$\frac{(\sqrt{a} + \sqrt{b}) \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c+dx]}{a^{1/4}}\right]}{2 a^{3/4} b^{3/4} d} - \frac{(\sqrt{a} - \sqrt{b}) \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c+dx]}{a^{1/4}}\right]}{2 a^{3/4} b^{3/4} d}$$

Result (type 7, 283 leaves):

$$\begin{aligned} & -\frac{1}{8d} \operatorname{RootSum}\left[b - 4b\#1^2 - 16a\#1^4 + 6b\#1^4 - 4b\#1^6 + b\#1^8 \&, \right. \\ & \left. \left(2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] - i \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] + 6 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^2 - \right. \right. \\ & \quad \left. \left. 3 i \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^2 + 6 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^4 - 3 i \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^4 + \right. \right. \\ & \quad \left. \left. 2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^6 - i \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^6 \right) / \left(-b\#1 - 8a\#1^3 + 3b\#1^3 - 3b\#1^5 + b\#1^7 \right) \& \left. \right] \end{aligned}$$

■ **Problem 408: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sec}[c+dx]}{a - b \sin[c+dx]^4} dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c+dx]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} + \sqrt{b}) d} + \frac{\operatorname{ArcTanh}[\sin[c+dx]]}{(a-b)d} - \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c+dx]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} - \sqrt{b}) d}$$

Result (type 7, 342 leaves):

$$\begin{aligned} & \frac{1}{8ad - 8bd} \left(-8 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \right. \\ & \quad \left. 8 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] - b \operatorname{RootSum}\left[b - 4b\#1^2 - 16a\#1^4 + 6b\#1^4 - 4b\#1^6 + b\#1^8 \&, \right. \right. \\ & \quad \left. \left. \left(2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] - i \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] - 10 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^2 + \right. \right. \right. \\ & \quad \left. \left. 5 i \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^2 - 10 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^4 + 5 i \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^4 + \right. \right. \\ & \quad \left. \left. 2 \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx] - \#1}\right] \#1^6 - i \operatorname{Log}\left[1 - 2 \cos[c+dx] \#1 + \#1^2\right] \#1^6 \right) / \left(-b\#1 - 8a\#1^3 + 3b\#1^3 - 3b\#1^5 + b\#1^7 \right) \& \left. \right] \end{aligned}$$

■ **Problem 409: Result is not expressed in closed-form.**

$$\int \frac{\operatorname{Sec}[c+dx]^3}{a - b \sin[c+dx]^4} dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$\frac{b^{3/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c+dx]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} + \sqrt{b})^2 d} + \frac{(a-5b) \operatorname{ArcTanh}[\sin[c+dx]]}{2(a-b)^2 d} +$$

$$\frac{b^{3/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c+dx]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} - \sqrt{b})^2 d} + \frac{1}{4(a-b)d(1-\sin[c+dx])} - \frac{1}{4(a-b)d(1+\sin[c+dx])}$$

Result (type 7, 529 leaves):

$$\frac{1}{4(a-b)^2 d} \left(-2(a-5b) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \right.$$

$$2(a-5b) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + b \operatorname{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8 \&, \right.$$

$$\left. \frac{1}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7} \left(2b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right] - i b \operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right] - 4a \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right] \right.$$

$$\left. \#1^2 - 6b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right] \#1^2 + 2i a \operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right] \#1^2 + 3i b \operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right] \#1^2 - \right.$$

$$4a \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right] \#1^4 - 6b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right] \#1^4 + 2i a \operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right] \#1^4 +$$

$$\left. 3i b \operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right] \#1^4 + 2b \operatorname{ArcTan}\left[\frac{\sin[c+dx]}{\cos[c+dx]-\#1}\right] \#1^6 - i b \operatorname{Log}\left[1-2\cos[c+dx]\#1+\#1^2\right] \#1^6 \right) \& \left. + \right.$$

$$\left. \frac{a-b}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{-a+b}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} \right)$$

■ **Problem 410: Result is not expressed in closed-form.**

$$\int \frac{\sec[c+dx]^5}{a-b\sin[c+dx]^4} dx$$

Optimal (type 3, 249 leaves, 7 steps):

$$\frac{b^{5/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c+dx]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} + \sqrt{b})^3 d} + \frac{(3a^2 - 6ab + 35b^2) \operatorname{ArcTanh}[\sin[c+dx]]}{8(a-b)^3 d} - \frac{b^{5/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c+dx]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} - \sqrt{b})^3 d} +$$

$$\frac{1}{16(a-b)d(1-\sin[c+dx])^2} + \frac{3a-11b}{16(a-b)^2 d(1-\sin[c+dx])} - \frac{1}{16(a-b)d(1+\sin[c+dx])^2} - \frac{3a-11b}{16(a-b)^2 d(1+\sin[c+dx])}$$

Result (type 7, 731 leaves):

$$\frac{1}{16 (a-b)^3 d} \left(-2 (3 a^2 - 6 a b + 35 b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right] + 2 (3 a^2 - 6 a b + 35 b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right] - \right.$$

$$2 b^2 \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7}\right.$$

$$\left(2 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] + 6 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] - i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] - 3 i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] - \right.$$

$$26 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^2 - 14 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^2 + 13 i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^2 +$$

$$7 i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^2 - 26 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^4 - 14 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^4 +$$

$$13 i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^4 + 7 i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^4 + 2 a \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^6 +$$

$$6 b \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x] - \#1}\right] \#1^6 - i a \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^6 - 3 i b \operatorname{Log}\left[1 - 2 \operatorname{Cos}[c+d x] \#1 + \#1^2\right] \#1^6 \& \right) +$$

$$\frac{(a-b)^2}{\left(\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right)^4} + \frac{(3 a - 11 b) (a-b)}{\left(\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right)^2} -$$

$$\frac{(a-b)^2}{\left(\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right)^4} +$$

$$\left. \frac{(a-b) (-3 a + 11 b)}{\left(\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right)^2} \right)$$

■ **Problem 420: Unable to integrate problem.**

$$\int \operatorname{Cos}[e+f x]^5 (a+b \operatorname{Sin}[e+f x]^4)^p dx$$

Optimal (type 5, 197 leaves, 8 steps):

$$\frac{\operatorname{Sin}[e+f x] (a+b \operatorname{Sin}[e+f x]^4)^{1+p}}{b f (5+4 p)} - \frac{1}{b f (5+4 p)}$$

$$(a-b (5+4 p)) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, -p, \frac{5}{4}, -\frac{b \operatorname{Sin}[e+f x]^4}{a}\right] \operatorname{Sin}[e+f x] (a+b \operatorname{Sin}[e+f x]^4)^p \left(1 + \frac{b \operatorname{Sin}[e+f x]^4}{a}\right)^{-p} -$$

$$\frac{2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, -p, \frac{7}{4}, -\frac{b \operatorname{Sin}[e+f x]^4}{a}\right] \operatorname{Sin}[e+f x]^3 (a+b \operatorname{Sin}[e+f x]^4)^p \left(1 + \frac{b \operatorname{Sin}[e+f x]^4}{a}\right)^{-p}}{3 f}$$

Result (type 8, 25 leaves):

$$\int \cos[e + f x]^5 (a + b \sin[e + f x]^4)^p dx$$

■ **Problem 421: Unable to integrate problem.**

$$\int \cos[e + f x]^3 (a + b \sin[e + f x]^4)^p dx$$

Optimal (type 5, 140 leaves, 7 steps):

$$\frac{\text{Hypergeometric2F1}\left[\frac{1}{4}, -p, \frac{5}{4}, -\frac{b \sin[e + f x]^4}{a}\right] \sin[e + f x] (a + b \sin[e + f x]^4)^p \left(1 + \frac{b \sin[e + f x]^4}{a}\right)^{-p}}{f} - \frac{\text{Hypergeometric2F1}\left[\frac{3}{4}, -p, \frac{7}{4}, -\frac{b \sin[e + f x]^4}{a}\right] \sin[e + f x]^3 (a + b \sin[e + f x]^4)^p \left(1 + \frac{b \sin[e + f x]^4}{a}\right)^{-p}}{3 f}$$

Result (type 8, 25 leaves):

$$\int \cos[e + f x]^3 (a + b \sin[e + f x]^4)^p dx$$

■ **Problem 423: Unable to integrate problem.**

$$\int \sec[e + f x] (a + b \sin[e + f x]^4)^p dx$$

Optimal (type 6, 158 leaves, 7 steps):

$$\frac{\text{AppellF1}\left[\frac{1}{4}, 1, -p, \frac{5}{4}, \sin[e + f x]^4, -\frac{b \sin[e + f x]^4}{a}\right] \sin[e + f x] (a + b \sin[e + f x]^4)^p \left(1 + \frac{b \sin[e + f x]^4}{a}\right)^{-p}}{f} + \frac{1}{3 f} \text{AppellF1}\left[\frac{3}{4}, 1, -p, \frac{7}{4}, \sin[e + f x]^4, -\frac{b \sin[e + f x]^4}{a}\right] \sin[e + f x]^3 (a + b \sin[e + f x]^4)^p \left(1 + \frac{b \sin[e + f x]^4}{a}\right)^{-p}$$

Result (type 8, 23 leaves):

$$\int \sec[e + f x] (a + b \sin[e + f x]^4)^p dx$$

■ **Problem 424: Unable to integrate problem.**

$$\int \sec[e + f x]^3 (a + b \sin[e + f x]^4)^p dx$$

Optimal (type 6, 239 leaves, 9 steps):

$$\frac{\text{AppellF1}\left[\frac{1}{4}, 2, -p, \frac{5}{4}, \sin[e+fx]^4, -\frac{b\sin[e+fx]^4}{a}\right] \sin[e+fx] (a+b\sin[e+fx]^4)^p \left(1+\frac{b\sin[e+fx]^4}{a}\right)^{-p}}{f} + \frac{1}{3f}$$

$$2 \text{AppellF1}\left[\frac{3}{4}, 2, -p, \frac{7}{4}, \sin[e+fx]^4, -\frac{b\sin[e+fx]^4}{a}\right] \sin[e+fx]^3 (a+b\sin[e+fx]^4)^p \left(1+\frac{b\sin[e+fx]^4}{a}\right)^{-p} +$$

$$\frac{1}{5f} \text{AppellF1}\left[\frac{5}{4}, 2, -p, \frac{9}{4}, \sin[e+fx]^4, -\frac{b\sin[e+fx]^4}{a}\right] \sin[e+fx]^5 (a+b\sin[e+fx]^4)^p \left(1+\frac{b\sin[e+fx]^4}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \sec[e+fx]^3 (a+b\sin[e+fx]^4)^p dx$$

■ **Problem 431: Unable to integrate problem.**

$$\int \cos[e+fx]^5 (a+b\sin[e+fx]^n)^p dx$$

Optimal (type 5, 226 leaves, 9 steps):

$$\frac{\text{Hypergeometric2F1}\left[\frac{1}{n}, -p, 1+\frac{1}{n}, -\frac{b\sin[e+fx]^n}{a}\right] \sin[e+fx] (a+b\sin[e+fx]^n)^p \left(1+\frac{b\sin[e+fx]^n}{a}\right)^{-p}}{f} -$$

$$\frac{2 \text{Hypergeometric2F1}\left[\frac{3}{n}, -p, \frac{3+n}{n}, -\frac{b\sin[e+fx]^n}{a}\right] \sin[e+fx]^3 (a+b\sin[e+fx]^n)^p \left(1+\frac{b\sin[e+fx]^n}{a}\right)^{-p}}{3f} +$$

$$\frac{\text{Hypergeometric2F1}\left[\frac{5}{n}, -p, \frac{5+n}{n}, -\frac{b\sin[e+fx]^n}{a}\right] \sin[e+fx]^5 (a+b\sin[e+fx]^n)^p \left(1+\frac{b\sin[e+fx]^n}{a}\right)^{-p}}{5f}$$

Result (type 8, 25 leaves):

$$\int \cos[e+fx]^5 (a+b\sin[e+fx]^n)^p dx$$

■ **Problem 432: Unable to integrate problem.**

$$\int \cos[e+fx]^3 (a+b\sin[e+fx]^n)^p dx$$

Optimal (type 5, 148 leaves, 7 steps):

$$\frac{\text{Hypergeometric2F1}\left[\frac{1}{n}, -p, 1+\frac{1}{n}, -\frac{b\sin[e+fx]^n}{a}\right] \sin[e+fx] (a+b\sin[e+fx]^n)^p \left(1+\frac{b\sin[e+fx]^n}{a}\right)^{-p}}{f} -$$

$$\frac{\text{Hypergeometric2F1}\left[\frac{3}{n}, -p, \frac{3+n}{n}, -\frac{b\sin[e+fx]^n}{a}\right] \sin[e+fx]^3 (a+b\sin[e+fx]^n)^p \left(1+\frac{b\sin[e+fx]^n}{a}\right)^{-p}}{3f}$$

Result (type 8, 25 leaves):

$$\int \cos[e+fx]^3 (a+b\sin[e+fx]^n)^p dx$$

■ **Problem 474: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^2}{\sqrt{a - a \sin[e + f x]^2}} dx$$

Optimal (type 3, 62 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}[\sin[e + f x]] \cos[e + f x]}{2 f \sqrt{a \cos[e + f x]^2}} + \frac{\tan[e + f x]}{2 f \sqrt{a \cos[e + f x]^2}}$$

Result (type 3, 142 leaves):

$$\frac{1}{4 f \sqrt{a \cos[e + f x]^2}} \operatorname{Sec}[e + f x] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + \right. \\ \left. \cos[2(e + f x)] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \right) - \right. \\ \left. \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] + 2 \sin[e + f x] \right)$$

■ **Problem 483: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^2}{(a - a \sin[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 106 leaves, 5 steps):

$$-\frac{\operatorname{ArcTanh}[\sin[e + f x]] \cos[e + f x]}{8 a f \sqrt{a \cos[e + f x]^2}} - \frac{\tan[e + f x]}{8 a f \sqrt{a \cos[e + f x]^2}} + \frac{\operatorname{Sec}[e + f x]^2 \tan[e + f x]}{4 a f \sqrt{a \cos[e + f x]^2}}$$

Result (type 3, 213 leaves):

$$\frac{1}{64 f (a \cos[e + f x]^2)^{3/2}} \operatorname{Sec}[e + f x] \left(3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + \right. \\ \left. 4 \cos[2(e + f x)] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \right) + \right. \\ \left. \cos[4(e + f x)] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \right) - \right. \\ \left. 3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] + 14 \sin[e + f x] - 2 \sin[3(e + f x)] \right)$$

■ **Problem 543: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sin[e + f x]^2)^p (d \tan[e + f x])^m dx$$

Optimal (type 6, 120 leaves, 3 steps) :

$$\frac{1}{d f (1+m)} \text{AppellF1} \left[\frac{1+m}{2}, \frac{1+m}{2}, -p, \frac{3+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a} \right] \\ \left(\cos[e+f x]^2 \right)^{\frac{1+m}{2}} (a+b \sin[e+f x]^2)^p \left(1 + \frac{b \sin[e+f x]^2}{a} \right)^{-p} (d \tan[e+f x])^{1+m}$$

Result (type 6, 260 leaves) :

$$\left(a (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{1+m}{2}, -p, \frac{3+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a} \right] (a+b \sin[e+f x]^2)^p \tan[e+f x] (d \tan[e+f x])^m \right) / \\ \left(f (1+m) \left(a (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{1+m}{2}, -p, \frac{3+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a} \right] + \right. \right. \\ \left. \left(2 b p \text{AppellF1} \left[\frac{3+m}{2}, \frac{1+m}{2}, 1-p, \frac{5+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a} \right] + \right. \right. \\ \left. \left. a (1+m) \text{AppellF1} \left[\frac{3+m}{2}, \frac{3+m}{2}, -p, \frac{5+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a} \right] \right) \sin[e+f x]^2 \right) \right)$$

■ **Problem 547: Unable to integrate problem.**

$$\int \cot [c+d x]^3 (a+b \sin [c+d x]^2)^p dx$$

Optimal (type 5, 95 leaves, 3 steps) :

$$-\frac{\csc [c+d x]^2 (a+b \sin [c+d x]^2)^{1+p}}{2 a d} + \frac{(a-b p) \text{Hypergeometric2F1} \left[1, 1+p, 2+p, 1 + \frac{b \sin [c+d x]^2}{a} \right] (a+b \sin [c+d x]^2)^{1+p}}{2 a^2 d (1+p)}$$

Result (type 8, 25 leaves) :

$$\int \cot [c+d x]^3 (a+b \sin [c+d x]^2)^p dx$$

■ **Problem 552: Result is not expressed in closed-form.**

$$\int \frac{\cot [x]^3}{a+b \sin [x]^3} dx$$

Optimal (type 3, 153 leaves, 11 steps) :

$$\frac{b^{2/3} \text{ArcTan} \left[\frac{a^{1/3}-2 b^{1/3} \sin [x]}{\sqrt{3} a^{1/3}} \right]}{\sqrt{3} a^{5/3}} - \frac{\csc [x]^2}{2 a} - \frac{\text{Log} [\sin [x]]}{a} - \\ \frac{b^{2/3} \text{Log} [a^{1/3} + b^{1/3} \sin [x]]}{3 a^{5/3}} + \frac{b^{2/3} \text{Log} [a^{2/3} - a^{1/3} b^{1/3} \sin [x] + b^{2/3} \sin [x]^2]}{6 a^{5/3}} + \frac{\text{Log} [a+b \sin [x]^3]}{3 a}$$

Result (type 7, 210 leaves) :

$$\frac{1}{24 a} \left(8 \operatorname{RootSum} \left[a + 3 a \#1^2 + 8 b \#1^3 + 3 a \#1^4 + a \#1^6 \&, \right. \right. \\ \left. \left. \left(-b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{x}{2} \right] \right] + a \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{x}{2} \right] \right] \#1 + 4 b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{x}{2} \right] \right] \#1^2 + 2 a \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{x}{2} \right] \right] \#1^3 + b \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{x}{2} \right] \right] \#1^4 + \right. \right. \\ \left. \left. a \operatorname{Log} \left[-\#1 + \operatorname{Tan} \left[\frac{x}{2} \right] \right] \#1^5 \right) / \left(a \#1 + 4 b \#1^2 + 2 a \#1^3 + a \#1^5 \right) \& \right] - 3 \left(\operatorname{Csc} \left[\frac{x}{2} \right]^2 + 8 \left(\operatorname{Log} \left[\operatorname{Sec} \left[\frac{x}{2} \right]^2 \right] + \operatorname{Log} [\operatorname{Sin}[x]] \right) + \operatorname{Sec} \left[\frac{x}{2} \right]^2 \right) \right)$$

- **Problem 554: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[x]}{\sqrt{a + b \operatorname{Sin}[x]^3}} dx$$

Optimal (type 3, 28 leaves, 4 steps):

$$-\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \operatorname{Sin}[x]^3}}{\sqrt{a}} \right]}{3 \sqrt{a}}$$

Result (type 3, 66 leaves):

$$-\frac{2 \sqrt{b} \operatorname{ArcSinh} \left[\frac{\sqrt{a} \operatorname{Csc}[x]^{3/2}}{\sqrt{b}} \right] \sqrt{\frac{b + a \operatorname{Csc}[x]^3}{b}}}{3 \sqrt{a} \operatorname{Csc}[x]^{3/2} \sqrt{a + b \operatorname{Sin}[x]^3}}$$

- **Problem 555: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + d x] \sqrt{a + b \operatorname{Sin}[c + d x]^4} dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \operatorname{Sin}[c + d x]^4}}{\sqrt{a}} \right]}{2 d} + \frac{\sqrt{a + b \operatorname{Sin}[c + d x]^4}}{2 d}$$

Result (type 3, 166 leaves):

$$\left(\sqrt{\operatorname{Cos}[c + d x]^4 \left(a + 2 a \operatorname{Tan}[c + d x]^2 + (a + b) \operatorname{Tan}[c + d x]^4 \right)} \right. \\ \left. \left(\sqrt{a} \left(\operatorname{Log} [\operatorname{Tan}[c + d x]^2] - \operatorname{Log} \left[a + a \operatorname{Tan}[c + d x]^2 + \sqrt{a} \sqrt{a \operatorname{Sec}[c + d x]^4 + b \operatorname{Tan}[c + d x]^4} \right] \right) \operatorname{Sec}[c + d x]^2 + \right. \right. \\ \left. \left. \sqrt{a \operatorname{Sec}[c + d x]^4 + b \operatorname{Tan}[c + d x]^4} \right) \right) / \left(2 d \sqrt{a \operatorname{Sec}[c + d x]^4 + b \operatorname{Tan}[c + d x]^4} \right)$$

- **Problem 556: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c + d x]^3}{\sqrt{a + b \operatorname{Sin}[c + d x]^4}} dx$$

Optimal (type 3, 89 leaves, 4 steps) :

$$-\frac{a \operatorname{ArcTanh}\left[\frac{a+b \sin [c+d x]^2}{\sqrt{a+b} \sqrt{a+b \sin [c+d x]^4}}\right]}{2(a+b)^{3/2} d} + \frac{\operatorname{Sec}[c+d x]^2 \sqrt{a+b \sin [c+d x]^4}}{2(a+b) d}$$

Result (type 4, 63448 leaves) : Display of huge result suppressed!

- **Problem 557: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+d x]}{\sqrt{a+b \sin [c+d x]^4}} dx$$

Optimal (type 3, 51 leaves, 3 steps) :

$$\frac{\operatorname{ArcTanh}\left[\frac{a+b \sin [c+d x]^2}{\sqrt{a+b} \sqrt{a+b \sin [c+d x]^4}}\right]}{2 \sqrt{a+b} d}$$

Result (type 4, 39909 leaves) : Display of huge result suppressed!

- **Problem 558: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+d x]}{\sqrt{a+b \sin [c+d x]^4}} dx$$

Optimal (type 3, 35 leaves, 4 steps) :

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sin [c+d x]^4}}{\sqrt{a}}\right]}{2 \sqrt{a} d}$$

Result (type 3, 142 leaves) :

$$\left(\sqrt{8 a+3 b-4 b \operatorname{Cos}[2(c+d x)]+b \operatorname{Cos}[4(c+d x)]} \left(\operatorname{Log}[\operatorname{Tan}[c+d x]^2] - \operatorname{Log}\left[a+a \operatorname{Tan}[c+d x]^2+\sqrt{a} \sqrt{a \operatorname{Sec}[c+d x]^4+b \operatorname{Tan}[c+d x]^4} \right] \right) \right. \\ \left. \operatorname{Sec}[c+d x]^2 \right) / \left(4 \sqrt{2} \sqrt{a} d \sqrt{a+2 a \operatorname{Tan}[c+d x]^2+(a+b) \operatorname{Tan}[c+d x]^4} \right)$$

- **Problem 559: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+d x]^3}{\sqrt{a+b \sin [c+d x]^4}} dx$$

Optimal (type 3, 70 leaves, 5 steps) :

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sin [c+d x]^4}}{\sqrt{a}}\right]}{2 \sqrt{a} d} - \frac{\operatorname{Csc}[c+d x]^2 \sqrt{a+b \sin [c+d x]^4}}{2 a d}$$

Result (type 3, 185 leaves) :

$$-\left(\sqrt{8 a+3 b-4 b \cos [2(c+d x)]+b \cos [4(c+d x)]}\right. \\ \left.\left(\sqrt{a}\left(\log [\tan [c+d x]^2]-\log \left[a+a \tan [c+d x]^2+\sqrt{a} \sqrt{a \sec [c+d x]^4+b \tan [c+d x]^4}\right]\right)\right) \sec [c+d x]^2+\right. \\ \left.\csc [c+d x]^2 \sqrt{a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4}\right) / \left(4 \sqrt{2} a d \sqrt{a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4}\right)$$

■ **Problem 561: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan [c+d x]^2}{\sqrt{a+b \sin [c+d x]^4}} d x$$

Optimal (type 4, 411 leaves, 4 steps) :

$$\frac{\cos [c+d x] \sin [c+d x] (a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4)}{\sqrt{a+b} d \sqrt{a+b \sin [c+d x]^4} (\sqrt{a}+\sqrt{a+b} \tan [c+d x]^2)} - \\ \left(a^{1/4} \cos [c+d x]^2 \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{(a+b)^{1/4} \tan [c+d x]}{a^{1/4}}\right], \frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right]\right. \\ \left. (\sqrt{a}+\sqrt{a+b} \tan [c+d x]^2) \sqrt{\frac{a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4}{(\sqrt{a}+\sqrt{a+b} \tan [c+d x]^2)^2}}\right) / \left((a+b)^{3/4} d \sqrt{a+b \sin [c+d x]^4}\right) + \\ \left(a^{1/4} \cos [c+d x]^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{(a+b)^{1/4} \tan [c+d x]}{a^{1/4}}\right], \frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right]\right) (\sqrt{a}+\sqrt{a+b} \tan [c+d x]^2) \\ \left.\sqrt{\frac{a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4}{(\sqrt{a}+\sqrt{a+b} \tan [c+d x]^2)^2}}\right) / \left(2(a+b)^{3/4} d \sqrt{a+b \sin [c+d x]^4}\right)$$

Result (type 4, 291 leaves) :

$$\begin{aligned}
& - \left(2 i \sqrt{2} \sqrt{a} \operatorname{Cos}[c + d x]^2 \right. \\
& \quad \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \operatorname{Tan}[c + d x] \right], \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \operatorname{Tan}[c + d x] \right], \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}} \right] \right) \\
& \quad \sqrt{1 + \left(1 - \frac{i \sqrt{b}}{\sqrt{a}} \right) \operatorname{Tan}[c + d x]^2} \sqrt{1 + \left(1 + \frac{i \sqrt{b}}{\sqrt{a}} \right) \operatorname{Tan}[c + d x]^2} \Big/ \\
& \quad \left((\sqrt{a} + i \sqrt{b}) \sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} d \sqrt{8 a + 3 b - 4 b \operatorname{Cos}[2(c + d x)] + b \operatorname{Cos}[4(c + d x)]} \right)
\end{aligned}$$

■ **Problem 562: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + b \operatorname{Sin}[c + d x]^4}} dx$$

Optimal (type 4, 162 leaves, 2 steps):

$$\begin{aligned}
& \left(\operatorname{Cos}[c + d x]^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{(a + b)^{1/4} \operatorname{Tan}[c + d x]}{a^{1/4}} \right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a + b}} \right) \right] \right) \\
& \quad (\sqrt{a} + \sqrt{a + b} \operatorname{Tan}[c + d x]^2) \sqrt{\frac{a + 2 a \operatorname{Tan}[c + d x]^2 + (a + b) \operatorname{Tan}[c + d x]^4}{(\sqrt{a} + \sqrt{a + b} \operatorname{Tan}[c + d x]^2)^2}} \Big/ \left(2 a^{1/4} (a + b)^{1/4} d \sqrt{a + b \operatorname{Sin}[c + d x]^4} \right)
\end{aligned}$$

Result (type 4, 195 leaves):

$$\begin{aligned}
& - \left(2 i \operatorname{Cos}[c + d x]^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \operatorname{Tan}[c + d x] \right], \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}} \right] \sqrt{1 + \left(1 + \frac{i \sqrt{b}}{\sqrt{a}} \right) \operatorname{Tan}[c + d x]^2} \right. \\
& \quad \left. \sqrt{2 + \left(2 - \frac{2 i \sqrt{b}}{\sqrt{a}} \right) \operatorname{Tan}[c + d x]^2} \right) \Big/ \left(\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} d \sqrt{8 a + 3 b - 4 b \operatorname{Cos}[2(c + d x)] + b \operatorname{Cos}[4(c + d x)]} \right)
\end{aligned}$$

■ **Problem 563: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cot}[c + dx]^2}{\sqrt{a + b \text{Sin}[c + dx]^4}} dx$$

Optimal (type 4, 477 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{\text{Cos}[c + dx]^2 \text{Cot}[c + dx] (a + 2a \text{Tan}[c + dx]^2 + (a + b) \text{Tan}[c + dx]^4)}{ad \sqrt{a + b \text{Sin}[c + dx]^4}} + \\
 & \frac{\sqrt{a + b} \text{Cos}[c + dx] \text{Sin}[c + dx] (a + 2a \text{Tan}[c + dx]^2 + (a + b) \text{Tan}[c + dx]^4)}{ad \sqrt{a + b \text{Sin}[c + dx]^4} (\sqrt{a} + \sqrt{a + b} \text{Tan}[c + dx]^2)} - \\
 & \left((a + b)^{1/4} \text{Cos}[c + dx]^2 \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{(a + b)^{1/4} \text{Tan}[c + dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a + b}}\right)\right] \right. \\
 & \left. (\sqrt{a} + \sqrt{a + b} \text{Tan}[c + dx]^2) \sqrt{\frac{a + 2a \text{Tan}[c + dx]^2 + (a + b) \text{Tan}[c + dx]^4}{(\sqrt{a} + \sqrt{a + b} \text{Tan}[c + dx]^2)^2}} \right) / (a^{3/4} d \sqrt{a + b \text{Sin}[c + dx]^4}) + \\
 & \left((a + b)^{1/4} \text{Cos}[c + dx]^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{(a + b)^{1/4} \text{Tan}[c + dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a + b}}\right)\right] (\sqrt{a} + \sqrt{a + b} \text{Tan}[c + dx]^2) \right. \\
 & \left. \sqrt{\frac{a + 2a \text{Tan}[c + dx]^2 + (a + b) \text{Tan}[c + dx]^4}{(\sqrt{a} + \sqrt{a + b} \text{Tan}[c + dx]^2)^2}} \right) / (2 a^{3/4} d \sqrt{a + b \text{Sin}[c + dx]^4})
 \end{aligned}$$

Result (type 4, 378 leaves):

$$\frac{\sqrt{8 a + 3 b - 4 b \cos[2(c + d x)] + b \cos[4(c + d x)]} \cot[c + d x]}{2 \sqrt{2} a d}$$

$$\left(\cos[c + d x]^4 \left(a \sec[c + d x]^4 \tan[c + d x] + b \tan[c + d x]^5 + 1 \right) / \left(\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \right) \left(i a + \sqrt{a} \sqrt{b} \right) \right. \\ \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + d x] \right], \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}} \right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + d x] \right], \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}} \right] \right) \right. \\ \left. \left. \sec[c + d x]^2 \sqrt{1 + \left(1 - \frac{i \sqrt{b}}{\sqrt{a}} \right) \tan[c + d x]^2} \sqrt{1 + \left(1 + \frac{i \sqrt{b}}{\sqrt{a}} \right) \tan[c + d x]^2} \right) \right) / \\ \left(a d \sqrt{\cos[c + d x]^4 (a + 2 a \tan[c + d x]^2 + (a + b) \tan[c + d x]^4)} \right)$$

■ **Problem 565: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sin[c + d x]^4)^p \tan[c + d x]^3 dx$$

Optimal (type 6, 279 leaves, 11 steps):

$$\frac{(a + b + 2 b p) \operatorname{Hypergeometric2F1}\left[1, 1 + p, 2 + p, \frac{a + b \sin[c + d x]^4}{a + b}\right] (a + b \sin[c + d x]^4)^{1 + p}}{4 (a + b)^2 d (1 + p)} + \frac{\sec[c + d x]^2 (a + b \sin[c + d x]^4)^{1 + p}}{2 (a + b) d} - \frac{1}{2 (a + b) d} \\ (a + b + 2 b p) \operatorname{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, \sin[c + d x]^4, -\frac{b \sin[c + d x]^4}{a}\right] \sin[c + d x]^2 (a + b \sin[c + d x]^4)^p \left(1 + \frac{b \sin[c + d x]^4}{a}\right)^{-p} + \\ \frac{1}{2 (a + b) d} b (1 + 2 p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \sin[c + d x]^4}{a}\right] \sin[c + d x]^2 (a + b \sin[c + d x]^4)^p \left(1 + \frac{b \sin[c + d x]^4}{a}\right)^{-p}$$

Result (type 6, 2007 leaves):

$$- \left(\left((1 - 2 p) \operatorname{AppellF1}\left[-2 p, -p, -p, 1 - 2 p, -\frac{(a + b) \sec[c + d x]^2}{-b + \sqrt{-a b}}, \frac{(a + b) \sec[c + d x]^2}{b + \sqrt{-a b}}\right] + \right. \right. \\ \left. \left. 2 p \operatorname{AppellF1}\left[1 - 2 p, -p, -p, 2 - 2 p, -\frac{(a + b) \sec[c + d x]^2}{-b + \sqrt{-a b}}, \frac{(a + b) \sec[c + d x]^2}{b + \sqrt{-a b}}\right] \sec[c + d x]^2 \right) \\ (a + b \sin[c + d x]^4)^p \tan[c + d x]^3 \left(\frac{-a + \sqrt{-a b} - (a + b) \tan[c + d x]^2}{b + \sqrt{-a b}} \right)^{-p} \left(\frac{a + \sqrt{-a b} + (a + b) \tan[c + d x]^2}{-b + \sqrt{-a b}} \right)^{-p}$$

$$\begin{aligned}
& \left(\cos [c+d x]^4 \left(a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4 \right) \right)^p \Bigg/ \left(4 d p(-1+2 p) \right. \\
& \left. \left(\frac{1}{2(-b+\sqrt{-a b})}(-1+2 p) \right) (a+b) \sec [c+d x]^2 \left[(1-2 p) \operatorname{AppellF1} \left[-2 p,-p,-p, 1-2 p,-\frac{(a+b) \sec [c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \sec [c+d x]^2}{b+\sqrt{-a b}} \right] + \right. \\
& \left. 2 p \operatorname{AppellF1} \left[1-2 p,-p,-p, 2-2 p,-\frac{(a+b) \sec [c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \sec [c+d x]^2}{b+\sqrt{-a b}} \right] \sec [c+d x]^2 \right) \\
& \tan [c+d x] \left(\frac{-a+\sqrt{-a b}-(a+b) \tan [c+d x]^2}{b+\sqrt{-a b}} \right)^{-p} \left(\frac{a+\sqrt{-a b}+(a+b) \tan [c+d x]^2}{-b+\sqrt{-a b}} \right)^{-1-p} \\
& \left(\cos [c+d x]^4 \left(a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4 \right) \right)^p - \frac{1}{2(b+\sqrt{-a b})(-1+2 p)} \\
& (a+b) \sec [c+d x]^2 \left[(1-2 p) \operatorname{AppellF1} \left[-2 p,-p,-p, 1-2 p,-\frac{(a+b) \sec [c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \sec [c+d x]^2}{b+\sqrt{-a b}} \right] + 2 p \operatorname{AppellF1} \left[1-2 p, \right. \right. \\
& \left. \left. -p,-p, 2-2 p,-\frac{(a+b) \sec [c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \sec [c+d x]^2}{b+\sqrt{-a b}} \right] \sec [c+d x]^2 \right] \tan [c+d x] \left(\frac{-a+\sqrt{-a b}-(a+b) \tan [c+d x]^2}{b+\sqrt{-a b}} \right)^{-1-p} \\
& \left(\frac{a+\sqrt{-a b}+(a+b) \tan [c+d x]^2}{-b+\sqrt{-a b}} \right)^{-p} \left(\cos [c+d x]^4 \left(a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4 \right) \right)^p - \frac{1}{4 p(-1+2 p)} \\
& \left(\frac{-a+\sqrt{-a b}-(a+b) \tan [c+d x]^2}{b+\sqrt{-a b}} \right)^{-p} \left(\frac{a+\sqrt{-a b}+(a+b) \tan [c+d x]^2}{-b+\sqrt{-a b}} \right)^{-p} \left(\cos [c+d x]^4 \left(a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4 \right) \right)^p \\
& \left(4 p \operatorname{AppellF1} \left[1-2 p,-p,-p, 2-2 p,-\frac{(a+b) \sec [c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \sec [c+d x]^2}{b+\sqrt{-a b}} \right] \sec [c+d x]^2 \tan [c+d x] + \right. \\
& \left. (1-2 p) \left(- \left(4(a+b) p^2 \operatorname{AppellF1} \left[1-2 p, 1-p,-p, 2-2 p,-\frac{(a+b) \sec [c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \sec [c+d x]^2}{b+\sqrt{-a b}} \right] \right) \right. \right. \\
& \left. \left. \sec [c+d x]^2 \tan [c+d x] \right) \Bigg/ \left((-b+\sqrt{-a b})(1-2 p) \right) + \frac{1}{(b+\sqrt{-a b})(1-2 p)} \right. \\
& \left. 4(a+b) p^2 \operatorname{AppellF1} \left[1-2 p,-p, 1-p, 2-2 p,-\frac{(a+b) \sec [c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \sec [c+d x]^2}{b+\sqrt{-a b}} \right] \sec [c+d x]^2 \tan [c+d x] \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 p \operatorname{Sec}[c+d x]^2 \left(\left(2 (a+b) (1-2 p) p \operatorname{AppellF1} \left[2-2 p, 1-p, -p, 3-2 p, -\frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b+\sqrt{-a b}} \right], \right. \right. \\
& \left. \left. \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \right) / \left((-b+\sqrt{-a b}) (2-2 p) \right) - \frac{1}{(b+\sqrt{-a b}) (2-2 p)} 2 (a+b) (1-2 p) p \operatorname{AppellF1} \left[\right. \\
& \left. \left. 2-2 p, -p, 1-p, 3-2 p, -\frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b+\sqrt{-a b}} \right] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \right) \right) - \\
& \frac{1}{4 (-1+2 p)} \left((1-2 p) \operatorname{AppellF1} \left[-2 p, -p, -p, 1-2 p, -\frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b+\sqrt{-a b}} \right] + 2 p \operatorname{AppellF1} \left[\right. \right. \\
& \left. \left. 1-2 p, -p, -p, 2-2 p, -\frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b+\sqrt{-a b}}, \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b+\sqrt{-a b}} \right] \operatorname{Sec}[c+d x]^2 \right) \left(\frac{-a+\sqrt{-a b}-(a+b) \operatorname{Tan}[c+d x]^2}{b+\sqrt{-a b}} \right)^{-p} \\
& \left(\frac{a+\sqrt{-a b}+(a+b) \operatorname{Tan}[c+d x]^2}{-b+\sqrt{-a b}} \right)^{-p} \left(\operatorname{Cos}[c+d x]^4 (a+2 a \operatorname{Tan}[c+d x]^2+(a+b) \operatorname{Tan}[c+d x]^4) \right)^{-1+p} \\
& \left(\operatorname{Cos}[c+d x]^4 (4 a \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]+4 (a+b) \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]^3) - \right. \\
& \left. \left. 4 \operatorname{Cos}[c+d x]^3 \operatorname{Sin}[c+d x] (a+2 a \operatorname{Tan}[c+d x]^2+(a+b) \operatorname{Tan}[c+d x]^4) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 566: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sin}[c+d x]^4)^p \operatorname{Tan}[c+d x] dx$$

Optimal (type 6, 141 leaves, 7 steps):

$$\frac{\operatorname{Hypergeometric2F1} \left[1, 1+p, 2+p, \frac{a+b \operatorname{Sin}[c+d x]^4}{a+b} \right] (a+b \operatorname{Sin}[c+d x]^4)^{1+p}}{4 (a+b) d (1+p)} + \frac{1}{2 d}$$

$$\operatorname{AppellF1} \left[\frac{1}{2}, 1, -p, \frac{3}{2}, \operatorname{Sin}[c+d x]^4, -\frac{b \operatorname{Sin}[c+d x]^4}{a} \right] \operatorname{Sin}[c+d x]^2 (a+b \operatorname{Sin}[c+d x]^4)^p \left(1 + \frac{b \operatorname{Sin}[c+d x]^4}{a} \right)^{-p}$$

Result (type 6, 466 leaves):

$$\begin{aligned}
& - \left(\left((-b + \sqrt{-ab}) (b + \sqrt{-ab}) (-1 + 2p) \operatorname{AppellF1} \left[-2p, -p, -p, 1 - 2p, -\frac{(a+b) \operatorname{Sec}[c+dx]^2}{-b + \sqrt{-ab}}, \frac{(a+b) \operatorname{Sec}[c+dx]^2}{b + \sqrt{-ab}} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx] (a+b \operatorname{Sin}[c+dx]^4)^p \left(-a + \sqrt{-ab} - (a+b) \operatorname{Tan}[c+dx]^2 \right) \left(a + \sqrt{-ab} + (a+b) \operatorname{Tan}[c+dx]^2 \right) \right) \right) / \\
& \left(2(a+b)^2 dp \left(b (-1 + 2p) \operatorname{AppellF1} \left[-2p, -p, -p, 1 - 2p, -\frac{(a+b) \operatorname{Sec}[c+dx]^2}{-b + \sqrt{-ab}}, \frac{(a+b) \operatorname{Sec}[c+dx]^2}{b + \sqrt{-ab}} \right] \operatorname{Sin}[2(c+dx)] + \right. \right. \\
& \quad \left. \left. 2p \left((b + \sqrt{-ab}) \operatorname{AppellF1} \left[1 - 2p, 1 - p, -p, 2 - 2p, -\frac{(a+b) \operatorname{Sec}[c+dx]^2}{-b + \sqrt{-ab}}, \frac{(a+b) \operatorname{Sec}[c+dx]^2}{b + \sqrt{-ab}} \right] + (b - \sqrt{-ab}) \operatorname{AppellF1} \left[1 - 2p, \right. \right. \right. \\
& \quad \left. \left. \left. -p, 1 - p, 2 - 2p, -\frac{(a+b) \operatorname{Sec}[c+dx]^2}{-b + \sqrt{-ab}}, \frac{(a+b) \operatorname{Sec}[c+dx]^2}{b + \sqrt{-ab}} \right] \right) \operatorname{Tan}[c+dx] \right) \left(a + 2a \operatorname{Tan}[c+dx]^2 + (a+b) \operatorname{Tan}[c+dx]^4 \right) \left. \right)
\end{aligned}$$

■ **Problem 568: Unable to integrate problem.**

$$\int \operatorname{Cot}[c+dx]^3 (a+b \operatorname{Sin}[c+dx]^4)^p dx$$

Optimal (type 5, 127 leaves, 6 steps):

$$\frac{\operatorname{Hypergeometric2F1} \left[1, 1+p, 2+p, 1 + \frac{b \operatorname{Sin}[c+dx]^4}{a} \right] (a+b \operatorname{Sin}[c+dx]^4)^{1+p}}{4ad(1+p)} - \frac{\operatorname{Csc}[c+dx]^2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Sin}[c+dx]^4}{a} \right] (a+b \operatorname{Sin}[c+dx]^4)^p \left(1 + \frac{b \operatorname{Sin}[c+dx]^4}{a} \right)^{-p}}{2d}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Cot}[c+dx]^3 (a+b \operatorname{Sin}[c+dx]^4)^p dx$$

■ **Problem 574: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sin}[c+dx]^n)^3 \operatorname{Tan}[c+dx]^m dx$$

Optimal (type 5, 306 leaves, 10 steps):

$$\begin{aligned}
& \frac{a^3 \operatorname{Hypergeometric2F1} \left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\operatorname{Tan}[c+dx]^2 \right] \operatorname{Tan}[c+dx]^{1+m}}{d(1+m)} + \frac{1}{d(1+m+n)} \\
& 3a^2 b (\operatorname{Cos}[c+dx]^2)^{\frac{1+m}{2}} \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), \operatorname{Sin}[c+dx]^2 \right] \operatorname{Sin}[c+dx]^n \operatorname{Tan}[c+dx]^{1+m} + \frac{1}{d(1+m+2n)} \\
& 3ab^2 (\operatorname{Cos}[c+dx]^2)^{\frac{1+m}{2}} \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{1}{2}(1+m+2n), \frac{1}{2}(3+m+2n), \operatorname{Sin}[c+dx]^2 \right] \operatorname{Sin}[c+dx]^{2n} \operatorname{Tan}[c+dx]^{1+m} + \\
& \frac{1}{d(1+m+3n)} b^3 (\operatorname{Cos}[c+dx]^2)^{\frac{1+m}{2}} \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{1}{2}(1+m+3n), \frac{1}{2}(3+m+3n), \operatorname{Sin}[c+dx]^2 \right] \operatorname{Sin}[c+dx]^{3n} \operatorname{Tan}[c+dx]^{1+m}
\end{aligned}$$

Result (type 6, 13001 leaves) :

$$\begin{aligned}
& \left(\left(\left(a^3 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \right) / \right. \\
& \quad \left((1+m) \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \quad \left(3 \times 2^n a^2 b (3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^n \right) / \\
& \quad \left((1+m+n) \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \quad \left(3 \times 2^{2n} a b^2 (3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{2n} \right) / \left((1+m+2n) \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right. \\
& \quad \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \quad \left. \left. 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \quad \left(2^{3n} b^3 (3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\tan\left[\frac{1}{2}(c+dx)\right] \left(-\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{-1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3n} \right) / \left((1+m+3n) \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right. \\
& \left((3+m+3n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+3n), m, 1+3n, \frac{1}{2}(3+m+3n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& \left. 2 \left((1+3n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), m, 2+3n, \frac{1}{2}(5+m+3n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), 1+m, 1+3n, \frac{1}{2}(5+m+3n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \left. \right) \\
& \left(a^3 \tan[c+dx]^m + 3 a^2 b \sin[c+dx]^n \tan[c+dx]^m + 3 a b^2 \sin[c+dx]^{2n} \tan[c+dx]^m + b^3 \sin[c+dx]^{3n} \tan[c+dx]^m \right) \left. \right) / \\
& \left(d \left(- \left(a^3 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{-1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \right) / \left((1+m) \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right. \\
& \left. \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& \left(a^3 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{-1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \right) / \\
& \left(2 (1+m) \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right. \\
& \left. \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& \left(a^3 (3+m) \tan\left[\frac{1}{2}(c+dx)\right] \left(-1 / (3+m) (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, m, 2, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] + 1 / (3+m)m(1+m) \text{AppellF1}\left[1 + \frac{1+m}{2}, 1+m, 1, 1 + \frac{3+m}{2}, \right. \\
& \left. \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-\frac{\text{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right)^m \Big/ \\
& \left((1+m) \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left(\text{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \left. \left. \left. m \text{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) - \\
& \left(3 \times 2^n a^2 b (3+m+n) \text{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
& \left. \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{\text{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right)^m \left(\frac{\text{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right)^n \right) \Big/ \\
& \left((1+m+n) \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \left((3+m+n) \text{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left((1+n) \text{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \left. \left. \left. m \text{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) + \\
& \left(3 \times 2^{-1+n} a^2 b (3+m+n) \text{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
& \left. \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{\text{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right)^m \left(\frac{\text{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right)^n \right) \Big/ \\
& \left((1+m+n) \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \left((3+m+n) \text{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left((1+n) \text{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \left. \left. \left. m \text{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(3 \times 2^n a^2 b (3+m+n) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-\frac{1}{3+m+n} (1+n)(1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), m, 2+n, 1+\frac{1}{2}(3+m+n), \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3+m+n} \right. \right. \\
& \quad \left. \left. m(1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), 1+m, 1+n, 1+\frac{1}{2}(3+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^n \Big/ \\
& \quad \left((1+m+n) \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) - \\
& \left(3 \times 2^{2n} a b^2 (3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{2n} \right) \Big/ \\
& \quad \left((1+m+2n) \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right)^2 \left((3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - 2 \left((1+2n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), m, 2(1+n), \frac{1}{2}(5+m+2n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), 1+m, 1+2n, \frac{1}{2}(5+m+2n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& \left(3 \times 2^{-1+2n} a b^2 (3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{2n} \right) \Big/ \left((1+m+2n) \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \\
& \quad \left((3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \quad \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) + \\
& \left(3 \times 2^{2n} a b^2 (3+m+2n) \tan \left[\frac{1}{2} (c+dx) \right] \left(-\frac{1}{3+m+2n} (1+2n) (1+m+2n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1+m+2n), m, 2+2n, \right. \right. \right. \\
& \quad \left. \left. 1 + \frac{1}{2} (3+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] + \frac{1}{3+m+2n} m (1+m+2n) \operatorname{AppellF1} \left[\right. \right. \\
& \quad \left. \left. 1 + \frac{1}{2} (1+m+2n), 1+m, 1+2n, 1 + \frac{1}{2} (3+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] \right) \\
& \quad \left(-\frac{\tan \left[\frac{1}{2} (c+dx) \right]}{-1 + \tan \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c+dx) \right]}{1 + \tan \left[\frac{1}{2} (c+dx) \right]^2} \right)^{2n} \Big/ \left((1+m+2n) \left(1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \\
& \quad \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \quad \left. 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) - \\
& \left(2^{3n} b^3 (3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \quad \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right]^2 \left(-\frac{\tan \left[\frac{1}{2} (c+dx) \right]}{-1 + \tan \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c+dx) \right]}{1 + \tan \left[\frac{1}{2} (c+dx) \right]^2} \right)^{3n} \Big/ \\
& \quad \left((1+m+3n) \left(1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \right)^2 \left((3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \tan \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left((1+3n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), m, 2+3n, \frac{1}{2} (5+m+3n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), 1+m, 1+3n, \frac{1}{2} (5+m+3n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \left(2^{-1+3n} b^3 (3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3n} \Big/ \left((1+m+3n) \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \\
& \left((3+m+3n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+3n), m, 1+3n, \frac{1}{2}(3+m+3n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& \quad 2 \left((1+3n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), m, 2+3n, \frac{1}{2}(5+m+3n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& \quad \quad \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), 1+m, 1+3n, \frac{1}{2}(5+m+3n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \Big) + \\
& \left(2^{3n} b^3 (3+m+3n) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-\frac{1}{3+m+3n} (1+3n) (1+m+3n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+3n), m, 2+3n, 1+\frac{1}{2}(3+m+3n), \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3+m+3n} m (1+m+3n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+3n), \right. \right. \right. \\
& \quad \left. \left. \left. 1+m, 1+3n, 1+\frac{1}{2}(3+m+3n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) \\
& \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3n} \Big/ \left((1+m+3n) \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \\
& \left((3+m+3n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+3n), m, 1+3n, \frac{1}{2}(3+m+3n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& \quad 2 \left((1+3n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), m, 2+3n, \frac{1}{2}(5+m+3n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& \quad \quad \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), 1+m, 1+3n, \frac{1}{2}(5+m+3n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \Big) + \\
& \left(a^{3m} (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{-1+m} \right. \\
& \quad \left. \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2)^2} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2)} \right) \right) \Big/ \\
& \left((1+m) \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \Bigg) + \\
& \left(3 \times 2^a a^2 b m (3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right. \\
& \left. \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{-1+m} \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^n \left(\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} - \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \\
& \left((1+m+n) \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \left. \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \left(3 \times 2^{2n} a b^2 m (3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{-1+m} \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{2n} \\
& \left. \left(\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} - \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \left((1+m+2n) \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right. \\
& \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \left. \left. 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \left. \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \left(2^{3n} b^3 m (3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right. \\
& \left. \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{-1+m} \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{3n} \left(\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} - \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left((1+m+3n) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \left((3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. - \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left((1+3n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), m, 2+3n, \frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), 1+m, 1+3n, \frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \left(3 \times 2^n a^2 b n (3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right. \\
& \quad \left(- \frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{-1+n} \left(- \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \\
& \left((1+m+n) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \left(3 \times 2^{1+2n} a b^2 n (3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \quad \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(- \frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{-1+2n} \\
& \quad \left(- \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \left((1+m+2n) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right. \\
& \quad \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \quad 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(3 \times 2^{3n} b^3 n (3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right. \\
& \left. \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{-1+3n} \left(-\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \\
& \left((1+m+3n) \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \left((3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left((1+3n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), m, 2+3n, \frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) - \right. \\
& \quad \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), 1+m, 1+3n, \frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) - \\
& \left(a^3 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \right. \\
& \left. \left(-2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \quad (3+m) \left(-\frac{1}{3+m} (1+m) \operatorname{AppellF1} \left[1+\frac{1+m}{2}, m, 2, 1+\frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \quad \left. \frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[1+\frac{1+m}{2}, 1+m, 1, 1+\frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) - \\
& \quad 2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left(-\frac{1}{5+m} 2 (3+m) \operatorname{AppellF1} \left[1+\frac{3+m}{2}, m, 3, 1+\frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\right. \right. \\
& \quad \left. \left. \frac{1}{2} (c+dx) \right] + \frac{1}{5+m} m (3+m) \operatorname{AppellF1} \left[1+\frac{3+m}{2}, 1+m, 2, 1+\frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - m \left(-\frac{1}{5+m} (3+m) \operatorname{AppellF1} \left[1+\frac{3+m}{2}, 1+m, 2, 1+\frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m} (1+m) (3+m) \operatorname{AppellF1} \left[1+\frac{3+m}{2}, 2+m, 1, 1+\frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) / \left((1+m) \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(c+dx)\right]^2 - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \Big)^2 \Big) - \\
& \left(3 \times 2^n a^2 b (3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
& \tan\left[\frac{1}{2}(c+dx)\right] \left(-\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{-1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^n \\
& \left(-2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \Big) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \\
& \quad (3+m+n) \left(-\frac{1}{3+m+n} (1+n) (1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), m, 2+n, 1+\frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3+m+n} m (1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), 1+ \right. \right. \\
& \quad \left. \left. m, 1+n, 1+\frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \Big) - \\
& \quad 2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left((1+n) \left(-\frac{1}{5+m+n} (2+n) (3+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+n), m, 3+n, 1+\frac{1}{2}(5+m+n), \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{5+m+n} m (3+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+n), \right. \right. \\
& \quad \left. \left. 1+m, 2+n, 1+\frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \Big) - \\
& \quad m \left(-\frac{1}{5+m+n} (1+n) (3+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+n), 1+m, 2+n, 1+\frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{5+m+n} (1+m) (3+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+n), \right. \right. \\
& \quad \left. \left. 2+m, 1+n, 1+\frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \Big) \Big) \Big) \Big) / \\
& \left((1+m+n) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - m \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) \Big) -
\end{aligned}$$

$$\begin{aligned}
& \left(3 \times 2^{2n} a b^2 (3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{2n} \\
& \left(-2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \\
& \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + (3+m+2n) \left(-\frac{1}{3+m+2n} (1+2n) (1+m+2n) \operatorname{AppellF1} \left[1+\frac{1}{2} (1+m+2n), m, 2+2n, 1+\frac{1}{2} (3+m+2n), \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{3+m+2n} m (1+m+2n) \operatorname{AppellF1} \left[1+ \right. \right. \\
& \quad \left. \left. \frac{1}{2} (1+m+2n), 1+m, 1+2n, 1+\frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) - \\
& 2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left((1+2n) \left(-\frac{1}{5+m+2n} 2(1+n) (3+m+2n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+2n), m, 1+2(1+n), 1+\frac{1}{2} (5+m+2n), \right. \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m+2n} m (3+m+2n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+ \right. \right. \\
& \quad \left. \left. 2n), 1+m, 2(1+n), 1+\frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) - \\
& \quad \left. m \left(-\frac{1}{5+m+2n} (1+2n) (3+m+2n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+2n), 1+m, 2+2n, 1+\frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m+2n} (1+m) (3+m+2n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+2n), \right. \right. \\
& \quad \left. \left. 2+m, 1+2n, 1+\frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) \Big/ \\
& \left((1+m+2n) \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Big) -
\end{aligned}$$

$$\begin{aligned}
& \left(2^{3n} b^3 (3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{3n} \\
& \left(-2 \left((1+3n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), m, 2+3n, \frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), 1+m, 1+3n, \frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right. \\
& \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + (3+m+3n) \left(-\frac{1}{3+m+3n} (1+3n) (1+m+3n) \operatorname{AppellF1} \left[1+\frac{1}{2} (1+m+3n), m, 2+3n, 1+\frac{1}{2} (3+m+3n), \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{3+m+3n} m (1+m+3n) \operatorname{AppellF1} \left[1+ \right. \right. \\
& \quad \left. \left. \frac{1}{2} (1+m+3n), 1+m, 1+3n, 1+\frac{1}{2} (3+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) - \\
& 2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left((1+3n) \left(-\frac{1}{5+m+3n} (2+3n) (3+m+3n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+3n), m, 3+3n, 1+\frac{1}{2} (5+m+3n), \right. \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m+3n} m (3+m+3n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+ \right. \right. \\
& \quad \left. \left. 3n), 1+m, 2+3n, 1+\frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) - \\
& \quad \left. m \left(-\frac{1}{5+m+3n} (1+3n) (3+m+3n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+3n), 1+m, 2+3n, 1+\frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m+3n} (1+m) (3+m+3n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+3n), \right. \right. \\
& \quad \left. \left. 2+m, 1+3n, 1+\frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) \Big/ \\
& \left((1+m+3n) \left(1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \left((3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left((1+3n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), m, 2+3n, \frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), 1+m, 1+3n, \frac{1}{2} (5+m+3n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \Big) \Big)
\end{aligned}$$

Problem 575: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b \sin[c + dx]^n)^2 \tan[c + dx]^m dx$$

Optimal (type 5, 215 leaves, 8 steps):

$$\frac{a^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan[c + dx]^2\right] \tan[c + dx]^{1+m}}{d(1+m)} + \frac{1}{d(1+m+n)}$$

$$2ab (\cos[c + dx]^2)^{\frac{1+m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), \sin[c + dx]^2\right] \sin[c + dx]^n \tan[c + dx]^{1+m} + \frac{1}{d(1+m+2n)}$$

$$b^2 (\cos[c + dx]^2)^{\frac{1+m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2}(1+m+2n), \frac{1}{2}(3+m+2n), \sin[c + dx]^2\right] \sin[c + dx]^{2n} \tan[c + dx]^{1+m}$$

Result (type 6, 8343 leaves):

$$\left(2^{1+m} \tan\left[\frac{1}{2}(c + dx)\right] \left(-\frac{\tan\left[\frac{1}{2}(c + dx)\right]}{-1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}\right)^m \left(\left(a^2(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right]\right) / \right.$$

$$\left. \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c + dx)\right]^2\right) + \right.$$

$$\left. \left(2^{1+n} ab(3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \left(\frac{\tan\left[\frac{1}{2}(c + dx)\right]}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}\right)^n\right) / \right.$$

$$\left. \left((1+m+n) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - \right.\right.$$

$$\left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - \right.\right.$$

$$\left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c + dx)\right]^2\right) + \right.$$

$$\left. \left(4^n b^2(3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \left(\frac{\tan\left[\frac{1}{2}(c + dx)\right]}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}\right)^{2n}\right) / \right.$$

$$\left. \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - \right.\right.$$

$$\left. 2 \left((1+2n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), m, 2(1+n), \frac{1}{2}(5+m+2n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - \right.$$

$$\begin{aligned}
& m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \Bigg) \\
& \left(a^2 \operatorname{Tan} [c+dx]^m + 2ab \operatorname{Sin} [c+dx]^n \operatorname{Tan} [c+dx]^m + b^2 \operatorname{Sin} [c+dx]^{2n} \operatorname{Tan} [c+dx]^m \right) \Bigg) / \left(d \right. \\
& \left. \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right. \\
& \left. \left(-\frac{1}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} 2^{1+m} \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \right. \right. \\
& \left. \left(\left(a^2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) / \left((1+m) \right. \right. \right. \\
& \left. \left. \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) - m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) + \\
& \left(2^{1+n} ab (3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^n \right) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \right. \right. \\
& \left. \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), \right. \right. \right. \\
& \left. \left. \left. 1+m, 1+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) + \left(4^n b^2 (3+m+2n) \right. \\
& \left. \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{2n} \right) / \\
& \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \right. \right. \\
& \left. \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) + \\
& \frac{1}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} 2^m \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\left(a^2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) / \left((1+m) \right. \right. \\
& \left. \left. \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \left(2^{1+n} a b (3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^n \right) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), \right. \right. \right. \right. \right. \\
& \left. \left. \left. 1+m, 1+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \left(4^n b^2 (3+m+2n) \right. \\
& \left. \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{2n} \right) / \\
& \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \left. \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \Bigg) + \\
& \frac{1}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} 2^{1+m} m \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{-1+m} \left(\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{(-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2)^2} - \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 (-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2)} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(a^2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) / \left((1+m) \right. \right. \\
& \quad \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \\
& \quad \left(2^{1+n} a b (3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^n \right) / \\
& \quad \left((1+m+n) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), \right. \right. \right. \right. \\
& \quad \left. \left. \left. 1+m, 1+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) + \left(4^n b^2 (3+m+2n) \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{2n} \right) / \\
& \quad \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) + \\
& \quad \frac{1}{1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} 2^{1+m} \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{-1+\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(\left(a^2 (3+m) \left(-1 / (3+m) (1+m) \operatorname{AppellF1} \left[1+\frac{1+m}{2}, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. m, 2, 1+\frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + 1 / (3+m) m (1+m) \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1+m}{2}, 1+m, 1, 1+\frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) / \left((1+m) \right. \\
& \quad \left. \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(c+dx)\right]^2 - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \Big) + \\
& \left(2^{1+n} a b (3+m+n) \left(-\frac{1}{3+m+n} (1+n) (1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), m, 2+n, 1+\frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3+m+n} m (1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), 1+m, 1+n, \right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^n \right) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& \left(4^n b^2 (3+m+2n) \left(-\frac{1}{3+m+2n} (1+2n) (1+m+2n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+2n), m, 2+2n, 1+\frac{1}{2}(3+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3+m+2n} m (1+m+2n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+2n), 1+m, 1+2n, \right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1}{2}(3+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{2n} \right) / \\
& \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+2n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), m, 2(1+n), \frac{1}{2}(5+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), 1+m, 1+2n, \frac{1}{2}(5+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& \left(2^{1+n} a b n (3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
& \quad \left. \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{-1+n} \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2}{\left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) + \\
& \left(2^{1+2n} b^2 n (3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \\
& \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{-1+2n} \left(-\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \Bigg) / \\
& \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) - \\
& \left(a^2 (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(-2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \right) \\
& \quad \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + (3+m) \left(-\frac{1}{3+m} (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, m, 2, 1 + \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1+m, 1, \right. \right. \\
& \quad \left. \left. 1 + \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) - 2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \\
& \left(-\frac{1}{5+m} 2 (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, m, 3, 1 + \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \quad \left. \frac{1}{5+m} m (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1+m, 2, 1 + \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - m \left(-\frac{1}{5+m} (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1+m, 2, 1 + \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m} (1+m) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 2+m, 1, 1 + \frac{5+m}{2}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \tan \left[\frac{1}{2} (c+dx) \right]^2 \Bigg)^2 \Bigg) - \\
& \left(4^n b^2 (3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\frac{\tan \left[\frac{1}{2} (c+dx) \right]}{1 + \tan \left[\frac{1}{2} (c+dx) \right]^2} \right)^{2n} \right. \right. \\
& \left. \left(-2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\right. \right. \right. \right. \\
& \quad \left. \frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] + \\
& \quad (3+m+2n) \left(-\frac{1}{3+m+2n} (1+2n) (1+m+2n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1+m+2n), m, 2+2n, 1 + \frac{1}{2} (3+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] + \frac{1}{3+m+2n} m (1+m+2n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1+m+2n), 1+m, \right. \right. \\
& \quad \left. \left. 1+2n, 1 + \frac{1}{2} (3+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] \right) - 2 \tan \left[\frac{1}{2} (c+dx) \right]^2 \Bigg) \\
& \left((1+2n) \left(-\frac{1}{5+m+2n} 2(1+n) (3+m+2n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (3+m+2n), m, 1+2(1+n), 1 + \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m+2n} m (3+m+2n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (3+m+2n), 1+m, \right. \right. \\
& \quad \left. \left. 2(1+n), 1 + \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] \right) \Bigg) - \\
& \quad m \left(-\frac{1}{5+m+2n} (1+2n) (3+m+2n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (3+m+2n), 1+m, 2+2n, 1 + \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m+2n} (1+m) (3+m+2n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (3+m+2n), \right. \right. \\
& \quad \left. \left. 2+m, 1+2n, 1 + \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] \right) \Bigg) \Bigg) \Bigg) / \\
& \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2(1+n), \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c+dx) \right]^2 \right)^2 \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 576: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \sin[c + dx]^n) \tan[c + dx]^m dx$$

Optimal (type 5, 124 leaves, 6 steps):

$$\frac{a \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan[c + dx]^2\right] \tan[c + dx]^{1+m}}{d(1+m)} + \frac{1}{d(1+m+n)}$$

$$b (\cos[c + dx]^2)^{\frac{1+m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), \sin[c + dx]^2\right] \sin[c + dx]^n \tan[c + dx]^{1+m}$$

Result (type 6, 5184 leaves):

$$\begin{aligned} & \left(2^{1+m} \tan\left[\frac{1}{2}(c + dx)\right] \left(-\frac{\tan\left[\frac{1}{2}(c + dx)\right]}{-1 + \tan\left[\frac{1}{2}(c + dx)\right]^2} \right)^m \left(\left(a(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) / \right. \right. \\ & \quad \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, \right. \right. \right. \right. \\ & \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) + \\ & \quad \left(2^n b(3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \left(\frac{\tan\left[\frac{1}{2}(c + dx)\right]}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2} \right)^n \right) / \\ & \quad \left((1+m+n) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - \right. \right. \right. \\ & \quad \left. \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) \right) \\ & \left. (a \tan[c + dx]^m + b \sin[c + dx]^n \tan[c + dx]^m) \right) / \left(d \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) \\ & \left(-\frac{1}{\left(1 + \tan\left[\frac{1}{2}(c + dx)\right]^2 \right)^2} 2^{1+m} \sec\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right]^2 \left(-\frac{\tan\left[\frac{1}{2}(c + dx)\right]}{-1 + \tan\left[\frac{1}{2}(c + dx)\right]^2} \right)^m \right. \\ & \quad \left(\left(a(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) / \left((1+m) \right. \right. \\ & \quad \left. \left. \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, \right. \right. \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \Big) + \\
& \left(2^n b (3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^n \right) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) + \\
& \frac{1}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} 2^{1+m} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \left(\left(a(3+m) \left(-1/(3+m)(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. m, 2, 1+\frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 1/(3+m)m(1+m) \operatorname{AppellF1}\left[\right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1+m}{2}, 1+m, 1, 1+\frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) \Big) / \left((1+m) \right. \\
& \quad \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) + \\
& \left(2^n b (3+m+n) \left(-\frac{1}{3+m+n} (1+n)(1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), m, 2+n, 1+\frac{1}{2}(3+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3+m+n} m(1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), 1+m, 1+n, \right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1}{2}(3+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^n \right) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) +
\end{aligned}$$

$$\begin{aligned}
& \left(2^n b n (3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \left. \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{-1+n} \left(-\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) - \\
& \left(a (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(-2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \right) \\
& \quad \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + (3+m) \left(-\frac{1}{3+m} (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, m, 2, 1 + \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1+m, 1, \right. \right. \\
& \quad \left. \left. 1 + \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) - 2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \\
& \left(-\frac{1}{5+m} 2 (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, m, 3, 1 + \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \quad \left. \frac{1}{5+m} m (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1+m, 2, 1 + \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - m \left(-\frac{1}{5+m} (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1+m, 2, 1 + \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \right) \\
& \quad \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m} (1+m) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 2+m, 1, 1 + \frac{5+m}{2}, \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) / \left((1+m) \right. \\
& \left. \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(2^n b (3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\frac{\tan \left[\frac{1}{2} (c+dx) \right]}{1 + \tan \left[\frac{1}{2} (c+dx) \right]^2} \right)^n \right. \\
& \left(-2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), \right. \right. \right. \\
& \quad \left. \left. \left. 1+m, 1+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \quad (3+m+n) \left(-\frac{1}{3+m+n} (1+n) (1+m+n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1+m+n), m, 2+n, 1 + \frac{1}{2} (3+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \quad \quad \left. \left. -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] + \frac{1}{3+m+n} m (1+m+n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (1+m+n), \right. \right. \\
& \quad \quad \left. \left. 1+m, 1+n, 1 + \frac{1}{2} (3+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] \right) - \\
& \quad 2 \tan \left[\frac{1}{2} (c+dx) \right]^2 \left((1+n) \left(-\frac{1}{5+m+n} (2+n) (3+m+n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (3+m+n), m, 3+n, 1 + \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \quad \left. \left. -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m+n} m (3+m+n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (3+m+n), \right. \right. \\
& \quad \quad \left. \left. 1+m, 2+n, 1 + \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] \right) - \\
& \quad m \left(-\frac{1}{5+m+n} (1+n) (3+m+n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (3+m+n), 1+m, 2+n, 1 + \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \quad \quad \left. \left. -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] + \frac{1}{5+m+n} (1+m) (3+m+n) \operatorname{AppellF1} \left[1 + \frac{1}{2} (3+m+n), 2+m, \right. \right. \\
& \quad \quad \left. \left. 1+n, 1 + \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] \right) \right) \right) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& \quad \quad \left. \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 585: Unable to integrate problem.**

$$\int \cot [c+dx]^3 (a+b \sin [c+dx]^n)^p dx$$

Optimal (type 5, 136 leaves, 7 steps) :

$$\frac{\text{Hypergeometric2F1}\left[1, 1+p, 2+p, 1+\frac{b\sin[c+dx]^n}{a}\right] (a+b\sin[c+dx]^n)^{1+p}}{a d n (1+p)} - \frac{\text{Csc}[c+dx]^2 \text{Hypergeometric2F1}\left[-\frac{2}{n}, -p, -\frac{2-n}{n}, -\frac{b\sin[c+dx]^n}{a}\right] (a+b\sin[c+dx]^n)^p \left(1+\frac{b\sin[c+dx]^n}{a}\right)^{-p}}{2 d}$$

Result (type 8, 25 leaves) :

$$\int \cot[c+dx]^3 (a+b\sin[c+dx]^n)^p dx$$

■ **Problem 591: Result unnecessarily involves higher level functions.**

$$\int \frac{a+b\sin[e+fx]^2}{(g\cos[e+fx])^{5/2} \sqrt{d\sin[e+fx]}} dx$$

Optimal (type 4, 107 leaves, 7 steps) :

$$\frac{2(a+b)\sqrt{d\sin[e+fx]}}{3dfg(g\cos[e+fx])^{3/2}} + \frac{(2a-b)\text{EllipticF}\left[e-\frac{\pi}{4}+fx, 2\right]\sqrt{\sin[2e+2fx]}}{3fg^2\sqrt{g\cos[e+fx]}\sqrt{d\sin[e+fx]}}$$

Result (type 5, 120 leaves) :

$$\left(2\left(-2(2a-b)\cos[e+fx]^2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+fx]^2\right] + (a+b+(2a-b)\cos[e+fx]^2)(\sin[e+fx]^2)^{1/4}\right)\tan[e+fx]\right) / \left(3fg^2\sqrt{g\cos[e+fx]}\sqrt{d\sin[e+fx]}(\sin[e+fx]^2)^{1/4}\right)$$

■ **Problem 592: Result more than twice size of optimal antiderivative.**

$$\int (c\cos[e+fx])^m (d\sin[e+fx])^n (a+b\sin[e+fx]^2)^p dx$$

Optimal (type 6, 137 leaves, 3 steps) :

$$\frac{1}{df(1+n)} c \text{AppellF1}\left[\frac{1+n}{2}, \frac{1-m}{2}, -p, \frac{3+n}{2}, \sin[e+fx]^2, -\frac{b\sin[e+fx]^2}{a}\right] (c\cos[e+fx])^{-1+m} (\cos[e+fx]^2)^{\frac{1-m}{2}} (d\sin[e+fx])^{1+n} (a+b\sin[e+fx]^2)^p \left(1+\frac{b\sin[e+fx]^2}{a}\right)^{-p}$$

Result (type 6, 279 leaves) :

$$\left(a (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, \frac{1-m}{2}, -p, \frac{3+n}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \right. \\ \left. (c \cos[e+fx])^m (d \sin[e+fx])^n (a+b \sin[e+fx]^2)^p \tan[e+fx] \right) / \\ \left(f (1+n) \left(a (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, \frac{1-m}{2}, -p, \frac{3+n}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] + \left(2 b p \operatorname{AppellF1}\left[\frac{3+n}{2}, \frac{1-m}{2}, 1-p, \frac{5+n}{2}, \right. \right. \right. \right. \\ \left. \left. \left. \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] - a (-1+m) \operatorname{AppellF1}\left[\frac{3+n}{2}, \frac{3-m}{2}, -p, \frac{5+n}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \right) \sin[e+fx]^2 \right) \right)$$

■ **Problem 593: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + (c \cos[e+fx] + b \sin[e+fx])^2} dx$$

Optimal (type 4, 79 leaves, 3 steps):

$$\frac{\operatorname{EllipticE}\left[e+fx + \operatorname{ArcTan}\left[\frac{b}{c}\right], -\frac{b^2+c^2}{a}\right] \sqrt{a + (c \cos[e+fx] + b \sin[e+fx])^2}}{f \sqrt{1 + \frac{(c \cos[e+fx] + b \sin[e+fx])^2}{a}}}$$

Result (type 4, 325 leaves):

$$\left(\text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\sqrt{(b^2+c^2)^2 + (b^2-c^2) \cos[2(e+fx)]} - 2bc \sin[2(e+fx)]}}{\sqrt{(b^2+c^2)^2}} \right], \frac{2\sqrt{(b^2+c^2)^2}}{2a+b^2+c^2+\sqrt{(b^2+c^2)^2}} \right] \right. \\
\left. \sqrt{2a+b^2+c^2 + (-b^2+c^2) \cos[2(e+fx)] + 2bc \sin[2(e+fx)]} (2bc \cos[2(e+fx)] + (b^2-c^2) \sin[2(e+fx)]) \right] / \\
\left(\sqrt{2} \sqrt{(b^2+c^2)^2} f \sqrt{\frac{2a+b^2+c^2 + (-b^2+c^2) \cos[2(e+fx)] + 2bc \sin[2(e+fx)]}{2a+b^2+c^2+\sqrt{(b^2+c^2)^2}}} \sqrt{\frac{(2bc \cos[2(e+fx)] + (b^2-c^2) \sin[2(e+fx)])^2}{(b^2+c^2)^2}} \right)$$

- **Problem 594: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a + (c \cos[e+fx] + b \sin[e+fx])^2}} dx$$

Optimal (type 4, 79 leaves, 3 steps):

$$\frac{\text{EllipticF} \left[e+fx + \text{ArcTan} \left[b, c \right], -\frac{b^2+c^2}{a} \right] \sqrt{1 + \frac{(c \cos[e+fx] + b \sin[e+fx])^2}{a}}}{f \sqrt{a + (c \cos[e+fx] + b \sin[e+fx])^2}}$$

Result (type 6, 529 leaves):

$$\frac{1}{b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} f} \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \operatorname{Sin}\left[2(e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right]}{2 a+b^2+c^2-b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}}}\right],$$

$$\frac{2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \operatorname{Sin}\left[2(e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right]}{2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}}}] \operatorname{Sec}\left[2(e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right]$$

$$\sqrt{\frac{b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \left(-1+\operatorname{Sin}\left[2(e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right]\right)}{2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}}}} \sqrt{\frac{b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \left(1+\operatorname{Sin}\left[2(e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right]\right)}{2 a+b^2+c^2-b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}}}}$$

$$\sqrt{2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \operatorname{Sin}\left[2(e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right]}$$

Test results for the 9 problems in "4.1.8 (a+b sin)^m (c+d trig)^n.m"

- Problem 2: Result more than twice size of optimal antiderivative.

$$\int \frac{A+B \operatorname{Cos}[x]}{1+\operatorname{Sin}[x]} dx$$

Optimal (type 3, 19 leaves, 5 steps):

$$B \operatorname{Log}[1+\operatorname{Sin}[x]] - \frac{A \operatorname{Cos}[x]}{1+\operatorname{Sin}[x]}$$

Result (type 3, 42 leaves):

$$2 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \frac{2 A \operatorname{Sin}\left[\frac{x}{2}\right]}{\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]}$$

Test results for the 19 problems in "4.1.9 trig^m (a+b sin^n+c sin^(2 n))^p.m"

- Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin[x]^4}{a + b \sin[x] + c \sin[x]^2} dx$$

Optimal (type 3, 323 leaves, 12 steps):

$$\frac{x}{2c} + \frac{(b^2 - ac)x}{c^3} - \frac{\sqrt{2} \left(b^3 - 2abc - \frac{b^4 - 4ab^2c + 2a^2c^2}{\sqrt{b^2 - 4ac}} \right) \text{ArcTan} \left[\frac{2c + (b - \sqrt{b^2 - 4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} \right]}{c^3 \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} -$$

$$\frac{\sqrt{2} \left(b^3 - 2abc + \frac{b^4 - 4ab^2c + 2a^2c^2}{\sqrt{b^2 - 4ac}} \right) \text{ArcTan} \left[\frac{2c + (b + \sqrt{b^2 - 4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2 - 2c(a+c) + b\sqrt{b^2 - 4ac}}} \right]}{c^3 \sqrt{b^2 - 2c(a+c) + b\sqrt{b^2 - 4ac}}} + \frac{b \cos[x]}{c^2} - \frac{\cos[x] \sin[x]}{2c}$$

Result (type 3, 410 leaves):

$$\frac{1}{4c^3} \left(4b^2x + 2c(-2a+c)x - \left(4 \left(ib^4 - 4iab^2c + 2ia^2c^2 + b^3\sqrt{-b^2+4ac} - 2abc\sqrt{-b^2+4ac} \right) \operatorname{ArcTan} \left[\frac{2c + (b - i\sqrt{-b^2+4ac}) \operatorname{Tan} \left[\frac{x}{2} \right]}{\sqrt{2}\sqrt{b^2 - 2c(a+c) - ib\sqrt{-b^2+4ac}}} \right] \right) \right) /$$

$$\left(\sqrt{-\frac{b^2}{2} + 2ac} \sqrt{b^2 - 2c(a+c) - ib\sqrt{-b^2+4ac}} \right) -$$

$$\left(4 \left(-ib^4 + 4iab^2c - 2ia^2c^2 + b^3\sqrt{-b^2+4ac} - 2abc\sqrt{-b^2+4ac} \right) \operatorname{ArcTan} \left[\frac{2c + (b + i\sqrt{-b^2+4ac}) \operatorname{Tan} \left[\frac{x}{2} \right]}{\sqrt{2}\sqrt{b^2 - 2c(a+c) + ib\sqrt{-b^2+4ac}}} \right] \right) /$$

$$\left(\sqrt{-\frac{b^2}{2} + 2ac} \sqrt{b^2 - 2c(a+c) + ib\sqrt{-b^2+4ac}} \right) + 4bc \operatorname{Cos}[x] - c^2 \operatorname{Sin}[2x]$$

■ **Problem 2: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[x]^3}{a + b \operatorname{Sin}[x] + c \operatorname{Sin}[x]^2} dx$$

Optimal (type 3, 298 leaves, 10 steps):

$$-\frac{bx}{c^2} + \frac{\sqrt{2}b \left(b - \frac{ac}{b} - \frac{b^2}{\sqrt{b^2-4ac}} + \frac{3ac}{\sqrt{b^2-4ac}} \right) \operatorname{ArcTan} \left[\frac{2c + (b - \sqrt{b^2-4ac}) \operatorname{Tan} \left[\frac{x}{2} \right]}{\sqrt{2}\sqrt{b^2 - 2c(a+c) - b\sqrt{b^2-4ac}}} \right]}{c^2 \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2-4ac}}} +$$

$$\frac{\sqrt{2}b \left(b - \frac{ac}{b} + \frac{b^2}{\sqrt{b^2-4ac}} - \frac{3ac}{\sqrt{b^2-4ac}} \right) \operatorname{ArcTan} \left[\frac{2c + (b + \sqrt{b^2-4ac}) \operatorname{Tan} \left[\frac{x}{2} \right]}{\sqrt{2}\sqrt{b^2 - 2c(a+c) + b\sqrt{b^2-4ac}}} \right]}{c^2 \sqrt{b^2 - 2c(a+c) + b\sqrt{b^2-4ac}}} - \frac{\operatorname{Cos}[x]}{c}$$

Result (type 3, 358 leaves) :

$$\frac{1}{c^2} \left(-bx + \frac{\left(i b^3 - 3 i a b c + b^2 \sqrt{-b^2 + 4 a c} - a c \sqrt{-b^2 + 4 a c} \right) \text{ArcTan} \left[\frac{2 c + (b - i \sqrt{-b^2 + 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) - i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} \sqrt{b^2 - 2 c (a + c) - i b \sqrt{-b^2 + 4 a c}}} + \right. \\ \left. \frac{\left(-i b^3 + 3 i a b c + b^2 \sqrt{-b^2 + 4 a c} - a c \sqrt{-b^2 + 4 a c} \right) \text{ArcTan} \left[\frac{2 c + (b + i \sqrt{-b^2 + 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) + i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} \sqrt{b^2 - 2 c (a + c) + i b \sqrt{-b^2 + 4 a c}}} - c \text{Cos} [x] \right)$$

■ **Problem 3: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sin}[x]^2}{a + b \text{Sin}[x] + c \text{Sin}[x]^2} dx$$

Optimal (type 3, 253 leaves, 9 steps) :

$$\frac{x}{c} - \frac{\sqrt{2} \left(b - \frac{b^2 - 2 a c}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{2 c + (b - \sqrt{b^2 - 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) - b \sqrt{b^2 - 4 a c}}} \right]}{c \sqrt{b^2 - 2 c (a + c) - b \sqrt{b^2 - 4 a c}}} - \frac{\sqrt{2} \left(b + \frac{b^2 - 2 a c}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{2 c + (b + \sqrt{b^2 - 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) + b \sqrt{b^2 - 4 a c}}} \right]}{c \sqrt{b^2 - 2 c (a + c) + b \sqrt{b^2 - 4 a c}}}$$

Result (type 3, 310 leaves) :

$$\frac{1}{c} \left(x - \frac{\left(i b^2 - 2 i a c + b \sqrt{-b^2 + 4 a c} \right) \text{ArcTan} \left[\frac{2 c + (b - i \sqrt{-b^2 + 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) - i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} \sqrt{b^2 - 2 c (a + c) - i b \sqrt{-b^2 + 4 a c}}} - \frac{\left(-i b^2 + 2 i a c + b \sqrt{-b^2 + 4 a c} \right) \text{ArcTan} \left[\frac{2 c + (b + i \sqrt{-b^2 + 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) + i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} \sqrt{b^2 - 2 c (a + c) + i b \sqrt{-b^2 + 4 a c}}} \right)$$

■ **Problem 4: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sin}[x]}{a + b \text{Sin}[x] + c \text{Sin}[x]^2} dx$$

Optimal (type 3, 226 leaves, 8 steps) :

$$\frac{\sqrt{2} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \operatorname{ArcTan} \left[\frac{2c + (b - \sqrt{b^2-4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}}} \right]}{\sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}}} + \frac{\sqrt{2} \left(1 + \frac{b}{\sqrt{b^2-4ac}} \right) \operatorname{ArcTan} \left[\frac{2c + (b + \sqrt{b^2-4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) + b\sqrt{b^2-4ac}}} \right]}{\sqrt{b^2-2c(a+c) + b\sqrt{b^2-4ac}}}$$

Result (type 3, 268 leaves):

$$\frac{(i b + \sqrt{-b^2+4ac}) \operatorname{ArcTan} \left[\frac{2c + (b - i \sqrt{-b^2+4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) - i b \sqrt{-b^2+4ac}}} \right]}{\sqrt{b^2-2c(a+c) - i b \sqrt{-b^2+4ac}}} + \frac{(-i b + \sqrt{-b^2+4ac}) \operatorname{ArcTan} \left[\frac{2c + (b + i \sqrt{-b^2+4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) + i b \sqrt{-b^2+4ac}}} \right]}{\sqrt{b^2-2c(a+c) + i b \sqrt{-b^2+4ac}}}$$

$$\sqrt{-\frac{b^2}{2} + 2ac}$$

- **Problem 5: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{a + b \sin[x] + c \sin[x]^2} dx$$

Optimal (type 3, 221 leaves, 7 steps):

$$\frac{2\sqrt{2} c \operatorname{ArcTan} \left[\frac{2c + (b - \sqrt{b^2-4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}}} \right]}{\sqrt{b^2-4ac} \sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}}} - \frac{2\sqrt{2} c \operatorname{ArcTan} \left[\frac{2c + (b + \sqrt{b^2-4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) + b\sqrt{b^2-4ac}}} \right]}{\sqrt{b^2-4ac} \sqrt{b^2-2c(a+c) + b\sqrt{b^2-4ac}}}$$

Result (type 3, 233 leaves):

$$2 i c \left(\frac{\operatorname{ArcTan} \left[\frac{2c + (b - i \sqrt{-b^2+4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) - i b \sqrt{-b^2+4ac}}} \right]}{\sqrt{b^2-2c(a+c) - i b \sqrt{-b^2+4ac}}} - \frac{\operatorname{ArcTan} \left[\frac{2c + (b + i \sqrt{-b^2+4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) + i b \sqrt{-b^2+4ac}}} \right]}{\sqrt{b^2-2c(a+c) + i b \sqrt{-b^2+4ac}}} \right)$$

$$\sqrt{-\frac{b^2}{2} + 2ac}$$

- **Problem 6: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Csc}[x]}{a + b \sin[x] + c \sin[x]^2} dx$$

Optimal (type 3, 244 leaves, 10 steps):

$$\frac{\sqrt{2} c \left(1 + \frac{b}{\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[\frac{2c + (b - \sqrt{b^2-4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}}} \right] + \sqrt{2} c \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[\frac{2c + (b + \sqrt{b^2-4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) + b\sqrt{b^2-4ac}}} \right] + \frac{\text{ArcTanh}[\text{Cos}[x]]}{a}}{a \sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}} + a \sqrt{b^2-2c(a+c) + b\sqrt{b^2-4ac}}}$$

Result (type 3, 306 leaves):

$$\frac{1}{a} \left(\frac{c \left(-i b + \sqrt{-b^2+4ac} \right) \text{ArcTan} \left[\frac{2c + (b - i \sqrt{-b^2+4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) - i b \sqrt{-b^2+4ac}}} \right] + \sqrt{-\frac{b^2}{2} + 2ac} \sqrt{b^2-2c(a+c) - i b \sqrt{-b^2+4ac}}}{c \left(i b + \sqrt{-b^2+4ac} \right) \text{ArcTan} \left[\frac{2c + (b + i \sqrt{-b^2+4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) + i b \sqrt{-b^2+4ac}}} \right] + \sqrt{-\frac{b^2}{2} + 2ac} \sqrt{b^2-2c(a+c) + i b \sqrt{-b^2+4ac}}} + \text{Log} \left[\text{Cos} \left[\frac{x}{2} \right] \right] - \text{Log} \left[\text{Sin} \left[\frac{x}{2} \right] \right] \right)$$

■ **Problem 7: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Csc}[x]^2}{a + b \text{Sin}[x] + c \text{Sin}[x]^2} dx$$

Optimal (type 3, 271 leaves, 12 steps):

$$\frac{\sqrt{2} b c \left(1 + \frac{b^2-2ac}{b\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[\frac{2c + (b - \sqrt{b^2-4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}}} \right] + \sqrt{2} b c \left(1 - \frac{b^2-2ac}{b\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[\frac{2c + (b + \sqrt{b^2-4ac}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2c(a+c) + b\sqrt{b^2-4ac}}} \right] + \frac{b \text{ArcTanh}[\text{Cos}[x]]}{a^2} - \frac{\text{Cot}[x]}{a}}{a^2 \sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}} + a^2 \sqrt{b^2-2c(a+c) + b\sqrt{b^2-4ac}}}$$

Result (type 3, 388 leaves):

$$\left(\text{Csc}[x]^2 (-2a - c + c \cos[2x] - 2b \sin[x]) \right) \left(\frac{2c \left(-i b^2 + 2 i a c + b \sqrt{-b^2 + 4 a c} \right) \text{ArcTan} \left[\frac{2c + (b - i \sqrt{-b^2 + 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} \sqrt{b^2 - 2c(a+c) - i b \sqrt{-b^2 + 4 a c}}} \right) + \right.$$

$$\left. \frac{2 i c \left(-b^2 + 2 a c + i b \sqrt{-b^2 + 4 a c} \right) \text{ArcTan} \left[\frac{2c + (b + i \sqrt{-b^2 + 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2c(a+c) + i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} \sqrt{b^2 - 2c(a+c) + i b \sqrt{-b^2 + 4 a c}}} \right) + a \text{Cot} \left[\frac{x}{2} \right] - \right.$$

$$\left. \left. \left. \left. \left. 2 b \text{Log} \left[\text{Cos} \left[\frac{x}{2} \right] \right] + 2 b \text{Log} \left[\text{Sin} \left[\frac{x}{2} \right] \right] - a \text{Tan} \left[\frac{x}{2} \right] \right) \right) \right) \right) \right) / \left(4 a^2 (c + b \text{Csc}[x] + a \text{Csc}[x]^2) \right)$$

■ **Problem 8: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Csc}[x]^3}{a + b \sin[x] + c \sin[x]^2} dx$$

Optimal (type 3, 331 leaves, 14 steps):

$$\frac{\sqrt{2} c \left(b^3 - 3 a b c + \sqrt{b^2 - 4 a c} (b^2 - a c) \right) \text{ArcTan} \left[\frac{2c + (b - \sqrt{b^2 - 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b \sqrt{b^2 - 4 a c}}} \right]}{a^3 \sqrt{b^2 - 4 a c} \sqrt{b^2 - 2c(a+c) - b \sqrt{b^2 - 4 a c}}} +$$

$$\frac{\sqrt{2} c \left(b^3 - 3 a b c - \sqrt{b^2 - 4 a c} (b^2 - a c) \right) \text{ArcTan} \left[\frac{2c + (b + \sqrt{b^2 - 4 a c}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2c(a+c) + b \sqrt{b^2 - 4 a c}}} \right]}{a^3 \sqrt{b^2 - 4 a c} \sqrt{b^2 - 2c(a+c) + b \sqrt{b^2 - 4 a c}}} -$$

$$\frac{\text{ArcTanh}[\text{Cos}[x]]}{2 a} - \frac{(b^2 - a c) \text{ArcTanh}[\text{Cos}[x]]}{a^3} + \frac{b \text{Cot}[x]}{a^2} - \frac{\text{Cot}[x] \text{Csc}[x]}{2 a}$$

Result (type 3, 481 leaves) :

$$\frac{1}{16 a^3 (c + b \operatorname{Csc}[x] + a \operatorname{Csc}[x]^2)}$$

$$\operatorname{Csc}[x]^2 (-2 a - c + c \operatorname{Cos}[2 x] - 2 b \operatorname{Sin}[x]) \left(\frac{8 c \left(-i b^3 + 3 i a b c + b^2 \sqrt{-b^2 + 4 a c} - a c \sqrt{-b^2 + 4 a c} \right) \operatorname{ArcTan}\left[\frac{2 c + (b - i \sqrt{-b^2 + 4 a c}) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) - i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} \sqrt{b^2 - 2 c (a + c) - i b \sqrt{-b^2 + 4 a c}}} + \right.$$

$$\frac{8 c \left(i b^3 - 3 i a b c + b^2 \sqrt{-b^2 + 4 a c} - a c \sqrt{-b^2 + 4 a c} \right) \operatorname{ArcTan}\left[\frac{2 c + (b + i \sqrt{-b^2 + 4 a c}) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) + i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} \sqrt{b^2 - 2 c (a + c) + i b \sqrt{-b^2 + 4 a c}}} - 4 a b \operatorname{Cot}\left[\frac{x}{2}\right] +$$

$$\left. a^2 \operatorname{Csc}\left[\frac{x}{2}\right]^2 + 4 (a^2 + 2 b^2 - 2 a c) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] - 4 (a^2 + 2 b^2 - 2 a c) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right] - a^2 \operatorname{Sec}\left[\frac{x}{2}\right]^2 + 4 a b \operatorname{Tan}\left[\frac{x}{2}\right] \right)$$

■ **Problem 10: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[x]^2}{a + b \operatorname{Sin}[x] + c \operatorname{Sin}[x]^2} dx$$

Optimal (type 3, 230 leaves, 9 steps) :

$$-\frac{x}{c} - \frac{\sqrt{2} \sqrt{b^2 - 2 c (a + c) - b \sqrt{b^2 - 4 a c}} \operatorname{ArcTan}\left[\frac{2 c + (b - \sqrt{b^2 - 4 a c}) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) - b \sqrt{b^2 - 4 a c}}} \right]}{c \sqrt{b^2 - 4 a c}} +$$

$$\frac{\sqrt{2} \sqrt{b^2 - 2 c (a + c) + b \sqrt{b^2 - 4 a c}} \operatorname{ArcTan}\left[\frac{2 c + (b + \sqrt{b^2 - 4 a c}) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) + b \sqrt{b^2 - 4 a c}}} \right]}{c \sqrt{b^2 - 4 a c}}$$

Result (type 3, 314 leaves) :

$$\frac{1}{c} \left(-x + \frac{\left(i b^2 - 2 i c (a+c) + b \sqrt{-b^2+4ac} \right) \text{ArcTan} \left[\frac{2c + (b-i\sqrt{-b^2+4ac}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2-2c(a+c)-ib\sqrt{-b^2+4ac}}} \right]}{\sqrt{-\frac{b^2}{2}+2ac} \sqrt{b^2-2c(a+c)-ib\sqrt{-b^2+4ac}}} \right) + \frac{\left(-i b^2 + 2 i c (a+c) + b \sqrt{-b^2+4ac} \right) \text{ArcTan} \left[\frac{2c + (b+i\sqrt{-b^2+4ac}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2-2c(a+c)+ib\sqrt{-b^2+4ac}}} \right]}{\sqrt{-\frac{b^2}{2}+2ac} \sqrt{b^2-2c(a+c)+ib\sqrt{-b^2+4ac}}} \right)$$

■ **Problem 13: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[x]^2}{a + b \text{Sin}[x] + c \text{Sin}[x]^2} dx$$

Optimal (type 3, 324 leaves, 11 steps) :

$$\frac{\sqrt{2} b c \left(1 + \frac{b^2-2c(a+c)}{b\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[\frac{2c + (b-\sqrt{b^2-4ac}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}} \right]}{(a-b+c)(a+b+c) \sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}} - \frac{\sqrt{2} b c \left(1 - \frac{b^2-2c(a+c)}{b\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[\frac{2c + (b+\sqrt{b^2-4ac}) \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2-2c(a+c)+b\sqrt{b^2-4ac}}} \right]}{(a-b+c)(a+b+c) \sqrt{b^2-2c(a+c)+b\sqrt{b^2-4ac}}} + \frac{\text{Cos}[x]}{2(a+b+c)(1-\text{Sin}[x])} - \frac{\text{Cos}[x]}{2(a-b+c)(1+\text{Sin}[x])}$$

Result (type 3, 407 leaves) :

$$\begin{aligned}
& \frac{c \left(-i b^2 + 2 i c (a + c) + b \sqrt{-b^2 + 4 a c} \right) \operatorname{ArcTan} \left[\frac{2 c + (b - i \sqrt{-b^2 + 4 a c}) \operatorname{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) - i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} (a^2 - b^2 + 2 a c + c^2) \sqrt{b^2 - 2 c (a + c) - i b \sqrt{-b^2 + 4 a c}}} \\
& - \frac{c \left(i b^2 - 2 i c (a + c) + b \sqrt{-b^2 + 4 a c} \right) \operatorname{ArcTan} \left[\frac{2 c + (b + i \sqrt{-b^2 + 4 a c}) \operatorname{Tan} \left[\frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) + i b \sqrt{-b^2 + 4 a c}}} \right]}{\sqrt{-\frac{b^2}{2} + 2 a c} (a^2 - b^2 + 2 a c + c^2) \sqrt{b^2 - 2 c (a + c) + i b \sqrt{-b^2 + 4 a c}}} + \frac{\operatorname{Sin} \left[\frac{x}{2} \right]}{(a + b + c) \left(\operatorname{Cos} \left[\frac{x}{2} \right] - \operatorname{Sin} \left[\frac{x}{2} \right] \right)} + \frac{\operatorname{Sin} \left[\frac{x}{2} \right]}{(a - b + c) \left(\operatorname{Cos} \left[\frac{x}{2} \right] + \operatorname{Sin} \left[\frac{x}{2} \right] \right)}
\end{aligned}$$

- **Problem 14: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[x]^3}{a + b \operatorname{Sin}[x] + c \operatorname{Sin}[x]^2} dx$$

Optimal (type 3, 206 leaves, 10 steps):

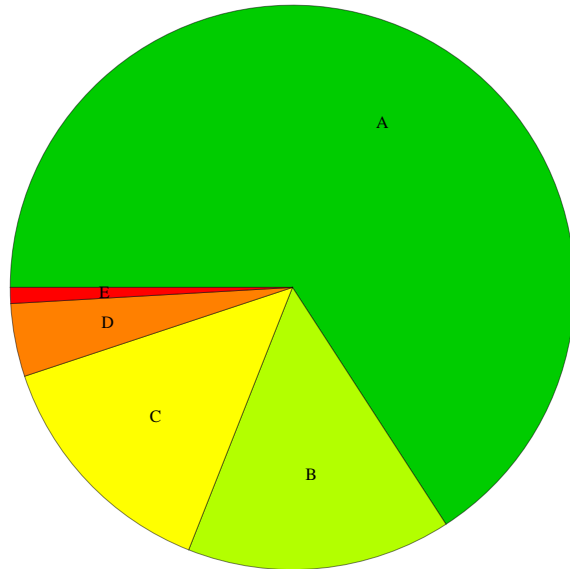
$$\begin{aligned}
& \frac{(b^4 + 2 c^2 (a + c)^2 - 2 b^2 c (2 a + c)) \operatorname{ArcTanh} \left[\frac{b + 2 c \operatorname{Sin}[x]}{\sqrt{b^2 - 4 a c}} \right]}{\sqrt{b^2 - 4 a c} (a^2 - b^2 + 2 a c + c^2)^2} - \frac{(a + 2 b + 3 c) \operatorname{Log}[1 - \operatorname{Sin}[x]]}{4 (a + b + c)^2} + \\
& \frac{(a - 2 b + 3 c) \operatorname{Log}[1 + \operatorname{Sin}[x]]}{4 (a - b + c)^2} + \frac{b (b^2 - 2 c (a + c)) \operatorname{Log}[a + b \operatorname{Sin}[x] + c \operatorname{Sin}[x]^2]}{2 (a^2 - b^2 + 2 a c + c^2)^2} - \frac{\operatorname{Sec}[x]^2 (b - (a + c) \operatorname{Sin}[x])}{2 (a - b + c) (a + b + c)}
\end{aligned}$$

Result (type 3, 481 leaves):

$$\frac{1}{4} \left(-\frac{8 i b^3 x}{(a-b+c)^2 (a+b+c)^2} + \frac{16 i b c (a+c) x}{(a-b+c)^2 (a+b+c)^2} + \frac{2 i (a-2b+3c) \text{ArcTan}[\text{Cot}[x]]}{(a-b+c)^2} - \frac{2 i (a+2b+3c) \text{ArcTan}[\text{Cot}[x]]}{(a+b+c)^2} - \right. \\ \left. \frac{4 b^4 \text{ArcTan}\left[\frac{\sqrt{-b^2+4ac}}{b+2c \sin[x]}\right]}{\sqrt{-b^2+4ac} (a^2-b^2+2ac+c^2)^2} - \frac{8 c^2 (a+c)^2 \text{ArcTan}\left[\frac{\sqrt{-b^2+4ac}}{b+2c \sin[x]}\right]}{\sqrt{-b^2+4ac} (a^2-b^2+2ac+c^2)^2} + \frac{8 b^2 c (2a+c) \text{ArcTan}\left[\frac{\sqrt{-b^2+4ac}}{b+2c \sin[x]}\right]}{\sqrt{-b^2+4ac} (a^2-b^2+2ac+c^2)^2} + \right. \\ \left. \frac{(a-2b+3c) \text{Log}\left[\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^2\right]}{(a-b+c)^2} - \frac{(a+2b+3c) \text{Log}[1-\text{Sin}[x]]}{(a+b+c)^2} + \frac{2 b^3 \text{Log}[2a+c-c \text{Cos}[2x]+2b \text{Sin}[x]]}{(a^2-b^2+2ac+c^2)^2} - \right. \\ \left. \frac{4 b c (a+c) \text{Log}[2a+c-c \text{Cos}[2x]+2b \text{Sin}[x]]}{(a^2-b^2+2ac+c^2)^2} + \frac{1}{(a+b+c) \left(\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right)^2} - \frac{1}{(a-b+c) \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^2} \right)$$

Summary of Integration Test Results

5809 integration problems



A - 3825 optimal antiderivatives

B - 880 more than twice size of optimal antiderivatives

C - 808 unnecessarily complex antiderivatives

D - 243 unable to integrate problems

E - 53 integration timeouts